KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

COLLEGE OF SCIENCE

INSTITUTE OF DISTANCE LEARNING

AN OPTIMAL PORTFOLIO SELECTION OF EPACK MUTUAL FUND, CASE STUDY: DATABANK FINANCIAL SERVICES LIMITED

BY

BOATENG SAMUEL

A Thesis submitted to the College of Science in partial fulfillment of the requirements for the degree of MSC in Industrial Mathematics

MAY, 2013

DECLARATION

This thesis is the true account of the candidate's own research work except for references to other people's work which have been fully acknowledged.

Mr Samuel Boateng ……………………… ………………………..

(Student) **Signature** **Date**

Mr K.F Darkwah …………………… ………………………..

(Supervisor) **Signature** **Date**

Prof S.K Amponsah ……………………. ………………..

(Head of Department) **Signature** **Date**

DEDICATION

I would like to dedicate this thesis to the glory of God and my wife Ellen Adjei, and my newly born daughter Nhyira Henewaa Oti Boateng.

ACKNOWLEGEMENT

My first thanks goes to the Almighty God who has graciously helped me to complete this thesis.

I am also grateful to my supervisor, Mr Kwaku Darkwah for his valuable suggestions, constructive criticisms, encouragement, corrections and expert advice he offered in producing this dissertation.

My profound gratitude goes to my wife (Miss Ellen Adjei) for her advice, encouragement and prayer support.

I finally wish to express my thanks to all, especially my fellow postgraduate students, Kumasi center, who helped in one way or the other to the successfully completion of this dissertation.

ABSTRACT

Risk plays a very important role in the decision-making process for both investors and companies. It is important that the level of risk associated with investment be quantified so that investors would be able to manage or control it.

The main objective of the study is to efficiently allocate funds to assets of Epack portfolio and quantify the risk associated with it by using Konno and Yamazaki model.

Databank financial services limited was selected for the case study. Its annual report of Epack mutual fund from 2000 to 2009 was collected for the analysis.

The study showed that an amount of GH¢25,998.48 should be allocated to Enterprise Insurance(EI), GH¢55,357.71 to Fan milk(FAM) GH¢18,643.81 to Aluworks(ALM) and no amount to Standard Chartered Bank(SCB), Guinness Ghana Ltd(GGL), Social Security Bank(SSB) and Unilever Ghana limited(UG). In the process the risk is reduced by GH¢37,908.69.

According to the study, it is highly recommended that more resources should be allocated to Fan milk which forms part of consumable assets and little or no amount to the banks.

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1.0 CHAPTER ONE

1.1 Introduction

When we talk about investing in "portfolios", many people are confused about the term. If you went back 50 years in a time, however, no one would have the slightest clue what you were talking about. The term investment portfolio did not exist until the late 1960s. The idea has now become so entrenched that we cannot imagine a world without them. Investopedia (2009)

Portfolio decisions play an important role in wealth accumulation, accounting for perhaps 90 percent of total returns. (Ibbotson and Kaplan, 2000).

Portfolio

An investment portfolio is a collection of income producing assets that have been bought to meet a financial goal. Portfolios are held directly by investors and /or managed by financial professionals.

Prudence suggests that investors should construct an investment portfolio in accordance with risk tolerance and investing objectives. One thinks of an investment portfolio as a pie that is divided into pieces of varying sizes representing a variety of asset classes and/or types of investments to accomplish an appropriate risk-return portfolio allocation.

For example, a conservative investor might favor a portfolio with large cap value stocks, broad-based market index funds, investment-grade bonds and a position in liquid, highgrade cash equivalents. In contrast, a risk loving investor might add some small cap growth stocks to an aggressive, large cap growth stock position, assume some high-yield bond exposure, and look to real estate, international and alternative investment opportunities for his or her portfolio. Investopedia (2009).

Diversification

Diversification is a risk management technique that mixes a wide variety of investments within a portfolio. The rationale behind this technique contends that a portfolio of different kinds of investments will, on average, yield higher returns and pose a lower risk than any individual investment found within the portfolio. Diversification strives to smooth out unsystematic risk events in a portfolio so that the positive performance of some investments will neutralize the negative performance of others. Therefore, the benefits of diversification will hold only if the securities in the portfolio are not perfectly correlated.

Studies and mathematical models have shown that maintaining a well-diversified portfolio of 25 to 30 stocks will yield the most cost-effective level of risk reduction. Investing in more securities will still yield further diversification benefits, albeit at a drastically smaller rate.

Most non-institutional investors have a limited investment budget, and may find it difficult to create an adequately diversified portfolio. This fact alone can explain why mutual funds have been increasing in popularity. Buying shares in a mutual fund can provide investors with an inexpensive source of diversification. (Wikipedia, 2009)

Capital Asset Pricing Model (CAPM)

Capital asset pricing model (CAPM) is a model of financial economics that is used to measure the rate of return of an asset in a well-diversified portfolio, and thus determine its value. CAPM is therefore a model used to determine the price of an asset.

CAPM was introduced by Treynor (1961), who built on the work of Harry Markowitz.

Asset Allocation

This is an investment strategy that aims to balance risk and reward by apportioning a portfolio's assets according to an individual's goals, risk tolerance and investment horizon. The three main asset classes - equities, fixed-income, and cash and equivalents have different levels of risk and return, so each will behave differently over time. There is no simple formula that can find the right asset allocation for every individual. However, the consensus among most financial professionals is that asset allocation is one of the most important decisions that investors make. In other words, your selection of individual securities is secondary to the way you allocate your investment in stocks, bonds, and cash and equivalents, which will be the principal determinants of your investment results.

Asset-allocation mutual funds, also known as life-cycle, or target-date, funds, are an attempt to provide investors with portfolio structures that address an investor's age, risk appetite and investment objectives with an appropriate apportionment of asset classes. However, critics of this approach point out that arriving at a standardized solution for allocating portfolio assets is problematic because individual investors require individual solutions. (Investopedia, 2009)

Pooled Fund

Funds from many individual investors that are aggregated for the purposes of investment is described as pooled fund. Investors in pooled fund investments benefit from economies of scale, which allow for lower trading costs per dollar of investment, diversification and professional money management. The enormous advantages of investing in pooled fund vehicles make them an ideal asset for many investors. There are added costs involved in the form of management fees, but these fees have been steadily declining for many years as competition has increased. The main detractor of pooled fund investments is that capital gains are spread evenly among all investors - sometimes at the expense of new shareholders. (Investopedia, 2009)

Marginal Efficiency of Capital

Marginal efficiency of capital describes the rate of discount which would make the present value of expected income from fixed capital assets equal to the present supply price of the asset. As investment increases, the rate of returns decreases because early investment was directed at the most lucrative possibilities; subsequent investment is channelled into less promising areas and the returns diminish. (Investopedia, 2009)

Arbitrage Pricing Policy

Arbitrage pricing policy refers to the practice of simultaneously trading the same asset (be it a currency, security or commodity) in different markets with different price levels so as to make a profit. The actions of players who deal in currencies, securities, or commodities eliminate price disparities between markets. (Investopedia, 2009)

Portfolio Selection Theory

This theory talks about how risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasizing that risk is an inherent part of higher reward. According to the theory, it's possible to construct an "efficient frontier" of optimal portfolios offering the maximum possible expected return for a given level of risk. (Wikipedia, 2009)

Modern portfolio theory

Modern portfolio theory (MPT) is a theory of investment which tries to maximize return and minimize risk by carefully choosing different assets. Although MPT is widely used in practice in the financial industry and several of its creators won a Nobel Prize for the theory. In recent years the basic assumptions of MPT have been widely challenged by fields such as behavioral economics, and many companies using variants of MPT have gone bankrupt in various financial crises.

MPT is a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that has collectively lower risk than any individual asset. This is possible, in theory, because different types of assets often change in value in opposite ways. For example, when the prices in the stock market fall, the prices in the bond market often increase, and vice versa. A collection of both types of assets can therefore have lower overall risk than either individually.

More technically, MPT models an asset's return as a normally distributed random variable, defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets so that the return of a portfolio is the weighted combination of the assets' returns. By combining different assets whose returns are not correlated, MPT seeks to reduce the total variance of the portfolio. MPT also assumes that investors are rational and markets are efficient.

MPT was developed in the 1950s through the early 1970s and was considered an important advance in the mathematical modeling of finance. Since then, much theoretical and practical criticism has been leveled against it. These include the fact that financial returns do not follow a Gaussian distribution and that correlations between asset classes are not fixed but can vary depending on external events (especially in crises). Further, there is growing evidence that investors are not rational and markets are not efficient.

Wikipedia (2009)

The risk-free asset

The risk-free asset is the (hypothetical) asset which pays a risk-free rate. In practice, short-term Government securities (such as treasury bills) are used as a risk-free asset, because they pay a fixed rate of interest and have exceptionally low default risk. The risk-free asset has zero variance in returns (hence is risk-free); it is also uncorrelated with any other asset (by definition: since its variance is zero). As a result, when it is combined with any other asset, or portfolio of assets, the change in return and also in risk is linear. Wikipedia (2009)

Capital Allocation Line

The Capital Allocation Line (CAL) is the line of expected return plotted against risk (standard deviation) that connects all portfolios that can be formed using a risky asset and a riskless asset. It can be proven that it is a straight line.

Bogey

A buzzword that refers to a benchmark used to evaluate a fund's performance. The benchmark is an index that reflects the investment scope of the funds investment. Comparing a fund's performance to a benchmark index gives investors an idea of how well the fund is doing compared to the market. Investopedia (2009)

Zero-Investment Portfolio

A group of investments which, when combined, create a zero net value. Zero-investment portfolios can be achieved by simultaneously purchasing securities and selling equivalent securities. This will achieve lower risk/gains compared to only purchasing or selling the same securities.

Zero-investment portfolios have many uses, including:

1. Reducing taxes, because they generate little or no interest income.
2. Reducing risk by protecting against unexpected shifts in the value of the held securities.
3. Protecting the overall value of the portfolio so that investment can be made at a later date.
4. Determining if the average portfolio returns are statistically different from zero.

Risk

Risk is defined as the standard deviation (or variance of a return of an investment. This measure quantifies the extent to which single – period monetary returns from an investment fluctuate. Having quantified risk, it was possible to build models which concentrate on risk as well as expected return, and to provide an understanding of the process of diversification.

Investment

Investment is the current commitment of money or other resources in the expectation of reaping future benefits. (Investopedia, 2009)

Investment opportunities in Ghana

Ghana was the first country south of the Sahara to be independent with a stable multi party democracy. It is regarded as the 'Gateway to Africa', endowed with natural resources and high literacy rate with friendly people. It has numerous opportunities in the area of investment. Some of the areas investors can access are enumerated below

Agro – processing

Agro-processing has emerged as one of the most attractive sectors for foreign investment. This entails the establishment of manufacturing industries to add value to local agricultural and fishery products, especially processing fruits into fruit juices, and purees etc, cocoa beans into cocoa products, rice into fragrant rice, flour etc., cassava into industrial starch, etc. The scope for export of these products to regional and international markets is quite significant.

Processing of raw agricultural products for local consumption and exports is very much encouraged by the government, however, priority is given to the processing of the following products: Cocoa, Pineapple, Cashew, Palm oil, Vegetable (tomatoes, chilies, etc.,) Cassava (Processed into starch under the Presidents Special Initiative), Floriculture, and Seafood, particularly tuna processing.

Real estate development

Ghana has immense opportunity in property development and construction. The potential investor interested in the Real Estate sub-sector may be looking at the construction of residential houses, Industrial and commercial houses as well as shopping centers. Residential accommodation, particularly hotels and hostels for tertiary institutions.

Experts in the construction sector recognize huge potential within the real estate sub-sector in Ghana, particularly among the ever-increasing middle- income earners who are eager to own houses as well as among the large number of Ghanaians living abroad. Opportunities also abound for investors interested in Export Processing Enclave real estate development, which has been fashioned to provide factory shells, office space and serviced plots to potential investors. Hostel needs for the teaming tertiary students in and around the country's Public and Private Universities/Polytechnics offer excellent opportunity to the foreign investor within the real estate sub-sector.

Pharmaceuticals and medical supplies

Ghana acts as gateway to a huge sub-regional market of over 250 million people for the manufacturing and marketing of pharmaceutical and other medical supplies. An entrepreneur can set up a chemical and pharmaceutical processing plant within the manufacturing sector as well as venture into the Health Service Delivery sub-sector within the infrastructural facilities sector. The need to attract investors into the pharmaceuticals sub-sector has become so imperative in view of challenges brought about by diseases such as HIV/AIDS, malaria, etc.

Information and Communication Technology (ICT)

Ghana has vast opportunity for the development of Information and Communication Technology (ICT). The potential investor within the sub-sector has the following aspects of ICT readily available for consideration: Provision of International Internet Protocol, (IP) based network/internet technology, Content development and services, Multimedia publishing, Computer software and packages production, etc.

1.2 Background of study

Over the past years Databank group Limited in Ghana has stood out as an established institution for providing good analysis of investment portfolios.

Databank has been influential in the development of the capital markets in Ghana, acting as advisor, placement agent and broker to private clients, government and corporations alike. Databank group consist of board of directors, managers and the research team. The board of director is the highest decision body. The managers take responsibility of day to day management of the group. The research team analyses their investments from time to time. They make use of computer aided programs such as investment management software in making their analysis. In Ghana, databank performs frequent analysis of stock exchanges and offer advice to prospective investors. Sometimes they manually study the trend of the investment and make predictions.

Databank was founded in April 1990 to provide corporate and public finance advisory services to companies in Ghana. It's mission is to provide innovative and responsive corporate finance, brokerage, fund management and research services to local and foreign individuals, multinational companies, institutions, and portfolio investors for the ECOWAS sub-region. Through the years, they have successfully expanded their operations and their presence in the Ghanaian market and beyond, building key relationships with the private sector, government and public corporations and educating their clients on the benefits of corporate advisory services.

Databank currently consists of Databank Brokerage Ltd, Databank Asset Management Services Ltd, Databank Corporate Finance Ltd, Databank Research and Information Ltd, and Databank Securities Ltd in The Gambia. Databank has branches in Tema, Kumasi and The Gambia. They have also extended into the asset management and private equity financing markets.

Databank has been involved in numerous groundbreaking transactions which have taken place on the Ghana Stock Exchange including advising the Ghanaian government on privatization deals, the private sale of 52% (US$21million) of the Social Security Bank (SSB), the US$25million packaged sale of government interests in seven listed companies; the first tender offer on the Ghana Stock Exchange (Enterprise Insurance Company Ltd); the US$35 million shelf offering of the HFC dollar-indexed bonds, and the first cross-border listing on the Ghana Stock Exchange (Trust Bank Ltd of The Gambia). (Databank, 2009)

1.3 Problem Statement

Even though databank is making an effort to give investors advice on how to go about their investment in order to get a higher returns.

1. Fund managers face difficult task of allocating resources (assets) efficiently. Allocating resources is one of the challenges confronting investors in Ghana. Inefficient allocation of funds or resources always leads to loss of funds. So people are afraid to invest because they think they might lose their money.

2. Every investor wants to make good returns and have minimum risk on his/her investment. Risk plays a very important role in the decision-making process for both investors and companies, so it is important that the level of risk associated with investment be quantified. The problem is that the investors are not able to quantify risk that will give them higher returns. Ideally, for a higher return, you need to take higher risk. But how much risk is an investor ready to accommodate in order to maximize profit on their investment? Inability of investors to quantify risk adversely affects their decision on investment.

1.4 Objective of the study

1. To quantify the risks of investment of Epack portfolio in Databank Financial Services by using Konno and Yamazaki model.

2. To determine the best allocation of funds to Epack portfolio in order to gain optimal returns with tolerable investment risk or minimized investment risk.

1.5 Justifications

1. Asset allocation remains investor's most important decision. This study will provide a framework for determining the relative importance of active management and asset allocation in portfolio performance.

2. As more people do right kind of investment by allocating resources in viable market, the economy grows thereby reducing inflation rate which make people have value for their money.

3. This study will be used as academic reference book for further studies.

1.6 Methodology

The problem is to optimally select an asset of a portfolio which will yield better returns and reduce risk on investment. To achieve this, Konno and Yamazaki Model will be used. This model will lead to system of linear equations which will be solved by a computer program called The Management Scientist Version 5.0. The principle underlying the operation of this software is Revised Simplex Algorithm with LU decomposition which will be extensively discussed in chapter three.

Libraries and Internet facilities will be the source of information for this thesis.

The data consist of price of assets at the beginning and the end of the year. Thus prices at January and December for every year. The period under consideration is ten (10) consecutive years. (From 2000 to 2009).

1.7 Thesis Organization

Chapter one deals with introduction which includes background to the study, problem statement, objectives, justification and methodology of the study.

Chapter two talks about literature review.

Chapter three extensively deals with models and methodology.

Chapter four deals with data collection and analysis.

Finally chapter five deals with conclusion and recommendation.

2.0 CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Agents in financial markets operate in a world in which they make choices under risk and uncertainty. Portfolio managers, for example, make investment decisions in which they take risks and expect rewards. They choose to invest in a given portfolio because they believe it is ‘‘better’’ than any other they can buy. Thus the chosen portfolio is the most preferred one among all portfolios that are admissible for investment. Not all portfolio managers invest in the same portfolio because their expectations and preferences vary.

2.2 Developments in portfolio optimization

The study of decision making under risk has a long history, beginning with early decision models of resource allocation that maximized expected returns. Portfolio theory significantly improved our ability to analyze and identify optimal choices under risk by extension of the analysis to include variability, as well as expected returns. The theory of how choices under risk and uncertainty are made was introduced by Von Neumann and Morgenstern (1944). They gave an explicit representation of investor’s perferences in terms of an investor’s utility function. If no uncertainty is present, the utility function can be interpreted as a mapping between the available alternatives and real numbers indicating the ‘‘relative happiness’’ the investor gains from a particular alternative. If an individual prefers good A to good B, then the utility of A is higher than the utility of B.

Thus, the utility function characterizes individual's preferences. Von Neumann and Morgenstern (1944) showed that if there is uncertainty, then it is the expected utility that characterizes the preferences. The expected utility of an uncertain prospect, often called a lottery, is defined as the probability weighted average of the utilities of the simple outcomes.

Since von Neumann and Morgenstern (1944), many researchers have tried to model portfolio optimization problems within an expected utility maximization framework. Markowitz's (1952) paper "Portfolio Selection" sparked further interest in developing a mathematical approach to optimizing multi-asset portfolios. Markowitz's legendary study of portfolio optimization is regarded as the pioneering work of modern portfolio theory. In the Markowitz model, risk is stated in terms of the predicted variance of portfolio return, a function that is quadratic in the decision variables. All other functions and constraints are assumed to be linear (Sharpe, 1971).

The objective of the model is to form the efficient portfolios.

Sharpe (1971) claimed that if the essence of a portfolio analysis problem could be adequately captured in a form suitable for linear programming methods, the prospect for practical application would be greatly enhanced. Sharpe (1971), Stone (1973) tried to convert the portfolio problem into a linear programming model. Konno and Yamazaki (1991) proposed a new portfolio optimization model as an alternative to Markowitz mean-variance model. They employed L1 –Mean Absolute Deviation (MAD) - as a risk measure instead of variance, so they could overcome the problem of computational difficulty encountered in Markowitz Mean – Variance model. MAD model is said to be a viable alternative because it does not require the covariance matrix of the returns, and MAD portfolios have fewer assets (Simaan, 1997). It is also argued that as the number of the assets decreases the transaction costs of the portfolio will decrease either. MAD portfolio optimization model has 2T+2 rows where T is the time span of study.

Feinstein and Thapa (1993) reformulate the MAD portfolio optimization model so that the number of rows decreased to T + 2, which implies that the maximum number of the stocks invested in, decreases from 2T + 2 to T + 2.

Chang (2005) modified Feinstein and Thapa's model so that his model has fewer variables and the same number of constraints.

2.3 Limitation of Markowitz Portfolio model

Markowitz's portfolio optimization model, contrary to its historical reputation, has not been used extensively in its original form to construct a large – scale portfolio. One of the most significant reasons behind this is the computational difficulty associated with solving a large – scale quadratic programming problem with a dense covariance matrix. (Konno and Yamazaki, 1991)

Several authors tried to alleviate this difficulty by using various approximation schemes (Shape, 1967; 1971; Stone, 1973) in the early years of the history. Yet these efforts are largely discounted because of the popularity of equilibrium models such as Capital Asset Pricing Model (CAPM) and Asset Pricing Theory (APT) which are less computationally demanding.

However, one of the biggest criticisms of Markowitz's model, is that it does not produce portfolios that are adequately diversified. McLeod (1998) noted that portfolio managers believe that the Markowitz model gives unrealistic portfolios, which are not properly diversified. When the model was applied to a South African dataset he found only four out of seven indices were ever included, with one of them never having more than 3% of funds allocated.

Bowen (1984) noted that the Markowitz model required large volumes of data and found that it was difficult to estimate covariances. He doubted whether the predictions from the model would be reliable and concluded that 'semantic and statistical barriers exist that prevent the average businessman from coming to grips with the approach'.

In commenting on why the Markowitz optimization is not used more in practice, despite its theoretical success, Michaud (1989) gave the following possible reasons:

(i) the conceptually demanding nature of the theory;

(ii) the fact that most investment companies are not structured to use a mean-variance optimization approach

(iii) anecdotal evidence that portfolio managers find the composition of optimized portfolios counter-intuitive.

In a comparative study of the Markowitz model and the Sharpe model (using the critical line method), Affleck-Graves and Money (1976), noted interesting links between the two.

Their study used expected index portfolio returns and standard deviations and they observed that the results obtained with Sharpe's model became progressively better with every index that was added. It was further noted that if more portfolios are added, to the point that each share was its own portfolio, the model simulates the Markowitz model. Furthermore it was found that if very low upper boundaries (in terms of the percentage holding of any one share) were enforced on Markowitz's model, the one-index model was a close approximation of the optimal portfolio. Their study also found that Markowitz's model naturally limits the maximum weight invested in any one share to about 40 percent (if no upper boundaries are enforced) and has in the region of six shares in the efficient portfolio, which they felt gave it a natural diversification.

Sharpe (1963) simplified model for portfolio analysis, which he called the 'critical line method', showed that any set of efficient portfolios can be described in terms of a set of 'corner portfolios'. Adjacent corner portfolios are related in the following way: the one will contain either all the assets of the other, except for one, or all the assets of the other plus one additional one. This implies that moving along the minimum variance frontier from one corner portfolio to another will have the effect that one share is either added to the portfolio or removed from the portfolio. Individual securities can be examined to see whether they will either fall out of the portfolio or enter the portfolio. This radically reduced the computational effort of determining the portfolio with the maximum risk-reward ratio.

Tobin (1958) added risk-free asset to the set of risky assets re-defined the efficient frontier as a straight line. According to Tobin, all investors would select the optimal risky portfolio – the point where the straight line from the risk free rate is tangent to the efficient frontier. Individual investors will add more or less of the risk-free asset to their complete portfolios, according to their risk averseness. This implies that the only difference in approach amongst investors would be where they would position their portfolio along the straight line between the risk-free asset and the optimal risky portfolio. The straight line is therefore the capital allocation line (CAL).

3.0 CHAPTER THREE

PORTFOLIO MODELS AND METHODS

3.1 Measurement of risk

Risk plays a very important role in the decision-making process for both investors and companies, so it is important that the level of risk associated with investment can be quantified. Risk is measured by the standard deviation (σ) of return of a security, calculated using either the historical returns over time or the expected returns in the future.

3.2 Calculating risk and return using probability

The expected returns and standard deviation are given by the following formulae:

Let P_1…………..P_n be the probability of that n different outcomes, R_1…………..R_n are the corresponding returns associated with the outcomes then,

Expected return of a security $\left(\overline{R}\right)=\sum_{i=1}^{n} P_i \times R_i$ ……………………………..(3.1)

Standard deviation $(\sigma)=\sqrt{\sum P_i \times \left(R_i - \overline{R}\right)^2}$ ……………………………….(3.2)

Example

Table 3.1: details of returns of securities A and B

The probability of return on A is P_A and the corresponding return on A is R_A.

The probability of return on B is P_B and the corresponding return on B is R_B

Table 3.1 Returns of securities of A and B

SECURITY A		SECURITY B	
P_A	R_A(%)	P_B	R_B(%)
0.05	10	0.05	18
0.20	20	0.25	12
0.50	20	0.40	28
0.20	25	0.25	28
0.05	25	0.05	38
1.00		1.00	

The expected returns and standard deviations of the two securities are calculated as follows:

(i) Expected return of a security $\left(\bar{R}\right) = \sum_{i=1}^{n} P_i \times R_i$

Expected return of security of A:

$(0.05 \times 10) + (0.20 \times 12) + (0.50 \times 20) + (0.20 \times 25) + (0.05 \times 25) = 20.75$ percent

Expected return of Security of B:

$(0.05 \times 18) + (0.25 \times 12) + (0.40 \times 28) + (0.25 \times 28) + (0.05 \times 38) = 24$ percent

(ii) Standard deviation $(\sigma) = \sqrt{\sum P_i \times \left(R_i - \bar{R}\right)^2}$

Standard deviation of security A:

$((0.05 \times (10 - 20.75)^2) + (0.20 \times (20 - 20.75)^2) + (0.50 \times (20 - 20.75)^2)$

$+ (0.20 \times (25 - 20.75)^2 + (0.05 \times (25 - 20.75)^2)^{1/2} = 3.27$ percent.

Standard deviation of B:

$((0.05 \times (18 - 24)^2) + (0.25 \times (12 - 24)^2) + (0.40 \times (28 - 24)^2) + (0.25 \times (28 - 24)^2$

$+ (0.05 \times (38 - 24)^2)^{1/2} = 7.62$ percent.

Here we can see that while security B has a higher expected level of return compared to security A, it also has a correspondingly higher level of risk.

3.3 Calculating risk and return using historical returns

The mean and standard deviation of the annual returns of a security, calculated over a number of years (n), can be found using the following equations.

Mean return ($\bar{R}$) = $\dfrac{\sum_{i=1}^{n} R_i}{n}$

Standard deviation (σ) = $\sqrt{\dfrac{\sum_{i=1}^{n} (R_i - \bar{R})^2}{n}}$

Where n is the number of years and R_i is the expected returns for assets.

3.4 The concept of diversification

In order for investors to control and manage risk it is important for them to understand why risk exists in the first place. Therefore it is useful to consider that the overall level of risk that investors and companies face can be separated into systematic and unsystematic risk. Systematic risk also called (non – diversifiable, non-specific, unavoidable or market risk) represents how an investment return are affected by systematic factors such as business cycles, the application of tariffs, the possibilities of war. Systematic accounts for roughly 30 percent of an individual shares total risk.

Unsystematic risk (or diversifiable, specific, avoidable or non – market risk) is the risk specific to a particular security, that is the risk of the individual company performing badly or going into liquidation. This risk also accounts for approximately 70 percent of an individual share's total risk. Investor can progressively reduce unsystematic risk by spreading their investments over a larger number of different securities.

3.5 Portfolio possibilities set

Suppose an investor can invest in *n* different assets, associated with uncertain rates of return $R_1, \ldots\ldots\ldots\ldots, R_n$. The investor chooses a portfolio that combines the assets in a variety of proportions, or portfolios, or portfolio weights. The weights are nothing but the proportion of wealth invested in each available asset.

Let $w_1 \ldots\ldots\ldots.., w_n$ denote the weights. Note that the investor does not have to diversify in all available assets. However, if an asset is not in the portfolio, then its proportion is zero ($w_1 = 0$).

Short sales occur when an investor sells a security that he or she does not already own. If you have a short position in a security, then that security effectively has a negative weight in the portfolio ($w_1 < 0$) that is you hold a negative amount for this security. Therefore the rate of return on a portfolio is given as the weighted average of rates of return of the individual assets:

$$R_p = \sum_{i=1}^{n} w_i R_i = w_1 R_1 + ...+ w_n R_n$$

3.5.1 Expected return and risk for a portfolio of two investments

In developing the Markowitz model, we will take "risk" to mean standard deviation of return.

According to Markowitz, the expected (rate of return) and the corresponding variance of return of a portfolio of two investments, A and B are given as:

Let $E(R_P)$ = the expected return of the portfolio

$E(R_A)$ = the expected return of investment A

$E(R_B)$ = the expected return of investment B

σ_P^2 = variance of the portfolio return

σ_A^2 = variance of return for investment A

σ_B^2 = variance of return for investment B

σ_{AB} = the covariance between the returns of investment A and the returns of investment B

α = the proportion of the portfolio's value invested in investment A.

$$E(R_p) = \alpha E(R_A) + (1-\alpha)E(R_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.3)}$$

$$\sigma_p^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha)\sigma_{AB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.4)}$$

The parameter α has a value between 0 and 1. When α = 0, the funds are invested entirely in B, and when α = 1, the funds are invested entirely in A. When 0 < α < 1, the funds are invested partly in A and partly in B.

By definition, correlation coefficient (r_{AB}) = $\dfrac{\sigma_{AB}}{\sigma_A \sigma_B}$,

where σ_{AB} is covariance of returns A and B.

Replacing σ_{AB} by $\sigma_A \sigma_B r_{AB}$ where r_{AB} is the correlation coefficient between the returns of investment A and the returns of investment B, we have

$$\sigma_p^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha)\sigma_A \sigma_B r_{AB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

The variance of return and hence the standard deviation of return (risk) of the portfolio depends not only on the risk of the individual investments, but also on the extent to which their returns are correlated. The more negative the degree of correlation on, the greater the benefits of diversification and the lower the overall level of risk incurred. To illustrate this let us consider three situations:

(i) The returns of A and B are perfectly positively correlated (r_{AB} = + 1)

(ii) The returns of A and B are uncorrelated (r_{AB} = 0).

(iii) The returns of A and B are perfectly negatively correlated ($r_{AB} = -1$)

3.5.2 Perfectly positively correlated returns

From $\sigma_p^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha)\sigma_A \sigma_B r_{AB}$,

when r_{AB} = + 1, we get

$$\sigma_p^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha)\sigma_A \sigma_B$$

$= \{\alpha\sigma_A + (1-\alpha)\sigma_B\}^2$

$$\sigma_p = \alpha\sigma_A + (1-\alpha)\sigma_B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

Thus, in this case the risk of the portfolio as measured by the standard deviation of portfolio return is simply the value-weighted average of the individual risk of the component investments.

The expected return and risk of the portfolio in this case, for the range of values of α, are shown in the figure 3.1 below. There is a straight line joining A and B, indicating that there is risk averaging, but no benefits of diversification in terms of risk reduction. Expected return and risk both increases as α increases because B has a higher expected return and a higher risk.

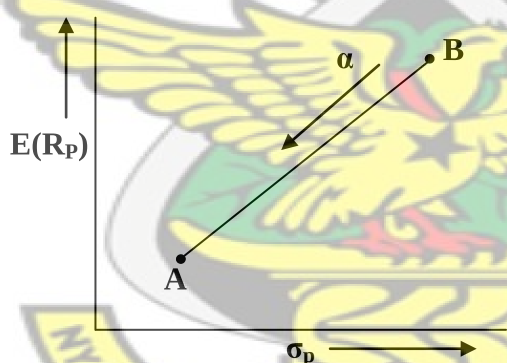

Figure 3.1: Positive correlation of returns

3.5.3 Uncorrelated returns

For uncorrelated returns $r_{AB} = 0$, we have

$$\sigma_p^2 = \alpha^2\sigma_A^2 + (1-\alpha)^2\sigma_B^2 + 2\alpha(1-\alpha)\sigma_A\sigma_B r_{AB},$$

but $r_{AB} = 0$

Hence $\sigma_p^2 = \alpha^2\sigma_A^2 + (1-\alpha)^2\sigma_B^2$

$$\sigma_p = \sqrt{\alpha^2\sigma_A^2 + (1-\alpha)^2\sigma_B^2}$$

This is less than $\sigma_p = \alpha\sigma_A + (1-\alpha)\sigma_B$

Except when α = 1 or α = 0, in which case the whole investment is undertaken in a single asset. Figure 3.2 below shows that a single line is obtained, but in this case the line is curved. As α decreases from 1, the portfolio risk is reduced initially, even though B is more risky investment than A, illustrating the benefits of diversification.

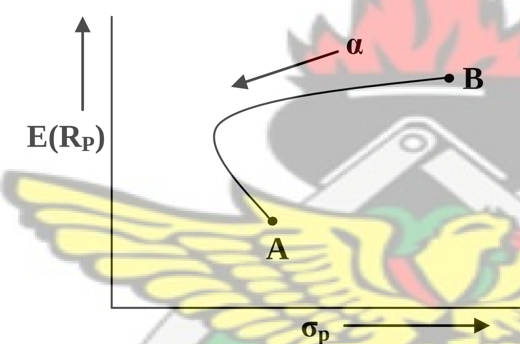

Figure 3.2: Uncorrelated returns

We can find the value of α for which the portfolio risk is minimized. This is the same as the value of α for which the variance of portfolio return is minimized.

Differentiating the equation; $\sigma_p^2 = \alpha^2\sigma_A^2 + (1-\alpha)^2\sigma_B^2$ with respect to α gives

$$\frac{\partial\sigma_p^2}{\partial\alpha} = 2\alpha\sigma_A^2 - 2(1-\alpha)\sigma_B^2$$

Setting $\frac{\partial\sigma_p^2}{\partial\alpha} = 0$ for minimum value, we obtain

$$0 = 2\alpha\sigma_A^2 - 2(1-\alpha)\sigma_B^2$$

$$0 = 2\alpha(\sigma_A^2 + \sigma_B^2) - 2\sigma_B^2$$

Hence

$$\alpha = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.7)$$

So this value of α gives the minimum risk portfolio.

3.5.4 Perfectly negatively correlated returns

For perfectly negatively correlated returns, $r_{AB} = -1$

Substituting it in the main equation (3.5),

$$\sigma_p^2 = \alpha^2\sigma_A^2 + (1-\alpha)^2\sigma_B^2 + 2\alpha(1-\alpha)\sigma_A\sigma_B r_{AB},$$

We get $\sigma_p^2 = \alpha^2\sigma_A^2 + (1-\alpha)^2\sigma_B^2 - 2\alpha(1-\alpha)\sigma_A\sigma_B,$

$$= \{\alpha\sigma_A - (1-\alpha)\sigma_B\}^2$$

and $\sigma_p = \{\alpha\sigma_A - (1-\alpha)\sigma_B\}$

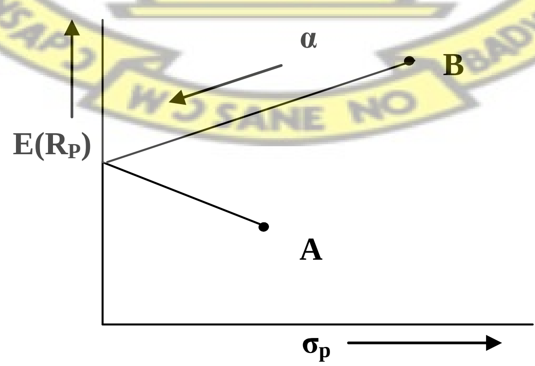

Figure 3.3: Negative correlated returns

The figure above shows that in this extreme and generally unrealistic situation, it is possible to choose a value of α for which $\sigma_p = 0$. For this value of α, the return from the portfolio is known with certainty because the variations in return from the two investments will exactly offset each other. The value of α for which $\sigma_p = 0$ is given by:

$$0 = \alpha\sigma_A - (1-\alpha)\sigma_B$$

$$\Rightarrow \quad 0 = \alpha(\sigma_A + \sigma_B) - \sigma_B$$

and $\alpha = \dfrac{\sigma_B}{\sigma_A + \sigma_B}$ ………………………………………….. (3.8)

3.5.5 Expected return and risk for a portfolio of many investments

The general formulae for the expected return and risk of a portfolio of *n* investments is given below:

Let $E(R_p)$ = the expected return of the portfolio

$E(R_i)$ = the expected return of the *i*th investment.

σ_p^2 = the variance of the portfolio return

σ_i^2 = the variance of the portfolio return

σ_{ij} = the covariance between the returns of *i*th and *j*th investments

x_i = the proportion of the portfolio's value invested in the *i*th investment.

$$\sum_{i=1}^{n} x_i = 1$$

and $x_i \geq 0$ if no short sales are allowed.

$$E(R_p) = \sum_{i=1}^{n} x_i E(R_i) \tag{3.9.1}$$

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} x_i x_j \sigma_{ij} \tag{3.9.2}$$

For n = 3, we have

$E(R_p) = x_1 E(R_1) + x_2 E(R_2) + x_3 E(R_3)$

$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + x_3^2 \sigma_3^2 + 2x_1 x_2 \sigma_{12} + 2x_1 x_3 \sigma_{13} + 2x_2 x_3 \sigma_{23}$

Where $x_1 + x_2 + x_3 = 1$.

There are now three variance terms and three covariance terms in the formula for the variance of portfolio return. Figure 3.4 below shows the expected return and risk of all possible portfolios consisting of three investments A, B and C, with each pair having small positive correlation, as is typical. Unlike the two investments case, where the possible portfolios lie on a single line, the possible portfolios cover the shaded area.

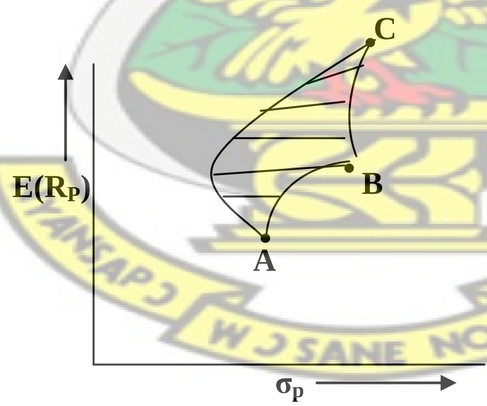

Figure 3.4 Expected return and risk for portfolios of A, B and C.

For n = 4, there are four variance terms and six covariance terms; for n = 5, there are five variance terms and ten covariance terms.

In general with n investments, there are n variances and $\frac{1}{2}\left(n^2 - n\right)$ covariance terms.

Therefore for a portfolio which are spread over a large number of investments, the number of covariance terms dominates the number of variance terms. Thus, the risk of the portfolio will depend more on the average covariance between investments than on the riskiness of the investments themselves.

3.6 Portfolio Optimization

Modern portfolio theory is based on the idea that investors seek high investment returns and wish to minimize their risk. Expecting higher returns within a lower level of risk is contradictory; therefore, constructing a portfolio requires a trade off between risk and return. Thus, investors must allocate their wealth among different securities, so called diversification.

Mean- variance optimization developed by Markowitz (1952) can be used in order to determine how an investor allocates his wealth among securities.

The proportion of securities in a portfolio depends not only on their means and variance, but on the interrelationships so called covariance. Thus, covariance between securities as well as returns and variances are calculated as an input in portfolio optimization. Markowitz portfolio theory uses equally weighted scheme calculating these input parameters. Once the input parameters are obtained, both the risk and the return on any portfolio consisting of security combinations are calculated as follows;

Let μ_p = portfolio return,

σ_p^2 = the variance on the portfolio

ρ_{ij} = the correlation coefficient between the assets i and j.

μ_i = return on asset i

$$\mu_p = \sum_{i=1}^{n} \mu_i x_i$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i x_i x_j \sigma_j \rho_{ij}$$

The goal of portfolio optimization is to find a combination of assets(x_i = portfolio weight of asset i) that minimizes variance of the portfolio return for any given level of expected return or identically, a combination of assets that maximizes the expected return of portfolio for any given level of risk.

3.7 Mathematical formulation of Markowitz model.

Markowitz' portfolio selection problem (Markowitz, 1952, 1959), also called the mean-variance optimization problem, can be formulated in three different ways. The Markowitz model, put forward in 1952, is a multi (two) objective optimization model which is used to balance the expected return and variance of a portfolio. Markowitz (1952) showed how rational investors can construct optimal portfolios under conditions of uncertainty. For an investor, the returns (for a given portfolio) and the stability or its absence (volatility) of the returns are the crucial aspects in the choice of portfolio. Markowitz used the statistical measurements of expectation and variance of return to describe, respectively, the benefit and risk associated with an investment. The objective is either to minimize the risk of the portfolio for a given level of return, or to maximize the expected level of return for a given level of risk.

Consider an investor who has a certain amount of money to be invested in a number of different securities with random returns. Let R_i denote the return in the next time period for each security i, $i = 1,2,..., n$, and estimates of its expected return, μ_i, and variance, σ_i^2 are given. Furthermore, for any two securities i and j, their correlation coefficient ρ_i^2 is also assumed to be known. If we represent the proportion of the total funds invested in security i by x_i , one can compute the expected return and the variance of the resulting portfolio $\mathbf{x} = (x_1, \ldots\ldots, x_n)$ as follows:

$$E[x] = x_1\mu_1 + \ldots\ldots\ldots\ldots\ldots\ldots + x_n\mu_n = \sum_{i=1}^{n} x_i\mu_i$$

$$Var[x] = \sum_{ij} \rho_{ij}\sigma_i\sigma_j x_i x_j = \sum_{i=1}^{n}\sum_{i=1}^{n} x_i x_j \sigma_{ij}$$

where

σ_{ij} = the coefficients of the (nxn) variance-covariance matrix, V, defined for stock i and stock j

($\sigma_{ii} = \sigma_i$ is the diagonal coefficients for the stock i)

The classical Mean-Variance (MV) model for minimizing variance and constraining the expected portfolio return yields at least a target value at the end of holding period is set out below (Markowitz, 1952, 1959).

Minimize $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}$ ………………………… …………………………(3.7.1)

Subject to $\sum_{i=1}^{n} x_i \mu_i \geq \beta$, β is the minimal rate of return required by investor………(3.7.2)

$\sum_{i=1}^{n} x_i = 1$, budget constraint, weight sum must be equal to 1…………….(3.7.3)

Or Maximize Z = $\sum_{i=1}^{n} x_i \mu_i$

Subject to $\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \leq \lambda$, λ is the minimum risk the investor is able to bear.

$0 \leq x_i \leq \mu_{x_i}$, $i = 1$, ………., n …………………………………………… (3.7.4)

μ_{x_i} is the maximum allowable amount for investment in stock i

Equation (3.7.2) indicates that the investor is expecting a return not below β. Equation (3.7.3) explains that the sum of fraction of each asset must be equal to 1.

Mathematically, this formulation produces a convex quadratic programming problem. There are non negativity constraints on the weights since short selling is expensive for individual investors and are not generally permissible for most institutional investors. Further, to ensure that the portfolio is diversified, the weight of a stock cannot exceed the upper limit. A feasible portfolio *x* is called efficient if it has the maximal expected return among all portfolios with the same variance, or alternatively, if it has a minimum variance among all portfolios that have at least a certain expected return. The collection of efficient portfolios forms the efficient frontier of the portfolio universe. Varying the desired level of return, β, in (3.10.2) and repeatedly solving the quadratic program identifies the minimum variance portfolio for each value of β . These are the efficient portfolios that compose the efficient set. The MV model form a quadratic programming problem and it requires the use of complex non-linear numerical algorithms to solve the portfolio problem. The practical application of such models was severely limited until computers were powerful enough to handle even the smallest problems. Sharpe (1971) commented that if the portfolio problem could be formulated as a linear programming problem, the prospect for practical application would be greatly enhanced.

3.8 Markowitz (Mean-variance model)

The Markowitz model, below, minimizes the variance of a given portfolio. It assumes that portfolios can be completely characterized by their mean return and variance (or risk). The portfolios solved for by this program map out the efficient frontier.

$$\text{Minimize } E[\sum_{j=1}^{n} x_j R_j - \sum_{j=1}^{n} r_j x_j]^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j \tag{3.8.1}$$

$$\text{Subject to } \sum_{j=1}^{n} r_j x_j = \rho M_o \tag{3.8.2}$$

$$\sum_{j=1}^{n} x_j = M_0 \tag{3.8.3}$$

$$\begin{gathered} x_j \leq u_j \\ j = 1,2...,n \end{gathered} \tag{3.8.4}$$

We are minimizing the variance in (3.8.1), where σ_{ij} is the covariance between assets *i* and *j*, x_i is the amount invested in asset *i*, x_j is the amount invested in asset *j*, and *n* is the number of assets in each portfolio. The constraints require that in (3.8.2) the total sum of the returns of each asset times the amount invested in that asset is equal to the minimum rate of return the investor wants times the total amount of money being invested, where

r_j is the average yearly return of asset *j*, ρ is the minimal rate of return required by the investor which is not portfolio dependent, M_o is the total amount of money being invested which is constant, and u_j is the maximum amount the investor wishes to place in a single stock.

In equation (3.8.3), the constraint says that the sum of the amount invested in each asset has to equal the total fund being invested. Equation (3.8.4) requires that the amount invested in each asset is less than or equal to the maximum amount the investor wants invested in each asset. Notice that we do not require that x_j is greater than or equal to 0, rather we want to allow short selling which is what x_j < 0 signifies.

3.9 Solving Markowitz

Using Lagrangian multipliers μ and λ we can find a solution of the Markowitz problem. The

formulation of the Lagrangian is:

$$L=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}w_i w_j \sigma_{ij}-\lambda\left(\sum_{i=1}^{n}w_i \bar{r}_i-\bar{r}\right)-\mu\left(\sum_{i=1}^{n}w_i-1\right), \qquad (3.9.1)$$

where $w_i=\frac{x_i}{M_0}$. Here, we changed variables from x_i to w_i for simplicity, but the solution does not depend on M_o. It follows that the upper bound on x_j is still u_j and there is no lower bound since we are allowing short-selling. Note that λ corresponds to constraint (3.8.2) and μ corresponds to (3.8.3) of the Markowitz formula.

After differentiating the Lagrangian with respect to w_i and setting the derivatives equal to zero, we get the following generalization: For a portfolio with *n* weights, two Lagrangian multipliers λ and μ, and mean rate of return $\bar{r}$, we have:

$$\sum_{j=1}^{n} \sigma_{ij} w_j - \lambda \bar{r}_i - \mu = 0 \quad \text{for } i=1,2,\dots,n \tag{3.9.1}$$

$$\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r} \tag{3.9.2}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{3.9.3}$$

Note: μ is not unrestricted in sign.

The solution to these equations produce a set of weights for an efficient portfolio with a mean rate of return $\bar{r}$, which is equivalent to ρM_o in equations (3.8.1)-(3.8.4). While there is not a closed form solution to (3.9.1) in linear algebra, it can be solved using numerical analysis techniques to solve systems of linear equations, such as LU matrix factorization.

3.10 Konno and Yamazaki model

Konno and Yamazaki (1991) introduced the L_1 risk function

(mean absolute deviation – MAD)

$$w(x) = E\left|\sum R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right| \tag{3.10.1}$$

instead of the L_2 risk (variance) function where R_j is a random variable representing the rate of return per period of the asset j. They proved that these two measures are the same if (R_1,…….,R_n) are multivariate normally distributed. So the Konno – Yamazaki MAD portfolio optimization model becomes as follows:

Minimize $$w(x) = E\left|\sum R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right| \quad (3.10.2)$$

Subject to $$\sum_{j-1}^{n} E[R_j]x_j \geq \rho M_0 \quad (3.10.3)$$

$$\sum_{j=1}^{n} x_j = M_0 \quad (3.10.4)$$

$$0 \leq x_j \leq u_j \qquad j = 1\ldots\ldots\ldots\ldots\ldots n$$

Konno and Yamazaki assumed that the expected value of a random variable can be approximated by the average from the data.

So:

$$r_j = E[R_j] = \frac{1}{T}\sum_{t=1}^{T} r_{jt} \quad (3.10.5)$$

Where r_{jt} is the realization of random variable R_j during period t (where t = 1…….T). Thus, w(x) is approximated by $\frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt} - r_j)x_j\right|$.

Denoting $a_{jt} = r_{jt} - r_j$ (j =1……..n and t = 1…………...T), model (3.10.2) can be expressed as follows.

Minimize $$\frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n} a_{jt} x_j\right| \quad (3.10.6)$$

Subject to $$\sum_{j=1}^{n} r_j x_j \geq \rho M_0 \quad (3.10.7)$$

$$\sum_{j=1}^{n} x_j = M_0 \quad (3.10.8)$$

$$0 \leq x_j \leq u_j \qquad j = 1\ldots\ldots\ldots\ldots\ldots n \quad (3.10.9)$$

Konno and Yamazaki replaced equation (3.10.2) with equation (3.10.4) which is equivalent to equation 3.10.3. Where

$y_t = \left| \sum_{j=1}^{n} a_{jt} x_j \right|$

Minimize $\dfrac{\sum_{t=1}^{T} y_t}{T}$ (3.10.10)

Subject to $y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0$ t = 1…………T (3.10.11)

$y_t - \sum a_{jt} x_j \geq 0$ t = 1…………T (3.10.12)

$\sum_{j=1}^{n} r_j x_j \geq \rho \, M_0$ (3.10.13)

$\sum_{j=1}^{n} x_j = M_0$ (3.10.14)

$0 \leq x_j \leq u_j$ j = 1……………n

We are minimizing the mean absolute deviation in (3.10.6), where a_{jt} is the yearly returns minus the average returns (r_{jt} - r_j) for asset j for each time t, where T is the time horizon.

The constraints require that in (3.10.7) the total sum of the returns of each asset times the amount invested in that asset is equal to the minimum rate of return the investor wants times the total amount of money being invested , where r_j is the average yearly returns of asset j, ρ is the minimal rate of return required by the investor which is not portfolio dependent, M_o is the total amount of money being invested which is constant, and μ_j is the maximum amount the investor wishes to place in a single asset.

In equation (3.10.8), the constraint implies that the sum of the amount invested in each asset should be equal to the total amount of money being invested.

Equation (3.10.9) requires that the amount invested in each asset is less than or equal to the maximum amount the investor wants invested in each asset.

According to Konno and Yamazaki the MAD portfolio optimization model's advantages over the Markowitz's model are

(i) this model does not use the covariance matrix which therefore does not need to be calculated.

(ii) the model is linear so solving this linear model is much easier than solving a quadratic model

(iii) the maximum number of assets that are invested in is $2T + 2$ while Markowitz's model may contain as many as n assets, where T is time period of historical data.

(iv) T can be used as a control variable to restrict the number of assets.

3.11 The efficient frontier

Every possible asset combination can be plotted in risk-return space, and the collection of all such possible portfolios defines a region in this space. The line along the upper edge of this region is known as the *efficient frontier* (sometimes "the Markowitz frontier"). Combinations along this line represent portfolios (explicitly excluding the risk-free alternative) for which there is lowest risk for a given level of return. Conversely, for a given amount of risk, the portfolio lying on the efficient frontier represents the combination offering the best possible return. Mathematically the **Efficient Frontier** is the intersection of the Set of Portfolios with Minimum Variance (MVS) and the Set of

Portfolios with Maximum Return. Formally, the efficient frontier is the set of maximal elements with respect to the partial order of product order on risk and return, the set of portfolios for which one cannot improve both risk and return.

The efficient frontier is illustrated below, with return μ_p on the *y*-axis, and risk σ_p on the *x*-axis; an alternative illustration from the diagram in the CAPM article is at right.

The efficient frontier will be convex – this is because the risk-return characteristics of a portfolio change in a non-linear fashion as its component weightings are changed. (As described above, portfolio risk is a function of the correlation of the component assets, and thus changes in a non-linear fashion as the weighting of component assets changes.) The efficient frontier is a parabola (hyperbola) when expected return is plotted against variance (standard deviation).

The region above the frontier is unachievable by holding risky assets alone. No portfolios can be constructed corresponding to the points in this region. Points below the frontier are suboptimal. A rational investor will hold a portfolio only on the frontier.

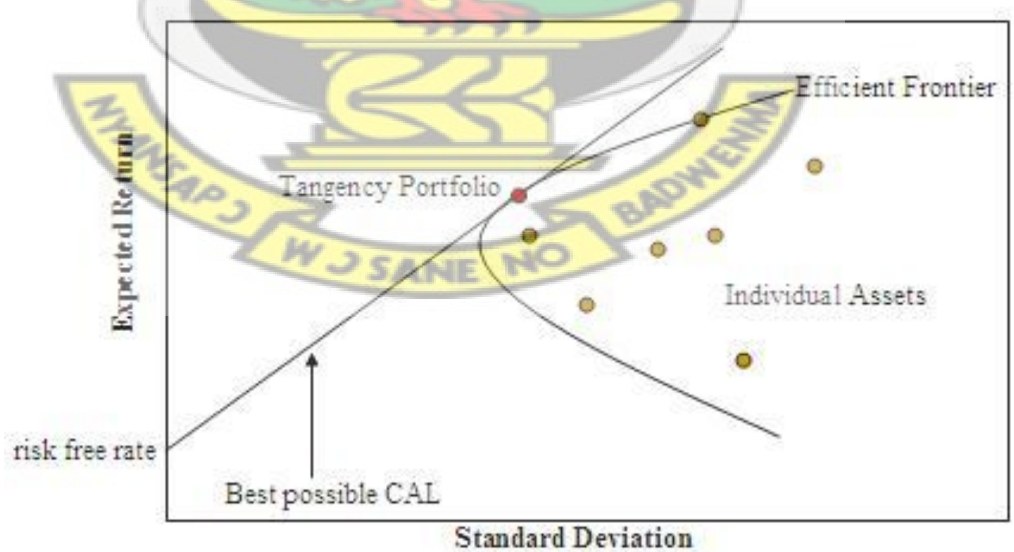

Figure 3.5 Efficient Frontier

3.12 Indifference curves and the optimum portfolio

An indifference curve is the locus of points at which the investor gets a particular level of satisfaction or utility from any combination of expected return and risk. For a risk-averse investor, indifference curves are concave, moving upward and to the right, indicating that the greater the amount of risk incurred by the investor, the greater the added expected return necessary to keep the investor equally satisfied. The steeper the slope of the curve, the more risk-averse the investor, because it indicates that a greater amount of extra return is required to compensate for an increase in risk. Clearly, investors will have their own set of indifference curves, depending on their individual trade-off between return and risk(i.e their own utility functions).

Fig 4 below shows three indifference curves U_1, U_2 and U_3. Curve U_1 gives the least amount of utility because it provides the highest risk for a given level of expected return or, alternatively, the lowest expected return for a given level of risk. Curve U_3 gives the greatest amount of utility.

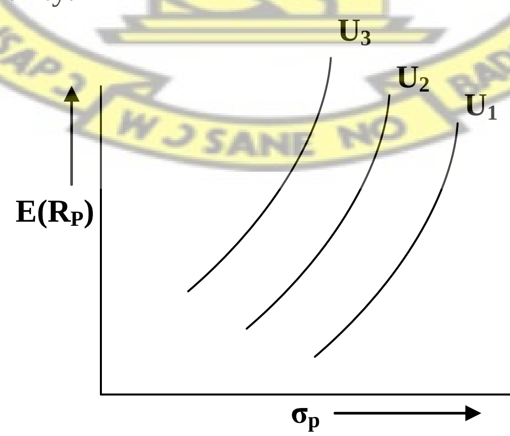

Figure 3.5 Indifference curves

3.13 The utility function

An utility function is a mathematical function that assigns a utility value to all possible portfolio returns. We use U(r) to denote the utility value associated with return R. This value measures the degree of subjecting satisfaction associated with return R.

Let w represents the investor's risk aversion, σ^2 be the variance E(r) be the expected return. The level of utility becomes

$$U = E(r) - w\sigma^2$$

If w = 0, the investor is risk-neutral, because the utility level the specific portfolio provides is independent from its risk. If the value of w approaches infinite, the investor will never invest in risky assets, and prefer the risk-free interest rate.

Instead of using variance, and therefore obtaining linear indifference curves upward sloping, one can use standard deviation and obtain standard parabola that open upwards.

3.14 The Revised Simplex Method

In many industrial applications, the simplex algorithm is used to solve Linear Programming problems with thousands of constraints and variables. It is an iterative procedure that provides a structured method for moving from one basic feasible solution to another, always maintaining or improving objective function until an optimal solution is obtained. However, this method is largely efficient when the variables involved are few. This is because, there are a lot of computation before the final optimal solution is obtained. An improvement of simplex method is *revised simplex algorithm* which seeks to address computational difficulties when the variables are many. (Amponsah, 2007)

Consider the following linear programming problem in standard matrix form:

Maximize $Z = C^T X$

Subject to $AX = B$

With $X \geq 0$

where X is the column vector of unknowns, including all slack, surplus, and artificial variables;

C^T is the row vector of corresponding costs; A is the coefficient matrix of the constraint equations; and B is the column vector of the right-hand side of the constraint equations. They are represented as follows:

$$X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}, C = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{pmatrix}, B = \begin{pmatrix} B_1 \\ B_2 \\ . \\ . \\ B_m \end{pmatrix}, 0 = \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & \dots & . \\ . & . & \dots & . \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

Let X_s = the column vector of basic variables, C_S^T = the row vector of costs corresponding to X_s, and S = the basis matrix corresponding to X_s.

3.14.1 Steps involve in the Revised Simplex Algorithm

Step 1: Entering Vector P_k:

For every nonbasic vector P_j, calculate the coefficient

$z_j - c_j = WP_j - c_j$ (maximization problem) or

$c_j - z_j = c_j - WP_j$ (minimization problem) where $W = C_S^T S^{-1}$.

The nonbasic vector P_j with the most negative coefficient becomes the entering vector (E.V), P_k.

If more than one candidate for E.V exists, choose one.

Step 2: Departing Vector P_r:

(a) Calculate the current basis X_s: $X_s = S^{-1}B$

(b) Corresponding to the entering vector P_k, calculate the constraint coefficients t_k:

$t_k = S^{-1} P_k$

(c) Calculate the ratio θ:

$$\theta = \min_i \left\{ \frac{(X_s)_i}{t_{ik}}, t_{ik} > 0 \right\}, i = 1, 2, \ldots, m$$

The departing vector (D.V.), is the one that satisfies the above condition.

Note: If all $t_{ik} \leq 0$, there is no bounded solution for the problem. Stop.

Step 3: New Basis:

$S_{new}^{-1} = E\, S^{-1}$, where $E = (u_1, \ldots, u_{r-1}, \eta, u_{r+1}, \ldots, u_m)$

Note, $\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \\ . \\ . \\ \eta_m \end{pmatrix}$, where $\eta_i = \begin{cases} -\dfrac{t_{ik}}{t_{rk}}, \text{if } i \neq r \\ \dfrac{1}{t_{rk}}, \text{ if } i = r \end{cases}$

and u_i is a column vector with 1 in the ith element and 0 in the other $(m - 1)$ elements.

Set $S^{-1} = S_{new}^{-1}$ and repeat steps 1 through 3, until the following optimality condition is satisfied.

$z_j - c_j \geq 0$ (maximization problem), or

$c_j - z_j \geq 0$ (minimization problem)

Then the optimal solution is as follows:

$X_s = S^{-1}B$; $Z = C_S^T X_s$

3.14.2 Illustration of the revised simplex algorithm

Maximize: $Z = 10x_1 + 11x_2$

Subject to $x_1 + 2x_2 \leq 150$

$3x_1 + 4x_2 \leq 200$

$6x_1 + x_2 \leq 175$

$x_1, x_2 \geq 0$

The problem is then put in standard form by introducing the slack variables x_3, x_4 and x_5

Maximize $Z = 10x_1 + 11x_2 + 0x_3 + 0x_4 + 0x_5$

Subject to

$$x_1 + 2x_2 + x_3 = 150$$

$$3x_1 + 4x_2 + x_4 = 200$$

$$6x_1 + x_2 + x_5 = 175$$

With all the variables nonnegative

$$P_1 = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix},\ P_2 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix},\ P_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, P_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, P_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},\ B = \begin{pmatrix} 150 \\ 200 \\ 175 \end{pmatrix}$$

Initialization: $X_s = (x_3, x_4, x_5)^T$; $C_S^T = (0, 0, 0)$

$$S = (P_3, P_4, P_5) = I = S^{-1} = \begin{pmatrix} 1\ 0\ 0 \\ 0\ 1\ 0 \\ 0\ 0\ 1 \end{pmatrix}$$

First iteration: The nonbasic vectors are P_1 and P_2

(a) Entering vector: $W = C_S^T S^{-1} = (0, 0, 0)I = (0, 0, 0)$

$(z_1 - c_1, z_2 - c_2) = W(P_1, P_2) - (c_1, c_2)$

$$= \quad (0,0,0)\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 6 & 1 \end{pmatrix} - (10,11) = (-10,-11)$$

Since the most negative coefficient corresponds to P_2, it becomes the entering vector (E.V).

(b) Departing Vector:

$$X_s = S^{-1}B = IB = B = \begin{pmatrix} 150 \\ 200 \\ 175 \end{pmatrix}$$

$$t_2 = S^{-1}P_2 = IP_2 = P_2 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$$

$$\theta = \min\left\{\frac{150}{2}, \frac{200}{4}, \frac{175}{1}\right\} = 50$$

Since the minimum ratio corresponds to P_4, it becomes the departing vector (D.V.)

(c) New basis:

$$\eta = \begin{pmatrix} \dfrac{-t_{32}}{t_{42}} \\ \dfrac{1}{t_{42}} \\ \dfrac{-t_{52}}{t_{42}} \end{pmatrix} = \begin{pmatrix} -2/4 \\ 1/4 \\ -1/4 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1/4 \\ -1/4 \end{pmatrix}; \quad E = (u_1, \eta, u_3)$$

$$S_{new}^{-1} = ES^{-1} = EI = E = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1/4 & 1 \end{pmatrix}$$

Summary of first iteration: $X_s = (x_3, x_2, x_5)^T$; $C_S^T = (0, 11, 0)$

Second iteration:

Now the nonbasic vectors are P_1 and P_4

(a) Entering Vector:

$$W = C_S^T S^{-1} = (0, 11, 0)\begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1/4 & 1 \end{pmatrix} = (0, 11/4, 0)$$

$$(z_1 - c_1, z_4 - c_4) = W(P_1, P_4) - (c_1, c_4)$$

$$= (0, 11/4, 0)\begin{pmatrix} 1 & 0 \\ 3 & 1 \\ 6 & 0 \end{pmatrix} - (10, 0) = (-7/4, 11/4)$$

Since the most negative coefficient corresponds to P_1, it becomes the entering vector (E.V).

(b) Departing Vector:

$$X_s = S^{-1}B = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1/4 & 1 \end{pmatrix}\begin{pmatrix} 150 \\ 200 \\ 175 \end{pmatrix} = \begin{pmatrix} 50 \\ 50 \\ 125 \end{pmatrix}$$

$$t_1 = S^{-1}P_1 = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1/4 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 3/4 \\ 21/4 \end{pmatrix}$$

$$\theta = \min\left\{-, \frac{50}{3/4}, \frac{125}{21/4}\right\} = 500/21$$

Since the minimum ratio corresponds to P_5, it becomes the departing Vector (D.V)

(c) New Basis:

$$\eta = \begin{pmatrix} \frac{-t_{31}}{t_{51}} \\ \frac{-t_{21}}{t_{51}} \\ \frac{1}{t_{51}} \end{pmatrix} = \begin{pmatrix} -\frac{-1/2}{21/4} \\ -\frac{3/4}{21/4} \\ \frac{1}{21/4} \end{pmatrix} = \begin{pmatrix} 2/21 \\ -1/7 \\ 4/21 \end{pmatrix}; \quad E = (u_1, u_2, \eta)$$

$$S_{new}^{-1} = ES^{-1} = \begin{pmatrix} 1 & 0 & 2/21 \\ 0 & 1 & -1/7 \\ 0 & 0 & 4/21 \end{pmatrix}\begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1/4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -11/21 & 2/21 \\ 0 & 2/7 & -1/7 \\ 0 & -1/21 & 4/21 \end{pmatrix}$$

Summary of the second iteration:

$$X_s = (x_3, x_2, x_1)^T; \quad C_S^T = (0, 11, 10)$$

Third Iteration:

Now the nonbasic vectors are P_5 and P_4.

(a) Entering Vector:

$$W = C_S^T S^{-1} = (0, 11, 0)\begin{pmatrix} 1 & -11/21 & 2/21 \\ 0 & 2/7 & -1/7 \\ 0 & -1/21 & 4/21 \end{pmatrix} = (0, 8/3, 1/3)$$

$$(z_5 - c_5, z_4 - c_4) = W(P_5, P_4) - (c_5, c_4)$$

$$= (0, 8/3, 1/3)\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} - (0,0) = (1/3, 8/3)$$

Since all the coefficients are non negative, the above step gives the optimal basis. The optimal values of the variables and the objective function are as follows:

$$\begin{pmatrix} x_3 \\ x_2 \\ x_4 \end{pmatrix} = S^{-1}B = \begin{pmatrix} 1 & -11/21 & 2/21 \\ 0 & 2/7 & -1/7 \\ 0 & -1/21 & 4/21 \end{pmatrix}\begin{pmatrix} 150 \\ 200 \\ 175 \end{pmatrix} = \begin{pmatrix} 1300/21 \\ 225/7 \\ 500/21 \end{pmatrix}$$

$$Z = C_S^T X_s = (0, 11, 10)\begin{pmatrix} 1300/21 \\ 225/7 \\ 500/21 \end{pmatrix} = 1775/3$$

3.15 Summary of the chapter

Among the models discussed so far, Konno and Yamazaki model for mean absolute deviation has been selected to be used in the formulation of the problem in chapter four.

4.0 CHAPTER FOUR

DATA ANALYSIS

4.1 Data collection

Databank Financial Services Limited has proven itself beyond every reasonable doubt that it is one of the institutions setting the pace when it comes to management of asset or investing in viable markets. The data for the thesis was collected from Databank Ghana Limited. It was annual report of Epack fund from 2000 to 2009. The data was secondary in cedis.

Let:	SCB	-	Standard Chartered ,
	GGL	-	Guinness Ghana Limited
	SSB	-	Social Security Bank,
	EI	-	Enterprise Insurance
	FAM	-	Fan Milk
	UG	-	Unilever Ghana Limited
	ALM	-	Alumworks

Below is a table showing the historical yearly return of each of the asset and the average return for each of the asset.

Table 4.1: Historical yearly returns of the Assets

YEAR	SCB	GGL	SSB	EI	FAM	UG	ALM
2000	0.2500	-0.0050	0.0060	0.0820	-0.0070	-0.1690	0.1860
2001	-0.1000	0.0000	0.0160	0.0350	0.0100	0.2140	-0.0050
2002	0.8200	0.0150	0.1770	0.1550	0.0840	0.2510	-0.0600
2003	3.2300	0.4600	1.7030	0.5900	0.2010	0.9230	0.0300
2004	10.9000	0.7050	0.6000	-0.2500	1.6200	0.7960	0.6000
2005	-0.6000	-0.1720	-1.0500	0.0410	-0.1000	-0.3500	-0.1370
2006	-0.6000	-0.1730	-1.0500	0.0410	-0.1000	-0.3500	-0.1370
2007	10.2000	0.3050	0.6500	0.4180	0.5900	0.6100	-0.1500
2008	12.0000	0.7700	0.1000	1.8400	2.1100	1.8900	-0.1000
2009	-8.0000	-0.6500	-0.9000	-0.9400	1.0500	-0.8750	0.1300
Average	**2.8100**	**0.1255**	**0.0252**	**0.2012**	**0.5458**	**0.2940**	**0.0357**

The average mean of each of the asset is then used to calculate the mean absolute deviation for the asset of each year.

The table below shows the mean absolute deviation of each of the asset in each year.

Table 4.2 Mean Absolute Deviation of the assets.

YEAR	SCB	GGL	SSB	EI	FAM	UG	ALM
2000	-2.5600	-0.1305	-0.0192	-0.1192	-0.5528	-0.4630	0.1503
2001	-2.9100	-0.1255	-0.0092	-0.1662	-0.5358	-0.0800	-0.0407
2002	-1.9900	-0.1105	0.1518	-0.0462	-0.4618	-0.0430	-0.0957
2003	0.4200	0.3345	1.6778	0.3888	-0.3448	0.6290	-0.0057
2004	8.0900	0.5795	0.5748	-0.4512	1.0742	0.5020	0.5643
2005	-3.4100	-0.2975	-1.0752	-0.1602	-0.6458	-0.6440	-0.1727
2006	-3.4100	-0.2985	-1.0752	-0.1602	-0.6458	-0.6440	-0.1727
2007	7.3900	0.1795	0.6248	0.2168	0.0442	0.3160	-0.1857
2008	9.1900	0.6445	0.0748	1.6388	1.5642	1.5960	-0.1357
2009	-10.8100	-0.7755	-0.9252	-1.1412	0.5042	-1.1690	0.0943

4.2 Data Analysis

The initial and final prices of each of the asset in a year were recorded as in Table 1. Thus the price of the asset in the January was recorded as initial and the price of the asset in December was recorded as final price in the year under consideration.

Historical yearly returns were then calculated for each of the asset. This was done by subtracting initial price from the final price of the asset in the year under consideration. Absolute deviations per year were then computed using yearly return of an asset and average return of the assets for all the years. Thus absolute deviation is the difference between yearly return of an asset and the average return of assets for all the years. (see the appendix)

4.3 Software used

This is a linear programming problem. Thus after linearizing Konno and Yamazaki absolute mean deviation model, optimization method is employed to get the desired result. Management Scientist Version 5.0 software is used to solve the problem. This is done by entering the coefficient of both the objective function and the constraints.

4.4 Model Formulation

Problem: Fund managers of Epack Investment plan to invest at most GH¢100,000.00 and demands a yearly return of at least 3% (GH¢3,000) and wishes that no asset will receive more than 75% of their budget (that is at most GH¢75,000.00).

Definition of variables

Let	x_1	=	Standard Chartered Bank
	x_2	=	Guinness Ghana Limited
	x_3	=	Social Security Bank
	x_4	=	Enterprise Insurance
	x_5	=	Fan Milk
	x_6	=	Unilever Ghana Limited
	x_7	=	Aluworks

From equation 3.10.3, Y_t is defined as

$y_t = \left| \sum_{j=1}^{n} \left(r_{jt} - \overline{r_j} \right) x_j \right|$ where $(r_{jt} - r_j)$ are the mean deviations. This implies that

$y_t = \sum_{j=1}^{n} \left(r_{jt} - \overline{r_j} \right) x_j$ or $y_t = -\sum_{j=1}^{n} \left(r_{jt} - \overline{r_j} \right) x_j$ for all j = 1, 2, ….., 7 and

t = (1, 2, ……, 10)

Now the formulation is as follows:

The objective function is to minimize the average absolute deviation i.e

Minimize $\frac{1}{10}\{Y_1 + Y_2 + Y_3 + \text{...........} + Y_{10}\}$

For the budget constraint,

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 100{,}000$

The return demand is formulated as

$2.81x_1 + 0.1255x_2 + 0.0252x_3 + 0.2012x_4 + 0.5438x_5 + 0.2940x_6 + 0.0357x_7 \geq 3{,}000.$

For the amount invested in each asset, the formulation is

$x_j \leq 75{,}000$, j = (1, 2 ……, 7).

So Minimize: $Z = \frac{1}{10}\sum_{t=1}^{10} y_t$

Subject to: $y_t - \sum_{j=1}^{7} P_{jt} x_j \geq 0$, j = (1, 2, …, 7), t = (1, 2, ..., 10) and P_{jt} are mean absolute deviations in **Table 4.2**

$y_t + \sum_{j=1}^{7} P_{jt} x_j \geq 0$, j = (1, 2, …, 7), t = (1, 2, ..., 10).

$\sum_{j=1}^{7} x_j = 100{,}000$

$\sum \pi_j x_j \geq 3{,}000$, π_j's are the average returns in **table 4.1** and j = (1, 2, …., 7)

$x_j \leq 75{,}000$, j = (1, 2, …., 7)

4.5 Computational Method

As stated earlier, The Management Scientist Version 5.0 will be used. It is a computer software package developed by David R. Anderson, Dennis J. Sweeney, and Thomas A. Williams. It is designed to solve the various quantitative models such as Linear programming problem, Transportation problem, Assignment problem and Integer linear programming problem for managerial decision making.

The specification of the computer used is 3.20 Ghz speed, 1 MB of memory and 80 GB hard disk size running on Service pack 2 operating system.

To sum up, the underlying principle behind this computer program is Revised Simplex Algorithm with LU decomposition which was discussed extensively in chapter **3.14**.

4.6 Result

Computer Output 1

OPTIMAL SOLUTION

Objective Function Value = 37908.6872

Variable	Value	Reduced Costs
X1	30898.5951	0.0000
X2	34740.4105	0.0000
X3	28549.5319	0.0000
X4	9085.3976	0.0000
X5	58255.4368	0.0000
X6	43134.7505	0.0000
X7	43134.7505	0.0000
X8	4621.1259	0.0000
X9	126666.8733	0.0000
X10	0.0000	0.0536
X11	0.0000	1.5169
X12	0.0000	0.0954
X13	0.0000	0.1622
X14	25998.4819	0.0000
X15	55357.7076	0.0000
X16	0.0000	0.1073
X17	18643.8105	0.0000

Computer Output 2

Constraint	Slack/Surplus	Dual Prices
1	0.0000	-0.1000
2	61797.1902	0.0000
3	0.0000	-0.1000
4	69480.8211	0.0000
5	0.0000	-0.1000
6	57099.0638	0.0000
7	0.0000	-0.1000
8	18170.7951	0.0000
9	116510.8736	0.0000
10	0.0000	-0.1000
11	0.0000	-0.1000
12	86269.5009	0.0000
13	0.0000	-0.1000
14	86269.5009	0.0000
15	9242.2519	0.0000
16	0.0000	-0.1000
17	253333.7466	0.0000
18	0.0000	-0.1000
19	0.0000	-0.0464
20	0.0000	0.0000
21	0.0000	-0.0178
22	0.0000	-1.0036
23	75000.0000	0.0000
24	75000.0000	0.0000
25	75000.0000	0.0000
26	49001.5181	0.0000
27	19642.2924	0.0000
28	75000.0000	0.0000
29	56356.1895	0.0000

Computer Output 3

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	0.0000	0.1000	1.0997
X2	0.0000	0.1000	0.7169
X3	0.0000	0.1000	0.5348
X4	0.0000	0.1000	0.2244
X5	0.0462	0.1000	0.1621
X6	0.0000	0.1000	0.5404
X7	0.0000	0.1000	0.5404
X8	0.0000	0.1000	0.2939
X9	0.0399	0.1000	0.1520
X10	0.0464	0.1000	No Upper Limit
X11	-1.5169	0.0000	No Upper Limit
X12	-0.0954	0.0000	No Upper Limit
X13	-0.1622	0.0000	No Upper Limit
X14	-0.0734	0.0000	0.0635
X15	-0.1951	0.0000	0.2252
X16	-0.1073	0.0000	No Upper Limit
X17	-0.0942	0.0000	0.1088

Computer Output 4

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	-30898.5951	0.0000	No Upper Limit
2	No Lower Limit	0.0000	61797.1902
3	-34740.4105	0.0000	No Upper Limit
4	No Lower Limit	0.0000	69480.8211
5	-28549.5319	0.0000	No Upper Limit
6	No Lower Limit	0.0000	57099.0638
7	-9085.3976	0.0000	No Upper Limit
8	No Lower Limit	0.0000	18170.7951
9	No Lower Limit	0.0000	116510.8736
10	-58255.4368	0.0000	No Upper Limit
11	-43134.7505	0.0000	No Upper Limit
12	No Lower Limit	0.0000	86269.5009
13	-43134.7505	0.0000	No Upper Limit
14	No Lower Limit	0.0000	86269.5009
15	No Lower Limit	0.0000	9242.2519
16	-4621.1259	0.0000	No Upper Limit
17	No Lower Limit	0.0000	253333.7466
18	-126666.8733	0.0000	No Upper Limit
19	-24632.0786	0.0000	0.0000
20	No Lower Limit	0.0000	0.0000
21	82038.7419	100000.0000	124820.9344
22	28841.3159	36000.0000	43881.7066
23	0.0000	75000.0000	No Upper Limit
24	0.0000	75000.0000	No Upper Limit
25	0.0000	75000.0000	No Upper Limit
26	25998.4819	75000.0000	No Upper Limit
27	55357.7076	75000.0000	No Upper Limit
28	0.0000	75000.0000	No Upper Limit
29	18643.8105	75000.0000	No Upper Limit

4.7 Discussion of Result

For clarity and consistency, x_1 …………….., x_{10} is used to represent y_1 ……….., y_{10} in the computer output, and x_{11} ………………, x_{17} is used to represent the seven assets as defined in **4.4**

The main decision variables are x_{11}…………, x_{17} which represents the seven assets of the portfolio. The objective function value is the optimal solution to the Epack Investment Fund. The solution minimize risk by GH¢37,908.69. Below the objective function value we have values of the decision variables in the optimal solution. $x_{11} = x_{12} = x_{13} = x_{16} = 0.0000$, $x_{14} = 25998.48$, $x_{15} = 55357.71$, $x_{17} = 18643.81$. This means that for optimal solution, fund manager of Epack Investment Fund should allocate an amount of GH¢25,998.48 to Enterprise Insurance(EI), GH¢55,357.71 to Fan milk(FAM) GH¢18,643.81 to Aluworks(ALM) and no amount to Standard Chartered Bank(SCB), Guinness Ghana Ltd(GGL), Social Security Bank(SSB) and Unilever Ghana limited(UG).

5.0 CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The study revealed that using historical data of asset returns can help one to determine the best allocation of resource and make appreciable returns. Analyzing the data collected from Databank financial services ltd from 2000 to 2009, the risk of allocating resources is quantified. The problem was minimizing variance (risk) of a portfolio. After solving the model, the value of the objective function in the optimal solution was GH¢37,908.69 which indicate the risk. Hence the risk has been quantified. From the optimal solution, it came up that an amount of GH¢25,998.48 should be allocated to Enterprise Insurance(EI), GH¢55,357.71 to Fan milk(FAM) GH¢18,643.81 to Aluworks (ALM) and no amount to Standard Chartered Bank(SCB), Guinness Ghana Ltd(GGL), Social Security Bank(SSB) and Unilever Ghana limited(UG).

5.2 Recommendation

Based on the results and findings of this study, the following recommendations are made.

- I recommend to the management of Databank Financial Service Limited at Adabraka, Accra to let their clients know the extent of risk (quantifiable risk) and to allay any fear entertained by clients before investing in a portfolio.
- About half of the amount invested was allocated to Fan Milk (GH¢55,357.71). I recommend that more funds should be allocated to consumable assets such as Fan milk and insurance companies such as Enterprise Insurance. However care

must be taking when investing in Banks.

- With changes in technology and dynamics in economic systems, I recommend that the services of experts should be engaged so that they will be come out with better analysis of past data before any investment is done.

REFERENCES

1. Affleck-Graves, J.F. and Money A.H. (1976). A comparison of two portfolio selection models. *The Investment Analysts Journal*, 7(4): 35-40.

2. Amponsah S.K.(2007). *Optimization Techniques*, UPK, KNUST, Kumasi page 42-137

3. Andrew, A., Booth P.M, Bloomfield D., England P. (1993). *Investment Mathematics and Statistics*, Graham and Trotman, U.K

4. Boron, R., Govindasami N. (1997). *Schaum's Outline of theory and problems of Operation Research*, 2nd edition, McGraw-Hill, U.S.A

5. Bowen P.A. (1984). A hypothesis: Portfolio theory is elegant but useless. The Investment Analysts Journal, 24(2): pg 17-21.

6. Chang, C. (2005), "A Modified Goal Programming Approach for the Mean Absolute Deviation Portfolio Optimization Model", Applied Mathematics and Computation, (Article In Press)

7. Denzil, W., Antony H. (2007). *Corporate Finance: Principles and Practice, Financial times/Prentice hall*, 4th edition, U.S.A

8. Feinstein, C.D. and Thapa, M.N. (1993), "Notes: A Reformulation of a Mean-Absolute Deviation Portfolio Optimization Model", Management Science, Vol.39, No.12, pp. 1552-1553

9. Konno,H. and Yamazaki H.(1991), "*Mean-Absolute Deviation Portfolio Optimization Model and its Applications to Tokyo Stock Market*", Management Science, Vol.37,No.5, pp.519-531

10. Markowitz, H.(1952), "Portfolio Selection", The Journal of Finance, Vol.7, No.1, pp.77

11. Michaud, R.O., (1989). The Markowitz optimization enigma: Is 'optimized' optimal? *Financial Analysts Journal*, 45(1): pg 31-42.

12. McLeod W. (1998). *Enhancements to the Markowitz Mean - Variance optimisation process of asset allocation*. Unpublished MSc dissertation. Cape Town: University of Cape Town.

13. Neumann, J.V. and Morgenstern O. (1944). *The theory of games and economic Behaviour*, Princeton, Princeton University press, U.S.A

14. Sharpe, W. (1971), "A Linear Programming Approximation for the General Portfolio Analysis Problem", Journal of Financial and Quantitative Analysis, pp.1263-1275

15. Stone, B. (1973) "A linear Programming Formulation of the General Portfolio Selection Problem", Journal of Financial and Quantitative Analysis, Vol 8, pp.621-636

16. Simaan, Y. (1997), "Estimation Risk in Portfolio Selection: The Mean Variance Model Versus the Mean Absolute Deviation Model, Management Science, Vol.43, No.10, pp.1437-1446

17. Tobin J. (1958). Liquidity Preference as Behavior Towards Risk. Review of Economic Studies 25 (1): pg 65–86.

18. Treynor, J. L. (1961). "Market Value, Time, and Risk". Unpublished manuscript dated 8/8/61, No. 95-209.

19. www.databankgroup.com

20. www.investopedia.com

21. www.ibbotson.com/research/papers/mutual.asp

22. www.wikipedia.org

23. www.osec.ch

APPENDIX

Prices of the asset from the beginning of the to the end of the year

	ASSET AND ITS MONTHY PRICE													
YEAR	**SCB**		**GGL**		**SSB**		**EI**		**FAM**		**UG**		**ALM**	
	JAN	DEC	JAN	DEC	JAN	DEC	JAN	DEC	JAN	DEC	JAN	DEC	JAN	DEC
2000	1.900	2.150	0.095	0.090	0.198	0.204	0.188	0.270	0.092	0.085	0.185	0.016	0.249	0.435
2001	2.150	2.050	0.090	0.090	0.204	0.220	0.270	0.305	0.085	0.095	0.016	0.230	0.435	0.430
2002	2.050	2.870	0.090	0.105	0.220	0.397	0.305	0.460	0.095	0.179	0.230	0.481	0.430	0.370
2003	2.870	6.100	0.105	0.565	0.397	2.100	0.460	1.050	0.179	0.380	0.481	1.404	0.370	0.400
2004	6.100	17.000	0.565	1.270	2.100	2.700	1.050	0.800	0.380	2.000	1.404	2.200	0.400	1.000
2005	17.00	16.400	1.270	1.098	2.700	1.650	0.800	0.841	2.000	1.900	2.200	1.850	1.000	0.863
2006	16.400	15.800	1.098	0.925	1.625	0.600	0.841	0.882	1.900	1.800	1.850	1.500	0.863	0.725
2007	15.800	26.000	0.925	1.230	0.600	1.250	0.882	1.300	1.800	2.390	1.500	2.110	0.725	0.710
2008	26.000	38.000	1.230	2.000	1.250	1.350	1.300	3.140	2.390	4.500	2.110	4.000	0.710	0.610
2009	38.000	30.000	2.000	1.350	1.350	0.450	3.140	2.200	4.500	5.550	4.000	3.125	0.310	0.440

Calculation of mean absolute deviation

Mean Absolute deviation = $r_{jt} - \overline{r_j}$, where j = 1, 2, ……., 7 and t = 1, 2, ……, 10 and $\overline{r_j}$ is the mean of an asset over the period.(10 years)

```
LINEAR PROGRAMMING PROBLEM

MIN 0.1X1+0.1X2+0.1X3+0.1X4+0.1X5+0.1X6+0.1X7+0.1X8+0.1X9+0.1X10

     S.T.

        1)  1X1-2.56X11-0.1305X12-0.0192X13-0.1192X14-0.5528X15-0.463X16+0.1503X17>0
        2)  1X1+2.56X11+0.1305X12+0.0192X13+0.1192X14+0.5528X15+0.463X16-0.1503X17>0
        3)  1X2-2.91X11-0.1255X12-0.0092X13-0.1662X14-0.5358X15-0.08X16-0.0407X17>0
        4)  1X2+2.91X11+0.1255X12+0.0092X13+0.1662X14+0.5358X15+0.08X16+0.0407X17>0
        5)  1X3-1.99X11-0.1105X12+0.1518X13-0.0462X14-0.4618X15-0.043X16-0.0957X17>0
        6)  1X3+1.99X11+0.1105X12-0.1518X13+0.0462X14+0.4618X15+0.043X16+0.0957X17>0
        7)  1X4+0.42X11+0.3345X12+1.6778X13+0.3888X14-0.3448X15+0.629X16-0.0057X17>0
        8)  1X4-0.42X11-0.3345X12-1.6778X13-0.3888X14+0.3448X15-0.629X16+0.0057X17>0
        9)  1X5+8.09X11+0.5795X12+0.5748X13-0.4512X14+1.0742X15+0.502X16+0.5643X17>0
       10)  1X5-8.09X11-0.5795X12-0.5748X13+0.4512X14-1.0742X15-0.502X16-0.5643X17>0
       11)  1X6-3.41X11-0.2975X12-1.0752X13-0.1602X14-0.6458X15-0.644X16-0.1727X17>0
       12)  1X6+3.41X11+0.2975X12+1.0752X13+0.1602X14+0.6458X15+0.644X16+0.1727X17>0
       13)  1X7-3.41X11-0.2985X12-1.0752X13-0.1602X14-0.6458X15-0.644X16-0.1727X17>0
       14)  1X7+3.41X11+0.2985X12+1.0752X13+0.1602X14+0.6458X15+0.644X16+0.1727X17>0
       15)  1X8+7.39X11+0.1795X12+0.6248X13+0.2168X14+0.0442X15+0.316X16-0.1857X17>0
       16)  1X8-7.39X11-0.1795X12-0.6248X13-0.2168X14-0.0442X15-0.316X16+0.1857X17>0
       17)  1X9+9.19X11+0.6445X12+0.0748X13+1.6388X14+1.5642X15+1.596X16-0.1357X17>0
       18)  1X9-9.19X11-0.6445X12-0.0748X13-1.6388X14-1.5642X15-1.596X16+0.1357X17>0
       19)  1X10-10.81X11-0.7755X12-0.9252X13-1.1412X14+0.5042X15-1.169X16+0.0943X17>0
       20)  1X10+10.81X11+0.7755X12+0.9252X13+1.1412X14-0.5042X15+1.169X16-0.0943X17>0
       21)  1X11+1X12+1X13+1X14+1X15+1X16+1X17=100000
       22)  2.81X11+0.1255X12+0.0252X13+0.2012X14+0.5438X15+0.294X16+0.0357X17>3000
       23)  1X11<75000
       24)  1X12<75000
       25)  1X13<75000
       26)  1X14<75000
       27)  1X15<75000
       28)  1X16<75000
       29)  1X17<75000
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