CHAPTER ONE: INTRODUCTION

Geodesy is concerned with the relative positioning of points and the gravity field of the earth. For this task, a well defined coordinate system is needed on which measurements are normally tied to a set of reference points called a geodetic datum (geoid or ellipsoid).

A control survey is a means of establishing precise horizontal and vertical positions of geodetic monuments. There are two types of survey controls: horizontal and vertical controls. Classically, these were separately determined. Horizontal controls are defined with respect to an ellipsoid of revolution whilst vertical controls are defined with reference to a local geoid. Horizontal and vertical terrestrial geodetic control networks are important and valuable for the accurate positioning of construction and engineering projects, since they serve as points of reference for correct positioning of such projects.

Controls are established by various methods. The classical (traditional) methods are traversing, triangulation and trilateration. Modern methods include the use of satellite techniques such as the Global Positioning System (GPS), and satellite altimeters. Satellite techniques can be used to establish and densify three-dimensional networks more rapidly, with greater accuracy and less difficulty than terrestrial techniques (Poku-Gyamfi and Gunter, 2006). Moreover, the use of classical methods is limited by such requirements as inter-visibility between the instrument stations and target stations, favourable weather and atmospheric conditions and accessibility of stations (nature of the terrain). In addition the accuracy levels are low. Hence classical geodetic (triangulation or trilateration) networks established by terrestrial methods are insufficient to contemporary requirements.

Presently the most accurate positioning technology is the Global Positioning System (GPS). The GPS gives accurately the three-dimensional position of a point (latitude, longitude, and ellipsoidal height) and can measure under all weather conditions.

In addition, it can measure when placed on any platform (static or dynamic). One major advantage of the GPS over the traditional methods is that inter-visibility is not a requirement. In addition to providing highly accurate data, it is easy to use, portable, less labour intensive, and its surveys are relatively less costly.

The coordinates of the GPS are referenced to the World Geodetic System (WGS 84), a global ellipsoid having its origin as the mass center of the earth, and height, referenced to the surface of the ellipsoid.

The mapping systems of various countries are based on their local coordinate systems. In the local coordinate systems, horizontal positions are referenced to the local ellipsoids that are defined differently by various countries to fit their topography, and heights are referenced to the local geoid (orthometric height, H).

For accurate location and mapping of the natural resources which are sometimes trans-boundary, there is a need to integrate data obtained in one system into another. This however cannot be done directly since the various systems are incompatible. The datums to which the systems are referenced can however be linked mathematically so as to make data compatible.

The mathematical relationships that are employed in accomplishing this task are referred to as transformation models. The process of mathematically converting data in one coordinate system to another is referred to as coordinate transformation. To achieve this, control points must exist that have coordinates in both reference frames. The transformation can be thee-dimensional (3-D), two dimensional (2-D), or even one-dimensional (1-D), depending on the given requirement.

There are several transformation models such as Bursa-Wolf, Molodensky-Badekas, Veis Model, Thomson-Krakiwsky Model, Helmert Similarity Transformation, Affine transformation. (Hofmann-Wellenhof et al,1997). Each of these models could be used to determine parameters that are necessary to convert data in a local geodetic frame into WGS84 frame and vice versa.

The choice of the most appropriate network transformation model is influenced by such factors as (Rizos, 1997):

- Whether the model is to be applied to a small area, or over a large region.
- Whether one (or both) networks have significant distortions.
- Whether the networks are three-dimensional (3-D) in nature, 2-D or even 1-D.
- The accuracy required.

In addition, the geoid must be well defined for accurate height conversions. Even though GPS usage in Ghana is fast growing, there is lack of standards to regulate its usage. This is due largely to the absence of transformation parameters and lack of a geoidal model for the Ghana datum (Fosu, 2006, Poku-Gyamfi and Gunter, 2006).

The reliability of the derived parameters is usually expressed in terms of standard deviation or variance. It is important that the parameters be as uncorrelated as possible when looking at the magnitudes and standard deviations of the different parameters. Where the standard deviations of particular parameters are equal to or larger than the parameters themselves, there is strong justification for omitting the parameters (Hoar, 1982).

This project is limited to the determination of transformation parameters for the study area in specific and for the whole of Ghana in general between the Ghana geodetic datum and WGS 84. This can be achieved if the two systems have common points of known coordinates. This is achieved by carrying out GPS surveys on trigonometric points already established by the classical methods and comparing the coordinates obtained by the two methods using standard mathematical models to determine the parameters. These parameters, together with the control points used in the survey, can then be used in further densification of the entire nation.

This research compares four transformation models and their associated parameters for the transformation of curvilinear coordinates from WGS 84 system to the Ghana National Grid, and vice versa. These include:

1. A seven-parameter conformal Bursa-Wolf transformation model (Bursa, 1962; Wolf, 1963) comprising of three origin shifts, three rotations about the x-, y- and z-axis, and a scale factor.
2. A five-parameter conformal transformation based on a curvilinear form of the Molodensky model, comprising of three origin shifts from the geo-centre, a difference in semi-major axis length and flattening.
3. A seven-parameter Molodensky-Badekas model comprising of three origin shifts, three rotations about the x-, y- and z-axis, a scale factor, the geo-centroid of ellipsoid .
4. A simple three parameter model comprising of three origin shifts.

1.1 Research Problem and Justification for the Project

Global Navigation Satellite System (GNSS) is one of the most useful gifts that science and technology has offered mankind due to its global coverage and free access (Poku-Gyamfi and Gunter, 2006). In Ghana GNSS services are normally obtained with the GPS.

There is a growing use of GPS surveys in Ghana due to its numerous advantages over classical methods. Growing applications include land and engineering surveying, GIS, and navigation. If GPS data is properly processed and used, GPS can be an effective tool to promote national development as the data can be used for planning of communities, exploration and exploitation of natural resources, correct positioning of engineering and construction works, scientific investigation of earthquake activities, enhancement of agricultural productivity and provision of services among other applications.

Notwithstanding its advantages and applications, GPS usage in Ghana is problematic due to a number of reasons notable among them being the lack of a well- defined coordinate system and the parameters needed to transform GPS coordinates into local coordinates, and vice versa (Fosu, 2006). Hence no generally accepted standards exist for its use in Ghana. Consequently users adopt different approaches in the post -processing of the GPS data. These generate a lot of confusion in the use of the data. In effect there is no consistency of GPS data from different sources.

An optimal set of transformation parameters between the Accra and WGS84 datum does not exist, making it difficult for Ghana to exploit the tremendous potential of GPS. To establish compatibility between the two coordinate systems and hence enable Ghana exploit the tremendous potential of GPS, there is a need to determine the transformation parameters between the Ghana datum and WGS84 datum.

This research therefore seeks to investigate and identify the most suitable transformation model and use it to determine the best transformation parameters for the study area. These parameters, when determined will help in transforming observed GPS data into the Ghana mapping system and vice versa. This will consequently boost Ghana's economy as it will help reduce the cost and duration of acquisition of geographic information data, increase the land management and delivery process; enhance the transportation system and the security system among many others.

1.2 Research Questions

The research will try to answer the following questions:

- Under what conditions do the models apply?
- Which of the models give more reliable results?
- Can a set of accurate and consistent parameters be proposed for the ten regions as a whole?
- Are derived parameters applicable outside the study area?

1.3 Aims and Objectives

The main aim of this research is to propose a transformation model for the study area for the integration of GPS data into the cadastral system of Ghana**.** To achieve this, the following objectives have been set:

1. To study or investigate the existing transformation models.
2. To determine transformation parameters for the study area.
3. Test the transformation parameters in and outside the study area.
4. Propose a transformation model for the study area.
5. Investigate the most suitable transformation model for Ghana.

1.4 Study Area and Scope of Research

The research covered five regions (Ashanti, G. Accra, Central, Western and Eastern) which form part of the national GPS-network referred to as the Golden triangle (figure 1.1). Raw GPS data were collected from the Survey Department, processed, and computations were carried out to determine the transformation parameters. In all three transformation models were studied, applied and their accuracies were compared. Based on the accuracies, an optimal set of transformation parameters and model was proposed, and recommendations were made. The quality of the transformation was tested based on standard error, and accuracy of the transformed coordinates.

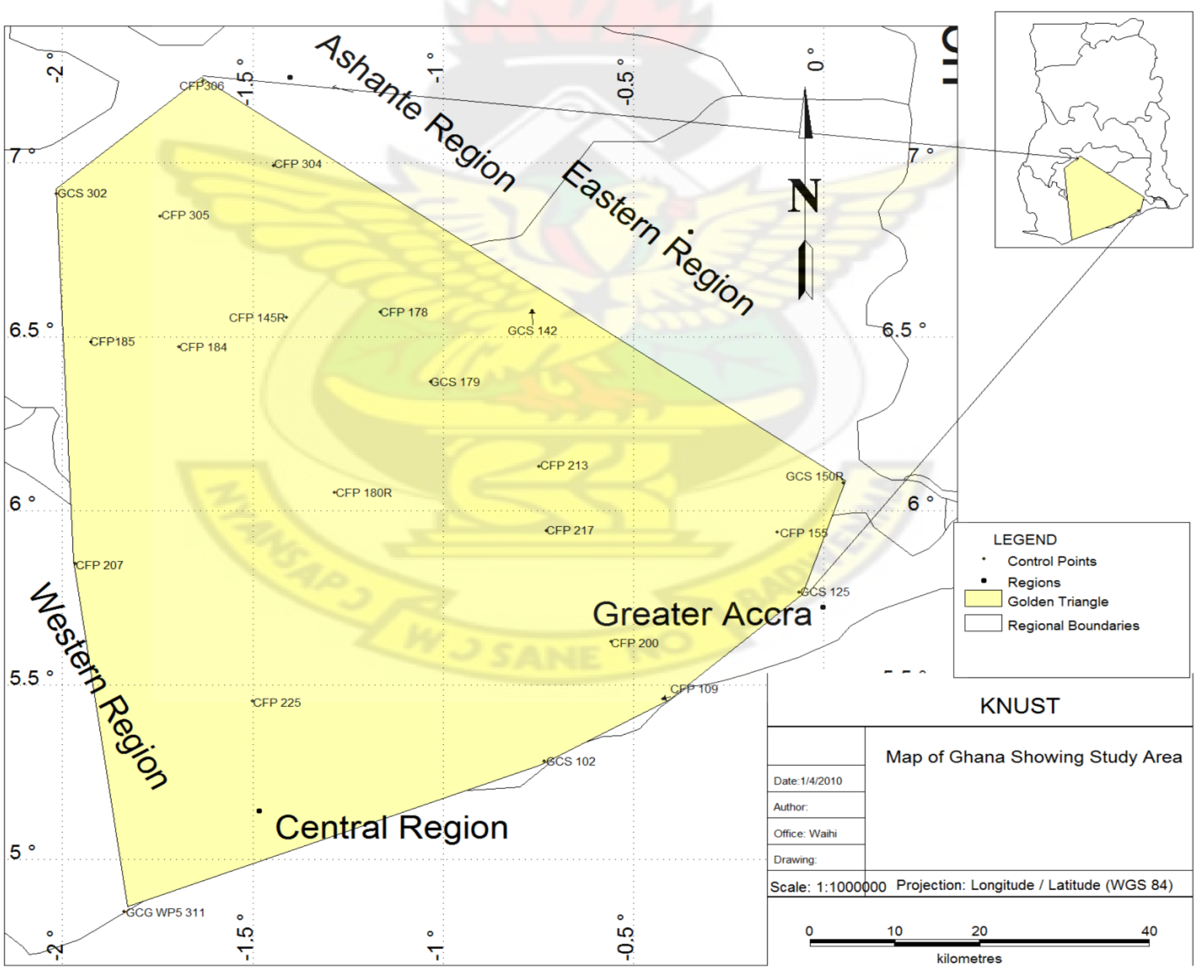

Fig. 1.1: Map of Ghana showing the study area

1.6 Outline of Thesis

The research is in five chapters. Chapter one is the introduction. It comprises of the background to the study, research problem and justification for the project, research questions, aims and objective, scope of research and thesis outline. Chapter two is literature review which examines coordinate systems, datums, and reference frames; and the cadastral framework of Ghana. It also reviews GPS co-ordinate systems and various models employed in the determination of transformation parameters. Chapter three contains the methodology and resources, data processing, and computations. Chapter four contains the results, analysis and discussion of results. Chapter five is the conclusion and recommendations.

CHAPTER TWO: CO-ORDINATES AND DATUM TRANSFORMATIONS

2.1 Coordinate Systems

One of the basic needs of geodesy and other physical sciences is to give reasonable descriptions for the positions of objects relative to each other. Scientifically, this is done in a mathematical language, by properly assigning numbers to each position in space. These numbers are called coordinates and the system defined by this procedure is referred to as a coordinate system (htpp://seds.org/~spider/spider/scholar/cords.html). A coordinate system can hence be defined as a set of numbers which identifies a position in space (Fosu, 2006).

A coordinate system is defined by three things: an origin, the directions of the axes and a scale, which is usually but not always the same for each axis. There are many different coordinate systems, based on a variety of geodetic datums, units, projections, and reference systems in use today. Coordinates can be plane or curvilinear, two-dimensional or three-dimensional, or even one-dimensional.

Many basic coordinate systems are used to represent points in two-dimensional or three-dimensional space. René Descartes introduced the system of coordinates based on orthogonal axes (Ayer and Fosu, 2008). These three-dimensional and two-dimensional systems used in analytical geometry are often referred to as Rectangular and Plane Cartesian systems respectively.

Coordinates may also be grouped as Global or Local coordinates depending on regions of coverage and applicability. Global coordinate systems have their origin at the geo-centre (Centre of mass of the earth) whereas local coordinate systems have their origin severally defined by the various Nations and organizations that adopt them.

Co-ordinates are normally tied to a set of reference points called a geodetic datum. Traditionally, two types of datum are used: horizontal, based on ellipsoids and vertical datum, based on the geoid.

2.2 Reference Figures of the Earth

The earth has a highly irregular and constantly changing solid surface. This is to a large extent the result of its rotation, which causes its equatorial bulge, and the competition of geologic processes such as the collision of plates and of volcanism, resisted by the earth's gravity field (Dana, 1997). Gravity similarly affects the liquid surface (dynamic sea surface topography) and the earth’s atmosphere. Because of these mass excesses and deficiencies within the earth, the shape of the earth is irregular and can be determined only approximately.

The figure of the earth was refined from a flat-earth model to ellipsoidal model of sufficient accuracy to allow global exploration, navigation and mapping (Dana, 1997). Ellipsoidal earth model represents the shape of the earth with an ellipse of well defined shape, size and orientation.

2.3 Geodetic Datums

A geodetic datum is any numerical or geometrical quantity or set of such quantities that serves as a reference or base for other quantities (Smith, 1997). Geodetic datums define the exact model of the earth: the size and shape of the earth and the origin and orientation of the coordinate systems used to map it. Hundreds of different datums have been used to frame position descriptions since the first estimates of the earth's size were made by Aristotle (Dana, 1997). Datums have evolved from those describing a spherical earth to ellipsoidal models derived from years of satellite measurements. Examples of datums are the geoid and mean sea levels used for height referencing, and the ellipsoid used for spherical coordinate definitions. The simplest mathematical approximation of the shape and size of the Earth, on which coordinate systems are based are ellipsoids and geoids (Ayer and Fosu, 2008).

Specifically, to define a datum, the following will be needed:

1. An indication in the field (usually via a monument) of where the datum's initial point (origin) is located, along with measures of the latitude and longitude coordinates of this origin.
2. The azimuth of a line connecting the datum's initial point to a secondary point. This secondary point must also be identifiable in the field (once again, usually via a monument).
3. The precise definition of the model of the earth upon which the datum is based. Usually, if a spheroid is being used to represent the Earth, this is specified by the radius and flattening of the spheroid.

4. A quantity known as the datum's geoid separation at the initial point. This value, which is usually (but not always) zero indicates the vertical distance between the actual surface of the Earth and the surface of the datum's model of the earth. It is zero when the spirit leveled height at the origin is adopted to be the height above the ellipsoid, implying that the geoid and ellipsoid coincide at the origin. One method of doing this is simply to adopt the observed astronomical latitude and longitude of the point. The implication of this method is that the ellipsoid beneath the point will be parallel to the horizontal plane at the point. Either the geodetic azimuth or the geodetic longitude may be chosen so as to satisfy the Laplace's equation (Hoar, 1982).

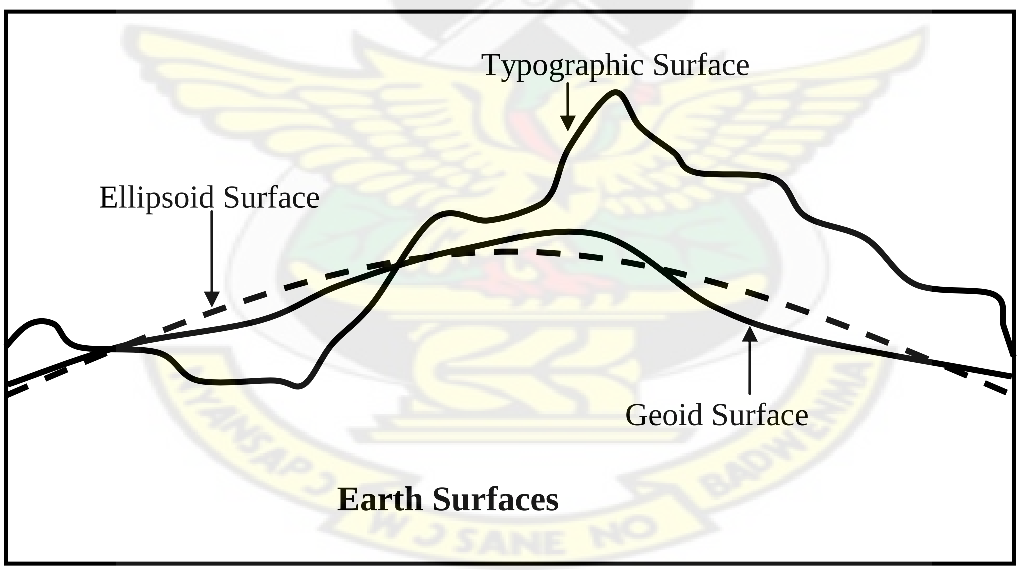

Fig. 2.1: Earth Surfaces (Modified from, Fosu, 2006)

Collectively, these four items allow a datum to establish a completely unambiguous set of references that can in turn be used to measure the location of any other point on the surface of the Earth.

Different nations and agencies use different datums as the basis for coordinate systems used to identify positions in geographic information systems, precise positioning systems, and navigation systems. The diversity of datums in use today and the technological advancements that have made possible global positioning measurements with sub-meter accuracies requires careful datum selection and careful conversion between coordinates in different datums.

2.3.1 Geoid

For height measurements, a reference 'zero height' surface is required. This height reference surface must be a level surface and for it to be used worldwide, it must be a closed shape like the shape of an ellipsoid. This equi-potential surface of the earth's gravity field which closely approximates mean sea level is referred to as the geoid. This means that at any point it is perpendicular to the direction of gravity (plumb line). The geoid is chosen as the reference level surface because every point on it has exactly the same potential, throughout the world.

The geoid has the following characteristics (Fosu, 2006):

1. It is a physical definition.
2. It is a complicated surface (irregular), hence cannot be represented by any mathematical equation.
3. It is defined by infinite number of parameters.
4. It allows users to know the direction in which water flows.
5. It can be sensed by instruments.

2.3.2 Reference Ellipsoid

While it is necessary to make observations and measurements on or near the physical surface of the earth, it would be quite impossible to perform detailed and extensive computations on a surface whose definition requires an infinite number of parameters.
Since the geoid is defined by infinite parameters, it is unsuitable as a mathematical model for computations. Hence a more regular figure must be adopted. It must be simple enough so that computations are not overly difficult, but must nowhere depart from the true figure of the earth by an amount which will give intolerable errors in the results. This figure is the ellipsoid of revolution which is chosen as the best mathematical model of the earth. The basic idea behind using the reference ellipsoids is that they fit the real shape of the earth as described by the geoid rather well and can thus be regarded as representative, yet simple, expression of the shape of the earth.

Reference ellipsoids are taken to be concentric with their coordinate system, geocentric or near-geocentric, with the axis of revolution coinciding with the Z-axis of the coordinate system. Reference ellipsoids are the horizontal surfaces to which the geodetic latitudes and longitudes are referenced. To serve in the above role, an ellipsoid (together with the associated Cartesian coordinate system) must be fixed with respect to the earth. Such an ellipsoid (fixed with respect to the earth) is often called a geocentric horizontal datum.

The ellipsoid has the following characteristics (Fosu, 2006):

1. It is a mathematical definition.

2. It is described by two parameters: the semi-major axis (for size) and the flattening (for shape).
3. It cannot be sensed by instruments.
4. It has been chosen to be as close as possible (but not exactly) to the earth's surface (or geoid) on a national, regional, or global point of view.

The complete datum definition consists of eight parameters- the 3-D location of the origin (three parameters), the 3-D orientation of the axes (three parameters), the size of the ellipsoid (one parameter) and the shape of ellipsoid. (Harvey,1986; Steed, 1990).

Because the ellipsoid shape does not fit the Earth perfectly, there are lots of different ellipsoids in use, some of which are designed to best fit the whole Earth, and some to best fit just a region. Thus various ellipsoids used in different regions differ in size, shape and orientation (Jackson, 1980; Smith, 1997).

The ellipsoid is formed by rotating an ellipse about its semi-minor axis (polar radius, b) with the semi-major axis (a) generating the equatorial plane. This ellipsoid of revolution approximates an oblate spheroid.

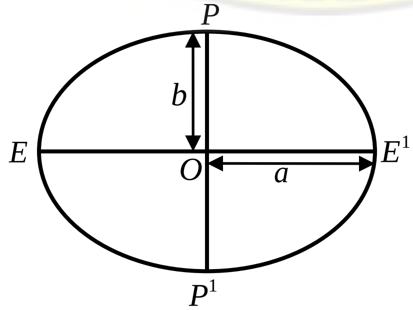

Fig. 2.2: A plan section of a reference ellipsoid (Fosu, 2006)

The parameters of the ellipsoid are defined as follows (Fosu, 2006):

The first eccentricity of the ellipsoid is defined by

$$e = \frac{\left(a^2 - b^2\right)^{\frac{1}{2}}}{a} \text{ …….2.1,}$$

and the flattening is defined by

$$f = \frac{a-b}{a} \text{ ……….2.2.}$$

The flattening and first eccentricity are related by the equation

$$e^2 = 2f - f^2 \text{ ……2.3.}$$

The next two other derived properties of the ellipsoid are the radius of curvature in the plane of the meridian (ρ) and the radius of curvature in the plane of the prime vertical (υ) defined respectively by

i. $$\rho = \frac{a\left(1 - e^2\right)}{\left(1 - e^2 \sin^2\phi\right)^{3/2}} \text{ ………..2.4}$$

ii. $$V = \frac{a}{\sqrt{\left(1 - e^2 \sin^2\phi\right)}} \text{ …….2.5.}$$

Where ϕ is the latitude of a point on the reference ellipsoid.

2.3.3 Conversion of Orthometric Heights (H) to Ellipsoidal Heights (h)

The height of a point above the geoid is referred to as the orthometric height and the height above the ellipsoid is the ellipsoidal height. With reference to figure 2.3, the ellipsoidal height (h), orthometric height (H) and the geoid height (N) are related by the basic equation (Hofmann-Welenhof et al., 1997; Ayer and Fosu, 2008 ; Yeboah, 2007).

$$h = H + N \text{ ………. 2.6}$$

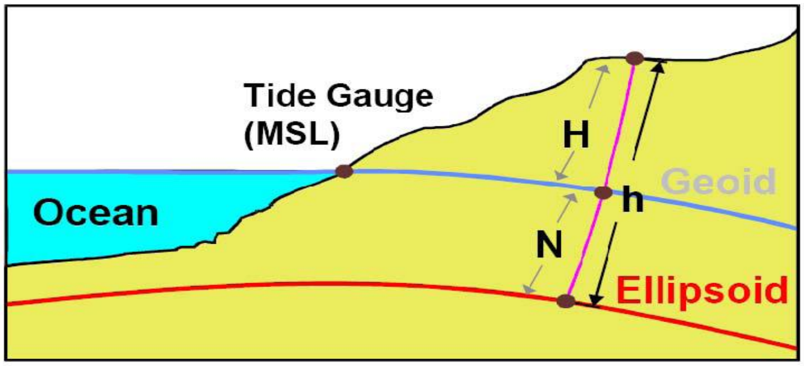

Figure 2.3: Relationships between the ellipsoidal height h and the orthometric height H (Source: WGS84 Implementation manual, 1998, see Yeboah, 2007)

For the purposes of converting the orthometric heights (H) to their corresponding ellipsoidal heights (h) on the War Office ellipsoid, the basic relationship between orthometric heights and ellipsoidal heights is used. For War Office we have

$h_{War} = H + N_{War} = H + \left(N_{WGS} - \Delta h\right)$ ……………2.7

And for WGS 84 reference system, we have

$h_{WGS84} = H + N_{WGS84}$ ………………………..2.8

Subtracting the two equations we have

$h_{WGS84} - h_{War} = N_{WGS84} - N_{War}$ ……..2.9

Putting $\Delta h = \Delta N$ we have $N_{War} = N_{WGS84} - \Delta h$ ……………….2.10

Using the GPS heights h_{WGS84} and orthometric heights (H) in the relationship, geoid heights N_{WGS84} on WGS 84 were obtained (Ayer and Fosu, 2008). The War Office geoid heights N_{War} are obtained as follows:

$N_{WAR} = N_{WGS84} - \Delta h$ ……………..2.11

The ellipsoidal heights above the War Office ellipsoid is then computed from,

$h_{War} = h_{WGS84} - \Delta h$ ………………………2.12

2.4 Geodetic Datum Classification and Types

Geodetic datums can be classified in two ways. Global or absolute datums have their origin as the centre of the earth. Regional ellipsoids approximate the earth surface only in their region of validity. Geodetic datum types include horizontal, vertical and complete datums which describe both vertical and horizontal systems.

2.4.1 GPS Coordinate Systems and WGS 84 Datum

GPS positioning is based on the World Geodetic System, 1984 (WGS 84) datum. This consists of a three-dimensional Cartesian coordinate system and an associated ellipsoid. WGS 84 positions can be defined as either XYZ Cartesian coordinates or geographic (latitude, longitude and ellipsoidal height) coordinates. The WGS 84 definition (D.M.A., 1987; 1991; Ayer and Fosu, 2008) includes the following items:

1. The WGS 84 Cartesian axes and ellipsoid are geocentric.
2. The orientation of the axes, and hence the orientation of the ellipsoid equator and prime meridian of zero longitude coincided with the equator and prime meridian of the Bureau Internationale de l’Heure at the moment in time 1984.0-that is , midnight on New Year’s Eve 1983.
3. Since 1984.0, the orientation of the axes and ellipsoid has changed but the average motion of the crustal plates relative to the ellipsoid is zero.

4. The shape and size of the ellipsoid is defined by the semi-major axis length a= 6378137.0 metres, and the reciprocal of flattening 298.257223563.

2.4.2 Horizontal Datum

Horizontal Datums define the relationship between the physical earth and horizontal coordinates such as latitude and longitude. A horizontal geodetic datum may consist of the longitude and latitude of an initial point (origin); an azimuth of a line (direction) to some other triangulation station; the parameters (radius and flattening) of the ellipsoid selected for the computations; and the geoid separation at the origin.

At its most basic level, the horizontal datum is a collection of specific points on the Earth that have been identified according to their latitude and longitude. To create a horizontal datum, surveyors marked each of the positions they had identified, typically with a brass, bronze, or aluminum disk (referred to as a monument). These markers were placed so that surveyors could see one marked position from another. A variety of methods such as triangulation have been used by surveyors to connect the horizontal monuments into a unified network.

2.4.3 Vertical Datums

The vertical datum is a collection of specific points on the Earth with known heights either above or below mean sea level. Vertical datums define level surfaces. Some vertical datums are based on sea-level measurements and leveling networks whereas others are based on gravity measurements.

2.5 Coordinate Systems and Frames

To define a coordinate system we need to define (Fosu, 2006):

i. Its origin (3 components)

ii. Its orientation (3 components, usually the direction cosines of one axis and one component of another axes, and definition of handedness)

iii. Its scale.

In all 7 quantities are needed to uniquely specify the frame. In practice these quantities are determined as the relationship between two different frames. The two coordinate types commonly used are ellipsoidal (geographic or curvilinear) coordinates (latitude, longitude and ellipsoidal height) and rectangular (Cartesian) coordinates - (XYZ).

2.5.1 Geographical Coordinates (latitude, longitude, height)

The most commonly used coordinate system today is the latitude, longitude, and height system. The Prime Meridian and the Equator are the reference planes used to define latitude and longitude. The geographical coordinates can be defined with reference to the ellipsoid or with reference to the geoid. For the ellipsoidal coordinates (ϕ, λ, h) the ellipsoid is the reference surface, and for the astronomical coordinates (Φ, Λ, H), the geoid is the reference surface. The astronomical coordinate system is defined in terms of the earth's gravity field.

2.5.2 Earth-Centered Earth Fixed Cartesian Coordinates (X, Y, Z)

Earth Centered, Earth Fixed Cartesian (ECEF) coordinate systems (figure 2.4) are also used to define three- dimensional positions with respect to the center of mass of the reference ellipsoid. The Z-axis points toward the North Pole. The X-axis is defined by the intersection of the plane defined by the prime meridian and the equatorial plane. The Y-axis completes a right- handed orthogonal system by a plane 90 degrees east of the X-axis and its intersection with the equator.

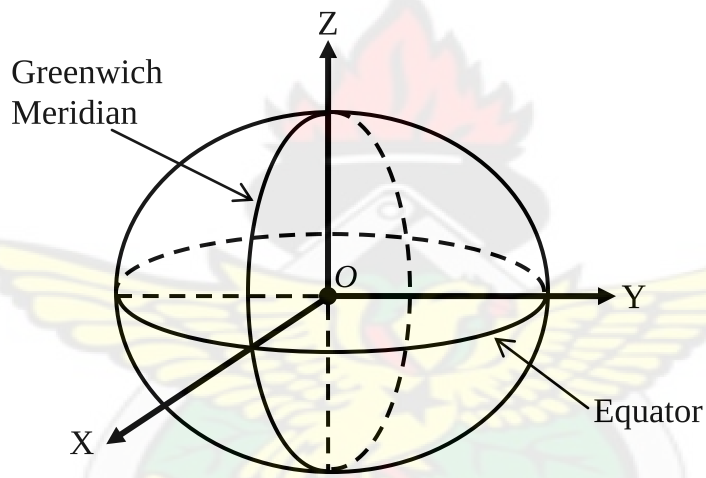

Fig. 2.4: ECEF coordinate system (Fosu, 2006)

2.6 Relationship between the Ellipsoidal and Cartesian Coordinates

Transformation between two different types of coordinate sets is unique and well defined. Mathematical algorithms exist for converting coordinates in one geodetic system to another. Ellipsoidal co-ordinates (φ, λ, h) can be converted to rectangular coordinates (X, Y,Z) using the following standard transformation equations (Fosu, 2006; Ayer and Fosu, 2008; Bowring, 1976; Hoar,1982;Marzooqi et al., 2005; Featherstone, 1997; Yeboah, 2007):

$$\begin{bmatrix} X = (V+h)\cos\phi\cos\lambda \\ Y = (V+h)\cos\phi\sin\lambda \\ Z = (V - e^2V + h)\sin\phi \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.13)$$

$$V = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \text{ and } e^2 = \frac{a^2-b^2}{a^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.14)$$

Where V = the radius of curvature in the prime vertical,

$h = ellipsoidal\ height$,

$a = semi - major\ axis\ of\ the\ reference\ ellipsoid$,

$b = semi - minor\ axis\ of\ the\ reference\ ellipsoid$ and

$e = first\ eccentricity\ of\ the\ reference\ ellipsoid$

Inverse computation from (X, Y, Z) to (ϕ, λ, h) is complicated for ϕ. If ϕ is known, λ and h can be computed using the Bowring's 1976 equations:

$$tan\lambda = \frac{Y}{X} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.15)$$

The latitude, ϕ is computed from the following expressions:

$$tan\ \phi = \frac{Z + e'^2 bsin^3\mu}{r - e^2 acos^3\mu} \ldots\ldots\ldots\ldots(2.16)$$

where

$$tan\mu = \frac{aZ}{br}$$

$$h = rcos\phi + Zsin\phi - a\sqrt{(1 - e^2 sin^2\phi)}$$

Where $r = \sqrt{x^2 + y^2}$,

and the second eccentricity of the ellipsoid, $\boldsymbol{e'^2 = \frac{e^2}{1-e^2}}$

2.7 Datum Transformations

Surveyors and Engineers are usually confronted with the task of integrating geodetic information based on two different incompatible geodetic datums. Two ellipsoidal datums can differ in terms of the position of the origin of their coordinates; orientation of the coordinate axes, and size and shape of the reference ellipsoids. It therefore becomes necessary to determine parameters that will mathematically convert data from one geodetic datum to another. This can be achieved if the two systems have at least three common points with known coordinates.

2.7.1 Types of Datum Transformations

Datum transformations are accomplished by various methods.

These methods can be classified either as Cartesian transformations (two-dimensional or three-dimensional) or position-dependent (curvilinear or polynomial) transformations.

2.7.2 Cartesian Transformations

Complete datum conversion is based on seven parameter transformations that include three translation parameters, three rotation parameters and a scale parameter.

Simple three parameter conversion between latitude, longitude, and height in different datums can be accomplished by conversion through Earth-Centered, Earth Fixed (ECEF) Cartesian coordinates. The parameters are related through three origin shifts (ΔX, ΔY, ΔZ), three rotation of the axes (R_X, $R_{Y,}$ R_Z) and a scale change Δs.

When taking the Cartesian approach to the determination of co-ordinate transformation parameters, the curvilinear co-ordinates must first be converted to ECEF co-ordinates of their respective reference ellipsoids using the Bowring's algorithms. The transformation parameters are then determined using any one of the many existing transformation models that are based on the Cartesian coordinate system.

The reverse transformation from Cartesian to curvilinear co-ordinates is more involving because the direct inversion of equations (2.13) requires some iteration for the solution of latitude (Bowring, 1985, see Featherstone, 1997). Closed (non-iterative) solutions to this problem do exist by using quartic equations (Borkowski, 1989, see Featherstone, 1997). For this research, Bowring's algorithm will be used for all computations. If the geoid separations of the common points are known, the orthometric heights can however be directly converted to ellipsoidal heights.

2.7.3 Three-Dimensional Cartesian Transformations

The three-dimensional transformation is generally expressed mathematically as (http://en.wikipedia.org/wiki/Helmert_transformation):

$$X_T = C + \mu RX \text{, where.............................} (2.17)$$

$$X_T = \text{the transformed (target) vector and } X = \text{the initial (source) vector.}$$

The parameters are:

i. C- A translation vector. It contains three translations along the coordinate axes.

ii. μ-A scale factor, which is dimensionless as it is usually expressed in parts per million, it, must be divided by 1000000.

iii. R- A rotation matrix. It consists of three rotations about the three axes. The rotational matrix is an orthogonal matrix. The rotations are measured in radians.

If the transformation parameters are unknown, they can be calculated with reference points (i.e. points whose coordinates are known in both systems). Since a total of seven (7) parameters have to be determined, knowledge of at least two points and one coordinate of a third point (e.g. a Z- Coordinate) are sufficient to determine the transformation parameters. This gives a system of linear equations with 7 equations and 7 unknowns, which can be solved.

In practice, it is best to use more points. Through this correspondence, a higher accuracy is obtained and a statistical assessment of the results becomes possible. In this case, the calculation is adjusted with the Gaussian least squares method. A numerical value for the accuracy of the transformation parameters is obtained by calculating the values at the reference points, and weighting the results relative to the centroid of the points.

Practically, these parameters are computed from the inclusion of at least three known points in the network. However, the inaccuracy of these points will affect the transformation parameters, as these points will contain observation errors. Therefore a ‘real world’ transformation will only be at best an estimate and should contain a statistical measure of its quality. If more points are known, a least squares adjustment can be performed to reduce the effect of errors in the given coordinates (Mikhail, 1976; Fan, 1997; Constantin-Octavian , 2006)

Once the curvilinear co-ordinates have been converted to their Cartesian counterparts using equation (2.13), the following mathematical models can be used to transform these Cartesian co-ordinates between the respective datums.

2.7.4 Helmert 7-Parameter Similarity Transformation

This is a 7-parameter similarity transformation comprising of three translational parameters (ΔX, ΔY, ΔZ), three rotation parameters (Rx, Ry, Rz), and a scale factor (Δs). This model relates two Cartesian coordinates. It is written in the form (Hoar, 1982, http://www.linz.govt.nz/geodetic/conversion-coordinates/geodetic-datum-conversion/datum-transformation-equations/index.aspx, http://en.wikipedia.org/wiki/Helmert_transformation):

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+\Delta S\times 10^{-6}) \begin{bmatrix} 1 & +R_Z & -R_Y \\ -R_Z & 1 & R_X \\ +R_Y & -R_X & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad \text{.............................}(2.18)$$

Where $\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$ = coordinates in the first datum

and $\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}$ = coordinates in the second datum.

Application of the Helmert Transformation Model

The Helmert transformation model is frequently used in geodesy to produce distortion-free transformations from one datum to another datum enabling regional surveying points to be converted into the WGS 84 locations used by GPS.

The similarity transformation is popular due to (Constantin-Octavian, 2006):

- The small number of parameters involved,
- The simplicity of the model, which is more easily implemented into software,
- The fact that it is adequate for relating two coordinate systems in the case when they are homogenous (no local distortion in scale or orientation).

2.7.5 The Bursa-Wolf Transformation Model

The Bursa-Wolf transformation model (Bursa, 1962; Wolf, 1963) is a modified form of the Helmert seven-parameter conformal transformation model for transforming three-dimensional Cartesian co-ordinates between two datums. According to Krakiwsky and Thomson, 1974 (see Marzooqi et al., 2005; Featherstone, 1997), this transformation model is more suitable to satellite datums on a global scale. According to Oliver, 1996, this method produces accuracies at ±1m level.

The transformation involves three geocentric datum shift parameters (ΔX, ΔY, ΔZ), three rotation elements (R_X, R_Y, R_Z) and scale factor ($1+\Delta S$). According to Hoar, 1982, Burford, 1985 (see Marzooqi et al., 2005; Featherstone, 1997), the Bursa-Wolf model for relating coordinates in system 1 to system 2 is given by

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \left[1 + \Delta S \times 10^{-6}\right] \begin{bmatrix} 1 & +R_Z & -R_Y \\ -R_Z & 1 & R_X \\ +R_Y & -R_X & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad \text{.................................... (2.19)}$$

Where $\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$ = coordinates vector in the first datum and

$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}$ = coordinates vector in the second datum.

Each station with coordinates in both systems provides such a set of equations, and the seven transformation parameters are estimated from the combined sets, where R_x, R_y ,R_Z are positive when rotations from the local geodetic to the geocentric systems are anticlockwise.

For transforming coordinates of points from War Office to WGS 84, equation 2.19 may be re-written as

$$\begin{bmatrix} X_{WGS} \\ Y_{WGs} \\ Z_{WGS} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 1+\Delta S & R_Z & -R_Y \\ -R_Z & 1+\Delta S & R_X \\ R_Y & -R_X & 1+\Delta S \end{bmatrix} \begin{bmatrix} X_{War} \\ Y_{War} \\ Z_{War} \end{bmatrix} \quad \text{…………… (2.20)}$$

Where;

X_{WGS}, Y_{WGS}, Z_{WGS} : are the global datum (WGS 84) Cartesian co-ordinates;

X_{war}, Y_{war}, Z_{war} : are the local datum (War Office) Cartesian co-ordinates;

(ΔX, ΔY, Δ Z): are the origin shifts;

(Rx, Ry, Rz): are the rotations of the axes about the origin to ensure parallelism of the axes of the two systems; and (ΔS): is a scale change.

Equation 2.20 is based on the following assumptions:

1. The origin is different and a vector offset is necessary between the coordinate systems.
2. There is a rotation about each axis.
3. There may be a scale change.

Limitations of the Bursa-Wolf Model

One problem with the Bursa-Wolf model is that the adjusted parameters are highly correlated when a network of points used to determine the parameters covers only a small portion of the earth.

2.7.6 The Molodensky-Badekas model

The Molodensky-Badekas (Molodensky et al., 1962; Badekas, 1969) is also a seven-parameter model for transforming three-dimensional Cartesian coordinates between two datums. This transformation model is more suitable for transformation between terrestrial and satellite datums, (Krakiwsky and Thomson, 1974). The transformation also involves three barycentric datum shift parameters $(\Delta X, \Delta Y, \Delta Z)$, three rotation elements (R_X, R_Y, R_Z) and scale factor (1+ΔS). This transformation model is theoretically identical to the Bursa-Wolf model. The molodensky-Badekas model in its matrix form, is given by Hoar, 1982, Krakwisky and Thomson (1974), Burford (1985) and Harvey (1986) (see Marzooqi et al., Featherstone, 1997), as

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} + \begin{bmatrix} 1+\Delta s & +R_Z & -R_Y \\ -R_Z & 1+\Delta S & R_Y \\ +R_Y & -R_X & 1+\Delta S \end{bmatrix} \begin{bmatrix} X_1 - X_m \\ Y_1 - Y_m \\ Z_1 - Z_m \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2.21)$$

Where $X_m = \frac{1}{n}\sum_{i=1}^{n} X_i$, $Y_m = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $Z_m = \frac{1}{n}\sum_{i=1}^{n} Z_i$

$X_i, Y_{i,} Z_i$ are the Cartesian coordinates in the source (first) datum and n is the number of common points. (ΔX, ΔY, ΔZ) are the shifts between the barycentre or centroid of the terrestrial networks, and the rotation matrix and scale change are theoretically identical to the Bursa-Wolf model, $X_m, Y_{m,} Z_m$ are the coordinates of the geometric centroid of points in the source datum as defined above.

$(X_{1,}, Y_1, Z_1) = coordinates\ of\ a\ common\ point\ in\ the\ source\ system$ and

$(X_2, Y_2, Z_2) = coordinates\ of\ a\ common\ point\ in\ target\ system$

For transforming coordinates of points from War Office to WGS 84, equation 2.21 may be re-written as

$$\begin{bmatrix} X_{WGS} \\ Y_{WGS} \\ Z_{WGS} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} + \begin{bmatrix} 1+\Delta S & R_Z & -R_Y \\ -R_Z & 1+\Delta S & R_X \\ R_Y & -R_X & 1+\Delta S \end{bmatrix} \begin{bmatrix} X_{War} - X_m \\ Y_{War} - Y_m \\ Z_{War} - Z_m \end{bmatrix} \quad \ldots\ldots\ldots\ \ldots\ldots\ (2.22)$$

Where the

$$(X_{war}, Y_{war}, Z_{war}) = coordinates\ of\ points\ on\ the\ War\ Office\ datum$$

$$(X_{WGS}, Y_{WGS}, Z_{WGS}) = coordinates\ of\ points\ on\ the\ WGS\ 84\ datum$$

$$(X_m, Y_m, Z_m) = coordinates\ of\ the\ geometric\ centroid\ of\ points\ in\ War\ Office$$

and the rest of the terms are as previously defined.

Therefore, the only conceptual difference between the Molodensky-Badekas and Bursa-Wolf models is the choice of the point about which the axial rotations and scale change are applied (Featherstone, 1997). As this point is the barycentre for the Molodensky-Badekas model, this model offers a more appropriate option for the transformation between terrestrial and satellite datums (Featherstone, 1997).Theoretically, the Bursa-Wolf and Molodensky-Badekas models should give the same results when the same data are used to determine the respective sets of transformation parameters.

The Molodensky-Badekas model removes the high correlation between parameters by relating the parameters to the centroid of the network.

2.7.7 The Simple Three-Parameter Model

This model simply applies a three-dimensional origin shift, with little regard for any scale changes or rotations (Featherstone, 1997). Therefore, it is coarse, but also extremely simple to implement. The Cartesian coordinates from the initial datum are simply added to the origin shift and then converted to curvilinear coordinates on the new datum. In vector form, this is given by (Featherstone, 1997):

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad \text{...(2.23)}$$

Where ΔX, ΔY, ΔZ, are the translational (Shift) parameters.

2.7.8 Two-Dimensional Transformation Models

Two-dimensional similarity transformation expresses relationship between two systems, two-dimensional coordinate system. The aim of the similarity transformation is to prevent deformation of shape. In this transformation model, the coordinate axes are perpendicular to each other in the two systems and it is assumed that scale factor is the same on the x-axis and y- axis in the two systems. There is shifting, rotation and scale difference between two coordinate systems as (U, V) and (X, Y).

In cases of relatively small networks, (less than 100 km by 100 km) two-dimensional transformation models may be sufficient. The initial 3-D Cartesian coordinates are converted to geodetic ones and, finally, to map projection coordinates. Then, the full 2-D similarity transformation, with two translation parameters X_o, Y_o, one rotation α and a scale parameter K= (1+k), known as Helmert transformation, is expressed as follows (Hofmann-Wellenhof et al., 1997).

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X_O \\ Y_O \end{bmatrix} + K \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \quad \text{.................. (2.24)}$$

2.7.9 Distortion- Free Transformations

Distortions in the local geodetic networks may be due to systematic errors in data or approximate reductions, and affect mainly the scale and orientation of the network (Featherstone, 1997). The best way to avoid this is to solve for a number of sets of local parameters, which better represent the local and regional areas, rather than determining one set of parameters for a country or continent. The curvilinear transformations are useful in removing these local distortions. The curvilinear transformations are conceptually simpler as they directly produce a coordinate change in degrees instead of converting through Cartesian coordinates.

2.7.10 Molodensky Transformation Models

Molodensky transformation can be classified either as a three (3)-parameter or five (5)-parameter transformation. The three-parameter transformation comprise three origin shifts of the geometrical centres of the reference ellipsoids associated with the datums, whereas the additional two parameters are changes in the semi-major axis (a), and flatenning (f) of the reference ellipsoids associated with the two datums. The Standard model simply applies a three-dimensional origin shift from the geocentre (ΔX, ΔY, ΔZ) in conjunction with a scale change provided by the difference in semi-major-axis length (Δa) and flattening (Δf) of the respective reference ellipsoids.

From Defense Mapping Agency (1991), http://sas2.elte.hu/tg/eesti_datums_egs9.htm, the standard Molodensky transformation is given by

$$\phi_W = \phi_A + \left[\begin{array}{l} (-\Delta X \sin\phi \cos\lambda + \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi) + \frac{\Delta a}{a} (v e^2 \sin\phi \cos\phi) \\ + \Delta f \left(\frac{\rho a}{b} + \frac{v b}{a} \right) \sin\phi \cos\phi \end{array} \right] (\rho + h) \quad \text{..........(2.25)}$$

$$\lambda_W = \lambda_A + \frac{-\Delta x \sin\lambda + \Delta y \cos\lambda}{(v + h) \cos\phi} \quad \text{..(2.26)}$$

Where the radius of curvature of the meridians (***ρ***) is given by

$$\rho = \frac{a(1 - e^2)}{(1 - e^2 \sin^2\phi)^{3/2}} \quad \text{..(2.27)}$$

and the numerical values of a and e refer to the reference spheroid associated with the initial datum.

2.7.11 Abridged Molodensky Formulas

This datum transformation model as used by the National Imagery and Mapping Agency (NIMA), the former Defense Mapping Agency (DMA) is a five parameter transformation model, which transforms three dimensional curvilinear co-ordinates between two datums. This model directly transforms geographical coordinates from one coordinate system to another using five parameters; three origin shifts (ΔX, ΔY, ΔZ) and two shifts in ellipsoidal parameters (Δa, Δf).

It simply applies the three dimensional geocentric datum shift parameters (ΔX, ΔY, ΔZ) in conjunction with the differences between the semi-major axis (Δa) and flattening (Δf) of the local geodetic system ellipsoid and the WGS84 ellipsoid respectively (WGS 84 minus local). The Abridged Molodensky transformation is given as (D.M.A., 1987; Molnar and Timar, 2005):

$$\Delta\phi'' = \frac{1}{\rho \sin 1''}\left[-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + (a\Delta f + f\Delta a)\sin 2\phi\right] \ldots (2.28)$$

$$\Delta\lambda'' = \left[\frac{-\Delta X \sin\lambda + \Delta Y \cos\phi}{V \cos\phi \sin 1''}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.29)$$

$$\Delta h = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi + (a\Delta f + f\Delta a)\sin^2\phi - \Delta a ....(2.30)$$

With $\rho = \frac{a(1-e^2)}{(1-e^2 \sin^2\phi)^{3/2}}$ and $V = \frac{a}{(1-e^2 \sin^2\phi)^{1/2}}$, and $e^2 = 2f - f^2$

Where ϕ, λ, h = geodetic coordinates with respect to the local ellipsoid

$\Delta\phi, \Delta\lambda, \Delta h$ = corrections to transform local datum coordinates to WGS 84 (Φ, λ, h).

$\Delta X, \Delta Y, \Delta Z$ = corrections to transform local datum coordinates to WGS 84 X, Y, Z.

$\Delta a, \Delta f$ = (WGS 84 minus local) semi-major axis and flattening respectively.

a = semi-major axis of the local geodetic system ellipsoid

f = flattening of the local geodetic system ellipsoid

ρ = radius of curvature in the meridian.

V = radius of curvature in the vertical.

e = first eccentricity of the reference ellipsoid.

The formulae deal only with a translation of the origin and changes in ellipsoidal size and shape, and assumes that the Cartesian coordinate axes of the two systems are parallel (Deakin, 2004; Mitsakaki, 2004)

The iterative approach of the Abridged Molodensky transformation uses all the three equations to obtain convergence for values of ΔX, ΔY and ΔZ. It starts with a first approximation of Δh $(\Delta h = 0)$, leading to progressive refinements in the values of Δh and the corresponding values of ΔX, ΔY and ΔZ (Ayer & Tienah,2008). This is implemented in a computer program capable of performing a large number of iterations leading to convergence of results.

2.8 Map Projections

A projection is a specific type of coordinate transformation, which is a systematic representation of curved surface on a plane. A curved surface is mapped into plane coordinates using distorting angles, azimuths, distances, or area, which minimizes distortions as much as possible.

Projections are of various types like conformal projection that minimizes the distortion of local angles; but large areas may be distorted. An equal-area projection preserves areas but angles and distances will be distorted. An azimuthal projection maintains the correct direction, or azimuth from the centre of the projection to every point in the map. Examples of conformal projection are: Lambert parallel, Transverse Mercator (or UTM) etc. Each different type of projection uses a set of parameters that define the mapping between ellipsoid and plane coordinates. Ghana uses the Transverse Mercator projection as its projection system (Philip and Muehrcke, 1998).

2.8.1 Projection from Geographical Coordinates to Ghana Grid Coordinates

The following mathematical formulae are used to convert geographical coordinates into a grid or a map (Survey Records, 1936):

$$X = X_O + Kw^2 \sin 2\phi + Ow^4 \quad \text{..}(2.31)$$

$$Y = 900{,}000 + (s.f)Mw\cos\phi + Nw^3 \quad \text{..}(2.32)$$

where

$X_0 = distance\ along\ the\ central\ meridian\ from\ the\ origin\ to\ latitude\ \phi$

$\lambda = longitude\ of\ a\ point,$

$w = deviation\ in\ longitude\ from\ the\ origin,$

$X = Northing\ coordinate\ of\ a\ point\ in\ War\ Office$

$Y = easting\ coordinate\ of\ a\ point\ in\ War\ Office$

$$T = \left(\frac{V}{3\rho}\right)\sin 1''\cos^2\phi, \quad K = \left(\frac{1}{4}\right)V\sin^2 1''$$

$$O = \left(\frac{1}{24}\right)V\sin^4 1''\cos^3\phi\,(5 - \tan^2\phi)\sin\phi$$

$$M = V\sin 1'', \quad N = \left(\frac{1}{6}\right)V\sin^3 1''\cos^3\phi\left(\frac{V}{\rho} - \tan^2\phi\right), \quad \rho = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}},$$

and $V = \dfrac{a}{(1-e^2\sin^2\phi)^{1/2}}$.

ω= λ + 1° (reduced to seconds), s.f = 0.99975, scale factor at origin.

Where, ρ and V are the principal radii of curvature along the meridian and at right angles to the meridian respectively, each reduced by 1/4000.

Also, $X_0 = (.f)(M_1 - M_0)$,

Where

$M_1 = a(A_1 + B_1 + C_1 + D_1)$, $M_0 = a(A_0 + B_0 + C_0 + D_0)$, $\phi_0 = 4°40'$, and a = semi-major axis of the War Office ellipsoid,

$$A_1 = \left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256}\right)\phi_1, \quad A_0 = \left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256}\right)\phi_0,$$

$$B_0 = -\left(\frac{3e^2}{8} + \frac{3e^4}{32} + \frac{45e^6}{1024}\right)\sin 2\phi_0, \quad B_1 = -\left(\frac{3e^2}{8} + \frac{3e^4}{32} + \frac{45e^6}{1024}\right)\sin 2\phi_1,$$

$$C_1 = \left(\frac{15e^4}{256} + \frac{45e^6}{1024}\right)\sin 4\phi_1, \quad D_1 = -\frac{35e^6}{3072}\sin 6\phi_1,$$

$$C_0 = \left(\frac{15e^4}{256} + \frac{45e^6}{1024}\right)\sin 4\phi_0, \text{ and } D_0 = -\frac{35e^6}{3072}\sin 6\phi_0$$

2.8.2 Conversion from Ghana National Grid Coordinates Easting and Northing (y, x) to Geographical Coordinates (ϕ, λ)

The following mathematical equations are used in executing this conversion (Survey Records 1936):

$$\phi = \phi_1 - AY^2 \tan\phi_1 + BY^4 \tan\phi_1 \quad (2.33)$$

$$w = CY \sec\phi_1 - DY^3 \quad (2.34)$$

Where,

$\phi_1 = latitude\ of\ the\ point\ on\ the\ central\ meridian\ X\ feet\ from\ the\ origin.$

$Y = y - 900{,}000,$

$A = \frac{1}{2}\rho V \sin 1'',$

$B = \frac{\left(1 + \tan^2 \phi_1\right)}{24\rho V^3 \sin 1''},$

$C = \frac{1}{V \sin 1''},$

$D = \frac{\left(V/\rho + 2\tan^2 \phi_1\right)}{6V^3 \sin 1'' \cos\phi_1},$ and

$E = \frac{\left(1 + \tan^2 \phi_1\right)}{3V^3 \sin 1''}.$

2.9 Realization of Ghana's Geodetic Reference System

The Ghana framework of Geodetic Networks based on the Accra Datum is made up of primary and secondary triangulation, primary, secondary and tertiary traverses and precise levels. The southern half of the country which is relatively mountainous or hilly is covered with triangulation points with baselines up to a maximum of about 55 miles and controlled by three measured bases (Yeboah, 2007).

The Geodetic reference survey of Ghana started in June 1904 with observation for latitude taken by Captain F. G. Guggisberg (who later became Governor of the then Gold Coast) from a pillar in the compound of the house of the Secretary for Native Affairs in

Accra. Observations were made to fifteen pairs of stars with a zenith telescope to give a final probable error of 0".360. This pillar was subsequently connected by traverse to GCS 547 in Accra. The longitude of GCS547 was determined by exchange of telegraphic signals with Cape Town in November and December 1904 (Clifford, 2007; Survey Records, 1936, Ayer and Fosu, 2008, Yeboah, 2007; Kwarteng and Yankson, 2007). The pillar GCS 547 was then connected to GCS 121, a pillar at Legon by triangulation and the values obtained were adopted as the basic latitude and longitude in Ghana for further triangulation to obtain a national geodetic framework. Most of the triangulation was completed between 1926 and 1929 (Survey Records,1936, Ayer and Fosu, 2008; Yeboah, 2007; Kwarteng and Yankson, 2007).

Heights in Ghana are based on the mean sea level determined in Accra by means of tidal observations from 9th April 1922 to 30th April 1923 (Ayer and Fosu, 2008; Yeboah, 2007). In order to transfer the heights to other established triangulation points, a double line of spirit levels was run to the Legon pillar GCS 121 and this gave a reduced level of H as 483.80feet to pillar GCS 121 (Ayer and Fosu, 2008). The trigonometric heights were then involved in the computation of the framework with the observed vertical heights weighted in inverse proportion to the respective distances of the lines to get the orthometric heights for the trigonometric points (Survey Records, 1936).

Due to the limitations in computational aids at the time of the original triangulation, the triangulation net was adjusted in smallish figures so that homogeneity or lack of homogeneity for the entire network could not be guaranteed. The primary traverses similarly were very good individually but could be out of sympathy collectively (Ayer and Fosu, 2008).

2.10 Accra Datum

Ghana's National horizontal datum is based on the War Office ellipsoid suggested by the British War Office. The parameters of this ellipsoid are; semi-major axis a=6378299.99899 m, semi-minor axis b=6356751.68824 m, flattening f=1/296, and feet to meter conversion factor of 0.304799706846 (Ayer, 2008; Ayer and Fosu, 2008, Yeboah, 2007). For the computation of planimetric coordinates, the Transverse Mercator projection is used for Ghana with the whole country placed at the same origin of the intersection of longitude 1° W and latitude 4°40' N. The values assigned to this origin are 0.000 feet for northings and 900,000 feet for eastings and a scale factor along central meridian of 0.99975 (Ayer, 2008; Ayer and Fosu, 2008; Yeboah, 2007; Kwarteng and Yankson, 2007; Clifford, 2007). The Ghana National Grid is therefore a horizontal coordinate system, consisting of geodetic datum that uses the war office ellipsoid called Accra Datum. The realization of this terrestrial reference framework is established using trigonometric pillars and a Transverse Mercator projection for deriving easting and northing coordinates. This is the coordinate system used to indicate positions of features on all survey maps in Ghana (Survey Records, 1936).

CHAPTER THREE: OBSERVATIONAL AND COMPUTATIONAL PROCESSES

It is clear from the preceding chapters that the task of determining transformation parameters between two datums involves both observational and computational processes. This chapter focuses on these procedures and the tests conducted to check the accuracy of the determined parameters.

3.1 Observational Procedure

The derivation of datum transformation parameters requires a comparison between the XYZ Cartesian coordinates of points common to both datums, the minimum number of common points depending on the type of transformation model applied. In practice it is best to use more points as this increases redundancy, leading to better results and provides a means of making statistical analysis of the parameters.

The coordinates of the common points on the WGS 84 ellipsoid were derived from latitude, longitude and ellipsoidal height delivered by the GPS receiver and the differential processing with the International Geodetic System (IGS) data. Those for the local system, which were established by traditional methods, were obtained from knowledge of the latitude and longitude of the points and their heights above the mean sea level (orthometric height).These values were obtained from the Ghana Survey records, 1936.

In order to obtain the WGS 84 co-ordinates of the points, a GPS survey was carried out on nineteen trigonometric stations within the Golden Triangle whose latitudes, longitudes and orthometric heights are known on the War Office ellipsoid using two dual-frequency GPS receivers ,one at the base site (reference) and the other on the field as rover. The observations were carried out in static mode, and processed as a single point averaged to more than one hour observation time relative to WGS 84 ellipsoid. The dual-frequency receivers were preferred to single-frequency receivers as they are capable of tracking both L1 and L2 frequency signals whereas the single-frequency receiver can track only the L1 frequency signal. The dual-frequency receivers can also effectively resolve baselines longer than 20km where ionospheric effects have larger impact on co-ordinate computations.

To obtain geographical co-ordinates of the common points in the WGS 84 system, the coded data obtained by the GPS receivers was post-processed using Spectrum survey software, version 4.13.

3.2 Computational Procedure

The process of determining transformation parameters requires a lot of computational tasks which will be practically impossible without the use of a computer programming language. For this task, a Matlab computer program was written to handle the various transformation models. The program follows the Combined Least Squares Adjustment (CLSA) technique. The input data consists of the ellipsoidal coordinates of the points in WGS 84 system $(\phi, \lambda, h)_{WGS}$ and their corresponding co-ordinates in the War Office system $(\phi, \lambda, H)_{War}$. The output for the Abridged Molodensky five parameter model is the three shift parameters, the ellipsoidal height differences between the two datums, residuals and standard deviation. The output for the models is the Cartesian (X, Y, Z) coordinates of the points in the two systems and the transformation parameters between the two systems with their standard deviation and residuals.

3.2.1 Transformation Procedure

The implementation of the simple three -parameter, the seven - parameter Bursa-Wolf and the seven –parameter Molodensky- Badekas' models requires that the coordinates of the points in both systems be in the Rectangular Cartesian coordinates (X, Y, Z). For the WGS 84 coordinates, since the ellipsoidal height is known in addition to the latitude and longitude, the Cartesian coordinates can be obtained directly using the standard equations (2.13 and 2.14).

However, for the War Office ellipsoid, the ellipsoidal height is not known. If the geoid separation is known, the orthometric heights could simply be used to determine the ellipsoidal heights using equation (2.7). The geoid separation for the War Office ellipsoid is however not known since the geoid is undefined.

Fortunately, the Abridged Molodensky transformation model provides a means for determining the approximate difference in ellipsoidal height, Δh, between the War Office and WGS 84 ellipsoids, using equation (2.12). Hence, the transformations are implemented by first applying the Abridged Molodensky model and use of the standard models (equations 2.13 and 2.14) to determine the transformation parameters using the Block shift, Bursa-wolf and Molodensky-Badekas' models.

A total of 19 stations tied to War Office datum were used for the transformation process. Tables 3.1 and 3.2 are lists of the stations and their coordinates in the WGS 84 and War Office coordinate systems respectively.

Table 3.1: Golden Triangle Processed GPS coordinates in WGS 84

POINT ID	LONG				LAT				ELLIP. HEIGHT(h/m)
CFP 109	0	25	24.81766	W	5	27	36.32569	N	78.2744
CFP 200	0	33	33.54116	W	5	37	32.87415	N	304.9379
CFP 225	1	30	3.96614	W	5	27	18.31243	N	275.1437
GCS 102	0	44	3.86162	W	5	16	57.87905	N	83.4515
CFP 155	0	07	19.18235	W	5	56	20.52243	N	524.5492
GCS 179	1	01	59.90579	W	6	22	19.62456	N	492.5083
CFP 180R	1	17	10.36042	W	6	03	13.64974	N	437.6990
CFP 217	0	43	46.93781	W	5	56	35.18511	N	311.0926
GCS 142	0	45	56.05923	W	6	34	32.86850	N	782.2084
CFP 213	0	44	56.05567	W	6	07	41.50938	N	327.0218
CFP 178	1	09	52.78589	W	6	34	16.88627	N	615.7568
CFP 185	1	55	30.56291	W	6	29	5.19173	N	643.5756
CFP 306	1	37	49.67431	W	7	14	9.09967	N	536.0062
GCS 302	2	01	0.32842	W	6	54	44.92864	N	560.8285
CFP 304	1	26	43.21595	W	6	59	31.95133	N	620.9316
CFP 305	1	44	36.30423	W	6	50	46.8373	N	417.0231
GCS 145R	1	24	42.82948	W	6	33	24.89911	N	503.7124
CFP 184	1	41	41.39178	W	6	28	17.60759	N	472.1430
CFP 207	1	57	58.15176	W	5	50	58.62504	N	399.3477

Table 3.2: Golden Triangle Test Point coordinates on War Office Ellipsoid

POINT ID	LONG				LAT				ORTH. HEIGHT(H/m)
CFP 109	0	25	25.84579	W	5	27	26.29465	N	182.0
CFP 200	0	33	34.55078	W	5	37	22.8541	N	917.7
CFP 225	1	30	4.874719	W	5	27	8.187017	N	819.5
GCS 102	0	44	4.840	W	5	16	47.85006	N	197.9
CFP 155	0	07	20.246	W	5	56	10.488	N	1641.6
GCS 179	1	02	0.868087	W	6	22	9.631232	N	1528.8
CFP 180R	1	17	11.27207	W	6	03	3.624034	N	1348.5
CFP 217	0	43	47.92012	W	5	56	25.19009	N	937.4
GCS 142	0	45	57.03492	W	6	34	22.92578	N	2478
CFP 213	0	44	57.03109	W	6	7	31.50499	N	988.6
CFP 178	1	09	53.7538	W	6	34	6.931122	N	1937
CFP 185	1	55	31.4628	W	6	28	55.21511	N	2019.9
CFP 306	1	37	50.57978	W	7	13	59.26309	N	1648
GCS 302	2	01	1.223389	W	6	54	35.04617	N	1756.8
CFP 304	1	26	44.12655	W	6	59	22.06127	N	1937.6
CFP 305	1	44	37.22302	W	6	50	36.9414	N	1275.3
GCS 145R	1	24	43.77802	W	6	33	14.9303	N	1563.7
CFP 184	1	41	42.30616	W	6	28	7.6459303	N	1464.2
CFP 207	1	57	59.05258	W	5	50	48.56716	N	1227.65

The flow chart of the transformation procedure is shown in figure 3.1. The transformation procedure is in the following steps:

1. Apply the Iterative Abridged Molodensky transformation to the geographic coordinates of common points on the WGS 84 and Ghana War Office ellipsoids (Table 3.1 and 3.2) to determine the five transformation parameters $(\Delta X, \Delta Y, \Delta Z, \Delta a, \Delta f)$. Note that Δa and Δf are the differences between the parameters of the two ellipsoids.
2. Use the parameters to determine the ellipsoidal heights of the common points on the War Office ellipsoid (equation 2.30).

3. Convert the geographic co-ordinates (Φ, λ, h) of the common points to rectangular Cartesian co-ordinates (X, Y, Z).
4. Apply the three-parameter, Bursa-Wolf, and Molodensky-Badekas' transformation models to determine the transformation parameter sets for each model.
5. Perform statistical tests on all the models.

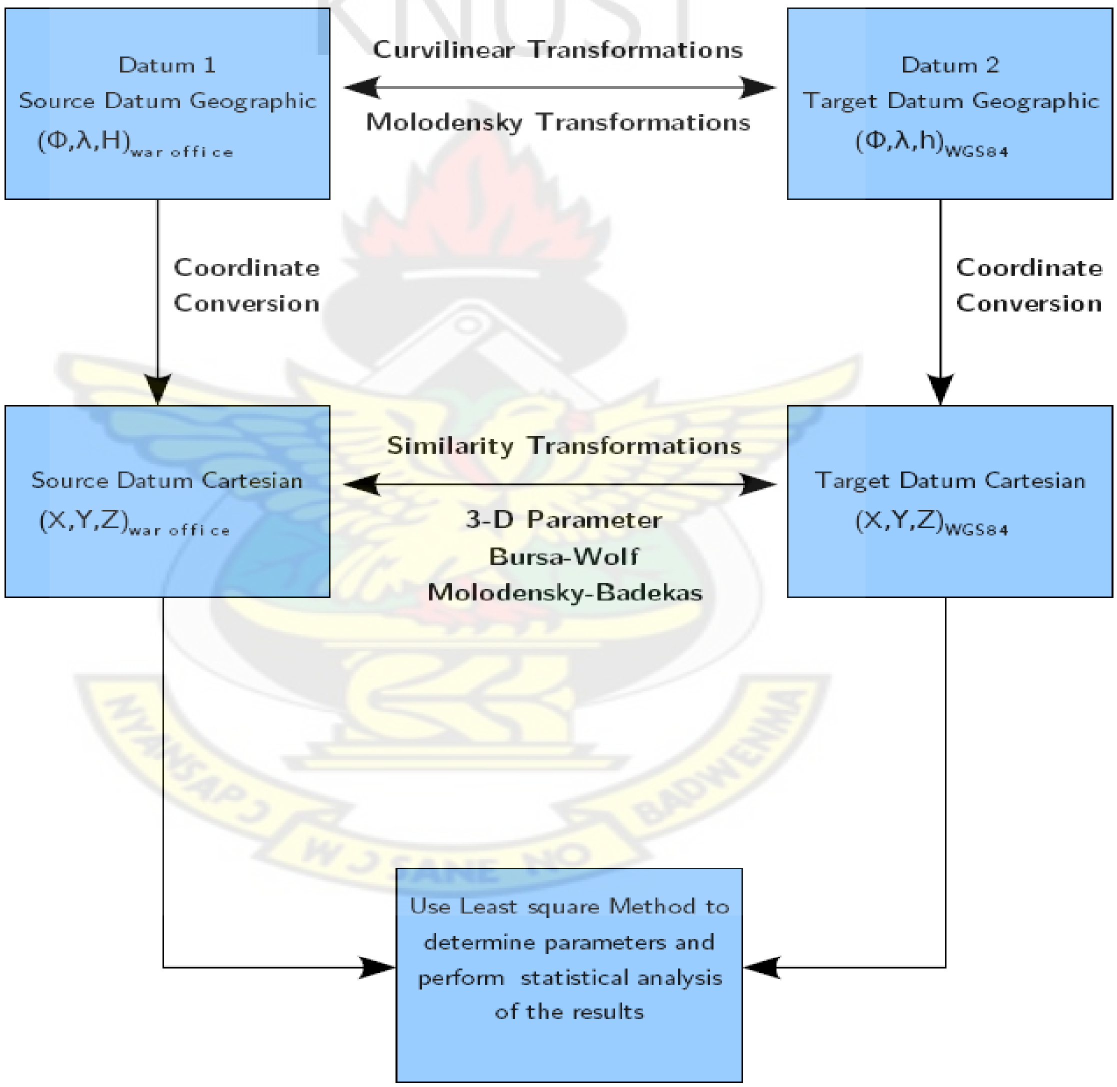

Fig.3. 1: Flow chart showing the transformation procedure.

3.2.2 Statistics of the Parameters

The variance of unit weight is given by $\sigma^2 = \frac{V^T V}{n-u}$ where n = number of measurements and u = number of unknown parameters. The standard deviation is given by $\sigma = \sqrt{\frac{V^T V}{n-u}}$ and the standard deviation of the individual quantities is given by $\sigma_x = \sqrt{\sigma^2 (q_{xx})}$, where q_{xx} is the diagonal matrix of the variance-covariance matrix $Q_{xx} = (A^T A)^{-1}$.

3.2.3 Implementation of the Transformation Models

a. The Abridged Molodensky Model

The abridged Molodensky equations (equations 2.28, 2.29, and 2.30) were re-arranged to give the following simplified equations:

$$\Delta X \sin\phi \cos\lambda + \Delta Y \sin\varphi \sin\lambda - \Delta Z \cos\phi = K \sin 2\phi - \Delta\phi''(\rho \sin 1'') \text{..................}(3.1)$$

$$with\, K = a\Delta f + f\Delta a$$

$$-\Delta X \sin\lambda + \Delta Y \cos\phi = \Delta\lambda''(\nu \cos\phi \sin 1'') \text{..}(3.2)$$

$$\Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi = \Delta h + \Delta a + (a\Delta f + f\Delta a)\sin^2\phi \text{.............}(3.3)$$

Equations (3.1) to and (3.3) were then written in matrix format as

$$\begin{bmatrix} \sin\phi\cos\lambda & \sin\phi\sin\lambda & -\cos\phi \\ -\sin\lambda & \cos\phi & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} K\sin 2\phi - \Delta\phi''(\rho \sin 1'') \\ \Delta\lambda''(\nu\cos\phi\sin 1'') \\ \Delta h + \Delta a + (a\Delta f + f\Delta a)\sin^2\phi \end{bmatrix} \text{........}(3.4)$$

Equation (3.4) is of the form AX=L+V……………… (3.5),

where

X is the matrix of unknown parameters, V is the matrix of residuals, A is the design matrix and L is the observation matrix. Hence the least squares solution is of the form:

$$X = \left(A^T A\right)^{-1} A^T L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.6)$$

Where

$$A = \begin{bmatrix} \sin\phi\cos\lambda & \sin\phi\sin\lambda & -\cos\phi \\ -\sin\lambda & \cos\phi & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}, X = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}, and\, L = \begin{bmatrix} K\sin 2\phi - \Delta\phi''\left(\rho \sin 1''\right) \\ \Delta\lambda''\left(\nu \cos\phi \sin 1''\right) \\ \Delta h + \Delta a + \left(a\Delta f + f\Delta a\right)\sin^2\phi \end{bmatrix}$$

The vector of residuals is given by V=AX-L.

The iterative approach for solving for X used all the three equations to obtain convergence for values of ΔX, ΔY, and ΔZ (Ayer, 2008; Ayer & Tiennah,2008). Since Δh is unknown, the program was started by first assuming that Δh =0 and then iterating until convergence was achieved. The computations were carried out using a Matlab programming language (code). The values of ΔX, ΔY, ΔZ and Δh, and the residuals and statistics were then generated from the program.

b. Conversion of Orthometric Heights to Ellipsoidal Heights

The orthometric heights of the test points were converted to their corresponding ellipsoidal heights on the War Office ellipsoid using equation 2.12. Hence Δh values obtained through the Abridged Molodensky transformation were used to obtain the ellipsoidal heights of the test points in War Office.

c. The Simple Block- Shift Model

For transforming from War Office to WGS 84, equation (2.23) was rewritten as

$$\begin{bmatrix} X_{WGS} \\ Y_{WGS} \\ Z_{WGS} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X_{War} \\ Y_{War} \\ Z_{War} \end{bmatrix} \quad \text{...} (3.7)$$

The least square solution of equation 3.7 is

$X = (A^T A)^{-1} A^T L$, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \text{and } L = \begin{bmatrix} X_{WGS} - X_{War} \\ Y_{WGS} - Y_{War} \\ Z_{WGS} - Z_{War} \end{bmatrix}$$

This model compares the Cartesian coordinates of the common points in the two systems. Hence the parameters with their residuals and statistics are obtained by first determining the ellipsoidal heights of the points in the War Office system through the Abridged Molodensky equations, and then comparing the coordinates.

d. Bursa-Wolf Transformation Model

Using least squares approach the Bursa-Wolf equation (equation 2.20) was written as (Marzooqi et al, 2005)

AX = L+V,

Where

$$A = \begin{bmatrix} 1 & 0 & 0 & X_{War} & 0 & -Z_{War} & Y_{War} \\ 0 & 1 & 0 & Y_{War} & Z_{War} & 0 & -X_{War} \\ 0 & 0 & 1 & Z_{War} & -Y_{War} & X_{War} & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta L \\ R_X \\ R_Y \\ R_Z \end{bmatrix} \text{ and } L = \begin{bmatrix} X_{WGS} - X_{War} \\ Y_{GWS} - Y_{War} \\ Z_{WGS} - Z_{War} \end{bmatrix}$$

The residual matrix is given by V=AX-L.

This model also requires that the coordinates in both datums be in the Earth Centered Earth Fixed Cartesian form. Hence the Cartesian coordinates of the War Office coordinates were determined by first of all applying the Abridged Molodensky Model to determine the ellipsoidal heights of the test points on the War Office ellipsoid (equation 2.12).Using these heights together with the geographical coordinates of the test points in War Office, the Cartesian coordinates $(X_{war,} Y_{war}, Z_{war})$ of the points in the War Office ellipsoid were obtained using Bowring's algorithms (equation 2.13).The Bursa-Wolf model was then applied using the least squares approach to solve for the unknown matrix X.

e. Molodensky – Badekas Model

The Molodensky- Badekas transformation equation (equation 2.22) was re-written using least squares approach (Marzooqi et al, 2005) as:

AX=L+V, where

$$A = \begin{bmatrix} 1 & 0 & 0 & \mu_x & 0 & -\mu_z & \mu_y \\ 0 & 1 & 0 & \mu_y & \mu_z & 0 & -\mu_x \\ 0 & 0 & 1 & \mu_z & -\mu_y & \mu_x & 0 \end{bmatrix}, X = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta S \\ R_X \\ R_Y \\ R_Z \end{bmatrix}, \; L = \begin{bmatrix} X_{WGS} - X_{War} \\ Y_{GWS} - Y_{War} \\ Z_{WGS} - Z_{War} \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix} = \begin{bmatrix} X_{War} - X_m \\ Y_{War} - Y_m \\ Z_{War} - Z_m \end{bmatrix}$$

Where

$(\Delta X, \Delta Y, \Delta Z) = shifts\ between\ the\ centroids\ of\ the\ centroids\ of\ the\ networks$

$(X_m, Y_m, Z_m) = the\ centroid\ of\ the\ War\ Office\ network$

The vector of residuals is given by V = AX-L

Similar to the Bursa-Wolf model, the Molodensky-Badekas model also requires that both coordinates be in Cartesian format. Hence the War Office Cartesian coordinates were first determined as discussed in the Bursa-Wolf method before the Molodensky-Badekas model was implemented to determine the unknown matrix, X. The Matlab program code for the determination of the transformation parameters is given in Appendix A.

3.2.4 Testing of the Transformation Parameters

The accuracies of the various transformation parameter sets were tested in the field. In all, twenty seven points were tested in the field, twenty within the Golden Triangle; the study area (Table 3.4) and seven within Ho municipality; outside the study area (Table 3.3). These points were used as check points to access the accuracy of each transformation model.

The seven points within Ho municipality were established through GPS survey connected to an original War Office pillar. The coordinates within Golden Triangle were the same coordinates used for the determination of the parameters plus the coordinates of an additional pillar.

Testing was carried out by comparing the War Office grid coordinates (Easting, Northing) of the points with the projected coordinates generated by transforming the GPS derived coordinates. This was achieved by substituting the determined transformation parameters into their various equations to convert the observed GPS coordinates $(X_{WGS,} Y_{WGS}, Z_{WGS})$ to War Office Cartesian coordinates $(X_{war,} Y_{war}, Z_{war})$.The War Office Cartesian coordinates were then converted to War Office geographical coordinates $(\phi, \lambda, h)_{war}$ and then projected to Ghana Transverse Mercator(TM) using the TM projection from ellipsoid to the plane coordinates (Easting, Northing).Fig. 3.2 is a flow chart showing the testing procedure.

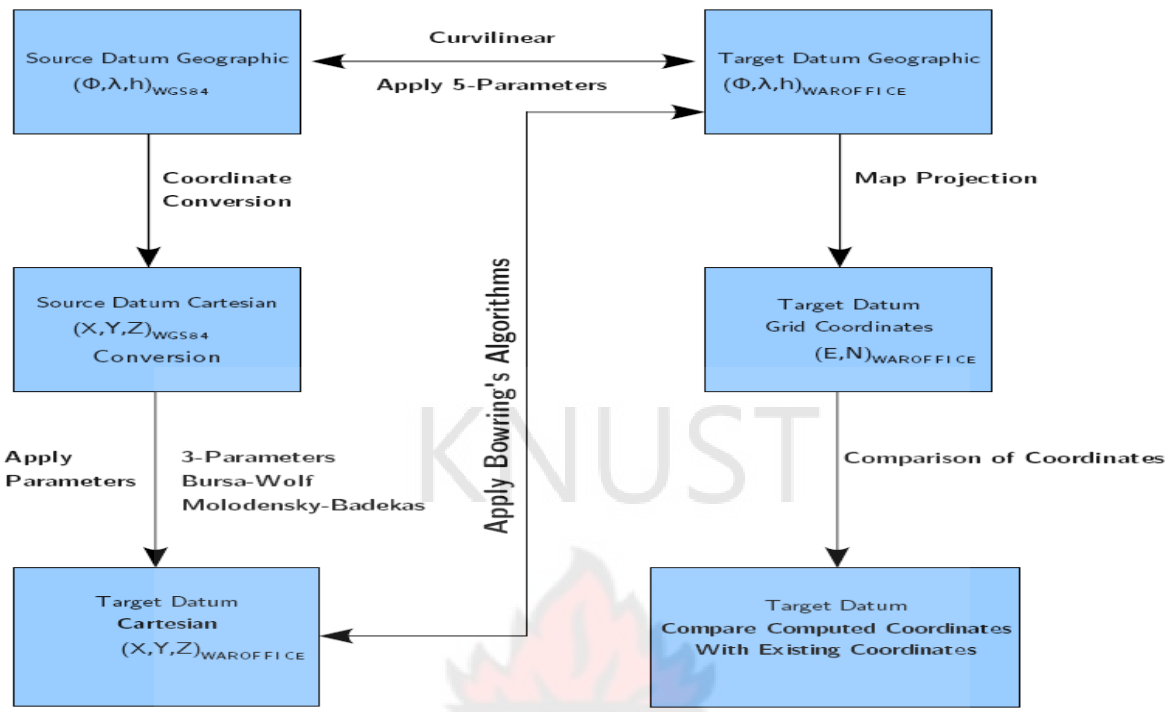

Fig.3. 2: Flow chart for the testing procedure.

Table 3.3: Ho test point Processed GPS coordinates in WGS84

POINT ID	LONG				LAT				ELLIPSOIDAL HEIGHT(m)
SGV/RS/09/1	0	28	00.96351	E	6	36	28.43680	N	509.519
SGV/GPS/15/00/1	0	27	30.17769	E	6	35	56.57307	N	555.752
SGV/10/03/3	0	24	01.93707	E	6	34	04.30323	N	373.666
SGV/10/03/1	0	24	00.50850	E	6	34	07.95973	N	365.901
SGV/4/02/102	0	27	34.73890	E	6	36	30.53945	N	560.146
SGV/6A/07/01	0	27	37.37868	E	6	36	31.48729	N	561.743
SGV/RS/09/02	0	27	59.37079	E	6	36	27.37823	N	513.610

The Ho test stations above were established through GPS survey tied to an existing War Office station.

Table 3.4: Golden Triangle Processed GPS coordinates in WGS84

POINT ID	LONG				LAT				ELLIP. HEIGHT(h/m)
CFP 109	0	25	24.81766	W	5	27	36.32569	N	78.2744
CFP 200	0	33	33.54116	W	5	37	32.87415	N	304.9379
CFP 225	1	30	3.96614	W	5	27	18.31243	N	275.1437
GCS 102	0	44	3.86162	W	5	16	57.87905	N	83.4515
CFP 155	0	07	19.18235	W	5	56	20.52243	N	524.5492
GCS 179	1	01	59.90579	W	6	22	19.62456	N	492.5083
CFP 180R	1	17	10.36042	W	6	03	13.64974	N	437.6990
CFP 217	0	43	46.93781	W	5	56	35.18511	N	311.0926
GCS 142	0	45	56.05923	W	6	34	32.86850	N	782.2084
CFP 213	0	44	56.05567	W	6	07	41.50938	N	327.0218
CFP 178	1	09	52.78589	W	6	34	16.88627	N	615.7568
CFP 185	1	55	30.56291	W	6	29	5.19173	N	643.5756
CFP 306	1	37	49.67431	W	7	14	9.09967	N	536.0062
GCS 302	2	01	0.32842	W	6	54	44.92864	N	560.8285
CFP 304	1	26	43.21595	W	6	59	31.95133	N	620.9316
CFP 305	1	44	36.30423	W	6	50	46.8373	N	417.0231
GCS 145R	1	24	42.82948	W	6	33	24.89911	N	503.7124
CFP 184	1	41	41.39178	W	6	28	17.60759	N	472.1430
CFP 207	1	57	58.15176	W	5	50	58.62504	N	399.3477
GCS 125	0	03	54.53046	W	5	45	58.98280	N	97.4343

CHAPTER FOUR: RESULTS AND ANALYSIS

To compute the various sets of transformation parameters for the study area, nineteen points within the Golden Triangle common to both the WGS 84 and War Office systems were selected for investigation using the Abridged Molodensky five-parameter, the simple three- parameter, the Bursa-Wolf and Molodensky – Badekas seven- parameter models. A least square approach was used to derive the transformation parameters, the residuals and the statistics of the parameters. The observations were assumed to carry equal weight. The derived parameters with their residuals and statistics are as shown in tables 4.1 to 4.4. The residuals and standard deviations relatively small compared with the parameters to warrant their acceptance.

The derived parameters were tested on twenty points within the Golden Triangle and seven points outside it. The test results are as shown in tables 4.5 and 4.6.Generally, the parameters gave better results when applied within the study area (Golden Triangle) than when applied outside it (Ho). Also, the standard deviations from the tests are smaller than those of the parameters necessitating their acceptance.

4.1 Results from the Procedure for the Derivation of Transformation Parameters

The parameters for transforming data from War office to WGS 84 using the Abridged Molodensky model and their statistics are as given in Table 4.1.

Table 4.1: Results for the Abridged Molodensky Model

Parameter	Value	Standard Error (m)
ΔX (m)	-196.7481	0.546
ΔY (m)	32.7059	0.549
ΔZ (m)	322.6385	0.546
Δa (m)	-162.996	
Δf	-2.5568E-5	
a (m)	6378299.996	
1/f	296	
Standard Deviation(m)	2.3802	

Generally, the standard errors are very small compared with their corresponding quantities. It can also be inferred from the results that there is about 67.8 percent probability that the parameters lie within the range of ±2.3802 from their values stated in table 4.1. Also, the standard errors for the shift parameters have the same values.

The parameters for transforming data from War Office to WGS 84 using Simple Three-parameter transformation model and their statistics are as given in Table 4.2.

Table 4.2: Results from Simple Three Parameter Model

Parameter	Value	Standard Error(m)
ΔX (m)	-196.862	0.569
ΔY (m)	32.518	0.569
ΔZ (m)	322.541	0.569
Standard Deviation(m)	2.482	

Again, the standard errors of the parameters are relatively small compared with the parameters. It can also be inferred from the statistics that there is about 67.8 percent probability that the parameters lie within the range of ±2.482 from their values. In addition, the standard errors for all the parameters have the same value.

The results from the Bursa-wolf transformation Model for transforming from War Office to WGS 84 as are given in Table 4.3.

Table 4.3: Results from the Bursa-Wolf Model

Parameter	Value	Standard Error
ΔX (m)	-157.367	0.551
ΔY (m)	31.934	0.551
ΔZ (m)	326.65	0.551
R_X (rad)	8.0824225E-7	6.6248889E-7
R_Y(rad)	-2.121908E-8	0.8239470E-8
R_Z (rad)	-7.371663E-9	0.9307370E-9
ΔS	-6.3ppm	1.0ppm
Standard Deviation	2.400	

The standard errors are generally relatively small compared with their corresponding quantities. The statistics also show that there is a 67.8 percent probability that the quantities are within the range of ±2.4 from the values given in table 4.3. The standard errors of the three shift parameters are equal.

Results from the Molodensky-Badekas model for transforming data from War Office to WGS 84 are as given in Table 4.4. It can be inferred from the table that the standard errors of the individual quantities are generally relatively small compared with the quantities themselves. In addition, the standard errors for the three shift parameters are equal. The results also show that there is a 67.8 percent probability that the quantities lie in the range of ±2.4 from the values given in Table 4.4

Table 4.4: Results from the Molodensky-Badekas Model

Parameter	Value	Standard Error
ΔX (m)	-196.772	0.558
ΔY (m)	33.296	0.558
ΔZ (m)	322.318	0.558
R_X (rad)	8.0824225E-7	6.3164956E-7
R_Y (rad)	-2.121908E-8	0.6411050E-8
R_Z (rad)	-7.371663E-9	1.1602807E-9
ΔS	-6.3ppm	1.07ppm
X_m (m)	6338744.908	
Y_m(m)	-121620.012	
Z_m (m)	690770.048	
Standard Deviation	2.4325	

Fig 4.1-a to Fig.4.1-d are scatter diagrams of the residuals generated from the computation of the transformation models. Fig 4.1-shows that except for the Abridged Molodensky model (the first 19 points) the residuals in X (V_X) are highly correlated.

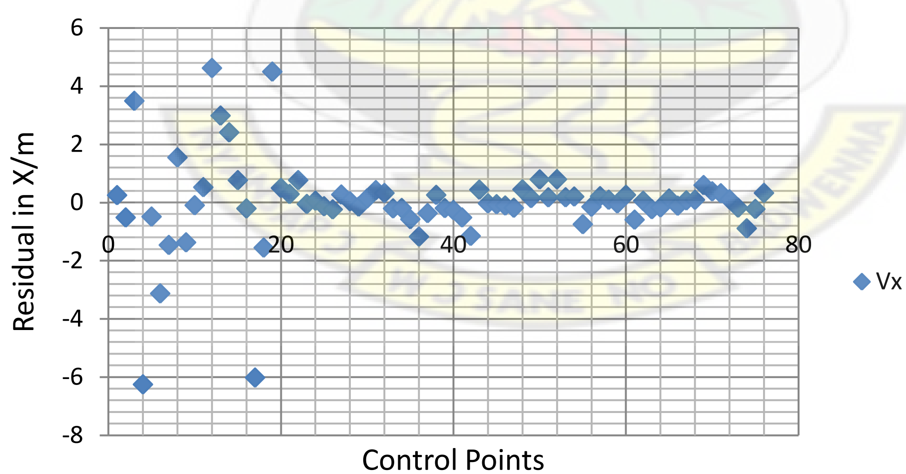

Fig. 4.1-a:Scatter Diagram of the Residuals in X

Fig 4.1-b also shows that the residuals in Y (V_Y) are randomly distributed for all the four models implemented.

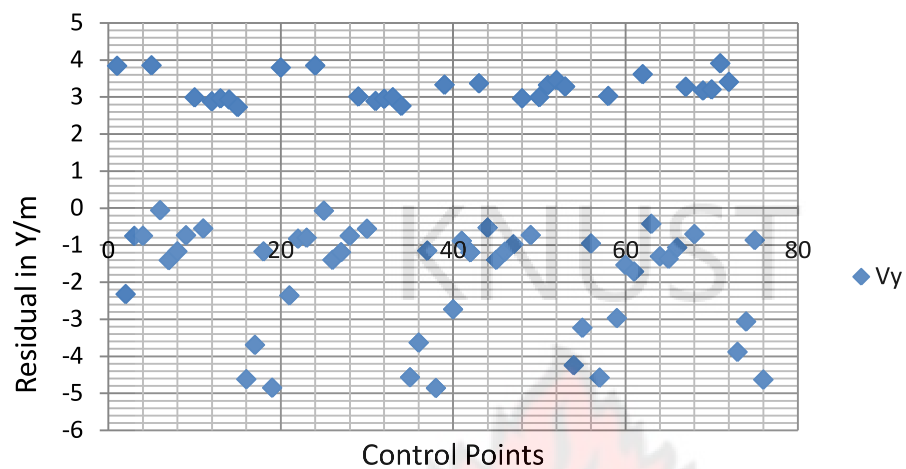

Fig. 4.1-b: Scatter Diagram of the Residual in Y

Also, Fig. 4.1-c shows that except for the Abridged Molodensky model the residuals in Z (V_Z) are randomly distributed.

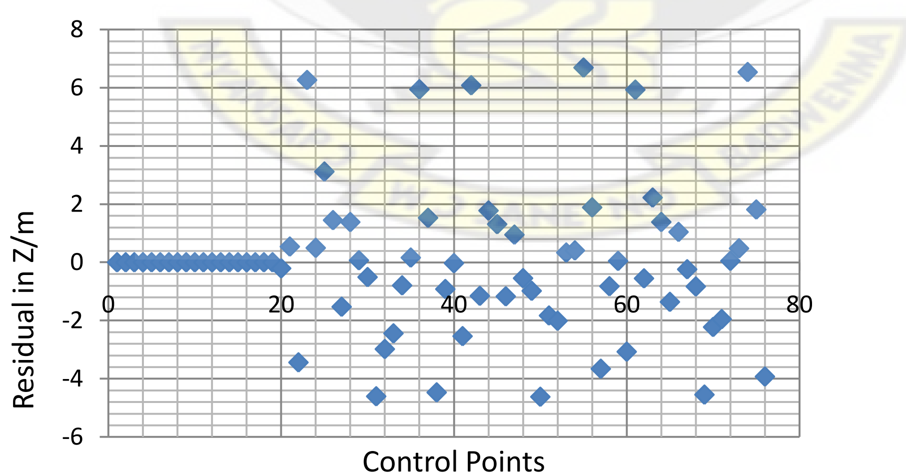

Fig.4.1-c : Scatter Diagram of the Residuals in Z

Fig 4.2-a to 4.2-e are graphs showing how residuals vary with the points. The graphs generally show that except for the Abridged for the Abridged Molodensky model, the residuals in X are the least while those in z are the largest. For the Abridged Molodensky five parameter model (Fig 4.2-a), the residuals in X are the largest while those in Z are the least.

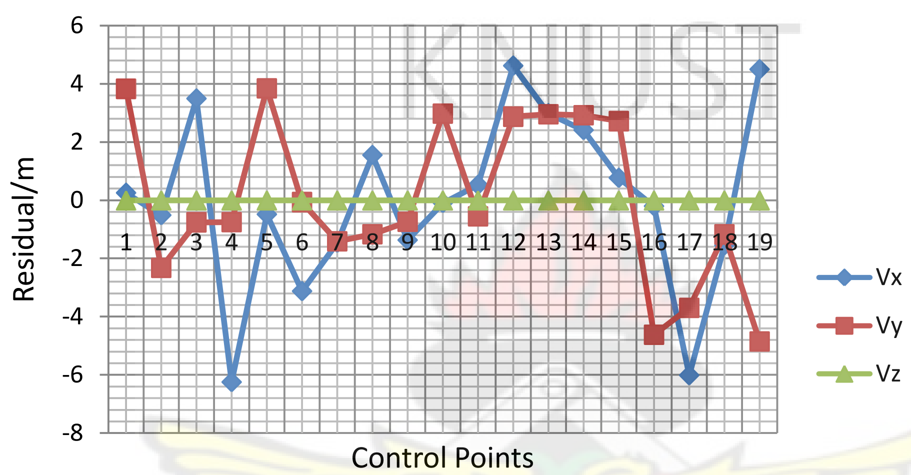

Fig. 4.2-a : Graph of Residuals for 5-Parameter Transformation

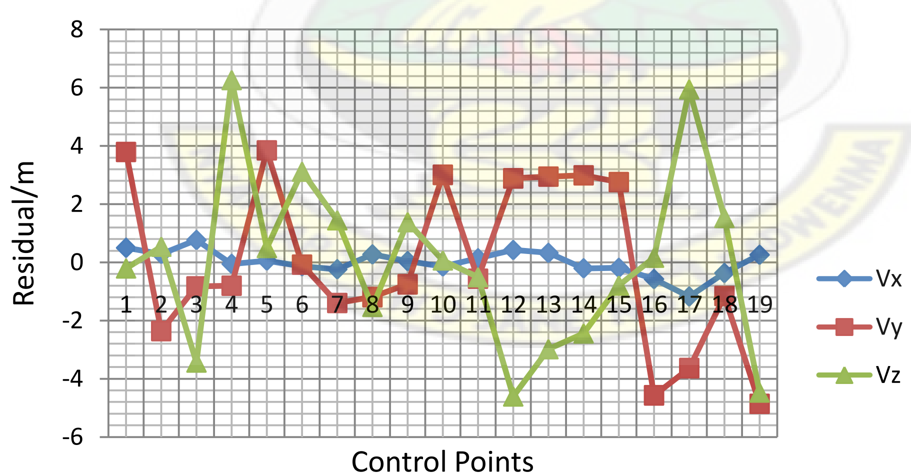

Fig. 4.2-b:Graph of Residuals for 3-Parameter Transformation

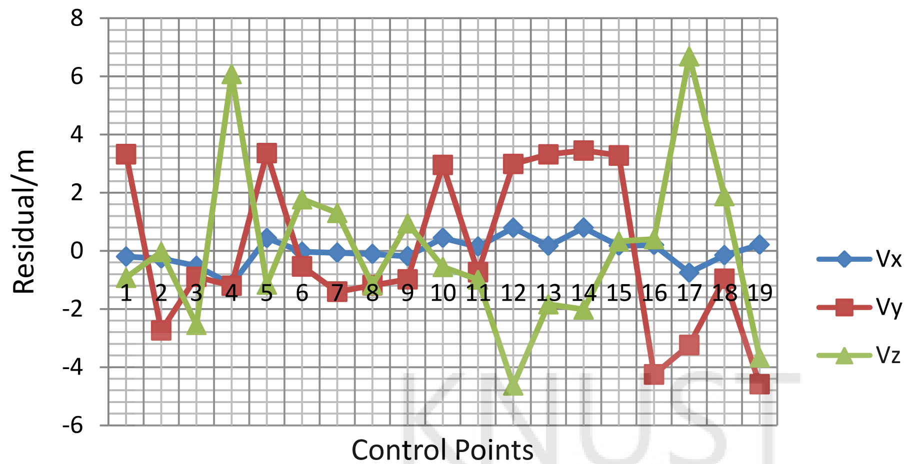

Fig.4.2-c : Graph of Residuals for Bursa-Wolf Transformation

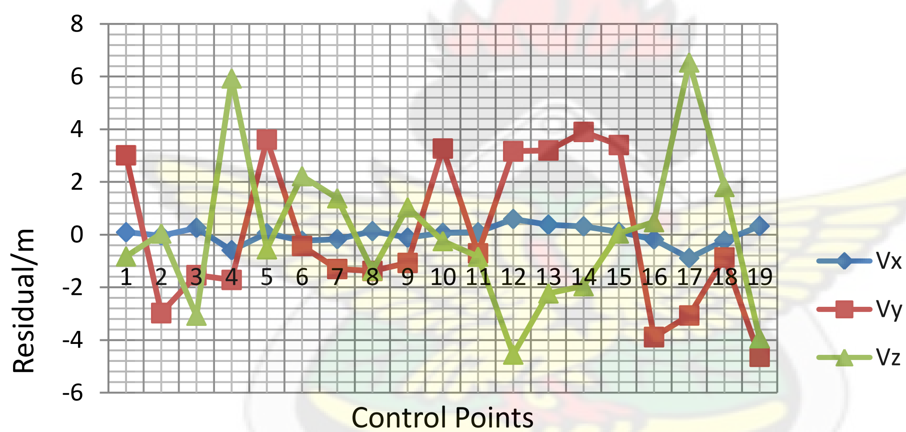

Fig. 4.2-d: Graph of Residuals for Molodensky-Badekas Transformation

Fig 4.2-e is a combined graph for the residuals of all the models. Generally, except for the Abridged Molodensky model, the residuals have smallest values in X and largest values in Z.

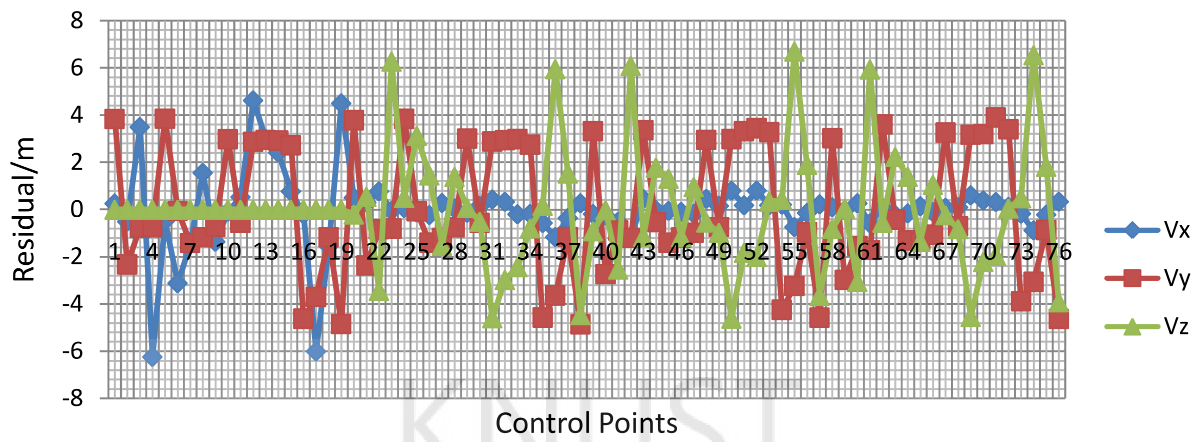

Fig. 4.2-e: Graph of Residuals for all the Transformation Models

4.2 Results from Testing

A summary of the statistics from the test results within Golden Triangle and Ho are given in Table 4.5 and Table 4.6 respectively. The detail results from the tests are given in appendix A-B. All the four models produced acceptable results. The ten- parameter produced slightly better results than the seven parameter model. The errors from the five parameter test are quite high, although within acceptable range. Again, the seven- and ten-parameter models produced very close results. The Molodensky-Badekas seven-parameter model gave the best results for points within Golden Triangle and the simple three-parameter model produced the best results for points in Ho.

Table 4.5: Summary of Test results within Golden Triangle

Results	Three Parameter	Five Parameter	Seven Parameter	Ten Parameter
Min. diff. in E (m)	-0.0561	0.0155	-0.1039	0.0838
Max. diff. in E (m)	1.9675	1.7794	1.5021	1.4579
Min. diff. in N (m)	0.0173	0.2106	0.0003	0.0006
Max. diff. in N (m)	4.0850	4.1602	-1.0875	1.7770
Standard error in Northing (m)	0.9728	2.1758	0.8498	0.8499
Standard error in Easting (m)	1.1097	0.9653	0.5625	0.5581
RMS (m)	1.4757	2.3803	1.0191	1.0168

Table 4.6: Summary of Test results from Ho

Results	Three Parameter	Five Parameter	Seven Parameter	Ten Parameter
Min. diff. in E (m)	0.0466	-0.0573	0.0597	0.1052
Max. diff. in E (m)	0.6770	-0.4822	-0.9406	-0.8952
Min. diff. in N (m)	-0.3200	-2.4613	-0.8995	-0.8982
Max. diff. in N (m)	-1.3670	-3.5061	-1.9791	-1.9782
Standard error in Northing (m)	1.1156	3.1955	1.6892	1.6883
Standard error in Easting (m)	0.3015	0.3132	0.4458	0.4129
RMS (m)	1.1556	3.2109	1.7471	1.7381

A graph comparing the results is shown in Fig 4.3 to 4.13.

4.2.1 Analysis of Test Results from Golden Triangle

Fig 4.3 is a graph for the residuals in northing for the Golden Triangle test points. As can be seen, the graphs for the ten-parameter and the seven-parameter models which give the most accurate results are super-imposed. The five-parameter model gave the least accurate results.

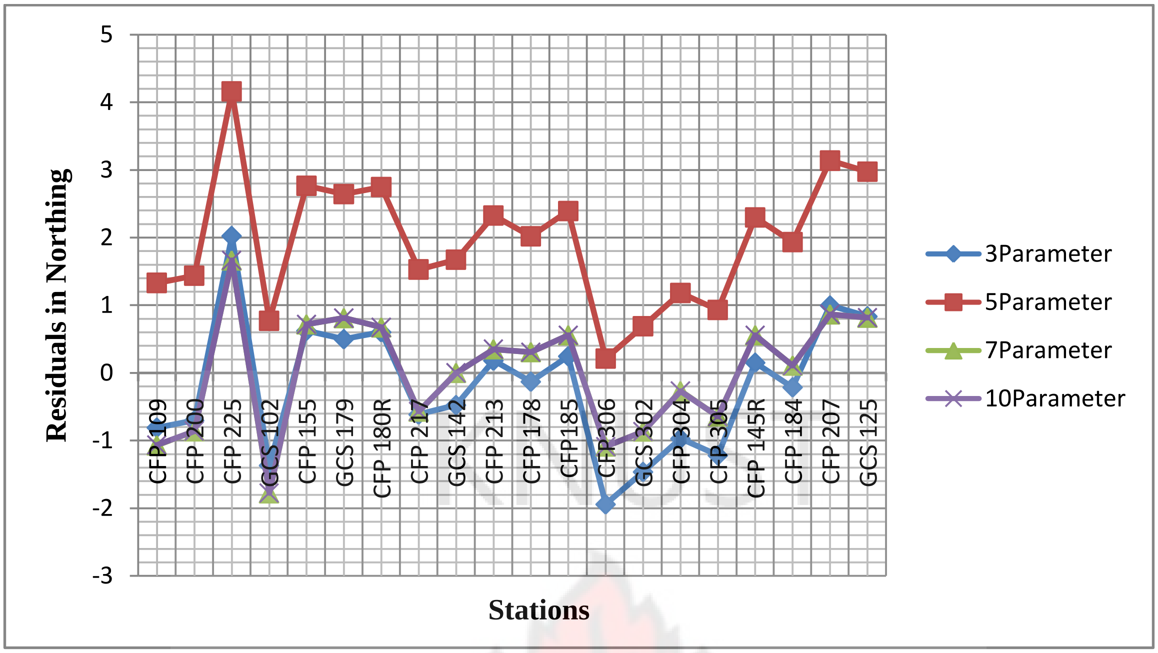

Fig 4.3: Graph of residual in Northing within Golden Triangle

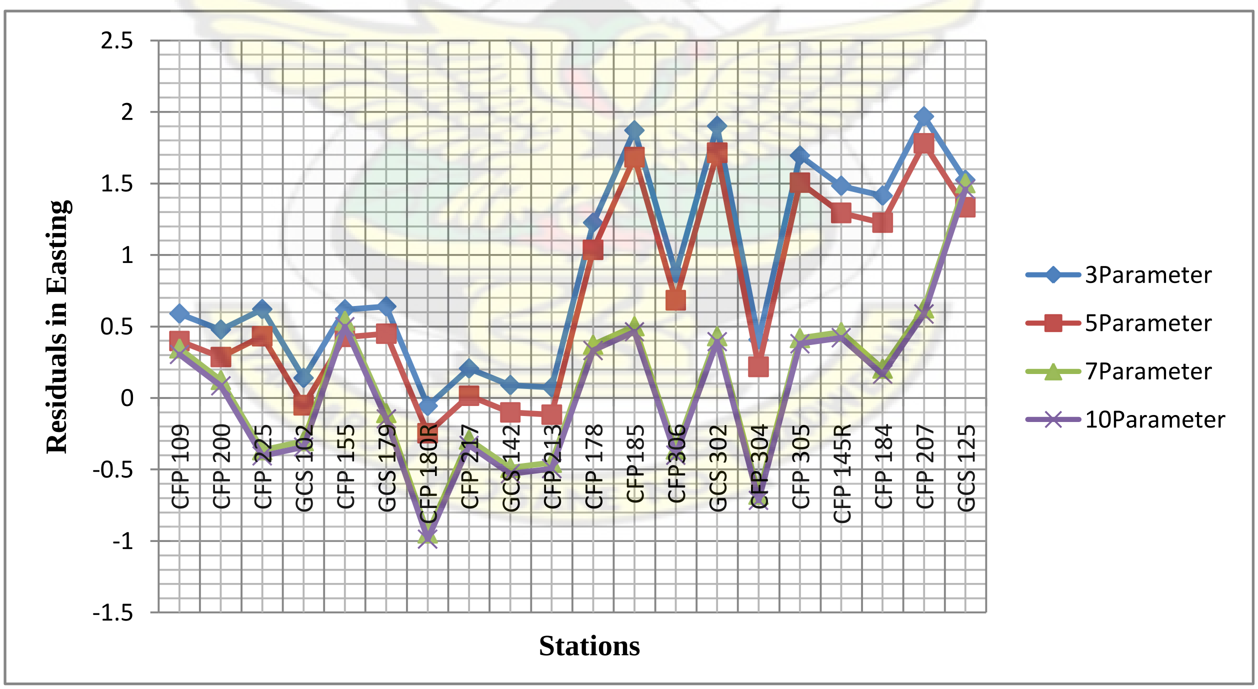

Fig 4.4: Graph of residual in Easting within Golden Triangle

Fig 4.4 is a graph showing the distribution of the residuals in easting for the Golden Triangle test points. Generally, the results from the seven- and ten- parameter models gave the best results, and the three-parameter model gave the least accurate results.

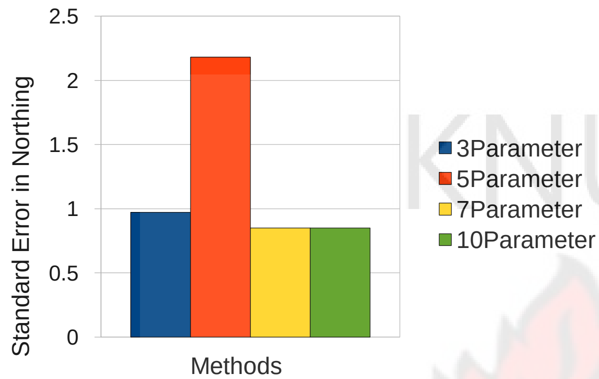

Fig 4.5: Graph of standard error in Northing within Golden Triangle

Fig. 4.5 is a histogram showing the standard errors in the northing of the test points of Golden Triangle stations for the four methods used. As can be, the Bursa-Wolf seven-parameter and Molodensky-Badekas ten-parameter gave the least errors, followed by the three parameter slight method. The Abridged-Molodensky method gave the highest standard error in the northing for the Golden Triangle test points.

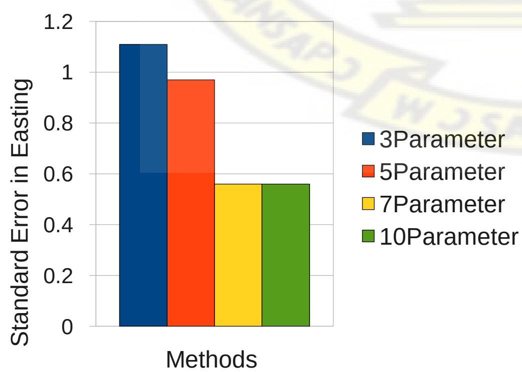

Fig 4.6: Graph of standard error in Easting within Golden Triangle

Fig. 4.6 is also a histogram showing the standard errors in the northing Golden Triangle test points. Again, both the Bursa- wolf seven-parameter and Molodesky-Badekas methods gave the least error, followed by the Abridged Molodensky five parameter method. The three parameter (block shift) method gave the highest error of about 1.1 m. Generally, by comparison of figures 4.5 and 4.6, it can be observed that the standard errors in easting are relatively smaller than the standard errors in the northing except for the three-parameter method.

Figure 4.7 is also a histogram, showing the root mean square errors for the testing of the transformation parameters on the Golden Triangle stations. Again, both the Bursa-wolf seven parameter and the Molodensky-Badekas ten-parameter models gave the least root mean square errors, followed by the three-parameter model. The Abridged Molodensky model gave the highest root mean square errors for the Golden Triangle test points.

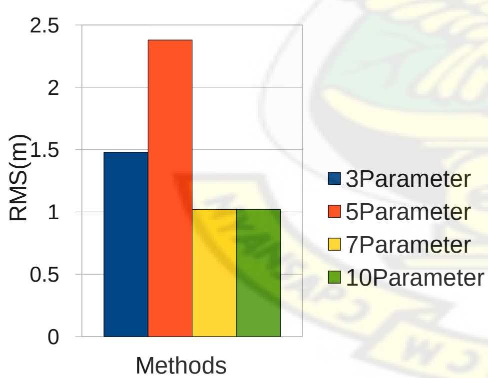

Fig 4.7: Graph of RMS within Golden Triangle

4.2.2 Analysis of Test Results from Ho

Fig 4.8 is a graph for the residuals in northing for the Ho test points. From the graph, the least errors were generated by the Abridged Molodensky five-parameter. The seven- and ten-parameter models gave approximately same results for the residuals in northing. From Fig 4.8, the three-parameter model gave the best results in northing for this area.

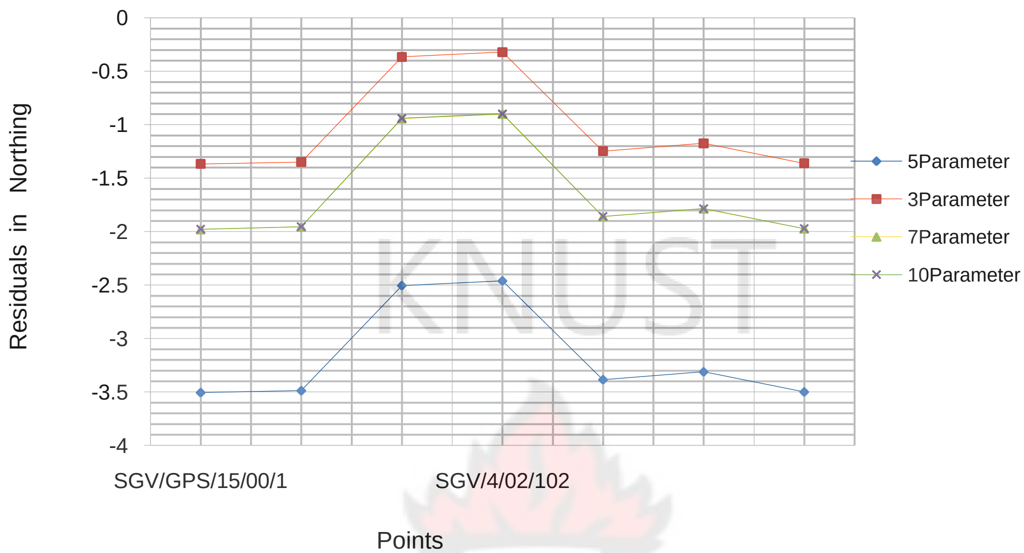

Fig 4.8: Graph of residual in Northing within Ho stations

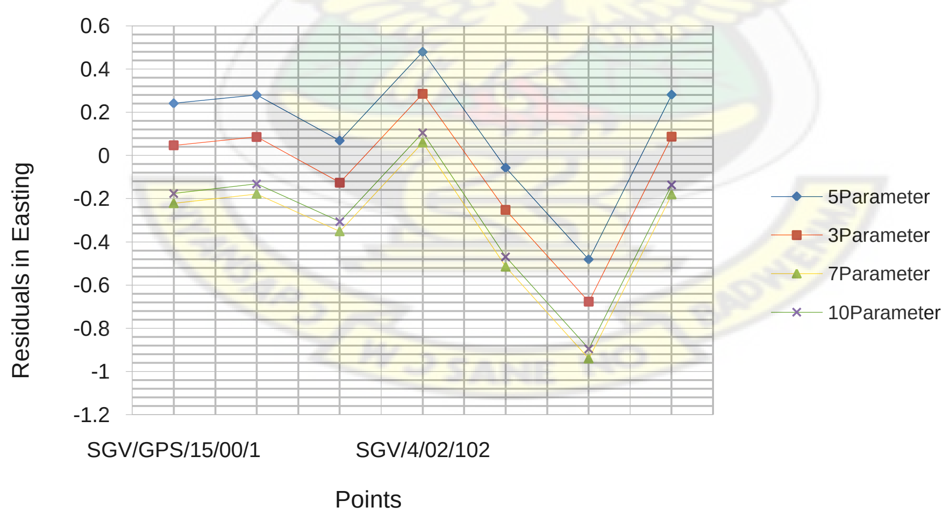

Fig 4.9: Graph of residual in Easting within Ho

Fig 4.9 is a graph for the residuals in easting for test points in Ho. From the graph, the maximum errors were obtained from the seven-parameter model. The seven- and ten-parameter models gave very close results in the easting and the three-parameter model gave the best results in easting for this area.

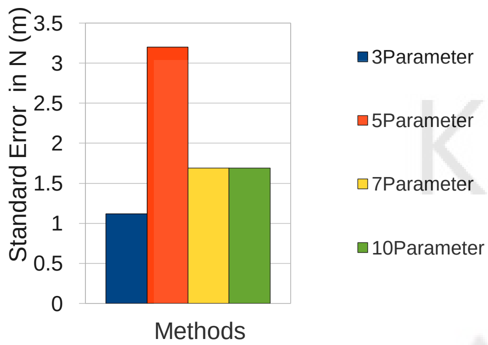

Fig 4.10: Graph of standard error in Northing within Ho

Fig 4.10 is a histogram of the standard error in the northing of the Ho test points. The three-parameter block shift method gave the least standard error of 1.2 m in the northing, followed by the Bursa-Wolf seven parameter model and Molodensky-Badekas seven parameter model which gave the same numerical value for the standard error of 1.6 m in the northing. The five- parameter Abridged Molodensky gave the highest standard error in the northing of about 3.2m. This is quite a relatively large error, certainly coming from the War Office Coordinates of the test point coordinates.

Fig. 4.11 is also a histogram of the standard errors in the easting of the Ho test points. Generally, the errors in the easting are relatively small compared to the errors in the northing. The three parameter model highest accuracy in the easting followed by the

Abridged Molodensky five parameter model, and the Molodesky-Badekas seven parameter model and the Bursa-Wolf seven parameter model.

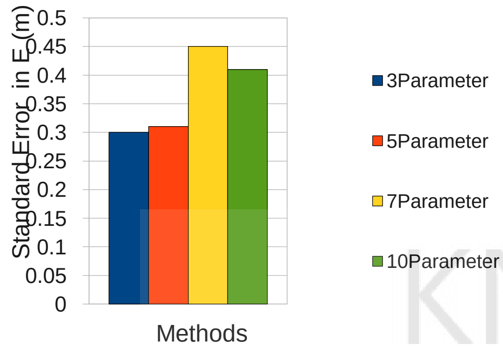

Fig 4.11: Graph of standard error in Easting within Ho

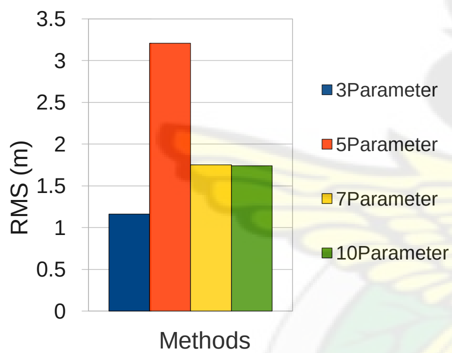

Fig 4.12: Graph of RMS error within Ho

Fig. 4.12 is a histogram showing the root-mean-square error of the test results of the various models for the Ho test points. The three-parameter block shift method gave the least error followed by the Molodensky-Badekas ten-parameter model and the Bursa-wolf seven-parameter model. The error for the Bursa-wolf model is just slightly greater than that of the Molodensky-Badekas model. The Abridged Molodensky model gave the highest error, relatively high compared with the other models.

4.2.3 Analytical Comparison of Test Results from Golden Triangle and Ho

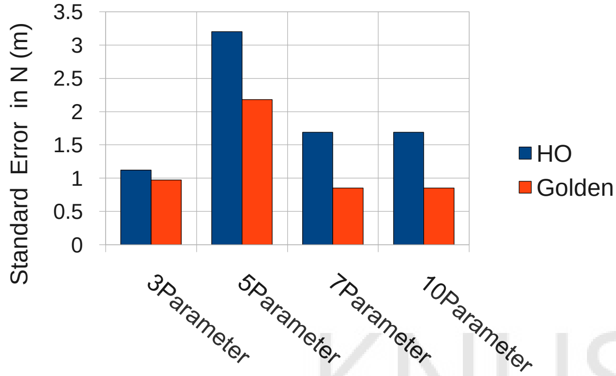

Fig 4.13: Graph of standard error in Northing within Ho and Golden Triangle

Fig. 4.13 is a histogram for comparing the standard errors in the northing of the Golden Triangle and Ho test points. It can be observed from the graph that the test results from the Golden Triangle stations gave better results for the northing coordinates for all the models than the test results from the Ho stations. For the Ho test coordinates, the three parameter model gave the best results for the northing whilst the five parameter model gave the least accurate result for the northing. The Bursa-wolf and Molodensky-Badekas models gave the same accuracy more accurate than the five parameter model and less accurate than the three-parameter model. For the Golden test points, the results show that the seven-parameter Molodensky-Badekas model is the most accurate whilst the five parameter model is the least accurate model for the study area. The three parameter model is only slightly less accurate compared to the seven-parameter Bursa-Wolf and Molodensky-Badekas models.

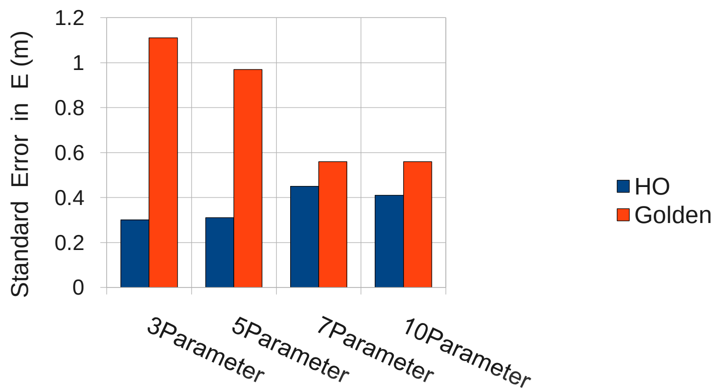

Fig 4.14: Graph of standard errors in Easting within Ho and Golden Triangle

Fig. 4.14 is a histogram of the standard errors in the easting for the models employed. By comparison with Fig. 4.13, it is clear that the test results for the easting are relatively more accurate that those of the northing. The results also show that, for the Ho study area, the most accurate results are produced by the three-parameter model whilst the least accurate results for the easting are produced by the seven-parameter model. The 5-parameter model gave slightly less accurate results than the three-parameter model. The ten-parameter model gave results that are slightly more accurate than the seven-parameter model. It can also be deduced from the histogram that the easting coordinates for Ho study area are generally more accurate than those of the Golden Triangle are. Also for the Golden Triangle study area, the ten-and seven-parameter models gave the best results whilst the three parameter model gave the least accurate results. The results of the five parameter model gave significantly larger values of errors in the easting than the seven- and ten-parameter models.

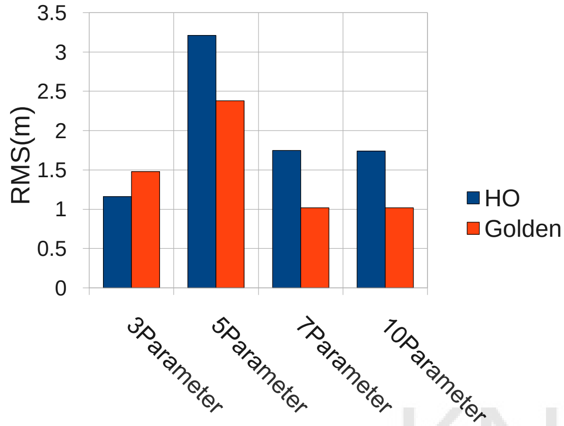

Fig 4.15: Graph of RMS within Ho and Golden Triangle

Fig.4.15 is a histogram representation of the root-mean-square errors of testing the various models at Ho and Golden Triangle study areas. The graph generally shows that except for the three-parameter model, the other models gave better results for the Golden Triangle area than the Ho area. For Ho area, the simple three- parameter model gave the best result whilst the least accurate model is the five-parameter model. For Golden Triangle study area, the seven-parameter Molodensky-Badekas model gave best results whilst the five-parameter model is the least accurate model.

Based on the test results the simple three-parameter model is recommended for the Ho area, and the seven parameter Molodensky-Badekas model is recommended for the Golden Triangle.

CHAPTER FIVE: CONCLUSION AND RECOMENDATIONS

5.1 CONCLUSION

In this research, four transformation models which are popularly used and implemented in most geodetic software were studied. GPS observations were carried out on nineteen control points within Golden Triangle to determine their geographic coordinates in the WGS 84 system. The corresponding coordinates of these points on the War Office ellipsoid were obtained from the Ghana survey records. These coordinates were substituted into a Matlab computer program and the parameters of the various models were derived. Table 5.1 is a summary of the derived parameters given in Table 4.1-4.4.

Table 5.1: A summary of the derived parameters

Parameter	Abridged Molodensky	Simple Three parameter	Seven Parameter Bursa-Wolf	Seven Parameter Molodensky-Badekas
ΔX (m)	-196.748±0.546	-196.862±0.569	-157.367±0.551	-196.772±0.558
ΔY (m)	32.706±0.549	32.518±0.569	31.934±0.551	33.296±0.558
ΔZ (m)	322.639±0.546	322.541±0.569	326.650±0.551	322.318±0.558
Δa (m)	-162.996	-	-	-
Δf	-2.5568E-5	-	-	-
R_X (rad)	-	-	8.082423E-7 ±6.624889E-7	8.082423 ± 6.316496E-7
R_Y (rad)	-	-	-2.121908E-8 ±0.823947E-8	-2.121908E-8 ± 0.641105E-8
R_Z (rad)	-	-	-7.371663E-9 ± 0.930737E-9	-7.371663E-9 ±1.160281E-9
ΔS	-	-	(-6.3±1.0) ppm	(-6.3± 1.07) ppm
Standard deviation(m)	2.380	2.482	2.400	2.433

These results (Table 5.1) compare very well with those published by the Ghana Survey Department and previous researchers (Ayer and Tiennah, 2008, Ayer and Fosu, 2008 and Yeboah, 2007). Also, the standard errors and standard deviations are within acceptable range as they are relatively smaller than the parameters themselves.

These parameters were tested on points within the study area (Golden Triangle) and outside the study area (Ho). The large errors in the transformed Ho points were due to systematic errors. These points are not original War Office points and the procedures followed in connecting them to the War Office station is likely to introduce these systematic errors. The differences between the transformed and existing coordinates should ideally be zero. However, this could only be achieved if the local network had been perfect. The results for all the models gave errors that are acceptable for GPS usage. For the Golden Triangle, the Molodensky-Badekas and Bursa-Wolf models gave very good results, almost the same error. However, by considering the distribution of the errors for the individual stations and the root mean square errors, the Molodensky-Badekas model is recommended for transforming coordinates within the Golden Triangle.

For the Ho area, the three parameter block-shift model gave the best of results. All the four models gave results that are within acceptable range. Generally, the results from the test stations show that the projected coordinates in easting are more accurate than the projected coordinates for the northing.

Even though the seven parameter models are rigorous models providing very high (sub-metre or even sub-centimeter) accuracy, this could not been realized due to the non-homogeneity of the War Office coordinate system. The errors could be due to distortions in scale and orientation of the War Office ellipsoid coordinates. The Ghana cadastral network is a mixture of two separated coordinates systems: a two-dimensional triangulation network and a one-dimensional height system. Therefore, the horizontal and vertical components are very likely to have different scale factors, hence introducing errors in the parameters since the models either assumed that there is no scale difference or scale differences exist for only the horizontal coordinates.

Furthermore, the Accra datum was determined to lower accuracy by a conventional terrestrial triangulation, measuring distances and angles, the local datum point being fixed on basis of astronomical observations. This local network has evolved over a time span of several decades. For these and other reasons the geometrical quality of the system might be affected by distortions, some of them are being quite local, others having a more systematic character (e.g. bias in the scale). Distortions could also be due to human activities such as building and construction, and crustal movements such as tectonic activities, landslides, etc., since most parts of Ghana are in fault zones.

Until a geoid model is determined for Ghana, among the models studied for transforming coordinates between the WGS84 and Ghana War Office, the Molodensky-Badekas model is recommended for Golden Triangle and the Simple Three-parameter Block-Shift model is recommended for Ho. The three and five parameters can also be used for hand-held

GPS and other applications that require them. The seven parameter model is recommended for the entire nation.

5.2 FURTHER RECOMMENDATIONS

From the findings it is recommended that future research into this subject should:

i. Consider other rigorous methods such as the multiple regressions, the 8- and 9-parmeter affine transformation models (described by Constantin- Octavian, 2006) and the zoning method described by Kheloufi, 2006 which turn to reduce errors due to distortions and scale differences between the vertical and horizontal networks.

ii. Carry out numerical investigations into the effect of crustal movements and tectonic activities on the quality of the control points used in the determination of the transformation parameters.

iii. Model the distortions by re-adjusting and transforming the Accra datum to the WGS 84 datum, using all the historical measurements.

iv. Test coordinates of each point for outliers. This will require a large number of data points that are evenly distributed, preferably covering the entire nation.

In addition, it is recommended that an accurate geoidal model be developed for the Accra datum. The lack of this geoidal model could be responsible for errors in height which eventually led to errors in the transformation parameters. When the geoid is determined and used to determine more accurate parameters, it is recommended that all existing maps and data be converted to the WGS 84 system and not the reverse as most GPS users may either not understand or may not be interested in the technicalities of transforming data from the WGS 84 to War Office system. Also, the WGS 84 system is global and users must be able to acquire and share data with the global community at all times and instantly without the need to transform.

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APPENDIX A: **Matlab Code for the Determination of the Transformation Parameters**

```
function [X,x,ho,v,dh,thevariance,all_mu]=Adensky2(wo,wgs,q,n,u)
%
format long

%-----INPUT--------------------
% wo = Waroffice
% wgs = WGS84
% q = Number of Iterations
% n = Number of measurements or points
% u = Number of unknowns

clc
%convert the various angles from degrees to radians
phi=wo(:,1)/180*pi;
lamda=wo(:,2)/180*pi;
phiw=wgs(:,1)/180*pi;
lamdaw=wgs(:,2)/180*pi;

hw=wgs(:,3);
Ho=wo(:,3);
dphi=(-wo(:,1)+wgs(:,1))*3600;
dlamda=(-wo(:,2)+wgs(:,2))*3600;

a=6378299.996;
da=-162.996;
f=1/296;
df=-2.56771292e-5;
e=sqrt(2*f-f^2);

k=a*df+f*da;
[m r]=size(wo);
aw = 6378137.0; % earth semimajor axis in meters
  fw = 1/298.257223563; % reciprocal flattening
  ew=sqrt(2*fw-fw^2);

  e2w = 2*fw -fw^2; % eccentricity squared
for j=1:m
    rhow(j)=a*(1-ew^2)/((1-ew^2*(sin(phiw(j)))^2)^(3/2));

rho(j)=a*(1-e^2)/((1-e^2*(sin(phi(j)))^2)^(3/2));
V(j)=a/(1-e^2*(sin(phi(j)))^2)^.5;
U(j)=a/(1-ew^2*(sin(phiw(j)))^2)^.5;

end
rho=rho';
rhow=rhow';
U=U';
V=V';
```

```
dh=0;

for num=1:q
    A=[sin(phi).*cos(lamda), sin(phi).*sin(lamda), -cos(phi) ;-
sin(lamda), cos(phi), (phi-phi);cos(phi).*cos(lamda),
cos(phi).*sin(lamda), sin(phi)];
    L=[k.*sin(2*phi)-dphi.*(rho*(sin(1/3600*pi/180)));
dlamda.*(V.*cos(phi)*(sin(1/3600*pi/180)));dh+da-
(a.*df+f.*da).*(sin(phi).^2)];
    Xt=inv(A'*A);
    X=Xt*A'*L;
    dX=X(1);
    dY=X(2);
    dZ=X(3);
    v=A*X-L;

dh=dX*cos(phi).*cos(lamda)+dY*cos(phi).*sin(lamda)+dZ*sin(phi)+(a
*df+f*da)*(sin(phi)).^2-da;
end

%----Variance
thevariance=(v'*v)/(3n-u)

% ---SD
sd=sqrt(thevariance)

% --- Individual Variance----
vXt=sd*sqrt(diag(Xt))


Nw= hw-Ho;
ho=hw-dh;
Xw = (U(j)+hw).*cos(phiw).*cos(lamdaw);
Yw = (U(j)+hw).*cos(phiw).*sin(lamdaw);
Zw = (rhow(j)+ hw).*sin(phiw);
Xo = (V(j)+ho).*cos(phi).*cos(lamda) ;
Yo = (V(j)+ho).*cos(phi).*sin(lamda);
Zo = (rho(j)+ ho).*sin(phi);
TRANSFORMEDo=[Xo,Yo,Zo]';
TRANSFORMEDw=[Xw,Yw,Zw]';



to_orig=TRANSFORMEDo;
tw_orig=TRANSFORMEDw;

[r,c]=size(TRANSFORMEDo);

TRANSFORMEDo=TRANSFORMEDo';
TRANSFORMEDw=TRANSFORMEDw';

% to=to_orig(:);
```

```
% tw=tw_orig(:);
to=TRANSFORMEDo(:);
tw=TRANSFORMEDw(:);

L=tw-to;

%-----WE BEGIN THE NEXT PART HERE------

[A_new]=doExtract(TRANSFORMEDo);

Xt1=inv(A_new'*A_new);
x1=Xt1*A_new'*L;
[k p]=size(x1);
v=A_new*x1-L;
%----Variance
thevariance1=(v'*v)/(n-k)

% ---SD
sd1=sqrt(thevariance1)

% --- Individual Variance----
vXt1=sd*sqrt(diag(Xt1))


%for i=1:18
%A=[1,0,0,Xoi,0,-Zoi,Yoi;0,1,0,Yoi,Zoi,0,-Xoi;0,0,1,Zoi,-
Yoi,Xoi,0];

%X=inv(A'*A)*A'*L
Xmu=mean(TRANSFORMEDo(:,1));
Ymu=mean(TRANSFORMEDo(:,2));
Zmu=mean(TRANSFORMEDo(:,3));

all_mu=[Xmu Ymu Zmu];

% TRANSFORMEDo_new
TRANSFORMEDo(:,1)=TRANSFORMEDo(:,1)-Xmu;
TRANSFORMEDo(:,2)=TRANSFORMEDo(:,2)-Ymu;
TRANSFORMEDo(:,3)=TRANSFORMEDo(:,3)-Zmu;

[A2]=doExtract(TRANSFORMEDo);

x2=inv(A2'*A2)*A2'*L;

Xt2=inv(A2'*A2);
x2=Xt2*A2'*L;
[k p]=size(x2);
v=A2*x2-L;
%----Variance
thevariance2=(v'*v)/(n-k);

% ---SD
```

```
sd2=sqrt(thevariance2);

% --- Individual Variance----
vXt2=sd*sqrt(diag(Xt2))




x=[x1 x2];

% -----Displays------
% this is the display for molodensky values

[r,c]=size(X);

%sdash='----------------------';
%ssbar='|';
%fprintf('\t\t %s \n',sdash)
%fprintf('\t\t %s \n','|ABRIDGED MOLODENSKY, X|')
%fprintf('\t\t %s \n',sdash)
%for i=1:r
%    fprintf('\t\t%s\t%.5f \t%s\n',ssbar,X(i),ssbar)
%end
%fprintf('\t\t %s\n',sdash)

%[r,c]=size(x);

%sdash1='-------------------------------';
%ssbar='|';
%fprintf('\t\t %s\n',sdash1)
%fprintf('\t\t%s\n','| BURSAWOLF | MOLODENSKY BADESKY|')
%fprintf('\t\t %s \n',sdash1)
%for i=1:r
%    fprintf('\t\t%s %.5f \t %s %.5f \t
%s\n',ssbar,x(i,1),ssbar,x(i,2),ssbar)
%end
%fprintf('\t\t %s\n',sdash1)

%[r,c]=size(ho);
%fprintf('\t\t %s \n',sdash)
%fprintf('\t\t %s \n','|  ELLIPSOIDAL HEIGHT  |')
%fprintf('\t\t %s \n',sdash)
%for i=1:r
%    fprintf('\t\t%s\t%.5f \t%s\n',ssbar,ho(i),ssbar)
%end
%fprintf('\t\t %s \n',sdash)

%[r,c]=size(dh);
%fprintf('\t\t %s \n',sdash)
%fprintf('\t\t %s \n','|    HEIGHT DIFF.      |')
%fprintf('\t\t %s \n',sdash)
%for i=1:r
```

```
%     fprintf('\t\t%s\t%.5f \t%s\n',ssbar,dh(i),ssbar)
%end
%fprintf('\t\t %s \n',sdash)

%[r,c]=size(v);
%fprintf('\t\t %s \n',sdash)
%fprintf('\t\t %s \n','|           RESIDUAL       |')
%fprintf('\t\t %s \n',sdash)
%for i=1:r
%     fprintf('\t\t%s\t%.5f \t%s\n',ssbar,v(i),ssbar)
%end
%fprintf('\t\t %s \n',sdash)

%fprintf('\t\t %s \n',sdash)
%fprintf('\t\t %s \n','|           VARIANCE      |')
%fprintf('\t\t %s \n',sdash)
%fprintf('\t\t%s\t%.5f \t%s\n',ssbar,thevariance,ssbar)
%fprintf('\t\t %s \n',sdash)
```

APPENDIX B: RESIDUALS FROM DERIVATION OF PARAMETERS

TABLE B-1: RESIDUALS FROM THREE- AND FIVE- PARAMETER

5 PARAMETER			3 PARAMETER		
Vx (m)	Vy (m)	Vz (m)	Vx (m)	Vy (m)	Vz (m)
0.250789211	3.834351913	0.000696333	0.499289827	3.790010751	-0.205873131
-0.51465985	-2.326390521	0.000696376	0.289795588	-2.36295434	0.541949993
3.490689203	-0.755647962	0.000696365	0.766866052	-0.83032207	-3.440651524
-6.25039815	-0.752394676	0.000696334	-0.05323609	-0.80447206	6.269788183
-0.4850066	3.848910718	0.000696307	0.059690706	3.844584348	0.497252517
-3.12650144	-0.062499475	0.000696338	-0.13065347	-0.07645304	3.121240952
-1.46153222	-1.412170106	0.000696448	-0.23847008	-1.40242003	1.450064026
1.548878695	-1.171360018	0.000696442	0.272389828	-1.19498468	-1.528239395
-1.37065135	-0.737938895	0.000696425	0.041398033	-0.75517381	1.384871686
-0.09521272	2.982815374	0.000696428	-0.14275087	3.005710852	0.068992582
0.525443892	-0.554922091	0.000696435	0.139324578	-0.56250754	-0.507571418
4.619819551	2.875866256	0.00069644	0.422204797	2.884483182	-4.607021536
2.985604958	2.955826685	0.000696375	0.325127661	2.9444506	-2.981294685
2.408983872	2.924079884	0.000696321	-0.20497334	2.986574976	-2.442858911
0.765787875	2.721703052	0.000696318	-0.19093426	2.754958161	-0.793408891
-0.20499693	-4.624040339	0.000696382	-0.57438046	-4.57047366	0.162565561
-6.01659514	-3.696455575	0.000696371	-1.17310412	-3.63622749	5.951204243
-1.55299896	-1.17453352	0.000696431	-0.36681899	-1.15353374	1.531533125
4.495285518	-4.85332114	0.000696409	0.259234602	-4.86125043	-4.472543376

TABLE B-2: RESIDUALS FROM BURSA-WOLF AND MOLODENSKY-BADEKAS

BURSA- WOLF			MOLODENSKY- BADEKAS		
Vx (m)	Vy (m)	Vz (m)	Vx (m)	Vy (m)	Vz (m)
-0.195225517	3.324185912	-0.918872525	0.091581966	3.020139882	-0.822755289
-0.260568759	-2.7360742	-0.029161525	-0.03348725	-2.97417081	0.046873549
-0.515922232	-0.88899788	-2.539072094	0.260345174	-1.53889935	-3.072445459
-1.149741062	-1.19886125	6.087714273	-0.59208736	-1.71654382	5.937159946
0.440216243	3.362681899	-1.153399057	0.056066371	3.609689485	-0.55128947
-0.03095863	-0.5324628	1.780196529	-0.2322046	-0.4295401	2.227437277
-0.060295569	-1.40152176	1.311403506	-0.16894224	-1.30717791	1.386970148
-0.105731379	-1.18142927	-1.1707388	0.13392197	-1.37324579	-1.360071364
-0.18280786	-0.9815046	0.951283804	-0.11214427	-1.06795248	1.044620025
0.448375548	2.958169413	-0.545173917	0.069322788	3.269610293	-0.238970827
0.148980899	-0.73305808	-0.980946967	0.091954838	-0.7060706	-0.830218238
0.792945518	2.987857407	-4.623361723	0.596241679	3.168675174	-4.549638089
0.174872217	3.317143337	-1.829565139	0.381622028	3.197123946	-2.22547187
0.796718565	3.448074332	-2.011074596	0.315334431	3.902658747	-1.952385524
0.176625869	3.278093861	0.335258796	0.106926906	3.401597122	0.049629611
0.203028413	-4.24667588	0.408882446	-0.17981264	-3.88698325	0.480196063
-0.747538718	-3.23616176	6.696678039	-0.88982882	-3.06669774	6.542561334
-0.147503633	-0.95900244	1.889835949	-0.2230761	-0.86726305	1.816007355

APPENDIX C- TEST RESULTS FROM GOLDEN TRIANGLE

TABLE C-1: THREE PARAMETER TEST RESULTS

WAR OFFICE COORDINATES								
	EXISTING COORDINATES		PROJECTED COORDINATES		RESIDUALS			
POINTNAME	Northings(ft)	Eastings(ft)	Northings(ft)	Eastings(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
CFP 109	286868.63	1109433.05	286865.971	1109434.983	-2.659	1.933	-0.810	0.589
CFP 200	346933.94	1060041.45	346931.627	1060043.016	-2.313	1.566	-0.705	0.477
CFP 225	285019.85	717756.06	285026.481	717758.096	6.631	2.036	2.021	0.621
GCS 102	222464.16	996471.72	222459.669	996472.179	-4.491	0.459	-1.369	0.140
CFP 155	460739.72	1218791.85	460741.758	1218793.879	2.038	2.029	0.621	0.618
GCS 179	617579.48	887815.7	617581.109	887817.798	1.629	2.098	0.497	0.639
CFP 180R	502139.98	795978.88	502141.95	795978.696	1.97	-0.18	0.600	-0.056
CFP 217	461992.4	998070.31	461990.379	998070.986	-2.021	0.676	-0.616	0.206
GCS 142	691483.15	984942	691481.585	984942.295	-1.565	0.295	-0.477	0.090
CFP 213	529124.19	991066.89	529124.778	991067.134	0.588	0.244	0.179	0.074
CFP 178	689861.56	840169.51	689861.129	840173.529	-0.431	4.019	-0.131	1.225
CFP185	658750.36	564228.3	658751.161	564234.436	0.801	6.136	0.244	1.870
CFP306	931057.32	671516.26	931050.945	671519.119	-6.375	2.859	-1.943	0.871
GCS 302	813987.32	531310.67	813982.515	531316.907	-4.805	6.237	-1.465	1.901
CFP 304	842589.26	738496.56	842586.068	738497.895	-3.192	1.335	-0.973	0.407
CFP 305	789811.15	630369.77	789807.146	630375.327	-4.004	5.557	-1.220	1.694
CFP 145R	684673.93	750479.52	684674.42	750484.385	0.49	4.865	0.149	1.483
CFP 184	653823.6	647795.79	653822.895	647800.428	-0.705	4.638	-0.215	1.414
CFP 207	428353.9	548934.64	428357.16	548941.095	3.26	6.455	0.994	1.967
GCS 125	398140.35	1239541.76	398143.082	1239546.761	2.732	5.001	0.833	1.524

TABLE C-2: FIVE PARAMETER TEST RESULTS

WAR OFFICE COORDINATES								
POINT	EXISTING COORDINATES		PROJECTED COORDINATES		RESIDUALS			
NAME	Northings(ft)	Eastings(ft)	Northings(ft)	Eastings(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
CFP 109	286868.63	1109433.05	286872.988	1109434.356	4.358	1.306	1.328	0.398
CFP 200	346933.94	1060041.45	346938.649	1060042.39	4.709	0.94	1.435	0.287
CFP 225	285019.85	717756.06	285033.499	717757.476	13.649	1.416	4.160	0.432
GCS 102	222464.16	996471.72	222466.681	996471.554	2.521	-0.166	0.768	-0.051
CFP 155	460739.72	1218791.85	460748.789	1218793.25	9.069	1.4	2.764	0.427
GCS 179	617579.48	887815.7	617588.153	887817.175	8.673	1.475	2.644	0.450
CFP 180R	502139.98	795978.88	502148.985	795978.074	9.005	-0.806	2.745	-0.246
CFP 217	461992.4	998070.31	461997.41	998070.361	5.01	0.051	1.527	0.016
GCS 142	691483.15	984942	691488.634	984941.67	5.484	-0.33	1.672	-0.101
CFP 213	529124.19	991066.89	529131.815	991066.509	7.625	-0.381	2.324	-0.116
CFP 178	689861.56	840169.51	689868.177	840172.907	6.617	3.397	2.017	1.035
CFP185	658750.36	564228.3	658758.209	564233.82	7.849	5.52	2.392	1.682
CFP306	931057.32	671516.26	931058.011	671518.501	0.691	2.241	0.211	0.683
GCS 302	813987.32	531310.67	813989.574	531316.292	2.254	5.622	0.687	1.714
CFP 304	842589.26	738496.56	842593.128	738497.275	3.868	0.715	1.179	0.218
CFP 305	789811.15	630369.77	789814.203	630374.709	3.053	4.939	0.931	1.505
CFP 145R	684673.93	750479.52	684681.469	750483.765	7.539	4.245	2.298	1.294
CFP 184	653823.6	647795.79	653829.942	647799.81	6.342	4.02	1.933	1.225
CFP 207	428353.9	548934.64	428364.191	548940.478	10.291	5.838	3.137	1.779
GCS 125	398140.35	1239541.76	398150.109	1239546.132	9.759	4.372	2.975	1.333

TABLE C-3: SEVEN PARAMETER BURSA-WOLF TEST RESULTS

WAR OFFICE COORDINATES								
POINT	EXISTING COORDINATES		PROJECTED COORDINATES		RESIDUALS			
POINT	Northings(ft)	Eastings(ft)	Northings(ft)	Eastings(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
CFP 109	286868.63	1109433.05	286865.118	1109434.189	-3.512	1.139	-1.070	0.347
CFP 200	346933.94	1060041.45	346931.108	1060041.867	-2.832	0.417	-0.863	0.127
CFP 225	285019.85	717756.06	285025.301	717754.866	5.451	-1.194	1.661	-0.364
GCS 102	222464.16	996471.72	222458.324	996470.734	-5.836	-0.986	-1.779	-0.301
CFP 155	460739.72	1218791.85	460742.075	1218793.627	2.355	1.777	0.718	0.542
GCS 179	617579.48	887815.7	617582.137	887815.359	2.657	-0.341	0.810	-0.104
CFP 180R	502139.98	795978.88	502142.185	795975.778	2.205	-3.102	0.672	-0.945
CFP 217	461992.4	998070.31	461990.527	998069.359	-1.873	-0.951	-0.571	-0.290
GCS 142	691483.15	984942	691483.151	984940.402	0.001	-1.598	0.000	-0.487
CFP 213	529124.19	991066.89	529125.338	991065.409	1.148	-1.481	0.350	-0.451
CFP 178	689861.56	840169.51	689862.569	840170.736	1.009	1.226	0.308	0.374
CFP185	658750.36	564228.3	658752.186	564229.949	1.826	1.649	0.557	0.503
CFP306	931057.32	671516.26	931053.752	671515.082	-3.568	-1.178	-1.088	-0.359
GCS 302	813987.32	531310.67	813984.48	531312.09	-2.84	1.42	-0.866	0.433
CFP 304	842589.26	738496.56	842588.378	738494.346	-0.882	-2.214	-0.269	-0.675
CFP 305	789811.15	630369.77	789809.04	630371.147	-2.11	1.377	-0.643	0.420
CFP 145R	684673.93	750479.52	684675.756	750481.037	1.826	1.517	0.557	0.462
CFP 184	653823.6	647795.79	653823.956	647796.466	0.356	0.676	0.109	0.206
CFP 207	428353.9	548934.64	428356.738	548936.698	2.838	2.058	0.865	0.627
GCS 125	398140.35	1239541.76	398143.027	1239546.688	2.677	4.928	0.816	1.502

TABLE C-4: SEVEN PARAMETER MOLODENSKY-BADEKAS TEST RESULTS

WAR OFFICE COORDINATES								
POINT	EXISTING COORDINATES		PROJECTED COORDINATES		RESIDUALS			
NAME	Northerns(ft)	Easterns(ft)	Northerns(ft)	Easterns(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
CFP 109	286868.63	1109433.05	286865.122	1109434.046	-3.508	0.996	-1.069	0.304
CFP 200	346933.94	1060041.45	346931.112	1060041.725	-2.828	0.275	-0.862	0.084
CFP 225	285019.85	717756.06	285025.306	717754.73	5.456	-1.33	1.663	-0.405
GCS 102	222464.16	996471.72	222458.33	996470.593	-5.83	-1.127	-1.777	-0.344
CFP 155	460739.72	1218791.85	460742.076	1218793.482	2.356	1.632	0.718	0.497
GCS 179	617579.48	887815.7	617582.135	887815.22	2.655	-0.48	0.809	-0.146
CFP 180R	502139.98	795978.88	502142.186	795975.641	2.206	-3.239	0.672	-0.987
CFP 217	461992.4	998070.31	461990.528	998069.218	-1.872	-1.092	-0.571	-0.333
GCS 142	691483.15	984942	691483.148	984940.261	-0.002	-1.739	-0.001	-0.530
CFP 213	529124.19	991066.89	529125.338	991065.268	1.148	-1.622	0.350	-0.494
CFP 178	689861.56	840169.51	689862.565	840170.598	1.005	1.088	0.306	0.332
CFP185	658750.36	564228.3	658752.183	564229.816	1.823	1.516	0.556	0.462
CFP306	931057.32	671516.26	931053.744	671514.947	-3.576	-1.313	-1.090	-0.400
GCS 302	813987.32	531310.67	813984.475	531311.958	-2.845	1.288	-0.867	0.393
CFP 304	842589.26	738496.56	842588.372	738494.21	-0.888	-2.35	-0.271	-0.716
CFP 305	789811.15	630369.77	789809.035	630371.013	-2.115	1.243	-0.645	0.379
CFP 145R	684673.93	750479.52	684675.753	750480.901	1.823	1.381	0.556	0.421
CFP 184	653823.6	647795.79	653823.954	647796.331	0.354	0.541	0.108	0.165
CFP 207	428353.9	548934.64	428356.74	548936.565	2.84	1.925	0.866	0.587
GCS 125	398140.35	1239541.76	398143.029	1239546.543	2.679	4.783	0.817	1.458

APPENDIX D- TEST RESULTS FROM HO

TABLE D-1: THREE PARAMETER TEST RESULTS

POINT NAME	EXISTING COORDINATES		PROJECTED COORDINATES		RESIDUALS			
	Northings	Eastings	Northings	Eastings				
	(ft)	(ft)	(ft)	(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
SGV/RS/09/1	704787.86	1432182.203	704792.345	1432182.05	-4.485	0.153	-1.367	0.047
SGV/GPS/15/00/1	701567.336	1429088.779	701571.766	1429088.499	-4.43	0.28	-1.35	0.085
SGV/10/03/3	690195.42	1408130.98	690196.614	1408131.392	-1.194	-0.412	-0.364	-0.126
SGV/10/03/1	690563.69	1407987.31	690564.74	1407986.373	-1.05	0.937	-0.32	0.286
SGV/4/02/102	704992.44	1429537.42	704996.531	1429538.247	-4.091	-0.827	-1.247	-0.252
SGV/6A/07/01	705088.99	1429801.81	705092.841	1429804.031	-3.851	-2.221	-1.174	-0.677
SGV/RS/09/02	704680.715	1432022.117	704685.18	1432021.832	-4.465	0.285	-1.361	0.087

TABLE D-2: FIVE PARAMETER TEST RESULTS

WAR OFFICE COORDINATES								
EXISTING COORDINATES			PROJECTED COORDINATES		RESIDUALS			
POINT	Northings	Eastings	Northings	Eastings				
NAME	(ft)	(ft)	(ft)	(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
SGV/RS/09/1	704787.86	1432182.203	704799.363	1432181.411	-11.503	0.792	-3.506	0.241
SGV/GPS/15/00/1	701567.336	1429088.779	701578.782	1429087.861	-11.446	0.918	-3.489	0.280
SGV/10/03/3	690195.42	1408130.98	690203.638	1408130.755	-8.218	0.225	-2.505	0.069
SGV/10/03/1	690563.69	1407987.31	690571.765	1407985.736	-8.075	1.574	-2.461	0.480
SGV/4/02/102	704992.44	1429537.42	705003.548	1429537.608	-11.108	-0.188	-3.386	-0.057
SGV/6A/07/01	705088.99	1429801.81	705099.857	1429803.392	-10.867	-1.582	-3.312	-0.482
SGV/RS/09/02	704680.715	1432022.117	704692.198	1432021.193	-11.483	0.924	-3.500	0.282

TABLE D-3: SEVEN PARAMETER BURSA-WOLF TEST RESULTS

	EXISTING COORDINATES		PROJECTED COORDINATES			RESIDUALS		
POINT	Northings	Eastings	Northings	Eastings				
NAME	(ft)	(ft)	(ft)	(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
SGV/RS/09/1	704787.86	1432182.203	704794.353	1432182.93	-6.493	-0.727	-1.979	-0.222
SGV/GPS/15/00/1	701567.336	1429088.779	701573.752	1429089.363	-6.416	-0.584	-1.956	-0.178
SGV/10/03/3	690195.42	1408130.98	690198.512	1408132.134	-3.092	-1.154	-0.942	-0.352
SGV/10/03/1	690563.69	1407987.31	690566.641	1407987.114	-2.951	0.196	-0.899	0.06
SGV/4/02/102	704992.44	1429537.42	704998.539	1429539.11	-6.099	-1.69	-1.859	-0.515
SGV/6A/07/01	705088.99	1429801.81	705094.849	1429804.896	-5.859	-3.086	-1.786	-0.941
SGV/RS/09/02	704680.715	1432022.117	704687.187	1432022.711	-6.472	-0.594	-1.973	-0.181

TABLE D-4: SEVEN PARAMETER MOLODENSKY-BADEKAS TEST RESULTS

	EXISTING COORDINATES		PROJECTED COORDINATES			RESIDUALS		
POINT	Northings	Eastings	Northings	Eastings				
NAME	(ft)	(ft)	(ft)	(ft)	dN(ft)	dE(ft)	dN(m)	dE(m)
SGV/RS/09/1	704787.86	1432182.203	704794.35	1432182.78	-6.49	-0.577	-1.978	-0.176
SGV/GPS/15/00/1	701567.336	1429088.779	701573.749	1429089.213	-6.413	-0.434	-1.955	-0.132
SGV/10/03/3	690195.42	1408130.98	690198.509	1408131.985	-3.089	-1.005	-0.942	-0.306
SGV/10/03/1	690563.69	1407987.31	690566.637	1407986.965	-2.947	0.345	-0.898	0.105
SGV/4/02/102	704992.44	1429537.42	704998.535	1429538.961	-6.095	-1.541	-1.858	-0.47
SGV/6A/07/01	705088.99	1429801.81	705094.846	1429804.747	-5.856	-2.937	-1.785	-0.895
SGV/RS/09/02	704680.715	1432022.117	704687.184	1432022.561	-6.469	-0.444	-1.972	-0.135