KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

Measuring The Investment Risk and Using Annuities to Determine The Expected Liability of Pension Funds:A case study of the Social Security and National Insurance Trust (SSNIT)

BY

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Declaration

I hereby declare that this submission is my own work towards the award of the MSc. degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

I dedicate this work to the Almighty God and my family especially to my sisters for their support and to my late Dad and Mum.

Abstract

Effective management of investment risk is essential for every institution which is exposed to investment risk. Pension funds in Ghana are especially exposed to investment risk due to increasing investment risk factors exposed to the market. Pension fund exist to provide benefits to its members, therefore members are mainly concerned with losses as far as it decreases the value of their benefits. These losses usually occur as a result of the pension fund investing in the financial market and portfolio mismatching, which makes the tradeoff between risk and return a topic that most pension fund (investors) must consider carefully before an investment decision is made. The study uses the concept and methodology of the “value at risk" risk measure which is a tool for measuring an entity’s exposure to market risk, to determine the maximum loss of the investment portfolio of SSNIT on the Ghana capital market. The maximum loss of the scheme’s investment is quantified under the variancecovariance method of computing value-at-risk using an implementation of portfolio consisting of twenty-three stocks for 30 time interval with the confidence level of 95%, 99% and 90%.The study further used the whole life annuity model with mortality data to determine the expected liability to be paid by the fund to its members. Based on the normality of the distribution of the portfolio risk factors at 95% confidence level,the maximum loss of SSNIT is quantified under the 95% confidence level.

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CONTENTS

Abbreviations

SSNIT .... Social Security National Insurance Trust

DB .... Defined Benefit

DC .... Defined Contribution

VaR ....Value at Risk

IMF .... International Monetary Fund OECD ....

Organisation for Economic Cooperation and Development ALM ....

Asset-Liability Management

ES ....Expected Shortfall CVaR

.... Conditonal Value at Risk

ISO .... International Organisation for Standardization

IT ....Information Technology

NAIC ....National Association Insurance Commisions

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CHAPTER 1

Introduction

1.1 Overview of the Study

This study seeks to quantify the amount of risk regarding the investment portfolio of pension providers in Ghana.Investment risk under pension fund is define as the trade-off between risk and return on the assets built up against their fund obligations.Pension providers are faced with the risk of investing in the market and the risk that pensioners will live longer than expected and since they have to pay monthly pension to the pensioners until their death,investment risk may affect the annuity provider's solvency.

The need to measure and manage investment risk has become very important as employers and employees become aware of their exposure to investment risk and their need to mitigate it.For individuals investment risk is the risk of suffering a loss of ones contribution(income),reduced care or a return to employment old age.For those institutions providing covered individuals with guaranteed retirement income,investment risk is the risk of being highly exposed to the capital market ,resulting to low returns than expected returns and unfunded pension liabilities (Franzen,2010).

According to the center for insurance policy and research of the National Insurance Commissions (NAIC,US) the key drivers to the growing need to address investment risk is the increasing exposure to market and economic volatility,responsibility of sufficient retirement income and uncertainty of governments benefits.

1.2 Background of the Study

Occupational pension funds works under a very simple procedure; Contributions are being paid into the fund, and these funds are then invested on the capital markets, which are paid out in the form of pension benefits. The operational variables used in this process are uncertain, therefore makes the procedure to be risk capsulated.Lange (2010) argues that investing in pension contributions in capital markets exposes the pensioner to risks. Investment returns are uncertain, and this holds true for salary and mortality trends as well. Pension funds are undeniable active risk lovers. Under the pension funds the two most important risk found are investment and longevity risk. For defined contribution (DC) pension funds, these risks are re-distributed to their participants, whiles defined benefit (DB) pension funds, give the employee the security of a pre-defined pension benefit, perform their task to give safe pension benefits by assuming and retaining the risk. DB pension funds can become complex risk-sharing institutions, as they may subsequently re-distribute risk between the different groups of stakeholders. The risks pension funds take need to be measured and managed. But managing risk is not equivalent to avoiding risk.

Holton (2004) defined investment risk as the chance that an investment's actual return will be different than expected. This includes the possibility of losing some or all of the original investment.With the development of financial markets throughout the years, the importance of investment risk measurement and management is increasing due global financial crisis in 2007 – 2008. The International Monetary Fund (IMF) concluded in an analysis of Risk management and the pension fund industry' that policymakers should introduce measures to encourage better risk management practices and to reduce the risk of another cycle of over and under-funding'. Stewart from the Organization for Economic Cooperation and Development (OECD) conducted a first assessment of pension fund risk management in 2009 concluding that after several golden decades of equity investments delivering adequate returns, the topic of risk management has returned to the fore front of the pension industry given the now challenging funding and investment environment (Stewart,2005). Risk measurement has stayed at the above cited fore front of the pension industry. The perfect pension storm set the stage for the risk management revolution to reach the doorsteps of pension funds.

Modern risk management tools analogous to those which are used in other sectors of the financial industry such as securities firms and banks are increasingly applied by pension funds. Nowadays, pension funds in many jurisdictions calculate Value-at-Risk (VaR), apply risk budgeting concepts and analyze fat tails. Asset-Liability-Management (ALM) is routinely applied as strategic risk management tool, albeit the quality of the models and the rigour in its application still vary. But it is questionable if the risk management approaches now being applied by DB pension funds are in all cases well suited to their needs.More fundamentally, the perception of risk appears to be currently in a state of flux. Unlike securities firms, banks or insurance companies, there is no consensus between pension funds, their sponsors, regulators and accountants on the significance of the different risk factors facing pension funds.

The main characteristics of pension funds are the importance of liabilities and the longterm Investment horizon. Pension funds are usually described as long-term investors. This conception provides the argument for higher investment in asset classes such as equity that are subject to higher volatility in the short-term but also reward higher returns in the long-term (Ryan and Fabozzi,2002). As liquidity risk is not significant, pension funds can take more market risk than short-term investors thus rendering the provision of pension more affordable. The paradigm of the long-term investor is changing. Pension fund regulation became more risk aware and safety-focused. Shortfall risk is perceived as central risk factor threatening benefit security. Many countries have introduced pension reforms in recent years encouraging higher funding ratios, so that in a balance sheet perspective assets are sufficient to cover liabilities. As funding levels are tracked over shorter periods of time the investment horizon turned more short-term as well.

1.3 Problem Statement

Pension providers are obliged to pay fixed amount to a pensioner on a monthly basis for as long as the pensioner remains alive.

Due to advances made in medical technology,people changing their lifestyles and other factors ,life expectancy have increased continually since the 1960s (Olansky *et al.* 2007). In addition contributions made to SSNIT has decreased from 18.5 % to 11.5%. Also,the guarantee period has increased from 12 years to 15 years.This situation exposes SSNIT to the likelihood that at future date,it may not be able to meet its financial obligations to pensioners.

Due to this,most pension funds invest heavily in stocks listed on the capital market with their aim of gaining higher returns,to meet their funding liabilities without considering the amount of risk they are highly exposed to in the capital market .

Since the financial crisis returns on these investment have fallen resulting to fund managers not meeting their funding liabilities. For instance in Ghana, “The Social Security and National Insurance Trust (SSNIT) had invested GHS 51, 600,000 of Ghanaian workers’ money and earned only GHS 820,000 out of it as of 2004 has raised eyebrows and concerns about the fate of the trust (Nkrumah,2010).

Pension fund managers are not distressed by gains but rather they like to focus on the promise of high returns, but they should also ask how much risk they must assume in exchange for these returns.

In this study, we measure and manage the amount of risk SSNIT must assume when investing in the stock market in exchange of their expected returns to meet their future liabilities.

1.4 Objectives of the Study

The objectives of the study are as follows:

1. To measure the investment risk of SSNIT pension fund using the Value at Risk method.

2. To estimate the expected liability of the said pension fund.

1.5 Justification of the Study

The scope of this study is to measure and manage investments under pension funds which are highly exposed to capital market risk. This will help fund managers to invest in profitable investment by making productive investment decision and ensure that the fund members do not suffer a loss,therefore meeting their future liabilities. It will also contribute to knowledge in pension fund management.

1.6 Limitations of the Study

The study did not take into account other risks to which pension funds are exposed to such as interest rate risk and adverse policy changes . Also,the study is limited in scope since other pension schemes are not covered. This is due to time and resource constraints. The effect of this limitation is however not to distort the eventual outcome of the study as SSNIT remains the single largest pension fund in the country.

1.7 Organization of the Thesis

The entire thesis is presented in five chapters.Chapter one highlights the rationale of the study ,objectives,justifications and limitations of the study.In Chapter two, we review existing literature on Pensions in Ghana,investment risk and measures used to measure investment risk.Chapter three explains the methodologies used in the study ,including nature and source of data,analytical tools used in the study and we present the results of the study in chapter four.

In the concluding chapter, we make conclusions and recommendations based on our findings.We also make recommendations for future studies.

CHAPTER 2

Literature Review

2.1 Introduction

In this chapter ,we review existing literature on investment risk , risk measurement and management for pensions and Pensions in Ghana.

2.2 Investment Risk

Investment risk is any potential risk that the value of an investment may decline due to economic changes or other events that impact market factors (stock prices, interest rates,or foreign exchange rates).

Investment risk is present in any product where the investor is exposed to financial losses if the maximum expected return is not attained.This often occurs when the actual return gained is less than the expected return or losing all investment or some of the investment made(Olsen,1997).

Investment risk is one of the main challenges facing life annuity providers and pensions scheme.Life annuity and pension providers have to pay the pensioner and annuity holder respectively for life.This could threaten the financial stability of the paying institution if the investment made are exposed largely to market risk.Market risk has been shown to improve over time due to the global financial crisis.

Financial market risk reflects the chance that the actual return on an asset or a portfolio of assets may be very different than the expected return. For this reason, a measure of market risk is necessary to carry through a successful risk management (Litterman,2004).

2.3 Development of Risk Management

One of the most general definitions of risk was defined by the International Organization for Standardization (ISO),according to this standard, risk is defined as the effect of uncertainty on objectives (ISO, 2009).

Kocken (2006) gave a broad definition for risk management as a process that starts on the strategic level, first, with analysing and defining the relevant risk factors for the pension fund and its stakeholders, second, deciding on the acceptable and desirable amount of risk to be taken, and which then continues on the operational level with the process of measuring and controlling risk.

Risk is understood as something subjective, linked to the individual profile of a pension fund and its stakeholders. This differs from the bulk of the investment-banking orientated body of literature on risk management which usually defines risk in an objective way not differentiating according to the needs of different investors or stakeholders. The conception of risk applied here comes closer to Balzer's remark that risk is relative rather than an absolute concept (Balzer,1994).

Over the past two decades, the financial world has evolved from return driven to a genuine risk management industry. The term risk management certainly is not confined to what is best denoted with risk control: Measuring risks, setting limits and ensuring adherence to these limits.This is necessarily part of the whole process of risk-return optimization. Risk management also compromises the decision making process of considering risk-return trade-offs and optimizing stakeholders' targets (Kocken,2006).

According to McNeil *et al.*(2005), a bank's attitude to risk is rather active than defensive, as banker actively and willingly take on risk in order to benefit from return opportunities. Risk management can be seen as the core competence of a bank. Bankers are using their expertise, market position and capital structure to manage risks by restructuring and transferring them to various market participants.

Crouhy *et al.* (2005) on one hand refer risk management to be widely acknowledged as one of the most creative forces in the world's financial markets. An example is the rapid development of the huge market for credit derivatives, which emphasize the dispersion of risk (i.e. the credit risk exposure) of an institution to those who are willing, and presumably able to bear it. On the other hand, Crouhy *et al.* (2005) mention extraordinary failures in risk management such as Long-Term Capital Management and the string of financial scandals associated with the millennial boom in equity and technology markets (e.g. Enron and WorldCom). These are only a few examples of where risk management has not been able to prevent market disruptions and business accounting scandals.

2.4 Classification of Risk

The basic risk factors relevant to financial institutions can be broadly clustered into market risk, credit risk, liquidity risk, underwriting risk, and operational risk. Market risk refers to changes in the value of an investment due to changes of market factors, such as interest rates, exchange rates or stock markets. Credit risk is defined as the potential that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms. Liquidity risk is the risk that a firm is not able to settle a position at market values due to liquidity disruptions in the markets (Herring *et al.*2005).

Finally, underwriting or actuarial risk refers to the risk that a financial company will be unable to fulfill their contractual obligations towards their customers. These different risk factors differ in their relevance for the different sectors of the financial industry. Their regulatory regimes differ accordingly. Market risk forms the most important risk category for securities firms whereas credit risk traditionally posed the central risk for banks. Insurance companies on the other hand are faced with underwriting risk. This risk is related to the correct assessment and pricing of the insured risk, which in the case of life insurance companies is constituted by longevity risk. It is suggested here that, basically the same holds true for DB pension funds.

Furthermore, different financial sectors are faced with different degrees of liquidity risk resulting from the different time horizon of their assets and liabilities ranging from high as for securities firms and investment banks to low in the case of life insurance companies. Herring *et al.* (2005) studies pointed out that insurance companies are unlikely to find it necessary to incur fire-sale losses on the liquidation of their assets and exacerbate market dislocations by selling assets in markets with falling prices .

This implies that this risk is even lower for pension funds. The focus of risk management systems has to adapt accordingly. Davies (2001) research points out that the nature of the liabilities is the key to understanding how institutions differ in their operations. In the banking and securities industry on the one hand, risk is in general perceived to be mainly on the active side of the balance sheet, in form of credit or market risk. On the other hand, the basic risk at insurance companies and DB pension funds is connected to the passive side. While in banking risk usually denotes risky assets or off-balance-sheet derivatives, in the jargon of a life insurance industry, the term risk is used to refer to a single contract on the liability side meaning a client .Whereas a bank aims at securing sufficient funding for the assets, insurance companies accumulate contributions in form of assets on the active side to secure the fulfillment of the liabilities.

'Liabilities differ in certainty and timing' Davies (2001), ranging from fixed amount and timing in the case of banks, to fixed amounts but unknown timing as for life insurance contracts to unknown amounts and unknown timing in the case of DB pension funds and more complex life insurance products.

2.5 Development of Modern Risk Management

Today's understanding of risk management in the financial industry is based on the pricing of risk. Risk management is a quantitative, computer-based process, blending the methodology developed by financial economic theory with the technology provided by the IT industry (Rosen, 2003). It is based on finance models depicting the behavior of market variables. The theoretical foundation of quantitative risk management as it is understood here is closely linked to the origin of financial economic theory. It can be put down to Markowitz (1952) publication where he introduced the concept of the mean-variance optimization of risk-based return as opposed to the traditional approach of return-only optimization. Further milestones included the development of the Capital Asset Pricing Model by Sharpe (1964), the formula on option pricing by Black and Scholes (1973), and Ross (1976) Arbitrage Pricing Theory.

The ideas developed by academics were first applied at the desks of brokers and dealers in the starting option markets in the early 1970s and found broader application in investment banking after the stock market crash of 1973/74 and the volatile economic environment that followed over the 1970s and the beginning of the 1980s (Bernstein,1999). The advancement of risk techniques accelerated at the beginning of the 1990s with the release of JP Morgan's RiskMetrics in 1994 which marked the beginning of the standardized use of VaR in measuring market risk (Rahl,2000).

The ‘real’ start of modern risk management is therefore often linked to this event. The development of modern risk management was linked to the development of capital markets, notably the option markets, so that the needs of those trading on these markets, mainly brokers/dealers and investment banks, initially informed the development of concepts and tools. Modern risk management evolved around market risks, the assets underlying these risks are tradable and valued at market prices. It takes the financial view on risk.

Risk management was not imposed on the markets by regulation but evolved as part of a process of adaptation to changing market conditions across national borders and regulatory regimes. But even though risk management as we know it today was not a regulatory invention, its evolution did not occur in a vacuum and was certainly shaped by regulatory events along the way (Mengle,2003). The 1988 Basel Accord is usually regarded as such a regulatory event. It was a response of the international banking regulators to a series of bank failures and bank crises. The Basel Accord set minimum capital standards for banks based on the total of a bank‘s risk-weighted assets. The 1988 Basel Accord represented the first step towards a risk-based regulation of banks. It was also the first time that a financial sector was regulated subject to international standards thereby creating the level playing field, a geographically even competitive surrounding. But the first Basel Accord referred to credit risk only which was traditionally the most relevant risk category for banks, but was in the late 1980s neither traded nor valued at market prices.

Therefore, it can be argued that it was not the first Basel Accord but the Basel Amendment 1996 which represented the first regulatory implementation of modern risk management as it referred to assets that were traded and valued at market prices applying the concepts of financial economic theory. As banks became increasingly involved in trading activities, they became increasingly exposed to market risk as well. The 1996 Basel Committee Amendment extended the risk-based regulatory approach to market risk. The release of JP Morgan‘s RiskMetrics in 1994 had marked the beginning of the standardised use of VaR in measuring market risk. The Basel Amendment firmly implemented VaR as risk measure in the banking industry by allowing besides the standard approach also internal models, which base the calculation of the banks required capital for market risk on VaR. The Basel II Accord, which is due for implementation,extends capital requirements to operational risk and applies all capital requirements to financial holding companies of internationally active banks.

2.6 Dissemination of Risk Management

Starting from the late 1980s insurance companies increasingly applied ALM mainly to manage interest rate risk (Santomero,1997). Asset-Liability-Models became important risk management tools especially for life insurance companies. The reasons are to be found firstly in the changing economic environment with increasingly volatile interest rates from the late 1970s onwards. Secondly, with the bundling of insurance and saving products, the business model of life insurance companies changed from underwriting pure actuarial risk into taking also speculative market risk (Scherer,2006). Thirdly, in many countries life insurance companies invested a higher percentage of their portfolio in equities from the 1990s onwards, thereby becoming more exposed to market risk. As the ALM models became more sophisticated integrating the financial view of risk with the traditional actuarial approach, asset and liability management systems became the strategic risk management tool at insurance companies as it combines the mean-variance efficiency analysis of assets with the precise liability constraints faced by insurance companies.

2.7 Approaches to Investment Risk Measurement

Existing approaches to measuring the risk of a financial position can be grouped into four different categories: the notional-amount approach; factor-sensitivity measures;risk measures based on the loss distribution; risk measures based on scenarios (McNeil *et al.* 2005).

1. Notional-amount approach

   This is the oldest approach to quantifying the risk of a portfolio of risky assets. In the notional-amount approach the risk of a portfolio is defined as the sum of the notional values of the individual securities in the portfolio,where each notional value may be weighted by a factor representing an assessment of the riskiness of the broad asset class to which the security belongs. Variants of this approach are still in use in the standardized approach of the Basel Committee rules on banking regulation. The advantage of the notional-amount approach is its apparent simplicity. However, from an economic viewpoint the approach is flawed for a number of reasons. To begin with, the approach does not differentiate between long and short positions and there is no netting. For instance, the risk of a long position in foreign currency hedged by an offsetting short position in a currency forward would be counted as twice the risk of the unhedged currency position. Moreover, the approach does not reflect the benefits of diversification on the overall risk of the portfolio. For example, if we use the notionalamount approach,it appears that a well-diversified credit portfolio consisting of loans to companies that default more or less independently has the same risk as a portfolio where the whole amount is lent to a single company. Finally, the notional-amount approach has problems in dealing with portfolios of derivatives, where the notional amount of the underlying and the economic value of the derivative position can differ widely.

2. Factor-sensitivity measures

Factor-sensitivity measures give the change in portfolio value for a given predetermined change in one of the underlying risk factors; typically they take the form of a derivative (in the calculus sense). Important factor sensitivity measures are the duration for bond portfolios and the Greeks for portfolios of derivatives. While these measures provide useful information about the robustness of the portfolio value with respect to certain well-defined events, they cannot measure the overall riskiness of a position. Moreover, factor-sensitivity measures create problems in the aggregation of risks

• For a given portfolio it is not possible to aggregate the sensitivity with respect to changes in different risk factors. For instance, it makes no sense to simply add the delta and the vega of a portfolio of options.

• Factor-sensitivity measures cannot be aggregated across markets to create a picture of the overall riskiness of the portfolio of a financial institution.

Hence these measures are not very useful for capital-adequacy decisions; used in conjunction with other measures they can be useful for setting position limits.

3. Risk measures based on loss distributions

Most modern measures of the risk in a portfolio are statistical quantities describing the conditional or unconditional loss distribution of the portfolio over some predetermined horizon. Examples include the variance, the Value-at-Risk and the expected shortfall, It is of course problematic to rely on any one particular statistic to summarize the risk contained in a distribution. However, the view that the loss distribution as a whole gives an accurate picture of the risk in a portfolio has much to commend it:

- losses are the central object of interest in risk management and so it is natural to base a measure of risk on their distribution;
- the concept of a loss distribution makes sense on all levels of aggregation from a portfolio consisting of a single instrument to the overall position of a financial institution; • if estimated properly, the loss distribution reflects netting and diversification effects; and, finally,
- loss distributions can be compared across portfolios.

For instance, it makes perfect sense to compare the loss distribution of a book of fixedincome instruments and of a portfolio of equity derivatives, at least if the time horizon is the same in both cases (Dowd,2001). There are two major problems when working with loss distributions. First, any estimate of the loss distribution is based on past data. If the laws governing financial markets change, these past data are of limited use in predicting future risk. The second, related problem is practical. Even in a stationary environment it is difficult to estimate the loss distribution accurately, particularly for large portfolios, and many seemingly sophisticated risk-management systems are based on relatively crude statistical models for the loss distribution (incorporating, for example, untenable assumptions of normality). However, this is not an argument against using loss distributions. Rather, it calls for improvements in the way loss distributions are estimated and, of course, for prudence in the practical application of risk-management models based on estimated loss distributions. In particular, risk measures based on the loss distribution should be complemented by information from hypothetical scenarios. Moreover, forward-looking information reflecting the expectations of market participants, such as implied volatilities, should be used in conjunction with statistical estimates (which are necessarily based on past information) in calibrating models of the loss distribution (Dowd and Blake,2006).

2.8 Investment Risk Measures

In financial economics, it is often assumed that the key factors influencing investment decisions are "risk" and "return". In practice, return is almost always interpreted as the expected investment return. However, there are many possible interpretations and different ways of measuring investment risk (ACTED,2013).

Risk measures relevant for investment risk are:

• Variance of Return: The variance of return assumes that investors make choices solely on the basis of the mean and variance of return. Hence it measures the uncertainty of returns. • Downside Semi-Variance of return: This measure seeks to quantify the view that investors dislike the probability of low returns.

• Expected Shortfall Probabilities: Expected Shortfall (ES), sometimes also referred to as Conditional Value-at-Risk (CVaR), is another approach to estimate risk by measuring the probability of returns falling below a certain level, where the benchmark level can be expressed as the return on a benchmark fund if it is more appropriate than an absolute level. • Value at Risk (VAR): Value at Risk is a single, summary, statistical measure of possible portfolio losses. Specifically, value at risk is a measure of losses due to "normal" market movements. Losses greater than the value at risk are suffered only with a specified small probability. Subject to the assumptions used in its calculation, value at risk aggregates all of the risks in a portfolio into a single number suitable for use in reporting to regulators, the boardroom, or disclosures in an annual report. It considers only negative deviations from expected results. It calculates the maximum loss expected (or worst case scenario) on an investment over a given time period and given a specified degree of confidence. VAR has three standard elements: a relatively high level of confidence (typically either 95 or 99),a time period (a day, a month or a year) and an estimate of investment loss(expressed either in amount or percentage terms).

There are three methods of calculating VAR: the historical method, the variance-covariance method and the Monte Carlo Simulation. The historical method simply re-organizes actual historical returns, putting them in order from worst to best. It then assumes that history will repeat itself, from a risk perspective. The Variance-Covariance method assumes that stock returns are normally distributed. It requires that we estimate only two factors:an expected (or average) return and a standard deviation, which allow us to plot a normal distribution curve. Monte Carlo simulation runs multiple hypothetical trails through the model .It refers to any method that randomly generates trails (Artzner *et al.* 1999).

- Scenario Stress Tests: Scenario stress tests unlike the VaR and ES are not based on normal market conditions, as they are a tool for quantifying the size of potential losses under stress events. Their ultimate purpose is to simulate stress events similar to the ones that have occurred during the financial crisis where multiple things went wrong at the same time.

2.9 The Need for Pensions

Pensions, in a broad sense , are regular payment made by the state or a pension fund to people of or above the retirement age and to some widows and disabled people.At retirement, salaries are no more paid hence a decline or a complete cut off of income.To sustain a living at retirement for employees most employers including government run a pension scheme.This pension scheme is meant to support employees who go on retirement for several reasons.Employees and employers make regular contributions to the scheme during their years of service and these contributions are invested (Barbone,1999).

It is obvious that pensions are necessary as in many cases it becomes the only source of livelihood for elderly people.

Employment based pension

A retirement plan is an arrangement to provide people with an income during retirement when they are no longer earning a steady income from employment. Often retirement plans require both the employer and employee to contribute money to a fund during their employment in order to receive defined benefits upon retirement. It is a tax deferred savings vehicle that allows for the tax-free accumulation of a fund for later use as a retirement income. Funding can be provided in other ways, such as from labor unions, government agencies, or self-funded schemes. Pension plans are therefore a form of "deferred compensation".

Benefits

Retirement plans may be classified as defined benefit or defined contribution according to how the benefits are determined. A defined benefit plan guarantees a certain payout at retirement, according to a fixed formula which usually depends on the member's salary and the number of years' membership in the plan.A traditional pension plan that defines a benefit for an employee upon that employee's retirement is a defined benefit plan (Davis,1991).

A defined contribution plan will provide a payout at retirement that is dependent upon the amount of money contributed and the performance of the investment vehicles utilized. Hence, with a defined contribution plan the risk and responsibility lies with the employee that the funding will be sufficient through retirement, whereas with the defined benefit plan the risk and responsibility lies with the employer or plan managers.Defined contribution plans allow the employer and employee to make contributions ,so that the final benefit depend on how much was in the account and the rate earned by the account's investment (Davis,1991).

Some types of retirement plans, such as cash balance plans, combine features of both defined benefit and defined contribution plans. They are often referred to as hybrid plans. Such plan designs have become increasingly popular in the US since the 1990s. Examples include Cash Balance and Pension Equity plans.

2.10 Pension in Ghana

In Ghana,the pensions industry is regulated by the National Pensions Regulation Authority(NPRA)through the National Pensions Act.There are few pension providers of which the Social Security and National Insurance Trust(SSNIT) is the largest.

For many years,Ghana operated the a pension scheme known as CAP 30 which was created in 1950 for all public servants .The name "CAP 30" was coined from chapter 30 of the pension ordinance of 1946.CAP 30 is a defined benefit scheme which gives members the option to choose between a lump sum payment on retirement or monthly pension until death.To qualify for a pension under CAP 30 scheme one must serve continuously for 10 years in the public service .Upon retirement,a member get 80% of his final salary as pension.The CAP 30 was a non-contributory scheme so members make no contributions to the scheme.It was funded by the government (Kumado and Goekel,2003).

2.11 Pension Reform in Ghana

Over the years, concerns have been raised and agitations made by public servants over inadequacies of the level of pensions to sustain a respectable life for retired public servants. A particular concern to most workers' groups has been the low pensions received by workers' under the Social Security and National Insurance Trust (SSNIT) Pension Scheme compared to those still under Chapter 30 of the 1950 British Colonial Ordinances (Pension Ordinance No. 42), popularly known as CAP 30.

In addition, pension schemes that have been operated in the country so far have, beside their limitations, also failed to consider the plight of workers in the informal sector, who constitute the bulk (about 85%) of the working population in Ghana. The concern rose to a peak in agitation and protests by workers' of organizations for the restoration of public service pensions to the level of the provisions still available to some public officers under CAP 30, in place of the SSNIT system that had been introduced in 1972 as the mandatory and universal pension scheme for all employees. In recognition of the need for reforms to ensure a universal pension scheme for all employees in the country, and to further address concerns of Ghanaian workers, the Government in July 2004 initiated a major reform of the Pension System in Ghana. The process started with the establishment of a Presidential Commission on Pensions under the chairmanship of Mr. T. A. Bediako.

The Bediako Commission was charged with the responsibility to examine existing pension arrangements and to make appropriate recommendations for a sustainable pension scheme(s) that would ensure retirement income security for Ghanaian workers, with special reference to the public sector.

The Commission submitted its Final Report in March 2006. The Government accepted almost all the recommendations of the Commission and issued a White Paper (W.P. No. 1/2006) in July, 2006.

The main recommendation of the Commission was the creation of a new contributory Three-Tier Pension System for Ghana, funded by direct contributions of employers and employees to, replace existing parallel pension schemes. The new contributory three-tier pension scheme comprises two mandatory schemes and a voluntary scheme as follows:

- A first tier mandatory basic national social security scheme which will incorporate an improved system of SSNIT benefits, mandatory for all employees in both the private and public sectors
- a second tier occupational (or work-based) pension scheme, mandatory for all employees but privately managed, and designed primarily to give contributors higher lump sum benefits than presently available under the CAP 30 and SSNIT pension scheme; and
- a third tier voluntary provident fund and personal pension schemes, supported by tax benefit incentives to provide additional funds for workers who want to make voluntary contributions to enhance their pension benefits and also for workers in the informal sector.

It is important to underline that provision has been made in the 3rd-Tier voluntary Personal Pension Scheme to cater for the peculiar needs of workers in the informal sector of the economy which covers about 85% of the working population (National Pension Regulatory Authority, NPRA).

2.11.1 Funding

A new pension regime came into force in Ghana on 4th December, 2008 when the National Pensions Act (Act 766) was enacted. The aims of the new pensions regime are to provide retirement income security for workers, ensure that retirement and related benefits are received as and when due, and establish uniformity in the rules and standards of pension administration

The Law put in place a mandatory basic national social security scheme (Tier 1), a mandatory fully-funded and privately managed occupation pension scheme (Tier 2) and a voluntary fully funded and privately managed provident fund and personal pension scheme (Tier 3). Total pension contributions were increased from 17.5% of workers salary to 18.5% with 5.5% being deducted from every workers salary monthly and employers contributing the remaining 13%. Out of this, 13.5% of salaries are paid to SSNIT and 5% to the second tier occupational pension scheme. Contributions to the third tier are voluntary.

Act 766 mandated SSNIT to operate the Basic National Social Security Scheme. The main functions of SSNIT under this law are:

- Have a fund into which contribution shall be paid;
- Administer the social security scheme;
- Provide social protection for workers through the provision of old age, invalidity and death benefits; and
- Take responsibility for the investment of the funds.

The Law exempts SSNIT from taxes. It sets the minimum entry age of fifteen (15) years and maximum of forty five (45) years. Scheme members are entitled to receive super annuation pensions on attainment of the compulsory retirement age of sixty years or on attainment of the voluntary retirement age of fifty five and contributing to the scheme for 180 months in aggregate. Pensions will range from a minimum of 50% of the three best year annual salary to a maximum of 80% of the three best year's salary.

The Second Tier Occupational Pension Scheme is a work-based scheme which provides a lump sum benefit on termination of service, death or retirement. This is funded by a contribution of five percent of the employee's salary and paid to approved trustees. Accrued benefits can be transferred to another approved trustee on leaving the employment of a company. Conditions for withdrawal from an occupational scheme include:

- Attaining retirement age;
- Self employed and attaining the age of 50 years;
- Medical grounds; and
- Payment to beneficiaries on death of contributor.

Contributions to Occupational Pensions are deductible for tax purposes. Benefits received from these schemes are also non-taxable. All investment income and capital gains are treated as deductible income for tax purposes.

The Third Tier Scheme is funded by contributions from contributors or contributors' employers. It is a defined contributions scheme that pays out lump sum benefits on retirement or on another prescribed event. Contributions to the scheme are voluntary. Contributions made and returns from investment are credited to the account of the contributor subject to any deductions of fees.

Contribution made by employers does not vest for the employee until the end of the vesting periods. Vesting occurs in the event of severance or in the event of liquidation of the employer. The employee may forfeit part or all of the employer's contribution if he/she leaves employment before the end of the vesting period. Withdrawals from Tier Three schemes are allowed on the attainment of retirement age, after ten years from the date of the first contribution for those employed in the formal sector and after five years from the date of the first contribution for informal sector contributors. Withdrawals are also allowable on medical grounds. Contributions to tier three schemes are deductible for income tax purposes. Contributions of up to 16.5% of the contributor's monthly income is tax exempt. Investment income on the fund and capital gains are also tax exempt. Accrued benefits are tax exempt if withdrawn on retirement, withdrawn after ten years of first contribution, withdrawn on medical grounds or by beneficiaries of an estate of a contributor to the scheme.

The aims of the new pension regime are laudable in that, it attempts to relieve the financial pain of old age. Its success will depend on the ability of the new regime, especially the tier three scheme, to increase the coverage of both self employed and informal workers. One of the challenges of the old regime was its low coverage of the total working population.

2.12 Benefits under SSNIT Pension Scheme

According to SSNIT benefits are paid to members of the scheme when they qualify.There are three main contingencies under which benefits can be paid.

These categories are listed below:

- Old Age Pension:This is a monthly payment made to retired members of the scheme.Members who retire at the normal pensionable age (age 60)and have made contributions of at least 180 months qualify for a full pension.Members who retire earlier than their normal pensionable age but have made contributions to the scheme for at least 180 months qualify for a reduced pension.
- Invalidity Pension: Members who for one reason or the other are incapable of working for a living and have contributed 12 months within the last 36 months before the unfortunate incidence.The member must provide a medical certificate to prove he or she is unable to be gainfully employed due to a disability (physical or mental). .
- Survivor's Lump sum Benefit: This is a lump sum paid to the beneficiary of a member of the scheme if the member dies in service or dies after retirement but before the age of 75.If a pensioner dies after the age of 75,nothing is paid to the beneficiary.
- Other Benefits: With the three tier scheme members would have access to multiple retirement income for members.

2.13 Life Annuities

A life annuity is an annuity, or series of payments at fixed intervals, paid while the purchaser (or annuitant) is alive. A life annuity is an insurance product typically sold or issued by life insurance companies. Pensions are paid as life annuities.

A predertimened amount of money is paid to the pensioner as long as he or she is alive. The payment stream from the issuer to the annuitant has an unknown duration based principally upon the date of death of the annuitant. At this point the contract will terminate and the remainder of the fund accumulated is forfeited unless there are other annuitants or beneficiaries in the contract. The valuation for pension annuity depends on the length of service, interest rate, the salary scale and mortality. The pension fund can calculate the Actuarial present value (or expected present value) of the annuity in order to estimate the value of its liabilities.

Annuities must be paid as long as the pensioner is alive. It is therefore prudent for pension funds to pursue investment strategies that will ensure return are sufficient to pay its liability.

CHAPTER 3

Methodology

3.1 Introduction

In this chapter we shall consider the data used for the study and also discuss the models used.

3.2 Data

Secondary data was obtained from the Social Security and National Insurance Trust(SSNIT).SSNIT is the biggest pension provider in Ghana with investment across the various sectors of the economy.The data obtained was well representative of Ghanaian Pensioners.It contains data on SSNIT investment portfolio of stocks listed on the Ghana Capital market from 2011 to 2013.

For the purpose of the study, opening price of stocks listed was ignored hence all analysis was carried out on the closing price of the stocks listed.

SSNIT mortality data and the total benefit paid out at the end of 2014 was also used for the analysis .

3.3 Value at Risk Model

The Value at Risk is calculated using the following procedure:

Let the Portfolio's current value be defined as p and it is known,the Portfolio's future value is not known in advance and is a random variable denoted by P.We need to estimate the distribution of P to calculate VaR.If we assume a standard distribution such as a normal distribution,the problem reduces from one estimating an entire distribution to that of estimating the parameters necessary to specify the distribution.

The risk factors such as the interest rates, volatilities being considered are then specified.If X is an N dimensional vector which contains the values of these risk factors in future .We need to make sure that the historical data is available for these risk factors.We then characterized the distribution of X based on the historical data.We then convert that characterization of the distribution of X into a characterization of the distribution of P.This is achieved by portfolio mapping function.Portfolio's future value can be expressed in terms of X by using a function θ called the portfolio mapping function.

$$P = X(\theta) \tag{3.1}$$

The relationship is called portfolio mapping.Portfolio mapping function θ maps the N dimensional space of the risk factors to the one-dimensional space of the portfolio's future market value.

Assuming X holds the prices of different stocks then it is a very simple portfolio mapping.However, if X holds many different risk factors such as prices,interest rates and implied volatilities,then the portfolio mapping function will be complicated. So we need to apply the portfolio mapping function θ to the entire distribution of X to obtain the entire distribution of P.

If θ is a linear polynomial and P is normally distributed then all we need to do is to calculate μ_p and σ_p for the portfolio.If we assume that X contains the prices of a set of stocks,then the portfolio's standard deviation can be computed from the asset level:

$$\sigma_p = \sqrt{hCh^T} = \sqrt{\sum_i \sum_j h_i h_j \sigma_{ij}} = \sqrt{\sum_i \sum_j h_i h_j P_{ij} \sigma_i \sigma_j} \tag{3.2}$$

where $h = N \times 1$ vector of asset weights

$C = N \times N$ covariance matrix for the asset returns and

$\sigma_{ij} = P_{ij}\sigma_i\sigma_j$ introduces the correlation coefficient

For any linear portfolio , we are able to compute its risk if we know the weights and the covariance matrix of the assets.

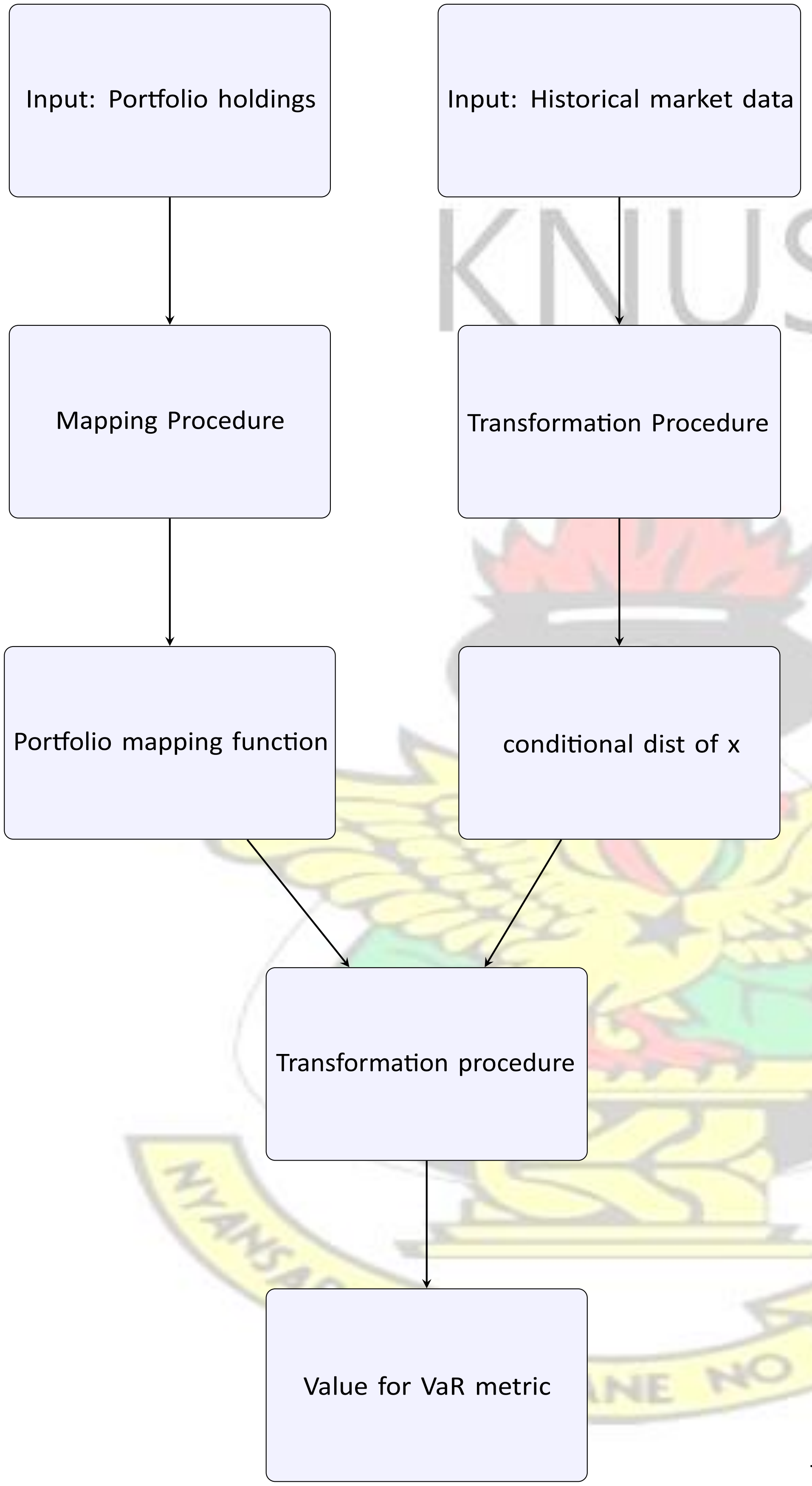
A linear mapping function θ is applied to a normal vector X.By mapping evenly spaced values for X through the mapping function θ. The output values for P after the mapping are also evenly spaced,indicating that the portfolio mapping does not cause any distortion.Therefore, X is normal and P is normally distributed.

If θ ,the portfolio mapping function, is not a linear polynomial.This is a non-linear, so we cannot compute σ_p,using (3.2).Therefore, P cannot assumed to be normally distributed. A non-linear mapping function θ is applied to a normal vector X.By mapping evenly spaced values for X through the mapping function θ.The output values for P after the mapping are not evenly spaced ,indicating how the portfolio mapping distort the distribution of P.

Therefore,P now has a non-normal distribution.

The mapping procedure accepts a portfolio's composition as an input and its output is the mapping function θ that defines P as a function of X. The inference procedure accepts historical data of the corresponding risk factors of the N-dimensional vector NX as its input. The purpose of the inference procedure is to characterize the probability distribution of X based on its input. The output of the inference is the characterization of the distribution of X. The transformation procedure then combines the outputs from the mapping procedure and the inference procedure and uses them to characterize the distribution of P . Based on the distribution of and the current portfolio value , the transformation procedure then determines the value of VaR.

Below is the Schematic representation of how the Value-at-Risk is calculated:



The VaR Measure The Schematic Representation for Calculating VaR .

3.3.1 The Variance-Covariance Value at Risk Method

The Variance-Covariance model is the best method to calculate VaR for portfolios with linear positions and whose distributions are close to the normal probability density function. The Variance-Covariance model may not be appropriate for portfolios with non linear positions such as options and non-normal distributions. In such cases, one should use Monte Carlo method to calculate the Value-at-Risk of the portfolio.

The Variance-Covariance method is analytical, it allows easy analysis of the VaR results using marginal and component VaR measures.

It is easy to implement since it involves a simple matrix multiplication. It is also, computationally fast, even with a large number of assets, because it replaces each position by its linear exposure. Portfolios that are linear combinations of normally distributed risk factors are themselves normally distributed. It only requires the market values and exposures of current positions, combined with risk data. Also, the Variance Covariance model provides adequate measurement of market risks. As a parametric approach, VaR is easily amenable to analysis, since measures of marginal and incremental risk are a by-product of the VaR computation. This method is important because it illustrates the "mapping" principle in risk management.

3.3.2 Computatation of Variance-Covariance Value at Risk

If the risk factors consist of prices of stocks,then we denote Portfolio's current return by p and it is known.We therefore denote the Portfolio's future return which is not known and it is a random variable by P. We need to estimate the distribution of P to calculate VaR. Now since the Variance-Covariance VaR model assumes a standard normal distribution, we assume a standard distribution such as a normal distribution for P. The problem reduces from one of estimating an entire distribution to that of estimating the parameters necessary to specify that distribution $\mu_p$and σ_p.

$X$is an N- dimensional vector which contains the values of these risk factors. Based on the historical data, we can characterize the distribution of X.

We then convert that characterization of the distribution of X into a characterization of the distribution of P. This is achieved by the portfolio mapping function. Portfolio's future value can be expressed in terms of X by using a function θ called the portfolio mapping function. Portfolio's mapping function θ maps the N-dimensional space of the returns of the stocks to the one-dimensional space of the portfolio's future market value where N corresponds to the number of stocks chosen.

$$P = X(\theta) \tag{3.3}$$

This relationship is called portfolio mapping. Now, since X holds the prices of the different stocks then it is a very simple portfolio mapping. So, we need to apply the portfolio mapping function θ to the entire distribution of X to obtain the entire distribution of P . θ is a linear polynomial and P is normally distributed and then all we need to do is calculate μ_p and σ_p for the portfolio. If we assume that X contains the prices of a set of stocks, then the portfolio's risk can be computed from asset level:

$$\sigma_p = \sqrt{hCh^T} = \sqrt{\sum_i \sum_j h_i h_j \sigma_{ij}} = \sqrt{\sum_i \sum_j h_i h_j P_{ij} \sigma_i \sigma_j} \tag{3.4}$$

where h = $N \times 1$ vector of asset weights

$C = N \times N$ covariance matrix for the asset returns and

$\sigma_{ij} = P_{ij}\sigma_i\sigma_j$ introduces the correlation coefficient

The output of the mapping procedure in the variance-covariance method is a linear mapping function θ that is applied to a normal vector X. The output values for P after the mapping are also evenly spaced, indicating that the portfolio mapping does not cause any distortion. Therefore, since X is normal, P now is normally distributed. The inference procedure accepts historical data of the stock returns of the N-dimensional vector$R$ as its input.Since the returns of the stocks are normally distributed, a linear combination of these is also normally distributed.

The output of the inference procedure is that the characterization of the distribution of X is a normal distribution. The transformation procedure then combines the outputs from the mapping procedure and the inference procedure and uses them to characterize the distribution of P. In the Variance-Covariance method, the transformation procedure determines that, the distribution of P is a normal distribution. Based on the distribution of P and the current portfolio value p , the transformation procedure then determines the value of VaR. Since P is normally distributed then the VaR for a target probability p^* is calculated:

$$VaR(p^*) = Z_{1-p^*}\sigma_p + (p - \mu_p) \quad (3.5)$$

With Z_{1-p^*} is equal to 1.645 for a target probability of 95%. Over a short time horizon, such as a day, it is reasonable to assume the portfolio's forecasted return equals to its current return. In such cases,VaR is calculated:

$$VaR(p^*) = Z_{1-p^*}\sigma_p \quad (3.6)$$

Figure 3.1 below shows the VaR at 95% confidence level.

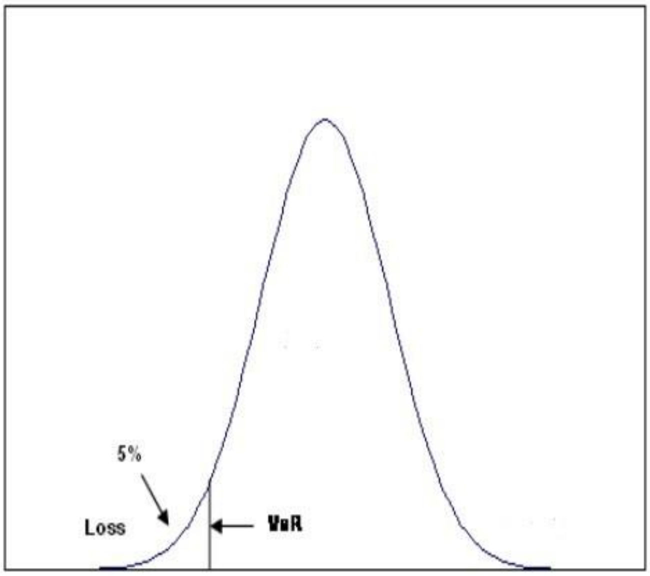


Figure 3.1: VaR at 95%

3.3.3 Assumption of the Variance-Covariance Method

The method assumes that the stock returns are normally distributed.

3.4 Annuity Model

3.4.1 Survival Probabilities

The probability that a life aged x survives to age $x + 1$ is given by p_x. It follows that the probability that a life aged x does not survive to age $x + 1$ is given by q_x

Therefore the probability of survival for n years is given by

$$ {}_np_x = p_x \times_2 p_x \times \ldots \times_n p_x = \prod_{i=1}^{n} {}_ip_x \quad (3.7)$$

${}_np_x$ can be obtained using the relation

$$\frac{l_x}{l_{x+n}}$$

where l_x = the number of lives aged x and l_{x+n} = the number of lives aged $x + n$

Using data obtained from SSNIT, ${}_np_x$ for and $x = 60$ was calculated. The obtained mortality table was used to calculate the expected present value of the annuity to be paid to the pensioners.

Whole life Annuity

Consider an annuity contract to pay GHS1 at the start of each future year provided a life now aged x is then alive.

If the life dies between ages $x+k$ and $x+k+1$ (k = 0,...,$\omega - x - 1$) which is to say, $K_x = K$, the present value at time 0 of the annuity payments which are made is $\ddot{a}_{\overline{k}|}$ (we define $a_{\overline{0}|} = 1$).

Therefore the present value of the annuity at time 0 is $\ddot{a}_{\overline{K_x}|}$.

Expected Present Value

The expected value of $\ddot{a}_{\overline{K_x}|}$ is:

$$E[\ddot{a}_{\overline{K_x}|}] = \sum_{k=0} \ddot{a}_{\overline{k}|} P(K_x = k) \qquad (3.8)$$

This is taken from the general formula for expectation

$$E[g(x)] = \sum_x g(x).P(X = x) \qquad (3.9)$$

So, the expectation of $\ddot{a}_{\overline{K_x}|}$ defines the actuarial value a_x, and:

$$E[\ddot{a}_{\overline{K_x}|}] = \sum_{k=0}^{\infty} a_{\overline{k}|}\, {}_{k|}q_x \qquad (3.10)$$

$$a_x = E[\ddot{a}_{\overline{K_x}|}] = \sum_{k=0}^{\infty} a_{\overline{k}|}\, {}_{k|}q_x = \sum_{k=1}^{\infty}\left(\sum_{j=0}^{k-1} v^{j+1}\right) {}_{k|}q_x \qquad (3.11)$$

This result holds since $a_{\overline{0}|} = 0$ and

$$a_{\overline{k}|} = v + v^2 + \ldots + v^k = \sum_{j=0}^{k-1} v^{j+1} \quad (3.12)$$

If we write out the sum more fully:

$$\ddot{a}_x = 0 \times {}_0|q_x + v \times {}_1|q_x + (v + v^2) \times {}_2|q_x + (v + v^2 + v^3) \times {}_3|q_x + \ldots \quad (3.13)$$

Now reversing the order of summation:

$$\ddot{a}_x = v[{}_1|q_x + {}_2|q_x + \ldots] + v^2[{}_2|q_x + {}_3|q_x + \ldots] + \ldots \quad (3.14)$$

$$= v\sum_{k=1}^{\infty} {}_k|q_x + v^2 \sum_{k=2}^{\infty} {}_k|q_x + \ldots \quad (3.15)$$

Hence

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^t\, {}_tp_x \quad (3.16)$$

Let S be the annual amount paid to a pensioner aged x. Then the expected present value of the pensioner's total annuity until death is given by

$$S\ddot{a}_x^{(m)}$$

where m is the frequency of the payments (usually monthly-12).

Now, let's consider n pensioners who retired in a particular year, each will be paid an annual annuity of S_i. The total amount to be paid by the pension fund to this particular group is given by

$$\sum_{i=1}^{n} S_i \quad (3.17)$$

Therefore the expected present value at time 0 of the total amount to be paid to the total number of pensioners from that group until death is given by

$$\sum_{i=1}^{n} S_i \ddot{a}_x^{(m)} \quad (3.18)$$

CHAPTER 4

Analysis

4.1 Introduction

The purpose of this chapter is to present and discuss the empirical findings of the study.The chapter is in three sections. We start with the descriptive statistics of the investment portfolio of SSNIT.The second section presents results obtained from the variance covariance Value at Risk measure of the investment portfolio.The third section present a simple annuity model based on the mortality rates.

4.2 Implementation of the Variance- Covariance Method

The data consist of 851 monthly closing price of 23 stocks dated from 2nd January, 2011 (01/02/2011) to 2nd December, 2013 (12/02/2013). SSNIT portfolio consisting of 23 stocks is listed:

Table 4.1: SSNIT Portfolio of the 23 Listed Stocks

Name

FML Fan Milk Ghana Limited
GGBL Guiness Ghana Brewery limited
TOTAL Total Ghana Limited SCB Standard
Charted Bnak
AGA Anlogold Ashanti ARD Anglgold
Depository
UNIL Uniliver Ghana Limited
MLC Mechanical Lloyd HFC HFC Bank Ghana
Limited
SG Soceite General Ghana Limited
EGL Enterprise Life Insurance Group Limited
GCB Ghana Commercial Bank
ALW Aluworks Ghana Limited PBC Produce
Buying Company
CPC Cocoa Processing Company
BOPP Benso Oil Palm Plantation CAL CAL
Bank Ghana Limited
GWEB Golden Webb Ghana
AYRTN Ayrton Drugs
GOIL Ghana Oil Company Limited
SIC State Insurance Company
ETI Ecobank Transitional

The monthly returns are computed as follows:

$$R_i(t) = (P_i(t+1) - P_i(t))/P_i(t) \qquad (4.1)$$

Where $P_i(t+1)$ is the value of the closing price of the stock at current time and $P_i(t)$ is the value of the closing price of the stock at previous time.

4.3 Descriptive Analysis

The data used for the analysis was monthly returns on SSNIT's portfolio of shares on the Ghana Stock exchange. The portfolio consists of 23 shares.

The graph below shows the monthly returns on the portfolio.

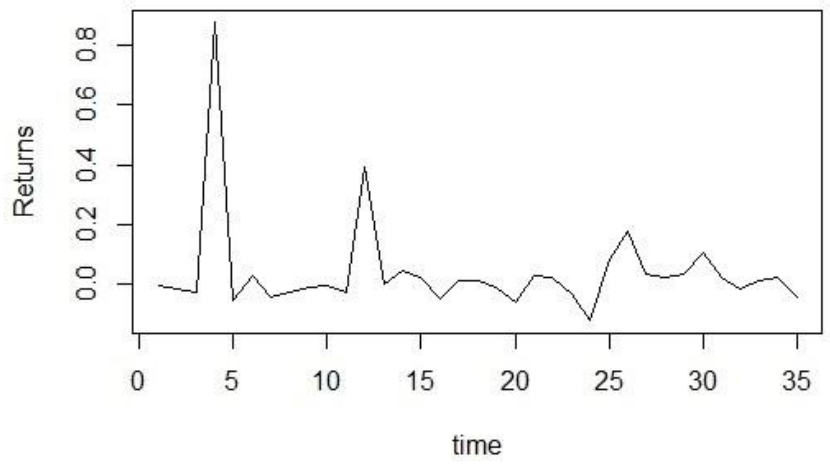


Figure 4.1: Monthly Returns of the Portfolio

The maximum return for the period was 0.8753468 (87.5%) and the minimum was -0.1208744 (-12.1%). The average return was 4.14% with a standard deviation of 0.167.

The monthly return as can be seen from the graph is very volatile and ocassionally goes below 0%

4.3.1 The VaR Model

The objective was to compute the Value at Risk (VaR) for the trading portfolio using the variance-covariance method.

The variance-covariance method for calculating VaR assumes that the returns are normally distributed. Before carrying on with the computation of VaR, we first test for normality of the returns.

The graph below is a histogram of returns on the portfolio .

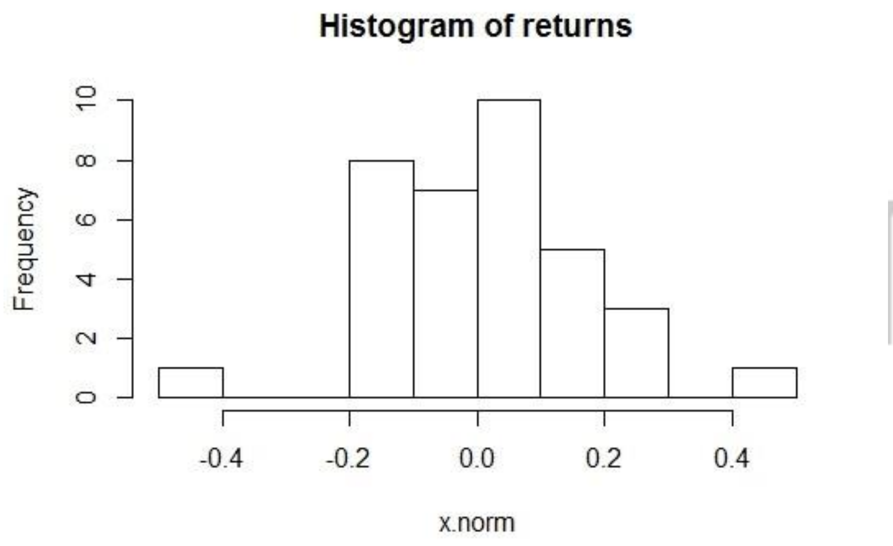


Figure 4.2: Histogram of Returns on the Portfolio

From the histogram we can observe that the returns are normal.

A density function was also fitted to the data

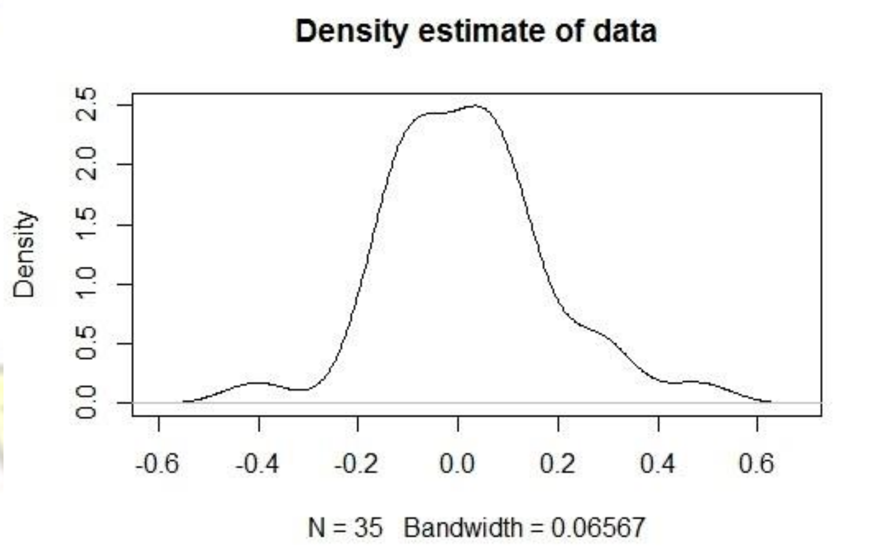


Figure 4.3: Density of Data

To confirm the normality of the returns, the Geary test for normality was done with the null hypothesis that the returns follow a normal distribution.

Results from the Geary test for normality on Portfolio returns

$d = 0.49699,\ p - value = 1$

The results showed that the $p - value = 1$ is greater than 0.05 ,this confirms the normality of the returns at the 95% confidence interval.

Since the returns are normal, we proceed with the variance-covariance computation of VaR. The parameters: mean, standard deviation and the covariance matrix of the monthly returns on the portfolio are computed as follows:

The mean of the portfolio P, is denoted by μ_p such that $\mu_p = E(R)$ and the standard deviation of the portfolio P is denoted, by σ_p such that $\sigma_p = \sqrt{Variance(R)}$.

$$R = w_1R_1 + \ldots + w_nR_n \tag{4.2}$$

on a portfolio with weights $w = [w_1,\ldots,w_n]$, the mean and the standard deviation are denoted by $\mu_p = mw^T$ and $\sigma_p = \sqrt{wCw^T}$ respectively. Since the data used was the monthly closing prices to compute the Value at risk,we compute the standard deviation through :

$\sigma_{p30days} = \sqrt{30} \times \sigma_{pdaily}$.

Hence, $\mu_p = 0.067274$ and $\sigma_p = 0.011896$

We compute the VaR at 95% 99% and 90% confidence interval. This represents the maximum loss that can be incured in a trading month.

The Value at Risk for a portfolio at a target probability of 95% is given by:

$$VaR(95\%) = 1.645\sigma_p + (P - \mu_p) \tag{4.3}$$

Again the Value at Risk for a portfolio at a target probability of 99% is given by:

$$VaR(99\%) = 2.33\sigma_p + (P - \mu_p) \tag{4.4}$$

And the value at risk for a portfolio at a target probability of 90% is given by:

$$VaR(90\%) = 1.282\sigma_p + (P - \mu_p) \quad (4.5)$$

For a GHS 11.7 billion investment in the portfolio of 23 stocks we calculate the VaR in Ghana cedis,that is $GHS11.7billion \times theVaRin\% = VaRGhanacedis$. The table below shows the results of the VaR calculated in cedis .

Table 4.2: VaR results in % and GHS

VaR %	VaR in GHS	alpha
-0.4979359	-582,585,003	95%
-0.7321129	-856,572,093	99%
-0.373097	-4,365,234,900	90%

From the table the 95% VaR was found to be GHS 582,585,003.This means that,under a normal market condition the most the portfolio of SSNIT can lose over a trading month is GHS 582,585,003 and we are 95% confident that the actual loss will not exceed it.

Again the 99% VaR was found to be GHS 856,572,093.This implies that, under a normal market condition the most the portfolio of SSNIT can lose over a trading month is GHS 856,572,093 and we are 99% confident that the actual loss will not exceed it.

Finally,the 90% VaR was found to be GHS 4,365,234,900 which is the amount that the portfolio of SSNIT can lose over a trading month and we are 90% confident that the actual loss will not exceed it.

4.4 Results of the Annuity Model

The parameters used to determine annuity paid was computed as follows:

$P = 1 - q$ where q is the probability of dying ${}_np_x$ is the probability that a life aged x survives for n years

$(1 + i)^{-n}$ is the n years discount factor with $i = 20\%$ (i is the interest rate) ${}_np_x \times v^n$ is the n year discount factor adjusted for probability of survival Table 4.2 shows the results of the calculated parameters.

Table 4.3: Parameters of the Annuity Model

Age	n	q	p=1-q	${}_np_x$	$(1+i)^{-n}$	${}_np_x \times V^n$	a_x	$a_x^{(m)}$
60	1	0.000959	0.999040	0.999040307	0.833333333	0.832533589	0.832533589	0.374200
61	2	0.001921	0.998078	0.997120921	0.694444444	0.692445084	1.524978673	1.066645
62	3	0.005774	0.994225	0.991362764	0.578703704	0.573705303	2.098683977	1.640350
63	4	0.040658	0.959341	0.951055662	0.482253086	0.458649528	2.557333505	2.099000
64	5	0.050454	0.949545	0.903071017	0.401877572	0.362923988	2.920257493	2.461924
65	6	0.044633	0.955366	0.862763916	0.334897977	0.28893789	3.209195383	2.750862
66	7	0.051167	0.948832	0.818618042	0.279081647	0.228461272	3.437656654	2.979323
67	8	0.044548	0.955451	0.782149712	0.232568039	0.181903025	3.619559679	3.161226
68	9	0.062576	0.937423	0.733205374	0.193806699	0.142100114	3.761659793	3.303326
69	10	0.060209	0.939790	0.689059501	0.161505583	0.111286956	3.872946749	3.414613
70	11	0.080779	0.919220	0.633397313	0.134587986	0.085247669	3.958194418	3.499861
71	12	0.051515	0.948484	0.600767754	0.112156655	0.067380102	4.025574519	3.567241
72	13	0.075079	0.924920	0.555662188	0.093463879	0.051934344	4.077508863	3.619175
73	14	0.072538	0.927461	0.515355086	0.077886566	0.040139238	4.117648101	3.659314
74	15	0.061452	0.938547	0.483685221	0.064905472	0.031393817	4.149041918	3.690708
75	16	0.111111	0.888888	0.429942418	0.054087893	0.023254679	4.172296598	3.713963
76	17	0.142857	0.857142	0.368522073	0.045073244	0.016610485	4.188907083	3.730573
77	18	0.177083	0.822916	0.303262956	0.037561037	0.011390871	4.200297954	3.741964
78	19	0.246835	0.753164	0.22840691	0.031300864	0.007149334	4.207447288	3.749113
79	20	0.584033	0.415966	0.095009597	0.026084053	0.002478235	4.209925523	3.751592
80	21	0.961344	0.038655	0.003672681	0.021736711	7.9832E-05	4.210005355	3.751672
81	22	0.974153	0.025846	9.49264E-05	0.018113926	1.71949E-06	4.210007074	3.751673
82	23	0.982794	0.017205	1.6333E-06	0.015094938	2.46545E-08	4.210007099	3.751673
83	24	0.988579	0.011420	1.86523E-08	0.012579115	2.3463E-10	4.210007099	3.7516737
84	25	0.992435	0.0075648	1.41101E-10	0.010482596	1.47911E-12	4.210007099	3.751673
85	26	0.994995	0.005004	7.06134E-13	0.008735497	6.16843E-15	4.210007099	3.751673
86	27	0.996692	0.003307	2.33573E-15	0.007279581	1.70031E-17	4.210007099	3.751673
87	28	0.997814	0.002185	5.10372E-18	0.006066317	3.09608E-20	4.210007099	3.751673
88	29	0.998557	0.001442	7.364E-21	0.005055264	3.72269E-23	4.210007099	3.751673
89	30	0.999047	0.000952	7.01444E-24	0.00421272	2.95499E-26	4.210007099	3.751673
90	31	0.999371	0.000628	4.41015E-27	0.0035106	1.54823E-29	4.210007099	3.751673

From the table,we can see that Using a limiting age of 90, the pension fund's estimated liability for a single pensioner who went on pension at age 60 and is paid GHS1 annually was found to be

GHS3.751673. Assuming a total of 5000 pensioners were paid GHS 1 per year, then the total liability was GHS18,758.37. However, all pensioners are not paid GHS1 monthly neither are they paid similar amounts. The amount paid depends on the salary scale and length of service.

We denoted S as the amount paid annually by the pension fund to all pensioners. Then the expected value of the fund's liability is GHS 18,758.37S. The total benefit paid by SSNIT at the end of 2014 was GHS 944,445,000. therefore the expected liability value to be paid is

$$18{,}758.37 \times 944{,}445{,}000 = GHS944{,}463{,}758.37$$

CHAPTER 5

Conclusion And Recommendation

5.1 Introduction

In this chapter we make conclusions and recommendations based on the results obtained from the study.

5.2 Conclusions

The objectives of this study was two fold: To measure the investment risk of pension funds under the Value at Risk metric and also to obtain the expected liability value to be paid.We analyzed investment and mortality data from SSNIT to determine the investment risk ,the expected liability value respectively .

The pension fund's liabilities is the expected present value of annuity payments the fund will make to pensioners. Using a limiting age of 90, the pension fund's estimated liability for a single pensioner who went on pension at age 60 and is paid GHS1 annually was found to be GHS 3.751673. For a total of 5000, pensioners paid GHS 1 per year, the total liability was GHS 18,758.37. Again,the expected present value of the fund's liability to be paid is estimated to be$GHS944,463,758.37$.

On the other hand,the returns of the stock data considered in this study are normally distributed. So based on the normality of the returns of the data the Variance Covariance method is used to compute the Value at Risk using the R software.

The method quantified the risk of SSNIT'S investment as $GHS582,585,003$ at 95% confidence level, $GHS856,572,093$ at 99% confidence level and $GHS4,365,234,900$ at 90% confidence level. It is observed that the three different confidence level of computing VaR give different results. Higher confidence level gives higher VaR and lower confidence level gives lower VaR.

Since,the study used 95% to test for the normality of the returns,we are certain that under a normal market condition the maximum loss SSNIT must expect is GHS 582,585,003 and we are 95% confident that the actual loss will not exceed it. Also, the maximum return of the portfolio for the period is greater than the Value at Risk with the 95% confidence level. Thus, any investor who would invest in this portfolio would go with the 95% confidence level of computing the Value-at-Risk.

Hence, the pension fund(SSNIT) needs to be monitored by knowing the expected liability value to be paid annually and when investing fund managers must quantify the Value at risk at an appropriate confidence level to know the amount of risk regarding the investment portfolio in a particular period.This will help reduce the rate of unfunded liabilities.

In conclusion, if fund managers know the value at risk of the investment portfolio and also know the expected liability to be paid out it will help them make the right investment decision.

5.3 Recommendations

Based on the conclusions made,we make some recommendations to pension funds,policy makers and institutions exposed to investment risk.

- SSNIT should ensure that the value of its investment portfolio is always greater than the value of its liabilities .
- We recommend the use of VaR to monitor the investment portfolio of pension funds this will ensure that the value of the portfolio does not fall below some minimum level that would expose the fund to insolvency.

References

Artzner,P.,Delben,F.,Eber,J.M.,&Heath,D.(1999).Coherent Measures of Risk.*Mathematical Finance*,9(3),203-228.

Barbone,L.(1999).Pension and Social Security In Sub-Saharan Africa Issues and Options.

Balzer, L. A. (1994). Measuring investment risk: A review. *The Journal of Investing, 3(3)*, 47-58.

Bernstein,P.L.(1999). *Capital Ideas: the improbable origins of modern wall street*. New York; London,Free Press.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The journal of political economy*, 637-654.

Carnes,B.A., & Olshansky,S.J.(2007).A Realistic View of Aging,Mortality and Future Longevity.*Population and Development Review*,33(2),367-381.

Crouhy, M., Galai, D., & Mark, R. (2005). *Essentials of Risk Management*.New York: McGraw-Hill.

Davis,J.T.(1991).Institutional Impediments to Workforce Retrenchment and Restructuring in Ghana,s State Enterprises.*World Development*,19(8),987-1005.

Davies,E.P.(2001).Portfolio-regulation of Life Insurance Companies and Pension Funds.*Working Paper*.

Dowd, K. (2001). Sharp Thinking. Risk, Risk Management for Investors Special Report, 34-S37.

Dowd,K.,& Blake,D.(2006).After VaR:The theory,Estimation and Insurance Applications of Quantile-Based Risk Measures.*Journal of Risk Insurance*,73(2),193-229.

Franzen, D. (2010). Managing investment risk in defined benefit pension funds.

Herring, R., C., Schuermann,Till (2005).Capital Regulation for Position Risk in Banks,Securities Firms and Insurance Companies.*Capital adequacy beyond Basel:banking,securities,and insurance*.H.S.Scott.Oxford,Oxford University Press:15-86.

Holton,G.A.(2004).Defining Risk.*Financial Analysts Journal,60(6),* 19-25.

Kocken, T. P. (2006). Curious Contracts: Pension Fund Redesign for the Future.
Kumado,K.,& Gockel,A.F.(2003).A Study of Social Security in Ghana.*University of Ghana*.

Lange,M.(2010).States and Economic Development,*The Handbook of Politics:States and Society in Global Perspective*,USA,Springer Publishers.

Litterman, B. (2004). Modern investment management: an equilibrium approach (Vol. 246). John Wiley & Sons.

Markowitz,H.(1952).Portfolio Selection. *Journal of Finance*,7(1),77-91.

McNeil, A. J., Frey, R., & Embrechts, P. (2005). *Quantitative Risk Management*.Princeton: Princeton University Press.

Mengle, D. (2003). Regulatory Origin of Risk Management. *Modern Risk Management: A History*, London: Risk Books:417-425

Nkrumah, D. (2010). Workers' future in danger. *Daily Graphic*, Accra. P 1, 3.

Olsen,R.A.(1997). Investment Risk: The Experts' Perspective.*Financial Analyst Journal*,53(2),62-66.

Rahl, L. (2000). Risk Budgeting:The Next Step of the Risk Management Journey-A Veteran's Perspective.

Rosen, D. (2003). The Development of Risk Management Software.*Modern Risk Management: A History*. London,Risk Books:611.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of economic theory, 13(3),* 341-360.

Ryan,R.J.,& Fabozzi,F.J.(2002).Rethinking Pension Liabilities and Asset Allocation.*The Journal of Portfolio Management*,28(4),7-15.

Santomero, A. M. (1997). Commercial bank risk management: an analysis of the process. *Journal of Financial Services Research, 12(2)*, 83-115.

Scherer,B.(2006).Introduction.Rethinking Asset Management: Consequences of the Pension Crisis. *Asset and Liability Management Tools:A Handbook for Best Practice*.B.Scherer.London,Risk Books:XV-XXII.

Sharpe, W.F. (1964), Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *The Journal of Finance*, Vol. XIX, No 3, September.

Stewart, F. (2005). Developments in Pension Fund Risk Management in Selected OECD and Asian countries. *OECD Working Paper on Insurance and Private Pensions.*

Appendix

Table 5.1: VaR in % for individual stocks

Stock	VaR
FML	-0.1492181
GGBL	-0.5532129
TOTAL	-0.7608949
SCB	-0.1498859
ANGLO.DEP	-0.007480066
UNIL	-0.3591301
MLC	-0.4464482
HFC	-0.2698911
SG	-0.3781795
EGL	-0.2729877
GCB	-0.2580934
ALW	-1
PBCL	-0.2579957
CPC	-0.4085149
BOPP	-0.3263641
CAL	-0.2705304
GWEB	-0.29173
AYRTN	-0.05453538
GOIL	-0.2424675
SIC	-0.6499284
ETI	-0.4231786

Table 5.2: Parameter Results for Individual Stocks

Stock	Mean	Variance	std dev	Weights	Wgtsmean
FML	0.01176644	0.009860586	0.097871625	0.039045727	0.000459429
GGBL	0.053000173	0.139825115	0.36855137	0.004385189	0.000232416
TOTAL	0.076157125	0.266587353	0.508891512	0.038694911	0.002946893
SCB	-0.004090474	0.008087656	0.088637354	0.009156276	-3.74535E-05
ANGLO DEP	1.0367E-05	2.13476E-05	0.00455386	0.050306893	5.2153E-07
UNIL	0.033731672	0.058723803	0.238843004	0.01852304	0.000624813

MLC	0.036203845	0.08863454	0.293431635	7.0163E-05	2.54017E-06
HFC	0.004773565	0.028703862	0.166984286	1.40326E-06	6.69856E-09
SG	-0.02903183	0.046382356	0.212266686	0.061848712	-0.001795581
EGL	0.025717976	0.033948565	0.181600128	0.075811155	0.001949709
GCB	0.010883328	0.027527322	0.163526228	0.021504969	0.000234046
ALW	0.537142857	10.39548701	3.177809481	0.104577997	0.056173324
PBCL	0.02217375	0.029865928	0.170330901	0.087352973	0.001936943
CPC	0.028571429	0.072689076	0.265729646	0.094544684	0.002701277
BOPP	0.017527685	0.044996449	0.209071365	0.03911589	0.000685611
CAL	0.01717489	0.031494177	0.174912389	0.065883086	0.001131535
GWEB	0.021428571	0.037313259	0.190386884	0.013120487	0.000281153
AYRTN	0.000560224	0.001154964	0.033495748	0.09654433	5.40864E-05
GOIL	0.009820265	0.024217369	0.153380065	0.085142838	0.000836125
SIC	0.051832248	0.187375424	0.426640176	0.003157336	0.000163652
ETI	-0.014335203	0.063598762	0.248559156	0.09121194	-0.001307542

Table 5.3: Stock Returns

Date	FML	GGBL	TOTAL	SCB	ANGLO DEP	UNIL
2013-02-01	0	0	0.003278689	-0.012776413	0	-0.065420561
2013-02-02	0	0	-0.001633987	0.003484321	0	-0.16
2013-02-03	0.00952381	0	0.006546645	0.005456349	0	0.011904762
2013-02-04	0	0	-0.001626016	0.00148002	0	-0.117647059
2013-04-05	0.050943396	0	-0.035830619	-0.014778325	0	0.007066667
2013-02-06	0.032315978	-0.015625	-0.10472973	-0.025	0	-0.003045148
2013-02-07	0.012173913	-0.126984127	0	-0.025641026	0	-0.030544489
2013-03-08	0.029209622	-0.2	0.018867925	-0.116315789	0	0.015068493
2013-02-09	0.227045075	-0.090909091	-0.007407407	-0.030970816	0	0.043184885
2013-02-10	0	0	-0.01119403	0.004302397	0	0.015523933
2013-02-11	0	0	0	-0.003059976	0	0.005095541
2013-02-12	-0.099319728	2.1	-0.045283019	0.111111111	0	1.320659062

2012-02-01	0.043806647	0	-0.007905138	-0.005524862	0	-0.011469143
2012-03-02	0.060781476	-0.032258065	0.055776892	0.011111111	0	0
2012-03-03	0.019099591	-0.016666667	0.226415094	0.095604396	0	-0.000552486
2012-02-04	0.001338688	-0.020338983	0	-0.048645938	0	-0.000552792
2012-02-05	-0.064171123	-0.034602076	-0.015384615	-0.051133368	0	-0.005530973
2012-02-06	-0.025714286	-0.091397849	0.021875	0.005555556	0	-0.021134594
2012-02-07	-0.046920821	-0.309664694	-0.032110092	0.005524862	0	0
2012-04-08	-0.233846154	-0.277142857	-0.034755134	0.001648352	0	-0.034090909
2012-02-09	-0.016064257	0.233201581	-0.001636661	0.001097093	0	-0.074117647
2012-02-10	0	0.009615385	0	0.035616438	0.019230769	-0.17407878
2012-03-11	0	0	0	0	0	0
2012-02-12	-0.275510204	-0.168253968	2.852459016	-0.391534392	-0.018867925	-0.344615385
2011-02-01	0.112676056	0.114503817	-0.045531915	0	0	0.177230047
2011-04-02	0.329113924	0.164383562	0.119037004	0.17826087	0	0.081754736
2011-04-03	0.043809524	0.044117647	0.139442231	0.182287823	0	0.126267281
2011-02-04	0.031021898	0.070422535	0.021678322	-0.007490637	0	0.104746318
2011-02-05	0.026548673	0.131578947	-0.050308008	-0.031446541	0	0.071851852
2011-03-06	0	0.027906977	0.501261261	-0.090909091	0	0.043538355
2011-02-07	0.037931034	0.018099548	0.005760922	0.035714286	0	0
2011-02-08	0.021594684	0.046666667	-0.875417661	-0.027586207	0	0.005298013
2011-02-09	0.035772358	0.157112527	-0.042145594	0.003546099	0	0.105401845
2011-02-10	0.04866562	0.135779817	0	0.033922261	0	0.072705602
2011-04-11	0	-0.01453958	0.006	0.023923445	0	0.016111111

Table 5.4: Stock Returns

Date	MLC	HFC	SG	EGL	GCB	ALW
2013-02-01	-0.035714286	-0.006666667	0.06	-0.062857143	-0.140893471	0.5
2013-02-02	-0.111111111	-0.161073826	0	-0.042682927	0.03	0
2013-02-03	-0.166666667	0.24	-0.028301887	0.031847134	0.034951456	-0.333333333

2013-02-04	0	0.103225806	-0.048543689	0.018518519	0.011257036	19
2013-04-05	-0.05	0.005847953	-0.030612245	0.03030303	-0.07606679	-0.85
2013-02-06	-0.052631579	-0.069767442	-0.042105263	0.147058824	-0.018072289	0
2013-02-07	-0.055555556	-0.125	-0.131868132	0.01025641	-0.100204499	0
2013-03-08	-0.058823529	-0.064285714	0.139240506	0.116751269	-0.088636364	-0.333333333
2013-02-09	-0.0625	-0.236641221	-0.111111111	0	-0.069825436	0.75
2013-02-10	0	-0.14	-0.0125	0	0.002680965	0.142857143
2013-02-11	0	-0.093023256	0.012658228	0.090909091	0.005347594	0
2013-02-12	1.533333333	0.217948718	-0.0625	-0.216666667	0.289893617	-0.375
2012-02-01	-0.078947368	-0.010526316	0.053333333	0.053191489	0.035051546	0
2012-03-02	0.114285714	0.170212766	0.443037975	0.252525253	-0.183266932	0
2012-03-03	-0.128205128	0.090909091	-0.122807018	-0.036290323	-0.002439024	0.2
2012-02-04	0	0.041666667	-0.34	-0.041841004	-0.019559902	0
2012-02-05	-0.147058824	0.16	0.409090909	-0.23580786	0.271820449	-0.166666667
2012-02-06	0.034482759	0.068965517	-0.107526882	0.028571429	0.068627451	0
2012-02-07	0.033333333	-0.129032258	0.060240964	0.005555556	-0.060550459	0
2012-04-08	0	0	0.113636364	-0.066298343	-0.109375	-0.2
2012-02-09	-0.161290323	0	-0.081632653	-0.023668639	0.120614035	-0.25
2012-02-10	0	-0.148148148	-0.055555556	-0.018181818	0.033268102	0.333333333
2012-03-11	0	0	0	0	0	0
2012-02-12	-0.423076923	-0.608695652	-1	-0.703703704	-0.602272727	0.25
2011-02-01	-0.133333333	-0.088888889	0	0.0625	0.257142857	0.2
2011-04-02	0.538461538	0.243902439	0	0.392156863	0.238636364	0.333333333
2011-04-03	0.15	0.019607843	0	-0.014084507	0	-0.125
2011-02-04	0	0.038461538	0	0.214285714	0.342507645	-0.142857143
2011-02-05	0.086956522	0.018518519	0	0	0.161731207	0
2011-03-06	0.04	0	0	0.329411765	-0.103921569	0
2011-02-07	0	0.036363636	0	0.336283186	0.083150985	-0.166666667
2011-02-08	0.115384615	0.140350877	0	0.145695364	0.086868687	0
2011-02-09	0.068965517	0	-0.011627907	0.069364162	-0.016728625	0.2
2011-02-10	0.161290323	0.061538462	-0.035294118	0	-0.054820416	0

2011-04-11	0.055555556	0.391304348	-0.085365854	0.027027027	-0.046	-0.166666667

Table 5.5: Stock Returns

Date	PBCL	CPC	BOPP	CAL	GWEB	AYRTN
2013-02-01	-0.083333333	0	0.004878049	-0.029411765	0	0
2013-02-02	0.181818182	0	0.033980583	0.01010101	0	0
2013-02-03	-0.076923077	-0.5	0.17370892	0.06	0	0
2013-02-04	0	0	0	-0.028301887	0	0
2013-04-05	0	0	0	0.067961165	-0.333333333	0
2013-02-06	-0.083333333	1	0	0	0	-0.055555556
2013-02-07	0	0	-0.16	-0.072727273	0	0.058823529
2013-03-08	-0.090909091	0	-0.00952381	-0.117647059	0	0
2013-02-09	0	0	-0.038461538	-0.066666667	0	0
2013-02-10	0	0	-0.05	-0.05952381	0	0
2013-02-11	0	0	-0.242105263	0.012658228	0	0
2013-02-12	0.7	0	0.111111111	0.2125	1	-0.055555556
2012-02-01	-0.058823529	0	0.015625	0.041237113	0	0
2012-03-02	0.0625	0	-0.030769231	-0.059405941	0	0
2012-03-03	0	0	-0.015873016	0	0	0.058823529
2012-02-04	-0.117647059	0	-0.193548387	-0.073684211	0	-0.055555556
2012-02-05	-0.066666667	0	-0.06	0	0	0
2012-02-06	-0.071428571	0	0.10212766	-0.022727273	0	0
2012-02-07	0.076923077	0	0.11969112	-0.011627907	0	0.058823529
2012-04-08	-0.214285714	-0.5	0.24137931	0.047058824	0	0
2012-02-09	0	1	0.033333333	0.033707865	-0.25	-0.055555556
2012-02-10	0.272727273	0	0.075268817	0.086956522	0	0
2012-03-11	0	0	0	0	0	0
2012-02-12	0.285714286	0	-0.65	-0.63	0.333333333	0
2011-02-01	0	0	0.05	0.540540541	0	0.058823529
2011-04-02	0.222222222	0	0.734693878	-0.052631579	0	-0.055555556
2011-04-03	-0.090909091	0	0.411764706	0.018518519	0	0

2011-02-04	-0.2	0	-0.233333333	0.181818182	0	0
2011-02-05	0.1875	0	0.014492754	0.246153846	0	0
2011-03-06	0.263157895	0	0.071428571	0.012345679	0	0
2011-02-07	-0.166666667	0	0.233333333	0.426829268	0	0
2011-02-08	0	0	0.040540541	-0.042735043	0	0.058823529
2011-02-09	-0.1	0	-0.020779221	-0.053571429	0	-0.055555556
2011-02-10	0	0	0.01061008	-0.056603774	0	0.058823529
2011-04-11	-0.055555556	0	-0.160104987	-0.02	0	0

Table 5.6: Stock Returns

Date	GOIL	SIC	ETI
2013-02-01	0.028571429	-0.351351351	0.130434783
2015-02-02	0	-0.083333333	0.038461538
2013-02-03	0.009259259	-0.090909091	0.111111111
2013-02-04	0.119266055	0	0.2
2013-04-05	0	0	0.027777778
2013-02-06	0.18852459	-0.05	-0.189189189
2013-02-07	0.089655172	-0.210526316	0
2013-03-08	0.17721519	0	0
2013-02-09	-0.284946237	-0.2	0
2013-02-10	0.015037594	0.083333333	-0.066666667
2013-02-11	0.022222222	0	-0.321428571
2013-02-12	-0.355072464	2	0.210526316
2012-02-01	0	0	-0.043478261
2012-03-02	-0.134831461	0.205128205	0.045454545
2012-03-03	0.064935065	0.106382979	-0.043478261
2012-02-04	0.085365854	-0.230769231	0
2012-02-05	0.011235955	0.075	0.181818182
2012-02-06	0.055555556	0	0.192307692
2012-02-07	0.042105263	-0.069767442	0.064516129

2012-04-08	0.02020202	0.025	-0.090909091
2012-02-09	-0.00990099	-0.024390244	0.133333333
2012-02-10	-0.01	0.05	0
2012-03-11	0	0	-0.647058824
2012-02-12	-0.373737374	-0.19047619	0
2011-02-01	0.129032258	-0.058823529	0.5
2011-04-02	0.314285714	0	0.166666667
2011-04-03	0.141304348	-0.0625	-0.19047619
2011-02-04	0.285714286	-0.1	-0.058823529
2011-02-05	0.014814815	-0.333333333	0.25
2011-03-06	-0.124087591	1.388888889	-0.05
2011-02-07	-0.333333333	-0.069767442	0
2011-02-08	0.0875	-0.05	-0.052631579
2011-02-09	0.034482759	-0.052631579	0
2011-02-10	0	0.083333333	0
2011-04-11	0.033333333	0.025641026	-1

R codes used to compute the variance-covariance VaR

```
wgt <- read.csv("C:/Users/Narttis/Desktop/m/wgt.txt", sep="")
> View(wgt)
> covariance=cov(wok)
> sigma=cov(wok)
> weights=wgt$Weights
> wgt <- read.delim("C:/Users/Narttis/Desktop/m/wgt.txt")
> View(wgt)
> mu=wgt$Mean
> weights=wgt$Weights
> sigma=cov(wok)
> VaR(R = NULL, p = 0.95, ..., method = c("modified", "gaussian",
+ "historical", "kernel"), clean = c("none", "boudt", "geltner"),
```

```
+ portfolio method = c("single", "component", "marginal"), weights = NULL,
+ mu = NULL, sigma = NULL, m3 = NULL, m4 = NULL, invert = TRUE)
Error: '...' used in an incorrect context
> > > VaR(wok,p=0.95,method="gaussian",clean="none",weights=weights,mu=mu,sigma=sigma,invert=

VaR -0.4979359
> VaR(wok,p=0.99,method="gaussian",clean="none",weights=weights,mu=mu,sigma=sigma,invert=TRU

VaR -0.7321129
> VaR(wok,p=0.90,method="gaussian",clean="none",weights=weights,mu=mu,sigma=sigma,invert=TRU

VaR -0.373097
```