KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

DEPARTMENT OF MATHEMATICS

INSTITUTE OF DISTANCE LEARNING

TOPIC: DETERMINATION OF OPTIMUM QUANTITY COST AND CYCLE TIME USING INVENTORY MODEL WITH STOCK LEVEL DEPENDENT DEMAND RATE AND VARIABLE HOLDING COST.

BY

BERNARD OSEI OWUSU

JUNE 2013.

DETERMINATION OF OPTIMUM QUANTITY COST AND CYCLE TIME USING INVENTORY MODEL WITH STOCK LEVEL DEPENDENT DEMAND RATE AND VARIABLE HOLDING COST.

CASE STUDY: MANTRAC GHANA LIMITED, ACCRA

BY

BERNARD OSEI OWUSU, B.ED. MATHEMATICS

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

Industrial Mathematics,

Institute of Distance Learning.

JUNE, 2013

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

Bernard Osei Owusu (PG3016809) ………………….........................25 /03/ 2013

Student Name &ID Signature Date

Tel.: 026 530 3167

Certified by:

Mr. K.F. Darkwah …………………… ……………………

Supervisor(s) Name Signature Date

Certified by:

Mr. K.F. Darkwah …………………… ……………………

Head of Dept. Name Signature Date

Certified by:

Professor I.K. Dontwi …………………… ……………………

Dean, IDL Signature Date

ABSTRACT

This thesis seeks to address the problem of inventory management Mantrac Ghana Limited. The objective of this study includes:

(a) To model Mantrac Ghana Limited's inventory cost as Retroactive Holding Cost problem.

(b) To determine optimal order quantity, reorder point and optimal total cost of Mantrac Ghana Limited using Retroactive Holding Cost increase.

Inventory models in which the demand rate depends on the inventory level are based on the common real-life observation that greater product availability tends to stimulate more sales.

This thesis considers the inventory policy for an item with a stock-level dependent demand rate and a storage-time dependent holding cost.

The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage.

Two time-dependent holding cost step functions are mentioned: Retroactive holding cost increase, and incremental holding cost increase. Procedures are developed for determining the optimal order quantity and the optimal cycle time using Retroactive holding cost increase.

TABLE OF CONTENTS

DEDICATION

THIS THESIS IS DEDICATED

TO

THE GLORY OF GOD,

AND MY LATE GRANDMOTHER

MAD. CHARITY AMENYEWU.

ACKNOWLEDGEMENT

It is appropriate to salute certain individuals who have helped me in divers ways over the years. However, some have contributed so much that their names literally jumped off the pages of my life and demand recognition.

My greatest gratitude and appreciation goes to Our Heavenly Father, whose abundant grace, mercies, guidance and protection has made my academic pursuance worthwhile.

Second and foremost, I offer my sincerest gratitude to my supervisor, Mr. K.F. Darkwah, who has supported me throughout my thesis with his patience and knowledge. I attribute the level of my Masters degree to his encouragement and effort, without him this thesis would not have seen the light of day.

I have been blessed with a friendly and cheerful group of fellow students and past school mates especially Mr. Victor Tekpor, Miss Stella of Ghana Police Service, Kumasi Central, Mr. Gaddiel Kweku Eshun of Mantrac Ghana Limited many others.

I am also most grateful to all the Lecturers in the Mathematics Department of Kwame Nkrumah University of Science and Technology, The Ministers of the New Apostolic Church, Kumasi Central for their encouragement and support.

Finally, I thank my dear mother, Madam Patience Pani, my dear brother Patrick Aidoo and last but not least, my better half, Mrs Linda Osei Owusu of Ghana Health Service, Kwadaso for their assistance, prayers and encouragement. May Our Heavenly Father richly bless you all.

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

1.1 Background

The raw materials, work-in-process goods and completely finished goods that are considered to be the portion of a business's assets that are ready or will be ready for sale is inventory. Inventory represents one of the most important assets that most businesses possess, because the turnover of inventory represents one of the primary sources of revenue generation and subsequent earnings for the company's shareholders/owners.

Possessing a high amount of inventory for long periods of time is not usually good for a business because of inventory storage, obsolescence and spoilage costs. However, possessing too little inventory isn't good either, because the business runs the risk of losing out on potential sales and potential market share as well.

Inventory management forecasts and strategizes, such as a just-in-time inventory system, can help minimize inventory costs because goods are created or received as inventory only when needed.

Inventory modeling is one of the most developed fields of operations management and much space has been devoted to this topic in the management science, operational research and practitioner oriented journals.

One of the basic implicit assumptions of inventory models has been the infinite shelf life of products while in storage, that is, a product once in stock remains unchanged and fully usable for satisfying future demand.

Management scientists have been applying quantitative methods to help inventory managers make two critical decisions: how much inventory to order, and when to order it. With low value items, the how much to order and when to order can be based on simple heuristics or rules of thumb.

In traditional inventory models, the demand rate is assumed to be a given constant. Various inventory models have been developed for dealing with varying and stochastic demand. All these models implicitly assume that the demand rate is independent, i.e. an external parameter not influenced by the internal inventory policy.

In real life, however, it is frequently observed that the demand for a particular product can indeed be influenced by internal factors such as price and availability. The change in the demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.

Most models that consider demand variation in response to item availability (i.e. inventory level) assume that the holding cost is constant for the entire inventory cycle. This thesis presents an inventory model with a stock-level dependent demand rate and a variable holding cost. In this model, the holding cost is an increasing step function of the time spent in storage. Two types of time-dependent holding cost increase functions rate considered: Retroactive increase and incremental increase. For each type, a simple algorithm that minimizes the total inventory cost (TIC) is developed for calculating the optimal order quantity and associated cycle time.

The step structure of the holding cost function may be unique since it is the representative of many real- life situations in which the storage times can be classified into different ranges, each with its distinctive holding cost. This is particularly true in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost. For example, three different holding cost rates may apply to short-term, medium term and long term food storage.

1.2 Brief History of Mantrac Ghana Limited

Mantrac Ghana Ltd headquartered in Accra is the authorized dealer for Caterpillar products in Ghana. It provides Caterpillar machines for wide and varied applications in the construction, agricultural and mining development sectors of the economy and a complete range of lift trucks and warehousing equipment for material handling needs.

They also provide Caterpillar engines and generators for the oil, industrial marine, power generation, agriculture and pump applications. In addition, they supply the Olympian range of generators for small-scale industries and residential applications.

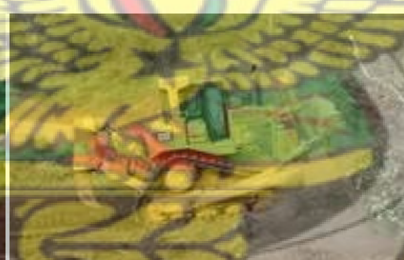
Mantrac Ghana also supplies Kenworth trucks, Ingersoll-Rand drilling equipment and Perkins engines. They provide full product support service, ranging from simple component repairs to complete rehabilitation of machines.

Mantrac Ghana Ltd. is the sole authorized dealer for Caterpillar Products in Ghana. Mantrac Ghana Ltd. distributes and supports the full range of CAT construction equipment including Wheel Loaders, Skid Steer Loaders, Dump Articulated Trucks, Backhoe Loaders, Excavators, Motor Graders, Track-Type Tractors and other products. Moreover, Mantrac Ghana Ltd. distributes Mining, Power Systems and Forklifts, Material-Handling & Warehousing equipment for a wide range of industries and applications. Mantrac Ghana Limited is also the sole approved supplier of genuine Caterpillar parts, which are available at competitive prices. Highly-qualified employees work through an extensive branch network that includes a head office in Accra and branches in Kumasi, Takoradi and Tarkwa. Mantrac service centers are equipped to perform total overhauls, there are also qualified service engineers, with necessary diagnostic and repair tools, can be dispatched at any time to customers'

Figure 1.0 Some types of Hydraulic Hoses

Construction Equipment Mining Equipment

Power Generation & Engines Material Handling & Warehousing Equipment



Construction Equipment

Figure 1.1 Some Equipment of Mantrac Ghana Limited

1.3 Problem Statement

The main problem facing the company especially in Ghana as an example, is the expected marginal holding cost incurred by maintaining excess inventory due to over-ordering; and the expected backlogging cost incurred by not satisfying demand on time due to under ordering.

The company currently faces the potential problem of setting optimum safety stock level for an inventory-level dependent demand rate and a time dependent holding cost. The company's strategy is good for safety stock, but they may order too many quantities which will lead to increase in holding cost and the risk of losses through obsolesce or damages or they may order too small which will also increase the risk of lost sales and unsatisfied customers.

This study would demonstrate the need to set up a master ordering schedule considering the imprecise nature of forecasts of future demands and the uncertain lead time of the manufacturing process.

1.3.0 Objectives

The objectives of the study are:

1. To model Mantrac Ghana Limited's inventory cost of Hydraulic Hoses as Retroactive Holding Cost problem.
2. To determine optimal order quantity and optimal total cost of Hydraulic Hoses in Mantrac Ghana Limited using Retroactive Holding Cost increase Solution Algorithm.

1.4 Methodology

The problem under study is how to determine the optimum quantity of goods to order, the cost and the period within which to order to maximize profit.

The application of inventory model will allow unit holding cost values to vary across different storage period. Variable unit holding costs are considered in the model in determining the optimal inventory policy.

The holding cost per unit is assumed to increase only when the storage time exceeds specified discrete values. In other words, the holding cost per unit time is an increasing step function of the storage time. Two types of holding cost step function are considered: Retroactive Holding Cost Model and Lot – size model, for forecast and simulation methodology will be used to get the optimal solution. In retroactive increase, the unit holding cost rate of the last storage period is applied to all storage periods.

Data was obtained from the inventory department of Mantrac Ghana Limited .The following data was obtained: Stock list, Cost per unit item, Data on demand, Data on supply, Inventory holding cost.

Mathematical software; Matlab was applied to solve the equations. Materials from the internet, books on inventory from KNUST library, papers journals on inventory were used in carrying out this thesis.

1.5 Justification

Most companies in Ghana incur much cost as a result of keeping excess inventory in the warehouse. The operating cost of these companies keeps on increasing as the years go by. Also, some of these companies do not deliver services to their customers on time as a result of inadequate stocks hence the need for proper investigation into inventory control.

Furthermore due to unavailability of factories which produce some essential goods that consumers need, some companies (retailers and wholesalers) have their goods run out of stock thereby affecting the economic and social survival of the population who depend solely on these companies. In addition, scientific research is an effective tool in finding antidotes to most problems facing both developed and developing nations hence the need for students to acquire the skills in academic research to assist in developing ones country.

1.6 Organization of the Thesis

This thesis is organized as follows; the Background of the study, the Problem Statement and the Objective of the study. This is followed by the Methodology and the Justification of the thesis.

Relevant literature is reviewed in Chapter 2 whilst chapter 3 contains the methodology which clearly explains the mathematical tools that are applied and chapter 4 deals with the analysis and modeling of data.

Chapter 5, finally, contains the conclusions and the recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

The development of modern inventory management principles began when Harris (1913) derived the Economic Order Quantity (EOQ) formula. EOQ assumes that demand occurs at known, constant rate and supply fulfills the replenishment order after a fixed lead time. Unfortunately, the real world is not as ideal as that. In reality, demand rate is rarely constant; hard-to-predict market is common in most practical situations.

Also, unpredictable events in supply systems can cause and create unpredictable delays in replenishments. Moreover, in current times when outsourcing is at the centre stage, complex and longer supply chains magnify the length and variability of lead times (Welborn, 2008). Although in the early days researchers acknowledged the necessity for considering uncertainties present in the real world, the rigorous work on inventory control models with stochastic features really began in 1950s. The classic book by Hadley and Whitin (1963), comprehends the research work done in this field to that date. This fundamental research done in those early days has had a pivotal effect on the subsequent developments in the field of inventory theory.

2.1 Inventory Models

The aim of inventory management is to minimize total operating costs while satisfying customer service requirement. In order to accomplish this objective, an optimal order policy will be determined by answering to questions such as when to order and how much to order. The operating costs taken into account, the procurement costs, the holding costs and the shortage costs which are incurred when the demand of the client cannot be satisfied (either lost sales costs or orders costs).

There exist different inventory policies namely: periodic review policy and the continuous review policy. The first policy implies that the stock level will be checked after a fixed period of time and an ordering decision will be made in order to complete the stock to an upper limit (order up to point), if necessary. In the second inventory policy, the stock level will be monitored continuously.

Deterioration refers to decay, damage or spoilage. In respect of items of foods, films, drugs, chemicals, electronic components and radio-active substances, deterioration may happen during normal period of storage and the loss is to be taken into account where we analyze inventory systems.

There have been various models proposed for stock-level dependent inventory systems. Baker and Urban (1988a) investigated a deterministic inventory system in which the demand rate depends on the inventory level described by a polynomial function. A non-linear programming algorithm is utilized to determine the optimal order size and the reorder point. Urban (1995) investigated an inventory system in which the demand rate during stock-out periods differs from the in-stock period demand by a given amount. The demand rate depends on both the initial stock and the instantaneous stock. Urban formulates a profit-maximizing model and develops a closed-form solution.

Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle. Paul et.al (1996) investigated a deterministic inventory system in which shortage are allowed and fully back logged. The demand is stock dependent to certain level and then constant for the remaining periods. A flow chart is provided to solve the general solution.

One of the terminal conditions used in the development of the Datta and Pal model was that the inventory level fall to zero at the end of the order cycle (i.e. $i = 0$ *when* $t = T$). In an inventory system that possesses an inventory-level-dependent demand rate, this may not provide the optimal solution. It may be desirable to order large quantities, resulting in stock remaining at the end of the cycle, due to the potential profits resulting from the increased demand. This phenomenon is discussed in Baker and Urban.

Pal et al. (1993) developed a deterministic inventory model assuming that the demand rate is stock dependent and that the items deteriorate at a constant rate θ .

Classical inventory models found in the existing literature generally deal with constant demand rate of the item or product. Evaluation of an inventory system with such a demand rate was first considered by F.W Harns (1915). He formulated the well-known known square root formula q = $\sqrt{2C3D/C1}$ for economic order quantity (EOQ) of the item; C_1, C_3, D are the holding cost, replenishment cost and demand rate, respectively. After the pioneering attempt by Harns, several other researchers have extended his constant demand –rate to many other interesting and realistic situations. A description of these models can be found in Naddar or any other standard literature on the subject.As time has progressed, inventory models have been developed in which the demand rate is not required to be constant. Such studies have been undertaken by Siver and Meal, Donaldson,Silver and Datta and Pal and Mukherjee.

Very recently, it has been observed that in some situations the demand may be influenced by the on-hand inventory; that is the demand rate may go up or down if the on-hand inventory level increases or decreases. Such a situation generally arises for a consumer- goods type of inventory. In this connection, it would not be out of place to refer to the observation made by Levin et al. around it. It is a common belief that large pile of goods displayed in a supermarket will lead the customer to buy more" .Later, Silver and Peterson also noted the sales at the retail level tend to be more proportional to the inventory displayed.

Among the important papers published so far with inventory-level-dependent demand rate. Mention should be made of works by Gupta and Vrat, Mandal and Phaujdar, Baker and Urban. Gupta and Vrat have discussed a situation where the demand rate has been assumed to depend on the order quantity, whereas Mandal and Phaujdar have discussed an inventory problem assuming the demand rate to be a linear function of the on-hand inventory level at that time. Baker and Urban have analyzed a similar situation assuming the demand rate to be dependent on the on-hand inventory i according to the relation R (i) = αi^{β} where $\alpha > 0$, $0 < \beta < 1$.

A number of authors investigated inventory systems with a two-stage demand rate. Baker and Urban (1988b) considered an inventory system with an initial period of level-dependent demand followed by a period of constant demand. The analysis conducted on this model imposes a terminal condition of zero inventories at the end of the order cycle. Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level-dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle. Paul et al. (1996) investigated a deterministic inventory system in which shortages are allowed and are fully back-logged. The demand is stock dependent to a certain level and then constant for the remaining periods.

Hwang and Hann (2000) constructed an inventory model for an item with an inventory-level dependent demand rate and a fixed expiry date. All units that are not sold by their expiry date are regarded as useless and therefore discarded. Separable programming is utilized to determine the optimal order level and order cycle length.

Ray and Chaudhuri (1997) take the time value of money into account in analyzing an inventory system with stock-dependent demand rate and shortages.

Shao et al. (2000) determined the optimum quality target for a manufacturing process where several grades of customer specifications may be sold. Since rejected goods could be stored and sold later to another customer, variable holding costs are considered in the model. Beltran and Krass (2002) analyzed the dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory. Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation :(a) a nonlinear function of storage time and (b) a nonlinear function of storage level. While Goh (1992) models a holding cost variation over time as a continuous nonlinear function, the storage time is divided into a number of distinctive periods with successively increasing holding costs. As the storage time extends to the next time period, the new holding cost can be applied either retroactively (to all storage periods) or incrementally (to new period only).

Zipkin (2000) provides a systematic discussion of inventory models with stochastic lead times. systems, parallel systems and limited-capacity systems. Exogenous sequential systems are essentially standard inventory systems with constant lead times replaced by stochastic lead times (Kaplan, 1970).In a parallel system, an infinite-server queue is used to model the supply process. With an unlimited capacity, the order lead times are independent and identically distributed random variables.

The aim of inventory management is to minimize total operating costs while satisfying consumer service requirements. In order to accomplish this objective, an optimal ordering policy will be determined by answering to questions such as when to order and how much to order. The operating costs taken into account are the procurement costs, the holding costs and the shortage costs which are incurred when the demand of a client cannot be satisfied (either lost sales costs or backorder costs).

There exist different inventory policies which are periodic-review policy and the continuous-review policy. The first policy implies that the stock level will be checked a fixed period of time and an ordering decision will be made in order to complete to an upper limit (order up to point), if necessary .In second policy, the stock level will be monitored continuously. A fixed quantity will be ordered when the stock level reaches a reorder point. The order quantity will only be delivered after a fixed lead time and shortage can exist if the inventory is exhausted before the receipt of the order quantity. Those basic policies can be adapted to take into account special situations such as stochastic demands and lost sales or backorder.

Research on Inventory Record Inaccuracy (IRI) has been taking place since 1960s with the report by (Rinehart, 1960). The author stated that this inaccuracy produces “deleterious” on operational performance. Following this, it was reported that this divergence between stock record and physical stock results in “warehouse denials” (Iglehart and Morey, 1972). Their research took into consideration the frequency the depth of inventory counts and stocking policy to minimize total inventory and inspection costs.

Moreover, focusing on the significance of measuring IRI, DeHoratius and Raman (2008) show that inventory counts may not impact record inaccuracy and additional buffer stock may not be equally necessary across all times in all stores. In fact, safety stock in the continuous- review lost-sales inventory models is one of the effective inventory management policies for mitigating long run total cost.

Ritchken and Sanker (1984) used a regression- based method to adjust the size of the stock by incorporating an additional safety stock requirement in order to estimate the risk in inventory problems. Persona et al. (2007) propose innovative cost-based analytical models for showing that one can reduce the occurrence of stock-outs by introducing a safety stock or pre-assembled modules or components. On considering the continuous-review lost sales inventory models with a Poisson demand, Hills (2007) shows that a base- stock policy is “economically” optimal and that computing the optimal base-stock and its corresponding cost is quite simple for a backorder model.

However for lost- sales model, this policy is not optimal. Hence, the author proposes three alternative policies. Two of these involve modifying the optimal base-stock policy by imposing a delay between the placements of successive orders. The third policy is to place orders at predetermined fixed and regular intervals. However these policies require a lot of complex calculations for lead-times under demand uncertainty.

In addition, quantitative measures were applied and it was found out that the quality of service-level declines in a continuous review (Q, R) inventory policy when there are inventory miscounts and variations in lead- time (Kumar and Arora, 1992). Even though most of the current research focusing on (Q, R) policy often proposes models of operational research, stimulation modeling is becoming an effective and timely tool and is capturing the cause and effect relationship in this field (Kang and Gershwin, 2004).

Urban (1995) investigated an inventory system in which the demand rate during stock out periods differs from the in-stock period demand by a given amount. The demand rate depends upon both the initial stock and the instantaneous stock. Urban formulates a profit-maximizing model and develops a closed form solution. Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, has a level dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle.

Paul et al. (1996) investigated a deterministic inventory system in which shortages are allowed and are fully back-logged. The demand is stock dependent to a certain level and then constant for the remaining periods. Hwang and Hahn (2000) constructed an inventory model for an item with an inventory-level dependent demand rate and a fixed expiry date. All units that are not sold by their expiry date are regarded as useless and therefore discarded.

The holding cost is explicitly assumed to be varying over time in only few inventory models. Shao et al. (2000) determined the optimum quality target for a manufacturing process where several grades of customers' specifications may be sold. Since rejected goods could be sold later to another customer, variable holding costs are considered in the model. Betran and Krass (2002) analyzed the dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory.

Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. While Goh (1992) models holding cost variation over time as a continuous nonlinear function, the storage time is divided into a number of distinct periods with successively increasing holding costs. As the storage time extends to the next time period, the new holding cost can be applied either retroactively (to all storage periods) or incrementally (to the new period only).

Montgomery et al. (1973) propose a continuous review inventory system where a fraction of the unfilled demand is backordered and the remaining fraction is lost. Both the cases of deterministic and stochastic demands are considered, but the stochastic demand case is treated heuristically. Rosenberg (1979) reformulates the above model by introducing "fictitious demand rate that simplifies the analysis of the partial backorder policy and gives an economic interpretation of the circumstances under which this policy is optimal.

Kim and Park (1985) extend the Montgomery et al.(1973) stochastic demand model to one in which the cost of a backorder is assumed to be proportional to the length of time for which the backorder exists. Assuming at most one order outstanding at any point in time and an arbitrary continuous destiny function of lead time demand, they derive the equations from which the optimal order quantity and the reorder point can be iteratively computed. Assuming Poisson demand and an exponential lead time, Woo and Sphicas (1991) formulate a partial backorder model that allows a finite number of orders to be outstanding.

Rabinowitz et al. (1995) analyze a (Q, r) inventory model where a fixed maximum number of backorders b is allowed. During the stock out period, the first b units of incoming demand are backordered and the remainder is lost. Under the assumption of Poisson demand and no more than a single order outstanding, they derive the expected annual cost function and employ an exhaustive search procedure to find the optimal values of Q,R and b. Chu et al. (2001) generalize the above model by dividing the lead time into two segments and use two backorder control limits, one for each time segment.

Posner et al. (1972) treat the case where backorder customers are willing to wait for a random period of time. The demand process is assumed to be Poisson, and the lead time and how long the customers are willing to wait are assumed to be exponentially distributed. Das (1977) uses an(S-1, S) policy and assumes that customers are willing to wait for a fixed amount of time before canceling their orders. Moinzadeh (1989) also considers an (S-1,S) inventory system with Poisson demand and a constant lead time. Smeitink (1990) proves that Moinzadeh’s results holds for an arbitrary lead time and that the steady-state net inventory probabilities depend on the mean of the lead time and not on the shape of its distribution. Chang and Dye (1999) consider a partial backordering system for deteriorating items with the backlogging rate dependent on the length of the waiting time for the next replenishment.

Moon and Gallego (1994) introduce the distribution-free procedures in the analysis of stochastic inventory models. They solve both the continuous review and the periodic review model with a mixture of backorders and lost sales using the minimax distribution-free approach. The treatment of the periodic review model is heuristic.

Porteus (1990) reviews stochastic periodic review models including one where a fraction of the excess demand is backordered. A myopic approximation to this model is provided by Nahmias (1979). For recent findings regarding the computation of optimal solutions to general (s, S0 inventory systems with a backorder policy (both periodic and continuous review systems)

During the lead time there is a cut off point. Before that, if shortage occurs incoming demands will be filled by emergency orders, and after that all unfilled demands are backordered. Backorders costs are usually time dependent, that is, they accumulate over time. DeCroix and Arreola-Risa (1998) and Cheung (1998) consider inventory systems that offer economic incentives (time –based price discount) to customers who are willing to wait longer than normal delivery times. Furthermore, Kim and Park (1985) and Park (1989) argue that the time duration of the backorder is a critical factor of the backorder costs and must be considered in an inventory system.

Given the importance of shortening the time duration of the backorder period, it is reasonable to let backorders occur close to the time when replenishment is due to arrive. Although inventory systems are typically customer driven, we do notice that there are many real systems controlled solely by the supplier In such cases ,emergency orders are often adopted instead of the lost sale policy (although they are mathematically the same) in order to maintain customer loyalty. Rabinowitz et al. (1995) consider a model for this type of inventory system. However, in their model, shortages are first backordered and the rest are filled by emergency orders. This may not be the most cost-effective because of the time –dependent cost of backorders. Furthermore, setting the time limit rather than the limit on backorders is operationally more convenient.

The assumption of no more than one outstanding order is commonly made in the existing inventory models with emergency orders or lost sales. The usage and plausibility of the assumption has been discussed in detail Hadley and Whitin (1963), Kim and Park (1985), and Cheung (1998). In particular, Hadley and Whitin (1963) discussed the difficulty in developing the exact solutions for the lost sales case when more than one outstanding order is allowed. Hadley and Whitin (1963,p. 198) argued that “ If $r < Q$, then there can never be more than a single order outstanding. In the lost sales case then, it is possible to stipulate that there is only a single order outstanding if one requires that $r < Q$.”

System Dynamics (SD) methodology aims to model real complex dynamic systems for understanding them and coming up with policies to change the problematic dynamic behavior. The real dynamic problems contain feedbacks, delays and random noise or uncertainties which make them “complex” (Größler,2004). Feedbacks and delays are the main reasons why human-decision- making behavior results in unwanted behavior in these systems (Sterman 1989a). In most cases, the problems that are which SD is interested in have problematic dynamic behaviour usually caused by not optimal decisions of humans. To achieve the aim of making valid models of dynamic systems, SD tries to capture human decision making behaviour together with feedbacks and delays which are all endogenously included in the model.

In other words, SD models should be able to represent “intended rationality" of human beings (Größler, 2004). The words intended rationality or bounded rationality is used to describe the decision making behaviour of humans in these complex dynamic systems which are far from optimal. This behaviour should not be interpreted as humans acting irrationally (Größer et al.2004). However, the rationality of decision maker is bounded or limited because of the complexity of many real dynamic systems (Sterman, 2000). Thus the modeler should represent the two bounded rationality of the decision maker for the model to be a valid representation of reality.

In order to model human decision- making behaviour in a certain system, one must first understand how people behave or decide in that system. Laboratory experiments are conducted where subjects play the role of the decision-maker in the model of the system to capture the behaviour of human beings. Then their decision behaviour is modeled with the help of certain heuristics and rules. Various studies work on generic systems such as stock management problem and use laboratory experiments to come up with decision-making behaviour formulation (Sterman 1989a., b.,Dogan and Sterman 2000,Barlas and Özevin 2001). Many of these studies base their formulations on anchor and adjustment heuristic which is first proposed by Tversky and Kahnman (1982).

Clark and Scarf (1960) considered a multiechelon serial system under continuous review. Svoronos and Zipkin (1991) study continuous review hierarchical inventory systems with exogenous stochastic replenishment lead times and one-for-one replenishment policy. By preserving the order of replenishment, the authors were able to approximate the study-state performance and to bring out the important role played by the lead time variance. Lee and Billington (1993) use a single-node periodic review model as a building block to analyze a decentralized supply chain with normally distributed demands and processing lead time.

More examples on supply chain models were proposed by Tayue et.al (1999). An extension of the standard periodic-review model is to impose a capacity limit at each stage on the maximum amounts of outputs per time unit. Glasserman and Tayur (1994) demonstrate that in a serial system with an echelon base-stock policy, the inventory and backorders are stable if the mean demand per period is less than the capacity at every node. Glasserman and Wang (1998) use a large deviations approach to obtain an asymptotic linear relationship between lead time and inventory as the fill rate approaches 100%.

Zipkin (2000) provides a systematic discussion of inventory models with stochastic lead times. Based on the system structure, the models are divided into three groups: exogenous sequential systems, parallel systems and limited-capacity systems. Exogenous sequential systems are essentially standard inventory systems with constant lead times replaced by stochastic lead times (Kaplan, 1970). In a parallel system, an infinite-server queue is used to model the supply process. With an unlimited capacity, the order lead times are independent and identically distributed random variables.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

The main objective of this thesis is to determine the optimum (i.e. minimum cost) inventory policy for an inventory system with inventory-level dependent demand rate and a time-dependent holding cost. Assuming the demand rate to be inventory level dependent means the demand is higher for greater inventory levels. Assuming the holding cost per unit of the item per unit time to be time dependent means the unit holding cost is higher for longer storage periods. The model that will be developed for the inventory system is based on allowing unit holding cost values to vary across different storage period. Variable unit holding costs are considered in the model in determining the optimal inventory policy.

The holding cost per unit is assumed to increase only when the storage time exceeds specified discrete values. In other words, the holding cost per unit time is an increasing step function of the storage time. Two types of holding cost step functions are considered: Retroactive increase, and incremental increase. In retroactive increase, the unit holding cost rate of the last storage period is applied to all storage periods. In incremental increase, the rate of each period, including the last period, is applied only to units stored in that particular period.

3.1 Inventory

The raw materials, work-in-process goods and completely finished goods that are considered to be the portion of a business's assets that is ready or will be ready for sale. Inventory represents one of the most important assets that most businesses possess, because the turnover of inventory represents one of the primary sources of revenue generation and subsequent earnings for the company's shareholders/owners.

Possessing a high amount of inventory for long periods of time is not usually good for a business because of inventory storage, obsolescence and spoilage costs. However, possessing too little inventory isn't good either, because the business runs the risk of losing out on potential sales and potential market share as well. Inventory management forecasts and strategies, such as a just-in-time inventory system, can help minimize inventory costs because goods are created or received as inventory only when needed.

3.1.1 Types of inventory

Several different types of inventories are conducted, depending upon the type of material involved and type of information needed.

Generally, inventory types can be grouped into four classifications. These are:

- Raw materials inventory
- Work-in-process (WIP) inventory
- Finished goods inventory
- Maintenance, repair, and operating supplies, or MRO goods

The figure below displays the types of inventory.

Figure 3.1 Types of Inventory (csun (2011) ie.California State University, Northridge ppt.).

3.2 Inventory Models

Inventory analysis has two problems of importance to the organization of stock items, namely:

- Deciding when to place an order for the replenishment of the stock.
- Deciding how large an order is to be placed.

Two types of uncertainties must be considered:

a. the quantity of items that will be demanded during a given period

b. the time that will elapse between placing an order and the actual delivery of the item.

A major problem of inventory is how we can establish optimal stock levels and this is difficult because of the uncertainty of supply and demand for the commodity. Using inventory models we could formulate policies to control the system.

In some cases such as retailer, wholesaler / distributor, where items are purchased externally, if the problem of inventory exists, then there are two main questions, which generally arise and face any organization. These are how many to order and when to order. Having too much inventory reduces both purchase and /or ordering costs, but it may tie up capital, which may lead to unnecessary holding cost and possibility of deteriorating items.

Whereas having too little inventory reduces the holding cost, but it can result in lost of customers, which may affect the reliability of the organization.

Answering these two questions will lead to the optimal level of inventory for any organization, which minimizes its total inventory cost.

Inventory costs, which are related to the operation of an inventory system, are caused by the actions or lack of actions that the organization is establishing.

The most common costs to an inventory system may include:

- The purchase cost of an item obtained from an external source.
- The order cost of issuing a purchase order to an outside source.
- The holding /carrying cost for keeping items in storage.

3.3 Inventory Level

This depends on the relative rates of flow in and out of the system.

Let

$y(t)$ be rate of input flow of items at time t

$Y(t)$ be the cumulative flow of items into the system

$z(t)$ be the rate of flow of items out of the system time t

$Z(t)$ be the cumulative flow of items out of the system ,

then the inventory level, $I(t)$ is the cumulative input less the cumulative output.

$$I(t) = Y(t) - Z(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.0$$

$$I(t) = \int_0^t [y(t) - z(t)]dt$$

$$I(t) = \int_0^t y(t)dt - \int_0^t z(t)dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots.3.1$$

The figure 3.3 below represents the inventory system when the rates vary with time. The figure might represent a raw material inventory. The flow out of inventory is relatively continuous activity where individual items are replaced into the production system for processing. To replenish the inventory, an order is placed to a supplier. After some delay time, called the lead time, the raw material is delivered in a lot of a specific amount. At the moment of delivery, the rate of input is infinite and other times it is zero. Whenever the instantaneous rates of input and output to a component is not the same, the inventory level changes. When the input rate is higher, inventory grows; when the output rate is lower, inventory declines. Usually the inventory level remains positive. This corresponds to the presence of *on hand inventory*. In situation where cumulative output exceeds the cumulative input, the inventory level is negative. This is what we call *a backorder or shortage condition*.

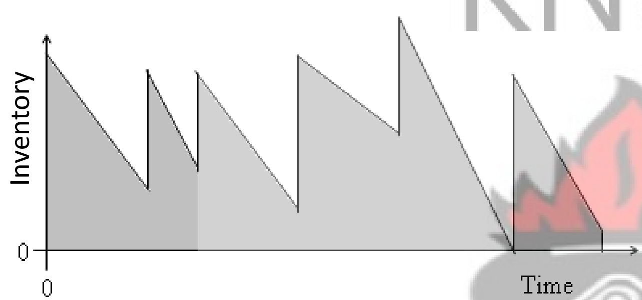


Figure 3.2 Inventory fluctuations as a function of time

3.4 Lot-size or Economic Production Quantity (EPQ)

3.4.1 Introduction

The lot-size refers to the number of units in an order. The Lot-size model is design for the production situations in which once supply begins, demand begins. During supply, demand would be reducing the inventory while supply would be adding the inventory. We assume that supply rate exceeds the demand rate during the supply run. The excess supply would cause a gradual inventory build-up during the supply period. When supply is completed, the continuing demand will cause the inventory to gradually decline until a new supply is started. The inventory pattern for this system is as shown below in figure 3.4 respectively.

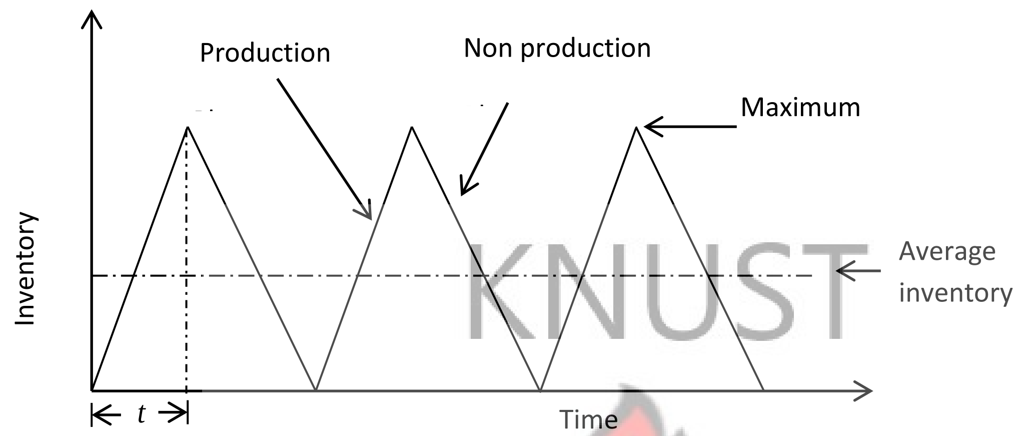


Figure 3.4 Inventory pattern for lot-size model

3.4.2 Assumptions for Lot-size

The following assumptions are considered:

- Average demand is fixed
- Demand pattern is periodic
- Average cost per order is constant
- Daily production rate is greater than daily demand rate during the production run.

3.4.3 Notations for lot-size

Let:

c_z = Ordering cost per order

C_h = Annual holding cost per unit

H_c = Annual Holding Cost

n = Number of orders per unit time

k = Annual Setup cost

t = Number of days for production

d = Demand rate

D = Annual demand

Q = Lot size

τ = Cycle time

T = Cost per time

T_c = Total annual holding cost

$\left(Q^*, \tau^*, T^*\right)$ = List of Optimal quantities

3.4.4 Inventory Level Trajectory of Lot-Size Model

The equation of the trajectory of the inventory pattern is of the form:

$$Q(t) = \frac{1}{T}(p - d)t \qquad \text{……………..3.2}$$

Where $p =$ daily arrival rate

$d =$ daily demand rate

$t =$ number of days for production

Since we are assuming that p will be larger than d, the daily inventory build-up rate during the production phase is $p-d$. If we run production for t days and place $p-d$ units in the inventory each day, the inventory at the end of production will be $(p-d)t$. From the diagram above, the inventory at the end of production is also the maximum inventory. Thus

Maximum inventory $= (p-d)t$

If we are aware of producing lot-size of Q units at a daily production of p units, then:

$Q = pt$ and the length of production t must be $t = \frac{Q}{p} days$

Thus,

$$\text{Maximum inventory} = (p-d)t$$

$$= (p-d)\left(\frac{Q}{p}\right)$$

$$= \left(1-\frac{d}{p}\right)Q \quad \text{……………………...3.3}$$

$$\textit{Average Inventory} \;\; = \frac{1}{2}\int_0^T Q(t)dt$$
$$= \frac{1}{2}\int_0^T \left[\frac{1}{T}(p-d)\frac{Q}{P}\right]dt$$
$$= \frac{1}{2}\left[\frac{1}{T}(p-d)\frac{Q}{P}\right]T$$
$$= \frac{1}{2}(p-d)\frac{Q}{P}$$

$$\therefore \textit{Average Inventory} = \frac{1}{2}\left(1-\frac{d}{p}\right)Q \quad \text{……………………..3.4}$$

3.4.5 Development of the Optimal Order Quantity for Lot-Size Model

We develop below the Lot-size model through the construction of the total inventory cost model.

Let the annual holding cost per unit be C_h, the equation for annual holding cost is

$$\text{Annual Holding Cost} = (\text{Average inventory})(\text{Annual cost per unit})$$

$$H_c = \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h \quad \text{……………………3.5}$$

If D is the annual demand for the product and k is the setup cost per production then the annual setup cost is

$$Annual\ Setup\ or\ Ordering\ Cost = (Number\ production\ per\ year)(Setup\ cost\ per\ production)$$

$$= \frac{D}{Q}k \quad \text{…………………..3.6}$$

Thus, the total annual cost T_c model is

$$T_c = \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h + \frac{D}{Q}k \quad \text{…………………3.7}$$

Suppose that facility operates 250 days per year .Then we write daily demand d in terms of annual demand D as follows:

$$d = \frac{D}{250}$$

Now let P denotes the annual production if the product were produced every day then,

$$P = 250p \qquad \text{and} \qquad p = \frac{P}{250}$$

Thus,

$$\frac{d}{p}=\frac{\left(\frac{D}{250}\right)}{\left(\frac{P}{250}\right)}=\frac{D}{P}$$

Therefore the total annual cost model can also be written as

$$T_c=\frac{1}{2}\left(1-\frac{D}{P}\right)QC_h+\frac{D}{Q}k \qquad \text{……………………….3.8}$$

Setting to zero the derivative of T_c with respect to Q, we obtain

$$\frac{dT_c}{dQ}=0$$

$$\Rightarrow\frac{1}{2}\left(1-\frac{D}{P}\right)C_hQ^2=2Dk$$

$$\Rightarrow Q^2=\frac{2Dk}{\left(1-\frac{D}{P}\right)C_h}$$

Solving for order quantity optimality policy we have

$$Q^*=\sqrt{\frac{2Dk}{\left(1-\frac{D}{P}\right)C_h}} \qquad \text{…………………3.9}$$

and

$$\tau^*=\frac{Q^*}{d} \qquad \text{…………………..3.10}$$

Substitute optimal lot-size, Q^*, into the total cost expression, T_c,

$$T_c^* = \frac{1}{2}\left(1 - \frac{D}{P}\right)Q^* C_h + \frac{D}{Q^*}k \qquad \text{……………...3.11}$$

Note: As the production rate p approaches infinity, $\frac{D}{p}$ approaches zero. That is $\left(1 - \frac{D}{p}\right)$ representing probability of no shortage.

At optimum, the total holding cost is equal to total ordering or set-up cost.

3.4.6 Stockout and service level

Stockout: Stockout occurs when there is insufficient stock to satisfy customers demand.

Service level $= 1 - p$ (stockout)

Taylor (2006), Anderson (2004)

3.4.7 Effective Inventory Cost Decision for Lot-Size Model

1) Holding cost, normal inventory $= \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h$

2) Minimum holding cost $= \frac{1}{2}\left(1 - \frac{D}{P}\right)Q^* C_h$

i.e $\frac{1}{2}\left(1 - \frac{D}{P}\right)Q^* C_h < \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h$

3) $Ordering\ \ Cost = \left(\frac{D}{Q}\right)k$

4) $Minimum\ Ordering\ Cost = \left(\frac{D}{Q^*}\right)k$

i.e $\left(\frac{D}{Q^*}\right)k < \left(\frac{D}{Q}\right)k$

3.4.8 Periodic Review Inventory System

An alternative to the continuous review system is the periodic review inventory system. With a periodic review, the inventory may be checked and orders placed on a weekly, bi-weekly, tri-weekly, monthly or some other periodic basis.

3.4.9 Replenishment Level (M)

It is inventory level at which the order quantity should be demanded at the review period. If the normal probability distribution is used then:

$M = d + zs$ where

d = mean demand

z = number of standard deviations necessary to obtain the acceptable stockout probability

s = standard deviation of the distribution

3.4.10 How-Much-To-Order Decision

The how-much-to-order decision at any review period is determined using the model

Let:

$q = M - X$ where q represents the order quantity at review period

M = replenishment level

X = the inventory on hand at review period which varies since demand is probabilistic.

Taylor (2006), Anderson (2004)

3.5 Retroactive Holding Cost and Incremental Increase Cost Models

There is no question that uncertainty plays in most inventory management situations. The retail merchant wants enough supply to satisfy customer demands but ordering too much increase holding costs and the risk of losses through obsolescence or spoilage. An order too small increases the risk of lost sales and unsatisfied customers. These situations are common, and answers one gets from deterministic analysis very often are not satisfactory when uncertainty is present.

The model that will be developed for the inventory system is based on allowing unit holding cost values to vary across different storage periods. Variable unit holding costs are considered in the model in determining the optimal policy. The holding cost per unit is assumed to increase only when the storage time exceeds specific discrete values. That is the holding cost per unit time is an increase step function of the storage time. Two types of holding cost step functions will be considered:

- Retroactive increase; the unit holding cost rate of the last storage period is applied to all storage period.
- Incremental increase; Higher storage cost rates is applied to storage in later periods.

3.5.1 Notations for Retroactive Holding Cost and Incremental Increase Cost

The following notations are adopted from Goh (1992) for the model under consideration for Mantrac Ghana Limited's Hydraulic Hoses inventory system.

$q(t)$: the on – hand inventory at time t

D : constant (base) demand rate

n : number *of* distinct time periods with different holding cost rate

t : time from the start of the cycle at $t = 0$

t_i : end time of period i, where $i = 1, 2, \ldots, n, t_0 = 0$, and $t_n = \infty$

k : ordering cost per order

h_i : holding cost of the item in period i

$h(t)$: holding cost of the item at time $t, h(t) = h_i$ if $t_{i-1} \leq t \leq t_i$

T : cycle time

β : demand parameter indicating elasticity in relation to the inventory level

3.5.2 Assumptions and Limitations for Retroactive Holding Cost and Incremental Holding Increase Cost

The following assumptions and limitations are considered:

- The demand rate R is an increasing step function of the inventory level q.
- The holding cost is varying as an increasing step function of time in storage.
- Replenishments are instantaneous.
- Shortages are not allowed.
- A single item is considered.
- The demand rate R dependence on the inventory level q is expressed as $R(q) = Dq^{\beta}$, $D > 0, 0 < \beta < 1$, $q \geq 0$ …………………3.12

3.6 The Models

The Total Inventory Cost (TIC) per unit time includes two components:

a. Ordering cost

b. Holding cost

Once ordering is made per cycle, the ordering cost per unit time is simply $\frac{k}{T}$. The total holding cost per cycle is obtained by integrating the product of the holding cost $h(t)$ and inventory $q(t)$ over the whole cycle.

That is,

The total holding cost per cycle = $\frac{k}{T}+\frac{1}{T}\int_0^T h(t)q(t)\,dt$

Hence,

$$\text{TIC}=\frac{k}{T}+\frac{1}{T}\int_0^T h(t)q(t)dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.13$$

Since the demand rate is equal to the rate at which the inventory level decrease, we can describe inventory level q by the following differential equation:

$$\frac{dq(t)}{dt}=-D[q(t)]^{\beta}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.14$$
$$D>0,\ \ 0\le t\le T,\ \ 0<\beta<1$$

The on-hand inventory level at time t, $q(t)$ can be evaluated by solving equation (3.14):

$$q^{-\beta}dq=-Ddt,$$

Integrating both sides:

$$\int_0^t q^{-\beta}dq=\int_0^t -Ddt \ \ where\ 0\le t\le T,$$

$$\Rightarrow \left.\frac{q^{1-\beta}}{(1-\beta)}\right|_0^t=-Dt,$$

$$\Rightarrow q^{1-\beta}(t)-q^{1-\beta}(0)=-D(1-\beta)t$$

$$\Rightarrow q^{1-\beta}(t)=-D(1-\beta)t+q^{1-\beta}(0)$$

However,

$$q^{1-\beta}(0) = Q^{1-\beta},$$

Thus,

$$q^{1-\beta}(t) = -D(1-\beta)t + Q^{1-\beta},$$

$$q(t) = \left[-D(1-\beta)t + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} \qquad \text{……………………...3.15}$$

The period T can be evaluated by substituting the inventory function $q(t)$ at T.

That is:

$$q(T) = 0$$

$$\Rightarrow q(T) = \left[-D(1-\beta)T + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} = 0$$

$$\Rightarrow -D(1-\beta)T - q^{1-\beta}(T) = -Q^{1-\beta}$$

$$T\left(-D(1-\beta) - q^{1-\beta}\right) = -Q^{1-\beta}$$

Hence, $T = \dfrac{Q^{1-\beta}}{D(1-\beta) + q^{1+\beta}}$ ……………….. ……3.16a

Or

$$Q = \left[D(1-\beta)T + q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} \qquad \text{……………………..3.16b}$$

3.6.1 Retroactive Holding Cost Increase

The holding cost is assumed to be an increasing step function of storage time, that is $h_1 < h_2 < h_3 ... < h_n$. Here, a uniform holding cost that depends on the length of storage is used. Specifically, the holding cost of the last storage period applies retroactively to all previous storage periods. Thus, if the cycle ends at in period, e, with $(t_{e-1} \le T \le t_e)$ then the holding cost rate h_e is applied to all periods $1, 2, ..., e$. In this case; the TIC per unit time can be expressed as

$$TIC = \frac{k}{T} + \frac{h_i}{T}\int_0^T q(t)dt, \qquad \text{…………………….. 3.17}$$
$$where \;\; t_{i-1} \le T \le t_i$$

Substituting the value of $q(t)$ from (3.1.5)

$$TIC = \frac{k}{T} + \frac{h_i}{T}\int_0^t \left[-D(1-\beta)t + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} dt,$$
$$= \frac{k}{T} - \frac{h_i}{D(2-\beta)T}\left[-D(1-\beta)\Big|_0^T + Q^{1-\beta}\right]^{\frac{(2-\beta)}{(1-\beta)}}$$

Thus,

$$TIC = \frac{k}{T} + \frac{h_i}{D(2-\beta)T} \times \left[\left[Q^{1-\beta}\right]^{\frac{(2-\beta)}{(1-\beta)}} - \left[-D(1-\beta)T + Q^{1-\beta}\right]^{\frac{(2-\beta)}{(1-\beta)}}\right]$$

Substituting the value of T from (4.2.4)

$$TIC = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)}, \qquad \text{……………………. 3.18}$$
$$where \;\; t_{i-1} \le T \le t_i$$

Setting the derivative of TIC with respect to Q equal to zero and solving for Q, we obtain:

$$Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}, \; t_{i-1} \le T \le t_i. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.19$$

3.6.2 Solution Algorithm

The optimum solution can be determined by using the following solution algorithm steps, Alfares (2007):

1. Beginning with the lowest holding cost h_1, using $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$, $t_{i-1} \le T \le t_i$ to determine Q and $T = \frac{Q^{1-\beta}}{D(1-\beta)}$ to determine T for each h_i until Q is realizable ie $(t_{i-1} \le T \le t_i)$. Call these values T_R *and* Q_R.
2. Use $Q = [D(1-\beta)T]^{\frac{1}{(1-\beta)}}$ to calculate all break-point values of Q, $Q_i = Q(t_i)$, $t_1 \le T < T_R$; each Q_i is obtained by substituting t_i into $Q = [D(1-\beta)T]^{\frac{1}{(1-\beta)}}$.
3. Use $TIC = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)}$, $t_{i-1} \le T \le t_i$ to calculate the TIC for Q_R andeach Q_i
4. Choose the value of Q that gives the lowest TIC.

3.6.3 Incremental Holding Cost Increase

The holding cost is now assumed to be an incremental step function of the storage time. According to this function, higher storage cost rate apply to storage in later periods. Thus, if the cycle ends in period, $e, with\ (t_{e-1} \le T \le t_e)$, then holding cost rate h_1 is applied to period 1, rate h_2 is applied to period 2, and so on; thus rate h_e is applied only to period e from time t_{e-1} up to time T. For this we reset the value of t_e as $(t_e = \mathrm{T})$, and then express the TIC per unit time as:

$$\mathrm{TIC} = \frac{k}{T} + \frac{h_1}{T}\int_0^{t_1} q(t)dt + \frac{h_2}{T}\int_{t_1}^{t_2} q(t)dt + \ldots + \frac{h_e}{T}\int_{t_{e-1}}^{t_e=T} q(t)dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.20$$

Substituting the value of $q(t)$ from eqn. (4.2.3), we obtain:

$$\begin{aligned}\mathrm{TIC} &= \frac{k}{T} + \sum_{i=1}^{e}\frac{h_i}{T}\int_{t_{i-1}}^{t_i}\left[-D(1-\beta)t + Q^{1-\beta}\right]^{\frac{1}{(1-\beta)}} dt,\\ &= \frac{k}{T} + \sum_{i=1}^{e}\frac{-h_i}{D(2-\beta)T}\left[-D(1-\beta)t\Big|_{t_{i-1}}^{t_i} + Q^{1-\beta}\right]^{\frac{(2-\beta)}{(1-\beta)}}\end{aligned}$$

Substituting the value of T from eqn. (4.2.4), and rearranging and simplifying terms gives:

$$\mathrm{TIC} = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1}\frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times \left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}} \qquad \ldots 3.21$$

To find the optimal order size Q^*, we set the derivative TIC with respect to Q equal to zero.

After simplification, we obtain:

$$\begin{aligned}&-\frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_1 Q}{(2-\beta)} + \sum_{i=1}^{e-1}(h_{i+1}-h_i)\left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{1}{(1-\beta)}} - \sum_{i=1}^{e-1}\frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \qquad \ldots\ldots\ldots\ldots 3.22\\ &\times\left[Q^{1-\beta} - D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}} = 0\end{aligned}$$

If the entire inventory cycle happens to fall into the first period $(0 \le \mathrm{T} \le t_1)$, then $e = 1$ and the summations over i in eqn. 3.22 are empty. In that case the optimum solution is simply obtained by substituting h_1 into eqn. 3.19 to calculate Q^*, and then substituting Q^* into eqn. 3.15 to calculate T. obviously, a simple closed form solution for Q^* and T^* can be determined only if $T \le t_1$.

In general, the optimum solution must be determined by the following solution algorithm steps, Alfares (2007).

3.6.4 Solution Algorithm

1. Substitute h_1 into $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$, $Q* = t_{i-1} \le T \le t_i$ to determine $Q_{\max}$, and then substitute $Q_{\max}$ into $T = \frac{Q^{1-\beta}}{D(1-\beta)}$ to determine $T_{\max}$. If $\mathrm{T}_{\max} \le \mathrm{t}_1$, stop; the solution $(Q_{\max,} T_{\max})$ is optimal.
2. Substitute h_n into $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$, $Q* = t_{i-1} \le T \le t_i$ to determine $Q_{\min}$, and Substitute $Q_{\min}$ into $T = \frac{Q^{1-\beta}}{D(1-\beta)}$ to determine $T_{\min}$.
3. Depending on the values of $T_{\min}$ and $T_{\max}$, determine the possible periods that T may fall into (i.e., all feasible values of e).

4. For each feasible value of e, solve

$$-\frac{kD(1-\beta)}{Q^{1-\beta}}+\frac{h_1Q}{(2-\beta)}+\sum_{i=1}^{e-1}(h_{i+1}-h_i)\left[Q^{1-\beta}-D(1-\beta)t_i\right]^{\frac{1}{(1-\beta)}}-\sum_{i=1}^{e-1}\frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)}$$
$$\times\left[Q^{1-\beta}-D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}}=0$$

numerically to determine the optimum value of Q. If Q corresponds to the correct period, it is considered realizable.

5. Using $\mathrm{TIC}=\frac{kD(1-\beta)}{Q^{1-\beta}}+\frac{h_i(1-\beta)Q}{(2-\beta)}+\sum_{i=1}^{e-1}\frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)}\times\left[Q^{1-\beta}-D(1-\beta)t_i\right]^{\frac{(2-\beta)}{(1-\beta)}}$,

   calculate TIC for Q_R and each $Q_i=Q(t_i)$

6. Choose the value of Q that gives the lowest TIC

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND MODELING

4.1 Data Collection and Description

The data for this project was obtained from Inventory Department of Mantrac Ghana Limited on stock, demand and supply of Hoses covering a period of six years on monthly basis.

Stock: The goods or merchandise kept on the premises of a business or warehouse and available for sale or distribution.

Demand: An economic principle that describes a consumer's desire and willingness to pay a price for a specific good or service. Holding all other factors constant, the price of a good or service increases as its demand increases and vice versa.

Supply: A fundamental economic concept that describes the total amount of a specific good or service that is available to consumers. Supply can relate to the amount available at a specific price or the amount available across a range of prices if displayed on a graph. This relates closely to the demand for a good or service at a specific price; all else being equal, the supply provided by producers will rise if the price rises because all firms look to maximize profits.

The data comprises the following:

- Monthly data on stock, demand and supply of Hoses from January 2005 to December 2010.

Table 4.1 Stock, demand and supply data of Hoses for January, 2005 to December, 2010.

YEAR	**2005**			**2006**			**…**	**2010**		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**	**…**	**STOCK**	**DEMAND**	**SUPPLY**
JAN	2400	1804	1804	3500	1405	1405	**…**	3200	1850	1850
FEB	2060	2045	2045	2600	1975	1975	**…**	3400	2135	2135
MARCH	2500	1750	1750	4000	4128	4000	**…**	3500	3590	3500
APRIL	1500	251	251	4500	2145	2145	**…**	4000	3250	3250
MAY	1400	468	468	4800	2694	2694	**…**	3840	2850	2850
JUNE	900	390	390	3000	1945	1945	**…**	3000	2000	2000
JULY	1000	1005	1000	4100	3056	3056	**…**	3010	3045	3010
AUG	2000	1065	1065	3500	2043	2043	**…**	4500	2800	2800
SEPT	935	50	50	1000	632	632	**…**	3000	900	900
OCT	3500	3567	3500	2000	1024	1024	**…**	2100	610	610
NOV	4000	1157	1157	1200	380	380	**…**	1490	780	780
DEC	3000	4309	3000	1500	1902	1500	**…**	1000	1200	1000
AVERAGE	**2099.58**	**1488.42**		**2975**	**1944.08**		**…**	**3003.33**	**2084.17**	

Average annual demand = 23182.32 units

Average annual stock = 32207.88 units

The stock, demand and supply data for January, 2007 to December 2010 is displayed in appendix A. The average stock and demand for each year was displayed in the last rows of the table. The average monthly stock for the six years period was **2684** and that of demand was **1932**.

- Inventory holding cost per unit per year and fixed ordering cost per inventory cycle as at the year 2010 are displayed in the table below.

Table 4.2 Data on cost components

COST	AMOUNT (GH¢)
Ordering cost per order (k)	58.00
Holding cost period one (h_1)	2.90
Holding cost period two (h_2)	3.90
Holding cost period three (h_3)	4.90
Holding cost period four (h_4)	5.90

The costs are in Ghana cedis.

4.2 Stock, Demand and Supply data compared

Figure 4.1 below displays the trajectory of stock, demand and supply data from January 2005 to December 2010.

A cursory look at the pattern of the graph shows that during most of the periods, the stock was more than the demand and supply. However, a careful observation of the pattern of the graph shows the incidence of periodicity with high and low points. During some months, the demand was more than the stock and supply and this will result into backorders and lost sales which is one of the problems of the company.

The high demands during this period are due to some factors such as the low humidity which cause the hose to wear off very early because those periods are in the harmattan. Also because the grounds are very hard during those periods, the hydraulic systems of the excavators which make use of the hose do wear off quickly because of the difficulties the excavators experienced when excavating.

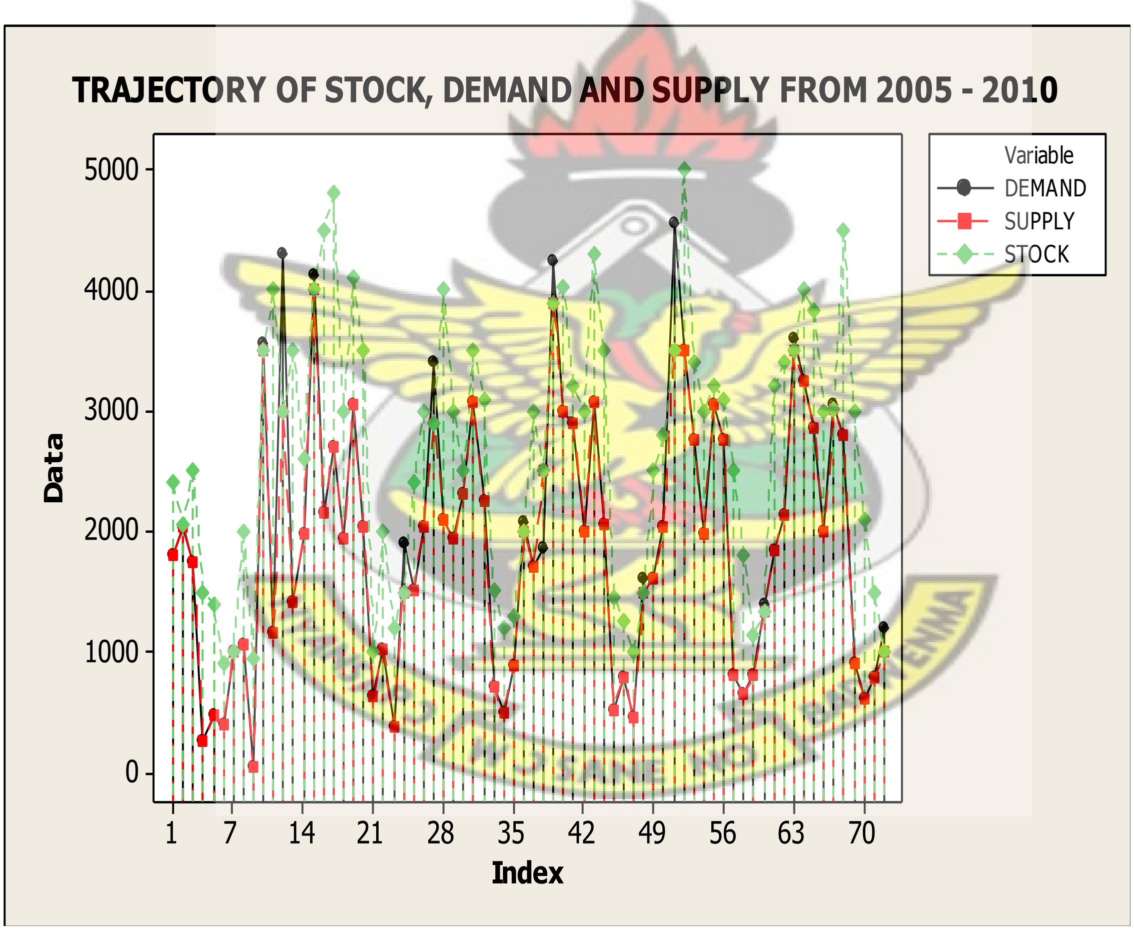


Figure 4.1 Trajectory of Stock, demand and supply from 2005 - 2010

4.2.1 Computational procedures

The following values were used in the computations.

Average annual demand (D) $= \frac{Total\ average\ demand}{6\ years} \times 12\ months$

$= \frac{11591.17}{6} \times 12$

= 1931.86 x 12

= 2683.985 units

Ordering cost per order (k) = GH¢58.00

Inventory holding cost per unit year (h) = GH¢2.90

According to H.K Alfares(2007), fraction of demand backordered during stock out period (β) = 0.1

Holding Cost for period one h_1 = GH¢2.90/ unit year $0< T\leq 0.2$, $t_1 = 0.2$ year

Holding Cost for period two h_2 = GH¢3.90/ unit year $0.2 < T\leq 0.4$, $t_2 = 0.4$ year

Holding Cost for period three h_3 = GH¢4.90/ unit year $0.4 < T\leq 0.6$, $t_3 = 0.6$ year

Holding Cost for period four h_4 = GH¢5.90/ unit year $0.6< T\leq 0.8$, $t_4 = 0.8$ year etc.

The following computations were made using Retroactive Holding Cost solution algorithm.

Alfares (2007):

- **Solution algorithm step 1**

Using $Q* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$, to compute Q, and $T = \frac{Q^{1-\beta}}{D(1-\beta)+q^{1+\beta}}$ to determine

T for each h_i until Q is realizable $(i.e.\ t_{i-1} \leq T \leq t_i)$. Call these values Q_R and T_R.

Begin with $h_1 =$ GH¢2.90

$$\Rightarrow \quad Q* = \left[\frac{58(2683.985)(1-0.1)(2-0.1)}{2.90}\right]^{\frac{1}{(2-0.1)}} = 409.2637 \text{ units}$$

The corresponding cycle time, $T = \frac{(409.2637)^{(1-0.1)}}{2683.985(1-0.1)} = 0.0928$ (*realizable*)

since $0 < T \leq 0.2$.

When $h_2 =$ GH¢3.90

$$\Rightarrow \quad Q* = \left[\frac{58(2683.985)(1-0.1)(2-0.1)}{3.90}\right]^{\frac{1}{(2-0.1)}} = 350.1741 \text{ units}$$

The corresponding cycle time, $T = \frac{(350.1741)^{(1-0.1)}}{2683.985(1-0.1)} = 0.0807$ (*not realizable*)

since $0.2 < T \leq 0.4$.

When h_3 = GH¢4.90

$$\Rightarrow \qquad Q* = \left[\frac{58(\,2683.985)(\,1-0.1\,)(2-0.1)}{4.90}\right]^{\frac{1}{(\,2-0.1)}} = 310.5343 \text{ units}$$

The corresponding cycle time, $T = \frac{(310.5343)^{(\,1-0.1)}}{2683.985(1-0.1)} = 0.0724$ (*not realizable*)

since $0.4 < T \leq 0.6$.

When h_4 = GH¢5.90

$$\Rightarrow \qquad Q* = \left[\frac{58(\,2683.985)(\,1-0.1\,)(2-0.1)}{5.90}\right]^{\frac{1}{(\,2-0.1)}} = 281.6171 \text{ units}$$

The corresponding cycle time, $T = \frac{(281.6171)^{(\,1-0.1)}}{2683.985(1-0.1)} = 0.0663$ (*not realizable*)

since $0.6 < T \leq 0.8$.

This is repeated until realizable Q_R and T_R are obtained for $h_1 < h_2 < h_3 < \ldots h_n$. From above, $Q_R = 409.2637$ and $T_R = 0.0928$.

The table below displays the results for Solution Algorithm step1:

Table 4.3 Results of Solution Algorithm step1

h_i	Q	T	$\left(i.e.\ t_{i-1} \le T \le t_i\right)$	Remark(s)
h_1= GH¢2.90	409.2637	0.0928	$0 < T \le 0.2$	Realizable
h_2 = GH¢3.90	350.1741	0.0807	$0.2 < T \le 0.4$	Not realizable
h_3= GH¢4.90	310.5343	0.0724	$0.4 < T \le 0.6$	Not realizable
h_4 = GH¢5.90	281.6171	0.0663	$0.6 < T \le 0.8$.	Not realizable

- **Solution Algorithm step2**

Calculating all break points of Q, $Q_i = Q\left(t_i\right)$, $t_1 \le T < T_R$; each Q_i is obtained by substituting t_i into $Q = \left[D\left(1-\beta\right)T\right]^{\frac{1}{(1-\beta)}}$

We have:

When t_1= 0.2

$\Rightarrow Q_1 = [2683.985(1-0.1)0.2]^{\frac{1}{(1-0.1)}} = 960.0208$ units

When t_2 = 0.4

$\Rightarrow Q_2 = [2683.985(1-0.1)0.4]^{\frac{1}{(1-0.1)}} = 2073.8$ units

When $t_3 = 0.6$

$$\Rightarrow Q_3 = [2683.985(1 - 0.1)0.6]^{\frac{1}{(1-0.1)}} = 3254.0 \text{ units}$$

When $t_4 = 0.8$

$$\Rightarrow Q_4 = [2683.985(1 - 0.1)0.8]^{\frac{1}{(1-0.1)}} = 4479.6 \text{ units}$$

The table below displays the result for Solution Algorithm step2:

Table 4.4 Results of Solution Algorithm step2

T	**Q_i**
$t_1 = 0.2$	$Q_1 = 960.0208$
$t_2 = 0.4$	$Q_2 = 2073.8$
$t_3 = 0.6$	$Q_3 = 3254.0$
$t_4 = 0.8$	$Q_4 = 4479.6$

From the table the minimum Q_i is $Q_1 = 960.0208$.

- **Solution Algorithm step3**

Using $TIC = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)}, \quad t_{i-1} \le T \le t_i$ to calculate the TIC using Q_R = 409.2637 units and each Q_1,

we have:

for Q_R = 409.2637 units,

$\Rightarrow$ TIC (409.2637) = $\frac{58(2683.985)(1-0.1)}{(409.2637)^{(1-0.1)}} + \frac{2.90(1-0.1)(409.2637)}{(2-0.1)}$ = 1186.9

For Q_1= 960.0208 units,

$\Rightarrow$ TIC (960.0208) = $\frac{58(2683.985)(1-0.1)}{(960.0208)^{(1-0.1)}} + \frac{2.90(1-0.1)(960.0208)}{(2-0.1)}$ = 1608.8

For Q_2 = 2073.8 units,

$\Rightarrow$ TIC (2073.8) = $\frac{58(2683.985)(1-0.1)}{(2073.8)^{(1-0.1)}} + \frac{3.90(1-0.1)(2073.8)}{(2-0.1)}$ = 3976.1

For Q_3 = 3254 units,

$\Rightarrow$ TIC (3254) = $\frac{58(2683.985)(1-0.1)}{(3254)^{(1-0.1)}} + \frac{4.90(1-0.1)(3254)}{(2-0.1)}$ = 7649.4

For Q_4 = 4479.6 units,

$\Rightarrow$ TIC (4479.6) = $\frac{58(2683.985)(1-0.1)}{(4479.6)^{(1-0.1)}} + \frac{5.90(1-0.1)(4479.6)}{(2-0.1)}$ = 1259.2

The table below displays the Q^* and TIC^* for solution algorithm step3:

Table 4.5 Results of TIC using Q_R and Q_i

Q^*	TIC^*
$Q_R^* = 409.0208$	1186.9
$Q_1^* = 960.0208$	1608.8
$Q_2^* = 2073.8$	3976.1
$Q_3^* = 3254$	7649.4
$Q_4^* = 4479.6$	1259.2

Solution Algorithm step 4

Choose the value of Q that gives the lowest TIC.

From table 4.6, the value of Q that gives the lowest Total Inventory Holding Costs (TIC) is 409.2637 and corresponding minimum Total Inventory Cost TIC^*= GH¢1186.9 the cycle period realizable for this order quantity Q^* and TIC^* is 0.2year.

4.2.2 Discussion of Results

From the Solution Algorithm step 1, a cycle time of 0.0928 gives a realizable value. This gives an optimum quantity of **409.0208** units and Total Inventory Cost of GH**¢ 1186.90**.

Low quantity costs with corresponding higher cycle times results in Low Total Inventory Cost as can be observed across the table. Also, the higher the optimum cost, the higher the Total Inventory Cost. The Total Inventory Cost also depends on the cycle time.

However, cycle times of 0.0807, 0.0724 and 0.0663 gives unrealizable values as can be seen in the table below.

Table 4.6 Quantity, Cycle time and Total Inventory Cost

Q∗	Cycle time, T	TIC (GH¢)	
409.0208	0.0928	1186.9	Realizable
960.0208	0.0807	1608.8	Not realizable
2073.8	0.0724	3976.1	Not realizable
3254	0.0663	7649.4	Not realizable

As more bundles of hoses remain in the warehouses with associated increases in holding costs, the Total Inventory Cost also increases .This will result in the company incurring more debts.

The Total Inventory Cost (TIC) often depend on the lead time demand, the expected shortage cost at the end of the period, the fraction of demand backordered during stock out period and the quantity demanded by the customers.

Shortage brings loss of goodwill and it is difficult to the exact amount of shortage cost. The same problem is experienced in the case of the ordering and holding costs hence in inventory, the decision maker may allow the flexibility in the cost parameter values in order to tackle the uncertainties which always fit the real situations.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

An inventory model to model Mantrac Ghana Limited's inventory cost using a Retroactive Holding Cost was used.

With the model, the optimal order quantity Q^* which minimizes the Total Inventory Cost (TIC) was determined to be **409 units.** The cycle period T^* for the quantity Q^* to be produced per cycle is 0.2year.

- Mantrac Ghana Limited could order **409** units per each order within the cycle period of 0.2 year. This would minimize the Total Inventory Cost to **GH¢1186.9**

5.2 Recommendations

Based on the findings so far arrived at, in order to ensure proper inventory control systems at Mantrac Ghana Limited, the following recommendations are made.

- To determine optimal quantity Q^*, optimal Total Inventory Cost (TIC^*) and cycle period T^*, companies who own storage facilities should use Retroactive Holding Cost model.
- To sustain the inventory of Mantrac Ghana Limited, stakeholders of the company should use the Retroactive Holding Cost Model to produce the quantity Q^*, of **409** units per each order within the cycle period of 0.2 year.
- There should be further study at Mantrac Ghana Limited, using another holding cost as increasing step function of the storage time, : that is: higher storage cost rates apply to storage of later periods (Incremental holding cost increase).

REFERENCES

[1] Alfares H.K (2007). Inventory model with stock-level dependent demand rate and variable holding cost. International Journal of Production Economics 108 . 259-265.

[2] Baker, R.C.and Urban, TL., 1988b. A deterministic inventory system with an inventory level dependent rate. Journal of the Operation Research Society 39 (9). 823 – 831.

[3] Beltran, J.L.and Krass, D., 2002. Dynamic lot sizing with returning items and disposals. IIE Transactions 34 (5). 437 – 448.

[5] Datta, T.K. and Pal, A.K.(1990). A note an inventory model with inventory level dependent rate. Journal of the Operation Research Society 41 (10). 971 – 975.

[6] Goh. M., 1992. Some results for inventory models having inventory level dependent demand rate. International Journal of Production Economics 27 (1). 155 – 160.

[7] Größler, A. 2004. A Content and Process View on Bounded Rationality in System Dynamics. Systems Research and Behavioral Science, 21, 319-330.

[8] Hadley, G and Whitin, T M (1963). Analysis of Inventory Systems, Prentice Hall, Inc., Englewood Cliffs, NJ.

[9] Hwang. H., Hahn. K.H., 2000. An optimal procurement policy for items with an inventory level dependent demand rate and fixed life time. European Journal of Operation Research 127 (3), 537 – 545.

[10] Iglehart. D., Morrey. R.C., 1972. Inventory systems with imperfect asset information. Management Science 18 (8), 388 – 394.

[11] Kang. Y and Gershwin. S.B., 2004. Information inaccuracy in inventory systems –stock loss and stock-out. IEE Transactions 37. 843 – 859.

[12] Kumar. S. Arora. S., 1992. Effects of inventory miscount and non inclusion of the lead time variability on inventory system performance. IIE Transactions 24 (2). 96 – 103.

[13] Nahmias S., 1997. Production and Operation analysis, volume third edition, McGraw – Hill International Editions.

[14] Pal et al. (1993).*A deterministic Inventory model for deteriorating with stock dependent demand rate. International Journal Production Economics* 32 (5), 291-299.

[15] Persona. A. Battini. D. Manzini. R. Pareschi. A. 2007. Optimal safety stock levels of subassemblies and manufacturing components International Journal of Production Economics. 110 (1 – 2). 147 – 159.

[16] Ray, J., Chaudhuri, K.S., 1997. An EOQ model with stock dependent demand, shortage, inflation and time discounting. International Journal of Production Economics 53 (2), 171–180

[17] Rinehart, R.F., 1960. Effects and causes of discrepancies in supply operations. Operations Research 8 (4). 543 – 564.

[18] Ritchken.P.and Sankar. R. 1984. The effect of estimation risk in establishing safety stock levels in an inventory model. Journal of Operational Research Society 35 (12).1091–1099.

[19] Shao. Y.E., Fowler. J.W.and Runger, G.C., 2000. Determine the optimal target for a process with multiple markets and variable holding costs. International Journal of Production Economics 65 (3), 229 – 242.

[20] Sterman, J.D., 2000. *Business Dynamics*. McGraw-Hill. USA.

[21] Urban, T.L., 1995. Inventory models with the demand rate dependent on stock and shortage levels. International Journal of Production Economics 40 (1), 2 – 28.

APPENDIXES

APPENDIX A DATA ON STOCK, DEMAND AND SUPPLY FROM 2006 - 2010

TABLE A1 Data on stock, demand and supply from Jan. 2005 – Dec. 2006

YEAR	2005			2006		
MONTH	STOCK	DEMAND	SUPPLY	STOCK	DEMAND	SUPPLY
JANUARY	2400	1804	1804	3500	1405	1405
FEBRUARY	2060	2045	2045	2600	1975	1975
MARCH	2500	1750	1750	4000	4128	4000
APRIL	1500	251	251	4500	2145	2145
MAY	1400	468	468	4800	2694	2694
JUNE	900	390	390	3000	1945	1945
JULY	1000	1005	1000	4100	3056	3056
AUGUST	2000	1065	1065	3500	2043	2043
SEPTEMBER	935	50	50	1000	632	632
OCTOBER	3500	3567	3500	2000	1024	1024
NOVEMBER	4000	1157	1157	1200	380	380
DECEMBER	3000	4309	3000	1500	1902	1500
AVERAGE	**2099.58**	**1488.42**	**1373.33**	**2975**	**1944.08**	**1899.92**

TABLE A.2 Data on stock, demand and supply from Jan. 2007 – Dec. 2008

YEAR	**2007**			**2008**		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**
JANUARY	2400	1504	1504	3000	1704	1704
FEBRUARY	3000	2040	2040	2500	1865	2500
MARCH	2900	3400	2900	3895	4234	3895
APRIL	4000	2100	2100	4022	3001	3001
MAY	3000	1934	1934	3200	2890	2890
JUNE	2500	2310	2310	3000	2003	2003
JULY	3500	3078	3078	4300	3070	3070
AUGUST	3100	2250	2250	3500	2065	2065
SEPTEMBER	1510	712	712	1450	520	520
OCTOBER	1200	500	500	1250	792	792
NOVEMBER	1300	890	890	1000	450	450
DECEMBER	2000	2068	2000	1500	1600	1500
AVERAGE	**2534.17**	**1898.83**	**1851.5**	**2718.08**	**2016.17**	**2032.5**

TABLE A.3 Data on stock, demand and supply from Jan. 2009 – Dec. 2010

YEAR	**2009**			**2010**		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**
JANUARY	2500	1607	1607	3200	1850	1850
FEBRUARY	2800	2045	2045	3400	2135	2135
MARCH	3500	4560	3500	3500	3590	3500
APRIL	5000	3500	3500	4000	3250	3250
MAY	3400	2750	2750	3840	2850	2850
JUNE	3000	1984	1984	3000	2000	2000
JULY	3200	3055	3055	3010	3045	3010
AUGUST	3100	2750	2750	4500	2800	2800
SEPTEMBER	2500	810	810	3000	900	900
OCTOBER	1800	655	655	2100	610	610
NOVEMBER	1145	798	798	1490	780	780
DECEMBER	1340	1400	1340	1000	1200	1000
AVERAGE	**2773.75**	**2159.5**	**2066.2**	**3003.33**	**2084.17**	**2057**

AVERAGE ANNUAL DEMAND (D) FOR THE PERIOD - 23182.32 units

AVERAGE ANNUAL STOCK - 32207.88 units

AVERAGE ANNUAL SUPPLY - 18757.26 units

APPENDIX B

Table B1 Data on cost components

COST	AMOUNT (GH¢)
Fixed ordering cost per inventory cycles (C)	58.00
Holding cost per item per year (h)	2.90

APPENDIX C

C.1 MATLAB OUTPUT FOR DETERMINATION OF OPTIMAL QUANTITY AND OPTIMAL TIC OF MANTRAC GHANA LIMITED'S INVENTORY OF HOSES USING RETROACTIVE HOLDING COST MODEL.

```
>> D=2683.985

D =

  2.6840e+003

>> k=58

k =

   58

>> h1=2.90
```

```
h1 =

    2.9000

>> b=0.1

b =

    0.1000

>> Q=(((k*D)*(1-b)*(2-b))/h1)^(1/(2-b))

Q =

  409.2637

>> T=(Q^(1-b))/(D*(1-b))

T =

    0.0928

>> h2=3.90

h2 =

    3.9000
```

```
>> Q=(((k*D)*(1-b)*(2-b))/h2)^(1/(2-b))

Q =

  350.1741

>> T=(Q^(1-b))/(D*(1-b))

T =

    0.0807

>> h3=4.90

h3 =

    4.9000

>> Q=(((k*D)*(1-b)*(2-b))/h3)^(1/(2-b))

Q =

  310.5343
```

```
>> T=(Q^(1-b))/(D*(1-b))

T =

    0.0724
>> h4=5.90

h4 =

    5.9000

>> Q=(((k*D)*(1-b)*(2-b))/h4)^(1/(2-b))

Q =

  281.6171

>> T=(Q^(1-b))/(D*(1-b))

T =

    0.0663
```

End of step 1

Step 2

```
>> D=2683.985

D =


  2.6840e+003


>> b=0.1


b =


    0.1000


>> T=0.2


T =


    0.2000


>> Q=(D*(1-b)*T)^(1/(1-b))
```

```
Q =

  960.0208

>> T=0.4

T =

    0.4000

>> Q=(D*(1-b)*T)^(1/(1-b))

Q =

  2.0738e+003

>> T=0.6

T =

    0.6000

>> Q=(D*(1-b)*T)^(1/(1-b))

Q =

  3.2540e+003
```

```
>> T=0.8

T =

   0.8000
>> Q=(D*(1-b)*T)^(1/(1-b))

Q =

  4.4796e+003
```

Step 3

```
>> k=58

k =

   58

>> D= 2683.985

D =

  2.6840e+003
```

```
>> b=0.1

b =

   0.1000
>> h1=2.90

h1 =

   2.9000

>> Q1=960.0208

Q1 =

  960.0208

>> TIC=((k*D)*(1-b))/(Q1^(1-b))+((h1)*(1-b)*Q1)/(2-b)

TIC =

  1.6088e+003
>> h2=3.90
```

```
h2 =

   3.9000

>> Q2=2073.8

Q2 =

  2.0738e+003

>> TIC=((k*D)*(1-b))/(Q2^(1-b))+((h2)*(1-b)*Q2)/(2-b)

TIC =

  3.9761e+003

>> h3=4.90

h3 =

   4.9000

>> Q3=3254

Q3 =

      3254
```

```
>> TIC=((k*D)*(1-b))/(Q3^(1-b))+((h3)*(1-b)*Q3)/(2-b)

TIC =

  7.6494e+003

>>
>> h4=5.90

h4 =

    5.9000

>> Q4=4479.6

Q4 =

  4.4796e+003

>> TIC=((k*D)*(1-b))/(Q4^(1-b))+((h4)*(1-b)*Q4)/(2-b)

TIC =
  7.6494e+003


>>
>> h4=5.90
```

```
h4 =

    5.9000

>> Q4=4479.6

Q4 =

  4.4796e+003

>> TIC=((k*D)*(1-b))/(Q4^(1-b))+((h4)*(1-b)*Q4)/(2-b)
TIC =
 1.2592e+004
```