KWAME NKRUMAH UNIVERISTY OF SCIENCE AND TECHNOLLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING

OPTIMIZING BANKS' LOAN PORTFOLIO IN GHANA

[A CASE STUDY OF NKORANMAN RURAL BANK, SUNYANI]

By

Jacob Agbenyega Awuitor (BSc. Physics, KNUST)

PG6318411

A Thesis submitted to the Department of Mathematics, Institute of Distance Learning, Kwame Nkrumah University of Science and Technology in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

[Industrial Mathematics]

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CERTIFICATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

The task of the thesis is to design a model to find the optimal solution for allocating funds for the various types of loans offered by Nkoranman Rural Bank, Sunyani, in order to maximize the net returns. The aims and the objectives of the project are to model the Loan Portfolio of Nkoranman Rural Bank, Sunyani using Linear Programming and solve the Linear Programming Model to maximize profit. A linear programming Model was formulated using the data collected from Nkoranman Rural Bank, Sunyani and Karmarkar's algorithm, an interior point method is used to solve it. The results show that Nkoranman Rural Bank Limited, Sunyani, will realize an optimal annual profit of GHc 117,123.51 on its loan portfolio if the management of the bank adopts the Model and implement it accordingly. This amount represents 35.5 % of the total fund to be disbursed by the bank for the year 2015, as against the 28.2 % profit made in 2014.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

DEDICATION

I dedicate this thesis to my parents, my dear wife, **Sarah Pokuah** and my lovely daughter, **Yesulikplim Amenuveve Awuitor**.

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I express my profound gratitude to my Creator for the life given to me. I also thank him for the knowledge, wisdom, protection and the strength given to me in life, especially during the period of this course.

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CHAPTER 1

INTRODUCTION

2.0. Introduction

This chapter discusses the general overview of the study including the background of the study, the statement of the problem and the objectives of the study. The chapter also looks at the summarized version of the methodology, significance of the study and the organization of the thesis.

1.1.0 Background of the study

Banks are licensed and regulated financial institutions that invest money received from customers, pay it out on request, give loans at an interest rate and exchange currency.

According to the International Monetary Fund's Gobat (2012), banks are institutions that match up savers and borrowers to help ensure that the economies function smoothly. Banks are intermediaries between depositors (who lend money to bank) and borrowers (to whom the bank lends money). The amount banks pay for deposits and the income they receive on their loans are both called interest.

For most banks across the globe, lending is a primary source of revenue or income among their core functions. Lending therefore has become the principal business activity for most banks in Ghana. In spite of the high interest rate charged on loans, borrowers still patronize various types of loans for various reasons and intentions.

The Loan Portfolio is a very important financial portfolio and as such, its effective management is fundamental to the safety and sustainability of banks. Most loan managers concentrate their effort on how to effectively approve loans and carefully monitor loan performance. It is therefore vital and prudent for financial establishments like banks to maximize the return on their Loan Portfolio.

1.1.1 Types of Banks

There are three main types of banks depending on the specific services they offer and the activities they undertake. The activities of banks can be grouped into: retail banking, investment banking, corporate banking, private banking and business banking. The three major types of banks are the Central banks, Investment banks and Retail banks.

The Central Bank

A central bank is an institution responsible for supervising the monetary system of a nation or a group of nations. Their usual duties are to issue currency, regulate the money supply and control rates. Their decisions affect the financial market in general and the currency markets in particular. Sometimes, they also supervise the commercial banking system in their country.
Some of the most important central banks are Federal Reserve Bank (US), European Central Bank, Bank of Japan, Bank of England, Bank of Canada, Reserve Bank of Australia and Reserve Bank of New Zealand (Tradimo, Inc., 2014). The Central Bank of Ghana is the Bank of Ghana (BoG).

Retail banks

Retail banks provide banking services directly to individual members of the public and small businesses. Examples of retail banking services:

- accepting salaries and wages into a current account,
- acting as intermediary to pay bills through direct debits and standing orders,
- providing cheque books, credit and debit cards,
- providing cash withdrawal facilities, ➢ savings and transactional accounts and ➢ managing savings.

Retail banks include Commercial banks, Community banks, Community development banks, Rural banks, Land development banks, Co-operative banks, Postal savings banks and Private banks.

Investment banks

Investment banks, sometimes called wholesale banks, provide financial services to other financial institutions and occasionally to countries. These banks raise investment capital from investors on behalf of corporations and governments and issue securities (both equity and debt capital).

The Investment banks issue securities for a listed company, either through an Initial- Public-Offering (IPO) or a secondary issue, in order to raise capital for expansion (Tradimo, Inc., 2014).

Investment banks also provide huge loans to companies that prefer debt as a means to raise capital, a banking activity known as merchant banking. Other tasks include providing investment advice to investors and broking on behalf of institutional investors. They provide investment management and advise corporations on capital market activities such as mergers and acquisitions. Investment banks are usually regulated by the central bank of the country in which they operate and/or a specific regulatory authority. These banks include 'the Investment banks', and Merchant banks.

1.1.2 Core Functions of Banks

The banks perform primary functions known as banking functions and secondary functions also called non-banking functions. They primary functions are the main functions of banks. These functions include:

- **Accepting Deposits** such as **Current Deposits, Fixed Deposits**, Saving **Deposits** and Recurring **Deposits** from the public;
- **Granting of Loans and Advances** such as Loans, Cash Credits, Overdraft and Discounting of Bill of Exchange.

The important secondary functions are:

- **Agency Functions** like Transfer of Funds, Collection of Cheques, Portfolio Management, Periodic Payments, Periodic Collections and acting as Trustees, Executors, Advisors as well as Administrators on behalf of their clients.
- **General Utility Functions** like Locker facility, Issue Letter of Credits and Drafts, Underwriting of Shares, Dealing in Foreign Exchange, Social Welfare Programmes and acting as Referees to financial standing of customers.

1.1.3 Banks in Ghana

In Ghana, there are several banks and other financial institutions whose activities are overseen and regulated by the Central bank which is Bank of Ghana (BoG). The central bank of Ghana which was established on the 4th March, 1957 according to the BoG (2014), traces its roots to the Bank of Gold Coast (BCG). The bank was formally established by the Bank of Ghana Ordinance (No. 34) of 1957, passed by the British Parliament. The BoG has since 1957 undergone various legislative changes and it currently operates under the Bank of Ghana Act, 2002 (Act 612).

The governing body of the Bank is the Board of Directors as stipulated in the Bank of Ghana Act 2002 (Act 612). The Board consists of the Governor, who is the Chairman, two Deputy Governors and nine Non-executive directors. The present Governor of the Bank of Ghana is Dr. Henry Kofi Akpenamawu Wampah and the two Deputy Governors are Mr. Millison Kwadwo Narh and Dr. Abdul-Nashiru Issahaku. The Bank of Ghana is mandated to maintain stability in the general level of prices, ensure efficient operation of banking and credit systems and to support general economic growth.

The Central Bank of Ghana performs the following functions (BoG, 2014):

- formulate and implement monetary policy aimed at achieving the objectives of the Bank;
- promote by monetary measure the stabilization of the value of the currency within and outside Ghana;
- institute measures which are likely to have a favourable effect on the balance of payments, the state of public finances and the general development of the national economy;
- regulate, supervise and direct the banking and credit system, and ensure the smooth operation of the financial sector;
- promote, regulate and supervise payment and settlement systems;
- license, regulate, promote and supervise non-banking financial institutions;
- act as banker and financial adviser to the Government;
- issue and redeem the currency notes and coins;
- ensure effective maintenance and management of Ghana's external financial services; and
- promote and maintain the relations with international banking and financial institutions and subject to the Constitution, implement international monetary agreements to which Ghana is a party.

The banks licensed in Ghana as at December, 2013 by the Bank of Ghana are Standard Chartered Bank Ghana Ltd, Ghana Commercial Bank (GCB) Ltd, SG-SSB Limited, UT Bank Limied, The Royal Bank Ltd, UniBank (Ghana) Ltd, National Investment Bank (NIB) Ltd, Agricultural Development Bank (ADB) Ltd, Prudential Bank Ltd, Merchant Bank (Ghana) Ltd, Ecobank Ghana Ltd, CAL Bank Ltd, HFC Bank, United Bank for Africa (UBA) Ghana

Ltd, Stanbic Bank Ghana Ltd, Bank of Baroda (Ghana) Ltd, Zenith Bank (Ghana) Ltd, Guaranty Trust Bank (Ghana) Ltd. Fidelity Bank Ghana Ltd, First Atlantic Bank Ltd, Bank of Africa (Ghana) Ltd, Banque Sahélo-Saharienne pour I'Investissement et le Commerce (BSIC) Ghana Ltd, Access Bank (Ghana) Ltd, Barclays Bank of Ghana Ltd, Energy Bank (Ghana) Ltd, International Commercial Bank Ltd and First Capital Plus Bank Ltd (BoG, 2014). The others include ARB Apex Bank and representative offices of Citibank and Ghana International Bank.

1.1.4 The ARB Apex Bank Limited and Rural Banks

The ARB Apex Bank Ltd is a mini central Bank in Ghana for the Rural and Community Banks (RCBs) financed mainly through the Rural Financial Services Project (RFSP), a project of Government of Ghana. The initiative is to holistically address the operational bottleneck of the rural financial sector with the aim of broadening and deepening financial intermediation in the rural areas. The ARB Apex Bank was established to bring the rural population into the banking system under rules designed to suit their socio-economic circumstances and the peculiarities of their occupation in farming and craft-making (ARB Apex Bank Ltd, 2014).

The Bank of Ghana set up a department at the head office called the Rural Banking Department. The department was to see to the establishment and supervision of rural banks in the country. The rural banks were to operate under the banking law of Ghana (amended in 1989, 2005), companies' code and the rules and regulations of the Bank of Ghana. The first rural bank to be set up in the early 1970's was the Nyarkrom Rural Bank at Agona Nyarkrom (Offei, 2009).

The ARB Apex Bank Limited was incorporated as a public limited liability company on 4th January, 2000 and was granted license by BoG on 23rd April, 2001 under the then Banking Law, 1989 (PNDCL 225). The bank began clearing services on 2nd July, 2002 in all the 11 clearing centres in Ghana at the 10 regional capitals and Hohoe. Its shareholders are the Rural and Community Banks (ARB Apex Bank Ltd, 2014).

There are about 143 rural and community banks in the ten regions of Ghana. The banks undertake a mix of micro finance and commercial banking activities structured to satisfy the need of the rural areas. They provide banking services by way of funds mobilization and credit to cottage industry operators, farmers, fishermen and regular salaried employees.

They also grant credits to customers for payment of school fees, acquisition and rehabilitation of houses as well as to meet medical expenses. Some of the banks have specific gender programmes focusing on women-in-development and credit-with-education activities for rural women. Rural and Community Banks are therefore, the main vehicle for financial intermediation, capital formulation and retention of rural dwellers in the rural areas (ARB Apex Bank Ltd, 2014).

The Bank of Ghana Annual Report (2013) stated that the balance sheet of RCBs grew by 21.6 per cent to GH¢1,852.9 million. The increase in assets reflected mainly in Loans and Advances (10.5%), Investment (16.6%) and Cash and Bank Balances (31.7%). The growth in assets was funded by Shareholders’ funds, Deposits and Borrowings which increased by 29.8 per cent, 15.8 per cent and 57.1 per cent respectively.

At the end of 2013, the number of Rural and Community Banks (RCBs) in Ghana stood at 140. This number has increased to 143 as at October, 2014 according to the statistics of the MiniCentral Bank of RCBs, ARB Apex Bank.

Table: 1.1 Regional Distributions of RCBs and Branches

REGION	NUMBER OF BANKS	NUMBER OF BRANCHES

Ashanti	27	154
Brong Ahafo	23	90
Central	19	108
Eastern	22	108
Greater Accra	6	29
Northern	8	18
Upper East	4	12
Upper West	10	15
Volta	11	42
Western	12	75
TOTAL	143	651
HEAD OFFICES PLUS BRANCHES		**794**

Source: ARB Apex Bank Ltd Website (2014)

1.1.5 Challenges Facing Rural and Community Banks in Ghana

The Rural and Community banks in Ghana are faced by a number of challenges that include:

- weak management as a result of the inability to attract qualified and competent personnel.
- low capital base, shareholders are general poor and therefore cannot make substantial investments in the rural banks;
- lack of modern technology and adequate communication facilities to promote modern banking operations;
- inadequate training due to the inability of the banks to afford the cost of training;
- high risk involved in farming or agricultural operations in rural areas (Offei, 2011).

1.2.0 Loans

A loan is a sum of money which is borrowed from a financial institution or bank and has to be repaid after specific period of time, usually together with an extra amount of money called interest. According to Wikimedia Inc. (2014), a loan is arrangement in which a lender gives money or property to a borrower and the borrower agrees to return the property or repay the money, usually along with interest, at some future point(s) in time. Usually, there is a predetermined time for repaying a loan, and generally the lender has to bear the risk that the borrower may not repay a loan.

Generally, a loan may be Secured or Unsecured**.** S**ecured** loans are loans that rely on assets (house, car, land, etc) as collateral for the loan. In the event of loan default, the lender can take possession of the asset and use it to cover the loan. **Unsecured loans** do not depend on an asset for collateral. Unsecured loans rely solely on the credit history and income to qualify the borrower for the loan. Interest rates for secured loans may be lower than those for unsecured loans.

There are number of conditions attached to a loan: for example, when it is to be repaid and the rate of interest to be charged on the sum of money loaned. The two major characteristics that vary among bank loans are the terms of the loan and the security or collate required getting the loan. For the loan term we have the long term and the short term, and for the security there is secured or unsecured debt (Offei, 2011).

According to Burton (2002) cited in Offei (2011), engaging in loan gives a greater amount of money to fulfill ones project. Some clients find it difficult to pay for these loans but they still want to apply for it due to financial constraints. Most people apply for loans because of the following underlying reasons:

- to start or develop an existing business;
- to pay for existing loan;
- to own a property such as car, house, etc;
- for educational purposes and others;
- to cater for unexpected emergency such as car repair, medical expense, etc.
- for recurring everyday expense such as rent, food, utilities, etc.

The Truth-in-lending law requires banks and other financial institutions state the rate of interest as an annual percentage rate (APR). The amount of interest charged on a loan is determined by three factors: Principal, rate and time. Total interest charge on a loan is called the finance charge (Brooks et al, 1988). The total interest on a loan depends on the timescale the interest is calculated on, because interest paid may be compounded. An extra charge by the loan lending institution for arranging the loan is called Loan origination fee.

When obtaining a loan, the borrower is usually required to sign a loan agreement called a promissory note or simply, a note. The promissory note is written promise that the money will be repaid on a certain date. Money borrowed on a single-payment loan is paid back plus interest in one lump sum whereas installment loan is repaid by series of periodic payments, usually monthly. Installment plan is commonly used for loans of large amounts of money that take a year or more to repay (Brooks et al, 1988).

1.2.1 Banks' Loan Portfolio and its Compositions

Loan portfolio of a bank or financial institution refers to the loans that have been made or bought and are being held for repayment. Loan portfolios are the major asset of banks, thrifts, and other lending institutions. Simply, loan portfolio is the loans that a lender is owed. The loan portfolio is listed as an asset on the lender's balance sheet. The value of a loan portfolio depends on both the principal and interest owed and average creditworthiness of the loans (that is the probability or likelihood that the principal and interest will be paid).

Banks are one major player in financial markets, distinguished through their ability to originate and manage loans. The loan portfolio also forms a significant part of their whole portfolio. Unfortunately, loan portfolio adjustments cannot happen instantaneously, like in most other asset categories. This is due to market frictions such as transaction costs and the illiquidity of loans arising from informational asymmetries between lender and borrower (Pfingsten and Rudolph, 2002).

A bank's loan portfolio may composed of the types or categories of loan its offers, the total amount of loan its gives, the total returns on the loans and the customers (borrowers). Harris et al (2013) stated that banks' loan portfolios consist primarily of real estate loans (the largest group), commercial and industrial (C&I) loans, and consumer loans. Other loans include loans to depository institutions, loans to farmers, loans to foreign governments and official institutions, loans to non-depository financial institutions, and lease financing receivables.

The categories of loans in Ghana includes Salary loan, Susu loan, Student loan, Small-andMedium Enterprises (SMEs) loan, Agriculture loan, Funeral loan, Vehicle loan, House loan, Real estate loan, Business loan and Personal loan. Banks loan portfolio composition may be influenced by their locations, loan policies and the models used to optimize profit. De Haas et al (2009) considered three main potential determinants of bank portfolio composition: bank ownership, bank size, and the legal environment.

1.2.2 Credit Risks and Losses on Banks' Loans

For most banks, lending is a primary source of value creation and also the largest and most obvious source of credit risk. The economic profitability of lending is determined by the yield charged relative to the cost of funds lent and credit risk realized. The loan yields are set based on expectations about uncertain future interest rates and credit risk outcomes and are a good predictor of expected interest income.

Arku (2013) explained that credit risk is one of the major risks in banking industry. It is the potential for a loss when a borrower cannot make payments as obligated to a lender. It is commonly measured and communicated as the probability of an individual borrower's default.

A risk is the potential that events, expected or unexpected, may have an adverse impact on the bank's earnings or capital. The following are nine categories of risk for bank supervision purposes. These risks are *credit, interest rate, liquidity, price, foreign exchange, transaction, compliance, strategic,* and *reputation*. Banks with international operations are also subject to *country risk* and *transfer risk*. These risks are not mutually exclusive; any product or service may expose the bank to multiple risks.

Even though the largest and the obvious risk is lending risk, there are other pockets of credit risk both on and off the balance sheet, such as the investment portfolio, overdrafts, and letters of credit. The risk of repayment, i.e., the possibility that an obligor will fail to perform as agreed, is either lessened or increased by a bank's credit risk management practices. A bank's first defense against excessive credit risk is the initial credit-granting process, sound underwriting standards, an efficient, balanced approval process, and a competent lending staff.

Because a bank cannot easily overcome borrowers with questionable capacity or character, these factors exert a strong influence on credit quality. Borrowers whose financial performance is poor or marginal, or whose repayment ability is dependent upon unproven projections can quickly become impaired by personal or external economic stress. Management of credit risk, however, must continue after a loan has been made, for sound initial credit decisions can be undermined by improper loan structuring or inadequate monitoring.

Traditionally, banks have focused on oversight of individual loans in managing their overall credit risk. While this focus is important, banks should also view credit risk management in terms of portfolio segments and the entire portfolio. Effective management of the loan portfolio's credit risk requires that the board and management understand and control the bank's risk profile and its credit culture. To accomplish this, they must have a thorough knowledge of the portfolio's composition and its inherent risks.

Banks engaged in international lending face country risks that domestic lenders do not.
Country risk encompasses all of the uncertainties arising from a nation's economic, social, and political conditions that may affect the payment of foreigners' debt and equity investments. Country risk includes the possibility of political and social upheaval, nationalization and expropriation of assets, governmental repudiation of external indebtedness, controls, and currency devaluation or depreciation. Unless a nation repudiates its external debt, these developments might not make a loan uncollectible. However, even a delay in collection could weaken the lending bank.

1.2.3 Loan Portfolio Management

According to the Comptroller of the Currency Administrator of National Banks (1998), loan portfolio management (LPM) refers to the process by which risks that are inherent in the credit process are managed and controlled. Assessing LPM involves evaluating the steps bank management takes to identify and control risk throughout the credit process. The assessment focuses on what management does to identify issues before they become problems. The loan portfolio is typically the largest asset and the predominate source of revenue. As such, it is one of the greatest sources of risk to a bank's safety and soundness. Effective management of the loan portfolio and the credit function is fundamental to a bank's safety and soundness.

LPM encompasses all systems and processes used by banks to adequately plan, direct, control, and monitor the institution's lending operations. The principal components of an effective LPM system include strategic portfolio planning, lending policies and procedures, loan underwriting standards, a reliable risk identification program, clearly defined limits for portfolio concentrations, and an internal credit and collateral review. Portfolio management is a continuous process that must include analysis of how business results were achieved, whether such results will continue, and how the institution can maximize its opportunities and provide the greatest benefits to its members. Because of the inherent risks in lending and the System's statutory limitations on lending authorities, each institution must effectively manage the loan portfolio (Derrick et al, 1998).

Effective loan portfolio management begins with oversight of the risk in individual loans. Prudent risk selection is vital to maintaining favorable loan quality. Therefore, the historical emphasis on controlling the quality of individual loan approvals and managing the performance of loans continues to be essential. To manage their portfolios, bankers must understand not only the risk posed by each credit but also how the risks of individual loans and portfolios are interrelated. These interrelationships can multiply risk many times beyond what it would be if the risks were not related. Until recently, few banks used modern portfolio management concepts to control credit risk. Now, many banks view the loan portfolio in its segments and as a whole and consider the relationships among portfolio segments as well as among loans (Comptroller of the Currency Administrator of National Bank, 1998).

In every institution, the LPM system must ensure that all material aspects of lending operations are adequately controlled relative to the institution’s risk-bearing capacity. Loan underwriting standards form the critical link between the institution’s strategic portfolio objectives and the individual loans in its portfolio. While the safety and soundness of the institution is ultimately determined by its portfolio management system, loan underwriting standards become the foundation that supports the quality, composition, size, and profitability of the portfolio (Derrick et al, 1998).

The nine elements below complement the fundamental loan management principles. The nine elements are:

- Assessment of the credit culture,
- Portfolio objectives and risk tolerance limits,
- Management information systems,
- Portfolio segmentation and risk diversification objectives,
- Analysis of loans originated by other lenders,
- Aggregate policy and underwriting exception systems,
- Stress testing portfolios,
- Independent and effective control functions,
- Analysis of portfolio risk or reward tradeoffs.

Loan portfolio management, like other management programs and operations, requires an effective system of checks and balances to ensure that the institution is meeting program objectives and is adequately protected from unnecessary risk exposure. An institution’s lending operations should be controlled by a number of internal control components, which will generally include a combination of both “preventive” and “detective” controls. In portfolio management, preventive controls ensure that transactions and activities are performed in compliance with board direction and objectives. An institution's loan portfolio management system and its internal control system depend on an adequate management information system (MIS).

An adequate MIS is one that provides sufficient, accurate, and timely information on the condition, quality, and performance of the loan portfolio to enable board and management to make informed and prudent decisions on credit extensions, controls, and risk exposure. Each institution must have an MIS capable of providing sufficient information, data, and reports to identify and monitor all primary business and credit risks. The minimum components for MIS are a comprehensive loan accounting system and a general ledger system that accurately tracks and reports the institution's financial condition and operating results.

An effective MIS should be linked to and facilitate the institution's process for periodically analyzing and determining the allowance for loan losses and capital adequacy. MIS information and summary reports on the loan portfolio, including compliance with underwriting standards, should be regularly extracted to determine loss exposure (probable, potential, or remote possibility) based on credit quality, collateral position, and other measurable portfolio risks.

To ensure effective and timely portfolio and risk management for the more complex institutions, the MIS should have the capability to conduct portfolio stress testing and to predict portfolio risks under a variety of changing conditions or within a combination of alternative scenarios. For example, any stress-testing model should enable testing of changes in key variables that can affect a borrower's repayment capacity, cash flow, or financial condition. At a minimum, stress-testing models should be able to predict the impact on a loan or group of loans. That impact could be the result of changes in the following: Interest rates, Production costs and operating expenses, Commodity prices, Production levels and Collateral values.

A common approach to portfolio risk analysis involves analyzing the present composition and performance of the existing loan portfolio. Typical goals may be to determine the number and volume of loans outstanding by predetermined categories, such as loan quality classes, primary commodity, branch office, loan officer, size of loan, or various financial or performance ratios. The loan portfolio "slicing and dicing" gives an overall characterization of the institution's principal assets, including the concentration of loans in specific ranges or commodity groups.

1.2.4 Loan Portfolio Diversification and Segmentation

In 1952, Nobel Laureate Harry Markowitz demonstrated mathematically why putting all your eggs in one basket is an unacceptably risky strategy and why diversification is the nearest an investor or business manager can ever come to a free lunch. The revelation touched off the intellectual movement that revolutionized Wall Street, corporate finance, and business decisions around the world; its effects are still being felt today (Bernstein, 1996).

Risk diversification is a basic tenet of portfolio management. Concentrations of credit risk occur within a portfolio when otherwise unrelated loans are linked by a common characteristic. If this common characteristic becomes common source of weakness for the loans in concentration, the loans could pose considerable risk to earnings and capital. Managing the loan portfolio includes managing any concentrations of risk. By segmenting the portfolio into pools of loans with similar characteristics, management can evaluate them in light of the bank's portfolio objectives and risk tolerances and, when necessary, develop strategies for reducing, diversifying, or otherwise mitigating the associated risks.

For many banks, "portfolio segmentation" has customarily meant dividing the loan portfolio into broad categories by loan types such as commercial and industrial loans, real estate loans, and consumer loans. As segmentation techniques became more sophisticated, banks identified industry concentrations. Although these divisions are a good starting point, the full benefits of portfolio segmentation can be realized only if the bank is able to form segments using a broader range of risk characteristics.

Segmenting a portfolio and diversifying risk require comprehensive management information systems. The MIS data base should include both on- and off-balance-sheet credit exposures. If a bank lacks adequate data on each loan or does not possess a system to "slice and dice" the data for analysis, management's ability to manage the loan portfolio is compromised. But identifying the concentration is only half the job; understanding the dynamics of the concentration and how it will behave in different economic scenarios is the other half (Derrick et al, 1998).

1.9 Profile of Nkoranman Rural Bank Limited

Nkoranman Rural Bank Limited was established in Seikwa, in the Tain District of the Brong Ahafo Region in 1987. The Bank has six (6) branches with the Head Office at Seikwa and total staff strength of 56 headed by the General Manager, Mr. Ali Maama. The Sunyani Branch of Nkoranman Rural Bank was established in 1990. The Branch Manager is Mrs. Salamatu Obosu and the Project or Loan Officer is Mr. Francis Adomako-Tawiah. Core functions of the Bank include Granting Loans, Micro Savings, Ordinary Savings and Current Account. Others are Fixed Deposits, Apex Transfer and Western Union Money Transfer.

1.10 Statement of the Problem

For most banks, lending is a primary source of revenue or income and as such, it is one of the two primary functions (banking functions) of banks. The story is no different in Ghana.

According to Bank of Ghana (2013), the growth in total assets of banks and Non-Bank Financial Institutions (NBFIs) reflected mainly in Loans & Advances and Investments which increased by 35.2 per cent and 47.2 per cent to GH¢18,639.3 million and GH¢12,135.5 million respectively.

Giving of loans and advances has therefore become the principal business activity for most banks in Ghana, despite the high rate of credit losses on Banks' loan portfolios. Nkoranman Rural Bank as lending institution is also faced by the problem of bad debts which differs from loan type to loan type. A decision not to give loans with high probability of bad debt may result in exodus of customers from the bank to other banks, since some customers bank in a particular bank because of its favourable and reliable loan policy.

The key issues for the Management and the Board of Nkoranman Rural Bank to address are the mechanisms for managing credit risks and how to select the optimal loan portfolio to maximize the returns on Loans. The task of the thesis is to quantitatively analyze and Model the Loan Portfolio of Nkoranman Rural Bank, Sunyani to find the optimal solution of disbursing funds for the types of loans offered by the bank in order to maximize profits.

1.11 Objectives of the Thesis

The aims and the objectives of the project are to:

- Model the Loan Portfolio of Nkoranman Rural Bank, Sunyani using Linear Programming
- Solve the Linear Programming Model using Karmarkar's algorithm.

1.12 Methodology

The problem at hand is to determine the optimal returns on a bank's loan portfolio. The researcher uses a Linear Programming Model to formulate a Model involving a linear objective function subject to a set of linear constraints. The Model will subsequently be solved using Karmarkar's Algorithm to determine the optimal annual returns on Loans given out by

Nkoranman Rural Bank, Sunyani. The Karmarkar's algorithm is an Interior Point Method for solving practical Linear Programming problems.

The data on the various types of loans given out by Nkoranman Rural Bank, Sunyani for the 2015 financial year, will be taken from the bank. The MATLAB Programming Language will be employed to solve for the optimal solution. The sources of materials and information in the thesis are relevant Text Books, PDFs, Internet and MATLAB Programming Language.

1.13 Significance of the Thesis or Justification

The model used in the thesis will assist:

- decision makers, policy makers, loan managers and other stakeholders at the Banks to formulate prudent and effective loan policies to maximize profits on their Loan Portfolios.
- banks especially Nkoranman Rural Bank, Sunyani to efficiently and effectively allocate funds available for loan in order to optimize the returns on their Loan Portfolios.

1.14 Organization of the Thesis

The thesis is arranged into five chapters. Chapter one, the introduction explains the background to the study and gives brief profiles of Nkoranman Rural Bank, Sunyani. The chapter also describes the problem, objectives, methodology, significance and the organization of the thesis.

Chapter two, the Literature Review discusses relevant literatures on loans and theses' abstracts on the use of mathematical models especially linear optimization to examine loan portfolios, rate of credit losses and other related issues.

Chapter three, the Methodology describes the method use in the Research. Chapter four analyses the collected data using the proposed model. Chapter five comprises of the conclusions and recommendations.

CHAPTER 2 LITERATURE REVIEW

2.0 Introduction

Similar works were done by other researchers using various kinds of mathematical models to solve and analyze practical issues relating to optimal loan portfolios. Some authors investigated very important areas of concern of loans such as its compositions, management and diversification. This chapter provides a vivid overview of the literature on loan portfolios and how to optimize the returns on loans and advances.

2.6 Loan Portfolio Management and Diversification

In a study, Imeokpararia (2013) focuses on the effect of loan management on performance of Nigerian banks. The data was obtained from a survey of some selected banks in Nigeria and analyzed by the use of regression. Some performance indicators such as profit after tax, earnings per share and dividend were used to measure the performance of the selected banks. The analyses reveal that effective management of loan portfolio and credit function is fundamental to banks safety and soundness. Although these activities continue to be mainstays of loan portfolio management, analysis of past credit problems, such as those associated with the banking sector, has made it clear that portfolio managers should do more. Banks, as custodians of depositors' fund therefore, are obliged to exercise due care and prudence on their

lending operations. The author concludes that loan management has not affected the performance of Nigerian banks.

Chodechai (2004) while investigating factors that affect interest rates, degree of lending volume and collateral setting in the loan decision of banks, says: Banks have to be careful with their pricing decisions as regards to lending as banks cannot charge loan rates that are too low because the revenue from the interest income will not be enough to cover the cost of deposits, general expenses and the loss of revenue from some borrowers that do not pay.

Almgren and Chriss (2001) distinguished four areas of importance from the point of view of practitioners and academicians that play a very important role in the value of the firm and the owners' wealth. These are cash management, accounts receivable and accounts payable management and inventory management.

Nnanna (2005) further stressed that "Bank lending decisions generally are fraught with a great deal of risks, which calls for a great deal of caution and tact in this aspect of banking operations. The success of every lending activity to a great extent therefore, hinges on the part of the credit analysts to carry out good credit analysis, presentation, structuring and reporting.

According to Donkor (2013), liquidity is a major factor that determines the health of a company. Profitability only is not enough. A firm can have successful sales and show profits at the end of the year, but might suffer from lack of cash in the short run, whenever, it is needed. This situation, which is observed very often among Ghanaian banks, could lead the management of a company in seeking external sources of funds, which eventually will increase their cost of capital and financial risk, hence lowering their profitability. In Ghana, the emphasis in liquidity has been relatively new, since in general, the emphasis in working capital management is recent. At present, though, it has become obvious, that not only the working capital management strategies which a firm follows affect its shareholders' wealth and the firm's value significantly, but also, that they can be used as a competitive advantage against the other companies of their industry.

Jahn et al (2013) in their discussion paper explained that banks face a trade-off between monitoring benefits and concentration risk. Banks with a concentrated loan portfolio are expected to have better monitoring abilities, which might lower the loan portfolio's credit risk, while they are confronted with increased credit risk due to industrial concentrations. If the riskreturn-profile of a loan were exogenous, i.e. outside the influence of a bank, the banks' credit portfolio risk would be higher for banks with lower diversification in the credit portfolio. However, the loan's risk-return-profile is to some extent endogenous, i.e. it can be influenced by a bank. Due to banks' monitoring activities, it is not per se clear whether diversified banks are less risky than concentrated banks.

Carletti et al (2006) in discussing multiple-lending stated that banks choose to share lending whenever the benefit of greater diversification, in terms of higher cost per project monitoring dominates the cost of free-riding and duplication of efforts.

Acharya et al (2006) empirically examine the impact of loan portfolio concentration versus diversification on performance indicators of Italian banks. The authors use the HerfindahlHirschman Index (HHI) as a measure of loan portfolio concentration across different industries and sectors. They find that industrial or sectoral diversification implies unaffected or marginally increased return and increased credit risk for banks with a moderate downside risk in the loan portfolio, whereas banks with a high credit risk in their loan portfolio experience decreased bank performance through diversification. The authors conclude that "diversification per se is no guarantee of superior performance or greater bank safety and soundness".

Behr et al (2007) adopt this improved toolkit of concentration measures and analyse all German banks to find out whether a concentration strategy is superior to a diversification strategy in terms of the banks' risk-return characteristics. The authors conclude that a concentrated loan portfolio brings a slightly higher return on assets. Measuring the bank's loan portfolio credit risk with the loan-loss provision (LLP) ratio and the non-performing loan (NPL) ratio, respectively, reveals that concentrated banks tend to have lower credit risk. However, when the standard deviation of the loan loss provision ratio and respective nonperforming loan ratio, as a more straightforward risk measure of a loan portfolio's unexpected credit risk, is used instead, concentrated banks are more risky than diversified banks. In sum, a bank's concentration strategy seems to reflect the typical risk-return trade-off.

Boeve et al (2010) analyse German cooperative banks and savings banks from 1995 until 2006. The authors separate the bank-specific selection and monitoring abilities defined by the actual over expected loss ratio and include average values of bank-specific and other controls as common determinants of the loan portfolio's credit risk. They observe that concentrated banks show, on average, a significantly higher monitoring quality. Comparing concentration benefits with associated concentration risks using a common credit risk model, the authors find strong evidence for cooperative banks that a higher concentration level is, on average, associated with a lower credit risk of the loan portfolio, whereas results for savings banks depend more on the applied diversification measure.

Csongor and Curtis (2005) explained that when a portfolio is diversified, the risk of individual securities (in the case of banks: the credit risk of an individual firm) is being diversified away.

This risk is called unsystematic risk. The risk that cannot be diversified away is called systematic risk, which is sometimes equated with the market risk. Systematic risk could be described as the uncertain tendencies of the market. A well diversified portfolio will have the same tendencies as the market, in other words nearly perfect correlation with the market. If the market happens to have a negative tendency (graphically the best fitted line is negative), then the loss of the portfolio will be equal to the loss of the market, and vice versa if the tendency is positive.

2.7 Markowitz Theory and Efficient Frontier

Markowitz (1952) dealt with portfolio selection and developed the basic portfolio model, in his thesis at the University of Chicago. Markowitz designed a way to measure the risk of securities statistically and thereby constructed desired portfolios based on one's overall risk-reward preferences. The statistical approach to plot the risk reward relation is preceded by assigning expected values, standard deviations and correlations to security's single-period returns (no annuities). Later with these statistical measures one can calculate the volatility and the expected return of the portfolio, which are used as measures for risk and reward respectively. With quadratic programming (optimization, minimization of a quadratic function subject to linear constraints) an investor is able to find a few portfolios (out of an almost infinite number of possible weights of the securities) that will give the optimal risk-reward combination the securities making up portfolios.

These portfolios make up the efficient frontier. The assumptions behind this quantification is that all market participants have the same expectations, investors are able to invest in a totally riskless assets yielding the risk free rate of interest and the cost of transactions, information and for management is zero on the market. Based on these assumptions, one should be able to construct an optimal portfolio for all investor preferences. Markowitz divided the portfolio selection process into two separate decisions, first off; find the portfolio with the maximum reward for least amount of risk taken, lowest possible standard deviation. Second; decide on how to allocate the funds between the riskless assets and the risky assets.

The basic model developed by Markowitz (1952), derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of risk under a reasonable set of assumptions and derived the formula for computing the variance of the portfolio. This portfolio variance formulation indicated the importance of diversification for reducing risk, and showed how to properly diversify. The Markowitz model is based on certain assumptions. Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

According to Best and Grauer (1991), the Full Variance Model developed by Markowitz is based on the assumption that the purpose of portfolio management is to minimize variance for every possible combination of the expected yield. It has been argued by Blume (1970) and Hodges and Brealey (1972) that the concept of the efficient frontier is basic to the understanding of portfolio theory. Assume that in the market place, there are a fixed number of common stocks in which a businessman can invest. Each of the securities has its own expected yield and standard deviation; others have the same standard deviation but vary in expected yield. The investor will select the security that offers the highest yield for a chosen level of risk exposure as presented by the standard deviation. It is assumed that investors try to minimize risk by minimizing the deviation from the expected yield, and this is done by means of portfolio diversification procedures, loan diversion and unwillingness to repay loans.

2.8 Credit Risks Management

For most banks, loans are the largest and most obvious source of credit risk. The economic profitability of lending is determined by the yield charged relative to the cost of funds lent and credit risk realized.

By examining the drivers of contract features in revolving bank credit agreements, Dennis et al (2000) also give a valuable insight into the relationship between maturity and credit risk. The authors extract data on global loan transactions and private placements from TR Deal scan and find that factors which can be related to an increased loan portfolio credit risk, such as less secured loans or a higher earnings variance, imply a shorter revolving loans’ maturity.

Aretz and Pope (2013) also decompose firms’ default risk into common factors such as global, country and industry effects by analysing firms from 24 countries and 30 industries. They find that around 61% of the systematic variance in changes in firms’ default risk is due to global and industry effects.

Donkor (2013) explained in his thesis that sound credit management is a prerequisite for a financial institution’s stability and continuing profitability, while deteriorating credit quality is the most frequent cause of poor financial performance and condition. The prudent management of credit risk can minimize operational risk while securing reasonable returns. Ensuring lending staffs comply with the credit union's lending license and by-laws, is the first step in managing risk. The second step is to ensure board approved policies exist to limit or manage other areas of credit risk, such as syndicated and brokered loans, and the concentration of lending to individuals and their connected parties (companies, partnerships or relatives).

Arku (2013) explained that credit risk is one of the major risks in banking industry. It is the potential for a loss when a borrower cannot make payments as obligated to a lender. It is commonly measured and communicated as the probability of an individual borrower's default.

Updegrave (1987) found that there were eight variables that affected consumer credit risk: the number of variables, the historic repayment record, bankruptcy history, work and resident duration, income, occupation, age and the state of savings account. Similar results were found by Steenackers and Goovaerts (1989) who collected data on personal loans in Belgian credit company and found out that age, resident and work duration, the number and duration of loans, district, occupation, phone ownership, working in the public sector or not, monthly income and housing ownership have a significant relationship with repayment behaviour.

In Ghana, Awotwi (2011) used a data from an international bank in Ghana, to study the risk of default. The data comprise of 9939 observations of customers of which 14% turned out to be defaulters. According to him, six variables were found to be highly significant in the default. These are Marital Status, Number of months the applicant has been in current employment, interest rate, tenure of loan or repayment period, income level and loan amount.

2.9 Interest Rate and Probability of Loss on Loan Portfolio

The Wikimedia Inc. (2014) defines interest rate as the rate at which interest is paid by borrowers (debtors) for the use of money that they borrow from lenders (creditors). It is often expressed as an annual percentage of the principal. Interest rate is the proportion of a loan that is charged as interest to the borrower, typically expressed as an annual percentage of the loan outstanding. Different interest rates exist parallel for the same or comparable time periods, depending on the default probability of the borrower, the residual term, the payback currency, and many more determinants of a loan or credit.

According to Offei (2011), interest rates on loans are the most important factor affecting repayment of agricultural loans. In agriculture, large rate of default in loan repayment has been a perennial problem, farming experience, and total application costs. Most of the defaults arise from par management.

Rheinberger and Summer (2009) in their credit risk portfolio models, stated three parameters that drives loan losses: The probability of default (PD) by individual obligors, the loss given default (LGD) and the exposure at default (EAD). While the standard credit risk models focus on modeling the PD for a given LGD, a growing recent literature has looked closer into the issue of explaining LGD and of exploring the consequences of dependencies between PD and LGD. Most of the papers on the issue of dependency between PD and LGD have been written for US data and usually find strong correlations between these two variables. The first papers investigating the consequences of these dependencies for credit portfolio risk analysis using a credit risk model suggested by Finger (1999) and Gordy (2000). The authors used a different credit risk model in the tradition of actuarial portfolio loss models and focus directly on two risk factors: an aggregate PD and an aggregate API as well as their dependence. The authors used this approach because their interest was to investigate the implications of some stylized facts on asset prices and credit risk that have frequently been found in the macro-economic literature for the risk of collateralized loan portfolios. The authors also believe that the credit risk model we use gives us maximal flexibility with assumptions about the distribution of systematic risk factors.

There are a variety of models that try to capture the dependence between PD and LGD. These models are developed in the papers of Jarrow (2001), Carey and Gordy (2003), and Bakshi et al (2001). Most of these papers look at bond data but some also cover loans.

Acharya et al (2003) investigated defaulted bonds and look into recoveries of US corporate credit exposures, Grunert and Weber (2005) investigated recoveries of German bank loans and summarizes existing knowledge about recoveries. While these papers show a nuanced picture of the determinants of recoveries that consists of many microeconomic and legal features such as the industry sector in which exposures are held or the seniority of a claim all papers find that macroeconomic conditions play a key role.

The study by Harris et al (2013) develops a timely and unbiased measure of expected credit losses. The study models an expected rate of credit losses (*ExpectedRCL*) as a linear combination of several credit-related measures currently disclosed by banks. More specifically, the information in reported credit losses, non-performing loans, average loan yield, duration and composition of the loan portfolio is used to estimate the *ExpectedRCL*. It may therefore serve as a benchmark for estimating the profitability of a loan yield when evaluating bank performance and value creation. *ExpectedRCL* also contains incremental information about future credit losses relative to ALLL and PLLL respectively. The evidence provided by the study is also relevant for policy deliberation as standard setters contemplate revising existing rules relating to loss provisioning and requiring the recognition of some expected credit losses.

Paddy (2012) uses historical data on payments, demographic characteristics and statistical techniques to construct logistic regression model (credit scoring models) and to identify the important demographic characteristics related to credit risk. The logistic regression model was used to calculate the probability of default. Customers' age, sex, occupation, number of dependent, marital Status and amount of loan collected were used. The results showed that default rate is higher in males than in females, 30—39 year olds have the highest rate of default. Married customers defaulted more than the customers who are not married (single) and the higher the number of dependents, the higher the default rate. The self employed clients defaulted more than salary earners. It was found out that the higher the amount of loan collected, the higher the probability of default. The predicting power of the model is 70%.

2.10 Linear Optimization of Loan Portfolio

Linear programming (LP) is recognized as a powerful tool to help decision making under uncertainty in financial planning. It shows how portfolio optimization problems with sizes measured in millions of constraints and decision variables featuring constraints on semivariance, skewness or nonlinear activity functions in the objective can be solved. Based on the LP model, more computer software programs are being developed to help investors maximize their expected rates of return on their stock and bond investments subject to risk, dividend and interest, and other constraints.

Linear programming (LP) is a highly versatile quantitative technique, which has found wide use in management and economics according to Offei (2011). It is used both as a research technique and a planning tool, particularly at the individual firm and industry levels. In general, LP is designed to maximize or minimize a linear objective function subject to a set of linear constraints. Other related techniques are goal programming, mixed integer programming and quadratic programming.

Lyn (2002) discussed how one can use linear programming to estimate the interest rates for the prices of bonds. He explained that in the personal sector finance where lending is far greater than the higher profile cooperate sector, linear programming can be used to develop credit scorecards. According to Al-Faraj et al (1990), decision makers are always faced with the problem of determine optimal allocation of limited resources. The problem is to determine the best combination of activities levels, which does not use more resources than are actually available and at the same time maximize output, revenue, service level or minimize cost.

Jao (2009) indicated that linear programming techniques have long been used in many areas of economic analysis and business administration. Their application to bank management appears to be a relatively new development. According to Pearson (2007), Linear Programming (LP) is useful in managerial decisions, product mix, make-buy, media selection, marketing research, loan Portfolio selection, shipping and transportation and multi-period scheduling.

Among all the optimization techniques, linear programming is perhaps the most used and best understood by the business and industrial community, Wu (1989) stated. Linear programming deals with optimization problems that can be modeled with a linear objective function subject to a set of linear constraints. The objective of these problems is either to minimize resources for a fixed level of performance, or to maximize performance at a fixed level of resources.

Amponsah et al (2006) modeled a banking policy for Atwima Kwanwoma Rural Bank. The bank's policy of granting loans were modeled as a linear programming problem with respect to profit and budget constraints on the loan portfolios. Their research showed greater profit and expansion of service if recommendation were to be implemented. The bank found their policy proposal suitable for implementation.

In his thesis, Donkor (2013) developed a linear programming model using the Karmarkar's projective scaling algorithm to help Capital Rural Bank Limited, Dominase-Agency (Sunyani) to maximize their profit on loans. The results from the model showed that, Capital Rural Bank Limited, Dominase-Agency (Sunyani) would be making annual profit of GH¢5,961,300.00 if they are to stick to the model. From the study, it was realized that the scientific method used to come out with this model can have a dramatic increase in the bank's profit on loans if put into practice.

A study by Offei (2011) was to: (i) come out with a quantitative model that will maximize the returns on loans and (ii) determine optimum loan portfolio for Juaben Rural Bank. Data including the type of loans, the interest rate and the probability of bad debt associated with each type of loan were collected from the bank. The data was then modeled as a linear programming problem and the Quantitative Manager for Windows software was used to solve the problem. It was observed that out of six million, five hundred thousand Ghana cedis (GH¢6,500,000.00) to be disbursed as loan in 2012 financial year, one million, one hundred and ten thousand, four hundred and sixteen Ghana cedis (GH¢1,110,416.00) and seven hundred and four thousand, one hundred and sixty-six Ghana cedis (GH¢704,166.00) should be given to agriculture and transport sectors respectively. The trading sector should be given one million, four hundred and eighty- nine thousand, five hundred and eighty four Ghana cedis (GH¢1,489,584.00), The cottage industry, one million, four hundred and thirty five thousand, four hundred and seventeen Ghana cedis (GH¢1,435,417.00) and salary loans one million, seven hundred and sixty thousand, four hundred and seventeen Ghana cedis (GH¢1,760,417.00). With these allocation the bank will make a maximum profit of one million, eight hundred and fifty- two thousand, four thundered eighty- five Ghana cedis eighty nine pesewas (GH ¢1,852,485.89).

To address revenue losses due to poor allocation of funds by most financial institutions to prospective loan seekers, Gyakwa (2013) developed a Linear Programming model to help Christian Community Microfinance Limited (CCML), Eastern Zone, to allocate their funds to prospective loan seekers in order for them to maximize their profits. The problem was modeled as a linear programming problem. Simplex algorithm was used to solve the problem. It was observed that, if CCML Eastern zone, disbursed a total of GH¢120,000.00 a profit of GH¢32,068.48 will be realized.

The decline of relevant portfolio planning models especially in Ghana is attributed mainly to the evolving dynamics of the Ghanaian banking industry where the regulatory controls have changed with a high frequency. A lot of banks had suffered substantial losses from a number of bad loans in their portfolio due to the models used in allocating funds to loans. As a result, most banks are not able to maximize their profit margin due to poor allocation of funds. Adu (2011) formulated a linear programming model using the Simplex algorithm to help Prudential Bank Limited and Asante Akyem Rural Bank to maximize their profit margin. The results from the model showed that, Prudential Bank and Asante Akyem Rural Bank would be making annual profit of GH¢8003572.5 and GH¢176750 respectively if they are to stick to the model. From the study, it was realized that the scientific method used to develop the propose model can have a dramatic increase in the two banks profit margin if put into practice.

According to Brushammar and Windelhed (2004), risk control and return maximization were important aspects of this need. The objective of their thesis is to devise and implement a method that allows the institutions to make optimal funding decisions, given a certain risk limit. They proposed a funding approach based on stochastic programming. The approach allows the Institution's portfolio manager to minimize the funding costs while hedging against market risk. The authors employ principal component analysis and Monte Carlo simulation to develop a multicurrency scenario generation model for interest and exchange rates. Each of the optimization models presents the optimal funding decision as positions in a unique set of financial instruments. By choosing between the optimization models, the portfolio manager can decide which financial instruments he wants to use to fund the loan portfolio. Results show that the optimization models have the potential to deliver sound and profitable funding decisions. In particular, they conclude that the utilization of one of our optimization models would have resulted in an increase in the institution's net income over the past 3.5 years.

Sebil (2005) designed a Linear Programming model and apply optimization techniques to the operations of real life problems of banks giving out loans to different clientele. The main objective of the study was to determine the maximum net returns which comprises of the difference between the revenue from interest and lost funds due to bad debts. The bad debts are not recoverable both as principal and interest, thus reducing the total revenue. It is also to use the optimum solution to determine among the different clientele, the ones that are recommended and the ones that are least attractive. Atwima Kwanwuma Rural Bank was chosen as the bank for the study. A linear programming model which is one of the quantitative analysis techniques was developed and the Simplex Method used to solve the problem. It was found out that the bank could make an optimum profit of 23.002 billion annually. That is about 37.1% of 62 billion, the total amount disbursed as loans to the above.

Gyakwa (2013) developed a Linear Programming model to help Christian Community Microfinance Limited (CCML), Eastern Zone, to allocate their funds to prospective loan seekers in order for them to maximize their profits. The problem was modeled as a linear programming problem. Simplex algorithm was used to solve the problem. It was observed that, if CCML Eastern zone, disbursed a total of GH¢120,000.00 a profit of GH¢32,068.48 will be realized.

CHAPTER 3 METHODOLOGY

3.0 Introduction

This chapter describes the models used, how they are formulated and the algorithms involved in the formulation. The chapter highlights on the step-by-step procedures and computer programmes or software employed.

3.3.0 Optimization Techniques

Optimization is a mathematical methodology used to make decisions for allocating finite resources to achieve an overall objective subject to the constraints imposed by the environment or nature. It is the technique of finding alternative solution or result with the most cost effective or highest achievable performance under the given constraints, by maximizing desired factors and minimizing undesired ones. The theory of optimization is well studied by academics in fields ranging from operations research to computer science and used across a range of industries or fields including the banking, production, transportation, agricultural, petroleum, energy, telecommunication, business, economics, etc.

Given a function $f(x)$ which is constrained by some factors defined by an equation $g(x) \leq 0$. The methods employed to determine the value of x that gives the extreme values of $f(x)$ are termed optimization techniques. The extreme values mean the maximum and minimum values satisfying the constraint, $g(x)$. The value of x that gives the extreme values of $f(x)$ is called the Optimal Solution and the extreme value of $f(x)$ is called the Optimal Objective function.

Generally, an optimization problem may be continuous or discrete. It may also be constrained or unconstrained. Some of the optimization techniques are linear programming (LP), Second order cone programming (SOCP), Semidefinite programming (SDP), Geometric programming, Quadratic programming, Nonlinear programming and Stochastic programming. Others include Integral programming, Fractional programming, dynamic programming, etc.

3.3.1 Linear Programming Model

Linear programming (LP) is a type of convex programming model in which both the objective or performance function and the constraints equations or inequalities are all linear. It is also known as Linear Optimization. It involves minimizing or maximizing a linear function over a convex polyhedron specified by linear and non-negativity constraints.

Linear programming (LP) or linear optimization is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationships. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. The feasible region is a convex polyhedron, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine function defined on this polyhedron. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such point exists (Wikimedia Foundation Inc., 2014).

According to Offei (2011) an objective function is a mathematical expression that combines the variables to express your goal and the constraints are expressions that combine variables to express limits on the possible solutions. A variable or decision variables usually represent things that can be adjusted or controlled. It is programming because it involves the movement from one feasible solution to another until the best possible solution is attained.

Linear programming is a considerable field of optimization for several reasons. Linear programming can be applied to various fields of study. Many practical problems in operations research can be expressed as linear programming problems. It is used in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, banking, agricultural, petroleum and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment and design.

3.3.2 Linear Programming Problem Formulation

According to Robere (2012), a linear program is a constrained optimization problem in which the objective function and each of the constraints are linear. It means that the degree of all variables should be one. Given the variables $x_1, x_2, x_3, x_4, \dots, x_n$, the objective or performance linear function can be written as

$$z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n,$$

where $c_1, c_2, c_3, \dots, c_n$, are constants and $z = f(x_1, x_2, x_3, x_4, \dots, x_n)$

The General problem is to optimize (maximize or minimize) the objective function

$$z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n,$$

Subject to m number of **mixed soft constraints**

$$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \cdots + a_{in}x_n \leq b_i, i = 1, 2, 3, \dots, d$$

$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \cdots + a_{in}x_n = b_i, i = d+1, d+2, d+3, \ldots, m$

And the **Non negativity or hard constraints**

$x_1, x_2, x_3, \ldots, x_n \geq 0$

This covers all type of linear constraints since any constraint containing " $\geq$ " can be converted to a constraint containing " $\leq$ " by multiplying by -1. For the **standard maximization** problem;

Maximize $z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$,

Subject to the constraints:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \leq b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n \leq b_3$

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$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \leq b_m$

And the Non negativity constraints

$x_1, x_2, x_3, \ldots, x_n \geq 0$,

For the **standard minimization** problem;

Minimize $z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$,

Subject to the constraints:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \leq b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n \leq b_3$ . .

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$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \leq b_m$

And the Non negativity constraints

$x_1, x_2, x_3, \dots, x_n \geq 0$,

3.3.3 Methods of Solution of Linear Programming Problems

Linear programming problems are solved using **graphical**, **simplex** and the **barrier** or **interior point** methods. The simplex methods and the interior point methods are the techniques that are in wide use today. Both techniques generate an improving sequence of trial solutions until a solution that satisfies the conditions for an optimal solution is attained.

3.3.4 The Simplex Method

The Simplex method is a method that proceeds from one Basic Feasible Solution (BFS) or extreme point of the feasible region of an LP problem expressed in tableau form to another, in such a way as to continually increase (or decrease) the value of the objective function until optimality is reached. The Simplex method moves from one extreme point to one of its neighboring extreme point. It is a matrix method used for higher dimensions (more than 2) of decision variables. The Simplex method is based on the Gauss -Jordan elimination i.e. to locate the pivot and eliminate both above and below the pivot. If the problem is feasible, then it finds the solution in a finite number of iterative steps.

The Simplex method or algorithm for solving linear programming problems was developed by George Dantzig in 1947. An example is an n-dimensional convex figure that has exactly (n+1) extreme points. For instance, a simplex in two dimensions is a triangle, and in three dimensions is a tetrahedron. The Simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem. It provides a structured method for moving from one BFS to another, always maintaining or improving the objective function until an optimal solution is obtained.

In Linear Programming if the initial Basic Feasible Solution (BFS) exists then the normal **Simplex Method** can be used. In cases where such an obvious initial BFS does not exist, the ***M*-Method** or **Two-Phase Simplex Method** can be used. The *M*-method is sensitive to roundoff error when being implemented on computers. The two-phase method is used to circumvent this difficulty.

In the tableau implementation of the primal simplex algorithm, the right-hand-side column is always nonnegative so the basic solution is feasible at every iterative step. The Simplex method will always terminate in a finite number of steps with an indication that a unique optimal solution has been obtained or that one of these special cases has occurred. These special cases are Alternative optimal solutions, Unbounded solutions and Infeasible solutions.

3.3.5 The Simplex Algorithm

The principle underlying the Simplex method involves the use of the algorithm which is made up of two phase, where each phase involves a special sequence of number of elementary row operations known as pivoting. The first phase of the algorithm, is finding an initial basic feasible solution (BFS) to the original problem and the second phase, consists of finding an optimal solution to the problem which begins from the initial basic feasible solution.

The following steps are involved in using the simplex algorithm to solve linear programming program:

Step 1: Convert the soft constraints into equalities by introducing slack or surplus (excess) variables.

Step 2: Set up an initial tableau.

Step 3: Test the coefficients of the objective function row to determine whether an optimal solution has been reached, i.e., whether the optimality condition that all coefficients are nonnegative in that row is satisfied. If not, select a currently non-basic variable to enter the basis.

Step 4: Determine the entering variables. Select the most negative value in the objective function row or entry and its variable becomes the leaving variable.

Step 5: Determine the leaving or departing variables. The leaving basic variable corresponds to the *smallest or minimum* positive ratio found by dividing constants of each constraint row by the current positive coefficients or entries of the entering non-basic variable.

Step 6: Perform a pivot operation. The idea of pivoting is to make a non basic variable to become a basic variable. This change of basis is done using the Gauss-Jordan elimination. If all the coefficients in objective function row are nonnegative or positive, then the basic solution is optimum.

Example

Consider the LP problem:

Max $z = 3x_1 + x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$x_1 + 2x_2 + x_3 \leq 6$

$2x_1 + 2x_2 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Solution

By adding slack variables x_4, x_5 and x_6, we obtain the following system of equations for soft constraints:

$$2x_1 + x_2 + x_3 \;\; + x_4 \qquad\quad = 2$$

$$x_1 \;\; + \; 2x_2 + \; x_3 + \quad x_5 \quad = 6$$

$$2x_1 + 2x_2 + 3x_3 \; + \qquad x_6 = 5$$

Initial Tableau:

Set up an initial tableau.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution	Ratio
x_4	2	1*	1	1	0	0	2	2*
x_5	1	2	3	0	1	0	5	2.5
x_6	2	2	1	0	0	1	6	3
z	-3	-1	-3	0	0	0	0	

The initial Basic Feasible Solution (BFS), x = [0, 0, 0, 2, 5, 6] and it is not optimum.

Second Tableau:

We choose x_2 as the entering variable since *any* non basic variable with negative coefficient can be chosen as entering variable. The smallest ratio is given by x_4 row. Thus x_4 is the departing (leaving) variable and its row is pivoted.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution	Ratio
x_2	2	1	1	1	0	0	2	2
x_5	-3	0	1*	-2	1	0	1	1*
x_6	-2	0	-1	-2	0	1	2	-
z	-1	0	-2	0	0	0	2	

The current Basic Feasible Solution, x = [0, 2, 0, 0, 1, 2] and it is not optimum.

Third Tableau:

The variable with the most negative coefficient is x_3 and it is chosen as the entering variable. The smallest ratio is given by x_5 row. Thus x_5 is the departing (leaving) variable and its row is pivoted.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution	Ratio
x_2	5*	1	0	3	-1	0	1	1/5*
x_3	-3	0	1	-2	1	0	1	-
x_6	-5	0	-4	-2	1	1	3	-
z	-7	0	0	-3	2	0	4	

The current Basic Feasible Solution x = [0, 1, 1, 0, 0, 3] and it is not optimum.

Fourth Tableau:

The variable with the most negative coefficient is x_1 and it is chosen as the entering variable. The smallest ratio is given by x_2 row. Thus x_2 is the departing (leaving) variable and its row is pivoted.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
x_1	1	1/5	0	3/5	-1/5	0	1/5
x_3	-3	0	1	-2	1	0	8/5
x_6	0	1	0	-1	0	1	4
z	0	7/5	0	6/5	3/5	0	27/5

The optimal Basic Feasible Solution (BFS) x* = [1/5, 0, 8/5, 0, 0, 4] and z* = 27/5 = 5.4. It means that the maximum value of 5.4 occurs at (1/5, 0, 8/5).

3.3.6 The Interior Point Methods

The Interior Point Methods (also referred to as **barrier methods**) are a certain class of algorithms to solve linear and nonlinear convex optimization problems. The interior point method was invented by John von Neumann. Von Neumann suggested a new method of linear programming, using the homogeneous linear system of Gordan (1873) which was later popularized by Karmarkar's algorithm, developed by Narendra Karmarkar in 1984 for linear programming. The methods include the Khachyan's ellipsoid method and Karmarkar's projective scaling method. Contrary to the Simplex method, it reaches an optimal solution by traversing the interior of the feasible region (Wikimedia Foundation, Inc, 2014).

An interior point method is a linear or nonlinear programming method that achieves optimization by going through the interior of the region rather than around its surface. Unlike Simplex Method, iterates are calculated not on the boundary, but in the interior of the feasible region. This algorithm is an iterative algorithm that makes use of projective transformations and a potential function (Karmarkar"s potential function) (Forsgren et al, 2002). The current iterate is mapped to the center of the special set in the interior feasible region using a projective transformation.

3.4.0 Karmarkar's Method

Karmarkar's algorithm is an algorithm introduced Narendra Karmarkar in 1984 for solving linear programming problems. It was the first reasonably efficient algorithm that solves these problems in polynomial time. It is also known as *Karmarkar's interior point LP algorithm*. It is an iterative algorithm that, given an initial point X_0 and parameter$q$, generates a sequence

$X_1, X_2, \dots, X_N$ which are the solutions of linear program (LP).

The method or algorithm starts with a trial solution and shoots it towards the optimum solution. The polynomial running-time of this algorithm combined with its promising performance created a tremendous excitement and spawned a flurry of research activity in interior – point method of linear programming that eventually transformed the entire field of optimization.

Karmarkar's still remains interesting because of its historical impact and Projective – scaling approach. Karmarkar's projective scaling method seeks the optimum solution to an LP problem by moving through the interior of the feasible region.

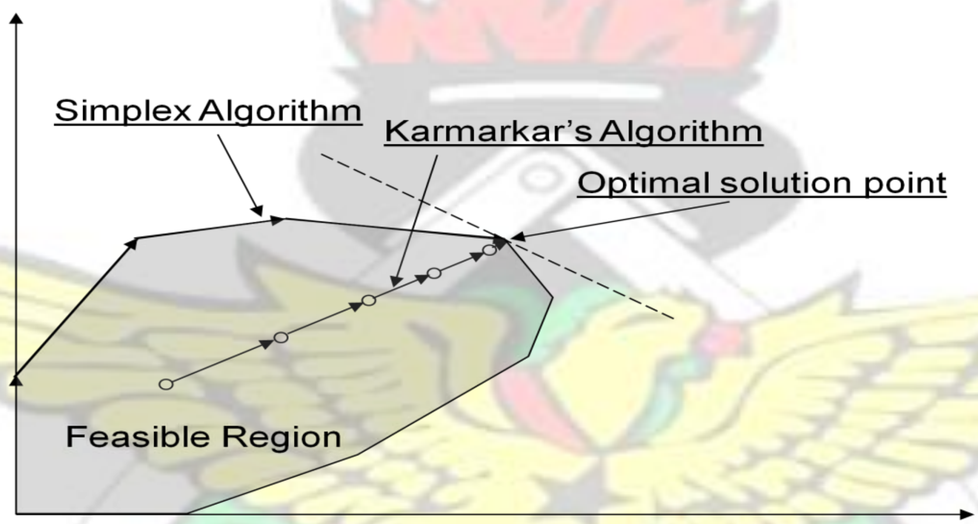

Fig. 3.1 Graphical Representation of Simplex and Karmarkar Algorithms

3.4.1 Karmakar's Projective Algorithm

The basic idea of iteration is to transform linear problem via an affine scaling, so that the current point x_k is transformed to a "central point" $e = (1,1,1,\dots,1)^T$, then to take a step along the steepest-descent direction in transformed space, and finally to map the resulting point back to the corresponding position in the original space. Conceptually, Karmarkar's algorithm is described in similar terms, except that a "projective scaling" transformation is used.

Karmarkar's Projective Scaling method follows iterative steps to find the optimal solution. It starts with a trial solution and shoots it towards the optimum solution.

In using the method, some special assumptions must be made.

1. The linear program has a strictly feasible point, and that the set of optimal points is bounded.
2. The linear program has a special "Canonical" form.

   Minimize $z = C^T x$

   Subject to $Ax = 0, \sum_{i=1}^{n} x_i = 1, x \geq 0$
3. The optimal objective value is zero (that is the value of the objective at the optimum is zero) $z^* = 0$.

Consider the general linear programming problem below.

Minimize $z = C^T x$

Subject to $Ax = b,$

$x \geq 0$

To use Karmarkar's method, the LP should be expressed in the following form

Minimize $z = C^T x$

Subject to $Ax = 0, \sum_{i=1}^{n} x_i = 1, x \geq 0$

Denote null space of A, $\Omega = \{x \in \mathbb{R}^n | Ax = 0\}$

Define simplex, $\Delta = \{x \in \mathbb{R}^n | e^T x = 1, x \geq 0\}, e = [1, \ldots, 1]^T$

Denote the center of the simplex $\Delta,\ a_0 = \frac{e}{n} = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]^T, a_0 \in \Delta$

It can also be rewritten as

Minimize $z = C^T x$

Subject to $x \in \Omega \cap \Delta$

Note that the constraint set (or feasible set), $\Omega \cap \Delta$ can be represented as

$\Omega \cap \Delta = \{x \in \mathbb{R}^n | Ax = 0, e^T x = 1, x \geq 0\}$

$\Omega \cap \Delta = \left\{x \in \mathbb{R} \middle| \begin{bmatrix} A \\ e^T \end{bmatrix} x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x \geq 0 \right\}$

$\Omega \cap \Delta = \left\{x \in \mathbb{R} \middle| B_0 x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x \geq 0 \right\}$

Algorithm Steps 1. Initialize: Set $k := 0;\ x_0 = a_0 = \frac{e^T}{n}$;

2. **Check stopping criterion**: Stop if $cx_k < q$where q is the termination parameter or tolerance of very small value or if$cx_k = 0$. If not go to the next step.
3. **Update**: Find a new point $x_{k+1} = \Psi(\mathrm{x_k})$ where Ψ is an update map.

First update step in the algorithm

1. Compute pre-specified step size such that,$\alpha \in (0,1)$ as $\alpha = \frac{(n-1)}{3n}$ though Karmarkar recommends a value of ¼ in his original and determine the radius of the largest sphere inscribed in the set$\{x | e^T x = 1, x \geq 0\}$ as $r = 1/\sqrt{n(n-1)}$
2. Compute the diagonal $D_0$of the feasible point x_0 as $D_0 = diag(x_0)$ and $B_0 = \begin{bmatrix} AD_0 \\ e^T \end{bmatrix}$
3. Compute the **orthogonal projector** of *c*onto the null space of B_0

$c_p = [I_n - B_0^T (B_0 B_0^T)^{-1} B_0] D_0 c$

4. Compute the **normalized orthogonal projection of c** onto the null space of B_0

$c_0 = \frac{c_p}{\|c_p\|}$ where c_0 is a unit vector in the direction of c_p.

5. Compute the **steepest-descent direction** vector$d_0 = -rc_0$.
6. Compute x_1' using $x_1' = x_0 + \alpha d_0 = x_0 - \alpha r c_0$

7. Compute x_1 by applying the inverse transformation,

$$x_1 = \frac{D_0 x_1'}{e^T D_0 x_1'}$$

General update steps in the algorithm

1. Compute pre-specified step size $\alpha = \frac{(n-1)}{3n}$ such that $\alpha \in (0,1)$ and determine the radius of the largest sphere inscribed in the set $\{x | e^T x = 1, x \geq 0\}$ as $r = 1/\sqrt{n(n-1)}$

2. Compute the matrices $D_k = \begin{bmatrix} x_{k,1} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & x_{k,n} \end{bmatrix}$ and $B_k = \begin{bmatrix} AD_k \\ e^T \end{bmatrix}$

   x_k is in general, not at the center of the simplex, so D_k whose diagonal entries are the components of the vector x_k is used to transform this point to the center.

3. Compute the **orthogonal projector** of c onto the null space of B_k.

   $c_p = [I_n - B_k^T (B_k B_k^T)^{-1} B_k] D_k c$.

4. Compute the **normalized orthogonal projection of c** onto the null space of B_k.

   $c_k = \frac{c_p}{\|c_p\|}$ where c_k is a unit vector in the direction of c_p.

5. Compute the **steepest-descent direction** vector $d_k = -r c_k$

6. Compute x_{k+1}' using $x_{k+1}' = x_k + \alpha d_k = x_k - \alpha r c_k$

7. Compute x_{k+1} by applying the inverse transformation, $x_{k+1} = \frac{D_k x_{k+1}'}{e^T D_k x_{k+1}'}$

- **Example**

  Minimize $z = 3x_1 + 3x_2 - x_3$

  Subject to $2x_1 - 3x_2 + x_3 = 0$

  $x_1 + x_2 + x_3 = 1$

  $x_1, x_2, x_3 \geq 0$

- **Solution**

$$c^T = [3 \quad 3 \quad -1], \qquad A = [2 \quad -3 \quad 1], \qquad e^T = [1 \quad 1 \quad 1] \; and \; n = 3$$

1. Initialize:

Set $k := 0;$

$e^T = [1 \quad 1 \quad 1]$

$n = 3$

$$x_0 = a_0 = \frac{e}{n} = [1/3 \quad 1/3 \quad 1/3]^T$$

2. Update: Set $x_{k+1} = \Psi(\mathrm{x_k})$ where Ψ is an update map

i. Compute pre-specified step size $\alpha = \frac{(n-1)}{3n} = \frac{3-1}{3\times 3} = 0.2222$

and $r = 1/\sqrt{n(n-1)} = \frac{1}{\sqrt{3(3-1)}} = 0.4082$

ii. Compute $D_0 = diag(x_0) = \begin{bmatrix} 0.3333 & 0 & 0 \\ 0 & 0.3333 & 0 \\ 0 & 0 & 0.3333 \end{bmatrix}$

and $B_0 = \begin{bmatrix} AD_0 \\ e^T \end{bmatrix} = \begin{bmatrix} 0.6667 & -1 & 0.3333 \\ 1 & 1 & 1 \end{bmatrix}$

iii. Compute the **orthogonal projector** of *c*onto the null space of B_0

$$c_p = [I_n - B_0^T(B_0B_0^T)^{-1}B_0]D_0c$$

$$c_p = \frac{1}{42}\begin{bmatrix} 16 & 4 & -20 \\ 4 & 1 & -5 \\ -20 & -5 & 25 \end{bmatrix}[1 \quad 1 \quad -0.3333]$$

$$c_p = \begin{bmatrix} 1.9048 \\ 0.4762 \\ -2.3810 \end{bmatrix}$$

iv. Compute the **normalized orthogonal projection of c** onto the null space of B_0,

$$c_0 = \frac{c_p}{\|c_p\|} = \begin{bmatrix} 0.6172 \\ 0.1543 \\ 0.7715 \end{bmatrix}$$

v. Compute x_1' using $x_1' = x_0 - \alpha r c_0$

$$x_1' = \begin{bmatrix} 0.3333 \\ 0.3333 \\ 0.3333 \end{bmatrix} - (0.2222 \times 0.4082) \begin{bmatrix} 0.6172 \\ 0.1543 \\ 0.7715 \end{bmatrix}$$

$$x_1' = \begin{bmatrix} 0.2773 \\ 0.3193 \\ 0.4033 \end{bmatrix}$$

vi. Compute $\bar{x}_1$ by applying the inverse transformation,

$$x_1 = \frac{D_0 x_1'}{e^T D_0 x_1'} = \begin{bmatrix} 0.2773 \\ 0.3193 \\ 0.4033 \end{bmatrix}$$

$z = 1.3867$

The iteration continues since $z > 0$

First Iteration ($k = 1$)

The first iteration is summarized below:

$$D_1 = \begin{bmatrix} 0.2773 & 0 & 0 \\ 0 & 0.3193 & 0 \\ 0 & 0 & 0.4033 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} AD_1 \\ e^T \end{bmatrix} = \begin{bmatrix} 0.5547 & -0.9580 & 0.4033 \\ 1 & 1 & 1 \end{bmatrix}$$

$$c_p = [I_n - B_1^T (B_1 B_1^T)^{-1} B_1] D_1 c = \begin{bmatrix} 1.9780 \\ 0.2199 \\ -2.1979 \end{bmatrix}$$

$$c_1 = \frac{c_p}{\|c_p\|} = \begin{bmatrix} 0.6671 \\ 0.0742 \\ -0.7413 \end{bmatrix}$$

$$x_2' = x_1 - \alpha r c_1 = \begin{bmatrix} 0.2728 \\ 0.366 \\ 0.5517 \end{bmatrix}$$

$$x_2 = \frac{D_1 x'_{(2)}}{e^T D_1 x'_{(2)}} = \begin{bmatrix} 0.0010 \\ 0.2503 \\ 0.4006 \end{bmatrix}$$

$$z = 1.1077$$

The iteration continues since$z > 0$. Similar iterations can be followed to get the final solution up to some predefined tolerance level.

3.4.2 Putting an LP in Standard Form for Karmarkar's Method

Karmarkar Method is applied to linear programming problems in the following form:

Minimize $z = C^T x$

Subject to $Ax = 0$

$\sum_{i=1}^{n} x_i, x \geq 0$

A linear programming problem in the form below must therefore be transformed into the Karmarkar's Standard form.

Maximize $z = C^T x$

Subject to $Ax \leq b,$

$x \geq 0$

1. Transform the primal maximization problem into the dual minimization problem.

   Minimize $w = by$

   Subject to $A^T y \geq C^T,$

   $y \geq 0$

2. Combine the primal and its dual problems. According to the **von Neumann Duality Principle**, the objective value *z* of the maximization problem in standard form has a maximum value if and only if the objective value *w* of a dual minimization problem has a minimum value. Therefore

   $C^T x = by$

   $C^T x - by = 0$

3. Convert the soft constraints of the primal and dual problems into equalities by inserting slack and surplus (excess) variables respectively.

4. Introduce a bounding constraint, M such that $\sum x + \sum y \leq M$, where M is sufficiently large number that include all feasible solutions of original problem. Add a slack variable to the bounding constraint to convert it into equality, $\sum x + \sum y + s = M$.

5. Homogenize the constraints which have non zero right-hand sides by introducing another dummy variable *d* (such that d = 1).

6. Replace the constraint $\sum x + \sum y + s = M$, with $\sum x + \sum y + s - Md = 0$ and write another
   equation $\sum x + \sum y + s + d = M + 1$

7. Make change of variables in the system of equations such that the RHS of the last equation becomes one, $x_i = (M + 1)a_i, i = 1, 2, 3, ...$

8. Add an artificial variable to the last constraint and a multiple of the artificial variable to each of the other constraints so that the sum of the coefficients of all variables in each constraint (except the last) will equal zero.

Example: Convert the LP problem into the Karmarkar's Standard form.

Max $\quad z = 3x_1 + x_2$

Subject to $\quad 2x_1 - x_2 \leq 2$

$$x_1 + 2x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

Transform the primal maximization problem into the dual minimization problem.

Min $\quad w = 2y_1 + 5y_2$

Subject to $\quad 2y_1 + y_2 \geq 3$

$$-y_1 + 2y_2 \geq 1$$

$$y_1, y_2 \geq 0$$

Combine the primal and its dual problems using the **von Neumann Duality Principle.**

$$3x_1 + x_2 - 2y_1 - 5y_2 = 0$$

$$x_1 - x_2 \leq 2$$

$$x_1 + 2x_2 \leq 5$$

$$2y_1 + y_2 \geq 3$$

$$-y_1 + 2y_2 \geq 1$$

$$x_1, x_2, y_1, y_2 \geq 0$$

Inserting slack and excess (surplus) variables into the set of constraints yields

$$3x_1 + x_2 - 2y_1 - 5y_2 \qquad = 0$$

$$2x_1 - x_2 + s_1 = 2$$

$$x_1 + 2x_2 + s_2 = 5$$

$$2y_1 + y_2 - e_1 = 3$$

$$-y_1 + 2y_2 - e_2 = 1$$

$x_1, x_2, y_1, y_2, s_1, s_2, e_1, e_2 \geq 0$

By inspection, it can be seen that the values of variables that yield the optimal primal solution and optimal dual solution will not any variable exceeding 10. Because we have 8 variables, the bounding constraint, M=10 (8) = 80.

$x_1 + x_2 + y_1 + y_2 + s_1 + s_2 + e_1 + e_2 \leq 80$

Adding a slack (dummy variable,d_1) to the bounding constraint yields

$x_1 + x_2 + y_1 + y_2 + s_1 + s_2 + e_1 + e_2 + d_1 = 80$

The new goal is to find a feasible solution to

$3x_1 + x_2 - 2y_1 - 5y_2 = 0$
$2x_1 - x_2 + s_1 = 2$

$x_1 + 2x_2 + s_2 = 5$

$2y_1 + y_2 - e_1 = 3$

$-y_1 + 2y_2 - e_2 = 1$

$x_1 + x_2 + y_1 + y_2 + s_1 + s_2 + e_1 + e_2 + d_1 = 80$

$x_1, x_2, y_1, y_2, s_1, s_2, e_1, e_2, d_1 \geq 0$

Introduce a new dummy variable d_2 (such that $d_2 = 1$) to "homogenize" the constraints which have non zero right-hand sides. This gives the following system of equations:

$3x_1 + x_2 - 2y_1 - 5y_2 = 0$

$2x_1 - x_2 + s_1 - 2d_2 = 0$

$x_1 + 2x_2 + s_2 - 5d_2 = 0$

$2y_1 + y_2 - e_1 - 3d_2 = 0$

$-y_1 + 2y_2 - e_2 - d_2 = 0$

$x_1 + x_2 + y_1 + y_2 + s_1 + s_2 + e_1 + e_2 + d_1 = 80$

$x_1 + x_2 + y_1 + y_2 + s_1 + s_2 + e_1 + e_2 + d_1 + d_2 = 81$

$x_1, x_2, y_1, y_2, s_1, s_2, e_1, e_2, d_1, d_2 \geq 0$

Make change of variables in the system of equations above. Make change of variables in the system of equations such that the RHS of the last equation becomes one.

$x_1 = (M+1)a_1, x_2 = (M+1)a_2, y_1 = (M+1)a_3, y_2 = (M+1)a_4, s_1 = (M+1)a_5,\ s_2 = (M+1)a_6, e_1 = (M+1)a_7, e_2 = (M+1)a_8, d_1 = (M+1)a_9\ and\ d_2(M+1)a_{10}$

This yields the following:

$3a_1 + a_2 - 2a_3 - 5a_4 = 0$

$2a_1 - a_2 + a_5 - 2a_{10} = 0$

$a_1 + 2a_2 + a_6 - 5a_{10} = 0$

$2a_3 + a_4 - a_7 - 3a_{10} = 0$

$-a_3 + 2a_4 - a_8 - a_{10} = 0$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 - 80a_{10} = 0$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} = 1$

$a_1, a_2, a_3, q_4, a_5, a_6, a_7, a_8, a_9, a_{10} \geq 0$

Introduce or define an artificial variable a_{11} to the last constraint and a multiple of a_{11} to each of the other constraints. The multiple is chosen so that the sum of the coefficients of all variables in each constraint (except the last) will equal zero. The artificial variable a_{11} is then minimized subject to the rest of the system of equations.

This yields an LP in the form for Karmarkar's Method.

Min $z = a_{11}$

Subject to:

$$3a_1 + a_2 - 2a_3 - 5a_4 + 3a_{11} = 0$$

$$2a_1 - a_2 + a_5 - 2a_{10} + a_{11} = 0$$

$$q_1 + 2q_2 + q_6 - 5q_{10} + q_{11} = 0$$

$$2q_3 + q_4 + q_7 - 3q_{10} + q_{11} = 0$$

$$-q_3 + 2q_4 + q_8 - q_{10} + q_{11} = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 + q_9 - 80q_{10} + 71q_{11} = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 + q_9 + q_{10} + q_{11} = 1$$

$$q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11} \geq 0$$

CHAPTER 4 DATA ANALYSIS AND RESULT DISCUSSION

4.0 Introduction

This chapter describes how the data collected from Nkoranman Rural Bank Limited, Sunyani, is solved using the proposed model.

4.1 Data on Types of Loans

The management of Nkoranman Rural Bank Limited, Sunyani is formulating a loan policy involving GH¢330,000.00 for the year 2015. The bank is obligated to give loans to different clientele. Table 4.1 provides the type of loans, the interest rate charged by the bank and the probability of bad debt as estimated from past experience.

Table 4.1: Data on Types of Loans, Interest rate and Probability of bad debt

S/N	LOAN TYPE	INTEREST RATE	PROBABILITY OF BAD DEBT
1	Salary	0.30	0.00
2	Susu	0.48	0.01
3	Agriculture	0.32	0.01
4	Funeral	0.10	0.01
5	Commercial	0.32	0.15
6	Personal	0.32	0.10
7	Microfinance	0.45	0.05

Source: Nkoranman Rural Bank Limited, Sunyani

According to the bank's policy, there are limits on how its loan funds are allocated. The policy requires that the bank observes the following conditions:

- Allocate at most 60% of the total funds to Salary, Susu and Microfinance loans.

- To help farmers in and around Sunyani, Agriculture loans must be at most 50% of Susu and Microfinance loans.

- The sum of Funeral loans, Commercial loans and Personal loans must be at most 30% of the total funds.

- The total ratio for bad debt on all loans must not exceed 0.10.

4.2 Linear Programming Model Formulation

The variables of the model are defined as follows in Ten Thousands of Ghana cedis:

x_1= Salary loans

x_2= Susu loans

x_3= Agriculture loans

x_4= Funeral loans

x_5= Commercial loans

x_6= Personal loans

x_7= Microfinance loans

The objective of the Nkoranman Rural Bank is to maximize its net returns. This is determined as the difference between the revenue from interest and lost funds due to bad debts. Since bad debts are unrecoverable both as principal and interest, the objective function may be written as

Maximize

$$.48(x_2 - 0.01x_2) + 0.32(x_3 - 0.01x_3) + 0.1(x_4 - 0.01x_4) + 0.32(x_5 -$$

$$_6 - 0.1x_6) + 0.45(x_7 - 0.05x_7) - 0.01x_2 - 0.01x_3 - 0.01x_4 - 0.15x_5 - x$$

$z = 0.3(x_1) + 0$

$0.15x_5) + 0.32($

$0.1x_6 - 0.05x_7$

This is simplified as

Maximize

$$z = 0.3x_1 + 0.4652x_2 + 0.3068x_3 + 0.089x_4 + 0.122x_5 + 0.188x_6 + 0.3775x_7$$

Subject to the conditions outlined by the bank

- The total fund allocated for loans is at most 33(in Ten Thousands of Ghana cedis).

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 33$$

- Allocate at most 60 % of the total funds to Salary, Susu and Microfinance loans.

$$x_1 + x_2 + x_7 \leq 0.6(33)$$

- To help farmers in and around Sunyani, Agriculture loans must be at most 50 % of Susu and Microfinance loans.

$$x_3 \leq 0.5(x_2 + x_7)$$

- The sum of Funeral, Commercial loans and Personal loans must be at most 30 % of the total funds.

$$x_4 + x_5 + x_6 \leq 0.3(33)$$

- The total ratio for bad debt on all loans must not exceed 0.10.

$$\frac{0.01x_2 + 0.01x_3 + 0.01x_4 + 0.15x_5 + 0.1x_6 + 0.05x_7}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7} \leq 0.1$$

The non-negativity constraints

$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

The standard LP problem is to:

Max

$$z = 0.3x_1 + 0.4652x_2 + 0.3068x_3 + 0.089x_4 + 0.122x_5 + 0.188x_6 + 0.3775x_7$$

Subject to:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 33$$

$$x_1 + x_2 + x_7 \leq 19.8$$

$$-0.5x_2 + x_3 - 0.5x_7 \leq 0$$

$$x_4 + x_5 + x_6 \leq 9.9$$

$$-0.1x_1 - 0.09x_2 - 0.09x_3 - 0.09x_4 + 0.05x_5 - 0.05x_7 \leq 0$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

4.3 Transforming the LP into Standard Form for Karmarkar's Method

The LP is converted into Standard form for Karmarkar's method as shown in Appendix B**.**

The linear programming problem now in standard form for Karmarkar's Method is as follows:

Minimize $z = a_{27}$

Subject to:

$$0.3a_1 + 0.4652a_2 + 0.3068a_3 + 0.089a_4 + 0.122a_5 + 0.188a_6 + 0.3775a_7 - 33a_{14} - 19.8a_{15} - 9.9a_{17} + 60.8515a_{27} = 0$$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 - 33a_{26} + 25a_{27} = 0$$

$$a_1 + a_2 + a_7 + a_9 - 19.8a_{26} + 15.8a_{27} = 0$$

$$-0.5a_2 + a_3 - 0.5a_7 + a_{10} - a_{27} = 0$$

$$a_4 + a_5 + a_6 + a_{11} - 9.9a_{26} + 5.9a_{27} = 0$$

$$-0.1a_1 - 0.09a_2 - 0.09a_3 - 0.09a_4 + 0.05a_5 - 0.05a_7 + a_{12} - 0.63a_{27} = 0$$

$$a_{14} + a_{15} - 0.1a_{18} - a_{19} - 0.3a_{26} - 0.6a_{27} = 0$$

$$a_{14} + a_{15} - 0.5a_{16} - 0.09a_{18} - a_{20} - 0.4652a_{26} + 0.0552a_{27} = 0$$

$$a_{14} + a_{16} - 0.09a_{18} - a_{21} - 0.3068a_{26} - 0.6032a_{27} = 0$$

$$a_{14} + a_{17} - 0.09a_{18} - a_{22} - 0.089a_{26} - 0.821a_{27} = 0$$

$$a_{14} + a_{17} + 0.05a_{18} - a_{23} - 0.122a_{26} - 0.928a_{27} = 0$$

$$a_{14} + a_{17} - a_{24} - 0.188a_{26} - 0.812a_{27} = 0$$

$$a_{14} + a_{15} - 0.5a_{16} - 0.05a_{18} - a_{25} - 0.3775a_{26} - 0.0725a_{27} = 0$$

$$\sum_{i=1}^{25} a_i - 99a_{26} + 74a_{27} = 0$$

$$\sum_{i=1}^{27} a_i = 1$$

$$a_i \geq 0, i = 1, 2, 3, \ldots, 27$$

4.4 Solution Procedure

C =[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]

0.3 0.4652 0.3068 0.089 0.122 0.188 0.3775 0 0 0 0 0 0 -33 -19.8 0 -9.9 0 0 0 0 0 0 0 0 0 60.8515

1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -33 25

1 1 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -19.8 15.8

0 -0.5 1 0 0 0 -0.5 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1

0 0 0 1 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -9.9 5.9

-0.1 -0.09 -0.09 -0.09 0.05 0 -0.05 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.63

A= 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 -0.1 -1 0 0 0 0 0 0 -0.3 -0.6

0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 -0.5 0 -0.09 0 -1 0 0 0 0 0 -0.4652 -0.0552

0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 -0.09 0 0 -1 0 0 0 0 -0.3068 -0.6032

0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 -0.09 0 0 0 -1 0 0 0 -0.089 -0.821

0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0.05 0 0 0 0 -1 0 0 -0.122 -0.928

0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 -1 0 -0.188 -0.812

0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 -0.5 0 -0.05 0 0 0 0 0 0 -1 -0.3775 -0.0725

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 -99 74

e = [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]

The Karmarkar Algorithm was written in MATLAB programming language code and ran for the matrices **C**, **A** and **e** with a tolerance of1.0×10^{-15} using a Pentium (R) Dual-Core CPU Toshiba computer with 500 GB capacity hard disk drive, processing speed of 2.30 GHz and a RAM of 3.00 GB. The iterative process converges at 1292.

4.5 Results

The iteration converges at 1292 and the optimal solutions are:

$a_1 = 0.0051$, $a_2 = 0.1663$, $a_3 = 0.0640$, $a_4 = 0.0043$, $a_5 = 0.0279$, $a_6 = 0.035$, $a_7 = 0.215$, $a_8 = 0.0054$, $a_9 = 0.0051$, $a_{10} = 0.0299$, $a_{11} = 0.0313$, $a_{12} = 0.2908$, $a_{13} = 0.2908$, $a_{14} = 0.0020$, $a_{15} = 0.0024$, $a_{16} = 0.0012$, $a_{17} = 0.0002$, $a_{18} = 0.0006$, $a_{19} = 0.0279$, $a_{20} = 0.0015$, $a_{21} = 0.0001$, $a_{22} = 0.0013$, $a_{23} = 0.0001$, $a_{24} = 0.0004$, $a_{25} = 0.0001$, $a_{26} = 0.0100$, $a_{27} = 0.0000$

The solutions for the primal are:

$a_1 = 0.0051$, $a_2 = 0.1663$, $a_3 = 0.0640$, $a_4 = 0.0043$, $a_5 = 0.0279$, $a_6 = 0.035$, $a_7 = 0.215$

Using the transformation $x_i = (M+1)a_i = 100a_i$, yields the optimal solutions for the basic variables in the primal LP.

$x_1 = 100(0.0051) = 0.51$, $x_2 = 100(0.1663) = 16.63$, $x_3 = 100(0.0640) = 6.40$

$x_4 = 100(0.0043) = 0.43$, $x_5 = 100(0.0279) = 2.79$, $x_6 = 100(0.0356) = 3.56$

$x_7 = 100(0.0215) = 2.15$

The Optimal Objective Solution is determined as

$z = 0.3x_1 + 0.4652x_2 + 0.3068x_3 + 0.089x_4 + 0.122x_5 + 0.188x_6 + 0.3775x_7$

$z = 0.3(0.51) + 0.4652(16.63) + 0.3068(6.40) + 0.089(0.43) + 0.122(2.79) + 0.188(3.56) + 0.3775(2.15)$

Therefore $z = 11.712351$

Table 4.2: Table of Results

BASIC VARIABLES	LOAN TYPES	AMOUNT TO BE ALLOCATED (GHc)	PERCENTAGE AMT. TO BE ALLOCATED (%)
x_1	Salary Loans	5,100.00	1.55
x_2	Susu Loans	166,300.00	50.39
x_3	Agriculture Loans	64,000.00	19.39
x_4	Funeral Loans	4,300.00	1.30
x_5	Commercial Loans	27,900.00	8.45
x_6	Personal Loans	35,600.00	10.79
x_7	Microfinance Loans	21,500.00	6.52
Net Profit on Loans		**z**	**117,123.51**

4.6 Discussion of Results

According to the results, Nkoranman Rural Bank Limited, Sunyani, must allocate GHc 5,100.00 for Salary loans, GHc 166,300.00 for Susu loans, GHc 64,000.00 for Agricultural loans, GHc 4,300.00 for Funeral loans, GHc 27,900.00 for Commercial loans, GHc 35,600.00 for Personal loans and GHc 21,500.00 for Microfinance loans. If the allocation is done effectively and efficiently, the bank will realize a maximum return of GHc 117,123.51 on its loan portfolio by adopting this Model. This amount represents 35.5 % of the total fund to be disbursed by the bank, as against the 28.2 % profit made in 2014.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The data collected on the loan portfolio composition of Nkoranman Rural Bank, Sunyani was modeled successfully using Linear Programming Model. The model was solved using Karmarkar's Algorithm. The results show that Nkoranman Rural Bank Limited, Sunyani, will realize an optimal annual profit of GHc 117,123.51 on its loan portfolio if the management of the bank adopts the Model and implement it accordingly. This amount represents 35.5 % of the total fund to be disbursed by the bank for the year 2015, as against the 28.2 % profit made in 2014. To achieve this, the bank must disburse or allocate GHc 5,100.00 for Salary loans, GHc 166,300.00 for Susu loans, GHc 64,000.00 for Agricultural loans, GHc 4,300.00 for Funeral loans, GHc 27,900.00 for Commercial loans, GHc 35,600.00 for Personal loans and GHc 21,500.00 for Microfinance loans.

5.2 Recommendations

It is apparent that the use of the LP as Optimization Model and the Karmarkar's Algorithm as a method of solution for allocating funds for the loan portfolio of Nkoranman Rural Bank, Sunyani yields maximum annual returns 35.5 % of the total fund to be disbursed by the bank.

I recommend that Nkoranman Rural Bank, Sunyani adopt and implement the Model strictly and effectively in their allocation of funds for loans. Managers of Banks and other Lending institutions should employ mathematicians and computer scientists to help them use scientific methods to disburse funds for their loans. I also recommend that students and researchers compare Simplex method to Karmarkar's method.

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APPENDICES

APPENDIX A: QUESTIONAIRE

KWAME NKRUMAH UNIVERISTY OF SCIENCE AND TECHNOLLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING

Topic: Optimizing Banks' Loan Portfolio in Ghana

QUESTIONNAIRE

SECTION A: GENERAL INFORMATION OF THE BANK

1. Name of Bank: ....
2. Name of General Manager: ....
3. Headquarters of Bank: ....
4. Number of Branches: ....
5. Total Staff Strength: ....
6. Location: ....
7. Date Established and Registered: ....
8. Metropolitan/ Municipal/ District Assembly: .... 9. Region: ....

SECTION B: GENERAL INFORMATION OF THE BRANCH

1. Name of Branch: ....
2. Name of Branch Manager: ....
3. Name of Loan Manager/Officer: ....
4. Location: ....
5. Date Established and Registered: ....

6. Metropolitan/ Municipal/ District Assembly: .... 7. Region: ....

SECTION C: LOAN COMPOSITION (TYPES OF LOANS)

Tick the box beside Yes if the bank gives the type of Loan or No if the bank does not.

1. Salary Loan Yes ☐ No ☐
2. Susu loan Yes ☐ No ☐
3. Student loan Yes ☐ No ☐
4. Agriculture loan Yes ☐ No ☐ 5. Funeral loan Yes No
6. Vehicle loan Yes ☐ No ☐
7. House loan Yes ☐ No ☐
8. Commercial loan Yes ☐ No ☐
9. Personal loan Yes ☐ No ☐
10. Others (Specify): ☐ ☐ ...., ...,

SECTION D: DATA ON THE TYPES OF LOANS

S/N	LOAN TYPE	INTEREST RATE	PROBABILITY OF BAD DEBT
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

❖ Total **FUNDS** allocated for **LOANS** in **2015**: ....

SECTION E: OTHER CORE FUNCTIONS OF THE BANK ASIDE GIVING LOANS

1. ....
2. ....
3. ....
4. ....
5. ....

THANK YOU AND GOD BLESS YOU.

APPENDIX B: TRANSFORMING LP INTO STANDARD FORM FOR KARMARKAR'S METHOD

The dual of the primal LP problem above is written as

Min

$w = 33y_1 + 19.8y_2 + 9.9y_4$

Subject to

$y_1 + y_2 - 0.1y_5 \geq 0.3$

$y_1 + y_2 - 0.5y_3 - 0.09y_5 \geq 0.4652$

$y_1 + y_3 - 0.09y_5 \geq 0.3068$

$y_1 + y_4 - 0.09y_5 \geq 0.089$

$y_1 + y_4 + 0.05y_5 \geq 0.122$

$y_1 + y_4 \geq 0.188$

$y_1 + y_2 - 0.5y_3 - 0.05y_5 \geq 0.3775$

$y_1, y_2, y_3, y_4, y_5 \geq 0$

Combining the primal problem and its dual, yields

$0.3x_1 + 0.4652x_2 + 0.3068x_3 + 0.089x_4 + 0.122x_5 + 0.188x_6 + 0.3775x_7 - 33y_1 - 16.5y_2 - 9.9y_4 = 0$

Inserting slack variables ($x_8, x_9, x_{10}, x_{11}, x_{12}$) and excess (surplus) variables ($y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}$) into the set of constraints for both the primal and the dual problems, gives

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 33$

$x_1 + x_2 + x_7 + x_9 = 19.8$

$-0.5x_2 + x_3 - 0.5x_7 + x_{10} = 0$

$x_4 + x_5 + x_6 + x_{11} = 9.9$

$-0.1x_1 - 0.09x_2 - 0.09x_3 - 0.09x_4 + 0.05x_5 - 0.05x_7 + x_{12} = 0$

$y_1 + y_2 - 0.1y_5 - y_6 = 0.3$

$y_1 + y_2 - 0.5y_3 - 0.09y_5 - y_7 = 0.4652$

$y_1 + y_3 - 0.09y_5 - y_8 = 0.3068$

$y_1 + y_4 - 0.09y_5 - y_9 = 0.089$

$y_1 + y_4 + 0.05y_5 - y_{10} = 0.122$

$y_1 + y_4 - y_{11} = 0.188$

$y_1 + y_2 - 0.5y_3 - 0.05y_5 - y_{12} = 0.3775$

All variables ≥ 0

Introduce a bounding constraint, M such that the sum of all independent variables is at most M.

$i.e \sum_{i=1}^{12} x_i + \sum_{i=1}^{12} y_i \leq M$

Adding a slack variable, x_{13} to the bounding constraint and taking M = 99, yields

$\sum_{i=1}^{13} x_i + \sum_{i=1}^{12} y_i = 99$

Introducing a new dummy variable *d* (such that d = 1) to "homogenize" the constraints which have non zero right-hand sides, gives the following system of equations:

$0.3x_1 + 0.4652x_2 + 0.3068x_3 + 0.089x_4 + 0.122x_5 + 0.188x_6 + 0.3775x_7 - 33y_1 -$

$19.8y_2 - 9.9y_4 = 0$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 - 33d = 0$

$x_1 + x_2 + x_7 + x_9 - 19.8d = 0$

$-0.5x_2 + x_3 - 0.5x_7 + x_{10} = 0$

$x_4 + x_5 + x_6 + x_{11} - 9.9d = 0$

$-0.1x_1 - 0.09x_2 - 0.09x_3 - 0.09x_4 + 0.05x_5 - 0.05x_7 + x_{12} = 0$

$y_1 + y_2 - 0.1y_5 - y_6 - 0.3d = 0$

$y_1 + y_2 - 0.5y_3 - 0.09y_5 - y_7 - 0.4652d = 0$

$y_1 + y_3 - 0.09y_5 - y_8 - 0.3068d = 0$

$y_1 + y_4 - 0.09y_5 - y_9 - 0.089d = 0$

$y_1 + y_4 + 0.05y_5 - y_{10} - 0.122d = 0$

$y_1 + y_4 - y_{11} - 0.188d = 0$

$y_1 + y_2 - 0.5y_3 - 0.05y_5 - y_{12} - 0.3775d = 0$

$\sum_{i=1}^{13} x_i + \sum_{i=1}^{12} y_i - 99d = 0$

$\sum_{i=1}^{13} x_i + \sum_{i=1}^{12} y_i + d = 100$

All variables ≥ 0

Transform the variables, such that the RHS of the last equation is equal to 1.

$x_i = (M+1)a_i, i = 1, 2, 3, \ldots, 13$

$y_i = (M+1)a_{13+i}, i = 1, 2, 3, \ldots, 12$

$d = (M+1)a_{26}$

Using the transformation of the variables, the system of equations becomes

$$100(0.3a_1 + 0.4652a_2 + 0.3068a_3 + 0.089a_4 + 0.122a_5 + 0.188a_6 + 0.3775a_7 - 33a_{14} - 19.8a_{15} - 9.9a_{17}) = 0$$

$$100(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 - 33a_{26}) = 0$$

$$100(a_1 + a_2 + a_7 + a_9 - 19.8a_{26}) = 0$$

$$100(-0.5a_2 + a_3 - 0.5a_7 + a_{10}) = 0$$

$$100(a_4 + a_5 + a_6 + a_{11} - 9.9a_{26}) = 0$$

$$100(-0.1a_1 - 0.09a_2 - 0.09a_3 - 0.09a_4 + 0.05a_5 - 0.05a_7 + a_{12}) = 0$$

$$100(a_{14} + a_{15} - 0.1a_{18} - a_{19} - 0.3a_{26}) = 0$$

$$100(a_{14} + a_{15} - 0.5a_{16} - 0.09a_{18} - a_{20} - 0.4652a_{26}) = 0$$

$$100(a_{14} + a_{16} - 0.09a_{18} - a_{21} - 0.3068a_{26}) = 0$$

$$100(a_{14} + a_{17} - 0.09a_{18} - a_{22} - 0.089a_{26}) = 0$$

$$100(a_{14} + a_{17} + 0.05a_{18} - a_{23} - 0.122a_{26}) = 0$$

$$100(a_{14} + a_{17} - a_{24} - 0.188a_{26}) = 0$$

$$100(a_{14} + a_{15} - 0.5a_{16} - 0.05a_{18} - a_{25} - 0.3775a_{26}) = 0$$

$$100(\sum_{i=1}^{13} a_i + \sum_{i=14}^{25} a_i - 99a_{26}) = 0$$

$$100(\sum_{i=1}^{26} a_i) = 100$$

The above system of equalities can be simplified as

$$0.3a_1 + 0.4652a_2 + 0.3068a_3 + 0.089a_4 + 0.122a_5 + 0.188a_6 + 0.3775a_7 - 33a_{14} - 19.8a_{15} - 9.9a_{17} = 0$$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 - 33a_{26} = 0$$

$a_1 + a_2 + a_7 + a_9 - 19.8a_{26} = 0$

$-0.5a_2 + a_3 - 0.5a_7 + a_{10} = 0$

$a_4 + a_5 + a_6 + a_{11} - 9.9a_{26} = 0$

$-0.1a_1 - 0.09a_2 - 0.09a_3 - 0.09a_4 + 0.05a_5 - 0.05a_7 + a_{12} = 0$

$a_{14} + a_{15} - 0.1a_{18} - a_{19} - 0.3a_{26} = 0$

$a_{14} + a_{15} - 0.5a_{16} - 0.09a_{18} - a_{20} - 0.4652a_{26} = 0$

$a_{14} + a_{16} - 0.09a_{18} - a_{21} - 0.3068a_{26} = 0$

$a_{14} + a_{17} - 0.09a_{18} - a_{22} - 0.089a_{26} = 0$

$a_{14} + a_{17} + 0.05a_{18} - a_{23} - 0.122a_{26} = 0$

$a_{14} + a_{17} - a_{24} - 0.188a_{26} = 0$

$a_{14} + a_{15} - 0.5a_{16} - 0.05a_{18} - a_{25} - 0.3775a_{26} = 0$

$\sum_{i=1}^{25} a_i - 99a_{26} = 0$

$\sum_{i=1}^{26} a_i = 1$

$a_i \geq 0, i = 1, 2, 3, \dots, 26$

Introduce an artificial variable a_{27} into the last constraint and a multiple of a_{27} to each of the other constraints. The multiple is chosen so that the sum of the coefficients of all variables in each constraint (except the last) will equal zero. The artificial variable a_{27} is then minimized subject to the rest of the system of equations. This yields a linear programming problem in a standard form for Karmarkar's Method as follows:

Minimize $z = a_{27}$

Subject to: $0.3a_1 + 0.4652a_2 + 0.3068a_3 + 0.089a_4 + 0.122a_5 + 0.188a_6 + 0.3775a_7 - 33a_{14} - 19.8a_{15} - 9.9a_{17} + 60.8515a_{27} = 0$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 - 33a_{26} + 25a_{27} = 0$

$a_1 + a_2 + a_7 + a_9 - 19.8a_{26} + 15.8a_{27} = 0$

$-0.5a_2 + a_3 - 0.5a_7 + a_{10} - a_{27} = 0$

$a_4 + a_5 + a_6 + a_{11} - 9.9a_{26} + 5.9a_{27} = 0$

$-0.1a_1 - 0.09a_2 - 0.09a_3 - 0.09a_4 + 0.05a_5 - 0.05a_7 + a_{12} - 0.63a_{27} = 0$

$a_{14} + a_{15} - 0.1a_{18} - a_{19} - 0.3a_{26} - 0.6a_{27} = 0$

$a_{14} + a_{15} - 0.5a_{16} - 0.09a_{18} - a_{20} - 0.4652a_{26} + 0.0552a_{27} = 0$

$a_{14} + a_{16} - 0.09a_{18} - a_{21} - 0.3068a_{26} - 0.6032a_{27} = 0$

$a_{14} + a_{17} - 0.09a_{18} - a_{22} - 0.089a_{26} - 0.821a_{27} = 0$

$a_{14} + a_{17} + 0.05a_{18} - a_{23} - 0.122a_{26} - 0.928a_{27} = 0$

$a_{14} + a_{17} - a_{24} - 0.188a_{26} - 0.812a_{27} = 0$

$a_{14} + a_{15} - 0.5a_{16} - 0.05a_{18} - a_{25} - 0.3775a_{26} - 0.0725a_{27} = 0$

$\sum_{i=1}^{25} a_i - 99a_{26} + 74a_{27} = 0$

$\sum_{i=1}^{27} a_i = 1$

$a_i \geq 0, i = 1, 2, 3, \ldots, 27$

APPENDIX C: MATLAB CODES FOR KARMARKAR‟S ALGORITHM

```
C=input('Enter the Matrix C:')

A=input('Enter the Matrix

A:') e=input('Enter the Matrix

e:')  mn=size(A) m=mn(1)

n=mn(2)

x1=([e]/n)'

I=eye(n)

r=1/sqrt(n*(n-

1)) a=(n-

1)/(3*n)

x=([e]/n)'

q=10^(-15) k=0
```

```
while(C*x>q)

D=diag(x)

B=[A*D;e]

Cp=(I-B'/(B*B')*B)*C'

Co=Cp/norm(Cp)

Xn=x1-(r*a)*Co

x=(D*Xn)/(e*D*Xn

) Z=C*x k=k+1

end
```