KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

APPLICATION OF QUEUING MODEL AT THE DRIVER AND VEHICLE LICENSING AUTHORITY-KUMASI(EYE-TEST SECTION)

BY

AKWASI ADU GYAMFI

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DECLARATION

I hereby declare that this submission is my own work towards the award of the M.SC degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

Akwasi Adu Gyamfi	....	....
Student	Signature	Date

Certified by:

Dr. E. Osei-Frimpong	....	....
Supervisor	Signature	Date

Certified by:

Prof. S. K. Amponsah	....	....
Head of Department	Signature	Date

DEDICATION

This work is dedicated to my family.

ABSTRACT

In general customers do not like to wait for a long period before services are rendered. Most customers prefer to give up or leave instead of waiting in queues. Over the years, the importation of vehicles into the country has led to massive queues and congestion at the DVLA premises. The DVLA is a government agency solely responsible for issuing license and evaluating drivers and cars in Ghana. This authority is responsible for ensuring safety on the road.

The major problem facing the DVLA is that, the number of applications have increased, whiles the DVLA premises still remains the same so drivers have to queue for a long time before services are rendered.

These inconveniences might cost both parties, that is the driver who would be driving without valid documents and the DVLA which is a business entity for the nation would also loose some revenue if drivers fail to patronize the services rendered to them by the authority due to long queues.

Therefore understanding queues and knowledge on how to manage them is one of the most essential fields in operations management.

This work analysis the $M/M/1$ queuing model at the eye-test section of the DVLA(Kumasi). Due to insufficient number of Optometrist, queues are always formed at the eye-test section of the DVLA. The $M/M/1$ queuing model provides details on the arrival rates, performance measures, busy and idle period of the Optometrist, for which management of the DVLA could use and provide information to drivers on the amount of time they are expected to spend both in the queue and in the system during the five working days at the eye-test section of the DVLA.

In all Monday and Friday recorded the highest arrival rates and performance measures as compared to the other three working days.

Customers can therefore choose to go through the eye-test examination on Tuesdays, Wednesdays and Thursdays to avoid long queues and congestions since these days recorded the least arrival rates and performance measures.

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CONTENTS

LIST OF ABBREVIATION

DVLA ....Driver and Vehicle licensing Autority

FCFS ....First Come First Serve

LCFS .... Last Come First Serve PDF

....Probability Density Function

PH ....Phase-Type Distribution RSS

.... Random Selection for Service

PR ....Priority

GD ....General Discipline

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CHAPTER 1

INTRODUCTION

All over the world queuing in one way or the other has become part of the activities people go through before services are rendered. In Ghana, people queue in the banks to cash money, in various schools to pay school fees, even at road sides to buy food, just to mention a few cases. In a nut shell, queues are inevitable in our communities virtually in any corner one may find himself or herself. The occurrence of queues in the provision of services motivated the researcher to pursue this work on queuing.

A queuing situation is basically characterized by a flow of customers arriving at one or more service facilities.

Queues help facilitate institutions and business to provide services in an orderly manner. Queues forming can be beneficial to society if it can be managed so that both parties namely those that wait in the queue and the other that serves derive maximum benefit. In general nobody would like to wait, but reduction of the waiting time usually require extra investments. To decide whether or not to invest, it requires knowledge of the effect of investment on the waiting time(Adan and Resing, 2002). In view of this, queuing theory is looked at.

Queuing theory describes basic phenomena such as the waiting, the throughput, the losses, the number of queuing items etc(Acheampong, 2013). In addition, queuing theory is described as mathematical study of waiting lines or queues. A model is thus constructed so that queue lengths and waiting times can be predicted. When resources are scarce, limited queues are formed to cater for people or customers and this brings about queuing systems.

According to Klienrock (1975)any system in which arrivals place demand upon finite-capacity, resources can broadly termed as queuing system. A queuing system can therefore be described as customers arriving for services, waiting for services, if it is not immediate and having waited for services, leaving the system after being served(Lin, 1972). There is an adage that 'Time is money' when people spend ample time in queues just to be attended to for a service, they sometimes get frustrated. In my opinion they could have used that wasted time for something beneficial.

The DVLA is one institution in Ghana where queues are always formed and seen. The queues at the DVLA will scare anyone who wishes to benefit from any of the services the institution provides, if it is the first time the person is being at their premises. Almost all the sections /departments at the DVLA do have queues. It is very obvious that whenever the demand for services exceeds its supply, queues are definitely inevitable. Generally, when customers spend very long time at a place or any institution for service then the institution is seen as delivering poor service or services and needs to improve upon it's performance.

This study seeks to help the management of the DVLA to ensure employee adequacy, to be able to inform customers on the average time they need to spend at the eye-test section from Monday-Friday for services.

This chapter looks at the background to the study, discusses the problem statement, the objectives of the study and its significances. It also highlights on limitations and organization of the study.

1.1 BACKGROUND TO THE STUDY

Generally, Queuing theory is considered a branch of Operation Research. The results from Queuing theory are often used to make decisions about the resources needed to provide a service. Basically a model is constructed so that queue lengths and waiting times can be predicted(Sundarapandian, 2009). According to Anokye et al. (2013) the issue of queuing has become a subject of scientific debate, since every society is confronted with the problem of queuing.

In our part of this world, queues are a frequent sight because of limited resources.

In Ghana, every driver is required to be of sound mind and he or she must be eighteen years or above. Again one has to make sure that the vehicle is licensed and all the necessary documents on the vehicle should be valid. These are the questions a qualified driver should ask him or her self;

- Do I have a driver's license?
- Has my vehicle been registered?
- Has my vehicle been insured and provided with road worthy sticker?

If the answers to any of these questions is no, then the driver using such a vehicle could be arrested and prosecuted by the laws of the land. It is only the DVLA which has been authorized by the Government of Ghana to issue driver's license and other documents on vehicles. Before a vehicle sets off on the road all requisite documents must be valid or else the law will take it course on the culprit; that is either the culprit would be fined or in some cases sentenced.

In view of all these some drivers try to obtain the necessary documents illegally. If it happens that there are long queues at the premises of the DVLA, drivers get worried and frustrated. Some might even leave the queue with the hope of coming back later to find no queue. These inconveniences might cost both parties, that is the driver who would be driving without valid documents and the DVLA which is a business entity for the nation would also loose some revenue.

There is an overwhelming need for business and institution to value customers' time and install more efficient procedures to improve the efficiency of queues and waiting lines. Lewin (1943) suggested that early field theory when perceived, waiting time is longer, the effect on customers' responses is more negative when the wait occurs further from the goal state of the service encounter or subsequent to goal achievement than close to the goal state. Delays affect customers at different stages of the waiting period. This is where queuing theory becomes very important.

Acheampong (2013) states that, queuing theory enables mathematical analysis of related process, including arriving at the (back of the) queue, waiting in the queue(essentially a storage process) and being served at the front of the queue. The mean waiting time in the queue, waiting time in the queue plus service times, mean utilization of the service facility, distribution of the number of customers in the system among others are some of the issues queuing theory tries to address. Queuing theory deals with the most unpleasant experiences of life which is waiting. The first queuing theory was treated at the beginning of the 20th century and it was designed for telecommunication system (Erlang, 1909). Vohhra (2010) claims that queuing theory permits the derivation and calculation of several performance measures including the service and the probability of encountering the system in certain states such as empty, full or having an available server, or having to wait in a certain time to be served. In most cases where queues are formed which is referred to as service center, only a limited number of customers serve the service facility and the waiting line.

We note that at any service center, if a customer arrives and the service is loaded, the customer gets onto a waiting line and waits until the service facility becomes available. This brings about the interarrival times of customers and the service times are assumed to be random in most cases.

1.1.1 Significance of Transportation and Driver's License

Motor vehicles are a primary mode of travel, providing an unprecedented degree of mobility throughout the world (Arnett, 2002). In motorized countries cars provide important economic, social, financial, and educational opportunities for families (Hirsch, 2003). Intikhab et al. (2008) reviewed that, efficient transportation system plays an important role in catering for the daily necessities in the lives of the citizens. These include access to amenities and services that are central to the lives of all individuals such as employment, education, health services and leisure.

Katz et al. (1991) put forward that airlines, companies, banks, manufacturing firms etc, try to minimize the total cost and the cost of providing service to their customers. Therefore the speed of service is highly becoming a very important competitive parameter. The provision of ever-faster service, with the ultimate goal of having zero customers waiting time has recently received managerial attention for several reasons (Davis and Magged, 1990). According to Anokye et al. (2013) the role of transportation in human life cannot be over emphasized. At the individual level Wane (2001) elaborates transportation as a crucial factor for urban insertion, since it gives access to economic activities, facilitates family life and helps in spinning social networks. Technology advancement such as computers, Internet etc have provided firms with the ability to provide faster services.

In an attempt to acquire vehicle to enable individuals traveling faster and carry out daily activities easily, cities in the world now witness tremendous motorization during the recent times, especially since 1998 global car population have exceeded 400 million (Walsh, 1990). Privately owned vehicles are essential where population settlement patterns have decentralized communities separating people by distance from schools, jobs and services (Hanna, 2012).

All over the world, a variety of terms are used to describe the people who choose to drive or ride a motor vehicle without a valid license. Some of these common terms used include; unlicensed driver, unauthorized driver, disqualified driver, suspended driver, revoked driver, canceled driver and never licensed driver (Watson, 2004). Unlicensed driving still remains a major problem in various countries, though there are ongoing improvements in traffic law. Driving without a valid license remains a serious problem in many jurisdictions (Sweedler and Stewart, 2007). According to Watson (2003), it does not play a direct causative role in road crashes, it undermines driver licensing systems and is linked to other high risk driving behaviors. Watson (2003) stated that roadside license check surveys represent the most direct means of estimating the prevalence of unlicensed driving. Since distance plays a greater role especially where public transportation is not available or inadequate and being mobile requires access to car and possessing a driver license (Patel et al., 2000).

According to WHO (2009), motor vehicle transport is estimated to kill 1.2 million people each year accounting for 25 percent of all deaths from injury. Worldwide, between 20 million and 50 million people are injured or disabled each year in road traffic crashes and this probably underreported. Unlicensed driving seems to be a major problem for road safety in the sense that Watson (2003) claims that, the crashes involving unlicensed drivers tend to be more severe than those involving licensed drivers, resulting in higher rates of fatality and serious injury.

The issue of unlicensed driving arises when the developmental and mobility needs of obtaining a license to drive that can occur before and long after the age of becoming eligible to be licensed (Tsai et al., 2008). With reference from WHO 2009 data, traffic crashes globally are the single greatest killer of 15-24 years-olds in motorized countries, in which majority are unlicensed drivers. Watson (2004) reviewed that those drivers who have let their license expire are less likely to be involved in serious crashes than those who have never held a license. When unlicensed drivers take to the road, it is unknown unless they are involved in an accident reported to the police (Mayhew, 2007).

Hanna et al. (2012) defined a driving license as an official document that allows a person to operate a motorized vehicle legally on a public roadway. The laws, requirements, ages, and difficulty of obtaining a driver's license vary from country to countries. A large number of people drive without valid licenses. This include those who have had their license expired and also those who have never held a license. Basically, to qualify and prepare young people to drive, an organized sequence of education, training, and practice provide basic knowledge and skills to prepare for a license is usually provided (Vernick et al., 1990) (Curry et al., 2012). In Ghana, before one can obtain a driver's license he or she must be 18 years and above. In some countries for instance adults supervise driving just for the young person to gain experience and maturity prior to obtaining a full license (Dee et al., 2005). Even though unlicensed driving is illegal, it is not always consistently enforced or detected in different settings (DeYoung and Gebers, 2004). If licensing requirements become too restrictive to get a timely license or if viewed as unfair, young people may alternatively use other forms of transport such as driving unlicensed cars or motorcycles (Simons-Morton et al., 2006). Though training in this manner does not guarantee a safer driver, it brings about parental control and legal oversight from authorities to their onroad behavior over a given period (Preusser and J., 2007). Imai and Mansfield (2008) suggested that by setting community standards, laws, and practices that are clearly communicated, modeled by parents, and consistently enforced to young people can define and direct driving behaviors of young people. Watson (2004) also argued that roadside license check surveys provide the most direct means of estimating the community-wide prevalence of unlicensed driving, both in general and among different traffic offender types.

1.1.2 Driver license Acquisition at the DVLA

Before an eighteen years and above can acquire a driver's license at the DVLA, the person must go through some stages and procedures describe below.

FIRST STAGE

Customers must;

Step 1

Present photocopy of national ID and passport pictures to room 5 for physical assessment and approval.

Step 2

Undertake an on line registration and generation of invoice at the client service unit.

Step 3

Proceed to the bank for payment for a scratch card.

Step 4

Testing of the eyes.

Step 5

Scheduling for learners permit(Examinations)

Step 6

Undertake the learners permit examination at the examination hall and seek approval for learners permit given in room 5.

Step 7

Undertake computer based test in room 17.

SECOND STAGE

Step 1

Make payment for in-traffic examination at the bank

Step 2

Proceed to room 5 for In-traffic exams

Step 3

Pay capture fee at the bank

Step 4

Proceed to the capture room for processing of Temporary license.

1.1.3 Eye-test section

The DVLA provides various forms of services to it's customers. One of such services which is also very vital for every driver to under go is at the eye-test section. Every driver who is seeking a driver's license for the first time or those renewing an expired license cannot escape this section. Even when a driver uses the back door to obtain other documents, the eye-test and the capturing sections require the physical presence of the customers themselves. Match out middle men who are referred to as 'Goro boys' cannot have their way. When 'Goro boys' get to this eye-test section, they bring the documents back to these prospective drivers for them to go through those processes themselves. This situation brings pressure and long queues. The eye is an important organ on our body which aid in driving. One must have good eye sight before he or she set off on the road to drive. Management at the DVLA has created the eye-test section where applying drivers eyes are tested. This eye-test section has a qualified doctor (an optometrist) who examines every applicant eyes before they are allowed to proceed with the other stages. The eye-test section at the DVLA always has a long queue.

1.1.4 Sections where queuing models could be applied at the DVLA

i. Road Sign section

This is one section at the DVLA that also has a very long queue. Customers queue and wait for their turn to be examined on road signs. Though one can apply queuing models here, it is a bit cumbersome. The reason is that those officials examining the drivers are the same people who issue and endorses learners license. Along the line, these officials conduct driving test for all the drivers who have passed the road sign test. Customers who are yet to undergo the road sign test wait in a queue for their share of the cake.

ii. Driving Test Section

This is a practical section. Here the applicants exhibit their driving capabilities , then show if they can really drive. There is at least a car provided by management of the DVLA for drivers to use for the driving test. Drivers who come along with their own cars have the choice to use them. Even though queues are formed, the limitation here is that the queue is often not ordered. Any driver who is ready for the driving test get into the provided car or get into his or her own car and then the driving test starts.

iii. The Capturing Section

Queuing model could also be applied at the capturing section. The capturing section is the last stage one goes through after passing through all the stages enumerated above successfully. Queues are also inevitable here. A passport size photograph of the driver is captured and used for the driver's license identity card. The limitation here is that, the room is also used for other purposes other than capturing.

1.2 PROBLEM STATEMENT

The DVLA of Ghana is a government agency solely responsible for issuing license and evaluating drivers and cars in Ghana. The DVLA was established in 1996 by Act 569 of Ghana's parliament. The act allowed the authority to have a semi-autonomous status in the public sector organization under the Ministry of Transport. The authority is responsible for ensuring safety on the road.

The laws of Ghana allows individuals eighteen years or above to drive. Residents of the country can apply for a three months learner's license. The major problem facing the DVLA is that the number of applications have increased, whiles the DVLA premises still remains the same and drivers have to queue for a long time for services rendered.

The DVLA seeks to

- provide license to drivers who are eighteen years and above.
- evaluate and issue road worthy certificate to vehicles
- register and give out registration numbers to new cars.

These services the DVLA provides will naturally yield queues and pressure since a driver without the necessary documents could be arrested and prosecuted by the law.

Most of the applicants are commercial vehicle drivers. They complain of spending precious and long hours in queues waiting for services including eye-testing to be rendered.

1.3 OBJECTIVES

As customers, they generally do not like to wait more than the expected wait time. Therefore, managers of any establishment which the DVLA is not an exception do not like long waiting times. According toOpara-Nadi (2005) waiting may adversely affect customers' loyalty, thereby affecting revenue. The purpose of this study is

- to find the performance measure at the eye-test section from Monday-Friday.
- to find the arrival rate and service rate when eyes are being tested by the optometrist.
- use the appropriate queuing model

1.4 SIGNIFICANCE OF THE STUDY

The findings of this study is hoped to help the management and administrators of the DVLA to

- give a fair idea to drivers on the amount of time they would be expected to spend at eye-test section

1.5 RESEARCH METHODOLOGY

The data for this study is a primary data obtained from the DVLA(Kumasi). A stop watch was used to calculate each service time and arrival time as well as waiting time of customers. The data was collected at five working days of the week.(Monday-Friday).

In addition, First come first serve(FCFS) was used as the queuing discipline.

1.6 SCOPE OF THE STUDY

The study focused on the drivers at eye-test section of the DVLA(Kumasi). It looked at the number of drivers who came for their eyes to be tested. The queue served two purposes which are;

- those who are acquiring the driver's license for the first time.
- those who are renewing the expired driver's license.

The time and date for the study were 9:00am-12:00 noon and 16th-20th Feb 2015.

1.7 LIMITATION OF THE STUDY

The researcher is limited only to the eye-test section. This is because it was the only section which was free from the 'goro boys'.

The 'goro boys' are the unqualified agents who collect monies from customers and provide them with the needed services. Even though they have not been employed by management of the DVLA, these 'goro boys' always have their way through all the section at the DVLA. Upon interviews with some of the 'goro boys',it was clear that only the eye-test section that they could not maneuver. The reason is that the eye-test section demands the presence of the driver and no one else. This situation limited the researcher to looked at only the queue at the eye-test section without considering the other queue at the other section at the DVLA simply because of the presence of these 'goro boys' at the various section at the DVLA.

Further more, data collection was limited to drivers who arrived at the section before 12:00 noon, so all those who came after 12:00 noon were not considered. Again, the data gathered should have been for a whole month but not for only the five working days as stated in this thesis. But due to time and financial constraints, the researcher was limited to drivers who arrived at the eye-test at 12:00 noon during the five working days.

However, it is hoped that these short falls would not affect the findings of the study.

1.8 ORGANIZATION OF THE STUDY

The study is put into five chapters. Chapter one looks into the background, problem statement, objectives, significance, research methodology, scope and limitations. Chapter two deals with the review of pertinent literature. Whiles chapter three addresses research methodology. Chapter four entails data collection and analysis. Finally chapter five presents summary of findings, conclusion and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, an overview of previous work falling within the scope of the thesis is presented. The aim of this chapter is in three folds. First to give the historical background of queuing theory. Secondly, to look at queuing theory and queuing models. Finally the findings of related work that will aid the researcher and serve as a guide for future research.

2.2 HISTORICAL BACKGROUND OF QUEUING THEORY

In the past queuing theory has been effectively used in such areas of policy making staff scheduling among others. Queuing theory's history can be traced back almost 100 years ago. Johannsen seen to be the first person who wrote on queuing theory (Johannsen, 1907), though the method used in his paper was not mathematically exact. The paper that had historical importance and exact treatment of the methods of queuing theory was from A.K. Erlang. In his paper Erlang (1909) lays foundation for the place of Poisson (and hence, exponential)distribution in queuing theory. His works done in the next 20 years were made up of the most important concepts and techniques which were the notation of statistical equilibrium and the methods of writing down balance of state equations which was later refereed to as 'Chapman-Kolmogorov' equation. In one of Erlang's work titled 'On the Rational Determination of Number of the Circuits' an optimization problem in queuing theory was treated for the first time. The problem of congestion in the twenties and thirties motivated Erlang and others to looked into queuing theory.

These congestion problems made many theoreticians like Molina, Fry, Crommelin, Feller, Jensen, Khintchine, Kolmogorov, Palm and Pollaczek became interested in these problems and developed general models which could be used in more complex queuing situations.

Kolmogorov's and Feller's works on purely discontinuous processes paved way for foundation for the theory of Markov processes and the inadequacy of the equilibrium theory in many queue situations (Syski, 1960). In addition, Pollaczek (1965) began investigations of the behavior of the system during finite time interval. He made queuing theory seen to be a useful area for researcher who wanted to do fundamental research on stochastic processes involving mathematical models.

In 1951, Kendall published his work on embedded Markov chains, which is the basis for computation involving queuing systems under a fair and general input conditions. From the 20th century a large majority of queuing theory results used in practice are those derived under the assumption of statistical equilibrium. But the process involved are not simple and therefore mathematical analysis and procedures are required.

Mathematical modeling is a process of approximation. In the 1960's, several authors looked into the studies on the role of approximations in the analysis of queuing systems. Lindley (1952) derived integral equations for waiting time distributions. His findings and investigations contributed to the use of renewal theory in queuing systems analysis in the 1960's. During the latter stages in the 1960's a number of the basic queuing systems that could be considered as reasonable models of real world phenomena had been analyzed and the papers that came out dealt with only small variations of the systems without delving deep into methodology.

Queuing theory was boosted by the introduction of computers and digitalization after the second World war. This made the likes of Jackson, Whittle and Kingman use computer technology in the analysis of queuing theory. Jackson (1957) had the first article on queuing network. It was a mathematical foundation for the analyzing networks. In the advanced and developed counties where standard of living are high, time becomes more valuable and precious as a commodity which make customers less willing to wait for service.

In conclusion queuing theory has it's origin in research by Agner Kamp Erlang when he created models to describe the Copenhagen telephone exchange. His idea has since seen applications including telecommunication, traffic engineering, computing and the design of factories, shows, offices and hospitals.

2.3 QUEUING THEORY

Queuing theory deals with problems which involve queuing (or waiting) (Tadj and Choudhury, 2005). Gross and Harris (1997)defined queuing theory as the mathematical study of waiting lines. These queuing models systems in which one or more servers at one or more service stations process arriving customers requests (Terekhov, 2013).

Queuing Theory is one of many sub-discipline of operations research. Much of the initial work on queuing theory is attributed to Erlang (Komenda, 2013). According to Stewart (2009) much of the theory is devoted to the derivation of performance measures evaluating characteristics such as the throughput, probability of delay, number of queuing items and the expected waiting time of customers in the queue. Queuing theory tries to answer questions like the average waiting time in the queue plus service times, the utilization of service facility, number of servers to be employed, expected number of customers in the system among others (Grassman et al., 2001).

Brown (2012)highlighted that, queuing theory provides long-run steady state performance measures and it gives a good fit for making long-term strategic decisions.

His work further stated that queuing theory provides a unique type of analytic model in which it puts together some of the stochastic nature of the production system by providing expected steady-state values. Alecu (2004) explains the main advantage of queuing theory resides in determining the important information about waiting times, arrivals and service stations characteristics and about the systems discipline. A wait for service is defined by Taylor (1994) as the time that a customer is ready to receive service until the time the service commences. Kocas (2000) further elaborates that customers who cannot immediately access a server experience a costly wait if they choose to stay or not to stay in line. Larson et al. (1991) explain that the length of wait in line increases, perceptions of waiting time increases and customers satisfaction at the end of wait tends to decrease. Carmon et al. (1995) emphasized that people typically do not like to wait, as it causes them to experience a broad range of unpleasant responses such as boredom, irritation, anxiety, tension, helplessness, and sometimes even humiliation.

Queuing theory uses queuing models to represent the various types of queuing systems that arise in practice (Nafees, 2007). The models enable finding an appropriate balance between the cost of service and the amount of waiting. In a nut shell, waiting line models comprises of mathematical formulas and relations used to determine the operating characteristics of these lines which in turn determines the effectiveness of operations of staff (Olusola et al., 2013).

2.4 QUEUING SYSTEM

Opara-Nadi (2005) defines queuing system as customers who enter a system, perhaps wait in a queue, served or leave the system not served. A queuing system is classified into finite queue length and infinite queue length. A finite queue length is a situation where customers joining a queue is restricted to some particular number, no customer can join the queue when it has reach it's limit. For instance if the space at a mechanical station can only cater for 20 cars at a

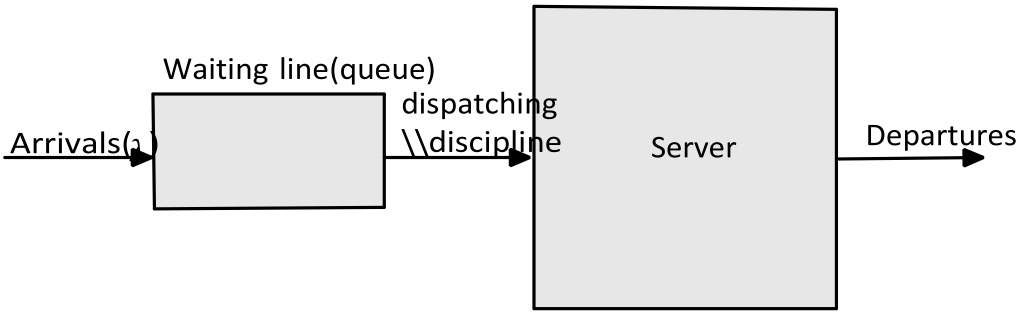

Figure 2.1: Queuing System

given period, any extra car would have to leave on it's arrival.

On the other hand, an infinite queue length has no restriction on the queue, a customer can wait for his turn to be served. Waiting lines or queues aid organizations, firms or businesses render service in an orderly fashion. Prabhu (1997) explains that a queue occurs when potential customers arrive at a system that offers a certain facility or service that the customers wish to use.

Queuing systems therefore comprises of a pivot tool in modeling and performance analysis. Some examples of systems include telecommunication systems and computer systems among others. In queuing systems the resources are called servers. A queuing system combines one or more servers that provide service to arriving customers. The manner customers join the system is called input. In a waiting system every arrival joins the system. A system with a finite capacity means that no customer would be able to join the system whiles an infinite system gives room for every customer to join.

2.4.1 Basic Queuing Systems

A queuing system is defined to include the waiting line and service channels. The number of customers in the system at any point in time is given by the number in the queue together with the number in service.

Customers requiring service are generated over time by an input source. These services are then performed for customers by the service mechanism, after which the customer leaves the queuing system (Nafees, 2007). We could have the following queuing systems:

- single-queue and a single server
- single-queue and multiple servers
- multiple-queue and multiple servers

The queuing system consists of three major components. Which may be characterized as follows;

2.5 ARRIVAL PROCESS

All queuing systems must be based on elements/entities, which include arriving customers, service facility and customers departing. These customers must first of all be present in the system before they can be processed. The arrivals of these customers depend solely on the environment. They could be in an unpredictable manner, smooth, in clumps(e.g.bus loads) more so arriving customers could also be independent and according to some kind of correlation and the involvement of probability distributions. There are three entities associated with queues, which are;

- Balking;

  This is a situation where a customer who is about to join a system, sees many people in the queue and instead of joining the queue he leaves simply because he would not like to spend a lot of time before he will be served.
  That is the arrival balks meaning he does not join the queue.

- Reneging;

  On the other hand the term reneging means a customer joins a queue for awhile and later finds out that the queue moves at a slower pace, he then steps out of the queue and leave just to save time.

- Jockeying;

  This term simply refer to more than one queue where a customer switch from one queue to another when he sees that a particular queue moves faster than the one he is currently in. If a customer finds himself in that situation he or she is then jockeying. The arrivals at a system may be taken from a finite(limited) or an infinite(unlimited) population. These two components are very important because equations for their solutions are based on different premises.

2.5.1 Customer arrivals

- Finite Population

  This refers to the limited-size entities extract which use service and sometimes form a queue. The reason this finite classification is important is that when an entity leaves its position as a member for the population, the size of the user group is reduced by one, which reduces the probability of the next occurrence. In addition, when an entity is served and gets back to the user group, the population increases and the probability of a user requiring service also increases.

- Infinite Population

  An infinite population is large enough in relation to the service system so that the population size caused by subtraction or addition to the population does not significantly affect the system probabilities.

2.5.2 Arrivals Distribution

The manner customers arrive in a queue for service should be critically defined. We could have a constant arrival distribution. The arrivals at a service facility can be in two folds. First, we can analyze the time between consecutive arrivals and check if it follows some statistical distribution. Mostly the assumption is that, times between arrivals are exponentially distributed. Secondly some time (T) could be set and see how many arrivals can happened within that time (T).

This assumption is refereed to as Poisson distribution.

2.6 SERVICE PROCESS

When entities enter the system, they must be served. The kind of services rendered to entities depends on the system. Several service processes include patients going through medical treatment, customers at a supermarket in the checkout process, tellers attending to customers at a bank and so on. In most cases, what we do care about is service time being long or short, and whether they are regular or not.

2.6.1 The service capacity

There could be a single server or multiple servers providing services to customers in a queue.

2.7 QUEUE DISCIPLINE

A queuing system consists basically of a waiting line(s) and servers. The factors to consider with waiting lines include number of queues, queue length and queue discipline.

- Number of queues

  A single line means one line only, whiles multiple lines refers to the singlelines which are formed in front of two or more servers.

- Queue length

The queue length in a system simply refers to a finite line and an infinite line. As the name implies a finite line is a queue which has limited line capacity. Arrivals here would have to sought for service elsewhere when there is not enough space to accommodate. An example is parking lots, where arrivals are denied entry because of lack of space. An infinite line is a queue which has a wider capacity. Here queues are very long. An example of an infinite line is, a customer after shopping from a super market joins a waiting line behind a counter to pay up for his or her items purchased.

- Queue discipline

A queue is an important component of a queuing system. There are twofolds of queue; an unlimited queue which could take care of as many customers as it can and a queue which deals with limited number of entities at any given time. Most of the arrivals that come in this particular queue when it is full are rejected (e.g.customers get a busy signal when trying to dial into a call center) (Hopp, 1999). Customers may also balk at joining the queue when it is too long, even if the queue is not limited. The discipline of a queuing system implies all other factors regarding the rules of conduct of the queue.

A queue discipline is therefore a procedure of a rule for giving out the order of service to customers in a queue.

The most common used order in which entities wait for service include;

- First come first served(FCFS)

In this queue discipline customers are served in a chronological arrival. This is popularly accepted as the fairest rule.

- Last come first served(LCFS)

With this queue discipline, the last customer would be attended to first. For instance when people are submitting documents to an office, those documents which are on top are taken first even though they were last to come.

- Priorities

Priority which involves; rush orders first, shortest processing time first, reservations first, emergencies first, largest order first, deposits first(in the banks), highest-profit first among others are examples of priority rules.

- Random order

For this queue discipline customers are served in no peculiar order. Service to customers are done randomly.

2.8 BASIC CONCEPTS FROM PROBABILITY THEORY

Queuing theory which is based on describing the arrival and service(departure) patterns by appropriate probability distributions.

This part describes some useful probability distributions which provide details for explaining random variables that are connected to arrivals, interarrivals times and service times of customers.

These are a few of the probability distributions associated to some queuing models;

2.8.1 Random variable

Elements contributing to the underlying processes in queuing systems can be modeled as random variables.

The outcome of arrivals of customers, interarrival times and services times of customers are uncertain, so we associate these entities with random variables. These random variables can take on several values with uncertain probabilities. The expected value of a random variable X is denoted by $E(X)$ and its variance by $\sigma^2(X)$

where $\sigma(X)$ is the standard deviation of X. The coefficient of variation of the positive random variable X is defined as

$$c_x = \frac{\sigma(X)}{E(X)} \tag{2.1}$$

2.8.2 Geometric distribution

A geometric random variable X with parameter p has probability distribution

$$P(X = n) = (1 - p)p^n, n = 0,1,2,... \tag{2.2}$$

2.8.3 Erlang distribution

A random variable X has an Erlang-k (k=1,2,...) distribution with mean $\frac{k}{\mu}$ if X is the sum of k independent random variables $X_1,...X_k$ having a common exponential distribution with mean $\frac{1}{\mu}$. The common notation is $E_k(\mu)$. The density of an $E_k(\mu)$ distribution is given by

$$f(t) = \mu\frac{\mu t^{k-1}}{k-1!}e^{-\mu t}, t > 0 \tag{2.3}$$

The distribution function equals

$$F(t) = 1 - \sum_{j=0}^{k-1}\frac{(\mu t)^j}{j!}e^{\mu t}, t >= 0 \tag{2.4}$$

The parameter μ is called the scale parameter, k.

The mean, variance and squared coefficient of variation are given as;
$E(X) = \frac{k}{\mu}$, $\sigma^2(X) = \frac{k}{\mu^2}$, $c_X^2 = \frac{1}{k}$

2.8.4 Hyper-exponential distribution

A random variable X is hyper-exponential distributed if X is with probability p_i, $i = 1,...,k$ an exponential random variable X_i with mean $\frac{1}{\mu_i}$. The notation H_k is used. The density function is given by

$$f(t) = \sum_{i=1}^{k} p_i \mu_i e^{-\mu_i t}, t > 0 \tag{2.5}$$

2.8.5 Phase-type distribution

The notation for this distribution is PH. The distribution is characterized by a Markov chain with states that is $1,2...,k$ and has a transition probability matrix P which is transient. P^n tends to zero as n tends to infinity. The residence time in state i is exponentially distributed with mean $\frac{1}{\mu_1}$, and the Markov is entered with probability p_i in state i, $i = 1,...,k$. Then the random variable has a phasetype distribution if X is the total residence time in the the preceding Markov chain, X is the total time elapsing from start in the Markov chain till departure from the Markov chain (Adan and Resing, 2002).

2.9 KENDALL NOTATION FOR REPRESENTING QUEUING MODELS

Kendall (1953) introduced a useful shorthand notation $A/B/X/Y/Z$, to characterize queuing models. These notations describe the first three characteristics given above; namely, arrival distribution, service distribution and number of parallel service channels. The complete notation thus appears in the following symbolic form above, where

A-is the inter-arrival distribution of the customers

B-is the service time distribution

X-the number of parallel servers

Y-the maximum number of customers allowed in the system

Z-queuing strategy

In most cases the first three symbols $A/B/X$ are required because they are the three most important characteristics. Y is left out if no restriction($Y = \infty$) and Z is the queuing discipline which could be FCFS, LCFS etc.

Table 2.1: Tabular form of Kendall notation

Characteristics	Symbols	Explanation
Inter arrival-time distribution(A) Service-time distribution(B)	*M* *D* E_k H_k *PH* *G*	Exponential Deterministic Erlang type k(k = 1,2,...) Mixture of k exponent Phase type General
Number of parallel servers(X)	1,2,.....	
Restriction of system capacity(Y)	1,2,.....	
Queue discipline(Z)	*FCFS* *LCFS* *RSS* *PR* *GD*	First come first served Last come first served Random selection for service Priority General discipline

2.9.1 Models For Single Server Queuing System

M/G/1 QUEUE MODEL

The $M/G/1$ queue model has an exponential arrival distribution and a general service time distribution.

The $M/G/1$ queue model shows how variability drives the performance of a queuing system (Hopp, 1999). The arrival of customers in this queuing model is also according to independent Poisson process, where the M refers to Markov process, but service times are general represented by G. This means that service times can take on any probability distribution, as long as they are independent of one another.

Notation for M/G/1 Queue Model

λ: arrival rate

$E(W)$: expected waiting time in the queue

$E(S)$: expected service time
$E(T)$: sojourn time

$E(N)$: expected number of customers in the system

$E(N_q)$: expected number of customers in the queue

$E(N_s)$: expected number of customers in the system ρ:

utilization of the server

The equation for finding the utilization of the server in the $M/G/1$ queuing model is given as

$$\rho = \lambda E(S) \tag{2.6}$$

Performance measure of the M/G/1 queue model

$$E(T) = E(W) + E(S) \tag{2.7}$$

$$E(N) = E(N_q) + E(N_s) \tag{2.8}$$

The expected number of customers in the queuing system with general service time was derived by Pollaczek and Khinchin (Nelson, 1995). This equation is given below as

$$E(N) = \rho + \frac{\rho^2(1 + C_s^2)}{2(1-\rho)} \tag{2.9}$$

where C_s^2 is the coefficient of variation which is defined as $C_s^2 = \frac{\sigma_s^2}{E(S)^2}$ and also σ_s^2 is the variance for service time.

The expected number of customers in the queue is given as

$$E(N_q) = \frac{\rho^2(1 + C_s^2)}{2(1-\rho)} \tag{2.10}$$

Little's theorem gives us a way of deriving at the expected waiting time in the queue. Little's theorem states that

$$N_q = \lambda E(W) \tag{2.11}$$

Therefore combining equation (2.10) gives equation for E(W) as

$$E(W) = \frac{1}{\lambda}\frac{\rho^2(1+C_s^2)}{2(1-\rho)} = \frac{\rho(E(S))(1+C_s^2)}{2(1-\rho)} \tag{2.12}$$

Gaver (1968) analysis of the virtual waiting time of an M/G/1 queue is one of the initial efforts using diffusion approximation for a queuing system. For quality of service, the assessment on the quantity of interest is the expected response time which is known as the sojourn time. The sojourn time is a sum of two components; the expected queuing time and the service time.

G/G/1 QUEUE MODEL

According to Bjournstad (2006), a queue with general inter-arrival and service time distribution is denoted as $G/G/1$. Analytically when treating such queuing model is somehow very difficult. However, several approximations in view of queuing length and average waiting time have been proposed.

A queue with general inter-arrival and service time distribution is denoted as $G/G/1$ Bjournstad (2006). In his work applying this queuing model is somehow difficult. Larson and Odoni (1981) stated that there is no exact result that exit for these queues due to the difficulty in obtaining the transition probabilities between queues. There are however some approximations on the performance measures for this queue model.

Performance measure of the G/G/1 queue model

$$E(W) \approx \frac{\rho E(S)(C_s^2 + C_a^2)}{2(1-\rho)} \tag{2.13}$$

The total system time is a combination of both waiting in queue and service time which gives

$$E(T) \approx \frac{\rho E(S)(C_s^2 + C_a^2)}{2(1-\rho) + E(S)} \qquad (2.14)$$

M/D/1 QUEUE MODEL

This queuing model has M indicating Markov(memoryless), which has exponential interarrival distribution, D refers to deterministic or constant length service and as usual 1 means single server.

Performance measure of the M/D/1 queue model

We computing the average number in the system to be;

$$L_s = \frac{\rho^2}{2(1-\rho)} + \rho \qquad (2.15)$$

The computation for the average number of customers in the queue in the $M/D/1$ queue model is also given as;

$$L_q = \frac{\rho^2}{2(1-\rho)} \qquad (2.16)$$

The average time a customer spends in the system for this queuing model is calculated as;

$$W_s = \frac{\mu(2-\rho)}{2(1-\rho)} \qquad (2.17)$$

For the average waiting time in the queue calculation is;

$$W_q = \frac{\rho\mu}{2(1-\rho)} \qquad (2.18)$$

2.9.2 Model For Single Queue And Multiple Servers

The figure above shows a single queue and multiple servers.

M/M/C QUEUE MODEL

This has the same features of the $M/M/1$ queue, but the only difference is the C, which is the parallel identical servers. The utilization rate per server,

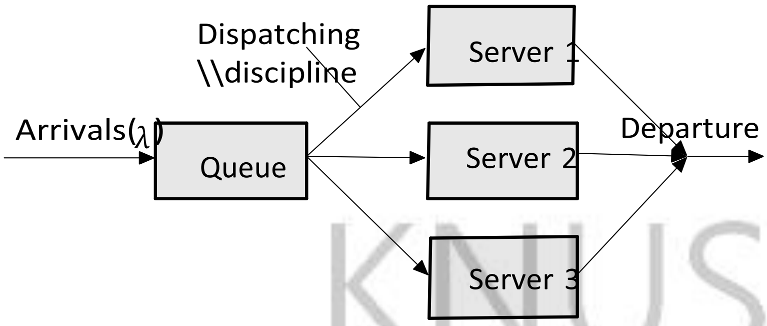

Figure 2.2: Multiple servers

$$\rho = \frac{\lambda}{C\mu} \tag{2.19}$$

where C is the number of servers in the system and $\rho < 1$

$$\lambda_n = \lambda$$

$$\mu_n = n\mu \tag{2.20}$$

When $n < C$, then equation (3.17) is used, where n is the number of customers in the system

$$\mu_n = C\mu \tag{2.21}$$

When n≥C, then equation (3.18) is used.

The State of the M/M/C queue model

The general expression for finding probability of customers in the system is

$$P_n = \rho P_0$$

$$P_n = \frac{\lambda^n}{\mu^n} P_0$$

Therefore to find the exact value for the probability of customers in a multiple server, two equations are involved

$$P_n = (\frac{\lambda}{\mu})^n \frac{1}{n!} P_0 \tag{2.22}$$

We use equation (3.20) when $n < C$

$$P_n = \frac{\lambda^n}{C! \mu^n C^{n-c}} P_0 \tag{2.23}$$

We use equation (2.23) when n≥C. Moreover, we know that all probabilities sum up to 1. So this gives us

$$P_0 + P_1 + P_2 + ... P\infty = 1$$

Finding Probability of zero customers in the queue

$$\sum_{n=0}^{C-1} \frac{\rho}{n!} P_0 + \sum_{n=c}^{\infty} \frac{\rho^n}{C! C^{n-c}} P_0 = 1$$

$$P_0 [\sum_{0}^{c-1} \frac{\rho^n}{n!} + \sum_{n=c}^{\infty} \frac{\rho^c}{c!} \frac{\rho^{n-c}}{c^{n-c}}] = 1$$

$$P_0 [\sum_{c-1}^{0} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} (1 + \frac{\rho}{c} + (\frac{\rho}{c})^2 + ...\infty)] = 1 \tag{2.24}$$

we see from equation 2.24

$$1 + \frac{\rho}{c} + (\frac{\rho}{c})^2 + ...\infty$$

is a geometric series so we have

$$P_0 [\sum_{c-1}^{0} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \frac{1}{1 - \frac{\rho}{c}}] = 1$$

$$P_0 = \frac{1}{\sum_0^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \frac{1}{(1 - \frac{\rho}{c})}}$$

Performance Measure of the M/M/C queue model

Finding the expected number of customers in the queue

$$L_q = \sum_{n=c}^{\infty}(n-c)\rho_n$$

$$L_q = \frac{\rho^{c+1}P_0}{C!C}\frac{1}{(1-\frac{\rho}{c})^2}$$

$$L_q = \frac{\rho^{c+1}P_0}{(C-1)!(C-\rho)^2} \tag{2.25}$$

Equation (2.25) is the expected number of customers in a multiple server queue. Finding the expected number of customers in the system

$$L_s = L_q + \rho \tag{2.26}$$

Therefore to find the expected time a customer spends in a queue is given in equation (2.27)

$$W_q = \frac{L_q}{\lambda} \tag{2.27}$$

CHAPTER 3

RESEARCH APPROACH

3.1 INTRODUCTION

In this chapter, we apply the $M/M/1$ queue model and the performance measures of the model which are probabilistic. We consider arrivals of drivers into the system, the waiting time of drivers, utilization of the server(s), idle time of the server and the busy time of the server(Optometrist) in the queuing model. Basically arrivals of customers, interarrival times and service times determines the outcome of results.

The $M/M/1$ queue model is analyzed and explained in this chapter.

3.2 AXIOMATIC DERIVATION OF THE $M/M/1$ QUEUE MODEL

In this section, arrival and departure distributions for Poisson queues are derived. The basic axioms governing this type of queue model are stated below;

Axiom 1:

Given $N(t)$ the number of arrivals or departures during the time interval $(0,t)$, the probability process describing $N(t)$ has stationary independent increments. This axiom is explained as follows. Given h, a positive increment then for $t_0 < t_1 < \dots t_k, (k+1)$ points in time, the two random variables $(N(t_{i+1}) - N(t_i))$ and

$(N(t_{i+1} + h) - N(t_i + h))$, $i = 0,1,\dots,k-1$, are independent and identically distributed.

Axiom 2

In any interval of time $h > 0$, there is a positive probability of arrival (departure) but this probability is not certain; that is, $0 < P(N(h) = 1) < 1$.

Axiom 3

In a sufficiently small interval of time, at most one arrival (departure) can occur; that is, $P(N(h) \geq 2) = 0$.

3.2.1 Poisson Arrivals Distribution(Pure Birth)

In this model it is assumed that only arrivals are allowed at rate λ per unit time. This in most cases referred to as the pure birth model. The assumption here is that, there is no one in the system at $t = 0$.

We usually assume and try to determine how many arrivals might enter the system within some time(t).

The arrivals of the customers can be said to follow Poisson distribution. That is $n(t)$ is the number of events or arrivals that have occurred from time 0 to time t. The Poisson distribution is given below as

$$P_n(t) = \frac{(\lambda t)^n}{n!}e^{-\lambda t} \tag{3.1}$$

For Poisson distribution it holds when;

$E(n) = \sigma^2(n) = \lambda$

where

- t describes the period 0 to t
- n is the total number of arrivals in the period 0 to t
- λ indicates the total average arrival rate

3.2.2 Exponential Interarrival Times Distribution

The basic definition of interarrival times are defined as the time intervals between two successive arrivals. Given that the arrival distribution is Poisson, one can derive the distribution of interarrival time as follows. Let $f(t), t > 0$, be the probability density function of interarrival time and define $F(t)$ as;

$$F(t) = \int_0^t f(u)du \tag{3.2}$$

3.2.3 Exponential Service Times distribution(Pure Death model)

This model also assumes that there are N customers in the system at $t = 0$. Departures occur at the rate μ per unit time.

In the case where arrivals at a service facility occur in a purely a random manner, a plot of interarrival times yields exponential distribution.

The $M/M/1$ queuing model assumes that the interarrival and service times are exponentially distributed. So if λ is the mean arrival rate, then the probability density function (PDF) for the time between successive arrivals would be

$$P_n(t) = \lambda e^{-\lambda t}, \tag{3.3}$$

Service time distribution is also given as;

$$P_\lambda = \int_0^{t_0} \lambda e^{-\lambda t} dt = 1 - e^{-\lambda t_0}$$

(3.4) For these distribution we have

$E(n) = \frac{1}{\lambda}$, $\sigma^2(n) = \frac{1}{\lambda^2}$

where λ is the average arrivals per time period.

3.2.4 Properties of Poisson and Exponential distribution

There are three important properties of Poisson and exponential distributions used in this thesis:

i. Memoryless Property

If service time are exponentially distributed, then the probability that a customer's service at some future time is independent of how long the customer has already been in service.(Bastani, 2007) ii. Additive Property

The sum of n independent Poisson process with parameter λ_i for $i = 1,2,\ldots,n$ is a Poisson process with parameter $\lambda_1 + \lambda_2 + \ldots + \lambda_n$. iii. Decomposition Property

Suppose that $n(t)$ is a Poisson process with average rate λ and that each arrival is marked with probability p, independent of all other arrivals. Let $n_1(t)$ and $n_2(t)$ respectively denote the number of marked and unmarked arrivals in $[0,t]$. Then $n_1(t)$ and $n_2(t)$ are two independent Poisson process with respective rates λp and $\lambda(1 - p)$ (Bastani, 2007).

3.3 M/M/1 QUEUE MODEL

This is the classical queue model in queuing theory. This is because of it's computational and conceptual simplicity. Moreover it is the queuing model that novice queuing theorists first get acquainted with (Bjournstad, 2006). It is also one of the earliest systems to be analyzed.

The arrivals in the $M/M/1$ are assumed to be Poisson with rate λ, that is interarrival times are independent exponentially distributed random variables with parameter λ. The service times also assumes to be independent and exponentially distributed with parameter μ. More so, the 1 in the model refers to only one server. It also has the $FCFS$ queue discipline. Bjournstad (2006) stated that, in this model we can derive simple computational methods to investigate its performance under different conditions.

This queue model assumes the number of arrivals in an interval of time which follows a Poisson distribution. The $M/M/1$ is a good approximation for a large number in a queuing system. The first M indicates Markov arrival process, which refers to the situation where entities arrive one at a time and the time between arrivals are exponential. The arrival process is memoryless meaning that the likelihood of an arrival within the next minutes is the same no matter how long it has been since the last arrival (Hopp, 1999). In addition, the second M also refers to a Markov service process, in which customers are processed one at a time in $FCFS$ order and service times are independent and exponential. Again as in Markov arrivals, a Markov service process is also memoryless, meaning that the expected time until a customer has completed service is still the same whether it has been in service for a long time. Again the memoryless property gives a very important assumption that; one event takes place in a small interval h, that is either there is an arrival of a customer or a customer is being served. This queue model assumes infinite queue length and infinite population length. To add up, the $M/M/1$ is a suitable queuing model if the arrival process meet the following assumptions;

1. The number of customers in the system is very large
2. The impact of a customer for the performance of the system is very small, thus, a single customer make use of a very small percentage of the system resources.

3. All customers are independent, that is decisions to use the system are independent of other users.

These three conditions listed above indicate a Poisson arrival process with FCFS queue discipline.

The average time between arrivals (customers) is denoted by ($\frac{1}{\lambda}$). The average amount of time the server is busy is (τ). Therefore the fraction of time the server is busy which is called the utilization of the server is given below as;

$$\rho = \frac{\tau}{\frac{1}{\lambda}} = \frac{\lambda}{\mu} \tag{3.5}$$

where $\tau = \frac{1}{\mu}$

It should be noted that $\rho < 1$ if we have an infinite queue length. Moreover, if $\rho > 1$ then utilization is above 100% and therefore there is a finite queue length. This makes server becomes saturated, working 100% of the time, therefore resulting in queue becoming longer and then congestion. We should note again that over a longer period of time, the service rate should always exceed an arrival rate. That is $\mu > \lambda$ should always be the case.

3.3.1 Parameters For M/M/1 Queue Model

λ :the average arrival rate (λ/minutes) τ

:the average service time of customers

μ :the average service rate of customers (μ/minutes) ρ

:utilization of the server

L_s :expected number of customers in the system(i.e.,in service or waiting in queue)

L_q :expected number of customers in queue

W_s :expected waiting time of customers in the system

W_q :expected waiting time of customers in the queue

3.3.2 The State of the M/M/1 queue model

We are much interested to find the probability of customers in the system at any given period of time.

Since both interarrival and service times are exponentially distributed, and memoryless, there is no need to know how long the current customer has been in service or how long it has been since a customer arrived into the system. This situation enables one to know the number of entities in the queue as the system state. Of course arrivals come into the system one at a time and customers are served one at a time, the state of the system either increases (arrival of customers) or decreases (customers leave when service is completed). This is called the birthdeath process, which implies increases in the state as ’Births’ and decreases as ’Deaths’.

The birth-death process can be use to calculate the long term average probabilities of finding the system in any given state. It is also clear that when a customer is joining a queue, he or she is interested in four major things. Which are

1. the length of the system -L_s
2. the length of the queue - L_q
3. waiting time of the system -W_s
4. waiting time of the queue -W_q

For these reasons, the state of the system then gives the probability of customers in the queue.

P_0= no customer in the queue

P_1= 1 customer in the queue P_2= 2

customers in the queue

.

.

.

P_n= n customers in the queue

We note that the probability of arrival (λh) and no arrival = 1. The probability of no arrival is then given as $(1 - \lambda h)$.

Moreover, the probability of a person being served (μh) and no service =1. Therefore the probability of no service is also given as $(1 - \mu h)$.

3.3.3 Determination of Probabilities of customers in a system

$P_n(t+h)$ =P_{n-1} × probability of one arrival and no service + P_{n+1} × one service and no arrival + $P_n(t)$ × no arrival and no service

This is given as

$P_n(t+h) = P_{n-1}(t)\times\lambda h(1-\mu h)+P_{n+1}(t)\times\mu h(1-\lambda h)+P_n(t)(1-\lambda h)(1-\mu h)$

$\Longrightarrow$ $P_n(t+h) = P_{n-1}(t)\lambda h-\lambda\mu h^2 + P_{n+1}(t)\mu h-\lambda\mu h^2 + P_n(t)(1-\lambda h-\mu h+\lambda\mu h^2)$ leaving all second order terms out we have;

$$\Longrightarrow P_n(t + h) = P_{n-1}(t)\lambda h + P_{n+1}(t)\mu h + (\mu h) + P_n(t)(1 - \lambda h - \mu h)$$

$$\Longrightarrow P_n(t + h) = P_{n-1}(t)\lambda h + P_{n+1}(t)\mu h + P_n(t)(1 - \lambda h - \mu h)$$

$$\Longrightarrow P_n(t + h) = P_{n-1}(t)\lambda h + P_{n+1}\mu h + P_n(t) - P_n(t)\lambda h - P_n(t)\mu h$$

$$\Longrightarrow \frac{P_n(t+h) - P_n(t)}{h} = \frac{h(P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\lambda+\mu)}{h}$$

$$\Longrightarrow \frac{P_n(t+h) - P_n(t)}{h} = P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\lambda+\mu)$$

At the steady state condition

$$\Longrightarrow \frac{P_n(t+h) - P_n(t)}{h} = 0$$

$$\Longrightarrow 0 = P_{n-1}(0)\lambda + P_{n+1}(0)\mu - P_n(0)(\lambda + \mu)$$

$$\Longrightarrow 0 = \lambda P_{n-1} + \mu P_{n+1} - P_n(\lambda + \mu)$$

$$P_n(\lambda + \mu) = \lambda P_{n-1} + \mu P_{n+1} \tag{3.6}$$

To find the probability of no customer in the queue we have;

$P_0(t+h)=P_1(t)$ × one service and no arrival + $P_0(t)$ × no arrival and no service

$$\Longrightarrow P_0(t+h) = P_1(t)(1-\lambda h)\mu h + P_0(t)(1-\lambda h)1$$

$$\Longrightarrow \frac{P_0(t+h) - P_0(t)}{h} = P_1(t)\mu - P_0(t)\lambda$$

Again applying the steady state condition we have

$$\Longrightarrow \frac{P_0(t+h) - P_0(t)}{h} = 0$$

t also tends to 0

$$0 = P_1(0)\mu - P_0(0)\lambda$$

$$\mu P_1 = \lambda P_0 \tag{3.7}$$

$$P_1 = \frac{\lambda}{\mu} P_0 \tag{3.8}$$

Having one customer in the system ($n = 1$), from equation (3.6) we see that

$$\lambda P_0 + \mu P_2 = (\lambda + \mu)P_1 \lambda P_0 +$$

$$\mu P_2 = \lambda P_1 + \mu P_1$$

from equation (3.7) we have

$$\lambda P_0 + \mu P_2 = \lambda P_1 + \lambda P_0$$

$$P_2 = \frac{\lambda}{\mu} P_1$$

from equation (3.8) we have

$$P_2 = \frac{\lambda}{\mu}\frac{\lambda}{\mu} P_0$$

$$P_2 = (\frac{\lambda}{\mu})^2 P_0 \tag{3.9}$$

substituting equation (3.5) into (3.9)

$P_1 = \rho P_0$

$P_2 = \rho P_1 = \rho_2 P_0$

$P_3 = \rho P_2 = \rho_3 P_0$

.

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.

$P_n = \rho_n P_0$

We also know that all probabilities sum up to 1, which is $P_0 + P_1 + P_2 + \ldots\infty = 1$.

Therefore this gives us

$P_0 + \rho P_0 + \rho^2 P_0 + \ldots\infty = 1$

$$P_0[1 + \rho + \rho^2 + \ldots\infty] = 1 \tag{3.10}$$

Equation (3.10) becomes a geometric series, therefore applying infinity geometric series summation formula

$$P_0 = [\frac{1}{1-\rho}] = 1$$

$$P_0 = 1 - \rho \tag{3.11}$$

$$P_1 = \rho P_0 \tag{3.12}$$

substituting equation (3.11) into equation (3.12) we get

$$P_1 = \rho(1 - \rho)$$

.

.

.

$$P_n = \rho_n P_0$$

$$P_n = \rho^n(1 - \rho) \tag{3.13}$$

This is the equation for finding the probability of (n) customers in a system.

3.3.4 Performance measure of the M/M/1 queue

Finding the expected number of entities or customers in a system

$$L_s = \sum_{j=0}^{\infty} jP_j$$

Where *j* is the number of customers in the system.

P_j is the steady state probability that there are so many people in the system.

$$L_s = Xj\rho_j P_0$$

$$L_s = P_0\rho Xj\rho_{j-1}$$

$$L_s = P_0\rho \sum \frac{d}{d\rho}\rho^j$$
$$L_s = P_0\rho \frac{d}{d\rho} \sum \rho^j$$

$$L_s = P_0\rho \frac{d}{d\rho}[1 + \rho + \rho^2 + ...\infty]$$

$$L_s = P_0\rho \frac{d}{d\rho} \frac{1}{1-\rho}$$

$$L_s = P_0\rho \frac{1}{(1-\rho)^2} \tag{3.14}$$

Substituting equation (3.11) into equation (3.14) we have;

$$L_s = \frac{(1-\rho)\rho}{(1-\rho)^2}$$

Finally, the expected number of customers in the system is

$$L_s = \frac{\rho}{1-\rho} \tag{3.15}$$

We could also see that, the expected number of customers in a queue together with a customer being served would give us the expected number of customers in the system. This is shown in the equation below;

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = L_q + \rho$$

Therefore the expected number of customers or entities in a queue is given by;

$$L_q = L_s - \rho \tag{3.16}$$

3.3.5 Little Theorem

Little's theorem put forward a very important relation between the average number of customers in the system (L_s), the average service time (τ) and the arrival rate of customers(λ). Little's law states that

$$L_s = \lambda W_s \tag{3.17}$$

$$L_q = \lambda W_q \tag{3.18}$$

From equations (3.17) and (3.18) we obtain

$$W_s = \frac{L_s}{\lambda} \tag{3.19}$$

$$W_q = \frac{L_q}{\lambda} \tag{3.20}$$

Where W_s and W_q are the expected waiting time in the system and in the queue respectively.

CHAPTER 4

APPLICATION AND ANALYSIS

4.1 INTRODUCTION

This chapter looks at the analysis and modeling of the data collected using the $M/M/1$ queuing model. The raw data used for the study would be found in the appendix A of this work.

Moreover, data collection, results, findings and summary of analysis of data were outlined and presented.

The next chapter highlights on the conclusion and recommendations drawn from the study.

4.2 ANALYSIS OF DATA

The data for the analysis were collected on Monday, Tuesday, Wednesday, Thursday, and Friday from the eye-section of the DVLA-Kumasi, from the 23rd-
27th February 2015.(9:00am-12:00 noon)

The mean arrival rates and the mean service rates would be calculated from the data collected and their results would be used to measure the performance of the entire system as outlined in chapter three.

4.2.1 Analysis of data collected on Monday

Table 4.1: Monday's Analysis

Time-Interval	Number of drivers	Arrival rate
$9:00 - 9:30$am<	7	0.2333
$9:30 - 10:00$am<	6	0.2
$10:00 - 10:30$am<	9	0.3
$10:30 - 11:00$am<	7	0.2333
$11:00 - 11:30$am<	11	0.3667
$11:30 - 12:00$noon<	17	0.5667
	total=57	total=1.9

From equation (3.5) in section(3.3), the mean arrival rate on Monday is
$\lambda = \frac{1.9}{6}$

= $0.3167 \approx 3$ drivers per 10 minutes

The total service time is 168.9 minutes.

Whiles the expectation of service time is $\frac{168.9}{57} = 2.9632$ minutes

Hence the service rate on Monday is

$\mu = \frac{1}{2.9632}$

$= 0.3375$

1. From equation (3.5) in section (3.3.3), the utilization or busy period of the the optometrist is

   $\rho = \frac{0.3167}{0.3375}$

   $= 0.9383$

2. From equation (3.11) in section (3.3.3), the idle period of the optometrist is

   $P_0 = 1 - 0.9383$

   $= 0.0617$

3. From equation (3.12) in section(3.3.3), the probability that there is no queue on Monday is

   $P(\text{no queue}) = P_0 + P_1$

   $= 0.0617 + (0.9383 \times 0.0617)$

   $= 0.0617 + 0.0579$

   $= 0.1196$

4. From equation (3.13) in section(3.3.3), the probability that there are 15 drivers in the system is

   $P_{15} = (0.9383)^{15} \times 0.0617$

   $= 0.3847 \times 0.061$

   $= 0.0237$

5. Probability that there are two or more drivers in the system is

   $P(n \geq 2) = 1 - (P_0 + P_1)$

   $= 1 - (0.0617 + 0.0579)$

   $= 0.8804$

4.2.2 Performance measure of the system on Monday

1. From equation (3.15)in section (3.3.4), the expected number of drivers waiting in the system is

   $L_s = \frac{0.9383}{0.0617}$

   $= 15.2075$

2. From equation (3.16) in section (3.3.4), the expected number of drivers waiting in the queue is

   $L_q = 15.2075 - 0.9383$

   $= 14.2693$

3. From equation (3.19)in section (3.3.5), the expected waiting time spent by a driver in the system is

   $W_s = \frac{15.2075}{0.3167}$

   $= 48.0186$ minutes

4. From equation (3.20) in section (3.3.5), the expected waiting time of a driver in the queue is

   $W_q = \frac{14.2693}{0.3167}$

   $= 45.0562$ minutes

4.2.3 Summary of Results on Monday

The results show that on Monday, the optometrist would be busy 93.83% of the time and would be idle 6.17% of the time. Also, 11.96% of the time there would be no queue, this means that any driver who comes in during this period joins the queue as the first person. Again 2.37% of the time there would be 15 drivers in the system, that is 14 of them in the queue and 1 being served. More so, 88.04% of the time there would be more than one driver in the system.

The results further shows that the average number of drivers in the queue and the system are approximately 14 and 15 drivers respectively. In addition, the average time spent by a driver in the queue and in the system is 45.0562 minutes and 48.0186 minutes respectively.

4.2.4 Analysis of data collected on Tuesday

Table 4.2: Tuesday's Analysis

Time-Interval	Number of drivers	Arrival rate
9 : 00 − 9 : 30am<	9	0.2368
9 : 30 − 10 : 00am<	6	0.1579
10 : 00 − 10 : 30am<	6	0.1579
10 : 30 − 11 : 00am<	6	0.1579
11 : 00 − 11 : 30am<	4	0.1053
11 : 30 − 12 : 00noon<	7	0.1842
	total=38	total=1

The mean arrival rate on Tuesday is

$\lambda = \frac{1}{6}$

= 0.1667 ≈ 2 drivers per 10 minutes

The total service time is 87.82 minutes.

Whiles the expectation of service time is $\frac{87.82}{38}$ =2.3111 minutes.

Hence the service rate on Tuesday is

$\mu = \frac{1}{2.3111}$

= 0.4327

1. The busy period of the optometrist is

   $\rho = \frac{0.1667}{0.4327}$

   = 0.3853

2. The free period of the optometrist is

   $P_0 = 1 - 0.3853$

   = 0.6147

3. Probability that there is no queue is

P(no queue)= $P_0 + P_1$
$0.6147 + (0.3853 * 0.6147)$

$= 0.7494$

4. Probability that there are 8 drivers in the system is

$P_8 = (0.3853)^8 \times 0.6147$

$= 0.000299$

5. Probability that there are two or more drivers in the system is

$P(n \geq 2) = 1 - (0.6147 + 0.2368)$

$= 0.1485$

4.2.5 Performance measure of the system on Tuesday

1. Expected number of drivers waiting in the system is
$L_s = \frac{0.3853}{0.6147}$

$= 0.6268$

2. Expected number of drivers waiting in the queue is

$L_q = 0.6268 - 0.3853$

$= 0.2415$

3. Expected waiting time spent by drivers in the system is
$W_s = \frac{0.6268}{0.3853}$

1.6268 minutes

4. Expected waiting time of drivers in the queue is
$W_q = \frac{0.2415}{0.3853}$

$= 0.6268$ minutes

4.2.6 Summary of Results on Tuesday

The results show that on Tuesday, the optometrist would be busy 38.58% of the time and would be idle 61.47% of the time. Also, 74.94% of the time there would be no queue, this means that any driver who comes in during this period joins the queue as the first person. Again 0.0299% of the time there would be 8 drivers in the system, that is 7 of them in the queue and 1 being served by the optometrist. More so, 14.85% of the time there would be more than one driver in the system. The results further shows that the average number of drivers in the queue and the system are approximately 0.2415 and 0.6268 drivers respectively. In addition, the average time spent by a driver in the queue and in the system is 0.6268 minutes and 1.6268 minutes respectively.

4.2.7 Analysis of data collected on Wednesday

Table 4.3: Wednesday's Analysis

Time-Interval	Number of drivers	Arrival rate
$9:00 - 9:30$am<	4	0.1333
$9:30 - 10:00$am<	8	0.2667
$10:00 - 10:30$am<	7	0.2333
$10:30 - 11:00$am<	7	0.2333
$11:00 - 11:30$am<	6	0.2
$11:30 - 12:00$noon<	10	0.3333
	total=42	total=1.3999

The mean arrival rate on Wednesday is

$\lambda = \frac{1.3999}{6}$

= 0.2333 ≈ 2 drivers per 10 minutes

The total service time is 119.76 minutes.

Whiles the expectation of service time is $\frac{119.79}{42} = 2.8514$ minutes Hence the service rate on Wednesday is

$\mu = \frac{1}{2.8514}$

= 0.3507 driver per minute which is equivalent to 3.507

1. The busy period of the optometrist is

   $\rho = \frac{0.2333}{0.3507}$

   = 0.6653

2. The free period of the optometrist is

   $P_0 = 1 - 0.6653$

   = 0.3347

3. Probability that there is no queue is

   P(no queue)= $P_0 + P_1$
   $0.3347 + (0.6653 \times 0.3347)$

$= 0.5574$

4. Probability that there are 10 drivers in the system is

   $P_{10} = (0.6653)^{10} \times 0.3347$

   $= 0.005687$

5. Probability that there are two or more drivers in the system is

   $P(n \geq 2) = 1 - (0.3347 + 0.2227)$

   $= 0.4426$

4.2.8 Performance measure of the system on Wednesday

1. Expected number of drivers waiting in the system is
   $L_s = \frac{0.6653}{0.3347}$

   $= 1.9878$

2. Expected number of drivers waiting in the queue is

   $L_q = 1.9878 - 0.6653$

   $= 1.3225$

3. Expected waiting time spent by drivers in the system is
   $W_s = \frac{1.9878}{0.2333}$

   $= 8.8520$ minutes

4. Expected waiting time of drivers in the queue is
   $W_q = \frac{1.3225}{0.2333}$

   $= 5.6687$ minutes

4.2.9 Summary of Results on Wednesday

The results show that on Wednesday, the optometrist would be busy 66.53% of the time and would be idle 33.47% of the time. Also, 55.74% of the time there would be no queue, this means that any driver who comes in during this period joins the queue as the first person. Again 0.5687% of the time there would be 10 drivers in the system, that is 9 of them in the queue and 1 being served by the optometrist. More so, 44.26% of the time there would be more than one driver in the system.

The results further shows that the average number of drivers in the queue and the system are approximately 2 and 1 drivers respectively. In addition, the average time spent by a driver in the queue and in the system is 5.6687 minutes and 8.8520 minutes respectively.

4.2.10 Analysis of data collected on Thursday

Table 4.4: Thursday's Analysis

Time-Interval	Number of drivers	Arrival rate
$9:00-9:30$am<	8	0.2667
$9:30-10:00$am<	7	0.2333
$10:00-10:30$am<	5	0.1667
$10:30-11:00$am<	5	0.1667
$11:00-11:30$am<	3	0.1
$11:30-12:00$noon<	4	0.1333
	total=32	total=1.0667

Therefore mean arrival rate on Thursday is

$\lambda = \frac{1.0667}{6}$

= 0.1778 ≈ 2 drivers per 10 minutes

The total service time is 95.32 minutes.

Whiles the expectation of service time is $\frac{95.32}{32}$ =2.9788 minutes Hence the service rate on Thursday is

$\mu = \frac{1}{2.9788}$

= 0.3375

1. The busy period of the optometrist is

   $\rho = \frac{0.1778}{0.3357}$

   = 0.5297

2. The free period of the optometrist is

   $P_0 = 1 - 0.5297$

   = 0.4703

3. Probability that there is no queue is

   P(no queue)= $P_0 + P_1$
   $0.4703 + (0.5297 \times 0.4703)$

$= 0.7194$

4. Probability that there are 10 drivers in the system is

   $P_{10} = (0.5297)^{10} \times 0.4703$

   $= 0.000818$

5. Probability that there are two or more drivers in the system is

   $P(n \geq 2) = 1 - (0.4703 + 0.2491)$

   $= 0.2806$

4.2.11 Performance measure of the system on Thursday

1. Expected number of drivers waiting in the system is
   $L_s = \frac{0.5297}{0.4703}$

   $= 1.1263$

2. Expected number of drivers waiting in the queue is

   $L_q = 1.1263 - 0.5297$

   $= 0.5966$

3. Expected waiting time spent by drivers in the system is
   $W_s = \frac{1.1263}{0.1778}$

   $= 6.3346$ minutes

4. Expected waiting time of drivers in the queue is
   $W_q = \frac{0.5966}{0.1778}$

   $= 3.3554$ minutes

4.2.12 Summary of Results on Thursday

The results show that on Thursday, the optometrist would be busy 52.97% of the time and would be idle 47.03% of the time. Also, 71.94% of the time there would be no queue, this means that any driver who comes in during this period joins the queue as the first person. Again 0.0882% of the time there would be 10 drivers in the system, that is 9 of them in the queue and 1 being served by the optometrist. More so, 28.06% of the time there would be more than one driver in the system.

The results further shows that the average number of drivers in the queue and the system are approximately 0.6 and 1 drivers respectively. In addition, the average time spent by a driver in the queue and in the system is 3.3554 minutes and 6.3346 minutes respectively.

4.2.13 Analysis of data collected on Friday

The mean arrival rate on Friday is

Table 4.5: Friday’s Analysis

Time-Interval	Number of drivers	Arrival rate
9 : 00 − 9 : 30am<	13	0.2333
9 : 30 − 10 : 00am<	10	0.2
10 : 00 − 10 : 30am<	10	0.3
10 : 30 − 11 : 00am<	10	0.2333
11 : 00 − 11 : 30am<	11	0.3667
11 : 30 − 12 : 00noon<	12	0.5667
	total=66	total=2.1999

The mean arrival rate on Friday is

$\lambda = \frac{2.1999}{6}$

= 0.3667 ≈ 4 drivers per 10 minutes

The total service time is 175.8 minutes.

Whiles the expectation of service time is $\frac{175.8}{66}$ =2.9632 minutes

Hence the service rate on Friday is

$\mu = \frac{1}{2.6636}$

= 0.3375

1. The busy period of the optometrist is

   $\rho = \frac{0.3667}{0.3754}$

   = 0.9767

2. The free period of the optometrist is

   $P_0 = 1 - 0.9767$

   = 0.0233

3. Probability that there is no queue is

   P(no queue)= $P_0 + P_1$

   $0.0233 + (0.9767 \times 0.0233)$

$= 0.0461$

4. Probability that there are 15 drivers in the system is

   $P_{15} = (0.9767)^{15} \times 0.0233$

   $= 0.0164$

5. Probability that there are two or more drivers in the system is

   $P(n \geq 2) = 1 - (0.0233 + 0.0228)$

   $= 0.9539$

4.2.14 Performance measure of the system on Friday

1. Expected number of drivers waiting in the system is

   $L_s = \frac{0.9767}{0.0233}$

   $= 41.9185$

2. Expected number of drivers waiting in the queue is

   $L_q = 41.9185 - 0.9767$

   $= 40.9418$

3. Expected waiting time spent by drivers in the system is

   $W_s = \frac{41.9185}{0.3667}$

   $= 114.3128$ minutes

4. Expected waiting time of drivers in the queue is

   $W_q = \frac{40.9418}{0.3667}$

   $= 111.6493$ minutes

4.2.15 Summary of Results on Friday

The results show that on Friday, the optometrist would be busy 97.67% of the time and would be idle 2.33% of the time. Also, 4.61% of the time there would be no queue, this means that any driver who comes in during this period joins the queue as the first person. Again 1.64% of the time there would be 15 drivers in the system, that is 14 of them in the queue and 1 being served by the optometrist. More so, 95.39% of the time there would be more than one driver in the system. The results further shows that the average number of drivers in the queue and the system are approximately 41 and 42 drivers respectively. Lastly, the average time spent by a driver in the queue and in the system is 111.6493 minutes and 114.3128 minutes respectively.

4.2.16 Results Of the Findings during the Week

- Comparing the mean arrival rates of the drivers during the days the study was conducted, table (4.6) showed that Friday had the highest arrival rate with 0.3667 drivers per 30 minutes. With Monday, Wednesday and Thursday in that order with respective arrival rates of 0.3167, 0.2333 and 0.1778. Whiles Tuesday recorded the least arrival rate of 0.1667 drivers per 30 minutes.

  In addition to this, Tuesday recorded the highest service rate of 0.4327 drivers, followed by Friday, Wednesday and Monday with service rates of 0.3754, 0.3507, and 0.3375 drivers per hour respectively. Again Thursday had the least service rate of 0.3357.

- From table (4.7), the Optometrist was very busy on Friday with 97.67% and idle 2.33% of the time. This follows on Monday with busy period of the Optometrist to be 93.83% and idle period of 6.17% of the time. Next are Wednesday and Thursday with busy period of 66.53% and 52.97% and idle period of 33.47% and 47.03% respectively. Tuesday recorded the least busy period of the Optometrist with $38.53.07\%$ and idle period of 61.47% of the time.

- Table (4.8), finally showed the performance measures during the week. Friday had the highest measure during the five working days when the study was conducted. It had 42 drivers in the system, 41 drivers in the queue, with 114.3128 and 111.6493 minutes of time spent by the drivers in the system and in the queue respectively for service. Monday also had the second highest measure during the week with 15 drivers in the system 14 in the queue, 48.0186 minutes spent in the system and 45.0562 minutes spent in the queue. Wednesday, Thursday and Tuesday followed in that order with 2, 1, 0.6268 drivers in the system respectively. More so, 1, 0.5966, 0.2415 drivers in the queue respectively on Wednesday, Thursday and Tuesday. In all Tuesday had the least minutes for a driver to wait in the system which was 0.6268 minutes and 0.2415 minutes in the queue for services.

In conclusion, we see from table (4.6) that, the mean arrival rate (λ) is always less than the mean service rate (μ), which shows the utilization period of the Optometrist which is $\rho < 1$.

Arrival Rate and Service Rate during the Week

Table 4.6: Comparison of the arrival rate and service rate during the week

DAYS	ARRIVAL RATE	SERVICE RATE
Monday	0.3167	0.3375
Tuesday	0.1667	0.4327
Wednesday	0.2333	0.3507
Thursday	0.1778	0.3357
Friday	0.3667	0.3754

Busy period and Idle period during the Week

Table 4.7: Comparison of the busy period and idle period of the Optometrist during the week

DAYS	BUSY PERIOD	IDLE PERIOD
Monday	0.9383	0.0617
Tuesday	0.3853	0.6147

Wednesday	0.6653	0.3347
Thursday	0.5297	0.4703
Friday	0.9767	0.0233

Performance Measures during the Week

Table 4.8: Comparison of the Performance Measures during the week

DAYS	L_s	L_q	W_s	W_q
Monday	15 drivers	14 drivers	48.0186 minutes	45.0562 minutes
Tuesday	0.6268 driver	0.2415 driver	1.6268 minutes	0.6268 minutes
Wednesday	2 drivers	1 driver	8.8520 minutes	5.6687 minutes
Thursday	1 driver	0.5966 driver	6.3346 minutes	3.3554 minutes
Friday	42 drivers	41 drivers	114.3128 minutes	111.6493 minutes

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter summarizes the results of the study, discusses the findings, conclusions arrived at, by the researcher and recommendations are made to guide drivers, management and administrators at the DVLA.

5.1.1 Conclusion

The objective of any firm is to increase revenue and improve upon customer service and care. Customers satisfaction is very key and essential in building up any entity or organization.

With knowledge of probability theory and stochastic process, the $M/M/1$ queuing model was appropriate to deal with the queue at the eye-test section at the DVLA. It was observed from the study that Monday and Friday had the highest utilization factor of 93.83% and 97.67% respectively of the time by the Optometrist. Again these days(Monday and Friday) had the highest arrival rates of 31.67% and 36.67% respectively.

Nevertheless, the performance measures from the study also showed again that Monday and Friday recorded the highest number of drivers of 14 and 41 drivers in the queue respectively, including the number of drivers in the system to be 15 and 42 respectively on Monday and Friday.

5.1.2 Recommendations

Based on the findings and conclusion of this study, the following recommendations are suggested for stakeholders of the DVLA.

It is therefore recommended to the management and administrators of the DVLA;

- to display the results of the study conducted on their notice board for drivers to know the average amount of time the drivers are expected to spend on each day at the eye-test section.
- from the study conducted, Monday and Friday recorded the highest arrival rates of 31.67% and 36.67% respectively as compared to the other three working days. Therefore, drivers should be made to be aware that, since Mondays begin a new working week and Fridays end another working week, many customers troop to the premises of the DVLA for services during those days, hence resulting in long queues on Mondays and Fridays.
- customers can therefore choose to go through the eye-test examination on Tuesdays, Wednesdays and Thursdays to avoid long queues and congestion, since those days recorded the least arrival rates and performance measures.

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APPENDIX A

Raw data collected on Monday

Arrivals	Arrival time	Departure Time	Service Time
1	09 : 00	09 : 02 : 58	0 : 02 : 58
2	09 : 05	09 : 08 : 40	0 : 03 : 40
3	09 : 09	09 : 12 : 49	0 : 03 : 49
4	09 : 15	09 : 18 : 50	0 : 03 : 50
5	09 : 19	09 : 21 : 40	0 : 02 : 40
6	09 : 25	09 : 27 : 55	0 : 02 : 55
7	09 : 30	09 : 32 : 59	0 : 02 : 59
8	09 : 33	09 : 36 : 59	0 : 03 : 59
9	09 : 37	09 : 41 : 20	0 : 04 : 02
10	09 : 41	09 : 44 : 10	0 : 03 : 10
11	09 : 45	09 : 48 : 09	0 : 03 : 09
12	09 : 55	09 : 59 : 00	0 : 04 : 00
13	09 : 59	10 : 02 : 53	0 : 03 : 53
14	10 : 03	10 : 04 : 59	0 : 01 : 59
15	10 : 05	10 : 07 : 45	0 : 02 : 45
16	10 : 10	10 : 13 : 44	0 : 03 : 44
17	10 : 15	10 : 18 : 55	0 : 03 : 55
18	10 : 19	10 : 21 : 59	0 : 02 : 59
19	10 : 22	10 : 25 : 05	0 : 03 : 05
20	10 : 25	10 : 27 : 47	0 : 02 : 47
21	10 : 27	10 : 30 : 55	0 : 03 : 55
22	10 : 30	10 : 34 : 05	0 : 04 : 05
23	10 : 34	10 : 35 : 44	0 : 01 : 44
24	10 : 37	10 : 38 : 55	0 : 01 : 55
25	10 : 40	10 : 43 : 07	0 : 03 : 07
26	10 : 44	10 : 46 : 59	0 : 02 : 59
27	10 : 50	10 : 53 : 45	0 : 03 : 45
28	10 : 54	10 : 55 : 39	0 : 01 : 39
29	10 : 56	10 : 57 : 57	0 : 01 : 57
30	11 : 01	11 : 03 : 44	0 : 02 : 44
31	11 : 04	11 : 07 : 05	0 : 03 : 05
32	11 : 07	11 : 09 : 59	0 : 02 : 59
33	11 : 10	11 : 13 : 06	0 : 03 : 06
34	11 : 13	11 : 16 : 44	0 : 03 : 44
35	11 : 16	11 : 18 : 58	0 : 02 : 58
36	11 : 18	11 : 21 : 44	0 : 03 : 44
37	11 : 21	11 : 24 : 57	0 : 03 : 57
38	11 : 24	11 : 26 : 38	0 : 02 : 38
39	11 : 26	11 : 29 : 09	0 : 03 : 09
40	11 : 29	11 : 31 : 59	0 : 02 : 59
41	11 : 31	11 : 33 : 08	0 : 03 : 08
42	11 : 33	11 : 36 : 07	0 : 03 : 07
43	11 : 36	11 : 38 : 47	0 : 02 : 47
44	11 : 38	11 : 40 : 58	0 : 02 : 58
45	11 : 40	11 : 43 : 44	0 : 03 : 44
46	11 : 43	11 : 47 : 00	0 : 04 : 00
47	11 : 47	11 : 52 : 00	0 : 05 : 00
48	11 : 52	11 : 55 : 43	0 : 03 : 43
49	11 : 55	11 : 58 : 15	0 : 03 : 15
50	11 : 58	12 : 01 : 18	0 : 03 : 18
51	12 : 01	12 : 04 : 20	0 : 03 : 20
52	12 : 04	12 : 07 : 44	0 : 03 : 44

53	12 : 07	12 : 10 : 18	0 : 03 : 18
54	12 : 10	12 : 13 : 17	0 : 03 : 17
55	12 : 13	12 : 15 : 57	0 : 02 : 57
56	12 : 15	12 : 16 : 58	0 : 01 : 58
57	12 : 16	12 : 18 : 49	73 0 : 02 : 49

The total number of arrivals at the eye-test section on Monday is 57 and the total service time is 168.9 minutes.

Raw data collected on Tuesday

Arrivals	Arrival Time	Departure Time	Service Time
1	9 : 00	09 : 02 : 03	0 : 02 : 03
2	9 : 10	09 : 12 : 01	0 : 02 : 01
3	9 : 12	09 : 13 : 57	0 : 01 : 57
4	9 : 15	09 : 17 : 45	0 : 02 : 45
5	9 : 17	09 : 20 : 00	0 : 03 : 00
6	9 : 20	09 : 22 : 10	0 : 02 : 10
7	9 : 22	09 : 24 : 13	0 : 02 : 13
8	9 : 25	09 : 27 : 55	0 : 02 : 55
9	9 : 30	09 : 33 : 15	0 : 03 : 15
10	9 : 42	09 : 46 : 20	0 : 04 : 20
11	9 : 46	09 : 48 : 45	0 : 02 : 45
12	9 : 50	09 : 51 : 45	0 : 01 : 45
13	9 : 55	09 : 56 : 35	0 : 01 : 35
14	9 : 58	10 : 00 : 23	0 : 02 : 33
15	10 : 00	10 : 02 : 45	0 : 02 : 45
16	10 : 05	10 : 07 : 08	0 : 02 : 08
17	10 : 08	10 : 09 : 59	0 : 01 : 59
18	10 : 10	10 : 11 : 45	0 : 01 : 45
19	10 : 13	10 : 16 : 05	0 : 03 : 05
20	10 : 25	10 : 27 : 09	0 : 02 : 09
21	10 : 28	10 : 29 : 57	0 : 01 : 57
22	10 : 40	10 : 42 : 35	0 : 02 : 35
23	10 : 45	10 : 47 : 23	0 : 02 : 23
24	10 : 50	10 : 53 : 01	0 : 03 : 01
25	10 : 54	10 : 56 : 33	0 : 02 : 33
26	10 : 57	10 : 58 : 49	0 : 01 : 49
27	11 : 00	11 : 02 : 07	0 : 02 : 07
28	11 : 15	11 : 16 : 55	0 : 01 : 55
29	11 : 21	11 : 23 : 53	0 : 02 : 53
30	11 : 26	11 : 28 : 41	0 : 02 : 41
31	11 : 30	11 : 33 : 01	0 : 03 : 01
32	11 : 35	11 : 37 : 55	0 : 02 : 55
33	11 : 38	11 : 40 : 13	0 : 02 : 13
34	11 : 41	11 : 43 : 52	0 : 02 : 52
35	11 : 44	11 : 46 : 45	0 : 02 : 45
36	11 : 50	11 : 52 : 55	0 : 02 : 55
37	11 : 56	11 : 59 : 09	0 : 03 : 09
38	12 : 00	12 : 02 : 50	0 : 02 : 50

The total number of arrivals at the eye-test section on Tuesday is 38 and the total service time is 87.82 minutes.

Raw data collected on Wednesday

Arrivals	Arrival Time	Departure Time	Service Time
1	09 : 00	09 : 01 : 33	0 : 01 : 33
2	09 : 05	09 : 07 : 23	0 : 02 : 23
3	09 : 10	09 : 11 : 58	0 : 01 : 58
4	09 : 20	09 : 22 : 36	0 : 02 : 36
5	09 : 35	09 : 36 : 42	0 : 01 : 42
6	09 : 37	09 : 40 : 37	0 : 03 : 37
7	09 : 40	09 : 43 : 14	0 : 03 : 14
8	09 : 44	09 : 48 : 11	0 : 04 : 11
9	09 : 48	09 : 50 : 51	0 : 02 : 51
10	09 : 51	09 : 53 : 59	0 : 02 : 59
11	09 : 55	09 : 58 : 00	0 : 03 : 00
12	09 : 58	10 : 01 : 10	0 : 03 : 10
13	10 : 01	10 : 06 : 57	0 : 05 : 57
14	10 : 07	10 : 08 : 50	0 : 01 : 50
15	10 : 11	10 : 12 : 26	0 : 01 : 26
16	10 : 13	10 : 14 : 44	0 : 01 : 44
17	10 : 14	10 : 15 : 27	0 : 01 : 27
18	10 : 17	10 : 20 : 57	0 : 03 : 57
19	10 : 25	10 : 26 : 50	0 : 01 : 50
20	10 : 33	10 : 37 : 00	0 : 04 : 00
21	10 : 37	10 : 41 : 22	0 : 04 : 22
22	10 : 41	10 : 46 : 40	0 : 05 : 40
23	10 : 46	10 : 49 : 33	0 : 03 : 33
24	10 : 49	10 : 51 : 10	0 : 02 : 10
25	10 : 51	10 : 53 : 05	0 : 02 : 05
26	10 : 55	10 : 58 : 21	0 : 03 : 21
27	11 : 05	11 : 09 : 14	0 : 04 : 14
28	11 : 11	11 : 14 : 54	0 : 03 : 54
29	11 : 15	11 : 17 : 39	0 : 02 : 39
30	11 : 20	11 : 23 : 34	0 : 03 : 34
31	11 : 23	11 : 24 : 21	0 : 01 : 21
32	11 : 30	11 : 33 : 27	0 : 03 : 27
33	11 : 33	11 : 34 : 24	0 : 01 : 24
34	11 : 38	11 : 40 : 29	0 : 02 : 29
35	11 : 40	11 : 42 : 08	0 : 02 : 08
36	11 : 42	11 : 46 : 43	0 : 04 : 43
37	11 : 47	11 : 52 : 03	0 : 05 : 03
38	11 : 52	11 : 57 : 07	0 : 05 : 07
39	11 : 57	12 : 00 : 45	0 : 03 : 45
40	12 : 01	12 : 03 : 57	0 : 02 : 57
41	12 : 04	12 : 06 : 55	0 : 02 : 55
42	12 : 07	12 : 09 : 00	0 : 02 : 00

The total number of arrivals at the eye-test section on Wednesday is 42 and the total service time is 119.76 minutes.

Raw data collected on Thursday

Arrivals	Arrival Time	Departure Time	Service Time
1	09 : 00	09 : 03 : 14	0 : 03 : 14
2	09 : 05	09 : 08 : 15	0 : 03 : 15
3	09 : 08	09 : 10 : 55	0 : 02 : 55
4	09 : 11	09 : 14 : 15	0 : 03 : 15
5	09 : 15	09 : 18 : 05	0 : 03 : 05
6	09 : 20	09 : 23 : 12	0 : 03 : 12
7	09 : 25	09 : 27 : 59	0 : 02 : 59
8	09 : 28	09 : 30 : 55	0 : 02 : 55
9	09 : 31	09 : 33 : 54	0 : 02 : 54
10	09 : 34	09 : 38 : 10	0 : 04 : 10
11	09 : 39	09 : 43 : 02	0 : 04 : 02
12	09 : 44	09 : 48 : 05	0 : 04 : 05
13	09 : 50	09 : 53 : 56	0 : 03 : 56
14	09 : 55	09 : 57 : 50	0 : 02 : 50
15	10 : 00	10 : 03 : 15	0 : 03 : 15
16	10 : 10	10 : 13 : 25	0 : 03 : 25
17	10 : 15	10 : 17 : 58	0 : 02 : 58
18	10 : 20	10 : 23 : 09	0 : 03 : 09
19	10 : 25	10 : 27 : 58	0 : 02 : 58
20	10 : 28	10 : 31 : 10	0 : 03 : 10
21	10 : 32	10 : 35 : 15	0 : 03 : 15
22	10 : 37	10 : 40 : 18	0 : 03 : 18
23	10 : 45	10 : 46 : 49	0 : 03 : 49
24	10 : 50	10 : 53 : 01	0 : 03 : 01
25	11 : 00	11 : 02 : 55	0 : 02 : 55
26	11 : 10	11 : 12 : 48	0 : 02 : 48
27	11 : 20	11 : 22 : 33	0 : 02 : 33
28	11 : 30	11 : 32 : 14	0 : 02 : 14
29	11 : 35	11 : 37 : 50	0 : 02 : 50
30	11 : 45	11 : 48 : 03	0 : 03 : 03
31	11 : 50	11 : 53 : 09	0 : 03 : 09
32	12 : 00	12 : 02 : 55	0 : 02 : 55

The total number of arrivals at the eye-test section on Thursday is 32 and the total service time is 95.32 minutes.

Raw data collected on Friday

Arrivals	Arrival Time	Departure Time	Service Time
1	09 : 00	09 : 02 : 30	0 : 02 : 30
2	09 : 03	09 : 05 : 10	0 : 02 : 10
3	09 : 06	09 : 09 : 07	0 : 03 : 07
4	09 : 09	09 : 12 : 45	0 : 03 : 45
5	09 : 12	09 : 15 : 00	0 : 03 : 00
6	09 : 15	09 : 17 : 05	0 : 02 : 05
7	09 : 17	09 : 18 : 45	0 : 01 : 45
8	09 : 19	09 : 21 : 01	0 : 02 : 01
9	09 : 21	09 : 23 : 00	0 : 03 : 00
10	09 : 24	09 : 25 : 55	0 : 01 : 55
11	09 : 26	09 : 27 : 46	0 : 01 : 46
12	09 : 28	09 : 29 : 59	0 : 01 : 59
13	09 : 30	09 : 32 : 15	0 : 02 : 15
14	09 : 32	09 : 34 : 33	0 : 02 : 33
15	09 : 35	09 : 37 : 45	0 : 02 : 45
16	09 : 37	09 : 39 : 59	0 : 02 : 59
17	09 : 40	09 : 42 : 46	0 : 02 : 46
18	09 : 43	09 : 44 : 59	0 : 01 : 59
19	09 : 45	09 : 47 : 38	0 : 02 : 38
20	09 : 47	09 : 49 : 58	0 : 02 : 58
21	09 : 50	09 : 52 : 19	0 : 02 : 19
22	09 : 53	09 : 56 : 10	0 : 03 : 10
23	09 : 57	10 : 00 : 07	0 : 03 : 07
24	10 : 01	10 : 04 : 14	0 : 03 : 14
25	10 : 05	10 : 07 : 55	0 : 02 : 55
26	10 : 07	10 : 11 : 09	0 : 03 : 09
27	10 : 12	00 : 00 : 00	0 : 00 : 00
28	10 : 12	10 : 15 : 02	0 : 03 : 02
29	10 : 16	10 : 19 : 10	0 : 03 : 10
30	10 : 20	10 : 22 : 56	0 : 02 : 56
31	10 : 23	10 : 25 : 55	0 : 02 : 55
32	10 : 25	10 : 28 : 45	0 : 03 : 45
33	10 : 29	10 : 32 : 09	0 : 03 : 09
34	10 : 33	10 : 35 : 45	0 : 02 : 45
35	10 : 36	10 : 38 : 55	0 : 02 : 55
36	10 : 39	10 : 42 : 09	0 : 03 : 09
37	10 : 42	10 : 45 : 15	0 : 03 : 15
38	10 : 46	10 : 49 : 45	0 : 03 : 45
39	10 : 49	10 : 52 : 56	0 : 03 : 56
40	10 : 52	10 : 55 : 44	0 : 03 : 44
41	10 : 55	10 : 57 : 58	0 : 02 : 58
42	10 : 57	11 : 00 : 15	0 : 03 : 15
43	11 : 00	11 : 02 : 51	0 : 02 : 51
44	11 : 02	11 : 04 : 58	0 : 02 : 58
45	11 : 05	11 : 08 : 05	0 : 03 : 05
46	11 : 08	11 : 11 : 02	0 : 03 : 02
47	11 : 11	11 : 14 : 01	0 : 03 : 01
48	11 : 14	11 : 16 : 47	0 : 02 : 47
49	11 : 16	11 : 18 : 35	0 : 02 : 35
50	11 : 19	11 : 21 : 15	0 : 02 : 15
51	11 : 21	11 : 22 : 58	0 : 01 : 58
52	11 : 23	11 : 25 : 56	0 : 02 : 56
53	11 : 26	11 : 29 : 15	0 : 03 : 15

54	11 : 30	11 : 33 : 03	0 : 03 : 03
55	11 : 33	11 : 35 : 45	0 : 02 : 45
56	11 : 36	11 : 39 : 45	0 : 03 : 45
57	11 : 40	11 : 42 : 44	79 0 : 02 : 44
58	11 : 42	11 : 45 : 15	0 : 03 : 15
59	11 : 45	11 : 48 : 10	0 : 03 : 10
60	11 : 48	11 : 51 : 05	0 : 03 : 05

The total number of arrivals at the eye-test section on Friday is 66 and the total service time is 175.8 minutes.