KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

A SURVIVAL ANALYSIS APPROACH TO ESTIMATING FUNDING LIQUIDITY RISK – THE CASE OF A GHANAIAN BANK

BY

Edem Boadum Ahiale

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FUFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF

MSc. ACTUARIAL SCIENCE

May 2016

DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc. Actuarial Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

Edem Boadum Ahiale Student(PG8729812)	....	....
Student	Signature	Date

Certified by:

Mr. Derrick Asamoah Owusu	....	....
Supervisor	Signature	Date

Certified by:

Prof. S. K. Amponsah	....	....
Head of Department	Signature	Date

DEDICATION

I dedicate this work to my family and Catherine Otoo for their support during the preparation of this thesis.

ABSTRACT

This paper presents a non-parametric survival model approach to estimating the run off profile of a bank product with uncertain cash flows. The most practical approach to measuring funding liquidity risk in banks is based on the individual bank's balance sheet (items of assets and liabilities) where inflows and outflows are compared to determine the cumulative cash shortfalls over future time periods. Steps are then taken to address any resulting funding gaps. The difficulty bank's face, however, is in assigning future cash flows related to products with indeterminate maturity. The focus of this study is to contribute to addressing this challenge using the product limit estimator developed by Kaplan and Meier. In view of the subject of the study being in monetary terms, measures are developed to address areas of possible divergence from the normal application of the product limit estimator. The paper then illustrates the framework using data set from a Ghanaian bank to estimate the empirical run off profile of a savings product over a 30 day period.

ACKNOWLEDGMENTS

I acknowledge the effort of the following towards the success of this study. First, to my supervisor, Mr.Derrick Asamoah Owusu for his immense contribution, criticisms and patience. And to all my course mates and family.

CONTENTS

DECLARATION i

DEDICATION ii

ACKNOWLEDGMENT iv

ABBREVIATION vii

LIST OF TABLES ix

LIST OF FIGURES x

1 INTRODUCTION 1

1.1 Background of the Study 1

1.2 Problem Statement 3

1.3 Research Objective 4

1.4 Methodology 5

1.4.1 Data Source 5

1.4.2 Population, Sample Size and Sampling Techniques 5

1.5 Significance of Study 5

1.6 Thesis Organization 6

2 LITERATURE REVIEW 7

2.1 Introduction 7

2.2 Funding Liquidity Risk 7

2.3 Solvency and liquidity 8

2.3.1 Bank for International Settlements on Liquidity Measurement and Run Off Rates 8

2.4 Liquidity Risk – monitoring framework 9

2.5 Basel Revised Run Off Rates 11

2.6 Stable and Volatile Balances 12

2.7 Customer Deposit Withdrawal Behaviour 12

2.8 Methods Employed in Addressing the Issue of Products with indeterminate Maturity 13

2.8.1 Replicating Portfolio Model 13
2.8.2 The Option Adjusted Spread (OAS) Models 14
2.8.3 Statistical Time Series Run Off Models 16
2.8.4 Stochastic Models 17
2.8.5 Retention Curve/Survival Models 17
2.9 Survival Analysis 18
3 METHODOLOGY 20
3.1 Introduction 20
3.2 Specifying the Subject of Study 20
3.3 Parametric, Non Parametric Arguments 21
3.3.1 Parametric Estimation Method 22
3.3.2 How Suitable is a Parametric Model 24
3.3.3 Non-Parametric Estimators 25
3.3.4 Justification for Model Choice 26
3.3.5 Suitability of Survival Analysis 26
3.4 The Kaplan – Meier Estimator 27
3.4.1 Assumptions 29
4 ANALYSIS and RESULTS 31
4.1 Introduction 31
4.2 Illustration of Processing Account Level Data 31
4.2.1 Obtaining Aggregate Level Data 32
4.2.2 Obtaining Individual Level Start Time 33
4.2.3 Dealing with Multiple Start Times on an Account 34
4.3 Obtaining Aggregate Level Start Time 35
4.4 Estimating Aggregate Level Run-Off Profile 35
4.5 Estimating the Funding Liquidity Risk 37
4.6 Benchmarking 38
4.6.1 Gaps of Liquidity 38
4.6.2 Considerations for Non-Maturing Liabilities 40

4.6.3 Principles for Managing Liquidity Risk 43
5 CONCLUSION and RECOMMENDATION 45
5.1 Summary of Results 45
5.2 Conclusions 45
5.3 Recommendations 46
APPENDIX 53

LIST OF ABBREVIATION

NoMALS .... Non Maturing Asset and Liability

PV .... Present Value OAS

.... Option Adjusted Spread

BIS ....Bank for International Settlement IMF

....International Monetary Fund

LCR .... Liquidity Coverage Ratio

ALM ....Asset Liability Management

NSFR .... Net Stable Funding Ratio

LIST OF TABLES

1.1 Funding Liquidity Risk(Balance Sheet) 2
1.2 Funding Matrix 3
4.1 Obtaining Aggregate Level Data 33
4.2 Individual Level Start Times 34
4.3 Survival Data 35
4.4 Summarised Kaplan Meier Table 36
4.5 Specimen Liquidity Matrix (in Run off mode) 38
5.1 Kaplan Meier Table 53
5.2 Mean Survival Time 53
5.3 Quantiles Estimation 53

LIST OF FIGURES

2.1 Basel III Run off rates (revised) for deposit products 11
2.2 Mechanism of an OAS model 15
2.3 Italian Deposit Run off Profile 17
2.4 Illustration of survival data 19
4.1 Individual Account Starting Point 34
4.2 The corresponding Kaplan-Meier Curve 37

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

The form of banking most popular in the world is Fractional Reserve Banking the practice where banks accept deposits from their customers (surplus units) and extend credit or make loans to other customers in need of funds (deficit units). In this practice, banks need to keep reserves to meet withdrawal requests of depositors (that are usually less than the amounts originally deposited).

Most often commercial banks earn little or nothing on reserves; meanwhile being profit seeking entities, the motivation to create credit and earn interest is rife. This means an inherent liquidity imbalance between their assets (typically mid to long term loans and overdrafts) and their liabilities (typically retail deposits and capital market debt). Any loss of confidence could trigger bank runs - unexpected increase in customer withdrawals from a depository financial institution at the same time because it is believed that the financial institution is, or might become, insolvent. During the early liquidity phase of the global financial crisis starting in 2007, many banks – despite meeting the existing capital requirements – experienced difficulties because they did not prudently manage their liquidity (BIS, 2014) . Ghana had its share of banks going bankrupt even before the global meltdown. In the year 2000, the Government of Ghana and the Central Bank closed down the Bank for Housing and Construction as well as the Corporative Bank on the back of losses and liquidity issues. The Social Security and National Insurance Trust had to bail out Meridian BIAO Bank when it went bankrupt. These developments reveal the fact that local banks are not completely shielded from future crisis stemming from an inability to meet their maturing obligations. Table 1.1 provides a general framework for assessing liquidity risk in banks. Funding liquidity risk arises from the liability side for either on balance sheet or off balance sheet items.

Table 1.1: Funding Liquidity Risk(Balance Sheet)

BALANCE SHEET	
Asset	Debt
Cash/cash like items eg.Tills	Unsecured Funding
Other Unencumbered Asset	Secured Funding
Other Encumbered Assets	
	Equity New Issues

Off BALANCE SHEET	
Asset	Liabilities
Derivatives	Derivatives
	Guarantee for Clients
	Commercial Papers

Focusing on the balance sheet items, liabilities can be classified into stable or volatile where these terms refer to the predictability of cash flows. Equity is the most stable although expensive. Debt follows – secured debt is more stable followed by unsecured debt. In the unsecured debt category retail deposits are more stable than money market deposits - under tight liquidity conditions, investors on the money market are more likely to exercise their option to call in their deposits than to roll over same. This information is an integral part of cash flow analysis by banks.

Banks use several quantitative methods/metrics to measure their liquidity risk including liquidity indices and other peer group comparisons such as borrowed funds/Total Assets, Deposit to Loan Ratio, Funding Gaps, Stress Tests, and Liquidity Coverage Ratio (Basel III). Most practical methods, however, start with forecasting daily inflows and outflows of cash. The process then considers unsecured funding sources and the liquidity characteristics of the asset inventory. Finally the information is then put together in a strategic perspective. This starts from current assets and liabilities as well as contingencies. The information is used to build a funding matrix. Any gaps should be covered by plans to raise additional funds either through borrowing, disposal of asset or further equity injection (depending on the time). Table 1.2 shows a hypothetical funding matrix and excludes off balance sheet items.

Table 1.2: Funding Matrix

Time Buckets(Upper Limits)		0N	1W	2W	1M	3M	6M	1Y	$>1Y$	Total
Main Inflows	Loans	24	48	66	102	150	240	390	540	1560
	Securities	330	60	60	30			30	30	900
	Cash and others	21								35
Main Outflows	Deposits	-6	-18	-30	-54	-120	-210	-354	-390	-1970
	Other funding	-18	-18	-18	-30	-36	-30	-66	-48	-460
	Bond				-78	-96	-120	-180	-462	-1590
NetFunding Requirement(NFR)		351	72	78	-30	-102	-120	-180	-330	-1495
Cumulative FundRequired(CFR)		351	423	501	471	369	249	69	-261	

Generating a maturity ladder (funding matrix) is no easy task especially of liabilities with indeterminate maturities. It would appear easy to arrive at it if the timing of cash flows associated with the inputs in all instances is known in advance. However, in practice this is not the case. The presence of certain items on a bank's balance sheet with uncertain cash flow timing presents a forecasting challenge. These items are referred to as Non Maturing Assets and Liabilities (NOMALs) or in other quarters items having indeterminate maturity.

1.2 Problem Statement

The major challenge faced by banks in assessing funding liquidity risk lies in how to assign cash flows to future time brackets especially for products with uncertain cash flows. Considering the sheer size of retail deposits on a bank's balance sheet; it is surprising that the banking sector is yet to come to a consensus on how to measure the retention rates (alternatively the run-off rates) of core deposits. This is in contrast to the approach taking by banks in valuing derivatives for instance. However in measuring funding liquidity risk, a bank's understanding of the liquidity risk features of products with uncertain cash flows is of significant importance (Jarrow and van Deventer, 1998). A bank should put in place liquidity management processes that are sufficient to meet its daily funding needs and cover both expected and unexpected deviations from regular activity. The question that needs to be answered is how are banks supposed to measure the run-off rates of products with uncertain cash flow timings? The answer to this question is the main focus of this study. Existing literature on quantitative models for measuring liquidity risk that attempted cash flow timing have mainly inferred run off profiles of unsecured debt products by studying the time series to show how the position evolved over time and not necessarily the time the product position stayed on the bank's books (Neu ,2007 and Vento and La Ganga, 2009). Other studies have tried using other financial instruments to mimic the cash flows inherent in bank savings product (Bardenhewer, 2006) or adopted estimation methods established for valuing other products (Jarrow and van Deventer, 1998). Yet there is another group that have attempted a time to event approach but have either assumed a distribution for account decrements (Poorman and Stern, 2012) or rather adopted a scenario based approach. This study adopts a simple non parametric time to event approach to measuring funding liquidity risk and avoids the complexity of a replicating portfolio or of a scenario based method.

1.3 Research Objective

The objectives of this study are:

1. To extend the boundaries of survival analysis to estimating the run-off profile of a bank liability product with uncertain cash flows in a bid to enhance funding liquidity risk management.

2. The study seeks to use observed decrements on a deposit product to obtain an empirical estimate of the survival distribution function – no prior assumption about the shape or form of the distribution is made.

1.4 Methodology

1.4.1 Data Source

The research will rely basically on primary data. Data will be obtained from accounts operated with the selected Ghanaian bank. Daily account transactions and balances collected will be processed to yield the information needed to arrive at thesis conclusion(s).

1.4.2 Population, Sample Size and Sampling Techniques

The first step of the study would focus on the selection of Thirty (30) accounts from the Ghanaian bank chosen. These accounts are selected purposively by the researcher. The thirty accounts were basically chosen to represent each of the 30 main branches of the said Ghanaian bank. To achieve this, simple random sampling will be adopted to select an account from each of the 30 branches.

1.5 Significance of Study

The study seeks to develop a framework for measuring the run off rate of nonmaturing bank products. This is of practical importance to the banking industry as funding liquidity risk is a key part of bank risk management and as such placed within the ambit of one of the most important committees in bank management - the Asset and Liability Committee (ALCO). Non maturing assets and liabilities account for a large portion of bank's balance sheet. The Bank of Ghana's Summary of Economic Data (2015) put total deposits of the banking industry at GHS35.8bn representing 64% of the industry's total balance sheet as at May 2015. Secondly, the study will also serve as a source of reference material to students, private and school libraries, as well as banking industry regulators. And also serve as a research paper for further research work.

1.6 Thesis Organization

This study is in five Chapters. Chapter one considers the Introduction of the study, its background, the framework for assessing funding liquidity risk, the problem Statement and the objective of the study. It also considers the justification for the study, the methodology and the thesis organization. Chapter two covers the review of available literature that is relevant to the study. Chapter three is devoted to the research methodology. Chapter four focuses on an illustration of the framework developed in chapter three. Chapter five then concludes.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The chapter is organized into the following sections: Funding Liquidity Risk; methods developed in the literature to address the issue of Products with Indeterminate Maturity, and Survival Analysis. This chapter highlights work already done on the subject and situates the objective of the paper.

2.2 Funding Liquidity Risk

There are three main liquidity notions - Central Bank Liquidity, Market Liquidity and Funding Liquidity (Nikolaou, 2009). Central Bank liquidity refers to the regulator's ability to provide the liquidity needed to get the entire financial system working effectively. Market Liquidity on the other hand refers generally to the ability of market participants to trade assets at short notice at minimal cost and without affecting the price significantly.

Whilst both central bank liquidity and market liquidity can be looked at from the macro level; Funding Liquidity is more of a micro level phenomenon and generally refers to an entity's (in this case a bank's) ability to meet their liabilities, unwind or settle their positions as they become due (BIS, 2010). Nikolaou, (2009) further noted that the three notions of liquidity are very much interconnected.

Risk connotes the probability that actual realisations of an economic agent will deviate from the expected (Machina and Rothschild, 1987). Thus the inability of a bank to service their future obligations as they fall due can be referred to as funding liquidity risk (IMF, 2008). The Basel Committee on Banking Supervision also introduces the element of efficiency of payment and post payment stability of the firm. The foregoing definitions imply liquidity as a time concept - the probability of an economic agent becoming illiquid is measured ahead with the liquidity characteristics of different time horizons usually being different (Matz and Neu, 2006). Banks by virtue of their activity of accepting deposits and granting credit are exposed to this risk. Gauthier et al. (2014) highlight how vulnerable leveraged institutions are to low cash holdings and short term debt.

2.3 Solvency and liquidity

Montes-Negret (2009) suggest no direct relationship between liquidity and solvency – an insolvent bank can be liquid or illiquid whilst a solvent bank be illiquid at times. The line between solvency and liquidity is blurred; however solvency can be viewed as a long run phenomenon whilst liquidity can be seen as a short run phenomenon.
Both concepts relate however to the ability to cover obligations with resources.

2.3.1 Bank for International Settlementson Liquidity Measurement and Run Off Rates

The Basel Committee on Banking Supervision (2010) released its framework for measuring and monitoring liquidity risk with a focus on two key ratios - Liquidity Coverage ratio and the Net Stable Funding Ratio – following its omission in Basel I and II. The purpose of the two (2) standard measurements was to introduce some robustness into liquidity profile of banks as well as promote long term resilience of banks. The liquidity coverage ratio (LCR) considers how banks can survive a 30 day liquidity stress scenario and requires that banks hold assets with high liquidity enough to cover net cash flows over the 30 day period. The Net Stable Funding Ratio reinforces asset liability management and requires that banks fund longer term resources with more stable liability. But the standard received criticism from industry players who maintained that it was not prudent to be 100% LCR compliant during periods of stress. Noting that it was more practical for banks to reach into their stock of high quality assets to address tight liquidity situations. The committee in response to the concerns released a statement in early 2012 relaxing the terms of compliance in times of liquidity stress. Subsequently, the Basel Committee on Banking Supervision introduced revisions to the LCR urging local regulators to see to a phased introduction of the standard from 2015. The standards makes prescriptions as to which asset categories qualify to be classified as high quality and further made uniform propositions regarding run-off and inflow rates for asset and liability classes during liquidity stress periods.

2.4 Liquidity Risk – monitoring framework

The earlier framework by the Basle Committee on Banking Supervision proposed a monitoring framework which are currently operational in many banks:

- Funding Matrix

This is the basis of the current study. Gaps are identified between inflows and outflows based on a bank's balance sheet and off balance sheet positions. The subject of interest is the cumulative gaps identified. Banks are then required to have contingency plans to address these gaps. Banks are required to report contractual cash and security flows in the relevant time bands based on their residual contractual maturity. Local Regulators are responsible for prescribing the the specific template, including required time bands, by which data must be reported. Supervisors should define the time buckets so as to be able to understand the bank's cash flow position. Possibilities include requesting the cash flow mismatch to be constructed for the overnight, 7 day, 14 day, 1, 2, 3, 6 and 9 months, 1, 2, 3, 5 and beyond 5 years buckets. Cash flows from existing derivative positions such as forwards and swaps should be integrated to the extent that their contractual maturities are relevant to the understanding of the cash flows. For implementation purposes banks are required to submit raw data to the regulator without any behavioural considerations and plans to bridge any gaps

- Concentration of Funding:

The idea behind this monitoring tool is to identify key sources of liquidity for the bank and ensure the sources are quite diversified. Banks are to identify large funding sources that are of such significance that withdrawal could be the cause of unstable operations. The monitoring is via three key metrics:

1. Funding liabilities sourced from significant counterparty to total balance sheet ratio which will allow the bank know the total proportion of its balance sheet being funded from the most significant counterparties

2. Funding liabilities sourced from each significant counterparty or product to total balance sheet ratio which will allow the bank have an overview of funding concentration from a single source or product

3. List of assets or liability amounts by significant currency The metrics require separate reporting at the minimum monthly intervals. The standard defines what significant counterparties, assets and currencies are. The proposal is a little relaxed in its implementation as it recognizes the difficulty banks and regulators will have trying to identify the actual counterparties behind certain types of debt.

- Available unencumbered assets

Places a focus on the bank's list of unencumbered assets with details on location, volumes, amounts and currency denomination. These assets serve a strategic purpose in that they can be used in securing funds in addressing cumulative funding gaps. Banks are required to report these assets under two main headings – unencumbered assets eligible for obtaining funds on the secondary market and those eligible for securing funding from the regulator.

securities received from customers that are eligible for assignment as security should be made. An estimation of relevant haircuts on eligible securities for secondary market and central bank borrowing is also required. It is strongly advised that banks and regulators do not use this metric as standalone but in conjunction with the funding gap report as by itself it does not provide an insight as to the level of secured debt against unencumbered assets.

• LCR by Significant Currency

This represents the ratio of high quality liquid asset in each significant currency to the total net cash outflow over a period of thirty (30) days. The framework provides a definition of what high quality liquid assets and significant currencies are. Regulators are encouraged to set minimum monitoring limits as the tool is not an international standard.

• Market Related Monitoring Tools

Banks and regulators are encouraged to be on top of current developments in the market as it relates to liquidity- market wide information and information on the financial markets as well as bank specific information. Banks are expected to be circumspect in their use of information and are encouraged to put the right interpretations on same. This monitoring mechanism is to be used on a continuous basis.

2.5 Basel Revised Run Off Rates

The revised framework further clarifies issues regarding application of the standard including regularity of reporting, issues of multiple currency as well as cross-border banking. Proposed run off rates are shown below:

Retail Deposits		
	Stable deposits fully insured by a robust , prefunded deposit insurance scheme	3%
	Stable deposits fully insured by other deposit insurance scheme	5%
	Less stable deposits (potentially including uninsured deposits from sophisticated or high net worth individuals, internet deposits and foreigncurrency deposits	10%
Unsecured Wholesale Funding		
	Stable deposits	5%
	Less stable deposits and other funding sources	10%

Figure 2.1: Basel III Run off rates (revised) for deposit products

2.6 Stable and Volatile Balances

Amounts placed in an accounts at a bank by customers is termed deposits and this is a liability which the bank has to pay upon demand (for demand deposits) or expiration of an agreed period. Deposits are used to fund assets and this means the need to match asset longevity with that of the liability that financed same is critical. The notion of stable (core) or unstable (volatile) deposits is central to the whole notion of funding liquidity risk especially in relation to products with uncertain cash flows. Core deposits are Low cost deposits not subject to frequent changes especially by way of withdrawals and as such does not re-price as quickly as other funding sources. Some banks have strict definitions as to what is core and what is volatile deposits; however these definitions do not in any way suggest that all deposits classified as core are necessarily stable. Secondly, what may pass as core deposit during normal times may not be easily obtained during periods of liquidity stress. It is therefore important for management to have clearly established procedures and policies for determining the volatility and composition of deposit structure. This ensures efficiency in the use of funds whilst making provisions for withdrawals. Deposit management programs aimed at increasing the volume of deposits must be pursued.

2.7 Customer Deposit Withdrawal Behaviour

In a web based survey; Takemura and Kozu (2010) concluded that the pattern of a customer's deposit can be inferred from some economic and psychological characteristics of the depositor. Using information from the Japanese market for instance, the researchers conclude that customer propensity to withdraw declined if customer was aware that deposits were insurance covered Using a multinational survey based on conjoint analysis, Boyle et al. (2015) concluded that depositors in a country with explicit deposit insurance had characteristics distinct from those of customers coming from countries without insurance cover. The paper also concluded that the introduction of deposit cover during a period of crisis could only address the fear of bank runs only partially. This study focuses on determining the run off profile of a deposit product without consideration of the social, economic or psychological forces driving such profiles.

2.8 Methods Employed in Addressing the Issue of Products with indeterminate Maturity

The literature on products with indeterminate maturity or non-maturing assets and liabilities (NoMALs) is quite varied. NoMALs represent a challenge to banks as the inflows and outflows are not certain. The presentation below is not in any way intended to suggest superiority in the order of presentation but rather an appreciation of efforts undertaken to address the challenge bank's face in measuring funding liquidity risk. The Replicating Portfolio Model and the Option Adjusted Spread Model are first presented and followed by other relevant works:

2.8.1 Replicating Portfolio Model

Using embedded options present in savings products, Bardenhewer, (2006) used plain vanilla instruments – money market instruments and bonds which are liquidly traded to construct a replicating portfolio deemed to have analogous features such as the timing of cash flows – i.e. the cash flows of the replicating portfolio match those of the savings product except for a certain margin. Assets are put into maturity buckets such that maturing assets in the replicating portfolio are replaced by another contract at par. The model aims at minimizing the margin volatility. A sample period is selected and the average of the tracking error between cash flows from the replicating portfolio and of the underlying savings product is calculated. This serves as an estimate for the expectation of the minimum margins volatility. The expectation can directly be used as a trend. The form of the trend function could be linear, quadratic or exponential and determination of the form can be estimated or be based on expert judgement. Bardenhewer specified a linear trend function as:

$$V_t = \beta_0 + \beta_1.\Delta_t + \sum k_i.(r_{i,t} - \bar{r}_i) + \delta.(cr_t - \bar{cr}) + \varepsilon_i \quad (2.1)$$

where $i \epsilon 1, \ldots, I$ represents maturity of buckets in months; V_t is total volume at time t; $r_{i,t}$ is the rate with maturity i at time t; $\bar{r}_i$ is the average interest rate with maturity i over estimation period; cr_t is the customer's rate at time t; $\bar{cr}$ is the average customer rate over estimation period; Δ_t is the time in months between time 0 and t; β, k_i and δ are parameters to be estimated and ε_i is the residual Once the trend function is determined the weights (non-negative and summing up to 1) of the different buckets are defined such that the yield of the replicated portfolio is similar to that of the savings product. An optimization formula is mostly used in determining the weights assigned to the buckets; however, Frauendorfer and Schurle (2007) minimized the expected downside deviation of the spread between yield on the replicating portfolio and that of the underlying product. Maes and Timmermans (2005) limited themselves to the use of the standard deviation of the spread. There are two approaches to defining the weights assigned to the buckets: The static approach which is criticized for failure to adjust weights as time passes and also assumes that future interest rates will evolve in the same manner as it did in the past. Second is the dynamic modeling approach which sought to address the short comings of the static approach – i.e. the approach meant that portfolio weights change over time with changes in NoMALs volume and yields (Frauendorfer and Schurle, 2007).

2.8.2 The Option Adjusted Spread (OAS) Models

The option adjusted spread is the residual of the market yield after deducting the bench mark interest rate (i.e. the yield spread added to a benchmark yield curve).

It is used to discount a security's payment such that it is equivalent to its market price taking into account embedded options via a dynamic pricing mechanism. The mechanism of an OAS model is indicated below:

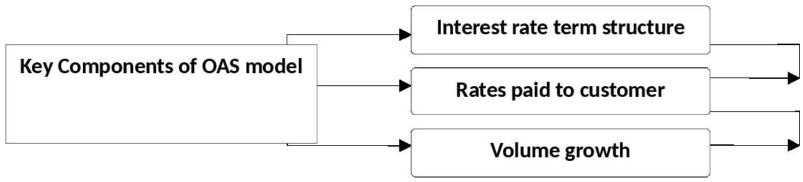


Figure 2.2: Mechanism of an OAS model

The models are premised on the fact that NoMALs have embedded options thus option pricing theory is applicable to them i.e. the embedded option is part of the present value (PV) of a NoMAL. Jarrow and van Deventer (1998) provide an approach to compute the PV of a savings product assuming no arbitrage and complete markets. They valued the savings product as an interest rate swap and showing that the present value can be written as ;

$$V(0) = \tilde{E}_0 \left(\int_0^{\tau} \frac{D(t)[r(t) - i(t)]}{B(t)} dt \right) \qquad (2.2)$$

where V(0) is the present value of the savings product at time 0 and $\tilde{E}_0(.)$ represents the unique martingale measure generated by the term structure of interest rate. The model assumes that deposit rate and volume depends only on money market information and suggest a positive relationship between money market rates and present value of the savings product. The approach specifies a strategy to hedge deposit liability on banks using investments in money market instruments and rolling over same – akin to the replicating portfolio method in this regard. The authors then define the withdrawal process and uses partial adjustment (an auto regressive process) to determine deposit rate stickiness.

Janosi et al. (1999) conducted an investigation into the JvD Model using deposit data over an 8 year period. The researchers concluded on the validity of the model after subjecting the data to time series analysis and with a term structure of interest rates modelled by the extended Vasicek model

2.8.3 Statistical Time Series Run Off Models

These models try to estimate a product with indeterminate maturity's run-of profile from the statistical distribution Neu (2007) and Vento and La Ganga (2009) . The focus is to estimate the stable portion NoMALs balance using a log linear time series regression to determine product run off at a given confidence level. Using Italian Banks' data, Vento and La Ganga (2009) used a mathematical framework of extremities to estimate maximum cash outflow for a product with uncertain maturity (sight deposit). Their approach was to estimate the core and volatile portion of current balance. The core balance is first determined and the volatile part is stated as the residual of the current balance after accounting for the core level. Core level of deposits, $\log V_{tt}$ is modelled using the equation;

$$LogV_t = \alpha + \beta.t - \sigma\sqrt{t}.\Phi^{-1}(cl) \qquad (2.3)$$

where $\alpha + \beta$ are estimates via linear regression of the amount time series; σ is the volatility of the balance sheet time series, $\Phi^{-1}(x)$ is the inverse cumulative normal distribution and cl is the confidence level.

Current value of NoMALs is assumed to be lognormally distributed and the geometric return of NoMALs also normally distributed. This information is used to determine a critical value for deposit growth for a given confidence level and used in conjunction with the geometric return to find the critical value corresponding to the core NoMALs amount. The resulting core deposits for the Italian banking system as at 31/05/2008 depicted a histogram in a run off mode.

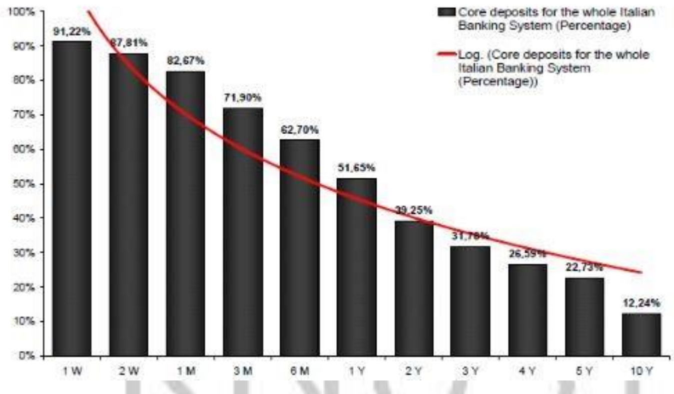


Figure 2.3: Italian Deposit Run off Profile

2.8.4 Stochastic Models

In their paper, Risk Management of Non Maturing Liabilities, Kalkbrener and Willing (2004) proposed a stochastic three factor model for liquidity and interest rate risk management of NoMALs founded on the three building blocks of Market Rates, Deposit Rates and deposit volumes. They proceed to state that the volume of deposits is specified by a stochastic process V(u) and define the process of minima as;

$$M(t) = \min_{0 \le u \le t} V(u) \tag{2.4}$$

Account decrements are assumed to follow a normal distribution (a suggested alternative is the lognormal distribution) and this is used to forecast future account balances. Several paths of account balance processes are simulated and in each scenario, M(t) is used to specify the minimal value in [0,t]. That is for each account, there is a simulated account balance path and at each t, the run off of the account path is obtained at a certain confidence level.

2.8.5 Retention Curve/Survival Models

So far the works of Poorman and Stern (2012) , Matz (2013) and Musakawa (2013) have attempted a time to maturity approach to addressing the problem of non maturing products on banks' books. In their study dubbed A New Approach To Analyzing Core Deposit Behaviour; Poorman and Stern (2012) relied on deposit pricing models, ALM models for estimating liquidity, income, and value metrics as well as U.S. GAAP to value core deposits. They grouped core deposits into product cohorts and use survival techniques to measure core deposits. They suggest the life of a deposit follows a Weibull distribution. Using South African Bank data and developed a scenario approach based on survival analysis to measure proportion retained on a savings account product. Both a normal liquidity state and a stressed state were superimposed and survival distribution obtained over a 30 day period using account level data. Matz, (2013) also used the proportion of account balances held over time to develop a retention curve.

2.9 Survival Analysis

Survival Analysis relates to data analysis methods that looks at time to the occurrence of some event of interest (Gardiner, 2010) . That is to say it refers to the measurement of time between two events. Time to event situations occur in several fields including demography, engineering, economics, and biostatistics and the terms failure time analysis, reliability analysis, duration analysis all refer to similar group of techniques. It is often possible in survival analysis for the event of interest not to be observed in all subjects. For instance patients in a clinical trial may be lost to follow-up, or the event may not have occurred at the end of the trial period (Dias et al. 2011) . These are censored cases but are still useful as they provide a lower bound for the actual non-observed survival time (Billingham et al. 1999) . Censoring is an integral part of survival analysis and its causative factors play a key role in inferential statistics. Notable censoring assumptions include Right Censoring – where the trial period ends before the desired event occurs in a subject. For instance in a trial where the event of interest is death; then a subject is considered censored if he is alive after the trial period. Left Censoring also relates to when the event start times are difficult to observe. Interval censoring is where it is only possible to indicate the interval in which the event of interest occurred and not actual observed time. Figure 2 below illustrates this; the other feature of survival studies is that observations may not all start at the same time this is the phenomenon of staggered entry.

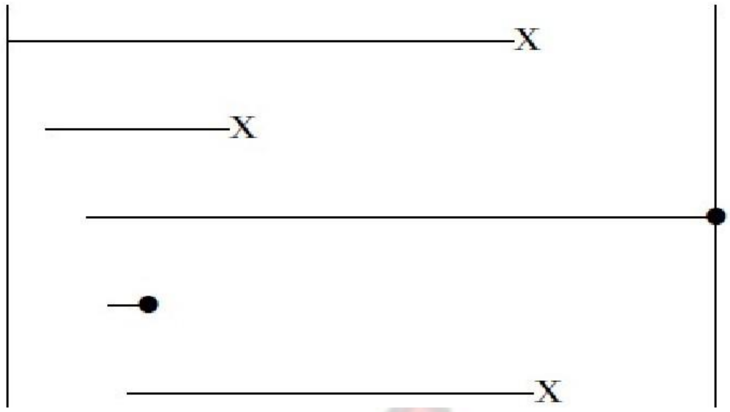


Figure 2.4: Illustration of survival data

• Censored cases

X Event

CHAPTER 3

METHODOLOGY

3.1 Introduction

Undoubtedly, funding liquidity risk is scenario based (BIS, 2010). However, the method adopted in this study does not seek to impose different scenarios on the measuring framework. The relevance of a complicated model in a stress situation is minimal; rather what is required is the availability of a robust contingency plan and the ease of applying all available liquid resources to manage a liquidity squeeze (Bardenhewer, 2006). The approach is to first define the subject of study followed by a brief on the Parametric and non-Parametric debate. Then a justification for the method of choice – Product Limit Estimator by Kaplan and Meier, (1958) - is provided; followed by specification of how the run-off profile will be estimated using the product limit estimator.

3.2 Specifying the Subject of Study

The focus of the study is a bank financial product as such subjects will be measured in monetary terms. Let a subject of study be $\frac{1}{100}$ of GHS1 (GHS is the ISO code for the Ghanaian Cedi – this implies one-hundredth of a GHS, which is 1 Pesewa). Such that if X_i denotes the balance on account *i* ; then if *i* has a balance of GHS25, then $X_i = 2500$ or if *i* has a balance of GHS15.25, then X_i This is similar to the approach adopted by Musakwa (2013).

Since funding liquidity risk is from the liability side of a bank balance sheet; a liability product with indeterminate maturity is used in the study – in this case Savings Account. Thus for each savings account the interest is the time that each subject stays on the bank's book denoted by T. Let X_t be the total amount outstanding (balance) on a savings product at time t. Also let $X_{i,t}$ be the balance on account i at time t. The equation below shows how the two variables are related:

$$X_t = \sum_i X_{i,t} \tag{3.1}$$

Some amount of money is normally expected to be on an account except for some unusual happening – for instance a bank run. Thus the need to define a constantly decreasing function of the total account balances from the start of the trial; this is done below:

$$X_t = \min_{0 \le s \le t} X_s \tag{3.2}$$

Equation(3.2) is similar in structure to that of Kalkbrener and Willing (2004) and Musakwa (2013). However they differ in their respective uses – whilst Kalkbrener and Willing used their version of the equation to determine the run off profile of simulated future account balances, Musakwa used it to determine run off profile on individual account balances. This study uses survival analysis to estimate the proportion S(t) of the total outstanding whose time on the bank's books exceeds t without imposing a distribution on S(t) . S(t) is defined as ;

$$S(t) = P(T \ge t) \tag{3.3}$$

3.3 Parametric, Non Parametric Arguments

The survival function of data which are right censored or interval censored can be estimated using a parametric estimator. In real life cases, however, a non-parametric method may be appropriate as the actual distribution is usually unknown (Zhao, 2008). Non-parametric estimator unlike the parametric case does not assume data follows a specified distribution. Parametric approaches are seen as biased estimators of the survival function whilst the non parametric estimators have been criticized for being too variable (Klein and Goel, 2013). The product limit estimator by Kaplan and Meier is the most commonly used non parametric method.

3.3.1 Parametric Estimation Method

There are possibly two ways to fit a parametric model to survival data. The first option may be to split the data and fit an individual or piecewise parametric model. The second option is to keep data in one piece and to and fit a parametric model to the entire dataset. A non-exhaustive list of parametric models are presented below:

Exponential Distribution

This is a relatively simple model as it involves the use of hazard function that remains same (fixed over time).The exponential distribution therefore has a single parameter, λ. The exponential model is a proportional hazards model, which implies the hazard of the event for an individual in one group at any time point is proportional to the hazard of asimilar individual in the other group – the treatment effect is measured as a hazard ratio.In order to be certain of the propriety of using this model, it is important to ascertain whether the hazard is likely to remain constant over an entire lifetime.

Weibul Distribution

The Weibull distribution can be parameterised either as a proportional hazards model or an accelerated failure time model. In an accelerated failure time model when two treatment groups are compared the treatment effect is in the form of an acceleration factor which acts multiplicatively on the time scale. Weibull models depend on two parameters– the shape parameter and the scale parameter. The Weibull distribution is more flexible than the exponential because the hazard function can either increase or decrease monotonically, but it cannot change direction. When considering the applicability of a Weibull distribution the validity of monotonic hazards must be considered.

Gompertz Distribution

The Gompertz distribution is akin to the Weibull on two grounds – first they both have two parameters (shape and scale parameters for the Gompertz); secondly, both distributions increase or decrease monotonically.The two distributions however have points of differences – the Gompertz distribution has a log-hazard function which is linear with respect to time, whereas the Weibull distribution is linear with respect to the log of time. Also, the Gompertz model can only be parameterised as a proportional hazards model. In order to be certain of the propriety of using this model, it is important to ascertain whether the hazard is monotonic over an entire lifetime.

Log-Logistic

As used in the literature on valuing products with indeterminate maturity, the loglogistic distribution is a failure time model and has a hazard function which can be non-monotonic with respect to time. It has two parameters. In order to be certain of the propriety of using this model, it is important to ascertain whether the hazard is non monotonic over an entire lifetime. Owing to their functional form, log-logistic models often result in long tails in the survivor function, and this must also be considered if they are to be used.

Log Normal

The log normal distribution is very similar to the log-logistic distribution, and has two parameters. The hazard increases initially to a maximum, before decreasing as time increases. As with log-logistic models, when considering the applicability of the log normal distribution the validity of non-monotonic hazards must be considered, and the validity of potentially long tails in the survivor function must be considered.

Generalised Gamma

The Generalised Gamma distribution is a flexible three-parameter model. It is a generalisation of the two parameter gamma distribution and it is useful because it includes the Weibull, exponential and log normal distributions as special cases,which means it can help distinguish between alternative parametric models.

Other Models

There are other less robust but flexible models available – such as Royston and Parmar's spline-based models. These are flexible parametric survival models that resemble generalised linear models with link functions. In simple cases these models can simplify to Weibull, Log-logistic or log normal distributions – which demonstrates their flexibility and usefulness in discriminating between alternative parametric models. Jackson et al. (2010) discuss and implement other flexible parametric distributions, such as the Generalised F - which has four parameters and which simplifies to the Generalised Gamma distribution when one of those parameters tends towards zero – as well as Bayesian semi-parametric models which allow an arbitrarily flexible baseline hazard, and which are extrapolated by making assumptions about the future hazard (ideally based upon additional data or expert judgement).

3.3.2 How Suitable is a Parametric Model

Quite a number of methods are available to test the suitability of a fitted model to the survival data. The easiest or straight forward approach is by visual inspection. That is to say for instance how closely does the fitted model follow the Kaplan Meier by mere observation? Whilst it is an easy approach; it is however subject to inaccuracy. An alternative is to use log cumulative hazard plots which can be constructed to show the observed hazards in the trial. Whether or not hazards are monotonic can be assessed using these plots well as test the reasonableness of the proportionality assumption under the proportional hazards method. Another alternative is the use of Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC)provide to statistically test the relative fit of alternative parametric models (Collett,2003).

3.3.3 Non-Parametric Estimators

When the empirical data is incomplete (truncated or censored), raw empirical estimators will not produce good results. In this scenario, there are two techniques available to determine the distribution function based on the data. The KaplanMeier product limit estimator can be used to generate a survival distribution function. The Nelson-Aalen estimator can be used to generate a cumulative hazard rate function.The Kaplan-Meier is the most commonly used estimator of the survival function, while the Nelson-Aalen is an alternative estimator for the same function. The Nelson–Aalen estimator is a non-parametric estimator of the cumulative hazard rate function in case of censored data or incomplete data. It is used in survival theory, reliability engineering and life insurance to estimate the cumulative number of expected events. An "event" can be the failure of a non-repairable component, the death of a human being, or any occurrence for which the experimental unit remains in the "failed" state (e.g., death) from the point at which it changed on. The estimator is given by

$$\tilde{H}(t) = \sum_{t_i \le t} \frac{d_i}{n_i}$$

with d_i the number of events at t_i and n_i the total individuals at risk at t_i The Kaplan–Meier estimator, also known as the product limit estimator, is a nonparametric statistic used to estimate the survival function from lifetime data. In medical research, it is often used to measure the fraction of patients living for a certain amount of time after treatment. In other fields, Kaplan–Meier estimators may be used to measure the length of time people remain unemployed after job loss the time-to-failure of machine parts. An important feature of the Kaplan–Meier curve is that the method can take into account some types of censored data, particularly right-censoring, which occurs if an object of study withdraws from a study, is lost to follow-up, or exists.

3.3.4 Justification for Model Choice

There are many asymptotic results for these estimators in the literature. In particular, it is known that they are asymptotically equivalent. On the other hand empirical results comparing these estimators are difficult to obtain. However, Colosimo et al. (2002) used Monte Carlo simulations to compare both estimators and concluded that for percentile estimation the Kaplan-Meier estimator presents a better performance for decreasing failure rates relative to the Nelson-Aalen estimator.

The curvature of the Nelson–Aalen estimator also gives an idea of the hazard rate shape. A concave shape is an indicator for infant mortality while a convex shape indicates wear out mortality. This feature of the Nelson-Aalen estimator will make for difficult interpretation/application to non-life data. The shape of the Kaplan Meier curve is not subject to different interpretations hence its adoption.

There are however limitations to the Kaplan–Meier estimator. A potential error is if a competing event were to rise. For example, if a doctor was using the Kaplan–Meier estimator to follow a patient with a malignant brain tumor and the patient were to die, the estimator could no longer be used (Jager et al. 2008). The event of interest is clearly defined and will not be subject to a competing event in non-life situations.

3.3.5 Suitability of Survival Analysis

The study adopts survival analysis approach to measure funding liquidity risk; however, survival analysis models were originally developed to model lifetime data not cash flows. It is therefore important to examine how suitable it is to apply survival analysis to model cash flows.

As indicated by Musakwa (2013), there are similarities between life time modelling and cash flow analysis that allow for applying survival analysis to both. They both model the length of time it takes for a subject to remain in a particular state. In lifetime modelling the variable of interest may be time to ’death’ whilst in cash flow modelling this may be time before withdrawal.

Secondly, both cash-flow and lifetime modelling are censored: that is, only partial information on the ’survival’ time is known for some of the subjects under observation. Generally, censorship is endemic in an experimental design where ’survival’ times are observed over a limited experiment period.

There are however areas of divergence – a key difference centres around the time origin to base 'survival' analysis for cash flow modelling purposes is unclear unlike the case with lifetime modelling. Section 3.4.1 shows the approach adopted to handle the time origin problem in the context of modelling cash-flow timing.

3.4 The Kaplan – Meier Estimator

The Kaplan-Meier estimator (also known as the product limit estimator) is widely used in medical studies to estimate patient survival rates. It uses information on those who die during the trial period and those who survive during the trial period and based on a mathematical formula derives estimates of subjects alive at any point in time (Kaplan and Meier 1958). The subjects under study need not stay their lives through the entire period of the trial (Right Censoring). The estimate is plotted over a period of time - Kaplan Meier Curve. Whilst the original trial related to actual survival with death being the event; the "event" may be any event of interest (Lex et al, 2012) .

The KM Survival estimate is mostly summarised using the median. However for certain areas of study (eg. Health Economics) it is appropriate to estimate the mean. Miller (1981) suggests three approaches to computing the mean:The area under the curve approach, the restricted mean approach, and the variable upper limits approach. Let $t_1 < t_2 < t_3 \ldots$ denote the ordered times subjects leave a bank's books via withdrawal. Also let $d_1, d_2, d_3 \ldots$ denote the number of subjects leaving the bank's books via withdrawal and let $x_1, x_2, x_3 \ldots$ be the corresponding number of subjects remaining (balance) on the bank's books such that $x_2 = x_1 - d_1, x_3 = x_2 - d_2$, etc.

Then $S(t_2) = P(T > t_2)$ the "Probability of a subject staying on a bank's book beyond time t_2" depends conditionally on $S(t_1) = P(T > t_1)$ "Probability of surviving beyond time t_1. Also $S(t_3) = P(T > t_3)$ "Probability of a subject staying on a bank's book beyond time t_3" depends conditionally on $S(t_2) = P(T > t_2)$ "Probability of a subject staying on a bank's book beyond time t_2". This recursive relationship could be used to derive a numerical estimate $\hat{S}(t)$ of the true survival function S(t).

For any $t\epsilon[t_0, t_1),\ S(t) = P(T > t) = 1$, because there has been no withdrawal. Thus for all t in this interval, let S(t)=1

Also for any two events A and B, $P(A^{\mathrm{T}}B) = P(B|A) * P(A)$ If it is assumed that A = "subject stays on bank's books to time t = 1 " and B = "subject stays on bank's books from time t_1 to beyond some time t before t_2 i.e."$T > t$".

For any $t\epsilon[t_1, t_2)$

$$S(t) = P(T > t)$$

$$= P(stay\ on\ books\ in[0,t_1))*P(stay\ on\ books\ in[t_1,t]|stay\ on\ book\ in[0,t_1)) \quad (3.4)$$

| {z } | {z }

where, S(t) = p(stay on banks books beyond some time t) The estimator S(t) is given by:

$$\hat{S}(t) = 1 * \frac{x_1 - d_1}{x_1} \quad (3.5)$$

or

$$\hat{S}(t) = 1 - \frac{d_1}{x_1} \quad (3.6)$$

similarly $t\epsilon[t_2, t_3)$,

$$S(t) = P(T > t)$$

$$= P(stay\ on\ books\ in[t_1,t_2))*P(stay\ on\ books\ in[t_2,t]|stay\ on\ book\ in[t_1,t_2))$$

| {z } | {z }

(3.7)

Thus, the estimator of S(t) is given by

$$\hat{S}(t) = 1 - \frac{d_1}{x_1} * \frac{x_2 - d_2}{x_2} \quad (3.8)$$

or

$$\hat{S}(t)\left(1 - \frac{d_1}{x_1}\right)\left(1 - \frac{d_2}{x_2}\right) \quad (3.9)$$

In general, for $t\epsilon[t_j, t_{j+1})$, j=1,2,3,...,

$$\hat{S}(t)\left(1-\frac{d_1}{x_1}\right)\left(1-\frac{d_2}{x_2}\right)...\left(1-\frac{d_j}{x_j}\right)=\prod_{J=1}\left(1-\frac{d_j}{x_j}\right) \tag{3.10}$$

This is the Kaplan-Meier estimate of the survival function S(t).

3.4.1 Assumptions

Below is a summary of four key assumptions underlying the product limit estimator:

1. only two mutually exclusive and collectively exhaustive states exist: "censored" or "event" (the "event" can also be referred to as "failure"). In this study, a case of withdrawal from the account is considered "failure". Any other case of exit from an account is considered a censored case. Therefore transfer to other product classes or charges on account are considered censored cases.

2. Event and censorship times should be clearly defined and measurable. The product limit method requires the survival time to be recorded precisely rather than simply recording whether the event occurred within some predefined interval.

3. As much as possible, left-censoring should be minimized or avoided. Leftcensoring occurs when the starting point of an experiment is not easily identifiable. Unlike in life experiments where the start date is easy to determine; this study focusses on cash flow modeling using multiple account information. Example start times in life studies include date of birth, experiment start date, or date on which a certain state is achieved. For cash flow modeling using multiple account data, the start date may be different for each account. The analysis is at aggregate level therefore an aggregate level start date is required which will take into consideration as many observations as possible. Let t_{0i} denote the time origin of subjects in account i from base date bt (base date in this study is the 180^{th} day as the study covers daily

account data over a 6 month period) and let $X_{i,t_{0i}}$ be the balance on account i at time t_{0i} The start time for account i occurs at $X_{i,t_{0i}}$ defined as

$$X_{i,t_{0i}} = Max(X_{i,t_1}, X_{i,t_2}, X_{i,t_3}, \ldots, X_{i,t_{180}}) \tag{3.11}$$

Where X_{i,t_1} represents historical balances on account i from and including the base date and $j = 1,2,3,\ldots,180$. with the start time t_{0i} for account i and the corresponding outstanding amount $X_{i,t_{0i}}$ at that time obtained; let M be the total number of accounts used in the study such that $i = 1,2,3,\ldots,M$. Then the average start $\bar{t}_0$ which corresponds to the aggregate level start time is;

$$\bar{t_0} = \frac{\sum_{i=1}^{M} t_{0_i} X_{i,t_{0_i}}}{\sum_{i=1}^{M} X_{i,t_{0_i}}} \tag{3.12}$$

The aim of defining the aggregate level start time from several account start times is to allow for as many observations of the event of interest as possible.

4. Assumption 4: The Kaplan-Meier estimator assumes independence of the subjects of study (Breslow and Crowley, 1974 ; Gill, 1980). This study also assumes independence of subjects under study. However with subjects grouped into accounts and such accounts having the same owner; subjects within an account may tend to be correlated. The effect of such intra-account correlations, however, is not to alter the ‘survival’ function estimates, but its variance (Williams, 1995; Williams, 2000). It is however safe or reasonable to assume independence in the case of subjects belonging to different account owners (Musakwa , 2013).

CHAPTER 4

ANALYSIS and RESULTS

4.1 Introduction

The overall aim of this chapter is to illustrate how to use the framework developed so far to estimate the run-off profile over a thirty day period for a retail savings product in a Ghanaian bank. The chapter begins with a section that focusses on how data on each account was collected and subsequently organized to obtain aggregate level data. Attention is then focused on how to obtain aggregate level start time from account level start times and finally how aggregate level data is used to achieve the product run off profile. This estimated profile should only be seen as a run off profile estimation method and should not be deemed applicable to other product classes with indeterminate maturity. It is also not applicable to the run off profile for similar products of other banks in that customer characteristics – income level, number of dependants, and other economic ingredients - differ across banks.

4.2 Illustration of Processing Account Level Data

Simple random sampling was used to select thirty accounts each representing one of the thirty main branches of the Ghanaian Bank chosen for the Study. The principle of simple random sampling is that every object has the same probability of being chosen. Each individual is chosen randomly and entirely by chance, such that each individual has the same probability of being chosen at any stage during the sampling process, and each subset of k individuals has the same probability of being chosen for the sample as any other subset of k individuals. In small populations and often in large ones, such sampling is typically done "without replacement", i.e., one deliberately avoids choosing any member of the population more than once. Although simple random sampling can be conducted with replacement instead, this is less common and would normally be described more fully as simple random sampling with replacement. For a small sample from a large population, sampling without replacement is approximately the same as sampling with replacement, since the odds of choosing the same individual twice is low.

The IT unit of the bank generated the list of personal call accounts that had been in existence over 6 months for each of the 30 branches in Microsoft Excel. For each branch the accounts were numbered from 1 to the last. Using Excel's RANDBETWEEN () function, an account was randomly selected for that branch. This was repeated for all the 30 branches.

Daily account data - deposits, withdrawals, and balances were obtained covering a period of six months (including weekends). The data was obtained with the help of staff of the Credit Risk Department of the subject bank. 180 days' data was collected because as a practice, it is expected that even newly established accounts will see some considerable amount of transactions thereon during this period. It is therefore a local norm for call accounts to have been operated for at least 6 months to enable a new loan applicant be considered for the loan.

4.2.1 Obtaining Aggregate Level Data

Account level data were then summed up to obtain aggregate level data across all the 180 days observed using Equation 3.1. That is the balance on one account on a particular day is added to balances on other accounts of study for that same day. Similarly all items of credit and items of debit were added across accounts for each day. The totals or aggregate were then converted to 1/100th of a monetary unit as defined. Data on credits or increments on the account, though not the subject of this paper,were collected for control purposes to ensure accuracy in account balance data collection. Any form of withdrawal that saw cash leave the bank in question is observed as an incident of the event. Such that withdrawal via cheque, ATM and transfers to accounts in different banks were considered occurrence of the event. Transfers to other accounts, or other bank charges were classified as censored events. All incidences of censoring were noted during the data collection stage and were separated from event data. Table 4.1 below illustrates the process.

Table 4.1: Obtaining Aggregate Level Data

		BRANCH. 1		BRANCH.2	, . . . ,	BRANCH.30	in 1/100 terms
		#XXXX000024		#XXXX000011	, . . . ,	#XXXX000078	AGGREGATE.
		Balance		Balance		Balance	Balance
		74.00		895.49	+, . . . ,+	126.40	3470640
	Day 1	74.00	+	895.49	+, . . . , +	126.40	3626246
	Day 2	74.00	+	895.49	+, . . . , +	126.40	3636246
	Day 3	74.00	+	895.49	+, . . . , +	126.40	3631246
	...	...	...	...	...	...	...
	Day 178	47.05	+	1068.99	+, . . . , +	3091.48	7142382
	Day 179	47.05	+	1068.99	+, . . . , +	3091.48	7142382
	Day 180	47.05	+	1068.99	+, . . . , +	3091.48	7142045

4.2.2 Obtaining Individual Level Start Time

Starting from the 180^{th} day (base day) and working backwards (that is using historical data), the maximum balance and the time recorded on each account is obtained. This serves as the starting time for each account in accordance with Equation 3.11. The first branch recorded its maximum balance of GHS374.15 on day 50. The second branch illustrated also recorded its local maxima of GHS1,751.32 on day 62. The process is repeated until all accounts of study are exhausted. Undoubtedly, this process will be quite arduous considering the number of individual accounts held at banks. Thus investments in computational power will have to be made. That notwithstanding; inferring the aggregate level start time from individual account start times ensures that more observations of the event of interest - in this case withdrawals - are captured in the estimation process as well as improves the credibility of the process.

4.2.3 Dealing with Multiple Start Times on an Account

In processing account level start time, it is possible to have in certain situations the maximum balance spanning different time periods or occurring at different times. In such instances the the most recent time at which the maximum balance occurs is the most suitable. This is because the impact of certain economic variables such as inflation is minimal over the short term than the long term. This section underscores the risk in estimating the runoff profile over long periods; it is important for purposes of relevance and credibility to work with the most recent data (in situations where multiple starting points could occur). This is illustrated in figure 4.1 below.

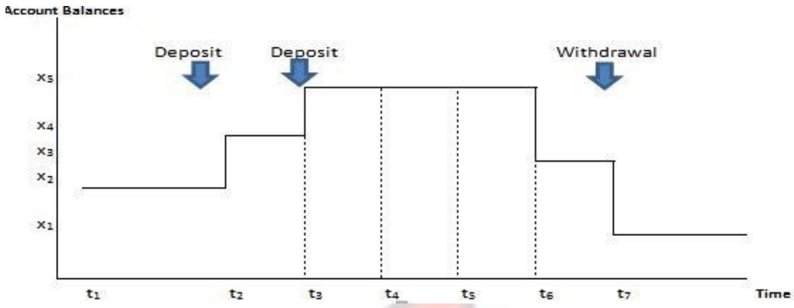


Figure 4.1: Individual Account Starting Point

In this case the maximum balance on the account is X_5 which occurs at times t_3, t_4, t_5, and t_6. Per the process outlined above; t_6 is the starting point of choice (considering the historical account balance data from $t_1 t_o$ the base date). Table 4.2 below illustrates the combined output of section 4.2.2 and 4.2.3 as outlined above.

Table 4.2: Individual Level Start Times

Branch #	1	2	3	4	5	6	7	8	9	10
Balance ($X_{it_{0i}}$)	374.15	1751.32	6797.17	1291.11	127.49	7146.45	5764.89	8625	300	1829.4
Day (t_{0i})	50	62	175	43	164	97	107	76	117	1
Branch #	11	12	13	14	15	16	17	18	19	20
Balance ($X_{it_{0i}}$)	258.26	4795.5	281.23	1721.68	4400.92	641.35	1902	2272.12	4231.05	1245.95
Day (t_{0i})	1	71	17	29	1	164	17	164	21	2
Branch#	21	22	23	24	25	26	27	28	29	30
Balance ($X_{it_{0i}}$)	10521.19	1092.62	4345.13	3325.45	2569.89	1939.53	2765.29	435.76	755.17	3551.9
Day (t_{0i})	136	118	58	176	1	23	171	120	134	78

4.3 Obtaining Aggregate Level Start Time

From sections 4.2.2 and 4.2.3 a maximum balance and its corresponding start time is obtained for each of the thirty (30) accounts sampled. The weighted aggregate start time using Equation 3.12 represents the aggregate level start time. From the individual level start times and balances indicated in Table 4.2 above, the aggregate level start time corresponds to the 89th day.

4.4 Estimating Aggregate Level Run-Off Profile

Incorporating the aggregate level start time and the corresponding aggregate level balance as well as decrements over a thirty day period yields the table below:

Table 4.3: Survival Data

Time	At Risk	Withdrawal (Failure)	Censored
1	4976794	500	0
2	4976294	150000	0
3	4826294	100402	0
4	4725892	109000	250
5	4616642	105	0
6	4616537	110450	0
9	4506087	283527	0
10	4222560	20000	35
16	4202525	244720	0
18	3957805	316000	0
19	3641805	107000	0
23	3534805	85364	0
24	3449441	11400	0
25	3438041	240570	0
26	3197471	100250	0
27	3097221	150500	0
29	2946721	1000	0
30	2945721	118285	2827436

Corresponding to the experiment start time is the total individuals (monetary units) at risk of withdrawal – 4,976,794. These are gradually reduced as time elapses as signified by the number of withdrawals and censored items. In all a total of 2,149,073 withdrawals (failures) were recorded during the thirty days of observation whilst 285 were censored. In addition a total of 2,827,436 did not record the event of interest (i.e. withdrawal) at the end of the study period and were thus considered censored. In a discrete time framework, as shown in Table 4.3 above, censoring times often coincide with the withdrawal times. At such times, this study adopts the convention of assuming that withdrawals precede censoring. The run off profile of the savings product over the 30 day period is estimated using the Kaplan-Meier estimator as defined in Equation(3.10). The K-M estimate of the survival distribution function of aggregate level data is depicted in the table below:

Table 4.4: Summarised Kaplan Meier Table

Time	Survival rate	Survival distribution function	Confidence Interval (Cl=95%)
1	1.000	1.000	(1, 1)
2	0.970	0.970	(0.97, 0.97)
3	0.979	0.950	(0.949, 0.95)
4	0.977	0.928	(0.927, 0.928)
5	1.000	0.928	(0.927, 0.928)
6	0.976	0.905	(0.905, 0.906)
9	0.937	0.848	(0.848, 0.849)
10	0.995	0.844	(0.844, 0.845)
16	0.942	0.795	(0.795, 0.796)
18	0.920	0.732	(0.732, 0.732)
19	0.971	0.710	(0.71, 0.711)
23	0.976	0.693	(0.693, 0.693)
24	0.997	0.691	(0.691, 0.691)
25	0.930	0.643	(0.642, 0.643)
267	0.969	0.622	(0.622, 0.623)
27	0.951	0.592	(0.592, 0.592)
29	1.000	0.592	(0.592, 0.592)
30	0.960	0.568	(0.568, 0.568)

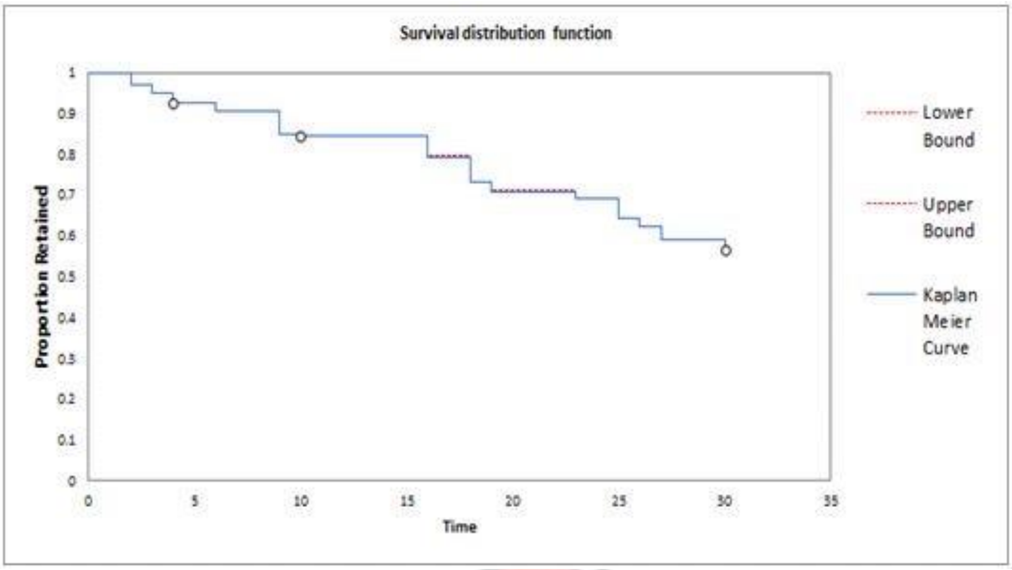


Figure 4.2: The corresponding Kaplan-Meier Curve

Table 4.4 and figure 4.2 summarizes the survival distribution function estimates and confidence intervals. Estimates for the survival distribution are reported as point estimate together with an associated confidence interval. The Kaplan-Meier survival curve is shown as a solid line, and the 95% confidence limits are shown as dotted lines in figure 4.2. For all the time periods, the point estimates of the survival distribution function almost coincide with either the lower bound or the upper bound of the confidence interval. The mean survival time is estimated at 23.995 CI 95% (23.987, 24.003). Details in Table 5.1 and 5.2 of the appendix. Incidents of the event are lowest at the beginning and larger mid period. Retention rates decreased with time.

4.5 Estimating the Funding Liquidity Risk

Incorporating the survival distribution function estimates above into a funding liquidity matrix, we define the time buckets up to the 30th day (one month) which is a standard time for liquidity measurement purposes. Using information from the survival curve and knowledge that proportion withdrawn up to time t = 1 – proportion retained up to time t, a run-off profile equivalent to the estimate of the funding liquidity risk on the deposit product is provided below:

Table 4.5: Specimen Liquidity Matrix (in Run off mode)

Specimen Liquid Matrix	Overnight	1 Week	2 Weeks	1 Month
Cash Flow(in run-off mode)	-149,303.82	-323,491.61	-303,584.43	-1,373,595.14

4.6 Benchmarking

The framework for measuring funding liquidity risk in the subject bank includes among others the following: The framework came into force in 2012 and is established first and foremost to cover the monitoring and the follow-up of:

- The gaps of liquidity
- The volumes and structure of the external funding necessary for the smooth running of the bank and its allocation in the main business lines
- The regulatory Basel 3 requirements: LCR and Net Stable Funding Ratio(NSFR)
- The Internal stress tests.

Funding liquidity risk is defined by the bank as the risk of the bank not being able to meet its commitments on their contractual or probable maturity date without undergoing unacceptable losses.This risk finds expression in the inability of the treasurer to cover the positions in central bank and in nostri accounts and to find resources to cover cash outflows.The liquidity risk results from the gap between the maturity date of the transactions in the liabilities(payable) and that of assets (available).

4.6.1 Gaps of Liquidity

The bank identifies two types of gaps – the static liquidity gap and the dynamic liquidity gap. The static liquidity gap is the focus of this research. Below is a brief description of the bank's gap measures:

The Static Liquidity Gap

The static gap represents the bank's accumulated balance of forecasted outgoing and incoming payments,at different future dates, calculated using the expected maturities of existing transactions (i.e. on the balance sheet). The liquidity gap allows for visualisation of the liquidity risk and to ensure that for every maturity there sources are sufficient to fund the uses. To reduce the funding liquidity risk thus means reducing the liquidity gaping deficit.A deficit static liquidity gap at a future date indicates that the maturity of asset transactions (sources of future incoming payments) is greater than the maturity of liability transactions (sources of future outgoing payments). This situation means that the bank carries out "maturity transformation". In other words, it lends longer than it borrows. Maturity transformation is standard practice for banks, and even has a historically central role in their financing of the economy over the long term.

However, it is becoming increasingly risky in a situation of more difficult access to liquidity and increasing pressures from regulators to maintain very liquid and flexible balance sheets.When a bank is in a position of excessive transformation, it is effectively betting that it will always find the commercial or market resources necessary to refinance its existing long-term assets. A bet which can be challenged by a systematic liquidity crisis (like the summer of 2011 with the drying up of the dollar funding market for banks in the euro area) or specific to the bank (e.g. a rumour of difficulties that triggers massive withdrawals of deposits by commercial and private customers). Hence the importance of the static gap as an indicator of liquidity risk which must be strictly controlled and closely monitored.

The Dynamic Gap

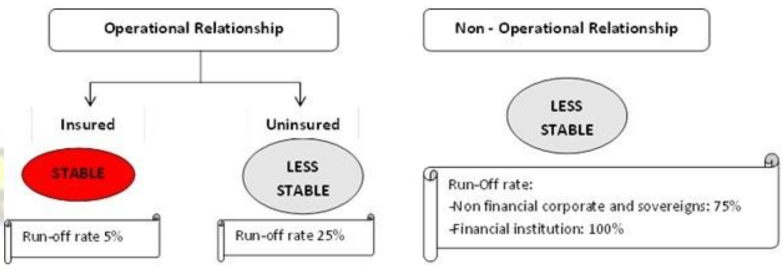
Dynamic scheduling takes into account the existing transactions as in the case of the static gap but with integration of the customers' behavior (or embedded options such as early repayments, etc.). This type of scheduling is calculated in normal conditions as well as under conditions of stress.The dynamic gap is not the object of a limit as such, but serves to calibrate those concerning the raising of external funding and those concerning its allocation within the bank.

4.6.2 Considerations for Non-Maturing Liabilities

In preparing its gap measures; the bank recognises the challenges with non maturing obligations and provides as follows:

Wholesale Deposits

The stability of the Wholesale deposits except Small Business Centres(SBC) is determined according to the hierarchy of the following criteria:



The 2 stages above allow classifying the deposits of Wholesale (except SBC) into stable/less-stable:

1. Detect the operational accounts

   Operational relation: the eligible deposits in the operational relation are defined by the following criteria:

   - Relative to the operations of clearing, custody or cash management
   - Paid below the current market conditions

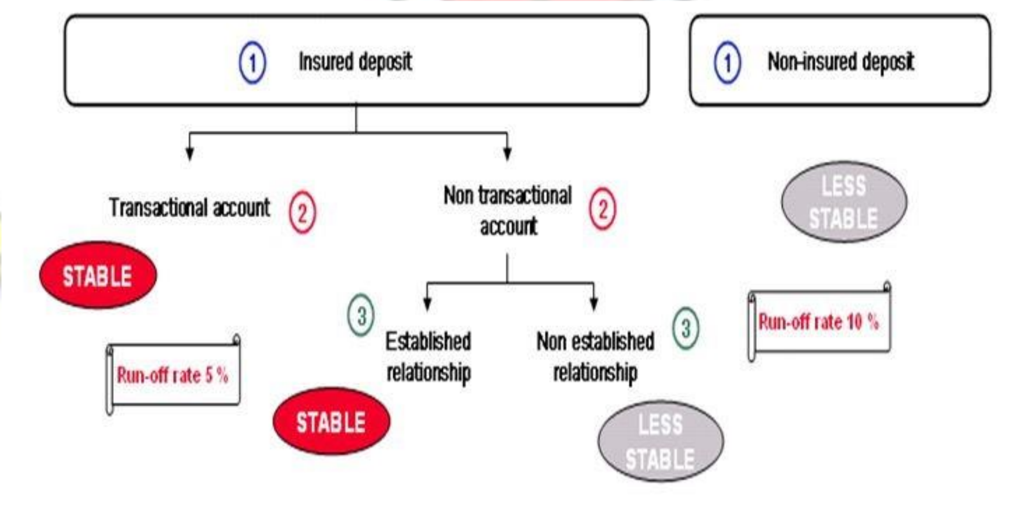
- Held in specific accounts

2. Distinguish the guaranteed and not guaranteed deposits

   Insured/guaranteed deposit: a guaranteed deposit is a deposit which is totally covered by an effective insurance or by a public guarantee bringing an equivalent protection.

Retail and Small Business Center (SBC) Deposits

The stability of the Retail and SBC deposits is determined according to the hierarchy of following criteria:



The 3 stages above allow classifying the deposits of the customers (Retail and SBC) into stable/less-stable:

1. Distinguish the guaranteed and not guaranteed deposits

   Insured/guaranteed deposit: a guaranteed deposit is a deposit which is totally covered by an effective insurance or by a public guarantee bringing an equivalent protection.

2. Detect the transactional accounts

   Transactional account: the deposits are in transactional accounts if the following conditions are respected:

- Recurring flows (i.e.: automatic debits relative to the payments of invoices, to the credit card payments) and key flows (i.e. domiciled salaries, loan repayments)
- A minimum of cash flows is essential to prove that the customer has really a transactional account

3. Detect the customers in established relation with the bank

   Established relation: the depositors established a relation with their bank which makes strongly improbable the withdrawal of the deposits

   - Marketing studies: on the customer "loyalty", the continuity of the attractiveness of products, seniority of the relation of the customer with the bank
   - A customer has several types of relations with the bank in terms of contracts, products (number and types of held products: loans, savings accounts).

Additional criteria

Except the criteria presented in previous two chapters, the following additional criteria are to be taken into account during the evaluation of the stability of the deposits:

1. Legal Right of withdrawal

   - Concerned deposits: Retail and Wholesale fixed-term deposits
   - The customers can remove their deposits at any time before the term

   a. Local legislation concerning anticipated withdrawals term accounts

   a. If there is no precision in the legislation, to refer to the contract

2. Existence of a dissuasive penalty

   - Concerned deposits: Retail and SBC fixed-term deposits

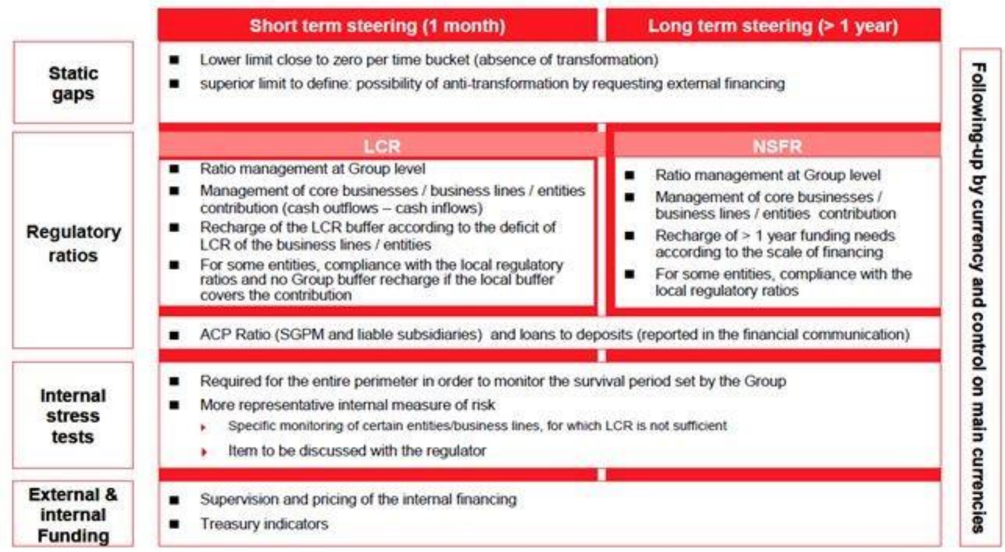
- The customers have to pay a "dissuasive" penalty in case of anticipated withdrawal for a deposit considered as a fixed-term deposit

3. ”Callable” fixed-term deposit

   - Concerned deposits: fixed-term deposits Wholesale except SBC
   - Option which can be exercised in the discretion of the investor on a horizon of 30 calendar days.

4.6.3 Principles for Managing Liquidity Risk

To satisfy internal and regulatory requirements for liquidity risk management, the bank has defined business principles in connection with the implementation of the liquidity metrics:



The regulatory requirements, and the related banking impacts, also resulted in an adaptation of the bank’s standards, procedures, models and support processes for measuring and managing liquidity risks.

Treasury Indicators Follow-up

Treasury indicators are set up in the bank including:

- Overnight Loan Indicator

  A high level of overnight loan indicates a more important dependence for the short-term funding and the potential weaknesses in the management/the planning of liquidity risks.

- Peaks of borrowings with the Central bank Indicator

  This allows assessing the level of dependence towards the central bank and its adequacy in terms of management of the liquidity.

- Cross Currency Swaps Indicators

  Evaluate the disparities of liquidity in foreign currencies and the dependence of the entity to the Cross Currency Swaps market.

- Unsecured Wholesale short-term funding concentration (single names) The Group tries to avoid any excessive concentration of its sources of funding, likely to make its liquidity vulnerable to a sudden and massive withdrawal of funds by a depositor.

- Unsecured short-term funding by maturity The Group tries to avoid any excessive concentration in the structure of its financing.

- Yearly Long Term Funding Program

  The Group compares the amount of its annual program of long-term refinancing with its peers.

- Long-term funding by maturity and type The Group tries to avoid a concentration of the maturities of its long-term funding by nature of instrument, by currency and by investor’s geographical zone.

These indicators are the object of a reporting on weekly basis.

CHAPTER 5

CONCLUSION and RECOMMENDATION

5.1 Summary of Results

The product limit estimator developed by Kaplan and Meier (1958) was used to estimate the run-off profile of the savings product. The approach developed was illustrated using data from a Ghanaian bank over a thirty day period.

Results from estimating individual and aggregate level start time aid in analysing cash flow timings in a survival analysis context. Findings from the application of the Product Limit Estimator to estimate the run-off profile of the bank's deposit and comparisons to current practices in the bank offer evidence to suggest that the current practice by the bank of using Basel III proposed run-off rates estimates a much lower funding liquidity risk than the bank assumes. The results of the analysis shows run-off rates for shorter time buckets are not uniform as regulatory requirements appear to indicate. Overall deposit run-off rate to thirty days stood at 43.2%. For retail and small business centre deposits; internal practice by the bank suggests a maximum run off rate of 10% which is quite low relative to the results achieved via survival analysis.

5.2 Conclusions

This study has developed a straightforward quantitative framework for measuring funding liquidity risk associated with bank products with indeterminate maturity (focus on a bank savings product). Using survival analysis approach the study focussed on determining the run-off profile of a bank deposit product.

The suitability of applying survival techniques to cash flow modelling was questioned. Using the weighted average of individual account starting positions aided in addressing a key area of divergence between lifetime modelling and cash flow modelling.

The technique used in this study also assumed constantly decreasing account balances. This is one of the reasons why a scenario based approach is not adopted as by this assumption; a stress situation had been introduced where during the trial period increments were ignored. This approach is consistent with general risk management practices where adverse but plausible situations are the subject of interest.

The framework developed in this paper provides a simple method for estimating the run off profile for a bank deposit product for managing funding liquidity risk. It minimizes the bias and sophistication introduced by parametric and scenario based approaches. This profile gives the probability of subjects (monetary units) staying on the bank's books beyond a certain time and aids in addressing the problem of cash flow timing uncertainty when measuring bank funding liquidity risk.

The study is however limited by the relatively small sample of accounts used in the analysis and the assumed independence of intra and inter accounts subjects of study. These notwithstanding; the paper can contribute to the ongoing debate as to whether Basel III (revised)/ regulator proposed run of rates for both insured and unsecured deposit categories are suitable for implementation.

5.3 Recommendations

This paper provides grounds for further study. Primary areas to consider will include resorting to in-house approach to estimating the run-off profile rather than to standards proposed by the regulator as this presents a more localised approach to dealing with funding liquidity risk.

Another area for further studies could be estimating the run off profile of different product classes or business lines (branches) for strategic risk management of individual banks. Further work could also be to model the evolution of a product's balance by separately projecting the run-off of existing and future business. Lastly the point on independence of subjects in the same account or across accounts could be explored.

Reference

Bank of Ghana (2015), Summary of Economic Data - May Edition

Bardenhewer, M. M. (2006). Modeling non-maturing products. Liquidity Risk Measurement and Management: A Practitioner's Guide to Global Best Practices, 408, 220

"Basle Committee. (2008). Principles for sound liquidity risk management and supervision. BIS, Basel."

Basle Committee on Banking Supervision. (2010). Basel III: International framework for liquidity risk measurement, standards and monitoring. Bank for International Settlements.

Basle Committee on Banking Supervision. (2014). Basel III: The Net Stable Funding Ratio. Bank for International Settlements

Billingham, L. J., Abrams, K. R., & Jones, D. R. (1999). Methods for the analysis of quality-of-life and survival data in health technology assessment

Boyle, G., Stover, R., Tiwana, A., &Zhylyevskyy, O. (2015). The impact of deposit insurance on depositor behavior during a crisis: A conjoint analysis approach. Journal of Financial Intermediation, 24(4), 590-601.

Breslow, N.E. and Crowley, J.J. (1974), A large sample study of the life table and the product limit estimates under random censorship. Ann. Statist. 2, 437-453 Collett, D. (2003). Modelling survival data in medical research (2nd ed.), boca raton. Chapman & Hall/CRC.

Colosimo, E., Ferreira, F. V., Oliveira, M., & Sousa, C. (2002). Empirical comparisons between Kaplan-Meier and Nelson-Aalen survival function estimators. Journal of Statistical Computation and Simulation, 72(4), 299-308.

Dias, S., Welton, N. J., Sutton, A. J., & Ades, A. E. (2011). NICE DSU Technical Support Document 2: a generalised linear modelling framework for pairwise and network meta-analysis of randomised controlled trials. National Institute for Health and Clinical Excellence, London, UK

Frauendorfer, K., &Schürle, M. (2007). Dynamic modelling and optimization of non-maturing accounts. Liquidity Risk Measurement and Management: A Practitioner's Guide to Global Best Practices, 327-359.

Gardiner, J. C. (2010). Survival analysis: overview of parametric, nonparametric and semiparametric approaches and new developments. In SAS Global Forum 2010. Statistics and Data Analysis

Gauthier, C., Souissi, M., & Liu, X. (2014). Introducing Funding Liquidity Risk in a Macro Stress-Testing Framework. International Journal of Central Banking, 10(4), 105-142

Gill, R. D. (1980). Censoring and Stochastic Integrals, Mathematical Centre Tracts 124. Amsterdam: Mathematisch Centrum. Mathematical Reviews (MathSciNet): MR596815 Zentralblatt MATH, 456.

International Monetary Fund Staff. (2008). Global Financial Stability Report: Containing Systemic Risks and Restoring Financial Soundness, April 2008. International Monetary Fund.

Jackson, C.H., Sharples, L.D., Thompson, S.G. (2010). Survival models in health economicevaluations: Balancing fit and parsimony to improve prediction. The InternationalJournal of Biostatistics, page 6;1;34.

Jager, K.; Dijk, P.; Zoccali, C.; Dekker, F. (2008). "The Analysis of Survival Data: The Kaplan–Meier Method". Kidney International74 (5): 560–565. doi:10.1038/ki.2008.217

Janosi, T., Jarrow, R. A., &Zullo, F. (1999). An empirical analysis of the Jarrowvan Deventer model for valuing non-maturity demand deposits. The Journal of Derivatives, 7(1), 8-31.

Jarrow, R. A., & Van Deventer, D. R. (1998). The arbitrage-free valuation and hedging of demand deposits and credit card loans. Journal of Banking & Finance, 22(3), 249-272.

Kalkbrener, M., & Willing, J. (2004). Risk management of non-maturing liabilities. Journal of Banking & Finance, 28(7), 1547-1568.

Kaplan E. and Meier P(1958). Nonparametric estimation from incomplete observations. Journal of the American Statistical Association 53:457–481

Klein, J. P., & Goel, P. (Eds.). (2013). Survival analysis: state of the art (Vol. 211). Springer Science & Business Media.

Lex, A., Streit, M., Schulz, H. J., Partl, C., Schmalstieg, D., Park, P. J., & Gehlenborg, N. (2012, June). StratomeX: Visual Analysis of LargeScale Heterogeneous Genomics Data for Cancer Subtype Characterization. In Computer graphics forum (Vol. 31, No. 3pt3, pp. 1175-1184). Blackwell Publishing Ltd

Machina, M.J., and M. Rothschild. (1987), .Risk..In The New Palgrave Dictionary of Economics, edited by J. Eatwell, M. Millgate, and P. Newman, pp.203.5. London, UK: MacMillan

Maes, K., & Timmermans, T. (2005). Measuring the interest rate risk of Belgian regulated savings deposits. NBB Financial Stability review, 136-151

Matz, L. (2013). A self Paced Guide to Asset Liability Management.Sheshunoff Information Services Inc. Austin, Texas

Matz, L., &Neu, P. (Eds.). (2006). Liquidity risk measurement and management: a practitioner’s guide to global best practices (Vol. 408). John Wiley & Sons.

Miller, R. (1981). Survival Analysis. Willey.

Montes-Negret, F. (2009). The heavenly liquidity twin: The increasing importance of liquidity risk. World Bank Policy Research Working Paper Series, Vol.

Musakwa, F. T. (2013). Measuring Bank Funding Liquidity Risk.

Neu, P. (2007). Liquidity risk measurement. Liquidity Risk Measurement and Management: A Practitioner’s Guide to Global Best Practices, 15-36.

Nikolaou, K. (2009). Liquidity (risk) concepts: definitions and interactions.

"Poorman, F & Stern, HS (2012). A new approach to analyzing core deposit behaviors. Bank Asset/Liability Management, 28(3):3–5"

Reynaud J. and Vauday J. (2008). IMF lending and geopolitics

Rich, J. T., Neely, J. G., Paniello, R. C., Voelker, C. C., Nussenbaum, B., & Wang, E. W. (2010). A practical guide to understanding Kaplan-Meier curves. Otolaryngology-Head and Neck Surgery, 143(3), 331-336

Takemura, T., & Kozu, T. (2010). Modeling individuals' deposit-withdrawal behaviors after receiving information on financial turmoil. The Research in Business and Economics Journal, 3, 58-74.

Takemura, T., Kozu, T., & Kobayashi, T. (2012). Modeling Deposit-Withdrawal Behavior with Risk Tolerance. Journal of Management Policy and Practice, 13(5), 116-127.

Vento, G. A., & La Ganga, P. (2009). Bank liquidity risk management and supervision: which lessons from recent market turmoil. Journal of Money, Investment and Banking, 10, 79-126.

Williams, R. L. (1995). Product-limit survival functions with correlated survival times. Lifetime data analysis, 1(2), 171-186.

Williams, R. L. (2000). A note on robust variance estimation for cluster correlated data. Biometrics, 56(2), 645-646.

Zhao, G. (2008). Nonparametric and parametric survival analysis of censored data with possible violation of method assumptions. ProQuest.

APPENDIX

Table 5.1: Kaplan Meier Table

Time	Failed	Censored.	Prop.failed	Surv.rate	Surv. dist	SE	LB (95%)	UB (95%)
1	500	0	0.000	1.000	0.99989953	0.000004	0.99989033	0.99990796
2	150000	0	0.030	0.970	0.96975965	0.000077	0.96961340	0.96990521
3	100402	0	0.021	0.979	0.94958562	0.000098	0.94940276	0.94976783
4	109000	250	0.023	0.977	0.92768397	0.000116	0.92747257	0.92789477
5	105	0	0.000	1.000	0.92766287	0.000116	0.92745145	0.92787370
6	110450	0	0.024	0.976	0.90546866	0.000131	0.90523565	0.90570113
9	283527	0	0.063	0.937	0.84849577	0.000161	0.84822828	0.84876284
10	20000	35	0.005	0.995	0.84447690	0.000162	0.84420781	0.84474557
16	244720	0	0.058	0.942	0.79530161	0.000181	0.79501952	0.79558337
18	316000	0	0.080	0.920	0.73180295	0.000199	0.73151799	0.73208767
19	107000	0	0.029	0.971	0.71030182	0.000203	0.71001863	0.71058480
23	85364	0	0.024	0.976	0.69314834	0.000207	0.69286739	0.69342910
24	11400	0	0.003	0.997	0.69085757	0.000207	0.69057696	0.69113798
25	240570	0	0.070	0.930	0.64251620	0.000215	0.64224559	0.64278668

26	100250	0	0.031	0.969	0.62237145	0.000217	0.62210630	0.62263648
27	150500	0	0.049	0.951	0.59212921	0.000220	0.59187350	0.59238483
29	1000	0	0.000	1.000	0.59192827	0.000220	0.59167262	0.59218382
30	118285	2827436	0.040	0.960	0.56815947	0.000222	0.56791217	0.56840669

Table 5.2: Mean Survival Time

Mean survival time (Time<0)	Standard deviation	Lower bound (95%)	Upper bound (95%)
23.995	0.004	23.987	24.003

Table 5.3: Quantiles Estimation

Quantile	Estimate	Lower bound (95%)	Upper bound (95%)
75%			
50%			
25%	18.000	0.000	30.000