KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

QUEUING IN HEALTH CARE CENTRES.

A CASE STUDY OF THE OUTPATIENT DEPARTMENT OF THE NORTH SUNTRESO HOSPITAL, KUMASI.

BY

ACHEAMPONG, LINDA

PG6186511

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MPHIL MATHEMATICS

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DECLARATION

I hereby declare that this submission is my own work and that no part of it has been accepted for the reward of any other degree of the University, except where due acknowledgement has been made in the text.

Acheampong, Linda ……………………. …………………

PG 6186511 Signature Date

Certified by:

Prof. S. K. Amponsah ……………….. ..……………….

Supervisor Signature Date

Prof. S. K. Amponsah ……………… ……………......

Head of department Signature Date

DEDICATION

This thesis is dedicated to the Alpha and Omega God, who is the beginning and the end of my life. It is also dedicated to my parents, Mr. and Mrs. I. K. Acheampong, my siblings, Mavis, Philophatra and Francis and also to my husband, David Nkrumah.

ACKNOWLEDGEMENT

My ultimate gratitude is owed to the Most High God for the wisdom, knowledge, understanding and the protection given to me to go through this work.

My gratitude also goes to my supervisor, Prof. S. K. Amponsah. I say, thank you very much for your time, instructions, directions and corrections that made this project to come to a successful end. God richly bless you.

I am grateful to the management and staff of the North-Suntreso Hospital for allowing me to use their hospital as a case study for this work.

Finally, my special thanks go to my dad for his encouragement and all my course mates.

ABSTRACT

Queuing theory is the mathematical study of waiting lines. The theory calculates the average time a customer spends in a system, the average time a customer spends in a line, and the average time a customer spends in service. The Out-Patient (OPD) department in any hospital is considered to be the shop window of the hospital and it is the most important service provided by all hospitals as it is the point of contact between a hospital and the community. The objective of the study was to find out the average number of arrivals entering the Out-Patient Department (OPD) of the North-Suntreso Hospital, the average service time of patients and the probability that a patient has to wait for a doctor. It also found out the average time a patient spends waiting for a doctor and the average number of patients who were present at the department. A stop watch was used to calculate the number of minutes spent by each patient from the record, history and the consulting room sections. Data were collected on Monday to Friday from the hours of 8:00 am to 12:00 pm. The number of patients from each section was taken. Queuing model was used to analyze the data gathered on patients" arrival and service times. It was realized that Wednesday recorded the highest arrival rates of patients. The consulting room recorded the highest number of patients waiting in the queue on Wednesday. On the average, a patient spent the following number of minutes in the department before leaving the hospital;

38.6525 on Monday, 16.2712 minutes on Tuesday, 47.9112 minutes on Wednesday, 18.436 minutes on Thursday and 31.0427 minutes on Friday. The results, thus, shows that patients queued for long in the entire system before receiving treatment, which made them uncomfortable.

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CHAPTER ONE

1.0 INTRODUCTION

Queuing theory describes basic phenomena such as the waiting time, the throughput, the losses, the number of queuing items, etc. in queuing systems. Following Kleinrock (1975), any system in which arrivals place demands upon a finite-capacity resource can be broadly termed a queuing system.

A queuing process consists of customers arriving at a service facility, waiting in a line (queue) if all servers are busy, then receiving service and finally departing from the facility. A queuing system is a set of customers, a set of servers and an order where by customers arrive and processed. Queues (waiting lines) are a part of everyday life. We all wait in queues to buy a movie ticket, make a deposit, mail a package, obtain food in cafeteria, pay for goods, etc. queues are formed because resources are limited.

Time is always a valuable asset for patients in seeking treatment at any healthcare centre, either public or private, and even more valuable for patients who are in critical conditions. Doctors" and specialists need to maximize their service time since some of them are assigned with administrative works, reading medical reports, and keep moving from one department to another. Waiting idly in the waiting room is not a productive situation where patients can spend their waiting time to do other activities that might benefit them rather than sitting for nothing.

Whenever the demand for a service exceeds its supply then queues are formed.

Long waiting time in any hospital is considered as an indicator of poor quality and needs improvement. Managing waiting lines create a great problem for managers seeking to improve upon quality healthcare delivery and patient satisfaction. Patients dislike waiting for a long time. For many patients, queuing or waiting in lines is annoying (Obamiro, 2003) or negative experience (Scotland, 1991). If the waiting time and service time is high, customers may leave the queue prematurely and this in turn results in customer dissatisfaction. This would reduce patients demand and eventually reduce revenue and profits gained by hospitals.

The study is designed to help the management of North-Suntreso Hospital about the employee adequacy and also help to reduce patient waiting time for services.

Hence, this chapter critically discusses the background to the study. The chapter also looks at the statement of the problem, the objective of the study and the significance of the study. It also highlights on the research methodology used, the scope, limitations and the organization of the study.

1.1 BACKGROUND TO THE STUDY

A queue is a waiting line, whether of people, signals or things (Ashhley, 2000). Queuing time is the amount of time a person, signal or a thing spends before being attended to for services.

Queuing theory is the Mathematical study of waiting lines. Queuing theory is generally considered as a branch of operations research because its results are often used when making business decisions about the resources needed to provide services. The theory seeks to determine how best to design and operate a system usually under conditions requiring allocation of scarce resources.

Queuing models are those where a facility performs a service. A queuing problem arises when the current service rate of a facility falls short of the current flow rate of customers. If the size of the queue happens to be a large one, then at times it discourages customers who may leave the queue and if that happens, then a sale is lost by the concerned business unit. Hence, the queuing theory is concerned with the decision making process of the business unit which confronts with queue questions and makes decisions relative to the numbers of service facilities which are operating.

The earliest use of queuing theory was in the design of a telephone system. A queue can be studied in terms of the source of each queued item, how frequently items arrive on the queue, how long the item can or should wait, whether some items should jump ahead in the queue, how multiple queues might be formed and managed, and rules by which items are queued or de-queued.

The queuing theory enables mathematical analysis of several related processes, including arriving at the (back of the) queue, waiting in the queue (essentially a storage process), and being served at the front of the queue.

The queuing theory permits the derivation and calculation of several performance measures including the service, and the probability of encountering the system in certain states such as empty, full, having an available server or having to wait a certain time to be served. (Vohra, 2010).

The subject of queuing theory can be described as follows: consider a service centre and the population of customers, which at sometimes enter the system in order to obtain service. It is often the case that the service centre can only serve a limited number of customers. If a new customer arrives and the service is exhausted, he/she enters a waiting line and waits until the service facility becomes available. So we can identify three main elements of service centre: a population of customers, the service facility and the waiting line.

Queues are formed because of limited resources and they are experienced almost every day. Thus queuing theory calculates the average time a customer spends in a system, the average time a customer spends in a line, and the average time a customer spends in service.

Queuing theory is applicable to intelligent transportation systems, call centres, advanced telecommunications.

Queuing Theory tries to answer questions like e.g. the mean waiting time in the queue, the mean system response time (waiting time in the queue plus service times), mean utilization of the service facility, distribution of the number of customers in the queue, distribution of the number of customers in the system and so forth. These questions are mainly investigated in a stochastic scenario, where e.g. the inter arrival times of the customers or the service times are assumed to be random.

1.1.1 The Out-Patient Department (OPD)

A hospital is an integral part of a social and medical organization, the function of which is to provide for the population complete healthcare, both curative and preventive, and whose out-patient services reach out to the family and its home environment. The out-patient department in any hospital is considered to be the shop window of the hospital and it is the most important service provided by all hospitals as it is the point of contact between a hospital and the community.

Nowadays, Out-Patient Department (OPD) services of the majority of hospitals are having queuing and waiting time problems. Patients‟ waiting time refers to the time from the registration of the patient for appointment with doctor till they enter the doctors‟ chamber.

The main objective of the out-patient department of a hospital should include the reduction of patients‟ time in the system, improvement on customer service, better resource utilization and reduction of operating costs.

Waiting time in out-patient departments is a problem throughout the world. One consistent feature of patient dissatisfaction has been expressed with the lengths of waiting time in the out-patient department.

Various functions affecting the services of an OPD are:

(i) Arrival pattern or input rate of patients at the central waiting room.

(ii) Services time at various clinics of OPD.

(iii) Queue lengths at waiting rooms of clinics of OPD.

The out-patient department provides health care to infants, children, adolescents, adults, and geriatric patients in need of non-emergency physician care.

The waiting time is particularly important for a hospital, since the “customers” are patients. Long wait create customer dissatisfaction on one hand and resources inefficiencies on the other hand. (Bharali, 2010).

1.1.2 Areas where queuing theory could be applied in health care centers

(i) Public health

Queuing models can be used for public health. For example, the resources needed for Mass Vaccination camp in a particular area, facility and resource planning for emerging or changing disease profiles or changing demographics.

(ii) Health care resource and infrastructure planning for disaster management Any type of disaster cause significant human and economic damage and they all demand a crisis response. It demands immediate rescue of people, provision of medical services needed and containment of the damage to people and property. In such scenarios, queuing models are frequently used in conjunction with simulation to answer the "what-if" questions, to plan, organize and be prepared for the calamities.

(iii) Hospital pharmacy and pharmacy stores

In pharmacy, queuing theory can be used to assess a multitude of factors such as prescription fill time, patient waiting time, patient counseling- time and staffing levels. The application of queuing theory may be a particular benefit in pharmacies with high volume outpatient workloads and or those that provide multiple points of service. By better understanding queuing theory, service managers can make decisions that increase the satisfaction of all relevant groups - customers, employees and management.

(iv) Walk-in-patients in physician offices, out-patient clinics and out-patient surgeries in hospitals

The management of healthcare facilities such as out-patient clinics is very complex and demanding to manage. The most common objectives of studies on the clinics have included the reduction of patient"s time in the system (out-patient clinic), improvement on customer service, better resource utilization and reduction of operating costs. (Bharali, 2010).

1.2 STATEMENT OF THE PROBLEM

Before the introduction of the National Health Insurance Scheme, the Ghana Health Service operated on the „Cash and Carry" system. This system made it compulsory for all Ghanaians to pay money immediately before and after treatment in the hospitals or clinics etc. As a result of this, most Ghanaians were not going to the hospitals and thus resulting in many deaths. The National Health Insurance Scheme was introduced in the year 2003 and the government of Ghana used this to replace the „Cash and Carry System".

With the introduction of the Health Insurance Scheme, there has been a remarkable increase in attendance at both the government and private hospitals. The major problem facing the hospital is that attendance has increased, the hospitals/clinics capacity still remains the same and patients have to queue a long time for service.

The North-Suntreso Hospital is one of the hospitals in Kumasi.

Its out-patient department seeks to:

(i) provide quality healthcare

(ii) increase access to health service

(iii) to improve upon its health service delivery.

(iv) reduce patients waiting time.

1.3 OBJECTIVES

The main purpose of this study is to look at the outpatient department of the North-Suntreso Hospital. Specifically, this theory seeks to find:

(i) the average number of arrivals entering the outpatients departments of the hospital (ii) the average service time of customers at various sections of the outpatient department.

(iii) the probability that a patient has to wait for a doctor.

(iv) the average time a patient spends waiting for a doctor.

(v) the average number of patients present at the outpatient department of the hospital.

1.4 SIGNIFICANCE OF THE STUDY

The findings of this study is hope to assist the hospital administrators and management to reduce the waiting time of patients in the system (outpatient clinic), improve on customer service, and maximize the utilization of its resources (doctors, nurses, hospital beds, etc). The findings could also be used for appropriate staffing and facilities design.

1.5 RESEARCH METHODOLOGY

The data used for this study was obtained from the North-Suntreso Hospital. A stop watch was used to calculate patients‟ waiting time from the records section, history section and the consulting room section. The data was gathered on five different days (from Monday to Friday).

The queuing discipline used for this study was the first come first served discipline.

1.6 SCOPE OF THE STUDY

The study looked at the outpatient department of the North-Suntreso Hospital in Kumasi. It focused on the number of patients who came to the outpatient department ward for health care delivery on the 8th to the 12th of October 2012 between the hours of 8:00 am – 12:00 pm.

It looked at the waiting and service times of patients to see how much time they spend in the outpatient department before they leave to their homes. These were done to achieve the objectives of the study and to know the number of minutes or hours the service providers in the hospital spend on their patients.

1.7 LIMITATIONS OF THE STUDY

The research is limited only to the out - patient department of the North – Suntreso Hospital. The data collection is limited to patients who arrived at the department before 12:00 pm.

The data gathered should have been on every patient who attended the hospital during the time that the study was conducted but due to time and financial constraints, the researcher was limited to patients who arrived at the hospital at 12:00 pm on the number of days that the study was conducted.

The researcher could not follow the patients" whiles they moved from one section of the department to the other.

Preferably, the number of patients whose data were collected in the record section should have been the same number of patients whose data were gathered in the other two sections. Thus, the patients used in the record section are not the same patients used in the history and consulting room sections. This happened because of the time limit put on the study.

It is however hoped that these shortfalls would not affect the findings of the study.

1.8 ORGANIZATION OF THE STUDY

The study is organized into five chapters. Chapter one deals with the background, problem statement, objectives, scope, limitations and research methodology of the study.

Chapter two presents the review of pertinent literature. In chapter three, we shall put forward the research methodology of the study.

Chapter four is devoted for data collection and analysis. Chapter five which is the final chapter of the study presents summary of findings, conclusions and recommendation.

1.9 SUMMARY

In this chapter, we considered the background, problem statement and objectives of the study. The significance, research methodology, scope and limitations of the study were also put forward.

The next chapter presents relevant and adequate literature on the application of queuing theory.

CHAPTER TWO

LITERATURE REVIEW

2.0 OVERVIEW

This chapter reviews relevant and adequate literature on queuing theory. It contains other research findings that will assist the researcher and serve as a guide for future research. The chapter also highlights on the history behind the theory being used and the basics of queuing theory.

2.1 RELATED WORKS

Queuing theory application is an attempt to minimize the cost of providing health care services through minimization of efficiencies and delays in the system (Singh, 2006).

McClain (1976) reviewed research on queuing models for evaluating bed assignment policies on utilization, waiting time, and the probability of turning away patients. Katz et al., (1991) put forward that hospitals, airline companies, banks, manufacturing firms etc., try to minimize the total waiting cost, and the cost of providing service to their customers. Therefore, speed of service is increasingly becoming a very important competitive parameter.

Green (2006a) presented the theory of queuing as applied in health care. She discusses the relationship among delays, utilization and the number of servers; the M/M/s models, its assumptions and extensions and the application of the theory to determine the required number of servers.

Nosek and Wilson (2001) reviewed the used queuing theory in pharmacy applications with particular attention to improving customer satisfaction. Customer satisfaction is improved by predicting and reducing waiting time and adjusting staffing.

The provision of ever-faster service, with the ultimate goal of having zero customer waiting time, has recently received managerial attention for several reasons (Davis et al., 2003).

In the more highly developed countries, where standards of living are high, time becomes more valuable as a commodity and consequently, customers are less willing to wait for service.

This is a growing realization by organizations that the way they treat their customers today significantly impact on whether or not they will remain loyal customers tomorrow.

Advances in technology such as computers, internet etc., have provided firms with the ability to provide faster services. For these reasons hospital administrators, physicians and managers are continuously finding means to deliver faster services, believing that the waiting will affect after service evaluation negatively.

Understanding the inefficiencies in the hospital and improving them is crucial for making health care policy and budgeting decisions (Wilson and Nguyen, 2004).

Agnihothri and Taylor (1991) sought the optimal staffing a hospital scheduling department that handles phone calls whose intensity varies throughout the day. There are known peak and non peak periods of the day. The authors grouped the periods that receive similar call intensity and determines the necessary staffing varies dynamically with call intensity. As a result of redistributing server capacity over time, customer complaints immediately reduced without an addition of staff.

Davis and Vollman (1990) stressed that patients‟ evaluation of service quality is affected not only by the actual waiting time but also by the perceived waiting time. The act of waiting has significant impact on patients‟ satisfaction. The authors concluded that the amount of time customers must spend waiting can significantly influence their satisfaction. Shimshak et al., (1981) considered a pharmacy queuing system with preemptive service of prescription order suspends the processing of lower priority prescriptions. Different costs are assigned to wait-times for prescriptions of different priorities.

Queuing theory uses queuing models or mathematical models and performance measures to assess and hopefully improve the flow of customers through a queuing system (Gorney, 1981; Bunday, 1996).

A good patient flow means that the patient queuing is minimized while a poor patient flow means patients suffer considerable queuing delays (Hall, 1999).

Siddhartan et al., (1996) analyzed the effect on patient waiting times when primary care patients use the Emergency Department. The authors proposed a priority discipline for different categories of patients and then a first-in-first-out discipline for each category. The authors found that the priority discipline reduces the average wait time for all patients: however, while the wait time for higher priority patients reduces, lower priority patients endure a longer average waiting time.

2.2 HISTORICAL PERSPECTIVE

Queuing theory as part of probability theory has evolved from classic teletraffic engineering in the last decades. In 1909, Erlang, a Danish teletraffic engineer published a paper called The Theory of Probabilities and Telephone conversations.

In the early 1920s Erlang developed the famous model to evaluate loss probabilities of multi-channel point-to-point conversations. The author observed that a telephone system was generally characterized by either (i) Poisson input (number of calls), Exponential holding (service) time, and multiple channels (servers) or (ii) Poisson input, constant holding time and a single channel. The Erlang model was extended to allow for calculation in finite source input situations by Engset several years later leading to the Engest model. The

“Application of the Theory of Probabilities to Telephone Trucking Problems” was soon published by Molina in 1927. The uses of the queuing theory to telephone were soon used by many.

A year after Molina‟s publications, Thorton Fry, published a book on “Probability and its Engeneering Uses”. This book expands much of the work done by Erlang.

Further works were done on the Poisson input, arbitrary output, and single and multiple channel problems by Felix Pollazeck in the 1930s. Names such as Kolmogorov and Khintchine in Russia, Coromnelin in France and Palm in Sweden also started in the same field of Felix Pollazeck.

In 1951, Kendall published his work about embedded Markov chains, which is the base for the calculation of queuing systems under fairly general input conditions. The author also defined a naming convention for queuing systems which is still used. Nearly at the same time Lindley developed an equation allowing for results of a queuing system under fairly general input and service conditions.

In 1957, Jackson started the investigation of networked queues thus leading to so called queuing network models. With the appearance of computers and computer networks, queuing systems and queuing networks have been identified as a powerful analysis and design tool for various applications.

It was only after the Second World War, however, that queuing theory was boosted mainly by the introduction of computers and the digitalization of the telecommunications infrastructure. For engineers, the two volumes by Kleinrock (1975, 1976) are perhaps the most well-known, while in applied mathematics, apart from the penetrating influence of Feller (1970, 1971), the Single Server Queue of Cohen (1969) is regarded as a landmark.

Since Cohen‟s book, which incorporates most of the important work before 1969, a wealth of books and excellent papers have appeared, an evolution that is still continuing today.

2.3 A QUEUING SYSTEM

Examples of queuing abound in daily life: queuing situations at a ticket window in the railway station or post office, at the cash points in the supermarket, the waiting room at the airport, train or hospital, etc. In telecommunications, the packets arriving at the input port of a router or switch are buffered in the output queue before transmission to the next hop towards the destination. In general, a queuing system consists of (i) arriving items (packets or customers), (ii) a buffer or waiting room, (iii) a service center and (iv) departures from the system.

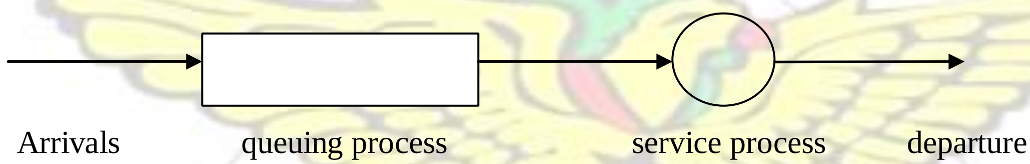


Figure 2.1 General Queuing Processes

The main processes as illustrated in Figure 2.1 are stochastic in nature. Initially in queuing theory, the main stochastic processes were described in continuous time, while with the introduction of the Asynchronous Transfer Mode (ATM) at the late eighties; many queuing problems were more effectively treated in discrete time, where the basic time unit or time slot was the minimum service time of one ATM cell. In the literature, there is unfortunately no widely adopted standard notation for the main random variables, which often troubles the transparency.

2.4 BASICS OF QUEUING THEORY

For application of queuing models to any situation we should first describe the input and output processes. The table below gives some examples of input and output processes.

Table 2.1: Examples of input and output processes

Setting	Input	Output
Hospital	Arrival of patients	Assessment, triage, provision of services, discharge
Bank	Arrivals of customers	Provision of services by tellers
Supermarket	Shoppers	Checkout centers

2.5 CHARACTERIZATION

A queuing system may be described as a system, where customers arrive according to an arrival process to be serviced by a service facility according to a service process. Each service facility may contain one or more servers. It is generally assumed that each server can only service one customer at a time. If all servers are busy, the customer has to queue for service. If a server becomes free again, the next customer is picked from the queue according to the rules given by the queuing discipline. During service, the customer might run through one or more stages of service, before departing from the system. In queuing theory, models are used to describe the characteristics of a queuing system. Some of the more commonly considered characteristics are discussed below.

(i) **Queue length**: the average number of customers in the queue waiting to get service. Large queues may indicate poor server performance while small queues may imply too much server capacity

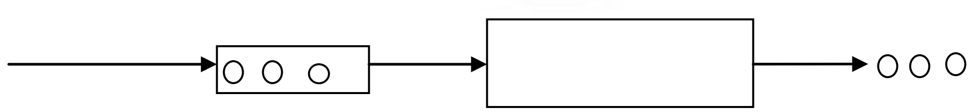
(ii) **System length** – the average number of customers in the queue in the system, those waiting to be and those being serviced. Large values of this statistic imply congestion and possible customer dissatisfaction and potential need for greater service capacity.

(iii)**Waiting time in the queue** – the average time that a customer has to wait in the queue to get service. Long waiting times are directly related to customer dissatisfaction and potential loss of future revenues, while very small waiting times may indicate too much service capacity.

(iii) **Total time in the system** – the average time that a customer spends in the system, from entry in the queue to completion of service. Large values of this statistic are indicative of the need to make adjustment in the capacity.

(iii) **Service idle time** – the relative frequency with which the service system is idle. Idle time is directly related to cost. However, reducing idle time may have adverse effects on the other characteristics mentioned.

It is important to mention here that the results obtained from various models are based on the assumption that the service the service system is operating under equilibrium or steady state conditions. For many systems, the operating day begins in transient state with no customers in the system. It takes some initial time interval for enough customers to arrive such that a steady state does not mean that the system will reach a point where the number of customers in the system never changes. Even when the system reaches equilibrium, fluctuations will occur. A steady state condition really implies that various system performance measures (the operating characteristics) would reach stable values.



Arrival process Queue Service mechanism Output process

Figure 2.2 General Structure of Queuing System

A queuing system is characterized by four components:

(i) Arrival process

(ii) Output process

(iii) Service mechanism and

(iv) Queue discipline. (Vohra, 2010).

2.5.1 Arrival process (input process)

Arrivals may originate from one or several sources referred to as the calling population. The calling population can be limited or 'unlimited'. An example of a limited calling population may be that of a fixed number of machines that fail randomly. The arrival process consists of describing how customers arrive to the system. Arrivals from the calling population may be classified on different bases as follows:

(i) **According to source** – the source of customers for a queuing system can be infinite or finite. For example, all people of a city could be potential customers at a supermarket. The number of people being very large, it could be taken to be infinite. On the other hand, there are many situations in business and industrial conditions where we cannot consider the population to be infinite.

(iii) **According to numbers** – the customers may arrive for service individually or in groups. Single arrivals are illustrated by customers visiting a beautician. On the other hand, families visiting restaurants, ship discharging cargo at a dock are examples of bulk, or batch, arrivals.

(iii) **According to time** – customers may arrive in the system at known (regular or otherwise) times, or they might arrive in a random way. The queuing models wherein customers" arrival times are known with certainty are categorized as deterministic models and are easier to handle. On the other hand, a substantial majority of the queuing models are based on the premise that the customers enter the system stochastically, at random points in time.

With random arrivals, the number of customers reaching the system per unit time might be described by a probability distribution. Although the arrivals might follow any pattern, the frequently employed assumption, which adequately supports many real world situations, is that the arrivals are Poisson distributed. (Vohra, 2010).

2.5.2 Output process

To describe the output process we usually specify the service time distribution (service mechanism). The service mechanism is the way customers receive service once they are selected from the queue. The output process tells the time that a customer leaves the system after going through all the service mechanisms.

2.5.3 Service mechanism

The service mechanism of a queuing system is specified by the number of servers (denoted by s), each server having its own queue or a common queue and the probability distribution of customer's service time. Basically there are about four types of service mechanism and different combinations of the same can be used for very complex networks. They are:

(i) **A single service facility**: in this system there is only one queue and only one server. Here customers in the queue wait till the service point is ready to take them for servicing.

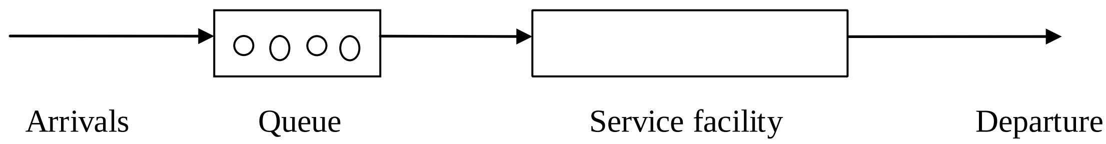


Figure 2.3 Single Channel Facility

(ii) **Multiple, parallel facilities with single queue**: that is, there is more than one server. The term parallel implies that each server provides the same type of facility. Example, booking at a service station that has several mechanics, each handling one vehicle at a time.

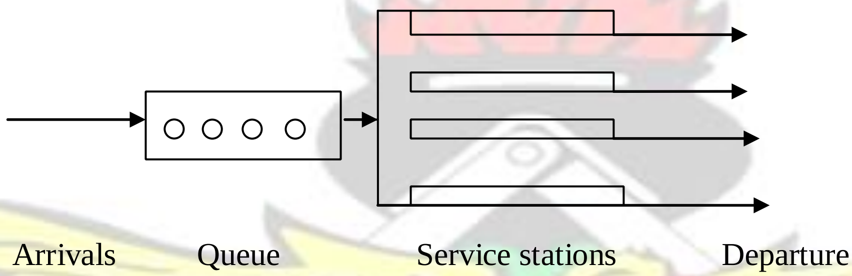


Figure 2.4 Multiple, Parallel Facilities with Single Queue

(iii) **Multiple, parallel facilities with multiple queues:** this model is different from the multiple, parallel facilities with single queue in that each of the servers has different queue. Example is, different cash counters in an account office where students can make payment of their school fees.

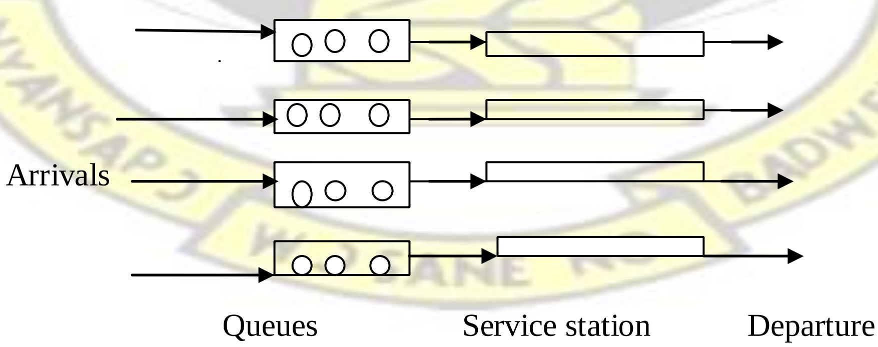


Figure 2.5 Multiple, Parallel Facilities with Multiple Queues

(iv) **Service facilities in a series**: in this, a customer enters the first station and gets a portion of service and then moves on to the next station, gets some service and then again moves on to the next station and so on, and finally leaves the system, having received the complete service. For example, machining of a certain steel item may consist of cutting, turning, knurling, drilling, grinding and packaging operations, each of which is performed by a single server in a series.

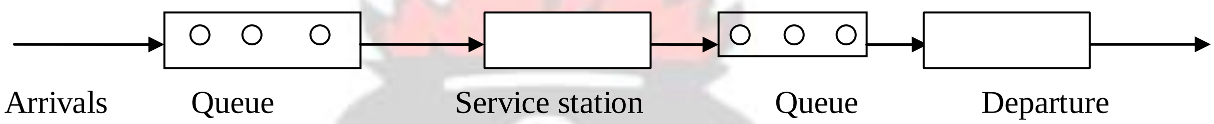


Figure 2.6 Service Facilities in Series

2.5.4 Queue Discipline

Discipline of a queuing system means the rule that a server uses to choose the next customer from the queue (if any) when the server completes the service of the current customer. The most common used queue disciplines are:

(i) **First-Come-First-Serve (FCFS)** - Customers are served on first-come-first-served basis. This is the most common discipline used in queuing systems. For example, with a queue at the bus stop, the people who came first will have their tickets first and board the bus first.

(ii) **First-In-First-Out (FIFO)** - Customers are served on a first-in first-out basis. For example, with a queue at the hospital, a patient who comes in first is served and leave before the others.

(iiI) **Last-Come-First-Serve (LCFS)** – That is customers who come in last are served first. Here, the customers are serviced in an order reverse of the order in which they enter so that the ones who join the last are served first. For example, the people who join an elevator last are the first ones to leave it. Also, assume that letters to be typed, or order forms to be processed accumulate in a pile, each new addition being put on the top of them. Thus, a just arriving task would be the next to be serviced provided that no fresh task arrives before it is picked up.

(iv) **Priority** – the customers in a queue might be rendered service on a priority basis. Thus, customers may be called according to some identifiable characteristics for service. Example is treatment given to a very important personality in preference to other patients in a hospital.

McQuarrie (1983) showed that it is possible, when utilization is high, to minimize waiting times by giving priority to clients who require shorter service times. This rule is a form of the shortest processing time rule that is known to minimize waiting times. It is found infrequently in practice due to the perceived unfairness (unless that class of customers is given a dedicated server, as in supermarket check-out systems) and the difficulty of estimating service times accurately.

2.6 THE SYSTEM CAPACITY (SIZE OF THE SOURCE POPULATION)

The system capacity is the maximum number of customers, both those in service and those in the queue(s), permitted in the service facility at some time. Whenever a customer arrives at a facility that is full, the arriving customer is denied entrance to the facility. Such a customer is not allowed to wait outside the facility (since that effectively increases the capacity) but is forced to leave without receiving service.

A system that has no limit on the number of customers permitted inside the facility has infinite capacity. Examples of infinite capacities include shoppers arriving at a supermarket, cars arriving at a highway toll booth, students arriving to register courses at a large University. Most queuing models assume such infinite arrival population.

A system with a limit on the number of customers has finite capacity.

2.7 ATTITUDE OF CUSTOMERS IN THE QUEUING SYSTEM

Customers in the queuing system can be classified as being patient or impatient. If a customer joins a queue if it exists, and wait till they enter the service station for getting service, they are called patient customers. On the other hand, the queuing systems may enjoy customer behavior in the form of defections from the queue. There may be jockeying among the many queues that is a customer may switch to the queues which are moving fast. Reneging is also possible in the queue. Here a customer stands in the queue for some time and then leaves the system because it is working slowly. The probability that a patient reneges usually increases with the queue length and the patient‟s estimate of how long he must wait to be served. In systems where demand exceeds server capacity, reneging is the only way that a system attains a "state of dysfunctional equilibrium" (Hall et al., 2006). Some customers on the other hand, may decide not to join the queue for some reason and may decide to return for the service later and this situation is known as balking.

Blocking occurs when a queuing system places a limit on queue length. For example, an outpatient clinic may turn away walk-in patients when its waiting room is full. In a hospital, where in-patients can wait only in a bed, the limited number of beds may prevent a unit from accepting patients.

McManus et al., (2004) presented a medical-surgical Intensive Care Unit where critically ill patients cannot be put in a queue and must be turned away when the facility is fully occupied. This is a special case where the queue length cannot be greater than zero, which is called a pure loss model (Green, 2006a).

Koizumi et al., (2005) found that blocking in a chain of extended care, residential and assisted housing facilities results in upstream facilities holding patients longer than necessary. They analyze the effect of the capacity in downstream facilities on the queue lengths and waiting times of patients waiting to enter upstream facilities. System-wide congestion could be caused by bottlenecks at only one downstream facility.

2.8 QUEUING MANAGEMENT

Queuing management refers to the control of queues and waiting lines. Queues and waiting lines could be controlled by two techniques;

(i) the operations management approach and

(ii) the psychology approach

2.8.1 The Operation management approach

This approach deals with the management of how customers, queues and servers could be coordinated towards the goal of rendering effective service at the least cost. It has a way of reducing the length of the queue which helps to reduce customers waiting time. Increasing productivity by training existing staff or employing more staff is the way to achieve the operation management approach to satisfy customers‟ demands. (katz and Martin, 1989)

2.8.2 The Psychology approach

The psychology approach is used to improve upon customer satisfaction in relation to queuing. It plays with the mind of customers by manipulating their perceptions and expectations. (katz and Martins,1989)

The approach is founded on the premise that customers see what they actually want to see. That is, they rely on what their minds tell them is true and look down upon the reality on the ground. Customers are satisfied when they see a fast moving queue because their minds tell them that the service providers in that system offer quick service. They however become unsatisfied when they see a long, slow – moving queue. (katz and Martins, 1989).

Customers‟ evaluation of service quality is affected not only by the actual waiting time but also, by the perceived waiting time. The act of waiting has great impact on customers‟ satisfaction.

Source: (Davis and Vollman, 1990)

One of the issues in queuing management is not only the actual amount of time the customer has to wait (Davis and Heineke, 1994).

The gap between the customer‟s perceptions of what happened during the service transaction and the customers‟ expectations of how the service transaction should have been performed is represented by the SERVQUAL model proposed by Parasuraman et al., (1985).

Mathematically, the model is presented as;

Satisfaction (S) = Perception (P) – Expectation (E) …. (2.1)

2.9 CUSTOMER EXPECTATIONS

Kano, (1984) suggested three categories of customer expectations. They are;

(i) **Satisfiers:** these are the characteristics which customers say they want in a service. The presence of these characteristics when provided leads to the satisfaction of customers. Example is experienced when more nurses are employed to take medical histories of patients.

No matter how serious a patient sickness is, he/she is satisfied with this kind of service. They are satisfied because they have in mind that the greater the number of nurses, the lesser their waiting their waiting times.

Thus customers are pleased when their perceptions of performances are equal to their expectations. That is, Perception (P) = Expectation (E) → Satisfied (S) customers. We could say from equation (2.1) that the characteristic, satisfaction (S) is zero (0) when customers are satisfied.

(i) **Dissatisfies**: they are the expected characteristics in a product or service and their absence leads to customer dissatisfaction. They are termed as the” must – be”s” in a product or service and no matter what the quality of that product is, they must be present to satisfy customers. Their absence would make customers go elsewhere. A laboratory centre must have a wash room to make patients take samples of say their urine for diagnosis. The absence of this would customers dissatisfied no matter how good the quality of service provided to them is. Customers are dissatisfied when their perceptions of performance fall below their expectations.

Thus, Perceptions (P) < Expectation (E) →Dissatisfied customers. From equation (2.1), Satisfaction (S) is negative (S < 0) when customers are dissatisfied.

(iii) Delighters or Exciters: customers generally do not expect to see these characteristics in a product or service because they are new to them. They are the unexpected qualities or bonuses that customers receive from their service providers. Example is the bonus customers enjoy when they recharge their mobile-phones with top-up cards. They are delighted or excited when their perceptions of performance exceed their expectations.

Thus, Perception (P) > Expectation (E) → Delighted or excited customers. From equation (2.1), Satisfaction (S) is positive (S > 0) when customers are delighted or excited.

To minimize cost and maximize profit, the psychology approach provides a great benefit in that it is less expensive to apply as compared with the operation management approach. Service providers must at all times provide quality services with those characteristics which would make customer satisfied and excited always. Management should not over rely on the psychology approach since it plays with the mind of its customers, though it is good. The reason is when customers get to know the reality on the ground; they might turn away without coming back.

2.10 MEASURING THE PERFORMANCE AND THE QUALITY OF QUEUING SYSTEMS

Queuing system can be studied in so many ways to see the rate at which it is performing and also used to judge the quality of services. We judge the quality of the service at least in parts – by the time customers have to wait and the length of the queue which depends on three things:

(i) The rate at which customers arrive

(ii) The time taken to serve each customer

(iii) The number of servers available

The performance of the queue can also be measured by:

(i) The time customers spend waiting

(ii) The average number of customers in the waiting line

(iii) The utilization rate of the server

(iv) Cost of operating the system

Management uses these to make decisions or plans for improving upon the waiting line operations.

2.11 THE ROLE OF EXPONENTIAL DISTRIBUTION

In most queuing situations, the arrivals of customers occur in a totally random fashion. Randomness here means that the occurrence of an event (e.g. arrival of a customer or completion of a service) is not influenced by the length that has elapsed since the occurrence of the last event. Random inter-arrival and service time are described quantitatively in queuing models by the negative exponential distribution.

A random variable, t, has an exponential distribution with parameter, λ, then the density function of t is given by;

F (t) = $\lambda e^{-\lambda t}, t > 0$

Then, E (t) = $\frac{1}{\lambda}$

P (t $\leq$ T) = $\int_0^T \lambda e^{-\lambda t} dt$

$= 1 - e^{-\lambda t}$ ... (2.2)

From this definition

E (t) shows that λ is the rate per unit time at which events (arrivals or departures) are generated.

2.12 FORGETFULNESS OR LACK OF MEMORY (MEMORYLESSNESS)

The number of arrivals occurring in any bounded interval of time t is independent of the number of arrivals occurring before time t. the fact that the exponential distribution is completely random is illustrated by the following examples; if the time now is 8:20 A.M and the last arrival has occurred at 8:02 A.M, if the probability that the next arrival will occur by 8:29 is a function of the interval from 8:20 to 8:29 only, and is totally independent of the length of time that has elapsed since the occurrence of the last event (8:02 to 8:20). This result is referred to as the forgetfulness or lack of memory of the exponential.

Let the exponential distribution, f (t); represent the time, t, between successive events. Also, let s be the interval since the occurrence of the last event, then lack of memory is given as;

$P(t > T + s/t > S) = P(t > T)$ … (2.3)

Lack of memory of the exponential distribution is essential because it implies that if we want to know the probability distribution of the time until the next arrival, then it does not matter how long it has been since the last arrival.

2.13 SUMMARY

In this chapter, we discussed the relevant and adequate literature on the application of queuing theory, history behind queuing theory, queuing systems, the basics of queuing theory and the characterization of queues. The system capacity of queues, attitude of customers in queues and the operation management and psychology approaches to managing queues have also been discussed.

The next chapter presents the research methodology to be used for the study.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This chapter looks at and gives report on the population from which the sample was drawn, instruments and procedures used for the collection and analysis of the data.

3.1 SOURCES AND THE TYPE OF DATA

The data for this study was collected from the North-Suntreso Hospital which is situated in Kumasi. The data collected is purely primary data.

The North-Suntreso Hospital has three sections within its out-patient department. They are the record section, history section and the consulting room section where patients see the doctors for treatment. Before a patient sees a doctor he or she has to join a queue from each of the sections.

3.2 DATA COLLECTION PROCEDURE

The researcher collected an introductory letter from the department of Mathematics, Kwame Nkrumah University of Science and Technology and visited the sample hospital and personally communicated with the administrator of the hospital. The researcher was taken through all the various section of the hospital. She sat among the patients at the various sections and recorded the data.

A stop watch was used to calculate the number of minutes spent by each patient from the recorded section where patients collected their folders through to the last section (the consulting room section). Data was collected on Monday to Friday from the hours of 8:00 am to 12:00 pm. The number of arrivals (i.e. number of patients) from each section was taken.

3.3 FORMULATION OF METHODS

3.3.1 The mean arrival rate

Let λ be the mean arrival rate and let n be the number of patients that entered the system between 8:00am-12:00pm. Also, let h be the number of hours between 8:00am-12:00pm. Then, the mean arrival rate is given by the formula, $\lambda = \frac{n}{h}$ arrival per hour … (3.1)

3.3.2 The mean service rate

3.3.2.1 The mean service rate at the records section

The record section is where patients" folders are sorted out. The service time used for sorting out the folders was taken within 1 hour and the folders were sorted out in batches.

Let $S_1, S_2 \ldots S_n$ be the observed service times when patients folders are sorted out in batches. Let n be the number of folders sorted out and let b be the start service time for sorting out the folders and let e be the finished service time for sorting out the folders.

From these defined letters

$$S_1 = \frac{e_1 - b_1}{n_1}$$

$$S_2 = \frac{e_2 - b_1}{n_2}$$

$$\vdots$$

$$S_n = \frac{e_n - b_1}{n_n}$$

Therefore the average service rate is given by

$$\mu = \frac{1}{n}\sum_{i}^{n} S_i \text{ arrival per hour, } \ldots (3.2)$$

3.3.2.2 The mean service rate at the history section and the consulting room

At the history and the consulting rooms sections, patients were attended to one after the other. The service time for each patient was recorded when he or she is attended to. Let $S_1, S_2 \ldots S_n$ be the observed service time of patients. Let b be the start service time of patients and e be the finished service times of patients.

Then,

$$S_1 = e_1 - b_1$$

$$S_2 = e_2 - b_2$$

$$\vdots$$

$$S_n = e_n - b_n$$

Hence, the mean service rate is given by;

$$\mu = \frac{1}{n}\sum_{i}^{n} S_i \text{ arrival per hour } \ldots (3.3)$$

3.3.3 Little's formula

The Little"s formula is given by;

$$L = \lambda W \quad (3.4)$$

This formula would be used to determine the amount of time that a patient would spend in the system. From this formula, L is defined as the average number of patients present in the queuing system and λ is defined as the mean arrival rate. W is also defined as the expected time a patient spends in the queuing system.

We could say from little"s formula that;

If equation (3.4) is Little"s second flow of equation

Then, $$W_q = \frac{L_q}{\lambda} \quad (3.5)$$

Equation (3.5) which follows directly from Little"s second flow equation was used for the single-channel and the multiple-channel waiting line models.

The general expression that applies to waiting line models is that the average time in the system, W, is equal to the average time in the waiting line, W_q plus, the average service time. A system with a mean service rate μ, the average or mean service time is 1/ μ.

Hence, we have the following general relationships;

$$W = W_q + \frac{1}{\mu}$$

$$L = \lambda W$$

$$L_q = \lambda W_q \quad (3.6)$$ and

$$L_s = \lambda W_s \quad (3.7)$$

Where equation equations (3.6) and (3.7) are defined as the average number of patients in the waiting line and the average number of patients in the system respectively.

3.4 THE OPERATING CHARACTERISTIC OF M/M/1 MODEL (SINGLE SERVER-INFINITE POPULATION)

$P = \frac{\lambda}{\mu}$ (3.8), is defined as the average utilization of the system i.e. the probability that the system is busy where $P < 1$

P (n customers during period T) = $\frac{e^{-\lambda t}(\lambda t)^n}{n!}$ (3.9)

When the time taken to serve different customers is independent then;

P (not more than T time period needed to serve a customer) = $1 - e^{-\mu T}$ ... (3.10)

1. Probability that the system is idle i.e. the probability that there are no customers in the system;

   $P_o = 1 - P$

   $= \left(1 - \frac{\lambda}{\mu}\right)$ (3.11)

2. The probability of having exactly n customers in the system

$P_n = P_n P_o$

$= \left(1 - \frac{\lambda}{\mu}\right)$

$= P^n (1 - P)$ (3.12)

3. Expected number of customers in the system $L_s = \sum_n nP_n$ (3.13)

This can be solved as;

$$L_s = \frac{\lambda}{\mu - \lambda} \quad (3.14) \quad or \quad \frac{P}{1-P} \quad (3.15)$$

4. Expected number of customers in the queue

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$= \frac{\lambda}{\mu - \lambda} - \frac{\lambda}{\mu}$$

$$= \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$= \frac{P^2}{1-P} \quad (3.16)$$

Equation (3.16) can also be noted as the average length of all queues including empty queues.

5. The average length of non empty queues (i.e. those which contain at least one customer)

$$L_n = \frac{\mu}{\mu - \lambda}$$

$$= \frac{1}{1-P} \quad (3.17)$$

6. Expected waiting time in the queue

$$W_q = \frac{1}{\lambda} L_q \quad (3.18)$$

Substituting equation (3.14) into equation (3.18)

We have, $$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \qquad (3.19)$$

7. Expected time a customer spends in the system

$$W_s = \frac{1}{\lambda} L_s \qquad (3.20)$$

Putting equation (3.14) into equation (3.20), we have;

$$W_s = \frac{1}{\mu - \lambda} \qquad (3.21)$$

Since the mean service rate is μ, the average (expected) time for completing the service is 1/μ.

Therefore, the expected time a customer would spend in the system would be equal to the expected waiting time in the queue plus the average servicing time.

Thus,

$$W_s = W_q + \frac{1}{\mu}$$

$$= \frac{\lambda}{\mu(\mu - \lambda)} + \frac{1}{\mu}$$

1

$$\qquad (3.22)$$

Which is the same as equation (3.21), as shown earlier.

8. The probability that a customer spends more than t units of time in the system

$$W_s(t) = e^{-t/W_s} \qquad (3.23)$$

9. The probability that a customer spends more than t units of time in the queue

$$W_q = Pe^{-\frac{t}{W_s}} \qquad (3.24)$$

3.5 SINGLE SERVER MODEL – FINITE POPULATION

This model is based on similar assumptions as that of M/M/1 model except that the input population is finite. For this model, the system structure is such that we have a total of M customers, a customer is either in the system (consisting of a queue and a single server) or outside the system and in a sense, arriving.

When a customer is in the arriving condition, then the time it takes him to arrive is a random variable having an exponential distribution with mean equal to 1/λ.

When there are n customers in the system, then there is M – n customers in the arriving state. From this, the total average rate of arrivals in the system is λ (M – n).

The single server model with finite population is self-regulating. This means that when the system gets busy, with many customers in the queue, then the rate at which additional customers arrive is reduced thus lowering the congestion in the system. In this model there is a depending relationship between arrivals.

For the reason of dependency relationships between arrivals, the Poisson probability distribution law cannot be strictly applied when the input population is finite. Instead of the arrivals" statement as an average for the population, we classify them as an average of a unit time.

Hence, the exponential distribution with mean = 1/λ.

3.6 THE OPERATING CHARACTERISTICS OF A SINGLE SERVER – FINITE POPULATION MODEL

The number of customers in the source population = M.

Average inter-arrival between successive arrivals = 1/λ

Service rate = λ

1. Probability that the system would be idle;

$$P_o = \frac{1}{\sum_{i=0}^{M} \frac{M!}{(M-i)!}\left(\frac{\lambda}{\mu}\right)^i} \quad (3.25)$$

2. Probability of n customers in the system,

$$P_n = P_o \left(\frac{\lambda}{\mu}\right)^n \frac{M!}{(M-n)!}, \quad 0 \le n \le M, \text{ or}, n \le M \quad (3.26)$$

3. Expected length of the queue,

L_q □M□ □ _____ □□ □1□P_o □□(3.27)

□

4. Expected number of customers in the system,

L_s □ L_q □ (1□ P_o)

□

□ M □ —□1□ P_o □□(3.28)

□

5. Expected waiting time of a customer in the queue, L_q

W_q □ ________

□(1□P_o)

1 □ M □□□□□□

□ □□ (1□P_o) □ □ □□□□(3.29)

□□

6. Expected time a customer spends in the system, 1

W_s □W_q □—

□

1 □ M □□□□□ □

□ □□1□P_o□□ □ □1□□□(3.30) □□

3.7 THE OPERATING CHARACTERISTICS OF A MULTIPLE SERVER MODEL-INFINITE POPULATION (M/M/K/∞)

The model is based on the following assumptions;

(i) The arrival of customers follows Poisson probability distribution

(ii) The service time has an exponential distribution

(iii) There are k servers, each of which provides identical services

(iv) A single waiting line is formed

(v) The input population is infinite

(vi) The service is on a first-come-first-served basis

(vii) The arrival rate is smaller than the combined service rate of all k service facilities

3.8 OPERATING CHARACTERISTIC OF THE MULTIPLE SERVER MODEL WITH INFINITE POPULATION

Average rate of arrivals = λ … (3.31)

Number of servers = k … (3.32)

Mean combined rate of all servers $= k\mu$ … (3.33)

Utilization factor of the entire system = $P = \frac{\lambda}{k\mu}$ (3.34)

Probability that the system shall be idle

1. $P_o = \left[\sum_{i=0}^{k-1} \frac{(\lambda/\mu)^i}{i!} + \frac{(\lambda/\mu)^k}{k!(1-P)}\right]^{-1}$ (3.35)

The probability that there would be exactly n customers in the system,

2. $P_n = P_o \frac{(\lambda/\mu)^n}{n!}$, when $n \leq k$ … (3.36)

$n!$

And,

P_n P_o k $!$ k n 1,

when n > k … (3.37)

3. The expected number of customers in the waiting line

k

P

L_q 2 P_o (3.38) $k!$ 1

P

4. The expected number of customers in the system,

L_s L_q (3.39)

5. The expected waiting time in the queue

L

W_q

(3.40)

q

6. The expected time a customer spends in the system,

$$W_s = W_q + \frac{1}{\mu} \quad (3.41)$$

3.9 THE ERLANG DITRIBUTION

The Erlang distribution is used to model situations where the inter-arrival times do not appear to be exponential. It is a continuous random variable (T) whose density function is specialized by two parameters; a scale parameter, μ, and a shape parameter k (where k is a positive integer).

A shape parameter and a scale parameter are kinds of a numerical parameter of a parametric family of probability distributions. The larger the scale factor, the more spread out the distribution. The scale parameter is useful in modeling applications since they are flexible enough to model a variety of data sets. A shape parameter allows a distribution to take on a variety of shapes, depending on the value of the shape parameter.

The erlang distribution (E_K (μ)) is given by;

F (t) = $\frac{\mu(\mu t)^{k-1}}{(k-1)!}e^{-\mu t}, t > 0 \ldots (3.42)$

The distribution function equals

F (t) = $1 - \sum_{j=0}^{k-1}\frac{(\mu t)^{k-1}}{j!}e^{-\mu t}, t > 0 \ldots (3.43)$

Applying integration by parts, the mean, variance and squared coefficients of variation are equal to;

$E(T) = \frac{k}{\mu}$, $\quad \sigma^2 = \frac{k}{\mu}$, $\quad C^2(T) = \frac{1}{K}$

3.10 POISSON PROCESS

The Poisson process is an extremely useful process for modeling purposes in many practical applications such as, e.g.; to model arrival process for queuing models or demand processes for inventory systems. It could be used to approximate stochastic processes. In the Poisson probability distribution, the observer records the number of events in a time interval. The Poisson process is a continuous-time process.

Let N (t) be the number of arrivals in [0, t] for a Poisson process with rate, λ, i.e. the time between successive arrivals is exponentially distributed with parameter, λ, and independent of the past. Then, the Poisson distribution with parameter, λt for N (t) can be calculated as;

P [N (t) =k] = $\frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0,1,2,\ldots, (3.44)$

Applying integration by parts gives

$E[N(t)] = \lambda t$, $\qquad \sigma^2[N\ (t)] = \lambda t$

The mean of the Poisson Process is the same as the variance. (Wayne,1991).

3.11 PURE BIRTH MODEL AND DEATH MODELS

The pure birth-death process is used to study the number of customers in a queue. It is a special case of continuous stochastic process where the state transitions are of only two types: "births" which increase the state variable by one and "deaths" which decrease the state by one.

When a birth occurs, the process goes from state n to n+1. When a death occurs, the process goes from n to state n-1. The process is specialized by birth rates

□□i□,i □ 0,1,2,3,□□ and

Death rates □□i□,i □1,2,3,□□

Poisson process is a pure birth process where $\lambda i = \lambda$ for all $i \geq 0$

Birth process is the same as arrival process and death process is the departure process. The birth-death process describes probabilistically how the number of customers, N (t) in the queuing system changes as time (t) increases.

The birth-death process is governed by these assumptions:

(i) Given N (t) = n, the current probability distribution of the remaining time until the next birth (arrival) is exponential with parameter λn.

(ii) Given N (t) = n, the current probability distribution of the remaining time until the next death (service completion) is exponential with parameter μn.

(iii) The random variable of assumptions one and the random variable of assumption two are mutually independent. The next transition in the state of the process is either $n \rightarrow (n+1)$ or $n \rightarrow (n-1)$ depending on whether the former or the latter random variable is smaller.

(Wayne, 1991).

3.12 THE TRAFFIC INTENSITY(ρ)

An important parameter in any queuing system is the traffic intensity also called the load or the utilization, defined as the ratio of the mean service time $E(X) = \frac{1}{\mu}$ over the mean inter-arrival time $E(\tau) = \frac{1}{\lambda}$

$$\rho = \frac{E(x)}{E(\tau)} = \frac{\lambda}{\mu} \ldots (3.45)$$

Where λ and μ are the mean inter-arrival and service rate, respectively.

Clearly, if $\rho > 1$ or $E[x] > E[\tau]$, which means that the mean service time is longer than the mean inter-arrival time, then the queue will grow indefinitely long for large t, because packets are arriving faster on average than they could be served. In this case ($\rho > 1$), the queuing system is unstable or will never reach a steady-state. The case where $\rho = 1$ is critical. In practice, therefore, mostly situations where $\rho < 1$ are of interest.

If $\rho < 1$, a steadystate can be reached. These considerations are a direct consequence of the law of conservation of packets in the system.

3.13 QUEUING MODEL

3.13.1 Kendall's notation

In the year 1953, Kendall introduced a notation that is commonly used to describe and classify the type of a queuing model that a queuing system corresponds to.

The general syntax is A/B/n/K/m, where A specifies the inter-arrival process, B the service process, n the number of servers, K the number of positions in the queue and m restricts the number of allowed arrivals in the queuing system. Examples for both the interarrival distribution A and the service distribution B are M (memoryless or Markovian) for the exponential distribution, G for a general distribution and D for a deterministic

distribution.

When other letters are used besides these three common assignments, the meaning will be defined. For example, M/G/1 stands for a queuing system with exponentially distributed Inter-arrival times, a general service distribution and 1 server. If one of the two last identifiers K and m is not written, they should be interpreted as infinity. Hence, M/G/1 has an infinitely long queue and no restriction on the number of allowed arrivals.

A queue is therefore described in a shorthand notation by A/B/C/K/N/D or the more concise A/B/C. in the concise version, it is assumed that $K=\infty$, $N = \infty$ and D = FCFS (first come, first served).

Table 3.1: Description of Kendall's notation

Letter	Description
A	The arrival process
B	The service time distribution
C	The number of servers
K	The number of channels in the system
N	The calling population
D	The queue discipline

For Kendall"s notation, the letters A and B can also be described by the following processes

Table 3.2: Description of Kendall's notation

Letter	Description
M	Exponential distribution
D	Deterministic inter-arrival times
E^K	Erlang distribution (k=shape parameter)
G	General distribution

3.13.2 Examples of queuing models

(i) M/M/1 Model

In this model, random arrivals and exponentially distributed service times are assumed. Poisson distribution is used to define the random arrivals. The M/M/1 Model has only one server serving customers on first come, first served bases. The population here is infinite, so arriving customers are unaffected by the queue size. The parameters given for M/M/1 model are λ (the average arrival rate), μ (the average service rate) which may be calculated from the average service time.

(ii) M/M/M/∞

This describes a queuing system with m number of servers and infinite number of waiting lines.

(iii) M/M/1/-/K

This describes a queuing system with a single server, infinite number of waiting lines and finite customer population k.

(iv) M/E_2/2/K

This describes a queuing system with Poisson arrivals, Erlangian of order 2 service time distribution, 2 servers, and maximum number of k in a queue

(v) M/D/5/40/200/FCFS

Where M is the exponential distributed times, D is the deterministic service times, 5 is the number of servers, 40 is the number of buffers (35 for waiting), a total population of 200 customers and FCFS is the service discipline (first come, first served).

3.14 DETERMINISTIC QUEUING MODELS

Queuing models can be categorized as being deterministic if customers arrive at known intervals and the service time is known with certainty.

Suppose that customers come to bank"s teller counter every 5 minutes. Thus the interval between the arrivals of any two successive customers in exactly 5 minutes, suppose that the banker takes exactly 5 minutes to serve each customer. Here the arrival and the service rates are each equal to 12 customers per hour. That is, 60minutes/5 minutes = 12 customers per every 5 minutes. In this situation, there shall be never a queue and the banker shall always be busy with work.

Now, suppose that the banker can serve 15 customers per hour, the consequence of this higher service rate would be that the banker would be busy $4/5^{th}$ of the time and idle in $1/5^{th}$ of the time. The teller shall take 4 minutes to serve a customer and wait for the next customer to come. Here also, there shall never be a queue as before.

If on the other hand, the banker can serve only 10 customers per hour, then the result would be that he would be always busy and the queue length will increase continuously without limit with the passage of time. It is easy to see when the service rate is less than the arrival rate, the service facility cannot cope with all the arrivals and eventually the system leads to explosive situations. This problem can be resolved by providing additional service station(s).

Symbolically let λ be the arrival rate of customers per unit time and μ be the service rate per unit time.

Then,

If $\lambda > \mu$, the waiting line shall be formed which will increase indefinitely; the service facility would always be busy; and the service system will eventually fail.

If $\lambda \geq \mu$, there shall be no waiting time; the proportion of time the service facility would be idle is 1- λ/μ. Where $\lambda/\mu = P$ is called the average utilization or the traffic ratio, or the clearing ratio. This indicates the proportion of time or the probability that the service station is busy.

From this model,

If $P > 1$, the arrivals come at a faster rate than the server can accommodate. The expected queue increases without limit and a steady state does not occur. This would make the system to fail ultimately.

If $P \leq 1$, the system would work and P is the utilization factor of the system, that is, the proportion of time the system busy.

Also, if $P < 1$, then steady-state probabilities would occur.

The deterministic queuing model may exist when we are dealing with, for example movements of items for processing in a highly automated plant. However, generally and more particularly when human beings are involved, the arrivals and servicing time are variable and uncertain. Thus, variable arrival rates and servicing times are the more realistic assumptions. The probabilistic queuing models are based on these assumptions.

3.15 PROBABILISTIC QUEUING MODELS

The vast majority of the queuing models are based on the assumption that one or more elements of the queuing system can be expressed only in probabilistic terms. All most all queuing models are of probabilistic type.

The models to be considered under the probabilistic queuing models are;

(i) Poisson- exponential, single server model-infinite population (ii)

Poisson –exponential, single server model-finite population and

(iii) Poisson-exponential, multiple server model-infinite population.

In each of this case, Poisson-exponential indicates that the customer arrivals follow Poisson distribution while the service time is distributed exponentially.

If the arrivals are independent with the average arrival rate equal to λ per period of time, then, according to the Poisson probability distribution, the probability of x arrivals in a specific time period is defined as,

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, for\ x = 0,1,2, \ldots$$

Where x = the number of arrivals in the period, λ= the average or mean number of arrivals per period and e = 2.71828.

3.16 SUMMARY

In this chapter, the research methodology adopted for the study, sources and type of data collection procedure were discussed. Formulation of methods, the operating characteristics of queuing models and probability distributions behind queuing theory were put across.

Analysis of the collected data shall be presented in the next chapter.

CHAPTER FOUR

DATA ANALYSIS

4.0 INTRODUCTION

This chapter looks at the analysis and the modeling of the data collected. Tables would be used to show the data collected and the findings would be used to answer the research questions that were formulated in chapter one.

4.1 ANALYSIS AND PRESENTATION OF DATA

This section deals with the analysis of data collected from all the three sections (records, history and consulting room) of the out-patient department of the North-Suntreso Hospital. The data for this analysis were collected on Monday, Tuesday, Wednesday, Thursday and Friday.

The mean arrival rates and the mean service rates would be calculated from the data collected and their results would be used to measure the performance of the entire system as stated in chapter two.

4.1.1 Presentation of data collected on Monday From equation (3.31), mean arrival rate, $\lambda = \frac{200}{4}$

= 50 patients per hour

Table 4.1: Service times of patients at records section on Monday

Number of folders (patients)	Service time (minutes)
14	17
07	11
10	13
09	12

From the Table 4.1, the mean service rate is;

$$u = \frac{number\ of\ folders(patients)}{total\ number\ of\ hours\ spent}$$

Total number of minutes = 17 + 11 + 13 + 12

= 53minutes = 0.8833 hour

Hence, $\mu = \frac{40}{0.8833}$

= 45.2847 patients/hour

From equation (3.32), the total number of servers (k) = 2

1. From equation (3.34), the utilization factor of the entire system is;

$P = \frac{50}{2(45.2843)}$

= 0.5521

2. From equation (3.35), the probability that the system would be idle is;

$$P_0 = \left[\sum_{i=0}^{2-1} \frac{\left({}^{50}/_{45.2847}\right)^i}{i!} + \frac{\left({}^{50}/_{45.2847}\right)^2}{2!(1-0.5521)}\right]^{-1}$$

50 i

1 i 0 45. i 2847! 1.1041 0! 0 1.1041 1! 1

2.1041

and

$50\ ^{2}$

$45.2847\ 1.3608$

$2!\ 1\ 0.5521$

$P_o\ 2.1041\ 1.3608^{1}\ 0.2886$

3. From equation (3.38), the expected number of patients waiting in the queue is;

$50\ _{2}\ 0.5521$

$L_q\ 452!\ .12847\ 0.5521\ 0.2886$

2

0.4841

4. From equation (3.39), the expected number of patients waiting in the system is;

$L_s\ 0.4841\ 5045.2847$

1.5882

5. From equation (3.42), the expected waiting time of a patient in the queue is;

$W_q\ 0.484150$

$0.0097 hour$

$0.5809 \min utes$

6. From equation (3.41), the expected time a patient spends in the system is;

$W_s\ 0.0097\ 145.2847$

$0.0318 hour$

$1.9059 \min utes$

The results show that the server would be busy 55.21% of the time and idle 28.86% of the time. Also, the average number of patients in the waiting queue is 0.4841 and the average number of patients waiting in the system is 1.5882. More so, the average time a patient spends in the queue is 0.5809 minutes and average a patient spends in the system is 1.9059 minutes.

Table 4.2: Service time of patients at the history section on Monday

Patient number	Service time (minutes)		
	Nurse 1	**Nurse 2**	**Nurse 3**
1	3	3	4
2	3	4	5
3	4	4	3
4	3	5	3
5	2	5	4
6	2	4	3
7	3	3	3
8	3	3	4
9	4	2	5
10	6	2	3
11	3	3	3
12	3	3	4
13	3	3	3
14	2	2	3
15	3	4	5
16	3	6	3

From Table 4.2, the mean service rate is;

□□ 482.7333

$= 17.5612 \, patients/hr$

Number of servers (k) = 3

1. The utilization factor of the system is;

$$P = \frac{50}{3(17.5612)}$$

$= 0.9491$

2. The probability that the system would be busy;

$$P_o = \left[\sum_{i=0}^{2} \frac{(50/17.5612)^i}{i!} + \frac{(50/17.5612)^3}{3!(1-0.9491)} \right]^{-1}$$

$= [7.9004 + 75.5762]^{-1}$

$= 0.012$

3. Expected number of patients waiting in the queue;

$$L_q = \frac{(50/17.5612)^3 (0.9491)(0.012)}{3!(1-0.9491)^2}$$

$= 16.9107$

4. Expected number of patients waiting in the system;

$L_s = 16.9107 + 50/17.5612$

$= 19.7579$

5. Expected time a patient spends in the queue;

$$W_q = \frac{16.907}{50}$$

$= 0.3381 hrs$

$= 20.2884 \min s$

6. Expected time a patient spends in the system

$W_s = 0.3381 + 1/17.5612$

$= 0.395 hrs$

$= 23.726 \min utes$

The results show that the server would be busy 94.91% of the time and idle 1.2% of the time. Also, the average number of patients in the waiting queue is 16.9107 and the average number of patients waiting in the system is 19.7579. More so, the average time a patient spends in the queue is 20.2884 minutes and average a patient spends in the system is 23.7026 minutes.

Table 4.3: Service time of patients at the consulting rooms on Monday

Patients number	**Service time (minutes)**			
	Doctor 1	**Doctor 2**	**Doctor 3**	**Doctor 4**
1	5	9	10	5
2	5	7	9	5
3	4	5	6	3
4	6	3	5	4
5	5	3	3	7
6	5	3	3	5
7	3	2	4	6
8	4	5	4	3
9	4	4	3	3
10	3	3	3	4

11	5	3	2	3
12	2	4	3	2
13	4	5	3	5
14		2	2	8

From Table 4.3, the number of servers (k) = 4

$$\mu = \frac{55}{3.9667}$$

$$= 13.8654\ patients/hour$$

1. The utilization factor of the system is;

$$P = \frac{50}{4(13.8654)}$$

$$= 0.9015$$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{3}\frac{\left(\frac{50}{13.8654}\right)^i}{i!} + \frac{\left(\frac{50}{13.8654}\right)^{4}}{4!(1-0.9015)}\right]^{-1}$$

$$= \left[18.9237 + 71.5325\right]^{-1}$$

$$= 0.0111$$

3. Expected number of patients waiting in the queue;

$$L_q = \frac{\left(\frac{50}{13.8654}\right)^4 (0.9015)(0.0030)}{\quad^{2}}$$

$$4!(1 - 0.9015)$$

$$= 7.267$$

4. Expected number of patients waiting in the system;

$$L_s = 7.267 + 50/13.8654$$

$$= 10.8731$$

5. Expected time a patient spends in the queue;

$$W_q = 7.267/50$$

$$= 0.1453 hrs$$

$$= 8.7204 \min utes$$

6. Expected time a patient spends in the system;

$$W_s = 0.1453 + \frac{1}{13.8654}$$

$$= 0.2174 hr$$

$$= 13.044 \min utes$$

The results show that the server would be busy 90.15% of the time and idle 1.11% of the time. Also, the average number of patients in the waiting queue is 7.267 and the average number of patients waiting in the system is 10.87311. More so, the average time a patient spends in the queue is 8.7204 minutes and average a patient spends in the system is 13.044 minutes.

4.1.2 Presentation of data collected on Tuesday

Mean arrival rate;

□□165/4

□ 41.25*patients /hr*

Table 4.4: Service time of patients at the record section on Tuesday

Number of folders (patients)	**Service time (minutes)**
14	16
08	11
15	19
06	10

From Table 4.4, the mean service rate is;

$\frac{43}{9333}$

□□

0.

□ 46.0731*patients /hr*

Number of servers (k) = 2

1. The utilization factor of the entire system;

P □ $\frac{41.25}{2(46.0731)}$

□ 0.4477

2. The probability that the system would be idle;

i 2 □1

$$P_o = \left[\sum_{i=0}^{1}\frac{\left(\frac{41.25}{46.0731}\right)^i}{i!} + \frac{\left(\frac{41.25}{46.0731}\right)^2}{2!\left(1-\frac{41.25}{2\times 46.0731}\right)}\right]^{-1}$$

$$= \left[1.8953 + 0.7257\right]^{-1}$$

$$= 0.3815$$

3. Expected number of patients waiting in the queue;

$$L_q = \frac{\left(\frac{41.25}{46.0731}\right)^2 \times 0.4477}{2!(1-0.4477)^2} \times 0.3815$$

$$= 0.2244$$

4. Expected number of patients waiting in the system;

$$L_s = 0.2244 + \frac{41.25}{46.0731}$$

$$= 1.1197$$

5. Expected waiting time of a patient in the queue;

$$W_q = \frac{0.2244}{41.25}$$

$$= 0.00544 hrs$$

$$= 0.3264 \min utes$$

6. Expected time a patient spends in the system;

$$W_s = 0.00544 + 1/46.0731$$

$$= 0.0271 hr$$

$$= 1.626 \min utes$$

The results show that the server would be busy 44.77% of the time and idle 38.15% of the time. Also, the average number of patients in the waiting queue is 0.2244 and the average number of patients waiting in the system is 1.1197. More so, the average time a patient spends in the queue is 0.3264 minutes and average a patient spends in the system is 1.626 minutes.

Table 4.5: Service time of patients at the record section on Tuesday

Patient number	Service time (minutes)		
	Nurse 1	Nurse 2	Nurse 3
1	3	2	4
2	3	3	3
3	4	5	3
4	2	5	2
5	2	3	3
6	3	2	2
7	2	4	5
8	3	5	4
9	3	2	3
10	2	3	3
11	3	5	2
12	5	3	5
13	4	3	6
14	4	2	3
15	3	2	4
16	3	4	3

From table 4.5, the number of servers (k) = 3

Mean service rate;

$$\mu = \frac{48}{2.6}$$

$$= 18.4615 \, patient/hr$$

1. The utilization factor of the entire system;

$$P = \frac{41.25}{3(18.4615)}$$

$$= 0.7448$$

2. Probability that the system would be idle

$$P_o = \left[\sum_{i=0}^{2} \frac{\left(\frac{41.25}{18.4615}\right)^i}{i!} + \frac{\left(\frac{41.25}{18.4615}\right)^3}{3!(1-0.7448)} \right]^{-1}$$

$$= (5.7306 + 7.2854)^{-1}$$

$$= 0.0768$$

3. Expected number of patients waiting in the queue;

$$L_q = \frac{\left(\frac{41.25}{18.4615}\right)^3 (0.7448)(0.0768)}{3!(1-0.7448)^2}$$

$$= 1.6329$$

4. Expected number of patients waiting in the system;

$L_s = 1.6329 + 41.25/18.4615$

$= 3.867$

5. Expected waiting time of a patient in the queue;

$W_q = 1.6329/41.25$

$= 0.03959 hr$

$= 2.3751 \min utes$

6. Expected waiting time of a patient in the system;

$$W_s = 0.03959 + \frac{1}{18.4615}$$

$= 0.09376 hr$

$= 5.6254 \min utes$

The results show that the server would be busy 74.48% of the time and idle 7.68% of the time. Also, the average number of patients in the waiting queue is 1.6329 and the average number of patients waiting in the system is 3.867. More so, the average time a patient spends in the queue is 2.3751 minutes and average a patient spends in the system is 5.6254 minutes.

Table 4.6: Service time of patients at the consulting room on Tuesday

Patient's number	**Service time (minutes)**			
	Doctor 1	**Doctor 2**	**Doctor 3**	**Doctor 4**
1	6	2	3	5
2	5	3	5	5
3	4	4	6	3
4	8	6	3	6
5	6	3	7	2
6	4	4	4	7
7	4	2	5	5
8	5	6	3	6

9	5	3	4	5
10	3	6	2	6
11	6	4	9	9
12	5	8	6	4
13	2	6	10	
14	2	3		
15	3			

From Table 4.6, the number of servers (k) = 4

Service rate;

$$\mu = \frac{54}{4.3}$$

$$= 12.5581$$

1. The utilization factor of the entire system;

$$P = \frac{41.25}{4(12.5581)}$$

$$= 0.8212$$

2. Probability that the system would be;

$$P_o = \left[\sum_{i=0}^{3} \frac{\left(\frac{41.25}{12.5581}\right)^i}{i!} + \frac{\left(\frac{41.25}{12.5581}\right)^4}{4!(1-0.8212)}\right]^{-1}$$

$$= \left[15.586 + 27.1282\right]^{-1}$$

$$= 0.0234$$

3. Expected number of patients waiting in the queue;

$$L_q = \frac{41..254!12(1.55810.8212^4)0.8212\ 0.0234}{2}$$

$= 2.9154$

4. Expected number of patients waiting in the system;

$L_s = 2.9154 + 41.25/12.5581$

$= 6.2001$

5. Expected waiting time of a patient in the queue;

$W_q = \frac{2.9154}{41.25}$

$= 0.0707hr$

$= 4.2406 \min utes$

6. Expected waiting time of a patient in the system;

$W_s = 0.0707 + \frac{1}{12.5581}$

$= 0.1503hr$

$= 9.0198 \min utes$

The results show that the server would be busy 82.12% of the time and idle 2.34% of the time. Also, the average number of patients in the waiting queue is 2.9154 and the average number of patients waiting in the system is 6.2001. More so, the average time a patient spends in the queue is 4.2406 minutes and average a patient spends in the system is 9.0198 minutes.

4.1.3 Presentation of data collected on Wednesday

Table 4.7: Service time of patients at the record section on Wednesday

Number of folders (patients)	Service time (minutes)
18	21
09	7
12	17
13	10

From Table 4.7, the number of servers (k) =2

Arrival rate of patients;

$$\lambda = \frac{210}{4}$$

$$= 52.5 \, patients/hr$$

Mean service rate of the system;

$$\mu = \frac{52}{0.9167}$$

$$= 56.7252 \, patients/hr$$

1. Utilization factor of the system;

$$P = 52.5/2(56.7252)$$

$$= 0.4628$$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{1} \frac{\left(52.5/56.7252\right)^i}{i!} + \frac{\left(52.5/56.7252\right)^2}{2!(1-0.4628)} \right]^{-1}$$

$= \left[1.9255 + 0.7973\right]^{-1}$

$= 0.3672$

3. The expected number of patients in the queue;

$$L_q = \frac{\left(\frac{52.5}{56.7252}\right)^2 0.4628}{2!(1-0.4628)^2} \times 0.3672$$

$= 0.5450$

4. The expected number of patients in the system;

$L_s = 0.5450 + \frac{52.5}{56.7252}$

$= 1.4705$

5. The expected waiting time of a patient in the queue;

$$W_q = \frac{0.5450}{52.5}$$

$= 0.028hr$

$= 1.6805 \min utes$

6. The expected waiting time of a patient in the system;

$W_s = 0.028 + \frac{1}{56.7252}$

$= 0.0456hr$

$= 2.7383 \min utes$

The results show that the server would be busy 46.28% of the time and idle 36.72% of the time. Also, the average number of patients in the waiting queue is 0.5450 and the average number of patients waiting in the system is 1.4705. More so, the average time a patient spends in the queue is 1.6905 minutes and average a patient spends in the system is 2.7383 minutes.

Table 4.8: Service time of patients at the history section on Wednesday

Patient number	Service time (minutes)		
	Nurse 1	Nurse 2	Nurse 3
1	5	3	2
2	5	4	4
3	3	5	3
4	2	3	5
5	4	3	3
6	3	4	3
7	3	4	4
8	2	5	5
9	3	2	2
10	3	2	3
11	2	3	2
12	2	3	2
13	2	2	3
14	3	4	2
15	2	3	5
16	3	2	3

From Table 4.8, the number of servers (k) =3

Mean service rate;

$$\frac{48}{2.5}$$

□□

□19.2

1. Utilization factor of the system;

P □ $\frac{52.5}{3(19.2)}$

□ 0.9115

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{2}\frac{\left(\frac{52.5}{19.2}\right)^i}{i!} + \frac{\left(\frac{52.5}{19.2}\right)^3}{3!(1-0.9115)}\right]^{-1}$$

$$= \left[7.4728 + 38.5017\right]^{-1}$$

$$= 0.0218$$

3. The expected number of patients in the queue;

$$L_q = \frac{\left(\frac{52.5}{19.2}\right)^3 0.9115}{3!(1-0.9115)^2} \times 0.0218$$

$$= 8.6251$$

4. The expected number of patients in the system;

$$L_s = 8.6251 + \frac{52.5}{19.2}$$

$$= 11.3595$$

5. The expected waiting time of a patient in the queue; 0.

$$W_q = \frac{8.6251}{52.5}$$

$$= 0.8573hr$$

$$= 9.8573 \min utes$$

6. The expected waiting time of a patient in the system;

$$W_s = W_q + \frac{1}{\mu}$$

$$= 0.1643 + \frac{1}{19.2}$$

$$= 0.2164hr$$

$$= 12.983 \text{ minutes}$$

The results show that the server would be busy 91.15% of the time and idle 2.18% of the time. Also, the average number of patients in the waiting queue is 8.6251 and the average number of patients waiting in the system is 11.3595 More so, the average time a patient spends in the queue is 9.8573 minutes and average a patient spends in the system is 12.983 minutes.

Table 4.9: Service time of patients at the consulting room on Wednesday

Patient number	**Service time (minutes)**			
	Doctor 1	**Doctor 2**	**Doctor 3**	**Doctor 4**
1	10	7	4	3
2	5	5	3	4
3	4	4	6	5
4	3	3	3	6
5	6	2	4	4
6	6	8	5	8
7	4	3	8	3
8	3	2	10	7
9	2	5	7	3
10	4	6	3	2
11	3	3	2	3
12	3	4		4
13	3	7		3
14	3			
15	2			

6. The expected waiting time of a patient in the system;

$W_s =$

From Table 4.9 the number of servers (k) = 4

Service rate;

$$\mu = \frac{52}{3.8167}$$

$$= 13.6243 \, patients/hr$$

1. Utilization factor of the system;

$$P = \frac{52.5}{4(13.6243)}$$

$$= 0.9633$$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{4-1} \frac{\left(\frac{52.5}{13.6243}\right)^i}{i!} + \frac{\left(\frac{52.5}{13.6243}\right)^4}{4!(1-0.9633)} \right]^{-1}$$

$$= \left[21.814 + 250.3220\right]^{-1}$$

$$= 0.0037$$

3. The expected number of patients in the queue;

$$L_q = \frac{\left(\frac{52.5}{13.6243}\right)^4 (0.9633)(0.0037)}{3!(1-0.9633)^2}$$

$$= 24.3106$$

4. The expected number of patients in the system;

$$L_s = 24.3106 + 52.5/13.6243$$

$$= 28.164$$

5. The expected waiting time of a patient in the queue;

$$W_q = \frac{24.3106}{52.5}$$

$$= 0.4631hr$$

$$= 27.7835 \min utes$$

$$0.4631 + \frac{1}{13.6243}$$

$$0.5365hr$$

$$= 32.1899 \min utes$$

The results show that the server would be busy 96.33% of the time and idle 0.37% of the time. Also, the average number of patients in the waiting queue is 24.3106 and the average number of patients waiting in the system is 28.164. More so, the average time a patient spends in the queue is 27.7835 minutes and average a patient spends in the system is 32.1899 minutes.

4.1.4 Presentation of data collected on Thursday

Table 4.10: Service time at the record section on Thursday

Number of folders (patients)	Service time (minutes)
16	18
14	16
10	13
09	11

6. The expected waiting time of a patient in the system;

$W_s =$

From Table 4.10, the number of servers (k) = 2

Arrival rate of patients;

$$\lambda = \frac{194}{4}$$

$$= 48.5 \, patients/hr$$

Service rate of patients;

Service rate of patients;

$$\mu = \frac{49}{0.9667}$$

$$= 50.6879$$

1. Utilization factor of the system;

$$P = \frac{48.5}{2(50.6879)}$$

$$= 0.4784$$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{1} \frac{\left(\frac{48.5}{50.6879}\right)^i}{i!} + \frac{\left(\frac{48.5}{50.6879}\right)^2}{2!\left(1 - \frac{48.5}{2(50.6879)}\right)} \right]^{-1}$$

$$= \left[1.9568 + 0.8776\right]^{-1}$$

$$= 0.3528$$

3. The expected number of patients in the queue;

$$L_q \square \frac{\square 48.5250!(1.6879\square\, 0.4784\square^{2}\, \square 0).4784\, \square 0.3528}{2}$$

$\square$ 0.2840

4. The expected number of patients in the system;

$L_s \square$ 0.2840$\square$ 48.550.6879

$\square$1.2408

5. The expected waiting time of a patient in the queue; 0.

$$W_q \square \frac{2840}{48.5}$$

$\square$ 0.0058*hr*

$\square$ 0.3513min *utes*

$$0.0058\square\frac{1}{50.6879}$$

0.0255*hr*

$\square$1.53min *utes*

The results show that the server would be busy 47.84% of the time and idle 35.28% of the time. Also, the average number of patients in the waiting queue is 0.2840 and the average number of patients waiting in the system is 1.2408. More so, the average time a patient spends in the queue is 0.3513 minutes and average a patient spends in the system is 1.53minutes.

Table 4.11: Service time of patients at the history section on Thursday

Patient number	**Service time (minutes)**		
	Nurse 1	**Nurse 2**	**Nurse 3**

6. The expected waiting time of a patient in the system;

W_s

1	3	4	5
2	2	3	3
3	5	2	4
4	3	3	3
5	5	5	2
6	2	3	2
7	3	4	3
8	4	4	5
9	3	3	3
10	3	2	2
11	2	5	4
12	3	3	5
13	4	2	3
14	5	3	2
15	5	5	3

From Table 4.11, the total number of servers (k) = 3

Mean service rate;

$$\frac{47}{5333}$$

2.

18.5526

1. Utilization factor of the system;

P 48.53(18.5526)

0.8714

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{2} \frac{\left(\frac{48.5}{18.5526}\right)^i}{i!} + \frac{\left(\frac{48.5}{18.5526}\right)^3}{3!(1-0.8714)} \right]^{-1}$$

$= \left[7.0312 + 23.1539\right]^{-1}$

$= 0.0331$

3. The expected number of patients in the queue;

$$L_q = \frac{\left(\frac{48.5}{18.5526}\right)^3 (0.8714)}{3!(1-0.8714)^2} \times 0.0331$$

$= 5.1931$

4. The expected number of patients in the system;

$$L_s = 5.1931 + \frac{48.5}{18.5526}$$

$= 7.8073$

5. The expected waiting time of a patient in the queue;

$$W_q = \frac{5.1931}{48.5}$$

$= 0.1071hr$

$= 6.4245$ min*utes*

$$0.1071 + \frac{1}{18.5526}$$

$0.161hr$

$= 9.66$ min*utes*

The results show that the server would be busy 87.14% of the time and idle 3.31% of the time. Also, the average number of patients in the waiting queue is 5.1931 and the average number of patients waiting in the system is 7.8073. More so, the average time a patient

6. The expected waiting time of a patient in the system;

W_s

spends in the queue is 6.4245 minutes and average a patient spends in the system is 9.66 minutes.

Table 4.12: Service time of patients at the consulting room on Thursday

Patients number	Service time (minutes)			
	Doctor 1	**Doctor 2**	**Doctor 3**	**Doctor 4**
1	5	4	8	4
2	7	3	5	3
3	3	10	3	10
4	4	2	4	5
5	2	3	2	7
6	5	5	5	4
7	4	3	3	3
8	8	3	4	2
9	6	2	3	4
10	2	4	2	5
11	3	6	5	2
12	2	3	6	4
13	5	2	3	3
14		2	2	2
15			3	

From Table 4.12 the total number of servers (k) = 4

Mean service rate;

$\frac{56}{7333}$

3.

15.0001

1. Utilization factor of the system;

$P = \frac{48.5}{4(15.0001)}$

$= 0.8083$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{3} \frac{\left(\frac{48.5}{15.0001}\right)^i}{i!} + \frac{\left(\frac{48.5}{15.0001}\right)^4}{4!(1-0.8083)}\right]^{-1}$$

$= \left[15.094 + 23.1274\right]^{-1}$

$= 0.0262$

3. The expected number of patients in the queue;

$$L_q = \frac{\left(\frac{48.5}{15.0001}\right)^4 0.8083}{4!(1-0.8083)^2} \times 0.0262$$

$= 2.6243$

4. The expected number of patients in the system;

$L_s = 2.6243 + \frac{48.5}{15.0001}$

$= 5.8576$

5. The expected waiting time of a patient in the queue;

$W_q = \frac{2.6243}{48.5}$

$= 0.0541 hr$

$= 3.2466$ min *utes*

$0.0541 + \frac{1}{15.0001}$

$0.1208 hr$

$= 7.246$ min *utes*

6. The expected waiting time of a patient in the system;

W_s □

□

The results show that the server would be busy 80.83% of the time and idle 2.62% of the time. Also, the average number of patients in the waiting queue is 2.6243 and the average number of patients waiting in the system is 5.8576. More so, the average time a patient spends in the queue is 3.2466 minutes and average a patient spends in the system is 7.246 minutes.

4.1.5 Presentation of data collected on Friday

Table 4.13: Service time of patients at the record section on Friday

Number of folders (patients)	Service time (minutes)
20	22
07	10
16	16
10	13

From Table 4.13 the total number of servers (k) = 2

The mean arrival rate;

$$\frac{186}{4}$$

□□

□ $46.5 \, patients/hr$

The mean service rate;

$\frac{53}{0667}$

1.

$= 52.1311\, patients/hr$

1. Utilization factor of the system;

$P = 46.5/2(52.1311)$

$= 0.4459$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{1} \frac{(46.5/52.1311)^i}{i!} + \frac{(46.5/52.1311)^2}{2!(1-0.4459)}\right]^{-1}$$

$= [1.892 + 0.718]^{-1}$

$= 0.3831$

3. The expected number of patients in the queue;

$$L_q = \frac{(46.5/52.1311)^2 (0.4459)(0.3831)}{2!(1-0.4459)^2}$$

$= 0.2213$

4. The expected number of patients in the system;

$L_s = 0.2213 + 46.5/52.1311$

$= 1.1133$

5. The expected waiting time of a patient in the queue; 0.2213

6. The expected waiting time of a patient in the system;

W_s

$$W_q = \frac{\quad}{52.1311}$$

$= 0.0042hr$

$= 0.2547 \min utes$

$$0.0042 + \frac{1}{52.1311}$$

$0.0234hr$

$= 1.4029 \min utes$

The results show that the server would be busy 44.59% of the time and idle 38.31% of the time. Also, the average number of patients in the waiting queue is 0.2213 and the average number of patients waiting in the system is1.1133. More so, the average time a patient spends in the queue is 0.2547 minutes and average a patient spends in the system is 1.4029 minutes.

Table 4.14: Service time of patients at the history section on Friday

Patient's number	Service time (minutes)		
	Nurse 1	**Nurse 2**	**Nurse 2**
1	4	5	4
2	3	3	3
3	4	3	2
4	3	4	2
5	2	4	5
6	5	3	5
7	3	5	4
8	3	2	4
9	5	2	4
10	3	3	3
11	4	5	5

12	4	4	3
13	5	3	2
14	3	2	2
15	2	2	3

From Table 4.14, the total number of servers (k) =3

The mean service rate;

45

5333

2.

17.7632

1. Utilization factor of the system;

P 46.53(17.7632)

0.8726

2. The probability that the system would be idle;

46.5 i 46.5 3 1

P_o i20 17i!.7632 4!(1 170..76328083)

7.0442 23.4687 1

0.0328

3. The expected number of patients in the queue;

6. The expected waiting time of a patient in the system;

W_s □

□

□46.5317!(1.7632□ 0.8726□^{3} □0).8726 □0.0328

L_q □ $_2$

□ 5.2724

4. The expected number of patients in the system;

L_s □ 5.22724□ 46.517.7632

□ 7.8902

5. The expected waiting time of a patient in the queue; 5.

W_q □ $\frac{2724}{46.5}$

□ 0.1697*hr*

□ 6.8031min *utes*

0.1134 □$\frac{1}{17.7632}$

0.1697*hr*

□10.1818min *utes*

The results show that the server would be busy 87.26% of the time and idle 3.28% of the time. Also, the average number of patients in the waiting queue is 5.2724 and the average number of patients waiting in the system is 7.8902. More so, the average time a patient spends in the queue is 6.8031 minutes and average a patient spends in the system is 10.1818 minutes.

Table 4.15: Service time of patients at the consulting room on Friday

Patient's number	Service time (minutes)			
	Doctor 1	Doctor 2	Doctor 3	Doctor 4

1	9	10	8	4
2	5	6	7	4
3	4	3	6	3
4	5	4	6	4
5	6	5	5	3
6	2	2	3	5
7	5	5	4	10
8	4	4	5	7
9	3	7	6	6
10	2	5	7	5
11	4	4	2	3
12	4	3	3	8
13	3			
14	2			

From Table 4.15 the total number of servers (k) =4

The mean service rate;

$$\mu = \frac{50}{4}$$

$$= 12.5$$

1. Utilization factor of the system;

$$P = \frac{46.5}{4(12.5)}$$

$$= 0.93$$

2. The probability that the system would be idle;

$$P_o = \left[\sum_{i=0}^{3} \frac{\left(\frac{46.5}{12.5}\right)^i}{i!} + \frac{\left(\frac{46.5}{12.5}\right)^4}{4!(1-0.93)} \right]^{-1}$$

6. The expected waiting time of a patient in the system;

W_s □

□

□□20.2190 □113.9889□□1

□ 0.0075

3. The expected number of patients in the queue;

□464.!5(121□.50□.⁴93□)0.93□0.0075

L_q □ 2

□11.3582

4. The expected number of patients in the system;

L_s □11.3582 □ 46.5/12.5

□15.0782

5. The expected waiting time of a patient in the queue;

W_q □ $\frac{11.3582}{46.5}$

□ 0.2443*hr*

□14.6557min *utes*

0.2443□$\frac{1}{12.5}$

0.3243*hr*

□19.458min *utes*

The results show that the server would be busy 93% of the time and idle 0.75% of the time. Also, the average number of patients in the waiting queue is 11.3582 and the average number of patients waiting in the system is 15.0782. More so, the average time a patient spends in the queue is 14.6557 minutes and average a patient spends in the system is 19.458 minutes.

Table 4.16: The operating characteristics at all the various sections on Monday

Operating characteristics	Records	History	Consulting room
The mean arrival rate(λ) [patients/hr]	50.0000	50.0000	50.0000
The mean service rate(μ) [patients/hr]	45.2847	17.5612	13.8654
Utilization factor of the system(P) in percentage	55.2100	94.9100	90.1500
The probability that the system would be idle(P_o) in percentage	28.8600	1.2000	1.1100
The expected number of patients in the queue(L_q)	0.4841	16.9107	7.2670
The expected number of patients in the system(L_s)	1.5882	19.7579	10.8731
The average time a patient spends in the queue(W_q) in minutes	0.5809	20.2884	8.7204
The average time a patient spends in the system(W_s) in minutes	1.9059	23.7026	13.0440

From T

able 4.16, the busiest of all the sections is the history section. Its utilization factor is 91.91% followed by the consulting room section with a utilization factor of 90.15%. The record section recorded the least of 55.21%. The Table also shows that the history section has more patients waiting in the queue than that of the consulting room and the record section. Also, the number of minutes a person spends before his or her history is taken from the history section is far more than that of the other two sections. On the average, a patient spends about 23.7026 minutes in the entire system of the history section and 13.044 minutes in the consulting room and 1.9059 minutes at the record section.

Table 4.17: The operating characteristics at all the various sections on Tuesday

Operating characteristics	Records	History	Consulting room
The mean arrival rate(λ) [patients/hr]	41.2500	41.2500	41.2500
The mean service rate(μ) [patients/hr]	46.0731	18.4615	12.5581
Utilization factor of the system(P) in percentage	44.7700	74.4800	82.1200
The probability that the system would be idle(P_o) in percentage	38.1500	7.6800	2.3400
The expected number of patients in the queue(L_q)	0.2244	1.6329	2.9154
The expected number of patients in the system(L_s)	1.1197	3.8670	6.2001
The average time a patient spends in the queue(W_q) in minutes	0.3264	2.3751	4.2406
The average time a patient spends in the system(W_s) in minutes	1.6260	5.6254	9.0198

From T
able 4.17, the busiest of all the sections is the consulting room section. Its utilization factor is 82.12% followed by the history section with a utilization factor of 74.48%. The record section recorded the least of 44.77%. The Table also shows that the consulting room section has more patients waiting in the queue than that of the history section and the record section. Also, the number of minutes a person spends before he or she sees a doctor at the consulting room section is far more than that of the other two sections. On the average, a patient spends about 9.0198 minutes in the entire system of the consulting room section and 5.6254 minutes at the history section and 1.626 minutes at the record section.

Table 4.18: The operating characteristics at all the various sections on Wednesday

Operating characteristics	**Records**	**History**	**Consulting room**
The mean arrival rate(λ) [patients/hr]	52.5000	52.5000	52.5000
The mean service rate(μ) [patients/hr]	56.7252	19.2000	13.6243
Utilization factor of the system(P) in percentage	46.2800	91.1500	96.3300
The probability that the system would be idle(P_o) in percentage	36.7200	2.1800	0.3700
The expected number of patients in the queue(L_q)	0.5450	8.6251	24.3106
The expected number of patients in the system(L_s)	1.4705	11.3595	28.1640
The average time a patient spends in the queue(W_q) in minutes	1.6805	9.8573	27.7835
The average time a patient spends in the system(W_s) in minutes	2.7383	12.9830	32.1899

From T

able 4.18, the busiest of all the sections is the consulting room section. Its utilization factor is 96.33% followed by the history section with a utilization factor of 91.15%. The record section recorded the least of 46.28%. The Table also shows that the consulting room section has more patients waiting in the queue than that of the history section and the record section. Also, the number of minutes a person spends before he or she sees a doctor at the consulting room section is far more than that of the other two sections. On the average, a patient spends about 32.1899 minutes in the entire system of the consulting room section 12.983 minutes at the history section and 2.7383 minutes at the record section.

Table 4.19: The operating characteristics at all the various sections on Thursday

Operating characteristics	Records	History	Consulting room
The mean arrival rate(λ) [patients/hr]	48.5000	48.5000	48.5000
The mean service rate(μ) [patients/hr]	50.6879	18.5526	15.0001
Utilization factor of the system(P) in percentage	47.8400	87.1400	80.8300
The probability that the system would be idle(P_o) in percentage	35.2800	3.3100	2.6200
The expected number of patients in the queue(L_q)	0.2840	5.1931	2.6243
The expected number of patients in the system(L_s)	1.2408	7.8073	5.8576
The average time a patient spends in the queue(W_q) in minutes	0.3513	6.4245	3.2466

From T

The average time a patient spends in the system(W_s) in minutes	1.5300	9.6600	7.24600

able 4.19, the busiest of all the sections is the history section. Its utilization factor is 87.14% followed by the consulting room section with a utilization factor of 80.83%. The record section recorded the least of 47.84%. The Table also shows that the history section has more patients waiting in the queue than that of the consulting room and the record section. Also, the number of minutes a person spends before his or her history is taken from the history section is far more than that of the other two sections. On the average, a patient spends about 9.66 minutes in the entire system of the history section and 7.246 minutes in the consulting room and 1.53 minutes at the record section.

Table 4.20: The operating characteristics at all the various sections on Friday

Operating characteristics	**Records**	**History**	**Consulting room**
The mean arrival rate(λ) [patients/hr]	46.5000	46.5000	46.5000
The mean service rate(μ) [patients/hr]	52.1311	17.7632	12.5000
Utilization factor of the system(P) in percentage	44.5900	87.2600	93.0000
The probability that the system would be idle(P_o) in percentage	38.3100	3.2800	0.7500
The expected number of patients in the queue(L_q)	0.2213	5.2724	11.3582
The expected number of patients in the system(L_s)	1.1133	7.8902	15.0782

From T

The average time a patient spends in the queue(W_q) in minutes	0.2547	6.8031	14.6557
The average time a patient spends in the system(W_s) in minutes	1.4029	10.1818	19.4580

able 4.20, the busiest of all the sections is the consulting room section. Its utilization factor is 93.00% followed by the history section with a utilization factor of 87.26%. The record section recorded the least of 44.59%. The Table also shows that the consulting room section has more patients waiting in the queue than that of the history section and the record section. Also, the number of minutes a person spends before he or she sees a doctor at the consulting room section is far more than that of the other two sections. On the average, a patient spends about 19.458 minutes in the entire system of the consulting room section and 10.1818 minutes at the history section and 1.4029minutes at the record section.

Table 4.21: The mean arrival rates of patients at the various sections from Monday to Friday

Days	Mean Arrival Rate
Monday	50.0000
Tuesday	41.2500
Wednesday	52.5000
Thursday	48.500
Friday	46.5000

From Table 4.21, the hospital outpatient department on Wednesday received the highest number of patients. The least arrival rate was recorded on Tuesday.

From T

Table 4.22: The mean service rate of servers at the various sections from Monday to Friday

Mean service rate (patients/hour)	Monday	Tuesday	Wednesday	Thursday	Friday
Records	45.2847	46.0731	56.7252	50.6879	52.1311
History	17.5612	18.4615	19.2000	18.5526	17.7632
Consulting room	13.8654	12.5581	13.6243	15.0001	12.5000

From Table 4.22, the server at the record room section served more patients as compared to the other sections. In every one hour, it was able to serve 56.7252 patients on the average on Wednesday.

Table 4.23: The utilization factor of the servers at the various sections from Monday to Friday

Utilisation factor	Monday	Tuesday	Wednesday	Thursday	Friday
Records	55.2100	44.7700	46.2800	47.8400	44.5900
History	94.9100	74.4800	91.1500	87.1400	87.2600
Consulting room	90.1500	82.1200	96.3300	80.8300	93.0000

From Table 4.23, the busiest of all the three sections was the consulting room. It recorded the highest number of utilization factor on Wednesday. On Monday also, the history section was the busiest among the three sections.

4.2 SUMMARY

In this chapter, the data collection, results and analysis of data were presented.

The next chapter presents the summary of major findings, conclusions and recommendations as drawn from the study.

CHAPTER FIVE

SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS 5.0 INTRODUCTION

This chapter summarizes the results of the study, discusses the conclusion arrived at by the researcher and recommendations are made to guide policy makers.

5.1 SUMMARY OF FINDINGS

Comparing the mean arrival rates of patients per hour on the days that the study was undertaken (Table 4.22), it was observed that Wednesday had the highest mean arrival of 52.5 patients/hour and Tuesday had the least arrival rate of 41.25 patients/hour.

From Table 4.24, analysis of the system utilization factor of the servers at all the three sections shows that among all the five number of days, the server at consulting room section is the busiest of all on Wednesday with a utilization factor of 96.33.

On Monday also, the history section recorded the highest utilization factor of 94.9100 which means its server was the busiest on that day.

Again, the server at the record section on Monday recorded the highest utilization factor of 55.2100.

Comparing Table 4.22 to Table 4.23, the mean arrival rate of patients on Wednesday is greater than the mean service rate of patients at the consulting room section on that same day (i.e. $52.5000 > 13.6243$ ($\lambda > \mu$)).

What this means is that the waiting line would be formed which would increase indefinitely; the service facility would always be busy. This accounted for the reason why the utilization factor at the consulting room section recorded the greatest value on Wednesday as compared to the other two sections on that same day.

In general, the mean arrival rates of patients at both the history and consulting room sections on Monday to Friday are all greater than the mean service rates. This is the reason why the utilization factor of the servers at the history and consulting room sections recorded the higher percentages as compared to those recorded at the records section. This situation would make the system congested and crowded since the queue length at consulting room and the history section would be too long.

Suppose the number of servers in the consulting rooms on Wednesday is increased from four to five, the system utilization of the servers would be 77.07 percent (i.e. it is reduced from 96.33 to 77.07). The number of patients in the queue reduces from 24.3106 to 1.9822; the number of patients in that system would also reduce from 28.164 to 5.8356. Again, the average time a patient spends in the queue declines from 27.7835 to 2.2654 minutes and the average that a patient would have spent in the entire system reduces from 32.1899 to 6.6659 minutes.

Also, suppose that the mean service rate on Wednesday at the consulting rooms is increased from 13.6243 patients/hour to 14.6243 patients/hour, then, the utilization factor of that system would reduce from 96.3300 to 89.7500 percent (a difference of 6.58 percent).

The number of patients in the queue also reduces from 24.3106 to 6.8574 (a difference of 17.4532), the number of patients in the entire system would also reduce from 28.1640 to 10.4473 (a difference of 17.7167).

Again, the average time that a patient would spend in the queue moves from 27.7835 minutes to 7.8370 minutes (the difference here is 19.9465 minutes). The average time a patient would have spent in the system moves from 32.1899 minutes to 11.9388 minutes (a difference of 20.2511 minutes).

More so, supposing on Wednesday, the mean service rate at the consulting room is increased from 13.6243 patients/hour to 14.6243 patients/hour and the number of servers is also increased from four to five; the utilization factor of the system would change from 96.3300 percent to 71.8000 percent (a difference of 24.53). The number of patients in the queue also reduces from 24.33106 to 1.03626 (a difference of 23.2743) and the number of patients in the system would change from 28.164 to 4.6262 (the difference here is 23.5378).

The average time that a patient would have spent in the queue changes from 27.7835 minutes to 1.1843 minutes (a difference of 26.5992 minutes) and the time to spent in the system by a patient would have reduced from 32.1899 minutes to 5.2848 minutes (a difference of 26.9051 minutes).

Increasing the number of servers and the mean service rate of servers are methods used to improve upon waiting lines and reducing patients waiting time (i.e. they are used to make a system reach its steady state) as shown on Wednesday.

Increasing the mean service rates of servers to make a system reach a steady state is cost effective as compared to increasing the number of servers to make a system reach a steady state.

Proper diagnosis of patients is a requirement in a hospital setting. So, it would be of a great help if the average service rate is rather reduced to enable the doctors have much time for their patients.

The analysis of the entire system of the North-Suntreso Hospital shows that queue length increases when the system is very busy. That is, when the utilization factor of the system is very high, it tends to increase patients waiting time.

Thus, on Wednesday and Friday, patients at the consulting rooms spent 32.1899 minutes and 19.4580 minutes respectively.

On the average the total number of minutes a patient spends in the queue before seeing a doctor are as follows; 20.8693 minutes on Monday, 2.7015 minutes on Tuesday, 11.5378 minutes on Wednesday, 6.7758 minutes on Thursday and 7.0578 minutes on Friday. Again, the average number of minutes a patient spent in the entire waiting queue before leaving the hospital after treatment are as follows; 29.5897 minutes on Monday, 6.9421 on Tuesday, 39.3213 minutes on Wednesday , 10.0224 minutes on Thursday and 21.7135 minutes on Friday.

Moreover, the entire waiting time of a patient in the system before seeing a doctor is 25.6085 minutes on Monday, 7.2514 minutes on Tuesday, 15.7213 minutes on Wednesday, 11.19 minutes on Thursday and 11.5847 minutes on Friday.

Also, on the average a patient spent the following number of minutes in the hospital before leaving after treatment; 38.6525 minutes on Monday, 16.2712 minutes on Tuesday, 47.9112 minutes on Wednesday, 18.436 minutes on Thursday and lastly, 31.0427 minutes on Friday.

5.2 CONCLUSIONS

Patients‟ satisfaction is very important to hospital management because the patients are the people who sell the good image of the hospital to others which help to increase the revenue of the hospital. The objective of every hospital is to help reduce patients‟ waiting time, increase revenue and improve upon customer services and care.

The study looked at the queuing system at all the various sections of the North-Suntreso outpatient department. It looked at patients‟ arrival rates, service rates and the utilization factor of the whole system. These three parameters were then used to measure the waiting time of patients in the available queues and in the entire system. They were also used to find the number of patients in the queue and in the whole system.

From the analysis of the study, it was shown that Wednesday recorded the highest arrival rates of patients and Tuesday recorded the least. Wednesday also had the highest number of patients in the waiting line and the in the whole system for consultation.

The study again, revealed that the history of patients taken by the nurses took much of the patients‟ time because, most of the nurses instead of them to work were chatting. This served as dissatisfaction for the patients.

It was also found out from the study that doctor- to- patients‟ ratio is very small. This tends to put the doctors under stress and tension and hence makes the doctors to dispose of patients without in-depth probing or treatment, which often leads to patients‟ dissatisfaction.

In total, it was found out that patients‟ had to wait for long in the queues of the history and consulting room sections of the North-Suntreso Hospital. Patients‟ did not queue for long at the record section because of the two servers which were available and always active to serve patients.

5.3 RECOMMENDATIONS

Based on the findings of this study, the following recommendations have been made to help management improve upon patients‟ satisfaction and also help reduce their waiting times for health care. Patients are uncomfortable with long queues at hospitals.

It is therefore recommended that patients with special ailments should be given a specific number of days to attend the hospital to avoid long queues. Patients should also be separated by the type of services required to help avoid congestion in the hospital premises.

Again, patients who require shorter service times should be given priority over those who require much service times which would help to avoid reneging.

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APPENDICES

APPENDIX 1: FORMAT FOR RECORDING SERVICE TIMES OF PATIENTS AT THE RECORD SECTIONS

Number of folders	Service time (minutes)

APPENDIX 2: FORMAT FOR RECORDING SERVICE TIMES OF PATIENTS AT THE HISTORY SECTIONS

Patient number	Service time (minutes)		
	Nurse 1	Nurse 2	Nurse 3
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			

APPENDIX 3: FORMAT FOR RECORDING SERVICE TIMES OF PATIENTS AT

THE CONSULTING ROOMS

Patient number	Service time (minutes)			
	Doctor 1	Doctor 2	Doctor 3	Doctor 4
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				

APPENDIX 4: FORMAT FOR RECORDING THE OPERATING CHARACTERISTICS

AT ALL THE VARIOUS SECTIONS

Operating characteristics	Records	History	Consulting room
The mean arrival rate(λ) [patients/hr]			
The mean service rate(μ) [patients/hr]			
Utilization factor of the system(P) in percentage			
The probability that the system would be idle(P_o) in percentage			
The expected number of patients in the queue(L_q)			
The expected number of patients in the system(L_s)			

APPENDIX 5: FORMAT FOR RECORDING THE MEAN ARRIVAL RATES OF PATIENTS

Days	Mean arrival rates
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	

APPENDIX 6: FORMAT FOR RECORDING THE MEAN SERVICE RATES OF PATIENTS

Mean service rate (patients/hour)	Monday	Tuesday	Wednesday	Thursday	Friday
Records					
History					
Consulting room					

APPENDIX 7: FORMAT FOR RECORDING THE UTILISATION FACTOR OF THE SERVERS AT THE VARIOUS SECTIONS

Utilisation factor	Monday	Tuesday	Wednesday	Thursday	Friday
Records					
History					
Consulting room					