KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

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DEPARTMENT OF MATHEMATICS

OPTIMIZING TRANSPORTATION COST OF SOLID WASTE

A CASE STUDY IN THE SUNYANI MUNICIPALITY

BY

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DECLARATION

I hereby declare that this submission is my own work towards the M.Sc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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DEDICATION

To my dear wife, Patricia Nyaaba Naninja (Mrs), and children Gloria, Bright and Pearl.

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ABSTRACT

Ridding our cities, towns and communities of filth has been one of the major concerns of various governments in Ghana. Metropolitan, Municipal and District Assemblies have the responsibility of ensuring that the populace is not engulfed with filth to the detriment of their health.

Owing to the increasing prices of petroleum products and high maintenance cost of vehicles used in the collection and transportation of solid waste by Zoomlion in the Sunyani Municipality. This thesis sought to determine the number of trips each type of vehicle (skip, compactor and roll-on) used in the transportation should go per day to minimize operational cost.

The problem was formulated as an Integer Linear Programming problem and was solved using the software called Linear programming Software (LIPS).

The results show that if only one skip vehicle makes 14 trips per day, the expected quantum of solid waste needed to be hauled to the landfill site at Asufufuo, 5km away, per day, can be achieved at a minimum cost of GH199.92.

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ABBREVIATIONS

BFS Basic Feasible Solution

BMW Biodegradable Municipal Waste

ISWN Integrated Solid Waste Management

LI Legislative Instrument

LIP Linear Integer Programming

LIPS Linear Programming Solver

LP Linear Programming

MILP Mixed Integer Linear Programming

MMDAs Metropolitan, Municipal and District Assemblies

MSW Municipal Solid Waste

NESPoCC National Environmental Sanitation Policy Coordinating Council

SW Solid Waste

SWM Solid Waste Management

WHO World Health Organisation

CHAPTER 1

INTRODUCTION

Solid waste management is a growing environmental and financial problem in countries throughout the world (Hari et al, 1994). Despite significant efforts made some countries in recent decades to improve solid waste management services, most municipalities and Metropolitan cites in developing countries still face major challenges in properly handling the growing volume of waste produced in their cities (Majani, 2000; Kasevaand Mbuligwe, 2003; Kitbuah et al., 2009). Increasing population, economic activities, urbanization and industrialization especially in developing countries such as those in Africa, have drastically increased the amount of waste generated (Taylor, 1999). Waste collection systems vary widely between different countries and regions. In Ghana, waste collection services are often provided by local government authorities or by private waste management companies (Kitbuah et al., 2009). The two collection methods practiced in Ghana are the communal and the franchised methods. The communal method of waste collection has several waste collection points called transfer stations located in the communities where all the waste are gathered from households and from other public institutions before they are transported to the disposal sites. In the franchise method of waste collection, the waste is collected from homes, institutions and in public places and transported to the disposal sites. The communal collection is used in areas where the residential houses are not well planned and there are no good road networks to ensure house to house collection.

1.0 DEFINITION OF WASTE LOGISTICS

The concept of waste logistics has not been widely used within the waste management studies. However, the term Reverse logistics is used rather frequently to refer partly to waste.

In this study, the term waste logistics will be used throughout the text. Logistics is the flow of material, information and money between consumers and suppliers or disposal such as in case of waste (Thomas, 2005). It incorporates the planning and execution of activities to move products from their origin to destination (Frazella, 2002). Logistics is defined as the management, control of the flow, storageof materials, information, financial services between suppliers and consumers (Bowersox, 2002). It includes all the activities undertaken to move the product fromits origin to its destination. In this regard, waste logistics can be understood as all the activities undertaken to move waste from its source of generation to the final disposal point (Roel et al., 2010).The quest to satisfy human wants and needs results in the conversion of resources; both natural and anthropogenic to suitable forms for human consumption and survival. Wastes, which are the unwanted material, results as such from the transformational as well as the consumption of these resources. The 1995 Environmental Act of UK defines waste as „any substance or object which the holder discards or intends to discard‟. A „holder‟ means the producer of the waste or the person who is in possession of it (Williams, 1998).With continuous economic development and an increase in living standards, the demand for goods and services is increasing quickly, resulting in an increase in per capita generation of solid waste (Narayana, 2008). The large quantities of waste generated in the Subin Sub-metro necessitate a system of collection, transportation and disposal. It requires knowledge of what the wastes are comprised of, and how they need to be collected and disposed.

Solid waste is any material which comes from domestic, commercial, and industrial sources arising from human activities which has no value to people who possess it and is discarded as useless. In the early days, waste disposal did not pose difficulty as habitations were sparse and land was plentiful. Waste disposal became problematic with the rise of towns and cities where large numbers of people started to congregate in relatively small areas in pursuit of livelihoods (Shafiul and Mansoor, 2003). While the population densities in urbanized areas and per capita waste generation increased, the available land for waste disposal decreased proportionately. Solid waste management thus emerged as an essential, specialized sector for keeping cities healthy and liveable. The collection, transfer and disposal of waste have been generally assumed by metropolitan governments in both developed and developing world.

This constitutes a basic and expected government function. The format varies in most urban areas where solid waste is collected either by a government agency or private contractor.

Despite the fact that developing countries do spend about 20 to 40 per cent of metropolitan revenues on waste management, they are unable to keep pace with the scope of the problem (Zerbock, 2003). In fact, when the governments of African countries whereby the World Health Organization (WHO) to prioritize their environmental health concerns, the results revealed that solid waste was identified as the second most important problem after water quality (Senkoro, 2003 cited by Zerbock, 2003).Waste handling is one of the greatest challenges facing humankind in modern times in spite of the numerous technological achievements that have been well documented. Technology alone has not been able to effectively control waste generated in communities worldwide. Rather, it appears that new technologies bring new types of waste into the environment to add to the complex accumulation puzzle" (Kwawe, 1995)."Recent events in major urban centres in Africa have shown that the problem of waste management has become a monster that has aborted most efforts by city authorities, states and federal governments, and professionals alike" (Onibokun, 1999).

For the first time in the history of mankind, we are witnessing an unprecedented phenomenon in the development of places of habitat: the balance of human settlement patterns have shifted from more people inhibiting rural areas to more people living in cities (Rabinovitch, 1998, UNFPA, 2001). This is especially so in developing countries such as Ghana. Urbanization introduces society to a new way of life: cars, pre-packed foods; it allows for economies of scale in the production of goods and services, and in the transportation of the finished products for human consumption (UNFPA, 2001: 32). In countries around the world, one major environmental problem that confronts municipal authorities is solid waste disposal as observed by Pacione (2005). Most city governments are confronted by mounting problems regarding the collection and disposal of solid waste. In most developing countries, the problems usually centre on the difficulties and high cost of disposing of the large volume of waste generated by households and businesses.

1.1 BACKGROUND TO THE STUDY

The Sunyani Municipality covers a land area of 110 square kilometers. It shares boundaries on the north-east with East Techiman Municipality, on the south-east with Bechem District, Tepa on the east and Berekum on the west and Odumase on the north. In essence, the area of the land stretches from Fiapri to Abesim to Yawahima to Watchman to Domsesie and to Nkwabeng. The compactness of the land size compared to the large volume of waste generated puts a tremendous responsibility on the planners of the municipality to keep refuse at the landfill site in a more hygienic condition and to manage waste efficiently.

The 2000 Population and Housing Census put the estimated population of the municipality at 156,750 for the year 2010 (Statistical Service Department – Brong-Ahafo Regional Office). Sunyani, the regional and municipal capital of the Sunyani Municipality harbors over 65% of the entire population in the district. The remaining 52 settlements have smaller population sizes which do not normally measure up to the population thresholds required for the provision of essential socio-economic services.

Household size in the municipality is 10.9 and this is the highest in the region. The Sunyani municipality has 67.1% of its inhabitants living in room(s) in compound houses which is higher than the regional average of 43.1%. The district has 11.3% of households living in separate houses. The high population density in the municipality underscores the need to manage waste properly in order to avert any health hazard.

The Sunyani Municipal Assembly has managed the waste generated in the municipality singlehandedly for some decades .It was recently that the Government of Ghana embraced the idea of private companies to partake in the management of waste. The Sunyani Municipal Assembly is one of the four municipals established under the Local Government Act 462 of 1988.The municipality was granted its present status by the Legislative Instrument (LI) 1426 of 1988.

Sunyani Municipal Assembly has contracted Zoomlion Ghana Limited to take charge of solid waste generated while the Assembly concentrates on liquid waste management. Zoomlion is a private waste management company. It started its operations in the country in 1995. Disposal of municipal solid waste in sanitary landfills is still the main waste management method employed by the Sunyani Municipal Assembly as in most municipalities in Ghana and the developing nations. The amount of solid waste generated in the municipality is collected from various locations and transported to the only landfill site at Asufofuo. The landfill site was constructed by the Assembly and commissioned in 1993 with an expected life span of 35 years. It comprises of a tipping bay and paved vehicle turning bays. The station is manned by six (6) trained staff and equipped with a bulldozer for spreading and compacting the waste dumped at the site (Technical Report, 2010).

Waste management is a major challenge to all cities and towns all over the world especially in developing countries. Waste collection and treatment in urban areas are a real concern to local governments. Following increasing ecological consciousness, not only individuals but also industrial companies have started to lend their support (Alves et al., 2010).Many districts in Ghana do not have adequate and good road infrastructure for the collection and transportation of waste generated. This has resulted in cities lagging woefully behind in terms of cleanliness. This puts the population of cities and towns in a precarious situation. Outbreaks of cholera and other preventable communicable diseases in unplanned suburbs in some cities and towns clearly underscore the connection between the proper management of waste and the health of the populace.

1.2 PROFILE OF ZOOMLION GHANA LIMITED

Zoomlion Ghana Limited is a giant in waste management as well as environmental sanitation business in Ghana and Africa as a whole. The Company was formed under the company‟s Act with registration number CA22256 in January 2006.The Company which was formed in 2006 as Zoomlion Ghana Limited with a few members of staff has now grown over the past four years with eight (8) subsidiaries. Zoomlion also operates in other African countries such as Togo, Angola and Guinea while negotiations are far advanced for the company to start operations in other African countries such as Nigeria, Sierra Leone and Liberia (www.zoomlionghana.com).

1.3 TYPES OF WASTE

There are various ways by which waste can be classified, depending on whether it is biodegradable or non-biodegradable, toxic or nontoxic, and the composition. However, the categorisation is largely by composition. The categorisation by composition includes;

1.3.1 Municipal Waste (including Household and Commercial)

Municipal waste is generated by households, commercial activities and other sources whose activities are similar to those of households and commercial enterprises. It does not include other waste arising e.g., from mining, industrial or construction and demolition processes. Municipal waste is made up of residual waste, bulky waste, secondary materials from separate collection (e.g., paper and glass), household hazardous waste, street sweepings and litter collections. It is made up of materials such as paper, cardboard, metals, textiles, organics (food and garden waste) and wood (European Environment Agency, 2009).

1.3.2 Industrial Waste (including manufacturing)

Manufacturing industry waste comprises many different waste streams arising from a wide range of industrial processes. Some of the largest waste generating industrial sectors in Western and Central Europe include the production of basic metals, food, beverage and tobacco products, wood and wood products and paper and paper products. The manufacturing industry has a central role to play in the prevention and reduction of waste as the products that they manufacture today become the wastes of tomorrow (European Environment Agency, 2009).

1.3.3 Hazardous Waste

Hazardous waste arises from a wide range of different sources including households, commercial activities and industry. Wastes are classified as being hazardous depending on whether they exhibit particular characteristics. The main disposal route for hazardous waste is landfill, incineration and physical or chemical treatment. On the recovery side, a significant proportion of hazardous waste is recycled or burned as a fuel. Hazardous waste is typically the subject of special legislation and requires special management arrangements to ensure that hazardous waste is kept separate from and treated differently to non-hazardous waste (European Environment Agency, 2009).

1.3.4 Construction and Demolition Waste

Construction and demolition waste is made up of two individual components (construction waste and demolition waste). It arises from activities such as the construction of buildings and civil infrastructure, total or partial demolition of buildings and civil infrastructure, road planning and maintenance. In some countries even materials from land leveling are regarded as construction and demolition waste. It is made up of numerous materials including concrete, bricks, wood, glass, metals, plastic, solvents, asbestos and excavated soil, many of which can be recycled in one way or another (European Environment Agency, 2009).

1.3.5 Waste from Electrical and Electronic Equipment

Waste from electrical and electronic equipment (commonly referred to as WEEE) consists of end of life products and comprises of a range of electrical and electronic items such as: Refrigerators, Information Technology and Telecommunication equipment, Freezers, Electrical and electronic tools, Washing machines, Medical equipment Toasters, Monitoring and control instruments, Hairdriers, Automatic dispensers, Televisions, etc. Thus, sources are all users of electrical and electronic equipment from householders to all kinds of commercial and industrial activities (European Environment Agency, 2009).

1.3.6 Biodegradable Municipal Waste

Biodegradable Municipal Waste (BMW) is waste from households and commercial activities that is capable of undergoing biological decomposition. Food waste and garden waste, paper and cardboard are all classified as biodegradable municipal waste. A range of options are used to treat BMW. Alternatives to landfill include composting, mechanical-biological pre-treatment recycling and incineration (European Environment Agency, 2009).

1.3.7 End-Of-Life Vehicles (Elvs) and Tyres

End-of-life vehicles are defined as cars that hold up to a maximum of eight passengers in addition to the driver, and trucks and Lorries that are used to carry goods up to a maximum mass of 3.5 tonnes. Thus their sources range from households to commercial and industrial uses. Cars are composed of numerous different materials. Approximately 75% of the weight of a car is made up of steel and aluminium, most of which is recycled. Other materials present include lead, mercury, cadmium and hexavalent chromium, in addition to other dangerous substances including anti-freeze, brake fluid and oils that, if not properly managed, may cause significant environmental pollution. The remainder is composed of plastic which is recycled, incinerated or landfilled (European Environment Agency, 2009).

1.3.8 Agricultural Waste

Agricultural waste is composed of organic wastes (animal excreta in the form of slurries and farmyard manures, spent mushroom compost, soiled water and silage effluent) and waste such as plastic, scrap machinery, fencing, pesticides, waste oils and veterinary medicines (European Environment Agency, 2009).

1.4 WASTE AND WASTE MANAGEMENT IN GHANA

With regards to decision making, a coordinating council, the National Environmental Sanitation Policy Coordinating Council (NESPoCC) has been put in place since January 2000 to expedite the implementation of the National Sanitation Policy. The national laws, specifically the Criminal Code (Act 29), 1960, and Revised Bye-laws of all the 110 MMDA‟s have enough laws to support the Environmental Sanitation Service delivery and enforce the compliance of sanitation rules. It is however noted that these laws are not deterrent enough and logistical problems make MMDA‟s impotent in ensuring clean, safe and healthy environment (Sanitation2004-Ghana).

1.5 SOLID WASTE

General waste management in Ghana is the responsibility of the Ministry of Local Government and Rural Development, which supervises the decentralized Metropolitan, Municipal and District Assemblies (MMDAs). However, regulatory authority is vested in the Environmental Protection Agency (EPA) under the auspices of the Ministry of Environment and Science. The Metropolitan, Municipal and District Assemblies are responsible for the collection and final disposal of solid waste through their Waste Management Departments (WMDs) and their Environmental Health and Sanitation Departments. The only guidelines, which indirectly discourage unsustainable practices and promote Sustainable consumption and production, are those on the Environmental Impact Assessment (Sanitation2004-Ghana).

1.6 PROFILE OF STUDY AREA

Sunyani Municipal Assembly is one of the twenty-two administrative districts in the Brong Ahafo Region of Ghana. It lies between Latitude 7 ^{0}20‟N and 7 ^{0}50‟N and longitudes 2^{0} 10‟W and shares boundaries with Sunyani West District to the North, Dormaa District to the West, Asutifi District to the East there effective economic and social interactions with the neighbouring districts which promote resource flow among these districts.

The Municipality has a total land area of 829.3 square kilometers (320.1 square miles). Sunyani also serves as a Regional Capital for Brong Ahafo. One third of the total land area is not in habited or cultivated which provides arable lands for future investment in crop farming. The Sunyani West District lies between latitude 7 ^{0}19‟N and 7 ^{0}35‟N and longitude 72 ^{0}08‟W and 2 $_{0}$31‟W.

It shares boundaries with Wenchi Municipality to the North East, Tan District to the North, Berekum and Dormaa East to the West, Sunyani Municipal to the South East and to the South East and to the Eastern boundaries of the District are Tano North and of in so North District.Sunyani West District has total land area of 1. 658.7square kilometers (SMA, 2010)

1.6.1 RELIEF AND DRAINAGE

The Sunyani Municipality and Sunyani and Sunyani West district lie within the middle belt of Ghana with height from 229 meters to 376 meter above sea level. The topography of the Municipality is fairly flat thus suitable for large scale agricultural mechanization. Cost of constructing houses and road is relatively minimal due to the nature of the topography. The drainage is basically dendrite with several stream and rivers, notably Tano, Amoma, Kankam ,Benu, Yaya and Bisi. Most of the water bodies area seasonal (S.M.A, 2010).

This often creates water shortages in the municipality during the dry season for both domestic and agricultural purposes.

1.6.2 CLIMATE AND VEGETATION

The study areas fall within the Semi-Equatorial climatic zone of Ghana. The monthly temperature varies between 23^{0}c with the lowest around August and the highest being observed around March and April. The relative humidifies are high averaging between 75 and 80 percent during the rainy seasons and 70and 80 percent during, the dry seasons of the year which is ideal for luxurious vegetative growth (S.M.A, 2010)

The average rainfall for Sunyani Municipality and Sunyani West District is about 88.987cm. These areas experience double maxima rainfall pattern. The main rainy season is between March to September with the minor between October to December. This offers two farming seasons in a year which support agricultural production n the municipality and the District. However, the rainfall pattern is changing over the years as a result of the forestation and depletion of water bodies resulting from human activities

(S.M.A, 2010). The study area falls largely within the moist-semi Deciduous Forest Vegetation Zone. This vegetation zone also contains most of the valuable timber species such as Wawa, Odum, and Mahogany. As indicated by the characteristics of the vegetation cover, tree crops such as Cocoa and Citrus thrive well in this zone. As a result of lumbering and farming practices, most of the forest areas have been degraded.

1.6.3 POPULATION

In 2000 the population of Sunyani Municipality was 101,145. Currently, with a growth rate of 3.8 percent the estimated population is 147,301. The growth rate of Sunyani compared with the regional population growth rate of 2.5 and the national growth rate of 2.7 percent indicates a high growth rate (Ghana Statistical Service, Sunyani, 2010). This has contributed to pressure on the available facilities and also generates more solid waste in the municipality.

Table 1.1 Population for Sunyani Municipal and Sunyani West District.

Name of Locality	1984	Growth rate	2000	Growth rate	2010
Sunyani Municipal	-	-	101,145	3.8	47,301
SunyaniWest	-	-	78,165	3.8	
Sunyani (combined)	98,183	3.5	179,115	3.8	

Source: Ghana Statistical Service, Sunyani

1.6.4 SPATIAL DISTRIBUTION AND DENSITY

The population density of the Municipality is 122 persons per square kilometer (S.M.A Computation 2010). In comparing this to the population density of the region which is 45.9/ sq. km and that of the nation of 76/sq. km. the municipality is densely populated resulting in pressure on social facilities and generating solid waste. From socio – economic survey conducted the densely populated areas in the municipality include zongo, new Dormaa and area 2 that order. On the average these areas have 18 persons per house

Nkwabeng, Abesim and Nkrankrom constitute the medium densely populated area with an average of 13 persons per house. The low density areas are Estate, South Ridge, Airport Area, Atronie and Boaakoniaba with an average of 8 persons per house.

The densely populated areas are mostly in the high and medium income groups (S.M.A, 2010). The distribution of population in the Sunyani West District is generally skewed. The four largest localities (Nsoatre, Chiraa, Odomase and Fiapre) hold a significant proportion of the District‟s total population with 59. 24 percent distributed among the other settlement (SWDA, 2010). The concentration of population in the four major settlements has increased demands for utility service such as water, sanitary facilities, electricity and other services which has resulted in the increase of solid waste in the municipality. Waste generation has also increased in the municipality especially in these four communities. Other major settlements in the municipality that are densely populated include: Asuakwaa, Dumasua Kwatire, Adantia, Tanom and Kobedi. The population density District (Sunyani West) 68.8 person per square kilometer (SMA, 2010).

1.6.5 ECONOMY OF THE STUDY AREA

In Sunyani Municipality, agricultural activities including crop farming, animal husbandry and fishing constitute the highest proportion of workers in the municipality forming 45.9 percent, followed by industry (carpentry, bricks and block laying, timber related industries and construction workers), 14.7 percent. Service and Administration (Government workers, financial institutions communication workers, Hairdressers and Seamstresses), constitute 9.6 percent, professional and Technical (Engineers, Consultants and Pharmacist), 9 percent, commerce, 8.6 percent and whereas others such a head potters, track pushers, mining form 1.4 percent (G SS, 2000).

1.7 PROBLEM STATEMENT

Extensive commercial activities are carried out at various locations in the Sunyani municipality which generate several tons of waste. These activities include selling of both perishable and nonperishable goods, particularly, on Wednesdays which happens to be the market day for the Municipality. This extensive commercialization tends to generate a lot of refuse. Convenience packing of consumer products is one significant source of increase volume (Ferrel and Hizlan, 2001).

Solid waste collection in the Municipality is done by the Zoomlion Company limited at centralized locations and then transported finally to the landfill site by the use of the following vehicles: skip, compactor, and roll-on-roll-off.

However, owing to financial constraints, increase in fuel cost, high cost of labour and vehicle maintenance, and poor road network, some communities are usually not covered during the solid waste collection on daily basis.

Even though they have relied on experts who apply some relevant scientific theories to manage waste, the application of operations research using optimization techniques is minimal or nearly absent.

Large chunk of the municipality"s budget is used for the collection and the transportation of municipal solid waste .So if there is a way of minimizing the collection and transportation cost, it will go a long way to helping the Assembly and Zoomlion Company limited and that is what this study seeks to address.

1.8 OBJECTIVE OF THE STUDY

The objective of the research is to find the optimal solution to the linear programming problem which will

1. Model cost of transporting waste materials to landfill site as ILP problem

2. Minimize the transportation cost of solid waste material to the land fill site by the branch and bound method

1.9 METHODOLOGY

The problem in this study was modeled as an Integer Linear Programming problem. In this work the Branch and Bound algorithm was used to solve the Integer Linear Programming problem.

The data which is a secondary data was obtained from the office of the Sunyani Municipal Assembly and the regional office of Zoomlion Ghana Ltd. for the year 2010. The model was solved using the software Lips (Linear programming solver) by typing the inequalities into a file. Lips is a comprehensive and powerful algebraic modeling language for linear and non-linear optimization problems, in discrete or continuous variables (Fourer et al, 2003). Information was sourced from the internet, Sunyani Polytechnic Library and Sunyani Municipal Library at Sunyani.

1.10 JUSTIFICATION

The profit gain as a result of minimizing the transportation cost will enable Zoomlion Ghana Limited to contribute to its continuous projects and programmes such as

- Poverty reduction through job creation for the un-employed youths in collaboration with the ministry of manpower, youth and employment.
- The creation of jobs in the local assembly of tricycles and waste containers.
- Contribution to skills development through training in the sector.
- Removal of the condition that support mosquito and fly breeding in drain and markets thereby protecting and promoting public health.
- The improvement in the aesthetics of the environment there by making the urban centers attractive to investors and tourists
- 100% target collection of waste generated in allocated zones by introduction of precollection services door-to-door with tricycles.

1.11 ORGANISATION OF THESIS

Chapter 1 looked at the quantum of solid waste generated in the Sunyani Municipality which has a relatively small land size but with high population density especially in Sunyani, the municipal and regional capital. It also considered the challenges the Sunyani Municipal Assembly and Zoomlion Ghana Ltd. are encountering in the collection and transportation of solid waste to the land fill site at Asufofuo. In chapter 2,   researches of authors whose work have a direct bearing on how to manage solid waste and how utilizing operations research methods will optimize the transportation cost were considered. The results of their findings were critically analyzed in order to have appropriate guide for this research work. In chapter 3, the methodology adopted for the thesis was considered. The problem of transporting solid waste generated in the municipality at a minimum cost was formulated as an integer programming problem. Linear Programming software tool call Lips was used.

In chapter 4, the optimal solution which was found using the computer software was analyzed and interpreted. Comparative analysis was made using the optimized results and the existing values. In chapter 5, a summary of the entire thesis work was considered and some suggestions and recommendations were given.

1.12 SUMMARY

It is the desire of the Sunyani Municipal Assembly and Zoomlion Ghana Ltd. to use their limited resources to manage effectively and judiciously the solid waste generated in the municipality. This is to ensure good and vibrant health for the inhabitants thereby increasing the productivity of the country. Optimization technique is a powerful tool that can be harnessed in this regard.

CHAPTER 2

LITERATURE REVIEW

Disposal of municipal solid waste is a problem commonly confronting all industrialized countries around the world and that transportation is an important component that needs to be carefully investigated (Mohd, 2004). According to Rathi (2007), municipal assemblies of the developing countries are not able to handle the increasing quantity of waste, which leads to uncollected waste on roads and other public places. By inference, the problem of disposing off solid waste by considering an effective way of minimizing the transportation cost in the developing countries cannot be gainsaid.

The growing concern for the greenhouse effect and the sustainability of the planet earth has engendered in various governments, well-meaning citizens and institutions, the need to manage waste efficiently. Improved advertizing techniques which have resulted in the production of attractive packaged goods and high levels of consumption of goods generate a lot of waste. There are numerous definitions of what constitutes waste. According to the European Environmental Protection Act (1990) "waste is any substance which constitutes scrap material any substance or article, which requires to be disposed of as being broken, worn out, contaminated or otherwise spoiled."

According to Tchobanoglous et al. (1993), the Solid Waste Management process can be commonly classified into generation, collection, storage, processing, transportation and disposal. Waste may be categorized with respect to the source that generated it. Waste types distinguished under this classification are (i) municipal solid waste which is generated by households as well as organic waste, glass, paper and other recyclable materials, (ii) residual waste that is generated by waste treatment facilities like composting units and incineration plants, (iii) industrial waste which is generated by industrial sectors, (iv) construction waste which is generated by construction and demolition sectors, (v) contaminated soil and (vi) other wastes which is a diverse set of smaller types of categories.

In his book „Waste Market, Dijkgfaaf (2008) wrote that Municipal Solid Waste flow accounts for about 40% of all waste that requires treatment and that this waste category presents perhaps the greatest waste management problems in the Netherlands. He also highlighted how the issue concerning waste is at the heart of many nations. He said that in 2006, he was one of the participants at a conference on Local Government Reform in Barcelona and noticed that researchers from Spain, Scandinavian countries, the UK and the USA were also studying issues concerning waste. He asserted that in 1972, a Dutch household paid 44 euro per year on the average for the collection and treatment of waste, and in 1990, the real costs were two times higher and then in 2003 a household paid more than five times as much. Due to this sharp rise in costs, several policy measures were introduced all with the view of minimizing waste collection costs.

According to Allen et al. (2007), recent outbreaks of cholera and other preventable communicable diseases in unplanned suburbs in some cities and towns clearly demonstrate the connection between the proper management of solid waste and the health of the populace. The rapid growth of population and urbanization decreases the non-renewable resources and disposal of effluent and toxic waste indiscriminately, are the major environmental issues posing threats to the existence of humans.

According to Nishanth et al. (2010), solid waste management is a global environmental problem in today‟s world. The increase in commercial, residential and infrastructure development due to population growth has negative impact on the environment. Urban solid waste management is considered as one of the most serious environmental problems confronting municipal authorities in the developing countries. One of these impacts is due to location of dumping site in unsuitable areas. Solid waste models that have been developed in the last two decades are varied in goals and methodologies. Solid waste generation prediction, facility site selection, facility capacity expansion, facility operation, vehicle routing, system scheduling waste flow and overall system operation have been some of these goals (Badran and El-Hagger, 2006).

Ghose et al. (2000) say that some of the techniques that have been used include linear programming, integer programming, mixed- integer programming, non-linear programming, dynamic programming, goal programming, grey programming, fuzzy programming, quadratic programming, stochastic programming, two-stage programming, interval-parameter programming, geographic information system. The enormity of waste management problems facing many municipalities has resulted in the emergence of a global consensus to develop local level solutions and community participation for better Municipal Solid Waste Management. A number of studies have been carried out to compare different methods of waste disposal and processing for different places. Some researchers have riveted their attention on vehicle routing systems.

Solid waste collection routing is one of the main components of solid waste management. Agha (2006) argued that despite the fact that the collection process constitutes 74% of SWM cost, it has been given little attention. Cost reduction strategies in Solid Waste routing may include optimizing the collection routing, thus minimizing the travelled distance and travelling time. A second strategy involves minimizing the number of truck trips to landfill sites. A third strategy involves maximizing the number of fully loaded trips to the disposal sites. Using the maximum extent of the available equipment is another strategy. Agha (2006) used mixed-integer programming model to optimize the routing system for Deir El- Balah in the Gaza Strip. The model minimized the total distance travelled by the collection vehicles. The results revealed that the application of the model improved the collection system by reducing the total by 23.4% which translated into a savings of US $1140 per month. He further argued that in spite of the fact that the collection process constitutes 74% of SWM cost, it has been given little attention.

Nuortio et al. (2006) described the optimization of vehicle routes and schedules for collecting municipal solid waste in Eastern Finland. They used a recently developed guided variable neighborhood threshold metaheuristic method which has been adapted to solve real-life waste collection problems to generate solutions. This approached was used to study the collection of waste in two regions in Eastern Finland. The results showed that a significant change can be obtained by using this approach compared with the existing practice. Other researchers have proposed an integrated method to solve waste management problems.

Rathi (2007), who carried out his research work in Mumbai, India, argued that there is no unique method that is capable of solving the problem of waste management. He was therefore a proponent of the idea that an integrated method with the aim to minimize environmental and social costs associated with waste management is the best. He formulated a linear programming model to integrate different options and stakeholders involved in Municipal Solid Management in Mumbai. Also, he asserted that municipal assemblies of the developing countries are not able to handle the increasing quantity of waste, which leads to uncollected waste on roads and other public places.

Prawiradinata (2004) developed an analytical model to solve for optimal waste management policies in the state of Ohio, USA. In the research, an analytical model of a single landfill and integrated solid waste management were formulated as optimization models. The simple analytical model was designed using control theory to answer questions about the optimal lifespan and replacement of a landfill and about the opportunities to substitute alternative disposal methods for landfill. The ISWM model which was formulated as a mixed-integer program helped waste management authorities to plan for the long-term future of facilities and the possible implementation of recycling and composting options.

Ferrell and Hizlan (1997) asserted that constructing an integrated and coherent municipal solid waste management plan is increasingly important because both the volume of municipal solid waste and the costs associated with traditional solutions are rising at much faster rate than in the past. Solid waste generation prediction has been the preoccupation of some researchers. A study conducted in Kuala Lumpar, Malaysia by Saeed et al. (2009) presented a forecasting study of municipal solid waste generation rate and the potential of recyclable components. The generation rates and the composition of solid wastes of various classes like street cleaning, industrial and constructional, institutional, residential and commercial were analyzed. The study showed that increased solid waste generation of Kuala Lumpar is alarming.

The attention of other researchers was captured by the escalating cost of transportation of municipal solid waste. They therefore undertook some studies that would optimize the transportation cost of solid waste generated in our communities. Wang et al.(1995) employed mixed –integer programming to determine an optimal transportation system model for two competing transportation models for the movement of recovered paper from selected sources in the state of Iowa, USA to anticipated markets within the state and the optimum number of intermediate processing stations to minimize transportation cost. They concluded that truck transportation is the most practical mode because of its flexibility in routing and unit hauling capacity.

Jalilzadeh and Parvaresh (2004) investigated the total time spent for collection and transportation of solid waste by various types of vehicles (van, mini truck, truck FAUN and compactor) used in the city of Urmia, Iran. The result of the study illustrated that van is the most economic vehicle for solid waste collection system in Urmia city. Moreover, these authors say that more than 60% of solid waste management cost is usually allocated for the purpose of collection and transportation of generated solid waste in the city of Urmia, Iran.

An insightful study on the optimization of transportation cost of municipal solid waste was conducted by Kulcar (1996) who used operations research method with system engineering to minimize the waste transportation cost in a major urban area, Brussels. Due to the complexity of the real situation, he developed a model to consider a set of points for the collection routes. Several means of transportation were used– transportation by vehicle, rail and canal were evaluated. Collector vehicles resided overnight in depots and daily evacuated out to an

incinerator.

Rhoma et al. (2009) used one of the populated cities in Germany – Duisburg in Nord-Rhine Westphalia. Municipal Solid Waste mathematical model was presented as capable of estimating the collection and the transportation cost as well as the environmental impact. The model presented a reasonably effective way to predict the fuel consumption and distance travelled for waste collection in different areas with different collection intervals within alternative network option. They contend that collection of solid waste has become more complex because of high fuel and labor cost from the total amount of money spent for collection and that 50% to 70% of the transport and disposal of solid waste was spent on collection phase. These challenge researchers to search for more efficient solid waste management methods.

Nganda (2007) proposed Integer Linear Programming and Mixed Integer Linear Programming models to tackle the problem of solid waste management at Kampala city, Uganda. The study confirmed the models to be valid and robust. The performance of the models was studied using a Hypothetical case study and other smaller models like LIPS. The Mixed Integer Linear Programming model was found to be more precise in measuring waste flow amounts among various modes in the model and total daily costs incurred in the management of waste. However, in the Ugandan situation where the technology to measure the amount of weight per truck is nonexistent, the Integer Linear Programming model was found to be appropriate. The coefficients of the decision variables were replaced with the total cost per trip from waste collection points.

Martagan et al. (2006) conducted a study to look at the problem of transporting metal waste from 17factories to 5 intermediate containers and finally to a disposal centre in an organized region of automobile parts suppliers in Turkey. They applied the classic mixed-integer programming model for the two-stage supply chain to minimize the transportation cost. After building the model and reaching the optimal solution for minimizing the total cost, Java program was coded in order to visualize the solution. The visualization of the optimal provided several interesting insights that would not be easily discovered otherwise.

The study conducted by Badran and El-Haggar (2005) in Port Said in Egypt had interesting results. They used operational research methodologies which had not been applied in any Egyptian governorate to study the optimization of solid waste management system. Mixed integer programming was used to model the proposed system and the solution was performed using LIPS software V4.2. The results revealed that the best model would include 27 collection stations of 15-ton daily capacity and 2 collection stations of 10-ton daily capacity. The results also showed that transfer of waste between the collection stations and landfill site should not occur. The profit generated by the proposed model was equivalent to US $8418.23.

Detofeno (2010) characterized the waste collection problem in the city of Joinville, Brazil as arcs coverage and employed the Teitz and Bart heuristics methods to obtain p-medians, adopted Gillelt and Johnson algorithms and applied Chinese Postman Algorithm to minimize the distances in the collection of waste. The application of these mathematical algorithms and computer implementation for the optimization of routes in a problem of arcs covering minimized the total distance covered daily by approximately 8.6%.

Also, Carvalho (2001) in Detofeno (2010) asserts that cleaning services absorb between 7% and15% of the resources of a municipal budget, of which 50% are used extensively to collect and transport waste and hence its optimization can lead to a significant saving of public fund.

In a study conducted in Illorin in the Kwara State of Nigeria, Ajibade(2008), used Integer Linear Programming to minimize the collection cost of solid waste. He made use of TORA Optimization model to solve an integer linear programming problem and GIS method for the location of collection containers and the optimum distribution and utilization of the vehicles in the various zones. The research suggested the introduction of smaller containers to be strategically and spatially positioned for manual collection and the relocation of some of the vehicles.

Xiangyou (1997) developed an inexact nonlinear programming and applied to municipal solid waste management planning under uncertainty. The model was applied to the planning of waste management activities in the Hamilton-Wentworth Region of Canada. The results provided decision support for the region‟s waste management activities and useful information for examining the feasibility and applicability of the developed methodology.

Additionally, Lu et al. (2009) were concerned about the impact of greenhouse gas emission on climate change and therefore developed an inexact dynamic optimization model which is grounded upon conventional mixed-integer linear programming approaches for the cost minimization of solid waste disposal and the mitigation of greenhouse gas emission. The results indicated that desired waste-flow patterns with a minimized system cost and greenhouse emission amount could be obtained. The proposed model could be regarded as a useful tool forrealizing comprehensive municipal solid waste with regard to mitigating climate change impacts.

Apaydin and Gonullu (2007) carried out a study in Trazbon city located in the northeast side of Turkey using the shortest path model to optimize solid waste collection processes by aiming at minimum distance. To use route optimization process, data relating to spending, truck type, capacity, solid waste production, number of inhabitants and Global Positioning System receiver data for each route were collected and all the data were analyzed with each other. The results of the study had 4-59% success for distance and 14-56% for time and this eventually contributed a benefit of 24% in total cost.

Filipiak et al. (2009) conducted a study in the township of Millburn, New Jersey, USA to look at the curbside collection of municipal solid waste using operations research tools. The research work identified some network models, especially the Chinese Postman Problem algorithm as one of the approaches that could be used to calculate the truck routes and the optimum sequence of each of the vehicles needed to collect the generated waste.

Tavares et al. (2009) used Geographic Information Systems 3D route modeling software for waste collection and transportation to investigate the optimization of fuel consumption. The model was first applied to route waste collection vehicles in Praia, the capital of Cape Verde and secondly to route the transport of waste from different municipalities of Santiago Island to incineration plant. The results yielded a minimization of fuel consumption by 8% for Praia and 12% fuel reduction for Santiago Island.

Loumas et al. (2007) used Genetic Algorithm for the identification of optimal routes in the collection of municipal solid waste. They argue that approximately 60-80% of total money spent on managing solid waste is used for the collection stage and that a small percentage improvement in the collection operation can result in a significant saving in the overall cost. The municipal solid waste management system they proposed was based on geo-reference spatial database which was supported by a geographic information system that took into consideration static and dynamic data. These include the positions of waste bins, road network and related traffic, population density of the area under study, waste collection schedules, truck capacities and their characteristic Nasserzadeh et al. (1991) employed mathematical modeling – Lagragian model, mathematical model of the finite difference type to design a number of modifications and changes in operational conditions which have a major influence on the overall performance of the large municipal solid waste incinerator plant in Sheffield, England. Calculations made from the results from the modeling work indicated significant savings about 18% in the running and maintenance costs.

Ogwueleka (2009) proposed a heuristic method to generate a feasible solution to an extended Capacitated Arc Routing Problem on undirected network, inspired by the refuse collection problem in Onitsha, Nigeria. The heuristic procedure consists of route first, cluster second method. The adoption of the proposed heuristic in Onitsha resulted in reduction of the number of existing vehicles, a 22.88% saving in refuse collection and 16.31% reduction in vehicle distance traveled per day.

Alidi and Al-Faraj (1990) developed a dynamic mixed-integer linear programming model for managing and planning the municipal solid waste systems of Dammam Metropolitan area, Saudi Arabia. The model was tested by applying it to a hypothetical municipal solid waste system. The model developed is general in structure and can be used to address other problems such as the problem of hazardous waste management, petroleum products distribution and food processing industries.

A research conducted in a small part of Attica's prefecture a suburb of Athens by Karadimasetal. (2006) used an innovative model for the estimation of urban solid waste productivity using an intelligent system based on fuzzy logic. The study considered several parameters of waste production such as population density, maximum building density, commercial traffic, area and type of shops, road network and its relative information linked with allocation of waste bins. The results showed the effectiveness of the system in the estimation process of the optimal number of waste bins in each region. The proposed system can be used as a tool by planners and decision makers in the process to estimate precisely the solid waste generation.

Chang and Wang (1996) applied multi-objective mixed integer programming techniques for reasoning the potential conflict between environmental and economic goals and for evaluating sustainable strategies for waste management in a metropolitan region in the city of Kaohsiung in Taiwan. The results provided a set of total solutions for long term waste stream allocation, siting, resource recovery and tipping fees evaluation.

Minciardi et al. (2008) modeled the problem of sustainable municipal solid waste with a nonlinear, multi-objective formulation. They exemplified the application of their procedure by considering the interaction with two different decision makers that are assumed to be in charge of planning the MSW system in the city of Genova in Italy. The foregone arguments and assertions made by the cited authors give credence to the fact that operations research methods could be employed to study the problem of optimizing the transportation cost of solid waste in the Sunyani Municipality. The results of this study could furnish decision makers and planners in the Sunyani Municipal Assembly and Zoomlion Ghana Ltd. reliable information to make prudent and sound decisions.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

This part of the work reviews relevant fundamentals that will help us to come out with an appropriate linear model and the best way it will be solved.

3.1 LINEAR PROGRAMMING

Linear programming is a mathematical technique that deals with the optimization (maximizing or minimizing) of a linear function known as objective function subject to a set of linear equations or inequalities known as constraints. It is a mathematical technique which involves the allocation of scarce resources in an optimum manner, on the basis of a given criterion of optimality. The technique used here is linear because the decision variables in any given situation generate straight line when graphed. It is also programming because it involves the movement from one feasible solution to another until the best possible solution is attained.

A variable or decision variables usually represent things that can be adjusted or controlled. An objective function can be defined as a mathematical expression that combines the variables to express your goal and the constraints are expressions that combine variables to express limits on the possible solutions.

Generally we have constrained problems and unconstrained optimization.

3.1.1 UNCONSTRAINED OPTIMIZATION

Unconstrained optimization finds the highest point (or lowest point) on an objective function. For optimization to be required there must be more than one solution available, any point on the function is a solution, and because the single variable is real- valued function, there are an infinite number of solutions. Some kind of optimization process is then required in order to choose the very best solution from among those available. Best solution can mean the solution that provides the most profit or consumes the least of some limited resource.

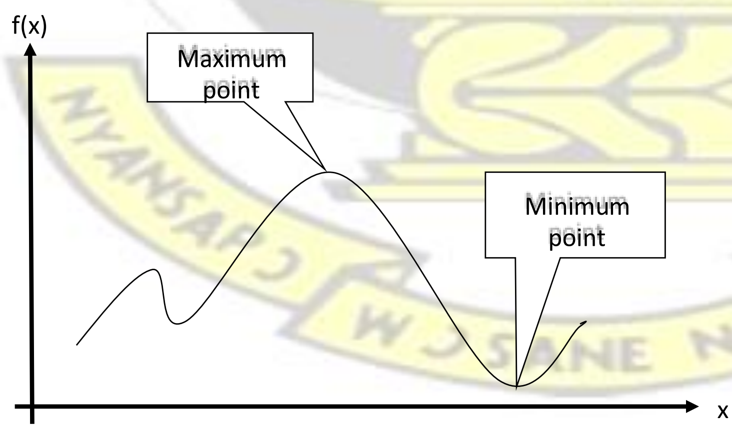

Figure 3.1: Simple unconstrained Optimization.

3.1.2 CONSTRAINED OPTIMIZATION

Constrained optimization is much harder than unconstrained optimization. In constrained optimization you still have to find the best point of the function, but have to respect various constrains while doing so. Unlike unconstrained problems the best solution may not occur at the top of the peak or at the bottom of the valley, the best solution might occur halfway up a peak when a constraint prohibits movement further up.

3.2 METHODS OF SOLVING LINEAR PROGRAMMING

Basically, there are two methods of solving a linear programming problem. These are

i. The graphical (Geometrical) Method ii.

The simplex (Algebraic) Method

3.2.1 THE GRAPHICAL METHOD

This method of solving Linear Programming Problem is applicable to problems involving only two decision variables. The following steps can be followed in solving Linear Programming Problem using the graphical approach;

STEP 1

Locate and identify or define the decisions variables in accordance with problem given.

STEP 2

Formulate the problem in a standard Linear Programming model. The standard Linear Programming model consists of the objective function which is either to maximize or minimize the constraints which are either inequality or an equation.

Generally, if the problem is of maximized type, the inequality used is the less than or equal to

Unless otherwise specified. On the other hand, minimization problem goes with greater than or equal to (□) unless otherwise stated. The non negativity constrain must also be stated

STEP 3

Consider each of the inequality as an equation and plot each equation on the graph as each will geometrically represents a straight line.

STEP 4

Mark the appropriate region.

If the inequality constraint corresponding to that line is less than or equal to, then the region below the line lying in the first quadrant (due to the non negativity of the decision variables) is shaded. For the inequality constraint corresponding with greater than or equal to, the region above the line in the first quadrant is shaded.

STEP 5

The points lying in common region will satisfy all the constraints simultaneously. The common region thus obtained is called the feasible region. Feasible Region also referred to as Feasibility Polygon is the region common to all constraints in any given problem. It contains all the feasible or possible solutions to the problem. Points in the feasible region do not contravene any of the constraints.

There may be a situation where a constraint may not touch the feasible region; such constraint is known as redundant constraint.

The edges or vertex of the feasible region is called extreme points or corner points and these are the points used to obtain the optimal solution. The optimal solution is the solution that maximizes or minimizes the objective function as the case may be.

STEP 6

To obtain the optimum solution theoretically, a line of equal profits or line of equal cost is drawn to represent the objective function after assigning a value say zero for the objective function so as to for a straight line passing through the origin. Stretch the objective function line till the extreme points of the feasible region.

In the maximization case this line will stop farthest from the origin or the last extreme point the line touches before it completely leaves the feasible region gives the optimal solution.

In the case of minimization, this line will stop nearest to the origin and passing through at least one corner of the feasible region or the first extreme point it touches before it enters the feasible region is the optimum solution.

In practice however, we determine the coordinates of the feasibility polygon and then substitute these coordinates into the objective function. If the problem is a maximum case, the one that gives the maximum value is the optimum solution; otherwise the one that gives the minimum value will give the optimal solution.

STEP 7

Draw the necessary conclusion.

3.2.2 TYPES OF GRAPHICAL SOLUTION

As the Linear Programming Model is based on the use of linear inequalities, there is the likelihood that, in solving the LP problem, there may be an instance when one may come across different forms of solutions.

3.2.2.1 A UNIQUE OPTIMAL SOLUTION

This is where the solution to the problem occurs at one and only one extreme point of the feasible region. That is, the combination that gives the highest contribution or profit or the minimum cost or time depending on the problem at hand.

3.2.2.2 INFINITELY MANY SOLUTIONS

This is where the optimal solution to the problem is obtained at more than one extreme point. This implies that there is no unique solution to the problem. When this happens, the assumption made is that the graph of the objective function is parallel to at least one of the constraints binding the feasible region. Thus two or more different points may give the same value. Thus all points on this line will give an optimal solution.

Figure 3.2 shows graphical representation of infinitely many solutions.

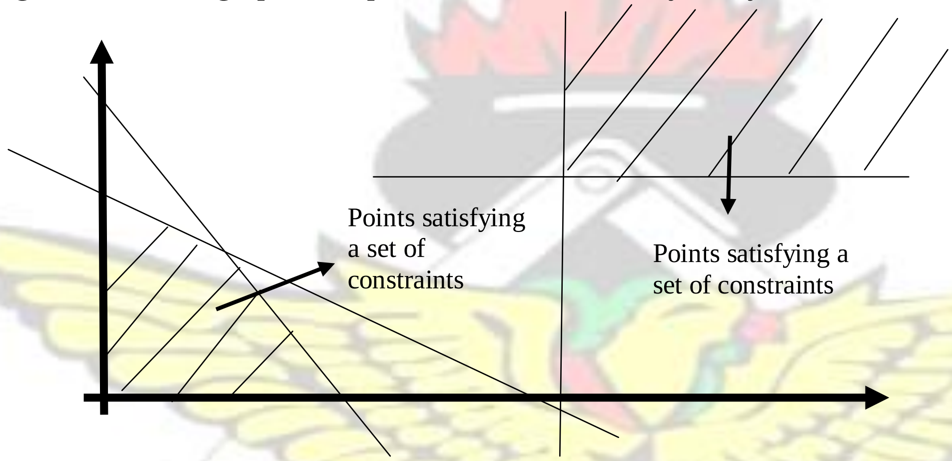

Figure 3.2 graphical representations of many solutions.

3.2.2.3 UNBOUNDED SOLUTION

This is a situation where the feasible region is not enclosed by constraints. In such situation, there may or may not be an optimal solution.

However, in all cases if the feasible region is unbounded, then there exists no maximum solution but rather a minimum solution.

To illustrate unbounded solution, let us consider a numerical example.

Maximize $Z=20x_1+10x_2$

Subject to $x_1 \geq 2$

$x_2 \leq 5$

$x_1 \geq 0$, $x2 \geq 0$

Graphing the feasible region as shown in Figure 3.3, it is part of the feasible region that is shown since the feasible region extends indefinitel

y in the

direction

of the x_1-

axis.

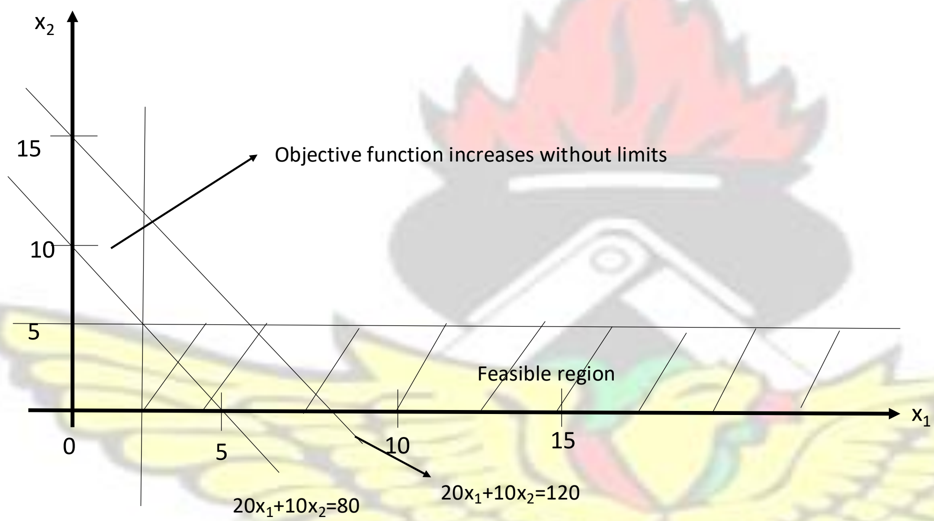

Figure 3.3: the feasible region of unbounded solution.

3.2.2.4 NO SOLUTION

There may also be a situation where there is no solution to the problem at hand. In such case, there will be no feasible region hence; the bounded area will be empty.

3.2.3 EXAMPLE OF A GRAPHICAL METHOD SOLUTION

A bicycle company produces two kinds of bicycles by hand. These were mountain bikes and street racers. The company wishes to determine the rates at which each type of bicycle should be produced in order to maximize profits on the sales of the bicycles on the assumption that all the bicycles produced will be sold.

Two mountain bikes and three racers are produced per day respectively and producing each type requires the same amount of time on the metal finishing machine, this machine can process at most a total of four bicycles a day of either type. The profit generated on the mountain bikes and the racers are GH¢15 and GH¢12 respectively. The above problem is formulate as follow

x_1 =Number of mountain bikes produce per day

x_2 =Number of racers produced per day

Maximize $Z = 15x_1 + 10x_2$ (in GH¢ per day)

$x_1 \leq 2$(constraint for mountain bikes per day)

$x_2 \leq 3$(constraint for racers per day)

$x_1 + x_2 \leq 4$ (production limit for metal finishing machine per day)

$x_1 \geq 0$and $x_2 \geq 0$

A graph of the constraints is plotted as follows

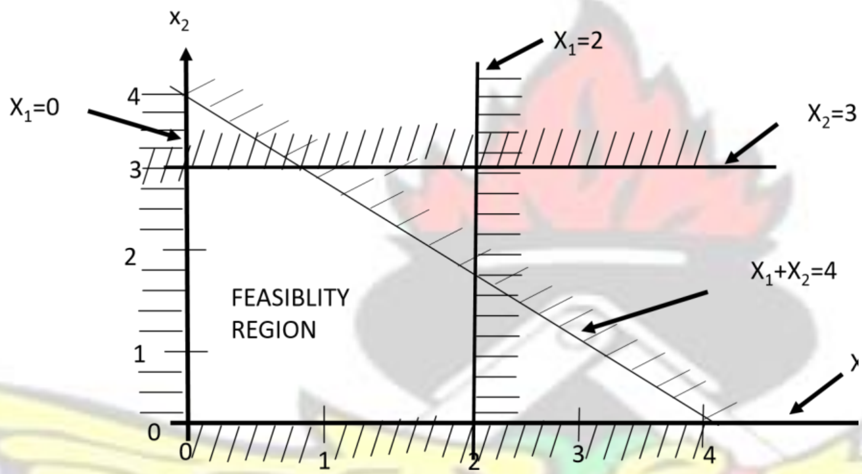

Figure 3.4: the feasible region of the bicycle company.

The limiting value of each of the constraint is shown as a line. Each constraint eliminates part of the plane. For example the vertical line labelled $x_1 = 2$ is the limiting value of the inequality $x_1 \leq 2$ All points to the right of the line violate the constraint (i.e. the infeasible region). The areas eliminated by the constraints are shaded. The unshaded area represents points that are not eliminated by any constraint, and is called feasible region.

To find a point in the feasible region gives the largest valued of the objective function. One way to do this is to randomly choose feasible points and to calculate the value of the objective function at those points, keeping the point that gives the best value of the objective.

Because there are an infinite number of points in the feasible region, this is not very effective because there is no guarantee that the best point will be found, or even that an objective function value that is close to the best possible value will be found.

An efficient search technique based on a couple of simple observations is developed. A line of equal profits is drawn to represent the objective function after assigning a value say zero for the objective function so as to get a straight line passing through the origin. The objective function line is stretched till the extreme points of the feasible region.

Figure 3.5 shows the constant profit line drawn and indicates the optimal solution

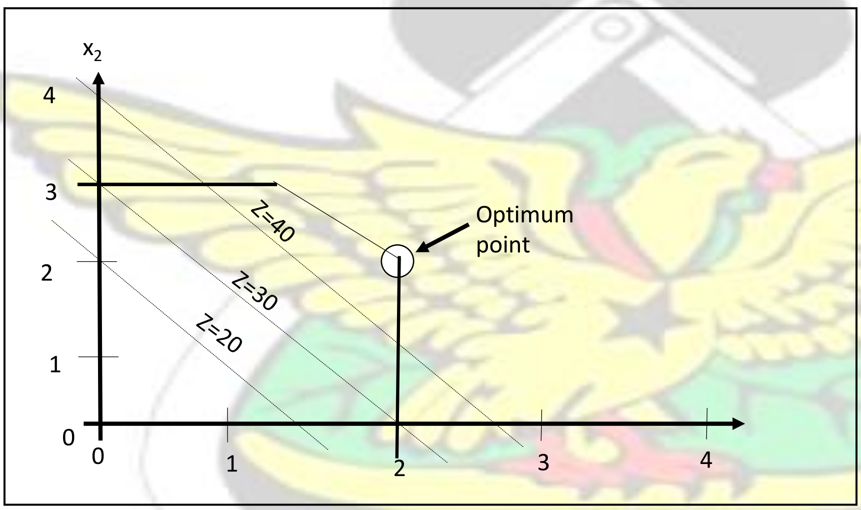

Figure 3.5: Constant Profit Lines for Bicycle Company.

As shown in Figure 3.5 the points are having the same value of Z (value of the objective function) form a line.

This is easy to understand if Z is replaced by specific value that can be plotted like $Z = 15x_1 + 10x_2$ becomes the line $15x_1 + 10x_2 = 20$ plotted in Figure 3.5.

Figure 3.5 also shows that all of the constant – profit lines are parallel. This is because all of the constant – profit line equations differing only by the selected value of Z. If the slope of any constant – profit line is to find, the constant will disappear, the slope of the entire constant – profit lines are the same.

Another observation is that the value of Z is higher for the constant – profit lines towards the upper right in Figure 3.5 and the last point is(2,2) with Z=50. This is the solution to the linear programming, the feasible point that has the best value of the objective function.

In some cases, the objective function has exactly the same slope as a face of the feasible region and the first contact is between the objective function and this face, as in Figure 3.6.

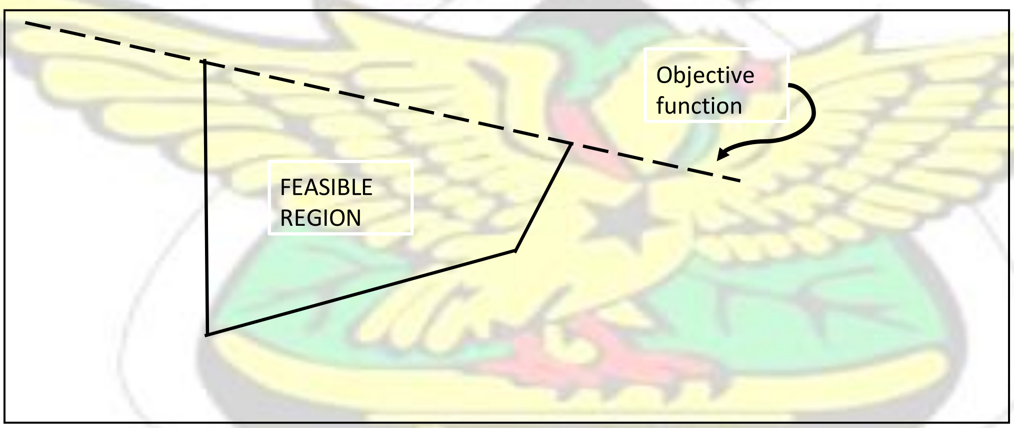

Figure 3.6: the slope of the objective function exactly matches the slope of the face of the feasible region.

This means that all of the points on that face have the same value of the objective function, and all are optimum, that is there are multiple optima. Though, if a face has first contact, then the corner points of the face also have first contact.

The important idea is that first contact between the objective function and the feasible region always involves at least one corner point. Hence, an optimum solution to the linear programming is always at a corner point or extreme point.

3.2.4 THE STANDARD FORM LINEAR PROGRAMMING

Linear programs can have objective functions that are to be maximized or minimized, constraints that are of three types (□,□,□) , and variables that have upper and lower bounds. An important subset of the possible LPs is the standard form LP. A standard form LP has these characteristics:

- The objective function must be maximized,
- All constraints are $\leq$type,
- All constraints right hand side are nonnegative,
- All variables are restricted to non-negativity.

A standard form LP is the simplest form of linear program and most significant property of a standard form LP is that the origin (all variables set to zero) is always a feasible

Corner point. This is because all standard form LPs have the kind of shape illustrated in Figure 3.7.

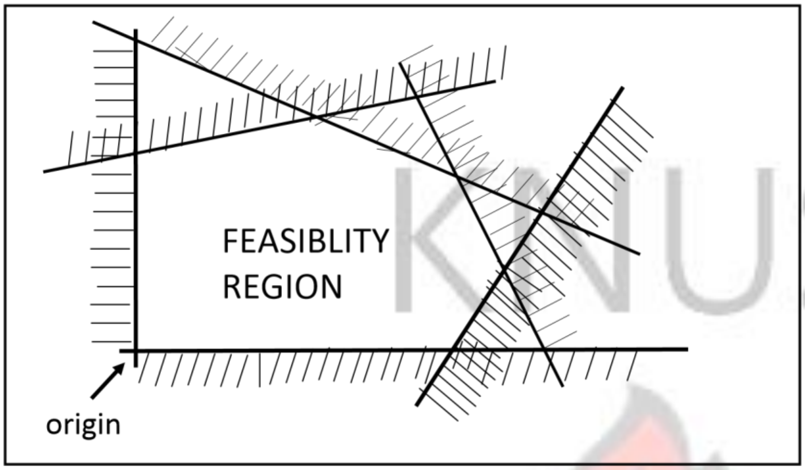

Figure 3.7: the origin is always a feasible extreme point or corner point in a standard form LP.

3.3 SLACK AND SURPLUS VARIABLES

A slack variable is associated with the (□) constraint and represents the amount by which the right-hand side of the constraint exceeds its left-hand side. For constraints of the type (□), the right-hand side normally represents the limited resource, whereas it‟s left-hand side represents the usage of this limited resource by the different activities (variables) of the model. In this regard, the slack variable represents the unused amount of the resource.

A surplus variable is identified with a (□) constraint and represents the excess of the left-hand side over the right-hand side. Constraints of the type (□) normally set minimum specification requirements, in which case the surplus variable would represent excess amount by which the minimum specification is satisfied.

To illustrate the slack and surplus variables, let us consider the following problem.

Minimize $Z = 3x_1 + 2x_2$

Subject to

$$x_1 + x_2 \le 4$$
$$2x_1 + x_2 \ge 2$$
$$x_1, x_2 \ge 0$$

The optimal solution to the problem above is (1,0). Then substituting the values $x_1 = 1$ and $x_2 = 0$ into the above constraints, we have

$1 + 0 = 1 \le 4$…………………… (1)

$2(1) + 0 = 2 \ge 2$………………. .(2)

From the constraints equations (1) and (2), the slack variable with respect to equation (1) is 41=3 and the surplus with respect to equation (2) is 2-2=0.

3.4 SIMPLEX METHOD

The simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. A simplex is an n-dimensional convex figure that has exactl n+1 extreme points. For example, a simplex in two dimensions is a triangle, and in three dimensions is a tetrahedron. The simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem.

The solution algorithm is an iterative procedure having fixed computational rules that leads to a solution to the problem in a finite number of steps (i.e., converges to an answer). The simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure.

The principle underlying the simplex method involves the use of the algorithm which is made up of two phase, where each phase involves a special sequence of number of elementary row operations known as pivoting. A pivot operation consist of finite number of m elementary row operations which replace a given system of linear equations by an equivalent system in which a specified decision variables appears in only one of the system and has a unit coefficient.

The algorithm has two phases, the first phase of the algorithm, is finding an initial basic feasible solution (BFS) to the original problem and the second phase, consists of finding an optimal solution to the problem which begins from the initial basic feasible solution.

3.4.1 FORMULATION OF THE PROBLEM

The objective function to be Maximized or Minimized is given by

$$Z = c_1x_1 + c_2x_2 + \ldots c_n x_n$$

Subject to the m constraints given by

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n} x_n \square b_1$$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2$

$.a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m$

The Non negativity constraints

$x_1 \ge 0, x_2 \ge 0 \ldots x_n \ge 0$

Where c_j, a_{ij} and b_j are all known constants and greater than zero and. $i = 1, 2, 3...., m$ and $j = 1, 2, 3... n$

3.4.2 ALGORITHM FOR SIMPLEX METHOD

A basic feasible solution to the system of m linear constraint equations and n variables is required as a starting point for the simplex method. From this starting point, the simplex successively generates better basic feasible solutions to the system of linear equations.

We proceed to develop a tabular approach for the simplex algorithm. The purpose of the tableau form is to provide an initial basic feasible solution that is required to get simplex method started. It must be noted that basic variables appear once and have coefficient of positive one.

3.4.2.1 SETTING UP INITIAL SIMPLEX TABLEAU

In developing a tabular approach we adopt these notations as used in the initial simplex tableau.

c_j □ objective function coefficients for variable j

b_i □ right – hand side coefficients (value) for constraint i

a_{ij} □ coefficients variable j in constraint i

c_B □objective function coefficients of the basic variables

C_j □Z_j □the net evaluation per unit of j □th variable

□A□matrix = the matrix (with m rows and n columns) of the coefficients of the variables in the constraint equations.

Table 3.1: General form – Initial Simplex Tableau

		Decision variables				Slack Variables						
C_j		C_j	C_j		C_j		0	0		0	Solution	(objective function coefficients)
C_n	Basic Variables	X_1	X_2		X_n		s_1	S_2		S_m		(Headings)
0	S_1											
…	S_2											
0	…											
	S_m											
	Z_j	Z_1	Z_2	…	Z_{mn}	…	Z_{11}	Z_{12}		Z_{1m}	Current value of objective function	
	c_j-Z_j	C_1-Z_1	C_2-Z_2	…	C_{mn}-Z_{mn}	…	C_{11}-Z_{11}	C_{22}-Z_{22}	…	C_{1n}-Z_{1n}		Reduced cost(Net contribution/unit)

Example 3.1

Maximize $Z = 6x_1 + 8x_2$

Subject to

$$5x_1 + 10x_2 \le 60$$

$$4x_1 + 4x_2 \le 40 \quad x_1, x_2 \ge 0$$

The above example can be restated in the standard form as follows:

Maximize $Z = 6x_1 + 8x_2 + 0s_1 + 0s_2$

Subject to

$$5x_1 + 10x_2 + s_1 = 60$$

$$4x_1 + 4x_2 + s_2 = 40$$

$$x_1, x_2, s_1, s_2 \ge 0$$

Transferring to the initial simplex tableau, we have table 3.1.1

Table 3.1.1: The Initial Tableau (Example 3.1)

				Pivot column			
		C_j	6	8	0	0	
	C_B	Basic variable	X_1	X_2	S_1	S_2	Solution
Pivot row	0	S_1	5	10	1	0	60
	0	S_2	4	4	0	1	40
		Z_j	0	0	0	0	0
		C_j-Z_j	6	8	0	0	

PIVOTING ELEMENT

The current basic variables always form an identity matrix within the simplex tableau. Note that the basic variables form a basis matrix that is an identity matrix (I).

From the initial tableau, the solution values can be read directly in the rightmost column. The values of Z_j row are calculated by multiplying the elements in the c_B column by the corresponding elements in the columns of the A matrix and summing them. Each value in the $(C_j - Z_j)$ row represents the net profit or net contribution that is added by producing one unit of product (if $C_j - Z_j$ is positive) or the net profit or net contribution that is subtracted by producing one unit of product (if $C_j - Z_j$ is negative).

Since all the Z_j values (j = 1,..,4) are equal to zero in the simplex tableau, we proceed to generate a new basic feasible solution (extreme point) that yields a better value for the objective function. This is accomplished by selecting one of current non – basic variables to be made basic and one of the current basic variables to be made non – basic in such a fashion that the new basic feasible solution yields an improved value for the objective function. This process is called changing the basis or iterating.

3.4.2.2 IMPROVING SOLUTION

The criteria for which a variable should enter or leave basis is summarized as follows:

Variable Entry Criteria: The variable entry criterion is based upon the values in the ($C_j - Z_j$) row of the simplex tableau. For a maximization problem, the variable selected for entry is the one having the largest (most positive) value of ($C_j - Z_j$). When all values of ($C_j - Z_j$) are zero or negative, the optimal solution has been obtained.

Variable Removing Criterion: The variable removal criterion is based upon the ratios formed as the values (b_i) in the "right-hand-side" column are divided by the corresponding values (a_{ij} coefficients) in the column for the variable selected to enter the basis. Ignore any a_{ij} values in the column that are zero or negative (i.e., do not compute the ratio). The variable chosen to be removed from the basis is the one having the smallest ratio. In the case of ties for the smallest ratio between two or more variables, break the tie arbitrarily (i.e., simply choose one of the variables for removal). This variable removal criterion remains the same for both maximization and minimization problems.

Applying the variable entry and removal criteria to our present maximization problem x_2 is chosen as the variable to enter basis and s_1 leaves the basis. Thus the current basic variable s_1 is replaced by non – basic variable (x_2).

Now that we have determined the new elements in the basis and that not in the basis we proceed to determine the new solution through pivoting x_2 into basis and pivoting s_1 out of basis. The pivoting process involves performing elementary row operations on the rows of the simplex tableau to solve the system of constrain equations in terms of the new set of basic variables. We initiate the pivoting processing by identifying the variable, x_2 to be entered into the basis by denoting its corresponding column as the pivot column in Table 3.1.1. Similarly, we identify the variable, s_1 to be removed from the basis by specifying the pivot row which is the row it corresponds as in Table 3.1.1. The element at the intersection of the pivot column and pivot is referred to as pivot element. The two – step pivoting process proceeds as follows:

Step I: Convert the pivot element to one by dividing all values in the pivot row by pivot element

(10). This new row is entered in the next tableau, Table 3.1.2.

Step II: The objective of the second step is to obtain zeros in all the elements of the pivot column, except, of course for the pivot element itself. This is done by elementary row operations involving adding or subtracting the appropriate multiple of the new pivot row or from the other rows. Performing these calculations, the results are as presented in Table 3.1.2

Table 3.1.2: Second Simplex Tableau (Example 3.1)

				Pivot column			
		C_j	6	8	0	0	
	C_B	Basic variable	X_1	X_2	S_1	S_2	Solution
Pivot row	8	X_2	$\frac{1}{2}$	1	$\frac{1}{10}$	0	6
	0	S_2	2	0	$-\frac{2}{5}$	1	16
		Z_j	4	8	$\frac{4}{3}$	0	48
		C_j-Z_j	2	0	$-\frac{4}{3}$	0	

Pivot element

The second simplex tableau can be constructed as shown in Table 3.1.2. Notice that the columns that correspond to the current basic variables x_2 (real variable) and s_2 (slack variable) form a basis (B) which is identity matrix. The values in the Z_j row and $(C_j - Z_j)$ row are computed in the same way as in the initial simplex tableau. Observe that $C_j - Z_j = 2$ (> 0) and so the optimal solution has not been obtained and continue the iteration since we are maximizing.

We continue the process by determining the variables leaving the basis and which is entering the basis using the variable entry and removing criteria stated earlier. The outcome is summarized in Table 3.1.3.

Table 3.1.3: Third Simplex Tableau (Optimal Solution)

	C_j	6	8	0	0	
C_B	Basic variable	X_1	X_2	S_1	S_2	Solution
8	X_2	0	1	$\frac{1}{3}$	$-\frac{1}{4}$	2
6	X_1	1	0	$-\frac{1}{5}$	$\frac{1}{2}$	8
	Z_j	6	8	$\frac{2}{5}$	1	64
	C_j-Z_j	0	0	$-\frac{2}{5}$	-1	

Observe that in this third simplex tableau all C_j □Z_j values are either zero or negative. We have thus obtained the optimal solution with $x_1 = 8$, $x_2 = 2$, $s_1 = 0$, $s_2 = 0$ and the optimal value of $Z = 64$.

The optimal solution suggests that the profit will be maximized when eight products of x_1 and two products of x_2 are produced.

3.4.3 SIMPLEX METHOD WITH MIXED CONSTRAINTS

Some Linear Programming problem may consists of a mixture of ≤, =, and ≥ sign in the constraints and wish to maximized or minimized the objective function. Such mixture of signs in the constraints is referred to as mixed constraints.

The following procedure is followed when dealing problem with mixed constraints

STEP1: Ensuring that the objective function is to be maximized. If it is to be minimized then we convert it into a problem of maximization by

Max W = -Min (-Z)

STEP2: For each constraints involving „greater or equal to‟ we convert to „less than or equal to‟ that is, constraints of the form

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$$

Is multiplied by negative one to obtain

$$-a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n \leq -b_2$$

STEP 3: Replace constraints

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

by

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$ and

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$$

Where the latter is written as

$$-a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n \leq -b_2$$

STEP 4: Form the initial simplex tableau

STEP 5: If there exist no negative entry appearing on the right hand side column of the initial tableau, proceed to obtain the optimum basic feasible solution

STEP 6: If there exist a negative entry on the Right Hand Side column of the initial tableau,

i. identify the most negative at the Right Hand Side , this row is the pivot row

ii. Select the most negative entry in the pivoting row to the left of the Right Hand Side. This entry is the pivot element

iii. Reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation

STEP 7: Repeat step 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exists on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.

3.5 DUALITY

Corresponding to any given linear programming problem called the primal problem, is another linear programming problem called the Dual Problem. Since a given linear programming problem can be stated in several forms (standard form, canonical form, etc), it follows that the forms of the dual problem will depend on the form of the primal problem. A fundamental of the primal dual-relationship is that the optimal solution to either the primal or the dual problem also provides optimal solution to the other. A maximization problems with all the less-than or equal to constraint and the non-negative requirement for the decision variables is said to be in canonical form as in example 3.3 used below. If the dual problem has optimal solution, then the primal also has an optimal solution and vice versa. The values of the optimal solution to the dual and primal are equal

These are rules for converting the primal problem in any form into its dual

Table 3.2: Converting of primal problem to dual form

PRIMAL PROBLEM	DUAL PROBLEM
Maximization	Minimization
Coefficient of objective function	Right hand sides of constraint
Coefficient of i^{th} constraint	Coefficient of i^{th} variable
i^{th} constraint is an inequality of the form $\leq$	i^{th} variable satisfies ≥ 0
i^{th} constraint is an equality	i^{th} variable is unrestricted
i^{th} variable is unrestricted	i^{th} constraint is an equality
i^{th} variable satisfies ≥ 0	i^{th} constraint is an inequality of the type $\geq$
Number of variables	Number of Constraints
Number of Constraint	Number of variables

Note: Tableau can be read both ways

Example 3.2

(a) Find the dual of the LP problem.

Maximize $z \square 3x_1 \square x_2 \square 4x_3$

Subject to

$3x_1 \square 3x_2 \square x_3 \square 18$

$2x_1 \square 2x_2 \square 4x_3 \square 12$

$x_1, x_3 \square 0, x_2$ *unrestricted*

The dual is given by

Minimize $u \square 18w_1 \square 12w_2$

Subject to

$3w_1 \square 2w_2 \square 3$

$3w_1 \square 2w_2 \square 1$

$w_1 \square 4w_2 \square 4$

$w_1 \square 0, w_2$ *unrestricted*

3.6 UNCONSTRAINT VARIABLES

In many practical situations, we may want to allow one or more of the decision variables, the x_j to be unconstrained in sign, that is either positive or negative.

We have already noted that the use of the simplex method requires that all the decision variables must be non negative at each iteration. However, by some simple algebraic manipulations, we can convert a linear programming problem involving variables that are unconstrained in sign into an equivalent problem having only non negative variables. This is accomplished by expressing each of the unconstrained variables as the difference of two non negative variables.

Assume the variable x_1 to be unconstrained in sign.

Define two new variables $x'_j \geq 0$ and $x''_j \geq 0$

Let. $x = x'_1 - x_1''$

Thus, when $x'_1 \geq 0$ and $x''_1 \geq 0$

$x'_1 \geq x_1''$ then $x_1 \geq 0$, and the desired result has been achieved. The unconstrained variable must be replaced by the two new variables wherever it appears in the linear programming model that is in both the objective function and the constraint set.

3.7 DEGENERACY

A linear program is said to be degenerate if one or more basic variables have a value zero. This occurs whenever there is a tie in the minimum ratio prior to reaching the optimal solution. This may result in cycling, that is the procedure could possibly alternate between the same set of non optimal basic feasible solutions and never reach the optimal solution.

In order to overcome this problem, the following steps may be used to break the tie between the key row tie

1. Select the rows where the ties are found for determining the key row.
2. Find the coefficient of the slack variable and divide each coefficient by the coefficients in the key column in order to break the tie. If the ratios at this stage do not break the tie, find the similar ratios for the coefficient of the decision variables.
3. Compare the resulting ratio column by column
4. Select the row which has the smallest ratio and this now becomes the key row.

3.8 TYPES OF SIMPLEX METHOD SOLUTIONS

The simplex method will always terminate in a finite number of steps with an indication that a unique optimal solution has been obtained or that one of three special cases has occurred. These special cases are:

1. Alternative optimal solutions
2. Unbounded solutions

3. Infeasible solutions

3.8.1 ALTERNATIVE OPTIMAL SOLUTIONS

The simplex method provides a clear indication of the presence of alternative or multiple, optimal solutions upon its termination. These alternative optimal solutions can be recognized by considering the ($C_j - Z_j$) row. Assume that we are maximizing and remember that when all ($C_j - Z_j$) values are all negative, we know that an optimal solution has been obtained. Now, the presence of an alternative optimal solution will be indicated by the fact that for some variable not in the basis, the corresponding ($C_j - Z_j$) value will equal zero.

Thus, this variable can be entered into the basis, the appropriate variable can be removed from the basis, and the value of the objective function will not change. In this manner, the various alternative optimal solutions can be determined.

3.8.2 UNBOUNDED SOLUTIONS

In the case of an unbounded solution, the simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity (+ ∞). Specifically, for a maximization problem we will encounter a simplex tableau having a non basic variable whose ($C_j - Z_j$) row value is strictly greater than zero.

a^{ij} elements in its column will be zero or negative value

And for this same variable all of the

(i.e. every coefficient in the pivot column will be either negative or zero). Thus, in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators. Negative numbers in the denominators cannot be considered since this will result in the introduction of a basic variable at a negative level (i.e. an infeasible solution would result). Zeros in the denominator will produce a ratio having an undefined value and would indicate that the entering basic variable should be increased indefinitely (i.e. infinitely) without any of the current basic variables being driven from the basis.

Therefore, if we have an unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value.

Generally, arriving at an unbounded solution indicates that the problem was originally misformulated within the constraint set and needs reformulation.

3.8.3 INFEASIBLE SOLUTION

An indication that no feasible solution is possible will be given by the fact that at least one of the artificial variables, which should be driven to zero by the simplex method will be present as a positive basic variable in the solution that appears to be optimal. For example, assume we are solving a maximization problem in which artificial variables are required. Then, at some iteration we achieve a solution in which all the $(C_j - Z_j)$ values are zero or negative, but which has one or more artificial variables as positive basic variables.

When an infeasible solution is indicated the management science analyst should carefully reconsider the construction of the model, because the model is either improperly formulated or two or more of the constraints are incompatible. Reformulation of the model is mandatory for cases in which the no feasible solution condition is indicated.

3.9 SENSITIVITY ANALYSIS

Suppose that you have just completed a linear programming solution which has a major impact. How much will the result change if your basic data is slightly wrong? Will that have a minor impact on your result? Will it give a completely different outcome, or change the outcome only slightly?

These are the kind of questions addressed by sensitivity analysis. It allows us to observe the effect of changes in the parameters in the LP problem on the optimal solution. It is also useful when the values of the problem parameters are not known. Formally, the question is this; is my optimum solution sensitive to a small change in one of the original problem coefficient. This sort of examination of impact of the input data on output results is very crucial. The procedure and algorithm of mathematical programming are important but the problems that really appear in practice are usually are associated with data: getting it all, and getting it accurate. What is required in sensitivity analysis is which data has significant impact on your results.

There are several ways to approach sensitivity analysis. If your model is small enough to solve quite quickly, you can simply change the initial data and solve the model again to see what results you get. At the extreme, if your model is very large and takes a long time to solve, you can apply formal methods of classical sensitivity analysis. The classical methods rely on the relationships between the initial tableau and any later tableau to quickly update the optimum solution when changes are made to the coefficient of the original tableau. Finally on the state of sensitivity analysis: you are typically limited to analyzing the impact of changing only one coefficient at a time. There are few accepted techniques for changing several coefficients at once.

3.9.1 CHANGE OBJECTIVE FUNCTION COEFFICIENT

A change of the coefficients of the objective function does not affect the values of the variables directly. So as we change the values of the objective function coefficients we should ensure that the optimality conditions are not violated. The range of values over which an objective function coefficient may vary without any change in the optimal solution is known as the range by those coefficient values that maintain ($C_j - Z_j \leq 0$) The computation for the range of optimality can be categorized into two; that for the basic variable and non-basic variable.

Example 3.3

Maximize $z = 50x_1 + 40x_2$

Subject to

$$3x_1 + 5x_2 \le 150$$
$$x_2 \le 20$$
$$8x_1 + 5x_2 \le 300$$
$$x_1, x_2 \ge 0$$

Table 3.3: Final Simplex Tableau (Example 3.3)

		X_1	X_2	S_1	S_2		
Basic variable	C_j	50	40	0	0	0	Solution
X_2	40	0	1	$\frac{8}{25}$	0	$-\frac{3}{25}$	12
S_2	0	0	0	$-\frac{8}{25}$	1	$-\frac{3}{25}$	8
X_1	50	1	0	$-\frac{5}{25}$ -	0	$\frac{5}{25}$	30

	Z_j	50	40	$\frac{14}{5}$	0	$\frac{26}{5}$	1980
	C_j-Z_j	0	0	$-\frac{14}{5}$	0	$-\frac{26}{5}$	

For any non-basic variable, the range of optimality will be $(-\infty, c_j - Z_j)$ in the maximization problem.

For the basic variable x_1 and x_2 the lower and upper limits of the coefficient within which the different solutions remains optimal can be computed by finding the ratio of $(C_j - Z_j)$ to x_j values in the final simplex tableau. The smallest positive value for the ratio gives the extent to which it can be increased and the negative value with the smallest absolute value gives the extent to which it can be decreased. This is illustrated below.

Table 3.3.1 The ratio of Cj-Zj.

X_1	1	0	$-\frac{5}{25}$	0	$\frac{5}{25}$
C_j-Z_j	0	0	$-\frac{14}{5}$	0	$-\frac{26}{5}$
$(C_j$-$Z_j)/x_1$			14		-26

The range of optimality for c_1 is $24 \leq c_1 \leq 64$

3.9.2 CHANGING A RIGHT HAND SIDE CONSTRAINT.

Right hand side constraints normally represent a limitation on the resources, and are likely to change in practice as business conditions change. An overall procedure for examining proposed changes to the right hand side of constraints is to check whether the proposed changes is within the allowable range of changes for the right hand side of the constraint. So an optimal tableau will continue satisfying the optimal conditions regardless of the altered values of the right hand side coefficients. The change in value of the objective function per unit increase in the constraints right hand side value is known as shadow price. When Simplex methods is used to solve LP problem, the values of the shadow price are found in the Z_j of the final simplex tableau

Let us again consider the example 3.3

Maximize $z = 50x_1 + 40x_2$

Subject to

$3x_1 + 5x_2 \le 150$

$x_2 \le 20$

$8x_1 + 5x_2 \le 300$

$x_1, x_2 \ge 0$

Basic	C_B	50	40	0	0	0	Solution
X_2	40	0	1	$\frac{8}{25}$	0	$-\frac{3}{25}$	12
S_2	0	0	0	$-\frac{8}{25}$	1	$\frac{3}{25}$	8
X_1	50	1	0	$-\frac{5}{25}$	0	$\frac{5}{25}$	30

	Z_j	50	40	$\frac{14}{5}$	0	$\frac{26}{5}$	1980
	C_j-Z_j	0	0	$-\frac{14}{5}$	0	$-\frac{26}{5}$	

The shadow prices with respect to each of the constraints are the z_j values of the variables s_1,s_2 and s_3 respectively: since they represent the unused resources. s_2 has an optimal value of 8 which means that the second constraint has an excess and so additional resources are unnecessary hence shadow price is 0.

The constraint with $s_1 = 0$ the user is to pay the right hand side up to $\frac{14}{5}$. This means that in the problem above it would not allow any slack to occur in the first constraint unless it is worth more than $\frac{14}{5}$.

For maximization problem with a greater than or equal to constraint, the value of the shadow price will be less or equal to zero because one unit increase on the right hand side cannot help. It makes it more difficult to satisfy the constraint. As a result, the optimal value of the objective function will decrease when the right hand value of the greater than constraint is increased. The shadow price gives a negative number because of the decrease. The shadow price is given by the negative of z_j values of the corresponding artificial variable in the final Simplex tableau.

The range of feasibility for a less than or equal to constraint is given by

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad \begin{pmatrix} a_{1j} \\ a_{2j} \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \vdots \\ b_m \end{pmatrix} + \begin{pmatrix} \vdots \\ \vdots \\ a_{mj} \end{pmatrix} \ge \begin{pmatrix} \vdots \\ \vdots \\ 0 \end{pmatrix}$$

b = current optimal solution

a_{ij} = the column corresponding to the slack variable associated with the constraint.

The range can be established by the maximum of the lower limits and the minimum of the upper limits.

From the above final simplex tableau suppose that the range of constraints 1 is to be determined then we have

$$\begin{pmatrix} 12 \\ 8 \\ 38 \end{pmatrix} + \begin{pmatrix} 8/25 \\ -8/25 \\ -5/25 \end{pmatrix} \Delta b_1 = \begin{pmatrix} 12 + 8/25\,\Delta b_1 \\ 8 - 8/25\,\Delta b_1 \\ 30 - 5/25\,\Delta b_1 \end{pmatrix} \ge \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$12 + 8/25\,\Delta b_1 \ge 0 \Rightarrow \Delta b_1 \ge -37.5$

$8 - 8/25\,\Delta b_1 \ge 0 \Rightarrow \Delta b_1 \le 25$

$30 - 5/25\,\Delta b_1 \ge 0 \Rightarrow \Delta b_1 \le 150$

Applying the conditions we will have $-37.5 \le \Delta b_1 \le 25$ the initial amount on the right hand side is 150 it therefore follows that

$-37.5+150 \le b_1 \le 25+150$

$112.5 \le b_1 \le 175$ Which is the range of optimality for b_1

3.9.3 Simultaneous Changes

If two or more objective functions coefficient or resources (right-hand-side) are to be changed then for each coefficient or constraint, compute the percentage or allowable increase or decrease represented by the change. If the sum of the percentage for all changes does not exceed 100%, then it satisfies 100% rule and the changes are allowable.

Other methods of solving Linear Programming Problems have been discussed below.

3.10 The Two – Phase Simplex Method

Given an LP problem

Maximize $Z = C^T X$

Subject to

$AX \le b$

$X \ge 0$

In Linear Programming if the initial Basic Feasible Solution (BFS) exists then the normal **Simplex Method** can be used. In cases where such an obvious initial BFS does not exist, the **Two-Phase Simplex Method** can be used.

Steps:

- Convert each inequality constraint to the standard form

- If constraint *i* is a ≥ or = constraint, add an artificial variable a_i. Also add the sign restriction $a_i \geq 0$

- For now, ignore the original LP‟s objective function. Instead solve an LP whose objective function is **min *w*** (sum of all the artificial variables). (**Phase I LP).**

- The act of solving the Phase I LP will force the artificial variables to be zero.

- Because each ai ≥ 0, solving the Phase I LP will result in one of the following ***three cases***

  Case 1. The optimal value of ***w*** is greater than zero. The original LP has no feasible solution.

  Case 2. The optimal value of **w** is equal to zero, and no artificial variables are in the optimal Phase I basis. Drop all columns in the optimal Phase I tableau that correspond to the artificial variables, combine the original objective function with the constraints from the optimal Phase I tableau (Phase II LP). The optimal solution to the Phase II LP is the optimal solution to the original LP.

  Case 3. The optimal value of w is equal to zero and at least one artificial variable is in the optimal Phase I basis. drop from the optimal Phase I tableau all nonbasic artificial variables and any variable from the original problem that has a negative coefficient in row 0 of the optimal Phase I tableau combine the original objective function with the constraints from the optimal Phase I tableau (Phase II LP). The optimal solution to the Phase II LP is the optimal solution to the original LP.

Example

Consider the problem

$\min\ z = 4x_1 \square x_2 \square x_3$

Subject to

$2x_1 \square\ x_2 \square\ 2x_3 \square\ 4$

$3x_1 \square\ 3x_2 \square\ x_3 \square\ 3$ x_1, x_2

$, x_3 \square\ 0$

Solution

There is no basic feasible solution so the two-phase method is used.

Phase I : The auxiliary form is written below:

$\min\ w = y_1 \square\ y_2$

subject to

$2x_1 \square\ x_2 \square\ 2x_3 \square\ y_1 \square\ 4$

$3x_1 \square\ 3x_2 \square\ x_3 \square\ y_2 \square\ 3$

$x_1, x_2, x_3, y_1, y_2 \square\ 0$

Table 3.4 The starting tableau (in non standard form)

BASIS	x_1	x_2	x_3	y_1	y_2	RHS
y_1	2	1	2	1	0	4
y_2	3	3	1	0	1	3
(-z)	4	1	1	0	0	0
(-w)	0	0	0	1	1	0

The tableau is converted to standard form by zeroing out the coefficients of the basic variables in the *w*-row:

Table 3.4.1 second tableau converting into standard forms

BASIS	x_1	x_2	x_3	y_1	y_2	RHS	Min Ratio
y_1	2	1	2	1	0	4	2
y_2	3	3	1	0	1	3	1
(-z)	4	1	1	0	0	0	
(-w)	-5	-4	-3	0	0	-7	

The coefficient of x_1 in the *w*-row is the most negative so it is introduced in to the basis. The minimum ratio test is **min{4/2, 3/3}=1**so y_2 leaves the basis**.** The pivot element is circled. Using $R_{12} = (-2/3)R_2+R_1$, $R_{22} = (1/3)R_2$, $R_{32} = (-4/3)R_2+R_3$, $R_{42} = (5/3)R_2+R_4$ the table becomes:

Table 3.4.2 Third tableau of two-phase simplex

BASIS	x_1	x_2	x_3	y_1	y_2	RHS	MinRatio
y_1	0	-1	1.33	1	-0.67	2	1.5
x_1	1	1	0.33	0	0.33	1	3
(-z)	0	-3	-0.33	0	-1.33	-4	
(-w)	0	1	1.33	0	1.67	-2	

The coefficient of x_3 in the *w*-row is the most negative so it is introduced in to the basis.

The minimum ratio test is **min{2/(4/3), 1/(1/3}=1.5** so $y_1$leaves the basis**.** The pivot element is circled. Using $R_{13} = (3/4)R_{12}$, $R_{23} = (-1/4)R_{12}+R_{22}$, $R_{33} = (1/4)R_{12}+R_{32}$, $R_{43} = R_{12}+R_{42}$, the table becomes:

Table 3.4.3 Fourth tableau of two-phase simplex

BASIS	x_1	x_2	x_3	y_1	y_2	RHS
x_3	0	-0.75	1	0.75	-0.5	1.5
x_1	1	1.25	0	-0.25	0.5	0.5
(-z)	0	-3.25	0	0.25	-1.5	-3.5
(-w)	0	0	0	1	1	0

All the coefficients in the *w*-row are non negative, *w* =0, and there are no artificial variables in the basis, so the phase I is completed. Phase II begins with the tableau shown below:

Table 3.4.4 Fifth tableau of two-phase simplex

BASIS	x_1	x_2	x_3	RHS	Min Ratio
x_3	0	-0.75	1	1.5	

x_1	1	**1.25**	0	0.5	0.4
(-z)	0	-3.25	0	-3.5	

The coefficient of x_2 in the z-row is the most negative so it is introduced in to the basis.

The minimum ratio test is **min{0.5/1.25} = 0.4** so $x_1$leaves the basis**.** The pivot element is bolded.

Using $R_{12} = 0.6R_2+R_1$, $R_{22} = 0.8R_2$, $R_{32} = 2.6R_2+R_3$, the table becomes:

Table 3.4.5 Final tableau of two-phase simplex

BASIS	**x_1**	**x_2**	**x_3**	**RHS**
x_3	0.6	0	1	1.8
x_2	0.8	1	0	0.4
(-z)	2.6	0	0	-2.2

All the coefficients in the z-row are non negative so the Phase II is also completed. The optimum solution is $\boldsymbol{x = (0, 0.4, 1.8)}$ and $\boldsymbol{z = 2.2}$

3.10.1 The Dual Simplex Method

In the tableau implementation of the primal simplex algorithm, the right-hand-side column is always nonnegative so the basic solution is feasible at every iteration. The basis for the tableau is *primal feasible* if all elements of the right-hand side are nonnegative. Alternatively, when some of the elements are negative, we say that the basis is *primal infeasible*.

For the primal simplex algorithm, some elements in row 0 will be negative until the final iteration when the optimality conditions are satisfied. In the event that all elements of row 0 are nonnegative, we say that the associated basis is *dual feasible*.

The Algorithm

Given a standard LP problem

Maximize $Z = C^T X$

Subject to

$$AX \leq b$$

$$X \geq 0$$

The following are the steps involved in the algorithm:

***Step* 1 (*Initialization*)**

Start with a dual feasible basis and let $k = 1$. Create a tableau for this basis in the simplex form. If the right-hand side entries are all nonnegative, the solution is primal feasible, so stop with the optimal solution.

Step* 2 *(Iteration k)

- *Select the leaving variable*. Find a row, call it ***r***, with a negative right-hand-side constant; i.e.,$b_r < 0$.Let row ***r***be the pivot rowand let the leaving variable be $x_{B(r)}$. A common rule for choosing r is to select the most negative RHS value; i.e.,$b_r = \min\{b_i : i = 1, \ldots, m\}$.
- *Determine the entering variable*. For each negative coefficient in the pivot row, compute the negative of the ratio between the reduced cost in row 0 and the structural coefficient

in row ***r***. If there is no negative coefficient, $a_{rj} < 0$, stop; there is no feasible solution. Let the column with the minimum ratio, designated by the index ***s***, be the pivot column; let x_s is the entering variable. The pivot column is determined by the following ratio test.

$$\frac{\bar{c}_s}{a_{rs}} = \min\left\{ \frac{\bar{c}_j}{a_{rj}} : a_{rj} < 0,\ j = 1,\ldots,n \right\}$$

❖ *Change the basis*. Replace $x_{B(r)}$ by x_s in the basis. Create a new tableau by performing the following operations (these are the same as for the primal simplex algorithm).Let $\mathbf{a}_i$ be the vector of the i^{th} row of the current tableau, and let $\mathbf{a}_{i\,new}$ be the i^{th} row in the new tableau.

Let b_i be the RHS for row i in the current tableau, and let $b_{i\,new}$ be the RHS of the new tableau. Let a_{is} be the element in the i^{th} row of the pivot column***s***.

The pivot row in the new tableau is:

$$a_{rnew} = \frac{a_r}{a_{rs}} \quad \text{and} \quad b_{rnew} = \frac{b_r}{a_{rs}}$$

The other rows in the new tableau are:

$$a_{inew} = (-a_{is} \times a_{rnew}) + a_i \ \text{and}\ b_{inew} = (-a_{is} \times b_{rnew}) + b_i \quad \text{for } i = 0,1,\ldots,m,\ i \neq r$$

***Step* 3 (*Feasibility test*)**

If all entries on the right-hand side are nonnegative the solution is primal feasible, so stop with the optimal solution. Otherwise, put $k \leftarrow k+1$ and return to Step 2.

Example

Consider the standard LP problem below to which slack variables z_1 and z_2 are already added:

minimize $w = 2x_1 + 3x_2 + 4x_3$

subject to

$x_1 + 2x_2 + x_3 - z_1 = 3$

$2x_1 - x_2 + 3x_3 - z_2 = 4$

$x_1, x_2, x_3, z_1, z_2 \geq 0$

Solution

The primal simplex algorithm would have to use two phases, since the initial solution where $z_1 = -3$ and $z_2 = -4$ is not feasible. On the other hand, $c \geq 0$, so the dual solution with $\pi_1 = \pi_2 = 0$ satisfies $c^T - \pi^T A \geq 0$ and is therefore feasible. The following tableau is obtained:

Table 3.5 Initial tableau for primal of two-phase simplex method

	-1	-2	-1	1	0	-3 -4
z_1 z_2	-2	1	3	0	1	

w	2	3	4	0	0	0

In the dual simplex algorithm the pivot is selected by picking a row i such that $a_{i0} < 0$ and a column $j \in \{j : a_{ij} < 0\}$ that minimizes $-a_{0j}/a_{ij}$. Pivoting then works just like in the primal algorithm.

Pivoting on a_{21}, the tableau becomes:

Table 3.5.1 Second tableau for primal of two-phase simplex method

z_1	0	$-\frac{5}{2}$	$-\frac{5}{2}$	1	$-\frac{1}{2}$	-1 2
x_1	1	$-\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{2}$	
w	0	4	7	0	1	-4

Pivoting on a_{12} changes the tableau to reach optimality:

Table 3.5.2 Final tableau for primal of two-phase simplex method

x_2	0	1	1	$-\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
x_1	1	0	-1	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{11}{5}$
w	0	0	3	$\frac{8}{5}$	$\frac{1}{5}$	$\frac{28}{5}$

Hence the optimal solution is $\{x_1 = \frac{11}{5}, x_2 = \frac{2}{5}, x_3 = 0, w = \frac{28}{5}\}$. Also $\lambda_1 = \frac{8}{5}$ and $\lambda_2 = \frac{1}{5}$

3.11 Integer Programming

There are several ways of solving pure integer programming problems. The best known ones are Cutting Planes method and Branch and bound method. The enumeration of all possible integer solutions to a linear programming problem would not be a practicable approach to solving the problem because of the tremendously high number of solutions (Amponsah, 2007)

For example, a problem with 30 (0 – 1) variables has 230 i.e. over one billion possible variable solutions. The number of solutions grows exponentially with the number of variables. The Branch and Bound is a method of implicit enumeration. This means that only a small fraction of the solutions are explicitly examined.

Integer Programming problems are essentially Linear Programming problems with additional requirements regarding the integrality of some or all the variables. If these integrality requirements are relaxed, the relaxed Linear Programming problem could serve as a bound on the best possible value for the optimal objective function value of the Integer Programming problem. If it is minimization, this would be a lower bound.

Example below is a formulation of integer linear programming problem

Formulation of the problem

$$
\begin{aligned}
&MaxZ = 3x_1 + 4x_2 \\
&s.t.\ 2x_1 + 8x_2 \le 16 \quad 7x_1 + 2x_2 \\
&\qquad \le 14 \quad x_1, x_2 \ge 0 \\
&\qquad x_1, x_2 \text{ int } eger
\end{aligned}
$$

$$Max\,Z = 3x_1 + 4x_2 + 0S_1 + 0S_2$$

$$s.t.\ 2x_1 + 8x_2 + S_1 = 16\quad 7x_1 + 2x_2 + S_2 = 14\quad x_1, x_2, S_1, S_2 \geq 0$$

$$x_1, x_2\ are\ \text{integer}$$

3.11.1 Branch and Bound Method

The first Branch and Bound algorithm was developed in 1960 by A. Land and G. Doig says Taha (2007) for the general mixed and pure Linear Programming. He further added that in 1965, E. Balas developed the additive algorithm for solving Linear Programming with pure binary (zero or one) variable. The additive algorithms computations were so simple that it was hailed as a possible breakthrough in the solution of general Integer Linear Programming.

The Branch and Bound method employs a strategy in which the feasible region is divided into smaller sub-groups (or nodes), each being examined for integer feasibility. The information regarding the bounds on the objective function value is constantly updated and used to guide us in a selective examination of nodes. The Branch and Bound method can be described as an iterative algorithm that involves two operations: (i).Branching (ii) Bounding.

Branching involves the subdivision of a linear programming feasible region into two sub regions.

Every time we branch we create two nodes (feasible sub regions) corresponding to the introducing of two mutually exclusive constraints. Some constraints impose bounds on variables.

The bounds have integer values and are of the form $X_k \leq [X_k^*]$ and $X_k \geq [X_k^*] + 1$where $[X_k^*]$ is the largest integer that is less than the non-integer value X_k^*

For example if X_k* is equal to 3.7, then the two constraints which are imposed by branching are $X_k \leq 3$ and $X_k \geq 4$. The imposition of such integer valued bounds on variables X_k results in the elimination of non-integer points. Bounding involves solving the linear programming problem defined at a given node. Further partitioning of a given feasible sub-region (node) can only give worse values for the value of the objective function. This is because with every subsequent partition we solve linear programming problem with smaller feasible regions. So the optimal value of the objective function at a given node is a lower bound for the optimal value of the objective function of subsequent nodes that are created by branching from it. At any step we choose to branch at the node with lowest lower bound if the solution at that node does not satisfy the integrality requirements. The process is terminated when a solution of a node reached does not satisfy the integrality requirements and has the lowest value for the lower bound than any of the existing nodes.

3.11.1.1 Algorithm Steps for Branch and Bound

The Integer Linear Programming problem to be solved is:

Minimize $c^T x$..........................1

Subject to

Ax□*b*..................2

x□0.......................3 *xj* integer for

some *j* € I

Step 1: Relax the integrality requirements (4). The resulting Linear Programming problem is referred to as Node 1. The optimal value of the objective function is the initial Lower bound for the objective function value. If the relaxed LP is found infeasible, the integer programming problem is infeasible, so stop.

Step 2: Compare the lower bound values for any currently defined nodes. If the solution at the node with the lowest Lower bound value satisfies the integrality requirements, stop, that solution is the optimal. If the lowest Lower bound is undefined (∞), stop, the problem is infeasible.

Step 3: Branch at the node with the lowest Lower bound value, by imposing two mutually exclusive constraints on the value of the variable X_k whose present value (X_{k^*}) violates the integrality requirements. Example below is solved using algorithm steps for branched and bound

$$
\begin{aligned}
&Max Z = 3x_1 + 4x_2 \\
&s.t.\ 2x_1 + 8x_2 \le 16 \\
&\quad 7x_1 + 2x_2 \le 14 \ x_1, x_2 \ge \\
&\quad 0 \ x_1, x_2 \text{ integer}
\end{aligned}
$$

$$
\begin{aligned}
&Max Z = 3x_1 + 4x_2 \\
&s.t.\ 2x_1 + 8x_2 \le 16 \\
&\qquad 7x_1 + 2x_2 \le 14 \ x_1, x_2 \ge 0 \\
&\qquad x_1, x_2 \text{ integer}
\end{aligned}
$$

Solution

Formulation of the problem

$$
\begin{aligned}
&Max Z = 3x_1 + 4x_2 + 0S_1 + 0S_2 \\
&s.t. \qquad 2x_1 + 8x_2 + S_1 = 16 \\
&\qquad\qquad 7x_1 + 2x_2 + S_2 = 14 \\
&\qquad\qquad x_1, x_2, S_1, S_2 \ge 0 \\
&\qquad\qquad x_1, x_2 \text{ are integer}
\end{aligned}
$$

TABLE 3.6.1 THE INITIAL TABLEAU FOR THE BRANCHED AND BOUND							
	C_j	3	4	0	0		
C_B	BV	x_1	x_2	S_1	S_2	RHS	Ratio
0	S_1	2	8	1	0	16	2
0	S_2	7	2	0	1	14	7
	Z_j	0	0	0	0	0	
	C_j □ Z_j	3	4	0	0		

□ $2R_1$ □ R_2 □ R_2□

TABLE 3.6.2 FINAL TABLEAU FOR THE BRANCHED AND BOUND							
	C_j	3	4	0	0		
C_B	BV	x_1	x_2	S_1	S_2	RHS	Ratio
4	x_2	0	1	$\frac{13}{26}$	$\frac{1}{□26}$	$\frac{21}{13}$	
3	x_1	1	0	$\frac{1}{□26}$	$\frac{2}{13}$	$\frac{20}{13}$	
	Z_j	3	4	$\frac{9}{26}$	$\frac{8}{26}$	$11\frac{1}{13}$	
	C_j □ Z_j	0	0	$\frac{9}{□26}$	$\frac{8}{□26}$		

x_1 □ $\frac{20}{13}$

x_2 □ $\frac{21}{}$

13

$Z = 11\frac{1}{13}$

Optimal solution:

$x_2 \leq 1$

$x_2 \geq 1 + 1 = 2$

Hence

$1 \geq x_2 \geq 2$

Maximize: $z = 3x_1 + 4x_2$

$2x_1 + 8x_2 \leq 16$

Subject to $7x_1 + 2x_2 \leq 14$ $x_2 \geq 2$

$x_1, x_2 \geq 0, x_1, x_2$ *are* int

Maximizes $z = 3x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1$

$2x_1 + 8x_2 + S_1 = 16$

Subject to $7x_1 + 2x_2 + S_2 = 14$ $x_2 - S_3 +$

$A_1 = 2$

$x_1, x_2, S_1, S_2, S_3, A_1 \geq 0$, x_1, x_2 *are* int

		TABLE 3.6.3 INITIAL TABLEAU FOR BRANCHED AND BOUND							
	C_j	3	4	0	0	0	$-M$		
C_B	BV	x_1	x_2	S_1	S_2	S_3	A_1	RHS	Ratio

0	S_1	2	8	1	0	0	0	16	2
0	S_2	7	2	0	1	0	0	14	7
$-M$	A_1	0	1	0	0	-1	1	2	2
	Z_j	0	$-M$	0	0	$-M$	$-M$	$-2M$	
	$C_j - Z_j$	3	$4+M$	0	0	$-M$	0		

$$-2R_1 + R_2 \to R_2'$$

$$-R_1 + R_3 \to R_3'$$

TABLE 3.6.4 FINAL TABLEAU FOR BRANCHED AND BOUND

	Cj	3	4	0	0	0	$-M$		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	A_1	RHS	Ratio
4	x_2	$\frac{9}{4}$	1	0	0	8	-8	2	
0	S_2	6	0	0	1	-2	2	10	
0	S_1	2	0	1	0	8	-8	0	
	Zj	9	4	0	0	32	-32	8	
	$Cj - Zj$	-6	0	0	0	-32	$-M+32$		

$$R_3 + R_1 \to R_1''$$

$$-\frac{1}{4}R_3 + R_2 \to R_2''$$

Optimal solution: $x_1 = 0$, $x_2 = 2$, $Z = 8$

Maximizes $Z = 3x_1 + 4x_2$

$2x_1 + 8x_2 \le 16$

Subject to $7x_1 + 2x_2 \le 14$

$x_2 \le 1$

$x_1, x_2 \ge 0$

Max $Z = 3x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3$

$2x_1 + 8x_2 + S_1 = 16$

Subject to $7x_1 + 2x_2 + S_2 = 14$

$x_2 + S_3 = 1$

$x_1, x_2, S_1, S_2, S_3 \ge 0$

TABLE 3.6.5 INITIAL TABLEAU FOR THE BRANCHED AND BOUND								
	Cj	3	4	0	0	0		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	RHS	Ratio
0	S_1	2	8	1	0	0	16	2
0	S_2	7	2	0	1	0	14	7
0	S_3	0	1	0	0	1	1	1
	Zj	0	0	0	0	0	0	
	$Cj - Zj$	3	4	0	0	0		

Pivoting column

$-8R_3 + R_1 \rightarrow R_1'$

$-2R_3 + R_2 \rightarrow R_2'$

TABLE 3.6.6 FINAL TABLEAU FOR THE BRANCHED AND BOUND								
	Cj	*3*	*4*	*0*	*0*	*0*		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	*RHS*	*Ratio*
0	S_1	*0*	*0*	*1*	$-\frac{2}{7}$	$-\frac{54}{7}$	$\frac{32}{7}$	
3	x_1	*1*	*0*	*0*	$\frac{1}{7}$	$-\frac{2}{7}$	$\frac{12}{7}$	
4	x_2	*0*	*1*	*0*	*0*	*1*	*1*	
	Zj	*3*	*4*	*0*	$\frac{3}{7}$	*4*	$9\frac{1}{7}$	
	$Cj - Zj$	*0*	*0*	*0*	$-\frac{3}{7}$	-4		

$-2R_2 + R_1 \rightarrow R_1$

Optimal solution $x_1 = 1\frac{5}{7}$

$x_2 = 1$

$Z = 9\frac{1}{7}$

$\lfloor x_1 \rfloor = 1$

$\lceil x_2 \rceil = 1 \le 2$

hence

$1 \le x_1 \le 2$

Therefore we branch.

Maximize $Z = 3x_1 + 4x_2$

$$
\text{Subject to} \quad
\begin{aligned}
2x_1 + 8x_2 &\le 16 \\
7x_1 + 2x_2 &\le 14 \\
x_2 &\le 1 \\
x_1 &\ge 2 \\
x_1, x_2 \ge 0,&\ x_1, x_2 \ \textit{are} \text{ int}
\end{aligned}
$$

Maximize $Z = 3x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4 - MA_1$

$$
\text{Subject to} \quad
\begin{aligned}
2x_1 + 8x_2 + S_1 &= 16 \\
7x_1 + 2x_2 + S_2 &= 14 \\
x_2 + S_3 &= 1 \\
x_1 - S_4 + A_1 &= 2
\end{aligned}
$$

$x_1, x_2, S_1, S_2, S_3, S_4, A_1 \geq 0$

TABLE 3.6.7 INITIAL TABLEAU FOR THE BRANCHED AND BOUND										
	Cj	3	4	0	0	0	0	$-M$		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	S_4	A_1	RHS	Ratio
0	S_1	2	8	1	0	0	0	0	16	8
0	S_2	7	2	0	1	0	0	0	14	2
0	S_3	0	1	0	0	1	0	0	1	∞
$-M$	A_1	1	0	0	0	0	-1	1	2	
	Zj	$-M$	0	0	0	0	M	$-M$	$-2M$	
	$Cj - Zj$	$3+M$	4	0	0	0	$-M$	0		

$\uparrow$

Pivoting column we break the tie arbitrary

TABLE 3.6.8 FINAL TABLEAU FOR THE BRANCHED AND BOUND										
	Cj	3	4	0	0	0	0	$-M$		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	S_4	A_1	RHS	Ratio
0	S_1	0	$\frac{52}{7}$	1	$\frac{-2}{7}$	0	0	0	12	
3	x_1	1	$\frac{2}{7}$	0	$\frac{1}{7}$	0	0	0	2	
0	S_3	0	1	0	0	1	0	0	1	
$-M$	A_1	0	$\frac{-2}{7}$	0	$\frac{-1}{7}$	0	-1	1	0	
	Zj	3	$\frac{2M}{7} + \frac{6}{7}$	0	$\frac{1}{7}M + \frac{3}{7}$	0	M	$-M$	6	
	$Cj - Zj$	0	$\frac{-2M}{7} + \frac{22}{7}$	0	$\frac{-1}{7}M - \frac{3}{7}$	0	$-M$	0		

$-2R_2 + R_1 \rightarrow R_1$

$-R_2 + R_3 \rightarrow R_3$

Optimal Solution

$x_1 = 2$

$x_2 = 0$

$Z = 6$

$$Maximize\ Z = 3x_1 + 4x_2$$

$$\text{Subject to}\quad \begin{aligned} 2x_1 + 8x_2 &\le 16 \\ 7x_1 + 2x_2 &\le 14 \\ x_2 &\le 1 \\ x_1 &\le 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

$$\text{Maximize } Z = 3x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

$$\text{Subject to}\quad \begin{aligned} 2x_1 + 8x_2 + S_1 &= 16 \\ 7x_1 + 2x_2 + S_2 &= 14 \\ x_2 + S_3 &= 1 \\ x_1 + S_4 &= 1 \\ x_1, x_2, S_1, S_2, S_3, S_4 &\ge 0 \end{aligned}$$

TABLE 3.6.9 INITIAL TABLEAU FOR THE BRANCHED AND BOUND

	Cj	3	4	0	0	0	0		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	S_4	RHS	Ratio
0	S_1	2	8	1	0	0	0	16	8
0	S_2	7	2	0	1	0	0	14	2
0	S_3	0	1	0	0	1	0	1	1
0	S_4	1	0	0	0	0	1	1	∞

	Zj	0	0	0	0	0	0	0	
	$C_j - Z_j$	3	4	0	0	0	0		

Pivoting Column↑

TABLE 3.6.10 FINAL TABLEAU FOR THE BRANCHED AND BOUND									
	Cj	3	4	0	0	0	0		
CB	*BV*	x_1	x_2	S_1	S_2	S_3	S_4	RHS	Ratio
0	S_1	0	0	1	0	-8	-2	6	
0	S_2	0	0	0	1	-2	-7	5	
4	x_2	0	1	0	0	1	0	1	
3	x_1	1	0	0	0	0	1	1	
	Zj	3	4	0	0	4	3	7	
	$C_j - Z_j$	0	0	0	0	-4	-3		

$-2R_4 + R_1 \to R_1'$

$-7R_4 + R_2 \to R_2'$

Optimal Solution

$x_1 = 1$

$x_2 = 1$

$Z = 7$

A SUMMARY SOLUTION OF BRANCHED AND BOUND

$$\begin{aligned} Max Z &= 3x_1 + 4x_2 \\ s.t. \quad & 2x_1 + 8x_2 \leq 16 \\ & 7x_1 + 2x_2 \leq 14 \\ & x_1 \geq 0, x_2 \geq 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

Node 1.Subproblem

$$\begin{aligned} Max Z &= 3x_1 + 4x_2 \\ s.t. \quad & 2x_1 + 8x_2 \leq 16 \\ & 7x_1 + 2x_2 \leq 14 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Solution

$x_1 = \frac{20}{30}$

$x_2 = \frac{21}{13}$

$Z = 11\frac{1}{}$

$x_2 \geq 2$

Node 2.Subproblem

$$\begin{aligned} Max Z &= 3x_1 + 4x_2 \\ s.t. \quad & 2x_1 + 8x_2 \leq 16 \\ & 7x_1 + 2x_2 \leq 14 \\ & x_2 \geq 2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Solution

$x_1 = 0$

$x_2 = 2$

$Z = 8$

Optimal Solution

$x_2 \leq 1$

Node 3.Subproblem

$$\begin{aligned} Max Z &= 3x_1 + 4x_2 \\ s.t. \quad & 2x_1 + 8x_2 \leq 16 \\ & 7x_1 + 2x_2 \leq 14 \\ & x_2 \leq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Solution

$x_1 = \frac{12}{7}$

$x_2 = 1$

$Z = 9\frac{1}{7}$

$x_2 \geq 2$

Node 4.Subproblem

$$\begin{aligned} Max Z &= 3x_1 + 4x_2 \\ s.t. \quad & 2x_1 + 8x_2 \leq 16 \\ & 7x_1 + 2x_2 \leq 14 \\ & x_2 \leq 1 \\ & x_1 \geq 2 \end{aligned}$$

Solution

$x_1 = 2$

$x_2 = 0$

$Z = 6$

x_1

le 5.Subproblem

$$\begin{aligned} Max Z &= 3x_1 + 4x_2 \\ s.t. \quad & 2x_1 + 8x_2 \leq 16 \\ & 7x_1 + 2x_2 \leq 14 \\ & x_2 \leq 1 \\ & x_1 \leq 1 \end{aligned}$$

Solution

$x_1 = 1$

$x_2 = 1$

$Z = 7$

3.12 OTHER METHODS OF SOLVING INTEGER LINEAR PROGRAMMING PROBLEMS

There are other methods that can be used to solve linear programming problems among which are discussed below.

3.12.1 Gomory's Cutting - Plane Method

In mathematical optimization, the **cutting-plane method** is an umbrella term for optimization methods which iteratively refine a feasible set or objective function by means of linear inequalities, termed *cuts*. Such procedures are popularly used to find integer **solutions to** mixed integer linear programming (MILP) problems, as well as to solve general, not necessarily differentiable convex optimization problems.

Let an integer programming problem be formulated as

$$\begin{aligned} &\text{Maximize } c^T x \\ &\text{Subject to } Ax \le b, \\ &\qquad x \ge 0, x_i \text{ all integers} \end{aligned}$$

The method proceeds by first dropping the requirement that the x_i be integers and solving the associated linear programming problem to obtain a basic feasible solution. Geometrically, this solution will be a vertex of the convex polytope consisting of all feasible points. If this vertex is not an integer point then the method finds a hyperplane with the vertex on one side and all feasible integer points on the other. This is then added as an additional linear constraint to exclude the vertex found, creating a modified linear programming program. The new program is then solved and the process is repeated until an integer solution is found.

Using the simplex method to solve a linear program produces a set of equations of the form

$$x_i + \sum \bar{a}_{i,j} x_j = \bar{b}_i$$

Where x_i is a basic variable and the x_j's are the nonbasic variables. Rewrite this equation so that the integer parts are on the left side and the fractional parts are on the right side:

$$x_i + \sum \lfloor \bar{a}_{i,j} \rfloor x_j - \lfloor \bar{b}_i \rfloor = \bar{b}_i - \lfloor \bar{b}_i \rfloor - \sum \left(\bar{a}_{i,j} - \lfloor \bar{a}_{i,j} \rfloor \right) x_j$$

For any integer point in the feasible region the right side of this equation is less than 1 and the left side is an integer, therefore the common value must be less than or equal to 0. So the inequality

$$\bar{b}_i - \lfloor \bar{b}_i \rfloor - \sum \left(\bar{a}_{i,j} - \lfloor \bar{a}_{i,j} \rfloor \right) x_j \leq 0$$

must hold for any integer point in the feasible region. Furthermore, if x_i is not an integer for the basic solution x,

$$\bar{b}_i - \lfloor \bar{b}_i \rfloor - \sum \left(\bar{a}_{i,j} - \lfloor \bar{a}_{i,j} \rfloor \right) x_j = \bar{b}_i - \lfloor \bar{b}_i \rfloor > 0$$

So the inequality above excludes the basic feasible solution and thus is a cut with the desired properties. Introducing a new slack variable x_k for this inequality, a new constraint is added to the linear program, namely

$$x_k - \sum_j \left(\lfloor a_{i,j} \rfloor - a_{i,j}\right) x_j = \lfloor b_i \rfloor - b_i, \quad x_k \ge 0, \; x_k \text{ an integer}$$

Example

Consider the integer optimization problem

Maximize

$x_1 + x_2 + x_3$

Subject to

$x_1 + x_2 \le 1$

$x_1 + x_3 \le 1$

$x_2 + x_3 \le 1$

$x_i \ge 0, x_i$ an integer for $i = 1,2,3$

Introduce slack variables x_i, i=4, 5, 6 to produce the standard form

Maximize

$x_1 + x_2 + x_3$

Subject to

$x_1 + x_2 + x_4 = 1$

$$x_1 + x_3 + x_5 = 1$$

$$x_2 + x_3 + x_6 = 1$$

$$x_i \geq 0, x_i \text{ an integer for } i = 1,2,...,6$$

Solving this with the simplex method gives the solution $x_1 = x_2 = x_3 = \frac{1}{2}$ and the equations

$$x_1 + \frac{1}{2}x_4 + \frac{1}{2}x_5 - \frac{1}{2}x_6 = \frac{1}{2}$$

$$x_2 + \frac{1}{2}x_4 - \frac{1}{2}x_5 + \frac{1}{2}x_6 = \frac{1}{2}$$

$$x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{1}{2}x_6 = \frac{1}{2}$$

Each of these equations produces the same cutting plane, and with the introduction of a new slack variable x_7 it can be written as a new constraint

$$-\frac{1}{2}x_4 - \frac{1}{2}x_5 - \frac{1}{2}x_6 + x_7 = -\frac{1}{2}$$

An analysis of the updated linear program quickly shows that the original three constraints are now redundant and the corresponding slack variables can be eliminated, leaving the simplified problem

Maximize

$x_1 \square x_2 \, x_3$

Subject to

$x_1 \square x_2 \square x_3 \square x_7 \square 1$

$x_i \geq 0$, x_i an integer for i=1, 2, 3, 7.

This is easily solved giving three solutions (x_1, x_2, x_3) = (1, 0, 0), (0, 1, 0), or (0, 0, 1).

3.13 SUMMARY

In this chapter, Linear Programming and Simplex method were discussed. The analysis on the Linear Programming and Simplex method also form part of the discussion in this section of the work. In the next chapter, the data collected from the company will be used to formulate the linear model and solve using the integer method.

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.1 Data Collection

The data for the write up was obtained from the Regional Office of Zoomlion at Sunyani. Other relevant information was sought for at the Sunyani municipal assembly. These include the cost of fueling each type of vehicle for a month, cost of repairs and maintenance for each type of vehicle during a month, monthly salaries of janitors who accompany the vehicles, and the capacities of each type of vehicle. The data is a secondary source from the Regional Office of Zoomlion Company, Ghana Ltd.

The Zoomlion Office in the Sunyani Municipality has three types of vehicles, skip, compactor and roll-on-roll-off, for the transportation of solid waste. The vehicles work for 7 days in a week, Monday to Sunday. There are 40 collection containers positioned at vantage points in the municipality. The vehicles are stationed at the office and move to the designated locations and convey the solid waste to the landfill site at Asufofuo, 5 kilometers away.

The vehicles are reasonably filled to capacity before transporting the waste to the landfill site.

The average monthly operational costs for the vehicles are shown in Table 4.1 below. This includes fuel cost, repairs and maintenance costs, and the salaries of the janitors accompanying each vehicle. The amounts have been quoted in Ghana cedis (GH¢).

Table 4.1 Monthly operational costs quoted in Ghana Cedis (GH¢)

Vehicle	Fuel in liters	Maintenance and Repairs	Salary	Total
Skip	774.67	300	850	2699.34
Compactor	967.11	500	1550	3948.22
Roll-On	580.23	300	850	2310.46
Total	2322.01	1100	3250	

Source: Zoomlion Ghana Ltd.(Regional Office,Sunyani,2013)

Table 4.2 Monthly cost of fuel converted to Ghana cedis (GH¢)

Vehicle	Fuel in liters	Converted to GH¢
Skip	774.67	1549.34
Compactor	967.11	1934.22
Roll-On	580.23	1160.46
Total	2322.01	4644.02

Source: Zoomlion Ghana Ltd.(Regional Office,Sunyani,2013)

Table 4.3 Monthly budget quoted in Ghana Cedis (GH¢)

Item	Total
Fuel in liters	3750(GH¢7500)
Maintenance	2500
Salary	3750
Total	13750

Source: Zoomlion Ghana Ltd.(Regional Office,Sunyani,2013)

Table 4.4 Number of Janitor accompanying a vehicle type per trip

Vehicle	Number of Vehicles	Number of Janitors	Number of Trips Per Week	Average number of trips per day
Skip	4	1	40	7
Compactor	2	3	12	2

Roll-On	1	1	20	3

Source: Zoomlion Ghana Ltd.(Regional Office,Sunyani,2013)

Table 4.5 Capacities of Vehicles

Vehicle	Capacity in Cubic Meters
Skip	15m3
Compactor	25m3
Roll-On	12m3
Total volume to be evacuated per day (at least)	210 m^3

Source: Zoomlion Ghana Ltd.(Regional Office,Sunyani,2013)

4.2 Data Analysis

Making use of the fact that the vehicles work for seven days in a week, Monday to Sunday, and making the assumption that there are 27 working days in a month, the cost per day was computed for each type of vehicle and the results displayed in Table 4.5

Table 4.6 Operational Cost per day quoted in Ghana Cedis (GH¢)

Vehicle	Fuel	Maintenance	Salary	Cost Per day
Skip	57.38	11.11	31.48	99.97
Compactor	71.64	18.52	57.41	147.57
Roll-On	42.98	11.11	31.48	85.57
Total	172.00	40.74	120.37	333.11

Average cost per trip was computed by dividing Table 4.6 by the average number of trips per day

Table4.7 Average cost per trip (quoted in Ghana cedis)

Vehicle	**Fuel**	**Maintenance**	**Salary**	**Total**
Skip	8.200	1.59	4.49	14.28
Compactor	35.82	9.26	28.71	73.79
Roll-On	14.33	3.70	10.49	28.52
Total	58.35	14.55	43.69	

Daily Budget per Trip is calculated for by dividing monthly budget by 27 working days. **Table 4.8 Daily Budget per trip quoted in Ghana cedis**

Item	Amount (GH¢)
Fuel	277.78
Maintenance	92.59
Salary	138.89
Total	509.26

4.3 Model Formulation

The objective function is the total cost incurred in the transportation of the solid waste from the 40 containers to the landfill site at Asufofuo. This comprises of the fuel cost, repair and maintenance cost and the cost of labor. The decision variables are x_1, x_2, *and* x_3; the number of trips that the skip, compactor and roll-on respectively make in a day.

Objective Function

Considering Table 4.7, the total amount spent per trip on a skip is GH¢ 14.28, GH¢73.79 on compactor and GH¢ 28.52 on a roll-on. It was desired to find the number of trips that will minimize the transportation cost. Therefore the objective function is: minimize C=

$14.28x_1 + 73.79x_2 + 28.52x_3$ ………………………………………………………… 1

Fuel Constraint

The budgetary constraints are considered for the fixed working capital. From Table 4.8, the total cost of fuel per day for all the vehicles should not be more the budgetary allocation for vehicles, GH¢277.78 (Ghana cedis). If the number of trips made per day by skip, compactor and roll-on are x_1, x_2 *and* x_3 respectively, then the cost for fuel used per day, then from Table 4.7, the inequality will be $8.20\ x_1 + 35.82\ x_2 + 14.33\ x_3 \leq 277.78$………………………………………..2

Maintenance Constraint

The budgetary provision of capital per day for maintenance and repairs should not be more than GH¢ 92.59 (Ghana cedis) as depicted in Table 4.8. Using Table 4.7, the total amount spent on maintenance should satisfy the inequality $1.59\ x_1 + 9.26\ x_2 + 3.70\ x_3 \leq 92.59$…………………..3

Salary Constraints

The budgetary allocation for the janitors on the vehicles should not be greater than

GH¢138.89 (Ghana cedis). Picking values from both Tables 4.7 and 4.8, the total expenditure for the janitors will satisfy the inequality $4.49\ x_1 + 28.71\ x_2 + 10.49\ x_3 \leq 138.89$……..………….. 4

Capacity Constraints

The capacity constraint requires that the volume of solid waste hauled to the landfill site should be at least 210 cubic meters. From Table 4.5, the total volume of waste will satisfy the inequality

$15\,x_1 + 25\,x_2 + 12\,x_3 \geq 210$………………………………………………………. 5

The problem of determining the number of trips that each type of vehicle operated by Zoomlion in the Sunyani Municipality should make in a day in order to minimize the transportation cost of hauling the solid waste from 40 collection containers to the landfill site was formulated as an Integer Programming problem, since number of trips will assume only integral values.

Summarizing the Objective Function Equations and Constraints

Minimize $C = 14.28x_1 + 73.79x_2 + 28.52x_3$. …………………………………………………… 1

Subject to

$8.20x_1 + 35.82x_2 + 14.33x_3 \leq 277.78$…………………………………………..……………….. 2

$1.59x_1 + 9.26x_2 + 3.70x_3 \leq 92.59$……………………………………………………….…….. 3

$4.49x_1 + 28.71x_2 + 10.49x_3 \leq 138.89$……..……………………………………………………. 4

$15x_1 + 25x_2 + 12x_3 \geq 210$……………………………………………………………………. 5

4.4 Computational Procedure

The solution to the optimization problem which was formulated as linear integer programming model was obtained using the Lips software.

Linear Program Solver (LiPS) is an optimization package intended for solving linear, integer, Branch and bound and goal programming problems.

4.5 Specifications of Computer Used

The solution to the model was obtained by using Pentium (R) Dual-Core Toshiba laptop computer with processor 2.30GHz, 2.30 GHz, memory of 4.00GB(3.87GB usable) RAM and hard disk space of 300GB.

4.6 Results

The results from Table 4.9 shows an optimal solution found to be: C= 199.92,

x_1=14, x_2 =0.and x_3 =0. This means that if only one Skip vehicle makes 14 trips per day, it will be able to haul the expected volume of solid waste to the landfill site at a cost of GH¢199.92. For a day, it is expected that at least 210m^3 of solid waste to be lifted to the landfill site.

Table 4.9 Optimal solution found minimum =199.92

Variable	Value	Objective Constraint
X_1	14	14.28
X_2	0	73.79
X_3	0	28.52

4.7 Discussion

(a) Currently, Zoomlion operates 4 skips, 2 compactors and 3 roll-on, vehicles. A skip makes 7 trips, compactor 2 trips and a roll-on 3 trips for the haulage of solid waste from the city to the landfill site at Asufufuo on daily basis. If these values are used in the computations, the total operational cost for a day is C=4*14.28*7+2*73.79*2+3*28.52*3= GH¢951.68

The corresponding volume (V) of waste that will be hauled,

V=4*15*7+ 2*25*2 + 12*3*3= 628m^3 .

(b) Solution x_1= 14

Even if 2 Skip vehicles out of the four are used and the current number of trips per day, 7, is maintained, it means that each vehicle will make 7 trips a day and the operational cost will be GH¢ 199.92; which is far less than what they have stated.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This research work sought to utilize optimization techniques to find the optimal solution to the integer programming model in order to minimize operational cost. With the aid of Lips software an optimal solution was obtained for the model.

It could be inferred from the results obtained that if only one skips vehicle makes 14 trips per day, the quantum of solid waste that is required to be transported to the landfill site a day can be achieved at a minimum cost of GHc199.92

5.2 Recommendations

- Taking cognizance of the current financial constraints and the quantum of solid waste generated and therefore should be evacuated from the communities to the landfill site at Asufufuo in the Sunyani municipality, it will be prudent on the part of management if additional Skip vehicles are procured for cost effective waste management in the municipality to maintain and improve upon the aesthetics of the communities.
- A visit to the landfill site at Asufufuo reveals that solid waste is being transported and dumped at the site; waste is not being managed properly. The obnoxious scent that emanates from the site, coupled with the rodents and flies that have infested the area, pose a serious health threat to the nearby residents and to the entire community. Further research may be encouraged on how waste generated at this site can be converted into wealth

❖ The research can be replicated in other municipalities.

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APPENDICES

APPENDIX I

Table 4.10 Time Spend Per Trip in Minutes

Vehicle	Time
Skip	30
Compactor	60
Roll-on	30
Time expected per trip	500

Table 4.11 Labourers and their monthly average salary

Work Type	Number of Labourers	Average Salary Per Month
Operation Monitor	1	350
Mechanics	10	3000
Tricycle Supervisor	1	350
Municipal Opt. Super.	1	1000
NYEP (Riders)Staff	342	34200

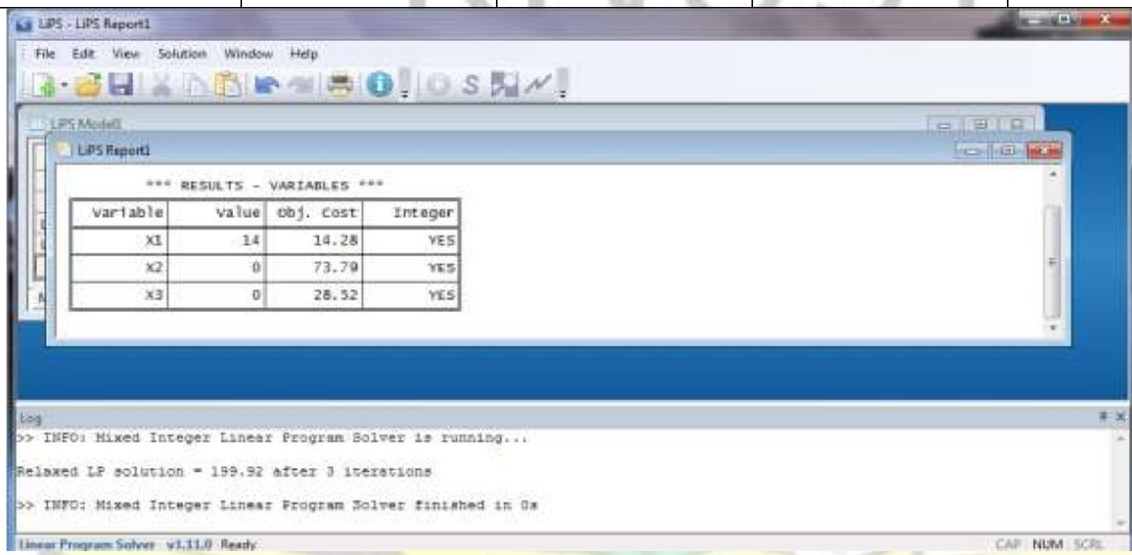
Source:zoomlion Ghana Ltd.(Regional Office,Sunyani,2013) **Table 4.12 Average Administration Cost**

Description	Average Cost Per Month
Pickers	684.00
Log Books	70.00
Wheel Barrows	650.00
Shovels	180.00
Nose Mask	991.80
Foot Fork	240
Ali Brooms	513.08

Source: Zoomlion Ghana Ltd.(Regional Office,Sunyani,2013)

TABLE 4.13 Optimized total costs and volume for different numbers of roll-on vehicles

Number of Skips	Number of Compactors	Number of Rollon Vehicles	Optimized total cost in GH¢	Volume of waste transported in (m^3)
1	0	0	215.88	210
2	0	0	431.76	420
3	0	0	647.64	630
4	0	0	863.52	840



Optimal solution found minimum =199.92

Fig. 4.1 Optimal solution found minimum =199.92