# COMBINING DATA ENVELOPMENT ANALYSIS WITH MACHINE LEARNING ALGORITHMS FOR PREDICTIONS 



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COMBINING DATA ENVELOPMENT ANALYSIS WITH MACHINE LEARNING
ALGORITHM FOR PREDICTIONS

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## DECLARATION

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma at Kwame Nkrumah University of Science and Technology, Kumasi or any other educational institution, except where due acknowledgment is made in the thesis.

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## DEDICATION

This work is dedicated with the deepest respect and honour to my late Uncle, Hon. Charles Yeboah (Former District Chief Executive Atwima), my parents Miss. Sussana Yeboah and Mr.

Alex Opoku Mensah all of blessed memory; my son, Ac e Opoku Appiahene, my daughter, Mercedes Yeboah Appiahene and my lovely and understanding wife Sandra Serwah, for their support, understanding and immense contribution to my life and education.


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#### Abstract

Comparative to other methods, DEA is an improved method to organize and analyze data. However, it is very difficult to use only DEA to predict the efficiency and performance of other or new Decision Making Units (DMU). The main objective of this study is to build a high accuracy machine learning predictive models for predicting the efficiencies of banks by combining DEA with Machine Learning algorithm. The study built four Machine Learning Models namely; DEA-DT, DEA-RF, DEA-NN and DEA-LR to predict the efficiencies of banks. The study used $33 \%$ of the total bank branches in Ghana, largely in the nine regions. A two-stage DEA was used to determine the efficiencies of all bank branches and these banks were grouped based on a proposed algorithm, Bank Classification Algorithm (BC Algorithm). In building the predictive models, $70 \%$ of the banks dataset were used to train and validate the models. The developed models were used to predict the efficiencies of the other $30 \%$ banks. A 10 -fold Cross-Validation was applied to check the performance of all predicting models on each case dataset. All experiments were executed within a simulation environment and conducted in R studio using R programming language. Standardized Machine Learning evaluation metrics were used to compare the models. The results suggested a very good performance of all the machine learning models proposed by the study. However, a comparison among them clearly indicated a much better performance by the DEA-RF for predicting banks' efficiency in collecting deposit and DEA-DT for predicting banks' efficiency in investing deposits. This study has demonstrated that combing two models improve the performance, predictions and classification accuracies suggested by previous studies. In conclusion, the study proposed the usage of the proposed BC Algorithm for classifying banks based on their efficiencies in deposit stage and investment stage.


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## LIST OF SYMBOLS, ABBREVIATIONS AND NOMENCLATURE

| \%PL | Rate of Performing Loans |
| :--- | :--- |
| AC | Overall accuracy |
| AI | Artificial Intelligent |
| ANN | Artificial Neural Network |

AUC
BCC
BOG Bank of Ghana
CAR Capital Adequacy Ratio
CCR

CPU
CRS

CV
DEA
DEA-DT
DEA-LR
DEA-NN
DEA-RF
DMU
DT
EFA
EFA
ERP
FORTRAN
GDP
GDP
GPT

Constant Returns To Scale
Charnes, Cooper and Rhodes
Central Processing Unit

Cross-Validation
Data Envelopment Analysis
Data Envelopment Analysis Decision Tree Model
Data Envelopment Analysis Logistic Regression Model
Data Envelopment Analysis Neural Network Model
Data Envelopment Analysis Random Forest Model
Decision making Unit
Decision Tree
Exploratory Factor Analysis
Exploratory factor analysis
Enterprise Resource Planning
Formula Translation
Gross Domestic Product
Gross domestic product
General Purpose Technologies
International Conference on Applied Sciences and Technology
International Conference on Data Mining
Information and communication Technology
Institute of Electrical and Electronic Engineering Area Under Curve

Banker, Charnes and Cooper

ICAST
ICDM
ICT
IEEE

| IMF | International Monetary Fund |
| :--- | :--- |
| IS | Information Systems |
| IT | Information Technology |

KNN

| LR |  |
| :---: | :---: |
| MAE |  |
| MAPE | Mean Absolute Percentage Error |
| MARS | Multivariate Adaptive Regression Splines |
| MARS | Multivariate Adaptive Regression Splines |
| MDA | Mean Decrease of Accuracy |
| MENA | Middle East and North African |
| MIS | Management Information System |
| MLP | Multilayer Layer Perceptron |
| MLR | Multinomial Logistic Regression |
| MRA | Multinomial Regression Analysis |
| MRA | Multinomial Regression Analysis |
| MSE | Mean Squared Error |
| NA | Not Available |
| NAICS | North America Industry Classification System |
| NN | Neural Networks |
| OCAM Office, Computing and Accounting Machinery |  |
| PCA | Principal Component Analysis |
| PCBS Palestinian Central Bureau of Stat |  |
|  | Performing Loans <br> probabilistic neural network |
|  | Random Access Memory |
|  | Relative Bias |
|  | K-Nearest Neighbors Logistic |
|  | Regression |
|  | Mean Absolute Error |

PL
PNN RAM
rBIAS

| RF | Random Forests |
| :--- | :--- |
| RF | Random Forest |
| RF | Random Forest |

RMSE

| RMSPE | Root Mean Square Percentage Error |
| :--- | :--- |
| ROC | Receiver Operating Characteristic |
| RST | Rough Set Theory |
| RWA | Risk-Weighted Assets |
| SBU | Strategic Business Unit |
| SFMC | State Insurance Corporation |
| SIC | Self-Organizing Feature Map Medium Scale Enterprise |
| SME | Social Security and National Insurance Trust |
| SOFM | Support Vector Machines |
| SSNIT | Vniversity of Energy and Natural Resources |
| SVM | Variable Returns to Scale |
| UENR | Variable Returns to Scale |
| VRS | West African Currency and divided the Board |
| VRS |  |

## CHAPTER ONE

## INTRODUCTION

### 1.0 BACKGROUND TO THE STUDY

A significant growth in performance and efficiency cements the basis for improvement in the standard of living (Majeed \& Ayub, 2018 and Niebel \& Mannheim, 2014). The prosperity of any country in terms of its economic fortunes is ultimately based on performance and efficiency of its important institutions like the banks (Cardona, Kretschmer, \& Strobel, 2013; Leung \& Zhang, 2016 and Stanley, Doucouliagos, \& Steel, 2018). This means that the survival and continuous existence of banks within the financial industry is of great importance to every economy and all of its citizens (Antonija et al., 2017; Navapan, Liu, \& Sriboonchitta, 2017 and Sahoo, 2014). As far as financial institutions such as banks play a significant role of financial mediations, the determination of their efficiency is essential and needs to be given enough attention. The significance of the banking industry is also premised on the ground that these financial institutions happen to be key channels of investments and allocations of credit in an economy (Sufian et al., 2016). The banking industry offers vital financial intermediation role by transforming deposits into prolific investments. Unlike countries like the USA where financial markets and the banking industry works in harmony to channel capitals, in countries like Ghana, financial markets are stunted and sometimes absolutely absent (Sufian et al., 2016). It is therefore the responsibility of the banking industry to close the gap between investors and debtors. The banking industry controls most of the financial flows and accounts of the financial system's total assets (Sufian et al., 2016). Therefore, it is prudent to presume that an efficient banking industry may help ensure an effective financial system which is conducive to economic growth and development.

This concept of efficiency is an old one and also quantifiable as it can be calculated through the assessment of the useful output ratio to total input. The traditional efficiency measurement of bank performance using the Data Envelopment Analysis (DEA) model usually accounts for
input and desirable output only (Hamid et al., 2017). According to a report by the Development \& Research Center in 2004, firms that effectively utilize their resources in their business process and operations experience greater efficiency and performance. This would lead them to greater competitiveness that promotes sustainable economic growth. Most banks and their management have little or no idea on how to predict the real efficiency of their organizations.

Even though assessing the concept of efficiency of banks has been extensively studied, a study by Burki \& Dashti (2003) cited by (Hamid et al., 2017) assessed the cost efficiency of Kuwaiti banks and suggested the cost inefficiency was found within the Kuwait banks. This according to the researchers was caused by locative inefficiency while the score for technical inefficiency was very high (Hamid et al., 2017). The analysis of the study was based on traditional Data Envelopment Analysis (DEA) measures of organizations' performance (Hamid et al., 2017). Thus efficiency as its best suits the requirements of management (Lefley, 2015; Wong \& Dow, 2011) A DEA is a linear-programming-based method which is used to determine the comparative efficiency of homogenous organizational units such as banks, school, governmental and non-governmental agencies, tax offices etc. A DEA model as a nonparametric technique has been used as a single method (Alexander et al., 2007; Álvarez et al., 2016; Necmi, 2006; Cao \& Yang, 2011 and Paço \& Pèrez, 2015) or combined
(Alinezhad, 2016; Anthanassopoulos \& Curram, 1996; Azadeh et al., 2011; Lee, 2010; Razavi et al., 2013 and Wu et al., 2006) with others extensively in previous literatures to measure the efficiency of an organization. Data Envelopment Analysis (DEA) has also been suggested to be a good qualitative measure of organizational efficiency. For instance , Cao \& Yang (2011); Chen et al. (2006); Madjid et al. (2009); Wang et al. (1997); Hatefi \& Fasanghari (2014) and Appiahene et al. (2019) applied DEA to assess the impact of IT on the performance of firms and concluded a positive impact. According to Aggelopoulos \& Georgopoulos (2017) ; Wanke et al. (2016), compared to other methods, DEA is superior method to analyze and ascertain
productivity since it enables effectiveness to change after some time. It also requires no earlier supposition on the detail of the best practice frontier.

Traditionally, DEA has been one of the most popular tools to assess a firm's performance. Machine Learning Algorithms have also been used for the prediction of bank efficiency and even its bankruptcy and such studies include (Chang et al., 1996; Dash et al., 2006; Jardin , 2018 and Mai et al., 2018). Neural Networks (NN), for instance have recognized their role as data analysis tools in different areas. Decision tree algorithm as used (Chen, 2016; Santos et al., 2017 and Wu , 2006) and Random Forest algorithms have also been found to be an efficient prediction method. Various techniques and machine learning algorithms have been applied in prediction studies but combining DEA with Machine learning algorithms to predict the efficiency of banks is really scarce especially using data from developing country.

Comparison of bank branches' efficiency across developing countries are also completely lacking in literature (Mohd, 2001 and Tra et al., 2018).

This study differs from previous studies (Hamad \& Anouze ,2015; Kwon \& Lee, 2015 and Wu 2006) in the following aspects: first and foremost, this study combines a two-stage DEA model with different Machine Learning Algorithms namely Decision Tree, Random Forest, Artificial Neural Network and Logistic Regression, where the efficiency at each stage (thus efficiency in collecting deposit from customers, Deposit efficiency and efficiency in investing the deposit, Investment efficiency) is also considered as input for the next stage using a huge dataset a from a developing country. The study also classified the various banks based on their efficiency scores using a proposed algorithm, Bank Classification Algorithm suggested by the study. This efficiency classes (Class1, Class2 Class 3 and Class 4) were used as the response variable making, the response variable for the study categorical. For the development of the four models, the study considered predictor variables which were both internal and external and directly influence bank performance and efficiency as suggested by (Thanassoulis et al., 2008).

The development and performance of the proposed in this current study was done on big dataset compared to Wu (2006) work which was done on a small dataset from existing literature (Wang et al., 1997). Kwon \& Lee (2015) also used a total of 181 DMUs as a dataset. Which accooidng to this same Kwon \& Lee (2015), a large data set is often favored for the generalization of models. A K-fold cross-validation was used to better the performnace of the models as compared to Wu (2006) works, which was based on V-fold cross validation.The data used in this study is not a cross-country bank-level data compared to Hamad \& Anouze (2015). In this study , $70 \%$ of the data set (444 DMUs) was used for training and $10 \%$ validation and $30 \%$ for testing the models compared to Kwon \& Lee, (2015) study of combining DEA with Back Propagation Neural Network which was based on a ratio of 60:20:20 for training, validating and testing respectively. This is also a single study that has build and proposed four different models that has yieded fovorable classification accuracy rate.These models were developed by combining DEA with Decision Tree (DEADT),Random Forest (DEA-RF), Artificial Neural Network (DEA-NN) and Logistic

Regression (DEA-LR). In the case of the DEA-DT, the model perfomed better than Wu (2006) work by giving an accuracy of not less than $90 \%$ compraed to Wu (2006) $69.44 \%$. Comparing the DEA-RF to that of Hamad \& Anouze (2015), the study also suggested a fovuarble classification accuracy of $90 \%$ compared to Hamad \& Anouze (2015) work that also yieded $79.4 \%$ and finaly, the DEA-NN suggeseted a favouable Mean Absolute Percentage Error (MAPE) of about $24.6 \%$ compared the BPNN MAPE of $36.9 \%$ also suggsetd by (Kwon \& Lee, 2015). Compared to the works of Hamad \& Anouze (2015); Kwon \& Lee, (2015) and Wu (2006), this study demonstrated the performance of the four proposed models by using five standard machine learning evaluation metrics namely; Root Mean Square Error(RMSE),Mean Absolute Percentage Error(MAPE),Mean Absolute Error(MAE),Root Mean Square Percentage Error (RMSPE), and rBIAS.

### 1.1 PROBLEM STATEMENT

The assessment of banks efficiencies creates serious problems for banks and their managers (Adusei, 2016; Sufian et al., 2016 and Titko, Stankevičienė, \& Lāce, 2014). Due to the critical significant role of banks in the national economy, their efficiencies and performances are a hotly debated topic in the academic and business environments (Titko et al., 2014). Models and frameworks designed for this assessment are normally based on econometric analysis (Brynjolfsson, 1993; Dedrick et al., 2013; Dedrick et al., 2003; Kılıçaslan et al., 2017; Nations, 2008 and Roghieh et al., 2004) and other parametric methods. Traditionally, financial institutions such as banks have concentrated on numerous profitability measures as a means to assess their efficiency and even performance by using multiple ratios based on different aspects of the operations (Dash et al., 2006; Wanke et al.,2016). Nevertheless, ratio analysis suggests comparatively insignificant amount of information when considering the approximation of overall efficiency measures of firms (Tavana et al., 2016). As an option to this traditional method of efficiency assessment, frontier efficiency analysis allows one to empirically recognize best practices in operational environments.

Non-parametric techniques such as Data Envelopment Analysis (DEA) have been suggested in literature as technique to evaluate bank efficiency (Lampe \& Hilgers, 2015). Compared to other methods, DEA is an improved method to organize and analyze data since it allows efficiency to change over time (Lampe \& Hilgers, 2014). It also does not involve any previous postulation on the bases of best practice frontier (Aggelopoulos \& Georgopoulos,

2017; Fallahpour et al., 2017 and LaPlante \& Paradi, 2014). However, such studies (Chen et al., 2006; Chen \& Zhu, 2004a; Izadikhah et al., 2017; Madjid et al., 2009; Tavana et al., 2017 and Wang et al., 1997) that used only non-parametric methods such as pure DEA and its models suffer from weak discrimination power (Ashoor, 2012; Chen, 2016; Santos et al., 2017 and $\mathrm{Wu}, 2006$, 2009) and also sensitive to the presence of outliers and statistical noise (Da et al., 2018 and Dash et al., 2006). It is also very difficult to use only DEA to predict the efficiency and performance of other or new Decision Making (Emrouznejad \& Yang, 2017;

LaPlante \& Paradi, 2014 and Wanke et al., 2016). Literature on DEA and Machine Learning Algorithms using data from Ghanaian banks is also scarce.

To overcome this problem, more promising and novel methods and procedures for the prediction and classification of bank efficiencies are necessary (Sreedhara et al., 2018). Combined DEA and Machine Learning Algorithms can offer these suitable and scientific methods. A lot of machine learning algorithms have been developed and utilized in the business and financial sectors for predictions and forecasting. For instance, Chen \& Hao (2017) used Feature Weighted Support Vector Machine and K-Nearest Neighbor Algorithm for stock market indices prediction using the two well-known Chinese stock market indices. There are other studies such as Abellán \& Castellano (2017); Ahn et al. (2000); Anandarajan et al. (2001); Caggiano et al. (2014); Caggiano et al. (2016); Göçken et al. (2016); Janek et al. (2016); Kim \& Han (2000); Patel et al. (2014b, 2014a); Qiu et al. (2016) and Tsai \& Wu, (2008) that have implemented machine learning algorithms in the business and financial sectors in forecasting and prediction studies.

According to literature, Machine Learning Algorithms used to build predictive models have been suggested as the one of finest and best predicting methods with a high accuracy and validity in the field business and financial forecasting (Göçken et al., 2016). Nevertheless, a combined high performance machine learning model for prediction and classification of bank branches' using their efficiency across developing countries are also completely lacking in literature (Mohd, 2001 and Tra et al., 2018) especially using a combined DEA and Machine Learning Algorithms. The results of models built in these studies were also weak in terms of performance (Yang et al., 2015). Combining two or more models has gained a lot of attentions and interest across various disciplines comprising Management and Decision

Sciences, Applied Mathematics, Data Science, Machine Learning and Artificial Intelligent (Yang et al., 2015 and Zheng et al., 2004). According to Yang et al. (2015) and Zheng et al.
(2004), combined models consistently outperform individual models for classification and prediction tasks. This current work uses a primary data gathered from more than four hundred (400) DMUs of bank branches.

### 1.2 MAIN AND SPECIFIC OBJECTIVES OF THE STUDY

The main objective of this study is to build a strong and robust predictive model for predicting the efficiency of banks by combining DEA with Machine Learning Algorithms using data from the Ghanaian banking sector as a case study. The output of this study is expected to be predictive models that can be used to predict the efficiency of Ghanaian banks accurately. This main objective was realized through the following specific objectives:

### 1.2.1 SPECIFIC OBJECTIVES

1. To use a Decision Tree algorithm to build a model for predicting bank efficiencies using dataset from the Ghanaian banking sector.
2. To use a Random Forest algorithm to build a model for predicting bank efficiencies using dataset from the Ghanaian banking sector.
3. To use an Artificial Neural Network to build a model for predicting bank efficiencies using dataset from the Ghanaian banking sector.
4. To use a Logistic Regression Algorithm to build a model for predicting bank efficiencies using dataset from the Ghanaian banking sector.
5. To adapt an existing algorithm to determine the efficiencies of the selected banks and use an algorithm to classify the banks based on their efficiencies in taking deposits and investing the deposits.
6. To develop and propose a high performance models for predicting bank efficiencies by combining DEA with: Decision Tree Algorithm (DEA-DT), Random Forest Algorithm (DEA-RF), Artificial Neural Network (DEA-NN) and Logistic Regression Algorithm (DEA-LR).

### 1.3 SIGNIFICANCE AND BENEFITS OF THE STUDY

The significance and benefits of this study include but not limited to the following:

1. Researchers can also use the proposed framework and models in the study to achieve high prediction accuracies in their studies.
2. The general Ghanaian banking industry and other firms may benefit from the results of this thesis as they may have a better understanding of relationship between resources and banks performance and efficiency.
3. Customers' and investors may benefit from the findings of this study as the finding would add value to their knowledge of how efficient their various bank branches are in terms of managing their capitals.
4. The predictive models build in the study can be used to assess and predict the efficiency of new banks going forward.
5. The proposed Bank Classification Algorithm (BC Algorithm) by the study can be used to classify banks in Ghana based on their efficiency in both deposit stage and investment stage.

### 1.4 SCOPE OF STUDY

The study was undertaken basically to classify and predict the efficiencies of banks by combining DEA with Machine Learning Algorithms. The case study firms were commercial bank branches in Ghana. Due to the large number of universal bank branches operating in the country as of the time of this study (2016), the study was limited to $33 \%$ (comprising 17 universal banks), of the total bank branches in Ghana mostly in Greater Accra, Ashanti, Western, Brong Ahafo, Eastern, Northern, Upper East, Volta and Central Regions. Part of the data for the study was also audited 2016 financial statements from the various banks. With respect to factors that affect bank's deposit collection, there were other several factors that were not taken into consideration because of inadequate data on these factors. This study is therefore limited to Bank's Fixed Asset, IT expenditure and Total Number of Employees as factors that can impact deposit mobilization by Banks.

### 1.5 ORGANISATION AND STRUCTURE OF THE THESIS

This thesis consist of six different chapters which have been structured in a logical sequence starting from the introductory chapter containing the background information of the topic, the problem statement, the objectives, the study's significance and scope. Chapter One is followed immediately by Chapter Two; literature review which also discusses the explanation and description of some concepts used in the study. It also contains information on extensive literature review of the topic, related works and finally conclusions drawn from these literature reviews. The Chapter Three which discusses the methodology describes comprehensively the various scientific methods, techniques and tools used to accomplish the study. The Chapter Four also deals with the presentation and discussions of the study results taking into consideration the five specific objectives.

Chapter Five also presents the general discussions and analysis of the study results. Finally the last chapter which is Chapter Six contains the conclusion drawn from the study, recommendations to various stakeholders who can use the study results and suggestions for future studies.

## CHAPTER TWO

## LITERATURE REVIEW

### 2.0 Introduction

The purpose of this study is to have a better understanding of the banks' efficiency measurement and how to combine machine learning algorithms with Data Envelopment Analysis (DEA) to build a high accuracy models for predicting the efficiency of banks and also classify these banks using their efficiencies. To provide the necessary background, this chapter will review literature on machine learning models for predicting banks' efficiency, DEA for assessing banks' efficiency, and previous applications of combined DEA and machine learning
algorithms models for predictions. The review is done in a way as to identify gaps in existing literature and present how this thesis will address these gaps.

### 2.1 OVERVIEW OF DATA ENVELOPMENT ANALYSIS (DEA)

The fundamental efficiency is a proportion of output over input. To enhance efficiency, one needs to either increase the output or reduces the input. If there is an increase in both cases, the rate of increment for output ought to be more prominent than the rate of increment for inputs. On the other hand, if the two are diminishing, the rate of abatement for outputs has to be lower than the rate of reduction for inputs. Another approach to accomplish higher efficiency is to present innovative technologies like IT or to reengineer business process-lean administrationwhich may decrease sources of inputs or increase the capacity to deliver more outputs (Adusei, 2016).

Data Envelopment Analysis (DEA) models can create new other options to enhance efficiency compared with different techniques. Linear programming is the backbone procedure that is based on enhancement platforms. Henceforth, what separates the DEA from different techniques is that it distinguishes the ideal methods for performance as opposed to the mean.

Distinguishing proof of ideal performance prompts benchmarking in a normative way. Utilizing DEA, bank managers can recognize top performers, as well as find alternative approaches to goad banking firms into getting to be extraordinary compared to other banks. Since the fundamental work of Charnes, Cooper, \& Rhodes (1978), DEA has been responsible for endless research literature within both the non-benefit and revenue driven sectors . Not long ago, the utilization of DEA within the banking sectors has been restricted to conferences and journals. Thus, bank managers have not accepted DEA as a standard instrument for benchmarking and decision making. This is attributed to complications in formulation and the disappointment of DEA experts to adequately close the theory gap. DEA is a linear approach
for distinguishing performance or its parts by considering various assets that are utilized to accomplish output or results in the banking sector (Peter Wanke et al., 2015).

These assessments can be done not just at the firm's level, yet additionally in subunits, for example, departmental comparisons, where numerous areas of improvement in saving specific input assets or techniques to expand the output can be recognized. In short, according to

Ascarya et al. (2008), DEA can enable bank directors to achieve the following:

- Review their banks relative efficiency.
- Recognize top performance in firms advertise, and
- Recognize approaches to enhance their efficiency, if their bank is not one of the best performing banks.


### 2.1.2 The Data Structures of DEA

Data Envelopment Analysis determines productive and economic performance of a set $j=1,2$, ... , n observed DMUs and in our case banks in Ghana. These observations transform a vectors of $i=1,2, \ldots, m$ inputs $x \in \mathbb{R}^{m}++$ into a vector of $i=1,2, \ldots, s$ outputs $y \in \mathbb{R}_{+}^{s}+$ using the technology represented by the following constant returns to scale (CRS) proposed by Charnes et al., (1978) productivity possibilities set: $P_{C R S}=\{(x, y) \mid \mathrm{x} \geq \mathrm{X} \lambda, \mathrm{y} \leq \mathrm{Y} \lambda, \lambda \geq$ $0\}$, where $X=(x)_{j} \in \mathbb{R}^{s X n}, Y=(y)_{j} \in \mathbb{R}^{m X n_{++}}$and $\lambda=\left(\lambda_{1}, \ldots, \lambda_{n}\right)$ is a semi-positive vector. Data are considered as regular vectors and matrices, which makes up the inputs of the approximation functions. The set of the approximation functions produce a structure that contains field with the approximation output as well as the input of the approximation function (Álvarez et al., 2016).

We can directly access the field and the complete structure can also serve as an input to other functions that output results. According to Álvarez et al. (2016), the following are some of the filed structures:

- X, Y and Yu: contain the inputs, outputs and undesirable outputs variables, respectively.
- n and neval: number of DMUs, and number of evaluated DMUs.
- $\mathrm{m}, \mathrm{s}$ and r : number of inputs, outputs and undesirable outputs.
- model, orient, rts: strings containing the model type, the orientation, and the returns to scale assumption.
- eff: computed efficiency measure.
- slackX, slackY, slackYu: computed input, output and undesirable output slacks.
- names: names of the DMUs.


### 2.1.3 Data Envelopment Analysis (DEA) Algorithm

The traditional strategy for applying Data Envelopment Analysis (DEA) to dataset is repeatedly solving as many Linear Programmes (LPs) as there are entities (Dulá, 2011). The measurements of these LPs are typically dictated by the amount of dataset, and they retain their magnitudes as each decision-making unit is scored. This approach can be computationally difficult, particularly with huge dataset. To avoid this, Dulá (2011) proposed an algorithm based on a two-stage phase procedure where the first stage finds the extreme efficient entities, the frame of the production possibility set. The frame is then used in the second stage to score the rest of the entities. The new technique applies to any of the four standard DEA returns to scale. It likewise confers adaptability to a DEA study on the grounds that it puts off the choice about introduction, benchmarking estimations, and so forth, until after the frame has been identified. Comprehensive computational testing on large data sets confirms and authenticates the algorithm that it is computationally efficient.

Algorithm 2.1 : Two-Phase Algorithm BuildHull
[Phase 1: Frame Identification]
Input. $\mathcal{A} \quad \geq$. The DEA data set. I.
Output. $\mathcal{F}: \quad \backslash$. The Frame of the DEA data set..$\backslash$.
Step 0. Initialization.

1. $\boldsymbol{R} \leftarrow \mathcal{A}, \boldsymbol{l} \leftarrow 1 \backslash \boldsymbol{R}$ is a temporary work-space array. $\backslash$.
2. Find one generator from $\boldsymbol{R}$ that is an extreme element of the DEA hull; remove it from $\boldsymbol{\Re}$ and place

Step 1. Iteration. While $\boldsymbol{R} \neq \emptyset$ Do:

1. Select some $\boldsymbol{\alpha} \in \boldsymbol{R}$.
2. Set $\boldsymbol{b} \leftarrow \boldsymbol{\alpha}^{\prime}$ and solve $\mathrm{LPs} \mathrm{P}^{\mathrm{CRS} / \mathrm{D}} \boldsymbol{\iota}^{\mathrm{CRS}}$.
$\left(\pi^{*}, \beta^{*}\right) \quad \leftarrow \quad$ Optimal dual solution.
3. If $\left(\pi^{*}, b\right)+\beta^{*}>0$ Then:
a. Find $a^{*} \in \boldsymbol{\Re}$ such that:

$$
\alpha^{*}=\arg \max _{\{\mathrm{aj}} \in \boldsymbol{R} \mid\left(\boldsymbol{\pi}^{*}, a j\right)+\beta^{* *}>0 ;\left(\boldsymbol{\pi}^{*}, \mathrm{a}_{\mathrm{j}}\right)+\beta^{*} .
$$

In the CRS case use Result 4B to find $a *$.In case of a tie, set $a^{*}$ to one extreme point among generators in support set.
b. $\boldsymbol{R} — \boldsymbol{R} \backslash \mathrm{a}^{*} ; \boldsymbol{\mathcal { A }}^{\boldsymbol{t}} \leftarrow a^{*}$.
c. $l-l+1$.

Else $\left(\pi^{*}, \mathrm{~b}\right)+\beta^{*} \leq 0$ :
a. $\boldsymbol{R}-\boldsymbol{\Re} \backslash \mathrm{a}^{*}$.

End Do. Step 2. Conclusion. $\mathcal{F} \leftarrow \mathcal{A}^{\boldsymbol{L}}$. End Phase 1.
[Phase 2. Score Dmus.]
Classify and score all points in $\boldsymbol{\mathcal { A }} \backslash \boldsymbol{\mathcal { F }}$ using appropriate DEA LP.
End.

Mehrabiana (2013) also proposed another DEA algorithm for classification of DMUs efficient and inefficient units. Mehrabiana (2013) algorithm depended on non -Archimedean Charnes-Cooper-Rhodes (CCR) framework. The model also applies affirmation value for the non Archimedean utilizing just basic calculations on inputs and outputs of DMUs.

The merging and effectiveness of the new algorithm demonstrate the benefit of this algorithm contrasted with the Thrall's algorithm.

### 2.1.4 Advantages and Disadvantages of DEA

### 2.1.4.1 Advantages of DEA

DEA is a model for analysis which obliges an exhaustive perspective of a firm's efficiency. It is a good and standard tool for assessing the efficiency of a firm. This is partly because of the way
that a large number of subjective elements influence the quality and efficiency of the provision that should be managed well. In spite of the fact that there is no reasonable connection between inputs (expended) and output (created) in DEA, Ashoor (2012) recognized the following:

- Each DMU can be described exclusively.
- Inefficient DMUs are enhanced by projecting them on the efficient frontier (envelopment).
- DMU encourages influencing inductions for each DMU among the surmising on the DMUs' general to profile.
- Different input and numerous outputs can be dealt with in different DMUs estimations.
- An attention on a practice frontier, rather than on focal propensities.
- No confinements are forced on the useful frame relating inputs to outputs
- A focus on a best-practice frontier, instead of on central-tendencies


### 2.1.4.2 Disadvantages of DEA

Although conventional DEA models are viewed as a ground-breaking apparatus for productivity appraisal, numerous impediments have been distinguished:

- As DEA is an extraordinary point strategy, it is exceptionally sensitive to noise (notwithstanding for symmetrical noise with a zero mean) that may cause critical mistakes in effectiveness estimations.
- Statistical hypothesis tests are troublesome in light of the fact that DEA is a nonparametric.
- The standard definition of DEA depends on isolated linear programs for each DMU, which is computationally difficult.
- Cold-heartedness to impalpable and downright parts (e.g. benefit quality in a bank office).
- Troubles in accumulating diverse parts of effectiveness particularly at whatever point DMUs performs multiple activities.
- Cold-heartedness to impalpable and downright parts (e.g. benefit quality in bank offices).
- There are crucial problems related to mixing multiple dimensions in the analyses
- There are critical issues identified with blending numerous dimensions in the analyses.
- It is difficult to rank effective units totally on the grounds that all efficient DMUs should have efficiency score of $100 \%$ score.
- There is no precisely vigorous methodology for assessing or testing the suitability of a set of influences in an efficiency study.


### 2.1.5 Application of Data Envelopment Analysis (DEA)

Data envelopment analysis (DEA) is a non-parametric technique that creates a relative ratio of weighted outputs to inputs for each decision making unit i.e. a relative efficiency score (Avkiran, 2006). DEA attempts to address some of the explicit flaws of the growth accounting approach (LaPlante \& Paradi, 2014). By enveloping the observed input-output combinations , DEA attains an estimate of the production frontier and uses it to detect the role of technological revolution productivity growth (Kılıçaslan et al., 2017). Data envelopment analysis is one of the topmost techniques used in efficiency measurement of firms (Titko et al., 2014). DEA has so many application areas but according to Paradi, Vela, \& Zhu, (2010), bank branch studies have been and will continue to be one of the most predominant application areas of DEA. Current DEA application studies in banks include that of Sharif et al. (2019); Silva et al. (2018); Stewart et al. (2015); Vidyarthi (2018); Wang et al. (2019); Wang et al. (2018); Zha et al. (2016) and Zhou et al. (2018).

As noted by LaPlante \& Paradi (2014), little attention have been given to assessing the growth potential of individual bank branches. Based on this, the authors presented five different models for assessing the efficiencies of branch network of one of Canada's top five
banks using DEA. The study suggested that two of the proposed models were able to successfully identified best performing branches using their efficiency scores. Avkiran (2014) applied a Dynamic Network Data Envelopment Analysis (DN- DEA) using a commercial bank as a case study with emphasis on testing the banks heftiness. The author used 16 external banks in China as benchmarked against 32 local banks for the post-2007 era that follows major reforms. The results of the study which was an illustrative one suggested there was no statistical
significant difference between the Chinese local and foreign banks performance based on mean overall efficiency estimates. They anticipated the need for progressively more sophisticated analysis tools such as DN-DEA in order to explore extensively bank performance.

Fukuyama \& Weber (2014) measured the Japanese bank performance using a dynamic Network DEA approach and suggested that for a 3 year dynamic window, inefficiency in banks ranges from $19.5 \%$ of average outputs and inputs in 2007-2009 to $21.5 \%$ of average outputs and inputs in 2008-2010. The authors indicated that a lot of banks in the data sample can improve their efficiency by collecting more deposits from customers in the first stage and then using the deposits to generate a larger collection of loans and securities in the second stage of operation (Fukuyama \& Weber, 2014).

Similar to Fukuyama \& Weber (2014), Duygun et al. (2015) also used a Network Data Envelopment Analysis approach which consisted of two stages to assess the efficiency of various European Airlines and suggested that most of the inefficiencies were generated during the first stage of the analysis. Kaffash et al. (2017) in a study proposed a new version of the modified semi-oriented radial DEA measure. To illustrate their proposed model they employed two widely used selections of inputs and outputs to estimate the efficiency scores for a sample of banks operating in Persian Gulf Council Countries (GCC) over the period of 2002-2011. The finding shows that banks operating in environment with a relatively lower risk of a banking crisis were more efficient (Kaffash et al., 2017). Empirical results by(Muhammad et al., 2018) also revealed a mix trend among Saudi banks 2008-2016 in achieving technical, pure technical and scale efficiency. As the Banking industry plays a critical role in the economic development of a country (Hamid et al., 2017). Hamid et al. (2017) applied DEA to measure the efficiency of the banking sector in the presence of Nonperforming Loan in Malaysia and suggested that, modelling the efficiency in the absence of objectionable outputs can give misleading results and biased assessment. Hamid et al. (2017) compared both results between the domestic and
foreign banks and suggested that the DEA technical efficiency score for domestic banks was slightly greater than the foreign banks.

Havidz \& Setiawan (2015) investigated the efficiency of Indonesian Islamic Banks by employing Data Envelopment Analysis (DEA) approach. The authors' data covered the periods of January 2008 - September 2014 which was based on the quarterly-published report from Indonesian Central Bank. The findings of their study show that none of the banks was consistently efficient for all periods of research by Overall Technical Efficiency (OTE), Pure Technical Efficiency (PTE), and Scale Efficiency (SE). The overall results also suggest that efficiency of Banks was significantly affected by Return On Assets (ROA), Operational Efficiency Ratio (OER), and Inflation Rates (INF), while Financing to Deposit Ratio (FDR), Capital Adequacy Ratio (CAR), size, and GDP growth rate have insignificant effect on bank efficiency. Titko et al. (2014) contributed to the existing analytical data on bank performance in Latvia by applying DEA to measure the efficiency of selected banks in Latvia. Based on their study, the authors' developed fourteen alternative model specifications by using the results of earlier conducted correlation analysis.

The growing investments and application of IT in organizations using various models, techniques and methods to achieve competitive advantage has generated the IT impact productivity debate popular called "Productivity Paradox"(Han et al., 2011). Several scholars, Brynjoifsson (1996); Brynjolfsson (1993) and Ko \& Osei-Bryson (2004) in an attempt to address the measurement issue of IT impact on productivity either used only DEA ,Cao \& Yang (2013); Chen et al. (2006); Chen \& Zhu (2004); Madjid et al. (2009); Sigala 2003 and Wang et al. (1997) or combined it with other methods, (Ko \& Osei-Bryson, 2006; Ngai et al., 2009; Shao \& Lin, 2001; Wu, 2009). For example, Paço \& Pèrez (2015) attempted to evaluate the impact of ICT on the productivity of hotels in Portugal through Data Envelopment Analysis (DEA). The study did not only demonstrated how important ICT is in realizing advanced levels of productivity but also
discussed other explicit concerns which should be taken into consideration so that the positive returns of the investment in ICT can be achieved. Paço \& Pèrez (2015) found that the accessibility of ICT does not alone lead to optimal performance. Using data from Iranian manufacturing businesses during 2002-2006, Abri \& Mahmoudzadeh (2014) applied the method DEA to study the subject matter. Results show that IT has positive and significant impact on the productivity of manufacturing companies. This positive impact would be experienced in high IT-intensive businesses more than the others. Wang et al. (1997) through their DEA model and methodology also evaluated the marginal benefits of IT using 36

DMUs financial institutions data. The study suggested that for a collection of IT investment, IT impacted substantially on organizations revenues but they were quick to contest their own results due to their beliefs that there was no relapse between IT investment and profits to performance and that IT was exclusively used in stage one . Using twenty Iran, conventional power plants data, Madjid et al. (2009) assessed the IT impact on productivity in conventional power plants and suggested a DEA model that permits the incorporation of production performance and investment performance.

Chen \& Zhu (2004) also used DEA model on a 27 banks. They considered IT investment, fixed asset and number of employees as inputs in stage I that can be utilized to collect funds in a form of deposits from customers. The outputs of this two-stage DEA were also profit and fractions of loans recovered from customers. Chen et al. (2006) used 27 DMUs of banks suggested only three firms units as efficient in the two efficient calculation phases. Their results suggest that the bank's investment in IT, assets and employee should be assigned to only one specific stage. Chen et al. (2006) used a two-stage DEA model with three inputs (IT investment ,fixed assets and number of Employees) and overall output as profit generated from investing the deposit from customers in securities and also given out loans to customers and fractions of loans recovered as was done by (Chen \& Zhu, 2004 and Wang et al., 1997).

Finally, Sheng et al.,(2002), also employed DEA to propose a framework that can be used to measure the productivity of organizational IT investment.

### 2.2 OVERVIEW OF MACHINE LEARNING ALGORITHMS

### 2.2.1 Background

In order to deal with a task on a computer, we must have a series of commands that should be followed to process the data to information popularly known as computer algorithm. For example, one can design a bubble sort algorithm for sorting numbers. The numbers in this case are going to be the inputs while the preferred ordered list would be the output. There can be many different algorithms to be designed to do the same work but computer scientist would be much interested in efficient algorithms that use less memory and space. On the other hand, there are situations that do not follow normal algorithm design. For instance, to sort email messages into authentic and spam, the input is an email text that in the simplest form is a file of characters.

The output is either YES/NO to specify whether the message is spam or authentic. Transforming input to output in such scenarios is very difficult because, what would be spam in email varies from user to other. So to deal with such situations, one can use the data available by simply compiling large volumes of emails dataset where we know some of these dataset to be spam easily. We can therefore "learn" the characteristics or feature that makes an email spam from the spam sample in the dataset. Thus, the computer called "machine" would be used to automatically extract its own algorithms (model) for this kind of task. According to Omary \& Mtenzi (2010) computer is a machine for executing or aiding calculation; it receives data, process them and output information based on an algorithm on how the data has to be process the data.

Whenever there is design of an algorithm for a computer task such as sorting, there is no need to spend time to lean sorting of numbers (Alpaydın, 2010). Nevertheless there are many other applications that designing an algorithm for all would be difficult or not even possible. In this
case, we can construct a good and important approximation even though this explanation may not take care of all the data but one can be sure of accounting for some part of the dataset. This is achieved by detecting certain patterns or characteristics and using same for new dataset .This method or process is termed as Machine Learning and has so many applications retail, finance, manufacturing, medicine, telecommunication, agriculture, sports, etc.

When Machine Learning is applied to large dataset, it becomes data mining but is not just about database; but also part of artificial intelligence (Alpaydın, 2010). Machine learning is ,therefore ,about programming computers (Machines) to optimize a performance criterion using example data or past experience (Alpaydı, 2010). We develop a model well-defined to some few parameters, and learning is the operation of a computer programme to advance the parameters of the model utilizing the training data or past experience. The model can be predictive to make predictions in the future, or descriptive to gain knowledge from data, or both. Machine learning utilizes the hypothesis of statistics in building mathematical models, , in light of the fact that the core mandate is making induction from dataset sample (Alpaydın, 2010).

### 2.2.2 Types of Machine Learning

According to Shalev-Shwartz \& Ben-david (2014), there are three (3) different types of Machine Learning discussed as follows:

- Supervised Learning: As the name implies, this type of machine learning algorithm is done under the supervision or assistance of a supervisor or teacher (Alpaydın, 2010). Thus, the entire learning process depends on the teacher or under the care of the teacher.

The algorithm is made up of dataset which contains both response and predictor variables (Smola \& Vishwanathan, 2008). We, therefore, build a function that processes the inputs to our desired output. There is a training process that proceeds until the point when the model accomplishes a coveted level of exactness on the training data which is normally percentage of the entire dataset. Examples are Regression, Decision Tree, Random Forest,

KNN, Logistic Regression, Neural Network etc. (Breiman, 2001a; Mashat et al., 2012; Omary \& Mtenzi, 2010 and Scornet, 2010).

- Unsupervised Learning: Unlike the previous supervised algorithms, these types do not have or contain any response or target variable of interest to predict (Alpaydın, 2010). This type of learning is independent and without any supervisor or teacher (Chao, 2011). It is normally used for clustering a sample population in different categories with the purpose of segmenting for a particular intervention (Braaten, 2010; Chang et al., 1996; Cutler et al., 2011; Krishnapuram et al., 2005; Liaw \& Wiener, 2002; Navot, 2006). Examples of such algorithms include Apriori algorithm and K-means clustering algorithm.
- Reinforcement Learning: As its names suggest, it is used to fortify, reinforce or better the function in order to generate the desired results (Bradtke \& Duff, 1995 and Chao, 2011). Thus, the machine (computer) is trained to make specific decisions. It is similar to supervised learning but this one lacks certain information about the dataset (Alpaydın, 2010). The machine normally trains itself repeatedly with little help using trial and error mechanism Shalev-Shwartz and Ben-david (2014) and typical example is the Markov Decision Process (Bradtke \& Duff, 1995).


### 2.2.3 Topmost Machine Learning Algorithms

There are so many Machine Learning Algorithms which can be applied to so many dataset problems in areas such as health, engineering, mathematics, business etc. but Le (2018) listed the following as the most topmost and most commonly used Machine Learning

Algorithms:

1. Linear Regression
2. Logistic Regression
3. Decision Tree
4. SVM
5. Naive Bayes
6. KNN
7. K-Means
8. Random Forest

9. Dimensionality Reduction Algorithms
10. Gradient Boosting algorithms (GBM XGBoost, LightGBM,CatBoost)

Wu et al. (2008) presented a list of top 10 data mining algorithms identified by the IEEE International Conference on Data Mining: C4.5, K-Means, SVM, Apriori, EM, PageRank, AdaBoost, KNN, Naive Bayes, and CART. According to the Wu et al. (2008) these top 10 algorithms are amongst the best powerful data mining algorithms in the scientific research environment. These 10 algorithms cover classification, clustering, statistical learning, association analysis, and link mining, which are all among the most important topics in data mining research and development (Wu et al., 2008).

For the purpose of this study, the focus would be on Decision Tree (C5.0), Random Forest (Breiman and Cutler algorithm) and Neural Network (Back propagation algorithm) which are among the topmost algorithms used in Data Science.

### 2.2.3.1 Logistic Regression

Logistic regression is one of the supervised Machine Learning classification algorithm used to assign observations to a discrete set of classes (Anouze, 2019; Lim \& Yu-Shan, 2000; Ream \& Rumberger, 2008 and Wickramasinghe \& Karunasekara, 2016). Examples of such classification problems are Email spam or not spam online transactions Fraud or not Fraud, Tumor Malignant or Benign. Logistic regression converts its output using the logistic sigmoid function (Sigmoid function) illustrated in the figure below to return a probability value.


Figure 2.1: A Graph of a Logistic Regression Sigmoid Function (Wickramas inghe \& Karunasekara, 2016)

It is a predictive analysis algorithm and based on the concept of probability (Mousavi et al., 2019)y. The Sigmoid function is an S-shaped curve that can take any real-valued number and map it into a value between 0 and 1 , but never exactly at those limits.

$$
1 /\left(1+e^{\wedge} \text {-value }\right)
$$

Where e is the base of the natural logarithm s (Euler's number or the $\operatorname{EXP}()$ function in your spreadsheet) and value is the actual numerical value that you want to transform.

## Types of Logistic Regression

Usually, logistic regression means binary logistic regression having binary target variables, but there can be a scenario where two or more groupings of target variables that can be predicted by it. Based on those numbers of groupings, Logistic regression can be divided into following;

- Binary or Binomial: With this type of classification, a dependent variable will have only two possible types either 1 or 0 . For example, these variables may represent success or failure, yes or no, win or loss etc.
- Multinomial: For multinomial classification, dependent variable can have 3 or more possible unordered types or the types having no quantitative significance. For example, these variables may represent "Type A" or "Type B" or "Type C".
- Ordinal: With this type of classification, dependent variable can have 3 or more possible ordered types or the types having a quantitative significance. For example, these variables may represent "poor" or "good", "very good", "Excellent" and each category can have the scores like $0,1,2,3$.


## Representation Used for Logistic Regression

Logistic regression uses an equation as the illustration similar to linear regression. Input values
(x) are combined linearly using weights or coefficient values (Beta) to predict an output value
(y) (Zhiyu, 2016). A key difference from linear regression is that the output value being modeled is a binary value ( 0 or 1 ) rather than a numeric value. Below is an example logistic regression equation:

$$
\begin{equation*}
y=e^{\wedge}(b 0+b 1 * x) /\left(1+e^{\wedge}(b 0+b 1 * x)\right) \tag{1}
\end{equation*}
$$

Where y is the predicted output, b 0 is the bias or intercept term and b 1 is the coefficient for the single input value ( x ). Each column in your input data has an associated b coefficient (a constant real value) that must be learned from your training data. The actual representation of the model that you would store in memory or in a file is the coefficients in the equation (the beta value or b's).

## Learning the Logistic Regression Model

The coefficients (Beta values b) of the logistic regression algorithm must be protected from your training data. This is done using Maximum-likelihood estimation. The Maximumlikelihood estimation is a common learning algorithm that makes assumptions about the distribution of your data (Kaitlin et al., 2018). The best coefficients would result in a model that would predict a value very close to 1 (e.g. female) for the default class and a value very
close to 0 (e.g. male) for the other class. The intuition for maximum -likelihood for logistic regression is that a search procedure seeks values for the coeff icients (Beta values) that minimize the error in the probabilities predicted by the model to those in the data (e.g. probability of 1 if the data is the primary class).

### 2.2.3.2 Decision Tree

A decision tree is a graphical framework representing decisi ons and their potential outcomes as shown in the Figure 2.1 (Shalev-Shwartz \& Ben-david, 2014). It basically consists of three categories of nodes as discussed below.


Figure 2.2: A Classic Example of Decision Tree (Shalev-Shwartz \& Ben-david, 2014)

1. Decision node: These are normally denoted by squares describing decisions that can be suggested. Lines springing from a square indicate all different preferences offered at a node (Jansson, 2016).
2. Chance node: This also is symbolized by circles displaying chance outcomes. Chance outcomes are events that can happen but are outside the capability of the decision maker to control (Alpaydın, 2010).
3. Terminal node: This last type of node is represented by triangles or by lines having no further decision nodes or chance nodes. Terminal nodes depict the final outcomes of the decision making process (Steinke \& Etten, 2017).

Decision Trees are commonly used for classification and regression tree analysis. Decision Trees are becoming increasingly more popular for data mining because they are easy to understand and interpret, require little data preparation, handle numerical and categorical data, and they perform very well with a large data set in a short time ( Tso \& Yau, 2007) . Decision trees produce excellent visualizations of results and their relationships. Although there are many specific Decision Tree Algorithms, the ID3, C4.5, C5.0, C\&RT, and CHAID and QUEST algorithms are the most commonly used ones (Delen et al., 2013). For this study the C5.0 DT algorithm proposed by Ross (1993) was adopted as it offers numerous improvements on C4.5. This C5.0 Decision Tree algorithm has the following features which make it different from other DT algorithms like the ID3, CART and even its immediate successor C4.5 (Delen et al., 2013).

- It normally suggests a dualistic (binary) tree or multi-branches tree
- It also uses Information Gain Entropy, as its splitting principles.
- C5.0 pruning method employs the Binomial Confidence Limit method.
- In an instance of treating omitted values, the C5.0 algorithm agrees whether to approximate the Not Available (NA) values as a function of other attributes or allocates the case statistically among the results.
- The tree produces C5.0 algorithms and its and rulesets are typically minor than the tree that C4.5 will produce.
- In terms of Boosting while the stochastic gradient boosting machines deviates from the exiting adaboost algorithm, C5.0 will do approximately analogous to adaboost. After generating the first tree, weights are calculated and consequent iterations create weighted trees or rulesets. The Subsequent trees (or rulesets) are controlled to be about the same size
as the initial model. The final prediction is a simple average of class probabilities created from each tree or ruleset (i.e. no stage weights).
- For Winnowing- A feature choosing step done before the model is built. The data set is divided (split) randomly into half and an initial model is fit. Each predictor is removed in turn and the effect on model performance is determined (using the other half of the random split). Predictors are flagged if their removal does not increase the error rate. The final model is fit to all of the training set samples using only the unflagged predictors. R has a library that contains caret function train that has attachments to C 5.0 which can tune over the model type, winnowing and even its boosting


## Other Features of C5.0

- It is allowed to vary the confidence factor for pruning and also possible to attune the least number of cases in a terminal node
- Turning on possible global pruning algorithm is also permitted C 5.0 can avoid boosting if it's considered to be unproductive.
- It is also possible to assign unequal costs to precise errors types
- It is more memory efficient faster than its predecessor, C4.5
- It produces analogous outcomes to that of C 4.5 with significantly lesser DT
- It's possible to weight distinct cases and misclassification types. Boosting in C5.0

As a method to reduce errors in the predictions made by the Decision Trees, a boosting algorithm can be implemented (Jansson, 2016). According to Jansson (2016), the fundamental idea of boosting algorithms is to achieve the following:

- Choose and implement a classifier.
- Split the dataset into a training set (of size $n$ ) and a validation set (of size $m$ ). The samples in these sets are chosen randomly in the first iteration. Note that both sets need to contain
samples from all classes.
- Train the classifier using the training set. This classifier is said to be a weak classifier. Test the classifier by classifying the testing set and create weights to indicate the flaws in the weak classifier.
- Repeat steps 2 and 3, for a given number of iterations, storing each weak classifier constructed this way. Note that at each iteration, the selection of samples is reflected by the assigned weights.
- Combine all weak classifiers into one strong classifier.

```
Algorithm 2.2 AdaBoost for binary classification
    1.
    procedure ADABOOST
    2.
    \(\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{y}_{\mathbf{1}}\right), \ldots \ldots,\left(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{y}_{\boldsymbol{n}}\right) \leftarrow \mathrm{n}\) observations, where \(\boldsymbol{x}_{\boldsymbol{j}}\), is a vector of attributes
    and \(\boldsymbol{y}_{\boldsymbol{j}}\), is the class.
    3. \(\boldsymbol{w}_{\boldsymbol{t}}(\boldsymbol{i}) \leftarrow\) weighting function. Initialized as \(1 / \boldsymbol{n}\) for \(\mathrm{i}=1, \ldots, \boldsymbol{n}\).
    4. \(\quad \mathrm{T} \leftarrow\) number of trials
    5. for \(\mathrm{t} \leftarrow 1\) to T do
    6.
    \(\boldsymbol{f}_{t}\left(\boldsymbol{x}_{\boldsymbol{i}}\right) \leftarrow\) weak classifier
        \(\epsilon_{\mathrm{t}}=\sum_{y_{i \neq f} f_{t}\left(x_{i}\right) W_{t}(i)}\)
8. \(\quad \alpha_{\mathrm{t}}=-\frac{1}{\log }\left(\frac{1-\epsilon_{l \mathbf{l}}}{}\right)_{2 \in t}\)
9. \(\quad\) for \(\mathrm{i} \leftarrow 1\) to \(\boldsymbol{n}\) do
    \(\boldsymbol{w}_{t+1}(\boldsymbol{i})=\boldsymbol{w}_{t} \boldsymbol{e}^{-\alpha l_{t}} \leftarrow\) if correctly classified
    11. \(\boldsymbol{w}_{t+1}(\boldsymbol{i})=\boldsymbol{w}_{t} \boldsymbol{e}^{\alpha_{l}} \leftarrow\) if incorrectly classified
    12.
    end
    13. \(w_{t}(i)=\Sigma ـ \quad n i=w_{1 t w}(i)(i) \leftarrow\) normalize \(\boldsymbol{w}_{t}\)
    14.
        end
15.
    \(\mathrm{F}(\mathrm{x})=\operatorname{sign}\left(\sum^{T}{ }_{t=1} \boldsymbol{\alpha}_{t} \boldsymbol{f}_{t}(\boldsymbol{x})\right) \leftarrow\) the final classifier
    16. end procedure
```

In C5.0 the particular boosting algorithm implemented is based on the idea of Adaptive Boosting or AdaBoost for short. This work made great impact in the field of machine learning due to the ability of combining several weak classifiers into a stronger one, while retaining the robustness to over-fitting of the weak classifiers.

The main idea of adaptive boosting is to weigh the data points in each successive boosting iteration during the construction of a classifier. These weights for each sample in the training data are distributed such that the algorithm would be focused to correctly classify the data points which were misclassified by the previous classifiers. The differences between the algorithm used in C5.0 and AdaBoost are as follows according to (Jansson, 2016):

- C5.0 tries to maintain a tree size similar to the initial one (which is generated without boosting taken into account). This is correlated with the amount of terminal nodes, which increase in number as the tree grows.
- C5.0 calculates class probabilities for all boosted models and within these models, weighted averages are calculated. Then, from these models, C5.0 chooses the class which has the maximum probability within the group.
- The boosting procedure ends if $\sum_{\left\{i: y i \neq F_{t}\left(x_{i}\right)\right\}} \omega_{t}(i)<0.1$ OR

(1)


## $\left|W_{m}\right|$

Where $W_{m i s}$ the cardinality of the set of weights associated with misclassified observations. The C5.0 weighting algorithm

```
Algorithm 2.3: The weighting procedure in C5.0 Algorithm
```


## 1. procedure

2. $\quad \mathrm{N} \leftarrow$ Number of samples in training set.
3. $\quad \boldsymbol{N}_{-} \leftarrow$ Number of misclassified samples
4. $\quad \mathrm{T} \leftarrow$ number of boosting iteration.
5. $\quad \boldsymbol{w}_{i, t} \leftarrow$ Weight of the i-th sample during t-th round boosting.
6. $\quad \boldsymbol{S}_{+} \leftarrow$ Sum over all weights associated with correctly classified samples
7. $\boldsymbol{S}_{-} \leftarrow$ Sum over all weights associated with correctly misclassified samples
8. $\quad$ for $\mathrm{t} \leftarrow 1$ to $\boldsymbol{T}$ do
9. 
10. 
11. 
12. 
13. 

$\boldsymbol{w}_{i, t}=\overline{\boldsymbol{W}_{i, t-1} S_{+- \text {midpoint }}-} \leftarrow$ weight if misclassified
14.
end 15. end 16. end procedure

The Decision Tree is a one of the topmost machine learning algorithms which suggests a graphical or diagrammatical illustration of a technique for classifying, predicting and evaluating an item of importance or concern (Jain et al., 2016a). It is an easy and commonly used classification method. It deals with decision analysis by employing a tree-like structure of decisions and their relative potential outcomes (Hu \& Wang, Shuaiwei 2016).It has nodes where at each node in a Decision Tree an attribute must be selected to divide the node's instances into subgroups. They are also a type of supervised learning method which splits dataset into more standardized clusters as possible from the variable to be predicted. Decision Tree accepts input set of well-ordered data, and output shaft is delivered in which each end node (leaf) is a decision (a class) and each non-end node (middle) shows a test (Hssina et al., 2016). They are normally used for acquiring facts to aid in decision making. It normally begins with a root node for the users to take the necessary actions then from the node the user riven individual node recursively based on an adopted DT algorithm .The end product is a tree with each branch denoting a potential scenario of the decision and its conclusion (Ogunde \& Ajibade, 2014).

The most commonly used algorithms in DT are: ID3, CART, CHAID and C4.5 with its extension C5.0. For this study, the C4.5 Algorithm which is an extension of the ID3 proposed by Ross Quinlan in 1994 (Pandya \& Pandya, 2015) was adopted and implemented in R studio using R codes with package "RWeka" (Hornik et al., 2019).

### 2.2.3.3 Random Forest

Random Forests (RF), on the other hand, are an algorithm for classification proposed by Breiman in 2001 and cited by Chi et al.(2011) and Cutler et al.(2011) that utilize an ensemble of classification trees. Random Forest is a supple, not complex to use Machine Learning Algorithm that creates, even without hyper-parameter tuning, a good result (Cutler, 2010). It happens to be one of the topmost and most used algorithms, because it is simple and can also be applied in both classification and regression tasks (Le, 2018). It is also one of the supervised Machine Learning Algorithm. As the name implies, it produces a forest and makes it random, "Forest". This makes it a combination of tress and most at time trained with the "bagging" method (Breiman, 2001a; Chao, 2011; Liaw \& Wiener, 2002; Scornet, 2010). The main idea behind this bagging method is that, a combination of learning models will increase the overall result (Breiman, 2001a; Chao, 2011) as shown in the Figure 2.2.


Figure 2.3: A Classic Example of Random Forest (Cutler et al., 2011)

Random Forests (RF) are algorithm for classification developed by Breiman in 2001 and cited by Chi et al. (2011) that utilize an ensemble of classification trees (Cutler et al., 2011; Cutler, 2010 and Gislason et al. (2006). RF is an ensemble machine learning algorithm (Cutler et al., 2011).

The fundamental principle of the RF algorithm is that constructing a smaller DT with limited characteristics is inexpensive process in terms of computation (Cutler et al., 2011). T hus, it is possible to construct numerous small, weak Decision Trees in parallel and merge these smaller trees to form one strong learner by using their mean performance or even or selecting the popular one. In terms of application and practicability, RF dgorithms are considered to be most precise learning algorithms to date (Cutler et al., 2011).

The RF algorithm adopted for this study was Leo Breiman and Adele Cutler Random Forest Algorithm Breiman (2001) and Cutler et al. (2011) illustrated in algorithm 2.7.This was implemented in R using R codes with "randomForest" package (Liaw \& Matthew, 2018 and

Tang, 2016). According to literature Random Forest has achieved a lot of good classifications and other prediction. A Random Forest model has a better ability in modeling and predicting. An important feature of Breiman's algorithm according to Livingston (2005) is the variable importance calculation. The success of the RF model is attributed to its generality ability to forecast the output for new data after the RF has been trained with a similar dataset. Literature also shows that RF as a Machine Learning Algorithm delivers higher classification and predictive accuracies than statistical procedures. Finally random forest algorithm utilizes the bagging method for building an ensemble of Decision Trees. Bagging is known to reduce the variance of the algorithm.

Random Forest can be used for any variable being it categorical target variable normally considered as "classification", or a continuous target variable, also called "regression". The same applies to the independent or predictor variables (Cutler et al., 2011).

Random Forest is a tree-based ensemble where each individual tree is subject to an assembly of random variables (Cutler et al., 2011). Thus, for instance, using a $p$-dimensional random vector $X=\left(X 1, \ldots, X_{p}\right)^{T}$ indicating the independent or predictor variables and a random variable $Y$ denoting target or dependent or response variable, one can suggest an unidentified combined distribution $P_{X Y}(X, Y)$. The purpose is to select a forecast function $f(X)$ for predicting $Y$. The prediction function is assessed by a loss function $L(Y, f(X))$ and formulated to reduce the anticipated value of the loss
$E_{X Y}(L(Y, f(X)))$
where the XY as subscripts represent expectancy with deference to the combined distribution of $X$ and $Y$.

Instinctively, $L(Y, f(X))$ is a quantity of how near $f(X)$ is to $Y$; it disciplines values of $f(X)$ that are far from $Y$. Archetypal selections of $L$ are squared error loss $L(Y, f(X))=(Y-f(X))^{2}$ for regression and zero-one loss for classification:
$L(Y, f(x))=I(Y \neq f(x))=\{0$ if $Y=f(x)$

It happens that reducing $E_{X Y}(L(Y, f(X)))$ for squared error loss gives the provisional anticipation $f(x)=E(Y \mid X=x)$

Else recognized as the regression function. In case of the classification, if the list of likely values of $Y$ is designated by $\mathcal{Y}$, reducing $E_{X Y}(L(Y, f(X)))$ for zero-one loss gives $f(x)=\arg ^{\max _{y \in y}} P(Y=y \mid X=x)$ popularly called the Bayes rule.

Ensembles build $f$ using an assembly of "base learners" $h_{1}(x), \ldots, h_{J}(x)$ and the base learners are joined to provide the "ensemble predictor" $f(x)$. With respect to regression, the mean of the base learners used

$$
\begin{equation*}
f(x)=\frac{1}{J} \sum_{j-1}^{J} h_{j}(x) \tag{6}
\end{equation*}
$$

Whereas in classification $f(x)$ is the class with the highest predictions (voting)
$f(x)=\arg ^{\max }{ }_{y \in y} \Sigma_{j-1}^{J} I\left(y=h_{j}(x)\right)$
The $j$ th base learner $j$ th in RF is a tree designated $h j(X, \odot j)$, where $\odot j$ is a assembly of random variables and the $\odot j$ 's are independent for $j=1, \ldots, J$.

### 2.2.3.4 Artificial Neural Network

Artificial Neural Network popular called ANN or Neural Network (NN) is a computational framework built on the architecture and also operates like the human biological neural networks (Lam, 2004). The flow of information through the network impact on the structure of the ANN because a neural network changes or learns, based on the input and output (Emrouznejad \& Shale, 2009; Gal, 2016; Ko, 2004; Krishnapuram et al., 2005). They are classified as nonlinear statistical data modeling tools where the complex relationships between inputs and outputs are modeled or patterns are found (Cutler et al., 2011 and Ngai et al., 2009). Artificial Neural Network (ANN) is a type of artificial intelligence technique that mimics the behavior of the
human brain (Lam, 2004). A Neural Network is a massively parallel distributed processor made up of simple processing units that have a natural tendency for storing experiential knowledge and making it available for us. ANNs can be grouped into two major categories: feed-forward and feedback (recurrent) networks. In the former network, no loops are formed by the network connections, while one or more loops may exist in the latter. The most commonly used family of feed-forward networks is a layered network in which neurons are organized into layers with connections strictly in one direction from one layer to another (Shamisi et al., 2011). The basic system of NN without the hidden layer consists of only two layers, the input and output layer. This normally called the skip layer because it is made up of a straightforward linear regression modeling in a ANN design .The input layer communicates directly with the output layer without involving the hidden layer. The figure 3.6 below shows a classical one layer NN with a hidden layer (Anantwar \& Shelke, 2012).


Inputs
Hidden
Outputs
Figure 2.4: Classical NN model (Shamisi et al., 2011)

## Multilayer Layer Perceptron (MLP)

Neural Networks consist of a large class of different architectures. In many cases, the issue is approximating a static nonlinear, mapping $f(x)$ with a neural network $f n N(x)$, where $x \in \mathbb{R}^{K}$ (Koivo, 2008). The most common and important ANN in function approximations are Multilayer Layer Perceptron (MLP) and Radial Basis Function (RBF) networks (Koivo, 2008; Mostafa, 2009; Shamisi et al., 2011). The MLP network is employed in the present study.

The MLP comprises an input layer with two or more hidden layers, and an output layer. Node i, also called a neuron, in a MLP network is shown in Figure 3.7 below. This also comprises of a summation $\left(\sum\right)$ and a nonlinear activation function $g$. It is important to note that NN where the hidden neurons have sigmoidal activation function and the output neurons the sigmoidal or identity function are called Multi-Layer Perceptron (Shamisi et al., 2011).


Figure 2.5 : Single node in a MLP network source (Koivo, 2008)
The inputs $x_{k}, k=1, \ldots, K$ to the neuron are multiplied by weights ${ }_{W k i}$ and summed up together with the constant bias term $\theta_{i}$. The resulting $n_{i}$ is the input to the activation function $g$. The activation function was initially selected to be a transmit fundon, but mathematical expediency a hyperbolic tangent (tanh) or a sigmoid function are most commonly used. The Hyperbolic tangent is formulated as:

$$
\begin{equation*}
\tanh (x)=\frac{1-e-x}{1+e}-x \tag{8}
\end{equation*}
$$

The output of node $i$ is given by,
$y_{i}=g_{i}=g\left(\sum^{K}{ }_{j=1} w_{j i x}+\theta i\right)$

Linking two or more nodes in parallel and also in series gives a MLP network with an example shown in the figure below. The same activation function $\quad g$ is used in both layers. The superscript of $n, \theta$ or $w$ refers to the layer, first or second. The output $y_{i, i} i=1,2$, of the MLP network becomes
$y_{i}=g\left(\sum_{3 j=1} w_{j i 2} g\left(n_{j 1}\right)+\theta_{j 2}\right)=g\left(\sum_{j 3=1} w_{j i 2} g\left(\sum_{k k=1} w_{k j 1} x_{k}+\theta_{j 1}\right)+\theta_{j 2}\right)$


Figure 2.6: A MLP network with one hidden layer source (Koivo, 2008)

From (3) we can conclude that a MLP network is a nonlinear parameterized map from input space $\mathbf{x} \boldsymbol{\epsilon} \mathbf{R}^{\boldsymbol{K}}$ to output space $\mathbf{y} \boldsymbol{\epsilon} \mathbf{R}^{m}$ (where $m=3$ in this case). The parameters are the weights $w_{j i}{ }^{k}$ and the $\operatorname{biases} \theta_{j}{ }^{k}$. Activation function $g$ is normally presumed to be equal in all layer and acknowledged in advance. In the figure the same activation function $g$ is used in all layers.

Given dataset $\left(x_{i}, y_{i}\right), i=1, \ldots, N$, calculating the best MLP network is expressed as a data fitting problem. The parameters to be determined are $\left(w_{j i^{k}}, \theta_{j}{ }^{k}\right)$. Building and processing of the ANN depends on the following building blocks.

Fixing the structure of MLP Network Topology: the number of hidden layers and neurons (nodes) in each layer.

Activation Functions: The activation functions for each layer are also chosen at this stage, that is, they are assumed to be known.

The unknown parameters to be estimated are the weights and biases, $\left(w_{j i^{k}}, \theta j^{k}\right)$.

There are many existing alg orithms for determining these network parameters but according to literature on Neural Network the algorithms are called learning or teaching algorithms
(Tsai et al., 2009). In system identification ,they belong to parameter estimation algorithms (West, 2000). The most well-known are back-propagation and Levenberg-Marquardt

Algorithms (Koivo, 2008).

Back-propagation is a gradient based algorithm, which has many variants (Shamisi et al., 2011). Levenberg-Marquardt is usually more efficient but needs more computer memory (Koivo, 2008).

### 2.2.3.5 The Back-Propagation Algorithm

A back propagation network is the most popular network because of its capability to find nonlinear solutions to undefined complex problems (Kriesel, 2005). The errors of the output of a back propagation network are propagated to back by means of the same connections used in feed forward mechanism by the derivation of the feed forward transfer function (Hinton, 2018). The learning function in this network is based on a basic two-way memory incorporation (Ekici \& Aksoy, 2009). The algorithm gives a prescription for changing the weights $w_{i j}$ in any feedforward network to learn a training set of input output pairs $\{x d, t d\}$.


Let us consider a simple two-layer MLP network shown Figure 2.5.

Figure 2.7: A two-layer MPL Network
Given the pattern $\mathrm{X}_{\mathrm{d}}$ the hidden unit j receives a net input,
$n e t_{j}{ }^{d}=\sum_{k=1}^{5} w_{j k} x_{k}{ }^{d}$
And gives out the output, $V_{j}^{d}=f\left(\right.$ net $\left._{j}{ }^{d}\right)=f\left(\sum_{k=1}^{5} w_{j k} x_{k}^{d}\right)$.
The output unit in this case receives
$n e t_{i}{ }^{d}=\sum_{j=1}^{3} W_{i j} V_{j}{ }^{d}=\sum_{j=1}^{3}\left(W_{i j} \cdot f\left(\sum_{k=1}^{5} w_{j k} x_{k}{ }^{d}\right)\right)$
and generate the final output
$O_{i}{ }^{d}=f\left(\right.$ net $\left._{j}{ }^{d}\right)=f\left(\sum_{j=1}^{3} W_{i j} V_{j}^{d}\right)=f\left(\sum_{j=1}^{3}\left(W_{i j} . f\left(\sum_{k=1}^{5} w_{j k} x_{k}{ }^{d}\right)\right)\right)$
The out usual error function is $\mathrm{E}\left[w^{\overrightarrow{ }}\right] \equiv \frac{1}{2} \sum_{d \in D}\left(t_{d}-O_{d}\right)^{2}$
For $i$ outputs and $m$ inputs pairs $\left\{\mathrm{X}_{\mathrm{d}}, \mathrm{t}_{\mathrm{d}}\right\}$
$\mathrm{E}\left[w^{\rightarrow}\right]^{=\frac{1}{2}} \sum_{d=1}^{m} \sum_{i=1}^{l}\left(t_{i}^{d}-O_{i}^{d}\right)^{2}$
Applying the example in this case, E will be formulated as
$\mathrm{E}\left[w^{\overrightarrow{ } \rightarrow}\right]^{=\frac{1}{2}} \sum_{d=1}^{m} \sum_{i=1}^{2}\left(t_{i}^{d}-O_{i}^{d}\right)^{2}$
$\mathrm{E}\left[w^{\rightarrow}\right\rceil^{=\frac{1}{2}} \sum_{d=1}^{m} \sum_{i=1}^{2}\left(t_{i}^{d}-f\left(\sum_{j}^{3} W_{i j} \cdot f\left(\sum_{k=1}^{5} w_{j k} x_{k}^{d}\right)\right)\right)^{2}$
If $\mathrm{E}\left[W^{\omega}\right]$ is differentiable given f is differentiable then gradient descent can be applied For hidden-to-out connections the gradient decent rule gives:
$\left.\Delta W_{i j}=-\eta \frac{\vartheta E}{}=-\eta \sum_{d=1}^{m}\left(t_{i}^{d}-O_{i}^{d}\right) f^{\prime}{ }_{\vartheta W_{i j}\left(\text { net }_{i} d\right.}\right) \cdot\left(-V_{j}{ }^{d}\right)$
$\Delta W_{i j}=\eta \sum^{m} d=1\left(t_{i}^{d}-O_{i}{ }^{d}\right) f^{\prime}\left(\right.$ net $\left.\left._{i}{ }^{d}\right) \cdot V_{j}{ }^{d}\right)$
$=f^{\prime}\left(n e t_{i}^{d}\right)\left(t_{i}^{d}-O_{i}^{d}\right)$
$\Delta W_{i j}=\eta \sum^{m}{ }_{d=1} \delta_{i}{ }^{d} V_{j}{ }^{d}$

We can use the chain rule to differentiate with respect to $W_{i j}$ to obtain the input-to hidden connection $W_{i j}$

$\Delta W_{j k}=\eta \sum_{m d=1}^{\sum 2 i=1\left(t_{i d}-O_{i d}\right) f^{\prime}(n e t i d) . W_{i j} f^{\prime}(n e t i d) . x_{k d}, ~}$
$\delta_{i}^{d}=f^{\prime}\left(\right.$ net $\left._{i}{ }^{d}\right)\left(t_{i}{ }^{d}-O_{i}{ }^{d}\right)$
$\Delta W_{j k}=\eta \sum m d=1 \sum 2 i=1 \delta_{i d} . W_{i j} f^{\prime}\left(n^{2} t i d\right) . x_{k d}$
$\delta_{j d}=f^{\prime}($ netid $) \sum 2 i=1 W_{i j} \delta i d$
$\Delta W_{j k}=\eta \sum^{m}{ }_{d=1} \delta_{i}^{d} \cdot x_{k}^{d}$
$\Delta W_{i j}=\eta \sum^{m}{ }_{d=1} \delta_{i}{ }^{d} V_{j}{ }^{d}$
$\Delta W_{j k}=\eta \sum^{m_{d=1}} \delta_{j}{ }^{d} \cdot x_{k}{ }^{d}$

This will result in the same form with different definitions of $\delta$
Generally, with an arbitrary number of layers, the back-propagation update rule has always the formulae $\Delta W_{i j}=\eta \sum^{m}{ }_{d=1} \delta_{\text {output }} . V_{\text {input }}$ (31) Where output and input refers to the connection under consideration. V stands for the appropriate input (hidden unit or real input, $\mathbf{x}_{\mathrm{d}}$ ) and $\delta$ depends on the layer under consideration.

Equation (41) will help us to calculate for a given hidden unit $V_{j}$ in terms of the $\delta^{\prime} s$ of the unit $O_{i}$. The coefficient are usual forward, but the errors $\delta$ are propagated backward as shown in the figure 2.6 below using dotted lines.


Figure 2.8 NN model with back -propagated errors
We have to use a nonlinear differentiable activation function such as
$f(x)=\sigma(x)=\frac{1}{1+e^{(-\alpha x)}}$
$f^{\prime}(x)=\sigma^{\prime}(x)=\alpha \cdot \sigma(x) \cdot(1-\sigma(x)=\tan (\alpha \cdot x)$
$f(x)=\alpha .\left(1-f(x)^{2}\right)$

### 2.2.4 Evaluation of Machine Learning Algorithms Models

The evaluation of any Machine Learning Algorithm being it supervised, unsupervised or a reinforcement is made a critical aspect of any computer experiment (Kotsiantis, 2007). For instance, if a particular model produces a substantial outcome when evaluated using an accuracy-score may not necessary means that the same model when evaluated against other metrics logarithmic-loss may also give a good result. This means that the selection of metrics influences how the performance of machine learning algorithms is measured and compared. They influence how one sees the importance of different characteristics in the results and your ultimate choice of which algorithm to choose (Kuncheva \& Whitaker, 2003).

In most literatures, authors normally use classification accuracy to assess model performance.

Nevertheless, this is not enough to really assess the performance of a model. In this section, the thesis discussed the various metrics that can be used to assess or evaluate the performance of a machine learning algorithm or model. There are so many metrics for evaluating machine learning algorithms but for the purpose of this review, the thesis focuses on the following: Glassification Accuracy: Classification Accuracy is actually the meaning of the term
"Accuracy" in machine learning performance measure (Osisanwo et al., 2017).
Mathematically, it is defined as the ratio of the number of perditions done correctly by the machine learning algorithm to the total dataset that was used as inputs.


The Total number of predicitions made

This would normally work accurately whenever there are equal numbers of sample belonging to each class. For instance, if there are $98 \%$ observations in a particular class, say class 1 and $2 \%$ observations in class 2 in our training set, then, it is most likely that the proposed model can easily predict at an accuracy of $98 \%$. This is just a matter of predicting all observations in the training dataset belonging to class 1.In another scenario, if the same machine learning model is applied on a dataset with $70 \%$ observations of class1 and $30 \%$ observations of class 2, then the accuracy would most likely reduce to $60 \%$. Classification Accuracy is a good measure but at times it generates poor sense of realizing high accuracy (Osisanwo et al., 2017). The reality would be much expensive when the cost of misclassification of the minor
(in our case class 2 ) is very high.

- Logarithmic Loss: Logarithmic Loss or Log Loss also operates by disciplining the false classifications (Kubat et al., 1998). It performs better for multi-class classification (Kuncheva \& Whitaker, 2003). With this approach the classifier would allocate probability to each class for all the samples. Assuming, there are N samples in each M classes then the Log Loss is calculated as below:

$$
\begin{equation*}
\log \text { Loss }=\frac{-1}{-} \sum_{i=1}^{N} \sum_{N^{M}}{ }_{j=1} y_{i j} * \log \left(p_{i j}\right) \tag{36}
\end{equation*}
$$

Where
$y_{i j}$, indicates whether sample i belongs to class j or not
$p_{i j}$ indicates the probability of sample i belonging to class j

Log Loss has no upper bound and it exists on the range $[0, \infty]$. Log Loss closer to 0 indicates higher accuracy, whereas if the Log Loss is not close to 0 then it shows poor accuracy.

General, reducing Log Loss offers better accuracy for the classifier.

- Confusion Matrix: Confusion Matrix as also presents a matrix as output and defines the comprehensive performance of the model. There are 4 important terms:

True Positives: The scenario where the model predicted YES and the real response was also YES.

True Negatives The scenario where the model predicted NO and the real response was YE S.

False Positives: The scenario where the model predicted YES and the real response was NO.

False Negatives: The scenario where the model predicted NO and the real response was also NO.

In this case the accuracy for the matrix can be evaluated by findig the mean of the values lying across the "main diagonal" i.e.

Accuracy $=$ TruePositive

+ FalseNegatives
(37)

Total Number of Samples
The confusion Matrix forms the basis for the other types of metrics.
$\square$ Area Under Curve: The Area Under Curve (AUC) is the most extensively used metrics for evaluating machine learning algorithms (Bradley, 1997). It is applied in binary classification problem. AUC of a classifier is the same as the probability that the classifier will rank a randomly chosen positive example higher than a randomly chosen negative
example (Daisuke \& Perez, 2017). The following two basic terms are normally used in AUC:

True Positive Rate (Sensitivity): True Positive Rate is defined as $T P /(F N+\mathrm{TP})$. True Positive Rate represent the proportion of positive data points that are correctly considered as positive, with respect to all positive data points.

TruePositiveRate $=$ $\qquad$ TruePositive

FalseNegative + TruePositive

False Positive Rate (Specificity): False Positive Rate is defined as $F P /(\mathrm{FP}+\mathrm{TN})$. False
Positive Rate represents the proportion of negative data points that are incorrectly considered as positive, with respect to all negative data points.

TruePositiveRate $=$ $\qquad$ FalsePositive
(39)

FalsePositive+TrueNegaive

For instance, Daisuke \& Perez ( 2017) in an attempt to predict future firms performance using Machine learning and dataset from Japan utilize the ROC curve as to evaluate the predictive performance of the model. According to the authors, their tasks of binary exit, growth, and profit growth classification require the setting of thresholds for which predicted probabilities surpassing this level will indicate a positive binary outcome. Given a fixed model, the ROC curve plots the true and false positive rates corresponding to the varying of this threshold value. Without any predictors (i.e. random guess), the curve should trace the 45 -degree line, and curves closer to the top-left corner are desirable (maximize true positive rate and minimize false positive rate). With this motivation, it is conventional to also summarize the ROC curve by the area under the curve, called AUC (Daisuke \& Perez, 2017).

- F1 Score: This is used to evaluate a test's accuracy. Is the Harmonic Mean between precision and recall. The range for F1 Score is [0, 1]. It determines how precise the classifier is as well as how robust it is. A higher precision but lower recall, offers an enormous accuracy, but it then misses a large number of instances that are difficult to classify. The greater the F1 Score, the better is the performance of our model. F1 Score attempts to find the balance between precision and recall. Mathematically:

$$
\begin{equation*}
F=2 * \frac{1}{\frac{1}{\text { precsion recall }}+\frac{1}{2}} \tag{40}
\end{equation*}
$$

Precision: It is the number of correct positive results divided by the number of positive results predicted by the classifier.

```
Precision =- TruePositive
    (41)
        TruePositive+FalsePositives
```

Recall: It is the number of correct positive results divided by the number of all relevant samples (all samples that should have been identified as positive). Specificity is the exact opposite of Recall.

Mean Absolute Error: Mean Absolute Error is the mean of the difference between the Original Values and the Predicted Values. It offers the measure of how the predictions deviated from the actual output but do not suggest an information about the direction of tle error; Thus whether we are under predicting the data or over predicting the data.

MeanAbsoluteError $\quad=\frac{1}{-} \sum_{j=1}^{N}\left|y_{j}{ }_{N} \quad y_{j}\right|$

- Mean Squared Error (MSE): This is almost the same us Mean Absolute Error but the only difference being that MSE takes the mean of the square of the difference between the original values and the predicted values (Patel et al., 2014b). The advantage of MSE is that
it is easier to compute the gradient, whereas Mean Absolute Error requires complicated linear programming tools to compute the gradient (Fallahpour et al., 2017). As, we take square of the error, the effect of larger errors become more pronounced then smaller error, hence the model can now focus more on the larger errors (Tavana et al., 2016).

$$
\begin{equation*}
\text { MeanSquaredError }=\frac{1}{-} \sum_{j=1}^{N}\left(y_{j}-y_{j}\right)_{2} \tag{43}
\end{equation*}
$$

Yousaf (2016) in a study to assessed factors that are likely to predict the employee turnover built a Machine Learning Algorithm to do that task. To evaluate the models performance, the author used overall accuracy, kappa, sensitivity, specificity and ROC Curve as the model's evaluation metrics. According to the authors these machine learning performance metrics are considered best practices for doing such classification task (Yousaf, 2016).

### 2.2.5 Application of Machine Learning Algorithms

Álvarez (2016); Prakash et al. (2017); Bani-Hani et al. (2009); Corrado et al. (2014); Ko \& Kweku-Muata (2014); Ko \& Osei-Bryson (2004) and Koellinger (2005) have since embarked on studies that are based on Machine Learning Algorithms. For example, Ko \& Osei-Bryson (2004) used new method called Multivariate Adaptive Regression Splines (MARS) which they suggested will provide supplementary understandings on the characteristics of the impact of IT investments on productivity. The results of this new study were compared with findings from previous works with the same data set. While the results of the former study suggested a positive but uniform impact of IT on productivity, this new study suggested that IT impact on productivity was not uniform but is dependent on other supporting factors. Ko \& Osei-Bryson (2004) concluded that there was a complementary relationship that exists between IT and nonIT related investments and stated that additional investment may not results in higher organizational productivity.

Ko \& Kweku-Muata (2014) revisited this issue and reconsidered the impact of IT capital on hospital productivity by using two data mining methods. According to the authors, this new approach gave them the strength to discover interactions between the input parameters as well as provisional impacts. The results suggested a complex relationship between IT investment and productivity. The descriptive and regression analysis of data on Small and Medium Enterprises from the two selected countries in Africa (Kenya and Tanzania) suggested that investment in ICT is one vital element of total factor productivity once a definite threshold is passed (Wolf, 2001). Commander et al. (2011) also used a regression analysis on a data on manufacturing companies in both Brazil and India suggested a strong positive relationship between ICT investment and productivity in both countries. Alinezhad (2016); Lee (2010) and Rocha \& Júnior (2010) all in one way or the other have applied a DT to make classifications or predictions regarding firms.

Carmona et al. (2018) predicted bank failures in US using an extreme gradient boosting approach which was based on 156 banks and their financial ratios. The results indicated that smaller values for financial ratios such as retained earnings to average equity, pretax return on assets, and total risk- based capital ratio were linked with a higher risk of bank failure. Chi et al. (2011) in a research proposed a credit rating forecasting model using market-based information as an independent variable. The Moody's KMV (KMV) model was used to assess the market-based information of each business. In order to validate the suggested technique, the authors used a hybrid model, which comprises random forests (RF) and Rough Set Theory (RST) to excerpt valuable data for credit rating. The results demonstrated that market-based data provides significant information in credit rating forecasts. Creamer (2009) utilized random forest and logistic regression for performance forecast of Latin America ADRs and banks. The study also arranges in order of magnitude accounting and corporate variables based on their impact on performance. According to Creamer (2009), Random Forest was intelligent to specify the most significant variables. Creamer (2009) finally concluded that elucidation of
predictive models for smaller dataset enhanced when the ability of Random Forest to rank and predict with the parameters of a logistic regression was integrated. Booth et al. (2014) proposed a special framework that utilizes new Machine Learning methods to forecast the price return over these seasonal events, and then use the predictions to build a profitable trading strategy. Kartasheva \& Traskin (2011) utilized an adapted Random Forest classification algorithm to predict insolvency of insurers. The study disclosed that RF methodology provides higher quality of prediction compared to other existing methods. Daisuke \& Perez (2017) in another study with the main aim to predict new companies efficiency with Machine Learning procedures used data on a large number of Japanese firms with supply-chain linkage information provided by a credit reporting agency. Using Random Forest Random, the study showed high accuracy in prediction. The authors suggested that the evidence of theory of the study offers real-world usage of Machine Learning methods in firm efficiency forecast.

Bia et al. (2006) discussed a new numerical method to predict growth in manufacturing from firm-survey responses. The authors based their forecasts on a predicting algorithm motivated by the Random Forest which is fast, strong to noise and permits for the handling of missing values. Tanaka et al. (2016) presented an original Random Forests-based early warning system for forecasting bank failures. The results indicated that the RF outpaces ordinary methods regarding forecast precision. Biau et al. 2006; Luca et al. (2010) and Patel et al. (2014) all have contributed to the Random Forest and organizations literature.

According to literature, NN has been utilized in different ways for classification and predictions because it has a high computational power and performance (Gemino \& Sauer, 2010 and Lam, 2004). For instance, Boritz \& Kennedy (1995) examined the efficiency and performance of a number of neural networks in predicting bankruptcy filing. The authors considered two methods for training Neural Networks namely: Back-Propagation and Optimal Estimation Theory. For the back-propagation training technique, four distinct models namely, BackPropagation, Functional Link Back-Propagation With Sines, Pruned BackPropagation,
and Cumulative Predictive Back-Propagation (Kriesel, 2005) were are tested. The authors did a comparative analysis of Neural Networks against the ordinary bankruptcy prediction practices such as Discriminant Analysis, Logit, and Probit.

The granting of loans to customers is one of the main functions of a credit union. Consequently, the application of technological tools that can support such business process is necessary and could be a key element in credit management (Sousa \& Figueiredo, 2014). Sousa \& Figueiredo (2014) developed two models that comprise Neural Network and Decision Tree models to analyze the capability of a credit union's customers to settle their debts. The results were evaluated through cross-validation of ten sets, repeated in ten simulations. The authors concluded that the models have statistically similar results and may help in a cooperative's decision-making process especially in credit management.

Lam (2004) also investigated the ability of Neural Networks, specifically back propagation algorithm and suggested a high accuracy of NN in predictions.

To improve the accuracy and the computation speed of credit scoring models, (Sang, Nam, \& Nhan, 2016) suggested a credit scoring model which was based on parallel Random Forest classifier and feature selection method to evaluate the credit risks of applicants. By integrating Random Forest into feature selection process, the authors indicated that importance features can be accurately evaluated to remove irrelevant and redundant features. The authors proposed an algorithm to select best features by using the best average and median scores and the lowest standard deviation as the rules of feature scoring. They experimentally assessed the performance of their algorithm by using two public datasets from Australia and Germany .The results of their study suggest an improved accuracy of their model comparable to other commonly used feature selection methods particularly in terms of their prediction accuracy, $76.2 \%$ with a significantly reduced running time of 72 minutes on German credit dataset and the highest average accuracy of $89.4 \%$ with the running time of only 50 minutes on Australian credit dataset (Sang et al., 2016).

According to Roy \& Urolagin (2019), the assessment of credit risk within the banking sector has become an important issues. Roy \& Urolagin, (2019) therefore, proposed a methodology made up of both Random Forest and Support Vector Machine that performs a two-level data processing in order to accurately predict creditworthiness of the clients. According to the authors, the proposed methodology would aid to attain results with minimized false positives. Despite the countless benefits of Automated Teller Machines (ATMs) to customers within the banking sector, ATM lacks the provisioning of security measures against frauds (Bajaj et al., 2019). Video surveillance has also been suggested to be one of the protuberant measures against ATM frauds (Bajaj et al., 2019). Bajaj et al. (2019) presented a method that can be applied for activity recognition in small premises such as ATM rooms by encoding the motion in images. The authors utilized Gradient-based descriptor (HOG) to extract features from image sequences and classified the extracted features using random forest classifier. Bajaj et al. (2019) concluded that their method was positive in detecting abnormal and normal human activities both in case of single and multiple personnel with an average accuracy of $97 \%$. A combine Random Forests and Logistic Regression was used by (Germán Creamer, 2009) for performance prediction of Latin American ADRS and Banks and suggested that the most important variables that affect banks are size, long term assets to deposit, number of directors and the efficiency of the legal system.

Finally, Emrouznejad \& Shale (2009) ; Hsu et al. (2016) ; Ma et al. (2018) ; Nami \& Shajari
(2018) ; Portas \& Abou-Rizk (1997) ; Toloo et al. (2015) ; Tsai et al. (2009) ; Tzeng (2014) ;

West (2000) ; Wu (2009) and Yeo \& Grant (2018) have all contributed to the Machine Learning Algorithm literature.

### 2.3 APPLICATION OF COMBINED DEA AND MACHINE LEARNING ALGORITHM

According to literatures, most studies either combine two or more Machine Learning Algorithms or integrate it with other models such as the popular non-parametric DEA model to detect or predict an outcome. For instance, Sakouvogui (2019) applied an integrated DEA with Support Vector Machines (SVM) on three-monthly dataset of US Agricultural Banks and
indicated a strong evidence that the efficiency measures of the selected banks in the dataset were strong before the financial crisis (2005-2006), during the financial crisis (20072009) and even after the financial crisis (2010-2016). The author further suggested that the integrated DEA-SVM had a lower performance during 2007-2009. Lee (2010) applied a combined DEA and Decision Tree for efficiency analysis and suggested appropriate guidelines for controls in Business- to-Consumer (B2C) applications. The author indicated that retail firms and information service providers implement B2C controls more efficiently than financial firm and suggested the possibility of using Decision Trees for controls assessment in B2C applications.

According to (Jain et al., 2016b), in most cases, the choice of input and output used for DEA assessment was based on the researchers' own judgments. Which according to them, most researchers lack the rigorous justification relating to the suitability of such input/output factors to measure the true technical efficiency of the DMUs. They investigated the relationship between computed efficiency scores and a single performance measure of DMUs by using decision tree (DT) analysis. Using the relative efficiency scores of 57 DMUs, the results indicated that 2 out of the 11 models are the most appropriate to measure relative efficiency when intermediary approach was considered to evaluate the banks (Jain et al., 2016b). Da et al. (2018) assessed the efficiency of peripheral European domestic banks in order to examine the effects of bank-risk determinants on their performance using 2007-2014. The authors applied both Data Envelopment Analysis and a Truncated Regression and suggested that financial ratios such as liquidity and credit risk were negatively affecting banks productivity, whereas capital and profit risk had a positive impact on their performance (Da et al., 2018). Kwon \& Lee (2015) implemented a two-stage DEA and Neural Network and suggested that the effectiveness of an integrated performance model was empirically supported by its practical application to the financial banking operations across U.S. banks.

Wu et al. (2006) and Wu (2006 and 2009) in separate studies applied both DEA and Machine

Learning Algorithm with other models to make classifications and predictions. In the case of Wu (2006), the author combined DT with a two-stage DEA model in a study to detect the IT impact on 36 firms performance. The authors used Wang et al. (1997) dataset and categorized both the CCR and BCC DEA scores of the various DMUs into four classes under two separate scenarios .Using a hybrid model that consist of two modules, Wu (2006) and cited by Chen (2016) and Santos et al. (2017), DT model was able to predict the classes originally given to each DMU per the CCR and BCC scores at an accuracy of $69.44 \%$ and
$64 \%$ respectively for the different "cutoff point" for efficient DMU. The author finally concluded that DT was easy to use and understood and suggested comparison studies of

DEA-DT and DEA-neural networks (Wu et al., 2006).
However, for the evaluation of the DEA scores, the author did not consider the efficiency score at each stage of the two - stage DEA model.

In another similar study, Delen et al. (2013) in determining the firm performance using a set of financial ratios adopted a two-step analysis methodology: first, using exploratory factor analysis (EFA) to recognized and confirmed fundamental dimensions of the financial ratios, followed a predictive modeling approaches to determine the possible associations between the firm performance and financial ratios. The authors tested their dataset using four popular decision tree algorithms (CHAID, C5.0, QUEST and CART) and performed information fusion-based sensitivity analyses to measure the relative importance of predictor variables. Delen et al. (2013) suggested that the CHAID and C5.0 decision tree algorithms produced the best prediction accuracy. Emrouznejad \& Anouze (2010) proposed a framework to combine DEA with C\&R for measuring the efficiency and productivity of DMUs. The authors suggested a set of rules that can be used by policy makers to discover reasons behind efficient and inefficient DMUs. Hamad \& Anouze (2015) in a study presented a three-stage combined system comprising DEA, RF, and Logistic Regression to assess and forecast the effect of environmental factors on banks' efficiency. The model recognized five significant factors and their impacts on bank performance when illustrated with 151 banks in the Middle East and

North African (MENA) countries from 2008-2010. Hamad \& Anouze (2015) utilized these variables to study their relationship with banks efficiency through logistic model. The authors concluded that the proposed model can be utilized by bank directors, and other stakeholders to better manage their banks.

Grzybowska \& Karwański (2014) exhibited a technique for variable choice in light of Random Forest and gradient boosting approach and its application to companies ranking in DEA technique. The study recommended that application of RF methods appears to be a good method to variable choice for the needs of DEA. Finally they suggested that Random forests and gradient boosting can be relied upon to enhance the automation of procedures to evaluate the status of companies by banks and other financial institutions. Grzybowska \& Karwański (2014) demonstrated that random forests yield higher outcomes in terms of both efficiency and expectation precision compared to other ensemble procedures. Using a recent sample of large corporate failures in the United States, Premachandra et al. (2009) examined the capability of DEA in assessing corporate bankruptcy by comparing it with logistic regression (LR). This study suggested that DEA outperforms LR in evaluating bankruptcy out-of-sample.

Sreekumar \& Mahapatra (2011) presented an integrated approach by combining Data Envelopment Analysis (DEA) and Neural Network (NN) for assessment and prediction of Indian B-schools' performance. The study suggested that with a total of 49 Indian B-schools chosen for benchmarking purpose. The average score of efficiency was 0.625 with a standard deviation of 0.175 when Charnes, Cooper and Rhodes (CCR) model is used. Similarly, when the Banker, Charnes and Cooper (BCC) model was used the average score was 0.888 with a standard deviation of 0.063 . According to the authors, the work offers a modest but inclusive methodology for improving performance of B-schools in India. Anthanassopoulos \& Curram (1996) in a comparative analysis study to assess the efficiency of DMUs also used DEA and NN. The results of the study suggested that, irrespective of the differences between the two
models, both methods offer a useful range of information regarding the assessment of performance.

In attempt to assess the efficiencies of personnel, Azadeh et al. (2011) suggested that personnel specifications have greatest impact on total efficiency and proposed an algorithm for such a task. The proposed algorithm assesses the impact of personnel efficiency attributes on total efficiency through Data Envelopment Analysis (DEA), Artificial Neural Network (ANN) and Rough Set Theory (RST). According to Azadeh et al. (2011), the proposed algorithm was superior to the conventional and existing models and algorithms. Dash et al. (2006) combined data envelopment analysis (DEA) and Neural Networks (NNs) to assess the relative branch efficiency of Canadian bank. Dash et al. (2006) compared the results with the ordinary DEA results. The authors finally concluded that the two are analogous. Saher et al. (2019) in a study determined the firm and sector efficiency using data envelopment analysis for 121 listed firms, from 2004 to 2016. Saher et al. (2019) concluded that all firms were not equally efficient. The study also used a Logit/ Probit Regression model, and the results indicated that the brand value and type of sector has a positive impact on firm efficiency (Saher et al., 2019).

In conclusion, the study made a search for DEA-related studies in popular academic databases like Taylor and Francis, Elsevier, IEEE, Scopus, EBSCO and Science Direct. The results showed more than 1000 studies. These numbers of studies are impressive and confirm the fact that the issues regarding DEA application at measuring efficiency are still on the agenda and require the further investigation. DEA is widely used in efficiency measurement studies (Ascarya, 2007; Yumanita et al., 2008; Emmanuel, 1999; Grmanová \& Ivanová,

2018; Halkos \& Salamouris, 2004; Havidz \& Setiawan, 2015; Jemric \& Vujcic, 2002; Kamarudin, Sufian, Loong, \& Anwar, 2017; Lampe \& Hilgers, 2015; LaPlante \& Paradi, 2014; Maletić, Kreća, \& Maletić, 2013; Nand \& Archana, 2015; Sarifuddin, Ismail, \& Kumaran, 2015; Shibu \& Ayekpam, 2018; Sreekumar \& Mahapatra, 2011; Sufian et al., 2016 and Titko et al., 2014). However, studies on bank efficiency with DEA techniques in developing countries are limited particularly in Africa. This also means that the methodological
and informational gap in studies on bank efficiency conducted by local researchers using DEA should be an important issue. From the literature review also, most two-stage DEA model did not consider the efficiency score in each of the stages as an input variable in the next stage. Conceptually, the author consider outputs from organizations as a result of a sequences of important events where resources are applied at all the two stages as in the scenarios used by previous authors (Chen \& Zhu, 2004; Wang et al., 1997). Thus, for the banks to have enough funds to invest in the second stage will depend on how efficient they are in collecting deposit. The current study uses a two-stage DEA concept that explicitly consider the efficiency score in all the stages i.e. both stages I and II as. This is acknowledged by using the efficiency of stage I and the total deposit that was realized at stage I as the two main inputs for stage II and its outputs as Percentage of Performing Loans Greenidge \& Grosvenor (2010) and Profit. Finally, the overall efficiency was calculated by using the efficiency of stage II, Fixed Asset, IT budget and the Number of Employees as inputs and Percentage of Performing Loans and Profit accrued from investing the deposits in securities as the two main outputs of the overall stage. This proposed model does not only analyze the IT impact on the two stages, but also ensures that efficiency that was calculated at each stage is also used as inputs.

### 2.4 CHAPTER SUMMARY

This Chapter reviews DEA, Machine Learning Algorithms and their applications either individually or a combined DEA and Machine Learning Algorithm. For the DEA, the Chapter reviews DEA in general including background information, and applications of DEA in efficiency measurements. With regard to background information section, the definition of DEA, its data structures, algorithms used in DEA, advantages and disadvantages of DEA as well as major application of DEA for efficiency measurements particular in the business area are addressed. From the review of the literature, it can be concluded that the application of DEA in measuring the efficiency of firms have many benefits such as, DEA does not consider the relationship among various organization performance measures in performance measurement, DEA measurement one does not worry about the relationship among several performance
measures and DEA is an improved method to organize and analyze data since it allows efficiency to change over time (Lampe \& Hilgers, 2014). It also does not involve any previous postulation on the requirement of the best practice frontier. However, the application of only pure DEA and its models suffer from weak discrimination power Ashoor (2012) and sensitive to the presence of outliers and statistical noise (Dash et al., 2006). It is also very difficult to use only DEA to predict the efficiency and performance of other or new Decision Making Units (LaPlante \& Paradi, 2014; Peter Wanke et al., 2015).

The second part of the Chapter Two also focuses on Machine Learning Algorithms its applications and previous studies on combined DEA and Machine Algorithms especially in the financial industry. In background information aspect, the origin and definition of Machine Learning Algorithms and types of Machine Learning Algorithms are discussed. It also introduces the topmost/most commonly used Machine Learning Algorithms and evaluation or performance measurements of Machine Learning Algorithms. Based on the review, four among the ten topmost Machine Learning Algorithms namely, Decision Tree, Random Forest, Neural Network and Logistic Regression were extensively discussed in this chapter and were used for the study. Finally application of Machine Learning Algorithms whether as a single tool or combined with others showing different cases studies are presented. From the review of the literature on Machine Learning Algorithms, it can be suggested that a lot of Machine Learning

Algorithms have been developed and utilized in the business and financial sectors for predictions and forecasting. Nevertheless, little attention have been given to prediction and classification of bank branches using their efficiency scores across developing countries Mohd (2001) especially using a combined two-stage DEA and Machine Learning Algorithms, where efficiency score at each stage is also used as an input for the next stage. With this, we can have a study that combines a two -stage DEA model with different Machine Learning Algorithms, where the efficiency at each stage is also considered as input for the next stage using dataset from a developing country.


## CHAPTER 3

## METHODOLOGY

### 3.0 Introduction

This chapter discusses the research methodology and the methods used in carrying out the study. The four Machine Learning Algorithms namely; Decision Tree, Random Forest Artificial Neural Networks and Logistic Regression and the four proposed models that were
realized by combining DEA with Machine Learning Algorithm is used for making predictions in a greater part of this thesis. These four proposed models are: Data Envelopment AnalysisDecision Tree (DEA-DT), Data Envelopment Analysis-Random Forest (DEA-RF), Data Envelopment Analysis-Neural Network (DEA-NN) and Data Envelopment Analysis-Logistic Regression (DEA-LR).

### 3.1 DATA

### 3.1.1 Study Area and Data Description

The main case study area of the study is the Ghanaian banking sector. The study was undertaken basically to study efficiency of Ghanaian banks. Due to the large number of commercial bank branches operating in the country as of the time of this study 2016 (Ghana, 2017), the study was limited to $33 \%$ of the total commercial (universal banks) branches in Ghana. These banks were mostly in Greater Accra, Ashanti, Western, Brong Ahafo, Eastern, Northern, Upper East, Volta and Central Regions. The study was limited to these areas because of the availability of information and logistics. Data for the study was also the audited 2016 financial statements from the various bank branches. The map of the study area is shown below in Figure 3.1.


Figure 3.1: Map of the case study area showing the distribution of bank branches used for the study (Author's construct).

### 3.1.2 Data Collection and Sample Size

Arrangement was made and the necessary protocols were followed to collect data (Fixed Assets excluding IT, Number of Employees, IT expenditure, total deposit, percentage of performing loans and profits accrued from investing deposit) from the selected banks branches using their 2016 audited financial statement. The sample size for the study data was calculated using minimum sample size formula in equation (10) at confidence level of $95 \%$ and $5 \%$ margin of error. This formula was used because the study is a cross-sectional survey and the response variable is also qualitative (Cochran, 1977) •
$\begin{gathered}2(\mathrm{Z} \alpha / 2) \cdot \mathrm{P}(1-\mathrm{P}) \\ (1.96) 2 .(0.5)(0.5)\end{gathered}$
Sample size $(\mathrm{n})=$ $\qquad$
$\qquad$ _2 = $=\quad 2=367.32 \approx 367$
E
(0.05)

Where $\alpha$-level of significance which in this case was selected to be $5 \%$, meaning $\left(\mathrm{Z}_{\alpha / 2}\right)=$ 1.96 at ( $95 \%$ confidence level). P is the estimated population proportion which was chosen to be 0.5 . This is the possessing attribute of interest based on previous studies or pilot studies. If no approximation of P is known, $\mathrm{P}=0.5$ is used which will give a sample size sufficiently large to guarantee accurate results (Bluman, 2009). Based on this minimum sample size value of 367, requests were made to collect data from about 650 universal bank branches in the selected study area and follow-ups were made to explain the project to the banks and also to have some firsthand information about the operation of the banks.

A total of 472 requests were granted (thus 472 bank branches data were supplied given a response rate of 73\%). After analyzing and cleaning the data, the total observation of the dataset came to 444 which were used for the study. This number is also consistent with Charter (1999) and cited recently by (Akena et al., 2018), that the total number for generalizability and reliability research studies is fairly individualistic but a minimum of 400 observations is recommended.

The Cedi value of IT budget data, data on the percentage of performing loans on the various bank branches, total number of employees at each branch, profit accrued from the banks investments and the Cedi value of the total deposit were obtained from the various branches. This study is different from previous works by Wang et al. (1997) and Wu (2006) which was cited by Chen (2016) and Santos et al. (2017) in the sense that the study used the percentage of performing loans (non-performing loans rate minus 100\%). The data in Appendix A shows the complete set of 444 observations which gave us the dataset for this study. This data sample
contains 444 observations on DMUs which is about $33 \%$ (comprising 17 universal banks) of the total commercial bank branches in Ghana.

### 3.2 THE PROPOSED BANK EFFICIENCY PREDICTION FRAMEWORK

The framework suggested in this study which was used to build the predictive models consists of three different stages, data collection stage, data preprocessing stage and the predictor development model stage (Decision Tree/Random Forest Classifier model). This means that the dataset for the model development goes through three different stages. It was used to build the predictive models used for predicting the efficiency of banks. Figure 3.2 is the proposed the framework used for the entire study. The framework also has a Neural Network version which was also used for developing the NN predictive models for the study.


Figure 3.2: The Banks efficiency prediction framework (Author's construct)

### 3.2.1 Dataset for the Model Development

### 3.2.1.1 Determining the Response Variables (Bank Efficiency Scores and Classes)

The response variables (efficiency scores of the bank branches) of the study were determined using a two-stage Data Envelopment Analysis (DEA).

## The Basics of DEA

Data Envelopment Analysis is a non -parametric method that produces a comparative ratio of weighted outputs to inputs for each Decision Making Unit (DMU) under consideration, i.e. relative efficiency score (Banker et al., 1984 and Cooper et al., 1978). This score is normally recorded as a figure between the percentages of $0-100 \%$ or a unit that falls between $0-1$. A

DMU with a score less than $100 \%$ or less 1 unit is classified as ineffective comparable to similar units in the sample (Avkiran, 2006). According to Wang et al. (1997), it is a scientific programming method used to compute the relative specialized efficiencies of gainful units. The DMUs employ several inputs to generate several outputs where the inputs cost and output cost are assumed not to be known. The framework was formerly used to examine the relative efficiency of non-profit organizations which was quickly followed by other profit making organization such as financial institutions, hospitals, the US Air Force, airports, schools and courts because of its strength (Avkiran, 2006).

For instance, Coelli \& Rao (2005); Jagoda et al. (2013); Lusigi (1997); Oeij, Looze, Have, Rhijn (2011) and Syverson (2011) used this model in their various studies. The model shown in Figure 3.3 was originally suggested by Charnes et al. (1978), CCR which have seen several extensions including the one by Banker et al. (1984), BCC which is considered a significant extension (Wang et al., 1997). The CCR, Constant Returns to Scale, CRS Charnes et al. (1978) was adopted for the study in reference to other similar models.


Figure 3.3: The Classical two-stage DEA model taking two (2) inputs adapted from Wu (2006).

Based on data matrix ( $\mathrm{X} ; \mathrm{Y}$ ), one can calculate the input oriented efficiency of each observation "o" ( in this case DMU) by calculating $n$ number of times the subsequent linear programming problem proposed by Charnes et al. (1978) and popularly known as DEA CCR model: $\min \theta$

Subject to

$$
\begin{aligned}
& \theta x_{o} \geq X \lambda \\
& Y \lambda \geq y_{o} \\
& \lambda \geq 0 .
\end{aligned}
$$

The optimum answer to this program is based on CRS technology denoted by $\theta_{\text {CRS }}$.
The restrictions entail the DMU $\left(\theta_{C R S} x_{0}, y_{o}\right)$ to belong to $P_{C R S}$, while the purpose is to pursue the minimum $\theta_{C R S}$ that decreases the input vector $x_{o}$ radially to $\theta_{C R S} x_{0}$ while still in $P_{C R S}$. A possible answer signaling radially efficiency is $\theta_{C R S^{*}}=1$, therefore if $\theta_{C R S^{*}}<1$, the DMU is radially inefficient and $(\lambda X, \lambda Y)$ surpasses $\left(x_{o}, y_{o}\right)$.

With regard to this property, we express the additional input excesses and output shortfalls by the following slack vectors: $s^{-} \epsilon \mathbb{R}^{m}$ and $s^{+} \epsilon \mathbb{R}^{s}$, respectively. Therefore: $s^{-}=$ $\theta_{C R s^{*}} x_{o}-, X \lambda$, and $s^{+}=Y \lambda-y_{o}$ with $s^{-} \geq 0$ and $s^{+} \geq 0$ for any possible solution( $\left.\theta, \lambda\right)$. To get the probable input excesses and output correct, the second stage program that integrates the optimal value $\theta_{C R S^{*}}$ and fix radial inefficiency is calculated as:
max

$$
\omega=e s^{-}+e s^{+}
$$

$\lambda, s^{-}, s^{+}$

Subject to:

$$
\begin{aligned}
& s-=\theta C R S . * x_{0}-X \lambda \\
& s^{+}=Y \lambda-y_{0}
\end{aligned}
$$

$$
\lambda \geq 0, s^{-} \geq 0, s^{+} \geq 0,
$$

Where $e(1, \ldots, 1)^{T}$ so $e s^{-}=\sum^{m_{i=1}} S_{i}^{-}$and $e s^{+}=\sum^{s_{i=1}} S_{i}{ }^{+}$. This means that a DMU is efficient technically on condition that the optimum solution $\left(\theta_{C R S}^{*}, \lambda^{*}, s^{-*}, s^{+*}\right)$ of the two above program satisfy $\theta_{C R S^{*}} \quad=1, s^{-*}=0$ and $s^{+*}=0$, so no
equi-proportional contraction of inputs, and individual inputs reduction and outputs growths are possible (Álvarez et al., 2016).

The analysis of technical efficiency assuming Variable Returns to Scale (VRS) proposed by Banker et al. (1984) and cited by Álvarez et al. (2016) called DEA BCC model, also consider this production possibility set $P_{V R S}=\{(x, y) \mid \mathrm{x} \geq \mathrm{X} \lambda, \mathrm{y} \leq \mathrm{Y} \lambda, \mathrm{e} \lambda=1, \lambda \geq 0\}$. Thus the only distinction with that of the CCR model (CRS) is the adjunction of the condition Azahyzigethe. Value Return to Scale (VRS) efficiency of each DUM under discussion alongside the succeeding second stage program similar to equation (12)
 VRS technology if and only if the optimum solution of the two programs satisfy $\theta_{V R S}=0$ and $s=0 . \quad{ }^{*}=1, s^{-*}$

## The DEA Model and Notation

As stated earlier, the operations of the various bank branches were considered to be a twostage operation. In this case, banks in Ghana use IT infrastructure, Number of staff and their fixed asset to collect money from their customers called deposit in the stage I. The Cedi value of the total deposit was used as the output variable while the Cedi equivalent of their IT (IT expenditure), Number of employees and fixed assets were used as the input variable. To evaluate the bank performance and efficiency, it is incumbent to align its resources to its fundamental business goals. For instance, IT facilities link Computers installed with needed software, mobile phones, Automated Teller Machines (ATMs), internet facilities etc. are vital tools that banks use to collect or gather deposit and also use it to invest their deposits into securities through electronic transfers and process loans for their customers. The study therefore, noted both stages I and II as resource-related value-added events and considered the efficiency of both stages as very important.

A bank's profit after all the various deductions including tax is centered on the revenues accrued from the investments and the loans given out in stage II. In this case it will be totally improper
to only assess a bank's performance and efficiency only on their profit margins without looking at how their loans given out are performing as regards percentage of performing loans. This is because a bank with weak percentage of performing loans stands a high probability of losing net profit and can also threaten the bank's existence. The study therefore denoted this situation by the variable, Percentage of Performing Loans.

The classical two-stage DEA for efficiency computing and analysis by Wang et al. (1997) and Wu (2006) cited by Chen (2016) and Santos et al. (2017) to compute the efficiency score of each DMU was adapted in this study. In this DEA model, the various units (banks) performance or efficiency measures were grouped into inputs and outputs. Using a bank as an example to derive the model ,the business processes of the bank and activities is viewed as a two-stage, wherein the first stage (Deposits Stage) of the model consists of collection of funds (Deposits) in Ghana Cedis from customers using their fixed asset, number of workers (Employees at each unit) and IT facilities (thus IT expenditure). In the next stage (Investment Stage), the banks use the deposits accumulated in stage I to invest in securities and also give loans to their customers and returns (Profits) generated from the investment in securities and percentage of performing loans, which are a good indicators of risk status are used as two outputs in stage II. The various variables used in the proposed model are defined as follows:

## Deposit Stage I Input:

- Fixed Assets (billions of GH) denoted as A
- Total IT expenditure (billions of GH) denoted as I
- Total number of Employees denoted as E Output:
- Deposit Investment Stage II Input:
- The efficiency of stage I denoted as $D_{0}$.
- The deposit also denoted as D.


## Output:

Percentage of Performing loans (PL)

Profit accrued from investing in securities (R)

## Input

Efficiency of stage II denoted as No

## KNUST

Fixed Assets (billions of GH) denoted as A
Total IT expenditure (billions of GH) denoted as I
Total number of Employees denoted as E

## Output:

Percentage of Performing Loans (PL).
Profit accrued from investing in securities (R)

After using the classical CCR model of DEA, the efficiency score for each DUM in each stage resulted in the three types of efficiency score as follows:

Efficiency of stage I, Do
Efficiency of stage II, No
Overall efficiency, G

DEA models used at the deposit stage, I produce stage I efficiency for eah DUM and the DEA applied at the investment stage, II also gives stage II efficiency for each DMU. DEA models employed at overall stage finally give us the overall efficiency for each DMU. The proposed

## Overall Stage

DEA model for the banks two-stage operation is depicted in the Figure 3.4. The DEA Model for the Bank's Dual Role Operations


Figure 3.4: The Proposed Dual Role DEA Model adopted from Appiahene et al. (2019)

## Efficiency Calculations

For each DMU, the technical efficiencies in both stages and their cor responding overall efficiencies were analyzed using CCR DEA algorithm proposed by Mehrabiana (2013) which depended on non-Archimedean Charnes-Cooper-Rhodes (CCR) framework below.

## Algorithm 3.1 : Two-Phase Algorithm BuildHull

## Step 0:

Input problem data, $k=1$.
$f l a g(j)=1$ if $\mathrm{DMU}_{\mathrm{j}}$ classified, else 0.
$\operatorname{flag}(j)=0, j=1, \ldots, n$.
Compute an assurance value $\overline{\boldsymbol{\varepsilon}}, \overline{\boldsymbol{\varepsilon}} \leftarrow \overline{\boldsymbol{\varepsilon}}$.

## Step 1:

Solve problem $E\left(P_{k}\right)$.
case $\left(\boldsymbol{\theta}_{k}<1\right)$ if $\quad\left(\left(\boldsymbol{\lambda}_{k}, \boldsymbol{\omega}_{k}\right)\right.$ is an SCSC pair),
$\mathrm{DMU}_{\mathrm{k}} \in \mathrm{U}_{\mathrm{M}}$.
if $\left(\mathbf{1}^{\mathrm{T}} \mathbf{S}_{\mathrm{k}}=0\right)$, if $\left(\lambda^{k_{k}}>0, \lambda^{\ell_{j}}=0\right.$ for $\left.j \neq k\right)$,
$\mathrm{DMU}_{\mathrm{k}} \in N E$.
else,
$\mathrm{DMU}_{\mathrm{k}} \in N E^{\prime}$,
if $\left(\left(\boldsymbol{\theta}_{k}, \boldsymbol{\lambda}_{k}, S_{k}\right)\right.$ is unique $\boldsymbol{\&}\left(\boldsymbol{\lambda}^{\boldsymbol{k}_{\boldsymbol{k}}}>\boldsymbol{0}, \boldsymbol{\lambda}_{\boldsymbol{\ell}}=\mathbf{0}\right.$ for some $\left.\boldsymbol{\ell} \neq k\right)$,
DMU $\boldsymbol{\ell} \in E, \operatorname{flag}(\boldsymbol{\ell})=1$ endif
else,

## $\mathrm{DMU}_{\mathrm{k}} \in N F$,

if $\left(\left(\theta_{k}, \lambda_{k}, S_{k}\right)\right.$ is unique $\&\left(\lambda_{k}>\mathbf{0}, \lambda_{j}=0\right.$ for some $\left.\boldsymbol{\ell} \neq \mathbf{k}\right)$, $\mathbf{D M U}_{\ell} \in \mathbf{E}, \operatorname{flag}(\boldsymbol{\ell})=1$.

$$
\text { endif } \quad \operatorname{case}\left(\boldsymbol{\theta}_{k}=1\right) \text { if }
$$



$$
\begin{aligned}
& \text { else, } \mathrm{DMU}_{\mathrm{k}} \in F \text {. } \\
& \text { endif }
\end{aligned}
$$

## Step 2:

$k \leftarrow \boldsymbol{k}+\mathbf{1}$ if $(k>n)$,
stop.
else, if ( $\operatorname{flag}(k)=1)$, go to Step 2
else, go to Step 1.
This algorithm has its package Robust Data Envelopment Analysis (rDEA) Version $1.2 \quad-5$
Simm (2016) and was implemented in R using R studio with screen shot shown in Figure 3.5 and the results are shown in appendix B.

The efficiency of stage I was calculated using IT expenditure ( $\mathrm{GH} \phi$ ), Fixed Asset (GH\&) and the Number of employees at each DMU as inputs and deposi $t$ as the main output. Regarding stage II, the deposit ( $\mathrm{GH} \phi$ ) realized from stage I and the efficiency value of stage I were used as inputs with bank profit and the percentage of performing loan as outputs.

The efficiency of the overall stage was also calculated using the efficiency value from stage II, Fixed assets, Number of employees and IT expenditure while their output was the banks profit and the percentage of performing loans.

## The R Studio



Figure 3.5: Screen shot of R studio used for the programming (Author's construct).

### 3.2.1.1.2 Classification of the Banks' Efficiencies Scores

Using the overall efficiency as the basis, the efficiency of both stage I (banks efficiency in collecting deposit from customers ) and the stage II ( the banks efficiency in investing their deposit into securities and given out loans) were categorized into classes as was done by Wang et al. (1997) and Wu (2006) cited by Chen (2016) and Santos et al. (2017) using the proposed Bank Classification Algorithm, BC Algorithm (Appiahene \& Missah, 2020) below:

## The Proposed Bank Classification Algorithm (BC Algorithm)

- Let DMU represent Bank's
- Let A represent Bank's fixed Asset excluding their IT
- Let I represent Bank's IT expenditure
- Let E represent Bank's total number of Employees
- Let D represent Bank's total deposits
- Let Do represent Bank's efficiency score in collecting deposit from customers
- Let No represent Bank's efficiency score in investing the deposits collected from customers

Let G represent Bank's overall efficiency score in doing business
Let PL represent Bank's percentage of performing loans
Let $R$ represent Bank's profits accrued from investing their deposit.
Let Efficiency Bank be a Bank with efficiency score $\geq 0.8$ comparative to similar units in the sample
// calculating the Efficiency of the Banks in the Deposit stage (Do), Investment stage (No) and their corresponding Overall stage (G)

1. Start
2. Input $A, I, E, D, P L, R$ of a $D M U$
3. Use A,I and $\boldsymbol{E}$ as input variables and $\boldsymbol{D}$ as output variables and determine the DMU's Do using the CCR Two-Phase DEA BuildHull Algorithm
4. Use Do and D as input variables and PL and $\boldsymbol{R}$ as output variables and determine the DMU's No using the algorithm suggested in STEP 8.
5. Use A, I, E and No as input variables and PL and R as output variables to determine DMU's $G$ using the STEP 8 algorithm.

## // Classifying the Banks based on their Do, No scores

// Categorizing the Banks based on their Do on the basis on their overall efficiency

- Let Class 1 be Banks efficient in collecting deposit but was not able to attain overall efficiency.
- Let Class 2 be Banks that realized overall efficiency even though they were not efficient in collecting deposit.
- Let Class 3 be Banks that were efficient collecting deposit and still had overall efficiency.

6. If DMU's Do $\geq 0.8 \& \& G<0.8$

Class $1<$ DMU
else

If DMU's $G \geq 0.8 \& \& D o<0.8$

Class 2 <----------- DMU
Else

If DMU's $D o \geq 0.8 \& \& G \geq 0.8$
Class 3 <---------- DMU
else
Class 4 <---------- DMU
// categorizing the Banks based on their No on the basis on their overall efficiency

Let Class 1 be Banks efficient in investing deposit but was not able to attain overall efficiency.

Let Class 2 be Banks that realized overall efficiency even though they were not efficient in investing deposit.

Let Class 3 be Banks that were efficient in investing deposit and still had overall efficiency.

- Let Class 4 be Banks inefficient in collecting deposit and cannot also achieve overall.
- 
- 
- Let Class 4 be Banks inefficient in investing deposit and cannot also achieve overall efficiency

7. If DMU's No $\geq 0.8 \& \& G<0.8$

Class $1<---------$ DMU else

If DMU's $G \geq 0.8$ \&\& No $<0.8$
Class 2 <----------- DMU
else

If DMU's $N o \geq 0.8 \& \& G \geq 0.8$
Class 3 <----------- DMU
else
Class 4 <---------- DMU
8. End

### 3.2.1.2 Predictor Variables

These are variables also called independent variables or experimental variables employed in statistical analysis to forecast or predict another variable called target or dependent variable (Chi et al., 2011; El-habil, 2012 and Tsai et al., 2009). For this study, the predictor variables were: Fixed Assets excluding IT, IT expenditure and Number of Employees.

### 3.3 THE COMPUTER PROGRAMMING PLATFORM

The various computer programming presented in this thesis were done using the RMiner. The R can be regarded as a computer programming language with a huge inbuilt library of predefined functions that can be utilized to implement several jobs (TutorialPoint, 2016). R allows for the incorporation of codes written in the C, C++, .Net, Python or FORTRAN languages for efficiency. R is open source software accessible under the GNU General Public License, and has pre-compiled binary versions which are provided for various operating systems like Linux, Windows and Mac. The RMiner offers a set of comprehensible functions (e.g. mining, saveMining) for classification and regression tasks. In particular, the library has the caret, C5.0, randomForest, neuralnet and kernlab (SVM) packages. It is important to mention that all the programming codes used for the entire study was code in R using the R studio. The codes were run on an intel® Celeron® CPU N2840 @ 2.16 GHz with an installed memory (RAM) of 2.00 GB . The machine is also a 64 -bit operating system, x 64 based processor with windows 8 .

### 3.4 BUILDING THE PREDICTIVE MODELS FOR PREDICTING BANKS EFFICIENCIES

### 3.4.1.1 The C5.0 Algorithm

The C5.0 Decision Tree algorithm is an extension or improvement of both ID3 and C4.5 algorithms, and therefore shares the same basic principles with the previous algorithms ID3 and C4.5 (Ananda \& Wibisono, 2014).Th is algorithm for tree construction uses the greedy algorithm Ross (1994) and Yuan \& Lin (2006) principle that construct tree from top to down (top down) recursively by divide and conquer (Ananda \& Wibisono, 2014). The C5.0 algorithm is shown in algorithm 3.2.

## Algorithm 3.2 : Generate_decision_tree

Input: data training samples; list of attributes; attributes_selection_method.
Output: decision tree.

## Method:

1. create a node $N$,
2. 

if samples has the same class, $C$, then
3. return $N$ as leaf node with class $C$ label;
4.

If list of attributes is empty then
5.
return $N$ as leaf node with class label that is the most class in the samples.
6.

Choose test-attribute, that has the most GainRatio using attributes_selection_method;
give node $N$ with test_attribute label;
for each $a_{i}$ pada test-attribute;
Add branch in node $N$ to test-attribute $=a_{i}$;
7.
8.
9.
10. $\quad$ Make partition for sample $s_{i}$ from samples where test-attribute $=a_{i}$;
11. If si is empty then
12. attach leaf node with the most class in samples;
13. else attach node that generate by Generate_decision_tre

| 14. | endfor |
| :--- | :--- |
| 15. | return $N$; |

Using the C4.5 algorithm as shown above, a DT model can be constructed as follows. For instance, if there is single set of training dataset S that consist of the attributes ( $\mathrm{P} 1, \mathrm{P} 2, \mathrm{P} 3, .$. and classes also comprising of (M1, M2, M3,... ). The C4.5 algorithm would work as follows:

1. If S is not free and all the dataset samples contain the equal class of Mi , then the Decision

Tree for S is a leaf node with label Mi.
2. Else if the attribute is free then the Decision Tree has a leaf node with label Mj where Mj is the uppermost class in the training samples $S$.
3. If S comprises a sample that has a dissimilar class of the partition S into $\mathrm{S} 1, \mathrm{~S} 2, \mathrm{~S} 3, \ldots \mathrm{Sn}$. Training sample S partitioned by different values of attribute Pm , which at the time assumes the role of a parent node. Suppose Pm involves 3 categories of values that are n1, n2, n3, then S will be partitioned into three subsets, namely the value of $\mathrm{Pm}=\mathrm{n} 1, \mathrm{n} 2=\mathrm{Pm}$, and $\operatorname{Pm}=\mathrm{n} 3$. This process continues recursively with the base case of step 1 and step 2. Attribute that would serve as the parent node or attribute that would partition the data is done by calculating the gain. Gain is used to select the attributes to be tested based on information theory concepts of entropy.

Entropy: Information entropy (or Shannon Entropy) defined by Shannon (1948) is an assessment which is founded on the possibility that is employed to calculate the quantity of doubt (Ananda \& Wibisono, 2014; Olaru \& Wehenkel, 2003 and Pandya \& Pandya, 2015).

Let $S$ be a random variable with number
$s_{i}, \quad i \in\{1, . . . n\}, \quad$ and probability mass function $p$.
Entropy of $S, H(S)=\sum^{n_{i=1}}-p\left(s_{i}\right) \log _{2}\left(p\left(s_{i}\right)\right)$
Note: For the purpose of entropy calculation, $\log _{2}(0)=0$
The entropy function $H(S)$ in the above formulae satisfies the following:

- $H(S)$ is continuous in $p\left(s_{i}\right), i \in\{1, . . . n\}$.
- If $p\left(s_{i}\right)={ }_{n}{ }^{1}$ for $i=1$,. . . $n$, then $H(S)$ is monotonically increasing function.
- $H(S, Q)=H(S)+H(Q)$ is addictive for any choice of two independent random variables $S$ and $Q$.


## Information Gain

Information gain in C5.0 algorithm is the variation in entropy that ensues just when after segregating the data based on an attribute. With an understanding of a random variable $S$, one can suggest that there are $N$ attributes $\left\{a_{i}\right\}^{N_{i=1}}$ connected to each observation, where $a_{i}=a_{i}(S)$ for all $i=1, \ldots, N$. This suggests observations on the form $\left(s_{i}, a i\left(s_{i}\right), \ldots, a_{N}(s i)\right)$, where each attribute $a i$ is in itself a realization of a random variable.

Assuming $\left\{s_{i}\right\}^{n_{i=1}}$ is the set of possible outcomes of the random variable S with entropy $\mathrm{H}(\mathrm{S})$.

Let A be the set of all possible values of the N -tuple $\left(a i\left(s_{i}\right), . ., a_{N}(s i)\right), i=1, \ldots$
, n. For each function ai, let $\mathcal{A}_{i}$ denote the set of possible outcomes of ai.

For a set of realizations S of the random variable S and attributes A , then the information gain $a$ split of S on attribute $a i\left(s_{i}\right)$ is given by $\operatorname{IG}\left(S, \mathcal{A}_{i}\right)=H(S)-\sum_{\alpha \epsilon \mathcal{A} i}| | S^{S^{\alpha} \mid} \mid H\left(S_{\alpha}\right)$

Where $S_{\alpha}=\left\{s \in S \mid a i\left(s_{i}\right)=\alpha\right.$

### 3.4.2 The Random Forest (RF) Algorithm

Algorithm 3.3: Binary Recursive Partitioning

Let $\quad \mathfrak{D}=\left\{\left(x_{1}, y_{1}\right), \ldots,\left(x_{N}, y_{N}\right)\right\}$ represent the training data, with $x_{i}=$ $T$
$\left(x_{i, 1}, \ldots, x_{i, p}\right)$.

1. Begin with all observations $\left\{\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{y}_{\mathbf{1}}\right), \ldots,\left(\boldsymbol{x}_{N}, \boldsymbol{y}_{N}\right)\right.$ in a single node.
2. Reiterate the following steps recursively for each unsplit node until the ending condition is met:
a. Discover the finest binary split amongst all binary splits on all $p$ predictors.
b. Split the node into two offspring nodes using the best split (step 2a).
3. At each prediction at $x$, forward $x$ down the tree till it settles in a terminal node. Let $k$ denote the terminal node and let $\boldsymbol{y}_{\boldsymbol{k} \mathbf{1}}, \ldots, \boldsymbol{y}_{\boldsymbol{k} \boldsymbol{n}}$ represent the target variables of the training data in node $k$. Predicted values of the target variable are

- $\hbar(\boldsymbol{x})==^{-} \boldsymbol{y} \overline{\boldsymbol{k}}={ }_{n}{ }^{\mathbf{1}} \boldsymbol{y} \boldsymbol{k} \boldsymbol{k}$ for regression
- $\hbar(x)=\operatorname{argmax} y \sum^{n_{i=1} I} I\left(y k_{i}=y\right)$ for classification where $I\left(y k_{i}=y\right)=1$

$$
\text { if } y k_{i}=y \text { and } 0 \text { otherwise }
$$

Algorithm 3.4 Random Forest

Let $\quad \mathfrak{D}=\left\{\left(x_{1}, y_{1}\right), \ldots,\left(x_{N}, y_{N}\right)\right\}$ represent the training dataset, with $\mathrm{x}_{\mathrm{i}}=$ $T$
$\left(x_{i, 1}, \ldots, x_{i, p}\right)$. For $\boldsymbol{j}=1$ to $J$

1. Select a bootstrap sample $\mathfrak{D}_{j}$ of size $\boldsymbol{N}$ from $\mathfrak{D}$.
2. Utilizing the bootstrap sample $\mathscr{D}_{j}$ in step(2) as the training dataset, fit a tree by using binary recursive partitioning :
a. Begin with all observations $\left\{\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{y}_{\mathbf{1}}\right), \ldots,\left(\boldsymbol{x}_{N}, \boldsymbol{y}_{N}\right)\right.$ in a single node.
b. Reiterate the following steps recursively for each unsplit node till the ending condition is met:
i. Pick $m$ predictors at random from the $p$ available predictors.
ii. Discover the finest binary split amongst all binary splits on all $m$ predictors iii. Split the node into two offspring nodes using the split from step ii. To make a prediction at a new point $x$

- $\mathcal{F}^{(x)}=\frac{1}{J} \sum_{j=1}^{J} \hbar_{j}(x)$ for regression
- $\mathcal{F}(x)=\operatorname{argmax}_{y} \sum_{j=1}^{J} I\left(\hbar_{j}(x)=y\right)$ for classification where $\hbar_{j}(x)$ is the prediction of the target variables at $\boldsymbol{x}$ using the jth tree.


## Utilizing the Out-Of-Bag Data

It is most likely that when taking a bootstrap sample from the dataset as illustrated in Algorithm 2.8 above, some observations would not find their way into the bootstrap sample. They are denoted as are "out-of-bag data", and may be very important in approximating generality error and predictor variable significant or importance. To approximate generality error, it is important to mention that if the trees are enormous, predictions trustingly attained by means of all the trees would be excessively positive if they are used to predict the target variable for observations that were in the training set D . Because of this, prediction of the target variable for observations found in the training dataset is accomplished by utilizing trees whose observation is out-of-bag. These types of predictions are referred to as out-of-bag predictions as illustrated in algorithm 3.5.

## Algorithm 3.5 Out-of-bag predictions

Let
$\mathcal{D}_{j}$ denote the jth bootstrap dataset and $\hbar_{j}(x)$ represent the prediction at
$X$ from the j th tree, for $\mathrm{j}=1, \ldots, \mathrm{~J}$. For $\mathrm{i}=1$ to N :

1. Let $\boldsymbol{g}_{i}=\left\{\boldsymbol{j}:\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{y}_{1}\right) \notin \mathfrak{D}_{j}\right.$ and let $\boldsymbol{j}_{i}$ be the cardinality of $\boldsymbol{g}_{i}$
2. State the out-out-of bag prediction at $\boldsymbol{x}_{\boldsymbol{i}}$ to be

- $\mathcal{F o o b}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)={ }^{1} \sum_{j=1}^{J} \hbar_{j}(\boldsymbol{x})_{J}$ for regression
- $\mathcal{F o o b}\left(x_{i}\right)=\operatorname{argmax}{ }_{y} \sum_{j=1}^{J} I\left(\hbar_{j}(x)=y\right)$ for classification where $\hbar_{j}(x)$ is the prediction of the target variables at $\boldsymbol{x}$ using the jth tree

In the case of regression with squared error loss, generality error is normally approximated through the out-of-bag mean squared error (MSE):

$$
\left.\begin{array}{rl}
=\frac{1}{-} \sum_{i=1}^{N}\left(y_{i}-{ }_{2}\right. & \\
& \text { MSEOob } \tag{48}
\end{array} \quad{ }_{N} \mathcal{F o o b}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)\right)
$$

where $\boldsymbol{\mathcal { F }} \boldsymbol{o o b}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$ is the out-of-bag prediction for observation i .
In the case of classification with zero one loss, generality error is also estimated by applying out-of-bag error rate:
$E_{\text {oob }}=\frac{1}{-1} \sum_{i=1}^{N} I_{N}\left(y_{i} \neq \boldsymbol{\mathcal { F o o b }}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)\right)$
Most misconceptions could be that the out-of-bag error rate is attained by calculating the outofbag error rate for individual tree, and finding the mean terror rates to suggest the out-ofbag error rate for the forest Cutler et al. (2011). Rather, we apply the error rate of the out-ofbag predictions. This gives us an opportunity to find a class-wise error rate for individual class, and an out-of-bag "confusion matrix" Cutler et al. (2011); Cutler (2010); Gislason et al.
(2006) by cross-tabulating $\boldsymbol{y}_{\boldsymbol{i}}$ and $\mathcal{F}$ oob $\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$.

## Permutation Variable Importance

Assessment of the most important and significant predictor variables is valuable for variable selection for analysis and explaining the fitted forest (Cutler et al., 2011). While it is standard in numerous applications to run a principal component analysis (PCA) to lessen dimensionality before fitting a classifier, it is conceivable that the principal component analysis capture the important information for prediction issue. For this situation, it might be desirable over acquiring variable significance straight from the algorithm and after that re-fit utilizing just the most vital predictors. Random Forests utilize an uncommon but instinctive measure of variable significance and importance. To do this measurement of the significance of variable $k$, the algorithm denoted as algorithm 3.6 is implemented for each tree.

[^0]
## To find the importance of variable $k$, for $k=1$ to $p$ :

1. (Find $\hat{\mathrm{y}}_{\mathrm{i} . \mathrm{J}}$ ) For $\mathrm{i}=1$ to N :
a. Let $\boldsymbol{g}_{i}=\left\{\mathrm{j}:\left(\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}\right) \notin \mathfrak{D}_{j} \quad\right\}$ and Let $\mathrm{j}_{\mathrm{i}}$ be the cardinality of $\boldsymbol{g}_{i}$ (Algorithm 2). b. Let $\hat{\mathrm{y}}_{\mathrm{i} . \mathrm{J}}=\hat{\mathrm{h}}_{\mathrm{J}}\left(\mathrm{x}_{\mathrm{i}}\right)$ for all $\mathrm{j} \in \boldsymbol{g}_{\boldsymbol{i}}$
2. (Find $\hat{y}_{\mathrm{i} . \mathrm{J}}^{*}$ ) For $\mathrm{j}=1$ to J :
a. Let $\mathfrak{D}_{j}$ be the jth bootstrap sample (Algorithm 2).
b. Let $\boldsymbol{g}_{i}=\left\{\mathrm{i}:\left(\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}\right) \notin \mathfrak{D}_{j}\right\}$.
c. Randomly permute the value of variable k for the data points $\left\{\mathrm{x}_{\mathrm{i}}: \mathrm{i} \in \boldsymbol{g}_{i}\right\}$ to give $\boldsymbol{\mathcal { P }}_{\boldsymbol{j}}$ $\left\{\mathrm{x}_{\mathrm{i}}{ }^{*}: \mathrm{I} \in \boldsymbol{g}_{\boldsymbol{i}}\right\}$
d. Let $\hat{\mathrm{y}}^{*}{ }_{\mathrm{i}, \mathrm{j}}=\hat{\mathrm{h}}_{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}^{*}\right)$ for all $\mathrm{i} \in \boldsymbol{g}_{\mathrm{i}}$.
3. For $\mathrm{i}=1$ to N :

- For classification:

$$
\operatorname{Imp}_{\mathrm{i}}={\underset{j}{1}}_{1}^{j_{i}} \sum_{j \in g_{i}} I\left(\left(\mathrm{y}_{\mathrm{i}} \neq \hat{\mathrm{y}} *_{\mathrm{i}} \cdot \mathbf{j}\right)-\frac{1}{j_{\mathrm{i}}} \sum_{j \in g} \quad I\left(\mathrm{y}_{\mathrm{i}} \neq \hat{\mathrm{y}}_{\mathrm{i}} \cdot \mathbf{j}\right)\right) .
$$

- For regression:

|  | i <br> $\mathrm{Imp}=\sum$ <br> $\mathbf{j}_{\mathrm{i}}$ <br> $\boldsymbol{j} \in \boldsymbol{g}_{i}$$\left(\left(\mathbf{y}_{\mathrm{i}} \neq \hat{\mathrm{y}} *_{\mathrm{i}} \cdot \mathbf{j}\right)^{2}-\frac{1}{j_{i}} \sum_{j \in g_{i}}\right.$ | $\left.I\left(\mathbf{y}_{\mathrm{i}} \neq \hat{\mathrm{y}}_{\mathrm{i}} \cdot \mathbf{j}\right)^{2}\right)$. |
| :--- | :--- | :--- | :--- |

### 3.4.3 The Artificial Neural Network

This thesis only considers applying the algorithms, specifically the back propagation algorithm.
The pseudocode for the MLP activity proposed by (Koivo, 2008) is as follows:

1. The structure of the network is first defined. In the network, activation functions are chosen and the network parameters, weights and biases, are initialized.
2. The parameters associated with the training algorithm like error goals, maximum number of epochs (iterations), etc., are defined.
3. The training algorithm is called.
4. After the Neural Network has been determined, the result is first tested by simulating the
5. Final validation must be carried out with independent data.

Now let's consider a network with M layers $\mathrm{m}=1,2, \ldots, \mathrm{M}$

- $V_{i}{ }^{m}$ from the output of the ith unit of the mth layer
- $V_{i}{ }^{o}$ is a synonym for $\boldsymbol{x}_{\boldsymbol{i}}$ of the ith input and subscript m layers m's layers, not patterns
- $W_{i j^{m}}$ mean connection from $V_{j}^{m-1}$ to $V_{i}^{m}$.


## Algorithm 3.7 ANN Back Propagation

1. Initialize the weights to small random values
2. Choose a pattern $\boldsymbol{x}^{\boldsymbol{d}} \boldsymbol{k}$ and apply to the input layer $\boldsymbol{V}^{\boldsymbol{o}} \boldsymbol{k}=\boldsymbol{x}_{\boldsymbol{d}}^{\boldsymbol{k}}$ for all k
3. Propagate the signal through the network $V^{m_{i}}=\boldsymbol{f}\left(\boldsymbol{n e t}^{\boldsymbol{m}_{i}}\right)=\boldsymbol{f}\left(\sum_{j} \boldsymbol{W}^{m_{i j}} \boldsymbol{V}^{m_{j}-\mathbf{1}}\right)$
4. Compute the deltas for the output layer, $\boldsymbol{\delta}^{m_{i}}=\boldsymbol{f}^{\prime}\left(\boldsymbol{n e t}^{M_{i}}\right)\left(\boldsymbol{t}_{\boldsymbol{d}}^{\boldsymbol{d}}-\boldsymbol{V}^{M_{i}}\right)$
5. Compute the deltas for the preceding layer for $\mathrm{m}=\mathrm{M}, \mathrm{M}-1, . .2$,

$$
\delta_{m i-1}=f^{\prime}\left(\text { net }_{m i-1}\right) \sum_{j} W_{m i j} \delta_{m i}
$$

6. Update all connections
$\Delta W_{m i j}=\eta \delta_{m i} V_{m j-1} \quad \Delta W_{n e w i j}=W_{n e w i j}+\Delta W_{i j}$
7. Move to step 2 and repeat for the next pattern

The feed forward network is the easiest and common type of network (Boritz \& Kennedy, 1995 and Kotsiantis, 2007). Training the NN is the method of setting the best weights on the inputs output of the Neural Network with the measured input data. This is compared to the measured
outputs. of each of the units. Back propagation happens to be the most common technique for
calculating the error gradient for a feed forward network (Kotsiantis, 2007). Neural Networks
perform well in applications when the functional form is nonlinear. They are useful in situations
where normal mathematical computation and prior knowledge on the relationship between
inputs and outputs are anonymous. Moreover, an initial step of feature selection before learning
is needed. Multilayer Perceptron (MLP) is applied to predict the impact of IT investment on
banks performance using the study data sets. In this study, linear combinati on functions and sigmoid transfer functions were used. The S-shaped or Binary sigmoidal function is by far the most common transfer function (Geoffrey \& Yau, 2007). The formula for the sigmoid is given:

$$
\begin{equation*}
\operatorname{Sigmoid}(x)=1 \quad+^{1} e_{-x} \tag{50}
\end{equation*}
$$



Figure 3.6: The proposed framework (Author's construct)

### 3.6.3.3 Programming Environment and Dataset

In this part of the methodology, the study presents the data that were used and their descriptions.
The entire dataset was obtained from the dataset that was used in the previous two sections to
build both the DT and RF models. The codes for the NN model building was written in R using the RMiner studio with the "neuralnet" package (Fritsch et al., 2016).

## The Output of the NN model

For banks' performance or efficiency indicators, the study considered two cases (Case 2 and Case 4). Each output consists of four different classes (Classes 1, 2, 3 and 4).

## NN Model Inputs

The study also adopted the predictor variables in both cases (Case 2 and Case 4 ) that were used in the DT and RF model as the NN model inputs.

## Number of Hidden Layer and Neurons for the NN

This current study adopted the use of only one hidden layer with five (5) Neurons in a three (3) layer network. This number of hidden neurons was chosen based on the equation $\mathrm{Nh}=\mathrm{Ni} 1$ proposed by Tamura \& Tateishi (1997) and cited by Vujičić et al. (2016). Where Nh is the number of hidden neurons to be used and Ni is also the number of input neurons. For this study, the number of inputs were 6 , meaning $\mathrm{Ni}=6$.

### 3.4.4 The Logistic Regression Algorithm

Logistic regression algorithm which is the second topmost Machine Learning Algorithm was also used (Appiahene \& Missah, 2019). Logistic regression is named for the function used at the core of the method, the logistic function. It measures the correlation between the dependent variable and the one or more independent variables, by estimating probabilities using its core logistic function. These probabilities must then be converted into binary values in order to actually make a prediction. This is the responsibility of the sigmoid function which is an Sshaped curve that can take any real-valued number and map it into a value between the range of 0 and 1 , but never exactly at those limits. This value between 0 and 1 will then be transformed into either 0 or 1 using a threshold classifier. The figure below illustrates the steps that logistic regression goes through to give you the desired output (efficiency class).


Figure 3.7 : The Logistic Regression Steps (Donges, 2019)
It is important to understand that the goal of an analysis using logistic regression is the same as that of any model-building technique used in Machine Learning: To find the best fit and most parsimonious. What actually makes a logistic regression algorithm different from the first topmost Machine Learning Algorithm, Linear Regression Algorithm is the outcome variable. In the logistic regression model, the outc ome variable is binary or dichotomous. Logistic regressions work with odds rather than proportions. The odds are simply the ratio of the proportions for the two possible outcomes. The formula for the logistic regression is given by:

$$
\begin{equation*}
\log e\left(1 \quad-p_{p}\right)=\beta_{0}+\beta_{1} X_{i 1}+\beta_{2} X_{i 2}+\cdots+\beta_{k} X_{k}+, i=1, \ldots n \tag{51}
\end{equation*}
$$

Where $\beta s$ are the regression parameters, the Xs are the explanatory variables and n is the sample size.

The Logistic Regression Algorithm (Angel et al., 2016) adapted for this study is described below. This algorithm was implemented in R codes using the package "elrm" Zamar et al.
(2007) .

Algorithm :3.8 The Logistic Regression Algorithm absolute error (MAE), relative bias

## Input:

1. Training algorithm L (logistic regression)
2. Sample matrix $X$
3. Labels vector $\mathrm{y}=[1, \ldots \mathrm{~K}]$
4. Initial regresor parameters vectors $\boldsymbol{\theta}_{\boldsymbol{i}}$ Main:

For $\mathrm{i}=1: \mathrm{K}$
Create a new binary vector $\boldsymbol{y}_{\boldsymbol{i}}$ for each label where $\boldsymbol{y}_{\boldsymbol{i}=\mathbf{1}}$ if it belong to the label and $\boldsymbol{y}_{\boldsymbol{i}}=\mathbf{0}$ if it does not belong.

Apply L to X to find $\boldsymbol{\theta}_{\boldsymbol{i}}$

## Output:

$\boldsymbol{\theta} \boldsymbol{i}$ Parameters vector for each regresor.

To implement the Logistic Regression Algorithm in R, the study adopted the following steps:

- Data Pre-processing step
- Fitting Logistic Regression to the Training set
- Predicting the test result $\square \quad$ Test accuracy of the result
- Visualizing the test set result.


### 3.4.5 Metrics for Evaluating Machine Learning Algorithms

The comparison of machine learning algorithms models should normally be done using the standard machine learning evaluation metrics and some of the evaluation metrics (Yao et al., 2017) that are usually used include but not limited, the root mean squared error (RMSE), mean (rBIAS), relative mean separation (rMSEP), mean absolute percentage error (MAPE), and root
mean squared prediction error (RMSPE). These standard machine learning evaluation metrics
are defined as:

$$
\begin{align*}
& \text { RMSE }=\sqrt{\frac{1}{m} \sum_{i=1}^{m}\left(\widehat{z_{l}}-z_{i}\right)^{2}}  \tag{52}\\
& M A E=\frac{1}{-} \sum_{i=1}^{m}\left|\widehat{z_{l}}-_{m} \quad z_{i}\right|
\end{align*}
$$

(53) $r$ BIAS $=\frac{1}{-} \sum_{i=1}^{m}\left(\widehat{z}_{l}-z_{i}\right)$

$m z$
$r M S E P=\sum^{m_{i=1}(z i-z i)^{2} / \sum^{m_{i=1}}(z p-z i)^{2}}$
MAPE $=\left(\left.\left.\frac{1}{-} \sum_{i=1}^{m}\right|^{\underline{\hat{z}_{i}}-z_{i}} \right\rvert\,\right) 100$

RMSPE $=\sqrt{\frac{\sum_{i=1}^{m}\left(\tilde{\varkappa}_{i}-z_{i}\right)^{2}}{m}}$

Where, $m$ is the total number of observations we want to validate, $z_{i}$ is the data indexed by $i, z_{i}$ is the prediction value, $z$ and $z p$ are the arithme tic mean of the observations and predictions respectively.

## SANE

(54)

## CHAPTER 4

## RESULTS AND DISCUSSIONS

### 4.0 Introduction

This chapter present and discusses results on the following: the application Decision Tree Algorithms, Random Forest Algorithms and Artificial Neural Network models for predictions; (2) the use of DEA for efficiency measurement and classification of the banks into various classes using the proposed Bank Classification Algorithm (3) the application of the four proposed machine learning models in predicting the efficiency of banks.

### 4.1 DISCUSSIONS OF THE PREDICTION OF THE BANKS USING DECISION TREE ALGORITHM

## Case 2-The Deposit Stage

According to the result, the DT model gave a high prediction accuracy of $50.8 \%$ and kappa value of $29.6 \%$ with a P-Value of 0.003298 . Thus, the DT model was able to predict classify the banks using the classification classes as response variables at an accuracy rate of about 50\% in the Case 2. Using the RMSE, MAE, MAPE, RMSPE and rBIAS as the performance evaluation measurement of the model, the following results were suggested: $\mathrm{RMSE}=0.70181$, $\mathrm{MAE}=0.492537$, $\mathrm{MAPE}=49.2537313$, $\mathrm{RMSPE}=0$ and $\mathrm{rBIAS}=0.492537$.

The detailed results of the performance measurement of the DT model in classifying the various classes under Case 2 are shown in the confusion matrix generated by the Case 2 analysis below:

## Confusion Matrix and Statistics



```
                        Accuracy : 0.5075 95%
    CI : (0.4198, 0.5948)
    No Information Rate : 0.3881
P-Value [Acc > NIR] : 0.003298
```

$$
\text { Kappa : } 0.2958
$$

## Case 4 -The Investment Stage

Under the Case 4, the DT model had a very low accuracy of $38.8 \%$ and kappa value of $16.6 \%$ with a P-Value of 0.08513 as compared to Case 2. In this the case of the DT in case the model cannot be used since its P -Value is above the threshold for significance.

Using the RMSE, MAE, MAPE, RMSPE and rBIAS as the performance evaluation measurement of the model, the following results were suggested: RMSE=0.782266, MAE=0.61194, MAPE $=61.19403$, RMSPE $=0$ and rBIAS $=0.61194$. The detailed results of the performance measurement of the DEA-DT model in predicting the various classes under Case 4 are shown confusion matrix below:


### 4.2 DISCUSSIONS OF THE PREDICTION OF THE BANKS USING RANDOM FOREST ALGORITHM

## Case 2-The Deposit Stage

The out-of-bag estimates of the error rate given by Eoob $={ }^{1} \sum_{i=1}^{N} I_{N}\left(y_{i} \neq \boldsymbol{F}\right.$ oob $\left.\left(\boldsymbol{x}_{\boldsymbol{i}}\right)\right)$ were used to choose the optimum Random Forest parameters (formula $=$ Case2 $\sim$ data $=$ TrainSet, ntree $=$ 500, mtry $=2$, importance $=$ TRUE). According to the result, the RF model gave a high overall prediction accuracy of $58.2 \%$. This means that the Random Forest model was able to predict the efficiency of the new $30 \%$ bank branches at an overall accuracy rate of about $58 \%$ in the Case 2. Using the RMSE, MAE, MAPE, RMSPE and rBIAS as the performance evaluation measurement of the model, the following results were suggested: RMSE=0.64646, MAE $=0.41791$, $\mathrm{MAPE}=41.79104$, RMSPE $=0$ and $\mathrm{rBIAS}=0.41791$. The following is also the confusion matrix generated by Case 2 analysis:

Confusion Matrix and Statistics


| 1 | 2 | 1 | 5 | 1 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Class 2 | 7 | 16 | 4 | 14 |  |
| Class 3 | 6 | 2 | 16 | 1 |  |
| Class 4 | 3 | 8 | 4 | 44 |  |

Overall Statistics Accuracy : 0.5821 95\% CI : (0.4938, 0.6667) No Information Rate : 0.4478 P-Value [Acc > NIR] : 0.001219

Kappa : 0.3959
Mcnemar's Test P-Value : 0.138147

## Case 4-The Investment Stage

For Case 4 out-of-bag estimates of the error rate (Eoob) were used to select the optimum Random Forest parameters (formula $=$ Case $4 \sim$., data $=$ TrainSet, ntree $=500, \mathrm{mtry}=6$,
importance $=$ TRUE). Under the Case 4, the RF model had a lower prediction accuracy of $40.3 \%$ as compared to the Case 2 which had $58.21 \%$. Using the RMSE, MAE, MAPE, RMSPE and rBIAS as the performance evaluation measurement of the model, the following results were suggested: $\quad$ RMSE $=0.772667, \quad \mathrm{MAE}=0597015, \quad \mathrm{MAPE}=59.7015, \quad \mathrm{RMSPE}=0$ and rBIAS $=0597015$. Confusion Matrix and Statistics

| Reference |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Prediction | Class 1 | Class 2 | Class 3 | Class | 4 Class |
| 1 | 0 | 3 | 3 | 2 |  |
| Class 2 | 4 | 4 | 6 | 6 |  |
| Class 3 | 6 | 8 | 17 | 6 |  |
| Class 4 | 21 | 8 | 7 | 33 |  |

```
                        Accuracy : 0.403
            95% CI : (0.3192, 0.4911)
    No Information Rate : 0.3507
P-Value [Acc > NIR] : 0.120256
```

```
Kappa : 0.1615 Mcnemar's Test P-
```

Kappa : 0.1615 Mcnemar's Test P-
Value : 0.007651

```
    Value : 0.007651
```


### 4.3 DISCUSSIONS OF THE PREDICTION OF THE BANKS USING ARTIFICIAL NEURAL NETWORK

## Case 2-The Deposit Stage

When the NN model which was trained and validated using 310 (70\%) bank branches dataset was used on the other 134 bank branches, the NN model was able to classify the banks into their respective classes at an accuracy rate of $14.2 \%$. Using the RMSE, MAE, MAPE, RMSPE and rBIAS as the performance evaluation measurement of the model, the following results were suggested: $\mathrm{RMSE}=1.176902$, MAE $=1.0421235$, MAPE $=57.45151114$,

RMSPE= $=1.179884$ and rBIAS $=-0.01943$.

The following is also the confusion matrix of the NN analysis:

```
Confusion Matrix and Statistics
    Reference
    Prediction 1 2 3 4
        0 0 0
    1
    2 0}000
    3 24 27 19 64
    4 0 0 0 0
    Overall Statistics
Accuracy : 0.141791 95% CI :
    (0.0875778, 0.2125375)
```

```
No Information Rate : 0.4776119
    P-Value [Acc > NIR] : 1
```


## Case 4-The Investment Stage

Under Case 4, the NN model had an accuracy of $16.42 \%$ as compared to Case 2 which was a little higher. Using the RMSE, MAE, MAPE, RMSPE and rBIAS as the performance evaluation measurement of the model, the following results were suggested:

RMSE $=1.241123, \mathrm{MAE}=1.11657, \mathrm{MAPE}=65.49859, \mathrm{RMSPE}=1.227733$ and $\mathrm{rBIAS}=-$
0.02396. The following is also the confusion matrix of the NN analysis:

```
Confusion Matrix and Statistics
    Reference
        Prediction 1 2 3 4
\begin{tabular}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2
\end{tabular}
        33}223\quad22 5
                        0 0 0 0
                        Overall Statistics
            Accuracy : 0.1641791
        95% CI : (0.1058435, 0.2379513)
        No Information Rate : 0.4104478
            P-Value [Acc > NIR] : 1
```

                Kappa : -0.009009
    
### 4.4 DISCUSSIONS OF THE BANKS' EFFICIENCY SCORES AND CLASSIFICATION 4.2.1 Bank Efficiency Determination Using the Adapted DEA Two-Phase BuildHull

## Algorithm



For each Bank, the technical efficiencies in both stages (Deposit Stage, Investment Stage) and their corresponding overall efficiencies were analyzed using CCR DEA algorithm proposed by Mehrabiana (2013) and depended on non-Archimedean Charnes-Cooper-Rhodes (CCR). Using the 444 bank branches, the efficiency of each bank branch at each stage was analyzed using the scenarios as discussed one after the other below.

Scenario 1: When an efficient unit is defined as a unit with an efficiency score of 1unit or $100 \%$.

For banks efficiency in terms of utilizing their resources to collect deposit from customers , only $14(3.15 \%$ ) bank branches were efficient (had $100 \%$ efficiency ). Just 33(7.43\%) bank branches had an efficiency scores of between $80 \%$ to $99 \%, 1$ ( $0.23 \%$ ) bank branch also had efficiency score of between 70 to $79,21(4.73 \%)$ had efficiency score of between 60 to $69 ; 19$ ( $4.28 \%$ ) had between 50 to 59 and 356 ( $80.18 \%$ ) had an efficiency score below $50 \%$.This $356(80.18 \%)$ number of bank branches confirms the fact that a lot of Ghanaian banks are not efficient in using their resources to collect deposits from customers. This confirms the fact that most banks in Ghana are struggling to meet the minimum capital requirements set by the central bank (Bank of Ghana) in 2017 (Addison, 2017, 2018a and 2018b) .The results of the deposit stage efficiency is shown in Figure 4.1.


Figure 4.1: A graph showing the deposit efficiency scores of the 444 DMUs (Author's construct)

For banks efficiency with regarding investing customers' deposits shown in Figure 4.2, only

1bank (DMU200) was efficient in investing the deposit to generate profit for the banks while only 1 bank (DMU219) had an efficiency score of between $80 \%$ to $100 \%$. This result suggests that close to $99.5 \%$ of Ghanaian bank branches that were considered for the study were not efficient in investing customers deposits. This also confirms reports of crises that have hit the Ghanaian banking industry with their issues such as an alleged managers and board of directors squandering depositors' money without investing them Abubakar (2018), and has also lead to about seven (7) universal banks collapsing in 2017 and 2018( Addison, 2018a;

Addison, 2018b).


Figure 4.2: A graph showing the investment efficiency scores of the 444 DMUs (Author's construct).

For Overall efficiency in the entire banking operations also shown in Figure 4.3, 79(17.79\%) bank branches were efficient (had 100\% efficiency score) with the majority ( 290 representing, $65.32 \%$ ) of them having an efficiency score of between $80 \%$ to $99 \% .4(0.9 \%)$ bank branches had efficiency score of between 70 to $79 \%, 32(7.21 \%)$ had an efficiency scores of between 60 and $69 \%$ and finally $39(8.78 \%)$ branches had between 50 to $59 \%$. In terms of overall
efficiency, there was no bank branch that had less than $50 \%$ efficiency score evident in the Figure 4.3.


Figure 4.3: A graph showing the overall efficiency scores of the 444 DMUs (Author's construct)

This analysis means that even though most bank branches in Ghana do not experience a higher percentage of efficiency in collecting deposit and investing the deposit, they still enjoy higher overall efficiency. The results suggest that banks in Ghana should identify ways of improving their efficiencies in both the deposit stage and investment stage and should not only rely on their overall efficiency scores as means of measuring their performance and success.

In real a life situation, it is very difficult to have units or departments attaining $100 \%$ efficiency and bank branches in Ghana are no exception. Normally, in grading of Systems in terms of their efficiency, systems with efficiency value between $80 \%$ and $100 \%$ are also considered to be excellent (efficient) in their performance. This is also evident in the Ghanaian banking sector
where the central bank (Bank of Ghana) always has minimum capital requirement for banks and other financial institutions (Addison, 2017, 2018a, 2018b).

This means that there is always a "cutoff point" for banks to meet in order to be efficient and remain competitive in the banking sector. Based on this, the author also inferred and considered banks with an efficient value of $80 \%$ or more as efficient. The study therefore adopted and the efficiency "cutoff point" (DMU efficiency $\geq 0.8$ ) suggested by Wu (2006) and cited by Chen (2016) and Santos et al. (2017) to discussed the efficiency scores in all the stages as follows: Scenario

2: When an efficient unit is defined as a unit with an efficiency score of 0.8 or more (80-100\%).

Under this scenario, 47 (10.59\%) banks were efficient in collecting deposit from customers while only 2 banks (bank with DMU200 and bank with DMU219) were efficient in investing the collected deposit to generate profit. A good number of bank branches 369 (83.11\%) were efficient at the entire banking operation (overall stage).

In all, the analysis and discussion of the results, a certain trend was emerging as shown Figure 4.4. Thus, the banks with higher deposit efficiency were likely to have a higher overall efficiency and that deposit efficiency was contributing to overall efficiency of a bank as shown in the Figure 4.4.


Figure 4.4: A graph showing the efficiency scores of deposit, investment and overall stages of the 444 DMUs (Author's construct)

This is a graph representing the 444 banks used in the study. From Figure 4.4, banks such as DMU4, DMU10, DMU18 etc. had the highest deposit efficiencies and still had the highest overall efficiencies. What is also evident from the graph is that a lot of the banks were not efficient in investing deposit collected from customers, except bank DMU200 and bank DMU219, also had efficiency of $100 \%$ and $90 \%$ respectively.

### 4.2.2 Using the proposed Banks' Classification Algorithm (BC Algorithm) to classify

## Banks based on their Efficiencies Scores.

The various banks (DMUs) were classified into classes using the BC Algorithm proposed by this study using both the deposit stage efficiency and investment stage efficiencies. In this first
instance (Case 1 and Case 3), efficient bank was defined using DEA score as $100 \%$ (1 unit). As suggested earlier, it is very difficult to have units or departments attaining $100 \%$ efficiency in real life situation and bank branches in Ghana are no exception. The study, therefore , adopted the efficiency "cutoff point" (DMU efficiency $\geq 0.8$ ) suggested by Wu (2006) and cited by Chen (2016) and Santos et al. (2017) by classifying the bank branches based on their efficiency scores in the two stages using this "cutoff point". Using the overall efficiency as the basis, the efficiency of both stage I (banks efficiency in collecting deposit from customers) and the stage II ( banks efficiency in investing their deposit into securities and given out loans) were categorized into classes Wang et al. (1997) and without any order. In this case four scenarios were used and the results are shown in Appendix B.

Case 1: When stage I efficiency was categorized on the basis of overall efficiency and in both cases efficient unit is one with a $100 \%$ (1) efficiency value. The classifications are as follows:

- Class 1: Banks efficient in collecting deposit but were not able to attain overall efficiency.
- Class 2: Banks that realized overall efficiency even though they were not efficient in collecting deposit.
- Class 3: Banks that were efficient in collecting deposit and still had overall efficiency.
- Class 4: Banks inefficient in collecting deposit and cannot also achieve overall efficiency.

Case 2: When stage I efficiency was categorized on the basis of overall efficiency and in both cases efficient unit is one with $80 \%-100 \%$ (0.8-1) efficiency value.

The classifications are below:

- Class 1: Banks efficient in collecting deposit but were not able to attain overall efficiency.
- Class 2: Banks that realized overall efficiency even though they were not efficient in collecting deposit.
- Class 3: Banks that were efficient collecting deposit and still had overall efficiency.
- Class 4: Banks inefficient in collecting deposit and cannot also achieve overall efficiency.

Case 3: When stage II efficiency was categorized on the basis of overall efficiency and in
both cases efficient unit is one with $100 \%$ (1) efficiency value. The classifications are as follows:

- Class 1: Banks efficient in investing deposit but were not able to attain overall efficiency.
- Class 2: Banks that realized overall efficiency even though they were not efficient in investing deposit.
- Class 3: Banks that were efficient in investing deposit and still had overall efficiency.
- Class 4: Banks inefficient in investing deposit and cannot also achieve overall efficiency.

Case 4: When stage II efficiency was categorized on the basis of overall efficiency and in both cases efficient unit is one with $80 \%-100 \%(0.8-1)$ efficiency value. The classifications are as follows:

- Class 1: Banks efficient in investing deposit but were not able to attain overall efficiency.
- Class 2: Banks that realized overall efficiency even though they were not efficient in investing deposit.
- Class 3: Banks that were efficient in investing deposit and still had overall efficiency.
- Class 4: Banks inefficient in investing deposit and cannot also achieve overall efficiency. Using the proposed algorithm for the classification, the following were discovered:
4.2.2.1 For Cases 1 and 2: When the efficient value was $100 \%$ (1) , only 1 DMU (DMU 427) was in class 1 while the majority of the DMUs representing $82 \%$ were in class 4 , followed by class 2 and class 3 . This means that the majority (364) of the commercial bank's branches (DMU's) were not $100 \%$ efficient in collection deposit from customers and also in their overall efficiency. Only just about 13 DMUs were efficient in the collection of deposit from customers. The entire results of this case 1 analysis are shown in Table 4.1 with its corresponding bar chart also shown in Figure 4.5.

When the cutoff for efficient DMU was drop from $100 \%$ to $80-100 \%$, the majority ( $72.5 \%$ ) of the DMUs were now in class 2 which means that the majority of the DMUs even though were not efficient in collecting deposits (stage I), they were able to achieve overall efficiency. Just
about $10.6 \%$ of the DMUs were efficient in utilizing resources for collecting deposit from customers and also using the resources to achieve overall efficiency. The results of this are also presented in Table 4.2 and Figure 4.6.

Table 4.1.: Efficiency Classes (case 1) (Authors construct).

Efficiency classes (Case 1)

|  | Frequency | Percent | Valid Percent | Cumulative <br> Percent |
| :---: | ---: | ---: | ---: | ---: |
| Class 4 | 364 | 82.0 | 82.0 | 82.0 |
| Class 1 | 1 | .2 | .2 | 82.2 |
| Class 2 | 66 | 14.9 | 14.9 | 97.1 |
| Class 3 | 13 | 2.9 | 2.9 | 100.0 |
| Total | 444 | 100.0 | 100.0 |  |

## Bar Graph of Efficiency Classes (Case 1)



Figure 4.5: A bar graph showing the proposed efficiency classes (Case 1) and the number of DMUs (Authors construct).

Table 4.2: Efficiency classes (Case 2) (Authors construct).

Efficiency classes (Case 2)

|  | Frequency | Percent | Valid Percent | Cumulative <br> Percent |
| :---: | :---: | :---: | :---: | :---: |
| Class 4 | 75 | 16.9 | 16.9 | 16.9 |
| Class 2 | 322 | 72.5 | 72.5 | 89.4 |
| Class 3 | 47 | 10.6 | 10.6 | 100.0 |
| Total | 444 | 100.0 | 100.0 |  |

## Bar Graph of Efficiency Classes (Case2)



Figure 4.6: A bar graph showing the proposed efficiency classes (Case 2) and the number of DMUs (Authors construct).

For Cases 3 and 4: When the efficient DMU value was at $100 \%$ (1), only 1 DMU (DMU 201) was in class 1 while the majority of the DMUs representing $82 \%$ were not $100 \%$ efficient in investing deposit from customers and giving out loans and also in their overall efficiency
followed by class 2 and there was no DMU in class 3. Thus no DMU was $100 \%$ efficient in investing their deposit. 79 DMUs ( $17.8 \%$ ) were able to achieve overall efficiency even though there were not efficient in investing their deposits.

The entire results of this case 3 analysis are shown in Table 4.3 below with its corresponding bar chart also shown in Figure 4.7. When the cutoff for efficient DMU was drop from $100 \%$ to $80-100 \%$, majority 368 (82.9\%) of the DMUs were now in class 2 which means that the majority of the DMUs even though were not efficient in using their resources to invest their deposit (stage II), they were able to achieve overall efficiency. Just about $16.7 \%$ (74) of the DMUs were not efficient in investing their deposit from customers and also using resources to achieve overall efficiency. The results of this are also shown in Table 4.4 and Figure 4.8.

Table 4.3: Efficiency classes (Case 3) (Authors construct).
Efficiency Classes (Case3)

|  | Frequency | Percent | Valid Percent | Cumulative Percent |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Valid | Class 4 | 364 | 82.0 | 82.0 | 82.0 |
|  | Class 1 | 1 | .2 | .2 | 82.2 |
| Class 2 | 79 | 17.8 | 17.8 | 100.0 |  |
|  | Total | 444 | 100.0 | 100.0 |  |

## Bar Graph of Efficiency Classes (Case 3)



Figure 4.7: A bar graph showing the proposed efficiency classes (Case 3) and the number of DMUs. (Authors construct).

Table 4.4: Efficiency classes (Case 4) (Authors construct).

## Efficiency Classes (Case 4)

|  | Frequency | Percent | Valid Percent | Cumulative Percent |
| :---: | :---: | :---: | :---: | :---: |
| Class 4 | 74 | 16.7 | 16.7 | 16.7 |
| Class 1 | 1 | .2 | .2 | 16.9 |
| Class 2 | 368 | 82.9 | 82.9 | 99.8 |
| Class 3 | 1 | .2 | .2 | 200.0 |
| Total | 444 | 100.0 | 100.0 |  |

## Bar Graph of Efficiency Classes (Case 4)



Figure 4.8: A bar graph showing the proposed efficiency classes (Case 4) and the number of DMUs (Authors construct).

### 4.5 DISCUSSION OF THE PROPOSED BANK EFFICIENCY PREDICTION FRAMEWORK

The proposed framework that was developed for building the predictive model was done under two scenarios .The first scenario was when the model was developed taking into consideration Decision Tree and Random Forest Algorithms. In both Decision Tree and Random Forest Algorithms, the dataset used for building the predictive models consisted of predictor variables and target or response variables. The second scenario, the case of bu ilding a predictive model using Artificial Neural Network, the predictor variables used for both Decision Tree and Random Forest Algorithms were considered as input whiles its target or response variables as in the case of Decision Tree and Random Forest were also considered as output for the Neural Network model. The study, therefore, developed the model for building a predictive model under two cases considered above but they were all working using the same concept. The discussion of the proposed framework in this chapter takes into consideration the two scenarios;
thus the model for both Decision Tree/Random Forest Algorithm model development and Artificial Neural network predictive model development.

At the preliminary stage (stage I), raw data were collected from the banks. After data collection, it is preprocessed in stage II before finally entering the stage where the predictive model development takes place. In this case, the financial data such as the Cedi value of IT Expenditure (I), Fixed Asset(A),Total Deposit(D), Profit(R),Rate of Performing Loans(\%PL), and finally the number of employees (E) from banks were used to calculate the efficiencies of the various banks (DMUs) using the CRS technology. The efficiencies of the banks at both deposit and investment stages were classified into classes (Classes 1 to 4 ) without any order but on the bases of their overall efficiency using the proposed Bank Classification Algorithm. This then becomes the target or dependent variables.
Now the banks financial data such the IT expenditure, fixed Assets etc. were used as predictor variables to predict the efficiency scores (Classes) of each bank branch for building the models. In the case of the target variables, the efficiencies of the banks, grouped into classes (Classes 1-4) were based on scenarios or cases in the study, hence two cases were used where efficient unit is one with efficiency score greater than or equal to 0.8 .

This final dataset for building the model is then randomly divided into two, $70 \%$ were train or build and validate the model using K-fold cross-validation while the remaining 30\% (test dataset) was used to test the models. Thus in the case of the banks, $70 \%$ of the DMUs which were selected randomly from the total dataset were used to build and validate the model is now used to predict the efficiency of the other $30 \%$ banks (DMUs). During the model building, the dataset also goes through rule extraction and finally building the classifier. If the suggested model can work with as accepted or high accuracy, the model can be used to estimate the efficiencies of new banks.

### 4.5.1 DISCUSSIONS OF THE PREDICTION OF THE BANKS EFFICIENCIES USING THE DEA-DT MODEL

A total of 444 bank branches (DMUs) on two separate scenarios, Case 2 and Case 4 were implemented in a DEA-DT model using R codes in RMiner studio. In each, there were four dependent variables (Classes 1 to 4) and six (6) independent variables. Therefore, the target variables as performance (efficiency) measure of each category were predicted as either in Class 1, Class 2, Class 3 or Class 4. The performance measure of the DEA- DT model was done by using a confusion matrix (Delen et al., 2013), RMSE, MAE, MAPE, RMSPE and rBIAS. The dataset for this analysis in both cases was divided randomly into $70 \%$ (training and validation dataset) and $30 \%$ (test dataset). For the performance analysis of the proposed DEA-DT model, the test dataset was used.
To evaluate the prediction accuracy of the model, K-fold cross validation was implemented to lessen the bias of sampling data and ensuring model error randomness. K-fold cross validation randomly divides data into k subsets and one subset is used as testing data and $\mathrm{k}-1$ subsets are used as training data (Yousaf, 2016). This process is repeated k times to cover all data. The study used 10 -fold cross validation in the analysis. In addition to the k -fold cross validation the study also use repeated cross validation where data partitioning was done to split the dataset into training (70\%) and testing (30\%). To estimate the models' performance, RMSE, MAE, MAPE, RMSPE and rBIAS were used. Analysis of the prediction by the DEA-

DT model was done in two cases (Cases 2 and 4). In the first case, the Case 2 variables were the dependent variables and the result was compared with the classes designated by the CCR during the DEA stage. The entire results of the DEA-DT prediction of both cases are shown in Appendices C and D.

### 4.5.1 The Bank branches (132 DMUs) efficiency analysis using the DEA-DT model

## Result

### 4.5.1.1 Case 2 -The Deposit Stage

According to the result, the combined DEA-DT model gave a high prediction accuracy of
$91.04 \%$ and kappa value of $80.2 \%$ with a P-Value of $5.296 \mathrm{e}-07$. This means that the proposed DT model was able to predict the efficiencies of the $30 \%$ banks at an accuracy rate of about $91 \% \%$ in the Case 2. The detailed results of the performance measurement of the DEA-DT model in predicting the various classes under Case 2 are shown in the confusion matrix generated by the Case 2 analysis below:

## Confusion Matrix and Statistics

Reference


### 4.5.1.2 Case 4-The Investment Stage

Under the Case 4, the proposed DEA-DT model had the highest prediction accuracy of 97.8\% and kappa value of $92.7 \%$ with a P-Value of $1.48 \mathrm{e}-08$ as compared to Case 2 . The detailed results of the performance measurement of the DEA-DT model in predicting the various classes under Case 4 are shown confusion matrix below:

Confusion Matrix and Statistics


```
                Overall Statistics
                    Accuracy : 0.9776
    95% CI : (0.936, 0.9954)
No Information Rate : 0.8209
P-Value [Acc > NIR] : 1.48e-08
    Kappa : 0.9274
```

According to the study methodology, the analysis and discussions of the results, the following were suggested:

The proposed DEA-DT model performed very well in predicting the efficiencies of bank
branches in both cases as it was able to predict in both cases at accuracy of not less than $90 \%$ which is on the higher side compared to similar studies like ( $\mathrm{Wu}, 2006$ ).

The results also suggest that the DEA-DT model works best on predicting bank investment efficiencies as compared to deposit efficiencies.

The results also suggest that under both deposit and investment stage, the DEA- DT model performs well in both Case 2 and Case 4 with RMSE of 0.299253 and 0.14926 respectively.

### 4.5.2 RESULTS AND DISCUSSIONS OF THE BANK BRANCHES (132 DMUS) EFFICIENCY ANALYSIS USING THE DEA-RF ALGORITHM PREDICTIVE MODEL

### 4.5.2 The Bank branches (132 DMUs) efficiency analysis using the RF model Result

In each there were four dependent variables (Classes 1 to 4) and six (6) independent variables for both Case 2 and Case 4. Therefore, the target variables as performance (efficiency) measure
of each category were predicted as either is in Class 1, Class 2, Class 3 or Class 4. To evaluate the prediction of the RF model, k -fold cross validation was implemented to lessen the bias of sampling data and ensuring model error randomness. K-fold cross validation randomly divides data into k subsets and one subset is used as testing data and $\mathrm{k}-1$ subsets are used as training data (Yousaf, 2016) . This process is repeated $k$ times to cover all data. The study used 10 -fold Cross Validation in the analysis. In addition to the K-fold cross validation the study also used repeated cross validation where data partitioning was done to split the dataset into training (70\%) and testing ( $30 \%$ ). To estimate the model's performance, the following Machine Learning performance metrics were used RMSE, MAE, MAPE,

RMSPE and rBIAS. Analysis of the prediction by the DEA-RF model was done in two cases (Case 2 and Case 4). In the first case, Case 2 variables were the dependent variables and the result was compared with the classes designated by the CCR during the DEA stage. The entire results of the RF prediction in both cases are shown in appendix E and F .

### 4.5.2.1 Random Forest Order of Significant Predictor Variables

During the Case 2 DEA-RF model building and using the permutation variable importance algorithm proposed by Cutler et al. (2011), the DEA-RF estimated the following as important variables with respect to MeanDecraeseAccuracy (shown in Appendix I) in building the model which has been listed in order of magnitude ,thus, most important to the least :

Number of Employees, Total Deposits, IT Expenditure, Percentage of Performing Loans, Fixed Assets and Profits .
For MeanDecraeseGini also shown below in Appendix I, the order is as follows: Number of Employees, Total Deposits, IT Expenditure, Percentage of Performing Loans, Fixed Assets and Profits.

With respect to Case 4 and variable importance according to both MeanDecraeseAccuracy and MeanDecraeseGini are shown in the Appendix I. What was evidence in both cases was the variable Number of Employees and IT Expenditure were always leading the chart in both cases.

### 4.5.2.2 Case 2-The Deposit Stage

The out-of-bag estimates of the error rate given by $E O O \overline{\bar{b}}{ }_{N}{ }^{1} \mathcal{F}_{i \dot{j} \theta d}^{N}\left(x_{i}\right) y$ ) 表ere used to choose the optimum Random Forest parameters (formula $=$ Case $2 \sim$ data $=$ TrainSet, ntree $=500$, mtry $=$ 2, importance $=$ TRUE $)$. According to the result, the DEA-RF model gave a high overall prediction accuracy of $94.78 \%$. This means that the proposed Random Forest model was able to predict the efficiency of the new $30 \%$ bank branches at an overall accuracy rate of about $95 \%$ in the Case 2.

The results show that DEA-RF model was able to predict 93 out of 98 ( $94.9 \%$ accuracy) bank branches that were able to realize overall efficiency even though they were not efficient in collecting deposits from customers. The DEA-RF model was able to predict (92.3\%) bank branches in class 3. Finally, the model was able to predict at an accuracy of (95.6\%) 22 out of the 23 banks inefficient in collecting deposit and also inefficient in overall banking operations. The following is also the confusion matrix generated by Case 2 analysis:

```
randomForest(formula = Case2 ~ ., data = TrainSet, ntree =
    500, mtry = 6, importance = TRUE)
    Type of random forest: classification
            Number of trees: 500
    No. of variables tried at each split: 6
                    Reference
        Crediction Class 2 Class 3 Class 4 Class
            Overall Statistics
            Accuracy : 0.9478 95% CI
            :(0.8953, 0.9787)
            No Information Rate : 0.7313
        P-Value [Acc > NIR] : 8.591e-11
            Kappa : 0.8813
```


### 4.5.2.3 Case 4 -The Investment Stage

For Case 4 out-of-bag estimates of the error rate (Eoob) were used to select the optimum Random Forest parameters (formula $=$ Case $4 \sim$., data $=$ TrainSet, ntree $=500, \mathrm{mtry}=6$, importance $=$ TRUE). Under the Case 4, the proposed RF model had the highest prediction accuracy of $97.01 \%$ as compared to the Case 2 which had $94.78 \%$.

In the case of the Classes 1 and 3, there was no bank branch that was in the test dataset. The DEA-RF model was able to predict 110 out of 113 at an accuracy of $97.3 \%$ of banks that realized overall efficiency even when they were not efficient in investing deposit from customers.

Finally, it was able to predict 20 out of 21 bank branches, class 4 -banks efficient in investing deposits and cannot also achieve overall efficiency. The following is also the confusion matrix generated by the Case 4 analysis:

```
randomForest(formula = Case4 ~ ., data = TrainSet, ntree =
    500, mtry = 6, importance = TRUE)
    Type of random forest: classification
                            Number of trees: 500
        No. of variables tried at each split: 6
        Confusion Matrix and Statistics
```

| Reference |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Prediction Class | 1 | Class | 2 | Class | 3 | Class | 4 |
| Class |  |  |  |  |  |  |  |



According to the study methodology, the analysis and discussions of the results, the following were suggested:

The proposed DEA-RF model performed very well in predicting the bank branches efficiency in both cases as it was able to predict in both cases at accuracy of not less than $94 \%$.

Finally the results also suggest that the DEA-RF model works best on all the classifications with RMSE of 0.228558 and 0.167248 in Case 2 and Case 4 respectively which is also on a better side compared to similar study by (Hamad \& Anouze, 2015).

### 4.5.3 EMPIRICAL RESULTS AND DISCUSSIONS OF THE PREDICTIONS BY THE DEA-NN MODEL

The study constructed 3-layer networks. The number of hidden neurons (5) based on the equation $\mathrm{Nh}=\mathrm{Ni}-1$ proposed by Tamura \& Tateishi (1997) were used in both cases and were set to be the same as the back propagation learning algorithm which has been frequently used in business classification studies was adopted. The study also used the "neuralnet" package and implemented it in R (Fritsch et al., 2016) studio using R codes.

To evaluate the prediction of the model, K -fold cross validation was implemented to lessen the bias of sampling data and ensuring model error randomness. K-fold cross validation randomly
divides data into k subsets and one subset is used as testing data and $\mathrm{K}-1$ subsets are used as training data (Yousaf, 2016) . This process is repeated k times to cover all data. The study used 10 -fold cross validation in the analysis. In addition to the k -fold cross validation the study also use repeated Cross Validation where data partitioning was done to split the dataset into training ( $70 \%$ ) and testing ( $30 \%$ ). To estimate the model's performance, machine learning models such as RMSE, MAE, MAPE, RMSPE and rBIAS were used as the evaluating measures. A 10 -fold cross-validation (CV) was applied to check the performance of all predicting models in each case. Analysis of the prediction by the DEA- NN model was done in two cases (Cases 2 and 4). In the first case, Case 2 variables were the dependent variables and the result was compared with the classes designated by the CCR during the DEA stage. The entire results of the DEANN prediction both cases are shown in G and H .

### 4.5.3.1 The Bank Branches (132 DMUs) Efficiency Analysis Using the DEA-NN Model

Four hundred and forty-four (444) commercial bank branches in Ghana were involved in this Neural Network study where $70 \%$ banks branches dataset were used to train or build the DEA-NN model. The proposed NN model was used to predict the efficiency of the remaining $30 \%$ bank branches. The input and output variables used as mentioned earlier were adopted from the dataset used in the previous section during the constructing of the decision tree model and random forest model. Specifically, the cases 2 and 4 were considered.

For the performance analysis of the proposed DEA-NN model, the 30\% bank branches dataset normally called the test data were used. Analysis and prediction of banks' efficiencies by the DEA-NN model was done under two separate scenarios (Case 2 and Case 4). In both cases (Case 2 and Case 4), the DEA-NN predicted scores (Classes) were compared with the efficiency classes designated by the CCR during the DEA. It is also important to mention that for the purpose of this study, efficient unit is a unit with an efficiency score of 0.8 or more.

The entire results of the DEA-NN prediction in both cases are shown in Appendices G and H.

### 4.5.3.2 Case 2 -The Deposit Stage

When the proposed DEA-NN model which was trained using 310 (70\%) bank branches dataset was used on the other 134 bank branches, the DEA-NN model was able to predict the efficiencies and their respective classes assigned by the CCR during the DEA at an accuracy rate of $71.64 \%$. The number of inputs used in the case 2 was 6 with 5 hidden neurons on a 3 layer network. The main expected outputs for this scenario were Classes 1 to 4 . With respect to Case 2 test dataset, there was no bank branch that has the class 1 characteristics (banks efficient in investing deposit but was not able to attain overall efficiency).

There were 103 banks with the class 2 characteristics (banks that realized overall efficiency even though they were not efficient in investing deposit), 14 banks branches with the class 3 characteristics (Banks that were efficient in investing deposit and still had overall efficiency) and finally 17 banks with the class 4 characteristics (Banks inefficient in investing deposit and cannot also achieve overall efficiency).

According to the findings, DEA-NN model predicted 96 out 103 bank branches realized overall efficiency even though they were not efficient in investing deposit Class 2.This means that the DEA-NN model was able to detect banks in the class 2 at accuracy of $93.2 \%$. For banks in the class 3 and class 4 categories, the DEA-NN model was not able to predict them.

The following is also the confusion matrix of the DEA-NN analysis:

```
Confusion Matrix and Statistics
            Reference
            Prediction 2 3 4
            2 96 14 13
            3 7 0 4
    0 0}0
    Overall Statistics
    Accuracy : 0.7164179 95%
    CI : (0.632137, 0.79087)
No Information Rate : 0.7686567
```

```
P-Value [Acc > NIR] : 0.935099793
```


### 4.5.3.3 Case 4 -The Investment Stage

Under Case 4, the proposed DEA-NN model had the highest prediction accuracy of 80.59\% as compared to Case 2. There were a total of 134 bank branches with the following classes distributions:

- Class 1 had only 1 bank branch
- Class 2 had the highest number of bank branches of 110
- Class 3 had only 1 bank branch
- Class 4 the was second highest (22) in terms of the number of bank branches in the dataset For both class 1 and class 3, the DEA-NN model predicted all wrongly. This implies that for both classes 1 and 3, the DEA-NN model accuracy was $0 \%$ and this may be attributed to the fact that there was not enough Class 1 and Class 3 banks in the training dataset. It was therefore very difficult for the model to predict a variable whose characteristic is not well known and has not been properly learned by machine properly. This is also consistent with the definition of Machine learning given by Alpaydı (2010) - programming computers (Machines) to optimize a performance criterion using example data or past experience (Alpaydın, 2010). In this case, there was no better past experience of both classes 1 and 3 by the proposed DEA-NN model.

With regard to the 110 banks that obtained overall efficiency in their business operations, even though they were not efficient in investing the deposit collected at the stage I of the DEA model, the proposed DEA-NN model was able to detect or predict 108 out of 110 (accuracy rate of
$98.18 \%$ ) of these bank branches. This means that the DEA-NN model performs best when predicting banks that can achieve overall efficiency in their business operations even though they cannot have good efficiency score in investing their deposits or giving loans to customers.

Finally, the DEA-NN model predicted all the other classes of banks wrongly. The following is also the confusion matrix generated by the DEA-NN model:

Confusion Matrix and Statistics

| Reference |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Prediction | 1 | 2 | 3 | 4 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 108 | 1 | 21 |
| 3 | 1 | 2 | 0 | 1 |
| 4 | 0 | 0 | 0 | 0 |

```
Overall Statistics
Accuracy : 0.8059701
        95% CI : (0.7287708, 0.8691644)
        No Information Rate : 0.8208955 P-Value
            [Acc > NIR] : 0.7188772
```

$$
\text { Kappa : } 0.0460022
$$

According to the study, the analysis and discussions of the DEA -NN prediction results, the following were suggested:

The DEA-NN model performance was very low in predicting the efficiencies of bank branches in both cases as compared to the other two models, thus DEA-DT and DEA-RF. Finally the DEA-NN model for classifications suggested MAPE of $24.6 \%$ and $20.5 \%$ for stage 1 and stage 2 respectively. In the case of the stage 1 classification of the testing data set, the proposed DEA-NN model performed better than the Kwon \& Lee (2015) BPNN model with MAPE of 36.9 \% for their testing data set.

### 4.5.4 EMPIRICAL RESULTS AND DISCUSSIONS OF THE PREDICTIONS BY

## THE DEA-LR MODEL

To evaluate the prediction of the model, K-fold cross validation was implemented to lessen the bias of sampling data and ensuring model error randomness. K-fold cross validation randomly divides data into k subsets and one subset is used as testing data and $\mathrm{K}-1$ subsets are used as training data (Yousaf, 2016) . To estimate the model's performance, machine learning models such as RMSE, MAE, MAPE, RMSPE and rBIAS were used as the evaluating measures. Analysis of the prediction by the DEA- LR model was done in two cases (Cases 2 and 4). In the first case, Case 2 variables were the dependent variables and the result was compared with the classes designated by the CCR during the DEA stage.

### 4.5.3.1 The Bank Branches (132 DMUs) Efficiency Analysis Using the DEA-LR Model

Analysis and prediction of banks' efficiencies by the DEA-LR model was done under two separate scenarios (Case 2 and Case 4). In both cases (Case 2 and Case 4), the DEA-LR predicted scores (Classes) were compared with the efficiency classes designated by the CCR during the DEA. It is also important to mention that for the purpose of this study, efficient unit is a unit with an efficiency score of 0.8 or more.

### 4.5.3.2 Case 2 -The Deposit Stage

When the proposed DEA-NN model which was trained using 310 (70\%) bank branches dataset was used on the other 132 bank branches, the DEA-LR model was able to predict the efficiencies and their respective classes assigned by the CCR during the DEA at an accuracy rate of $83.33 \%$. According to the findings, DEA-LR model predicted 110 out 132 bank efficiency. The details of the actual class and predicted class by the DEA-LR model are shown in Appendix L. The following are the Coefficients, Std. Errors, Residual Deviance, PVALUES and ODDS RATIOS realized from the model development used for the predictions.

Call:
multinom(formula $=$ Class $\sim$ EMPLOYEES + IT + ASSET, data $=$ data $)$
Coefficients:
(Intercept) EMPLOYEES IT ASSET

|  | Std. Errors: |  |
| :--- | :--- | :--- | :--- |
|  | (Intercept) | EMPLOYEES $\quad$ IT ASSET |

Residual Deviance:
304.1911 AIC:
320.1911

P-VALUES:
(Intercept) EMPLOYEES IT ASSET
Class $3 \quad 0 \quad 00.20091690 .1214307$
Class $4 \quad 0 \quad 00.81596600 .8630356$

ODDS RATIOS:
(Intercept) EMPLOYEES IT ASSET
Class 3 3.135161e-03 1.697198 $0.9999994 \quad 1$
Class $41.533586 \mathrm{e}-1338.0652730 .9999999 \quad 1$

### 4.5.3.3 Case 4 _The Investment Stage

Under Case 4, the proposed DEA-LR model had the highest prediction accuracy of $92.42 \%$ as compared to Case 2 which also recorded $83.33 \%$. This means that the DEA -LR model build under the investment stage. Per the results of the analysis, the DEA -LR model built in the investment stage predicted 122 out 132 bank efficien cy correct. The details of the prediction realized by the DEA-LR in the investment stage are also shown in Appendix L.

Class 3 -5.765075 0.5289785-6.117662e-07 8.673473e-08
Class 4 -29.505998 3.6393024-1.073200e-07-1.869479e-08

Compared to the DEA-LR model built in the deposit stage, the DEA-LR model in the investment stage was better as the results of almost all the Machine Learning Evaluation

Metrics favored the investment stage model. The following information is also the Coefficients, Std. Errors, Residual Deviance, P-VALUES and ODDS RATIOS of the DEA-

LR model in the investment stage.
Call:
multinom(formula $=$ Class $\sim$ EMPLOYEES + IT + ASSET, data $=$ data1 $)$

# Coefficients: <br> (Intercept) EMPLOYEES IT ASSET <br> Class 2 44.59138-4.611388-7.521170e-08-7.398283e-07 <br> Class 3 44.57503-5.296252-2.736245e-06-8.076129e-07 <br> Class 4 15.21849-1.044336-3.581966e-07-6.803632e-07 

Std. Errors:
(Intercept) EMPLOYEES IT ASSET
Class 2 6.093456e-12 3.853595e-11 2.239300e-06 4.191700e-07
Class 3 7.205787e-12 4.876917e-11 5.527349e-06 5.460845e-07 Class 4 9.330176e-13 9.328377e-12 2.247739e-06 4.190773e-07

Residual Deviance:
129.7539 AIC:
153.7539

P-VALUES:

| (Intercept) | EMPLOYEES | IT | ASSET |  |
| :--- | :---: | :---: | :---: | :---: |
| Class 2 | 0 | 0 | 0.9732064 | 0.07756643 |
| Class 3 | 0 | 000.6205737 | 0.13916291 |  |
| Class 4 | 0 | 00.8733863 | 0.10448700 |  |

## ODDS RATIOS:

(Intercept) EMPLOYEES IT ASSET
Class $22.321626 \mathrm{e}+190.0099380150 .99999990 .9999993$
Class $32.283956 \mathrm{e}+190.0050103370 .99999730 .9999992$
Class $44.067313 \mathrm{e}+060.3519254640 .99999960 .9999993$

## CHAPTER 5

GENERAL DISCUSSION

### 5.0 INTRODUCTION

This study has explored the application of Machine Learning Algorithm in the study of the efficiency of banks using banks in Ghana as a case study. The study proposed a framework called Banks' Efficiency Prediction. This framework was used to build the four different models namely DEA-DT, DEA-RF, DEA-NN and DEA-LR for predicting the efficiency of banks. These combined models were empirically evaluated using real world dataset from a developing country; Ghana and their performances were assessed using RMSE, MAE, MAPE, RMSPE and rBIAS as model performance metrics.

The study started with the development of the four Machine Learning Algorithms models, Decision Tree, Random Forest, Neural Networks and Logistic Regression which were used to classify the banks in Ghana using two scenarios of (Case 2 and Case 4). This was followed by the determination of the various bank efficiencies using a two- stage Data Envelopment Analysis (DEA). Predictor variables were used together with the various efficiency classification classes of the bank branches as the dataset. The DEA scores were classified under four different scenarios using the proposed Banks' Classification Algorithm (BC Algorithm) and were considered as the response variables. Four models were developed and proposed by combing DEA with four Machine Learning Algorithms, Decision Tree, Random Forest, Neural Networks and Logistic Regression. These proposed models namely; DEA-DT, DEA-RF, DEA-NN and DEA-LR were used to predict the efficiency classes of the bank branches using two scenarios of the DEA scores (Case 2 and Case 4). The proposed models' classification results were compared with the classification results of the original Decision

Tree, Random Forest, Artificial Neural Network and Logistic Regression using RMSE, MAE, MAPE, RMSPE and rBIAS as Machine learning performance metrics. In this last but one chapter, the study generally discusses the results realized from the study, the study's contribution to knowledge and a list of publications available at the time of writing this thesis.

### 5.1 ANALYSIS OF THE MACHINE LEARNING ALGORITHMS

To estimate the performance of the four proposed models used for the classifications in the study, RMSE, MAE, MAPE, RMSE and rBIAS were considered as the evaluating measures. A 10-fold Cross-Validation (CV) was also applied to check overfitting and performance of all predicting models, Decision Tree (DT), Random Forest (RF), Neural Network (NN) and Logistic Regression (LR) on each case dataset. The values of the five machine learning evaluation metrics for each of the model are given in the tables below.

For the case 2 dataset, the proposed models namely ; DEA-DT, DEA-RF, DEA-NN and DEALR by the study performed better compared to the Decision Tree, Random Forest, Neural Network and Logistic Regression with DEA-RF performing best with MAPE of $5.22 \%$ followed by DEA-DT $8.95 \%$ and finally the DEA-LR with $16.67 \%$. The DEA-ANN was the worst performing model in the deposit stage with MAPE of $24.59 \%$.

Table 5.1: Performance of the Models Using Machine Learning Evaluation Metrics under the Deposit Stage

| CASE 2- Deposit Stage |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| MODELS | RMSE | MAE | MAPE | RMSPE | rBIAS |
| DT | 0.70181 | 0.492537 | 49.2537313 | 0 | 0.492537 |
| RF | 0.64646 | 0.41791 | 41.79104 | 0 | 0.41791 |
| ANN | 1.176902 | 1.042135 | 57.45151114 | 1.179884 | -0.019427972 |
| LR | 0.583874 | 0.340909 | 34.09091 | 0.583874 | 1 |
|  |  | - |  |  |  |
| PROPOSED MODELS |  |  |  |  |  |
| DEA-DT | 0.299253 | 0.089552 | 8.955223881 | 0 | 0.089552 |
| DEA-RF | 0.228558 | 0.052239 | 5.223880597 | 0 | 0.052239 |
| DEA-ANN | 0.685259 | 0.599699 | 24.59171999 | 0.707712 | -17.5888 |
| DEA-LR | 0.408248 | 0.166667 | 16.66667 | 0.408248 | 1 |




Figure 5.1: Graph showing the Performance of the Machine Learning Algorithm s models and the four Models proposed in Case 2 (Author's construct).

Figure 5.1 shows the plot for both Machine Learning Algorithms models and the proposed model under the deposit stage. The results did not show any level of good performance of the machine learning algorithms models compared to the proposed models by the study which is an indication of a better machine learning model. Without exceptions, all the models estimates are significant at the 0.05 level of significance .The study results also reveal that among the
four proposed models (DEA-DT, DEA-RF, DEA-NN and DEA-LR) for predicting the efficiency of banks at the deposit stage; the DEA-RF is suitable for modeling bank deposit efficiency data. As it was the model with the best RMSE, MAE, MAPE, RMPSE and rBIAS. Also, by inspecting the MAE values which is a measure of prediction accuracy, we observe that MAE values for proposed models are smaller than those for the DT, RF,NN and LR models. Hence the proposed model is selected for deposit efficiency analysis and classification of banks. The proposed framework is better when comparing its performance with a similar study by Hamad \& Anouze (2015) where the authors introduces a three-stage integrated framework consisting of data envelopment analysis (DEA), random forest, and logistic regression to examine and predict the impact of environmental variables on banks' performance and had $79.4 \%$ accuracy rate of classification using 151 banks in Middle East and North African (MENA) countries.

Table 5.2: Performance of the Models Using Machine Learning Evaluation Metrics under the Investment Stage

| CASE 4-Investment Stage |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | :---: |
| MODELS | RMSE | MAE | MAPE | RMSPE | rBIAS |  |
| DT | 0.782266 | 0.61194 | 61.19403 | 0 | 0.61194 |  |
| RF | 0.772667 | 0.597015 | 59.7015 | 0 | 0.597015 |  |
| ANN | 1.241123 | 1.11657 | 65.49859 | 1.227733 | -0.02396 |  |
| LR | 0.758787 | 0.575758 | 57.57576 | 0.758787 | 1 |  |
|  |  |  |  |  |  |  |
| PROPOSED MODELS |  |  |  |  | 0 |  |
| DEA-DT | 0.149626 | 0.022388 | 2.238806 | 0.022388 |  |  |
| DEA-RF | 0.167248 | 0.02985075 | 2.98507463 | 0 | 0.029851 |  |
| DEA-ANN | 0.647185 | 0.474046108 | 20.53436629 | 0.677297 | -0.05259 |  |
| DEA-LR | 0.275241 | 0.075758 | 7.575758 | 0.275241 | 1 |  |




Figure 5.2: Graph showing the Performance of the Machine Learning Algorithms models and the four Models proposed in -Case 4 (Author's construct)

Figure 5.2 shows the plot for both Machine Learning Algorithms models and the proposed model under the investment stage. The results did not show any level of good performance of the machine learning algorithms models (Decision Tree, Random Forest, Neural Network and Logistic Regression) compared the proposed models by the study which is an indication of a better machine learning model. Without exceptions, all the models estimates are significant at
the 0.05 level of significance. The study results also reveal that among the four proposed models (DEA-DT, DEA-RF, DEA-NN and DEA-LR) predicting the efficiency of banks at the deposit stage; the DEA-DT is suitable for modelling bank investment efficiency data. This is because; it was the model with the best RMSE, MAE, MAPE, RMPSE and rBIAS. Also, by inspecting the MAE values which is a measure of prediction accuracy, we observe that MAE values for proposed models are smaller than those for the DT, RF NN and LR models. Hence the proposed model is selected for deposit efficiency analysis and classification of banks. The proposed framework is better when comparing its performance with a similar study by Wu (2006) where the author detected the impact of IT on firm performance by proposing a generic model using Data Envelopment Analysis (DEA) Decision Trees (DTs) and 36 Canadian banks as DMUs.

### 5.3 Discussion

The assessment of banks efficiencies creates serious problems for banks and their managers (Adusei, 2016; Sufian et al., 2016 and Titko, Stankevičiené, \& Lāce, 2014). Due to the critical significant role of banks in the national economy, their efficiencies and performances are a hotly debated topic in the academic and business environments (Titko et al., 2014). Models and frameworks designed for this assessment are normally based on econometric analysis (Brynjolfsson, 1993; Dedrick et al., 2013; Dedrick, Gurbaxani, \& Kraemer, 2003;

Kılıçaslan, Sickles, Kayış, \& Gürel, 2017; Nations, 2008 and Roghieh, Saeed, \& Sang-Yong, 2004) and other parametric methods.

Non-parametric techniques such as Data Envelopment Analysis (DEA) have been suggested in literature as technique to evaluate bank efficiency (Lampe \& Hilgers, 2015). However, such studies (Chen et al., 2006; Chen \& Zhu, 2004a; Izadikhah, Tavana, \& Di, 2017; Madjid et al., 2009; Tavana, Izadikhah, Caprio, \& Saen, 2017; Wang, Gopal, \& Zionts, 1997) that used only non-parametric methods such as pure DEA and its models suffer from weak discrimination power (Ashoor, 2012; Chen, 2016; Santos et al., 2017; Wu, 2006, 2009) and also sensitive to the presence of outliers and statistical noise (Da, Stasinakis, \& Bardarova, 2018 and Dash et
al., 2006). It is also very difficult to use only DEA to predict the efficiency and performance of other or new Decision Making (Emrouznejad \& Yang, 2017; LaPlante \& Paradi, 2014 and Wanke et al., 2016).

Combined DEA and Machine Learning Algorithms can offer these suitable and scientific methods. A lot of machine learning algorithms have been developed and utilized in the business and financial sectors for predictions and forecasting. For instance, Chen \& Hao (2017) used Feature Weighted Support Vector Machine and K-Nearest Neighbor Algorithm for stock market indices prediction using the two well-known Chinese stock market indices. There are other studies such as Abellán \& Castellano (2017); Ahn, Cho, \& Kim (2000);

Anandarajan, Lee, \& Anandarajan (2001); Caggiano, Calice, \& Leonida (2014); Caggiano, Calice, Leonida, \& Kapetanios (2016); Göçken, Özçalıcı, Boru, \& Ayse (2016); Janek, Beyers, Pieter, \& Villiers (2016); Kim \& Han (2000); Patel, Shah, Thakkar, \& Kotecha (2014b, 2014a); Qiu, Song, \& Akagi, (2016) and Tsai \& Wu, (2008) that have implemented machine learning algorithms in the business and financial sectors in forecasting and prediction studies.

According to literature, Machine Learning Algorithms used to build predictive models have been suggested as the one of finest and best predicting methods with a high accuracy and validity in the field business and financial forecasting (Göçken et al., 2016). Nevertheless, prediction and classification of bank branches using with a high performance models across developing countries are also completely lacking in literature (Mohd Zaini Abd, 2001 and Tra et al., 2018) According to Yang et al. (2015) Zheng et al. (2004) and validated by the findings in this study combined models consistently outperform individual models for classification and prediction tasks. This current work uses a primary data gathered from more than four hundred (400) DMUs of bank branches.

In comparison with other studies such as Hamad \& Anouze (2015); Kwon \& Lee (2015) and Wu (2006), the novelty of this thesis lies in both the significance of the frameworks /models
(DEA-DT, DEA-RF, DEA-NN and DEA-LR) applied in the study ,the algorithm proposed and used and the key findings observed. First and foremost, this study combines a two-stage DEA model with different Machine Learning Algorithms namely Decision Tree, Random Forest, Artificial Neural Network and Logistic Regression, where the efficiency at each stage (thus efficiency in collecting deposit from customers, Deposit efficiency and efficiency in investing the deposit, Investment efficiency) is also considered as input for the next stage using a huge dataset a from a developing country. The study also classified the various banks based on their efficiency scores using a proposed algorithm, Bank Classification Algorithm suggested by the study. This efficiency classes (Class1, Class2 Class 3 and Class 4) were used as the response variable making, the response variable for the study categorical. For the development of the four models, the study considered predictor variables which were both internal and external and directly influence bank performance and efficiency as suggested by (Thanassoulis et al., 2008). The development and performance of the proposed in this current study was done on big dataset compared to Wu (2006) work which was done on a small dataset from existing literature (Wang et al., 1997). Kwon \& Lee (2015) also used a total of 181 DMUs as a dataset. Which accooidng to this same Kwon \& Lee (2015), a large data set is often favored for the generalization of models. A K-fold cross-validation was used to better the performnace of the models as compared to Wu (2006) works, which was based on V-fold cross validation.The data used in this study is not a cross-country bank-level data compared to Hamad \& Anouze (2015). In this study , $70 \%$ of the data set (444 DMUs) was used for training and $10 \%$ validation and $30 \%$ for testing the models compared to Kwon \& Lee, (2015) study of combining DEA with Back Propagation Neural Network which was based on a ratio of 60:20:20 for training,validating and testing respectively. This is also a single study that has build and proposed four different models that has yieded fovorable classification accuracy rate.These models were developed by combining DEA with Decision Tree (DEADT),Random Forest (DEA-RF), Artificial Neural Network (DEA-NN) and Logistic Regression (DEA-LR). In the case of the DEA-DT, the model perfomed better than Wu
(2006) work by giving an accuracy of not less than $90 \%$ compraed to Wu (2006) 69.44\%. Comparing the DEA-RF to that of Hamad \& Anouze (2015), the study also suggested a fovuarble classification accuracy of $90 \%$ compared to Hamad \& Anouze (2015) work that also yieded $79.4 \%$ and finaly, the DEA-NN suggeseted a favouable Mean Absolute Percentage Error(MAPE) of about $24.6 \%$ compared the BPNN MAPE of $36.9 \%$ also suggsetd by (Kwon \& Lee, 2015). Compared to the works of Hamad \& Anouze (2015); Kwon \& Lee, (2015) and Wu (2006), this study demonstrated the performance of the four proposed models by using five standard machine learning evaluation metrics namely; Root Mean Square Error(RMSE),Mean Absolute Performance Error(MAPE),Mean Absolute Error(MAE),Root Mean Square Performance Error, and rBIAS.

Even though it has been noted by earlier authors that combining DEA with machine learning algorithms will yield favorable results (Wu, 2006) in terms of classification. An important observation that can be made from the studies conducted is that most of these frameworks were not able to produce machine learning algorithm models that can suggest a higher classification accuracy of more than $80 \%$. Comparing the results of four machine learning models which were built by combining DEA with machine learning algorithms in a single study using a bigger data set (444 DMUs) is very scarce in literature. This thesis therefore contributes significantly to filling this particularly huge gap in the literature of bank efficiency prediction.

### 5.4 ORIGINALITY AND CONTRIBUTIONS TO KNOWLEDGE

It is well acknowledged that making an original contribution to knowledge is often a contentious issue among scholars, especially in doctoral studies due to the arbitrary nature of the concept of originality. However, there is no doubt that, this study has provided an insight and significant contribution to the body of knowledge as regards assessing efficiency and performance of firms using an improve machine learning model. In comparison with related studies in combining DEA with Machine Learning Algorithms (Hamad \& Anouze, 2015; Kwon \& Lee, 2015; Wu, 2006), the novelty of this thesis lies in both the significance of the
frameworks /models (DEA-DT,DEA-RF, DEA-NN and DEA-LR) applied in the study ,the algorithm proposed and used and the key findings observed. Even though it has been noted by earlier authors that combining DEA with machine learning algorithms will yield favorable results (Wu, 2006) in terms of classification. An important observation that can be made from the studies conducted is that most of these frameworks were not able to produce machine learning algorithm models that can suggest a higher classification accuracy of more than $80 \%$. Comparing the results of four machine learning models which were built by combining DEA with machine learning algorithms in a single study using a bigger data set (444 DMUs) is very scarce in literature. This thesis therefore contributes significantly to filling this particularly huge gap in the literature of bank efficiency prediction.

In relation to bank efficiency assessment and analysis, the following novel contributions to knowledge were achieved;

1. The development of a high accuracy machine learning models (DEA-DT, DEA-RF DEANN and DEA-LR) for predicting bank efficiency by combing DEA and four machine learning algorithms namely; Decision Tree algorithm, Random Forrest algorithm, Artificial Neural Network and Logistic Regression.
2. The development of framework called Banks' Efficiency Prediction which has also proved to be effective when it was empirically tested with real world numerical data.
3. The design of a Bank Classification Algorithm (BC Algorithm) that was used to classify banks in into classes based on their efficiencies.
4. The used of percentage of non-performing loans as an output variable in a two stage DEA for assessing the efficiency of banks.
5. The use of efficiency value in each of the stages as input variable in the next stage in DEA. Thus the efficiency score for the stage I was used together with the deposits collected as inputs for stage II and finally, the efficiency for stage II was also used with the fixed assets, IT budget and number of employees as input for the overall stage.
6. This study has also contributed to literature with its huge dataset on DMUs.

This study has also contributed to the existing literature through the following journal publication and conference presentation:

## Journal Articles

1. Appiahene, P., Missah, Y. M., \& Najim, U. (2020). Predicting Bank Operational

Efficiency Using Machine Learning Algorithm: Comparative Study of Decision Tree, Random Forest , and Neural Networks. Advances in Fuzzy Systems, 2020.
https://doi.org/https://doi.org/10.1155/2020/8581202.
2. Appiahene, P., \& Missah, Y. M. (2019a). Predicting the Operational Efficiency of Banks in the presence of Information Technology Investment. International Journal of Advances in Electronics and Computer Science, 6(11), 1-6. https://doi.org/IJAECS-IRAJ-DOI16504.
3. Appiahene, P., Missah, Y. M., \& Najim, U. (2019). Evaluation of information technology impact on bank's performance : The Ghanaian experience. International Journal of Engineering Business Management, 11, 1-10. https://doi.org/10.1177/1847979019835337.
4. Appiahene, P., Ussiph, N., \& Missah, Y. M. (2018). Information Technology Impact on Productivity : A Systematic Review and Meta- Analysis of the Literature. International Journal of Information Communication Technologies and Human Development, 10(3), 39-61. https://doi.org/10.4018/IJICTHD. 2018070104.
5. Predicting the Operational Efficiency of Banks using their Information Technology: Decision Tree Algorithm Approach. Under Review

## Conference Presentations and Proceedings

1. Predicting the Operational Efficiency of Banks in the Presence of Information Technology Investment Using Artificial Neural Network. Presented at the International

Conference on Artificial Intelligence and Soft Computing (ICAISC) Munich, Germany 3rd - 4th July 2019.
2. Appiahene, P., \& Missah, Y. M. (2020). Bank Classification Algorithm: Case Study of Ghanaian Banks. 2019 International Conference on Communications, Signal Processing and Networks (ICCSPN), 1-6. https://doi.org/10.1109/iccspn46366.2019.9150171.
3. Information Technology Impact on Organizational Productivity: A Survey of the Literature was presented at the 5th International Conference on Applied Sciences and Technology (ICAST) held on September 2018 in Kumasi, Ghana.

## CHAPTER 6 6.0 CONCLUSION AND RECOMMENDATIONS

Firstly, against the background of few studies on the assessment of bank efficiencies using data from Ghana, this study has been undertaken in an effort to close the gap in literature between Ghana and the rest of the world. Secondly, there are few studies that have combined DEA with Machine Learning Algorithm like Decision Tree, Random Forest, Neural Network and Logistic Regression to predict the efficiency of banks. These banks were classified based on their efficiency scores at the deposit and investment stages using a proposed Bank Classification Algorithm (BC Algorithm). The study also provided an empirical assessment of four Machine Learning Algorithms using Ghanaian banks as a case study. The evaluation undertaken in this study indicates that combined models consistently outperform individual models for classification and prediction tasks suggested by Yang et al. (2015) and Zheng et al. (2004). In this study, a framework was proposed; DEA was combined with four different Machine Learning Algorithms to predict banks' efficiencies using Ghanaian banks as a case study. The dependent variables were Case 2 (Deposit efficiency) and Case 4 (Investment efficiency)
efficiency scores classes that were realized from determining the efficiencies of the various bank branches.

Firstly, using existing literatures on the topic, the study identified the most commonly used and cited methodologies used to assess the efficiency and performance of an organization. This resulted in the integration of a non-parametric model - a two-stage DEA which has been proved to be a good measurer of organizational performance or efficiency Chen et al. (2006); Chen \& Zhu (2004b) with Machine Learning Algorithm. Data collected from the selected banks, the DMUs were divided randomly into two: training and validation dataset (70\%) and testing dataset ( $30 \%$ ). The predictor variables were the various input and output parameters that were used to calculate the efficiency scores of each DMU using the DEA. These efficiency scores were classified under the deposit and investment stages. The combination of the DEA scores, classes (response or dependent variable) under each case and predictor variables form the dataset for the study. For the prediction models, the study utilized four popular machine learning algorithms and compared the four algorithms using several performance metrics of algorithm measurements. The best performed Machine Learning Algorithm model in each case, cases 2 and 4 using several performance measures were determined based on the $30 \%$ testing dataset. The results of the predictive models indicated the following:

- Combing a non-parametric model like DEA with a machine learning algorithm work better than standalone Machine Learning Algorithm.
- A combination of Data Envelopment Analysis, and Machine Learning Algorithms such as Decision Tree, Random Forest, Neural Network, and Logistic Regression gives high performance accuracy in predictions which confirms Yang et al. (2015) and Zheng et al. (2004) suggestions.
- The best Machine Learning model for predicting the efficiency of banks in terms of collecting deposit from customers is the proposed DEA-RF.
- The best Machine Learning model for predicting the efficiency of banks in terms of investing customers deposit is the proposed DEA-DT.


### 6.1 LIMITATIONS OF THE FINDINGS

Ascertaining the precincts of any academic work helps improve its reliability and the generalizability applications of the findings. There are some possible limitations that should be borne in mind in the analysis and generalization of the study findings.

The focus of the numerical illustration aspects of this study was entirely based on the Ghanaian banking industry experience. Given that in practical terms various economic indicators may differ across countries, geographical regions or even continents; it is completely possible that there may be substantial differences and disparities in the findings if this study is simulated in other countries or geographical regions. However, hypothetically, it can be said that the banking industry in many developing countries especially in sub-Saharan Africa are considered to exhibit comparable practical and economic characteristics. Hence, this limitation stated above does not emasculate the validity of the study undertaken and potential application of its main findings in sister developing countries especially those in Africa. Furthermore, the convergence of the findings with general body of knowledge supported by the validation results further reinforces the credibility of the research findings.

It is very important to acknowledge the limitation of the relatively few population sample considered in the study imposes. As the study only considered 444 commercial bank (universal banks) branches in Ghana, expanding the sampling data to include other financial firms like insurance companies, savings and loans, rural banks, credit unions could have enriched the findings and increased its potential generalization. Nevertheless, this should not invalidate the findings and conclusions as the demographic profile of the 444 selected bank branches show a cross fertilization of almost all the other financial institutions in Ghana.

As suggested, most statistical analytical approaches and tools are affected by issues of multicollinearity, sampling inconsistencies, measurement errors, analytical bias which are likely to impact on the results and the potential conclusions to be drawn from the findings. However, notwithstanding, the potential of these highlighted above, it can be suggested that the
demographic profile of the respondents in terms of experience, knowledge \& understanding on the topic and consistencies registered in the statistical analysis indicate some degree of reasonable credibility and trustworthiness in the results from the data given.

Finally, with respect to factors that affect bank's deposit collection, there were other several factors that were not taken into consideration because of inadequate data on these factors. Some of these are listed below;

- Environment: Consider 2 banks, one located in Accra central and the other in a small town Sogakope may have the same inputs, but the deposit mobilization will be significantly different.
- Customers: The type of customers you have makes a big difference; small traders as against large businesses etc.
- Sector of operation. Giving loans to farmers will result in more defaulting loans. Loans to contractors not paid on time by the Government have caused defaulting loans even collapsing some banks.
- Bank Policies: This will restrict the amount of loans that can be given and the sectors.
- Large loans: Loans beyond the capacity of the local branch will require assistance and approval from the head office. All these factors can affect deposit mobilization as suggested by (Ambe, 2017; Gunasekara \& Kumari, 2018; Jembere, 2016; Joyce, 2013; Madebo, 2013; Mushtaq \& Siddiqui, 2017; Nahidul et al., 2019; Ostadi \& Sarlak, 2014; Pesa \& Muturi, 2015; Turhani \& Hoda, 2016; Vuong et al., 2020) in their studies. This study is therefore limited to Bank's Fixed Asset, IT expenditure and Total Number of Employees as factors that can impact deposit mobilization by Banks.


### 6.2 RECOMMENDATIONS

1. As the empirical testing of the proposed framework was done using dataset from banks in Ghana, these developed models, especially the DEA-DT and DEA-RF is therefore recommend to bank managers and other researchers for predicting and classifying banks.
2. The study recommends the usage of the proposed algorithm to classify banks in Ghana based on the efficiency in doing business like deposit collections and investing the deposits.
3. The Management board of banks and other stakeholders may detect new ways to better their efficiency through the use of innovative technologies using the result of this thesis.
4. The general Ghanaian banking industry and other firms may use the results of this thesis to help them have a better understanding of relationship between their resources and the performance and efficiency of their banks.
5. Customers or investors can use the findings of this study as it would add value to their knowledge of how efficient their various bank branches are in terms of managing their investments.
6. The proposed framework can be used by managers of various firms and researchers to understand and correlate the impact of their resources on their firm performance.
7. To improve banks efficiency and performance in order to remain competitive in this current banking crisis in Ghana managers and stakeholders should not over rely on their overall efficiency or performance as means of measuring their growth, but should critically find ways of improving their efficiency in deposit collections from customers and also investing the deposits. This study suggests that a lot of Ghanaian banks are inefficient in investing customers' deposits.
8. Finally the study recommends to the central bank (Bank of Ghana) to use the method and algorithm proposed in this study to classify the various banks in Ghana based on their efficiency in collecting deposits and investing deposits since these two (deposit and
investing deposits) have become hot and critical issue in the current banking crisis in Ghana (Abubakar, 2018; Addison, 2017, 2018a and Addison, 2018).

### 6.3 FUTURE STUDIES

- Future studies can also test the proposed framework and the bank classification algorithm using a different dataset from other dual role operating firms such as insurance companies and rural banks.
- The dataset used in this study can be analyzed using a different methodology in terms of how the DEA scores were determined and the results compared with this study.
- Future studies can also consider the liquidity ratio on banks as output variable in the determination of the efficiency of banks.
- Future studies can also incorporate other factors such as Environment, Customers, Sector of operation, Bank Policies and Large loans as inputs for Deposit Stage and compare the results with this current study. Thus how these factors can also impact the model.


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| $\begin{gathered} \text { DM } \\ \mathbf{U} \end{gathered}$ | $\begin{gathered} \text { No. of } \\ \text { Employee } \\ \text { s } \end{gathered}$ | \% Performing Loans | $\underset{\text { budget }(\mathbf{G H} \mathbf{C})}{\text { IT }}$ | Fixed Asset(GHc) | Deposit(GH¢ | Profit(GHc <br> ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 7 | 83.7537 | 60897.3621 | 1454093.72 | 72757137 | 316871.7 |
| 2 | 7 | 82.69211 | 13859.0755 | 1458937.49 | 17605016 | 4372073 |
| 3 | 7 | 83.11167 | 34710.1547 | 58751.02 | 10256530 | 1939064.7 |
| 4 | 7 | 82.54663 | 23696.8498 | 105086.5 | 68830240 | 4579780.7 |
| 5 | 7 | 81.09985 | 47438.3618 | 1768645.92 | 30798389 | 778894.6 |
| 6 | 7 | 83.27801 | 27897.1118 | 231642.89 | 52158533 | 1098903.5 |
| 7 | 7 | 84.39781 | 29038.052 | 886766.52 | 79999706 | 2169496.8 |
| 8 | 7 | 84.7925 | 66112.7017 | 610520.6 | 12158202 | 4546746.5 |
| 9 | 7 | 84.38961 | 25474.0981 | 96518.98 | 23429068 | 2726352.2 |
| 10 | 7 | 83.91438 | 19228.4336 | 591265.4 | 66972317 | 1608772.5 |
| 11 | 7 | 83.14883 | 90718.4762 | 983547.47 | 44330728 | 1412219.6 |
| 12 | 7 | 84.21455 | 63402.9388 | 899117.96 | 5509682 | 1999396 |
| 13 | 7 | 82.71137 | 24820.0458 | 1113390.03 | 63062442 | 3485935.7 |
| 14 | 7 | 82.12436 | 82198.1167 | 895869.7 | 42907341 | 4601286.4 |
| 15 | 10 | 81.28045 | 66849.9966 | 378360.87 | 47005611 | 2937952.7 |
| 16 | 10 | 82.86733 | 41662.9971 | 1291477.35 | 56411163 | 178020.6 |
| 17 | 10 | 83.40505 | 13954.9107 | 157818.84 | 40767268 | 2197418.9 |
| 18 | 10 | 81.38958 | 8618.368 | 321805.62 | 73669458 | 1849928.7 |
| 19 | 10 | 84.21942 | 65122.7957 | 74096.2 | 4045097 | 356569.6 |
| 20 | 10 | 82.80137 | 22501.4046 | 1195705.25 | 26949380 | 1249464.5 |
| 21 | 8 | 83.44796 | 74708.9867 | 1648052.42 | 30161464 | 4420331.2 |
| 22 | 8 | 82.39093 | 71708.5146 | 934887.48 | 34945738 | 1202004.4 |
| 23 | 8 | 81.89895 | 51230.8662 | 1312692.06 | 72541502 | 275771.1 |
| 24 | 8 | 81.72461 | 72807.1954 | 145703.12 | 33410697 | 1746629.5 |

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## APPENDICES

Appendix A: The DEA Inputs and Outputs data for the 444 Banks DMUs

| 25 | 8 | 82.24098 | 57001.2629 | 1343246.94 | 4555214 | 214198.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 26 | 8 | 81.9301 | 56761.132 | 617310.01 | 1917982 | 2766706.5 |
| 27 | 8 | 83.39029 | 61384.8137 | 1108729.48 | 51759575 | 881929.5 |
| 28 | 8 | 83.4618 | 75610.5927 | 731615.58 | 85077214 | 2686259.8 |
| 29 | 8 | 83.7893 | 41301.2897 | 1223918.68 | 51156498 | 3748499.9 |
| 30 | 8 | 81.61138 | 16147.4105 | 1241505.32 | 44833724 | 1984384.1 |
| 31 | 8 | 84.66789 | 64261.9116 | 651393.03 | 76198639 | 1725473.9 |
| 32 | 8 | 82.91391 | 74086.2025 | 862395.61 | 53541075 | 4735426.5 |
| 33 | 8 | 82.97651 | 37068.7217 | 364839.11 | 83902102 | 3064981.4 |
| 34 | 8 | 84.00542 | 35709.9788 | 659382.86 | 59601194 | 337238.8 |
| 35 | 7 | 84.48814 | 89137.2205 | 1217831.91 | 33364809 | 1792806.8 |
| 36 | 7 | 82.9628 | 22715.4057 | 456158.54 | 60340108 | 497863.4 |
| 39 | 7 | 83:96494 | 658384:5964 | 1324069.64 | 102858809 | 76468837:5 |
| 38 | 7 | 83.34448 | \$1889.760 | 1087680.88 | 49789886 | 3934884.6 |
| 41 | 7 | 84.73585 | 58948.1106 | 1563957.17 | 39143763 | 4601904.2 |
| 42 | 7 | 83.24943 | 530.0299 | 1721702.77 | 33800644 | 1364603.2 |
| 43 | 7 | 83.81039 | 61867.3149 | 2404364.27 | 61626273 | 60831.48 |
| 44 | 7 | 84.21065 | 27642.0198 | 1615440.56 | 60231082 | 383397.99 |
| 45 | 7 | 85.38675 | 43620.7308 | 1211423.16 | 86922361 | 94964.24 |
| 46 | 7 | 85.57497 | 65135.1257 | 317603.59 | 38406087 | 23864.06 |
| 47 | 7 | 85.73434 | 134827.1346 | 2306880.07 | 31366089 | 154732.67 |
| 48 | 7 | 83.97825 | 39624.0834 | 2312024.67 | 56303255 | 157540.13 |
| 49 | 7 | 84.61915 | 27492.8512 | 2113253.82 | 70861614 | 22598.56 |
| 50 | 7 | 86.72497 | 125135.3219 | 1812950.03 | 61112071 | 330762.65 |
| 51 | 7 | 85.71362 | 132067.1273 | 10488.58 | 36145271 | 262307.63 |
| 52 | 7 | 85.98953 | 120439.421 | 1138134.67 | 79288312 | 153185.81 |
| 53 | 7 | 84.39833 | 118992.3979 | 1884590.82 | 39256592 | 374745.47 |
| 54 | 7 | 85.70841 | 137779.1401 | 454888.96 | 49350727 | 87500.86 |
| 55 | 7 | 83.54879 | 127492.9429 | 1310507.91 | 42026118 | 263872.76 |
| 56 | 7 | 85.46946 | 109222.5359 | 1441672.79 | 10186527 | 10416.49 |
| 57 | 8 | 84.33161 | 131057.0528 | 2406397.4 | 15222349 | 254451.9 |
| 58 | 8 | 84.84177 | 107542.4125 | 2230702.51 | 67899842 | 376535.7 |
| 59 | 8 | 84.7416 | 114753.5643 | 486385.17 | 72277525 | 231460.49 |
| 60 | 8 | 86.83833 | 52398.3301 | 1945943.36 | 38314238 | 413824.09 |
| 61 | 8 | 85.13257 | 81484.6853 | 85604.18 | 33186431 | 32792.78 |
| 62 | 8 | 84.0898 | 52190.2872 | 1212942.5 | 57899684 | 141870.95 |
| 63 | 8 | 83.98471 | 29978.277 | 2175448.62 | 54740574 | 427959.06 |



| 64 | 8 | 84.59234 | 112863.8048 | 2467311.67 | 39364242 | 224161.31 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 65 | 8 | 84.71331 | 45121.2452 | 214377.29 | 2414738 | 376569.97 |
| 66 | 8 | 84.27656 | 92631.6484 | 1456254.17 | 73009061 | 129133.45 |
| 67 | 8 | 84.73299 | 61014.6942 | 568709.68 | 20918328 | 243528.33 |
| 68 | 8 | 84.18429 | 109359.861 | 2306232.74 | 15808106 | 179951.77 |
| 69 | 8 | 83.39657 | 87069.6778 | 233394.13 | 32962703 | 50624.38 |
| 70 | 8 | 85.69877 | 125409.982 | 2105899.64 | 9106922 | 214176.49 |
| 71 | 8 | 84.73139 | 1703.001 | 2227485.17 | 38741653 | 441839.07 |
| 72 | 8 | 83.76835 | 72075.1806 | 46138.3 | 75065899 | 93423.12 |
| 73 | 8 | 84.70759 | 6528.152 | 1811396.71 | 5944887 | 47912.09 |
| 74 | 10 | 85.39766 | 137866.8884 | 601675.05 | 21017254 | 411043.51 |
| 75 | 6 | 84.65154 | 85611.8689 | 1868360.33 | 17369357 | 377752.15 |
| 76 | 10 | 84.4557 | 27052.307 | 1046261.6 | 12494808 | 321648.99 |
| 77 | 10 | 85.14075 | 101813.8507 | 1473196.92 | 51589349 | 279664.51 |
| 78 | 10 | 84.60654 | 41963.3303 | 2250464.82 | 23130577 | 454686.37 |
| 83 | 10 | 84:82333 | 1428090.33694 | 49889433:23 | \$89283849 | 189827:94 |
| 84 | 10 | 82:89907 | 98988:30.9\% | 232347:05 | 55144211234 | 397428:36 |
| 85 | 6 | 84:54779 | 5373357229 | 234296354.88 | 44878849 | 43698\% 4.48 |
| 88 | 10 | 84:883361 | 10250071.4984 | 78988399:86 | 38963334 | 2585044:36 |
| 87 | 10 | 83.42265 | 71745.8199 | 2294327.81 | 85805284 | 440015.77 |
| 88 | 10 | 84.49259 | 433.6275 | 1597652.84 | 11242352 | 108958.59 |
| 89 | 10 | 86.42369 | 41508.8025 | 98280.42 | 10034482 | 311570.81 |
| 90 | 7 | 85.19818 | 137110.8555 | 1933128 | 47475678 | 61259.5 |
| 91 | 7 | 84.32041 | 6736.4367 | 868209.5 | 84691512 | 51813.97 |
| 92 | 7 | 85.07102 | 137866.7712 | 2451903.63 | 4680542 | 40803.27 |
| 93 | 7 | 84.75843 | 104523.8738 | 102616.68 | 64848764 | 279102.96 |
| 94 | 7 | 84.68759 | 63633.6001 | 1776428.59 | 62553116 | 409229.01 |
| 95 | 7 | 83.61287 | 93886.5298 | 1163921.8 | 38618660 | 471542.49 |
| 96 | 7 | 85.11578 | 119991.1816 | 1280388.63 | 42924666 | 360773.33 |
| 97 | 7 | 84.58379 | 146730.6044 | 496686.5 | 71278156 | 401675.41 |
| 98 | 7 | 84.45135 | 113908.6877 | 739820.95 | 50491550 | 320079.93 |
| 99 | 7 | 85.49803 | 81657.5728 | 1544049.95 | 22010521 | 73544 |
| 100 | 7 | 83.63567 | 43702.7095 | 2424035.2 | 8549690 | 322669.72 |
| 101 | 7 | 84.02338 | 25250.4116 | 1062570.62 | 57244312 | 437573.05 |



| 102 | 7 | 86.77478 | 118550.8003 | 1894949.44 | 5763210 | 422830.69 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 103 | 7 | 86.02005 | 34931.6889 | 2007830.7 | 32830717 | 451720.6 |
| 104 | 7 | 85.57658 | 111870.2223 | 2301530.17 | 14895089 | 235689.32 |
| 105 | 7 | 83.84095 | 30768.1097 | 542467.36 | 27911303 | 37370.9 |
| 106 | 7 | 85.46901 | 129297.9651 | 438451.89 | 47314637 | 238965.23 |
| 107 | 7 | 82.57352 | 90000.2404 | 1567939.23 | 71753702 | 186826.77 |
| 108 | 7 | 85.77748 | 20078.8769 | 2136184.04 | 79250751 | 53221.74 |
| 109 | 7 | 84.03513 | 1876.2385 | 1370517.52 | 66082441 | 153566.76 |
| 110 | 7 | 85.70363 | 60363.7279 | 225139.13 | 40460697 | 445369.2 |
| 111 | 7 | 86.00411 | 8589.9743 | 299801.69 | 1684319 | 123565.41 |
| 112 | 7 | 86.2237 | 113514.7764 | 1505380.23 | 53877282 | 470794.2 |
| 113 | 7 | 82.23805 | 58950.2078 | 1444318.8 | 83413731 | 306483.57 |
| 114 | 7 | 85.00876 | 81951.1983 | 941272.87 | 60223089 | 358067.28 |
| 115 | 7 | 86.0707 | 130391.9549 | 1961424.09 | 26043899 | 133689.59 |
| 116 | 7 | 86.03892 | 89617.2365 | 289939.02 | 25290064 | 291611.03 |
| 117 | 7 | 84.68415 | 119371.8028 | 1239981.13 | 60599844 | 458001.55 |
| 118 | 8 | 98.36131 | 321077.1 | 4105600.5 | 35013412 | 10510990.2 |
| 119 | 8 | 99.46096 | 2537551.9 | 1731176.1 | 35947326 | 1086271 |
| 120 | 8 | 97.28851 | 751690.7 | 6019263.6 | 72250087 | 7074081.1 |
| 121 | 8 | 98.47515 | 2133281.4 | 3232716.4 | 101952375 | 648978 |
| 127 | 8 | 98.888358 | $4592845.91$ | $5386082.3$ | 104258331 | 99142874.9 |
| 128 | 8 | $9^{96.055014}$ | 1957998.2 | $152651653.6$ | $404884958$ | $\begin{gathered} 103945686 \\ \hline \end{gathered}$ |
| 124 | 8 | 99:43064 | $25882528.9$ | 1026130:8 | 128951568 | 54468887.2 |
| 135 | 8 | 97.523396 | 245493882.3 | $5368870.1$ | 349492322 | 348993832.3 |
| 136 | 8 | 99.919393 | 24159934.8 | 35888293.2 | 45958351 | 794286899.3 |
| 132 | 7 | 98.25481 | 1209918.6 | 7371631.1 | 135912822 | 2646443 |
| 133 | 7 | 98 | 762624.1 | 813814.8 | 57914100 | 5252397.1 |
| 134 | 7 | 99.61498 | 501231.2 | 6283134.3 | 73438281 | 3076513.7 |
| 135 | 7 | 98.33 | 1072597.4 | 4577787.3 | 92001282 | 6874788.1 |
| 136 | 7 | 98.07267 | 768843 | 3373742.6 | 99321171 | 9196754 |
| 137 | 7 | 98.70586 | 1708760.9 | 5630683.7 | 43533557 | 8744041.1 |
| 138 | 7 | 97.05627 | 651801.4 | 6135158.9 | 7747081 | 8218472 |
| 139 | 7 | 98.23505 | 1224650.8 | 6801755.9 | 56694183 | 8881535 |

## KNUST

| 14 0 | $\left\lvert\, \begin{gathered} 799.9095 \\ 6 \end{gathered}\right.$ | 1562372.5 | 2171086.4 | 94516973 | $\begin{aligned} & 7034537 . \\ & 4 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 1 | $7 \begin{gathered}98.4551 \\ 4\end{gathered}$ | 843531.6 | 2359478 | 82212382 | $\begin{aligned} & 3279579 . \\ & 4 \end{aligned}$ |
| 14 2 | $\begin{gathered} 798.5443 \\ 4 \end{gathered}$ | 1585753.4 | 6513016.1 | $\begin{gathered} 13635962 \\ 1 \end{gathered}$ | $\begin{aligned} & 8404108 . \\ & 4 \end{aligned}$ |
| 14 3 | $7 \begin{gathered}99.5776 \\ 4\end{gathered}$ | 2512064.6 | 2664817 | 42779358 | $\begin{aligned} & 7687387 . \\ & 9 \end{aligned}$ |
| 14 4 | 7799.2556 | 481919.7 | 4295186 | 91371505 | $\begin{aligned} & 3223650 . \\ & 5 \end{aligned}$ |
| 14 5 | 7798.8579 | 380896.6 | 176459.2 | 78563706 | $\begin{aligned} & 8539613 . \\ & 5 \end{aligned}$ |


| 14 | 7798.2141 | 1474280 | 5880578.8 | $\begin{array}{\|c\|} \hline 12250198 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & 1800166 . \\ & 6 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 7 | 799.3602 | 1753489.4 | 4149595.9 | $\begin{array}{\|c} 11044626 \\ 2 \end{array}$ | $\begin{aligned} & 7668729 . \\ & 2 \end{aligned}$ |
| 14 8 | 7798.1459 | 1770096.9 | 1615703.2 | $\begin{array}{\|c} 15360839 \\ 5 \end{array}$ | $\begin{aligned} & 10568444 \\ & .9 \end{aligned}$ |
| 14 9 | 7798.1762 | 979183 | 4546131.3 | 37138741 | 629038.4 |
| 15 | 7798.3329 | 663485.8 | 126533.9 | $\begin{array}{\|c\|} \hline 11376335 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & 8598456 . \\ & 4 \end{aligned}$ |
| 15 1 | 7797.0015 | 2085136.4 | 961278 | 64878503 | $\begin{aligned} & 4058873 . \\ & 6 \end{aligned}$ |
| 15 2 | $7 \begin{gathered} 82.0002 \\ 6 \end{gathered}$ | 1108931.94 | $\begin{array}{r} 2506457.1 \\ 65 \end{array}$ | $\begin{gathered} 21362641 . \\ 8 \end{gathered}$ | $\begin{aligned} & 5206934 . \\ & 08 \end{aligned}$ |
| 15 3 | $\begin{gathered} 784.8554 \\ 6 \end{gathered}$ | 326481.57 | $\begin{array}{r} 3766395.7 \\ 82 \end{array}$ | $\begin{gathered} 52237080 . \\ 3 \end{gathered}$ | $\begin{aligned} & 5072298 . \\ & 79 \end{aligned}$ |
| 15 4 | $7 \begin{gathered} 84.1622 \\ 3 \end{gathered}$ | 751980.4 | 110919.21 | 239959.5 | $\begin{aligned} & 1846667 . \\ & 4 \end{aligned}$ |
| 15 | $7 \begin{gathered} 82.3900 \\ 6 \end{gathered}$ | 130032.52 | $\begin{array}{r} 4182459.5 \\ 37 \end{array}$ | $\begin{gathered} 10119667 \\ 3 \end{gathered}$ | $\begin{aligned} & 3578507 . \\ & 92 \end{aligned}$ |
| 15 6 | $\begin{gathered} 783.1142 \\ 8 \end{gathered}$ | 242556.34 | $\begin{array}{r} 3817606.2 \\ 02 \end{array}$ | $\begin{gathered} 43359028 . \\ 6 \end{gathered}$ | $\begin{aligned} & 75559.23 \\ & 8 \end{aligned}$ |
| 15 7 | 7784.4780 | 299437.14 | $\begin{array}{r} 3002470.8 \\ 55 \end{array}$ | $\begin{array}{\|c\|} 33934353 . \\ 2 \end{array}$ | $\begin{aligned} & 1081354 . \\ & 51 \end{aligned}$ |
| 15 8 | 7\| $\begin{gathered}83.6397 \\ 5\end{gathered}$ | 940722.56 | $\begin{array}{r} 1651442.1 \\ 91 \end{array}$ | $\begin{gathered} 40815109 . \\ 1 \end{gathered}$ | $\begin{aligned} & 140021.0 \\ & 86 \end{aligned}$ |


| 15 9 | $7{ }^{7} 84.3319$ |  | $\begin{array}{r} \hline 3597834.8 \\ 87 \end{array}$ |  | $\begin{aligned} & 2405554 . \\ & 54 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 16 0 | 7 782.8810 | 1222675.04 | $\begin{array}{r} 4404799.9 \\ 44 \end{array}$ | $\begin{gathered} 34235253 . \\ 7 \end{gathered}$ | $\begin{aligned} & 2129945 . \\ & 4 \end{aligned}$ |
| 16 1 | 782.2803 | 398132.84 | $\begin{array}{r} 4014768.3 \\ 78 \end{array}$ | $\begin{gathered} 32452348 \\ 4 \\ \hline \end{gathered}$ | $\begin{aligned} & 3481403 . \\ & 73 \end{aligned}$ |
| 16 2 | $7 \begin{gathered}71.5216 \\ 3\end{gathered}$ | 642158.85 | 302468.901 | $\begin{gathered} 16189601 \\ 7 \end{gathered}$ | $\begin{aligned} & 2297697 . \\ & 06 \end{aligned}$ |
| 16 3 | 7 $\begin{gathered}84.5035 \\ 9\end{gathered}$ | 342295.36 | $\begin{array}{r} 4210096.4 \\ 72 \end{array}$ | $\begin{gathered} 29084457 . \\ 8 \end{gathered}$ | $\begin{aligned} & 3358087 . \\ & 66 \end{aligned}$ |
| 16 <br> 4 | 7882.1840 | 612146.81 | $\begin{array}{r} 4138228.7 \\ 96 \end{array}$ | 356886.1 | $\begin{aligned} & 3356740 . \\ & 01 \end{aligned}$ |
| 16 5 | 785.3629 <br> 1 | 346850.82 | $\begin{array}{r} 2136418.3 \\ 64 \end{array}$ | $\begin{gathered} 26767732 \\ 3 \end{gathered}$ | $\begin{aligned} & 3903962 . \\ & 9 \end{aligned}$ |


| 16 | 783.8685 | 348692.61 | 3809241.0 | 52948924. | 3235949. |
| ---: | :---: | :---: | :---: | :---: | :--- |
| 6 | 6 |  | 96 | 8 | 68 |
| 16 | 784.1336 | 585061.92 | 257617.49 | 47837720. | 5357837. |
| 7 | 4 |  |  | 5 | 62 |
| 16 | 782.6606 | 183142.26 | 2735387.3 | 9297909. | 3114191. |
| 8 | 9 |  | 67 | 8 | 44 |
| 16 | 78.9925 | 2 | 416914.07 | 4327.485 | 37434551. |
| 9 | 2 |  | 151150.5 |  |  |
| 17 | 78.4726 | 8 |  | 3091436.36 | 41203646. |
| 0 |  | 4283152. |  |  |  |


| $\begin{array}{r} 17 \\ 1 \\ \hline \end{array}$ | 7 | $\begin{gathered} 82.802 \\ 9 \end{gathered}$ | 449039.2 | $\begin{array}{r} \hline 3833892.6 \\ 39 \\ \hline \end{array}$ | $\begin{gathered} 52844924 \\ .9 \end{gathered}$ | $\begin{aligned} & 4110030.6 \\ & 3 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 17 \\ 2 \end{array}$ | 7 | $\begin{gathered} 82.013 \\ 5 \end{gathered}$ | 891619.7 | $\begin{array}{r} \hline 2206469.4 \\ 13 \\ \hline \end{array}$ | $\begin{gathered} 10045253 \\ .9 \end{gathered}$ | $\begin{array}{\|l\|} \hline 3730017.7 \\ 3 \\ \hline \end{array}$ |
| $\begin{array}{r} 17 \\ 3 \end{array}$ | 8 | $\begin{array}{\|c} 83.210 \\ 33 \end{array}$ | 252109.17 | $\begin{array}{r} 4194378.5 \\ 98 \end{array}$ | $\begin{gathered} 48251213 \\ .6 \end{gathered}$ | $\begin{aligned} & 3010627.3 \\ & 4 \end{aligned}$ |
| 17 4 | 8 | $\begin{gathered} 83.167 \\ 17 \end{gathered}$ | 139858.09 | $\begin{array}{r} 4264789.4 \\ 12 \end{array}$ | $\begin{gathered} 3825606 . \\ 6 \end{gathered}$ | 7424.127 |
| $\begin{array}{r} 17 \\ 5 \end{array}$ | 8 | $\begin{gathered} 81.178 \\ 79 \end{gathered}$ | 164231.81 | $\begin{array}{r} 4328487.0 \\ 57 \\ \hline \end{array}$ | $\begin{gathered} 2508880 . \\ 7 \\ \hline \end{gathered}$ | $\begin{aligned} & 4264805.3 \\ & 8 \end{aligned}$ |
| $\begin{array}{r} 17 \\ 6 \\ \hline \end{array}$ | 8 | $\begin{gathered} 81.176 \\ 34 \end{gathered}$ | 342981.24 | 242238.615 | $\begin{gathered} 47507489 \\ .5 \end{gathered}$ | $\begin{array}{\|l\|} \hline 2225206.3 \\ 3 \\ \hline \end{array}$ |
| $\begin{array}{r} 17 \\ 7 \end{array}$ | 8 | $\begin{gathered} 82.751 \\ 98 \end{gathered}$ | 599384.12 | 2837806.6 | $\begin{gathered} 37492716 \\ .4 \end{gathered}$ | 3997277.8 |
| $\begin{array}{\|r\|} \hline 17 \\ 8 \\ \hline \end{array}$ | 8 | $\begin{array}{\|c\|} \hline 85.0789 \\ 2 \\ \hline \end{array}$ | 18187.86 | $\begin{array}{r} 1179946.1 \\ 05 \end{array}$ | $\begin{array}{\|c\|} \hline 50748985 . \\ 2 \\ \hline \end{array}$ | $\begin{aligned} & 2983732 . \\ & 24 \end{aligned}$ |
| $\begin{array}{r} 17 \\ 9 \end{array}$ | 8 | $\begin{gathered} 83.4482 \\ 7 \end{gathered}$ | 185988.24 | $\begin{array}{r} 3556427.5 \\ 53 \end{array}$ | 30686046 | $\begin{aligned} & 2620285 . \\ & 27 \end{aligned}$ |
| 18 <br> 0 | 8 | $\begin{array}{\|c\|} \hline 81.9460 \\ 2 \\ \hline \end{array}$ | 506371.18 | 509307.737 | $\begin{gathered} 18487116 . \\ 6 \\ \hline \end{gathered}$ | $\begin{aligned} & 1726386 . \\ & 71 \end{aligned}$ |
| $\begin{array}{\|r} \hline 18 \\ 1 \end{array}$ | 8 | $\begin{array}{\|c} 84.9081 \\ 8 \end{array}$ | 629024.6 | 88138.47 | $\begin{gathered} 48886883 . \\ 6 \\ \hline \end{gathered}$ | $\begin{aligned} & 113912.3 \\ & 52 \end{aligned}$ |
| 18 <br> 2 | 8 | $\begin{gathered} 82.9676 \\ 7 \end{gathered}$ | 779307.96 | 639475.288 | 5986640.1 | $\begin{aligned} & 4115126 . \\ & 49 \end{aligned}$ |
| $\begin{array}{r} 18 \\ 3 \\ \hline \end{array}$ | 8 | $\begin{array}{\|c\|} \hline 83.8923 \\ 6 \\ \hline \end{array}$ | 257910.43 | $\begin{array}{r} 2490002.9 \\ 43 \\ \hline \end{array}$ | 32447078 | $\begin{aligned} & 2996130 . \\ & 46 \end{aligned}$ |
| 18 <br> 4 | 8 | $\begin{array}{\|c\|} \hline 81.9891 \\ 4 \end{array}$ | 665520.5 | $\begin{array}{\|r} 2495753.2 \\ 19 \end{array}$ | 50423673 | $\begin{aligned} & 1045263 . \\ & 87 \end{aligned}$ |
| 18 <br> 5 <br> 18 | 8 | $\begin{array}{\|c} 82.5017 \\ 3 \\ \hline \end{array}$ | 744796.73 | $\begin{array}{r} 3776270.1 \\ 59 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline 33095830 . \\ 8 \\ \hline \end{array}$ | $\begin{aligned} & 5165246 . \\ & 79 \end{aligned}$ |
| 18 <br> 6 | 8 | $\begin{array}{\|c\|} \hline 82.0144 \\ 2 \\ \hline \end{array}$ | 284858.94 | $\begin{array}{r} 1840045.1 \\ 88 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline 43680874 . \\ 4 \\ \hline \end{array}$ | $\begin{aligned} & 1464142 . \\ & 17 \end{aligned}$ |
| 18 <br> 7 <br> 18 | 8 | $\begin{array}{\|c\|} \hline 82.1576 \\ 2 \end{array}$ | 809251.09 | 109092.966 | $\begin{array}{\|c\|} \hline 52524259 . \\ 6 \end{array}$ | $\begin{aligned} & 557581.9 \\ & 72 \end{aligned}$ |
| 18 <br> 8 | 8 | $\begin{array}{\|c} 84.0623 \\ 6 \end{array}$ | 504028.44 | $\begin{array}{r} 1127782.1 \\ 43 \end{array}$ | $\begin{gathered} 47865636 . \\ 7 \end{gathered}$ | $\begin{aligned} & 5135843 . \\ & 49 \end{aligned}$ |
| 18 <br> 9 | 8 | 83.2653 | 13260.05 | 2146417.36 | $\begin{gathered} 40086973 . \\ 3 \end{gathered}$ | $\begin{aligned} & 1382960 . \\ & 05 \end{aligned}$ |
| 19 0 | 8 | $\begin{array}{\|c\|} 81.5087 \\ 1 \end{array}$ | 835344.57 | $\begin{array}{r} 2354995.9 \\ 84 \\ \hline \end{array}$ | $\begin{gathered} 18127613 . \\ 6 \end{gathered}$ | $\begin{aligned} & 529845.9 \\ & 59 \end{aligned}$ |

$\left.\begin{array}{|r|c|c|c|r|r|l|}\hline 19 & 8 & 84.2615 & 591802.54 & 1364560.8 & 46215888 . & 1003650 . \\ 1 & 1 & 1 & & 79 & 5 & 35\end{array}\right]$

| 19 <br> 8 | 9 | $\begin{gathered} 83.3073 \\ 7 \end{gathered}$ | 914371.95 | $\begin{array}{\|r} \hline 1873206.1 \\ 84 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline 37833570 \\ 3 \end{array}$ | $\begin{aligned} & 1599196 . \\ & 53 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 9 | 9 | $\begin{gathered} 82.5892 \\ 4 \end{gathered}$ | 812478.77 | $\begin{array}{\|r} 4212139.3 \\ 83 \end{array}$ | $\begin{gathered} 22130239 \\ 5 \end{gathered}$ | $\begin{aligned} & 1555236 . \\ & 69 \end{aligned}$ |
| 20 <br> 0 | 9 | $\begin{array}{\|c\|} \hline 83.1944 \\ 6 \end{array}$ | 511622.81 | $\begin{array}{r} 3449691.5 \\ 39 \end{array}$ | 106844.8 | $\begin{aligned} & 3268024 . \\ & 41 \end{aligned}$ |
| 20 1 | 9 | $\begin{array}{\|c\|} \hline 84.2021 \\ 8 \end{array}$ | 873907.98 | 691358.26 | $\begin{gathered} 35342167 . \\ 7 \end{gathered}$ | $\begin{aligned} & 328620.6 \\ & 15 \end{aligned}$ |
| $\begin{array}{r} 20 \\ 2 \\ \hline \end{array}$ | 9 | $\begin{array}{\|c\|} \hline 81.6963 \\ 4 \end{array}$ | 697783.73 | $\begin{array}{r} 1312174.3 \\ 36 \\ \hline \end{array}$ | $\begin{gathered} 20997534 . \\ 4 \end{gathered}$ | $\begin{aligned} & 102770.4 \\ & 79 \end{aligned}$ |
| 20 <br> 3 <br> 20 | 7 | $\begin{array}{\|c} 83.3893 \\ 9 \end{array}$ | 920241.04 | $\begin{array}{r} 2339441.3 \\ \hline 47 \\ \hline \end{array}$ | $\begin{gathered} 36212390 . \\ 1 \end{gathered}$ | $\begin{aligned} & 1349861 . \\ & 49 \end{aligned}$ |
| $\begin{array}{\|r\|} \hline 20 \\ 4 \\ \hline \end{array}$ | 7 | $\begin{gathered} 83.7592 \\ 9 \end{gathered}$ | 1002644.98 | $\begin{array}{r} 2263893.5 \\ 44 \\ \hline \end{array}$ | $\begin{gathered} 53658663 . \\ 7 \end{gathered}$ | $\begin{aligned} & 4426091 . \\ & 6 \end{aligned}$ |
| $\begin{array}{r}20 \\ 5 \\ \hline 2\end{array}$ | 7 | $\begin{gathered} 82.2098 \\ 9 \end{gathered}$ | 1197804.19 | $\begin{array}{\|r} 3550038.3 \\ 04 \end{array}$ | $\begin{gathered} 34962145 \\ 5 \\ \hline \end{gathered}$ | $\begin{aligned} & 3380922 . \\ & 03 \end{aligned}$ |
| 20 <br> 6 | 7 | $\begin{array}{\|c\|} \hline 81.8764 \\ 1 \end{array}$ | 632321.45 | $\begin{array}{r} 2599828.9 \\ 17 \end{array}$ | $\begin{gathered} 30217487 . \\ 9 \end{gathered}$ | $\begin{aligned} & 1814431 . \\ & 76 \end{aligned}$ |
| 20 <br> 7 | 7 | $\begin{array}{\|c\|} \hline 81.5314 \\ 6 \\ \hline \end{array}$ | 131873.99 | $\begin{array}{r} 4006209.9 \\ 53 \end{array}$ | 3740852.1 | $\begin{aligned} & 2410676 . \\ & 62 \end{aligned}$ |
| $\begin{array}{r}20 \\ 8 \\ \hline 20\end{array}$ | 7 | $\begin{gathered} 82.4320 \\ 8 \\ \hline \end{gathered}$ | 819794.69 | $\begin{array}{r} 2158832.1 \\ 63 \\ \hline \end{array}$ | $\begin{gathered} 10874561 . \\ 6 \\ \hline \end{gathered}$ | $\begin{aligned} & 3406117 . \\ & 11 \end{aligned}$ |
| 20 <br> 9 <br> 21 | 7 | $\begin{array}{\|c} 83.4572 \\ 5 \end{array}$ | 276370.12 | 2884354.79 | 10300981 | $\begin{aligned} & 1846963 . \\ & 81 \end{aligned}$ |
| 21 <br> 0 <br> 21 | 7 | $\begin{array}{\|c\|} \hline 85.5392 \\ 6 \\ \hline \end{array}$ | 124126.24 | 682878.561 | $\begin{gathered} 26423785 . \\ 7 \\ \hline \end{gathered}$ | $\begin{aligned} & 3764212 . \\ & 24 \end{aligned}$ |
| 21 <br> 1 | 7 | $\begin{array}{\|c\|} \hline 82.8574 \\ 4 \end{array}$ | 438207.62 | $\begin{array}{r} 2358484.7 \\ 28 \\ \hline \end{array}$ | $\begin{gathered} 36034865 \\ 5 \end{gathered}$ | $\begin{aligned} & 4711171 . \\ & 22 \end{aligned}$ |
| 21 <br> 2 <br> 21 | 7 | $\begin{array}{\|c} 83.6501 \\ 8 \\ \hline \end{array}$ | 1149509.81 | $\begin{array}{r} 1578214.2 \\ 98 \\ \hline \end{array}$ | $\begin{gathered} 36312850 . \\ 8 \end{gathered}$ | $\begin{aligned} & 73941.14 \\ & 3 \end{aligned}$ |
| 21 <br> 3 <br> 21 | 7 | $\begin{gathered} 82.3529 \\ 1 \end{gathered}$ | 265045.14 | $\begin{array}{\|r} 2150124.3 \\ 31 \end{array}$ | $\begin{gathered} 40063069 . \\ 9 \\ \hline \end{gathered}$ | $\begin{aligned} & 1330385 . \\ & 29 \end{aligned}$ |
| $\begin{array}{r}21 \\ 4 \\ \hline\end{array}$ | 7 | $\begin{gathered} 83.0033 \\ 1 \end{gathered}$ | 1048314.37 | 80128.112 | $\begin{gathered} 16878809 \\ 5 \\ \hline \end{gathered}$ | $\begin{aligned} & 5258928 \text {. } \\ & 44 \end{aligned}$ |


| 215 | 7 | 81.35924 | 520558.49 | 1242748.596 | 9417916.2 | 5202649.24 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| 21 6 | 7 | $\begin{gathered} 82.6245 \\ 1 \end{gathered}$ | 838590.5 | 788708.032 | $\begin{array}{\|c\|} \hline 10478538 . \\ 1 \end{array}$ | $\begin{aligned} & 4456794 . \\ & 37 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 7 | 7 | $\begin{gathered} 82.5881 \\ 9 \end{gathered}$ | 482478.5 | 1851866.83 | $\begin{gathered} 45443835 . \\ 6 \end{gathered}$ | $\begin{aligned} & 2447043 . \\ & 69 \end{aligned}$ |
| 21 <br> 8 | 7 | 83.7637 | 841223.56 | $\begin{array}{\|r} 3920988.8 \\ 93 \end{array}$ | 28113694 | $\begin{aligned} & 2094294 . \\ & 89 \end{aligned}$ |
| 21 9 | 7 | $\begin{gathered} 82.1520 \\ 8 \end{gathered}$ | 123279.46 | $\begin{array}{r} 2460541.2 \\ 03 \end{array}$ | 172946.5 | 4996613 |
| 22 0 | 7 | $\begin{gathered} 82.9583 \\ 4 \end{gathered}$ | 98276.5 | $\begin{array}{\|r\|} \hline 2774433.4 \\ 51 \end{array}$ | $\begin{gathered} 25529915 . \\ 8 \end{gathered}$ | $\begin{aligned} & 3435995 . \\ & 46 \end{aligned}$ |
| 22 1 | 7 | $\begin{array}{\|c\|} \hline 81.9534 \\ 4 \end{array}$ | 85753.41 | $\begin{array}{\|r} 3841123.9 \\ 84 \end{array}$ | 9752038.2 | $\begin{aligned} & 1772478 . \\ & 39 \end{aligned}$ |
| 22 2 | 7 | $\begin{array}{\|c\|} \hline 83.2766 \\ 9 \end{array}$ | 449467.68 | $\begin{array}{r} 1113997.3 \\ 55 \end{array}$ | $\begin{array}{\|c\|} \hline 30401404 . \\ 5 \end{array}$ | $\begin{aligned} & 297482.6 \\ & 12 \end{aligned}$ |


| 22 3 | 7 | $\begin{gathered} 82.3587 \\ 7 \end{gathered}$ | 1150160.89 | 234618.706 | $\begin{array}{\|c\|} \hline 52839977 . \\ 7 \end{array}$ | $\begin{aligned} & 2223829 . \\ & 49 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22 4 | 7 | $\begin{gathered} 84.6407 \\ 4 \end{gathered}$ | 387618.89 | $\begin{array}{\|r} 3028238.4 \\ \hline 75 \end{array}$ | 27477685 | $\begin{aligned} & 4064979 . \\ & 71 \end{aligned}$ |
| 22 | 7 | $\begin{gathered} 84.2799 \\ 2 \end{gathered}$ | 261949.13 | $\begin{array}{\|r} 2606305.1 \\ 96 \end{array}$ | $\begin{gathered} 37363993 . \\ 8 \end{gathered}$ | $\begin{aligned} & 962988.6 \\ & 55 \end{aligned}$ |
| 22 6 | 7 | $\begin{gathered} 83.5457 \\ 3 \end{gathered}$ | 90203.64 | 17982.031 | 6080739.5 | $\begin{aligned} & 2018319 . \\ & 18 \end{aligned}$ |
| 22 7 | 7 | $\begin{gathered} 81.4545 \\ 6 \end{gathered}$ | 474819.59 | $\begin{array}{\|r} 2255684.7 \\ 31 \end{array}$ | $\begin{gathered} 40265326 . \\ 9 \end{gathered}$ | $\begin{aligned} & 3947044 . \\ & 78 \end{aligned}$ |
| 22 8 | 7 | $\begin{gathered} 85.1288 \\ 4 \end{gathered}$ | 840004.49 | $\begin{array}{r} 2254550.5 \\ 39 \\ \hline \end{array}$ | $\begin{gathered} 34606850 . \\ 9 \end{gathered}$ | $\begin{aligned} & 4928665 . \\ & 61 \end{aligned}$ |
| 22 9 | 7 | $\begin{gathered} 81.2554 \\ 4 \end{gathered}$ | 678625.15 | $\begin{array}{\|r} 2526119.1 \\ 52 \end{array}$ | $\begin{gathered} 28065738 . \\ 2 \end{gathered}$ | $\begin{aligned} & 4570385 \text {. } \\ & 49 \end{aligned}$ |
| 23 0 | 7 | $\begin{gathered} 82.8116 \\ 1 \end{gathered}$ | 902389.46 | $\begin{array}{\|r} 1904850.6 \\ 21 \end{array}$ | $\begin{gathered} 26998986 . \\ 7 \end{gathered}$ | $\begin{aligned} & 4020959 \text {. } \\ & 23 \end{aligned}$ |
| 23 1 | 7 | $\begin{gathered} 83.4063 \\ 4 \end{gathered}$ | 156073.58 | 900091.1 | 22485755 | $\begin{aligned} & 2151751 . \\ & 55 \end{aligned}$ |
| 13 2 | 7 | $\begin{gathered} 83.2535 \\ 4 \\ \hline \end{gathered}$ | 914438.94 | $\begin{array}{\|r} 2201175.2 \\ 06 \end{array}$ | $\begin{gathered} 21430936 . \\ 5 \end{gathered}$ | $\begin{aligned} & 59461.46 \\ & 5 \end{aligned}$ |
| 23 3 | 7 | $\begin{gathered} 83.5398 \\ 9 \end{gathered}$ | 961744.44 | 952767.904 | 3770364.1 | $\begin{aligned} & 2329108 . \\ & 98 \end{aligned}$ |
| 23 4 | 6 | $\begin{gathered} 83.4747 \\ 2 \end{gathered}$ | 772487.46 | $\begin{array}{\|r} 2981480.6 \\ 27 \end{array}$ | $\begin{gathered} 20293937 . \\ 7 \end{gathered}$ | $\begin{aligned} & 3688927 . \\ & 16 \end{aligned}$ |


| 23 5 | 6 | $\begin{array}{\|c\|} \hline 85.1578 \\ 3 \end{array}$ |  | 634778.963 | 41159930. 3 | $\begin{aligned} & 585836.4 \\ & 74 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 <br> 6 | 6 | $\begin{gathered} 83.9336 \\ 3 \end{gathered}$ | 664351.17 | $\begin{array}{\|r} 1486543.0 \\ 32 \end{array}$ | $\begin{gathered} 29264153 . \\ 4 \end{gathered}$ | $\begin{aligned} & 1999926 . \\ & 63 \end{aligned}$ |
| 63 <br> 7 | 6 | $\begin{array}{\|c\|} \hline 82.6949 \\ 7 \\ \hline \end{array}$ | 774861.5 | $\begin{array}{r} 1217578.7 \\ 69 \\ \hline \end{array}$ | $\begin{gathered} 36349045 . \\ 6 \\ \hline \end{gathered}$ | $\begin{aligned} & 3935233 . \\ & 36 \end{aligned}$ |
| 23 <br> 8 | 6 | 80.9521 | 103651.64 | 2547610.29 | $\begin{gathered} 28140925 . \\ 5 \end{gathered}$ | $\begin{aligned} & 779745.9 \\ & 29 \end{aligned}$ |
| 23 9 | 6 | $\begin{gathered} 81.9396 \\ 3 \end{gathered}$ | 945886.5 | $\begin{array}{\|r} 1630814.0 \\ 03 \end{array}$ | $\begin{gathered} 45479423 . \\ 3 \end{gathered}$ | $\begin{aligned} & 5193703 . \\ & 14 \end{aligned}$ |
| 24 <br> 0 | 6 | $\begin{gathered} 82.3811 \\ 1 \end{gathered}$ | 462033.61 | $\begin{array}{r} 3059495.9 \\ 27 \end{array}$ | $\begin{gathered} 12110707 \\ 6 \end{gathered}$ | $\begin{aligned} & 1596343 . \\ & 94 \end{aligned}$ |
| 24 1 | 6 | $\begin{gathered} 83.3475 \\ 1 \end{gathered}$ | 580796.31 | $\begin{array}{\|r} 2710078.3 \\ 41 \end{array}$ | 4594862.1 | $\begin{aligned} & 2000744 . \\ & 43 \end{aligned}$ |



| 25 9 | $\left\|\begin{array}{c} 78.1194 \\ 5 \end{array}\right\|$ | 238161.95 | $\begin{array}{r} 3179735.6 \\ \hline 32 \end{array}$ | $\left\|\begin{array}{c} 7871234 . \\ 6 \end{array}\right\|$ | $\begin{aligned} & 4391949 . \\ & 3 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 26 \\ 0 \end{array}$ | $\begin{gathered} 781.4922 \\ 6 \end{gathered}$ | 703713.94 | $\begin{array}{r} 1703513.7 \\ 67 \end{array}$ | $10197228 .$ | $\begin{aligned} & 2753317 . \\ & 88 \end{aligned}$ |
| $\begin{array}{r} 26 \\ 1 \\ \hline \end{array}$ | 783.5719 | 964185.25 | 173117.275 | $20671767 .$ | $\begin{aligned} & 3807263 . \\ & 09 \end{aligned}$ |
| $\begin{array}{r} 26 \\ 2 \\ \hline \end{array}$ | 785.7968 | 745040.93 | 761024.059 | $\begin{gathered} 38582157 \\ 2 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 2097520 . \\ & 38 \end{aligned}$ |
| $\begin{array}{r} 26 \\ \hline 3 \end{array}$ | $7 \begin{gathered} 83.8031 \\ 7 \end{gathered}$ | 443689.53 | $\begin{array}{r} \hline 3584855.3 \\ 98 \end{array}$ | $\begin{gathered} 22158169 \\ 4 \end{gathered}$ | $\begin{aligned} & \hline 2684134 . \\ & 75 \end{aligned}$ |
| $\begin{array}{r} 26 \\ \hline \end{array}$ | 782.9318 | 855807.12 | $\begin{array}{r} 3579748.6 \\ 18 \end{array}$ | $\begin{array}{\|c\|} \hline 51757115 . \\ 7 \\ \hline \end{array}$ | $\begin{aligned} & 2868105 . \\ & 54 \end{aligned}$ |
| $\begin{array}{r} 26 \\ 5 \end{array}$ | $7 \begin{gathered} 82.6034 \\ 7 \end{gathered}$ | 1137262.22 | $2933227.4$ | $\begin{array}{\|c\|} \hline 14033400 . \\ 2 \end{array}$ | $\begin{aligned} & 4185749 . \\ & 72 \end{aligned}$ |
| 26 6 | $\begin{gathered} 780.6826 \\ 5 \end{gathered}$ | 341899.69 | $\begin{array}{r} 4022778.0 \\ 52 \end{array}$ | $34404797$ <br> 1 | $\begin{aligned} & \hline 697822.2 \\ & 98 \end{aligned}$ |


| 26 7 | 783.3945 9 | 1169754.76 | 4212125.0 03 | \|34238714. | $\begin{aligned} & 2827136 . \\ & 5 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 26 8 | $\begin{gathered} 783.1541 \\ 3 \end{gathered}$ | 1082384.08 | $\begin{array}{r} 1056647.8 \\ 27 \end{array}$ | $\begin{gathered} 14838174 . \\ 4 \\ \hline \end{gathered}$ | $\begin{aligned} & 1106071 . \\ & 61 \end{aligned}$ |
| 86 9 | 782.6504 7 | 481415.96 | $\begin{array}{\|r\|} \hline 3486467.8 \\ 82 \\ \hline \end{array}$ | $\begin{gathered} 2101134 \\ 9 \end{gathered}$ | $\begin{aligned} & 2663515 . \\ & 07 \\ & \hline \end{aligned}$ |
| 27 0 | 781.3585 3 | 913673.6 | 61811.988 | $\begin{gathered} 44428938 . \\ 1 \\ \hline \end{gathered}$ | $\begin{aligned} & 4065947 . \\ & 85 \\ & \hline \end{aligned}$ |
| 27 1 | 781.9892 8 | 1203704.01 | 3105694.0 46 | $\begin{gathered} 15628244 . \\ 1 \end{gathered}$ | $\begin{array}{\|l\|} \hline 4182229 . \\ 66 \\ \hline \end{array}$ |
| 1 27 2 | 782.8806 8 | 302825.12 | $\begin{array}{\|r} \hline 3644499.5 \\ 99 \end{array}$ | $\begin{gathered} 33199078 . \\ 3 \end{gathered}$ | $\begin{aligned} & 541033.6 \\ & 11 \end{aligned}$ |
| 27 3 | $786.1263$ |  | $\begin{array}{\|r} 3997632.3 \\ 93 \end{array}$ | $\begin{gathered} 33979416 . \\ 7 \end{gathered}$ | $\begin{aligned} & 2067895 . \\ & 06 \end{aligned}$ |


| 27 4 | $9 \begin{gathered} 80.7715 \\ 5 \end{gathered}$ | 332821.3 | $\begin{array}{\|r} 3427811.1 \\ 28 \end{array}$ | $\begin{array}{\|c\|} 16371296 \\ 2 \end{array}$ | $\begin{aligned} & 5052526 . \\ & 93 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 27 \\ 5 \end{array}$ | $9 \begin{gathered} 98.8888 \\ 5 \end{gathered}$ | 62442.69 | $\begin{array}{r} 1982780.8 \\ 24 \\ \hline \end{array}$ | $\begin{gathered} 9407327 . \\ 8 \end{gathered}$ | $\begin{aligned} & 5340105 . \\ & 97 \end{aligned}$ |
| $\begin{array}{\|r\|} \hline 27 \\ 6 \end{array}$ | $\begin{gathered} 983.7944 \\ 9 \end{gathered}$ | 925738.49 | 693501.556 | $\begin{array}{\|c\|} \hline 9610551 . \\ 8 \\ \hline \end{array}$ | $\begin{aligned} & 3828292 . \\ & 88 \end{aligned}$ |
| $\begin{array}{\|r\|} \hline 27 \\ \hline \end{array}$ | $\begin{gathered} 981.0216 \\ 5 \\ \hline \end{gathered}$ | 388732.63 | 368678.755 | $\begin{array}{\|c\|} \hline 30004130 . \\ 1 \\ \hline \end{array}$ | $\begin{aligned} & 3632216 . \\ & 01 \end{aligned}$ |
| $\begin{array}{\|r\|} \hline 27 \\ 8 \end{array}$ | $\begin{gathered} 984.0644 \\ 2 \end{gathered}$ | 370392.39 | $\begin{array}{r} 3222356.0 \\ 68 \end{array}$ | $\begin{array}{\|c\|} \hline 24015155 . \\ 2 \\ \hline \end{array}$ | $\begin{aligned} & 2938275 . \\ & 35 \end{aligned}$ |
| 27 <br> 9 | 982.1673 | 60050.24 | $\begin{array}{r} 2233200.2 \\ 59 \end{array}$ | $\begin{gathered} 12855566 . \\ 4 \end{gathered}$ | 3508848 |
| 28 <br> 0 | $9 \begin{gathered}93.6479 \\ 1\end{gathered}$ | 50902.58 | $\begin{array}{r} 2588040.3 \\ 99 \end{array}$ | $\begin{gathered} 7265053 . \\ 7 \end{gathered}$ | $\begin{aligned} & 4546933 . \\ & 71 \end{aligned}$ |
| 28 <br> 1 <br> 28 | $\begin{gathered} 984.3864 \\ 5 \\ \hline \end{gathered}$ | 395405.46 | 462025.047 | $\begin{gathered} 24387809 . \\ 6 \\ \hline \end{gathered}$ | 4592257 |
| $\begin{array}{r} 28 \\ \hline 2 \\ \hline \end{array}$ | $\begin{gathered} 983.7828 \\ 6 \\ \hline \end{gathered}$ | 642798.47 | $\begin{array}{r} 1680267.4 \\ 61 \end{array}$ | $\begin{array}{\|c\|} \hline 16255169 \\ 6 \\ \hline \end{array}$ | $\begin{aligned} & 4584855 . \\ & 99 \end{aligned}$ |
| 28 <br> 3 <br> 28 | $9 \begin{gathered}94.2868 \\ 6\end{gathered}$ | 973107.52 | $\begin{array}{r} 2230074.1 \\ 33 \end{array}$ | $\begin{gathered} 34853119 . \\ 3 \end{gathered}$ | $\begin{aligned} & 5459637 . \\ & 64 \end{aligned}$ |
| $\begin{array}{r} 28 \\ 4 \end{array}$ | $\begin{gathered} 983.2182 \\ 9 \end{gathered}$ | 359358.36 | $\begin{array}{r} 1960264.3 \\ 07 \end{array}$ | $\begin{gathered} 34902857 . \\ 4 \end{gathered}$ | $\begin{aligned} & 739480.9 \\ & 64 \end{aligned}$ |
| 28 <br> 5 | $9 \begin{gathered}95.3206 \\ 2\end{gathered}$ | 622980.97 | 1808297.52 | $\begin{gathered} 25564092 . \\ 5 \\ \hline \end{gathered}$ | $\begin{aligned} & 3604868 . \\ & 08 \end{aligned}$ |
| $\begin{array}{r} \hline 28 \\ 6 \\ \hline \end{array}$ | $\begin{gathered} 982.4892 \\ 5 \\ \hline \end{gathered}$ | 107462.22 | $\begin{array}{r} 1361323.1 \\ 36 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline 3480338 . \\ 6 \\ \hline \end{array}$ | $\begin{aligned} & 3535390 . \\ & 52 \\ & \hline \end{aligned}$ |
| 28 <br> 7 | $9 \begin{gathered}92.7996 \\ 2\end{gathered}$ | 236161.33 | $\begin{array}{r} 1993227.3 \\ 24 \end{array}$ | $\begin{array}{\|c\|} 13071725 . \\ 2 \end{array}$ | $\begin{aligned} & 1417624 . \\ & 19 \end{aligned}$ |
| 28 8 | $9 \begin{gathered}93.9905 \\ 1\end{gathered}$ | 568465.45 | $\begin{array}{r} 4253858.5 \\ 15 \end{array}$ | $\begin{gathered} 3004631 . \\ 4 \end{gathered}$ | $\begin{aligned} & 3419813 . \\ & 85 \end{aligned}$ |
| 28 <br> 9 | $\begin{array}{\|c} 983.3023 \\ 3 \end{array}$ | 879398.98 | $\begin{array}{r} 3892819.1 \\ 31 \end{array}$ | $\begin{gathered} 45604350 . \\ 9 \end{gathered}$ | $\begin{aligned} & 2076367 . \\ & 02 \end{aligned}$ |
| $\begin{array}{r}29 \\ 0 \\ \hline 29\end{array}$ | $9 \begin{gathered}83.7857 \\ 5\end{gathered}$ | 976108.51 | $\begin{array}{r} 3086380.7 \\ 24 \end{array}$ | 45976917 | $\begin{aligned} & 2203147 . \\ & 08 \end{aligned}$ |
| 29 <br> 1 <br> 29 | $9 \begin{gathered} 83.3855 \\ 9 \end{gathered}$ | 720814.44 | 2676457.97 | 4734366 | $\begin{aligned} & 3983036 . \\ & 09 \end{aligned}$ |
| 19 29 | 983.3448 <br> 8 | 680451.54 | 357113.537 | $\begin{gathered} 34467885 . \\ 4 \end{gathered}$ | $\begin{aligned} & 1295516 . \\ & 85 \end{aligned}$ |


| 293 | 7 | 82.70173 | 408872.71 | 451000.472 | 47074090.6 | 2660984.45 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 294 | 7 | 84.04845 | 565370.41 | 1376975.921 | 676853 | 5358451.63 |
| 295 | 7 | 81.56268 | 537444.5 | 3347157.996 | 14595461.5 | 3997065.12 |
| 296 | 7 | 83.74976 | 952701.56 | 2928378.713 | 16890826.9 | 4167938.23 |
| 297 | 7 | 82.98392 | 1116979.42 | 918113.738 | 11019816.6 | 2278026.75 |
| 298 | 7 | 82.04645 | 366295.71 | 3307574.424 | 50570556.9 | 1309836.72 |
| 299 | 7 | 82.5939 | 596098.94 | 2768207.622 | 50449984.7 | 3800476.25 |
| $\begin{array}{r} 300 \\ 303 \\ \hline \end{array}$ | 7 | $\begin{aligned} & 83.12855 \\ & 81.33659 \\ & \hline \end{aligned}$ | $\begin{array}{r} 654830.08 \\ 626887.19 \\ \hline \end{array}$ | $\begin{aligned} & 949033.596 \\ & 1535792.94 \end{aligned}$ | $\begin{gathered} 1487381.4 \\ 51627144.6 \end{gathered}$ | $\begin{aligned} & 1446292.86 \\ & 4475277.91 \end{aligned}$ |
| $\begin{array}{r} 301 \\ 304 \\ \hline \end{array}$ | 7 | $\begin{array}{r} 84.57414 \\ 82.64096 \\ \hline \end{array}$ | $\begin{array}{r} 667355.8 \\ 39874.23 \end{array}$ | $\begin{aligned} & 4161709.929 \\ & 153840.895 \\ & \hline \end{aligned}$ | $\begin{array}{r} 43293061.6 \\ 23547727.4 \\ \hline \end{array}$ | $\begin{array}{r} 2603546.03 \\ 3511300.24 \\ \hline \end{array}$ |
| $\begin{array}{r} 302 \\ 305 \\ \hline \end{array}$ | $\begin{aligned} & 7 \\ & 7 \end{aligned}$ | $\begin{array}{r} 80.79402 \\ 82.86258 \\ \hline \end{array}$ | $\begin{array}{r} 191073.67 \\ 299551.58 \\ \hline \end{array}$ | $\begin{array}{r} 862205.143 \\ 47965.456 \\ \hline \end{array}$ | $\begin{aligned} & 40896636.1 \\ & 14759655.8 \end{aligned}$ | $\begin{array}{r} 887895.423 \\ 5514417.95 \end{array}$ |
| 306 | 7 | 83.40433 | 329617.39 | 961080.231 | 28465518.9 | 675634.81 |
| 307 | 7 | 83.56454 | 860246.09 | 1362560.353 | 38023045 | 4405423.36 |
| 308 | 7 | 81.91672 | 1021111.69 | 1614071.552 | 47464989.5 | 2200039.33 |
| 309 | 7 | 83.00303 | 271677.77 | 2380247.211 | 22854036.8 | 2830575.23 |
| 310 | 7 | 88.18147 | 57529.218 | 6814260.9 | 1162750 | 3087755 |
| 311 | 7 | 88.82126 | 113881.995 | 8338042.7 | 71804619 | 4734982.2 |
| 312 | 7 | 88.10815 | 36197.733 | 4049351.7 | 27281780 | 2496369.3 |
| 313 | 7 | 85.74068 | 14956.951 | 4903338.3 | 112920284 | 1353712.2 |
| 314 | 7 | 86.88837 | 131985.658 | 3840085.7 | 54810906 | 3809936.9 |
| 315 | 7 | 90.90149 | 143581.407 | 7526950.2 | 65157564 | 9520888.6 |
| 316 | 7 | 87.20899 | 30475.229 | 7106504.4 | 62740686 | 7091437.7 |
| 317 | 7 | 86.20344 | 118479.591 | 7613872.8 | 72482768 | 6877266 |
| 318 | 7 | 87.83032 | 110272.7 | 5675110.5 | 59200746 | 3210662.3 |
| 319 | 7 | 87.02104 | 107721.596 | 2836378.7 | 111281717 | 10327105.2 |
| 320 | 7 | 88.47132 | 74717.345 | 4004816.9 | 29077697 | 5225677.7 |
| 321 | 7 | 88.67773 | 15785.178 | 2688550.6 | 126579968 | 4989785 |
| 322 | 7 | 87.62832 | 92125.424 | 8191706.6 | 75842240 | 212354.6 |
| 323 | 7 | 90.77804 | 43607.025 | 3433756 | 1395954 | 9383740.1 |
| 324 | 7 | 89.14697 | 72408.06 | 8932922.2 | 134817952 | 4261790.9 |
| 325 | 7 | 88.95022 | 29387.476 | 1588144.9 | 28418913 | 10180621.8 |
| 326 | 7 | 88.35316 | 61663.767 | 8127159.5 | 49711145 | 9944369.7 |
| 327 | 7 | 87.65222 | 29442.597 | 304675.8 | 40708006 | 7568533 |
| 328 | 7 | 88.01796 | 83021.647 | 3247795 | 37382281 | 4644610.1 |
| 329 | 7 | 88.52633 | 4785.769 | 7345243.7 | 118206930 | 8640336.7 |
| 330 | 7 | 87.91572 | 39755.848 | 4256909.2 | 120574855 | 4742856.8 |



| 331 | 7 | 88.08205 | 87622.152 | 8705914.3 | 127056705 | 7978300.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 332 | 7 | 87.00328 | 26540.937 | 1764029.2 | 103102523 | 10705478.1 |
| 333 | 7 | 88.21321 | 110152.115 | 2471658.3 | 32613252 | 6801890.3 |
| 334 | 7 | 88.34005 | 80230.831 | 6487277.6 | 103856678 | 9176406.1 |
| 335 | 7 | 87.99089 | 32266.654 | 7878471.9 | 64888924 | 6484969.8 |
| 336 | 7 | 88.72984 | 43003.87 | 3644291.1 | 16884711 | 1981714.3 |
| 337 | 7 | 89.34905 | 90827.068 | 2653690.3 | 65273776 | 11494811.1 |
| 338 | 7 | 89.2151 | 62324.722 | 5557371 | 56481462 | 4545928.1 |
| 339 | 7 | 88.36396 | 107390.436 | 2545526.7 | 47762861 | 11177873.8 |
| 340 | 7 | 86.08986 | 139280.753 | 473953.4 | 121456915 | 7791173 |
| 341 | 7 | 88.07091 | 124803.696 | 5717592.1 | 33522947 | 7551053.4 |
| 342 | 7 | 89.03802 | 15173.06 | 5703553.2 | 72641946 | 5685400.8 |
| 343 | 7 | 89.55549 | 107843.116 | 8209848.2 | 118997440 | 5290858.3 |
| 347 | 7 | 86.91763 | 48969.767 | 5966379.8 | 44904134 | 2919564.8 |
| 344 | 7 | 87.81817 | 30192.939 | 9000459.6 | 45091528 | 3784712.3 |
| 348 | 7 | 87.87288 | 67577.916 | 6360816.4 | 112181724 | 5523929.5 |
| 345 | 7 | 87.03665 | 8617.745 | 3460863.6 | 93788019 | 3962122 |
| 349 | 7 | 88.99133 | 17420.574 | 4995224.2 | 62520698 | 6522045.5 |
| 346 | 7 | 90.22766 | 22037.28 | 7126431.5 | 131187063 | 5576918.3 |
| 350 | 7 | 88.47752 | 139362.052 | 3946491.2 | 37116207 | 5877867.4 |
| 351 | 7 | 89.60176 | 102961.61 | 4439344.5 | 98843902 | 7990385.6 |
| 352 | 7 | 88.18573 | 144212.282 | 3900710.2 | 100545363 | 6429800.4 |
| 353 | 7 | 89.18348 | 4344.305 | 881263 | 49073372 | 9479460.7 |
| 354 | 7 | 87.62891 | 71287.979 | 5682420 | 90869199 | 9970019 |
| 355 | 7 | 89.27501 | 108662.873 | 4921103.4 | 27701986 | 8191991.6 |
| 356 | 7 | 90.68646 | 91382.941 | 4794873.5 | 99246894 | 5794972.4 |
| 357 | 7 | 88.79263 | 60789.409 | 3846473.1 | 129517864 | 641414 |
| 358 | 7 | 88.60031 | 27117.877 | 5934454 | 23382186 | 8449265.3 |
| 359 | 7 | 87.08 | 20960.724 | 1336310.1 | 101511023 | 5168022.9 |
| 360 | 7 | 87.75207 | 12140.289 | 5808940.7 | 126524290 | 927586.6 |
| 361 | 7 | 87.04723 | 89216.777 | 5209111.8 | 22888632 | 2212172.6 |
| 362 | 7 | 90.44098 | 50038.79 | 5164824.8 | 69377163 | 5098233.3 |
| 363 | 7 | 88.93036 | 68395.145 | 702072.1 | 12770840 | 209690.1 |
| 364 | 7 | 88.78139 | 64418.333 | 8916933.5 | 88467601 | 3681876.3 |
| 365 | 7 | 87.73192 | 113377.864 | 7793217.5 | 101701011 | 379160.9 |
| 366 | 7 | 87.76707 | 116895.776 | 8558184 | 88357120 | 1418711.7 |
| 367 | 7 | 88.21759 | 135790.951 | 8107327.1 | 49850693 | 6690671.9 |
| 368 | 7 | 88.96932 | 7347.509 | 556407.5 | 102653072 | 8604361.3 |



| 369 | 7 | 88.41555 | 131118.104 | 4664984.3 | 123497500 | 1541643.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 370 | 7 | 87.59914 | 42452.078 | 8957092.3 | 101121441 | 2110555.7 |
| 371 | 7 | 86.84321 | 67025.852 | 6491318.7 | 108708979 | 9416577 |
| 372 | 7 | 87.74896 | 120673.903 | 1481114.5 | 93407662 | 9787958.8 |
| 373 | 7 | 90.39395 | 31545.729 | 4941682.2 | 93122299 | 7329519.8 |
| 374 | 7 | 88.37643 | 47275.584 | 4165454.6 | 126187617 | 9513223.4 |
| 375 | 7 | 87.52278 | 117837.67 | 1205160 | 51294475 | 5947601.9 |
| 376 | 7 | 88.70288 | 38989.361 | 7712295.1 | 123219883 | 7605241.7 |
| 377 | 7 | 88.326 | 47791.667 | 6254703.4 | 69884519 | 10028211.8 |
| 378 | 7 | 86.08146 | 129881.46 | 7154018.6 | 100072008 | 2129479 |
| 379 | 7 | 88.35381 | 56330.589 | 3522259.8 | 114008754 | 5772244.7 |
| 380 | 7 | 87.51088 | 49594.869 | 3718126.2 | 14452497 | 1047865.9 |
| 381 | 7 | 85.03622 | 130064.439 | 6466278.4 | 73361918 | 8042111.5 |
| 382 | 7 | 87.97838 | 97521.192 | 8693325.6 | 36235450 | 10301702.8 |
| 383 | 7 | 88.65683 | 11684.255 | 634660.4 | 112024280 | 3000582.9 |
| 384 | 7 | 87.82917 | 95230.442 | 2398947 | 134213045 | 5369305.9 |
| 385 | 7 | 86.88558 | 8939.539 | 5426567.6 | 105651387 | 9483516 |
| 386 | 7 | 87.9709 | 102281.938 | 1838620.2 | 90277797 | 4283052.2 |
| 381 | 7 | 80.93405 | 18803.393 | 20173259.96 | 79509735.3 | 9041535.74 |
| 388 | 7 | 81.75597 | $\begin{aligned} & 5440.698 \\ & 1243.328 \end{aligned}$ | $\begin{aligned} & 38811844.7 \\ & 497536.1 \end{aligned}$ | 76928322.5 | 823751889.4 |
| 389 | 7 | $98.37087$ | 447335.352 | $\begin{array}{r} 246441.7 .25 \\ 2938030.65 \end{array}$ | 220212599.1 | $8796473.14$ |
| 394 | 7 | $70.62557$ | 30482.9072 | 1679234.33 | $130504096.3$ | $8992386: 86$ |
| 395 | 7 | 80.72794 | 39645.416 | 4266073.01 | 53950406.1 | 12070358 |
| 396 | 7 | 81.91952 | 81112.667 | 4040858.77 | 58030283.9 | 4250493.71 |
| 397 | 7 | 80.71498 | 64015.098 | 3309232.69 | 45071763.8 | 4198286.42 |
| 398 | 7 | 82.33603 | 64708.074 | 1754929.96 | 127206574 | 7079372.27 |
| 399 | 7 | 80.31396 | 89276.39 | 4715484.72 | 110934347.6 | 10507609.3 |
| 400 | 7 | 80.16639 | 79915.014 | 4924611.74 | 38622027.7 | 11124397.6 |
| 401 | 7 | 81.67642 | 10993.11 | 2329276.19 | 22352279 | 3863904.88 |
| 402 | 7 | 81.06484 | 78848.642 | 4869448.16 | 77895161 | 1594868.86 |
| 403 | 7 | 80.40484 | 92107.396 | 2668479.6 | 24306423.2 | 12264077.5 |
| 404 | 7 | 81.59677 | 80823.793 | 4780490.72 | 134720666.4 | 12666307.8 |
| 405 | 7 | 81.41284 | 26051.429 | 3753404.06 | 136836955.9 | 5959751.84 |
| 406 | 7 | 79.66288 | 67535.018 | 394801.16 | 119377742.4 | 1676853.97 |

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| $\begin{array}{r}40 \\ 7 \\ \hline\end{array}$ | 781.0639 | 49488.862 | $\begin{array}{\|c\|} \hline 1385248.3 \\ 8 \\ \hline \end{array}$ | 21105952.1 | $\begin{aligned} & 9538501 . \\ & 23 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 70 8 | 779.7295 | 62937.242 | 87926.55 | 41915171.6 | $\begin{aligned} & 783654.1 \\ & 56 \end{aligned}$ |
| 80 9 | $\begin{gathered} 779.3397 \\ 6 \end{gathered}$ | 6761.809 | $\begin{gathered} 3102212.2 \\ 3 \end{gathered}$ | 945606.7 | 10625524 |
| 41 0 | 781.3837 | 98673.492 | 133188.77 | 62179660.8 | 9539.223 |
| 41 1 | 780.2668 | 37376.315 | $\begin{array}{\|c\|} 3921553.3 \\ 5 \\ \hline \end{array}$ | 32706981.3 | 12523469 |
| 1 41 2 | 780.2127 | 5488.605 | $\begin{gathered} 4668480.1 \\ 4 \end{gathered}$ | 42464145.7 | $\begin{aligned} & 12249290 \\ & .9 \end{aligned}$ |


| 41 | 778.5527 | 13789.468 | 2570646.3 | 68350940.2 | 10132872 |
| ---: | :---: | :---: | :---: | :--- | :--- |
| 3 | 3 |  |  | 86771.2 | 7857589.3 |


| 42 6 | $\begin{array}{\|c\|} \hline 782.2091 \\ 2 \end{array}$ |  | \|3009962.8 ${ }^{3} \mathbf{}$ \| | 59152270.7 | $\begin{aligned} & 2171312 . \\ & 52 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 42 7 | 780.1119 | 35109.236 | $\begin{array}{\|c\|} \hline 3828456.2 \\ 6 \end{array}$ | $\begin{aligned} & 142937323 \\ & .5 \end{aligned}$ | $\begin{aligned} & 2572535 . \\ & 77 \end{aligned}$ |
| 42 <br> 8 | 879.3795 9 | 40963.836 | $\begin{array}{\|c\|} \hline 3113795.7 \\ 2 \\ \hline \end{array}$ | $\begin{aligned} & 103480415 \\ & .2 \end{aligned}$ | $\begin{aligned} & 2841664 . \\ & 87 \end{aligned}$ |
| 42 <br> 9 | 880.3381 4 | 9241.361 | 450033.74 | 18488148.8 | $\begin{aligned} & 744584.7 \\ & 41 \end{aligned}$ |
| 43 <br> 0 | 878.4732 6 | 95514.649 | $\begin{array}{\|c} 2026933.7 \\ 2 \\ \hline \end{array}$ | $\begin{aligned} & 121083286 \\ & .2 \end{aligned}$ | $\begin{aligned} & 4517378 . \\ & 89 \end{aligned}$ |
| 43 <br> 1 <br> 4 | 879.4072 <br> 8 | 40734.913 | 4146713.8 <br> 5 | 38687936 | $\begin{aligned} & 12635199 \\ & .6 \end{aligned}$ |
| 1 <br> 43 <br> 2 | 879.4687 <br> 9 | 54716.294 | $\begin{array}{\|c\|} 4746306.0 \\ 1 \end{array}$ | $\begin{aligned} & 131641542 \\ & .9 \end{aligned}$ | $\begin{aligned} & 2837121 . \\ & 74 \end{aligned}$ |


| $\begin{array}{r} 43 \\ 3 \end{array}$ | $\begin{gathered} 881.6153 \\ 7 \end{gathered}$ | 33352.141 | 3495016 | 50978622.6 | $\begin{aligned} & 10030664 \\ & .1 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 43 4 | 879.1198 | 100984.657 | 234139.77 | 32532750.8 | $\begin{aligned} & 1372458 . \\ & 7 \end{aligned}$ |
| $\begin{array}{r} 43 \\ 5 \end{array}$ | $\begin{gathered} 879.2242 \\ 4 \end{gathered}$ | 19722.699 | $\begin{array}{\|c\|} \hline 4074335.8 \\ 4 \end{array}$ | $\begin{aligned} & 134677194 \\ & .8 \end{aligned}$ | $\begin{aligned} & 3781065 . \\ & 14 \end{aligned}$ |
| $\begin{array}{\|r} 43 \\ 6 \\ \hline \end{array}$ | $\begin{gathered} 879.5138 \\ 5 \end{gathered}$ | 84235.523 | $\begin{array}{\|c\|} \hline 4592595.7 \\ 1 \\ \hline \end{array}$ | $\begin{aligned} & 143252394 \\ & .3 \end{aligned}$ | $\begin{aligned} & 3689017 . \\ & 37 \end{aligned}$ |
| $\begin{array}{r} 43 \\ 7 \end{array}$ | $\begin{array}{\|c\|} \hline 881.8725 \\ 3 \\ \hline \end{array}$ | 79224.163 | $\begin{array}{\|c\|} \hline 2727703.8 \\ 8 \\ \hline \end{array}$ | 68764019.2 | $\begin{aligned} & 2922656 . \\ & 77 \end{aligned}$ |
| 43 8 | $\begin{gathered} 881.4153 \\ 9 \end{gathered}$ | 97877.402 | 97074.85 | 82780806.7 | $\begin{aligned} & 6435749 . \\ & 73 \end{aligned}$ |
| 43 9 | 880.7227 | 30039.757 | 752668.68 | 58804667.1 | $\begin{aligned} & 7007667 . \\ & 45 \end{aligned}$ |
| 44 0 | 879.8170 <br> 2 | 100250.677 | $\begin{array}{\|c\|} \hline 3401630.1 \\ 3 \\ \hline \end{array}$ | 3981263.3 | $\begin{aligned} & 6819100 . \\ & 49 \end{aligned}$ |
| 44 <br> 1 | $\begin{array}{\|c\|} 880.7546 \\ 2 \\ \hline \end{array}$ | 103683.672 | 3540653 | $\begin{aligned} & 100904004 \\ & .8 \end{aligned}$ | $\begin{aligned} & 13065497 \\ & .8 \end{aligned}$ |
| $\begin{array}{r}14 \\ \hline 2\end{array}$ | 782.0160 | 85055.573 | $\begin{array}{\|c\|} \hline 3897491.6 \\ 1 \\ \hline \end{array}$ | 1279402.4 | $\begin{aligned} & 10949571 \\ & .3 \end{aligned}$ |
| 44 3 | $\begin{gathered} 780.2817 \\ 2 \end{gathered}$ | 37589.204 | $\begin{gathered} 4571202.8 \\ 7 \end{gathered}$ | $\begin{aligned} & 134881414 \\ & .7 \end{aligned}$ | $\begin{aligned} & 13335066 \\ & .6 \end{aligned}$ |
| 44 <br> 4 | $\begin{gathered} 780.0472 \\ 4 \end{gathered}$ | 86669.745 | $\begin{array}{\|c\|} \hline 1232400.6 \\ 9 \end{array}$ | $\begin{aligned} & 142651983 \\ & .1 \end{aligned}$ | $\begin{aligned} & 6459629 . \\ & 17 \end{aligned}$ |


| DMU | CCR Efficiency |  |  | Class |  |  |  |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
|  | Stage 1 <br> Efficiency | Stage 2 <br> Efficiency | Overall <br> Efficiency | Case 1 | Case 2 | Case 3 | Case 4 |
| 1 | 0.51041 | 0.037362 | 0.863484 | Class 4 | Class 2 | Class 4 | Class 2 |
| 2 | 0.129048 | 0.008119 | 0.905399 | Class 4 | Class 2 | Class 4 | Class 2 |
| 3 | 0.189628 | 0.022087 | 0.965382 | Class 4 | Class 2 | Class 4 | Class 2 |


| 4 | 1 | 0.002175 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| 5 | 0.216073 | 0.003382 | 0.877624 | Class 4 | Class 2 | Class 4 | Class 2 |
| 6 | 0.483117 | 0.020072 | 0.951268 | Class 4 | Class 2 | Class 4 | Class 2 |
| 7 | 0.563366 | 0.004981 | 0.918409 | Class 4 | Class 2 | Class 4 | Class 2 |
| 8 | 0.085544 | 0.012226 | 0.909131 | Class 4 | Class 2 | Class 4 | Class 2 |
| 9 | 0.355439 | 0.006896 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 10 | 0.472407 | 0.009612 | 0.930906 | Class 4 | Class 2 | Class 4 | Class 2 |
| 11 | 0.311048 | 0.022303 | 0.855815 | Class 4 | Class 2 | Class 4 | Class 2 |
| 12 | 0.038721 | 0.01963 | 0.884938 | Class 4 | Class 2 | Class 4 | Class 2 |
| 13 | 0.444092 | 0.001807 | 0.923815 | Class 4 | Class 2 | Class 4 | Class 2 |
| 14 | 0.301292 | 0.003506 | 0.887972 | Class 4 | Class 2 | Class 4 | Class 2 |
| 15 | 0.275436 | 0.002221 | 0.71688 | Class 4 | Class 4 | Class 4 | Class 4 |
| 16 | 0.278064 | 0.065517 | 0.629334 | Class 4 | Class 4 | Class 4 | Class 4 |
| 17 | 0.566731 | 0.005019 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 18 | 0.830898 | 0.003071 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 19 | 0.048307 | 0.026739 | 0.68221 | Class 4 | Class 4 | Class 4 | Class 4 |
| 20 | 0.1343 | 0.004237 | 0.661872 | Class 4 | Class 4 | Class 4 | Class 4 |

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Appendix B: The CCR DEA efficiency and the classification of the 444 Bank Branches (DMUs)

| 21 | 0.185101 | 0.004791 | 0.783606 | Class 4 | Class 4 | Class 4 | Class 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22 | 0.214974 | 0.037837 | 0.76041 | Class 4 | Class 4 | Class 4 | Class 4 |
| 23 | 0.446158 | 0.00145 | 0.839717 | Class 4 | Class 2 | Class 4 | Class 2 |
| 24 | 0.336537 | 0.008111 | 0.786179 | Class 4 | Class 4 | Class 4 | Class 4 |
| 25 | 0.028006 | 0.023187 | 0.757515 | Class 4 | Class 4 | Class 4 | Class 4 |
| 26 | 0.01182 | 0.092298 | 0.773928 | Class 4 | Class 4 | Class 4 | Class 4 |
| 27 | 0.318369 | 0.002069 | 0.808117 | Class 4 | Class 2 | Class 4 | Class 2 |
| 28 | 0.523699 | 0.00126 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 29 | 0.314873 | 0.002396 | 0.813951 | Class 4 | Class 2 | Class 4 | Class 2 |
| 30 | 0.2881 | 0.002338 | 0.807853 | Class 4 | Class 2 | Class 4 | Class 2 |
| 31 | 0.469401 | 0.001427 | 0.931386 | Class 4 | Class 2 | Class 4 | Class 2 |
| 32 | 0.329428 | 0.002892 | 0.798275 | Class 4 | Class 4 | Class 4 | Class 4 |
| 33 | 0.556094 | 0.00148 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 34 | 0.367568 | 0.00181 | 0.827586 | Class 4 | Class 2 | Class 4 | Class 2 |
| $3{ }^{3}$ | $0.333879$ | 0.0123242 | 0.893469 | $\text { Class } 4$ | $\text { Class } 2$ | $\text { Class } 4$ | $\text { Class } 2$ |
| 36 | 0.444926 | 0:0129987 | 0.923673 | $\text { Class } 2$ | $\text { Class } 2$ | $\text { Class } 4$ | $\text { Class } 2$ |
| 40 | 0.204714 | 0.0039441 | 0.888666 | $\begin{aligned} & \text { Class } 4 \\ & \hline \end{aligned}$ | Class 2 | Class 4 | Class 2 |
| 41 | 0.274533 | 0.003844 | 0.914323 | Class 4 | Class 2 | Class 4 | Class 2 |
| 42 | 1 | 0.018177 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 43 | 0.431135 | 0.001747 | 0.919634 | Class 4 | Class 2 | Class 4 | Class 2 |
| 44 | 0.423596 | 0.043561 | 0.892051 | Class 4 | Class 2 | Class 4 | Class 2 |
| 45 | 0.610891 | 0.001262 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 46 | 0.288549 | 0.002862 | 0.96554 | Class 4 | Class 2 | Class 4 | Class 2 |
| 47 | 0.218608 | 0.00351 | 0.901991 | Class 4 | Class 2 | Class 4 | Class 2 |
| 48 | 0.394909 | 0.052855 | 0.863667 | Class 4 | Class 2 | Class 4 | Class 2 |
| 49 | 0.497868 | 0.001534 | 0.943356 | Class 4 | Class 2 | Class 4 | Class 2 |
| 50 | 0.426821 | 0.046258 | 0.86984 | Class 4 | Class 2 | Class 4 | Class 2 |
| 51 | 1 | 0.043299 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 52 | 0.555211 | 0.038674 | 0.864187 | Class 4 | Class 2 | Class 4 | Class 2 |
| 53 | 0.274186 | 0.067967 | 0.846584 | Class 4 | Class 2 | Class 4 | Class 2 |
| 54 | 0.346354 | 0.00223 | 0.931909 | Class 4 | Class 2 | Class 4 | Class 2 |
| 55 | 0.29399 | 0.066705 | 0.83778 | Class 4 | Class 2 | Class 4 | Class 2 |
| 56 | 0.071285 | 0.010776 | 0.871243 | Class 4 | Class 2 | Class 4 | Class 2 |
| 57 | 0.092978 | 0.007115 | 0.755028 | Class 4 | Class 4 | Class 4 | Class 4 |
| 58 | 0.415331 | 0.001605 | 0.818824 | Class 4 | Class 2 | Class 4 | Class 2 |
| 59 | 0.444651 | 0.001506 | 0.933609 | Class 4 | Class 2 | Class 4 | Class 2 |
| 60 | 0.235273 | 0.002911 | 0.830839 | Class 4 | Class 2 | Class 4 | Class 2 |



| 61 | 0.390082 | 0.003295 | 0.922585 | Class 4 | Class 2 | Class 4 | Class 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 62 | 0.356193 | 0.001865 | 0.817932 | Class 4 | Class 2 | Class 4 | Class 2 |
| 63 | 0.336715 | 0.00197 | 0.818176 | Class 4 | Class 2 | Class 4 | Class 2 |
| 64 | 0.240565 | 0.00276 | 0.797182 | Class 4 | Class 4 | Class 4 | Class 4 |
| 65 | 0.021206 | 0.045055 | 0.835801 | Class 4 | Class 2 | Class 4 | Class 2 |
| 66 | 0.447906 | 0.001482 | 0.847057 | Class 4 | Class 2 | Class 4 | Class 2 |
| 67 | 0.12892 | 0.005202 | 0.80453 | Class 4 | Class 2 | Class 4 | Class 2 |
| 68 | 0.096667 | 0.006839 | 0.759633 | Class 4 | Class 4 | Class 4 | Class 4 |
| 69 | 0.271929 | 0.003249 | 0.83642 | Class 4 | Class 2 | Class 4 | Class 2 |
| 70 | 0.055687 | 0.012085 | 0.756909 | Class 4 | Class 4 | Class 4 | Class 4 |
| 71 | 0.610006 | 0.03609 | 0.874879 | Class 4 | Class 2 | Class 4 | Class 2 |
| 72 | 1 | 0.001433 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 73 | 0.05434 | 0.018299 | 0.828893 | Class 4 | Class 2 | Class 4 | Class 2 |
| 74 | 0.103459 | 0.005673 | 0.645367 | Class 4 | Class 4 | Class 4 | Class 4 |
| 75 | 0.141514 | 0.006259 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 76 | 0.061679 | 0.00941 | 0.65225 | Class 4 | Class 4 | Class 4 | Class 4 |
| 77 | 0.253551 | 0.070107 | 0.6186 | Class 4 | Class 4 | Class 4 | Class 4 |
| 88 | 0:39388 | 0:005497 | 0:996839 | Class 4 | Class 4 | Class 4 | Class 4 |
| 83 | 0:386968 | 00000383 | 0.695559 | Class 4 | Class 4 | Class 4 | Class 4 |
| 84 | 0.048367 | 0.007293 | 0.6480837 | Class 4 | Class 4 | Class 4 | Class 4 |
| 85 | 00330146 | 0.057698 | 0.999251 | Class 4 | Class 2 | Class 4 | Class 2 |
| 86 | 0.297222 | 0.061168 | 0.630784 | Class 4 | Class 4 | Class 4 | Class 4 |
| 87 | 0.421406 | 0.037496 | 0.604399 | Class 4 | Class 4 | Class 4 | Class 4 |
| 88 | 0.406552 | 0.009652 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 89 | 0.127924 | 0.153432 | 0.708022 | Class 4 | Class 4 | Class 4 | Class 4 |
| 90 | 0.331269 | 0.002305 | 0.912381 | Class 4 | Class 2 | Class 4 | Class 2 |
| 91 | 0.849506 | 0.001279 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 92 | 0.032601 | 0.023342 | 0.853471 | Class 4 | Class 2 | Class 4 | Class 2 |
| 93 | 0.784393 | 0.023617 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 94 | 0.438281 | 0.041797 | 0.860842 | Class 4 | Class 2 | Class 4 | Class 2 |
| 95 | 0.270714 | 0.064075 | 0.848983 | Class 4 | Class 2 | Class 4 | Class 2 |
| 96 | 0.300407 | 0.063149 | 0.853652 | Class 4 | Class 2 | Class 4 | Class 2 |
| 97 | 0.499953 | 0.036739 | 0.861094 | Class 4 | Class 2 | Class 4 | Class 2 |
| 98 | 0.354291 | 0.054253 | 0.857441 | Class 4 | Class 2 | Class 4 | Class 2 |
| 99 | 0.154178 | 0.004989 | 0.898771 | Class 4 | Class 2 | Class 4 | Class 2 |
| 100 | 0.059928 | 0.012563 | 0.851934 | Class 4 | Class 2 | Class 4 | Class 2 |



| 101 | 0.403142 | 0.044161 | 0.909206 | Class 4 | Class 2 | Class 4 | Class 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 102 | 0.040252 | 0.019337 | 0.871198 | Class 4 | Class 2 | Class 4 | Class 2 |
| 103 | 0.23053 | 0.078954 | 0.894393 | Class 4 | Class 2 | Class 4 | Class 2 |
| 104 | 0.10392 | 0.007379 | 0.865807 | Class 4 | Class 2 | Class 4 | Class 2 |
| 105 | 0.196752 | 0.003858 | 0.924014 | Class 4 | Class 2 | Class 4 | Class 2 |
| 106 | 0.332308 | 0.061316 | 0.866975 | Class 4 | Class 2 | Class 4 | Class 2 |
| 107 | 0.50235 | 0.040183 | 0.829004 | Class 4 | Class 2 | Class 4 | Class 2 |
| 108 | 0.566247 | 0.00139 | 0.96277 | Class 4 | Class 2 | Class 4 | Class 2 |
| 109 | 1 | 0.024499 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 110 | 0.366391 | 0.046532 | 0.930789 | Class 4 | Class 2 | Class 4 | Class 2 |
| 111 | 0.019503 | 0.065577 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 112 | 0.37688 | 0.047941 | 0.865 | Class 4 | Class 2 | Class 4 | Class 2 |
| 113 | 0.585254 | 0.03208 | 0.849674 | Class 4 | Class 2 | Class 4 | Class 2 |
| 114 | 0.422802 | 0.044828 | 0.877959 | Class 4 | Class 2 | Class 4 | Class 2 |
| 115 | 0.181763 | 0.004244 | 0.900455 | Class 4 | Class 2 | Class 4 | Class 2 |
| 116 | 0.198369 | 0.004369 | 0.933981 | Class 4 | Class 2 | Class 4 | Class 2 |
| 117 | 0.424176 | 0.04194 | 0.849459 | Class 4 | Class 2 | Class 4 | Class 2 |
| 118 | 0.212278 | 0.009815 | 0.923333 | Class 4 | Class 2 | Class 4 | Class 2 |
| 119 | 0.206058 | 0.003553 | 0.874061 | Class 4 | Class 2 | Class 4 | Class 2 |
| 120 | 0.441629 | 0.003201 | 0.872267 | Class 4 | Class 2 | Class 4 | Class 2 |
| 121 | 0.580752 | 0.00124 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 126 | 0.462025 | 0.0931294 | $0.876388$ | $\begin{aligned} & \text { Class } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class 2 } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \hline \text { Class } \end{aligned}$ |
| $\frac{127}{123}$ | 0.632826 | $\begin{array}{r} 0.002867 \\ 0.00344 \end{array}$ | $0.8144119$ | $\begin{aligned} & \text { Class } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \text { Class } 2 \end{aligned}$ |
| $\begin{aligned} & 128 \\ & 128 \\ & 124 \end{aligned}$ | $\begin{array}{r} 0.447556 \\ 0.837549 \end{array}$ | $\begin{array}{r} 0.004341 \\ 0.00169 \end{array}$ | 0.917541 | $\begin{aligned} & C \text { rass } 4 \\ & \text { Class } 4 \\ & \text { Class } 2 \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \text { Class } 3 \end{aligned}$ | $\begin{aligned} & \text { Class } 4 \\ & \text { Class } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \text { Class } 2 \end{aligned}$ |
| $\begin{array}{r} 129 \\ 125 \\ \hline 129 \end{array}$ | 0.767617 0.041549 | $\begin{array}{r} 0.001589 \\ 0.018476 \end{array}$ | 0.856316 | $\begin{aligned} & \text { Class } \\ & \text { Class } 2 \\ & \text { Class } 4 \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \text { Class } 2 \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \text { Class } 4 \end{aligned}$ | $\begin{aligned} & \text { Class } 2 \\ & \text { Class } 2 \end{aligned}$ |
| 130 | 0.247678 | 0.004582 | 0.988177 | Class 4 | Class 2 | Class 4 | Class 2 |
| 131 | 0.314942 | 0.002769 | 0.988611 | Class 4 | Class 2 | Class 4 | Class 2 |
| 132 | 0.904469 | 0.000928 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 133 | 0.396777 | 0.002965 | 0.989482 | Class 4 | Class 2 | Class 4 | Class 2 |
| 134 | 0.502856 | 0.001742 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 135 | 0.615602 | 0.002443 | 0.992526 | Class 4 | Class 2 | Class 4 | Class 2 |
| 136 | 0.677172 | 0.003027 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 137 | 0.284082 | 0.006567 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 138 | 0.052723 | 0.034683 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 139 | 0.377066 | 0.005122 | 0.993743 | Class 4 | Class 2 | Class 4 | Class 2 |
| 140 | 0.620313 | 0.002433 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |



| 141 | 0.555336 | 0.001538 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 142 | 0.894108 | 0.002015 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 143 | 0.278496 | 0.005875 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 144 | 0.634499 | 0.001395 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 145 | 0.694601 | 0.003554 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 146 | 0.806759 | 0.00103 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 147 | 0.750817 | 0.00227 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 148 | 1 | 0.002249 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 149 | 0.249434 | 0.003395 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 150 | 1 | 0.002471 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 151 | 0.44025 | 0.002045 | 1 | Class 2 | Class 2 | Class 2 | Class 2 |
| 152 | 0.142928 | 0.007969 | 0.822753 | Class 4 | Class 2 | Class 4 | Class 2 |
| 153 | 0.362185 | 0.003175 | 0.871956 | Class 4 | Class 2 | Class 4 | Class 2 |
| 154 | 0.002284 | 0.450441 | 0.87093 | Class 4 | Class 2 | Class 4 | Class 2 |
| 155 | 0.07089 | 0.011561 | 0.852999 | Class 4 | Class 2 | Class 4 | Class 2 |
| 156 | 0.301519 | 0.002462 | 0.858944 | Class 4 | Class 2 | Class 4 | Class 2 |
| 157 | 0.234344 | 0.003197 | 0.861134 | Class 4 | Class 2 | Class 4 | Class 2 |
| 158 | 0.275184 | 0.002632 | 0.84212 | Class 4 | Class 2 | Class 4 | Class 2 |
| 159 | 0.061079 | 0.012061 | 0.847198 | Class 4 | Class 2 | Class 4 | Class 2 |
| 160 | 0.234374 | 0.003109 | 0.836424 | Class 4 | Class 2 | Class 4 | Class 2 |
| 161 | 0.225179 | 0.003507 | 0.828976 | Class 4 | Class 2 | Class 4 | Class 2 |
| 162 | 0.120276 | 0.013449 | 0.822938 | Class 4 | Class 2 | Class 4 | Class 2 |
| 163 | 0.202576 | 0.003775 | 0.857827 | Class 4 | Class 2 | Class 4 | Class 2 |
| 164 | 0.002465 | 0.307508 | 0.825274 | Class 4 | Class 2 | Class 4 | Class 2 |
| 160 | 0.2849198 | 0:004768 | 0.858685 | Class 4 | Class 2 | Class 4 | Class 2 |
| 169 | 0.367406 | 0:002544 | 0:857936 | Class 4 | Class 2 | Class 4 | Class 2 |
| 162 | 0,670048 | 00003664 | 0:852658 | Class 4 | Class 2 | Class 4 | Class 2 |
| 168 | 00296492 | 0:0014418 | 0.869811 | Class 4 | Class 4 | Class 4 | Class 4 |
| 164 | 0.023335 | 00042793 | 0.738749 | Class 3 | Class 7 | Class 4 | Class 4 |
| 175 | 0.015304 | 0.055576 | 0.73693 | Class 4 | Class 4 | Class 4 | Class 4 |
| 176 | 0.352062 | 0.006137 | 0.72641 | Class 4 | Class 4 | Class 4 | Class 4 |
| 177 | 0.224608 | 0.003486 | 0.729681 | Class 4 | Class 4 | Class 4 | Class 4 |
| 178 | 0.316832 | 0.002153 | 0.833344 | Class 4 | Class 2 | Class 4 | Class 2 |
| 179 | 0.186464 | 0.003492 | 0.764345 | Class 4 | Class 4 | Class 4 | Class 4 |
| 180 | 0.113557 | 0.005693 | 0.724325 | Class 4 | Class 4 | Class 4 | Class 4 |

## KNUST

| $\begin{array}{r}18 \\ 1 \\ \hline 18\end{array}$ | $\begin{array}{r} 0.49629 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.00223 \\ 1 \end{array}$ | $\begin{array}{r} 0.88232 \\ 3 \\ \hline \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 0.03649 | - 0.02247 | $0.73269$ | Class 4 | Clas <br> s 4 | Clas <br> s 4 | Clas <br> s 4 |
| 18 3 | 0.19716 <br> 1 | $\begin{array}{r} 0.00469 \\ 9 \end{array}$ | $\begin{array}{r} 0.75253 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| 18 4 | 0.30160 <br> 9 | 0.03170 | $\begin{array}{r} \hline 0.71908 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| 18 5 | 0.19793 | 0.00510 | 0.72728 4 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ |


| 18 6 | \| 0.26565 | 2.00241 | $\begin{array}{r} 0.75718 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 0.47664 | 0.03146 | 0.75708 | Class 4 |  |  |  |
| 7 | 4 | 7 | 6 |  | s 4 | s 4 | s 4 |
| 18 | 0.28933 | 0.00350 | 0.74351 | Class 4 | Clas | Clas | Clas |
| 8 | 9 | 8 | 4 |  | s 4 | s 4 | s 4 |
| 18 | 0.27473 | 0.00266 | 0.81596 | Class 4 | Clas | Clas | Clas |
| 9 | 1 | 8 | 8 |  | s 2 | s 4 | s 2 |
| 19 | 0.10777 | 0.00577 | 0.71677 | Class 4 | Clas | Clas | Clas |
| 0 | 4 | 5 | 3 |  | s 4 | s 4 | s 4 |
| 19 | 0.27817 | 0.03780 | 0.73940 | Class 4 | Clas | Clas | Clas |
| 1 | 5 | 2 | 6 |  | s 4 | s 4 | s 4 |
| 19 | 0.12608 | 0.00527 | 0.65648 | Class 4 | Clas | Clas | Clas |
| 2 | 9 | 2 | 7 |  | s 4 | s 4 | s 4 |
| 19 | 0.24201 | 0.00278 | 0.65362 | Class 4 | Clas | Clas | Clas |
| 3 | 5 | 7 | 4 |  | s 4 | s 4 | s 4 |
| 19 | 0.15729 | 0.00367 | 0.66997 | Class 4 | Clas | Clas | Clas |
| 4 | 7 | 5 | 2 |  | s 4 | s 4 | s 4 |
| 19 | 0.12587 | 0.00457 | 0.65434 | Class 4 | Clas | Clas | Clas |
| 5 | 7 | 6 | 5 |  | s 4 | s 4 | s 4 |
| 19 | 0.01898 | 0.03156 | 0.65291 | Class 4 | Clas | Clas |  |
| 6 | 7 | 5 | 8 |  | s 4 | s 4 | s 4 |
| 19 | 0.28615 | 0.00202 | 0.74554 | Class 4 | Clas | Clas | Clas |
| 7 | 7 | 3 | 9 |  | s 4 | s 4 | s 4 |
| 19 | 0.20051 | 0.02238 | 0.65041 | Class 4 | Clas | Clas | Clas |
| 8 |  | 2 | 4 |  | s 4 | s 4 | s 4 |
| 19 | 0.11767 | 0.00482 | 0.64572 | Class 4 | Clas | Clas | Clas |
| 9 | 6 | 3 |  |  | s 4 | s 4 | s 4 |
| 20 | 0.00057 | 1 | 0.65058 | Class 4 | Clas | Clas | Clas |
| 0 | 2 |  | 5 |  | s 4 | s 1 | s 1 |


| 20 1 | $\begin{array}{r} 0.19178 \\ 6 \end{array}$ | 0.00306 | 0.66515 | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas s 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 2 | $\begin{array}{r} 0.11230 \\ 4 \end{array}$ | $\begin{array}{r} \hline 0.00499 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.64105 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| $\begin{array}{r}20 \\ 3 \\ \hline 20\end{array}$ | $\begin{array}{r} 0.24381 \\ 8 \end{array}$ | $\begin{array}{r} 0.00295 \\ 7 \end{array}$ | $\begin{array}{r} 0.83808 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 20 4 | $\begin{array}{r} 0.36009 \\ 5 \end{array}$ | $\begin{array}{r} \hline 0.00269 \\ 7 \end{array}$ | $\begin{array}{r} 0.84156 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 20 5 | $\begin{array}{r} 0.23601 \\ 3 \end{array}$ | $\begin{array}{r} 0.00316 \\ 2 \end{array}$ | 0.82673 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| 20 6 | $\begin{array}{r} 0.20583 \\ 3 \end{array}$ | 0.00348 | $\begin{array}{r} 0.82399 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \hline \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| 20 | $\begin{array}{r} \hline 0.02615 \\ 2 \end{array}$ | $\begin{array}{r} \hline 0.02799 \\ 1 \end{array}$ | $\begin{array}{r} \hline 0.82496 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 8 | 0.07358 | 0.01024 | $\begin{array}{r} 0.82654 \\ 2 \end{array}$ | Class 4 | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 80 9 | $\begin{array}{r} \hline 0.07120 \\ 5 \end{array}$ | $\begin{array}{r} \hline 0.01040 \\ 5 \end{array}$ | $\begin{array}{r} \hline 0.83819 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 21 | $\begin{array}{r} 0.18537 \\ 3 \end{array}$ | $\begin{array}{r} 0.00465 \\ 7 \end{array}$ | $\begin{array}{r} 0.90712 \\ 6 \end{array}$ | Class 4 | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 21 1 | $\begin{array}{r} 0.24775 \\ 2 \end{array}$ | 0.00427 4 | 0.83478 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 1 2 | 0.24270 | $\begin{array}{r} 0.00295 \\ 8 \end{array}$ | $\begin{array}{r} 0.84013 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| 21 3 | $\begin{array}{r} 0.27775 \\ 6 \end{array}$ | 0.00264 | $\begin{array}{r} 0.85426 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| 21 4 | $\begin{array}{r} 0.19209 \\ 3 \end{array}$ | $\begin{array}{r} 0.01018 \\ 6 \end{array}$ | $\begin{array}{r} 0.90989 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 5 | 0.06476 4 | $\begin{array}{r} \hline 0.01806 \\ 1 \end{array}$ | $\begin{array}{r} \hline 0.81981 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 21 6 | \|r 0.07190 | $\begin{array}{r} 0.01390 \\ 6 \end{array}$ | 0.83309 | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 21 7 | $\begin{array}{r} 0.31236 \\ 2 \end{array}$ | $\begin{array}{r} 0.00233 \\ 4 \end{array}$ | $\begin{array}{r} 0.83786 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 21 8 | $\begin{array}{r} 0.19245 \\ 5 \end{array}$ | $\begin{array}{r} 0.00382 \\ 6 \end{array}$ | $\begin{array}{r} 0.84446 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 21 9 | $\begin{array}{r} 0.00120 \\ 5 \end{array}$ | $\begin{array}{r} 0.94456 \\ 5 \end{array}$ | $\begin{array}{r} 0.85730 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 3 \end{aligned}$ |
| 22 0 | $\begin{array}{r} 0.17794 \\ 1 \end{array}$ | 0.0044 | 0.87486 | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 22 1 | $\begin{array}{r} 0.06813 \\ 7 \end{array}$ | $\begin{array}{r} 0.01079 \\ 3 \end{array}$ | 0.84015 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 2 | $\begin{array}{r} 0.20982 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.00351 \\ 8 \end{array}$ | $\begin{array}{r} 0.84090 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 3 | $\begin{array}{r} 0.41748 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.00487 \\ 6 \end{array}$ | $\begin{array}{r} 0.83268 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 4 | $\begin{array}{r} 0.18913 \\ 6 \end{array}$ | $\begin{array}{r} \hline 0.00483 \\ 7 \end{array}$ | $\begin{array}{r} 0.85213 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 5 | $\begin{array}{r} 0.25868 \\ 3 \end{array}$ | $\begin{array}{r} 0.00289 \\ 7 \end{array}$ | $\begin{array}{r} 0.86770 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 6 | $\begin{array}{r} 0.15393 \\ 8 \end{array}$ | $\begin{array}{r} 0.03642 \\ 1 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 62 7 | $\begin{array}{r} 0.27648 \\ 8 \end{array}$ | $\begin{array}{r} 0.00320 \\ 5 \end{array}$ | $\begin{array}{r} 0.82129 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 8 | $\begin{array}{\|r\|} \hline 0.23388 \\ 2 \end{array}$ | $\begin{array}{r} 0.00465 \\ 6 \end{array}$ | $\begin{array}{r} 0.85524 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 22 9 | 0.19083 7 | $\begin{array}{r} 0.00532 \\ 4 \end{array}$ | $\begin{array}{r} 0.81712 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| 23 0 | 0.18218 3 | $\begin{array}{r} 0.00486 \\ 9 \end{array}$ | $\begin{array}{r} 0.83150 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 1 | $0.15737$ | $\begin{array}{r} 0.01199 \\ \hline \end{array}$ | $\begin{array}{r} 0.85673 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 23 2 | $\begin{array}{r} 0.14439 \\ 6 \end{array}$ | $\begin{array}{r} \hline 0.00498 \\ 9 \end{array}$ | $\begin{array}{r} 0.83544 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 23 3 | $\begin{array}{r} 0.02559 \\ 9 \end{array}$ | $\begin{array}{r} 0.02845 \\ 6 \end{array}$ | $\begin{array}{r} 0.83861 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 23 4 | $\begin{array}{r} 0.16101 \\ 7 \end{array}$ | $\begin{array}{r} 0.00594 \\ 3 \end{array}$ | $\begin{array}{r} 0.97629 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 23 5 | $\begin{array}{r} 0.33057 \\ 3 \end{array}$ | 0.04813 | $\begin{array}{r} 0.99575 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 23 6 | $\begin{array}{r} 0.23178 \\ 3 \end{array}$ | 0.00903 | $\begin{array}{r} 0.99054 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |
| $\begin{array}{r}6 \\ \hline 23 \\ \hline\end{array}$ | $\begin{array}{r} \hline 0.28658 \\ 9 \end{array}$ | 0.00354 | 0.97675 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| $\begin{array}{r}23 \\ 8 \\ \hline\end{array}$ | $\begin{array}{r} \hline 0.22844 \\ 6 \end{array}$ | $\begin{array}{r} 0.00369 \\ 4 \end{array}$ | $\begin{array}{r} \hline 0.98181 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 8 23 9 | 0.35495 3 | $\begin{array}{r} 0.00373 \\ 4 \end{array}$ | $\begin{array}{r} 0.95919 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 24 0 | 0.09716 | 0.00873 6 | $\begin{array}{r} 0.96496 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |


| 24 1 | $\begin{array}{r} 0.03655 \\ 6 \end{array}$ | $\begin{array}{r} 0.02329 \\ 6 \end{array}$ | $\begin{array}{\|l\|} \hline 0.97587 \\ \hline \end{array}$ |  | $\begin{array}{\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 2 | $\begin{array}{r} 0.23442 \\ 8 \end{array}$ | 0.00519 | $\begin{array}{r} 0.97765 \\ 9 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \hline \text { s } 4 \\ \hline \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 24 3 | $\begin{array}{r} 0.19946 \\ 9 \end{array}$ | $\begin{array}{r} 0.00706 \\ 4 \end{array}$ | $\begin{array}{r} 0.96493 \\ 8 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 24 4 4 | $\begin{array}{r} 0.14391 \\ 7 \end{array}$ | $\begin{array}{\|r\|} \hline 0.00734 \\ 2 \end{array}$ | $\begin{array}{\|r} 0.94925 \\ 2 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 24 5 | $\begin{array}{r} 0.07781 \\ 6 \end{array}$ | $\begin{array}{\|r} \hline 0.02237 \\ 6 \end{array}$ | $\begin{array}{r} 0.96551 \\ 6 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 24 6 | $\begin{array}{r} 0.38945 \\ 7 \end{array}$ | $\begin{array}{r} 0.00223 \\ 2 \end{array}$ | 1 | Class 2 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \hline \end{array}$ |


| 24 7 | $\begin{array}{r} 0.18841 \\ 6 \end{array}$ | $\begin{array}{r} 0.00570 \\ 7 \end{array}$ | $\begin{array}{r} 0.96937 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 8 | $\begin{array}{r} 0.00934 \\ 3 \end{array}$ | $\begin{array}{r} \hline 0.11400 \\ 8 \end{array}$ | $\begin{array}{r} \hline 0.97362 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 24 9 | $\begin{array}{r} 0.22400 \\ 2 \end{array}$ | $\begin{array}{r} \hline 0.00383 \\ 1 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
|  | 0.50801 | $\begin{array}{r} 0.00575 \\ 6 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 25 1 | $\begin{array}{r} 0.40996 \\ 1 \end{array}$ | $\begin{array}{r} 0.00283 \\ 6 \end{array}$ | $\begin{array}{r} 0.96005 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 25 2 | $\begin{array}{r} 0.28461 \\ 8 \end{array}$ | $\begin{array}{r} 0.01619 \\ 3 \\ \hline \end{array}$ | $\begin{array}{r} 0.98052 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ |
| 25 3 | $\begin{array}{r} 0.02888 \\ 9 \end{array}$ | $\begin{array}{r} 0.02587 \\ 6 \end{array}$ | $\begin{array}{r} 0.83966 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}25 \\ 4 \\ \hline\end{array}$ | 0.21602 6 | \|r.00343 | $\begin{array}{r} 0.87227 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 25 5 | 2.11888 | [0.00614 | $\begin{array}{r} 0.83109 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 25 6 | $\begin{array}{r} 0.30442 \\ 7 \end{array}$ | $\begin{array}{r} 0.00391 \\ 5 \end{array}$ | $\begin{array}{r} 0.91767 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 65 7 | 0.27305 7 | $\begin{array}{r} 0.00328 \\ 3 \end{array}$ | $\begin{array}{r} 0.90400 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ |


| 258 | 0.046753 | 0.022374 | 0.83153 | Class 4 | Class 2 | Class 4 | Class 2 |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| 259 | 0.054518 | 0.018242 | 0.850865 | Class 4 | Class 2 | Class 4 | Class 2 |
| 260 | 0.06945 | 0.010263 | 0.81828 | Class 4 | Class 2 | Class 4 | Class 2 |


| 26 1 | $\begin{array}{r} 0.17330 \\ 3 \\ \hline \end{array}$ | $\begin{array}{\|r} \hline 0.00602 \\ 1 \\ \hline \end{array}$ | $\begin{array}{\|r\|} \hline 0.84567 \\ 4 \\ \hline \end{array}$ | Class 4 | $\begin{array}{\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 26 | 0.26524 6 | 0.00285 <br> 6 | $\begin{array}{r} 0.86723 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{array}{ll} \hline \text { Clas } \\ \text { s } 4 \\ \hline \end{array}$ | Clas |
| 26 3 | [ 0.15281 | $0.00485$ $7$ | $\begin{array}{r} 0.84256 \\ 4 \end{array}$ | Class 4 | $\begin{array}{\|l} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 26 4 | $\begin{array}{r} 0.35282 \\ 9 \end{array}$ | $\begin{array}{\|r\|} \hline 0.00205 \\ 8 \end{array}$ | $\begin{array}{r} 0.83624 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & s 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 26 | $\begin{array}{r} 0.09425 \\ 4 \end{array}$ | $\begin{array}{\|r} \hline 0.00975 \\ 2 \end{array}$ | $\begin{array}{r} 0.82749 \\ 8 \\ \hline \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |
| 26 6 | $\begin{array}{r} 0.23912 \\ 9 \end{array}$ | $\begin{array}{r} 0.00301 \\ 2 \end{array}$ | $\begin{array}{r} 0.81766 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |
| 26 7 | $\begin{array}{r} 0.23302 \\ 2 \end{array}$ | $\begin{array}{r} 0.00312 \\ 8 \end{array}$ | $\begin{array}{r} 0.83953 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 26 8 | [0.10007 | $\begin{array}{r} 0.00719 \\ 7 \end{array}$ | $\begin{array}{\|r} 0.89328 \\ 8 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | Clas | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |
| 86 9 | 0.01445 9 | [0.05051 | $\begin{array}{r} 0.82977 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ |


| $\begin{array}{r}27 \\ 0 \\ \hline\end{array}$ | 0.57219 | $\begin{array}{r} 0.00299 \\ 2 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 27 | 0.10497 | 0.00874 | 0.82184 | Class 4 | Clas | Clas | Clas |
| 1 | 1 | 9 | 2 |  | s 2 | s 4 | s 2 |
| 27 | 0.23002 | 0.00320 | 0.84321 | Class 4 | Clas | Clas | Clas |
| 2 | 4 | 6 | 4 |  | s 2 | s 4 | s 2 |
| 27 | 0.23091 | 0.00325 | 0.87018 | Class 4 | Clas | Clas | Clas |
| 3 | 5 | 5 | 6 |  | s 2 | s 4 | s 2 |
| 27 | 0.08815 | 0.01009 | 0.64363 | Class 4 | Clas | Clas | Clas |
| 4 |  |  | 4 |  | s 4 | s 4 | s 4 |
| 27 | 0.05135 | 0.01855 | 0.67228 | Class 4 | Clas | Clas | Clas |
| 5 | 1 | 9 |  |  | s 4 | s 4 | s 4 |
| 27 | 0.05214 | 0.01302 | 0.65866 | Class 4 | Clas | Clas | Clas |
| 6 | 6 | 3 | 7 |  | s 4 | s 4 | s 4 |
| 27 | 0.17580 | 0.00395 | 0.65202 | Class 4 | Clas | Clas | Clas |
| 7 | 5 | 8 | 3 |  | s 4 | s 4 | s 4 |
| 27 | 0.12921 | 0.00776 | 0.65849 | Class 4 | Clas | Clas | Clas |
| 8 | 1 | 3 | 8 |  | s 4 | s 4 | s 4 |
| 27 | 0.07015 | 0.00892 | 0.67402 | Class 4 | Clas | Clas | Clas |
| 9 | 9 | 4 | 8 |  | s 4 | s 4 | s 4 |
| 28 | 0.03965 | 0.02046 | 0.68888 | Class 4 | Clas | Clas | Clas |
| 0 | 7 | 2 | 1 |  | s 4 | s 4 | s 4 |



| 28 7 | $\begin{array}{r} 0.07088 \\ 6 \end{array}$ | $\begin{array}{r} \hline 0.00813 \\ 5 \end{array}$ | $\begin{array}{r} 0.65205 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 28 8 | 0.01606 | 0.03723 6 | $0.65675$ | Class 4 | Clas | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| 28 9 | $\begin{array}{r} 0.24148 \\ 2 \end{array}$ | $\begin{array}{r} 0.00815 \\ 9 \end{array}$ | $0.64980$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 4 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| 29 0 | $\begin{array}{r} 0.24240 \\ 5 \end{array}$ | $\begin{array}{r} 0.00762 \\ 2 \end{array}$ | $\begin{array}{r} 0.65399 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \\ \hline \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| 29 1 | $\begin{array}{r} 0.02520 \\ 5 \end{array}$ | $\begin{array}{r} 0.02764 \\ 7 \end{array}$ | 0.65199 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ |
| 19 2 | $\begin{array}{r} 0.20359 \\ 4 \end{array}$ | $\begin{array}{r} 0.00310 \\ 5 \end{array}$ | $\begin{array}{r} 0.66870 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ |
| 29 3 | $0.33034$ | 0.0038 | $\begin{array}{r} 0.83590 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 29 4 | $\begin{array}{r} 0.00464 \\ 3 \end{array}$ | $\begin{array}{r} 0.25882 \\ 9 \end{array}$ | $\begin{array}{r} 0.84662 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 29 5 | $\begin{array}{r} 0.10012 \\ 8 \end{array}$ | $\begin{array}{r} 0.00895 \\ 3 \end{array}$ | $\begin{array}{r} 0.82030 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 29 6 | $\begin{array}{r} 0.11401 \\ 7 \end{array}$ | $\begin{array}{r} 0.00806 \\ 7 \end{array}$ | $\begin{array}{r} 0.83982 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \\ \hline \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 69 7 | $\begin{array}{r} 0.07498 \\ \hline \end{array}$ | $\begin{array}{r} 0.00967 \\ 1 \end{array}$ | $\begin{array}{r} 0.85338 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 29 8 | $\begin{array}{r} 0.34870 \\ 7 \end{array}$ | $\begin{array}{r} 0.00208 \\ 4 \end{array}$ | $\begin{array}{r} 0.83820 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 29 9 | 0.34423 | $\begin{array}{r} 0.00246 \\ 3 \end{array}$ | $\begin{array}{r} 0.83208 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 30 0 | 0.01018 | $\begin{array}{r} \hline 0.07177 \\ 7 \end{array}$ | $0.83308$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 30 1 | $\begin{array}{r} 0.29861 \\ 6 \end{array}$ | $\begin{array}{r} 0.00250 \\ 9 \end{array}$ | $\begin{array}{r} 0.84983 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |


| 30 2 | $\begin{array}{r} \hline 0.28582 \\ 5 \end{array}$ | $\begin{array}{r} \hline 0.04214 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.81089 \\ 8 \end{array}$ | Class 4 | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 30 3 | 0.35299 3 | 0.00283 4 | $\begin{array}{r} \hline 0.82032 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
|  | $\begin{array}{r} \hline 0.26216 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.00487 \\ 5 \end{array}$ | $\begin{array}{r} \hline 0.94981 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 30 | $\begin{array}{r} \hline 0.21109 \\ 2 \end{array}$ | $\begin{array}{r} \hline 0.01221 \\ 5 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 30 6 | 0.19764 | $\begin{array}{r} 0.00376 \\ 3 \end{array}$ | $\begin{array}{r} 0.84647 \\ 5 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \hline \text { s } 2 \\ \hline \end{array}$ |
| 30 | $\begin{array}{r} \hline 0.25750 \\ 8 \end{array}$ | 200378 | $\begin{array}{r} \hline 0.84619 \\ 5 \\ \hline \end{array}$ | Class 4 | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |
| 30 8 | $0.31895$ | $\begin{array}{r} 0.00221 \\ 6 \end{array}$ | 0.82718 | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 30 9 | 0.15827 | $0.00466$ | $\begin{array}{r} \hline 0.84020 \\ 5 \\ \hline \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| $\begin{array}{r}31 \\ 0 \\ \hline 31\end{array}$ | 0.00841 4 | 2.12663 | $\begin{array}{r} 0.91113 \\ 6 \\ \hline \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31 1 | 0.49957 | $\begin{array}{r} \hline 0.00215 \\ 6 \end{array}$ | $\begin{array}{r} 0.95373 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}31 \\ \hline 2\end{array}$ | $\begin{array}{r} 0.19145 \\ 9 \end{array}$ | $\begin{array}{r} 0.00905 \\ 2 \end{array}$ | 0.91573 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}31 \\ \hline 3\end{array}$ | $\begin{array}{r} 0.86233 \\ 7 \end{array}$ | $\begin{array}{r} 0.00936 \\ 9 \end{array}$ | $\begin{array}{r} 0.94031 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 31 4 | $\begin{array}{r} 0.38245 \\ 3 \end{array}$ | 0.00227 3 | $\begin{array}{r} 0.92823 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 31 5 | $\begin{array}{r} 0.47426 \\ 3 \end{array}$ | $\begin{array}{r} 0.00477 \\ 7 \end{array}$ | $\begin{array}{r} 0.98678 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ |
| $\begin{array}{r}31 \\ \hline 6\end{array}$ | $\begin{array}{r} 0.43989 \\ 5 \end{array}$ | $\begin{array}{r} 0.00369 \\ 5 \end{array}$ | $\begin{array}{r} 0.96056 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}61 \\ \hline 7 \\ \hline 7\end{array}$ | $\begin{array}{r} 0.5281 \\ \hline 7 \end{array}$ | $\begin{array}{r} \hline 0.00310 \\ 2 \end{array}$ | $\begin{array}{r} 0.92917 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}31 \\ \hline 8 \\ \hline 31\end{array}$ | 0.42210 <br> 3 | 0.00227 | $\begin{array}{r} 0.93708 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 31 9 | $\begin{array}{r} 0.77513 \\ 5 \end{array}$ | $\begin{array}{r} 0.00303 \\ 4 \end{array}$ | $\begin{array}{r} 0.98684 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| 32 0 | $\begin{array}{r} 0.20360 \\ 6 \end{array}$ | $\begin{array}{r} 0.00587 \\ 6 \end{array}$ | $\begin{array}{r} 0.94118 \\ 4 \end{array}$ |  | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r}32 \\ 1 \\ \hline\end{array}$ | $\begin{array}{r} 0.92788 \\ 2 \end{array}$ | $\begin{array}{r} \hline 0.00128 \\ 9 \end{array}$ | 1 | Class 2 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 3 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}32 \\ 2 \\ \hline\end{array}$ | $\begin{array}{r} 0.52813 \\ 8 \end{array}$ | $\begin{array}{r} 0.00148 \\ 4 \end{array}$ | $\begin{array}{r} 0.95389 \\ 8 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}32 \\ 3 \\ \hline\end{array}$ | $\begin{array}{r} 0.00976 \\ 5 \end{array}$ | $\begin{array}{r} 0.21977 \\ 2 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 32 4 | $\begin{array}{r} 0.94031 \\ 7 \end{array}$ | $\begin{array}{r} 0.00103 \\ 4 \end{array}$ | 1 | Class 2 | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 3 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 32 5 | $\begin{array}{r} 0.19983 \\ 9 \end{array}$ | $\begin{array}{r} 0.01171 \\ 2 \end{array}$ | 1 | Class 2 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |


| 32 6 | $\begin{array}{r} 0.34702 \\ 6 \end{array}$ | 0.00654 | $\begin{array}{r} 0.98188 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 32 7 | [ 0.31717 | $\begin{array}{r} 0.00607 \\ 9 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 7 32 8 | $\begin{array}{r} 0.26052 \\ 2 \end{array}$ | $\begin{array}{r} 0.00406 \\ 2 \end{array}$ | 0.93836 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r} 32 \\ 9 \end{array}$ | 1 | 0.00239 | 1 | Class 3 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 33 0 | $\begin{array}{r} 0.84755 \\ 5 \end{array}$ | $\begin{array}{r} 0.00128 \\ 6 \end{array}$ | 0.98353 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 33 1 | $\begin{array}{r} 0.88508 \\ 3 \end{array}$ | $\begin{array}{r} 0.00205 \\ 3 \end{array}$ | $\begin{array}{r} 0.98704 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | Clas | Clas |
|  | $\begin{array}{r} 0.72497 \\ 3 \end{array}$ | $\begin{array}{r} 0.00339 \\ 5 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas |
| 33 3 | $\begin{array}{r} 0.22742 \\ 4 \end{array}$ | $\begin{array}{r} 0.00681 \\ 9 \end{array}$ | 0.94042 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 33 4 | $\begin{array}{r} 0.74817 \\ 7 \end{array}$ | $\begin{array}{r} 0.00288 \\ 9 \end{array}$ | $\begin{array}{r} 0.99228 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 33 5 | $\begin{array}{r} 0.45457 \\ 4 \end{array}$ | $\begin{array}{r} 0.00326 \\ 7 \end{array}$ | $\begin{array}{r} 0.96595 \\ 4 \end{array}$ | Class 4 | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| 33 6 | 0.11807 | $\begin{array}{r} 0.02005 \\ 2 \end{array}$ | $\begin{array}{r} 0.91434 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | Clas |
| 6 33 7 | $\begin{array}{r} 0.45528 \\ 9 \end{array}$ | $\begin{array}{r} 0.00575 \\ 7 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 33 8 | $\begin{array}{r} 0.40272 \\ 9 \end{array}$ | $\begin{array}{r} 0.00263 \\ 1 \end{array}$ | 0.96506 | Class 4 | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 33 9 | $\begin{array}{r} 0.33302 \\ 5 \end{array}$ | $\begin{array}{r} 0.00765 \\ 1 \end{array}$ | $\begin{array}{r} \hline 0.98404 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 34 0 | - 0.85228 | $0.00209$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}34 \\ 1 \\ \hline 34\end{array}$ | $\begin{array}{r} 0.23903 \\ 6 \end{array}$ | $\begin{array}{r} 0.00736 \\ 4 \end{array}$ | 0.95098 | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| $\begin{array}{r}34 \\ 2 \\ \hline\end{array}$ | $\begin{array}{\|r} 0.56162 \\ 1 \end{array}$ | $\begin{array}{r} 0.00255 \\ 9 \end{array}$ | $\begin{array}{r} 0.98982 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 34 3 | $\begin{array}{r} 0.82811 \\ 5 \end{array}$ | $\begin{array}{r} 0.00145 \\ 4 \end{array}$ | $\begin{array}{r} 0.97592 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| $\begin{array}{r}34 \\ 4 \\ \hline\end{array}$ | $\begin{array}{r} 0.31619 \\ 3 \end{array}$ | $\begin{array}{r} 0.00274 \\ 4 \end{array}$ | $\begin{array}{r} \hline 0.95800 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 34 5 | $\begin{array}{r} 0.77991 \\ 2 \end{array}$ | $\begin{array}{r} 0.00138 \\ 1 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |


| $\begin{array}{r}34 \\ 6 \\ \hline 34\end{array}$ | $\begin{array}{r} 0.92345 \\ 7 \end{array}$ | 0.00139 | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 34 | 0.32252 | $0.00364$ | $0.93568$ | Class 4 | Clas | Clas | Clas |
| 34 8 | 0.80725 9 | 0.00161 | 0.96238 3 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| 34 9 | $\begin{array}{r} \hline 0.47174 \\ 9 \end{array}$ | 0.00341 | $\begin{array}{r} 0.98219 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 0 | 0.25924 | 0.00517 8 | $\begin{array}{r} 0.93430 \\ 5 \end{array}$ | Class 4 | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| 35 1 | $\begin{array}{r} 0.69498 \\ 5 \end{array}$ | $\begin{array}{r} 0.00264 \\ 3 \end{array}$ | $\begin{array}{r} 0.97906 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 35 | 0.70181 | $\begin{array}{r} 0.00209 \\ 1 \end{array}$ | 0.95431 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 35 3 | 0.62419 | $\begin{array}{r} 0.00631 \\ 5 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 35 4 | $\begin{array}{r} 0.64869 \\ 2 \end{array}$ | $\begin{array}{r} 0.00358 \\ 7 \end{array}$ | $\begin{array}{r} 0.99201 \\ 2 \end{array}$ | Class 4 | Clas | Clas | Clas |
| 35 | $\begin{array}{r} 0.19581 \\ 9 \end{array}$ | $\begin{array}{r} 0.00966 \\ 8 \end{array}$ | $\begin{array}{r} 0.97004 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 35 6 | 0.70089 | $\begin{array}{r} \hline 0.00190 \\ 9 \end{array}$ | 0.98757 | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 65 7 | $\begin{array}{r} \hline 0.90563 \\ 7 \end{array}$ | $\begin{array}{r} \hline 0.01865 \\ 6 \end{array}$ | $\begin{array}{r} 0.91120 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ |
| 35 8 | $\begin{array}{r} 0.17256 \\ 7 \end{array}$ | $\begin{array}{r} 0.01181 \\ 4 \end{array}$ | $\begin{array}{r} 0.97779 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 35 | 0.71557 5 | 0.00166 4 | $\begin{array}{r} 0.98695 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ |


| 36 0 | $\begin{array}{r} 0.99692 \\ 7 \end{array}$ | $\begin{array}{r} 0.01308 \\ 7 \end{array}$ | 0.96464 | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 3 \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | Clas s 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 36 1 | $\begin{array}{r} 0.16242 \\ 2 \end{array}$ | $\begin{array}{r} 0.01237 \\ 1 \end{array}$ | $\begin{array}{r} 0.91187 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| $\begin{array}{r}16 \\ \hline 2\end{array}$ | $\begin{array}{r} 0.49262 \\ 1 \end{array}$ | $\begin{array}{r} 0.00240 \\ 3 \end{array}$ | $\begin{array}{r} 0.98627 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | Clas |
| 36 3 | $\begin{array}{\|r} \hline 0.08980 \\ 8 \end{array}$ | $\begin{array}{r} 0.00894 \\ 3 \end{array}$ | $\begin{array}{r} 0.93734 \\ 2 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 36 4 | 0.61744 | $\begin{array}{r} 0.00136 \\ 1 \end{array}$ | $\begin{array}{r} 0.97682 \\ 6 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 4 \end{array}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ |
| 36 5 | $\begin{array}{r} 0.70758 \\ 5 \end{array}$ | $\begin{array}{r} 0.00110 \\ 8 \end{array}$ | 1 | Class 2 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|l} \hline \text { Clas } \\ \text { s } 2 \\ \hline \end{array}$ |


| $\begin{array}{r}36 \\ 6 \\ \hline 36\end{array}$ | $\begin{array}{r} \hline 0.61465 \\ 5 \\ \hline \end{array}$ | 0.00127 <br> 6 | $\begin{array}{r} 0.98143 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 <br> 36 <br> 7 | 0.34651 | [0.00438 | $0.93916$ | Class 4 | Clas | Clas <br> s 4 | Clas |
| $\begin{array}{\|r} \hline 36 \\ 8 \end{array}$ | 1 | 0.00274 | 1 | Class 3 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}36 \\ \hline 9\end{array}$ | 0.86986 | $\begin{array}{r} 0.00796 \\ 9 \end{array}$ | $\begin{array}{r} 0.91081 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| 37 0 | $\begin{array}{r} 0.70702 \\ 7 \end{array}$ | $\begin{array}{r} 0.00464 \\ 4 \end{array}$ | $\begin{array}{r} 0.93458 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 37 | 0.78347 4 | $\begin{array}{r} 0.00283 \\ 2 \end{array}$ | $\begin{array}{r} 0.97584 \\ 2 \end{array}$ | Class 4 | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 37 2 | $\begin{array}{r} 0.65324 \\ 6 \end{array}$ | $\begin{array}{r} 0.00342 \\ 6 \end{array}$ | $\begin{array}{r} 0.97246 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r} 37 \\ \hline 3 \end{array}$ | $\begin{array}{r} 0.66936 \\ 9 \end{array}$ | $\begin{array}{r} 0.00257 \\ 3 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}37 \\ \hline 4 \\ \hline 37\end{array}$ | $\begin{array}{r} 0.88589 \\ 4 \end{array}$ | $\begin{array}{r} 0.00246 \\ 5 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 37 5 | $\begin{array}{r} 0.35912 \\ 2 \end{array}$ | $\begin{array}{r} 0.00379 \\ 1 \end{array}$ | $\begin{array}{r} 0.93516 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| $\begin{array}{r}37 \\ \hline 6\end{array}$ | [0.86178 | $\begin{array}{r} 0.00201 \\ 8 \end{array}$ | $\begin{array}{r} 0.99153 \\ 4 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 37 7 | $\begin{array}{r} 0.50569 \\ 5 \end{array}$ | $\begin{array}{r} 0.00469 \\ 1 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 37 8 | $\begin{array}{r} 0.72549 \\ 9 \end{array}$ | 0.00397 | $\begin{array}{r} 0.89707 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}8 \\ \hline 37 \\ 9 \\ \hline 38\end{array}$ | 0.79629 | $\begin{array}{r} 0.00165 \\ 5 \end{array}$ | $\begin{array}{r} 0.98108 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 38 0 | $\begin{array}{r} 0.10101 \\ 6 \end{array}$ | $\begin{array}{r} 0.00777 \\ 6 \end{array}$ | $\begin{array}{r} 0.90770 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | Clas |
| 38 1 | $\begin{array}{r} 0.52759 \\ 2 \end{array}$ | $\begin{array}{r} 0.00358 \\ 4 \end{array}$ | $\begin{array}{r} 0.92166 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 38 | 0.25227 4 | $\begin{array}{r} 0.00929 \\ 5 \end{array}$ | $\begin{array}{r} 0.97310 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| $\begin{array}{r}38 \\ \hline 3\end{array}$ | $\begin{array}{r} 0.91052 \\ 5 \end{array}$ | $\begin{array}{r} 0.00140 \\ 5 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Class } \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 38 4 | $\begin{array}{r} 0.93676 \\ 9 \end{array}$ | $\begin{array}{r} 0.00130 \\ 8 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Class } \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| $\begin{array}{r}38 \\ \hline 5 \\ \hline 38\end{array}$ | $0.84227$ | $\begin{array}{r} 0.00293 \\ 5 \end{array}$ | $\begin{array}{r} 0.99680 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 3 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| $\begin{array}{r}38 \\ \hline 6 \\ \hline 38\end{array}$ | 20.63111 1 | $\begin{array}{r} 0.00155 \\ 1 \end{array}$ | $\begin{array}{r} 0.96933 \\ 9 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| 68 <br> 38 <br> 7 | 0.55615 | 0.00372 <br> 1 | $\begin{array}{r} 0.91795 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |
| $\begin{array}{r}38 \\ \hline 8\end{array}$ | 0.56441 7 | $\begin{array}{r} 0.00552 \\ 7 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ | $\begin{aligned} & \text { Class } \\ & 2 \end{aligned}$ |


| 38 | 0.15460 | 0.01287 | 0.87204 | Class 4 | 7 | Class | Class |
| ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- |
| 9 | 1 |  | Class |  |  |  |  |


| $\begin{array}{\|r} \hline 39 \\ 0 \end{array}$ | $\begin{array}{r} 0.91844 \\ 6 \end{array}$ | $\begin{array}{r} 0.00211 \\ 8 \end{array}$ | $\begin{array}{r} 0.96305 \\ 3 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 39 1 | $\begin{array}{r} 0.51203 \\ 6 \end{array}$ | $\begin{array}{r} \hline 0.00509 \\ 3 \end{array}$ | $\begin{array}{r} 0.96381 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | Clas | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 1 39 2 | [ 0.73130 | $\begin{array}{r} 0.00343 \\ 3 \end{array}$ | $\begin{array}{r} 0.92055 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
|  | $\begin{array}{r} 0.44865 \\ 9 \end{array}$ | $\begin{array}{r} \hline 0.00662 \\ 1 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 39 4 | $\begin{array}{r} 0.02838 \\ 5 \end{array}$ | $\begin{array}{r} 0.07410 \\ 9 \end{array}$ | $\begin{array}{r} 0.89871 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | Clas |
|  | 0.37927 4 | $\begin{array}{r} 0.00731 \\ 5 \end{array}$ | $\begin{array}{r} 0.96905 \\ 2 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 39 6 | $\begin{array}{r} 0.40642 \\ 9 \end{array}$ | $\begin{array}{r} 0.00239 \\ 5 \end{array}$ | $\begin{array}{r} 0.88641 \\ 6 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas |
| 6 <br> 39 <br> 7 | \| 0.31467 | $\begin{array}{r} \hline 0.00304 \\ 5 \end{array}$ | $\begin{array}{r} 0.87277 \\ 1 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r}79 \\ \hline 8\end{array}$ | $\begin{array}{r} 0.89128 \\ 8 \end{array}$ | 0.00182 | $\begin{array}{r} 0.95233 \\ 7 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 89 9 | $\begin{array}{r} 0.78276 \\ 5 \end{array}$ | $\begin{array}{r} \hline 0.00309 \\ 7 \end{array}$ | $\begin{array}{r} \hline 0.93680 \\ 8 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| $\begin{array}{r} 40 \\ 0 \end{array}$ | $\begin{array}{r} 0.27324 \\ 8 \end{array}$ | $\begin{array}{r} 0.00941 \\ 7 \end{array}$ |  |  | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | Clas s 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 40 \\ 1 \end{array}$ | $\begin{array}{r} 0.17645 \\ 2 \end{array}$ | $\begin{array}{r} 0.00565 \\ 2 \end{array}$ | $\begin{array}{r} 0.90356 \\ 2 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| $\begin{array}{r} 40 \\ 2 \end{array}$ | $\begin{array}{r} 0.55077 \\ 2 \end{array}$ | $\begin{array}{r} 0.00828 \\ 6 \end{array}$ | $\begin{array}{r} 0.84476 \\ 3 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 4 \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 40 3 | $\begin{array}{r} 0.16952 \\ 1 \end{array}$ | $\begin{array}{r} 0.01649 \\ 6 \end{array}$ | $\begin{array}{r} 0.96272 \\ 5 \end{array}$ | Class 4 | $\begin{array}{\|l\|} \hline \text { Clas } \\ \text { s } 2 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 40 4 | $\begin{array}{r} 0.95154 \\ 3 \end{array}$ | $\begin{array}{r} 0.00307 \\ 4 \end{array}$ | $\begin{array}{r} 0.99809 \\ 2 \end{array}$ | Class 4 | $\begin{array}{\|l\|l\|} \hline \text { Clas } \\ \text { s } 3 \end{array}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| 40 | $\begin{array}{r} 0.98212 \\ 3 \end{array}$ | $\begin{array}{r} 0.00142 \\ 4 \end{array}$ | 1 | Class 2 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 2 \end{aligned}$ |


| 40 6 | 0.84089 3 | 0.00439 4 | $\begin{array}{r} 0.86369 \\ 5 \end{array}$ | Class 4 | $\begin{aligned} & \text { Clas } \\ & \text { s } 3 \end{aligned}$ | $\begin{aligned} & \text { Clas } \\ & \text { s } 4 \end{aligned}$ | $\begin{aligned} & \hline \text { Clas } \\ & \text { s } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 0.14819 | 0.01477 | 0.91478 | Class 4 |  |  |  |
| 7 | 6 | 6 | 1 |  | s 2 | s 4 | s 2 |
| 40 | 0.54276 | 0.02422 | 0.88443 | Class 4 | Clas | Clas | Clas |
| 8 | 3 | 6 | 7 |  | s 2 | s 4 | s 2 |
| 40 | 0.00857 | 0.36737 | 0.94590 | Class 4 | Clas | Clas | Clas |
| 9 | 9 | 4 | 8 |  | s 2 | s 4 | s 2 |
| 41 | 0.69216 | 0.00168 | 1 | Class 2 | Clas | Clas | Clas |
| 0 | 6 | 1 |  |  | s 2 | s 2 | s 2 |
| 41 | 0.22904 | 0.01251 | 0.97737 | Class 4 | Clas | Clas | Clas |
| 1 | 7 | 9 |  |  | s 2 | s 4 | s 2 |
| 41 | 0.38678 | 0.00943 | 1 | Class 2 | Clas | Clas | Clas |
| 2 | 5 | 1 |  |  | s 2 | s 2 | s 2 |
| 41 | 0.50543 | 0.00484 | 0.92044 | Class 4 | Clas | Clas | Clas |
| 3 | 6 | 7 | 3 |  | s 2 | s 4 | s 2 |
| 41 | 0.05518 | 0.03118 | 0.87297 | Class 4 | Clas | Clas | Clas |
| 4 |  | 2 | 3 |  | s 2 | s 4 | s 2 |
| 41 | 0.7088 | 0.00408 | 1 | Class 2 | Clas | Clas | Clas |
| 5 |  | 1 |  |  | s 2 | s 2 | s 2 |
| 41 | 0.046 | 0.01571 | 0.82714 | Class 4 | Clas | Clas | Clas |
| 6 |  | 9 | 1 |  | s 2 | s 4 | s 2 |
| 41 | 0.55148 | 0.00517 | 0.96748 | Class 4 | Clas | Clas | Clas |
| 7 | 2 | 4 | 4 |  | s 2 | s 4 | s 2 |
| 41 | 0.29659 | 0.00613 | 0.9058 | Class 4 | Clas | Clas | Clas |
| 8 | 5 | 9 |  |  | s 2 | s 4 | s 2 |
| 41 | 0.63901 | 0.02077 | 0.82062 | Class 4 | Clas | Clas | Clas |
| 9 |  | 3 | 6 |  | s 2 | s 4 | s 2 |
| 42 | 1 | 0.00280 | 1 | Class 3 | Clas | Clas | Clas |
| 0 |  | 2 |  |  | s 3 | s 2 | s 2 |


| 421 | 0.408563 | 0.034481 | 0.802272 | Class 4 | Class 2 | Class 4 | Class 2 |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| 422 | 0.662579 | 0.002915 | 0.907404 | Class 4 | Class 2 | Class 4 | Class 2 |
| 423 | 1 | 0.001024 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |
| 424 | 0.507189 | 0.003053 | 0.902483 | Class 4 | Class 2 | Class 4 | Class 2 |
| 425 | 0.320207 | 0.026386 | 0.826011 | Class 4 | Class 2 | Class 4 | Class 2 |
| 426 | 0.470876 | 0.004456 | 0.913693 | Class 4 | Class 2 | Class 4 | Class 2 |
| 427 | 1 | 0.001517 | 0.896051 | Class 1 | Class 3 | Class 4 | Class 2 |
| 428 | 0.634814 | 0.000985 | 0.983647 | Class 4 | Class 2 | Class 4 | Class 2 |
| 429 | 0.175837 | 0.005581 | 0.902997 | Class 4 | Class 2 | Class 4 | Class 2 |
| 430 | 0.741483 | 0.00122 | 0.958075 | Class 4 | Class 2 | Class 4 | Class 2 |
| 431 | 0.236912 | 0.010678 | 0.852505 | Class 4 | Class 2 | Class 4 | Class 2 |
| 432 | 0.808532 | 0.000775 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 433 | 0.314216 | 0.006433 | 0.818634 | Class 4 | Class 2 | Class 4 | Class 2 |
| 434 | 0.266599 | 0.022619 | 0.730777 | Class 4 | Class 4 | Class 4 | Class 4 |
| 435 | 0.869321 | 0.000918 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 436 | 0.877852 | 0.000842 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 437 | 0.420615 | 0.001529 | 0.796429 | Class 4 | Class 4 | Class 4 | Class 4 |
| 438 | 0.934293 | 0.002542 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 439 | 0.36266 | 0.003896 | 0.783238 | Class 4 | Class 4 | Class 4 | Class 4 |
| 440 | 0.024276 | 0.055998 | 0.752649 | Class 4 | Class 4 | Class 4 | Class 4 |
| 441 | 0.614926 | 0.004233 | 0.880834 | Class 4 | Class 2 | Class 4 | Class 2 |
| 442 | 0.008945 | 0.279807 | 0.938082 | Class 4 | Class 2 | Class 4 | Class 2 |
| 443 | 0.954979 | 0.003232 | 1 | Class 2 | Class 3 | Class 2 | Class 2 |
| 444 | 1 | 0.00148 | 1 | Class 3 | Class 3 | Class 2 | Class 2 |

## KNUST

Appendix C: The Case 2 Results of the DT predictions on the $\mathbf{3 0 \%}$ random test dataset

| DMU | DEA Class | predictions |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 2 |
| 2 | Class 2 | Class 2 |
| 3 | Class 2 | Class 2 |
| 4 | Class 2 | Class 2 |


| 5 | Class 2 | Class 4 |
| :---: | :---: | :---: |
| 6 | Class 4 | Class 4 |
| 7 | Class 2 | Class 2 |
| 8 | Class 2 | Class 2 |
| 9 | Class 2 | Class 2 |
| 10 | Class 3 | Class 2 |
| 11 | Class 2 | Class 2 |
| 12 | Class 2 | Class 2 |
| 13 | Class 2 | Class 2 |
| 14 | Class 2 | Class 2 |
| 15 | Class 2 | Class 2 |
| 16 | Class 2 | Class 2 |
| 17 | Class 2 | Class 4 |
| 18 | Class 2 | Class 2 |
| 19 | Class 2 | Class 2 |
| 20 | Class 3 | Class 2 |
| 21 | Class 4 | Class 4 |
| 22 | Class 2 | Class 2 |
| 23 | Class 4 | Class 4 |
| 24 | Class 4 | Class 4 |
| 25 | Class 4 | Class 4 |
| 26 | Class 4 | Class 4 |
| 27 | Class 2 | Class 4 |
| 28 | Class 2 | Class 2 |
| 29 | Class 2 | Class 2 |
| 30 | Class 2 | Class 2 |
| 31 | Class 2 | Class 2 |
| 32 | Class 2 | Class 2 |


| 34 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 35 | Class 2 | Class 2 |
| 36 | Class 2 | Class 2 |
| 37 | Class 2 | Class 2 |
| 38 | Class 2 | Class 2 |
| 39 | Class 2 | Class 2 |
| 40 | Class 2 | Class 3 |


| 41 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 42 | Class 2 | Class 3 |
| 43 | Class 2 | Class 2 |
| 44 | Class 3 | Class 3 |
| 45 | Class 2 | Class 2 |
| 46 | Class 2 | Class 2 |
| 47 | Class 2 | Class 2 |
| 48 | Class 2 | Class 2 |
| 49 | Class 2 | Class 2 |
| 50 | Class 2 | Class 2 |
| 51 | Class 2 | Class 2 |
| 52 | Class 2 | Class 2 |
| 53 | Class 2 | Class 2 |
| 54 | Class 2 | Class 2 |
| 55 | Class 4 | Class 4 |
| 56 | Class 2 | Class 2 |
| 57 | Class 4 | Class 4 |
| 58 | Class 4 | Class 4 |
| 59 | Class 4 | Class 4 |
| 60 | Class 4 | Class 4 |
| 61 | Class 4 | Class 4 |
| 62 | Class 4 | Class 4 |
| 63 | Class 4 | Class 4 |
| 64 | Class 4 | Class 4 |
| 65 | Class 4 | Class 4 |


| 66 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 67 | Class 2 | Class 2 |
| 68 | Class 2 | Class 2 |

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| 103 | Class 2 | Class 2 |
| :--- | :--- | :--- |



| 104 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 105 | Class 2 | Class 2 |
| 106 | Class 2 | Class 2 |
| 107 | Class 3 | Class 3 |
| 108 | Class 2 | Class 2 |
| 109 | Class 2 | Class 2 |
| 110 | Class 2 | Class 2 |
| 111 | Class 2 | Class 2 |
| 112 | Class 2 | Class 2 |
| 113 | Class 3 | Class 3 |
| 114 | Class 2 | Class 2 |
| 115 | Class 3 | Class 3 |
| 116 | Class 2 | Class 2 |
| 117 | Class 2 | Class 2 |
| 118 | Class 2 | Class 2 |
| 119 | Class 2 | Class 2 |
| 120 | Class 2 | Class 2 |
| 121 | Class 3 | Class 3 |
| 122 | Class 2 | Class 3 |
| 123 | Class 2 | Class 2 |
| 124 | Class 2 | Class 2 |
| 125 | Class 2 | Class 2 |
| 126 | Class 2 | Class 2 |
| 127 | Class 2 | Class 2 |
| 128 | Class 3 | Class 3 |
| 129 | Class 2 | Class 2 |
| 130 | Class 2 | Class 2 |
| 131 | Class 2 | Class 3 |
| 132 | Class 3 | Class 3 |


| 133 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 134 | Class 3 | Class 3 |

## Appendix D: The Case 4 Results of the DT predictions on the $\mathbf{3 0 \%}$ dataset of 132 Bank

| DMU | DEA Class | predictions |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 2 |


| 2 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 3 | Class 2 | Class 2 |
| 4 | Class 2 | Class 2 |
| 5 | Class 2 | Class 4 |
| 6 | Class 4 | Class 4 |
| 7 | Class 2 | Class 2 |
| 8 | Class 2 | Class 2 |
| 9 | Class 2 | Class 2 |
| 10 | Class 2 | Class 2 |
| 11 | Class 2 | Class 2 |
| 12 | Class 2 | Class 2 |
| 13 | Class 2 | Class 2 |
| 14 | Class 2 | Class 2 |
| 15 | Class 2 | Class 2 |
| 16 | Class 2 | Class 2 |
| 17 | Class 2 | Class 4 |
| 18 | Class 2 | Class 2 |
| 19 | Class 2 | Class 2 |
| 20 | Class 2 | Class 2 |
| 21 | Class 4 | Class 4 |
| 22 | Class 2 | Class 2 |
| 23 | Class 4 | Class 4 |
| 24 | Class 4 | Class 4 |
| 25 | Class 4 | Class 4 |
| 26 | Class 4 | Class 4 |
| 27 | Class 2 | Class 4 |
| 28 | Class 2 | Class 2 |
| 29 | Class 2 | Class 2 |


| 30 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 31 | Class 2 | Class 2 |
| 32 | Class 2 | Class 2 |
| 33 | Class 2 | Class 2 |

## Branches (DMUs)

KNUST

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| 34 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 35 | Class 2 | Class 2 |
| 36 | Class 2 | Class 2 |
| 37 | Class 2 | Class 2 |
| 38 | Class 2 | Class 2 |
| 39 | Class 2 | Class 2 |
| 40 | Class 2 | Class 2 |


| 41 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 42 | Class 2 | Class 2 |
| 43 | Class 2 | Class 2 |
| 44 | Class 2 | Class 2 |
| 45 | Class 2 | Class 2 |
| 46 | Class 2 | Class 2 |
| 47 | Class 2 | Class 2 |
| 48 | Class 2 | Class 2 |
| 49 | Class 2 | Class 2 |
| 50 | Class 2 | Class 2 |
| 51 | Class 2 | Class 2 |
| 52 | Class 2 | Class 2 |
| 53 | Class 2 | Class 2 |
| 54 | Class 2 | Class 2 |
| 55 | Class 4 | Class 4 |
| 56 | Class 2 | Class 2 |
| 57 | Class 4 | Class 4 |
| 58 | Class 4 | Class 4 |
| 59 | Class 4 | Class 4 |
| 60 | Class 4 | Class 4 |
| 61 | Class 4 | Class 4 |
| 62 | Class 4 | Class 4 |
| 63 | Class 4 | Class 4 |
| 64 | Class 4 | Class 4 |
| 65 | Class 4 | Class 4 |


| 66 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 67 | Class 2 | Class 2 |
| 68 | Class 2 | Class 2 |

KNUST

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| 104 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 105 | Class 2 | Class 2 |
| 106 | Class 2 | Class 2 |
| 107 | Class 2 | Class 2 |
| 108 | Class 2 | Class 2 |
| 109 | Class 2 | Class 2 |
| 110 | Class 2 | Class 2 |
| 111 | Class 2 | Class 2 |
| 112 | Class 2 | Class 2 |
| 113 | Class 2 | Class 2 |
| 114 | Class 2 | Class 2 |
| 115 | Class 2 | Class 2 |
| 116 | Class 2 | Class 2 |
| 117 | Class 2 | Class 2 |
| 118 | Class 2 | Class 2 |
| 119 | Class 2 | Class 2 |
| 120 | Class 2 | Class 2 |
| 121 | Class 2 | Class 2 |
| 122 | Class 2 | Class 2 |
| 123 | Class 2 | Class 2 |
| 124 | Class 2 | Class 2 |
| 125 | Class 2 | Class 2 |
| 126 | Class 2 | Class 2 |
| 127 | Class 2 | Class 2 |
| 128 | Class 2 | Class 2 |
| 129 | Class 2 | Class 2 |
| 130 | Class 2 | Class 2 |
| 131 | Class 2 | Class 2 |
| 132 | Class 2 | Class 2 |


| 133 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 134 | Class 2 | Class 2 | KNUST

## Appendix E: The Case 2 Results of the RF predictions on the $\mathbf{3 0 \%}$ random test dataset

| DMU | DEA Class | RF <br> Predictions |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 2 |


| 2 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 5 | Class 2 | Class 2 |
| 7 | Class 2 | Class 2 |
| 19 | Class 4 | Class 4 |
| 22 | Class 4 | Class 4 |
| 23 | Class 2 | Class 2 |
| 24 | Class 4 | Class 4 |
| 31 | Class 2 | Class 2 |
| 38 | Class 2 | Class 2 |
| 39 | Class 2 | Class 3 |
| 40 | Class 2 | Class 2 |
| 42 | Class 3 | Class 3 |
| 43 | Class 2 | Class 2 |
| 47 | Class 2 | Class 2 |
| 54 | Class 2 | Class 2 |
| 62 | Class 2 | Class 2 |
| 65 | Class 2 | Class 4 |
| 67 | Class 2 | Class 4 |
| 70 | Class 4 | Class 4 |
| 71 | Class 2 | Class 2 |
| 73 | Class 2 | Class 2 |
| 78 | Class 4 | Class 4 |
| 85 | Class 2 | Class 2 |
| 86 | Class 4 | Class 4 |
| 87 | Class 4 | Class 4 |
| 92 | Class 2 | Class 2 |
| 95 | Class 2 | Class 2 |
| 96 | Class 2 | Class 2 |
| 97 | Class 2 | Class 2 |
| 101 | Class 2 | Class 2 |
| 102 | Class 2 | Class 2 |
| 103 | Class 2 | Class 2 |
| 105 | Class 2 | Class 2 |
| 106 | Class 2 |  |
| 107 | Class 2 | Class 2 |
|  |  |  |
| 4 |  |  |


| 110 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 119 | Class 2 | Class 2 |


| 125 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 126 | Class 2 | Class 2 |
| 130 | Class 2 | Class 2 |
| 132 | Class 3 | Class 3 |
| 133 | Class 2 | Class 2 |
| 138 | Class 2 | Class 2 |
| 139 | Class 2 | Class 2 |
| 140 | Class 2 | Class 2 |
| 141 | Class 2 | Class 2 |
| 143 | Class 2 | Class 2 |
| 148 | Class 3 | Class 3 |
| 152 | Class 2 | Class 2 |
| 156 | Class 2 | Class 2 |
| 157 | Class 2 | Class 2 |
| 163 | Class 2 | Class 2 |
| 165 | Class 2 | Class 2 |
| 167 | Class 2 | Class 2 |
| 171 | Class 2 | Class 2 |
| 174 | Class 4 | Class 4 |
| 178 | Class 2 | Class 2 |
| 182 | Class 4 | Class 4 |
| 183 | Class 4 | Class 4 |
| 184 | Class 4 | Class 4 |
| 187 | Class 4 | Class 4 |
| 192 | Class 4 | Class 4 |
| 198 | Class 4 | Class 4 |
| 200 | Class 4 | Class 4 |
| 202 | Class 4 | Class 4 |
| 206 | Class 2 | Class 2 |
| 219 | Class 2 | Class 2 |
| 220 | Class 2 | Class 2 |
| 226 | Class 2 | Class 3 |
| 235 | Class 2 | Class 2 |
| 240 | Class 2 | Class 2 |
| 1 |  |  |


| 244 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 246 | Class 2 | Class 2 |
| 247 | Class 2 | Class 2 |
| 251 | Class 2 | Class 2 |
| 252 | Class 2 | Class 2 |


| 254 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 260 | Class 2 | Class 2 |
| 268 | Class 2 | Class 2 |
| 273 | Class 2 | Class 2 |
| 275 | Class 4 | Class 4 |


| 277 | Class 4 | Class 4 |
| :---: | :---: | :---: |
| 278 | Class 4 | Class 4 |
| 287 | Class 4 | Class 4 |
| 291 | Class 4 | Class 4 |
| 292 | Class 4 | Class 4 |
| 297 | Class 2 | Class 2 |
| 299 | Class 2 | Class 2 |
| 300 | Class 2 | Class 2 |
| 303 | Class 2 | Class 2 |
| 305 | Class 2 | Class 2 |
| 307 | Class 2 | Class 2 |
| 315 | Class 2 | Class 2 |
| 323 | Class 2 | Class 2 |
| 326 | Class 2 | Class 2 |
| 327 | Class 2 | Class 2 |
| 330 | Class 3 | Class 3 |
| 336 | Class 2 | Class 2 |
| 340 | Class 3 | Class 3 |
| 342 | Class 2 | Class 2 |
| 348 | Class 3 | Class 3 |
| 350 | Class 2 | Class 2 |
| 351 | Class 2 | Class 2 |
| 354 | Class 2 | Class 2 |
| 357 | Class 3 | Class 3 |
| 361 | Class 2 | Class 2 |
| 365 | Class 2 | Class 2 |
| 370 | Class 2 | Class 2 |
| 371 | Class 2 | Class 2 |
| 376 | Class 3 | Class 3 |
| 378 | Class 2 | Class 2 |


| 380 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 381 | Class 2 | Class 2 |
| 388 | Class 2 | Class 2 |
| 395 | Class 2 | Class 2 |
| 396 | Class 2 | Class 2 |

SANE

| 405 | Class 3 | Class 3 |
| :--- | :--- | :--- |
| 406 | Class 3 | Class 3 |
| 408 | Class 2 | Class 2 |
| 410 | Class 2 | Class 2 |
| 413 | Class 2 | Class 2 |
| 414 | Class 2 | Class 2 |
| 415 | Class 2 | Class 2 |
| 422 | Class 2 | Class 2 |
| 426 | Class 2 | Class 2 |
| 429 | Class 2 | Class 2 |
| 430 | Class 2 | Class 3 |
| 431 | Class 2 | Class 2 |
| 436 | Class 3 | Class 3 |
| 437 | Class 4 | Class 2 |
| 438 | Class 3 | Class 2 |
| 444 | Class 3 | Class 3 |



Appendix F: The Case 4 Results of the RF predictions on the $30 \%$ test dataset of the

| DMU | DEA Class | RF <br> Predictions |
| :--- | :--- | :--- |


| 1 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 5 | Class 2 | Class 2 |
| 6 | Class 2 | Class 2 |
| 7 | Class 2 | Class 2 |
| 10 | Class 2 | Class 2 |
| 11 | Class 2 | Class 2 |
| 16 | Class 4 | Class 4 |
| 17 | Class 2 | Class 2 |
| 22 | Class 4 | Class 4 |
| 26 | Class 4 | Class 4 |
| 27 | Class 2 | Class 2 |
| 28 | Class 2 | Class 2 |
| 35 | Class 2 | Class 2 |
| 36 | Class 2 | Class 2 |
| 41 | Class 2 | Class 2 |
| 42 | Class 2 | Class 2 |
| 43 | Class 2 | Class 2 |
| 44 | Class 2 | Class 2 |
| 50 | Class 2 | Class 2 |
| 59 | Class 2 | Class 2 |
| 62 | Class 2 | Class 2 |
| 63 | Class 2 | Class 2 |
| 66 | Class 2 | Class 2 |
| 68 | Class 4 | Class 4 |
| 69 | Class 2 | Class 4 |
| 71 | Class 2 | Class 2 |
| 73 | Class 2 | Class 2 |
| 75 | Class 2 | Class 2 |
| 77 | Class 4 | Class 4 |
| 79 | Class 2 | Class 2 |
| 82 | Class 4 | Class 4 |
| 104 | Class 2 | Class 2 |
| 106 | Class 2 | Class 2 |
| 108 | Class 2 | Class 2 |
|  |  |  |
| 10 |  |  |


| 114 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 116 | Class 2 | Class 2 |
| 119 | Class 2 | Class 2 |

## Bank Branches (DMUs)



SANE

| 120 | Class 2 | Class 2 |
| :--- | :--- | :--- |


| 125 | Class 2 | Class 4 |
| ---: | :--- | :--- |
| 127 | Class 2 | Class 2 |
| 128 | Class 2 | Class 2 |
| 132 | Class 2 | Class 2 |
| 134 | Class 2 | Class 2 |
| 141 | Class 2 | Class 2 |
| 142 | Class 2 | Class 2 |
| 143 | Class 2 | Class 2 |
| 145 | Class 2 | Class 2 |
| 147 | Class 2 | Class 2 |
| 149 | Class 2 | Class 2 |
| 151 | Class 2 | Class 2 |
| 153 | Class 2 | Class 2 |
| 157 | Class 2 | Class 2 |
| 158 | Class 2 | Class 2 |
| 161 | Class 2 | Class 2 |
| 163 | Class 2 | Class 2 |
| 164 | Class 2 | Class 2 |
| 172 | Class 2 | Class 2 |
| 175 | Class 4 | Class 4 |
| 176 | Class 4 | Class 4 |
| 179 | Class 4 | Class 4 |
| 180 | Class 4 | Class 4 |
| 186 | Class 4 | Class 4 |
| 188 | Class 4 | Class 4 |
| 191 | Class 4 | Class 4 |
| 193 | Class 4 | Class 4 |
| 196 | Class 4 | Class 4 |
| 199 | Class 4 | Class 4 |
| 205 | Class 2 | Class 2 |
| 212 | Class 2 | Class 2 |
| 217 | Class 2 | Class 2 |
| 218 | Class 2 | Class 2 |
| 222 | Class 2 | Class 2 |
| 14 |  |  |


| 225 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 227 | Class 2 | Class 2 |
| 230 | Class 2 | Class 2 |
| 234 | Class 2 | Class 2 |
| 239 | Class 2 | Class 2 |

KNUST

| 242 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 246 | Class 2 | Class 2 |
| 263 | Class 2 | Class 2 |
| 264 | Class 2 | Class 2 |
| 265 | Class 2 | Class 2 |


| 271 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 283 | Class 4 | Class 4 |
| 288 | Class 4 | Class 4 |
| 289 | Class 4 | Class 4 |
| 293 | Class 2 | Class 2 |
| 297 | Class 2 | Class 2 |
| 298 | Class 2 | Class 2 |
| 299 | Class 2 | Class 2 |
| 300 | Class 2 | Class 2 |
| 302 | Class 2 | Class 2 |
| 306 | Class 2 | Class 2 |
| 309 | Class 2 | Class 2 |
| 310 | Class 2 | Class 2 |
| 332 | Class 2 | Class 2 |
| 338 | Class 2 | Class 2 |
| 339 | Class 2 | Class 2 |
| 340 | Class 2 | Class 2 |
| 346 | Class 2 | Class 2 |
| 350 | Class 2 | Class 2 |
| 352 | Class 2 | Class 2 |
| 353 | Class 2 | Class 2 |
| 354 | Class 2 | Class 2 |
| 359 | Class 2 | Class 2 |
| 363 | Class 2 | Class 2 |
| 366 | Class 2 | Class 2 |
| 367 | Class 2 | Class 2 |
| 370 | Class 2 | Class 2 |
| 371 | Class 2 | Class 2 |
| 377 | Class 2 | Class 2 |
| 378 | Class 2 | Class 2 |


| 380 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 383 | Class 2 | Class 2 |
| 384 | Class 2 | Class 2 |
| 385 | Class 2 | Class 2 |
| 387 | Class 2 | Class 2 |

KNUST

| 390 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 391 | Class 2 | Class 2 |
| 397 | Class 2 | Class 2 |
| 400 | Class 2 | Class 2 |
| 401 | Class 2 | Class 2 |
| 404 | Class 2 | Class 2 |
| 407 | Class 2 | Class 2 |
| 412 | Class 2 | Class 2 |
| 414 | Class 2 | Class 2 |
| 415 | Class 2 | Class 2 |
| 423 | Class 2 | Class 2 |
| 424 | Class 2 | Class 2 |
| 426 | Class 2 | Class 2 |
| 429 | Class 2 | Class 4 |
| 437 | Class 4 | Class 2 |
| 440 | Class 4 | Class 4 |



SAME

| DMU | DEA <br> Class | NN <br> Prediction |
| :--- | :--- | :--- |


| 1 | 2 | 2.46907306 |
| ---: | ---: | ---: |
| 3 | 2 | 2.46907306 |
| 9 | 2 | 2.46907306 |
| 10 | 2 | 2.46907306 |
| 16 | 4 | 2.46907306 |
| 17 | 2 | 2.46907306 |
| 18 | 3 | 2.46907306 |
| 25 | 4 | 2.46907306 |
| 26 | 4 | 2.987401446 |
| 31 | 2 | 2.46907306 |
| 37 | 2 | 2.46907306 |
| 40 | 2 | 2.46907306 |
| 48 | 2 | 2.46907306 |
| 49 | 2 | 2.46907306 |
| 52 | 2 | 2.46907306 |
| 54 | 2 | 2.46907306 |
| 56 | 2 | 2.46907306 |
| 59 | 2 | 2.46907306 |
| 62 | 2 | 2.46907306 |
| 69 | 2 | 2.46907306 |
| 77 | 4 | 2.46907306 |
| 78 | 4 | 2.46907306 |
| 80 | 4 | 2.46907306 |
| 85 | 2 | 2.46907306 |
| 89 | 4 | 2.46907306 |
| 91 | 3 | 2.46907306 |
| 92 | 2 | 2.298099763 |
| 102 | 2 | 2.46907306 |
| 103 | 2 | 2.46907306 |
| 104 | 2 | 2.46907306 |
| 106 | 2 | 2.46907306 |
| 10 | 2 | 2.46907306 |
| 2 | 2 | 2.46907306 |
| 2.46907306 |  |  |
| 16 | 2.298099763 |  |
| 10 |  |  |
| 10 | 2 | 2 |


| 128 | 2 | 2.46907306 | Appendix G: The Case 2 Results of the ANN predictions on |
| :---: | :---: | :---: | :---: |
| 129 | 2 | 2.46907306 | of the Bank Branches (DMUs) |
| 133 | 2 | 2.46907306 |  |



| 137 | 2 | 2.46907306 |
| :--- | :--- | :--- |


| 141 | 2 | 2.46907306 |
| :---: | :---: | :---: |
| 145 | 2 | 2.46907306 |
| 146 | 3 | 2.46907306 |
| 153 | 2 | 2.46907306 |
| 156 | 2 | 2.46907306 |
| 158 | 2 | 2.46907306 |
| 159 | 2 | 2.298099763 |
| 160 | 2 | 2.46907306 |
| 164 | 2 | 2.571432751 |
| 165 | 2 | 2.46907306 |
| 172 | 2 | 2.298099763 |
| 183 | 4 | 2.46907306 |
| 190 | 4 | 2.46907306 |
| 191 | 4 | 2.46907306 |
| 199 | 4 | 2.46907306 |
| 207 | 2 | 2.298099763 |
| 210 | 2 | 2.46907306 |
| 211 | 2 | 2.46907306 |
| 212 | 2 | 2.46907306 |
| 215 | 2 | 3.0459214 |
| 216 | 2 | 2.356619717 |
| 230 | 2 | 2.46907306 |
| 231 | 2 | 2.46907306 |
| 233 | 2 | 2.298099763 |
| 235 | 2 | 2.46907306 |
| 236 | 2 | 2.46907306 |
| 248 | 2 | 2.571432751 |
| 250 | 2 | 2.46907306 |
| 251 | 2 | 2.46907306 |
| 254 | 2 | 2.46907306 |
| 255 | 2 | 2.46907306 |
| 256 | 2 | 2.46907306 |
| 263 | 2 | 2.46907306 |
| 264 | 2 | 2.46907306 |


| 270 | 2 | 2.46907306 |
| ---: | ---: | ---: |
| 272 | 2 | 2.46907306 |
| 280 | 4 | 2.987401446 |
| 284 | 4 | 2.46907306 |
| 285 | 4 | 2.46907306 |

KNUST

| 288 | 4 | 2.987401446 |
| ---: | ---: | ---: |
| 290 | 4 | 2.46907306 |
| 303 | 2 | 2.46907306 |
| 304 | 2 | 2.46907306 |
| 307 | 2 | 2.46907306 |


| 308 | 2 | 2.46907306 |
| ---: | ---: | ---: |
| 312 | 2 | 2.46907306 |
| 313 | 3 | 2.46907306 |
| 316 | 2 | 2.46907306 |
| 318 | 2 | 2.46907306 |
| 320 | 2 | 2.46907306 |
| 322 | 2 | 2.46907306 |
| 323 | 2 | 2.571432751 |
| 326 | 2 | 2.46907306 |
| 332 | 2 | 2.46907306 |
| 335 | 2 | 2.46907306 |
| 338 | 2 | 2.46907306 |
| 342 | 2 | 2.46907306 |
| 346 | 3 | 2.46907306 |
| 347 | 2 | 2.46907306 |
| 348 | 3 | 2.46907306 |
| 349 | 2 | 2.46907306 |
| 350 | 2 | 2.46907306 |
| 358 | 2 | 2.356619717 |
| 360 | 3 | 2.46907306 |
| 362 | 2 | 2.46907306 |
| 364 | 2 | 2.46907306 |
| 365 | 2 | 2.46907306 |
| 366 | 2 | 2.46907306 |
| 372 | 2 | 2.46907306 |
| 373 | 2 | 2.46907306 |
| 378 | 2 | 2.46907306 |
| 381 | 2 | 2.46907306 |
| 383 | 3 | 2.46907306 |
| 384 | 3 | 2.46907306 |
|  |  |  |
| 36 |  | 2 |


| 385 | 3 | 2.46907306 |
| ---: | ---: | ---: |
| 388 | 2 | 2.46907306 |
| 394 | 2 | 2.987401446 |
| 396 | 2 | 2.46907306 |
| 397 | 2 | 2.46907306 |

KNUST

| 398 | 3 | 2.46907306 |
| ---: | ---: | ---: |
| 400 | 2 | 2.46907306 |
| 402 | 2 | 2.46907306 |
| 403 | 2 | 2.356619717 |
| 408 | 2 | 2.46907306 |
| 409 | 2 | 2.571432751 |
| 410 | 2 | 2.46907306 |
| 414 | 2 | 2.987401446 |
| 415 | 2 | 2.46907306 |
| 422 | 2 | 2.46907306 |
| 423 | 3 | 2.46907306 |
| 425 | 2 | 2.46907306 |
| 427 | 3 | 2.46907306 |
| 429 | 2 | 2.46907306 |
| 432 | 3 | 2.46907306 |
| 440 | 4 | 2.987401446 |



Appendix H: The Case 4 Results of the ANN predictions on the $30 \%$ test dataset of the

| DMU | DEA <br> Class | NN <br> Prediction |
| :--- | :--- | :--- |


| 1 | 2 | 2.349055512 |
| ---: | ---: | ---: |
| 3 | 2 | 2.349055512 |
| 9 | 2 | 2.349055512 |
| 10 | 2 | 2.349055512 |
| 16 | 4 | 2.349055512 |
| 17 | 2 | 2.349055512 |
| 18 | 2 | 2.349055512 |
| 25 | 4 | 2.349055512 |
| 26 | 4 | 2.987326403 |
| 31 | 2 | 2.349055512 |
| 37 | 2 | 2.349055512 |
| 40 | 2 | 2.349055512 |
| 48 | 2 | 2.349055512 |
| 49 | 2 | 2.349055512 |
| 52 | 2 | 2.349055512 |
| 54 | 2 | 2.349055512 |
| 56 | 2 | 2.349055512 |
| 59 | 2 | 2.349055512 |
| 62 | 2 | 2.349055512 |
| 69 | 2 | 2.349055512 |
| 77 | 4 | 2.349055512 |
| 78 | 4 | 2.349055512 |
| 80 | 4 | 2.349055512 |
| 85 | 2 | 2.349055512 |
| 89 | 4 | 2.349055512 |
| 91 | 2 | 2.349055512 |
| 92 | 2 | 2.297836691 |
| 102 | 2 | 2.349055512 |
| 103 | 2 | 2.349055512 |
| 104 | 2 | 2.349055512 |
| 106 | 2 | 2.349055512 |
| 10 | 2 | 2.349055512 |
| 2 | 2.349055512 |  |
| 2 | 2 | 2.349055512 |
| 16 |  | 297836691 |
| 10 |  |  |
| 10 |  | 2 |


| 128 | 2 | 2.349055512 |
| :--- | :--- | :--- |
| 129 | 2 | 2.349055512 |
| 133 | 2 | 2.349055512 |

KNUST

## SANE

| 137 | 2 | 2.349055512 |
| ---: | ---: | ---: |


| 141 | 2 | 2.349055512 |
| :---: | :---: | :---: |
| 145 | 2 | 2.349055512 |
| 146 | 2 | 2.349055512 |
| 153 | 2 | 2.349055512 |
| 156 | 2 | 2.349055512 |
| 158 | 2 | 2.349055512 |
| 159 | 2 | 2.297836691 |
| 160 | 2 | 2.349055512 |
| 164 | 2 | 2.2856747 |
| 165 | 2 | 2.349055512 |
| 172 | 2 | 2.297836691 |
| 183 | 4 | 2.349055512 |
| 190 | 4 | 2.349055512 |
| 191 | 4 | 2.349055512 |
| 199 | 4 | 2.349055512 |
| 207 | 2 | 2.297836691 |
| 210 | 2 | 2.349055512 |
| 211 | 2 | 2.349055512 |
| 212 | 2 | 2.349055512 |
| 215 | 2 | 3.046277829 |
| 216 | 2 | 2.356788118 |
| 230 | 2 | 2.349055512 |
| 231 | 2 | 2.349055512 |
| 233 | 2 | 2.297836691 |
| 235 | 2 | 2.349055512 |
| 236 | 2 | 2.349055512 |
| 248 | 2 | 2.2856747 |
| 250 | 2 | 2.349055512 |
| 251 | 2 | 2.349055512 |
| 254 | 2 | 2.349055512 |
| 255 | 2 | 2.349055512 |
| 256 | 2 | 2.349055512 |
| 263 | 2 | 2.349055512 |
| 264 | 2 | 2.349055512 |
| 270 | 2 | 2.349055512 |
| 272 | 2 | 2.349055512 |
| 280 | 4 | 2.987326403 |
| 284 | 4 | 2.349055512 |
| 285 | 4 | 2.349055512 |


| 288 | 4 | 2.987326403 |
| :--- | :--- | :--- |
| 290 | 4 | 2.349055512 |
| 303 | 2 | 2.349055512 |
| 304 | 2 | 2.349055512 |
| 307 | 2 | 2.349055512 |


| 308 | 2 | 2.349055512 |
| :---: | :---: | :---: |
| 312 | 2 | 2.349055512 |
| 313 | 2 | 2.349055512 |
| 316 | 2 | 2.349055512 |
| 318 | 2 | 2.349055512 |
| 320 | 2 | 2.349055512 |
| 322 | 2 | 2.349055512 |
| 323 | 2 | 2.2856747 |
| 326 | 2 | 2.349055512 |
| 332 | 2 | 2.349055512 |
| 335 | 2 | 2.349055512 |
| 338 | 2 | 2.349055512 |
| 342 | 2 | 2.349055512 |
| 346 | 2 | 2.349055512 |
| 347 | 2 | 2.349055512 |
| 348 | 2 | 2.349055512 |
| 349 | 2 | 2.349055512 |
| 350 | 2 | 2.349055512 |
| 358 | 2 | 2.356788118 |
| 360 | 2 | 2.349055512 |
| 362 | 2 | 2.349055512 |
| 364 | 2 | 2.349055512 |
| 365 | 2 | 2.349055512 |
| 366 | 2 | 2.349055512 |
| 372 | 2 | 2.349055512 |
| 373 | 2 | 2.349055512 |
| 378 | 2 | 2.349055512 |
| 381 | 2 | 2.349055512 |
| 383 | 2 | 2.349055512 |
| 384 | 2 | 2.349055512 |
| 385 | 2 | 2.349055512 |
| 388 | 2 | 2.349055512 |
| 394 | 2 | 2.987326403 |
| 396 | 2 | 2.349055512 |
| 397 | 2 | 2.349055512 |


| 398 | 2 | 2.349055512 |
| ---: | ---: | ---: |
| 400 | 2 | 2.349055512 |
| 402 | 2 | 2.349055512 |
| 403 | 2 | 2.356788118 |
| 408 | 2 | 2.349055512 |
| 409 | 2 | 2.2856747 |
| 410 | 2 | 2.349055512 |
| 414 | 2 | 2.987326403 |
| 415 | 2 | 2.349055512 |
| 422 | 2 | 2.349055512 |
| 423 | 2 | 2.349055512 |
| 425 | 2 | 2.349055512 |
| 427 | 2 | 2.349055512 |
| 429 | 2 | 2.349055512 |
| 432 | 2 | 2.349055512 |
| 440 | 4 | 2.987326403 |



Appendix I: The Results of the DT predictions on the 30\% test dataset of the Bank Branches (DMUs)

| DMU | ValidSet\$Class | predictions |
| :---: | :---: | :---: |
| 1 | Class 2 | Class 2 |
| 2 | Class 4 | Class 2 |
| 3 | Class 4 | Class 2 |
| 4 | Class 4 | Class 2 |
| 5 | Class 4 | Class 2 |
| 6 | Class 4 | Class 2 |
| 7 | Class 2 | Class 2 |
| 8 | Class 2 | Class 2 |
| 9 | Class 4 | Class 2 |
| 10 | Class 4 | Class 2 |
| 11 | Class 2 | Class 2 |
| 12 | Class 2 | Class 2 |
| 13 | Class 2 | Class 2 |
| 14 | Class 1 | Class 1 |
| 15 | Class 2 | Class 4 |
| 16 | Class 1 | Class 2 |
| 17 | Class 2 | Class 2 |
| 18 | Class 2 | Class 2 |
| 19 | Class 4 | Class 2 |
| 20 | Class 2 | Class 4 |
| 21 | Class 1 | Class 4 |
| 22 | Class 4 | Class 4 |
| 23 | Class 3 | Class 2 |
| 24 | Class 4 | Class 2 |
| 25 | Class 2 | Class 2 |
| 26 | Class 2 | Class 2 |


| 27 | Class 4 | Class 2 |
| ---: | :--- | :--- |
| 28 | Class 4 | Class 1 |
| 29 | Class 2 | Class 4 |
| 30 | Class 2 | Class 1 |
| 31 | Class 1 | Class 2 |
| 32 | Class 1 | Class 4 |
| 33 | Class 2 | Class 2 |

Case 2


| 34 | Class 3 | Class 2 |
| ---: | :--- | :--- |
| 35 | Class 1 | Class 4 |
| 36 | Class 3 | Class 1 |
| 37 | Class 2 | Class 2 |
| 38 | Class 2 | Class 2 |


| 65 | Class 4 | Class 4 |
| :--- | :--- | :--- |
| 66 | Class 2 | Class 4 |
| 67 | Class 1 | Class 4 |
| 68 | Class 4 | Class 4 |

KNUST

SANE

| 69 | Class 4 | Class 4 |
| :---: | :---: | :---: |
| 70 | Class 4 | Class 4 |
| 71 | Class 4 | Class 2 |
| 72 | Class 4 | Class 4 |
| 73 | Class 4 | Class 4 |
| 74 | Class 1 | Class 4 |
| 75 | Class 4 | Class 4 |
| 76 | Class 4 | Class 4 |
| 77 | Class 4 | Class 4 |
| 78 | Class 4 | Class 4 |
| 79 | Class 4 | Class 4 |
| 80 | Class 4 | Class 4 |
| 81 | Class 1 | Class 4 |
| 82 | Class 1 | Class 4 |


| 83 | Class 4 | Class 4 |
| :---: | :---: | :---: |
| 84 | Class 4 | Class 4 |
| 85 | Class 4 | Class 4 |
| 86 | Class 4 | Class 4 |
| 87 | Class 4 | Class 4 |
| 88 | Class 4 | Class 4 |
| 89 | Class 4 | Class 4 |
| 90 | Class 4 | Class 4 |
| 91 | Class 4 | Class 4 |
| 92 | Class 3 | Class 1 |
| 93 | Class 3 | Class 3 |
| 94 | Class 3 | Class 2 |
| 95 | Class 1 | Class 2 |
| 96 | Class 1 | Class 2 |
| 97 | Class 1 | Class 2 |
| 98 | Class 3 | Class 3 |
| 99 | Class 3 | Class 2 |
| 100 | Class 3 | Class 3 |
| 101 | Class 1 | Class 2 |
| 102 | Class 1 | Class 2 |



| 104 | Class 3 | Class 2 |
| :---: | :---: | :---: |
| 105 | Class 1 | Class 2 |
| 106 | Class 3 | Class 2 |
| 107 | Class 3 | Class 3 |
| 108 | Class 3 | Class 3 |
| 109 | Class 1 | Class 2 |
| 110 | Class 3 | Class 1 |
| 111 | Class 3 | Class 3 |
| 112 | Class 2 | Class 3 |
| 113 | Class 3 | Class 2 |
| 114 | Class 2 | Class 3 |
| 115 | Class 2 | Class 2 |
| 116 | Class 2 | Class 2 |
| 117 | Class 2 | Class 2 |
| 118 | Class 4 | Class 2 |
| 119 | Class 4 | Class 2 |
| 120 | Class 4 | Class 2 |
| 121 | Class 2 | Class 2 |
| 122 | Class 2 | Class 4 |
| 123 | Class 4 | Class 2 |
| 124 | Class 2 | Class 2 |
| 125 | Class 4 | Class 2 |
| 126 | Class 2 | Class 2 |
| 127 | Class 2 | Class 2 |
| 128 | Class 2 | Class 2 |
| 129 | Class 2 | Class 2 |
| 130 | Class 4 | Class 2 |
| 131 | Class 2 | Class 2 |
| 132 | Class 2 | Class 4 |


| 133 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 134 | Class 2 | Class 2 |

## KNUST



Case 2

| DMU | ValidSet\$Class | predictions |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 3 |
| 2 | Class 1 | Class 4 |


| 3 | Class 4 | Class 4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Class 1 | Class 4 |  |  |  |
| 5 | Class 4 | Class 4 |  |  |  |
| 6 | Class 4 | Class 2 |  |  |  |
| 7 | Class 2 | Class 2 |  |  |  |
| 8 | Class 2 | Class 2 |  |  |  |
| 9 | Class 4 | Class 4 |  |  |  |
| 10 | Class 4 | Class 1 |  |  |  |
| 11 | Class 2 | Class 3 |  |  |  |
| 12 | Class 2 | Class 1 |  |  |  |
| 13 | Class 2 | Class 3 |  |  |  |
| 14 | Class 4 | Class 4 |  |  |  |
| 15 | Class 2 | Class 4 |  |  |  |
| 16 | Class 4 | Class 1 |  |  |  |
| 17 | Class 2 | Class 2 |  |  |  |
| 18 | Class 2 | Class 1 |  |  |  |
| 19 | Class 4 | Class 2 |  |  |  |
| 20 | Class 2 | Class 4 |  |  |  |
| 21 | Class 4 | Class 4 |  |  |  |
| 22 | Class 4 | Class 4 |  |  |  |
| 23 | Class 2 | Class 2 |  |  |  |
| 24 | Class 4 | Class 2 |  |  |  |
| 25 | Class 2 | Class 2 | 33 | Class 2 | Class 3 |
| 26 | Class 2 | Class 2 | 34 | Class 2 | Class 2 |
| 27 | Class 4 | Class 2 | 35 | Class 4 | Class 4 |
| 28 | Class 4 | Class 4 | 36 | Class 2 | Class 4 |
| 29 | Class 2 | Class 4 | 37 | Class 2 | Class 3 |
| 30 | Class 2 | Class 4 |  |  |  |
| 31 | Class 4 | Class 3 |  |  |  |
| 32 | Class 4 | Class 4 |  |  |  |

## KNUST



| 38 | Class 2 | Class 4 |
| ---: | :--- | :--- |
| 39 | Class 2 | Class 4 |
| 40 | Class 2 | Class 3 |
| 41 | Class 1 | Class 3 |
| 42 | Class 3 | Class 3 |
| 43 | Class 3 | Class 3 |


| 44 | Class 2 | Class 3 |
| :---: | :---: | :---: |
| 45 | Class 1 | Class 3 |
| 46 | Class 1 | Class 3 |
| 47 | Class 2 | Class 1 |
| 48 | Class 1 | Class 2 |
| 49 | Class 2 | Class 4 |
| 50 | Class 4 | Class 4 |
| 51 | Class 4 | Class 4 |
| 52 | Class 2 | Class 4 |
| 53 | Class 1 | Class 4 |
| 54 | Class 1 | Class 4 |
| 55 | Class 4 | Class 4 |
| 56 | Class 2 | Class 4 |
| 57 | Class 4 | Class 4 |
| 58 | Class 1 | Class 4 |
| 59 | Class 1 | Class 4 |
| 60 | Class 1 | Class 4 |
| 61 | Class 1 | Class 1 |
| 62 | Class 4 | Class 4 |
| 63 | Class 2 | Class 4 |
| 64 | Class 4 | Class 4 |
| 65 | Class 4 | Class 1 |
| 66 | Class 3 | Class 4 |
| 67 | Class 1 | Class 4 |
| 68 | Class 4 | Class 4 |
| 69 | Class 1 | Class 4 |
| 70 | Class 1 | Class 4 |
| 71 | Class 4 | Class 3 |


| 72 | Class 1 | Class 4 |
| :--- | :--- | :--- |
| 73 | Class 1 | Class 4 |

KNUST


| 74 | Class 1 | Class 4 |
| :--- | :--- | :--- |
| 75 | Class 4 | Class 4 |
| 76 | Class 1 | Class 1 |


| 77 | Class 4 | Class 4 |
| :---: | :---: | :---: |
| 78 | Class 4 | Class 1 |
| 79 | Class 4 | Class 4 |
| 80 | Class 4 | Class 4 |
| 81 | Class 4 | Class 4 |
| 82 | Class 4 | Class 4 |
| 83 | Class 1 | Class 4 |
| 84 | Class 4 | Class 4 |
| 85 | Class 4 | Class 1 |
| 86 | Class 4 | Class 4 |
| 87 | Class 4 | Class 4 |
| 88 | Class 4 | Class 1 |
| 89 | Class 4 | Class 1 |
| 90 | Class 4 | Class 4 |
| 91 | Class 4 | Class 4 |
| 92 | Class 3 | Class 4 |
| 93 | Class 3 | Class 3 |
| 94 | Class 3 | Class 3 |
| 95 | Class 1 | Class 3 |
| 96 | Class 1 | Class 3 |
| 97 | Class 1 | Class 3 |
| 98 | Class 3 | Class 3 |
| 99 | Class 3 | Class 1 |
| 100 | Class 3 | Class 3 |


| 101 | Class 4 | Class 3 |
| ---: | :--- | :--- |
| 102 | Class 1 | Class 3 |
| 103 | Class 3 | Class 1 |
| 104 | Class 3 | Class 3 |
| 105 | Class 1 | Class 2 |
| 106 | Class 3 | Class 3 |
| 107 | Class 2 | Class 3 |
| 108 | Class 3 | Class 3 |



| DMU | ValidSet[, <br> 7] | predValid |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 2 |
| 3 | Class 4 | Class 4 |
| 6 | Class 2 | Class 2 |


| 8 | Class 4 | Class 2 |
| :---: | :---: | :---: |
| 109 | Class 4 | Class 4 |
| 110 | Class 3 | Class 4 |
| 111 | Class 2 | Class 3 |
| 112 | Class 3 | Class 3 |
| 113 | Class 3 | Class 3 |
| 114 | Class 3 | Class 3 |
| 115 | Class 3 | Class 4 |
| 116 | Class 3 | Class 3 |
| 117 | Class 3 | Class 3 |
| 118 | Class 1 | Class 3 |
| 119 | Class 1 | Class 2 |
| 120 | Class 1 | Class 3 |
| 121 | Class 3 | Class 1 |
| 122 | Class 2 | Class 4 |
| 123 | Class 1 | Class 3 |
| 124 | Class 3 | Class 2 |
| 125 | Class 4 | Class 3 |
| 126 | Class 2 | Class 3 |
| 127 | Class 2 | Class 3 |
| 128 | Class 3 | Class 3 |
| 129 | Class 3 | Class 1 |
| 130 | Class 4 | Class 3 |
| 131 | Class 2 | Class 1 |
| 132 | Class 3 | Class 4 |
| 133 | Class 3 | Class 2 |
| 134 | Class 3 | Class 4 |

Appendix J: The Results of the RF predictions on the 30\% test dataset of the Bank
Branches (DMUs)
Case 2


| 123 | Class 3 | Class 3 |
| :--- | :--- | :--- |



| 124 | Class 3 | Class 3 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 126 | Class 3 | Class 4 |  |  |  |
| 128 | Class 3 | Class 1 |  |  |  |
| 131 | Class 1 | Class 3 |  |  |  |
| 132 | Class 3 | Class 3 |  |  |  |
| 138 | Class 1 | Class 3 |  |  |  |
| 139 | Class 3 | Class 3 |  |  |  |
| 142 | Class 3 | Class 3 |  |  |  |
| 144 | Class 3 | Class 4 |  |  |  |
| 147 | Class 3 | Class 4 |  |  |  |
| 149 | Class 1 | Class 3 |  |  |  |
| 150 | Class 3 | Class 4 |  |  |  |
| 156 | Class 4 | Class 4 |  |  |  |
| 157 | Class 4 | Class 4 |  |  |  |
| 162 | Class 4 | Class 4 |  |  |  |
| 167 | Class 4 | Class 4 |  |  |  |
| 169 | Class 4 | Class 4 |  |  |  |
| 171 | Class 2 | Class 4 |  |  |  |
| 175 | Class 4 | Class 4 |  |  |  |
| 180 | Class 4 | Class 4 | 208 | Class 4 | Class 4 |
| 183 | Class 4 | Class 4 | 216 | Class 4 | Class 4 |
| 187 | Class 2 | Class 4 | 219 | Class 4 | Class 1 |
| 190 | Class 4 | Class 4 | 220 | Class 4 | Class 2 |
| 193 | Class 4 | Class 4 | 221 | Class 4 | Class 2 |
| 204 | Class 2 | Class 4 | 228 | Class 1 | Class 4 |
| 207 | Class 4 | Class 3 | 230 | Class 4 | Class 4 |
|  |  |  | 234 | Class 4 | Class 4 |
|  |  |  | 237 | Class 4 | Class 4 |

## KNUST

| 239 | Class 4 | Class 4 |
| ---: | :--- | :--- |
| 241 | Class 4 | Class 4 |
| 245 | Class 4 | Class 4 |
| 247 | Class 4 | Class 4 |
| 248 | Class 4 | Class 4 |
| 258 | Class 4 | Class 4 |


| 259 | Class 4 | Class 4 |
| :---: | :---: | :---: |
| 261 | Class 4 | Class 4 |
| 267 | Class 4 | Class 4 |
| 271 | Class 4 | Class 4 |
| 274 | Class 4 | Class 4 |
| 279 | Class 4 | Class 4 |
| 281 | Class 4 | Class 4 |
| 282 | Class 4 | Class 4 |
| 287 | Class 4 | Class 4 |
| 289 | Class 4 | Class 4 |
| 292 | Class 4 | Class 4 |
| 293 | Class 4 | Class 4 |
| 300 | Class 4 | Class 4 |
| 304 | Class 4 | Class 2 |
| 305 | Class 4 | Class 4 |
| 317 | Class 3 | Class 3 |
| 319 | Class 3 | Class 1 |
| 323 | Class 1 | Class 2 |
| 324 | Class 3 | Class 3 |
| 329 | Class 3 | Class 3 |
| 333 | Class 1 | Class 1 |
| 334 | Class 3 | Class 3 |
| 338 | Class 3 | Class 3 |
| 340 | Class 3 | Class 1 |
| 341 | Class 1 | Class 3 |
| 344 | Class 1 | Class 3 |
| 345 | Class 3 | Class 2 |


| 356 | Class 3 | Class 2 |
| ---: | :--- | :--- |
| 363 | Class 1 | Class 2 |
| 365 | Class 3 | Class 3 |

KNUST

SANE

| 367 | Class 3 | Class 3 |
| :---: | :---: | :---: |
| 370 | Class 3 | Class 3 |
| 372 | Class 3 | Class 1 |
| 377 | Class 3 | Class 3 |
| 379 | Class 3 | Class 2 |
| 381 | Class 2 | Class 3 |
| 385 | Class 3 | Class 3 |
| 388 | Class 2 | Class 3 |
| 389 | Class 4 | Class 2 |
| 390 | Class 2 | Class 2 |
| 392 | Class 2 | Class 2 |
| 396 | Class 2 | Class 2 |
| 403 | Class 4 | Class 2 |
| 409 | Class 4 | Class 2 |
| 410 | Class 2 | Class 2 |
| 411 | Class 4 | Class 2 |
| 413 | Class 2 | Class 2 |
| 417 | Class 2 | Class 2 |
| 418 | Class 4 | Class 2 |
| 419 | Class 2 | Class 2 |
| 428 | Class 2 | Class 2 |
| 435 | Class 2 | Class 2 |
| 437 | Class 2 | Class 4 |

Case 4

| DMU | ValidSet[, |  |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 2 |
| 3 | Class 4 | Class 4 |
| 6 | Class 2 | Class 4 |
| 8 | Class 1 | Class 2 |
| 9 | Class 4 | Class 4 |
| 12 | Class 4 | Class 2 |
| 15 | Class 4 | Class 2 |
| 17 | Class 4 | Class 4 |
| 18 | Class 2 | Class 4 |
| 19 | Class 4 | Class 4 |


| 28 | Class 2 | Class 3 |
| ---: | :--- | :--- |
| 37 | Class 4 | Class 4 |
| 38 | Class 2 | Class 2 |
| 44 | Class 2 | Class 3 |
| 46 | Class 4 | Class 4 |


| 47 | Class 4 | Class 4 |
| :---: | :---: | :---: |
| 49 | Class 2 | Class 3 |
| 50 | Class 2 | Class 4 |
| 56 | Class 4 | Class 2 |
| 58 | Class 2 | Class 4 |
| 59 | Class 2 | Class 4 |
| 60 | Class 4 | Class 4 |
| 62 | Class 2 | Class 2 |
| 65 | Class 4 | Class 4 |
| 68 | Class 4 | Class 4 |
| 70 | Class 4 | Class 4 |
| 71 | Class 4 | Class 4 |
| 75 | Class 4 | Class 4 |
| 79 | Class 4 | Class 4 |
| 88 | Class 4 | Class 4 |
| 95 | Class 4 | Class 2 |
| 97 | Class 2 | Class 4 |
| 99 | Class 4 | Class 3 |
| 100 | Class 4 | Class 3 |
| 103 | Class 4 | Class 3 |
| 108 | Class 2 | Class 3 |
| 114 | Class 2 | Class 1 |


| 119 | Class 4 | Class 2 |
| ---: | :--- | :--- |
| 120 | Class 3 | Class 3 |
| 123 | Class 3 | Class 1 |
| 124 | Class 3 | Class 4 |
| 126 | Class 3 | Class 1 |
| 128 | Class 3 | Class 2 |
| 131 | Class 4 | Class 4 |
| 132 | Class 2 | Class 2 |
| 138 | Class 1 | Class 2 |




| 248 | Class 1 | Class 4 |
| ---: | :--- | :--- |
| 258 | Class 1 | Class 4 |

KNUST


| 259 | Class 1 | Class 4 |
| :---: | :---: | :---: |
| 261 | Class 1 | Class 4 |
| 267 | Class 4 | Class 4 |
| 271 | Class 1 | Class 4 |
| 274 | Class 1 | Class 4 |
| 279 | Class 4 | Class 1 |
| 281 | Class 1 | Class 4 |
| 282 | Class 1 | Class 4 |
| 287 | Class 4 | Class 4 |
| 289 | Class 4 | Class 4 |
| 292 | Class 4 | Class 4 |
| 293 | Class 4 | Class 4 |
| 300 | Class 4 | Class 4 |
| 304 | Class 4 | Class 4 |
| 305 | Class 1 | Class 4 |
| 317 | Class 3 | Class 3 |
| 319 | Class 3 | Class 3 |
| 323 | Class 1 | Class 3 |
| 324 | Class 3 | Class 3 |
| 329 | Class 3 | Class 3 |
| 333 | Class 1 | Class 4 |
| 334 | Class 3 | Class 3 |
| 338 | Class 3 | Class 3 |
| 340 | Class 3 | Class 4 |
| 341 | Class 1 | Class 2 |
| 344 | Class 1 | Class 3 |
| 345 | Class 3 | Class 3 |


| 356 | Class 3 | Class 3 |
| ---: | :--- | :--- |
| 363 | Class 4 | Class 2 |
| 365 | Class 2 | Class 3 |
| 367 | Class 3 | Class 2 |
| 370 | Class 2 | Class 3 |
| 372 | Class 3 | Class 4 |
| 377 | Class 3 | Class 3 |
| 379 | Class 3 | Class 3 |
| 381 | Class 3 | Class 2 |



| 5 | 4 | 2.970113 |
| ---: | ---: | ---: |
| 6 | 2 | 2.970113 |
| 7 | 2 | 2.970113 |
| 9 | 4 | 2.970113 |
| 11 | 4 | 2.970113 |
| 16 | 2 | 2.970113 |


| 385 | Class 3 | Class 3 |
| :---: | :---: | :---: |
| 388 | Class 34 |  |
| 389 | Class 14 | 2.9aø933 |
| $\begin{array}{r}390 \\ 21 \\ \hline\end{array}$ | Class ${ }_{4}$ | ${ }_{2}{ }^{\text {Class }}$ 2 ${ }^{\text {a }}$ |
| 392 | Class 3 | Class 3 |
| 396 | Class 3 | Class 1 |
| 403 | Class 1 | Class 3 |
| 409 | Class 1 | Class 2 |
| 410 | Class 2 | Class 4 |
| 411 | Class 1 | Class 3 |
| 413 | Class 3 | Class 3 |
| 417 | Class 3 | Class 3 |
| 418 | Class 1 | Class 3 |
| 419 | Class 2 | Class 3 |
| 428 | Class 2 | Class 3 |
| 435 | Class 3 | Class 3 |
| 437 | Class 2 | Class 1 |

Appendix K: The Results of the ANN predictions on the 30\% test dataset of the Bank
Branches (DMUs)
Case 2


| 121 | 3 | 2.970113 |
| ---: | ---: | ---: |
| 122 | 1 | 2.970113 |
| 125 | 1 | 2.970113 |
| 127 | 3 | 2.970113 |
| 130 | 1 | 2.970113 |
| 131 | 1 | 2.970113 |
| 136 | 3 | 2.970113 |
| 138 | 1 | 2.970113 |
| 140 | 3 | 2.970113 |



| 141 | 3 | 2.970113 |
| ---: | ---: | ---: |
| 146 | 3 | 2.970113 |


| 150 | 3 | 3.220686 |
| ---: | ---: | ---: |
| 157 | 4 | 2.970113 |
| 161 | 4 | 2.970113 |
| 162 | 4 | 2.970113 |
| 163 | 4 | 2.970113 |
| 164 | 4 | 2.970113 |
| 165 | 1 | 2.970113 |
| 168 | 4 | 2.970113 |
| 169 | 4 | 3.220686 |
| 171 | 2 | 2.970113 |
| 172 | 4 | 2.970113 |
| 210 | 4 | 4 |


| 218 | 4 | 2.970113 |
| ---: | ---: | ---: |
| 226 | 4 | 3.220686 |
| 228 | 1 | 2.970113 |
| 229 | 4 | 2.970113 |
| 235 | 1 | 2.970113 |
| 237 | 4 | 2.970113 |
| 238 | 4 | 2.970113 |
| 239 | 4 | 2.970113 |



| 240 | 4 | 2.970113 |
| :---: | :---: | :---: |
| 244 | 4 | 2.970113 |
| 246 | 4 | 2.970113 |
| 248 | 4 | 2.970113 |
| 252 | 4 | 2.970113 |
| 253 | 4 | 2.970113 |
| 257 | 4 | 2.970113 |
| 258 | 4 | 2.970113 |
| 260 | 4 | 2.970113 |
| 265 | 4 | 2.970113 |
| 272 | 4 | 2.970113 |
| 275 | 4 | 2.970113 |
| 277 | 4 | 2.970113 |
| 279 | 4 | 2.970113 |
| 287 | 4 | 2.970113 |
| 292 | 4 | 2.970113 |
| 293 | 4 | 2.970113 |
| 301 | 4 | 2.970113 |
| 310 | 1 | 2.970113 |
| 314 | 3 | 2.970113 |
| 319 | 3 | 2.970113 |
| 321 | 3 | 2.970113 |
| 322 | 3 | 2.970113 |
| 325 | 1 | 2.970113 |
| 326 | 1 | 2.970113 |
| 327 | 1 | 2.970113 |
| 328 | 1 | 2.970113 |
| 336 | 1 | 2.970113 |


| 340 | 3 | 2.970113 |
| ---: | ---: | ---: |
| 346 | 3 | 2.970113 |
| 353 | 1 | 2.970113 |
| 354 | 3 | 2.970113 |
| 355 | 1 | 2.970113 |
| 358 | 1 | 2.970113 |
| 367 | 3 | 2.970113 |
| 372 | 3 | 2.970113 |



| 387 | 2 | 2.970113 |
| :--- | :--- | :--- |
| 388 | 2 | 2.970113 |
| 391 | 2 | 2.970113 |
| 397 | 4 | 2.970113 |
| 401 | 4 | 2.970113 |
| 403 | 4 | 2.970113 |
| 404 | 2 | 2.970113 |
| 409 | 4 | 2.970113 |
| 411 | 4 | 2.970113 |
| 412 | 4 | 2.970113 |
| 415 | 2 | 2.970113 |
| 421 | 2 | 2.970113 |
| 422 | 2 | 2.970113 |
| 427 | 2 | 2.970113 |
| 428 | 2 | 2.970113 |
| 436 | 2 | 2.970113 |
| 437 | 2 | 2.970113 |
| 441 | 2 | 2.970113 |
|  |  |  |

Case 4

| DMU | V1 | V2 |
| ---: | ---: | :--- |
| 1 | 2 | 2.827259 |
| 2 | 1 | 2.827259 |
| 3 | 4 | 2.827259 |
| 5 | 4 | 2.827259 |
| 6 | 2 | 2.827259 |
| 7 | 2 | 2.827259 |
| 11 | 4 | 2.827259 |
| 16 | 2 | 2.827259 |
| 17 | 4 | 2.827259 |


| 19 | 4 | 2.827259 |
| :---: | :---: | :---: |
| 21 | 1 | 2.827259 |
| 23 | 2 | 2.827259 |
| 29 | 3 | 2.827259 |
| 34 | 2 | 2.827259 |
| 37 | 4 | 2.827259 |
| 41 | 1 | 2.827259 |
| 45 | 2 | 2.827259 |
| 54 | 4 | 2.827259 |
| 56 | 4 | 2.827259 |
| 57 | 4 | 2.827259 |
| 66 | 2 | 2.827259 |
| 75 | 4 | 2.827259 |
| 76 | 4 | 2.827259 |
| 78 | 4 | 2.827259 |
| 81 | 4 | 2.827259 |
| 82 | 2 | 2.827259 |
| 84 | 4 | 2.827259 |
| 87 | 2 | 2.827259 |
| 91 | 2 | 2.827259 |
| 97 | 2 | 2.827259 |
| 99 | 4 | 2.827259 |
| 103 | 4 | 2.827259 |
| 104 | 4 | 2.827259 |


| 108 | 2 | 2.827259 |
| ---: | ---: | ---: |
| 120 | 3 | 2.827259 |

KNUST

SANE


| 197 | 2 | 2.827259 |
| ---: | ---: | ---: |
| 201 | 4 | 2.827259 |
| 202 | 4 | 2.827259 |
| 210 | 1 | 2.827259 |

KNUST

SANE

| 212 | 4 | 2.827259 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 216 | 1 | 2.827259 |  |  |  |
| 218 | 4 | 2.827259 |  |  |  |
| 226 | 4 | 2.220586 |  |  |  |
| 228 | 1 | 2.827259 |  |  |  |
| 229 | 1 | 2.827259 |  |  |  |
| 235 | 4 | 2.827259 |  |  |  |
| 237 | 1 | 2.827259 |  |  |  |
| 238 | 4 | 2.827259 |  |  |  |
| 239 | 1 | 2.827259 |  |  |  |
| 240 | 4 | 2.827259 | 252 | 4 | 2.827259 |
| 244 | 1 | 2.827259 | 253 | 4 | 2.827259 |
| 246 | 4 | 2.827259 | 257 | 1 | 2.827259 |
| 248 | 1 | 2.827259 | 258 | 1 | 2.827259 |
|  |  |  | 260 | 4 | 2.827259 |
|  |  |  | 265 | 1 | 2.827259 |
|  |  |  | 272 | 4 | 2.827259 |
|  |  |  | 275 | 1 | 2.827259 |
|  |  |  | 277 | 4 | 2.827259 |
|  |  |  | 279 | 4 | 2.827259 |
|  |  |  | 287 | 4 | 2.827259 |
|  |  |  | 292 | 4 | 2.827259 |
|  |  |  | 293 | 4 | 2.827259 |
|  |  |  | 301 | 4 | 2.827259 |
|  |  |  | 310 | 4 | 2.827259 |
|  |  |  | 314 | 3 | 2.827259 |
|  |  |  | 319 | 3 | 2.827259 |
|  |  |  | 321 | 3 | 2.827259 |
|  |  |  | 322 | 2 | 2.827259 |
|  |  |  | 325 | 1 | 2.827259 |


| 326 | 1 | 2.827259 |
| :--- | :--- | :--- |



| 327 | 1 | 2.827259 |
| :---: | :---: | :---: |
| 328 | 1 | 2.827259 |
| 336 | 4 | 2.827259 |
| 340 | 3 | 2.827259 |
| 346 | 3 | 2.827259 |
| 353 | 1 | 2.827259 |
| 354 | 3 | 2.827259 |
| 355 | 1 | 2.827259 |
| 358 | 1 | 2.827259 |
| 367 | 3 | 2.827259 |
| 372 | 3 | 2.827259 |
| 387 | 3 | 2.827259 |
| 388 | 3 | 2.827259 |
| 391 | 3 | 2.827259 |
| 397 | 1 | 2.827259 |
| 401 | 1 | 2.827259 |
| 403 | 1 | 2.827259 |
| 404 | 3 | 2.827259 |
| 409 | 1 | 2.827259 |
| 411 | 1 | 2.827259 |
| 412 | 1 | 2.827259 |
| 415 | 3 | 2.827259 |
| 421 | 2 | 2.827259 |
| 422 | 3 | 2.827259 |
| 427 | 2 | 2.827259 |
| 428 | 2 | 2.827259 |
| 436 | 3 | 2.827259 |
| 437 | 2 | 2.827259 |
| 441 | 3 | 2.827259 |

Appendix L: Prediction by the DEA-LR Case 2

| DMU | Predicted | Actual |
| ---: | :--- | :--- |
| 1 | Class 2 | Class 2 |
| 2 | Class 2 | Class 2 |


| 4 | Class 2 | Class 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | Class 2 | Class 2 |  |  |  |
| 6 | Class 2 | Class 4 |  |  |  |
| 7 | Class 2 | Class 2 |  |  |  |
| 8 | Class 2 | Class 2 |  |  |  |
| 9 | Class 2 | Class 2 |  |  |  |
| 10 | Class 2 | Class 2 |  |  |  |
| 11 | Class 2 | Class 2 |  |  |  |
| 12 | Class 2 | Class 2 |  |  |  |
| 13 | Class 2 | Class 3 |  |  |  |
| 14 | Class 2 | Class 2 |  |  |  |
| 15 | Class 2 | Class 2 |  |  |  |
| 16 | Class 2 | Class 2 |  |  |  |
| 17 | Class 2 | Class 4 |  |  |  |
| 18 | Class 2 | Class 2 |  |  |  |
| 19 | Class 2 | Class 3 |  |  |  |
| 20 | Class 4 | Class 4 |  |  |  |
| 21 | Class 4 | Class 4 |  |  |  |
| 22 | Class 4 | Class 4 |  |  |  |
| 23 | Class 4 | Class 4 |  |  |  |
| 24 | Class 4 | Class 2 |  |  |  |
| 25 | Class 2 | Class 2 | 32 | Class 2 | Class 2 |
| 26 | Class 2 | Class 2 | 33 | Class 2 | Class 2 |
| 27 | Class 2 | Class 2 | 34 | Class 2 | Class 2 |
| 28 | Class 2 | Class 2 | 35 | Class 2 | Class 2 |
| 29 | Class 2 | Class 2 | 36 | Class 2 | Class 2 |
| 30 | Class 2 | Class 2 | 37 | Class 2 | Class 2 |
| 31 | Class 2 | Class 2 | 38 | Class 2 | Class 2 |
|  |  |  |  |  |  |

## KNUST

| 39 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 40 | Class 2 | Class 2 |
| 41 | Class 2 | Class 2 |
| 42 | Class 2 | Class 3 |
| 43 | Class 2 | Class 3 |
| 44 | Class 2 | Class 2 |


| 45 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 46 | Class 2 | Class 2 |
| 47 | Class 2 | Class 2 |
| 48 | Class 2 | Class 2 |
| 49 | Class 2 | Class 2 |
| 50 | Class 2 | Class 2 |
| 51 | Class 2 | Class 4 |
| 52 | Class 2 | Class 4 |
| 53 | Class 2 | Class 4 |
| 54 | Class 2 | Class 4 |
| 55 | Class 2 | Class 2 |
| 56 | Class 2 | Class 4 |
| 57 | Class 4 | Class 4 |
| 58 | Class 4 | Class 4 |
| 59 | Class 2 | Class 2 |
| 60 | Class 2 | Class 2 |
| 61 | Class 2 | Class 2 |
| 62 | Class 2 | Class 2 |
| 63 | Class 2 | Class 2 |
| 64 | Class 2 | Class 2 |
| 65 | Class 2 | Class 2 |
| 66 | Class 2 | Class 2 |
| 67 | Class 2 | Class 2 |
| 68 | Class 2 | Class 2 |
| 69 | Class 2 | Class 2 |
| 70 | Class 2 | Class 2 |
| 71 | Class 2 | Class 2 |
| 72 | Class 2 | Class 2 |


| 73 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 74 | Class 2 | Class 2 |

KNUST


| 75 | Class 2 | Class 2 |
| :--- | :--- | :--- |


| 76 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 77 | Class 2 | Class 2 |
| 78 | Class 2 | Class 2 |
| 79 | Class 2 | Class 2 |
| 80 | Class 2 | Class 2 |
| 81 | Class 4 | Class 4 |
| 82 | Class 4 | Class 4 |
| 83 | Class 4 | Class 4 |
| 84 | Class 4 | Class 4 |
| 85 | Class 4 | Class 4 |
| 86 | Class 4 | Class 4 |
| 87 | Class 4 | Class 4 |
| 88 | Class 4 | Class 4 |
| 89 | Class 4 | Class 4 |
| 90 | Class 2 | Class 2 |
| 91 | Class 2 | Class 2 |
| 92 | Class 2 | Class 2 |
| 93 | Class 2 | Class 2 |
| 94 | Class 2 | Class 2 |
| 95 | Class 2 | Class 2 |
| 96 | Class 2 | Class 2 |
| 97 | Class 2 | Class 2 |
| 98 | Class 2 | Class 2 |
| 99 | Class 2 | Class 2 |
| 100 | Class 2 | Class 2 |
| 701 | Class 3 |  |
| 7 |  |  |


| 102 | Class 2 | Class 3 |
| ---: | :--- | :--- |
| 103 | Class 2 | Class 2 |
| 104 | Class 2 | Class 2 |
| 105 | Class 2 | Class 3 |
| 106 | Class 2 | Class 2 |
| 107 | Class 2 | Class 2 |
| 108 | Class 2 | Class 2 |
| 109 | Class 2 | Class 2 |



| 110 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 111 | Class 2 | Class 2 |
| 112 | Class 2 | Class 2 |
| 113 | Class 2 | Class 2 |
| 114 | Class 2 | Class 3 |
| 115 | Class 2 | Class 3 |
| 116 | Class 2 | Class 2 |
| 117 | Class 2 | Class 2 |
| 118 | Class 2 | Class 3 |
| 119 | Class 2 | Class 2 |
| 120 | Class 2 | Class 3 |
| 121 | Class 2 | Class 2 |
| 122 | Class 2 | Class 2 |
| 123 | Class 2 | Class 2 |
| 124 | Class 2 | Class 2 |
| 125 | Class 2 | Class 2 |
| 126 | Class 2 | Class 2 |
| 127 | Class 2 | Class 3 |
| 128 | Class 2 | Class 2 |
| 129 | Class 2 | Class 2 |
| 130 | Class 2 | Class 3 |
| 131 | Class 2 | Class 3 |
| 132 | Class 2 | Class 2 |

Case 4 Prediction by the DEA-LR

| DMU | Predicted | Actual |
| :---: | :---: | :---: |
| 1 | Class 2 | Class 2 |
| 2 | Class 2 | Class 2 |
| 3 | Class 2 | Class 4 |
| 4 | Class 2 | Class 2 |
| 5 | Class 2 | Class 2 |
| 6 | Class 2 | Class 2 |
| 7 | Class 2 | Class 2 |
| 8 | Class 2 | Class 2 |
| 9 | Class 2 | Class 2 |
| 10 | Class 2 | Class 2 |


| 11 | Class 2 | Class 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | Class 2 | Class 4 |  |  |  |
| 13 | Class 2 | Class 2 |  |  |  |
| 14 | Class 4 | Class 4 |  |  |  |
| 15 | Class 2 | Class 2 |  |  |  |
| 16 | Class 4 | Class 4 |  |  |  |
| 17 | Class 4 | Class 4 |  |  |  |
| 18 | Class 4 | Class 4 |  |  |  |
| 19 | Class 4 | Class 2 |  |  |  |
| 20 | Class 2 | Class 2 |  |  |  |
| 21 | Class 2 | Class 2 |  |  |  |
| 22 | Class 2 | Class 2 | 27 | Class 2 | Class 2 |
| 23 | Class 2 | Class 2 | 28 | Class 2 | Class 2 |
| 24 | Class 2 | Class 2 | 29 | Class 2 | Class 2 |
| 25 | Class 2 | Class 2 | 30 | Class 2 | Class 2 |
| 26 | Class 2 | Class 2 | 31 | Class 2 | Class 2 |
|  |  |  | 32 | Class 2 | Class 2 |
|  |  |  | 33 | Class 2 | Class 2 |
|  |  |  | 34 | Class 2 | Class 2 |
|  |  |  | 35 | Class 2 | Class 2 |
|  |  |  | 36 | Class 2 | Class 2 |
|  |  |  | 37 | Class 2 | Class 2 |
|  |  |  | 38 | Class 2 | Class 2 |
|  |  |  | 39 | Class 2 | Class 2 |
|  |  |  | 40 | Class 2 | Class 2 |
|  |  |  | 41 | Class 2 | Class 2 |
|  |  |  | 42 | Class 2 | Class 2 |
|  |  |  | 43 | Class 2 | Class 2 |
|  |  |  | 44 | Class 2 | Class 4 |

45 Class $2 \quad$ Class 4


| 46 | Class 2 | Class 4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 47 | Class 2 | Class 4 |  |  |  |
| 48 | Class 2 | Class 4 |  |  |  |
| 49 | Class 2 | Class 4 |  |  |  |
| 50 | Class 4 | Class 4 |  |  |  |
| 51 | Class 2 | Class 2 |  |  |  |
| 52 | Class 2 | Class 2 |  |  |  |
| 53 | Class 2 | Class 2 |  |  |  |
| 54 | Class 2 | Class 2 |  |  |  |
| 55 | Class 2 | Class 2 |  |  |  |
| 56 | Class 2 | Class 2 |  |  |  |
| 57 | Class 2 | Class 2 |  |  |  |
| 58 | Class 2 | Class 2 |  |  |  |
| 59 | Class 2 | Class 2 |  |  |  |
| 60 | Class 2 | Class 2 |  |  |  |
| 61 | Class 2 | Class 2 |  |  |  |
| 62 | Class 2 | Class 2 |  |  |  |
| 63 | Class 2 | Class 2 |  |  |  |
| 64 | Class 2 | Class 2 | 71 | Class 4 | Class 4 |
| 65 | Class 2 | Class 2 | 72 | Class 4 | Class 4 |
| 66 | Class 2 | Class 2 | 73 | Class 4 | Class 4 |
| 67 | Class 2 | Class 2 | 74 | Class 4 | Class 4 |
| 68 | Class 2 | Class 2 | 75 | Class 4 | Class 4 |
| 69 | Class 4 | Class 4 | 76 | Class 4 | Class 4 |
| 70 | Class 4 | Class 4 | 77 | Class 2 | Class 2 |
|  |  |  | 78 | Class 2 | Class 2 |
|  |  |  | 79 | Class 2 | Class 2 |
|  |  |  | 80 | Class 2 | Class 2 |

## KNUST

| 81 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 82 | Class 2 | Class 2 |
| 83 | Class 2 | Class 2 |
| 84 | Class 2 | Class 2 |
| 85 | Class 2 | Class 2 |
| 86 | Class 2 | Class 2 |


| 87 | Class 2 | Class 2 |
| :---: | :---: | :---: |
| 88 | Class 2 | Class 2 |
| 89 | Class 2 | Class 2 |
| 90 | Class 2 | Class 2 |
| 91 | Class 2 | Class 2 |
| 92 | Class 2 | Class 2 |
| 93 | Class 2 | Class 2 |
| 94 | Class 2 | Class 2 |
| 95 | Class 2 | Class 2 |
| 96 | Class 2 | Class 2 |
| 97 | Class 2 | Class 2 |
| 98 | Class 2 | Class 2 |
| 99 | Class 2 | Class 2 |
| 100 | Class 2 | Class 2 |
| 101 | Class 2 | Class 2 |
| 102 | Class 2 | Class 2 |
| 103 | Class 2 | Class 2 |
| 104 | Class 2 | Class 2 |
| 105 | Class 2 | Class 2 |
| 106 | Class 2 | Class 2 |
| 107 | Class 2 | Class 2 |
| 108 | Class 2 | Class 2 |
| 109 | Class 2 | Class 2 |
| 110 | Class 2 | Class 2 |
| 111 | Class 2 | Class 2 |
| 112 | Class 2 | Class 2 |
| 113 | Class 2 | Class 2 |
| 114 | Class 2 | Class 2 |


| 115 | Class 2 | Class 2 |
| ---: | :--- | :--- |
| 116 | Class 2 | Class 2 |

KNUST

| 117 | Class 2 | Class 2 |
| :--- | :--- | :--- |
| 118 | Class 2 | Class 2 |
| 119 | Class 2 | Class 2 |
| 120 | Class 2 | Class 2 |
| 121 | Class 2 | Class 2 |
| 122 | Class 2 | Class 2 |
| 123 | Class 2 | Class 2 |
| 124 | Class 2 | Class 2 |
| 125 | Class 2 | Class 2 |
| 126 | Class 2 | Class 2 |
| 127 | Class 2 | Class 2 |
| 128 | Class 2 | Class 2 |
| 129 | Class 2 | Class 4 |
| 130 | Class 2 | Class 2 |
| 131 | Class 2 | Class 2 |
| 132 | Class 2 | Class 2 |

## Appendix M: The Predictive models Diagrams

## Decision Tree Algorithm Predictive model for Case 2



Class: Class 2 Class: Class 3 Class: Class 4

| Sensitivity | 1.0000 | 1.00000 | 1.0000 |
| :--- | :--- | :--- | :--- |
| Specificity | 1.0000 | 1.00000 | 1.0000 |
| Pos Pred Value | 1.0000 | 1.00000 | 1.0000 |

KNUST
1.0000
1.00000
1.0000

Prevalence

| 0.7313 | 0.09701 | 0.1716 |
| :--- | :--- | :--- |
| 0.7313 | 0.09701 | 0.1716 |
| 0.7313 | 0.09701 | 0.1716 |
| 1.0000 | 1.00000 | 1.0000 |

Decision Tree Algorithm Predictive model for Case 4


Statistics by Class:
Class: Class 1 Class: Class 2 Class: Class 3 Class: Class

| Sensitivity | 0.000000000 | 1.0000000 | 0.000000000 | 1.0000000 |
| :--- | :---: | :---: | :---: | :---: |
| Specificity | 1.000000000 | 0.9166667 | 1.000000000 | 1.0000000 |
| Pos Pred Value | NaN | 0.9821429 | NaN | 1.0000000 |
| Neg Pred Value | 0.992537313 | 1.0000000 | 0.992537313 | 1.0000000 |
| Prevalence | 0.007462687 | 0.8208955 | 0.007462687 | 0.1641791 |
| Detection Rate | 0.000000000 | 0.8208955 | 0.000000000 | 0.1641791 |
| Detection Prevalence 0.000000000 | 0.8358209 | 0.000000000 | 0.1641791 |  |
| Balanced Accuracy | 0.500000000 | 0.9583333 | 0.500000000 | 1.000000 |

## Random Forest Algorithm Predictive model for Case 2



Class 2 Class 2 Class 2 Class 2 Class 2 Class 2 Class 2 Class 2

Statistics by Class:

Sensitivity
Specificity
Pos Pred Value
Neg Pred Value
Prevalence
Detection Rate
Detection Prevalence
Balanced Accuracy

| Class: Class 2 | Class: Class 3 | Class: Class 4 |
| ---: | ---: | ---: |
| 0.9490 | 0.92308 | 0.9565 |
| 0.9444 | 0.97521 | 0.9820 |
| 0.9789 | 0.80000 | 0.9167 |
| 0.8718 | 0.99160 | 0.9909 |
| 0.7313 | 0.09701 | 0.1716 |
| 0.6940 | 0.08955 | 0.1642 |
| 0.7090 | 0.11194 | 0.1791 |
| 0.9467 | 0.94914 | 0.9693 |

Random Forest Predictive order of significant predictor Variables for Case 2

## Case2



| Class $2 \quad$ Class | Class | 4 | MeanDecreaseAccuracy MeanDecreaseGini |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No..of.Employees | 63.9262892 | 8.822786 | 121.189088 | 100.808330 | 49.156927 |
| X..Performing.Loans | 0.3540681 | -2.606728 | 16.168407 | 13.648501 | 6.872839 |
| IT.budget.GH.. | 8.4985550 | 9.716305 | 20.06233 | 23.214884 | 12.890875 |
| Fixed.Asset.GH.. | 3.5797285 | 13.145312 | 3.661803 | 9.405381 | 8.518839 |
| Deposit.GH.. | 50.1183677 | 89.151192 | 27.192690 | 78.135977 | 52.531178 |
| Profit.GH.. | 1.9453950 | -2.994302 | 12.707079 | 10.690923 | 5.025936 |

## Random Forest Algorithm Predictive model for Case 4



Node 2 ( $\mathrm{n}=$ ! $\operatorname{Node} 5$ ( $\mathrm{n}=\operatorname{Node} 7$ ( $\mathrm{n}=\operatorname{Node} 8$ ( $\mathrm{n}=\operatorname{Node} 9$ ( $\mathrm{n}=8$ )
${ }_{0}^{0.8} \begin{aligned} & -4 \\ & 0\end{aligned}$
Class 1 Class 1
$\begin{aligned} 0.8 & - \\ 0.4 & - \\ 0 & =\end{aligned}$
Class 1

| 0.8 | - |
| ---: | ---: |
| 0.4 | - |
| 0 |  |

Class 1
$\begin{array}{r}0.8 \\ 0.4 \\ 0\end{array}=$
Class 1

Statistics by Class:

Class: Class 1 Class: Class 2 Class: Class 3 Class: Class 4

| Sensitivity | NA | 0.9735 | NA | 0.9524 |
| :--- | :---: | :---: | :---: | :---: |
| Specificity | 1 | 0.9524 | 1 | 0.9735 |
| Pos Pred Value | NA | 0.9910 | NA | 0.8696 |
| Neg Pred Value | NA | 0.8696 | NA | 0.9910 |
| Prevalence | 0 | 0.8433 | 0 | 0.1567 |
| Detection Rate | 0 | 0.8209 | 0 | 0.1493 |
| Detection Prevalence | 0 | 0.8284 | 0 | 0.1716 |
| Balanced Accuracy | NA | 0.9629 | NA | 0.9629 |

## Random Forest Predictive order of significant predictor Variables for Case 4

## Case4

| No..of.Employees |  | No..of.Employees |  |
| :---: | :---: | :---: | :---: |
| Deposit.GH.. | $\bigcirc$ | Deposit.GH.. |  |
| IT.budget.GH.. | - | X.Performing.Loans |  |
| X.Performing.Loans | - | IT.budget.GH.. |  |
| Profit.GH.. |  | Profit.GH.. |  |
| Fixed.Asset.GH.. |  | Fixed.Asset.GH.. |  |
|  | 11171 |  | 11111 |
|  | 080 |  | 040 |
| MeanD | crease | Mean | Decrea |



## Artificial Neural Network Predictive model for Case 2



Error: 96.20079 Steps: 39

```
Statistics by Class:
\begin{tabular}{lrrr} 
& Class: 2 & Class: 3 & Class: 4 \\
Sensitivity & 0.9320388 & 0.00000000 & 0.0000000 \\
Specificity & 0.1290323 & 0.90833333 & 1.0000000 \\
Pos Pred Value & 0.7804878 & 0.00000000 & NaN \\
Neg Pred Value & 0.3636364 & 0.88617886 & 0.8731343 \\
Prevalence & 0.7686567 & 0.10447761 & 0.1268657 \\
Detection Rate & 0.7164179 & 0.00000000 & 0.0000000 \\
Detection Prevalence & 0.9179104 & 0.08208955 & 0.0000000 \\
Balanced Accuracy & 0.5305355 & 0.45416667 & 0.5000000
\end{tabular}
```

Artificial Neural Network Predictive model for Case 4


Error: 93.613506 Steps: 35

Statistics by Class:

Sensitivity
Specificity
Pos Pred Value
Neg Pred Value
Prevalence

Detection Prevalence
Balanced Accuracy

Detection Rate 0.0000000000 .805970150 .0000000000 .0000000
Class: 1
Class: 2
Class: 3 Class: 4 0.0000000000 .981818180 .0000000000 .0000000 1.000000000 .083333330 .9699248121 .0000000 NaN 0.830769230 .000000000 NaN

$$
0.9925373130 .500000000 .9923076920 .8358209
$$

$$
0.0074626870 .820895520 .0074626870 .1641791
$$

$$
0.0000000000 .805970150 .0000000000 .0000000
$$

$$
0.0000000000 .970149250 .0298507460 .0000000
$$

$$
0.5000000000 .532575760 .4849624060 .5000000
$$

## Appendix N: $\mathbf{R}$ codes used for the various analyses

## Data Envelopment Analysis Codes

install.packages("rDEA") library(rDEA) data1<read.csv(file.choose())
\#\# inputs and outputs for analysis
\#\#Stage 1 Analysis(Input=Employee,IT,Asset:Output=Deposit)

```
Y = data1['DEPOSIT']
X = data1[c('Employees', 'IT', 'ASSET')]
## Naive input -oriented DEA score for the first 275 firms under variable returns -to-scale
firms=1:444
di_stage1 = dea(XREF=X, YREF=Y, X=X[firms,], Y=Y[firms,], model="input",
RTS="constant") di_stage1$thetaOpt write.csv(di_stage1,file = "stage1.csv")
## inputs and outputs for analysis
## Stage 2 Analysis(Input=di_stage 1,deposit:Output=Profit,%PL)
Y = data1[c('PROFIT','PL')]
X = data.frame(di_stage1$thetaOpt,data1['DEPOSIT']) firms=1:275
di_stage2 = dea(XREF=X, YREF=Y, X=X[firms,], Y=Y[firms,], model="input",
RTS="variable") di_stage2$thetaOpt write.csv(di_stage2,file = "stage2.csv")
\#\# inputs and outputs for analysis
\#\#Overall Stage Analysis(Input=di_stage2,Employee,IT,Asset::Output=Profit,\%PL)
##Y = data1[c('PROFIT','PL')]
##X = data.frame(di_stage2$thetaOpt,data1['Employees','IT','FIXED'])
##firms=1:275
##di_overall = dea(XREF=X, YREF=Y, X=X[firms,], Y=Y[firms,], model="input",
RTS="variable")
##di_overall$thetaOpt
##write.csv(di_overall,file = "overall.csv")
Decision Tree Codes Case 2 library(C50)
library (caret) dataset \(=\) read. \(\operatorname{csv}(\) file.choose () ,
header \(=\) TRUE \()\)
str(dataset) head(dataset) View(dataset) set.seed(100) train <- sample(nrow(dataset),
\(0.7 *\) nrow \((\) dataset \()\), replace \(=\) FALSE \()\)
```

TrainSet <- dataset[train,] ValidSet
<- dataset[-train,]
model $=\mathrm{C} 5.0($ TrainSet, TrainSet[,7],trials $=10$, rules $=\mathrm{T}$, control $=\mathrm{C} 5.0 \mathrm{Control}(\mathrm{minCases}=$ 10)) $\operatorname{dim}$ (TrainSet)
dim(ValidSet)
summary(model)
test_prediction $=$ predict.C5.0(model, newdata $=$ ValidSet, type $=$ "class") res
$=\operatorname{cbind}(V a l i d S e t[, 7]$, as.data.frame(test_prediction))
confusionMatrix(test_prediction,ValidSet[,7]) write.csv(res,file =
"predictedCase2.csv")

## Decision Tree Codes Case 4 library(C50)

library $($ caret $)$ dataset $=$ read. $\operatorname{csv}($ file.choose () ,
header $=$ TRUE $)$
$\operatorname{str}($ dataset) head(dataset) View(dataset) set.seed(100) train <-
sample(nrow(dataset), 0.7 *nrow(dataset), replace $=$ FALSE $)$
TrainSet <- dataset[train,] ValidSet
<- dataset[-train,]
model $=$ C5.0(TrainSet,TrainSet[,7],trials $=10$, rules $=\mathrm{T}$, control $=$ C5.0Control $($ minCases $=$ 10)) $\operatorname{dim}$ (TrainSet)
dim(ValidSet)
summary(model)
test_prediction $=$ predict.C5.0(model, newdata $=$ ValidSet, type ="class") res
$=$ cbind(ValidSet[,7],as.data.frame(test_prediction))
confusionMatrix(test_prediction,ValidSet[,7]) write.csv(res,file

```
= "predictedCase4.csv") Random Forest Codes Case 2
sink("rforest_Case2.doc") library(randomForest)
library(RWeka) library(e1071) library(caret) data1 <-
read.csv(file.choose(), header = TRUE)
```

head(data1) $\operatorname{str}($ data1)
summary (data1)
\# Split into Train and Validation sets
\# Training Set : Validation Set $=70: 30$ (random) set. seed(1000)
train $<-$ sample(nrow(data1), $0.7 *$ nrow(data1), replace $=$ FALSE $)$
TrainSet <- data1[train,] ValidSet
<- data1[-train,]
summary(TrainSet)
summary(ValidSet)
\# Create a Random Forest model with default parameters model1 <-
randomForest(Case4 $\sim$., data $=$ TrainSet, importance $=$ TRUE) model1
\# Fine tuning parameters of Random Forest model
model2 $<$ - randomForest(formula $=$ Case $2 \sim$., data $=$ TrainSet, ntree $=500$, mtry $=6$,
importance $=$ TRUE $)$ model 2
\# Predicting on train set predTrain <- predict(model1, TrainSet,
type $=$ "class") res $=$
cbind(data1[train,10], as.data.frame(predTrain))
\# Checking classification accuracy table(predTrain, TrainSet\$Case2) \#
Predicting on Validation set
predValid <-predict(model2, ValidSet, type = "class")
\# Checking classification accuracy mean(predValid

## KNUST

\# To check important variables importance(model2) res = cbind(ValidSet[,7],as.data.frame(predValid))
write.csv(res,file = "RF_predictedCase4.csv")
varImpPlot(model2) Plotting library(dplyr)
library(ggraph) library(igraph)
install.packages("ggraph") \#
install.packages("reprtree") tree_func <-
function(final_model,
tree_num) \{
\# get tree by index
tree <- randomForest::getTree(final_model,
k = tree_num,
labelVar = TRUE) $\%>\%$
tibble::rownames_to_column() \%>\%

## SANE

== ValidSet\$Case2)
table(predValid,ValidSet\$Case2) confusionMatrix(predValid,
ValidSet\$Case2)
\# make leaf split points to NA, so the 0 s won't get plotted mutate('split point`\(=\) ifelse(is.na(prediction),`split point ${ }^{\text {, NA }}$ )
\# prepare data frame for graph graph_frame <- data.frame(from = rep(tree\$rowname, 2),

$$
\text { to }=c(\text { tree } \$ ` \text { left daughter`, tree \$ right daughter` }) \text { ) }
$$

\# convert to graph and delete the last node that we don't want to plot
graph <- graph_from_data_frame(graph_frame) \%>\%

```
delete_vertices("0")
```

V(graph)\$node_label <- gsub("_", " ", as.character(tree\$`split var`))
V(graph)\$leaf_label <- as.character(tree\$prediction)
$\mathrm{V}($ graph $) \$$ split <- as.character(round(tree\$`split point`, digits $=2)$ )
\# plot
plot <- ggraph $($ graph, 'dendrogram' $)+$ theme_bw ()$+$ geom_edge_link ()$+$ geom_node_point ()$+$ geom_node_text(aes(label = node_label), na.rm $=$ TRUE, repel = TRUE $)+$ geom_node_label(aes(label $=$ split $),$ vjust $=2.5$, na.rm $=$ TRUE, fill $=$ "white") + geom_node_label(aes(label = leaf_label, fill = leaf_label), na.rm = TRUE, repel $=$ TRUE, colour $=$ "white", fontface $=$ "bold", show.legend $=$ FALSE $)+$ theme $($ panel.grid.minor $=$ element_blank ()$, \quad$ panel.grid.major $=$ element_blank( $)$,
panel.background $=$ element_blank(),
plot.background = element_rect(fill = "white"),
panel.border $=$ element_blank ()$, \quad$ axis.line $=$
element_blank(), axis.text. $x=$
element_blank(), axis.text. $\mathrm{y}=$
element_blank(), axis.ticks = element_blank(),
axis.title. $\mathrm{x}=$ element_blank( $), \quad$ axis.title. $\mathrm{y}=$
element_blank(), plot.title $=$ element_text(size
$=18)) \quad \operatorname{print}($ plot $)$
set node labels
\}
\# tree_num <- which(model2\$finalModel\$forest\$ndbigtree == min(model2\$finalModel\$forest\$ndbigtree)) tree_func(model2,
tree_num = 5)
\#plot 2 install.packages("party") library(party)
$X=$ ctree $($ Case $2 \sim$., data=TrainSet, controls=cforest_control $(m t r y=6$, ntree $=500))$ plot $(X)$

## Random Forest Codes Case 4

sink("rforest_Case4.doc")
library(randomForest) library(RWeka)
library(e1071) library(caret) data $<$ -
read.csv(file.choose(), header $=$ TRUE $)$
head(data1) $\operatorname{str}($ data1 $)$
summary(data1)
\# Split into Train and Validation sets
\# Training Set : Validation Set $=70: 30$ (random) set.seed(1000)
train <- sample(nrow(data1), $0.7 *$ nrow(data1), replace $=$ FALSE $)$
TrainSet <- data1[train,] ValidSet
<- data1[-train,]
summary(TrainSet) summary(ValidSet)
\# Create a Random Forest model with default parameters model1 <-
randomForest $($ Case $4 \sim$., data $=$ TrainSet, importance $=$ TRUE $)$ model 1
\# Fine tuning parameters of Random Forest model
model $2<$ randomForest(formula $=$ Case $4 \sim$., data $=$ TrainSet, ntree $=500$, mtry $=6$, importance $=$ TRUE $)$ model 2
\# Predicting on train set

## SANE

predTrain <- predict(model1, TrainSet, type = "class") res
$=\operatorname{cbind}($ data1[train,10], as.data.frame(predTrain))
\# Checking classification accuracy table(predTrain,
TrainSet\$Case4) \# Predicting on Validation set
predValid <-predict(model2, ValidSet, type = "class") \#
Checking classification accuracy mean(predValid
$==$ ValidSet\$Case4)
table(predValid,ValidSet\$Case4) confusionMatrix(predValid,
ValidSet\$Case4)
\# To check important variables importance $($ model 2$)$ res $=$ cbind(ValidSet[,7],as.data.frame(predValid)) write.csv(res,file = "RF_predictedCase4.csv")
varImpPlot(model2) Plotting library(dplyr)
library(ggraph) library(igraph)
install.packages("ggraph") \#
install.packages("reprtree") tree_func <-
function(final_model,

> tree_num) \{ \# get tree by
tree
randomForest::getTree(final_model,
$\mathrm{k}=$ tree_num,
labelVar = TRUE) $\%>\%$
tibble::rownames_to_column() \%>\%
\# make leaf splitpoints to NA, so the 0 s won't get plotted mutate('split point $=$ ifelse(is.na(prediction), `split point`, NA))
\# prepare data frame for graph graph_frame < data.frame $($ from $=$ rep $($ tree $\$$ rowname, 2$)$,

$$
\text { to } \left.=\mathrm{c}\left(\text { tree }{ }^{\prime} \text { left daughter`, tree\$` right daughter``}\right)\right)
$$

index
\# convert to graph and delete the last node that we don't want to plot
graph <- graph_from_data_frame(graph_frame) \%>\% delete_vertices("0")
\# set node labels
V(graph)\$node_label <- gsub("_", " ", as.character(tree\$`split var`))
\# plot
plot <- ggraph(graph, 'dendrogram') + theme_bw() + geom_edge_link() + geom_node_point() + geom_node_text(aes(label = node_label), na.rm = TRUE, repel = TRUE $)+$ geom_node_label(aes(label $=$ split), vjust $=2.5$, na.rm $=$ TRUE, fill $=$ "white") + geom_node_label(aes(label = leaf_label, fill = leaf_label), na.rm = TRUE, repel $=$ TRUE, colour $=$ " white", fontface $=$ "bold", show.legend $=$ FALSE) + theme(panel.grid.minor $=$ element_blank(), panel.grid.major $=$ element_blank(), panel.background = element_blank(),
plot.background = element_rect(fill = "white"),
panel.border $=$ element_blank ()$, \quad$ axis.line $=$
element_blank(), axis.text.x =
element_blank(), axis.text.y $=$
element_blank(), axis.ticks = element_blank(),
axis.title. $\mathrm{x}=$ element_blank(), axis.title. $\mathrm{y}=$
element_blank(), plot.title $=$ element_text(size
$=18)$ ) $\operatorname{print}($ plot $)$
\# tree_num <- which(model2\$finalModel\$forest\$ndbigtree ==
$\min ($ model2\$finalModel\$forest\$ndbigtree)) tree_func(model2,
tree_num = 5)
V(graph)\$leaf_label <- as.character(tree\$prediction)
V(graph)\$split <- as.character(round(tree\$’split point`, digits = 2))
\#plot 2 install.packages("party") library(party)
$\mathrm{X}=$ ctree $($ Case $4 \sim$., data=TrainSet, controls=cforest_control $(m$ try $=6$, ntree $=500)$ ) $\operatorname{plot}(\mathrm{X})$ Artificial Neural Network Codes for Case 2
sink('Case2_nn.doc')
library("neuralnet") library(RWeka)
library(e1071) library(caret)
\#Going to create a neural network to perform sqare rooting \#Type
?neuralnet for more information on the neuralnet library datal <-
read.csv(file.choose(), header $=$ TRUE $)$
\# data1 $=$ data1[,2:8] str(data1)
\# this checks for NAs in the data all(is.na.data.frame(data1))
\# converting the Case $\$ 4$ factor type to numeric
unique $($ data1 $\$$ Case 2$)$ data $\$$ Case2 $=$
factor(data1\$Case2,
levels $=c($ "Class 2","Class 3",'Class 4'),
labels $=c(2,3,4))$
data1 $\$$ Case2 $=$ as.numeric(as.character(data1\$Case2))
set.seed(10) train <- sample(nrow(data1), 0.7*nrow(data1),
replace $=$ FALSE $)$
TrainSet <- data1[train,]
ValidSet <- data1[-train,] \# Fitting the neural network nnModel <- neuralnet(Case2 ~
No..of.Employees + X..Performing.Loans + IT.budget.GH.. +
Fixed.Asset.GH.. + Deposit.GH.. + Profit.GH..,
data $=$ TrainSet, hidden $=5$,rep
$=3$, threshold $=0.01$, err.fct $=$ "sse", $\quad$ act.fct $=$ "logistic",likelihood $=$
T,linear.output=T)
print(nnModel) \#Plot the
neural network
plot(nnModel)
\#testing the neural network on the test dataset
test.results <- compute(nnModel, covariate = ValidSet[,1:6]) \#Run them through the neural network
\# calculating the RMSE for the training dataset
predict_testNN $=($ test.results\$net.result $*(\max ($ data1\$Case2) $\quad-\quad \min (d a t a 1 \$ C a s e 2)))+$ $\min ($ data1\$Case2)

RMSE.NN $=($ sum((ValidSet\$Case2 - predict_testNN)^2) / nrow(ValidSet) $){ }^{\wedge} 0.5$

## RMSE.NN

\# confusioin MAtrix and some self inverted appproach
testPred $=$ as.factor(round(test.results\$net.result) $)$
testPred $=$ factor $($ testPred,$\quad$ levels $=$
c("2","3","4"), labels =c(2,3,4))
ValidSetCase2 $=$ as.factor(ValidSet\$Case2)
\# Confusion Matrix
confusionMatrix(testPred, ValidSetCase2)
\# classification Accuracy mean(testPred $==$
ValidSet\$Case2) \#Lets display a better
version of the results test.result $=$
cbind(ValidSet\$Case2, testPred)
\# View(test.result) write.csv(test.result, file =
"Case2_nnTestR.csv") Artificial Neural
Network Codes for Case 4

```
# install.packages('neuralnet')
sink('Case4_nn.doc')
library("neuralnet") library(RWeka)
library(e1071) library(caret)
```

\#Going to create a neural network to perform sqare rooting \#Type
?neuralnet for more information on the neuralnet library data1 <-
read.csv(file.choose(), header $=$ TRUE $)$
\# data1 $=\operatorname{data1}[, 2: 8] \operatorname{str}($ data1 $)$
\# this checks for NAs in the data all(is.na.data.frame(data1))
\# converting the Case\$4 factor type to numeric
unique(data1 $\$$ Case4) data $1 \$$ Case4 $=$
factor(data1\$Case4,
levels $=c($ "Class 1","Class 2","Class 3",'Class 4'),
labels $=c(1,2,3,4))$
data1 $\$$ Case4 $=$ as.numeric $($ as.character(data1 $\$$ Case4 $)$ )
set.seed(10) train <- sample(nrow(data1), 0.7*nrow(data1),
replace $=$ FALSE $)$
TrainSet <- data1[train,]
ValidSet < - data1[-train,] \# Fitting the neural network nnModel < - neuralnet(Case4 ~
No..of.Employees + X..Performing.Loans + IT.budget.GH.. +
Fixed.Asset.GH.. + Deposit.GH.. + Profit.GH.., data $=$ TrainSet, hidden $=5$,rep $=3$, threshold $=0.01$,err.fct $=$ "sse",
act.fct = "logistic",likelihood = T,linear.output=T)
print(nnModel) \#Plot
the neural network
plot(nnModel)
\#testing the neural network on the test dataset
test.results <- compute(nnModel,covariate = ValidSet[,1:6]) \#Run them through the neural network
\# calculating the RMSE for the training dataset
predict_testNN $=($ test.results\$net.result * $(\max ($ data $1 \$$ Case4 $)-\min ($ data1 $\$$ Case4 $)))+$ $\min$ (data1\$Case4)

RMSE.NN $=\left(\operatorname{sum}\left((\text { ValidSet } \$ \text { Case4 }- \text { predict_testNN })^{\wedge} 2\right) / \text { nrow }(\text { ValidSet })\right)^{\wedge} 0.5$
RMSE.NN

```
# confusioin MAtrix and some self inverted appproach
testPred = as.factor(round(test.results$net.result)) testPred
=factor(testPred
    levels = c("2","3"),
labels =c(2,3))
ValidSetCase4 = as.factor(ValidSet$Case4)
# Confusion Matrix
confusionMatrix(testPred, ValidSetCase4)
# classification Accuracy mean(testPred ==
ValidSet$Case4) #Lets display a better
version of the results test.result =
cbind(ValidSet$Case4, testPred)
# View(test.result) write.csv(test.result, file =
"Case4_nnTestR.csv") Appendix O: Other
```


## Files

```
'data.frame': 444 obs. of 7 variables:
```

'data.frame': 444 obs. of 7 variables:
\$ NO..OF.EMPLOYEES: int 7 7 7 7 7 7 7 7 7 7 ...
\$ NO..OF.EMPLOYEES: int 7 7 7 7 7 7 7 7 7 7 ...
\$L : num 83.8 82.7 83.1 82.5 81.1 ...
\$L : num 83.8 82.7 83.1 82.5 81.1 ...
\$ IT : num 60897 13859 34710 23697 47438
\$ IT : num 60897 13859 34710 23697 47438
S ASSET : num 1454094 1458937 58751 105087 1768646
S ASSET : num 1454094 1458937 58751 105087 1768646
\$ DEPOSIT : num 72757137 17605016 10256530 68830240
\$ DEPOSIT : num 72757137 17605016 10256530 68830240
30798389 ...
30798389 ...
\$ PROFIT : num 316872 4372073 1939065 4579781

```
    $ PROFIT : num 316872 4372073 1939065 4579781
```

778895 ...
\$ Class : Factor w/ 4 levels "Class 1","Class 2",..:
$24424422442 \ldots$
=== Summary ===

Kappa statistic 0.5523
Mean absolute error
0.2072

Root mean squared error
0.3218

Relative absolute error
$61.1731 \%$
Root relative squared error
78.2667 ㅇ

Total Number of Instances
310

045012 | $0 \quad \mathrm{~b}=$ Class 2

0290120 | $\quad 0=$ Class 4
Confusion Matrix and Statistics

Reference
Prediction Class 1 Class 2 Class 3 Class 4

| Class 1 | 2 | 1 | 3 | 4 |
| :--- | ---: | ---: | ---: | ---: |
| Class 2 | 9 | 25 | 8 | 17 |
| Class 3 | 3 | 2 | 10 | 0 |
| Class 4 | 7 | 10 | 2 | 31 |

Overall Statistics

$$
\text { Accuracy : } 0.5075
$$

$$
95 \% C I:(0.4198,0.5948)
$$

No Information Rate : 0.3881
P-Value [Acc > NIR]: 0.003298
Kappa : 0.2958
Mcnemar's Test P-Value : 0.023312

Statistics by Class:

Class: Class 1 Class: Class 2 Class: Class
3 Class: Class 4
Sensitivity
0.09524
0.6579
0.43478
0.5962

Specificity
0.92920
0.6458
0.95495



```
0.6563
0.6404
```

'data.frame': 444 obs. of 7 variables:
\$ NO..OF.EMPLOYEES: int $\begin{array}{llllllllll}7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7\end{array}$
$\$ \mathrm{PL} \quad$ : num $83.8 \quad 82.783 .182 .5 \quad 81.1$
\$ IT
\$ ASSET
: num $6089713859347102369747438 \ldots$ : num 14540941458937587511050871768646

```
    $ DEPOSIT : num 72757137 17605016 1025653068830240
30798389 ...
    $ PROFIT : num 316872 4372073 1939065 4579781 778895
    $ Class : Factor w/ 4 levels "Class 1","Class 2",..:
2
[1] Class 2 Class 1 Class 4 Class 3
Levels: Class 1 Class 2 Class 3 Class 4
Confusion Matrix and Statistics
```


## Reference

Prediction 1234
10000
200012
$\begin{array}{lllll}3 & 33 & 23 & 22 & 53\end{array}$
40000

## Overall Statistics

```
Accuracy : 0.1641791
    95% CI : (0.1058435, 0.2379513)
```

    No Information Rate : 0.4104478
    P-Value [Acc > NIR] : 1
    Kappa : -0.009009
    Mcnemar's Test P-Value : NA
    Statistics by Class:

| Class: | Class: | Class: | Class: 4 |
| ---: | ---: | ---: | ---: | ---: |
| 0.0000000 | 0.00000000 | 0.95652174 | 0.0000000 |
| 1.0000000 | 0.97297297 | 0.01801802 | 1.0000000 |
| NaN | 0.00000000 | 0.16793893 | NaN |
| 0.7537313 | 0.82442748 | 0.66666667 | 0.5895522 |


| Prevalence | 0.2462687 | 0.17164179 | 0.17164179 | 0.4104478 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Detection Rate | 0.0000000 | 0.00000000 | 0.16417910 | 0.0000000 |
| Detection Prevalence | 0.0000000 | 0.02238806 | 0.97761194 | 0.0000000 |
| Balanced Accuracy | 0.5000000 | 0.48648649 | 0.48726988 | 0.5000000 |

'data.frame': 444 obs. of 7 variables:

$\$ \mathrm{PL} \quad:$ num $83.8 \quad 82.783 .182 .581 .1 \ldots$
\$ IT : num $608971385934710 \quad 2369747438 \ldots$
\$ ASSET : num $14540941458937587511050871768646 \ldots$
$\$$ DEPOSIT : num $\quad 727571371760501610256530 \quad 68830240$
30798389 ...
\$ PROFIT : num 316872437207319390654579781778895
\$ Class : Factor w/ 4 levels "Class 1","Class 2",..:

[1] FALSE
[1] Class 2 Class 4 Class 3 Class 1
Levels: Class 1 Class 2 Class 3 Class 4
Confusion Matrix and Statistics

## Reference



$$
\begin{aligned}
\text { Accuracy } & : 0.141791 \\
95 \% \text { CI } & :(0.0875778,0.2125375)
\end{aligned}
$$

No Information Rate : 0.4776119
P -Value $[$ Acc $>\mathrm{NIR}]$ : 1
Kappa : 0
Mcnemar's Test P-Value : NA

Statistics by Class:

|  | Class: 1 | Class: 2 | Class: | 3 | Class: 4 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Sensitivity | 0.0000000 | 0.0000000 | 1.000000 | 0.0000000 |  |
| Specificity | 1.0000000 | 1.0000000 | 0.000000 | 1.0000000 |  |
| Pos Pred Value | NaN | NaN | 0.141791 | NaN |  |
| Neg Pred Value | 0.8208955 | 0.7985075 | NaN | 0.5223881 |  |
| Prevalence | 0.1791045 | 0.2014925 | 0.141791 | 0.4776119 |  |
| Detection Rate | 0.0000000 | 0.0000000 | 0.141791 | 0.0000000 |  |
| Detection Prevalence | 0.0000000 | 0.0000000 | 1.000000 | 0.0000000 |  |
| Balanced Accuracy | 0.5000000 | 0.5000000 | 0.500000 | 0.5000000 |  |

NO..OF.EMPLOYEES PL IT ASSET DEPOSIT
PROFIT Class
$1 \quad 783.75370 \quad 60897.361454093 .7272757137$
316871.7 Class 2
2
782.6921113859 .081458937 .4917605016
4372073.0
Class 4
$3 \quad 783.1116734710 .15 \quad 58751.0210256530$
1939064.7
Class 4
$782.54663 \quad 23696.85 \quad 105086.5068830240$
4579780.7
Class 2
781.0998547438 .361768645 .9230798389
778894.6 Class 4
$6 \quad 783.2780127897 .11 \quad 231642.8952158533$
1098903.5 Class 2
Confusion Matrix and Statistics
Reference
Prediction Class 1 Class 2 Class 3 Class 4

| Class 1 | 2 | 1 | 5 | 1 |
| :--- | :--- | ---: | ---: | ---: |
| Class 2 | 7 | 16 | 4 | 14 |
| Class 3 | 6 | 2 | 16 | 1 |
| Class 4 | 3 | 8 | 4 | 44 |

Overall Statistics

```
Accuracy : 0.5821
    95% CI : (0.4938, 0.6667)
```

    No Information Rate : 0.4478
    $$
\text { P-Value }[A c c>N I R]: 0.001219
$$

Kappa : 0.3959
Mcnemar's Test P-Value : 0.138147

Statistics by Class:

Class: Class 1 Class: Class 2 Class: Class
3 Class: Class 4
Sensitivity
0.11111
0.5926
0.5517
0.7333

Specificity
0.93966
0.7664
0.9143
0.7973

Pos Pred Value
0.22222
0.3902
0.6400
0.7458

Neg Pred Value
0.7867
0.8807

|  | 0.13433 | 0.2015 | 0.2164 |
| :---: | :---: | :---: | :---: |
| valence | 0.01493 | 0.1194 | 0.1194 |
| $\left.\begin{array}{llll}\text { racy } \\ 0.7653 & 0.06716 & 0.3060 & 0.1866 \\ \text { MeanDecreaseGini } & 0.52538 & 0.6795 & \end{array}\right]$ |  |  |  |

NO. .OF.EMPLOYEES 14.25971

IT
ASSET
57.58200
52.62596

NO.OF.EMPLOYEES PL IT ASSET DEPOSIT PROFIT
Class
$1 \quad 7 \quad 83.75370 \quad 60897.361454093 .7272757137$
316871.7 Class 2
$2 \quad 782.6921113859 .081458937 .4917605016$ 4372073.0

Class 1
3
$783.1116734710 .15 \quad 58751.0210256530$ 1939064.7

Class 4
4
$782.54663 \quad 23696.85105086 .50 \quad 68830240$
4579780.7 Class 3
778894.6 Class 4
$6 \quad 7 \quad 83.27801 \quad 27897.11 \quad 231642.8952158533$
1098903.5 Class 2

Confusion Matrix and Statistics

Reference
Prediction Class 1 Class 2 Class 3 Class 4 ||

| Class 1 | 0 | 3 | 3 | 2 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Class 2 | 4 | 4 | 6 | 6 |
| Class 3 | 6 | 8 | 17 | 6 |
| Class 4 | 21 | 8 | 7 | 33 |

Overall Statistics

```
Accuracy : 0.403 95\% CI : (0.3192, 0.4911)
```

No Information Rate : 0.3507
P-Value [Acc > NIR] : 0.120256
Kappa : 0.1615
Mcnemar's Test P-Value : 0.007651

Statistics by Class:

Class: Class 1 Class: Class 2 Class: Class
3 Class: Class 4
Sensitivity

$$
0.0000 \quad 0.17391
$$

0.5152
0.7021

Specificity
0.9223
0.85586
0.8020
0.5862

Pos Pred Value
0.0000
0.20000
0.4595
0.4783

Neg Pred Value
0.7540
0.83333
0.8351
0.7846

Prevalence
0.2313
0.17164
$0.2463 \quad 0.3507$
Detection Rate
$0.0000 \quad 0.02985$
0.1269
0.2463

Detection Prevalence
0.0597
0.14925
0.2761
0.5149

Balanced Accuracy
0.4612
0.51488
0.6586
0.6442

MeanDecreaseGini


## Overall Statistics

$$
\begin{aligned}
\text { Accuracy } & : 0.9104 \\
95 \% \text { CI } & :(0.8488,0.9529)
\end{aligned}
$$

No Information Rate : 0.7388
P-Value [Acc > NIR] : 5.296e-07
Kappa : 0.802
Mcnemar's Test P-Value : NA

Statistics by Class:

|  | Class: Class 2 Class: Class 3 Class: Class |  |  |
| :--- | :---: | :---: | :---: |
| 4 | 0.8990 | 0.81818 | 1.0000 |
| Sensitivity | 0.9429 | 0.94309 | 0.9727 |
| Specificity | 0.9780 | 0.56250 | 0.8889 |
| Pos Pred Value | 0.7674 | 0.98305 | 1.0000 |
| Neg Pred Value | 0.7388 | 0.08209 | 0.1791 |
| Prevalence | 0.6642 | 0.06716 | 0.1791 |
| Detection Rate | 0.6791 | 0.11940 | 0.2015 |
| Detection Prevalence | 0.9209 | 0.88064 |  |
| Balanced Accuracy |  |  |  |

'data.frame': 444 obs. of 7 variables:
\$ NO..OF.EMPLOYEES: int $\begin{array}{lllllllllll}7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & \ldots\end{array}$
$\$ \mathrm{PL}:$ num $83.8 \quad 82.783 .182 .581 .1 \ldots$
\$ IT : num $6089713859347102369747438 \ldots$
\$ ASSET : num 14540941458937587511050871768646
\$ DEPOSIT : num 72757137176050161025653068830240
30798389 ..
\$ PROFIT : num $316872437207319390654579781 \quad 778895$
\$ Class : Factor w/ 4 levels "Class 1","Class 2",..:
2222222222
=== Summary ===

| Kappa statistic | 0.9194 |
| :--- | ---: |
| Mean absolute error | 0.0209 |
| Root mean squared error | 0.1023 |
| Relative absolute error | $14.5603 \%$ |
| Root relative squared error | $38.5556 \%$ |
| Total Number of Instances | 310 |

$===$ Confusion Matrix $===$


## Reference

| Prediction Class | 1 | Class | 2 | Class | 3 | Class |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | 4

Overall Statistics

Accuracy : 0.9776
95\% CI : (0.936, 0.9954)
No Information Rate : 0.8209
P-Value [Acc > NIR] : 1.48e-08
Kappa : 0.9274
Mcnemar's Test $P$-Value : NA

Statistics by Class:

Class: Class 1 Class: Class 2 Class:
Class 3 Class: Class 4
Sensitivity NA 0.9727 NA
1.0000

Specificity 1.0000

## Appendix P: Document for Data Collection

## COVER LETTER PHD THESIS DATA COLLECTION SURVEY

Dear Sir/Madam,
I am currently pursuing PhD in Computer Science at Kwame Nkrumah University of Science and Technology-Kumasi, doing a study on efficiency measurement focusing mainly on financial institutions in Ghana. The purpose of the study is to build a high performance machine learning Data Envelopment Analysis (DEA) and four machine learning algorithms. This result of the four models would be compare to see the one with the best accuracy. To design, empirically test and validate the proposed models, I would require personnel's like you to fill out questionnaires .I am therefore by this letter seeking your assistance in terms of the data for the success of my study which is for academic purpose only. Provided data from your institution will be strictly treated as confidential information and would be used solely for the purpose of this research and there is nowhere your name would be mentioned in the study.

I believe the results will be valuable to your individual firms when it comes to how to assess your efficiency. You and your branch is part of a representative sample of financial institutions Decision Making Units in Ghana considered for my study. I am ready to accept any terms and conditions that would be attached to it and hope my application would meet your kindest consideration Thank you for your time.

Sincerely,
Peter Appiahene
Signed
(PhD Candidate, Department of Computer Science-KNUST-Kumasi)


[^0]:    Algorithm 3.6 Permutation Variable Importance

