CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

Economic growth around the world has not been equal for a long time. Some economies grow faster than others. Economists have predicted that the slower growing economies will eventually converge to the faster growing economies at some point in time.

Mathematical Economists have long been interested in the factors which cause different countries to grow at different rates and achieve different levels of wealth. This issue is especially relevant today as it was in the 1940's where developmental economics was born. The historical record shows a broad range of outcomes in achieving sustained economic growth. Some countries, particularly in Eastern Asian have achieved very rapid rates of growth and catching up with already wealthy countries while others, particularly Sub-Saharan Africa, have achieved little or no growth. The reasons for these differences remain an important theoretical and empirical task.

A review of recent theoretical advances in growth theories is potentially relevant for policy development and analysis of the determinants of economic growth. Although neoclassical economic theory has become dominant in economic analysis, development economists have been reluctant to adopt it, as it predicts stable growth independent of policy decisions. Cherery [1986] makes the case for the inadequacy of the neoclassical equilibrium approach for developing countries as it does not take into account disequilibrium factors such as internal demand constraints, external market constraints, economies of scale, learning by doing, and imperfect factor markets. In recent years, economists working within neoclassical theory have provided models which address a number of issues raised by the development economists. In particular, new models of endogeneous economic growth have been developed to deal with general issues of growth with regards to policies such as the operation of financial markets, trade policies, and government expenditure and taxation.

Caraiani [2007] argued that a country with a higher saving rate will experience faster growth, e.g. Singapore had a 40% saving rate in the period 1960 to 1996 and annual GDP growth of 5-6%, compared with Kenya in the same time period which had a 15% saving rate and annual GDP growth of just 1%. This relationship between savings growth was considered in the Cobb-Douglas model. This was retained in the Solow model; however, in the very long-run capital accumulation appears to be less significant than technological innovation in the Solow model.

Economic growth always takes the center stage in most economic policies and it is necessarily associated with economic development as there can be no development without growth. However, growth does not necessarily imply development. The reason is that whereas growth merely refers to the growth of output, development refers to all the changes in the economy including the social, political and institutional changes that accompany changes in output.

Because the study dwells on Ghana mostly, a brief overview of Ghana's economy will be presented in the paragraphs below.

While there is no doubt about the fact that the economic growth record of the last two decades, following reforms, differed from that of the first two decades in terms of consistency, it is also clear that the factors behind the growth experiences of shorter periods in-between show remarkable similarity. Whenever there has been considerable capital injection into the economy, this has been followed by significant growth. It is the difficulty in making those injections consistently in the absence of structural change that has left the economy still fragile after five decades of independence.

1.2 STATEMENT OF THE PROBLEM

According to Aryeetey and Fosu [2005], ‘economic growth was turbulent during much of the period after mid-1960s and only began to stabilize after 1984. In 1966, 1972, 1975-1976, 1979 and 1983, the growth rate of real GDP was negative for Ghana,’ according to Aryeetey [2005]. The GDP growth has been negative for a number of years. The years 1974, 1977 and 1978 recorded positive growth. The lowest growth of -14% was experienced in 1975.

A critical look at the growth rate of Ghana indicates that the growth rate has been positive since 1984, though the rate of growth has been fluctuating. The fluctuating nature brings up the curiosity for some empirical analysis. Cobb-Douglas [1928], production function has been used in determining growth rates of the American economy. This production function was later modified by Solow in 1956 and the resulting model predicted the growth rate of the American economy and elsewhere. Though the model predicted growth rate of developed and some developing countries, it has not been used here in Ghana to predict the growth rate. It is against this background that the study sought to model, and tests a model of economic growth of Ghana with Solow’s production function as the bases to determine whether the model predicts the growth pattern of Ghana from 1990 to 2010.

1.4 Objectives of the Study

- The main objective of the study is to use the Solow’s model (Augmented Cobb-Douglas production function) as a basis to model the economic growth of Ghana during the period 1990 -2010.
- To determine whether the new model predicts the growth pattern of Ghana from 1990 to 2010.

- To estimate the economic growth of Ghana for the past transition periods (1990-2010), relative to the real dynamics of the economy.

1.5 Justification for the Study

- According to Caraiani [2007], the Solow model provides a good approximation of the dynamics of the economy, with respect to the dynamics of the aggregate Gross Domestic Product (GDP) and to the ratios of the main macroeconomics variables, like production per labour, capital-output ratio or capital per labour.
- Economic growth always takes the center stage in most economic policies and is necessarily associated with economic development as there can be no development without growth.
- It is also evident that economic growth has a direct relationship with standard of living and consequently national development.
- The study will equip policy makers with the relevant data for the design of appropriate policies that will result in higher and sustainable growth rates for the whole economy.

1.6 Methodology

The study used Solow's production model as a basis to model the economic growth.

The Solow model (production function) is one of the foundations of the modern macroeconomics, not only of the neoclassical and of the new theory of economic growth, but also of the modern theory of business cycles. Growth rate model are obtained using differential equations techniques.

The study also considered growth rates from 1990 to 2010.

The computations were done using Matlab, SPSS and Excel spreadsheet. The recording and computation of the data was done using Matlab, SPSS and Excel. The computations were Capital, Labour force, Total Factor Productivity, and Total Production and the results from models were compared with the real GDP growth figures and variations noted.

Data used for the study was generally secondary data and was collected from sources such as the Institute of Statistical Social and Economic Research (ISSER), Ghana Statistical Service, Ministry of Finance and Economic Planning, the International Monetary Fund (IMF) website, the Bank of Ghana, and others such as journals, articles, reports and other unpublished materials.

1.7 Organization of Thesis

The study is organized into five chapters.

Chapter one gives a general introduction. In this chapter, the background to the study, research problem and the objectives, justification, methodology and organization of the thesis is covered. Chapter two presents a review of the relevant literature about the study. The chapter contains theoretical literature on economic growth and growth models. The last section of chapter two reviews the various empirical studies on growth.

Chapter three is the methodology. It presents the process of Modeling and the model development. Provision of Data Collected and its analysis, as well as findings are done in the chapter four of the thesis. Chapter five is made up of the recommendations, and conclusions of the study.

CHAPTER TWO

REVIEW OF FUNDAMENTALS

The chapter is divided into five sections. The first section deals with economic growth. The second section deals with Ghana's growth and the third section looks at theoretical literature on the economic growth of Ghana. The fourth section looks at theoretical literature on the growth rate elsewhere. The final section looks at the empirical studies on economic growth, Differential Equation and Economic Analysis.

2.1 Economic Growth

Economic growth is defined as the growth in individual human welfare: - but on a practical level, it is a sustained increase in per capita or per worker product. We often use real per capita GDP as a proxy, not just total GDP, for measuring the rate of economic growth.

Alternative measures include life expectancy, average levels of education, infant mortality and nutrition, all of which are related to individual welfare. Economic growth is an increase in the full productive capacity of an economy to produce real output (DeLorme *et al.*, 1983). That is economic growth involves the process of increasing and humanizing the determinants of these productive capacities. According to Wallace (1988), the productive capacities are remarkably vital, but actual growth depends not only on that, but also on the degree to which those capacities are utilized. Hence, it involves an increase over time in actual output of goods and services as well as an increase in the economy's capacity to produce goods and services.

At http//en. Wikipedia.org/wiki/Economic-growth,(2010), Economic growth was defined as the increase in the amount of goods and service produced by an economy over time. Economic growth is conventionally measured as the percent rate of increase in real Gross Domestic Product (Eichengreen, 2007). Growth is usually calculated in real terms. (That is to net out the effect of inflation on the price of the goods and service produced by adjusting inflation terms). In economics “economy’s growth” or “economy’s growth theory” typically refers to growth of potential output (that is production at “full employment”), which is caused by growth in aggregate demand or observed output as year on year difference in potential output. But according to Slavin (2005), Economic Growth is measured as the annual percent change of gross domestic product (GDP) or for study purposes; Economy’s Growth is generally distinguished from development economy which is the study of how poor countries can catch up with rich ones.

2.2 Ghana’s Growth

Since independence in 1957, Ghana has tried a number of approaches to achieving acceptable rates of growth and development. Governments after governments tried various approaches to achieve economic growth that is acceptable, but most approaches couldn’t materialize because of political instability, weak commodity demand etc. When Ghana gained her independence she was the world's leading producer of cocoa and this supported a well-developed infrastructure to service trade, and enjoyed a relatively advanced educational system, Aryeetey and Fosu [2005].

The growth rate record of Ghana has been unstable when the post-reformed period is compared to the earlier period. With logically high GDP growth in the 1950s and early 1960s, the economy began to experience a reduction in GDP growth in 1964. According to Aryeetey and Fosu [2005], ‘growth was turbulent during much of the period after mid-1960s and only began to stabilize after 1984. In 1966, 1972, 1975-1976, 1979 and 1983, the growth rate of real GDP was negative for Ghana'. The GDP growth has been negative for a number of years. This is mainly due to political instability between these years, even though some years recorded some positive growth in 1974, 1977 and 1978. From 1984 to 2006, the GDP growth has averaged about 3.9 to 4.5 percent, Baafi [2010].

There has always been the view that the economy of Ghana could and should grow faster than it has done. The recent growth record is deemed inadequate for the desired transformation of the economy, Aryeetey and Fosu [2005]. Ghana in 1993 set itself the target of becoming an upper middle income country by 2020 under its *Vision 2020* programme according to Aryeetey and Fosu [2005]. To achieve the targeted *per capita* income by that year, using a simple Harrod-Domar type model, it was reckoned that the economy needed to grow at an average of 8% for the period as indicated in the economic review, 1992 by the Institute of Economic Affairs.

The government sought to use the apparent stability of the Ghanaian economy as a springboard for economic diversification and expansion and began the process of moving Ghana from a primarily agricultural economy to a mixed agricultural industrial one, Aryeetey and Fosu [2005]. But unfortunately, the price of cocoa collapsed in the mid-1960s, destroying the fundamental stability of the economy. Since then, Ghana has been caught in a cycle of debt, weak commodity demand, and currency overvaluation, which has resulted in the decay of unproductive capacities and a crippling growth rate.

Since then the economy has not shown a capacity to move towards the target. The performance of the economy and economic growth have been characterised by the non-attainment of macroeconomic targets. In particular, whereas the GDP was expected to grow between 7.1% and 8.3% in the period 1996-2000, actual growth was between 4.2% and 5.0%. The significant deviation between set targets and the actual results was translated into low *per capita* GDP growth and poor sectoral growth, (Cited in Baafi, 2010).

The quite positive growth performance in response to the reform has persisted since 1984 with relatively little variance particularly in the early 1990s.Despite a strong desire to achiever faster growth rates this has not been easy. The inability of growth to move beyond 6.0 percent per annum has been linked to the absence of structural transformation (Aryeetey and Fosu, 2004) and these are driven by the fact that macroeconomic policies have not been anchored in comprehensive and credible longer term development frameworks. Baafi [2010] suggested that, per capita GDP growth closely tracks that of GDP suggesting a seemingly stable growth of population. The slow rate of per capita income growth in the economy hovering below the 4 percent mark after 1984 is largely attributed to low productivity (O'Connell and Ndulu, 2000). Growth since 2001 has been rising slowly as a result of the recovery of agricultural production and general improvement in economic management, particularly in the area of fiscal and monetary policies (AfDB/OECD, 2003). However, this is not regarded as sufficient to drive the economy towards the achievement of the Millennium Development Goals (MDGs) (Aryeetey and McKay 2005).

To achieve sustained economic growth, increased production and productivity must be at the centre of an economic recovery strategy. To formulate strategies for achieving sustained increased production and the rapid growth necessary for poverty reduction, relevant information is absolutely necessary. It is therefore important to decompose the structure of Ghana's economy and its growth rate, to gain a better understanding of those factors that have produced differences in growth rates in the various periods. That is to model the economic growth of Ghana from 1990 to 2010, considering the Solow model on economic growth.

2.3 Literature on the economic growth of Ghana

Baafi [2010], using time series data, tested the validity of the hypothesis for Ghana and the Western European Countries (with the United Kingdom (UK), as a proxy). He determined how fast or slow this convergence process is by using the returns to scale concept on Ghana's economy and latter account for the factor that determines economic growth in sectors. The study supported the null hypothesis of convergence that is Ghana is catching up with the Western European countries and also showed that Ghana's growth accounting exhibits decreasing returns which is an implication of its relatively slow convergence and unbalanced growth path (this refers to the simultaneous, coordinated expansion of several sectors of the economy). The study showed a negative relationship between GDP and labour supply both in the long run and short run relationship. Again, the study showed a positive relationship between GDP and capital, Agric and Industrial sector. Lastly, the study showed a negative relationship between GDP and AID and Service in the long run and positive relationship in the short run.

Accounting for Ghana's growth, Aryeetey and Fosu [2005], used the aggregate production function model of growth accounting and Cobb-Douglas production function in the model formulation. The fallout of their estimation indicates that most of GDP growth seems to be accounted for by factors outside the model. Their study showed that the only significant variable is the economic liberalization dummy variable which has a positive coefficient. Labour has negative coefficient, though not statistically different from zero. The capital variable has a positive coefficient though it's not statistically significant. The study suggests that total factor productivity may have played a more important role in the observed pattern of GDP growth, and that total factor productivity is affected by political regimes. In particular, liberal regimes apparently positively contribute to total factor productivity and to growth in Ghana.

However, in accounting for Ghana's economic growth, the study uses the aggregate production function approach following Mansouri's [2005] and Rubina & Verma [2006] approach-, by taking the Ghanaian specificities into account.

Aryeetey *et al*. [2001] presented a relatively simplistic growth accounting model in the form of a Cobb-Douglas production function for Ghana's economy. The model which was developed using data from 1961-96 is specified as:

$$q = \alpha + \beta l + \delta k + d + \varepsilon \tag{2.3}$$

Where

q : GDP growth

l : labour growth

k : growth in capital (measured as investment to GDP ratio)

d : a dummy variable representing economic liberalization (d = 1 from 1969-72 and 1983-96; d= 0 from 1961-68 and 1973-1982)

ε : the error term

The results of the estimation, admittedly crude, indicated that most of GDP growth seemed to be accounted for by factors outside of the model. The results show that the only significant variable is the economic liberalization dummy variable, which has a positive coefficient. Growth in labour and capital has supposedly not contributed significantly to GDP growth.

The results suggested that total factor productivity (TFP) may have played a more important role in the observed pattern of GDP growth, and that TFP is affected by economic regimes. In particular, liberal regimes apparently positively contribute to TFP and to growth.

In addition, their results tell us that real investment to GDP as a proxy variable to measure capital in the economy significantly affects growth with the expected positive sign. This implies that capital affects output as it is used as a factor of production. Furthermore, the labour force and human capital variables have the same positive impact on growth.

However, like the two debt variables, they claimed their results were not statistically significant. Their results tallied with economic theories which emphasize that investment affects growth through the multiplier effect of capital, and labor force affects output as it is used as a factor of production.

2.4 Literature on the economic growth elsewhere

Mathematical Economist grasped the importance of economic growth centuries ago. The rate of economic growth was began to accelerate 200 years ago, -during which the early economists started paying attention to how nations could become wealthier and how living standards could be raised, Baafi [2010]. Various macroeconomist like Adam Smith, Paul Romer, Robert Lucas, Robert Solow, Nicholas Kaldor, Roy Harrod and Evsey Domar have contributed theoretically and empirically towards the advancement of the study of economic growth. According to Adam Smith's[1776] growth theory, made in his book the '*Wealth of Nations*' are such that; the augmentation of specialization and exchange harmonize economic growth, voluntary market transactions are due to individuals and firms making decisions that are compatible with the 'general welfare'; close association exist between specialization and the generation of new technology and finally the bottom line in judging the performance of an economy is human welfare throughout the entire population. Adam Smith, beside the theory, there was also the institutions, laws, norms, rules of just conduct and among others. He was of the view that

Economies will grow if they have the right kind of government. A model, formulated by Thomas Malthus was based on a hypothesis that output, Y is a function of labour, L and land, N; where the amount of land is fixed in quantity whilst labour can grow or contract depending on birth and death rates.

$$Y= f\,(L, N), \tag{2.1}$$

Because labour is combined with a fixed stock of land, production is subject to diminishing returns. This model of course has been widely criticized, because it failed to take into account changes in the level of technology. The model did not also consider the amount of capital in the economy.

Another growth model for economic planning, which became popular subsequent to the World War II is the Harrod-Domar model. The model assumes that the amount of unemployed labour available is unlimited; hence output can be increased without triggering price increase and that productive investment is always equal to saving. According to the model, for any given level of output and constant capital-output ratio, the amount of capital needed can be determined and a constant proportion of the income generated by production output, the level of output can be increased. The main criticism of the model is the level of assumption, one being that there is no reason for growth to be sufficient to maintain full employment; this is based on the belief that the relative price of labour and capital is fixed, and that they are used in equal proportions. The model explains economic boom and bust by the assumption that investors are only influenced by output (known as the accelerator principle); this is now widely believed to be false.

The neoclassical model such as Solow [1956] and Swan [1956], seek to analyze the case where capital and labour can be combined in varying proportion and have demonstrated a way out of the razor-edge growth path of the Harrod-Domar growth model.

In the 1950's, many economists undertook comparative studies of economic growth following World War II reconstruction. Many agreed that the path to long-term growth was achieved through investment in industry and infrastructure and in moving further and further into capital intensive automated production. Although there was always a concern about diminishing returns to this approach because of equipment depreciation, it was a widespread view that effective industrial policy to adopt after the war. Other economists pointed to the Soviet command economy as a model of high-growth through tireless re-investment of output in further industrial construction.

However, some economists took a different view to the two approaches discussed above and argued that greater capital concentrations would yield diminishing returns once the marginal return to capital had equalized with that of labour—and that the apparent rapid growth of economies with high savings rates would be a short-term phenomenon. This analysis suggested that improved labour productivity or total factor technology was the long-run determinant of national growth, and that only under-capitalized countries could grow per-capita income substantially by investing in infrastructure. This was supported by the fact that some of these undercapitalized countries were still recovering from the war and were expected to rapidly develop in this way on a path of convergence with developed nations. This Total Factor Productivity is what is referred to as the Solow residual (cited in Baafi, 2010).

The Solow residual is defined as per-capita economic growth above the rate of per-capita capital stock growth, so its detection indicates that there must be some contribution to output other than advances in industrializing the economy (cited in Baafi, 2010). The fact that the measured growth in the standard of living, also known as the ratio of output to labour input, could not be explained entirely by the growth in the capital/labour ratio was a significant finding, and pointed to innovation rather than capital accumulation as a potential path to growth. Solow assumed a very basic model of annual aggregate output over a year (*t*). He said that the output quantity would be governed by the amount of capital (the infrastructure), the amount of labour (the number of people in the workforce), and the productivity of that labour. He concluded that the productivity of labour was the factor driving long-run GDP increases.

Romer (1989) suggested five stylized facts that growth theorists should be able to explain.

- In cross-section, the mean growth rate shows no variation with the level of per capita income.
- The rate of growth of factor inputs is not large enough to explain the rate of growth of output; that is, growth accounting always finds a residual.
- Growth in the volume of trade is positively correlated with growth in output.
- Population growth rates are negatively correlated with the level of income.
- Both skilled and unskilled workers tend to migrate towards high-income countries.

However, the most famous model which has been the corner stone of most economic growth analysis is the Solow Model. Although, many economists felt that a much more sophisticated model was needed to accurately depict the complex process of economic growth. Solow's simple neoclassical model still dominates the economic growth literatures.

Succession of attempts has been to create models of economy's growth that could explain the variation in growth experienced by different regions of the world.

In the simple model proposed by Malthus in 1850, economy's growth could not exceed population growth, but it was not long before it became evident that it was doing so (cited in Blejer and Chu, 1989). The Harrod-Domar model and its successors assume that there would be sufficient economy's growth to enable some to go into growth-enhancing investments. In a latter development, the 1956 Solow model introduced the influence of the substitution of capital for labour resulting from investment in improved capital equipment (cited in Carbo et al, 1987). According to Webster [2003], the Cobb-Douglas functional form of production function which came into existence in the 1920's is widely used in economics to represent the relationship of an output to inputs in an economy. It can be expressed in the form;

$$Y(t) = AL^{\alpha} K^{\beta} \tag{2.2}$$

Where: Y is total production expressed in terms of monetary value of all goods produced in a year, L is the labour input, K is the capital input, A is total factor productivity, and α and β are the output elasticities of labour and capital, respectively (the values are constants determined by available technology).

Output elasticity measures the responsiveness of output to a change in level of either labour or capital used in production, ceteris paribus. For example if α=0.15, a 1% increase in labour would lead to approximately a 0.15% increase in output.

Further, if $\alpha + \beta = 1$, the production function has constant returns to scale. That is, if L and K are each increased by 20%, Y increased by 20%. If $\alpha + \beta < 1$, returns to scale and decreasing, and if

$\alpha + \beta > 1$, returns to scale are increasing. Assuming perfect competition α and β can be shown to be labour and capital's share of output.

Cobb and Douglas explained production function showing that labour and capital shares of total output were constant over time in developed countries; but this approach has led to doubts over regarding whether constancy over time exists.

As result of subsequent research, notably that of Paul Romer and Robert Lucas (1994), some of the factors believed to influence technological change, have been embodied in the growth models, which are termed endogenous growth models.

Turturrean [2005] in his paper used the classical and modern econometric methods to model economic growth rate dynamics in Romania, from 1990-2004. The output of his analysis was an econometric model which describes the dynamics of the economic growth process in Romania represents a starting point for short and long term predictions in economic analyses.

The rate of economic growth around the world has been asymmetrical centuries. Some economies experience a rapid growth whilst the growth of others is at very slow rate. Although economists have predicted that slower growing economies will eventually converge with the rapid growing ones at some point in the future. This is known as the convergence hypothesis.

Making an estimation of the Solow model for the Romanian economy, Caraiani [2007] simulated the model both for the 1990-2004 periods and in the long run using estimates of the parameters from other studies and showed that the model provides a good approximation of the dynamics of the Romanian economy for the 1990-2004 periods with respect to the dynamics of the aggregate GDP and to the ratios of the main macroeconomics variables like, production per worker, capital-output ratio or capital per worker. The simulation for the 2030 time horizon indicates a potential of growth of over 3%.

We have noticed from the previous section that, the neo-classical Solow model explains economic growth as resulting from the combination of two elements, namely Capital and Labour. Now the question arising is how much of the output growth can be attributed to other factors apart from capital and labour. To answer this question, Solow decomposes the growth in output into three components,- with each identifiable as contribution of one factor of production, that is labour, capital and total factor productivity. This type of measurement of total factor productivity is often referred to as the Solow residual. The term residual is appropriate because the estimate present the part of measured GDP growth that is not accounted for by the weighted-average measured growth of the factors of production (capital and labour). To account for this, Solow used the Cobb-Douglas production function and started from his simple growth equation. For simplicity, we repeat the equation as

$$Y(t) = f(K, L, A) \tag{2.4}$$

Where A : total factor productivity

L : labour

K : capital

Using Cobb-Douglas production function, Solow stated the following equation

$$Y(t) = AK^{\alpha}L^{\beta} \tag{2.5}$$

From this, Solow defined his other factor (total factor productivity) to be technology as noted earlier. Solow acknowledged the convenience of the Cobb-Douglas production function because it exhibits constant returns to scale which is consistent with his model. We should note that the variable A is not constant but varies with different production functions based on the factors studied. Different authors like Mansouri [2005] have used different factors to account for the total factor productivity.

In accounting for the determinants of Morocco's economic growth, Mansouri [2005] used the aggregate production function model. He used the aggregate production of the following general form: (2.4)

$$Y = f(A, L, K)$$

Where Y is real GDP, A is total factor productivity, and L and K stand for labour and capital inputs respectively. Mansouri [2005]argued that A is determined by economic factors and in the case of Morocco, *FDI and FDI interaction with trade openness (TR)* are the vehicle through which technology travels,- hence;

$$A = g(FDI\ , FDI * TR\ ) \qquad (2.6)$$

Substituting (2.4) into (2.6), gives:

$$Y = f(\ FDI, FDI*\ TR, L, K) \qquad (2.7)$$

To account for the isolated impact of trade openness on economic growth, Mansouri [2005] introduced *TR* as an explanatory variable. Considering the specificities of the Moroccan economy, Mansouri [2005] accounted for the impact of drought cycles on economic growth in the particular case of Morocco. Mansouri finally added a proxy for drought (DR) to equation (2.7), to yield:

$$Y = G(FDIt\ ,TRt\ , FDIt * TRt\ , Lt\ ,Kt\ ,DRt\ ) \qquad (2.8)$$

Where *DR* is a proxy for drought, is the inverse of the cereal yield per hectare. The operational model that was finally selected by Mansouri [2005] to explain Moroccan growth is:

$$In(Y) = \beta_0 + \beta In(FDI) + \beta_2 In(TR) + \beta_3 In(FDI) * TR + \beta_4 In(L) + \beta_5 In(K) + \beta_6 (DR) + \varepsilon \ldots \quad (2.9)$$

Because Robert Solow used the ‘marginalist’ thinking of the 19th century neoclassical economist, his model is usually referred to as Neoclassical Growth Model. The basic structure of the Solow model is quite simple. To differentiate his model from the Harrod-Domar model and its fixed capital-output ratio, Solow defined a production function that permits factors to be continuously substituted for each other such that the marginal product of each factor is variable; depending on how much of the factor is already used in production and how many other factors it is combined with. This continuous substitutability of the factors of production is what makes Solow's model neoclassical in nature (Hendricks Van den Berg, 2001).

Solow furthermore assumed that each factor of production is subject to diminishing returns. Meaning equal increments of one factor are added to a fixed amount of the other factors of production, output increases, but it increases by ever-smaller amounts.

Thomas Malthus [1798], had assumed that labour was subject to diminishing returns when it was combined with a fixed stock of agricultural land. Solow's aim was to show that the Harrod-Domar model was wrong in concluding that a constant rate of saving and investment could bring everlasting economic growth. Solow showed that, with diminishing returns, continuous investment could not, by itself, generate permanent economic growth because diminishing returns would eventually cause the gains in output from investment to approach zero. Solow's model thus clashed with what many development economist were advising policy makers to do in order to increase economic growth, which was to increase saving and investment any way possible. (Hendricks Van den Berg, 2001).

But if investment is not the determinants of an economy's long-run rate of growth, what is? Solow's identified that; long-run growth must come from another source: technological progress. Only if an economy keeps increasing the amount of output that it can produce from a given amount of input then can it avoid diminishing returns and keep it per capita output growing forever. (Hendrick Van den Berg, 2001).

2.5 Empirical Literature: Economic Growth Models

In the last decade, a vast literature has gone into investigating the growth rates of the economy (Barro and Sala-i-Martin, 1992; Mankiw, Romer and Weil, 1992).

Slavin [2005] expressed the view that, model of economy's growth was developed by economists, Robert Solow in 1957. In his model, he assumed that an economy wide production function can be written in the simple form:

$$Y=AK^{0.3}L^{0.7} \tag{2.10}$$

Where Y is aggregate output, A is a number based on the current state of technology, K is a quantitative measure of the size of the stock of manufactured capital, and L the quantity of labour used during the period of time. K and L are the only factors of production explicitly included in the model. Both capital and labour are needed for the production of output, with the exponents in the equation reflecting their relative contributions.

A is called total factor productivity, and includes all contributions to total production not already reflected in the levels of K and L. Often, total factor productivity has been interpreted as reflecting the way in which technological innovation allows capital and labour to be used in more effective and valuable ways. For example, the development of computer word-processing greatly increased efficiency compared to the use of typewriters. Typewriters, which seem antique to us today, were themselves a huge productive advance over clerical work using pen paper. This process of improved technological methods has resulted in an increase in labour productivity; more output can now be produced with fewer labour-hours.

After some mathematical manipulations, the production function above can be converted to an equation for the growth rate of output per worker as a function of total factor productivity and the growth rate of manufactured capital per worker:

Growth rate of output per worker = growth rate of total factor productivity + 0.3 (growth rate of manufactured capital per worker)

For example, if total factor productivity grows at 1% per year and capital per worker grows at 2% per year, this equation says that output per worker will grow at 1.6% per year (1 %+(0.3)2%=1.6%). This became known as the Growth Accounting Equation.

Output per worker is what is commonly referred to as labour productivity. While labour productivity and GDP per capita are not quite equivalent (some people in the population do not work, for example), they are closely related. Thus, this model implies that the way to raise income per capita to achieve economy's growth is to increase the amount of capital that each person works with (the second term) and improved technology (the first term).

The use of the Solow growth model served to highlight some important factors in economy's growth,- particularly the model led to much discussion of the role of savings in providing the basis for growing levels of manufactured capital per worker. In addition technological change also received attention, since it was thought to be the main driver behind growth in the value of "A". For many years, economists tended to treat growth primarily as a matter of encouraging savings.

2.6 Literature on Production Function

A function summarizes the process of conversion of factors into a particular commodity.

We might propose a production function for a good Y of the following general form, first proposed by Philip Wicksteed [1894] (cited in Slavin, 2005);

$$Y=f(x_1, x_{2\ldots,} x_m) \qquad (2.11)$$

Which relates a single output y to a series of factor of production x_1, $x_{2\ldots,}$ x_m. In writing production functions in this form, meant excluding joint production, that is a particular process of production yields more than one output, using Ragnar Frisch's[1965] terms, we are concentrating on "single-ware" production(cited in Slavin,2005).

A production decision – a feasible choice of inputs and output – is a particular point on or in the graph embodied by the function f (K, L). It will be "on" the hill if it is technically efficient and 'in' the hill if it is technically inefficient. Properly speaking, the production function Y=f(K, L) is only the surface (and not the interior) of the graph, and thus denotes the set of technologically efficient points of the production set {Y, L, K} (i.e. for a given configuration of inputs, K,L, output Y is the maximum feasible output).

Obviously, the hill-shape of the production function indicates that the more one make use of the factors, the greater output is going to be (at least up to the some maximum, the "top" of the hill).

Let there be 'm' factors of production and let vector $X= (x_1, x_2,..,x_m)$ denote a bundle of factor inputs. An input space would be defined as the acceptable set of inputs for the economy. Commonly, a bundle of factor inputs x is deemed "acceptable" if every entry in that vector, i.e. the quantity of every factor, is a non-negative, finite real number. Thus, any input bundle x lies in R_+^m, the non-negative orthant of m-dimensional Euclidian space. Thus, R_+^m become the input space.

Let y be output, which is assumed to be a single, finite number, i.e. $y \in R$. thus, a production function f maps acceptable input bundles to output values, i.e., $f: R_+^m \rightarrow R$.

More specifically, f(x) is the maximum output achievement for a given set acceptable inputs,

$x \in R_+^m$.

The following assumptions are often imposed on any generic production function $f: R_+^m \rightarrow R$.

- F(x) is finite, non-negative, real-valued and single-valued for all non-negative and finite x.
- F(0, 0, …,o)=0 (no infinite implies no output)
- If $x \geq x^{1}$, then $f(x) \geq f(x^{1})$ (monotonicity, i.e. an increase in inputs does not decrease output)
- F is continuous and twice-continuously differentiable everywhere in the interior of the production set.
- The set $V(y)=\{x / f(x) \geq y\}$ is a convex set (quasi-concavity of f)
- The set V(y) is closed and non-empty for any y> 0.

Assumption (1) simply defines the production function as a well-defined function of inputs

$f: R_{+}^{m} \rightarrow R$. nothing new there. Assumption (2) simply establishes that one cannot produce something from nothing. This is somewhat self-evident, at least for economists.. The monotonicity assumption (3) is also straightforward: increasing inputs leads to increases in output (or, more precisely, no decrease in output). Assumption (4) is made largely for mathematical case; Assumption (5), the quasi-concavity of the production function f, is the more interesting one. Finally,(6) is imposed as a mathematical necessity

2.7 Differential Equation and Economic Analysis

Applications of differential equations are now used in modeling motion and change in all areas of science. The theory of differential equations has become an essential tool for economics analysis particularly since computer has become commonly available. It would be difficult to comprehend the contemporary literature of economics if one does not understand basic concepts (such as bifurcations and chaos) and results of modern theory of differential equations.

A differential equation expresses the rate of change of the current state as a function of the current state. A simple illustration of this type of dependence is changes of the Gross Domestic Product (GDP) over time. Consider state x of the GDP of the economy. The rate of change of the GDP is proportion to the current GDP.

$$X\ (t) = qx\ (t), \tag{2.12}$$

Where t stands for time and X(t) the derivation of the function x with respect to t. If the growth rate is g at any time t, the GDP at t is given by solving the differential equation. The solution is

$$X\ (t) = x\ (o)\ e^{qt}. \tag{2.13}$$

The solution tells that the GDP decays (increases) exponentially in time g is negative (positive).

We can explicitly solve the above differential function when q is a constant. It is reasonable to consider that the growth rate is affected by many factors, such as the current state of the economic system, accumulated knowledge of the economy, international environment, and many other conditions. This means that the growth rate may take on a complicated form q(x, t). The economic growth is described by

$$X\ (t) = q\ (x(t),t)\ x(t)$$

In general, it is not easy to explicitly solve the above function. There are various established methods of solving different types of differential equations.

Ordinary differential equations are differential equations whose solutions are functions of one independent variable, which we usually denote by t. The variable t often stands for time, and solution we are looking for, x (t), usually stands for some economic quantity that changes with time. Therefore we consider x (t), as a dependent variable. For instance, $x\ (t) = t^2\ x\ (t)$ is an ordinary differential equation. Ordinary differential equations are classified as autonomous and non-autonomous. The equation

$$X^1 (t) = ax (t) + b,$$

Where a and b are parameters. This is an autonomous differential equation because the time variable t does not explicitly appear. If the equation specially involves t, we call the equation non-autonomous or time-dependent. For instance,

$$X'' = 3X' - 2X + 2,$$

Is a second order equation, where the second derivative x" (t), is the derivative x'(t). as shown late, the solution is

$$X (t) = A_1 e^{2t} + A_2 e^t + 1,$$

Where A_1 and A_2, are two constants of integration. The first derivation x is the only one that can appear in a first order differential equation, but it may enter in various powers: İ, İZ, and so on. The highest power attained by the derivation in the equation is referred to as the degree of the differential equation. For instance,

$$\ddot{x} - 3\dot{x} + 2 = 0,$$

is a second-degree first-order differential equation.

CHAPTER THREE

THE MODEL

OVERVIEW

This chapter looks at the presentation of the models of Economic Growth. We start by introducing the assumptions for the model as well as production Function of the Robert Solow's type, which is an augmented form of Cobb-Douglas production function. Definition of parameters used follows. We then went on to develop the Labour Supply Model, followed by the Total Factor Productivity model (TFP). Furthermore, we went on to develop the Capital Accumulation Model, Production Model using the Solow's Production Function model. The development of the Growth Model follows using the Solow's Production Function as an input.

MODEL DEVELOPMENT

3.0 Assumptions

The following are assumptions made;

- Savings and investment decisions are exogenous (no individual optimization).
- Factor accumulation and technological growth are generated outside the model.
- It is assumed that each factor of production is subject to diminishing returns. That is, as equal increments of one factor are added to a fixed amount of the other factors of production, output increases, but it increases by ever-smaller amounts.
- It is assumed also that labour supply grows by itself.
- The Economy is closed to external forces and government intervention.
- Depreciation of capital is ignored.

3.1 The Production Function

Solow begins with a production function in which, *Y*, is a function of quantity of capital, *K* and labour *L*:

$$Y = f\ (K, L) \tag{3.1.1}$$

Solow assumed that this production function exhibits constant returns to scale, which means that if all inputs are increased by a certain multiple, output will increase by exactly that same multiple. Specifically, if equation 2.1 represents a constant-returns-to scale production function, then for any positive constant *c* the following must also hold:

$$cY = F\ (cK, cL) \tag{3.1.2}$$

We now take advantage of this characteristics of constant-returns-to scale production functions and let $c = 1/L$, which give us

$$Y/L = F(K/L, 1) \tag{3.1.3}$$

Equation 3.1 can be conveniently rewritten as

$$y=f(k) \tag{3.1.4}$$

If we define *Y/L* and *K/L* as *y* and *k,* respectively, and let the function *f(k)* represent *F(k, 1).* Equation 3.1.3 describes output per worker as a function of capital per worker.

This representation of the production function in per-worker terms is quite appropriate given that we define economic growth as the change in per capita output. In judging whether welfare in society increases, output per person must increase. In terms of the variables defined above, economic growth requires an increase in *y*, not just *Y*.

In addition to assuming constant returns to scale, Solow further assumed positive but diminishing marginal returns to any single inputs. That is the slope of output continuously decreases because each additional increase in *K* relative to *L* causes smaller and smaller output (Hendrick *Van den*

Berg, 2001). This is the inherent characteristics of the Solow model that brings convergence to light.

The General production function, with physical capital K, labor L and knowledge or technology A: is given as

$$Y(t) = F(K, A, L) \tag{3.1.5}$$

Time is discrete: t = 1, 2, 3…

The Solow growth model can be described by the interaction of five basic macroeconomic equations:

- Macro-production function
- GDP equation
- Savings function
- Change in capital
- Change in workforce

We have so far specified a neoclassical production function with the general form

$Y=f(K, L)$, in which f represent the functional relationship between output and inputs. But such a general form has its limitations. We can reach many useful qualitative conclusions, but specific quantitative solutions are not possible. To reach more specific quantitative conclusion, Solow applied the Cobb-Douglas production to his model. The Solow model also identified total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it. In the technical language used by macroeconomists, long-run growth in the Solow framework is determined by some other factors apart from capital and labour that is exogenous to the model.

The Solow residual measures total factor productivity, but is normally attached to the labour variable in the macroeconomy because return on investment doesn't seem to change very much in time or between developing nations, and developed nations—not nearly as much as human productivity seems to change.

The intensive-form production function is assumed to have the following properties:

$$\left.\begin{array}{l} f(0) = 0 \\ f'(k) > 0 \\ f''(k) < 0 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1.6)$$

And Inada conditions:

$$\left.\begin{array}{l} lim_{k\to 0} f'(k) = \infty \\ lim_{k\to \infty} f'(k) = 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1.7)$$

In macroeconomics, the **Inada conditions** (named after Japanese economist Ken-Ichi Inada) are assumptions about the shape of a production function that guarantee the stability of an economic growth path in a neoclassical growth model.

The six conditions are:

- the value of the function at 0 is 0,
- the function is continuously differentiable,
- the function is strictly increasing in t,
- the derivative of the function is decreasing (thus the function is concave),
- the limit of the derivative towards 0 is positive infinity,
- the limit of the derivative towards positive infinity is 0.

According to Borelli, P and Pessoa, S [2003], it can be shown that the Inada conditions imply that the production function must be asymptotically Cobb–Douglas.

The production function is not different from the general production function 3.1.5 where

Y(t)= F (K , L , A).

Unlike the general production function, Solow coupled labour supply, L and Productivity, A as one factor. That is;

$$Y(t) = F(K, A, L). \quad (3.1.8)$$

Productivity in the Cobb-Douglas Production Function was constant, with a value of 1. In Solow's model, productivity was made to grow proportional to it.

We shall begin with the production function of Solow's (1956) type as follows;

$$Y(t) = K^{\alpha}(A_{.}L_{.})^{\beta} \quad (3.1.9)$$

If we couple productivity and labour supply, i.e. A(t)*L(t) = Z(t), then the production function will be

$$Y(t) = K^{\alpha}Z^{\beta} \quad (3.1.10)$$

Where

Y(t):Total output (GDP).

A (t): Total Factor productivity, TFP (i.e. Knowledge and Technological change)

K(t) :Capital accumulation

L(t):labour Supply

α and β: are the output elasticities of capital and labour.

3.2 Definition of Parameters

The models contain certain parameters that are important to mention them here in order to put the results into perspective.

(τ-δ): This parameter represents the rate of labour supply. This constant of proportionality may not exist in the real world. Labour may not grow exogeneously. The growth of Labour may vary according to certain natural or economic conditions. 'τ' shows that labour will grow upwards forever. 'δ' indicates that labour will grow downward forever, until a point of equilibrium is achieved. That is, the point at which '$\tau = \delta$'. 'δ' is the parameter for exit rate of labour in the economy.

β: Solow assumed that the marginal productivity of labour is proportional to the amount of production per unit of labour. The constant of proportionality represented as β. The constant of proportionality may vary from time to time. The valu e of β may not remain the same over a period of time.

φ: This is the constant of proportionality , depicting the rate of growth of Total factor Productivity. Solow assumed Total Factor productivity to be exogenous. That is the growth rate is not affected by the national income or amount of capital.

TFP and labour supply are assumed to be exogenously determined.

We derived expressions (models) for the determinants; Labour supply, TFP, Capital Accumulation, and Production Function.

3.3 The Labour supply model

From the assumption bullet four, labour supply can be written mathematically as;

$$\frac{dL_{.}}{dt} = \tau L_{.} - \delta L_{.} = (\tau - \delta)L_{.} \qquad (3.3.1)$$

Where

τ : the rate of labour supply and

δ: rate at which people leave the labour sector (exit rate)

Equation 3.3.1 is a separable differential equation. Therefore it can solve by separating variables and integrating.

$$\frac{dL_{.}}{dt} = (\tau - \delta)L_{.}$$

Dividing both sides by L and integrating both sides, we get

$$\int \frac{dL_{.}}{dt} = \int (\tau - \delta)dt$$

$$In(L_{.}) = (\tau - \delta)t + C$$

Taking natural logarithm on both sides

$$L(t) = e^{(\tau - \delta)t + c}$$

$$L(t) = e^{c}e^{(\tau - \delta)t}$$

Let, $L_0 = e^c$, (Initial capital stock of the economy)

Therefore; $$L(t) = L_0 e^{(\tau - \delta)t} \qquad (3.3.2)$$

This represents the supply of labour model.

3.4 Total Factor Productivity Model (TFP)

Diewert (1992) define productivity as the ratio of output index to an input index. The model again assumed that TFP is endogenous. That is it grows proportional to itself. Hence;

$$\frac{dA}{dt} = \varphi A(t) \qquad (3.4.1)$$

$$\frac{dA_{.}}{dt} = \varphi A(t)_{.}$$

Solving by separating variables

$$\int \frac{dA_{.}}{dt} = \int \varphi\, A(t)_{.} dt$$

$$\int \frac{dA_{.}}{A_{.}} = \int \varphi dt$$

$$In(A_{.}) = \varphi t + c$$

It follows that

$$e^{In(A_{.})} = e^{\varphi t + c}$$

Let $A_o = e^c$ (Initial level of productivity)

And consequently, $A(t)_{.} = A_0 e^{\varphi t}$ (3.4.2)

Where

φ: parameter, indicating the growth of productivity.

A_0: *initial level of productivity*

3.5 Capital Accumulation Model

Let $\frac{dK}{dt}$ denote change of stock of Capital with respect to the time (t). In a closed economy total output, denoted by Y is equal to Total Income in a closed economy. In this case, the fraction of income saved which is defined as investment constitutes change in capital. This change in capital is denoted by **'s'** and therefore the amount saved will be equivalent to $sY(t)$, where $Y(t) = F(K, A, L)$ by equation 3.1.8.

In a closed economy without government involvement, we assume investment is equal to savings, it follows that;

$$\frac{dK}{dt} = sF(K, L, A) \tag{3.5.1}$$

From equation 3.1.8, we know that

$$Y(t) = F(K, A, L)$$

This implies that

$$\frac{dK}{dt} = sY(t) = I \tag{3.5.2}$$

This equation can be written as

$$\frac{dK}{dt} = sK^{\alpha}(AL)^{\beta} \tag{3.5.3}$$

Furthermore substituting $L(t)$ for $L_0e^{(\tau-\delta)t}$ and $A(t)$ for $A_0e^{\varphi t}$ in this equation yields;

$$\frac{dK}{dt} = sK^{\alpha}(A_0e^{\varphi t}L_0e^{(\tau-\delta)t})^{\beta}$$

Dividing both sides by K^{α} and integrating

$$\frac{dk}{K^{\alpha}} = s(A_0e^{\varphi t}L_0e^{(\tau-\delta)t})^{\beta}dt$$

$$\int K^{-\alpha}dk = \int s(A_0e^{\varphi t}L_0e^{(\tau-\delta)t})^{\beta}dt$$

$$\frac{K^{1-\alpha}}{1-\alpha} = s(A_0L_0)^{\beta}\int(e^{\varphi t}e^{(\tau-\delta)t})^{\beta}dt$$

$$\frac{K^{1-\alpha}}{1-\alpha} = \frac{s(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)t}}{(\varphi+\tau-\delta)\beta} + C$$

But $1-\alpha = \beta$

$$\frac{K^{\beta}}{\beta} = \frac{s(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)t}}{(\varphi+\tau-\delta)\beta} + C$$

Simplifying;

$$K^{\beta} = \frac{S(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)\beta t}}{(\varphi+\tau-\delta)} + C \quad \ldots\ldots\ldots. \qquad (3.5.3)$$

If $K(0) = K_0$ (initial capital stock)

$$K_0^{\beta} = \frac{S(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)\beta(0)}}{(\varphi+\tau-\delta)} + C$$

$$C = K_0^{\beta} - \frac{s(A_0L_0)^{\beta}}{\varphi+\tau-\delta} \qquad (3.5.4)$$

Replacing C in the equation 3.5.3, we have

$$K_{.}^{\beta} = \frac{s(A_0 L_0)^{\beta} e^{(\varphi+\tau-\delta)\beta t}}{(\varphi+\tau-\delta)} + K_0^{\beta} - \frac{s(A_0 L_0)^{\beta}}{(\varphi+\tau-\delta)}$$

$$K_{.}^{\beta} = \frac{s(A_0 L_0)^{\beta}(e^{(\varphi+\tau-\delta)\beta t} - 1)}{(\varphi+\tau-\delta)} + K_0^{\beta}$$

This then follows that the model for capital accumulation is given by

$$K(t) = \left[\frac{s(A_0 L_0)^{\beta}\left(e^{(\varphi+\tau-\delta)\beta t} - 1\right) + (\varphi+\tau-\delta)K_0^{\beta}}{(\varphi+\tau-\delta)}\right]^{\frac{1}{\beta}}$$

$$K(t)_{.} = \left[\frac{s\mu^{\beta}(e^{\omega\beta t}-1)+\omega K_0^{\beta}}{\omega}\right]^{\frac{1}{\beta}} \qquad (3.5.5)$$

Where

μ= $A_0 L_0$

ω= $\varphi + \tau - \delta$

In the next model, we replace the determinants with their appropriate expressions (models) in equation 3.1.9.

3.6 **The Production Model**

Using Robert Solow's production function, thus equation 3.1.8

$$Y(t) = K^{\alpha}(A_{.}L_{.})^{\beta}$$

And substituting equation 3.3.2, 3.4.2 and 3.5.5, we have

$$Y(t)_{.} = \left\{\left[\frac{s(A_0L_0)^{\beta}\left(e^{(\varphi+\tau-\delta)\beta t}-1\right)+(\varphi+\tau-\delta)K_0^{\beta}}{(\varphi+\tau-\delta)}\right]^{1/\beta}\right\}^{1-\beta} \left[A_0L_0e^{(\varphi+\tau-\delta)t}\right]^{\beta}$$

$$Y(t) = \left[\frac{s(A_0L_0)^{\beta}\left(e^{(\varphi+\tau-\delta)\beta t}-1\right)+(\varphi+\tau-\delta)K_0^{\beta}}{(\varphi+\tau-\delta)}\right]^{\frac{1-\beta}{\beta}} \left[A_0L_0e^{(\varphi+\tau-\delta)t}\right]^{\beta}$$

$$Y(t) = \left[\mu^{\beta}e^{\omega\beta t}\right]\left[\frac{s\mu^{\beta}\left(e^{\omega\beta t}-1\right)+\omega K_0^{\beta}}{\omega}\right]^{\rho} \tag{3.6}$$

Where

μ= A_0L_0

ω= $(\varphi+\tau-\delta)$

ρ=$\frac{1-\beta}{\beta}$

This is the production model of the economy.

3.7 The Growth Model

The growth model is represented by

$$\Delta Y = Y_{.}(t) - Y_{.}(t-1) \tag{3.7.1}$$

$$\Delta t = 1 \tag{3.7.2}$$

Using the equation 3.7.1 and substitution in time, t_2= t and t_1= t-1, we have

$$\Delta Y = \left[\mu^{\beta} e^{\omega\beta t}\right]\left[\frac{s\mu^{\beta}(e^{\omega\beta t}-1)+\omega K_0^{\beta}}{\omega}\right]^{\rho} - \left[\mu^{\beta} e^{\omega\beta(t-1)}\right]\left[\frac{s\mu^{\beta}(e^{\omega\beta(t-1)}-1)+\omega K_0^{\beta}}{\omega}\right]^{\rho} \quad \ldots\ldots (3.7.3)$$

Therefore the percentage change (Q) of Total output over time, thus the rate of growth becomes;

$$Q_t = \left[\frac{Y_{.}(t)-Y_{.}(t-1)}{Y_{.}(t-1)}\right] \times 100\% \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7.4)$$

This is the equation of the economy's growth rate at the time (t).

CHAPTER FOUR

MODEL APPLICATION

This chapter applied the real data on Ghana from 1990 to 2010. It computed the results using the capital Model, the Labour Model, the TFP Model, and the Production Model and finally computed the Growth rate using the Growth Model. These Models are applied to 1990 year's data as a base year, and the models are simulated for the periods 1990 to 2010. The chapter concludes with the discussion and summary of the results.

4.1. Computations for the periods 1990 to 2010

Computations for the year 1991

Labour

Using equation 3.2.2 and substituting the values for L_0 =5,962,958, n-δ=0.032, and t= 1, into the Matlab code L (t) = 5962958*exp (0.032*t) yields 6,156,900.

Total factor Productivity

Using equation 3.3.2 and substituting the values for A_0=100, g=0.032 and t= 1; into the Matlab code; A(t)= 100*exp(0.032*t); yields 103.25.

Capital

Using equation 3.4.5 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, β=0.3, ω=0.064 and t=1;into the Matlab code K(t)=((s*(μ^β)*(exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^(1/β) yields 893,680,000.00.

Production

Using equation 3.5.2 which is and substituting the values for K_0=846773730; s= 0.06; μ=596295800, ω=0.064 , β=0.3, ρ=(1-β)/β and t=1;into the Matlab code;

Y(t)=((μ^β)*exp(ω*β*t))*(((s*(μ^β)*exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^ρ; yields 3,172,500,000.00.

Growth Rate

Using equation 3.6.3, and substituting the values for Y(t-1) =3,266,886,838.00 and Y (t)=3,172,500,000.00, into the Matlab code; Q (t)= ((Y(t)-Y(t-1))/Y(t-1))*100; yields 2.89.

Computations for the year 1992

Labour

Using equation 3.2.2 and substituting the values for L_0 =5,962,958, n-δ=0.032, and t= 2, into the Matlab code L (t) = 5962958*exp(0.032*t) yields 6,357,100.

Total factor Productivity

Using equation 3.3.2 and substituting the values for A_0=100, g=0.032 and t= 2; into the Matlab code; A(t)= 100*exp(0.032*t); yields 106.61.

Capital

Using equation 3.4.5 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, β=0.3, ω=0.064 and t=2;into the Matlab code K(t)=((s*(μ^β)*(exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^(1/β) yields 943,000,000.00.

Production

Using equation 3.5.and substituting the values for K_0=846773730; s= 0.06; μ=596295800, ω=0.064, β=0.3, ρ= (1-β)/β and t=2; into the Matlab code;

Y(t)=((μ^β)*exp(ω*β*t))*(((s*(μ^β)*exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^ρ; yields 3,303,200,000.00.

Growth Rate

Using equation 3.6.3, and substituting the values for Y(t-1) =3,172,500,000.00 and Y(t)=3,303,200,000.00, into the Matlab code; Q(t)= ((Y(t)-Y(t-1))/Y(t-1))*100; yields 4,12.

Computations for the year 1993

Labour

Using equation 3.2.2 and substituting the values for L_0 =5,962,958, n-δ=0.032, and t= 3, into the Matlab code L(t)=5962958*exp(0.032*t) yields 6,563,800.

Total factor Productivity

Using equation 3.3.2 and substituting the values for A_0=100, g=0.032 and t= 3; into the Matlab code; A(t)= 100*exp(0.032*t); yields 110.08.

Capital

Using equation 3.4.5 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, β=0.3, ω=0.064 and t=3; into the Matlab code K(t)=((s*(μ^β)*(exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^(1/β) yields 994,860,000.00.

Production

Using equation 3.5.2 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, ω=0.064, β=0.3, ρ= (1-β)/β and t=3; into the Matlab code;

Y(t) = ((μ^β)*exp(ω*β*t))*(((s*(μ^β)*exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^ρ; yields 3,440,100,000.00.

Growth Rate

Using equation 3.6.3, and substituting the values for Y (t-1) =3,303,200,000.00 and Y(t)= 3,440,100,000.00, into the Matlab code; Q(t)= ((Y(t)-Y(t-1))/Y(t-1))*100; yields 4.14%.

	Total Labour Force		Total Factor Productivity		Total Capital ($ US)	
Year	**Actual**	**Calculated**	**Assumed**	**Calculated**	**Actual**	**Calculated**
1990	5,962,958	5,962,958	100	100.00	846,773,730.10	846,773,730.10
1991	6,170,246	6,156,900	100	103.25	1,044,279,211.00	893,680,000.00
1992	6,378,379	6,357,100	100	106.61	816,711,991.40	943,000,000.00
1993	6,630,762	6,563,800	100	110.08	1,418,975,139.00	994,860,000.00
1994	6,890,063	6,777,200	100	113.66	1,228,167,337.00	1,049,400,000.00
1995	7,154,239	6,997,600	100	117.35	1,364,512,238.00	1,106,700,000.00
1996	7,422,616	7,225,100	100	121.17	1,405,789,969.00	1,166,900,000.00
1997	7,695,831	7,460,100	100	125.11	1,640,841,947.00	1,230,200,000.00
1998	7,974,689	7,702,700	100	129.18	1,671,456,113.00	1,296,700,000.00
1999	8,260,518	7,953,100	100	133.38	1,578,001,319.00	1,366,600,000.00
2000	8,554,240	8,211,800	100	137.71	1,149,706,878.00	1,440,000,000.00
2001	8,808,936	8,478,800	100	142.19	1,439,998,829.00	1,517,100,000.00
2002	9,068,519	8,754,500	100	146.81	1,156,455,431.00	1,598,100,000.00
2003	9,331,979	9,039,200	100	151.59	1,748,749,261.00	1,683,100,000.00
2004	9,585,053	9,333,100	100	156.52	2,517,616,120.00	1,772,400,000.00
2005	9,852,131	9,636,600	100	161.61	3,109,129,779.00	1,866,200,000.00
2006	10,120,320	9,949,900	100	166.86	4,411,164,569.00	1,964,600,000.00
2007	10,376,027	10,273,000	100	172.29	4,953,021,277.00	2,067,900,000.00
2008	10,647,454	10,608,000	100	177.89	6,119,680,499.00	2,176,400,000.00
2009	10,925,982	10,952,000	100	183.68	5,122,231,687.00	2,290,200,000.00
2010	11,211,796	11,309,000	100	189.65	5,424,443,356.53	2,409,700,000.00

Table 1: Actual and Calculated Estimations

Sources of actual data: http://data.worldbank.org/country/ghana, MOFEP.

	(Production)		(Production)		
	GDP (constant 2000 US$)		GDP Growth Rate		**Periods**
Year	Actual	**Calculated**	Actual	**Calculated**	**(t)**
1990	**3,266,886,838.00**	3,266,886,838.00	**3.33**	3.33	0
1991	**3,439,438,121.00**	3,172,500,000.00	**5.28**	2.89	1
1992	**3,572,868,343.00**	3,303,200,000.00	**3.88**	4.12	2
1993	**3,746,152,457.00**	3,440,100,000.00	**4.85**	4.14	3
1994	**3,869,775,489.00**	3,583,500,000.00	**3.27**	4.17	4
1995	**4,028,916,869.00**	3,733,600,000.00	**4.02**	4.19	5
1996	**4,214,346,194.00**	3,890,900,000.00	**4.6**	4.21	6
1997	**4,391,195,243.00**	4,055,600,000.00	**4.2**	4.23	7
1998	**4,597,598,580.00**	4,228,200,000.00	**4.57**	4.26	8
1999	**4,799,892,758.00**	4,409,200,000.00	**4.55**	4.28	9
2000	**4,977,488,790.00**	4,598,800,000.00	**3.74**	4.3	10
2001	**5,176,588,342.00**	4,797,700,000.00	**4.18**	4.33	11
2002	**5,409,534,817.00**	5,006,200,000.00	**4.46**	4.35	12
2003	**5,690,830,628.00**	5,224,900,000.00	**5.34**	4.37	13
2004	**6,009,517,143.00**	5,454,300,000.00	**5.58**	4.39	14
2005	**6,364,078,886.00**	5,695,000,000.00	**5.86**	4.41	15
2006	**6,771,379,934.00**	5,947,600,000.00	**6.43**	4.44	16
2007	**7,208,793,173.00**	6,212,800,000.00	**5.7**	4.46	17
2008	**7,816,530,776.00**	6,491,100,000.00	**7.23**	4.48	18
2009	**8,180,601,366.00**	6,783,400,000.00	**4.14**	4.5	19
2010	**8,664,892,967.00**	7,090,300,000.00	**5.92**	4.52	20

Table 2: Actual and Calculated Estimations

Sources of actual data: http://data.worldbank.org/country/ghana, MOFEP.

	GDP/Production per labour		Capital per Labour		Rate of Change of Capital		Rate of Change of Labour	
Year	**($ US)**		**($ US)**		**(%)**		**(%)**	
	Actual	**Calculated**	**Actual**	**Calculated**	**Actual**	**Calculated**	**Actual**	**Calculated**
1990	547.86	547.86	142.01	142.01	-	-	-	-
1991	557.42	515.28	169.24	145.15	23.32	5.54	3.48	3.25
1992	560.15	519.61	128.04	148.34	-21.79	5.52	3.37	3.25
1993	564.97	524.10	214.00	151.57	73.74	5.50	3.96	3.25
1994	561.65	528.76	178.25	154.84	-13.45	5.48	3.91	3.25
1995	563.15	533.55	190.73	158.15	11.10	5.46	3.83	3.25
1996	567.77	538.53	189.39	161.51	3.03	5.44	3.75	3.25
1997	570.59	543.64	213.21	164.90	16.72	5.42	3.68	3.25
1998	576.52	548.92	209.60	168.34	1.87	5.41	3.62	3.25
1999	581.06	554.40	191.03	171.83	-5.59	5.39	3.58	3.25
2000	581.87	560.02	134.40	175.36	-27.14	5.37	3.56	3.25
2001	587.65	565.85	163.47	178.93	25.25	5.35	2.98	3.25
2002	596.52	571.84	127.52	182.55	-19.69	5.34	2.95	3.25
2003	609.82	578.03	187.39	186.20	51.22	5.32	2.91	3.25
2004	626.97	584.40	262.66	189.90	43.97	5.31	2.71	3.25
2005	645.96	590.98	315.58	193.66	23.49	5.29	2.79	3.25
2006	669.09	597.75	435.87	197.45	41.88	5.27	2.72	3.25
2007	694.75	604.77	477.35	201.29	12.28	5.26	2.53	3.25
2008	734.12	611.91	574.76	205.17	23.55	5.25	2.62	3.26
2009	748.73	619.38	468.81	209.11	-16.30	5.23	2.62	3.24
2010	772.84	626.96	483.82	213.08	5.90	5.22	2.62	3.26

Table 3: Actual and Calculated Estimations

Sources of actual data: http://data.worldbank.org/country/ghana, MOFEP.

4.2 Discussion of Results

4.2.1 Lessons from Literature

The following bullet points were important information from the literature that helped in arriving at the models for analysis:

- The economists use GDP (Gross Domestic Product) to measure economic performance.
- It was assumed by literature that labour grows proportional to itself.
- It was assumed there were three factors of production in the economy, namely Labour, TFP (Total Factor Productivity) and Capital.
- It was assumed that change of capital is equal to savings, which is also referred to as Investment.
- It was assumed from literature that TFP grows proportional to itself just like labour supply.
- It was again assumed that neither labour nor capital or TFP can be reduced to zero.
- TFP is made up of change in the level of technology, level of education, efficiency of labour, etc.

4.2.2 Data Analysis

The table containing the data and the Matlab codes can be found at the Appendix. The source of actual data was: http://data.worldbank.org/country/ghana, Ghana Statistical Service, MOFEP (Ministry of Finance Economic Planning), the African Development Indicator 2007 CD Rom of the World Bank Group.

From Table 1 it was observed that the actual data for labour over the periods 1990 to 2010 seem to be building up over the period. But, the calculated figures of labour seemed to be growing faster than that of the actual labour supply.

The calculated figures for TFP seem to be growing over the period, indicating that in the long run when growth rate of capital is constant, growth in GDP may be as a result of change in the level education, change of technology, etc. This was not the case in Cobb-Douglas model, where TFP was assumed to be constant.

Both actual and calculated figures for capital were evenly spread from 1990 to 2003 and uneven from 2004 to 2010. This is a reflection of real life situation. It should be understood that capital acquisition and usage could not be consistent as it depends on the changing needs of the economy as it has pointed out earlier on by Baafi (2010). The calculated figure for the capital over the period seemed to be building up with time.

The actual Production (GDP) figures were evenly spread over the period. They were higher in certain periods and lower in other periods. The calculated figures for production consistently build up over the periods. It increases year after year; highest in the last year under consideration, 2010 with a figure of 7,090,300,000.00.

Considering the production/GDP growth rate, the following findings were apparent. The growth rates were evenly spread for most of the years under consideration, especially, 1997, 2001, and 2002.

The actual average growth rate from 1984 to 2010 according to http://data.worldbank.org/country/ghana and African Development indicator 2007 CD Rom for Ghana was 4.5% which is relatively close to the calculated average growth rate of 4.21%.

The graphs below show these trends.

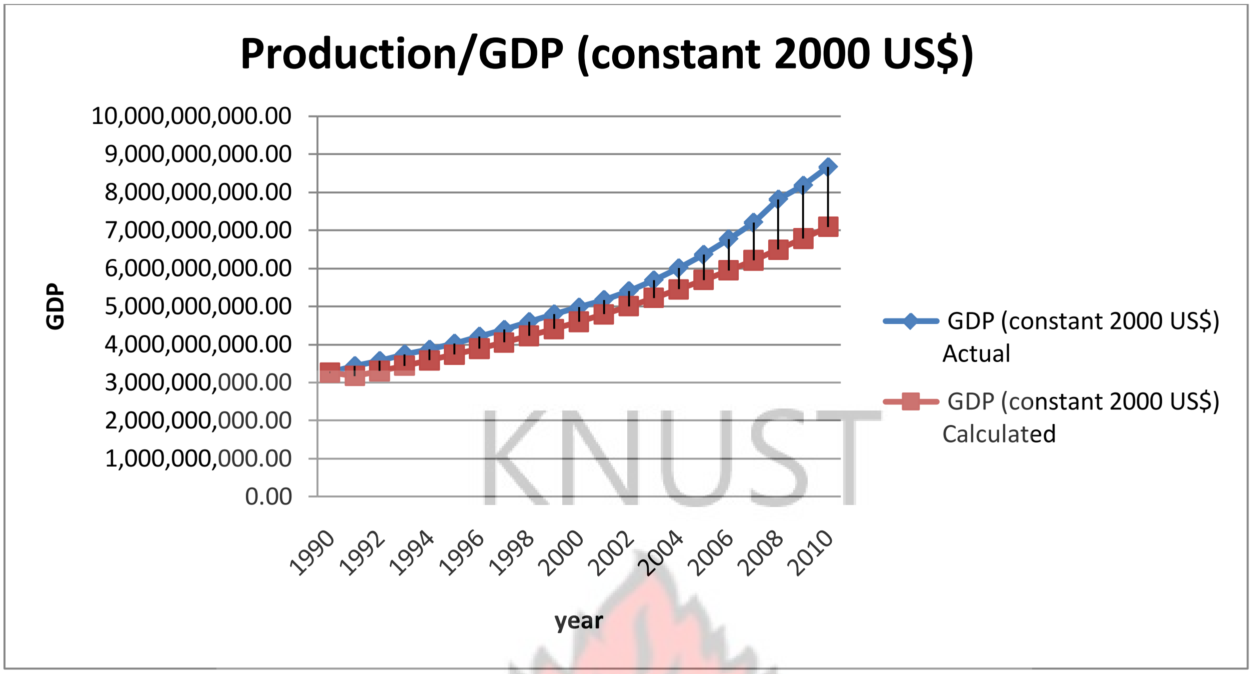

Figure 1: Production/GDP (constant 2000 US$

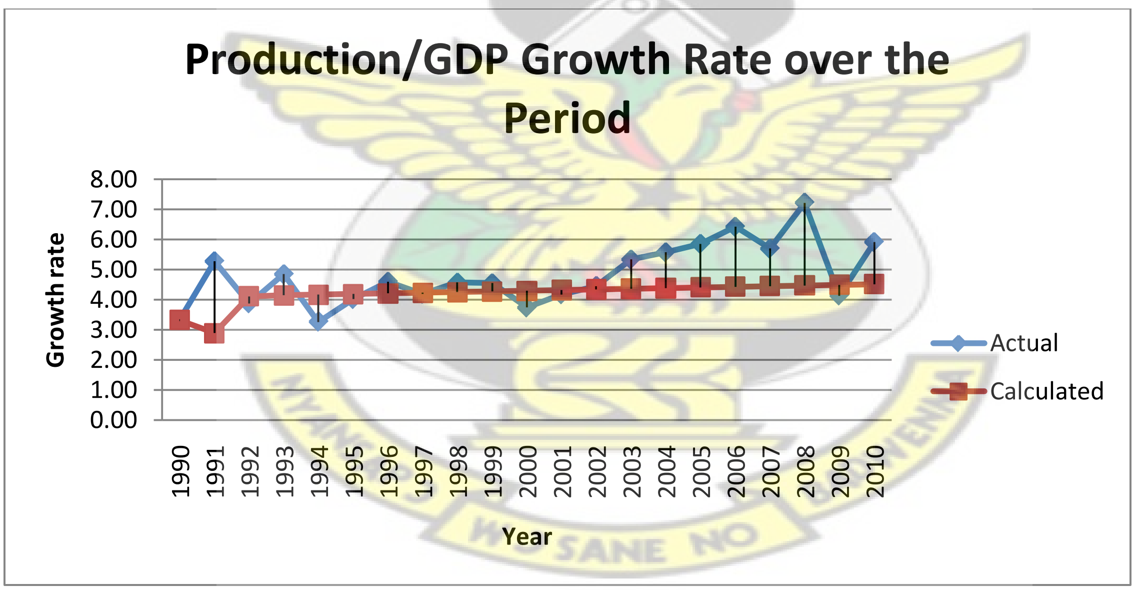

Figure 2: Production/GDP Growth Rate

Figure 1 depicts the production/GDP for the economy, which builds up from year to year. The actual production values were always above that of the calculated throughout the periods under consideration.

Figure 2 depicts the rate at which production builds up from year to year. From the graph it is apparent to see that growth rates were positive throughout the period under consideration.

It was also interesting to know that the difference between the growth rates were marginal. The actual growth rates for some periods were always above that of the calculated throughout the periods, except 1992, 1994, 2000 and 2009.

The actual average growth rate over the period was 4.5% as compared to the calculated value of 4.21%. From the graph the computed figures appear in red line. This is so because the variances between the rates were very small. This showed that the growth rate of Ghana has been slow as was noted by Aryeetey and Fosu (2004).

Production per Labour

The actual production per labour started from a high figure of $ 547.86 per labour in 1990 and kept increasing throughout the period..

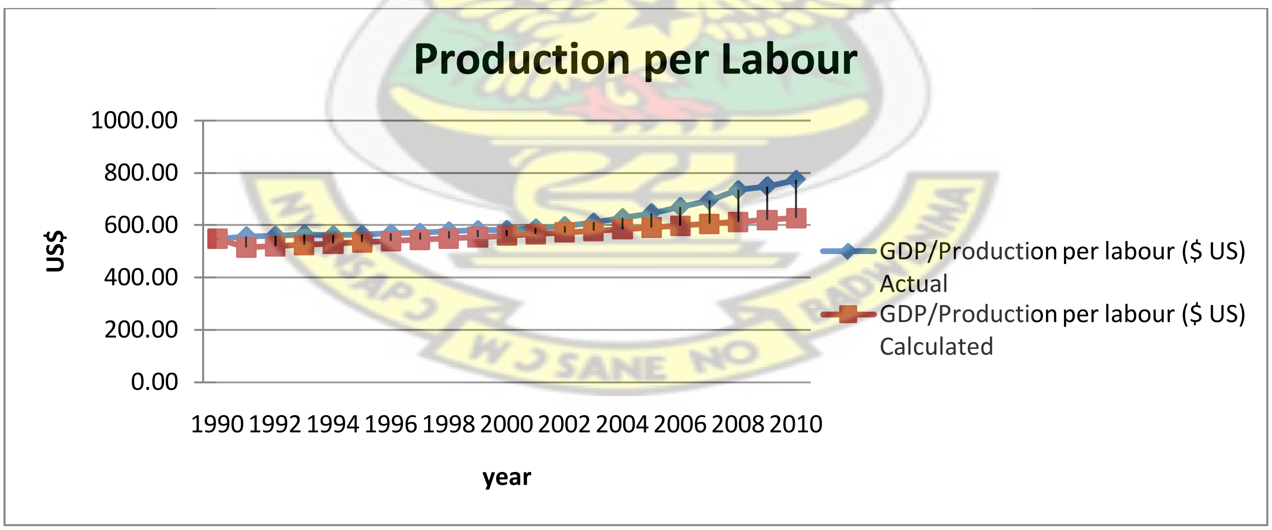

Figure 3: Production per Labour

The calculated production per labour started at $547.86 and reduced to $515.28 in 1991 and started increasing from1993. The variances between the figures were very small from 1990 t0 2004 and afterwards the actual figures started growing faster than the calculated.

Capital per Labour

Actual capital per labour was uneven over the periods under consideration as is depicted on the graph below. It increases and decreases throughout the period. It is neither increasing nor decreasing throughout the periods. The figure ranges between $142.01 per labour in 1990 and $483.82 in 2010.

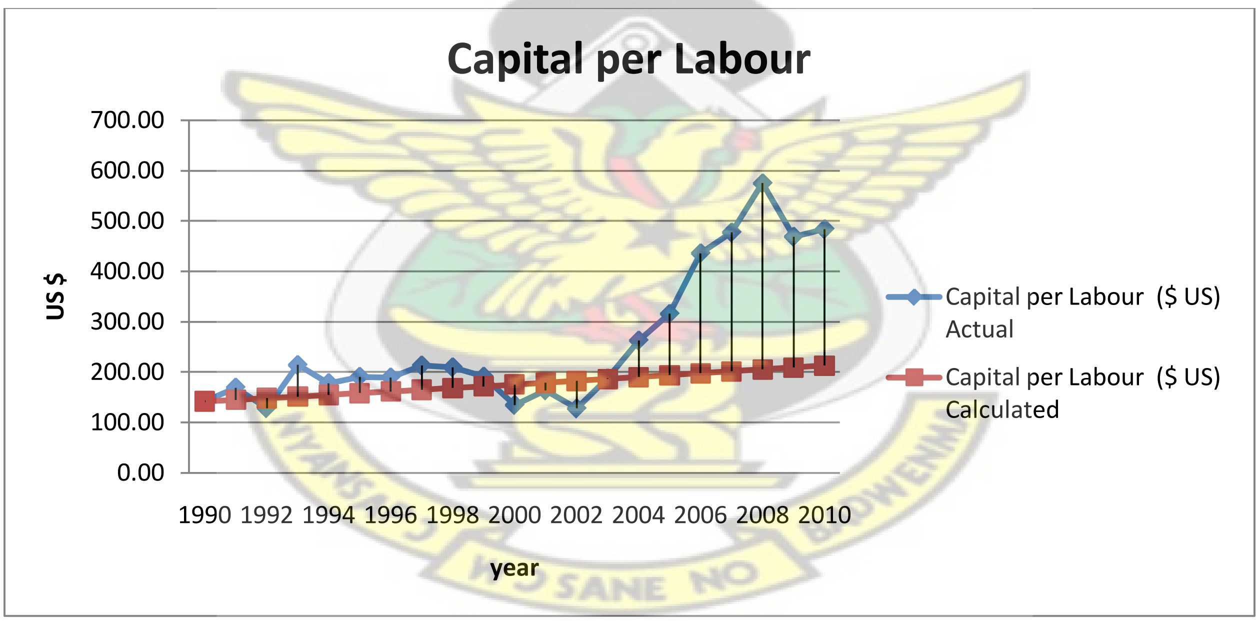

Figure 4: Capital per Labour

The calculated capital per labour started with a figure of $145.15 in 1991 grew steadily over the period to record $213.08 per labour in 2010.

Percentage change in Capital

As is depicted in figure 5, the actual percentage change was uneven over the periods. It ranges between a low figure -27.14 percent in 2000 to as high a figure of 73.74 percent in 1993. There were fluctuations in the rate of change of capital over the periods.

The calculated change in capital ranges between 5.22 percent in 2010 and 5.54 percent in 1990. But generally the percentage change lies between 0 and 100% making the graph smooth out around the horizontal axis.

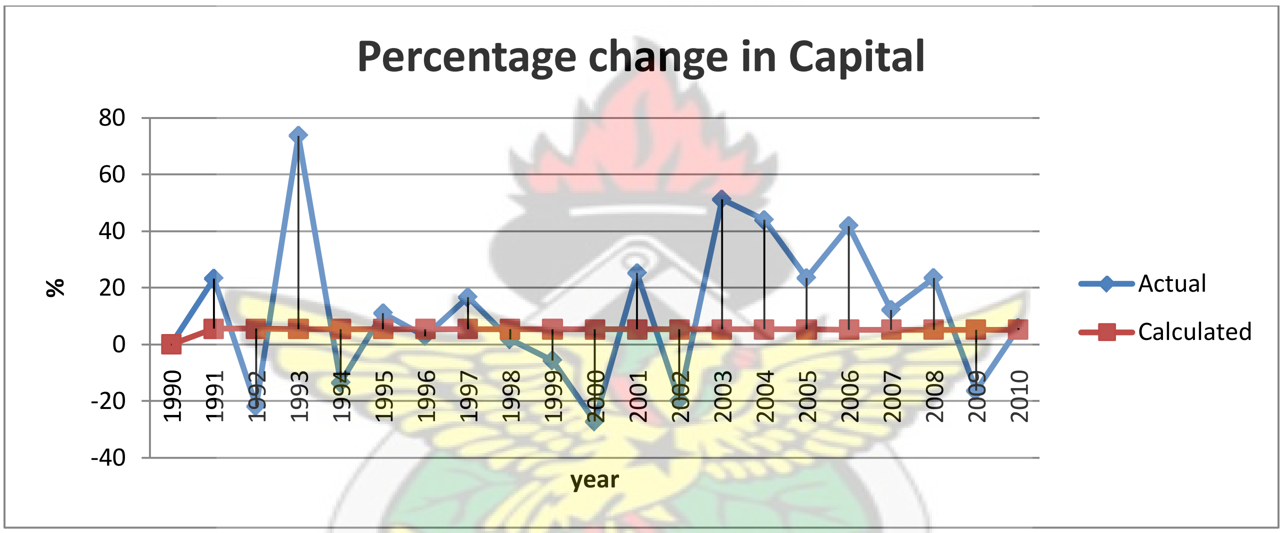

Figure 5: Percentage change in Capital

Rate of change of Labour Supply

It is amazing figure 6 showed similarities between actual and calculated rate of change in Labour Supply. The only difference was that the calculated rates of change were always higher for the periods 2001 to 2010 and vice versa for the periods before 2001.

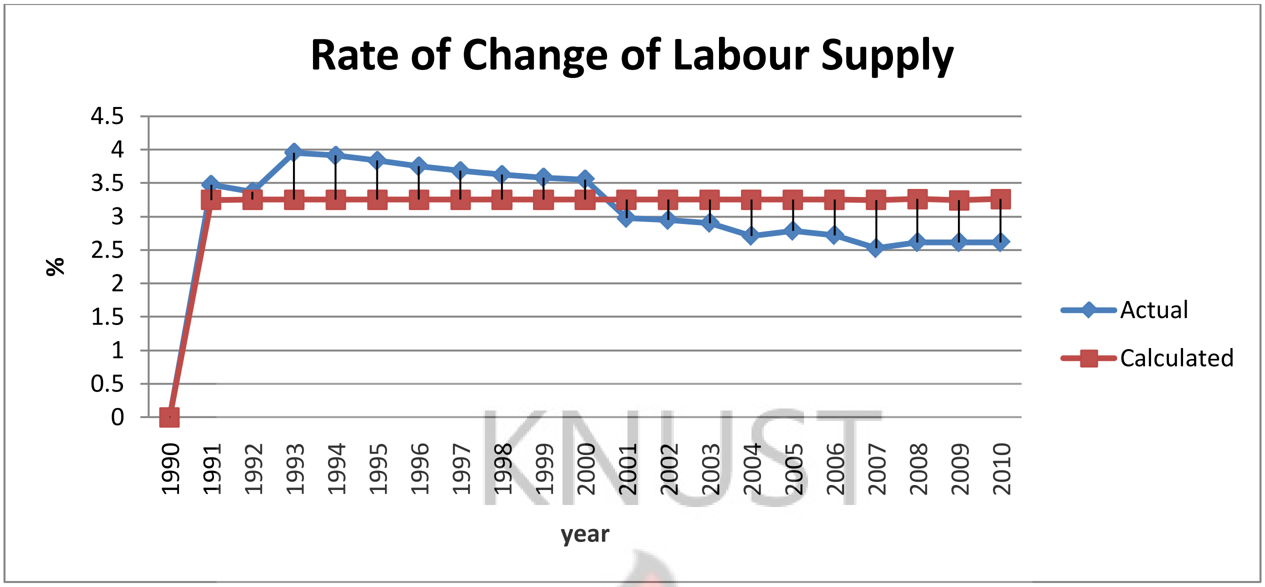

Figure 6: Rate of change of Labour Supply

Though the changes were uneven, the two lines showed similar patterns. The actual figure ranges between 1% and 5% whilst the calculated ranges between 2% and 5%.

Also, from appendix D, we observed that the correlation coefficient between the actual growth rates and calculated was 0.298, indicating that the two variables are correlated, but the strength of correlation is weak.

4.3 Summary of results

The aim of the research was to establish an appropriate economic growth model to be studied mathematically to measure economic growth.

The principal determinants of economy's growth were the labour supply, the active working population, Capital, thus the non-human elements that adds to labour for productivity and finally the level of technological advancement in the economy.

The conditions surrounding economy's growth would be an appropriate environment for both capital accumulation and healthy labour force coupled with the most appropriate technology.

The main findings of the study are summarized below:

The short-run relation results presented show that, there is a positive relation between the factors of production and real GDP, both for actual and the calculated. Thus as elements used in production increases, we expect the GDP also to increase.

The results on total factor productivity show that, there is a positive relation between real GDP growth and TFP. That is in the long run GDP growth may be as result of change in the level of technology, level of education etc Ceteris paribus.

The long-run relation results show that, there is a positive relationship between real GDP growth and capital, proxy by Gross domestic fixed capital formation. The results indicated that a percentage change in capital stock lead to a 0.4 percentage change in real GDP growth.

The long run relations results show a negative relation between GDP and labour proxy by labour force. The results indicate that, real GDP growth falls by -1.2 percent as labour force increases by one unit. Thus in the long run when GDP is constant or decreasing, a small increase in labour supply will reduce the GDP growth rate.

Furthermore, the correlation coefficient between the actual data and calculated was 0.298, which depicts a weak correlation between the two variables.

Finally, comparing the outcome of our model to the actual data gathered from the Ghana Statistical Service, World Bank Group and MOFEP. The results showed a very close relationship between the actual (4.5%) and calculated (4.25%) average growth rate over the periods 1990 to 2010.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Economic growth is defined as the increase in the amount of goods and services produced by an economy over time. Economy's performance is generally measured using GDP (Gross Domestic Product) by Economists. Being able to discern and measure progress more comprehensively that with GDP, is a key prerequisite for improved decision making.

The principal determinants of the economy's growth as used in this study were capital, labour and Total Factor productivity (T.F.P).

According to Slavin (2005), "growth is the quantitative increase in physical scale while development is qualitative improvement or the unfolding of potentiality. An economy can grow without developing, or develop without growing, or do both or neither".

The determinants have the following effects on the growth of the economy; α and β are the output elasticities of labour and capital respectively are constants determined by available technology. Output elasticity measures the responsiveness of output to a change in levels of either labour or capital used in the production, ceteris paribus. Cobb-Douglas model assumed that $\alpha = 0.7$ and $\beta = 0.3$, which was one of the assumptions used in the models. For example if: $\alpha = 0.2$, a 1% increase in labour would lead to approximately a 0.2% increase in output. Further, if: $\alpha + \beta = 1$, the production function has a constant returns to scale. That is, if L and K are each increased by 20%, Y increases by 20%. If: $\alpha + \beta < 1$, returns to scale are decreasing, and if: $\alpha + \beta > 1$, returns to scale is increasing. Assuming perfect competition, α and β can be shown to be labour capital's share of output.

Solow simplify that an economy-wide production function as

$Y = K^{\alpha}(A_{.}L_{.})^{\beta}$, as indicated in equation 3.1.8.

Solow assumed that TFP is proportional to itself, unlike Cobb-Douglas, which estimated TFP to be equal to 1 in their function. Solow again argued that, since technology changes over time and increases in the level of education also changes over time, it was prudent to allow TFP to also grow proportional to it.

A little work needs to be done on the parameters α, β from time to time in the production function.

The appropriate model is the model that answers all the questions of the system being described by the models. For example our production model of the economy's growth at equation 3.7.3, could be said to describe the system of economy's growth if it computes the intended output of the economy appropriately.

Finally, comparing the outcome of our model to the actual data we gathered from the Ghana Statistical Service, World Bank Group and MOFEP. The results showed a very close relationship between the actual (4.5%) and calculated (4.21%) average growth rate over the periods 1990 to 2010. This model predicted the growth pattern very well and it is our thinking that a little work needs to be done on the parameters to put the results into perspective. Therefore, Solow's model predicts the growth pattern of Ghana's economy.

5.2 Recommendations

Based on the results of the present study, the following recommendations were made:

To understand what is happening in the economy and be able to compare nation's well-being, there is the need for an exact solution to a model that gives a reasonable good approximation of the economy's growth.

As the economy strives to achieve middle income status, savings and investment rates should be encouraged to increase the parameter, s. This will lead to an increase in the capital stock and thus shift the rate of growth of real GDP from its current average of 4.5% to about 9% or higher. The study finds a significant positive relationship between real GDP and the level of the capital stock in both the short-run dynamic and the long-run static models.

Human development should be a core aim of every government as this will improve the quality of labour force to ensure that additional labour force contributes positively to GDP through Total factor Productivity.

More resources should be dedicated to the empirical studies of growth determents especially growth rate of various sectors of the economy and issues of convergence to establish the results of this study.

The model assumed that labour grows exponentially, though in reality it does not. The research focused on the quantitative aspects of growth modelling and therefore much emphasis was not placed on qualitative aspects.

We recommend that the students and researchers can extend this work to non-constant population.

Also, we recommend students and researchers can extend this work to include more than three determinants of production.

In light of these limitations of the model, it is recommended that further work is done on the parameters in order to bring the model into perspective.

References

1. AfDB/OECD (2003). “African Economic Outlook” Abidjan, Paris.

2. Adu G. (2008) Economic Growth in Ghana: Convergence and Determinants, Department of Economics, Kwame Nkrumah University of Science and Technology, Ghana.

3. Aryeetey, E. and A.K. Fosu (2005), Economic Growth in Ghana**:** 1960-2000, Draft Chapter for AERC Growth Project Workshop, Cambridge.

4. Aryeetey E., Fosu A., and Bawumia M.(2001). “Explaining Economic Growth Performance. The case for Ghana. African Economic Research Consortium Research Project.

5. Barnet R. A., Ziegler and Byllen K E (2005). Precalculus: Graphs and Models, 2^{nd} ed. McGraw-Hill Companies Inc. New York.

6. Bank of Ghana (2006). “Quarterly Economic Bulletin, October-December 2005” Prepared by the Research Department, and published by IDPS Department.

7. Barro, R. J. and X. Sala-i-Martin (1992). Convergence. *Journal of Political Economy*, 100 (2), 223-252.

8. ^Barelli, Paulo and Samuel de Abreu Pessoa (2003) "Inada Conditions Imply That Production Function Must Be Asymptotically Cobb-Douglas" Economics Letters 81(3) 361-

9. Barro, R.J. and Sala-i-Martín, X. 1995. Economic growth. McGraw-Hill, New York.

10. Barro, R.J. and X. Sala-i-Martin, (1995), *Economic Growth*, New York, McGraw-Hill.

11. Barro, R.J. and X. Sala-i-Martin, (2003), Economic Growth, 2nd Edition, MIT Press, Cambridge.

12. Buttrick, J. 1958. A Note on Professor Solow's Growth Model. The Quarterly Journal of Economics, Vol. 72, No. 4. 633-636.

13. Caraiani, P (2007). “Modeling the Economic Growth in Romania with Solow model”, Romania Journal of Economic Forecasting: 047, P47.

14. Cobb, C., and Douglas, P. (1928). "A Theory of Production." The American Economic Review 18(1): 139-165.
15. Fischer Stanley, Ratna Sahay and Carlos Vegh. 1998. ”From Transition to Market: Evidence and Growth Prospects”, *International Monetary Fund Working Paper 52/98*, Washington D.C.

16. Fosu, A.K (2001). “Economic Fluctuations and Growth in Sub-Saharan Africa: The Importance of Import Instability,” Journal of Development Studies, Vol. 37, No. 3, pp. 71-84.

17. Fosu, A.K (2002). “Political Instability and Economic Growth: Implications of Coup Events in Sub-Saharan Africa,” American Journal of Economics and Sociology, Vol. 61, No. 1, pp.329-48.

18. Fosu, A.K. (1992). “Political Instability and Economic Growth: Evidence from Sub-Saharan Africa,” Economic Development and Cultural sChange, Vol. 40, No. pp. 829-41.

19. Ghana Statistical Service. “2000 Population and Housing Census, Summary Report of Final Results, March 2002.

20. http//en. Wikipedia.org/wiki/Economic-growth, (Nov, 2010)

21. http://www.doingbusiness.org/economyrankings/,(Nov, 2010)

22. Inada, Ken-Ichi (1963) "On a Two-Sector Model of Economic Growth: Comments and a Generalization," The Review of Economic Studies, 30(2): 119-127

23. Institute for Social Statistics and Economic Reforms (1992), The State of the Ghanaian Economy.

24. Institute for Social Statistics and Economic Reforms (1994), The State of the Ghanaian Economy.

25. Institute for Social Statistics and Economic Reforms (1996), The State of the Ghanaian Economy.

26. Institute for Social Statistics and Economic Reforms (2000), The State of the Ghanaian Economy.

27. ISSER 2009, State of the Ghanaian Economy Report 2008, University of Ghana, Legon.

28. Levine Ross and David Renelt. 1992. “A Sensitivity Analysis of Cross-Country Growth Regressions”, *American Economic Review 82*: 942-963.

29. Mankiw, N. G., Romer, D. and D. N. Weil (1992), A Contribution to the Empirics of Economic Growth, Quarterly Journal of Economics, 107.

30. Mankiw, N.G., D. Romer, and D. Weil, (1992), “A Contribution to the Empirics of Economic Growth”,

Quarterly Journal of Economics, 107: 407-38.

31. Mansouri, B. (2005), The Iterative Impact of FDI and Trade Openness on Economic Growth: Evidence from Morocco, Economic Research Forum, Cairo.

32. O'Connell, S.A. and B.J. Ndulu (2000), "Africa's Growth Experience: A Focus on Sources of Growth." Framework Paper for AERC Collaborative Research Project on "Explaining African Economic Growth Performance."

33. Romer, P. M. (1994). "The Origins of Endogenous Growth." Journal of Economic Perspectives. vol.8, 1, 3-22.

34. Romer, Paul M. "The Origins of Endogenous Growth," Journal of Economic Perspectives, 8, 1994, 3–22

35. Smith, Adam, An Inquiry into the Nature and Causes of the Wealth of Nations (Chicago: Encyclopedia Britannica, 1990 [1776]).

36. Slavin S. L.(2005), Economics, 7th ed. McGraw-Hill Companies New York.

37. Solow Robert. 1956. "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics 70: 65-94*.

38. Solow, R. M. (1957), "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics 39: 312-320*.

39. Solow, R. M. 1956. A Contribution to the Theory of Economic Growth. The Quarterly Journal of Economics, Vol. 70, No. 1. 65-94.

40. Swan, T. W. (1956). Economic Growth and Capital Accumulation. *Economic Record*, 32 (63), 334-361.

41. *Total Factor Productivity Measures for the G7 Countries*, Jim Malley, Anton Muscatelli, And Ulrich Woiteck, University of Glasgow, Department of Economics, http://www.gla.ac.uk/economics/TFP/.

42. Turturrean, C.(2007),The Econometric Modeling of the Dynamics of the Romanian Economic Growth. Department Of Statistics, Faculty of Economy & Business Administration. Romania Journal of Economic Forecasting. Alexandru Ioan Criza University.

43. Van den Berg H. (2001), Economic Growth and Development, McGraw-Hill Companies, Inc, New York, US.
44. Webster T. J. (2003), Managerial Economics: Theory and Practice, Academic Press, UK.

45. World Bank. 2004. “World Development Indicators, CD-ROM.”

46. World Development Report (1996), Washington, D.C.: World Bank,

47. www.homepage.newschool.edu/het//home.htm.(Sept., 2010)

Appendices

Appendix A: Matlab Codes for the Models;

Matlab code for Total Factor productivity

```
for t=1:20
A(t)= 100*exp(0.032*t);
   disp ([A(t)])
end;
```

Matlab code for the Growth model

```
for t=1:20
     Q(t) = ((Y(t)-Y(t-1))/Y(t-1))*100
    disp ([Q(t)])
end;
```

Matlab code for Labour Supply

```
for t=1:20
   L(t)=5962958*exp(0.032*t);
   disp ( [L(t)] )
end;
```

Matlab code for Capital accumulation

```
β =0.3;
s= 0.06;
μ=596295800;
w= 0.064;
for t= 1:20
   K(t)=((s*(u^ B)*(exp(w*B*t)-1)+w*(846773730.1^B))/w)^(1/B);
   disp ( [K(t)])
end;
```

Matlab code for Production/GDP

```
B =0.3;
s= 0.06;
u=596295800;
w= 0.064;
p= (1-B)/B;
for t= 1:20
   Y(t)=((u^ B)*exp(w*B*t))*(((s*(u^ B)*exp(w*B*t)-1)+w*(846773730.1^B))/w)^p;
   disp ( [Y(t)])
end;
```

Appendix B: Correlation Coefficient.

		Calculated value from the model	Actual values from MOFEP, World bank Group.
Calculated value from the model	Pearson Correlation	1	.298
	Sig. (2-tailed)		.189
	N	21	21
Actual values from MOFEP, World bank Group.	Pearson Correlation	.298	1
	Sig. (2-tailed)	.189	
	N	21	21

Table 4: **Correlations**