KWAME NKRUMAH UNIVERSITY OF SCIENCE AND

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Longevity Risk Hedging And Annuity Pricing for Re-insurance: A case study of Social Security and National Insurance Trust (SSNIT)

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc. degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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DEDICATION

I dedicate this work to my parents Mr & Mrs. Roger and Mad. Akua Konadu and my siblings for their strong support and assistance throughout my study. I also dedicate this work to my wife to be Miss Gifty Asamoah for the moral support and to the fellows of Msc. Actuarial Science.

ABSTRACT

Predicting mortality trend and hedging of longevity risk in recent times has gained attention at a period when life expectancies are increasing unexpectedly. Due to advancement in technology, hygienic and medical practices, life insurance companies may achieve more profit because their liabilities will reduce since death benefit payments are paid whilst the annuity insurance scheme such as pensions may incur losses because of longevity improvement. The study seeks to investigate if there exists longevity risk pose to SSNIT and how they can reinsure the pensioners after the guarantee period of 15 years. Data collected from SSNIT covered the period 1991 – 2013 and Lee Carter model was used to predict future male mortality trend of the study. The study proposed that, SSNIT can insure the pensioners by acquiring life annuities from private life companies and individual longevity risk can be hedged through acquiring a life annuity from private life insurance companies since using Lee Carter model has proved that there is improvement in longevity.

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LIST OF ABBREVIATION

SSNIT ....Social Security and National Insurance Trust

WHO ....World Health Organisation

GHS .... Ghana Health Service

OECD .... Organization for Economic Cooperation and Development

LCfit .... Lee Carter fit model

OLS ....Ordinary Least Squares

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CHAPTER 1

INTRODUCTION

1.1 Background of the study

Due to advancement in technology and medical practices, life insurance companies may achieve more profit because their liabilities will reduce since death benefit payments are paid whilst the annuity insurance scheme such as pensions may incur losses because of longevity risk. In Ghana, life expectancy at birth has increased from 54 in 1988 to 60 in 2008 according to Ghana Health Service(GHS) fact and figures 2010 and 62.8 in 2015 predicted by WHO(2015). The increase in the life expectancy is called the longevity. This increment in the life expectancy suggests that pension payments have to be paid for a very longer period than the expected initially.

Longevity risk is a type of risk where the person's life expectancy varies from the expected according to MacMann et al.(2006). It has become a serious financial difficulty for these insurance companies and pension schemes (refer to Olivieri and Pitacco 2003, Cossette et al 2007, and Hari et al 2008; Yang et al., 2010). These companies have a problem with reserves and pricing an annuity product for a reason without bearing in mind enough mortality expansion.

Normally, actuaries use two approaches in view of the mortality expansion. Firstly, they make the assumption that mortality rates decrease with a certain number of percentage which is constant for each year for all ages. Secondly, the actuaries multiply a fixed life by a constant percentage. In this approach, the problem of mispricing is serious since it has a concept of fixed mortality and doesn't allow annual mortality improvement.

Countries like Taiwan, Japan and Korea have these mispricing problems since they have an official fixed life or annuity tables where the actuaries use to price their annuity products.

These official annuities and life tables are normally outdated since the data they used to setup the table might be many years ago. These tables are purposely used for valuation to calculate premiums since there is a lack of actual experience data. The best way of solving this problem is to setup a stochastic mortality models to aid life and annuity companies to hedge against longevity risk.

Several types of mortality models have been studied, discussed and proposed. (Carter and Lee 1992, Brouhns et al., 2002, Renshaw and Haberman, 2003, Koissi et al., 2006, Blake, 2006, Melnikov and Romaniuk, 2006, Cairns et al., 2006, Cairns et al., 2009 and Yang et al., 2010). Some of these models are hard to apply in practice because of market competition and its related challenges. Insurance companies will sell their annuity products using the unoriginal mortality rates since it would be too cheap or expensive in the market competition because other companies currently employ the idea of fixed mortality table in pricing their products. Apart from constructing stochastic mortality models, another best way is the capital market solution with three possible ways: Survivor swaps, Mortality securitization and Survivor bonds. A research by Huang and Wang, (2010) revealed that insurance companies could allocate their exposures to the capital markets which have more competitors and funding.

This aim of study is to use natural hedging in solving the problems in mispricing for longevity risk. There are few studies on natural hedging and other researchers investigated the impact of mortality deviations on annuities and life insurance separately. (Frees et al., 1996, Marceau and Gaillardetz, 1999, Milevsky and Promislow, 2001 and Cairns et al., 2004). Natural hedging is also another strategy actuaries used to get a solution to the mispricing problems about longevity risk. There is an optimal allocation of their life insurance and annuities to hedge against longevity risk. The natural hedging approach does not require market liquidity to hedge longevity risk and hence does not encourage transaction cost in large amount as in the case of mortality securitization.

This strategy is more practical and very suitable to implement internally to the insurance or the private pension funds.

Pension is a regular payment of a static sum of amount to an individual after his or her retirement from active service. There are various types of pensions which comprise Defined Contribution Plan, Defined Benefit Plans, Social and state, Disability etc. Defined Benefit (DB) plan is a plan which assures a certain amount of payment at retirement-using a fixed formula which is determined by the number of years as a member in the plan and member salary. A Defined Contribution plan is a plan which pays out at retirement due to the amount of money the member contributed and it also depend on the investment package used.

1.2 Problem statement

Pension scheme such as social and state schemes see increasing life expectancy at birth as a threat to their scheme since they would be paying a regular fixed amount to their members as long as the members survived after retirement.

The span of time people are anticipated to live in most OECD countries has received an increment by 25-30 years all through the last century. This has become a global phenomenon which confirming Ghana's life expectancy at birth increased from 54 in 1988 to 60 in 2008 according to GHS fact and figures 2010 and 62.8 in 2015 predicted by WHO, 2015. The increasing trend as depicted in Figure 1 above poses a risk to these pension companies in that there is possibility in future they might not be able to meet their financial obligations to the pensioners. The problem is how do the pension companies mitigate this risk of increasing life expectancies? The answer is the focus of this study.

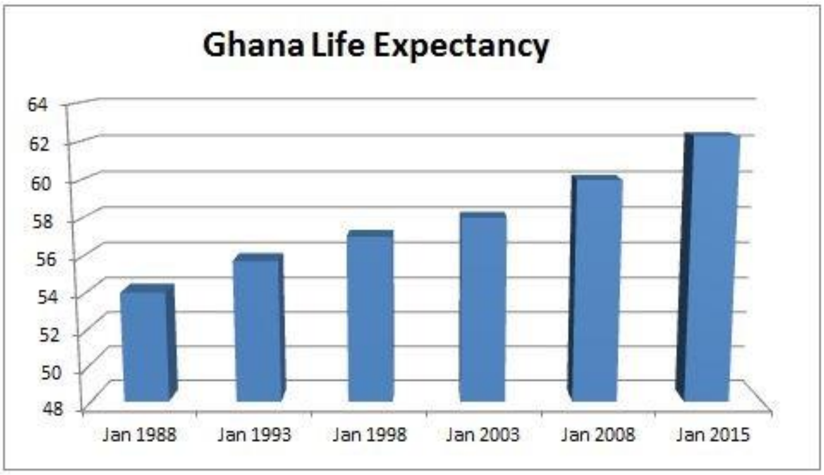

Figure 1.1: Life Expectancy

1.3 Objectives of the study

The main objective of this study is to project the mortality profile for male pensioners using lee carter model. The selection for male pensioners is due to lack of data on the part of the females. This projected mortality rate will then use in the natural hedging approach to hedge the longevity risk.

1.3.1 Specific objectives

1. To project mortality profile for male pensioner using lee carter model
2. To estimate annuity pricing for hedging longevity risk.

1.4 Justification of the study

The scope of this study is to forecast expected future mortality rate for male pensioners that are under Ghana's biggest pension scheme - SSNIT. This pension scheme which is highly exposed to longevity risk and the predicted mortality would make for better decision making and avoidance of mispricing problems. The research will explore data collection and analysis, using lee carter models to estimate future mortality rate.

1.5 Limitation of the study

This study focuses on addressing longevity risk challenges faced by pension funds using SSNIT as a case study. However, the study is limited in scope since other pension schemes are not covered. This is due to time and resource constraints. The effect of this limitation is however not to distort the eventual outcome of the study as SSNIT remains the single largest pension fund in the country.

1.6 Organization of the study

The study will be structured into five chapters as follows: Chapter One is the introduction which covers the associated background of the study.

In chapter two, relevant literature on Pensions (SSNIT) and natural hedging techniques in that respect was reviewed and the overview of the company under study was looked at. Related studies selected from various books and journals have been presented.

In chapter three, detailed description of the methodology employed discussed. This included the research questions, research instrument, secondary data and methods of analyzing the data. Based on this, the mortality rate will be examined and predicted. Chapter four gives the description of the Lee Carter model of the data collected, mainly devoted to the presentation of data analysis and discussion based on the information and responses received from respondents.

Chapter five finally gives the conclusions and recommendations for the natural hedging approach.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter presents the literature review on what is currently becoming a global impact on our pension schemes. Longevity risk is what most OECD countries are trying to hedge. This review will focused on the hedging longevity risk for pension companies, market development of longevity risk, models for forecasting future mortality rate, longevity modeling, longevity risk pricing models and various recommendations for these pension schemes.

2.2 Modeling of Longevity Risk

The main reason for developing market models for quantifying risk is the hedging of longevity risk. Even though, there are a life table and actuarial mortality models for forecasting and pricing pensions and its associated cash flows, which have developed over many years of which it has technology as deterministic based. Tuljapurkar and Boe (1998) study gives an in depth knowledge previous mortality trends and models for forecasting longevity. A further research was done by Stallard (2006) which also discussed the morbidity impact, forecasting longevity and the source of death on longevity. Lee and Carter (1992) came out with advancement in longevity for predicting morality trend for the future. Lee (2000) again made an assumption to their early studies that, the model takes age exact rate of mortality, which differs from time with differential impacts by age. Another assumption is that, the improved trend of mortality follows a random walk and the fitted past data follow a normal distribution for the stochastic factor. A further study to Lee Carter model was developed by Brouhns, Denuit and Vermunt (2002) which allow an assumption on Poisson errors and fitted a model using data from

Belgium. Renshaw and Haberman (2003) and (2006) studies have added two things to

Lee Carter's model. These are;

- A non-linear age factors fitted using Wales and England data.
- A cohort element which includes the birth year as a factor of impacting longevity risk. Cairns, Blake and Dowd (2006) postulated an expected model which looks identical to that of Lee Carter model using logit transformation for rate of mortality. Another model was proposed by Cairns, Blake, Dowd, Coughlan, Epstein, Ong, and Balevich (2007) which allow them to use a quadratic age effect, a cohort effect and an age effect to fit Wales, US and England mortality data even though the errors in the models were inadequate. A lot of research has shown that there is more need than a single factor to model improvement for the longevity risk. Models on pricing may execute better when using a single factor but in hedging it depends more on the improvement dynamics. Risk assessment and hedging for longevity require models that have multiples factors.

Woodbury and Manton (1977) studies also introduce an essential modeling technique that consider factors like physiology that cause longevity and further development was done by Manton, Yashin, and Vaupel (1985). The model they developed showed that impact on probabilities of survival should not depend on age. Lack of data made this model framework basically unexplored. A framework of financial estimation models as developed by Cairns, Blake, and Dowd (2006), Biffis (2005), Dahl (2004) and Dahl and Moller (2005) were used in interest rates calculation for term structure. The assumption to this model operates on arbitrage-free model and does not depend on market comprehensiveness since it is unable to absorb longevity product. This model is useful for pricing longevity since the market is incomplete based on hedging effectiveness using mean-variance approach. Pension benefits are index every year to better the life of pensioners and against inflation so as to improve and enhance their purchasing power. This indexation is done every year taken into consideration the economic fundamentals of the country(interest rates, inflation, the growth rate of the trust and the implication) to the survival of the trust.

Indexation is done in such a way that the benefits go to the pensioners in the lower income bracket.

2.3 Longevity Risk and its Market Development

Goshay and Sandor (1973) first published a paper on derivatives to introduce the subject of insurance risk securitization. The insurance companies became serious on this, when their assets could not curtail their liabilities in the late 1990s. Which led to finding of the study by Blake and Burrows (2001) that, mortality improvement poses a risk for the pension scheme because of the losses they will incur. Due to advancement in technology, hygienic and medical practices make individuals to live much longer than expected making it difficult for pension schemes to forecast any future mortality changes. This danger to predict future mortality is known as mortality risk. These create a need for the social and private pension schemes to share risks introduced by this mortality improvement.

There is a great financial pressure on the schemes and the government cause by the cash-based pension as suggested by Richard et al. (2006) studies. Also in research conducted by Blake shown that, there is a positive relation between capital market and longevity risk and suggested the risk can be transferred to private investors. In the same vain, persons can purchase some pension products that are safe to assured their life after retirement.

A definition on aggregate longevity risk was suggested and defined by Milevsky et al. (2006) that, life expectancy of an average cohort age surpasses the predicted number of years. This becomes a blockade to the pension houses to manage aggregate longevity risk effectively. The problems of mispricing and model predictions in the longevity risk market; was identified by Eric (2006) studies, which suggested that the failure to take into consideration the longevity improvement of the expected life expectancy. He later set an assumption in addition to the study; there can be a control on the uncertainty collection of the retiree cohort while there is still an unpredictable uncertainty for the future of the youth. Currently, there are a lot of models used in forecasting future mortality concerning the study of longevity risk. A study by Chen and Zhu (2009) put the models into two perspective; dynamic mortality and fixed mortality models. The Lee-Carter models, birth year effect models and generalized linear model (GLM) are good examples dynamic mortality models.

2.4 Longevity Risk Hedging

As life expectancy increases, the next thought is how it causes financial impact on the pension schemes. The issue at hand currently is that - there are no liquid forward markets in longevity risk. A research by Dowd, Blake, Cairns, and Dawson (2006) discussed longevity swap as a portfolio of forward contracts and priced longevity risk for mortality using financial market.

Hedging of longevity risk contracts are type of contracts where a pension scheme make financial arrangement to give a periodic static premiums or payment to its members, according to Belanger et al (2014), as shown in figure 2.1. The contract may be classified in the form of swap derivatives or insurance. The payment involves are normally two types;

1. payment based on mortality index and
2. payment based on pension plan's actual mortality experience.

Pension schemes will still bear the obligation of paying the pensioners unlike the buy-out where the annuitant fixed a claim against the insurer.

Figure 2.1: Structure of Longevity Risk Hedging Contract

Another way to solve the mispricing problems of longevity risk is the choice of natural hedging. The insurance companies may adjust the allocation of their life insurance and annuities to hedge against longevity risk. There is no need for market liquidity when using natural hedging strategy and this help to avoid large amount of transaction cost. The companies use this approach internally because it is more practical and convenient to implement as studied by Huang and Wang (2015).

The mutual ownership structure is another approach use to manage longevity risk as suggested by Sherris and Wills (2007). It is a commercial form of providing life insurance along with contributing contracts. Valdez and Detzel (2005) proposed an experience of risk sharing as a way to mitigate systematic adverse changes in Piggott. Risk sharing is a product which is potentially valuable in the life annuities product design. Lane and Beckwith (2007) came out with a discussion that, there is an increasingly affection for insurance risk through insurance securitizations and the alternative risk transfer market.

Apart from constructing stochastic mortality models, another best way is the capital market solution with three possible ways:

- Survivor swaps as suggested by Cairns et al.(2006) and Dowd et al. (2006)

, • Mortality securitization according to Cox et al. (2006)

- Survivor bonds, according to Blake and Burrows(2001) and Denuit et al. (2007). A research by Huang and Wang, (2015) revealed that insurance companies could allocate their exposures to the capital markets which have more competitors and funding.

2.5 Insurance and Reinsurance

Various studies have revealed that, there is low market share for the longevity insurance policies in the insurance (life) companies and it is supported by Milevsky's (2004) research; due to the following reasons;

- adverse selection exist - which explain that the price of annuity is less profound to one's life character. And those individual with a high force of mortality opt out of the market because they found the price costly.
- There is a reduction in the annuity market by the public pension and
- unwillingness on the part of the retiree to exchange property for annuity because it not regarded as an inheritance.

Some conclusions were drawn from the above factors and Purcal et al. (2003) confirmed it using some random models. The conclusions were that the highest rank was inheritance, government pension exclusion followed and adverse selection came last. Another study by Milevsky (2004) suggested a new insurance product about longevity and concluded that the price of the product should be very low so that young age individuals can buy with a little money.

A study of managing longevity risk in different ways was done by Blake et al. (2006) and it came out that the ways include buy - out of the pension plan and swapping non profit annuities for profit - share annuities. Currently, the existing methods and concepts are unable to fuel the demand for the market longevity insurance products - this fact was studied by Da (2008) and he does it by comparing the longevity risk insurance market abroad and home.

2.6 Longevity Risk Securitization Products

In 1970, the American Government National Mortgage Association delivered mortgage instruments certificate of guarantee where the first financial contract of longevity risk securitization was implemented. According to Cummins and Lewis (2003) studies, it suggested the need for financial market to adopt an operative way of financing to absorb the wealth deficiency to the borrowing funds of investors. The asset - backed securities and mortgage - backed securities extended $450 billion and $1.5 trillion respectively ever since 1970 when the financial contract was firstly issued in 2002. A lot of securitization products are in the market now after the longevity risk securitization was implemented officially in the U.S.

financial market. After 2002, the insurance market was opened to many risk management, including the mortality risk bonds. In Cox et al. (2005) studies, they discussed mortality risk securitization in annuity business.

To remove the risk of further rising costs, sponsors are increasingly looking to insure some or all of their pension scheme obligations with a specialist insurance company. This usually comes at a cost - usually but not always in the form of an up-front premium. These types of transactions are known as bulk annuity policies and can be structured in two ways; through either a buy-out or a buy-in. With a buy-out, the scheme's liabilities are transferred to the insurer and the sponsor's obligation to the members is extinguished. The terms of the insurance policy are required to precisely match the form of the members benefit under the scheme. Securing such a policy arrangement can be a long, protracted process. A buy-out normally precedes a wind-up of a scheme and involves the entire scheme membership being covered by the policy. A buy-out of only part of the membership is rare due to the fact that the scheme's trustees could be seen to be favouring one group of members by providing them with increased security (i.e. those covered by the bulk annuity policy) over the remaining members.

Under a buy-in, the policy is held by the trustees and is effectively a scheme asset which pays the member's benefits. In other words, the ultimate obligation to pay the members still remains with the scheme. A buy-in policy does not reduce the security of those members whose benefits are not insured by the policy, as income from the insurer can in theory be allocated across all the beneficiaries.

2.7 Future Mortality Prediction

As we have seen in various mortality derivatives discussed, a key component in designing a mortality derivative is forecasting future mortality. In this section, we will review models used in forecasting future mortality rates. There is a need for the insurance and pension industries to forecast future mortality, since predictions are required for pricing and reserving. Human mortality predicted ahead depends on the impact of such unknown factors future medical advances, new infectious diseases, and even disasters, both natural and man-made. No attempts are made to take these underlying factors into account and future mortality forecasts are attempted by extrapolating past trends.

There are a number of approaches to the problem. One of the oldest methods is based on the forecasting of parameters in some parametric model. Age-Period-Cohort (APC) models are a widely used methods for smoothing mortality tables. The classic reference is Clayton and Schifflers (1987). Lee and Carter (1992) introduced a simple bilinear model of mortality in which the time dependent component of mortality is reduced to a single index which was then predicted using time series methods. The model was fitted well by ordinary least squares (OLS) with the observed log mortality rates as dependent
variable.

Brouhns et al. (2002) improved on the OLS approach by modeling the number of deaths directly using a Poisson distribution and using maximum likelihood parameter estimation. De Boor (2001) constructed a two-dimensional regression basis as the Kronecker product of B-splines but neither author considers non normal data or the forecasting problem. Gu and Wahba (1993) and Wood (2003) use thin plate splines but again forecasting is not available. Curie et al(2002) used two-dimensional regression splines, specifically B-splines with penalties, usually known as P-splines (Eilers and Marx, 1996). Curie et al (2004) extended this work by using B-splines to construct a basis for bivariate regression. This construction gives a basis in two dimensions with local support and hence a fully flexible family of fitted mortality surfaces. The regression approach leads to a generalized linear model which is fitted by penalized likelihood. An important feature of this method is that forecasting is a natural consequence of the smoothing process. They considered future values as missing values;- the penalization then allows estimation of the future values simultaneously with the fitting of the mortality surface. The choice of penalty function, which can be of secondary importance in the smoothing of data, is now critical, since it is the penalty function that determines the form of the forecast.

2.8 Social Security and National Insurance Trust(SSNIT),Ghana

2.8.1 Pre-independecne Social Security

Before Ghana's independence, there was neither a national nor a uniform social security in the country. There were public and private schemes which catered for the security of various categories of workers. Kpessa (2010) finds that "the first programme was designed as a means of encouraging loyalty and efficiency without the colonial civil service". For example in 1940, the adoption by the country of the international Labour Organisation's (ILO) convention on workers made cash benefits payable to workers who suffered injury.

This retirement benefits was not one that was covered from deferred incomes by the retiree but rather, it was seen and considered as a reward for people who served the queen and the colony diligently till their old age. These public servants had no entitlement to such programmes unless they were judged by colonial administrators as meeting eligibility requirement of loyalty (Government of Ghana 2006). By the pension ordinance of 1946, a non-contributory pension scheme was established for workers categorized as African Senior Civil Servants which extended in a limited extent to their widows and orphans. This was to replace and unify the (European officer) ordinance (CAP-29, 1936) and the non-European officer ordinance (CAP-30, 1936) into a single non discriminating pension scheme that offered equal benefits to both expatriates and local workers (Government of Ghana, 2006).The scheme was referred to as (AP 30).

In 1955, by the teacher's pension ordinance, certified teachers were made coverable under the ordinance of 1946, CAP 30. Meanwhile, some organizations in the private sector, especially the major foreign trading and commercial firms were operating superannuation, pension and provident fund schemes under which benefits were paid out at the time of retirement. The CAP 30 scheme provided workers with the opportunity to go on voluntary retirement at the age forty five (45) years or compulsory retirement at the age of fifty (50) years. This is also meant for workers who were loyal to their work and worked continuously for ten (10) years without blemish. The scheme however created a lot of problems for the colonial government since it was fully funded by the government.

Due to the financial burden in the scheme, policy makers in the in the immediate postindependence opted for an establishment of a new and different scheme. The scheme was an arrangement for guaranteeing a reasonable, comfortable and decent life as well as the economic and social security of both pensionable and non-pensionable officers retiring from the colonial civil, through which the payment of pensions, gratuities or annual allowances (Adjei 1999).

2.9 Post Independence Social Security

On attending independence in 1957, Ghana embarked upon intensive industrial and educational programmes. The net result was a large influx of people to the cities and urban areas in search of white-collar jobs. The social, economic, and political challenges created by the rural urban migration where in unimaginable. The worker who has been alienated from his root and thus stripped of the security offered by the extended traditional family system faced a firm future, of that of insecurity. To address this unsatisfactory situation, the government of the first republic instituted the compulsory saving scheme which was later abolished due to lack of education on the scheme by the workers.

2.9.1 Social Security Scheme 1965

February 17th, 1965, the Parliament of the first republic passed a bill known as the social security ACT 279 of 1965 to establish a social security fund to provide for contributors, benefits under superannuation, invalidity, survivors among others (SSNIT Annual Report).

The ACT 279 of 1965 scheme was a provident fund under which lump sums were paid to qualified members. The fund started with a contribution of 7.5% by workers and fifteen percent(15%) by employers making a total of 22.5 percent. In less than a year, the contribution reduced from 7.5% to 5 percent of workers and from fifteen percent to 12.5% by employers after a huge public outcry of high contributions. The provident fund became popular with workers because inflation rates were very low. Successive governments also found the funds a ready source of capital for budget deficit financing. The fund was then managed by the state insurance cooperation (SIC) until 1972 were the NRC degree 127 was passed establishing the Social Security National Insurance Trust to take over the management of the fund. Out of the total contribution of 17.5%, 2% was set aside for life insurance and unemployment benefits. The scheme however was supposed to invest the rest of the 15.5% contribution into government bonds at an interest rate of 6 percent per annum and use 3% for administrative expenses and pay the following benefits: Invalidity, Emigration, Death and Survival, Sickness, Unemployment and Superannuation. This however was not sustainable as the gains made by the contributors account were eroded by the high inflation experienced in the country due to western economic conditions at the time and the constant 6% retained by government even when the banks interest rate has gone up over the 20 years period. Thus the erosion of the value of income paid to most retired pensioners in the 1980's serve as a catalyst to initiate a plan of action to convert the provident fund to a Defined Benefit Scheme. The scope of coverage by the NRCD
127 of 1972 provided for compulsory coverage for workers in establishments employing more than five workers including the self-employed. It however exempted some class of workers from membership; member of the Ghana Armed Force, Police Service, Prison Service, Ghana

National Fire Service, Foreigners in the diplomatic mission and senior member of the universities and research institutions. The PNDCL 247 was promulgated in 1991 as a Defined Benefits Scheme with the following mandate: Register employers and workers, collect contributions, manage the records of members, invest the funds of the scheme and process and pay the benefits to eligible members and declared dependents.

At the point of conversion in 1991, there were approximately twelve thousand (12,000) active employers and eight hundred and twenty thousand (820,000) workers belonging to the scheme. The conversion also paved way for workers in the informal sector to join the scheme as voluntary contributors. The law also exempted men and officers of the Ghana Armed Forces and other security agencies. The highlight of the new pension scheme included: Provision of three benefits,

- Old age pension,
- Invalidity pension,
- Death and survivors lump sum benefits pensions are paid monthly to qualified members. It was finance by employer and employee contribution of 12.5% and 5%of basic salaries respectively. The total of 17.5% is paid to SSNIT, full pension is earned at the age of sixty (60) years with a minimum of two hundred forty (240) months of contribution. Reduced pension is earned between age fifty five (55) and fifty nine (59) years with minimum contribution of two hundred and forty (240) months

A twenty five (25) percent lump sum payment option was available for both full and reduced pension. This is paid at the present value discounted at the prevailing Treasury bill rate. Invalidity pension earned after twelve (12) months of contribution within the last thirty six (36) months of contribution. Survivor’s lump sum was paid to a member who dies before reaching retirement or to a member once a pensioner die before attaining age seventy two (72).

Minimum accrual rate of fifty percent (50%) and maximum of eighty percent (80%) of three best year’s average salary of the member of the plan. A returned contribution accumulated at a prescribe interest rate is given to unqualified contributors. After operating for about fifteen (15) years as a pension scheme - workers especially the nonpensionable officers in the civil service started comparing their retirement benefits with the pensionable officers under CAP 30 of the 1946 pension ordinance, a non-contributory pension scheme and agitated for a change. Kpessa (2010) finds that "Benefits are calculated on the basic of 50 percent of the average of an employee's best three years' salary. Employees are awarded an additional 1.5% on top of the 50% minimum base pension formulae for every additional year of contribution beyond the two hundred and forty (240) months up to a maximum of pension benefits of 80% of the best three year's income". If an employee is not able to meet the minimum requirement of the two hundred and forty (240) months to qualify for the monthly pension benefits before retirement, will receive his or her accumulated contributions where interest is calculated at half the rate of the government's treasury bills

For the disability benefits, in order to qualify, the employee should have contributed not less than three years to the scheme and medically pass by the medical board in capable of working again. According to Dei (1997), dependants of deceased employees who contribute to the scheme for a total of two hundred and forty (240) months or more receive lump sum benefits equivalent to twelve years monthly retirement income. Under the social insurance scheme, if an employee passed away before reaching the two hundred and forty (240) months contribution threshold, the dependent receive a lump sum benefits equivalent to twelve (12) years retirement income proportional to the contribution made. In the event that a pensioner away before age seventy two (72), his or her dependants are paid lump sum benefits calculated up to age seventy two (72) of the deceased retiree while dependants of deceased pensioner age seventy two (72) and over are not entitled to any benefits. The most distinguishing feature of the social insurance plan has to do with the fact that it ensures intergenerational transfer and design to ensure solidarity and collectivisation of risk.Kpessa (2010). The running of two of the nation's pension scheme brought some level of dissatisfaction. Kumado and Goekel (2003), reported emphatically that clearly, retirement benefits under CAP 30 are undoubtedly better than those under the SSNIT scheme, which is why those who can keep themselves in the plan do so ,and others outside fighting to get on it. The problem is not only that there is great dissatisfaction among those workers who do not enjoy the superior coverage of CAP 30; it is also that largely unfunded nature of the plan is a drain on general revenue.

2.9.2 The National Pension Act 2008 (act 766)

A presidential commission set up in 2004 chaired by T.A Bediako came up with recommendation for a new national pension Act. The commission recommended the creation of a new contributory three - tier pension system for Ghana. In 2008, the Parliament of Ghana passed into law the new pension ACT 2008, (ACT 766) which received presidential assent on December 4, 2012. The new pension Act provided a new contributory three-tier pension scheme. The first tier - is a mandatory basic contributory social security scheme to be managed by SSNIT. The second tier - a mandatory fully funded and privately managed occupational scheme. The third tier, a voluntary fully funded and privately managed provident fund and personal pension plan. The new pension ACT also established the National Pension Regulatory Authority to oversee all pension activities in the country.

Features of the pension Act 2008 (ACT 766); The new pension Act saw an increase of both employer and employee contribution from 12.5 percent to 13 percent for employers and 5 percent to 5.5 percent for employees. A total of 18.5 percent contribution of which SSNIT received 13.5 percent and transfer 2.5 percent to the National Health Insurance Authority for member's health insurance and the remaining five percent are paid to the tier two. Three direct benefits are paid under the new pension ACT.; - superannuation pension, Invalidity pension and survivor's lump sum. The Act also gave an entry age of fifteen (15) years minimum and forty five 45 years maximum age. It however reduced the qualifying period for pension from twenty (20) years of contribution to fifteen (15) years. Guaranteed survivors benefits payment period increased from twelve (12) years to fifteen (15) years. Also increased the annuity period from twelve (12) to fifteen (15) years.

The age bracket to qualify for pension must be between fifty (50) and sixty (60) years.

The ACT also reduced the benefits payment, reducing the minimum pension right from Fifty percent(50%) to Thirty Seven and half percent(37.5%) and the maximum pension right from Eighty percent(80%) to Sixty percent(70%). Since SSNIT is partially funded, there is always the need to invest funds judiciously for maximum returns. This leads to the building of surplus fund that need to be re-invested. The investment portfolio of SSNIT scheme is divided into fixed and non-fixed income investment. The fixed include; registered stock, home finance, company index linked bond, fixed deposit, cooperate loans, treasury bills and government bonds. The non-fixed income investment include; real estate, commercial and residential development and equities. Actuarial valuation of the scheme in 2004 projected at two percent rate of return on investment and Three percent annual increase in the contributors, the scheme can be sustained over a Fifty (50) year period.Kpessa (2010).

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter discusses the methodology employed to achieve the objectives of the study. This research is an exploratory study designed to describe and forecast mortality pattern for male pensioners by using Lee Carter model. A suitable approach like natural hedging will be used to hedge against this risk the pension scheme is facing.

3.2 Source and type of Data

Raw data were obtained from the Social Security and National Insurance Trust (SSNIT). SSNIT is the nation parent pension scheme in Ghana with investment across various divisions of the economy. The data included the date of births, date of entry, date of retirement, date of death for only males. The data collected entails pensioners on SSNIT scheme from the 1991 to 2013. The data collected were analyzed to derive the number of deaths needed to forecast the mortality rate.

For the purpose of the study, early retirements were ignored, hence the analysis was carried out for pensioners who retire at the mandatory retirement age of 60. The data were structured in such a way that the mortality pattern for each cohort (a group of people retiring at the age of 60 in a particular year) was studied separately. This study clearly outlines three types of data that were collected:

1. Population data - this is essentially data on the target population of the scheme and which include information on the age and sex structure of the population as well as their contribution.

2. Price data - This comprises of information on the amount each member contribute under the scheme.

3. Income and expenditure data- This is data on the expenses that is incurred when life expectancy varies as expected in running the scheme.

3.3 Data analysis procedure

Raw data collected for a research are of no use, unless it is converted for the purpose of decision-making (Emory, Cooper, 2003). Data analysis usually comprises reducing the raw data into a practicable size, developing summaries and applying statistical inferences.

1. The data collected was validated for accuracy and appropriateness, i.e. Checking for data errors and missing data.

2. The result was divided into homogeneous groups, for example, by sex and age

3. Analyze the data to identify trends and necessary adjustments.

4. Lee Carter model was taken as an appropriate model for the mortality trend since it is the recent and influential stochastic model for predicting mortality.

5. Hedging longevity risk using annuity pricing approach.

3.4 Stochastic Mortality Model Setting: Lee-Carter Model

There are a lot of models that can be used to fit the mortality trend, but for the purpose of this study, Lee - Carter model will be used. Lee and Carter postulated this model of mortality in 1992. They fitted and predicted the ability of their model as simple and superior but one parameter model which is powerful. Many researchers have used their model since it is among the most popular stochastic mortality models and which can easily absorb any strong influence on the mortality level. This model does not use sex-specific factors but only age-specific mortality. The central death rate in t years for age x can be represented by:

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} \tag{3.1}$$

where

$D_{x,t}$ represents the number of deaths among each retiring year

$E_{x,t}$ represents the expected number of pensioners that reached the time period t Using the model to fit the central death $m_{x,t}$ rate is given as;

$$\ln(m_{x,t}) = \alpha_x + \beta_x K_t + \varepsilon_{x,t} \tag{3.2}$$

where; $m_{x,t}$ represents the central death rate for an individual aged x at period

t. α_x represents the average age - specific mortality factor.

β_x represents the age - specific improving factor which characterised the sensitivity of $m_{x,t}$ to the index K_t

K_t represents the time - varying effect index $\varepsilon_{x,t}$ represent an error term which is normally distributed with mean mean 0 and variance

$\sigma^2(\epsilon \sim N(0, \sigma^2))$

The K_t is predicted using an ARIMA (0, 1, 0) process. The general form of the ARIMA (p,d,q) is given as:

$$\Phi(B)(1 - B)^d k_t = (B)e_t \tag{3.3}$$

where B represent the back shift operator as $B_s k_t = k_{t-s}$

$$\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \tag{3.4}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_p B^p \tag{3.5}$$

where $e_t \sim N(0,\sigma^2)$ is IID

The time series k_t is stationary after differencing d-times. An ARIMA(0,1,0) model is effectively used for the dynamics of kt. This order (0,1,0) is preferred due to parsimony and it is an optimal ARIMA model for k_t. An ARIMA (0,1,0) is a random walk with

drift given as:

$$k_t = c + k_{t-1} + e_t \tag{3.6}$$

where c is the drift of the random walk process. The first derivative of k_t can also be expressed as:

$$\delta(k_t) = \mu_t \delta(t) + \sigma_k \delta(Z_k(t)) \tag{3.7}$$

where $(Z_k(t))$ is defined as standard Brownian motion.

The fitted Lee - Carter parameters also follow two popular approaches - these are Approximation and Singular Value Decomposition (SVD). In this study the parameter will be determined using the SVD approach then the Approximation approach will help the researcher to estimate the parameters of the model. The following steps show how the parameters for the analysis were obtained.

- The average of $ln(m_{x,t})$ at age x over time is equal to the fitted values of α_x

$$\alpha_x \frac{1}{n} \sum ln(m_{x,t}) \tag{3.8}$$

- Applying the standard controlling constraints on the β_x and k_t such that $\text{P}(b_x) = 1$ and $\text{P}(k_t) = 0$

$$\beta_x = \frac{\sum k_t Z_{x,t}}{\sqrt{\sum k_t Z_{x,t}}} \tag{3.9}$$

- k_t is eual to the sum of $ln(m_{x,t})$ - α_x over each age

$$k_t = \sum (ln(m_{x,t}) - \alpha_x) \qquad (3.10)$$

- The coefficient β_x is obtained with the dependent variables $ln(m_{x,t})$ - α_x and k_t, using regression models without intercept.

These estimated parameters; - α_x, β_x and k_t are used to forecast the future mortality rates.

Probability of death (mortality rate) among each cohort is given as $q_{x,t}$. This shows the probability that a pensioner aged x in year t belonging to a cohort retiring in certain year will die before attaining the age x + 1 for $x = x_0, x_1, \ldots x_m$ and $t = t_0, t_1, \ldots t_n$

$$q_{x,t} = \frac{d_{x,t}}{l_{x,t}} \qquad (3.11)$$

Probability of pensioner survival is given $P_{x,t} = 1 - q_{x,t}$

$${}_nP_{x,t} = \prod_{i=0}^{n} P_{x+i,t+i} \qquad (3.12)$$

The expected future life time of a pensioner aged x

$$e_{x,t} = \sum_{n \geq 1} {}_nP_{x,t} \qquad (3.13)$$

The central death rate is given as

$$m_{x,t} = \frac{d_{x,t}}{n_{x,t}} \qquad (3.14)$$

where; $q_{x,t}$ represents the probability of dying (that is, the number of death in the cohort over the number of pensioners that reached the time t).

$d_{x,t}$ represents the number of deaths among each cohort $l_{x,t}$ represents the number of pensioners that reached the time period t $n_{x,t}$ represents the total number of pensioners in the age cohort.

Their force of mortality is given us:

$$\mu_{x,t} = -\frac{1}{l_{x,t}}\frac{\delta(l_{x,t})}{\delta(x)} \tag{3.15}$$

where $q_{x,t}$ lies between 0 and 1 but $\mu_{x,t}$ and $m_{x,t}$ can be greater that 1
The relationship between $\mu_{x+\frac{1}{2}}$ and $q_{x,t}$ follows as:

$$1 - q_x = \frac{l_{x+1}}{l_x} \tag{3.16}$$

$$\frac{l_{x+1}}{l_x} = \ell^{-\int(\mu_x)\delta(x)} \tag{3.17}$$

$$\mu_{x+\frac{1}{2}} \text{w} -ln(l - q_x) \tag{3.18}$$

$$q_x = 1 - \varrho^{-\mu_{x+\frac{1}{2}}} \tag{3.19}$$

Some other approximation to help in the analysis are;

$$q_{x,t} = 1 - \%_{-m_{x,t}} \tag{3.20}$$

$$\Rightarrow m_{x,t} = -ln(l - q_{x,t}) \tag{3.21}$$

Using the principle of equivalency to calculate the premium for re - insurance that SSNIT will pay to these private life insurance companies. (Expected present value of money in = Expected present value of money out) Where Premiums will be equivalent to annuity and some loading factor. Mathematically,

$$P = \ddot{a}_x + l \tag{3.22}$$

where: $\ddot{a}_x$ is the series of payment to the pensioners.

l is some loading factor

P is the calculated premium that SSNIT have to pay.

3.5 Calculation of Annuity

Considering the amount paid to the retiree after the guarantee period of 15 - years is in the form of annuity (life). Assuming that life annuity is paid monthly. That is m - times in a year (where m = 12) Let T_x represent a random variable, the number of years the retirees will live before death.

Let $S^m = (\frac{1}{m}, \frac{2}{m}, \frac{3}{m}, ..., \frac{m-1}{m}, 1)$ which is the discrete random variable. Let Y^m represent the payment given to the retirees on the monthly bases, where

$$Y^m = \frac{1}{m}[1 + v^{\frac{1}{m}} + v^{\frac{1}{m}} + ... + v^{T_x + S^m - \frac{1}{m}}] \tag{3.23}$$

From the geometric series

$$Y^m = \frac{1}{m}[(v^{\frac{1}{m}})^0 + (v^{\frac{1}{m}})^1 + (v^{\frac{1}{m}})^2 + ... + (v^{\frac{1}{m}})^{T_x + mS^m - 1}] \tag{3.24}$$

$$Y^m = \frac{1}{m}\left(\frac{1 - v^{T_x + S^m}}{1 - v^{\frac{1}{m}}}\right) \tag{3.25}$$

Let the expectation of the present value be given as:

$$E(Y^{(m)}) = \ddot{a}_x^{(m)} = E\left[\frac{1}{m}\left(\frac{1 - v^{T(x) + S^{(m)}}}{1 - v^{\frac{1}{m}}}\right.\right. \tag{3.26}$$

!#

If there is an assumption that S_m and T_m are independent then:

$$\ddot{a}_x^{(m)} = \frac{1}{m}\left[\frac{1}{1 - v^{\frac{1}{m}}}\right][E(v^{s^{(m)} - 1})] \tag{3.27}$$

where $E(v_x^T + 1)$ is the benefit paid after death

$$A_x = E(v_x^T + 1) \tag{3.28}$$

where

$$v = \frac{1}{1+i} \tag{3.29}$$

Using the principle of equivalency, EPV of benefit + EPV of expenses = EPV of premiums

$$_{n}|\ddot{a}_{x} + exp = P \tag{3.30}$$

where $_{n}|\ddot{a}_{x} = V^{n} {}_{n}P_{x}\ddot{a}_{x+n}$

The EPV of the benefit is equal to the expected present value of the annuity as for a survivor to age $x + n$

CHAPTER 4

ANALYSIS OF RESULTS

4.1 Introduction

This part of the study presents a discussion and detailed results of the collected data that was analyzed. This chapter has three sections - preliminary results, results from Lee Carter Model and the annuity prices obtained. The R statistical package and LCfit online were used for the analysis

4.2 Preliminary Results

The number of males going on pension each year was obtained from SSNIT data. The result showed that the number of males retiring from active service is increasing. In 2010, the number of males who went on retirement was more than twice the number for 1991. The graph in Figure 4.3 gives a visual representation of the retirement data. The data from 1991 to 2013 was analysed to obtain the number of deaths observed each year among pensioners. Figure 4.1 and 4.2 shows graphs of the death patterns.

Table 4.1: Summary Statistics - male retirement

min	1st Qrt	Median	Mean	3rd Qtr	Max	Std.Dev.
1040	1504	2098	2295	2850	4626	1037.311

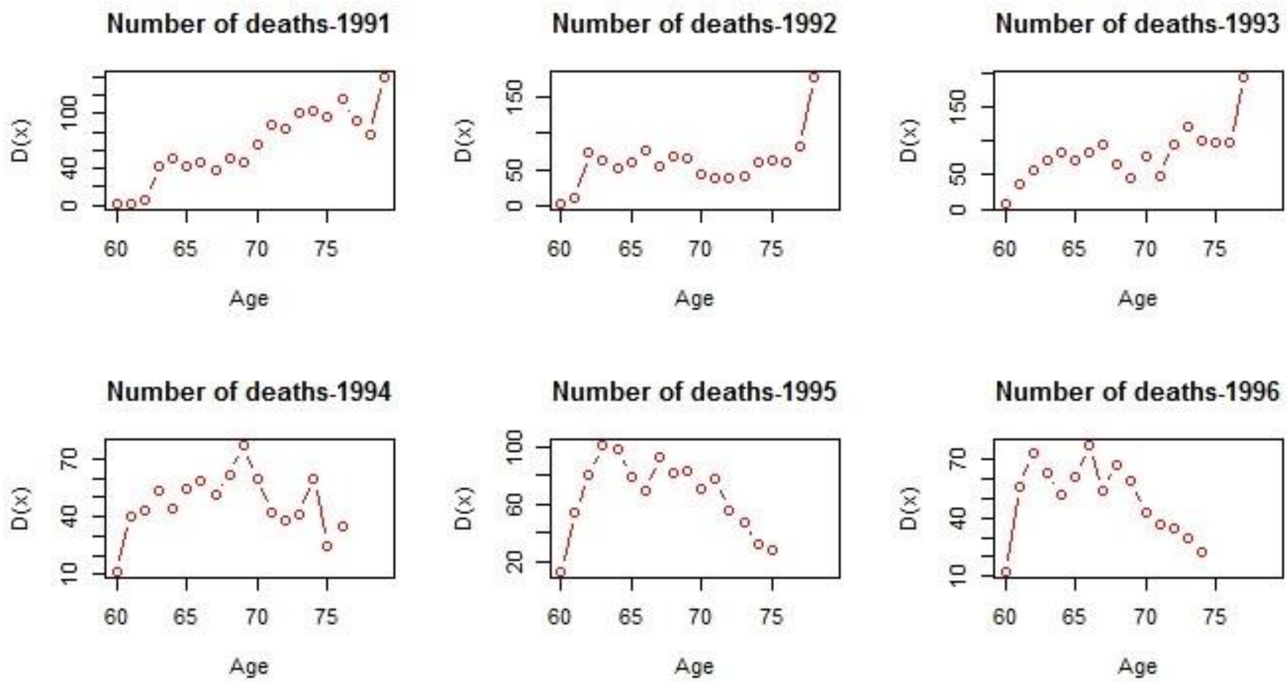

Figure 4.1: No. of Deaths Recorded from 1992 to 1996

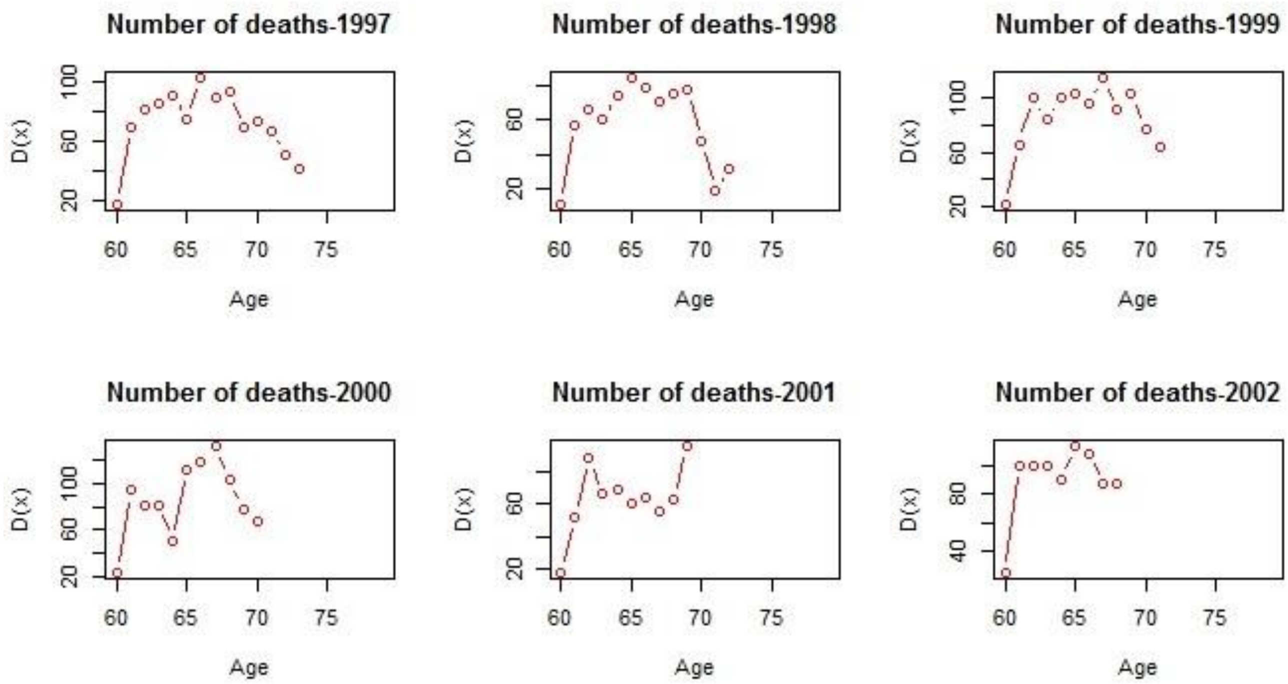

Figure 4.2: No. of Deaths Recorded from 1997 to 2002

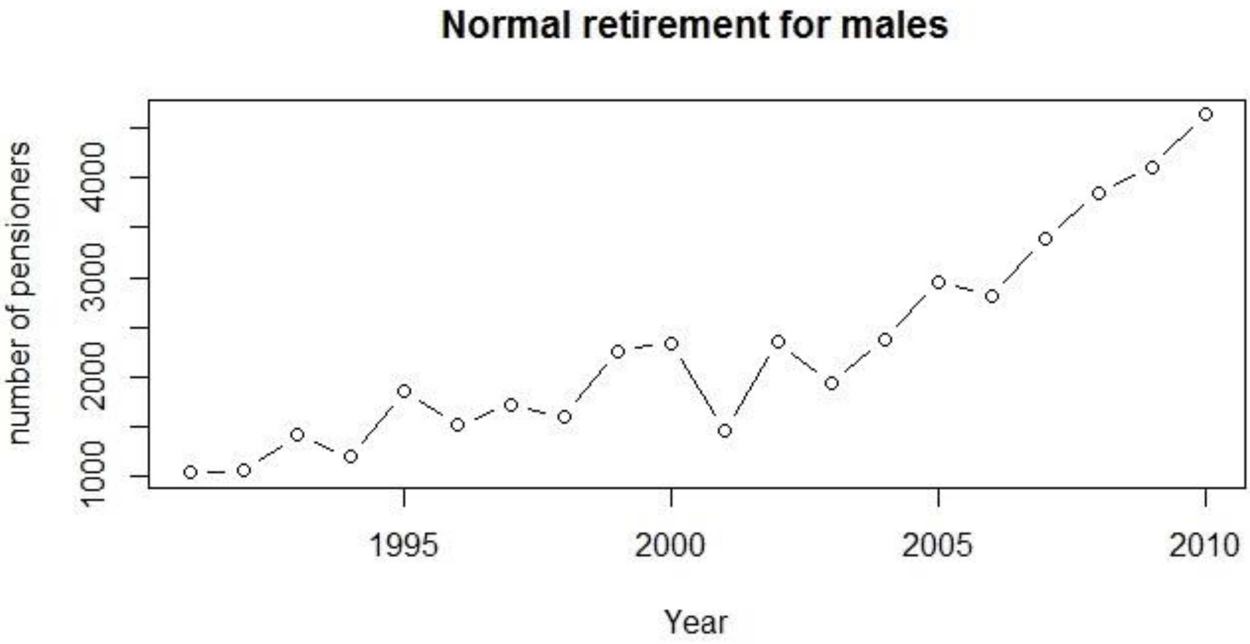

Figure 4.3: Male Pensioners

4.3 Lee Carter Model Report

These parameters β_x and SVD k_t were obtained after using the LCfit (Lee Carter fit) model software. The estimated parameters were used in the model to predict mortality for additional 5 years after the guaranteed period of 15 years since that is the ultimate requirement when talking about longevity hedging. The values of the estimated parameters after using the LCfit are in the table below.

Table 4.2: Age-Specific Estimated Parameters

Item	Age	α_x	β_x
	0	-4.873599965	0.108158123
	1	-4.873599965	0.108158123
	5	-8.538036676	0.106242842

	10	-8.564792501	0.072815113
	15	-8.004369805	0.049084924
	20	-7.839311925	0.054412704
	25	-7.647998748	0.059358241
	30	-7.34521564	0.05845539
	35	-6.950692746	0.051384768
	40	-6.506857698	0.047608094
	45	-6.034905621	0.041542662
	50	-5.598988594	0.041430654
	55	-5.164492547	0.040839968
	60	-4.693446867	0.04487984
	65	-4.17461322	0.05091936
	70	-3.603210491	0.056467574
	Total		1.000000000

The estimated parameter α_x shows the age pattern (the shape of the age) of the mortality by age. The outcome of the estimated parameter gives increasing values with age. This also shows that there is an increasing trend in the mortality. The estimated parameter β_x tells how the mortality at age x may change as the general level of mortality changes. The mortality rate at age x fluctuates when the value of β_x is larger as compared to the broad-spectrum of the mortality changes. It summation gives 1 as a requirement of the LCfit model. The estimated parameter K_t is the mortality index. Their values are shown in the Tables 4.3 and 4.4 below. This study took the Single Value Decomposition (SVD) of the K_t.The summation of the estimated SVD K_t values gives zero (0). The values of SVD K_t has a direct relationship with the mortality trend: - when the mortality trend declines as the years increases then it shows that the values of the estimated SVD K_t decreases over time.

Table 4.3: Estimated and Predicted k_t using ARIMA (0,1,0)

Item	Year	kt after second stage	kt from SVD	e0 from second stage kt	e0 from empirical nmx
	1991	9.366177574	9.734779551	72.3801048	72.11221701
	1992	8.805186884	9.376429608	72.78709888	72.36267958
	1993	8.120887179	7.842171282	73.25603498	73.41525943
	1994	8.102826637	7.603226264	73.26833096	73.57652832
	1995	7.15100285	7.105157794	73.91084718	73.91057558
	1996	6.870249879	6.90141413	74.09839782	74.04643138
	1997	6.460232567	6.082634228	74.37078753	74.58800852
	1998	6.677201407	6.056238842	74.22686684	74.60535538
	1999	5.765980747	5.45725338	74.82900141	74.99820348
	2000	5.319668753	4.660070867	75.12048222	75.51453382
	2001	6.039371523	5.13817447	74.64952965	75.20553147
	2002	5.130041689	4.581512542	75.24377804	75.5651229
	2003	4.997429593	4.510287394	75.32981417	75.61094583
	2004	4.007143465	4.069112307	75.96761591	75.89386869
	2005	4.125032533	3.924497157	75.89210553	75.98627704
	2006	3.558272469	3.607331651	76.25414741	76.18838788
	2007	3.292531383	3.02220226	76.42306985	76.55932379
	2008	2.821172062	2.578080885	76.72144939	76.83927288

	2009	3.429695651	3.092492893	76.3359439	76.5148925
	2010	3.58799838	2.470442494	76.23521931	76.90692259
	2011	1.693714435	2.435643853	77.42926145	76.92877688
	2012	1.781202504	1.683821568	77.37463919	77.39904595
	2013	1.505008133	1.704733418	77.54691804	77.38601316
	2014	0.941867452	1.328136269	77.89676176	77.62031436

Table 4.4: Estimated and Predicted k_t using ARIMA (0,1,0)

Item	Year	kt after second stage	kt from SVD	e0 from second stage kt	e0 from empirical nmx
	2015	0.862473883	1.060501114	77.94593477	77.78631477
	2016	0.300155354	1.104646136	78.29319115	77.75896251
	2017	0.564115082	0.733661615	78.13040469	77.98862907
	2018	-0.882121379	-0.726421298	79.01774095	78.88476849
	2019	-0.764641716	-0.615928908	78.94606811	78.81733867
	2020	-1.212869577	-0.28526213	79.21915546	78.61517583
	2021	-1.380028437	-1.049550508	79.3207441	79.08161632
	2022	-1.002734207	-2.802249919	79.09125315	80.14072204
	2023	-2.379273563	-2.267634061	79.92522554	79.81916944
	2024	-2.954618856	-3.129824315	80.25357091	80.33711656
	2025	-2.943873764	-3.387462118	80.24711644	80.47342815
	2026	-3.266130721	-2.816130347	80.44046744	80.14905357
	2027	-3.81946779	-3.155775624	80.7713849	80.35265522

	2028	-3.280872168	-3.404840495	80.44930103	80.48382718
	2029	-3.37826525	-3.233176313	80.50763806	80.38104755
	2030	-5.169196747	-3.953990732	81.57295071	80.81176085
	2031	-4.41600031	-3.703405455	81.12662607	80.66228108
	2032	-5.24989351	-4.040920018	81.62062386	80.86355325
	2033	-5.467237876	-5.087564197	81.7488842	81.48462125
	2034	-5.636812391	-4.79233928	81.84881259	81.30990751
	2035	-6.007326904	-6.622167286	82.0667201	82.38687262
	2036	-7.084175838	-7.147367735	82.69665432	82.69336386
	2037	-5.995304903	-7.143174118	82.05965903	82.69092127
	2038	-6.416987822	-7.266803445	82.30695823	82.76289802
	2039	-7.651216297	-7.802660406	83.02631827	83.07410938
	2040	-7.895612981	-8.040236595	83.16796458	83.21168726
	2041	-7.895985363	-8.531251603	83.1681802	83.49524075
	2042	-8.037464673	-7.94119917	83.25005575	83.15436578
	2043	-7.871611595	-8.917317894	83.15406578	83.7174331

Based on the estimated parameters, the central death rate was obtained as shown in Table 5.1 in the appendixA which helps for the mortality prediction. The graphs in the following pages show that at mortality at higher ages decreases. This study reveals that, in the nearby future mortality will receive a major improvement and this will expose a longevity risk to the pension fund which is SSNIT.

This longevity risk expose to the pension fund made the researcher to move further to calculate a yearly annuity assuming some interest rates - the delight of the researcher is that the social trustee to reinsure the pensioners after the guaranteed period to the life insurance companies to manage it.

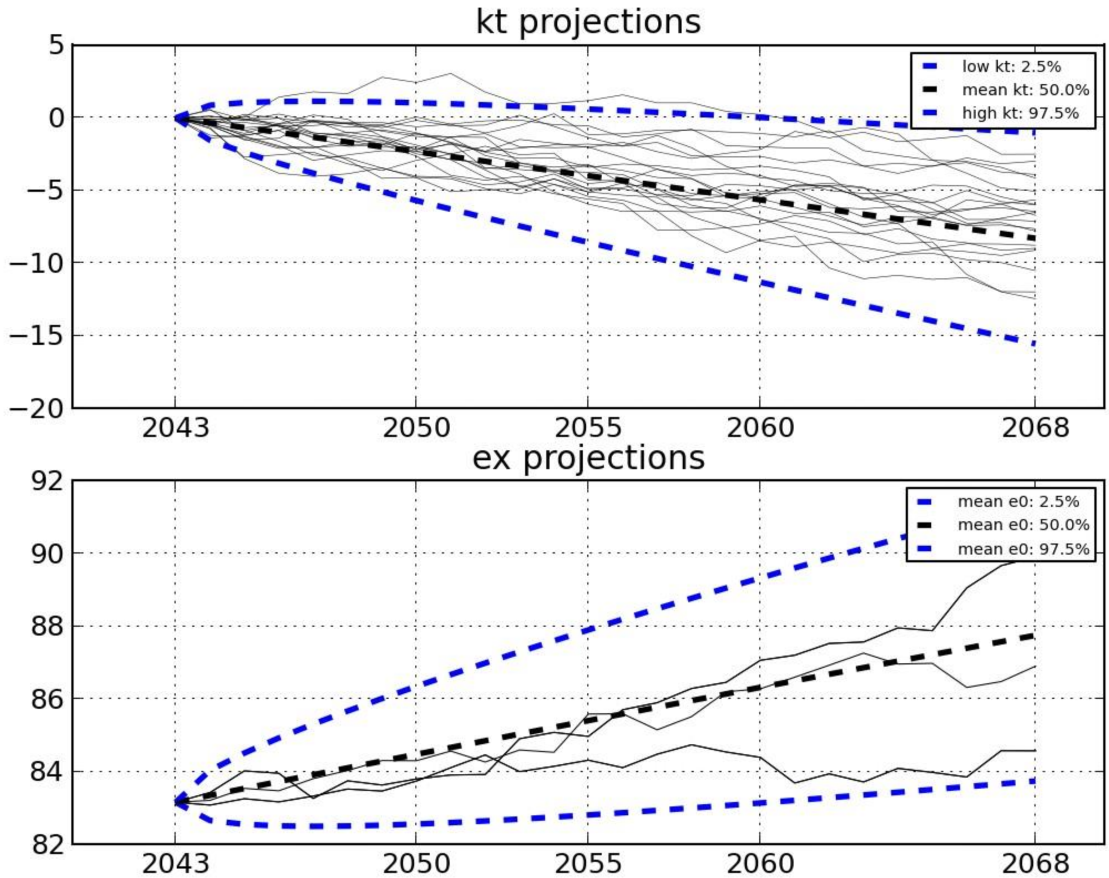

Figure 4.4: k_t and e_x projections

Figure 4.4 shows the projections for k_t and e_x. The projections were derived using the Lee-Carter model

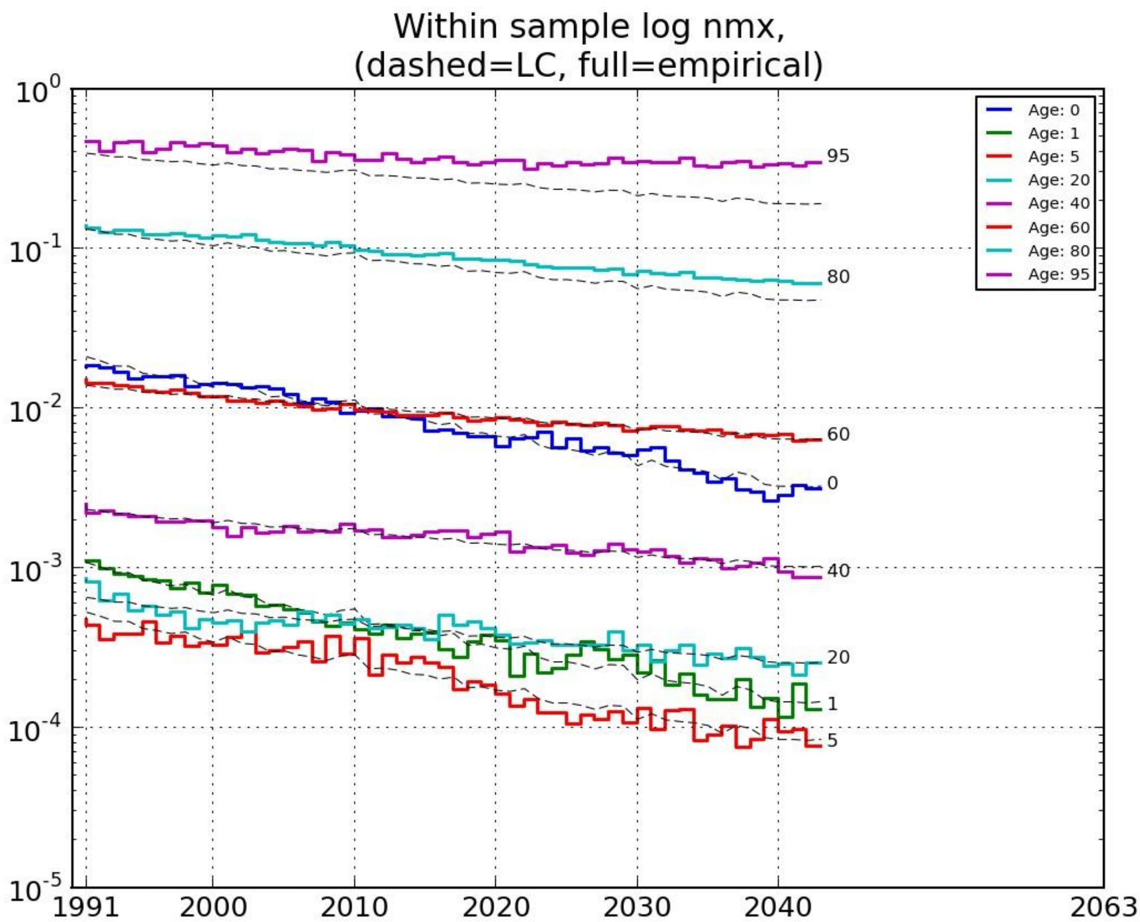

Figure 4.5: Estimated $ln(m_{(}x,t)$

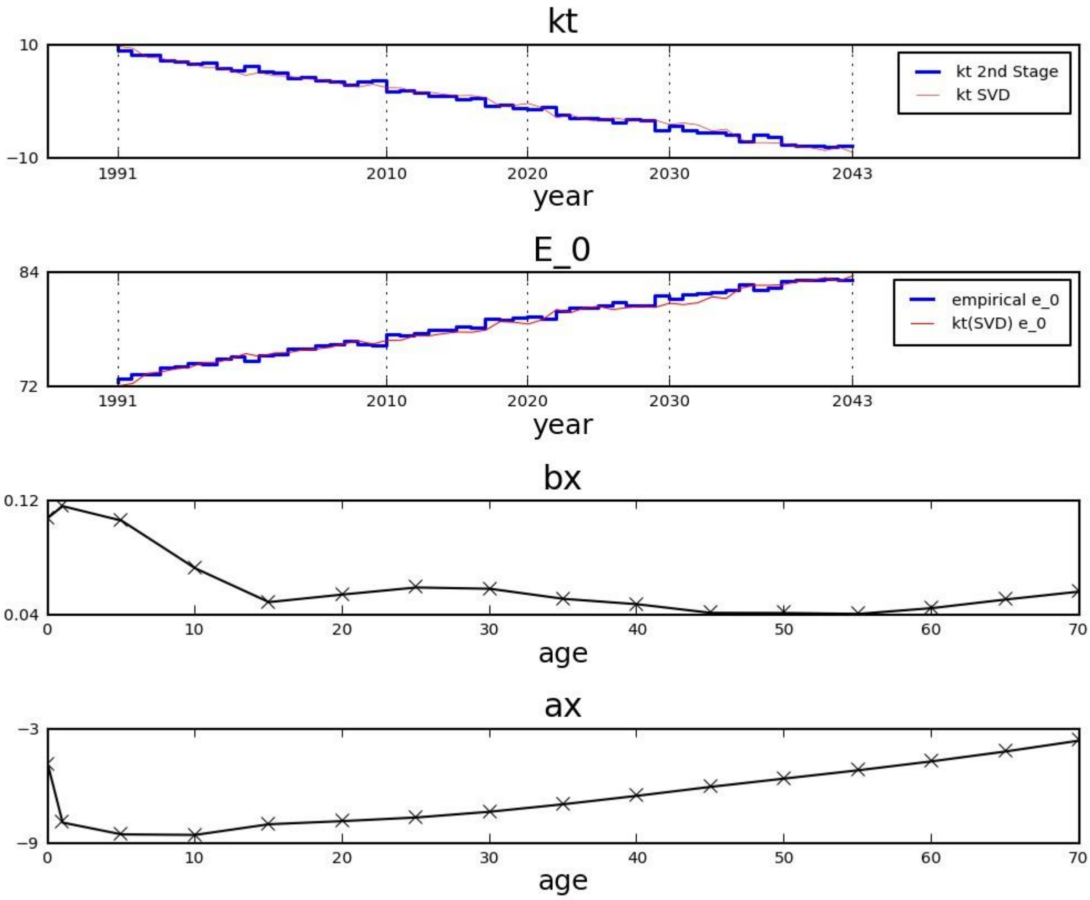

Figure 4.6: Various graphs showing the estimated parameters

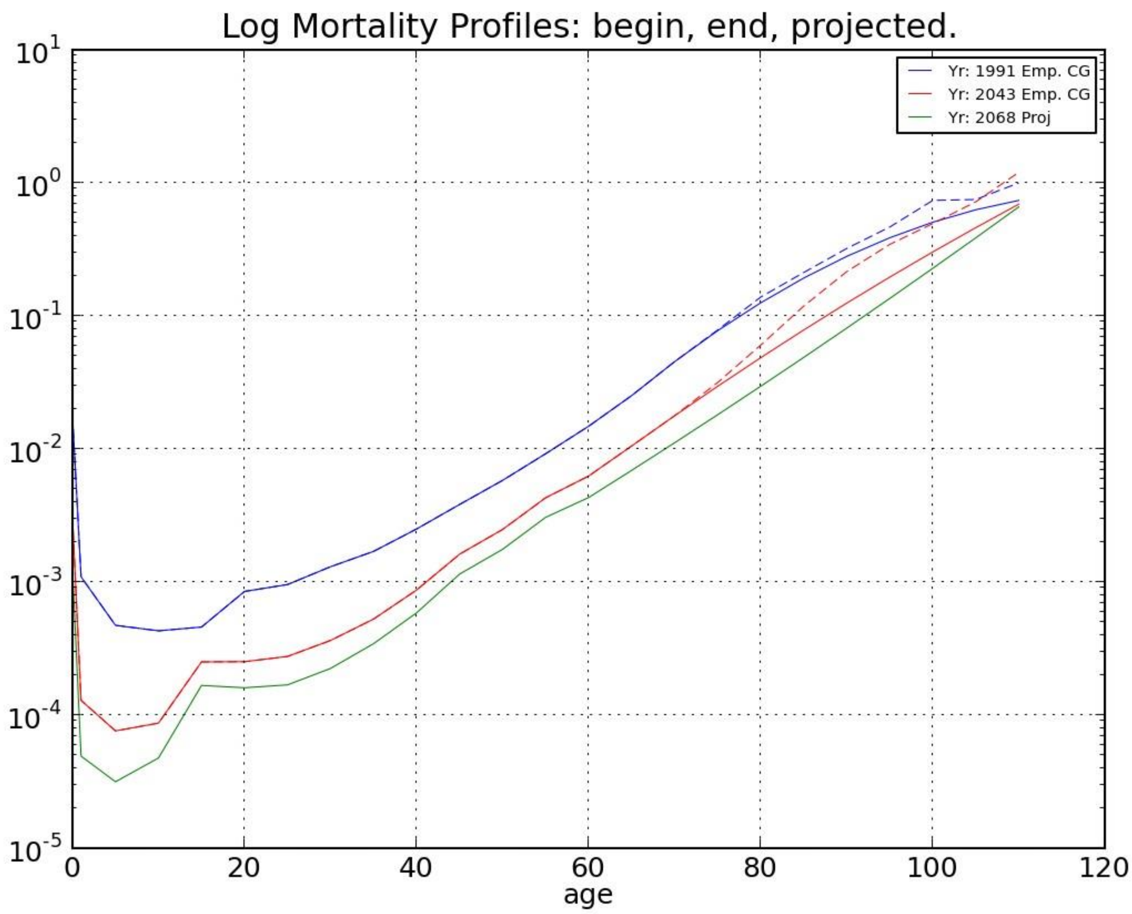

Figure 4.7: Projected Log Mortality Profile Table 4.5: Life Expectancy Projection

Item	Year (Projected)	Low Analytical e0	Median Analytical e0	High Analytical e0	Median Proj e0
	2043	83.15411681	83.15411681	83.15411681	83.15411681
	2044	82.65150402	83.34400361	84.03071979	83.20580123
	2045	82.54542513	83.53345345	84.50951847	83.53537109
	2046	82.50231269	83.72246022	84.92404101	83.47022448
	2047	82.49090648	83.91101733	85.30550757	83.80454874
	2048	82.49921766	84.09911779	85.66586632	84.02790842
	2049	82.521159	84.28675419	86.01116333	84.30284259
	2050	82.55315142	84.47391874	86.34493622	84.29750549

	2051	82.59288744	84.66060328	86.66945077	84.5631904
	2052	82.63878153	84.84679934	86.98625075	84.26175521
	2053	82.68969192	85.03249809	87.29643619	84.59377786
	2054	82.74476631	85.21769044	87.60081782	84.52916099
	2055	82.80335002	85.40236701	87.90000903	85.58220096
	2056	82.86492837	85.58651818	88.19448355	85.59027734
	2057	82.92908888	85.77013407	88.4846134	85.14878582
	2058	82.99549553	85.95320462	88.77069468	85.51219056
	2059	83.0638708	86.13571955	89.0529657	86.19591052
	2060	83.13398264	86.31766841	89.33161997	86.27805226
	2061	83.20563502	86.49904062	89.60681584	86.5977292
	2062	83.27866075	86.67982542	89.87868367	86.92772833
	2063	83.35291608	86.86001198	90.14733136	87.2612166
	2064	83.42827646	87.03958933	90.41284863	86.96066737
	2065	83.50463327	87.21854643	90.67531033	86.97941254
	2066	83.58189117	87.3968722	90.93477912	86.31587959
	2067	83.65996607	87.57455546	91.19130765	86.47662737
	2068	83.73878335	87.75158505	91.44494025	86.90210602

Probability of surviving beyond guarantee period ${}_{15}P_{60} = \prod_{i=1}^{15} {}_{i}P_{60}$ The Pension company will enter a deferred life annuity contract to insure each life after the guarantee period.

The deferred life annuity is given as $15|\ddot{a}_{60} = \ddot{a}_{60} - \ddot{a}_{60:\overline{15|}}$ The PV of the annuity $= V^{15}(\ddot{a}_{60}^{12} - \ddot{a}_{60:\overline{15|}}^{12})$

$$\Rightarrow Premium = \mathrm{V}^{15}(\ddot{a}_{60}^{12} - \ddot{a}_{60:\overline{15|}}^{12})$$

For each pensioner aged 60 a premium of P is paid by pension company to the insurer in exchange for a monthly income till death.

CHAPTER 5

CONCLUSION

5.1 Summary of Main Findings

From the LCfit model outputs, the Lee Carter model fitted well for the mortality rate of the male pensioners despite the irregularity of the mortality pattern. The predicted logarithm central death rate follows an increasing trend but start to decrease at the higher ages.

The predicted $K_{svd(t)}$ values show a slow downward trend in the mortality but the general mortality pattern depicts an increasing trend. This shows that the LCfit model gives more sensitive and dynamic prediction estimation of the mortality trend in the future. It has shown that, mortality is really improving and there is a high possibility that a lot of pensioners will still be alive after the 15 years of the guarantee period suggested by SSNIT. The study also tried to hedge this longevity risk by proposing a yearly premium that the SSNIT should adopt life insurance companies and pay to handle the pensioners after the guarantee period to another period of five years.

5.2 Conclusion

Using the Lee Carter model to project the male future mortality trend has shown that there is longevity improvement which confirm the study of Hangli et al (2007) that is most suitable to use Lee Carter model than Cairm Blake Dowd(CBD) model which only gives suitable graduation A fast growing in longevity is imposing a huge risk to SSNIT of which it trying to manage itself. This study with its objective to forecast male mortality trend using Lee Carter prediction model and how to hedge the longevity risk using annuities; - suggest that SSNIT should re-insured the pensioners after the guarantee period to private trustee and life companies to reduce pressure on the social scheme. It also suggests that, individual longevity risk can be hedged through acquiring a life annuity from private life insurance companies and that it should be affordable in the middle age class. In conclusion, predicting mortality trend and hedging of longevity risk in recent times is getting more attention at a period when life expectancy is increasing unexpectedly, confirming Oeppen & Vaupel (2002). And hence the use of Lee Carter model is the best fit model for the pension schemes and life insurance companies.

5.3 Recommendation

From the above discussion on the summary and conclusion, the researcher is recommending that, Ghanaians retirement age should be in synchronized with the life expectancy increment; - increasing the retirement age. This study also recommends that the hedging of longevity risk can also be done by using longevity bonds so any further study should take a look around it.

Reference

Aad, G., Abbott, B., Abdallah, J., Khalek, S. A., Abdinov, O., Aben, R., & Alpigiani, C. (2013). Search for the decay of the Standard Model Higgs boson in associated (W/Z) H production with the ATLAS detector. *Journal of High Energy Physics, 2015(1),* 1-89.

Agyepong, I. A., Kodua, A., Adjei, S., & Adam, T. (2012). When ’solutions of yesterday become problems of today’: crisis-ridden decision making in a complex adaptive system (CAS)-the Additional Duty Hours Allowance in Ghana. Health policy and planning, 27(suppl 4), iv20-iv31.

Avenell, A., Broom, J. I., Brown, T. J., Poobalan, A., Aucott, L. S., Stearns, S. C., & Grant, A. M. (2004). Systematic review of the long-term effects and economic consequences of treatments for obesity and implications for health improvement. Health technology assessment.

Belanger, H. G., Vanderploeg, R. D., Curtiss, G., & Warden, D. L. (2014). Recent neuroimaging techniques in mild traumatic brain injury.

Brouhns, N., Denuit, M., & Vermunt, J. K. (2002). Measuring the longevity risk in mortality projections. *Bulletin of the Swiss Association of Actuaries,* 2, 105-130.

Cairns, A. J., Blake, D., & Dowd, K. (2006). A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration. *Journal of Risk and Insurance, 73(4),* 687-718.

Cairns, A. J., Blake, D., Dowd, K., Coughlan, G. D., Epstein, D., Ong, A., & Balevich, I. (2009). A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. *North American Actuarial Journal, 13(1),* 1-35.

Cao, B., Li, X. W., Mao, Y., Wang, J., Lu, H. Z., Chen, Y. S., ... & Wang, C.

(2009). Clinical features of the initial cases of 2009 pandemic influenza A (H1N1) virus infection in China. *New England Journal of Medicine, 361(26),* 2507-2517.

Carter, L. R., & Lee, R. D. (1992). Modeling and forecasting US sex differentials in mortality. International *Journal of Forecasting, 8(3),* 393-411. Chapman, J. R., Dozois, E. J., Wolff, B. G., Gullerud, R. E.,& Larson, D. R. (2006). Diverticulitis: A progressive disease?: Do multiple recurrences predict less favorable outcomes?. *Annals of surgery, 243(6),* 876.

Chen, C. K., Chen, S. L., Mill, J., Huang, Y. S., Lin, S. K., Curran, S., ... & Asherson, P. (2003). The dopamine transporter gene is associated with attention deficit hyperactivity disorder in a Taiwanese sample. Molecular psychiatry, 8(4), 393-396.

Dahl, M., Melchior, M., & Moller, T. (2008). On systematic mortality risk and risk-minimization with survivor swaps. *Scandinavian Actuarial Journal, 2008(2-3),* 114-146.

Denuit, M., Devolder, P., & Goderniaux, A. C. (2007). Securitization of Longevity Risk: Pricing Survivor Bonds With Wang Transform in the Lee-Carter Framework. *Journal of Risk and Insurance, 74(1),* 87-113.

Dowd, K. (2003). Survivor bonds: a comment on Blake and Burrows. *Journal of Risk and Insurance, 70(2),* 339-348.

Dowd, K., Blake, D., Cairns, A. J., & Dawson, P. (2006). Survivor swaps. *Journal of Risk and Insurance, 73(1),* 1-17.

Frees, E. W., Carriere, J., & Valdez, E. (1996). Annuity valuation with dependent mortality. *Journal of Risk and Insurance*, 229-261.

Friedlingstein, P., Cox, P., Betts, R., Bopp, L., Von Bloh, W., Brovkin, V., & Zeng, N. (2006). Climate-carbon cycle feedback analysis: Results from the C4MIP model intercomparison. *Journal of Climate, 19(14),* 3337-3353.

Goshay, R. C., & Sandor, R. (1973). An inquiry into the feasibility of a reinsurance futures market. *Journal of Business Finance, 5(2),* 56-66.

Gu, G., Yuan, J., Wills, M., & Kasper, S. (2007). Prostate cancer cells with stem cell characteristics reconstitute the original human tumor in vivo. *Cancer research, 67(10),* 4807-4815.

Huang, D. W., Sherman, B. T., & Lempicki, R. A. (2008). Systematic and integrative analysis of large gene lists using DAVID bioinformatics resources. Nature protocols, 4(1), 44-57.

Koissi, M. C., Shapiro, A. F., & Högnäs, G. (2006). Evaluating and extending the Lee?Carter model for mortality forecasting: Bootstrap confidence interval. Insurance: Mathematics and Economics, 38(1), 1-20.

Kpessa, M. W. (2010). Ideas, Institutions, and Welfare Program Typologies: An Analysis of Pensions and Old Age Income Protection Policies in Sub-Saharan Africa. Poverty & Public Policy, 2(1), 37-65.

Kpessa, M. W. (2011). A comparative analysis of pension reforms and challenges in Ghana and Nigeria. *International Social Security Review, 64(2),* 91-109.

Law, D. G., Maclehose, R. F., & Longnecker, M. P. (2007). Obesity and time to pregnancy. *Human Reproduction, 22(2),* 414-420.

Lee, R. D., & Carter, L. R. (1992). Modeling and forecasting US mortality. *Journal of the American statistical association, 87(419),*659-671.

MacMinn, R., Brockett, P., & Blake, D. (2006). Longevity risk and capital markets. *Journal of Risk and Insurance*, 73(4), 551-557.

Marceau, E., & Gaillardetz, P. (1999). On life insurance reserves in a stochastic mortality and interest rates environment. Insurance: Mathematics and Economics, 25(3), 261-280.

Melnikov, A., & Romaniuk, Y. (2006). Evaluating the performance of Gompertz, Makeham and Lee-Carter mortality models for risk management with unit-linked contracts. Insurance: Mathematics and Economics, 39(3), 310-329.

Milevsky, M. A., Moore, K. S., & Young, V. R. (2006). ASSET ALLOCATION AND ANNUITY-PURCHASE STRATEGIES TO MINIMIZE THE PROBABILITY OF FINANCIAL RUIN. *Mathematical Finance, 16(4),* 647-671.

Milevsky, M. A., & Promislow, S. D. (2001). Mortality derivatives and the option to annuitise. Insurance: Mathematics and Economics, 29(3), 299-318.

Oeppen, J., & Vaupel, J. W. (2002). Broken limits to life expectancy. Science,296(5570), 1029-1031.

Piggott, J., Valdez, E. A., & Detzel, B. (2005). The simple analytics of a pooled annuity fund. *Journal of Risk and Insurance, 72(3),* 497-520.

Renshaw, A. E., & Haberman, S. (2003). Lee-Carter mortality forecasting with age-specific enhancement. Insurance: *Mathematics and Economics, 33(2),* 255-272.

Sabeti, P. C., Varilly, P., Fry, B., Lohmueller, J., Hostetter, E., Cotsapas, C., ... & Faggart, M. (2007). Genome-wide detection and characterization of positive selection in human populations. *Nature, 449(7164),* 913-918.

Stallard, E. (2006). Demographic issues in longevity risk analysis. *Journal of Risk and Insurance, 73(4),* 575-609.

Stevens, R., De Waegenaere, A., & Melenberg, B. (2010). Longevity risk in pension annuities with exchange options: The effect of product design. Insurance: *Mathematics and Economics, 46(1),* 222-234.

Sun, H. Longevity Risk Securitization of Housing Reverse Mortgages in Future China.

Tsai, J. T., Wang, J. L., & Tzeng, L. Y. (2010). On the optimal product mix in life insurance companies using conditional value at risk. Insurance: *Mathematics and Economics, 46(1),* 235-241.

Tuljapurkar, S., & Boe, C. (1998). Mortality change and forecasting: how much and how little do we know?. *North American Actuarial Journal, 2(4),* 13-47.

Wang, C. W., & Yang, S. S. (2013). Pricing survivor derivatives with cohort mortality dependence under the Lee-Carter framework. *Journal of Risk and Insurance, 80(4),* 1027-1056.

Wang, J. L., Huang, H. C., Yang, S. S., & Tsai, J. T. (2010). An optimal product mix for hedging longevity risk in life insurance companies: The immunization theory approach. *Journal of Risk and Insurance*, 77(2), 473-497.

Wills, S., & Sherris, M. (2010). Securitization, structuring and pricing of longevity risk. Insurance: *Mathematics and Economics, 46(1),* 173-185.

Woodbury, M. A., & Manton, K. G. (1977). A random-walk model of human mortality and aging. Theoretical Population Biology, 11(1), 37-48.

Wu, Z., & Huang, N. E. (2009). Ensemble empirical mode decomposition: a noise-assisted data analysis method. Advances in adaptive data analysis, 1(01), 1-41.

Yang, W., Lu, J., Weng, J., Jia, W., Ji, L., Xiao, J., ... & Zhu, D. (2010). Prevalence of diabetes among men and women in China. New England Journal of Medicine, 362(12), 1090-1101.

Yashin, A. I., Manton, K. G., & Vaupel, J. W. (1985). Mortality and aging in a heterogeneous population: a stochastic process model with observed and unobserved variables. *Theoretical population biology, 27(2),* 154-175.

APPENDIX A

Appendix A shows the number of death from 1991 to 2013 and number of people retiring at age 60 from 1991 t0 2013

Table 5.1: NUMBER OF DEATHS FROM 1991 - 2003

0	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1991	1	2	6	42	50	42	46	38	51	46	58	34	47
1992		2	10	43	53	44	54	59	51	62	41	60	42
1993			7	36	58	72	83	72	83	94	67	6	2 76
1994				11	40	86	70	86	59	72	83	78	83
1995					12	54	80	101	98	79	69	93	81
1996						12	56	73	63	52	61	77	54

1997							17	70	89	85	91	75	103
1998								11	57	77	60	74	84
1999									21	66	88	84	100
2000										23	94	107	82
2001											17	52	66
2002												24	100
2003													15

Table 5.2: NUMBER OF DEATHS FROM 2004 - 2013

0	2004	2005	2006	2007	2008	2009	2010
1991	42	33	56	64	68	78	139
1992	38	41	60	63	61	81	176
1993	49	61	79	114	105	107	193
1994	67	67	72	47	38	25	35
1995	87	71	77	56	47	32	28
1996	67	65	43	37	35	29	22
1997	90	94	87	73	67	51	42
1998	78	70	75	61	48	19	32
1999	103	96	115	91	76	77	64
2000	51	112	119	132	103	78	68
2001	66	69	61	64	56	63	66
2002	107	100	90	114	109	87	88
2003	65	90	82	80	87	81	70
2004	23	93	97	89	93	82	105
2005		23	96	122	100	97	98
2006			28	84	116	82	119
2007				29	99	95	108

2008					31	98	110
2009						32	101
2010							37

Table 5.3: MUMBER RETIRING AT AGE 60 FROM 1991 - 2003

1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1042	1041	1039	1033	991	941	899	853	815	764	718	660	626
	1065	1063	1053	1010	957	913	859	800	749	687	646	586
		1421	1414	1378	1320	1248	1165	1093	1010	916	849	787
			1198	1187	1147	1061	991	905	846	774	691	613
				1858	1846	1792	1712	1611	1513	1434	1365	1272
					1516	1504	1448	1375	1312	1260	1199	1122
						1728	1711	1641	1552	1467	1376	1301
							1613	1602	1545	1468	1408	1334
								2264	2243	2177	2089	2005
									2331	2308	2214	2107
										1469	1452	1400
											2352	2328
												1932

Table 5.4: NUMBER RETIRING AT AGE 60 FROM 2004 - 2011

2004	2005	2006	2007	2008	2009	2010	2011
579	537	504	448	384	316	238	99
544	506	465	405	342	281	200	24
711	662	601	522	408	303	196	3
530	463	396	324	277	239	214	179
1191	1104	1033	956	900	853	821	793
1068	1001	936	893	856	821	792	770
1198	1108	1014	927	854	787	736	694
1250	1172	1102	1027	966	918	899	867
1905	1802	1706	1591	1500	1424	1347	1283
2025	1974	1862	1743	1611	1508	1430	1362
1334	1268	1199	1138	1074	1018	955	889
2228	2121	2021	1931	1817	1708	1621	1533
1917	1852	1762	1680	1600	1513	1432	1362
2383	2360	2267	2170	2081	1988	1906	1801
	2947	2924	2828	2706	2606	2509	2411
		2818	2790	2706	2590	2508	2389
			3389	3360	3261	3166	3058
				3845	3814	3716	3606
					4098	4066	3965
						4626	4589

APPENDIX B

Appendix B shows the probability of death by age and year

Table 5.5: PROBABILITY OF DEATH BY AGE AND YEAR FROM 1991 - 1997

1991	1992	1993	1994	1995	1996	1997
0.000959693	0.00192123	0.005774783	0.040658277	0.050454087	0.044633369	0.051167964

	0.001877934	0.009407338	0.040835708	0.052475248	0.045977011	0.059145674
		0.004926108	0.025459689	0.042089985	0.054545455	0.06650641
			0.00918197	0.033698399	0.074978204	0.065975495
				0.006458558	0.029252438	0.044642857
					0.007915567	0.037234043
						0.009837963

Table 5.6: PROBABILITY OF DEATH BY AGE AND YEAR FROM 1998 - 2004

1998	1999	2000	2001	2002	2003	2004
0.044548652	0.062576687	0.060209424	0.080779944	0.051515152	0.075079872	0.07253886
0.068684517	0.06375	0.082777036	0.059679767	0.092879257	0.071672355	0.069852941
0.061802575	0.075937786	0.093069307	0.073144105	0.073027091	0.09656925	0.068917018
0.086781029	0.06519337	0.085106383	0.107235142	0.112879884	0.135399674	0.126415094
0.058995327	0.060831782	0.052214144	0.048117155	0.068131868	0.063679245	0.073047859
0.050414365	0.045818182	0.039634146	0.048412698	0.064220183	0.048128342	0.062734082
0.040911748	0.054235222	0.054768041	0.062031357	0.054505814	0.079169869	0.075125209
0.006819591	0.035580524	0.049838188	0.040871935	0.052556818	0.062968516	0.0624
	0.009275618	0.029424877	0.0404226	0.040210627	0.049875312	0.054068241
		0.00986701	0.040727903	0.048328817	0.038917893	0.025185185
			0.011572498	0.035812672	0.047142857	0.049475262
				0.010204082	0.042955326	0.048025135
					0.007763975	0.033907147
						0.0096517

Table 5.7: PROBABILITY OF DEATH BY AGE AND YEAR FROM 2005 - 2010

2005	2006	2007	2008	2009	2010
0.06145251 4	0.11111111 1	0.14285714 3	0.17708333 3	0.24683544 3	0.58403361 3

0.081027668	0.129032258	0.155555556	0.178362573	0.288256228	0.88
0.092145015	0.131447587	0.218390805	0.257352941	0.353135314	0.984693878
0.144708423	0.181818182	0.145061728	0.137184116	0.10460251	0.163551402
0.064311594	0.074540174	0.058577406	0.052222222	0.037514654	0.03410475
0.064935065	0.045940171	0.041433371	0.04088785	0.035322777	0.027777778
0.084837545	0.085798817	0.078748652	0.078454333	0.06480305	0.057065217
0.059726962	0.068058076	0.0593963	0.049689441	0.020697168	0.035595106
0.05327414	0.067409144	0.057196732	0.050666667	0.054073034	0.047512992
0.056737589	0.063909774	0.075731497	0.063935444	0.051724138	0.047552448
0.054416404	0.05087573	0.056239016	0.052141527	0.061886051	0.069109948
0.047147572	0.04453241	0.059036769	0.059988993	0.050936768	0.054287477
0.048596112	0.046538025	0.047619048	0.054375	0.053536021	0.048882682
0.03940678	0.042787825	0.041013825	0.044690053	0.041247485	0.055089192
0.007804547	0.032831737	0.043140028	0.036954915	0.037221796	0.039059386
	0.009936125	0.030107527	0.042867701	0.031660232	0.047448166
		0.008557096	0.029464286	0.029132168	0.034112445
			0.008062419	0.025694809	0.029601722
				0.007808687	0.024840138

					0.007998271