INVENTORY CONTROL SYSTEM FOR DETERMINING OPTIMAL QUANTITY; CASE STUDY: AGRICARE LIMITED, TANOSO-KUMASI

BY

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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DEDICATION

To the Almighty God, my family, all my friends and love ones whose presence around me provided the source of inspiration, the tonic and the comfort I needed all through my ordeals in life, I dedicate this piece of work.

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I would like to give my sincere thanks to the Almighty God for taking me through this programme successfully.

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ABSTRACT

This thesis addresses the problem of inventory management of an Agricare Limited, Tanoso-Kumasi. The objectives of this study includes: (a) to model poultry feed inventory as a Retroactive Holding Cost problem, (b) to determine optimal order quantity, optimal total inventory cost and cycle time of poultry feed inventory using Retroactive solution algorithm. We use real data (processing times, random yield factors, etc) from a poultry feed manufacturing company, (Agricare Limited) providing simultaneously the model validation and the evaluation of the relative performance of the company. The approach used to analyze this problem was simulation based optimization. The holding cost per unit of the item per unit time is assumed to be an increasing step function of the time spent in storage. Time-dependent holding cost increase step function model is considered, namely: Retroactive holding cost increase. Procedures were used for determining the optimal order quantity and the optimal cycle time for retroactive holding cost.

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CHAPTER ONE

INTRODUCTION

1.1 Background

The unused goods or materials available on hand at a particular time are known to be an Inventory. For instance, inventory may refer to tools, spare parts, equipments, amount of gases and oil still not refined, mines, water in dams, money in banks, and so forth. The term inventory as a problem is common to most organizations, since it is rare to find an organization that does not use, store, distribute, or sell materials of one kind or another. So, organizations are becoming increasingly aware of inventory, which is crucial to the success of many activities and is a part of their effective management.

1.1.1 Inventory

Like most management functions, effective inventory management does not just happen-it takes a lot of work especially under probabilistic demand conditions.

Successful inventory management involves balancing the costs of inventory with the benefits of inventory. Many small business owners fail to appreciate fully the true cost of carrying inventory which include not only direct costs of storage, insurance and taxes, but also cost of money tied up in inventory.

The occurrence of stock out in an inventory system is a phenomenon in real life situation. These situations are common, and answers one gets from deterministic analysis very often are not satisfactory when uncertainty is present. The decision maker faced with uncertainty does not act in the same way as the one who operates with perfect knowledge of the future.

Managers are often confronted with decisions on whether or not to hold inventories. If the demand realization turns out to be smaller than the available finished products, then, some processing cost has unnecessarily been incurred. But if the demand realization turns out to be larger than the available finished products, then some customers might buy in balk and their demand (and probably the future ones) could be lost.

Many demand histories behave like random walks that evolve over time with frequent changes in their direction and rate of growth or decline. Furthermore, as product life cycles get shorter, randomness and unpredictability of these demand processes become even greater.

In practice, for such demand processes, inventory managers often rely on forecasts based on a time series, of prior demand, such as a weighted moving average and stochastic (probabilistic) models. Typically, these forecasts are predicated on a belief that the most recent demand observations are the best predictors for future demands.

1.1.2 Brief History of the Company

Agricare Limited was established in 1968 as a wholly owned subsidiary of Pfizer international of New York, U.S.A. In 1975, Pfizer Corporation decided to shed off its interest in feed manufacturing. It ceded 25% of its equity holding to some private individuals. The Agricultural development Bank acquired 25% of the total share holding. In 1991, Pfizer Corporation sold off its remaining 50% shares of equity to other private individuals. Agricare profile (2008)

Agricare is currently a wholly owned Ghanaian company with its equity interest held by Ghanaians including Agricultural Development Bank of Ghana. They market all its products locally (in Ghana); serving every region of the nation, through a network of agencies and sales points. Agricare Limited is the market leaders and they envisage extending their influence beyond the frontiers of Ghana. Agricare profile (2008)

Agricare Limited, as premier feed mill firm in Ghana has 40 years experience in the feed milling industry. They have the mandate to provide a dependable source of quality nutritional needs for the poultry and livestock industry on a commercials basis in Ghana and beyond. Agricare has established state of art machinery with which high quality products are produced under strict hygienic conditions to meet the ever growing bio-security concerns of the livestock industry.

By their mission, they provide a dependable source of supply of nutrient needs of poultry and life stock on commercial basis, in support of the industry in Ghana and Africa, the company has received some awards namely;

- Asante's Golden Business Awards in 30th July 2007, December, 2007 and September 2009,
- Ghana's Top 100 companies award in 2008 by Ghana Investment promotion centre (G I P C) and
- Association of Ghana industries Award

1.1.3 Figures and Statistics of the Company

The company has seen a steady growth in her operations over the past eight years. Apart from 2000 where production and sales volume fell slightly, the ensuing years registered appreciable growth. Let

NT be Net Turnover

GP be Gross Profit

PBT be Profit before Tax.

CT be Company Tax

NP be Net Profit

DPT be Dividend Proposed Tax

DPS be Dividend per Share

The table below depicts the operational performance profile over the past eight years

Agricare profile (2008)

	2000	2001	2002	2003	2004
NT(GH)	871,383	1,568,041	1,834,633	2,182,902	3,131,986
GP(GH)	165,281	452,654	522,722	638,979	821,325
PBT(GH)	79,635	272,771	278,170	300,958	394,046
CT(GH)	20,034	75,082	75,803	85,818	144,638
NP(GH)	59,601	197,689	202,367	215,140	249,408
DPT(GH)	7,440	14,285	14,880	19,334	19,334
DPS(GH)	0.025	0.048	0.05	0.065	0.065

Table 1.0 Figure and Statistics of Agricare Limited

	2005	2006	2007	2008
NT(GH)	3,087,314	2,630,318	4,043,170	5,654,949
GP(GH)	598,262	490,186	749,490	1,206,115
PBT(GH)	190,208	59,461	-180,032	244,372
CT(GH)	35,445	14,983	46,070	-29,838
NP(GH)	154,763	44,478	-226,102	274,210
DPT(GH)	5,952	5,952	5,952	11,904
DPS(GH)	0.02	0.02	0.02	0.04

Table 1.1 Figure and Statistics of Agricare Limited continues

Agricare Limited has spacious warehouse where the finished goods are kept under very hygienic conditions.

The figure below shows the warehouse of Agricare Limited.

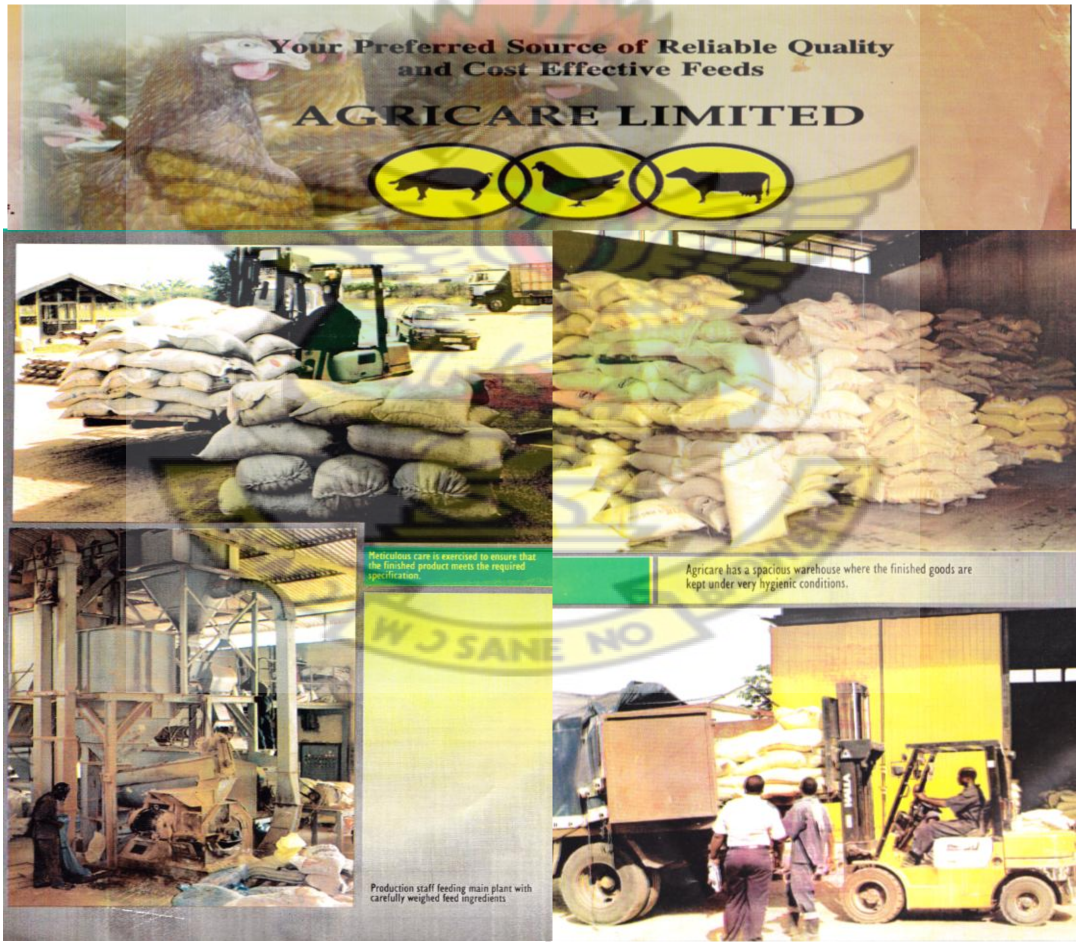

Figure 1.0 Warehouse Agricare Limited

1.2 Main problem of the company

The company currently faces the potential problem of setting optimum safety stock level for an inventory-level dependent demand rate and a time dependent holding cost. The company's strategy is good for safety stock, but they may order too much which will lead to increase in holding cost and the risk of losses through obsolesce or spoilage or they may order too small which will also increase the risk of lost sales and unsatisfied customers.

Inventory holding cost and safety stock inventory is crucial for the effective management of inventory and quantify the impact at the highest levels of many manufacturing and service industries. This study would demonstrates the need to set up a master production schedule considering the imprecise nature of forecasts of future demands and the uncertain lead time of the manufacturing process.

1.3 Objectives

The objectives of the study are:

1 To model poultry feed inventory as retroactive holding cost problem.

2 To determine optimal order quantity and optimal total cost using Retroactive solution Algorithm.

1.4 Methodology

In setting up optimum safety stock level, various inventory models such as stochastic model e.g. Retroactive Holding Cost Model and Lot-size model, for forecast and simulation methodology will be used to get the best optimal solution.

The data items that will be used for the execution of the models specified will be inventory data on 20% concentrate poultry feed from Agricare Limited which include:

- Stock list,
- Cost per unit item,
- Data on production,
- Data on demand,
- Inventory holding costs for three different lead times,
- Challenges faced in ordering and
- Challenges faced in sales

Mathematical software such as Derive 6, Minitab 16 and Matlab were applied to solve the equations and the algorithms involved. Materials from the internet, books on inventory from KNUST library, papers and journals on inventory were used in carrying out this project.

1.5 Justification

The study will unearth how effective and efficient of the current inventory system, which would provide the needed impulsion for internal control review which would help the firm to be at breast with the current control and corporate governance regulations. Organizational situation is not static and requires engendering high levels of motivation and commitment throughout the organization. This can only be achieved by generating a vision and leadership ability to appeal to higher ideals and values among employees using proper system of internal control. Where resources are misused because of failure of internal control process relating to inventory, could affect operations as well, resulting in retrenchment and far greater economic problems. The suppliers having business dealings with the firm would want to know the liquidity position of the company, to be convinced that they would be paid any money due them. Good control of funds would therefore, provide greater assurance to such suppliers.

"Safe feed-safe food" sums it all up. Keenly aware of the responsibility that this places on her, Agricare strictly adheres to the processes and assurance of the most rigorous sanitation and formulation requirements to ensure absolute safety in terms of product quality. The study will provide the needed impulsion for Customers' demands for agro feeds to meet profit and at lower cost, due to effective management of inventory of raw materials as well as finished products. In effect, capital would not be tied up in inventory and this would ensure liquidity for smooth operations.

The plant offers employment to a number of Ghanaians directly and indirectly through supply chain linkages; production of farm produce to the plant are processed into finished animal feeds. In view of this central role being played by the establishment as a pivot around which the livestock industry thrives, prudent management of inventory, which forms part of about 90% of its production cost cannot be toyed with; any stock management system that seeks to ensure that there are no stock-out when it comes to serving clients as against overstocking of both finished products and raw materials is quite imperative.

Among those stakeholders is the government of Ghana who benefits by way of corporate tax from the profit of the company and income taxes. Government again has vested interest in the operation of such an entity so established to deliver quality healthy feeds because they are invariably what we eat. It should never be forgotten that a healthy nation is a productive one; failure or mismanagement of such facility would be social cost to the nation as a whole. Therefore, government is keenly concerned about the internal processes and the finances of production.

Agricare has opened policy to researchers (for instance, research institutions such as the Animal Science Department of the Kwame Nkrumah University of Science and Technology, and The Animal Research Station of the University of Ghana, Legon, as well as the relevant faculties of the University of Cape Coast, and the University of Development Studies) and subsequent operational research findings has been its bed rock for success. The synergies existing between scientific research coupled with the experience of the employees over the years has helped to project its image above all other competitors by guaranteeing delivery of most efficient and effective services.

The financiers or investors put their financial resources with the hope of earning some good returns, and they are also very much interested in the performance of the firm. Persistent failure to earn dividend could not be tolerated. Additional resources for capacity enhancement by potential investors would depend on the track record of successes chopped. The general public is also interested in the operations of this company. It is proper inventory management among other things that would determine the long term survival and continue to be a market leader as it is presently.

In a nutshell, this project would help in demand forecast so as to produce feeds at optimal levels of cost and assists a decision management to determine the best demand planning for active demand period of a product. Products with expiring dates will be well monitored for necessary measures to be put in place to eliminate wastage and prevent losses through obsolescence, theft, insurance costs and warehousing and other hidden charges.

1.6 Organization of the Thesis

This thesis is composed of five chapters and appendices. The present chapter, Chapter one- **Introduction**, presents the background of the study and a brief history of the Poultry feed Company, Agricare limited. In the next chapter, Chapter **two-Literature Review** is presented to review the relevant literature, focusing on the issues of inventory control using various inventory models.

In Chapter three –**Methodology**, which would make use of the mathematical tools for the models presented. Chapter four – **Data Analysis and Results,** provides the descriptions of the production data and demand data from Agricare Limited, modeling and discussion of results.

The last chapter, Chapter five - **Conclusions & Recommendation**, presents a summary of the thesis and discusses a model for future research of poultry feed inventory of Agricare Limited.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

If we observe closely, inventories can be found everywhere. We don't know since when ants and squirrels are keeping inventories of their food supplies. And we do not know how they learned to keep an account of these inventories. Not only wildlife but also humans have been smart enough to realize the benefits of inventories. Since stone-ages we have been carrying inventories and managing them. But, the development of modern inventory management principles began when Harris (1913) derived the Economic Order Quantity (EOQ) formula. EOQ assumes that demand occurs at known, constant rate and supply fulfills the replenishment order after a fixed lead time. Unfortunately, the real world is not as ideal as that. In reality, demand rate is rarely constant; hard-to-predict market is common in most practical situations.

Also, unpredictable events in supply systems can cause unpredictable delays in replenishments. Moreover, in current times when outsourcing is at the centre stage, complex and longer supply chains magnify the length and variability of lead times (Welborn, 2008). Although in the early days researchers acknowledged the necessity for considering uncertainties present in the real world, the rigorous work on inventory control models with stochastic features really began in 1950s. The classic book by Hadley and Whitin (1963), comprehends the research work done in this field to that date. This fundamental research done in those early days has had a pivotal effect on the subsequent developments in the field of inventory theory.

2.1 Inventory Models

The aim of inventory management is to minimize total operating costs while satisfying customer service requirement. In order to accomplish this objective, an optimal order policy will be determined by answering to questions such as when to order and how much to order. The operating costs taken into account, the procurement costs, the holding costs and the shortage costs which are incurred when the demand of the client cannot be satisfied (either lost sales costs or orders costs).

There exist different inventory policies namely: periodic review policy and the continuous review policy. The first policy implies that the stock level will be checked after a fixed period of time and an ordering decision will be made in order to complete the stock to an upper limit (order up to point), if necessary. In the second inventory policy, the stock level will be monitored continuously.

Deterioration refers to decay, damage or spoilage. In respect of items of foods, films, drugs, chemicals, electronic components and radio-active substances, deterioration may happen during normal period of storage and the loss is to be taken into account where we analyze inventory systems. Dave and Patel (1983) put forward an inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no shortage. Roychowdhury and Chaudhury (1983) proposed an order level inventory model considering a finite rate of replenishment and allowing shortages. In their models Mishra (1975), Deb and Chaudhuri (1986) assumed that deterioration rate is time dependent. An extensive summary in this regard was made by Raafat (1991). Berrotoni (1962) discussed some difficulties of fitting empirical data to mathematical distribution. It may be said that the rate of deterioration increases with age. It may be inferred that the work of Berrotoni (1962) inspired Covert and Philip (1973) to develop an inventory model for deteriorating items with Weibull distribution by using two parameters. Mandal and Phaujdar (1989) however, assumed a production inventory model for deteriorating items with uniform rate of production and stock dependent demand. Some valuable works in this area were also done by Padmanabhan and Vrat (1995), Ray and Chaudhuri (1997), Mondal and Moiti (1999).

Today, inflation has become a permanent feature of the economy. Many researchers have shown the inflationary effect on inventory policy. Biermans and Thomas (1977), Buzacott (1975), Chandra and Bahner (1988), Jesse etal. (1983), Mishra (1979) developed their inventory models assuming a constant inflation rate. An inventory model with deteriorating items under inflation when a delay in payment is permissible is analyzed by Liao et al. (2000). Bhahmbhatt (1982) developed an EOQ model under a variable inflation rate and marked-up price. Ray and Chaudhuri (1997) presented an EOQ model under inflation and time discounting allowing shortages.

Both in deterministic and probabilistic inventory models of classical types; it is observed that payment is made to the supplier for goods just after getting the consignment. But actually nowadays a supplier grants some credit period to the retailer to increase the demand. In this respect Goyal (1985) just formulated an EOQ model under some conditions of permissible delay in payment. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005). The optimal replenishment policy for EOQ models under permissible delay in payments is also discussed by Chung et al. (2002) and Cung and Huang (2003). In recent times to make the real inventory systems more practical and realistic, Aggarwal and Jaggi (1995) extended the model with a constant deterioration rate. Hwang and Shinn (1997) determined lot-sizing policy for exponential demand when delay in payment is permissible. Shah and Shah (1998) then prepared a probabilistic inventory model with a cost in case delay in payment is permissible. After that Jamal et al. (1997) developed further following the lines of Aggarwal and Jaggi's (1995) model to take into consideration for shortage and make it more practical and acceptable in real situation.

The cost of holding an inventory is explicitly assumed to be varying over time in only few inventory models. Giri et al. (1996) developed a generalized EOQ models for deteriorating items with shortages, in which both the demand rate and holding cost are continuous functions of time. The optimal inventory policy is derived assuming a finite planning horizon and constant replenishment cycles. Ray and Chaudhuri (1997) took the time value of money into account in analyzing an inventory system with stock-dependent demand rate and shortages. Two types of inflation rates are considered. These are internal (company) inflation and external (general economy) inflation.

Various models have been proposed for stock-level dependent inventory system. Baker and Urban (1988a) investigated a deterministic system in which the demand rate dependence on the inventory level is described by a polynomial function. A non-linear programming algorithm is utilized to determine the optimal order size and recorder point. Urban (1995) investigated an inventory system in which the demand rate during stock-out periods differs from the in-stock period demand by a given amount. The demand rate depends on both the initial stock and instantaneous stock. Urban formulated a profit-maximizing model and develops a closed-form solution.

Many authors have also investigated inventory system with a two-stage demand rate. Baker and Urban (1988b) considered an inventory system with an initial period of level-dependent demand followed a period of level-dependent demand. The analysis conducted on this model imposes a terminal condition of Zero inventories at the end of the order cycle.

Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle. Paul et.al (1996) investigated a deterministic inventory system in which shortage are allowed and fully back logged. The demand is stock dependent to certain level and then constant for the remaining periods. A flow chart is provided to solve the general solution.

One of the terminal conditions used in the development of the Datta and Pal model was that the inventory level fall to zero at the end of the order cycle (i.e. $i = 0 \ when \, t = T$). In an inventory system that possesses an inventory-level-dependent demand rate, this may not provide the optimal solution. It may be desirable to order large quantities, resulting in stock remaining at the end of the cycle, due to the potential profits resulting from the increased demand. This phenomenon is discussed in Baker and Urban.

Pal et al. (1993) developed a deterministic inventory model assuming that the demand rate is stock dependent and that the items deteriorate at a constant rate θ.

The net profit over one production run is maximized by numerically solving two nonlinear equations, and the optimal solution is compared with the no deterioration ($\theta = 0$) case. Hwang and Halm (2000) also constructed an inventory model for an item with an inventory-level dependent demand rate and a fixed expiry date. All units that are not sold by their expiry date are regarded as useless and therefore discarded. Separated programming is utilized to determine the optimal order level and order cycle length.

Paknejad et al. (1995) presented a quality adjusted lot-sizing model with stochastic demand and constant lead time and studied the benefits of lower setup cost in the model. We note that the previous literature focuses on the issue of setup cost reduction in which information about lead-time demand, whether constant or stochastic, is assumed completely known. . Liang-Yuh and Hung-Chi (2000) modifies Paknejad et al.'s inventory model by relaxing the assumption that the stochastic demand during lead time follows a specific probability distribution and by considering that the unsatisfied demands are partially backordered. Also, instead of having a stock-out cost in the objective function, a service level constraint is employed.

Some research works also focus on inventory record inaccuracy (IRI) as far back in 1960_s with report by Rinehart (1960) on the case study of a Federal government supply facility. The author stated that this inaccuracy produces a "deleterious effect" on operational performance. Following this, Iglehart and Morey (1972) reported that this divergence between stock record and physical stock results in "warehouse denials". The research took into consideration the frequency and depth of inventory counts and stocking policy minimize total cost per time unit.

Studying a similar problem, $K\ddot{o}k$ and Shang (2004) have suggested implementing a cycle count program and carefully adjusting base stock levels across periods to minimize total inventory and inspection cost. Moreover, focusing on the significant of measuring IRI, Dehoratius and Raman (2008) showed that inventory counts may not impact record inaccuracy and additional buffer stock may not be equally necessary across all items in all stores. They also suggested that inventory density and product variety by substantial implication for identifying and eliminating the source of inventory record inaccuracy. However their study is only based on the retails stores of one firm and does not include all factors that might impact variation of IRI from one store to the next.

2.2 Stochastic (Probability) Inventory Models

Series of works also develop optimal inventory policies when demand distribution depends upon some unknown parameter and an estimate of the parameter is updated as actual demand is observed overtime. For demand distributions from the exponential family, Scarf (1959) formulates a single-product model as a dynamic program with a two-viable state space, and establishes that a base-stock policy based on a critical-fractal is optimal. Scarf (1960) shows how to reformulate this problem as a single-variable dynamic program, subject to some additional assumptions. Azoury (1985) and Miller (1986) generalize and extend these results to other classes of demand distributions. Lovejoy (1990) also shows that a critical-fractile inventory policy is optimal or near optimal for a more general class of demand distributions.

Dave (1986) developed a probabilistic scheduling period inventory model for continuously decaying items where lead-time was assumed to be deterministic. Wee et al. (2008) studied an inventory model with deteriorating items to develop an optimal replenishment inventory strategy. Hon (2006) derived an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time-discounting over a finite planning horizon, where the solution is obtained by minimizing the total cost function. In this connection mention may be made of the work of Kabak and Weinberg (1972), which is an extension of classical newsboy problem considering supply as a random variable, but newsboy suffers no decrease in expected revenue.

Time series forecasting models are increasingly to forecast demand and short-life product demand. Under an autoregressive moving average (ARMA) assumption, Kurawarwala and Matsuo (1998) estimated the seasonal variation of P.C. products demand using demand history of pre-section products and validated the models by checking the forecast performance with respect to actual demand-Miller and Williams (2003) incorporated seasonal factors in their model to improve forecasting accuracy while seasonal factors are estimated from multiplicative model. Hyndman (2004) extended Miller and Williams' (2003) work by applying various relationships between trend and seasonality under seasonal autoregressive integrated moving average (ARIMA) procedure.

In recent times, studies, de Alba (1993) derived an autoregressive model under Bayesian approach to forecast the quarterly GNP of Mexico and the quarterly unemployment rate for the United States. Huerta and West (1999) studied autoregressive models where Markov Chain Monte Carlo (MCMC) process is used to forecast from AR processes.

McCoy and Stephens(2004) extended Huerta and West's work(1999) and proposed ARMA models in which a frequency domain approach is adopted to identify the periodic behavior of time series.

Khouja (1996) studied an inventory problem involving second ordering opportunity. In his model, the ordering quantity is determined for a single period model with an emergency supply option, where he found that the total quantity emergency supply option is smaller than of the newsvender model. Liau and Lau (1997) studied the reordering strategies for a seasonal product under a newsvender models where a customer receives an order at the beginning of the season and places an additional order at some point during the season. They identified analytical conditions to maximize profits for using second ordering opportunity.

Eppen and Iyer (1997) cribbed on inventory problem of the fashion industry. They determined the initial inventory quantity for a season and adjusted the procurement quantity after information updates using Bayesian techniques. Gurnani and Tang (1999), and choi et al. (2003) investigated the optimal inventory quantity for seasonal products in which a retailer can order twice and ordering cost at the second time is a variable.

Goh (1994) apparently provided the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation: (a) a non-linear function of storage time and (b) a non-linear function of storage level. Hesham (2007) presented different functional form of holding cost time dependence. Two types of discontinuous steps functions were considered. The storage time was divided into a number of distinct periods with successively increasing holding costs. As the storage time extends to the next time period, the new holding cost can be applied either retroactively (to all storage periods), or incrementally (to all new periods).

The methodology for this project work was based on inventory models with stock-level dependent demand rate variable holding cost in the International Journal of Production Economics Research 108 (2008) 259-265 , (Alfares H.K (2007)) and lot-size model from Anderson et al. (2008)

CHAPTER THREE

METHODOLOGY

Inventory

3.0 Introduction

Any stored resource used to satisfy a current or future need (raw materials, work-in-process, finished goods, etc.) is what we called inventory. Inventory represents as much as 50% of invested capital in some companies, excessive inventory levels are costly and insufficient inventory levels lead to stock-outs. For maintaining the right balance between high and low inventory levels to minimize cost, Inventory planning and control should be as follows:

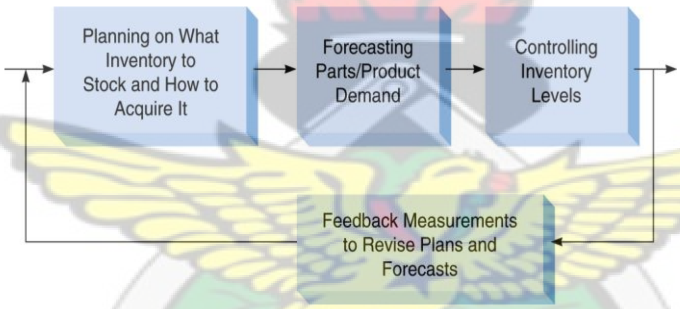

Figure 3.1 Inventory planning and control (Pearson education (2007))

3.1 Types of inventory

Several different types of inventories are conducted, depending upon the type of material involved and type of information needed.

Generally, inventory types can be grouped into four classifications. These are:

- Raw materials inventory
- Work-in-process (WIP) inventory
- Finished goods inventory

- Maintenance, repair, and operating supplies, or MRO goods

The figure below displays the types of inventory.

Figure 3.2 Types of Inventory (csun (2011) ie.California State University, Northridge ppt.).

3.2 Inventory Models

Inventory analysis has two problems of importance to the organization of stock items. Namely:

- Deciding when to place an order for the replenishment of the stock.
- Deciding how large an order is to be placed.

Two types of uncertainties must be considered:

a. the quantity of items that will be demanded during a given period

b. the time that will elapse between placing an order and the actual delivery of the item.

A major problem of inventory is how we can establish optimal stock levels and this is difficult because of the uncertainty of supply and demand for the commodity. Using inventory models we could formulate policies to control the system.

In some cases such as retailer, wholesaler / distributor, where items are purchased externally, if the problem of inventory exists, then there are two main questions, which generally arise and face any organization. These are how many to order and when to order. Having too much inventory reduces both purchase and /or ordering costs, but it may tie up capital, which may lead to unnecessary holding cost and possibility of deteriorating items.

Whereas having too little inventory reduces the holding cost, but it can result in lost of customers, which may affect the reliability of the organization.

Answering these two questions will lead to the optimal level of inventory for any organization, which minimizes its total inventory cost.

Inventory costs, which are related to the operation of an inventory system, are caused by the actions or lack of actions that the organization is establishing.

The most common costs to an inventory system may include:

- The purchase cost of an item obtained from an external source.
- The order cost of issuing a purchase order to an outside source.
- The holding /carrying cost for keeping items in storage.

3.3 Inventory Level

This depends on the relative rates of flow in and out of the system.

Let

$y(t)$ be rate of input flow of items at time t

$Y(t)$ be the cumulative flow of items into the system

$z(t)$ be the rate of flow of items out of the system time t

$Z(t)$ be the cumulative flow of items out of the system ,

then the inventory level, $I(t)$ is the cumulative input less the cumulative output.

$$I(t) = Y(t) - Z(t)$$

$$I(t) = \int_0^t [y(t) - z(t)]dt$$

$$I(t) = \int_0^t y(t)dt - \int_0^t z(t)dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

The figure 3.3 below represents the inventory system when the rates vary with time. The figure might represent a raw material inventory. The flow out of inventory is relatively continuous activity where individual items are replaced into the production system for processing. To replenish the inventory, an order is placed to a supplier. After some delay time, called the lead time, the raw material is delivered in a lot of a specific amount. At the moment of delivery, the rate of input is infinite and other times it is zero. Whenever the instantaneous rates of input and output to a component is not the same, the inventory level changes. When the input rate is higher, inventory grows; when the output rate is lower, inventory declines. Usually the inventory level remains positive. This corresponds to the presence of *on hand inventory*. In situation where cumulative output exceeds the cumulative input, the inventory level is negative. This is what we call *a backorder or shortage condition*.

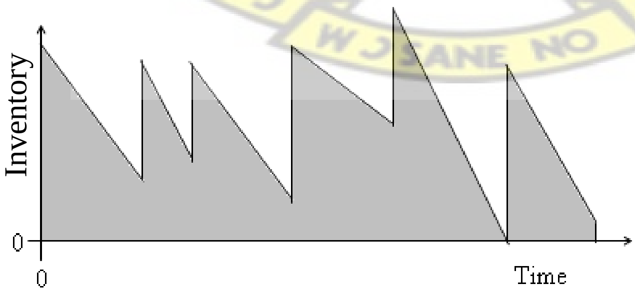

Figure 3.3 Inventory fluctuations as a function of time

3.4 Lot-size or Economic Production Quantity (EPQ)

3.4.1 Introduction

The lot-size refers to the number of units in an order. The Lot-size model is design for the production situations in which once supply begins, demand begins. During supply, demand would be reducing the inventory while supply would be adding the inventory. We assume that supply rate exceeds the demand rate during the supply run. The excess supply would cause a gradual inventory build-up during the supply period. When supply is completed, the continuing demand will cause the inventory to gradually decline until a new supply is started. The inventory pattern for this system is as shown below in figure1.4 respectively.

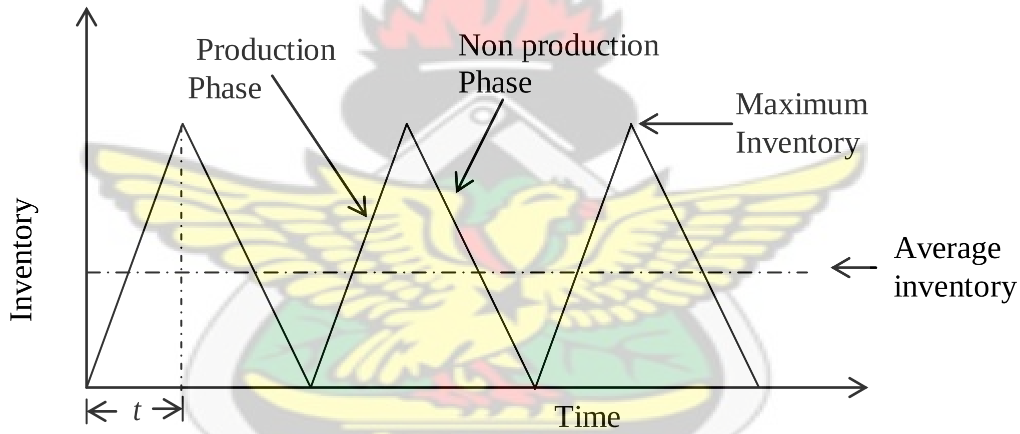

Figure 3.4 Inventory pattern for lot-size model

3.4.2 Assumptions for Lot-size

The following assumptions are considered:

- Average demand is fixed
- Demand pattern is periodic
- Average cost per order is constant
- Daily production rate is greater than daily demand rate during the production run.

3.4.3 Notations for lot-size

Let:

c_z = Ordering cost per order

C_h = Annual holding cost per unit

H_c = Annual Holding Cost

n = Number of orders per unit time

k = Annual Setup cost

t = Number of days for production

d = Demand rate

D = Annual demand

Q = Lot size

τ = Cycle time

T = Cost per time

T_c = Total annual holding cost

Q^*, τ^*, T^* = List of Optimal quantities

3.4.4 Inventory Level Trajectory of Lot-Size Model

The equation of the trajectory of the inventory pattern in fig. 1.1 below is of the form:

$$Q(t) = \frac{1}{T}(p-d)t \qquad \text{……………..3.2}$$

Where p = daily arrival rate

d = daily demand rate

t = number of days for production

Since we are assuming that p will be larger than d, the daily inventory build-up rate during the production phase is $p-d$. If we run production for t days and place $p-d$ units in the inventory each day, the inventory at the end of production will be $(p-d)t$. From the diagram above, the inventory at the end of production is also the maximum inventory. Thus

Maximum inventory $=(p-d)t$

If we are aware of producing lot-size of Q units at a daily production of p units, then:

$Q = pt$ and the length of production t must be $t=\dfrac{Q}{p}\,days$

Thus,

$$\text{Maximum inventory} = (p-d)t$$

$$= (p-d)\left(\frac{Q}{p}\right)$$

$$= \left(1-\frac{d}{p}\right)Q \qquad \text{…………………..3.3}$$

$$Average\ Inventory = \frac{1}{2}\int_0^T Q(t)\,dt$$

$$= \frac{1}{2}\int_0^T \left[\frac{1}{T}(p-d)\frac{Q}{P}\right]dt$$

$$= \frac{1}{2}\left[\frac{1}{T}(p-d)\frac{Q}{P}\right]T$$

$$= \frac{1}{2}(p-d)\frac{Q}{P}$$

$$\therefore\ Average\ Inventory = \frac{1}{2}\left(1-\frac{d}{p}\right)Q \qquad \text{…………………….3.4}$$

3.4.5 Development of the Optimal Order Quantity for Lot-Size Model

We develop below the Lot-size model through the construction of the total inventory cost model.

Let the annual holding cost per unit be C_h, the equation for annual holding cost is

$$\text{Annual Holding Cost} = (\text{Average inventory})(\text{Annual cost per unit})$$

$$H_c = \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots3.5$$

If D is the annual demand for the product and k is the setup cost per production then the annual setup cost is

$$\textit{Annual Setup or Ordering Cost} = (\textit{Number production per year})(\textit{Setup cost per production})$$

$$= \frac{D}{Q}k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.6$$

Thus, the total annual cost T_c model is

$$T_c = \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h + \frac{D}{Q}k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots3.7$$

Suppose that facility operates 250 days per year .Then we write daily demand d in terms of annual demand D as follows:

$$d = \frac{D}{250}$$

Now let P denotes the annual production if the product were produced every day Then,

$$P = 250p \qquad \text{and} \qquad p = \frac{P}{250}$$

Thus,

$$\frac{d}{p} = \frac{\left(\frac{D}{250}\right)}{\left(\frac{P}{250}\right)} = \frac{D}{P}$$

Therefore the total annual cost model can also be written as

$$T_c = \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h + \frac{D}{Q}k \qquad \text{……………………….3.8}$$

Setting to zero the derivative of T_c with respect to Q, we obtain

$$\frac{dT_c}{dQ} = 0$$

$$\Rightarrow \frac{1}{2}\left(1 - \frac{D}{P}\right)C_h Q^2 = 2Dk$$

$$\Rightarrow Q^2 = \frac{2Dk}{\left(1 - \frac{D}{P}\right)C_h}$$

Solving for order quantity optimality policy we have

$$Q^* = \sqrt{\frac{2Dk}{\left(1 - \frac{D}{P}\right)C_h}} \qquad \text{…………………3.9}$$

and

$$\tau^* = \frac{Q^*}{d} \qquad \text{…………………..3.10}$$

Substitute optimal lot-size, Q^*, into the total cost expression, T_c,

$$T_c^* = \frac{1}{2}\left(1 - \frac{D}{P}\right)Q^* C_h + \frac{D}{Q^*}k \qquad \text{……………...3.11}$$

Note: As the production rate p approaches infinity, $\frac{D}{p}$ approaches zero. That is $\left(1-\frac{D}{p}\right)$ representing probability of no shortage.

At optimum, the total holding cost is equal to total ordering or set-up cost.

3.4.6 Stockout and service level

Stockout: Stockout occurs when there is insufficient stock to satisfy customers demand.

Service level $=1-p$ (stockout)

Taylor (2006), Anderson (2004)

3.4.7 Effective Inventory Cost Decision for Lot-Size Model

1) Holding cost, normal inventory $=\frac{1}{2}\left(1-\frac{D}{P}\right)QC_h$

2) Minimum holding cost $=\frac{1}{2}\left(1-\frac{D}{P}\right)Q^*C_h$

i.e $\frac{1}{2}\left(1-\frac{D}{P}\right)Q^*C_h < \frac{1}{2}\left(1-\frac{D}{P}\right)QC_h$

3) $Ordering\ Cost = \left(\frac{D}{Q}\right)k$

4) $Minimum\ Ordering\ Cost = \left(\frac{D}{Q^*}\right)k$

i.e $\left(\frac{D}{Q^*}\right)k < \left(\frac{D}{Q}\right)k$

3.4.8 Periodic Review Inventory System

3.4.9 Introduction

An alternative to the continuous review system is the periodic review inventory system. With a periodic review, the inventory may be checked and orders placed on a weekly, bi-weekly, tri-weekly, monthly or some other periodic basis.

3.4.10 Replenishment Level (M)

It is inventory level at which the order quantity should be demanded at the review period. If the normal probability distribution is used then:

$M = d + zs$ Where

d = mean demand

z = number of standard deviations necessary to obtain the acceptable stockout probability

s = standard deviation of the distribution

3.4.11 How-Much-To-Order Decision

The how-much-to-order decision at any review period is determined using the model

Let:

$q = M - X$ Where

q Represents the order quantity at review period

M = replenishment level

X = the inventory on hand at review period which varies since demand is probabilistic.

Taylor (2006), Anderson (2004)

3.5 Retroactive Holding Cost and Incremental Increase Cost Models

There is no question that uncertainty plays in most inventory management situations. The retail merchant wants enough supply to satisfy costumer demands but ordering too much increase holding costs and the risk of losses through obsolescence or spoilage. An order too small increases the risk of lost sales and unsatisfied customers. These situations are common, and answers one gets from deterministic analysis very often are not satisfactory when uncertainty is present.

The model that will be developed for the inventory system is based on allowing unit holding cost values to vary across different storage periods. Variable unit holding costs are considered in the model in determining the optimal policy. The holding cost per unit is assumed to increase only when the storage time exceeds specific discrete values. That is the holding cost per unit time is an increase step function of the storage time. Two type of holding cost step functions will be considered:

- Retroactive increase; the unit holding cost rate of the last storage period is applied to all storage period.
- Incremental increase; Higher storage cost rates is applied to storage in later periods.

3.5.1 Notations for Retroactive Holding Cost and Incremental Increase Cost

The following notations are adopted from Goh (1992) for the model under consideration for Agricare inventory system.

$q(t)$: the on – hand inventory at time t

D: constant (base) demand rate

n: number of distinct time periods with different holding cost rate

t : time from the start of the cycle at $t = 0$

t_i : end time of period i, where $i = 1,2,\ldots,n, t_0 = 0$, and $t_n = \infty$

k : ordering cost per order

h_i : holding cost of the item in period i

$h(t)$: holding cost of the item at time t, $h(t) = h_i$ if $t_{i-1} \leq t \leq t_i$

T : cycle time

β : demand parameter indicating elasticity in relation to the inventory level

3.5.2 Assumptions and Limitations for Retroactive Holding Cost and Incremental Holding Increase Cost

The following assumptions and limitations are considered:

- The demand rate R is an increasing step function of the inventory level q.
- The holding cost is varying as an increasing step function of time in storage.
- Replenishments are instantaneous.
- Shortages are not allowed.
- A single item is considered.
- The demand rate R dependence on the inventory level q is expressed as

$$R(q) = Dq^{\beta},\ D > 0, 0 < \beta < 1,\ q \geq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

3.6 The Models

The Total Inventory Cost (TIC) per unit time includes two components:

a. Ordering cost

b. Holding cost

Once ordering is made per cycle, the ordering cost per unit time is simply $\frac{k}{T}$. The total holding cost per cycle is obtained by integrating the product of the holding cost $h(t)$ and inventory $q(t)$ over the whole cycle.

That is,

The total holding cost per cycle $= \frac{1}{T}\int_0^T h(t)\,q(t)\,dt$

Hence,

$$\text{TIC} = \frac{k}{T} + \frac{1}{T}\int_0^T h(t)\,q(t)\,dt \qquad \text{………………………..3.13}$$

Since the demand rate is equal to the rate at which the inventory level decrease, we can describe inventory level q by the following differential equation:

$$\frac{dq(t)}{dt} = -D\,[q(t)]^{\beta}, \qquad \text{……………………….3.14}$$

$$D > 0,\ \ 0 \le t \le T,\ \ 0 < \beta < 1$$

The on-hand inventory level at time t, $q(t)$ can be evaluated by solving equation (3):

$$q^{-\beta}dq = -Ddt\ ,$$

Integrating both sides:

$$\int_0^t q^{-\beta}dq = \int_0^t -Ddt \ \ where\ 0 \le t \le T,$$

$$\Rightarrow \left.\frac{q^{1-\beta}}{(1-\beta)}\right|_0^t = -Dt,$$

$$\Rightarrow q^{1-\beta}(t) - q^{1-\beta}(0) = -D(1-\beta)t$$

$$\Rightarrow q^{1-\beta}(t) = -D(1-\beta)t + q^{1-\beta}(0)$$

However,

$$q^{1-\beta}(0) = Q^{1-\beta},$$

Thus,

$$q^{1-\beta}(t) = -D(1-\beta)t + Q^{1-\beta},$$

$$q(t) = \left[-D(1-\beta)t + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} \qquad \text{.....................3.15}$$

The period T can be evaluated by substituting the inventory function $q(t)$ at T.

That is:

$$q(T) = 0$$

$$\Rightarrow q(T) = \left[-D(1-\beta)T + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} = 0$$

$$\Rightarrow -D(1-\beta)T - q^{1-\beta}(T) = -Q^{1-\beta}$$

$$T\left(-D(1-\beta) - q^{1-\beta}\right) = -Q^{1-\beta}$$

Hence,

$$T = \frac{Q^{1-\beta}}{D(1-\beta) + q^{1+\beta}} \qquad \text{.................. ......3.16a}$$

Or

$$Q = \left[D(1-\beta)T + q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} \qquad \text{........................3.16b}$$

3.6.1 Retroactive Holding Cost Increase

The holding cost is assumed to be an increasing step function of storage time, that is $h_1 < h_2 < h_3 ... < h_n$. Here, a uniform holding cost that depends on the length of storage is used. Specifically, the holding cost of the last storage period applies retroactively to all previous storage periods. Thus, if the cycle ends at in period, e, with $(t_{e-1} \le T \le t_e)$ then the holding cost rate h_e is applied to all periods $1, 2, \ldots, e$. In this case; the TIC per unit time can be expressed as

$$TIC = \frac{k}{T} + \frac{h_i}{T}\int_0^T q(t)\,dt, \qquad \text{where } t_{i-1} \le T \le t_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.17$$

Substituting the value of $q(t)$ from (3.1.5)

$$TIC = \frac{k}{T} + \frac{h_i}{T}\int_0^t \left[-D(1-\beta)t + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} dT,$$

$$= \frac{k}{T} - \frac{h_i}{D(2-\beta)T}\left[-D(1-\beta)\Big|_0^T + Q^{1-\beta}\right]^{\frac{(2-\beta)}{(1-\beta)}}$$

Thus,

$$TIC = \frac{k}{T} + \frac{h_i}{D(2-\beta)T} \times \left[\left(Q^{1-\beta}\right)^{\frac{(2-\beta)}{(1-\beta)}} - \left(-D(1-\beta)T + Q^{1-\beta}\right)^{\frac{(2-\beta)}{(1-\beta)}}\right]$$

Substituting the value of T from (4.2.4)

$$TIC = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)}, \qquad \text{where } t_{i-1} \le T \le t_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.18$$

Setting the derivative of TIC with respect to Q equal to zero and solving for Q, we obtain:

$$Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}, \; t_{i-1} \le T \le t_i. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.19$$

3.6.1.1 Solution Algorithm

The optimum solution can be determined by using the following solution algorithm steps, Alfares (2007):

1. Beginning with the lowest holding cost h_1, using $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{2-\beta}}$, $t_{i-1} \le T \le t_i$ to determine Q and $T = \frac{Q^{1-\beta}}{D(1-\beta)}$ to determine T for each h_i until Q is realizable $ie\ (t_{i-1} \le T \le t_i)$. Call these values T_R and Q_R.
2. Use $Q = [D(1-\beta)T]^{\frac{1}{(1-\beta)}}$ to calculate all break-point values of Q, $Q_i = Q(t_i)$, $t_1 \le T < T_R$; each Q_i is obtained by substituting t_i into $Q = [D(1-\beta)T]^{\frac{1}{(1-\beta)}}$.
3. Use $TIC = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)}$, $t_{i-1} \le T \le t_i$ to calculate the TIC for Q_R and each Q_i
4. Choose the value of Q that gives the lowest TIC.

3.6.2 Incremental Holding Cost Increase

The holding cost is now assumed to be an incremental step function of the storage time. According to this function, higher storage cost rate apply to storage in later periods. Thus, if the cycle ends in period, $e, with\ (t_{e-1} \le T \le t_e)$ then holding cost rate h_1 is applied to period 1, rate h_2 is applied to period 2, and so on; thus rate h_e is applied only to period e from time t_{e-1} up to time T. For this we reset the value of t_e as $(t_e = \mathrm{T})$, and then express the TIC per unit time as:

$$\mathrm{TIC} = \frac{k}{T} + \frac{h_1}{T}\int_0^{t_1} q(t)\,dt + \frac{h_2}{T}\int_{t_1}^{t_2} q(t)\,dt + \ldots + \frac{h_e}{T}\int_{t_{e-1}}^{t_e = T} q(t)\,dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.20$$

Substituting the value of $q(t)$ from eqn. (4.2.3), we obtain:

$$\mathrm{TIC} = \frac{k}{T} + \sum_{i=1}^{e}\frac{h_i}{T}\int_{t_{i-1}}^{t_i}\left[-D(1-\beta)t + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}}dt,$$

$$= \frac{k}{T} + \sum_{i=1}^{e}\frac{-h_i}{D(2-\beta)T}\left[-D(1-\beta)t\Big|_{t_{i-1}}^{t_i} + Q^{1-\beta}\right]^{\frac{(2-\beta)}{(1-\beta)}}$$

Substituting the value of T from eqn. (4.2.4), and rearranging and simplifying terms gives:

$$\mathrm{TIC} = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1}\frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times \left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}} \qquad \ldots 3.21$$

To find the optimal order size Q^*, we set the derivative TIC with respect to Q equal to zero.

After simplification, we obtain:

$$-\frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_1 Q}{(2-\beta)} + \sum_{i=1}^{e-1}(h_{i+1}-h_i)\left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{1}{(1-\beta)}} - \sum_{i=1}^{e-1}\frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \qquad \ldots\ldots\ldots 3.22$$

$$\times \left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}} = 0$$

If the entire inventory cycle happens to fall into the first period $(0 \le \mathrm{T} \le t_1)$, then $e = 1$ and the summations over i in eqn. 3.22 are empty. In that case the optimum solution is simply obtained by substituting h_1 into eqn. 3.19 to calculate Q^*, and then substituting Q^* into eqn. 3.15 to calculate T. obviously, a simple closed form solution for Q^* and T^* can be determined only if $T \le t_1$.

In general, the optimum solution must be determined by the following solution algorithm steps, Alfares (2007).

3.6.2.1 Solution Algorithm

1. Substitute h_1 into $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$, $Q* = t_{i-1} \le T \le t_i$ to determine $Q_{\max}$, and then substitute $Q_{\max}$ into $T = \frac{Q^{1-\beta}}{D(1-\beta)}$ to determine $T_{\max}$. If $T_{\max} \le t_1$, stop; the solution $(Q_{\max}, T_{\max})$ is optimal.

2. Substitute h_n into $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$, $Q* = t_{i-1} \le T \le t_i$ to determine $Q_{\min}$, and Substitute $Q_{\min}$ into $T = \frac{Q^{1-\beta}}{D(1-\beta)}$ to determine $T_{\min}$.

3. Depending on the values of $T_{\min}$ and $T_{\max}$, determine the possible periods that T may fall into (i.e., all feasible values of e).

4. For each feasible value of e, solve

$$-\frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_1 Q}{(2-\beta)} + \sum_{i=1}^{e-1}(h_{i+1} - h_i)\left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{1}{(1-\beta)}} - \sum_{i=1}^{e-1}\frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)}$$
$$\times \left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}} = 0$$

numerically to determine the optimum value of Q. If Q corresponds to the correct period, it is considered realizable.

5. Using

$$\text{TIC} = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1}\frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times \left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}},$$

calculate TIC for Q_R and each $Q_i = Q(t_i)$

6. Choose the value of Q that gives the lowest TIC

CHAPTER FOUR

DATA ANALYSIS AND RESULTS

4.1 Production and Demand data Description

The production and Demand data used for this study was obtained from the Accounts Department Agricare Limited, Tanoso Kumasi, Ghana.

The data comprises the following:

- Monthly Production and Demand data on 20% concentrate poultry feed of Agricare Limited from January, 2005 to December, 2010.

The table below displays the production and Demand data from January, 2005 to December, 2007.

	2005		2006		2007	
MONTH	**PRODUCTION**	**DEMAND**	**PRODUCTION**	**DEMAND**	**PRODUCTION**	**DEMAND**
JANUARY	723	248	123	114	317	276
FEBRUARY	37	156	4	88	373	205
MARCH	443	166	165	90	355	203
APRIL	180	145	172	61	169	194
MAY	242	88	40	113	293	127
JUNE	321	128	174	109	351	197
JULY	225	112	229	133	285	172
AUGUST	270	105	353	178	451	207
SEPTEMBER	32	133	297	132	162	173
OCTOBER	162	148	281	172	533	226
NOVEMBER	357	127	453	277	485	239
DECEMBER	224	98	445	233	393	271
AVEARGE	**268**	**137.833**	**228**	**141.667**	**347.25**	**207.5**

Table 4.1 Production and Demand Data of 20% Concentrate feed from Agricare Limited from 2005-2007.

The full Production and Demand data for the year 2005-2010 for the study is displayed at Appendix A.

- The cost per order and cost per unit item as of the year 2005-2010 is also obtained from Agricare Limited. The table 4.2 and table 4.3 display the data respectively.

YEAR	COST PER UNIT ITEM (GH□)
2005	Ø 20.31
2006	Ø 25.81
2007	Ø 28.58
2008	Ø 36.00
2009	Ø 63.88
2010	Ø 72.90

Table 4.2 Cost per unit item of 20% Concentrate Poultry feed from 2005-2010

YEAR	WA	TEMA
2005	□ 1.16	□ 83p
2006	□ 1.25	□ 90p
2007	□ 1.45	□ 1.04p
2008	□ 1.70p	□ 1.22p
2009	□ 1.70p	□ 1.22p
2010	□ 2.04p	□ 1.46p

Table 4.3 Cost per Order for 20% Concentrate Poultry Feed from 2005-2010.

The poultry feed is measured in tonnes.

The figure 4.1 below is a plot of Production and Demand data. The visual pattern of the plot of Production and Demand Data is periodic. Rises in Production correspond to rises in Demand. The average monthly production of 20% concentrate poultry feed was 175 tonnes. Production above this average represents high production and vice-versa. The average monthly Demand of 20% concentrate was 163 tonnes. Majority of the Demands on the graph within the 72 months were above the monthly average, indicating high demands and below the average correspond to low Demands.

The high and low Production and Demand recorded within the 72 months is based on the reason that, customers rare their fowls especially broilers, growers and Layers toward festive periods such as Christmas and for that matter they have to buy more feeds since these fowls (layers, broilers, growers) eat a lot. Low Production and Demand also is attributed to the fact that, customers keep new fowls in their farm that is chicks from the brooder house which do not eat much as compared with growers, broilers, layers etc. Another reason which might bring about these high and low Production and Demand is the form of formulation during production of feeds. The ratio of fish to soya beans, soya beans to cotton and other ingredients in the feed also account for the cost of the feed. Customers are unable to buy more bags in tonnes of 20% concentrate feed if the ratio of the formulation is high especially when customers are grooming new chicks.

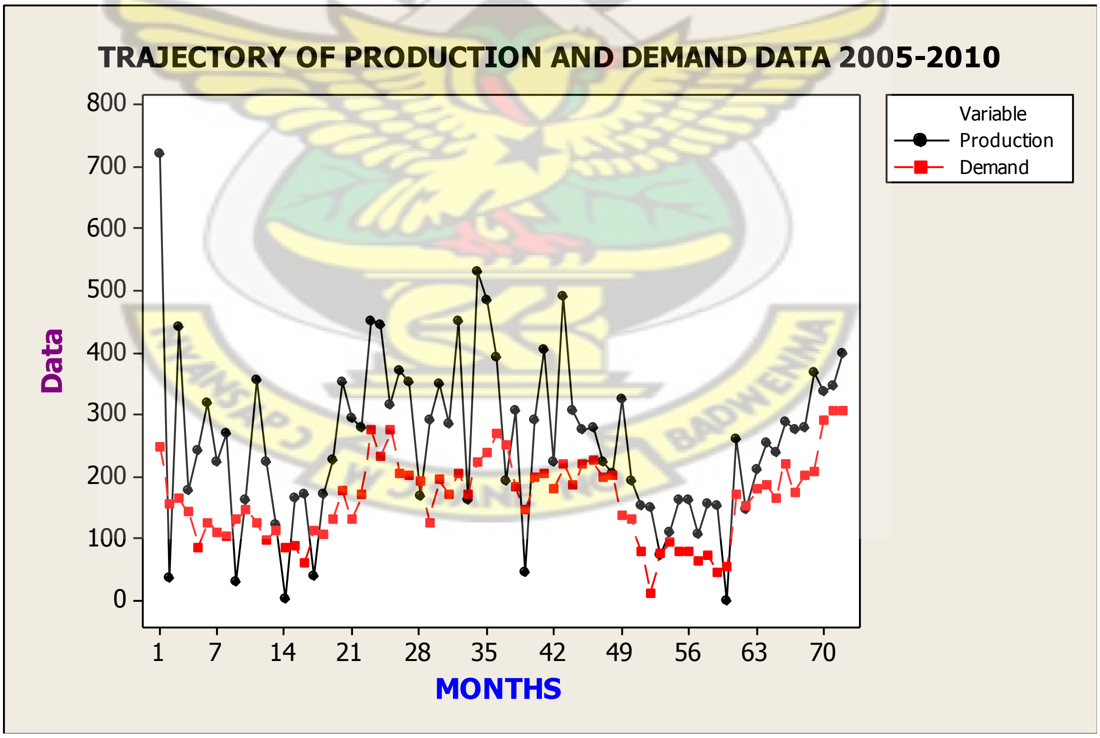

Figure 4.1 A plot of Production and Demand Data

4.2 Summary of Findings

4.2.1 The Trajectories of Production Data and Demand Data of Agricare Limited

The trajectories of Production and Demand data of Agricare Limited are both periodic and are linearly related. The quantity of poultry feed ordered from the company at any time t depends on the number of poultry feeds produced. The higher the poultry feeds that are produced in the company, the higher the demands and vice-versa. The optimal total inventory cost often depend on the availability of raw materials, type of feed formulation, lead time, environmental conditions that is festivity periods that dictate the amount of items demanded by customers.

4.3 Computational Procedure

Given the average demand per year, average cost of order and the average holding cost (see matlab results at Appendix D.2) the following parameters are used in determining the optimal solution for Agricare Limited.

That is:

Average Demand per year $= 1966$ units,

Average Set-up Cost per Order k=□ 438

$\beta = 0.1$

Holding Cost for period one h_1=□ 7/unit/year 0< T≤ 0.2, $t_1 = 0.2 year$

Holding Cost for period two h_2=□ 8/unit/year 0.2< T≤ 0.4, $t_2 = 0.4 year$

Holding Cost for period three h_3=□ 9/unit/year 0.4< T≤ 0.6, $t_3 = 0.6 year$ etc

The following computations were made using Retroactive Holding Cost solution algorithm, Alfares (2007):

- **Solution algorithm step 1**

Using $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{2-\beta}}$, to compute Q, and $T = \frac{Q^{1-\beta}}{D(1-\beta)+q^{1+\beta}}$

to determine T for each h_i until Q is realizable $i.e.\ t_{i-1} \leq T \leq t_i$. Call these values $Q_{R.}$ and T_R.

Begin with $h_1 = 7$

$$\Rightarrow Q* = \left[\frac{438(1966)(1-0.1)(2-0.1)}{7}\right]^{\frac{1}{2-0.1}} = 633.2215 \text{ units,}$$

The corresponding cycle time $T = \frac{633.2215^{(1-0.1)}}{1966(1-0.1)} = 0.1877 \text{ year } (\textit{realizable})$,

since $0 < T \leq 0.2$

When $h_2 = 8$

$$\Rightarrow Q* = \left[\frac{438(1966)(1-0.1)(2-0.1)}{8}\right]^{\frac{1}{2-0.1}} = 590.2477 \text{ units,}$$

The corresponding cycle time $T = \frac{590.2467^{(1-0.1)}}{1966(1-0.1)} = 0.1762 \text{ year } (\textit{not realizable})$,

since $0.2 < T \leq 0.4$

When $h_3 = \square\ 9$

$$\Rightarrow Q* = \left[\frac{438\ 1966\ \ 1-0.1\ \ 2-0.1}{9}\right]^{\frac{1}{2-0.1}} = 554.7677 \text{ units,}$$

The corresponding cycle time $T = \dfrac{554.7677^{(1-0.1)}}{1966\ 1-0.1} = 0.1667$ year *not realizable* ,

since $0.4 < \mathrm{T} \leq 0.6$

This is repeated until realizable Q_R and T_R are obtained for $h_1<h_2<h_3<\ldots h_n$. From above, $Q_R=633.2215$ and $T_R=0.1877$.

The table below displays the results for Solution Algorithm step1:

h_i	Q	T	*i.e.* $t_{i-1} \leq T \leq t_i$	Remark(s)
$h_1=7$	633.2215	0.1877	*i.e.* $t_{i-1} \leq T \leq t_i$	Realizable
$h_2=8$	590.2467	0.1762	*i.e.* $t_{i-1} \leq T \leq t_i$	Not realizable
$h_3=9$	554.7677	0.1667	*i.e.* $t_{i-1} \leq T \leq t_i$	Not realizable

Table 4.4 Solution Algorithm step1

- **Solution Algorithm step2**

Calculating all break points of Q, $Q_i = Q\ t_i\ , t_1 \leq T < T_R$; each Q_i is obtained by substituting t_i into $Q = \left[D\ 1-\beta\ T\right]^{\frac{1}{1-\beta}}$

We have:

When $t_1=0.2$

$$\Rightarrow Q_1 = \left[1966\ 1-0.1\ 0.2\right]^{\frac{1}{1-0.1}} = 679.3010 \text{ units}$$

When t_2=0.4

$$\Rightarrow Q_2 = \left[1966\ \ 1-0.1\ \ 0.4\right]^{\frac{1}{1-0.1}} = 1467 \text{ units}$$

When t_3=0.6

$$\Rightarrow Q_3 = \left[1966\ \ 1-0.1\ \ 0.6\right]^{\frac{1}{1-0.1}} = 2303 \text{ units}$$

The table below displays the result for Solution Algorithm step2:

T	Q_i
t_1=0.2	Q_1=679.3010
t_2=0.4	Q_2=1467
t_3=0.6	Q_3=2303

Table 4.5 Solution Algorithm step2

From the table the minimum Q_i is Q_1=679.3010.

- **Solution Algorithm step3**

Using $TIC = \dfrac{kD\ \ 1-\beta}{Q^{1-\beta}} + \dfrac{h_i\ \ 1-\beta\ \ Q}{2-\beta}$, $t_{i-1} \le T \le t_i$ to calculate the TIC using $Q_R = 633.2215$ units and each Q_i,

we have:

for $Q_R = 633.2215$ units,

$$\Rightarrow TIC\ \ 633.2215\ = \frac{438\ \ \ 1966\ \ \ 1-0.1}{633.2215^{\ 1-0.1}} + \frac{7\ \ 1-0.1\ \ (633.2215)}{2-0.1} = 4433,$$

For Q_1=679.3010 units,

$$\Rightarrow TIC\ \ 679.3010\ = \frac{438\ \ 1966\ \ 1-0.1}{679.3010^{\ 1-0.1}} + \frac{7\ \ 1-0.1\ \ (679.3010)}{2-0.1} = 4442\ \ \text{etc}$$

The table below displays the Q^* and TIC^* for solution algorithm step3:

Q	TIC
Q_R=633.2215	4432.6
Q_1=679.3010	4442.4
Q_2=1467.4	6655.7
Q_3=2302.4	10546

Table 4.6 TIC using Q_R and Q_i

- **Solution Algorithm step4**

Choose the value of Q that gives the lowest TIC.

From table 4.6, the value of Q that gives the lowest Total Inventory Holding Costs (TIC) is 633.2215 and corresponding minimum Total Inventory Cost TIC^*=□4432.6 the cycle period realizable for this order quantity Q^* and TIC^* is 0.2year.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

- A Retroactive Holding Cost Model was used to model the 20% concentrate poultry feed of Agricare Limited, Kumasi Ghana.

- With the model, the optimal order quantity Q^* which minimises the Total Inventory Cost (TIC) of 20% concentrate poultry feed was determined to be 633 tonnes. The cycle period T* for the quantity Q^* to be produced per cycle is 0.2year.

- Agricare Limited could release 633units per each order within the cycle period of 0.2 year. This would minimise the Total Inventory Cost (TIC) to □4.4326e+003.

5.2 Recommendations

Based on the findings so far arrived at, in order to ensure proper inventory system of Agricare Limited the following recommendations were made:

- To sustain the Agricare Company's inventory, managers of the company should produce the quantity Q^* of 633units of 20% percent Concentrate poultry feed per each order within the cycle period of every 0.2 year. This would minimize the Total Inventory Cost (**TIC***) to □ 4,432.6
- Companies who own storage facilities should use retroactive Holding Cost model to determine optimal quantity Q^*, optimal Total Inventory Cost TIC^* and cycle period T^*,

- The Inventory level during release scheduling period should be above 633 units to reduce the probability of stockout ie. $\left(1-\frac{D}{P}\right)$.
- There should be further research for Agricare in inventory using Incremental holding cost increase model (that is, higher storage cost rate apply to storage in later periods)

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APPEXDIX A

Production and Demand Data of 20% Concentrate Poultry Feed from Agricare Limited 2005-2010

A.1 Production and Demand Data 2005-2006

	2005		2006	
MONTH	**PRODUCTION**	**DEMAND**	**PRODUCTION**	**DEMAND**
JANUARY	723	248	123	114
FEBRUARY	37	156	4	88
MARCH	443	166	165	90
APRIL	180	145	172	61
MAY	242	88	40	113
JUNE	321	128	174	109
JULY	225	112	229	133
AUGUST	270	105	353	178
SEPTEMBER	32	133	297	132
OCTOBER	162	148	281	172
NOVEMBER	357	127	453	277
DECEMBER	224	98	445	233

Table A.1 Production and Demand 2005-2006

A.2 Production and Demand Data 2007-2008

	2007		2008	
MONTH	**PRODUCTION**	**DEMAND**	**PRODUCTION**	**DEMAD**
JANUARY	317	276	193	253
FEBRUARY	373	205	307	184
MARCH	355	203	47	147
APRIL	169	194	293	199
MAY	293	127	407	205
JUNE	351	197	226	182
JULY	285	172	491	221
AUGUST	451	207	308	189
SEPTEMBER	162	173	276	222
OCTOBER	533	226	281	229
NOVEMBER	485	239	225	200
DECEMBER	393	271	206	202

Table A.2 Production and Demand 2007-2008

A.3 Production and Demand Data 2009-2010

	2009		2010	
MONTH	**PRODUCTION**	**DEMAND**	**PRODUCTION**	**DEMAND**
JANUARY	326	138	261	173
FEBRUARY	194	134	149	153
MARCH	153	82	212	181
APRIL	151	14	256	187
MAY	74	77	239	166
JUNE	111	95	288	222
JULY	162	80	278	175
AUGUST	162	82	280	202
SEPTEMBER	108	66	369	208
OCTOBER	157	74	337	292
NOVEMBER	154	47	347	308
DECEMBER	1	57	401	307

Table A.3 Production and Demand 2009-2010

APPENDIX B

B.1 Cost per unit item of 20% Concentrate Poultry feed from 2005-2010

The cost per unit per a bag of 20% Concentrate Poultry feed was obtained within six year periods. The table B.1 shows cost per bag for 20% Concentrate Poultry feed form 2005-2010

YEAR	COST PER UNIT ITEM (GHø)
2005	**Ø 20.31**
2006	**Ø 25.81**
2007	**Ø 28.58**
2008	**Ø 36.00**
2009	**Ø 63.88**
2010	**Ø 72.90**

Table B.1 Cost per unit item of 20% Concentrate Poultry feed from 2005-2010

B.2 Cost per Order for 20% Concentrate Poultry Feed from 2005-2010.

YEAR	WA	TEMA	ACCRA
2005	Ø 1.16	Ø 83p	**Ø 60p**
2006	Ø 1.25	Ø 90p	**Ø 65p**
2007	Ø 1.45	Ø 1.04p	**Ø 76p**
2008	Ø 1.70p	Ø 1.22p	**Ø 89p**
2009	Ø 1.70p	Ø 1.22p	**Ø 89p**
2010	**Ø 2.04p**	**Ø 1.46p**	**Ø 1.07**

Table B.2 Cost per Order for 20% Concentrate Poultry Feed from 2005-2010.

APPENDIX C

Matlab Code for the trajectory of Production and Demand Data.

```
% matlab simple code for drawing graphs, figures of Agricare Limited inventory patterns
% n is the number of data points
% trajectory of production data
% trajectory of demand data
A=[];
for i=1:n
    a=input('enter values of [t,X]:');
    A=[A;a];
end
t=A(:,1);
X=A(:,2);
plot(t,X,'-')
title('TRAJECTORY OF DEMAND DATA OF 20% CONC. POULTRY FEEDS FROM JAN. 2005-DEC. 2010')
xlabel('TIME(MONTHS)')
ylabel('POULTRY FEED')
grid
```

APPENDIX D

D.1 Matlab Output for Determination of optimal quantity and optimal TIC of 20% Concentrate poultry feed released per cycle using Retroactive Holding Cost model.

Given the average demand per year, average cost of order and the average holding cost (see appendix B) the following parameters are used in determining the optimal solution for Agricare Limited.

That is:

Demand per year = 1966 units,

Set-up cost per order k = ȼ438,

```
>> k=438
k =
   438
>> D=1966

D =

     1966

>> k=438

k =

   438

>> h1=7
```

```
h1 =

     7

>> h2=8

h2 =

     8

>> h3=9

h3 =

     9

>> t1=0.2

t1 =

    0.2000

>> t2=0.4
```

t_2 =

0.4000

>> t_3=0.6

t_3 =

0.6000

>> β =0.1

β =

0.1000

>> Q=(((k*D*(1- β))*(2- β))/h_1)^(1/(2- β))

Q =

633.2215

>> T=((633.2215)^(1- β))/(D*(1- β))

T =

0.1877

```
>> Q=(((k*D*(1- β))*(2- β))/h2)^(1/(2- β))
```

Q =

590.2467

```
>> T=(( 590.2467)^(1- β))/(D*(1- β))
```

T =

0.1762

```
>> Q=(((k*D*(1- β))*(2- β))/h3)^(1/(2- β))
```

Q =

554.7677

```
>> T=((554.7677)^(1-β))/(D*(1-β))
```

T =

0.1667

>> Q_1=(D*(1- β)*(0.2))^(1/(1- β))

Q_1 =

679.3010

>> Q_2=(D*(1- β)*(0.4))^(1/(1- β))

Q_2 =

1.4674e+003

>> Q_3=(D*(1-β)*(0.6))^(1/(1- β))

Q_3 =

2.3025e+003

>> TIC=k*D*(1- β)/(633.2215)^(1- β)+(h_1*(1- β)*(633.2215))/(2- β)

TIC =

4.4326e+003

```
>> TIC=k*D*(1- β)/(679.3010)^(1- β)+(h1*(1- β)*(679.3010))/(2-β)

TIC =

  4.4424e+003
>> TIC=k*D*(1- β)/(1467.4)^(1- β)+(h2*(1- β)*(1467.4))/(2- β)

TIC =

  6.6557e+003

>> TIC=k*D*(1- β)/(2302.5)^(1- β)+(h3*(1- β)*(2302.5))/(2- β)

TIC =

  1.0546e+004
>> N=D/633.2215

N =

    3.1048

>> T=365/N
```

T =

  117.5615

D.2 Matlab Output for Determination of optimal quantity and optimal TIC of 20% Concentrate poultry feed released per cycle using Retroactive Holding Cost model when beta is increased β=0.2

```
>> b=0.2
b =
    0.2000
>> Q=(((k*D*(1-b))*(2-b))/h1)^(1/(2-b))
Q =
  823.6344
>> T=((823.6344 )^(1-b))/(D*(1-b))
T =
    0.1367
>> Q=(((k*D*(1-b))*(2-b))/h2)^(1/(2-b))
Q =
  764.7452
>> T=(( 764.7452)^(1-b))/(D*(1-b))
T =
    0.1289
>> Q=(((k*D*(1-b))*(2-b))/h3)^(1/(2-b))
```

```
Q =

  716.3061
>> T=(( 716.3061)^(1-b))/(D*(1-b))

T =

    0.1223

>> Q1=(D*(1-b)*(0.2))^(1/(1-b))

Q1 =

  1.3247e+003

>>  Q2=(D*(1-b)*(0.4))^(1/(1-b))

Q2 =

  3.1508e+003

>> Q3=(D*(1-b)*(0.6))^(1/(1-b))

Q3 =

  5.2304e+003

>> TIC=k*D*(1-b)/(823.6344)^(1-b)+(h1*(1-b)*(823.6344))/(2-b)

TIC =

  5.7654e+003

>> TIC=k*D*(1-b)/(1.3247e+003)^(1-b)+(h1*(1-b)*(1.3247e+003))/(2-b)

TIC =

  6.3113e+003
```

D.3 Matlab Output for Determination of optimal quantity and optimal TIC of 20% Concentrate poultry feed released per cycle using Retroactive Holding Cost model when cycle time is increased.

$h_1 = ø6/unit/year,\ 0 < T \le 0.25,\ t_1 = 0.25 years$

$h_2 = ø7/unit/year,\ 0.25 < T \le 0.5,\ t_2 = 0.5 years$

$h_3 = ø8 / unit / year,\ 0.5 < T \le \infty,\ t_3 = \infty$

We have:

```
>> D=1966

D =
       1966

>> k=438
k =

  438

>> β=0.1

   β =
         0.1000
>> h1=6

     h1 =
           6

>> h2=7

h2 =
      7

>> t1 = 0.25

t1 =
     0.2500

>> t2 =0.5

t2 =
   0.5000

>> Q* =((k*D*(1-b)*(2-b))/h1)^(1/(2-b))

Q* =
  686.7375

>> T=Q^(1-b)/(D*(1-b))

T =
```

0.2020 (Which is realizable since $0 < T \leq 0.25$)

>> Q^*=((k*D*(1-b)*(2-b))/h_2)^(1/(2-b))

Q^* =
633.2215

>> T=Q^ (1-b)/(D*(1-b))

T =

0.1877 (Which is not realizable since $0.25 < T \leq 0.5$)

>> Q_1=(D*(1-β)*t_1)^(1/(1-β))

Q_1 =
870.4424
>> Q_2=(D*(1-β)*t_2)^(1/(1-β))

Q_2 =

1.8803e+003

>> TIC^*=k*D*(1-β)/(686.7375)^(1-β)+(h_1*(1-β)*(686.7375))/(2-β)
TIC^* =

4.1204e+003

>> TIC^*=k*D*(1-β)/(870.4424)^(1-β)+(h_1*(1-β)*(870.4424))/(2-β)

TIC^* =

4.2259e+003

$$\tau^* = \frac{Q^*}{d}$$
τ=

127.4970

D.4 Matlab Output for Determination of optimal quantity and optimal TIC of 20% Concentrate poultry feed released per cycle using Lot-size model.

```
>> D=1966

D =
      1966

>> P=3092

P =

      3092

>> K=438

K =

   438

>> H=118.7904

H =

  118.7904

>> Q=sqrt(2*D*K/((1-(D/P))*H))

Q =

  199.5279

>> TC=(0.5*(1-(D/P))*Q*H)+((D/Q)*K)

TC =

  8.6315e+003

>> Number of orders per cycle N =D/Q

N =

    9.8533

>> Time period between orders T =365/N

T =

   37.0436
```

```
>> N=D/Q

N =

    2.8628

>> Stock=D/P

Stock =

    0.6358

>> Stockout=1-(D/P)

Stockout =

    0.3642
```