KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

OPTIMAL ALLOCATION OF HOSPITAL BEDS - A MATHEMATICAL PROGRAMMING APPROACH

(CASE STUDY OF PUBLIC HOSPITAL IN GHANA)

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc. degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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DEDICATION

This research work is dedicated to the glory of God, who is my source of wisdom, knowledge and strength and to my wife, Miss Angela O. Armah and my daughter, Blessing Okailey Obli.

ABSTRACT

This thesis provides application of a queuing theory and goal programming combination model to aid in the reallocation of beds in a public hospital in Ghana. The adoption of queuing theory and goal programming gives us a clear objective or target of this study. The queuing theory was able to determine the optimal number of beds required in each department or wards and successfully ensured that no patient is turned away from the wards and no loss of income generation to the hospital. Outcome from queuing theory are used to construct a multiobjective decision aiding model within goal programming framework. The model was tested with one year [January - December, 2010] data collected from a public hospital in Ghana.

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CONTENTS

LIST OF ABBREVIATION

GH ....Ghana

GP .... Goal Programming

LOS .... Length of Stay

MOP ....Multiple Objective Programming MOLP

.... Multiple Objectives Linear Programming

MCDA ....Multiple Criteria Decision Analysis MCDM

.... Multiple Criteria Decision Making

CGP .... Chebyshev Goal Programming

LGP .... Lexicographic Goal Programming WGP

....Weighted Goal Programming DSS

.... Decision Support System

QT .... Queuing Theory ED

....Exponential Distribution

PD .... Poisson Distribution

MDG ....Millenium Development Goal PTA

....Patient Turn Away PBO

.... Patient Bed Occupancy

PRA .... Patient Refusal Admision

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

The issue of quality health care is a great challenge faced by most governments in both developed and severely developing countries. Among the number of problems faced by the health sector, is the allocation of beds which forms core part of the health delivery system. Bed allocation in hospitals is a major challenge worldwide to many health sectors in different countries, due to increase number of patients, changes in our life style and other factors. This problem can be overcome, by implementing an effective and efficient bed management allocation systems within the health service. According to Gorunescu et al. (2002) an underprovision of hospital beds lead to patients in need of hospital care being turned away, consequently, patient dissatisfaction, build up of waiting list and stress characterize the hospital system.

The problem with bed management is a core operational challenge in all hospitals in Ghana, with public hospitals worse affected. Today, many private hospitals face great demands to reduce costs and improve quality of service by minimizing patient waiting times. There is also the need to have a means of measuring the performance of the activities that results in the availability of beds. Optimizing bed allocation is a critical to the efficient functioning of any hospital. Hospital resource management is concerned with the efficient and effective utilization of resources, i.e. operating rooms and beds, when and where they are needed. The understanding of patient in-flow and resource utilization is of great importance to all hospital managers and all ward heads.

In line with Governments effort to achieve the Millennium Development Goals (MDG), conscious effort has been made in addressing certain health matters though there are more to be done. One of it is availability of beds in our major and minor hospitals as well as clinics in Ghana. A central policy issue in many countries is how national(tax-based) funds should be allocated to localities. Rather than relying on arbitrary methods of solving this “resource allocation” problem. Nevertheless, shortages of beds in hospitals has brought a major challenge to health care system in Ghana, due to which most patients are found lying on bare floor, benches and chairs in an attempt to receive medical attention. Some even consult the physician, return to their homes and come another day as a result of shortage of beds.

In Ghana, many patients who go to the public health facilities are treated anyhow, to make way for others due to unavailability of beds and improper bed management system. However, most of our human resource in quest to receive proper medical care ends up getting infections, eventually leading to death; thereby reducing our manpower development as a country. As a result of this problem, Government spend huge sums of money in terms of providing fuel, since most patients are referred from one hospital to another at a distance. Unavailability of beds in our hospital facilities makes it difficult for emergency health workers to provide efficient service, since most cases being inter hospital transfers gets delayed for hours waiting an old patient to be discharged.

Due to the number of patient that need to be attended to, there is also a lot of burden on health care providers, this affect them mentally and psychologically. Working with a shortage in beds is highly unpleasant for both patients and staff.

It affects all types of qualities.

1.2 Problem Statement

In Ghana the problem of limited resources allocation in the health care sector have been identified as one of the major fundamental issues, in the management of hospital operations. A policy issue in the country is how national funds should be allocated to localities. Rather than relying on arbitrary methods of solving this “resource allocation” problem. Hospitals are under such pressure that, patients are turn-away because patients outnumber the available beds in most of the wards. A high rate of patient-bed ratio may indicate revenue loss to management, when beds are available elsewhere, and a low occupancy rate will result in waste of hospital resources as beds are often left unused. Therefore the penalty costs are directly transferred to the government. A balance needs to be found between the loss from patients turned away and unoccupied beds in each ward. A decision making model based on the combination of queuing theory and Goal Programming(GP) across the different wards to assess the potential of bed reallocation to improve the hospitals overall performance.

1.3 Objective of the Study

To build a decision making model to assess the potential of bed allocation to improve hospital management system.

The specific objectives of this thesis are as follows:

1. To balanced bed occupancy across entire hospital.
2. To find number of beds required to reach optimal target values.
3. Maximized daily income generation.
4. Minimized the number of patients unserved or unattended.

1.4 Methodology

The model we decided to use in studying the hospital bed allocation are both queuing theory and GP. The queuing theory captures stochastic arrival patterns and service times. Patient arrivals for specialty or ward can be approximated by Poisson process and patient service time following a phase type distribution. The model based on M/PH/m queuing model where the number of beds available is fixed to m and no queuing is allowed. With a Poisson arrival rate of λ per day and an LOS of τ days, the average number of arrivals during the day and average length of stay is therefore $\lambda\tau$. After formulation of queuing model, the goal programming methodology that uses the result of the queuing model as input to construct a multi-objective decision aiding model in the GP framework, taking account of targets and objectives related to customer service and income generation from the hospital manager and all wards heads.

1.5 Justification

This thesis will guide and assist hospital managers and ward heads in achieving the institution goals of optimum allocation of beds in improving the institution. The research will also assist research scientists, mathematicians and etc. to further develop suitable models to help public health professionals to devise better strategies for assessing potential improvements in the allocation of beds to different wards in hospital.

Finally, it will assist to measure the performance of the interventions in public hospitals in the country.

1.6 Thesis Organisation

This thesis is organized into five main chapters. Chapter one presents the Introduction of the thesis. This consists of the Background of the Study, the research Problem Statement, Objectives of the research, Methodology, Justification and Organization of the thesis. The second chapter deals with the Literature review, which looks at briefly work done by other researchers on the topic. Chapter three present the Formulation of the Mathematical Model. Data, Analysis and Discussions are considered in chapter four. Chapter five looks at Summary, Conclusion and Recommendations of the analyzed data.

CHAPTER 2

LITERATURE REVIEW

Diminishing government subsidies, competition, and the increasing influence of managed care means that hospitals are under enormous pressure to cut costs. In this regard, Harper and Shahani (2002) came up that, it is more important than ever for hospital managers to identify ways to deploy their resources more effectively. This chapter presents an overview of the work done on the bed capacity planning problem. It further reviews the use of stochastic modeling and, more specifically, of Monte Carlo simulation. Finally,it details operating room scheduling and gives some insight into scheduling techniques by providing a review of the relevant literature.

A number of researchers have investigated patient demand and bed capacity planning at a specific department within a hospital. de Bruin et al. (2010) reported on the dimensioning of hospital wards using the Erlang loss model. They argued that though most hospitals use the same target occupancy rate for all wards, sometimes an exception need be made for critical care and intensive units. They pointed out that this equity assumption is un realistic and that it might result in an excessive number of refused admissions particularly for smaller units. They used queuing theory to assess the impact of this assumption and also to develop a decision support system to evaluate the current size of the nursing unit. Tütüncü and Newlands (2009) showed that hospital beds planing and management was effective factor for Increase Patient Satisfaction and effectiveness units.

de Bruin et al. (2010) suggests various methods including ratios, discrete event analysis, queuing dynamics and stochastic simulation to solve the bed planning problem.

Gorunescu et al. (2002) applied ratio based methods using average treatment durations to determine required bed capacities. Green (2004) came up that the average required bed capacity is derived from a function of the length of stay (LOS) ratio.

Lapierre et al. (2005) present an approach on long and short term capacity decisions to be made based on target occupancy levels. These two methods tend not to take into account day to day and hour by hour admissions request fluctuations. This type of variation is due to the unpredictable nature of hospital admission rates and patients length of stays. Stochastic simulation models also have been used to determine the size of the required bed capacity based on the number of patients in the hospital.

Gorunescu et al. (2002) used a combination of simulation, queuing theory, statistical analysis and mathematical modeling to determine the bed requirements and to predict the probability that patients needing admission will be turned away or transferred. Li et al. (2009) developed a stochastic methodologies that combine queuing network analysis with integer programming models and discrete event simulation to balance bed occupancy rates against reduced delays, reduced patient blocking and reduced poor bed assignment.

Kao and Tung (1981) presented a network flow models that incorporate facility performance and budget constraints to determine the optimum size of bed capacity. The studies cited to a certain extent are theoretical and are arduous to apply by hospital management in a real life. This article proposes a decision analysis method that easily can be used by decision makers without specific technical modeling knowledge. The proposed approach enables hospital administration to choose the most appropriate method and remains simple to understand and apply. The project objective was to enable them to include their knowledge and insight in the solution process.

McClain has developed a stochastic model to forecast the allocation of nonobstetric patient-days to the obstetric unit and to predict the effect of such allocations on demand for obstetric beds McClain (1978). Dexter and Macario have modeled the distribution of patients at an obstetrical unit as a Poisson distribution and minimized the number of staffed beds subject to remaining below a specified probability of patient overflow Dexter and Macario (2001). Harris has developed a simulation model to aid decision making in the area of operating theater time tables and the resultant hospital bed requirements Harris (1986). Kembe M. M and D (2014) developed a queuing model for bed occupancy management and planning of hospitals. The model was used to describe the movements of patients through a hospital department and to determine the main characteristics of the access of patients to hospital, such as; mean bed occupancy and probability that a demand for hospital care is lost because all beds are occupied. They present a technique for optimizing the number of beds in order to maintain an acceptable delay probability at a sufficiently low level and finally they provide a way of optimizing the average cost per day of empty beds against costs of delayed patients. They established that bed emptiness is necessary to maintain service efficiency and provide more responsive and cost effective services. Bain et al. (2010) was argued the result of Bagust et al. (1999) which state that; risks are discern able when average bed occupancy rates exceed 0.85 and that acute hospital can expect regular bed shortages and periodic bed crises of average bed occupancy rises to 0.90 or more. They emphasizing that this conclusion can only apply to a particular queuing system. It was concluded that a queue model measures the inputs such as the arrival, service process and the number of beds. Green (2004) research on delay models, such as the M/M/s queuing model, which applied on capacity planning problems in hospitals. Arriving patients enter a queue when all beds are occupied. The probability of delay is an important performance measure in this type of study. In this approach the main outcome parameter is the probability of overload when the number of required beds exceeds the number of operational beds. Where diverting patients is a serious issue, its chose to incorporate turn away. For that reason, he adopted the Erlang loss queuing model.

CHAPTER 3

METHODOLOGY

3.1 Introduction

To achieve our objectives of creating a Decision System Support(DSS) to optimizing hospital bed allocation in hospitals, we adopted two (2) methods to help us with the model formulation are Queuing Theory (QT) and Goal Programing(GP).

The QT will help us solve inefficiencies in bed distribution in various wards in our hospitals and determine the number of beds required to increase daily income generation. Whereas GP will detect the number of incoming patients due to the bed occupancy to optimize daily income generation.

Results from the queuing models are used to construct a multi-objective decision aiding model in the GP framework, taking account of targets and objectives related to customer service and income generation from the hospital manager and all wards heads. The model is developed for use in public hospitals in Ghana.

3.1.1 Terminology

Admissions- Hospitals keep record of the number of admissions. The total amount of admissions is equal to the sum of a number of production parameters such as admissions, day treatments, and transfers (patients coming from another medical discipline).

Arrivals- Patients arrive to the system and are admitted if there is a free bed available. An arriving patient who finds all beds occupied is refused and leaves the system.

Beds- The capacity of a ward is measured in terms of operational beds. The number of operational beds, a management decision, is used to determine the available personnel budget. The number of operational beds is generally fixed and evaluated on a yearly basis. From day to day, the actual number of open or staffed beds slightly fluctuates (due to illness, holiday and patient demand)Green (2004).

Constraints- The set of equations or inequalities is called the constraints of the GLPP. $Ax(\leq,=,\geq)b$ is the set of constraints in the GLPP.

Criterion- A deviation variable measures the difference between the target level on a criterion and the value that is able to be achieved in a given solution. If the achieved value is above the target level then the difference is given by the value of the positive deviation variable. If the achieved value is below the target level then the difference is given by the value of the negative deviation variable. A decision problem which has more than one criterion is referred to as a Multi Criteria Decision Making(MCDM) or Multi Criteria Decision Aid (MCDA) problem.

Decision Maker's- The decision maker's refer to the person(s), organization(s), or stakeholder(s) to whom the decision problem under consideration belongs. A set of decision variables characteristics the decision to be made.

Decision Variable- A decision variable a factor over which the decision maker has control. An example is a manufacturing company which has to decide how many of a certain product to make in the next month.

Feasible Region- The set of solutions in decision space that satisfy all constraints and sign restrictions in a goal programming form the feasible region. Any solution to a GLPP which also satisfies the nonnegative restrictions of the problem is called a feasible solution to the GLPP. A feasible solution to the LP problem is a vector $x = (x_1,x_2,...,x_n)$ which satisfies the conditions

$$\sum_{j=1}^{n} C_j X_j(\leq,=,\geq)b_i, \quad i=1,\cdots,m \quad \text{and} \quad j=1,...,n; x_i \geq 0.$$

Goal- A goal in this work refers to a criterion and a numerical level, known as a target level, which the decision maker(s) desire to achieve on that criterion.

Length Of Stay- The time spent in the ward is entitled length of stay, often abbreviated as LOS, after which the patient is discharged or transferred to another ward. The LOS is easily derived from the hospital information system as both time of admission and time of discharge are logged on individual patient level. Length of stay is affected by congestion and delay in the care chain.

Objective Function- An objective in this work is referred to as a criterion with the additional information of the direction (maximise or minimise) in which the decision maker(s) prefer on the criterion scale. The linear function

$$z = \sum_{j=1}^{n} C_j X_j = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

which is to be maximized (or minimized) is called objective function of the General Linear programming Problem(GLPP).

Optimising- To optimise in the context of decision making means to find the decision which gives the best possible value of some measure from amongst the set of possible decisions.

Refused Admissions- If all beds are occupied the patient is turn away and is counted as a refused admission. In practice, a refused admission can result in a diversion to another hospital or an admission to a non-preferable clinical ward. Many wards have to deal with refused admissions due to unavailability of beds. The number of refused admissions can be interpreted as a service level indicator and is important for the quality of care. As this number is hard to measure we do not know how many patients are turned away, we demonstrate a method to approximate the number of refused admissions.

Solution Of GLPP - An n-tuple $(x_1, x_2, \ldots, x_n)$ of real numbers which satisfies the constrains of a GLPP is called the solution of GLPP.

3.1.2 Queuing Theory(QT)

Queuing theory is a method used to measure the probabilities and statistics for a wide range of waiting lines. QT examines every component of waiting in line to be served, including the arrival process, service process, number of servers, number of system places and the number of customers. It is also considered as operations research because the results are often used when making business decisions about the resources needed to provide better service. It is applicable in a wide variety of situations that may be encountered in business, commerce, industry, public service, engineering and health-care. In supermarkets and in banks queues form when there are insufficient server units to meet the demand for service.

It measures the flow of demands into and out of the waiting line and to make a decision on the minimum and maximum number of resources needed. Whenever the available resource is not sufficient to satisfy the demands, then the demanding units have to wait and are subjected to the queuing system.

3.1.3 Applications Of Queuing Theory

Real-life applications of queuing theory is used to develop more efficient queuing systems that reduce customer waiting times and increase the number of customers that can be served. The use of queuing theory, decision makers can design a system that will runs smoothly and efficiently. With minimum waiting time for customer and unused time for the system. The following cases was considered as some examples of a queuing system;

- Banks wants to determine how many teller booths.
- Restaurant wants to find how many tables and chairs.
- Ambulance crews to have in its district.
- Planes are waiting for their turn to land.
- An architect inquires how many parking spaces to have available in a shopping mall.
- A large grocery store want to known how many employees to assign to the delicatessen.

Similarly, in hospitals, the queues form when there are insufficient beds available to admit in patients. This is the area of study.

3.1.4 Kendall's Notation

David G. Kendall came up with notation to describe the characteristics of a queuing system. The notation has the form; A/B/c/K; where

A: the inter arrival time distribution B:

the service time distribution c:

number of servers

K: denotes the size of the system capacity.

For A and B the following abbreviations are very common:

M (Markov): denoted the exponential distribution with $A(t) = 1 - e^{-\lambda t}$ and $a(t) = \lambda \rho^{-\lambda t}$ where $\lambda > 0$ is a parameter. M stems from the fact that the exponential distribution is the only continuous distribution with the markov property, i.e. it is memoryless.

D(Deterministic): all values from a deterministic "distribution" are constant, i.e. have the same value. E_k (Erlang-k): Erlangian Distribution with k phases ($k \geq 1$). For the Erlang-k distribution we have

$$A(t) = 1 - e^{-k\mu t} \sum_{j=0}^{k-1} \frac{(k\mu t)}{j!} \quad (3.1)$$

H_k (Hyper-k): Hyper exponential distribution with k phases. Here we have

$$A(t) = \sum_{j=0}^{k} q_j(1 - e - \mu jt), \quad \mu > 0, \quad q_i > 0 \quad (3.2)$$

furthermore

$$\sum_{j=0}^{k} q_j = 1 \quad (3.3)$$

must hold.

G (General): general distribution, not further specified. In most cases at least the mean and the variance are known.

Exponentially distributed random variables are notated by M, meaning Markovain or memoryless. Furthermore, if the population size and the capacity is infinite, the service discipline M/M/1 denotes a system with Poisson arrivals, exponentially distributed service times and a single server. M/G/m denotes an m-server system with Poisson arrivals and generally distributed service times. M/M/r/K/n represents a system where the customers arrive from a finite-source with n elements where they stay for an exponentially distributed time, the service times are exponentially distributed, the service is carried out according to the requests arrival by r severs, and the system capacity is K.

3.1.5 Exponential Distribution(ED)

Following the ED, let consider a random variable t, and is continuous with t ≥0. Then the probability density t is F(t)=$\theta e^{-\theta t}$ and the cumulative distribution is given as $F(t)$ = $1 - e^{-\theta t}$

Then, the exponential variable t, expected value and the variance are given as;
$E(t) = \frac{1}{\theta}$, and $V(t) = \frac{1}{\theta^2}$

3.1.6 Poisson Distribution(PD)

The Poisson distribution of n is $P_n = \frac{\theta^n e^{-\theta}}{n!}, \quad n = 0, 1, 2, 3, \ldots$. The expectation and the variance $E(n)$ = θ, and $V(t)$ = θ respectively. The Poisson distribution can be also defined in unit of time t. Here the discrete variable n represents the number of occurrences in time t. The probability of n units in time t is also given by; $P_(n, t) = \frac{\theta t^n e^{-\theta} t}{n!}$

3.1.7 Little's Law

In 1961, John D.C. Little published a paper, which shows the relationship between the experted number of units in the system(L), the experted time in the system(W) and arrival rate(λ)

Hence; L=Wλ

Using Little's formula, the term Wλ is often referred to as the offered load to the system.

3.1.8 The Theoretical Queuing Model

To determine optimal bed numbers for a hospital system, we describe patient arrivals according to a Poisson process, hospital beds would be considered the servers and lengths of stay are modeled using phase-type distributions. Following Gorunescu et al. (2002) the M/PH/m model was adopted, where M denotes Poisson (Markov) arrivals, PH is the service distribution is phase-type, and m is the number of beds. There is no waiting area, which means that an arriving patient who finds all beds occupied is turn away. The length of stay of an arriving patient is independent and identically distributed with expectation. According to queuing theory, the probability of patients arrivals are lost because all the m beds are occupied can be calculated with the following formula,

$$T = \frac{(\lambda\tau)^m/m!}{\sum_{k=0}^{m}[(\lambda\tau)^k/k!]} \tag{3.4}$$

Where the probability of admitted patients was given below;

$$P^a = 1 - T \tag{3.5}$$

Therefore the bed occupancy rate is also defined as;

$$\rho = \lambda\tau(1 - T)/m \tag{3.6}$$

In order to apply the Erlang loss model and for purposes of validation we need to quantify the number of arrival rate(λ). Since the hospital information system only registers the number of admissions, while the number of refusal admissions is generally unknown we need to approximate λ. This can be accomplished by using expressions 3.6. First, the numerator in equation 3.6 is equal to the average number of occupied beds.This is given by the formula;

$$N = \lambda\tau(1 - T) \tag{3.7}$$

Substituting equations 3.4 into 3.7, the expression becomes;

$$N = \lambda\tau\left(1 - \frac{(\lambda\tau)^m/m!}{\sum_{k=0}^{m}[(\lambda\tau)^k/k!]}\right) \tag{3.8}$$

The average number of occupied beds, number of beds(m) and the average length of stay (τ) and λ, arrival rate are known variables and can be obtained from the hospital information system. After substitution in equation 3.4 then the probability of patient turn away(T) is the only parameter left unknown. Using clinical ward calculator, we determined numerically for each of the wards within the hospital.

Gorunescu et al. (2002) introduced a based stock inventory model to find the relationship between the average profit per day and the number of beds. The average profit(R^p) of the day is given by;

$$R^p = z\lambda\tau P^a - \pi\lambda T - f[m - \lambda\tau P^a] \tag{3.9}$$

Where

T, probability of patient turn away p^a,

probability of admitted patients ρ,

bed occupancy rate λ, arrival rate τ,

average length of stay m, number of

beds π, penalty cost

k, number of phases of service time R^p,

average profit of the day z , revenue per

day (minus direct cost) f , cost per idle

bed

N , average number of occupied beds

3.1.9 Goal Programming

Real world decision problems in management and engineering often involve multiple potentially conflicting performance objectives or goals. Such decision problems can generally be modeled as multiple objective optimization problem, the solution of which can be based on trade-off analysis among objectives. One of the common ways of acquiring and describing decision makers preferences is to set a target value for each objective and assign relative weights and/or priority levels to attain these target values. As one of the most popular multi objective optimization techniques, Goal Programming provides a pragmatic and flexible way to cater for the above preferences.

Goal programming is a branch of Multi-Objective Optimization techniques which involves solving problems containing several objectives or goals that are of concern to decision maker to achieve. Traditional Goal Programming techniques have been widely used to deal with Linear Multi Objective Optimization Problems. The Goal Programming, is to minimize the objective function directly as in the Linear Programming, the deviations between goals and what can be achieved within the given set of constraints. Three steps of goal programming model used include: Defining the decision variables; Objective function and Model constraints.

3.1.10 Goal Programming Variants

The decision maker must then decide which deviation variables are unwanted. These are penalized in an achievement function. The three major variants in terms of underlying distance metric used are introduced. These are Lexicographic, Weighted and Chebyshev Goal Programming.

Chebyshev Goal Programming(CGP)

It is known as CGP, because it uses the underlying Chebyshev (L∞) means of measuring distance. That is, the maximal deviation from any goal, as opposed to the sum of all deviations, is minimized. For this reason CGP is sometimes termed Min-max GP. CGP is the only major variant that can find optimal solutions for linear models that are not located at extreme points in decision space. This include a large number of application areas, especially those with multiple decision makers with each of whom has a preference to their own subset of goals that they regard as most important.

Chebyshev Goal Programming has the following algebraic format:

Min a = λ

s.t

$f_q(x) + n_q - p_q = b_q$ q=1,·,·,·,Q

$\frac{u_q n_q}{k_q} + \frac{v_q P_q}{k_q} \leq \lambda$ q=1,·,·,·,Q

$\underline{x} \in F$ q=1,·,·,·,Q $n_q, p_q \leq 0$

Lexicographic Goal Programming(LGP)

The Lexicographic Goal Programming is the existence of a number of priority levels. Each priority level contains a number of unwanted deviations to be minimized. The Lexicographic variant provides a very strong scheme for the decision maker to express their preferences. It can be the ideal choice when such a priority scheme exists in the mind of the decision maker. It can also be used to avoid direct unwanted comparisons between sensitive criteria. To formulate a generic lexicographic goal programming algebraically, we define the number of priority levels as L with corresponding index l = 1, ..., L. Each priority level is now a function of a subset of unwanted deviation variables which can be define as hl(n, p). This leads to the following formulation:

Lex Min a= $[h_1(\underline{n},p), h_2(\underline{n},p), \cdot,\cdot,\cdot h_L(\underline{n},p)]$

s.t

$f_q(\underline{x}) + n_q - p_q = b_q$ q=1,·,·,·,Q

$\frac{u_q n_q}{k_q} + \frac{v_q P_q}{k_q} \leq \lambda$ q=1,·,·,·,Q

$\underline{x} \in F$ q=1,·,·,·,Q

$n_q, p_q \leq 0$

where each $h_i(n_i, p_i)$ contains a number of unwanted deviation variables.

Weighted Goal Programming(WGP)

The WGP variant allows for direct trade-offs between all unwanted deviation variables by placing them in a weighted, normalised single achievement function. Weighted Goal Programming is sometimes termed non-preemptive GP. The weighted variant can be used when the decision maker wants to compare the deviations and investigate the trade-offs between them. These give the means of obtaining the decision maker's preferences for this variant. The weight technically contains two parts. The first is a preferential weight in the numerator that represents the relative importance of the penalization to the decision maker. A normalisation factor in the denominator is needed, that scales the deviations so they can be compared in the same units. It is vital that this part is not omitted if the goals are not measured in compatible units. Otherwise, the summation will be meaningless due to the problem of incommensurable.

The example will now be formulated as a WGP.

$\text{Min a}=\sum_{q=0}^{Q}(\frac{u_q n_q}{k_q} + \frac{v_q n_q}{k_q})$

s.t

$f_q(x) + n_q - p_q = b_q$ q=1,·,·,·,Q

$\frac{u_q n_q}{k_q} + \frac{v_q P_q}{k_q} \leq \lambda$ q=1,·,·,·,Q

$\underline{x} \in F$ $\quad\quad$ q=1,·,·,·,Q

$n_q, p_q \leq 0$

Illustration-Toy Problem 1

To illustrate goal programming (GP) we consider the decision maker regards to the penalisation of all unwanted deviations as equally important. The WGP with percentage normalisation is given as:

$$\text{Min a} = \frac{n_1}{120} + \frac{n_2}{7000} + \frac{n_3}{40} + \frac{n_4}{40}$$

s.t.

$4x_1 + 3x_2 + n_1 - p_1 = 120$ $100x_1 +$

$150x_2 + n_2 - p_2 = 7000$ $x_1 + n_3 -$

$p_3 = 40$ $x_2 + n_4 - p_4 = 40$

$2x_1 + x_2 \geq 50$

$2x_1 + 1.5x_2 \leq 75$ $x_1, x_2 \geq 0$, $n_q, p_q \geq 0$

q=1,·,·,·,Q

<u>SOLUTION</u>

We solved this GP problem using excel solver. Figure 3.1, shows detail of the solver spreed sheet results.

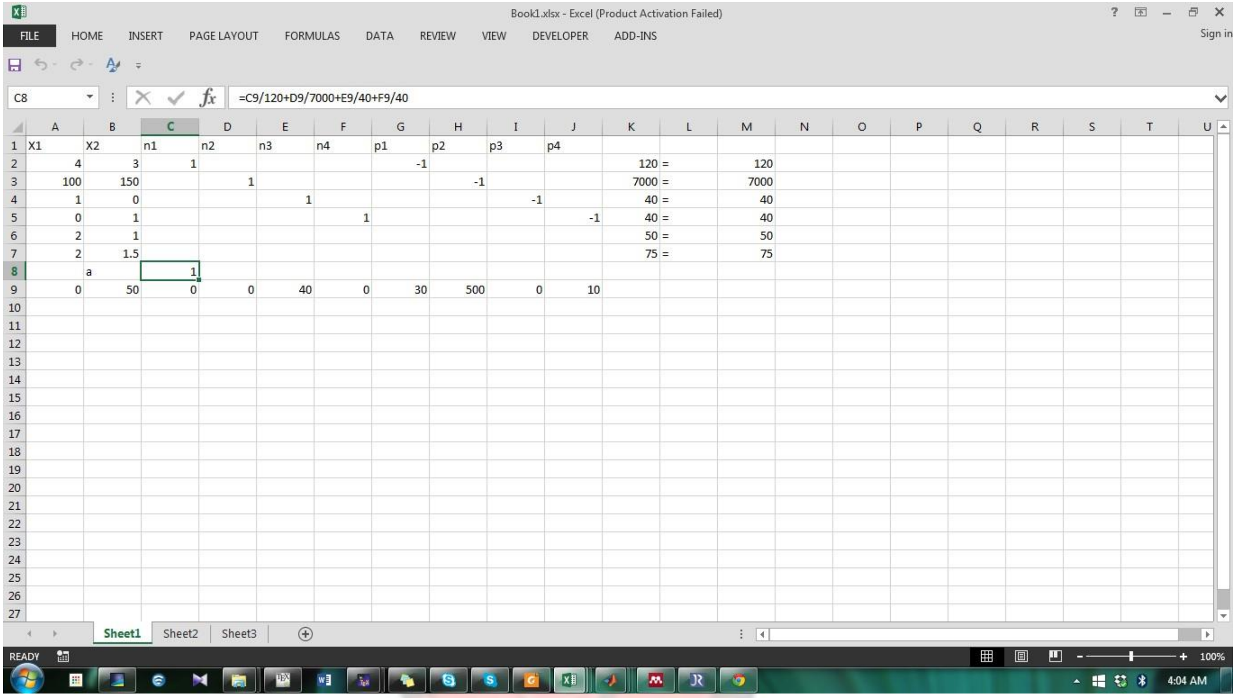


Figure 3.1: Excel solver

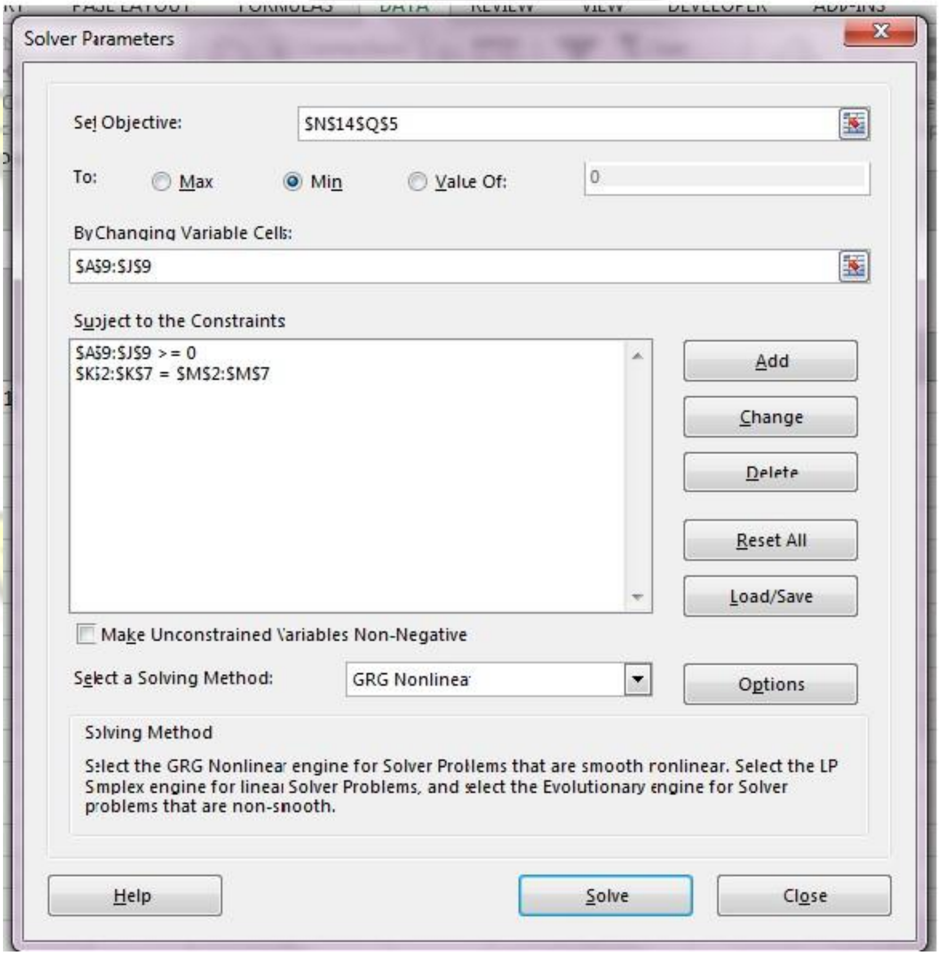


Figure 3.2: Constraints Solver Parameters The optimal solution for this goal programming model has; x_1 =0 x_2 =50

$p_1 = 30$ $n_1 = 0$

$p_2 = 500$ $n_2 = 0$

$p_3 = 0$ $n_3 = 40$

$p_4 = 10$ $n_4 = 0$

Solution of Toy-Problem 1 with percentage normalization. Goal number description target level with achieved value

1. Labour 120 No 150 → Overachieved
2. Profit 7000 No 7500 → Overachieved
3. Type A produced 40 Yes 40 → Achieved 4. Type B produced 40 Yes 40 → Achieved

The percentage deviation is convenient to apply and simple to understand.

Linear Minimax Goal Programming Models(LMGP)

Consider the standard linear multi objective optimization problem as follows.

$$\max f(X) \quad (3.10) \text{ s.t}$$

$$AX = b \tag{3.11}$$

where

$$f(x) = f_i(x) f_i(x) \quad = C^T X \quad (3.12) \text{ s.t.}$$

Ax=b, x≥ 0

$X = [x_1, x_2, \cdot, \cdot, \cdot, x_n]^T$, $A = (a_{ij})_{m \times n}$, $b = [b_1, b_2, \cdot, \cdot, \cdot, b_m]^T$ where $f_i(x) = C_i^T x$ is the ith objective.

Suppose a target value for an objective $\hat{f}_i(x)$ is provided and denoted by f_i. A reference point (goal) can be then denoted $\hat{f}$ as follows

$$\hat{f} = [\hat{f}_1, \hat{f}_2, \cdot, \cdot \hat{f}_k]^T \tag{3.13}$$

Linear minimax GP is based on ∞-norm and can be formulated as follows based on the minimax reference point approach by Yang (2000).

min r

s.t.

$$w_i[\hat{f}_1 - f_i(x)] \le r, \qquad i = 1,\cdot,k \tag{3.14}$$

Ax=b, x≥0 r≥0

where r is an auxiliary variable for minimization and w_i the relative weight of the objective $f(x)$. The preference assumption made in the above model is that the best compromise solution is the one that is nearest to the goal.

The equation 3.14 is a non-smooth programming problem. By replacing each objective constraint of problem 3.14 by two equivalent constraints as follows min r

s.t.

$$w_i[\hat{f}_i - f_i(x)] \le r \tag{3.15}$$

$-w_i[\hat{f}_i - f_i(x)] \le r$, $i = 1,\cdot,\cdot,k$ Ax=b, x≥0 , r≥0

Equation 3.15 is a linear programming problem and can be solved using simplex method.

To facilitate non-preemptive and preemptive GP, the equation then introducing deviation variables. Let d^+_i and d^-_i be the deviation variables for over and underachieves of degree that $f_i(x)$ of $\hat{f}_1$.Then the following deviation variable based minimax GP model equivalent to equation 3.14 can be formulated as follows min r

s.t.

$$w_i[d_{+i} + d_{-i}] \le r \tag{3.16}$$

$d^+_i - d^-_i = f - i(x) - \hat{f}_1$

$d_{+i} \times d_{-i} = 0, \quad d_{+i}, d_{-i} \geq 0, \quad i = 1,\cdot,k$

Ax=b, x≥0 r≥0

The equation is linear except for the complementarity conditions $d^{+}_{i} \times d^{-}_{i} = 0$ ($i = 1,\cdot,k$). Such a pseudo-linear programming problem can be solved using a slightly modified simplex algorithm with d^{+}_{i} and d^{-}_{i} not selected as basic variables simultaneously.

Equation 3.16 is more complex as it includes additional k nonlinear complementarity conditions and 2k deviation variables d^{+}_{i} and d^{-}_{i} ($i = 1,\cdot,k$).

Standard Traditional and Minimax Goal Programing

Traditional goal programming (TGP) is based on 1-norm, which can be formulated as

$$Min \quad P_1[\sum_{1\in(I)} {}_{=}(w_i d_{+i} + (w_i d_{+i}) + \sum_{i\in(I)} {}_{\geq}(w_i d_{-i}) + \sum_{j\in(I)} {}_{\leq}(w_j d_{+j}) +$$

$$P_2[\sum_{i\in(I)} {}_{\geq}(-w_i d^{+}_{i}) + \sum_{j\in(I)} {}_{\leq}(-w_j d^{i}_{j})] \quad (3.17)$$

where

$I^{=}$=i=1,·,k_1, $I^{\geq}$=i=k_1+1,·,k_2, $I^{\leq}$=i=k_2+1,·,k and $\bar{f} \geq 0$, (i=1,·,k). s.t.

$d^{+}_{i} - d^{-}_{i} = f - i(x) - \hat{f}_1, \quad i = 1,\cdot,k$

$d_{+i} X d_{-i} = 0, \quad i = 1,\cdot,k$

$d_{+i}, d_{-i} \geq 0, \quad i = 1,\cdot,k$ Ax=b, x≥0

Equations3.17 can be re-written as the following standard GP form, min r

s.t.

$\bar{A}\bar{x} = \bar{b}, \quad \bar{x} \geq 0 \; d^{+}_{i} x d^{-}_{i} =$

$0, \quad i = 1,\cdot,k$

where

$\bar{x} = [x^T d_1^+ d_1^- \cdots d_k^+ d_k^-]^T$

$$\bar{A} = \begin{bmatrix} C_1^T & -1 & 1 & \cdots & 0 & 0 \\ \vdots & 0 & 0 & \cdots & & \\ C_k^T & & & 0 & -1 & 1 \\ A & & & 0 & & \end{bmatrix} \tag{3.18}$$

$$\bar{b} = \begin{bmatrix} \hat{f}_1 & \cdots & \hat{f}_k & b^T \end{bmatrix}^T \tag{3.19}$$

With the TGP model all the goal values are included in the right hand constant $\bar{b}$ and weights (w_i) are only included in the objective function. This means that once a reference point is given changing weights will most likely lead to the identification of extreme corner solution.

Let assume that, $I^{=} = i = 1,\cdot,k_1$, $I^{\geq} = i = k_1+1,\cdot,k_2$, $I^{\leq} = i = k_2+1,\cdot,k$ and $\bar{f} \geq 0$, $(i=1,\cdot,k)$. Then minimum GP formulation can be transformed to the following standard GP form, min r

s.t.

$$\bar{A}\bar{\bar{x}} = \bar{b}, \quad \bar{\bar{x}} \geq 0 \quad d_i^{+} \times d_i^{-} = 0, \qquad i = 1,\cdot,k \tag{3.20}$$

Where

$$\bar{\bar{x}} = \begin{bmatrix} \bar{x}^r & rS_1 & \cdots & S_k \end{bmatrix}^T \tag{3.21}$$

$$\begin{bmatrix} \bar{A}_1 & 0 & 0 & \bar{h}_1 & \bar{I}_1 & 0 \\ 0 & \bar{A}_1 & 0 & \bar{h}_1 & 0 & \bar{I}_1 \\ 0 & 0 & \bar{A}_j & \bar{h}_j & 0 & 0 \\ 0 & \bar{A} & & 0 & & 0 \end{bmatrix} \tag{3.22}$$

$A^{-} =$ Γ_j

0

(3.23)

(3.24)

(3.25)

(3.26)

Where r being auxiliary variable and s_1 is the slack variable of the i th objective constraint. Both problems can be solved using a modified simplex algorithm with d_1^+ and d_1^- not selected as basic variables. Using this method to drop the complementarity condition.

$$\bar{\bar{b}} = \begin{bmatrix} 0 & 0 & 0 & \bar{b}^r \end{bmatrix}^r$$

$$\bar{A}_1 = \begin{bmatrix} \omega_1^+ & \omega_1^- & \cdots & & 0 & 0 \\ \vdots & \vdots & \ddots & & \vdots & \vdots \\ 0 & 0 & & \cdots & \omega_{k_1}^+ & \omega_{k_1}^- \end{bmatrix}, \quad \bar{h}_l = \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix}, \quad \bar{I}_l = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$\bar{A}_i = \begin{bmatrix} 0 & \omega_{k_1+1}^- & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \omega_{k_2} \end{bmatrix}, \bar{h}_i = \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix}, \bar{I}_j = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$\bar{A}_j = \begin{bmatrix} \omega_{k_2+1} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \omega_k & 0 \end{bmatrix}, \bar{h}_j = \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix}, \bar{I}_j = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

Objective Function

The Goal Programming model was to minimize all the negative deviations from income generation per day and admitting more patients.Weighted goal programing considers all the goals (i= 1, 2, ⋯, k) simultaneously and to minimize a weighted sum of the unwanted deviation. Deviations are weighted according to the relative importance of each goal for the decision makers. The weighted GP technique was used with the achievement function defined below.

$$min = \sum_{i=0}^{n} \left(\frac{w_i^{a1}}{g_i^{a1}} d_i^{a1} + \frac{w_i^{a2}}{g_i^{a2}} d_i^{a2} + \frac{w_i^{P1}}{g_i^{P1}} d_i^{P1} + \frac{w_i^{P2}}{g_i^{P2}} d_i^{P2} \right) \tag{3.27}$$

Where w values are the weights to the deviation variables in the objective function of a GP problem.

The Goal Constraints

This allow to determine how close a given solution comes to achieving the goals. The right hand side values of each goal constraint is the target value for the goal obtained from the queuing model result and represents the level of achievement that the decision maker want to obtain for the goal. The model contains twolevels target values of each objective of decision makers are stated by (3.30) to (3.39). The hard limit on the total number of available beds is model (3.40) and (3.41) and (3.42) explains number of beds chosen in each objectives for different department must be equal. Then equations (3.43) and (3.44) was used to add restriction to the value of the variable cannot exceed the number of beds x_{ij} and y_{ij} between breakpoint $(j-1)$ and j. The important constraints were identified through queuing model results and objective function were determined based on constraints. The important constraints included the total number of beds available in the hospital, income generation per day and minimized the number of patients unserved/unattended.

$$\sum_{j=1}^{b_i^a} c_{ij}^a x_{ij} + d_i^{a1} + d_i^{a2} - p_i^{a1} = g_i^{a1} \qquad i = 1 \cdots, n \qquad (3.28)$$

$$\sum_{j=1}^{b_i^a} c_{ij}^a x_{ij} + d_i^{a2} - p_i^{a2} = g_i^{a2} \qquad i = 1 \cdots, n \tag{3.29}$$

$$\left(\sum_{i=1}^{n} = \lambda_i\right)^{-1} \left[\sum_{i=1}^{n} \lambda_i \left(\sum_{j=1}^{b_i^a} c_{ij}^a x_{ij}\right)\right] + d_0^{a1} + d_0^{a2} - p_o^{a1} = g_o^{a1} \tag{3.30}$$

$$\left(\sum_{i=1}^{n} = \lambda_i\right)^{-1} \left[\sum_{i=1}^{n} \lambda_i \left(\sum_{j=1}^{b_i^a} c_{ij}^a x_{ij}\right)\right] + d_0^{a2} - p_o^{a2} = g_o^{a2} \tag{3.31}$$

$$\sum_{j=1}^{b_i^p} c_{ij}^p y_{ij} + d_i^{p1} + d_i^{p2} - p_i^{p1} = g_i^{p1} \qquad i = 1 \cdots, n \tag{3.32}$$

$$\sum_{j=1}^{b_i^p} c_{ij}^p y_{ij} + d_i^{p2} - p_i^{p2} = g_i^{p2} \qquad i = 1 \cdots, n \tag{3.33}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{b_i^p} c_{ij}^p y_{ij} + d_o^{p1} + d_o^{p2} - p_o^{p1} = g_o^{p1} \tag{3.34}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{b_i^p} c_{ij}^p y_{ij} + d_o^{p2} - p_o^{p2} = g_o^{p2} \tag{3.35}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{b_i^p} y_{ij} \leq B \tag{3.36}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{b_i^p} x_{ij} \leq B \tag{3.37}$$

$$\sum_{j=1}^{b_i^p} x_{ij} = \sum_{j=1}^{b_i^p} y_{ij} \tag{3.38}$$

$$x_{ij} \leq \bar{x}_{ij} \tag{3.39}$$

$$y_{ij} \leq \bar{y}_{ij} \tag{3.40}$$

$$Max g_o^{p1} = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^p y_{ij} \tag{3.41}$$

Where $d_i^Q \geq 0, \quad d_i^Q + \geq 0, \quad x_{ij}, \quad y_{ij}$ are non-negative and integer for all i and j.

$b^a{}_i$, number of breakpoints in patient admission probability function of department i.

$b^p{}_i$, number of breakpoints in profit per day function of department i.

x_{ij} , number of beds between breakpoints (j-1) and j of admitting patient in department i. y_{ij} ,number of beds between breakpoints (j-1) and j of profit per day in department i.

$c^a{}_{ij}$, unit value of patient admission probability for x_ij. $c^p{}_{ij}$, unit value of profit per day for y_{ij}. $g_i{}^Q$, target values of different objectives for decision maker i.

$d^Q{}_i$ − , negative deviations associated with the targets of the objectives for decision maker i.

$d^Q{}_i$ + positive deviations associated with the targets of the objectives for decision maker.

Q = $a1$, $a2$, $p1$, $p2$; and i=1,⋯ ,n; $a1$ and $a2$ represent the targets at the upper and lower-level of patient admission probability; $p1$ and $p2$ is the upper and lower-level of daily profit. With index i = 0 is from the hospital manager and i = 1,⋯ ,n.

Goal Programming Algorithms

The GP can be solve using two proposed algorithms. The methods are based on representing the multiple goals by single objective function. These methods are; 1. The weighted method: A single objective function is formed as the weighted sum of the goals of the problem. Let assume that the GP model has n goals and ith goal is given by;

minimize G_i, $i = 1,2,3,\cdots,n$

Then the combined objective function used in the weights method is define as; minimize , Z= $W_1G_1 + W_2G_2 + \cdots + W_nG_n$

The parameters W_i , i=1, 2, 3,⋯, n are positive weights that reflect the decision maker preferences regarding the relative importance of each goal. For example , W_i = 1, for all i, means that all goals carry equal weight.

2. The preemptive method: The decision maker must rank a the goals in order of importance. The objectives of the problem can be defined as.

minimize $G_1 = P_1$ (Highest priority)

...

minimize $G_n = P_n$ (Lowest priority)

Where the variables P_i is the component of the deviation variables, s^-_i and s^+_i are the goal i.

Excel solver Add-in software

Goal programming can be solved using simplex algorithm and programs such as QM solver or excel. The major PC-based spreadsheet used today, Lotus, Excel and Quarto all have built-in optimization tool called solver. These optimizers allow linear, non-linear and integer programs to be solved within the spreadsheet. The optimizer, which is available in the Analysis group on the data tab in Excel. One advantage of the spreadsheet approach to optimization is that, many optimization model can be represented in an understandable fashion in a spreadsheet. The spreadsheet copy command allows large models with many similar constraints to be created and solved quickly in a spreadsheet environment.

We applied these five main steps in solving GP problem in Excel.

1. Creating a spreadsheet which models the problem.
2. Specifying the location (cell) which contains the objective function.
3. Specifying the decision variables.
4. Specifying the cells which define the constraints
5. Solving the model.

The template in Excel serves as the means for bed assignment staff to input patient characteristics. The tool requires as inputs the patient information regarding the patients admitted in the unit. When the user requests the tool to generate a bed assignment, the Excel interface internally generates a data file that can be used by solver to solve the model presented in GP model. The solution is automatically read from solver and translated into a spreadsheet for the users to view. An example of the displayed results is shown in Figure 3 as per attached in appendix B.

CHAPTER 4

ANALYSIS

4.1 Introduction

This chapter gives the distribution on the analysis of beds reallocation, patients rate of admission, bed occupancy rate and the expected income generated per day. Our hospital is constantly threatened by the inadequacy of beds and bed occupancy problem across wards. The relationship between number of beds and system performance measures and the distribution fits to admission and length of stay data was collected across the wards. The parameters used include patient length of stay(LOS), arrival rate per day, number of beds and cost per bed. The parameters are shown in table 4.1. The sensitivity analysis of the system performance measures to changes in the number of beds began with the number of operational beds in each ward; 10, 40, 42 and 24 beds for the Emergency, Male, Maternity and Female wards respectively. The inadequacy of these beds across the wards has translated into a high number of patients being turned away and a high loss of income generation from the hospital.

Table 4.1: Hospital Data For Each Ward

Ward	No. of Beds	Arrival Rate	LOS	Charge Per Bed(GH₵)
Emergency	10	2.388	7.30	20
Male	40	1.428	20.50	15
Maternity	42	1.564	28.28	15
Female	24	1.575	18.69	15

4.2 Bed Distribution

The total operational beds available within the hospital numbered 116. However the model predicted that, the actual beds needed to satisfy the available patients numbered 150. All other ward needed beds to satisfaction ratio equal. Whilst other beds are available else where, male ward has an extra beds of ten (10), idle, which could be uses in other departments/wards.

The optimal number of beds for each ward is summarized in the table below.

Table 4.2: Optimal number of beds required in each ward

Ward	No. of beds in respect to 95% of patient admission	No. of beds in respect to Optimal profit/ day	Occupancy rate (%)
Emergency	22	26	90.1
Male	24	30	47.5
Maternity	46	51	87.8
Female	34	43	90.6

Table 4.3: Optimal Number Of Beds Needed To Satisfy The Available Patients

Ward	Beds Required	Occupancy Rate	Refusal Admission	Income Loss(GH₵)
Emergency	26	0.91	0.48	160
Male	30	0.48	0.00	0
Maternity	46	0.88	0.14	90
Female	29	0.91	0.26	90

Table 4.3, shows various system characteristics as a function of the number of beds for the emergency ward. From this table, it is observed that the more the number of beds, the lower the probability of a patient being turned away (lost demands) and the lower the bed occupancy. A graph of percentage of patients turned away against number of beds is provided in Figure 4.1. If the manager wishes to ensure that at most 15 of patients are turned away, there should be at least 18 beds in the ward.

Table 4.4: Distribution Of Beds In Emergency Ward

No. of Beds	Bed Occupancy (%)	Refusal Admission	No. Of PTA	Income Loss(GH₵)
10	91	0.48	8	160
12	75	0.38	6	120
14	65	0.30	5	100
16	57	0.22	3	600
18	50	0.15	2	400
20	45	0.09	1	200
22	41	0.06	0	0

24	38	0.03	0	0
26	34	0.01	0	0
28	32	0.00	0	0

The average number of patient arrivals during the day and average length of stay is therefore summarized in Table.4.4.

Table 4.5: Offered Load

Ward	N0. Of PBO	N0. Of PTA	Offered Load	Daily Income (GH)
Emergency	10	8	17	200
Male	29	0	29	435
Maternity	42	6	51	630
Female	24	6	28	360

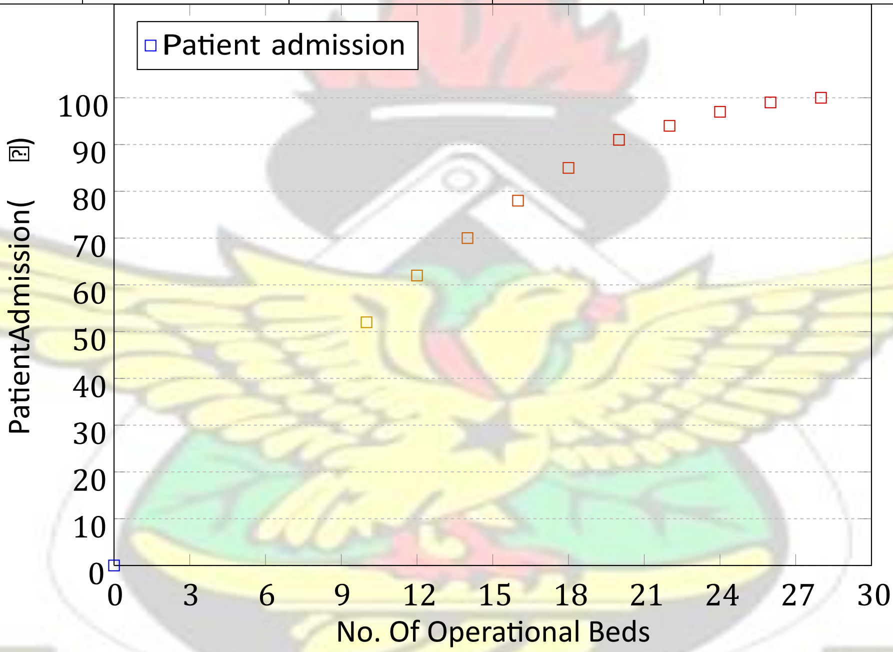


Figure 4.1: Graph of Beds Against Patient Admission In Emergency Ward

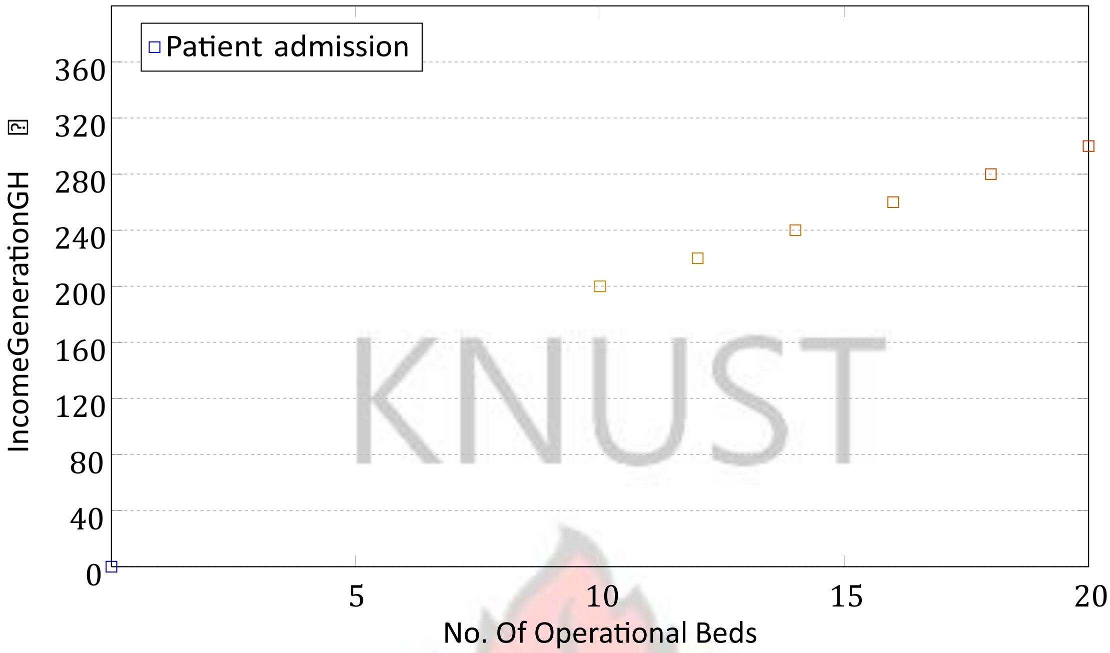


Figure 4.2: Graph of Beds Against Income Generation In Emergency Ward The inadequacy of these beds across the wards has translated into high delay probability of patients admission into the wards, a high number of patients being turned away from the wards and a high loss of income generation over the period of average length of stay in the wards. The details were shown in Figure 4.3 as predicts by the GP.

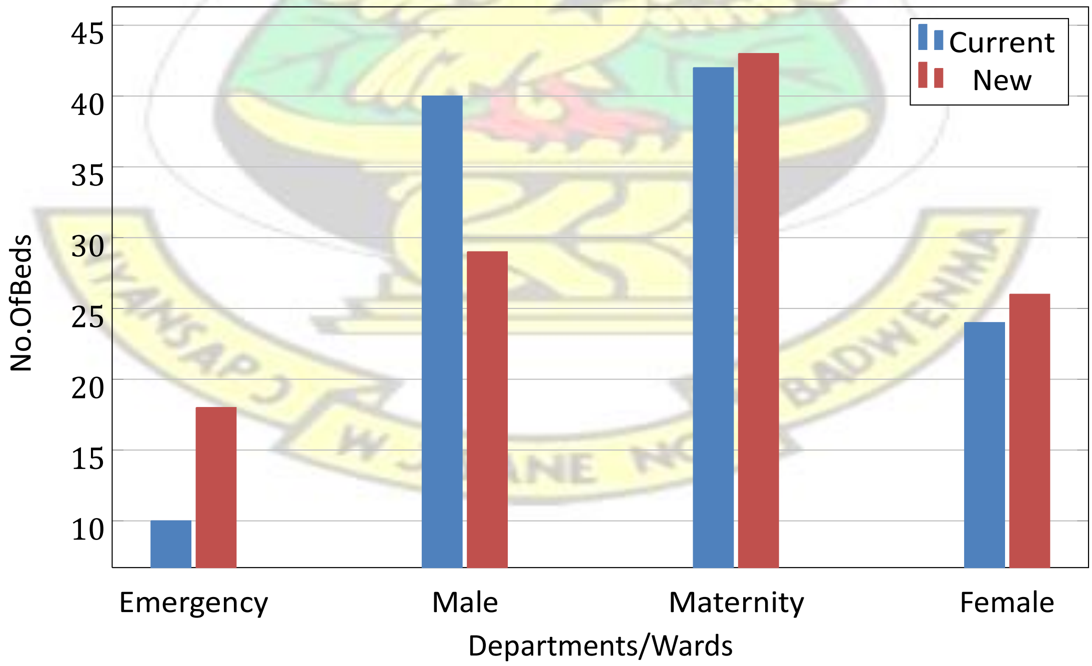


Figure 4.3: No. Of Operational Beds And New Beds Predict By GP

4.3 Patient Admission

The probability rate of upper and lower level target was set as 0.95 and 0.75 for all departments/wards. Outcome from GP model depicts that more patient can be admitted in Emergency, Male and Female wards, except Maternity ward.

In addition, all the departments/wards can not achieve the upper level target of 0.95.

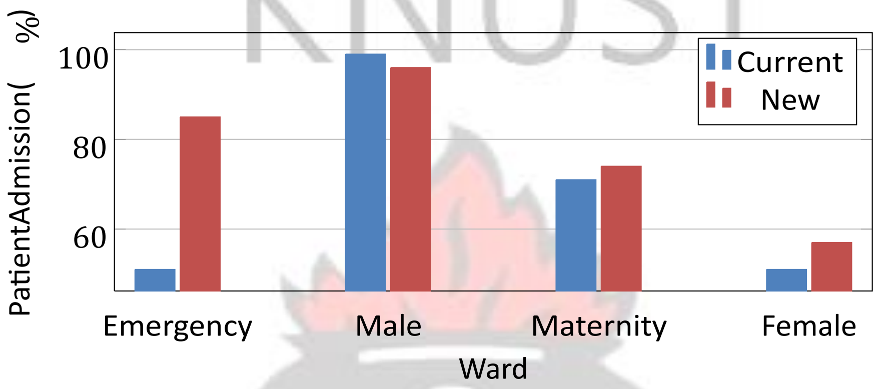


Figure 4.4: Patient Admission Probability

4.4 Income Generation

The optimum income generation based on Optimum number of beds required is refer to the table 4.4. The GP formulation give a clear view that only male ward

Table 4.6: Upper And Lower Level Target (GH□)

Wards	Upper Level Bound(GH□)	Lower Level Bound(GH□)
Emergency	520	440
Male	450	360
Maternity	690	630
Female	645	510

can reaches its income generation target in the new configuration. The results of the model for daily income generation are displayed in Figure 4.4. The rest of the wards are not able to achieve their upper level target, since the optimal number of beds exceeds the number of operational beds.

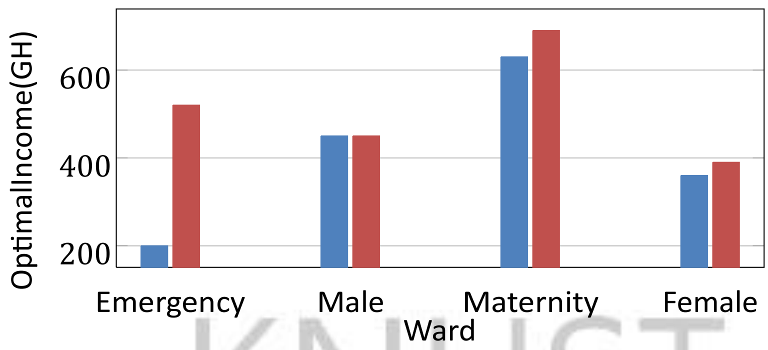


Figure 4.5: Optimal Income Generation

4.5 Bed Occupancy

The model shows that only male ward decreases it occupancy rate, whilst the rest increases. The ward who decrease their occupancy rate increase their income generation and vice versa. These findings illustrate that high bed occupancy rate are actually often not desirable for both patient admission probability and income generation. From Figure 4.5 shows that between 0.80 and 0.98 of the beds in the hospital are occupied in the new bed allocation.

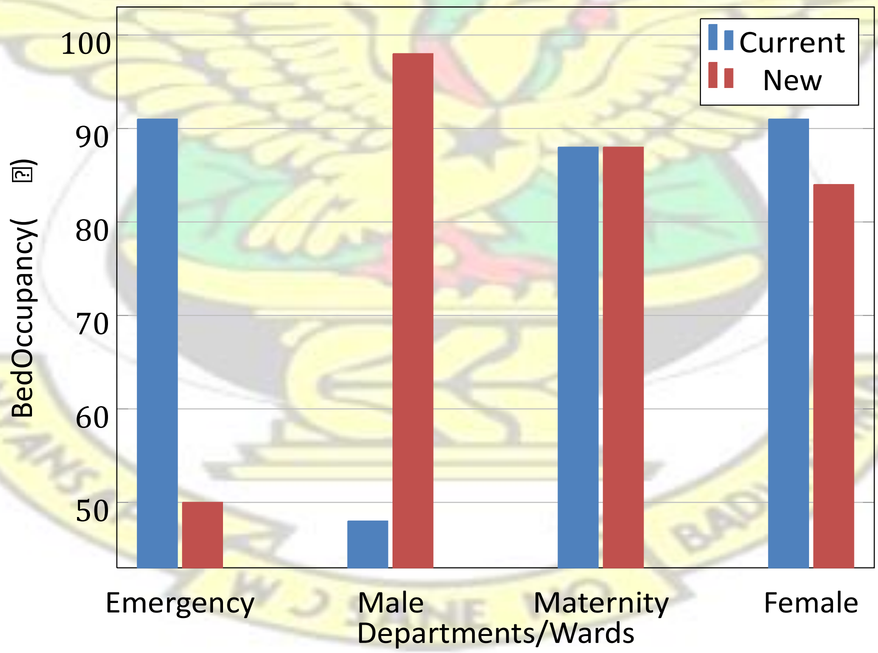


Figure 4.6: Bed Occupancy Rate

The optimal solution for each of the wards are presented in Appendix C. This shows detail of the solver spread sheet for the GP formulation.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The main objective or aim of this thesis is to give a fair view of reallocation of beds in each department or wards in a public hospitals in Ghana. being the strategic problem in an undisclosed hospital. The adoption of QT and GP gives us a clear fact and figures in arriving at a conclusive objective of this study. Data collected from the month of January to December 2010 was used for the analysis. The QT captures stochastic arrival patterns and service times. Patient arrivals for specialty or ward can be approximate by Poisson process follow by phase time distribution. The model base on M/PH/m queuing model, where the number of bed available is fixed to m and no queuing is allowed. With Poisson arrival rate of λ per day and an average LOS, τ days, the offered load in the hospital is given by $\lambda\tau$. The adoption of QT, help us to inefficiencies in bed distribution in various wards in our hospitals and determine the number of bed required in generating daily income. It is clear evident that bed assignments require more than the availability of a bed. Whereas GP would determine the incoming patient due to bed occupancy to maximize income generation.

The Multi-Objective Decision Aiding model being the outcome of GP gives expected targets and objective of the departmental heads. GP model with other constraints such as departmental specific upper and lower targets on the number of operational bed allocated. Our model can be use to estimate the average bed occupancy, patients refusal admission, income generation and also to determine an optimal number of beds for the departments/wards. In addition, the model enables the hospital manager to balance the cost of empty beds against the cost of patients refusal admission, thus facilitating a good choice of bed provision in order to have low cost and higher patients admission. It also calculating optimal bed numbers given an acceptable level of patient admitted. We also observed that, the mean number of occupied beds is the only performance measure that revealed a direct variation with the number of beds. This is because it increases as the number of beds increases and vise versa. The results demonstrated that all the goals formulated from the GP model depicts that more patients can be admitted in Emergency, Male and Female wards, except Maternity ward. In addition, all the wards can not achieve the upper level target goal.

5.2 Recommendation

This thesis when properly adopted and implemented in our policy formulation would help us in obtaining the maximum use of resources available. The use of QT and GP makes use of limited resources in achieving the targeted goals, taking away waste and does not give room for idle material. Adopting these model will bring sanity in our educational resources, hotels, banking, airports, port and harbor just to mention a few.

The modeling framework presented may also serve as a starting point for the development of capacity planning models for organization in same condition.

REFERENCES

Bagust, A., Place, M., and Posnett, J. W. (1999). Dynamics of bed use in accommodating emergency admission: Stochastic simulation model. *Bmj*, 319:155–158.

Bain, C. A., Taylor, P. G., McDonnell, G., and Georgiou, A. (2010). Myths of ideal hospital occupancy. *Medical Journal of Australia*, 192:42–43.

de Bruin, A. M., Bekker, R., Van Zanten, L., and Koole, G. (2010). Dimensioning hospital wards using the erlang loss model. *Annals of Operations Research*, 178:23–43.

Dexter, F. and Macario, A. (2001). Optimal number of beds and occupancy to minimize staffing costs in an obstetrical unit? *Canadian Journal of Anesthesia*, 48:295 – 301.

Gorunescu, F., McClean, S. i., Millard, P. H., and Others (2002). A queueing model for bed-occupancy management and planning of hospitals. *Journal of Operational Reserch Society*, 53(1):19–24.

Green, L. V. (2004). Capacity planning and management in hospitals. *Intenational Series in Operational Research*, 70.

Harper, P. R. and Shahani, A. (2002). Modelling for planning and management of bed capacities in hospitals. *Journal of the Operational Research Society*, 53:11 – 18.

Harris, R. A. (1986). Hospital bed requirements planning. *European Journal of Operational Research*, 25(1):121 – 126.

Kao, E. P. and Tung, G. G. (1981). Bed allocation in a public health care delivery system. *Management science*, 27:507 – 520.

Kembe M. M, Agada, P. O. and D, O. (2014). A queuing model for hospital bed occupancy managment. *international Journal of Computational and Theoretical Statistics*, 1.

Lapierre, S. D., Goldsman, D., Cochran, R., and DuBow, J. (2005). Bed allocation techniques based on census data. *Socio-Economic Planning Sciences*, 39:183 – 192.

Li, X., Beullens, P., Jones, D., and Tamiz, M. (2009). An integrated queuing and multi - objective bed allocation model with application to a hospital in china. *Journal of the Operational Research Society*, 60:330–338.

McClain, J. O. (1978). A model for regional obstetric bed planning. *Hearth Services Research*, 13:378.

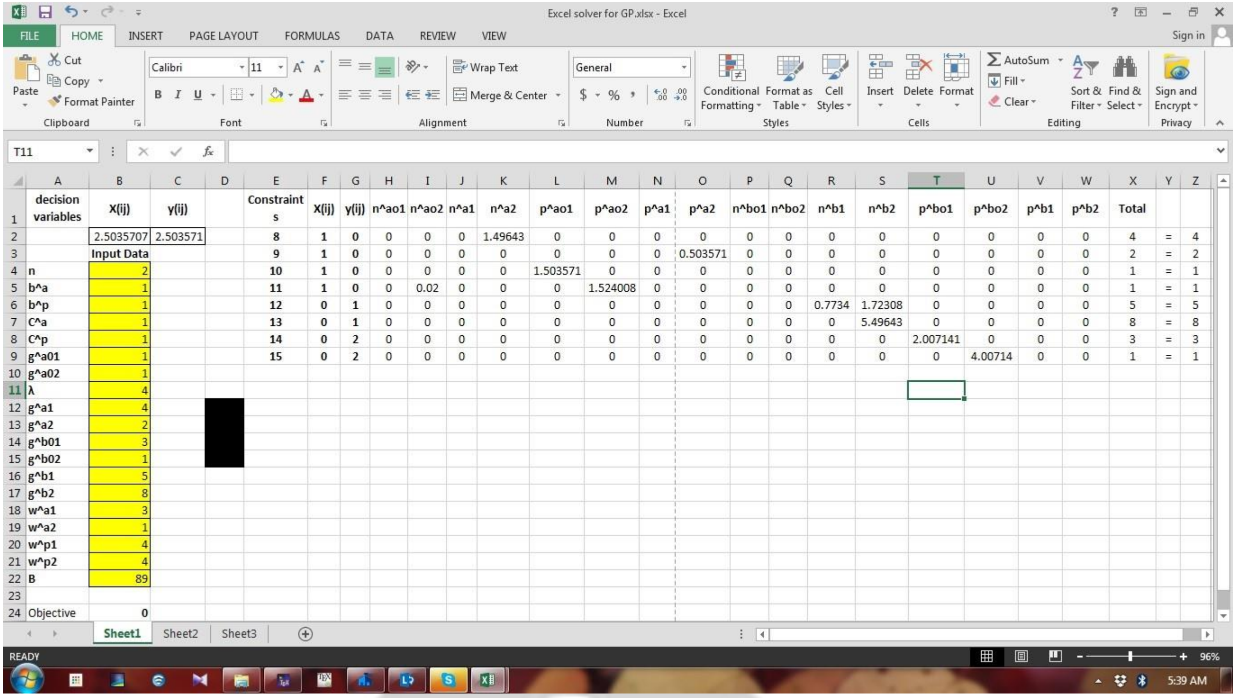
Tütüncü, Y. and Newlands, D. (2009). Hospital bed capacity and mix problem for state supported public and fee paying private wards. *IEG School of Management, CNRS-LEM (UMR 8179)*, 1:2–3.

APPENDIX A

Figure 5.1: Clinical Ward Calculator

• On line tools: http://www.vumc.nl/afdelingen/pica/software/. (Clinical Ward Calculator Based on the Erlang B Model).

APPENDIX B



decision variables	X(ij)	y(ij)
	2.5035707	2.503571
	Input Data	
n	2	
b^a	1	
b^p	1	
C^a	1	
C^p	1	
g^a01	1	
g^a02	1	
λ	4	
g^a1	4	
g^a2	2	
g^b01	3	
g^b02	1	
g^b1	5	
g^b2	8	
w^a1	3	
w^a2	1	
w^p1	4	
w^p2	4	
B	89	
Objective	0	

Constraints	X(ij)	y(ij)	n^ao1	n^ao2	n^a1	n^a2	p^ao1	p^ao2	p^a1	p^a2	n^bo1	n^bo2	n^b1	n^b2	p^bo1	p^bo2	p^b1	p^b2	Total		
8	1	0	0	0	0	1.49643	0	0	0	0	0	0	0	0	0	0	0	0	4	=	4
9	1	0	0	0	0	0	0	0	0	0.503571	0	0	0	0	0	0	0	0	2	=	2
10	1	0	0	0	0	0	1.503571	0	0	0	0	0	0	0	0	0	0	0	1	=	1
11	1	0	0	0.02	0	0	0	1.524008	0	0	0	0	0	0	0	0	0	0	1	=	1
12	0	1	0	0	0	0	0	0	0	0	0	0	0.7734	1.72308	0	0	0	0	5	=	5
13	0	1	0	0	0	0	0	0	0	0	0	0	0	5.49643	0	0	0	0	8	=	8
14	0	2	0	0	0	0	0	0	0	0	0	0	0	0	2.007141	0	0	0	3	=	3
15	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	4.00714	0	0	1	=	1

Figure 5.2: GP Formulation

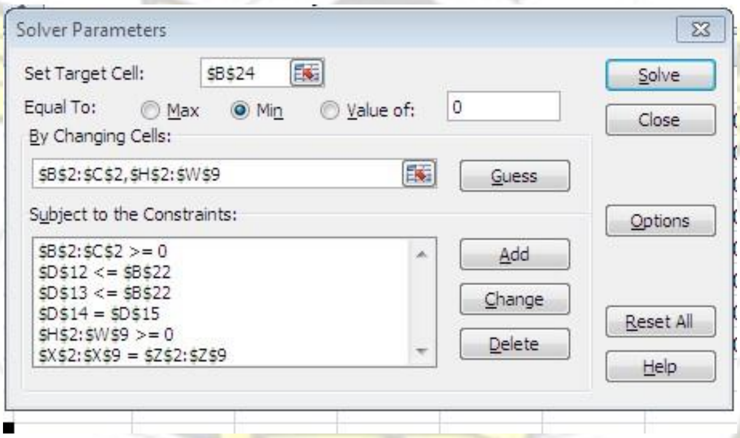
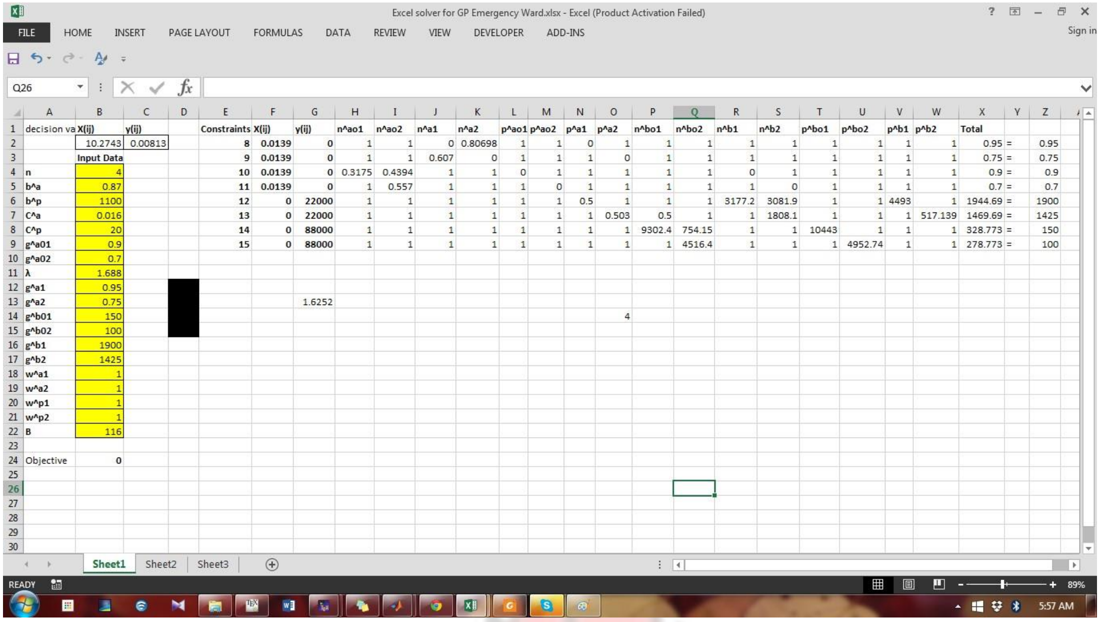


Figure 5.3: Solver Constraints

APPENDIX C

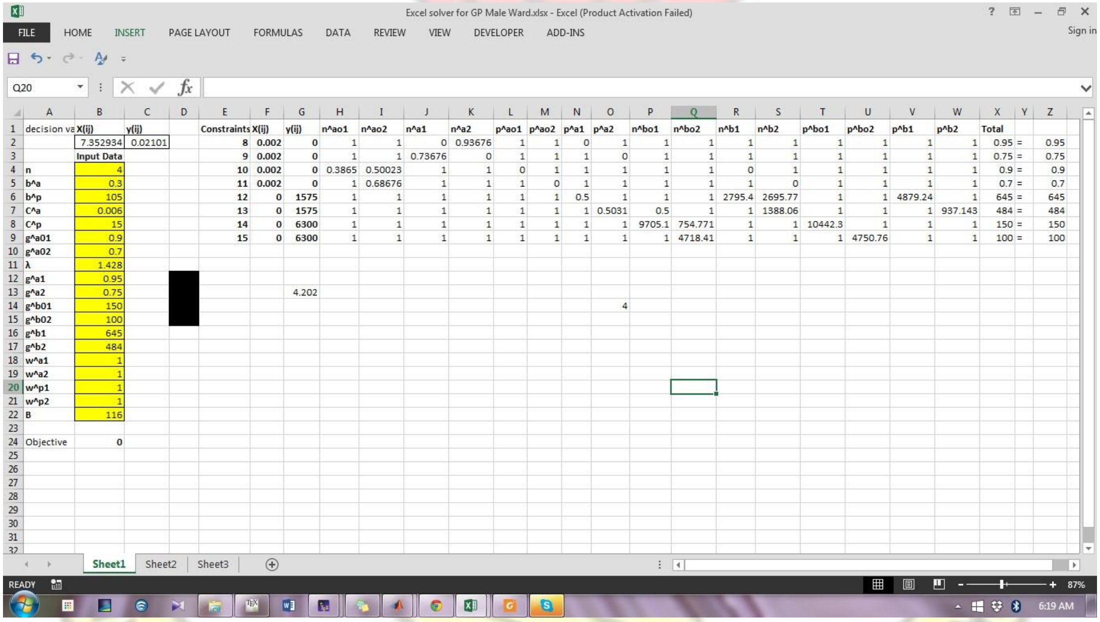


Row	decision va	X(ij)	y(ij)
2		10.2743	0.00813
3		Input Data	
4	n	4	
5	b^a	0.87	
6	b^p	1100	
7	C^a	0.016	
8	C^p	20	
9	g^a01	0.9	
10	g^a02	0.7	
11	λ	1.688	
12	g^a1	0.95	
13	g^a2	0.75	
14	g^b01	150	
15	g^b02	100	
16	g^b1	1900	
17	g^b2	1425	
18	w^a1	1	
19	w^a2	1	
20	w^p1	1	
21	w^p2	1	
22	B	116	
24	Objective	0	

[redacted]

Constraints	X(ij)	y(ij)	n^ao1	n^ao2	n^a1	n^a2	p^ao1	p^ao2	p^a1	p^a2	n^bo1	n^bo2	n^b1	n^b2	p^bo1	p^bo2	p^b1	p^b2	Total		
8	0.0139	0	1	1	0	0.80698	1	1	0	1	1	1	1	1	1	1	1	1	0.95	=	0.95
9	0.0139	0	1	1	0.607	0	1	1	1	0	1	1	1	1	1	1	1	1	0.75	=	0.75
10	0.0139	0	0.3175	0.4394	1	1	0	1	1	1	1	1	0	1	1	1	1	1	0.9	=	0.9
11	0.0139	0	1	0.557	1	1	1	0	1	1	1	1	1	0	1	1	1	1	0.7	=	0.7
12	0	22000	1	1	1	1	1	1	0.5	1	1	1	3177.2	3081.9	1	1	4493	1	1944.69	=	1900
13	0	22000	1	1	1	1	1	1	1	0.503	0.5	1	1	1808.1	1	1	1	517.139	1469.69	=	1425
14	0	88000	1	1	1	1	1	1	1	1	9302.4	754.15	1	1	10443	1	1	1	328.773	=	150
15	0	88000	1	1	1	1	1	1	1	1	1	4516.4	1	1	1	4952.74	1	1	278.773	=	100
		1.6252																			
										4											

Figure 5.4: GP Results For Emergency Department



Row	decision va	X(ij)	y(ij)
2		7.352934	0.02101
3		Input Data	
4	n	4	
5	b^a	0.3	
6	b^p	105	
7	C^a	0.006	
8	C^p	15	
9	g^a01	0.9	
10	g^a02	0.7	
11	λ	1.428	
12	g^a1	0.95	
13	g^a2	0.75	
14	g^b01	150	
15	g^b02	100	
16	g^b1	645	
17	g^b2	484	
18	w^a1	1	
19	w^a2	1	
20	w^p1	1	
21	w^p2	1	
22	B	116	
24	Objective	0	

[redacted]

Constraints	X(ij)	y(ij)	n^ao1	n^ao2	n^a1	n^a2	p^ao1	p^ao2	p^a1	p^a2	n^bo1	n^bo2	n^b1	n^b2	p^bo1	p^bo2	p^b1	p^b2	Total		
8	0.002	0	1	1	0	0.93676	1	1	0	1	1	1	1	1	1	1	1	1	0.95	=	0.95
9	0.002	0	1	1	0.73676	0	1	1	1	0	1	1	1	1	1	1	1	1	0.75	=	0.75
10	0.002	0	0.3865	0.50023	1	1	0	1	1	1	1	1	0	1	1	1	1	1	0.9	=	0.9
11	0.002	0	1	0.68676	1	1	1	0	1	1	1	1	1	0	1	1	1	1	0.7	=	0.7
12	0	1575	1	1	1	1	1	1	0.5	1	1	1	2795.4	2695.77	1	1	4879.24	1	645	=	645
13	0	1575	1	1	1	1	1	1	1	0.5031	0.5	1	1	1388.06	1	1	1	937.143	484	=	484
14	0	6300	1	1	1	1	1	1	1	1	9705.1	754.771	1	1	10442.3	1	1	1	150	=	150
15	0	6300	1	1	1	1	1	1	1	1	1	4718.41	1	1	1	4750.76	1	1	100	=	100
		4.202																			
										4											

Figure 5.5: GP Results For Male Department

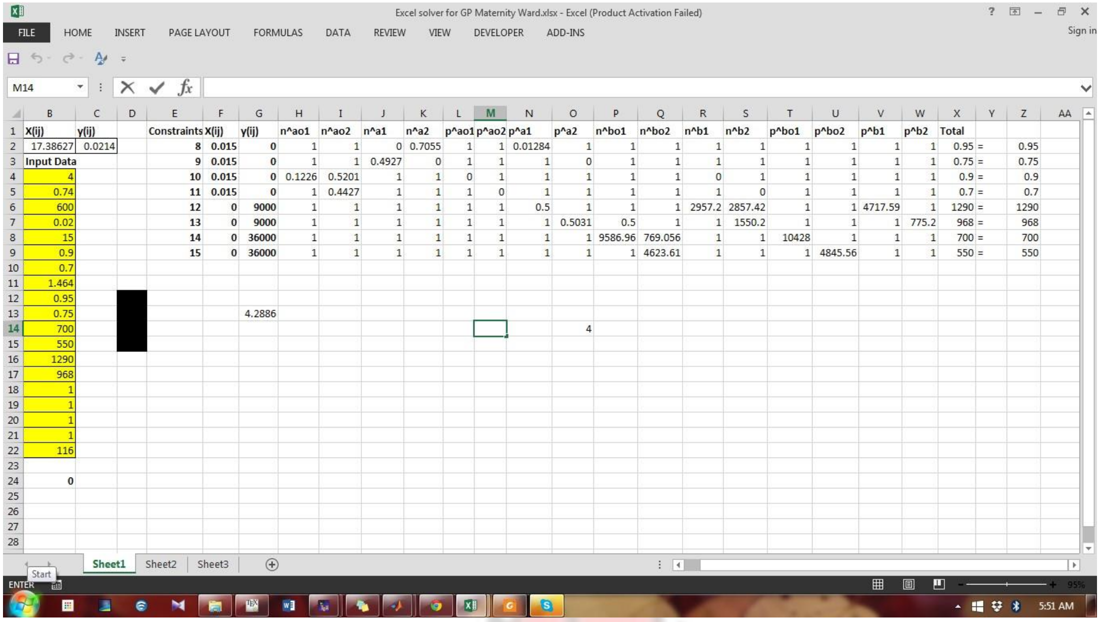


Figure 5.6: GP Results For Maternity Department

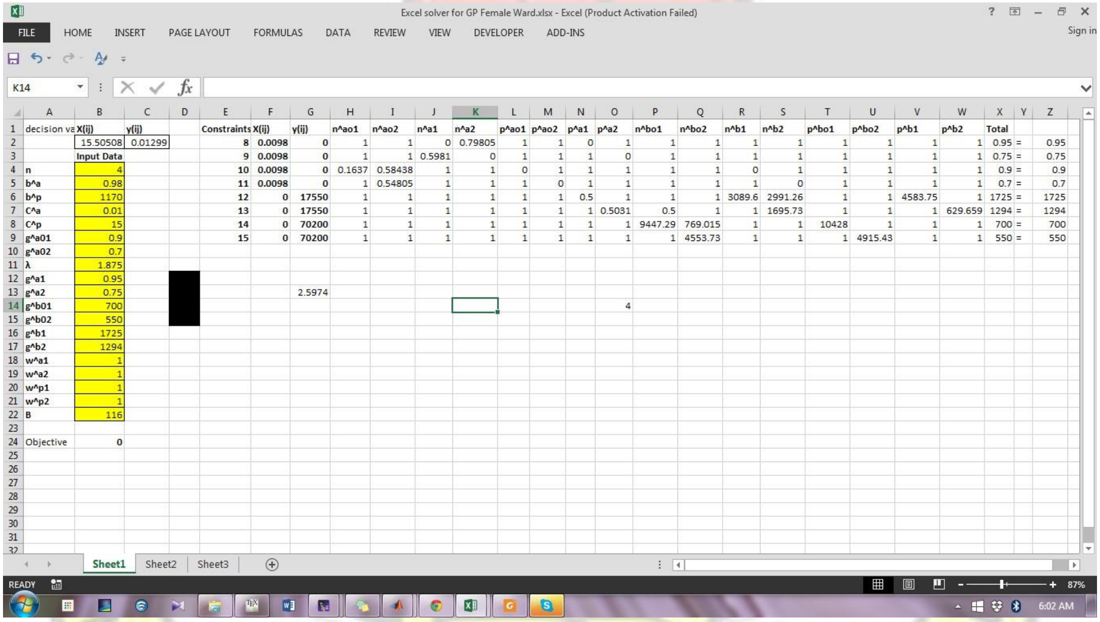


Figure 5.7: GP Results For Female Department