KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

GAME THEORY MODEL OF CONSUMERS RESPONSE TO SERVICE OFFERS. A CASE STUDY OF MTN-GHANA AND VODAFONE-GHANA IN THE TAMALE METROPOLIS

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A PROJECT REPORT SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Philosophy in Applied Mathematics (Mphil.)

APRIL, 2015

Declaration

I hereby declare that this submission is my own work towards the award of the M. Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

I humbly dedicate this work to my late parents: Afa Mohammed Abubakari and Hajia Mariam Abubakari, not forgetting my wife and children for their patience and prayers and all those who contributed toward my work.

Acknowledgements

First and foremost, I am very grateful to the Almighty God for the strength, endurance and above all the knowledge He bestowed in me throughout my life and also seeing me through this stage of my academic carrier.

I would like to express my sincere gratitude and appreciation to my supervisor Mr. K. F. Darkwa for his continuous support of my study and research, for his patience, motivation, enthusiasm and immerse knowledge. His guidance helped me in the time of research and writing of this thesis. Frankly, I could not have imagined having a better supervisor and mentor for my study. May God richly bless my supervisor.

I would also like thank all the lectures in the Mathematics Department for their support in seeing us through the programme. May the Almighty God bless all of them. My sincere thanks also goes to all my colleagues, especially, Mr. Jeffery and Oheneba-Osei N. Fidelis.

Abstract

Telecommunication and mobile phone usage for that matter have assumed a centre stage in the Ghanaian economy like any other economy the world over to a degree that its possession is increasingly becoming a necessity. The situation resulted to serious competition among Service Providers in the sector in their quest to control the market. The objectives of this thesis are to model consumers' response to the service offers employed by two Mobile Service Operators using game theory as well as to determine the optimal strategies and utilities of the considered firms. These operators included the MTN and Vodafone-Ghana.

We compared their selected service offers to determine how consumers react to them. The Lemke Howson Algorithm was used to solve the model and Nash Equilibria of the game were determined. The optimal strategies and utilities of the competing firms were obtained.

Table of Contents

List of Tables

List of Figures

List of Abbreviation

ITU - International Telecommunications Union

ICT - Information and Communication Technology

NCA - Ghana National Communication Authority

MTN - Mobile Telecommunications Network

GT - Ghana Telecommunication Company

KNUST - Kwame Nkrumah University of Science & Technology

CTI - Chinas Telecommunications Industry

MATLAB - Matrix Laboratory

S Us - Secondary Users

P Us - Primary Users

MPCR - Marginal Per Capita Returns

MGR - Marginal Group Returns

GSP - Generalized Second-Price Auction

PRP - Posted Reserve Price

NE - Nash Equilibrium

SPE - Subgame Perfect Equilibrium

RPP - Renegotiation-Proofness Principle

LQRM - Logit Quantal Response Model
ABM - Agent-Based Model

PSE - Payoff-Sampling Equilibrium

ASE - Action-Sampling Equilibrium

IBE - Impulse Balance Equilibrium

EWA - Self-Tuning Experience Weighting Attraction Learning

OPEC - Organization of Petroleum Exporting Countries

LPP - Linear Programming Problem

V CG - Voluntary Contribution Mechanism

AV G - Allocator Contribution Mechanism

ECU - Experimental Currency Unit

PDP - People Democratic Party

CSMA - Carrier Sense Multiple Access

NCUA - Non-Cooperative Carrier Sense Update Algorithm

Chapter 1

INTRODUCTION

1.1 Overview

Competition is a phenomenon that permeates all aspects of life. States, Institutions, Business Entities, Sports, among others compete in a manner for supremacy. In business, competition is the rivalry among market operators in their quest to achieve such goals as increasing gains, market shares and sales volumes by varying the elements of the marketing mix: pricing, product, distribution and promotion. In the business cycle, competition among marketers results in lower prices and promotes innovations. Among the sectors of the business domain, the telecommunication industry is very competitive. This is largely due to the fact that the companies in the industry render homogeneous services to consumers. Due to the competitive nature of the industry, the game theory as a methodology which influences almost every aspect of competition is adopted to determine optimal strategies for competitors. This introductory chapter of the research involves: the Background to the Study, Problem Statement, Objectives of the Study, Research Methodology, Justification of the Study, and the Organization of the Thesis.

1.2 Background to the Study

Telecommunication is the exchange of information between individuals over a significant distance by electronic means, particularly through electrical signals or electromagnetic waves, Wikipedia (2014). It encompasses not only the traditional areas of local and longdistance telephone service, but also advanced technology-based services including wireless communications, the internet, and among others. It is of interest to note that globally, the telecom industry is one of the fastest growing sectors especially in the developing nations. According to the International Telecommunications Union (ITU) report, there were approximately 6.9 billion mobile/cellular phone service subscriptions worldwide as of mid-2014 with the actual number of individuals holding the subscriptions of about 5.7 billion as many people hold more than one subscription. This, by the estimated world population of 7.125 billion people, Wikipedia (2014), indicates that about 80 persons hold subscriptions per 100 of global population, representing 80% penetration rate. It is reported that Ghana for the past two decades has been at the forefront in the ICT revolution in Africa. The liberalization and deregulation of the Ghana Telecom Industry in 1994 and the establishment of the National Communication Authority (NCA) by government has brought about a significant change in the sector. This action encouraged private sector participation which in effect improved public access in the rural and urban areas to telecom services through the provision of payphone facility, and expansion of mobile phone coverage, Emily and Lawrence, (2009). Today, the Ghana Telecom Industry can boast of two national fixed network operators and six mobile/cellular phone operators. Those operating in the public fixed lines are Vodafone Ghana and Airtel.

1.2.1 The Mobile Telecommunication Operators

There are six (6) mobile service providers in the Ghanaian market. They include the Mobile Telecommunications Network (MTN), Vodafone-Ghana, Airtel, Tigo, Expresso, and Glo-Ghana. Those selected for this exercise are the MTN-Ghana and the Vodafone-Ghana. The selection was based on their market shareholding in the Telecom Industry.The MTN is leading the market with 45.86% shareholding followed by Vodafone-Ghana with 22.65%. This can be verified in Table 1.1 of page 6 .

1. The Mobile Telecommunications Network (MTN)

The MTN Group, formally known as M-Cell, is a South African-based multinational telecommunication company operating in many European, Middle Eastern and African countries including Ghana. The company has its headquarters situated in Johannesburg, South Africa. It is one of the leading telecom service providers in the world. Records indicate that MTN as at June, 2013 controls 201.5 million subscribers across its catchment areas. Wikipedia, (2014).

In Ghana, MTN is the leading provider of telecommunication services over the years.

It got into the Ghanaian market following its acquisition of Investcom Limited in 2006, which owned the Scancom (GH) Limited, the operators of the then Areeba in the country. According to the NCA,(2014) Ghanas total mobile/cellular Voice Subscriber base, as at August 2014, stood at 29,531,488. Out of this, MTN is leading with 13,541,961 subscribers representing 45.86% market shareholding amidst fluctuations.

2. The Vodafone Group

The Vodafone Group is one of the world's leading mobile telecommunication companies. It, currently, operates in 31 countries across the globe and has a customer base of 315 million. The company is ranked among the top ten (10) global companies by market capitalization. It has its headquarters situated at London in England. The Vodafone Ghana hitherto called the Ghana Telecommunication Company (GT) joined the Vodafone Group following the successful acquisition of 70% share in the (GT) by Vodafone International in 2008. In the Ghanaian market, the company is the next contender to MTN in mobile telephony sector and a leader in providing of broadband services. (Wikipedia, 2014). With the mobile telephony segment it has approximately 6.7 million subscribers which represent 22.65% market shareholding, (NCA, 2014).

1.2.2 Telephone Service Penetration in Ghana

Statistics have shown that the total fixed lines on service as at August, 2014 stood at 265,289 with a penetration rate of 0.9986%. Of this, Vodafone Ghana alone controls 96.87% and that of Airtel's stands at 3.13%. For the mobile telephony segment, the total subscriptions as at August, 2014 stood at 29,531,488 with a penetration rate of 111.16%, NCA, (2014).

The penetration rate is the measure of the number of mobile phone voice subscriptions in the country. Below are the summaries of the report. Table 1.1 illustrates the trend of the telecom voice subscription of Mobile operators from February to August, 2014. Table 1.2 also shows the trends of the telecom voice subscription of Fixed Line Operators, monthly total market and the growth rates from February to August, 2014. Table 1.3 shows the market shareholdings of the Service Providers in the market from February to August 2014. Below is Table 1.1 which illustrates the trend of the telecom voice subscription of Mobile operators from February to August, 2014.

	Feb	March	April	May	June	July	Aug.
Operator							
EXPRESSO	153,727	154,704		143,379	127,505	122,356	123,82
TIGO	4,086,615	4,052,032	4,038,285	4,042,554	4,034,563	4,037,212	4,015,9
MTN	12,986,832	13,054,981	13,126,884	13,304,158	13,438,770	13,431,919	13,541,9
VODAFONE	6,413,376	6,480,434	6,644,825	6,732,555	6,678,141	6,749,504	6,688,7
AIRTEL	3,537,316	3,540,205	3,502,121	3,504,858	3,570,282	3,659,531	3,756,5
GLO	1,437,580	1,384,372	1,348,162	1,374,263	1,371,341	1,368,715	1,404,4
TOTAL	28,615,446	28,666,728	28,814,484	29,101,767	29,220,602	29,369,237	29,531,4
GROWTH	0.69%	0.18%	0.52%	1.00%	0.41%	0.51%	0.55%

TELECOM VOICE SUBSCRIPTION TRENDS 2014

Table 1.1: Mobile Operators

Table 1.2 shows the trends of the telecom voice subscription of Fixed Line Operators, monthly total market and the growth rates from February to August, 2014.

Table 1.2: Fixed Lines Operators

	Feb	March	April	May	June	July	Aug.
VODAFONE	259,093	258,313	257,765	258,899	257,802	256,816	256,984
AIRTEL	8,821	8,699	8,594	8,504	8,435	8,355	8,305
TOTAL FIXED	267,914	267,012	266,359	267,403	266,237	265,171	265,289
MTHLY GROWTH	-0.39%	-0.34%	-0.24%	0.39%	-0.44%	-0.40%	0.04%

Table 1.3 shows the trends of monthly total market from February to August, 2014.

	Feb	March	April	May	June	July	Aug.
MOBILE	28,615,446	28,666,728	28,814,484	29,101,767	29,220,602	29,369,237	29,531,4
FIXED	267,914	267,012	266,359	267,403	266,237	265,171	265,28
ACC. LINES	28,883,360	28,933,740	29,080,843	29,486,839	29,369,170	29,634,408	29,796,7
GROWTH	0.68%	0.17%	0.51%	0.99%	0.40%	0.50%	0.55%
POPTION	26,249,665	26,302,164	26,354,769	26,407,478	26,460,293	26,513,214	26,566,2

Table 1.3: Total Market

Table 1.4 shows the trends of monthly growth rates of Mobile Services from February to August, 2014.

Table 1.4: Growth Rates of Mobile Services

PENETRATIO N	Feb	March	April	May	June	July	Aug.

MOBILE	109.01 %	108.99 %	109.33 %	110.20 %	110.43 %	110.77 %	111.16 %
FIXED	1.02%	1.02%	1.01%	1.01%	1.01%	1.00%	1.00%
TOTAL	110.03 %	110.01 %	110.34 %	111.22 %	111.44 %	111.77 %	112.16 %
MARKET SHARE							
MOBILE	99.07%	99.08%	99.08%	99.09%	99.10%	99.11%	99.11%
FIXED	0.93%	0.92%	0.92%	0.91%	0.90%	0.89%	0.89%

Table 1.5 shows the trend of monthly market shares of Mobile Operators from February to August, 2014.

Table 1.5: Market Shares of Mobile Operators

	Feb	Mar	Apri	May	Jun	Jul	Aug
EXPRESSO	0.54%	0.54%	0.54%	0.49%	0.44%	0.42%	0.42%
TIGO	14.28%	14.13%	14.01%	13.89%	13.81%	13.75%	13.60%
MTN	45.38%	45.54%	45.56%	45.72%	45.99%	45.73%	45.86%
VODAFONE	22.41%	22.61%	23.06%	23.13%	22.85%	22.98%	22.65%
AIRTEL	12.36%	12.35%	12.15%	12.04%	12.22%	12.46%	12.72%
GLO	5.02%	4.83%	4.68%	4.72%	4.69%	4.66%	4.76%
TOTAL	100%	100%	100%	100%	100%	100%	100%

1.2.3 Service Offers

Service Offers are the special packages employ by the Mobile Service Providers to motivate customers to patronize their products. These offers were regarded as strategies in order to use game theory for analysis.

1.2.4 Game Theory

Game Theory is a mathematical theory that is used for the analysis and resolution of conflict situations in which participants have opposing interests.In our context, both the MTN and

Vodafone aimed at dominating the market irrespective of the other. The concept of game theory provides a language for formulating, analyzing and understanding strategic scenarios. It is used primarily in economics to address the functional relationship between selected strategies of individual players and their market outcome, which may either be profit or loss. It also seeks to find optimal decisions for operators in the decision making process.

1.3 Problem Statement

Considering the competitive nature of the Telecom Industry in Ghana, mobile phone service providers in the sector are always confronted with issues. That is, they are always concerned with how to retain their existing customers and how to woo frustrated customers of their opponents or better still, win new customers to their sides. As a result of the fierce competition, every service provider in the market is engaged in providing various forms of service offers that will be appealing to users in order to lure them to patronize their products. Records have shown that as at the end of August 2014, MTN Ghana had made a monthly increase of 110, 042 in their subscriber base, whiles both Vodafone Ghana and Tigo recorded losses of 60,721 and 21,266 respectively, NCA, (2014). With this fluctuating market trends, the mobile operators need to strategize in order to keep track in the business. Game theory as a discipline which influences almost every aspect of competition can be used to determine optimal strategies and better utilities for mobile operators in the market. Identifying the best strategies will also help improve the efficiency of the firms and customer satisfaction will be enhanced. Looking at the market trends currently, MTN is leading with 13,541,961subscribers representing 45.86% shareholding. The Vodafone Ghana is the next contender with 6,688,783 subscribers representing 22.65% shareholding, NCA, (2014).This can be verified in tables 1.1 and 1.5 of pages 5 and 6 respectively.It is against this backdrop that these two leading players in the market were selected for the study to examine the service offers they used and how consumers react to such offers. Each of these firms seeks to maximize the total number of consumers who adopt its products.

1.4 Objectives of the Study

The objectives of the study were to:

- Formulate a model of consumer response to mobile service offers using game theory.
- Determine the optimal strategies and payoffs for the two mobile operators.

1.5 Methodology of the Study

In the study, we formulated a linear programming of game theory to model consumers response to the service offers employed by MTN Ghana and Vodafone Ghana. This project made use of both primary and secondary data. For the primary data, structured questionnaires were administered to a sample of two hundred (200) respondents. The questionnaires were designed in such a manner that the respondents had the opportunity to choose their preferred network out of the two under study given a set of service offers from the two networks simultaneously matched. A mixture of non-probability sampling techniques (Quota and Purposive) were used to obtain the sample size. The assumption made in the selection of the respondents was that, 7 out of every 10 persons possess at least a mobile phone in Tamale. The Tamale Metropolis in the northern region of Ghana was chosen as the study area and the target population was customers who are eighteen (18) years and above, and are able to read and write (literates) in Tamale. Any person within the target population was eligible to be sampled. The customers were considered mature and are able to make decisions on their usage of mobile telecom products and services.

The secondary data were sourced from the Internet and confirmed at the respective outfits of the two competing firms. The data consisted of the service offers employed by the two network companies respectively to influence customers in the market. These service offers were regarded as strategies in order to use game theory for analysis. The Lemke Howson Algorithm was used to solve the model for the firms' optimal strategies and their associated payoffs/utilities were also calculated. The algorithm was coded in MATLAB, a programmable software, to solve the model to obtain optimal strategies and the expected utilities for both companies. The resources used for information on the study included the KNUST library and the internet.

1.6 Justification

There is no doubt that mobile phone has become an indispensable tool in our lives. The emergence of the device has made communication of all forms easier and faster for users. The increase in the number of service providers coupled with the massive patronage of the mobile phone services in the country has brought about a severe competition in the sector. This engineered our interest and the zeal to carry out this research.

In effect, the outcome of this thesis will:

- Reveal to mobile phone users the best and efficient service offers employed by the selected service providers.
- Help service providers to identify and play their optimal strategies in order to remain competitive in the market.
- Aids the National Communications Authority (NCA) for that matter, government to monitor the services render by the mobile service operators to the consuming public, in order to ensure satisfactory services.
- Add to the existing literature of game theory applications.

1.7 Thesis Organization

This report consists of five chapters.

Chapter One deals with Introduction, which focuses on the Background to the Study, Problem Statement, Objectives, Methodology of the Study, Justification and the Organization of the Thesis.

Chapter Two highlights on the historical background of game theory and reviews game theory applications. It also highlights on the historical background of the Lemke-Howson Algorithm and it's used as a solution method for bimatrix games.

Chapter Three presents the Methodology of the Study; it contains explanations to some basic concepts of game theory and relevant mathematical formulations.

Chapter Four is entitled Data Collection, Analysis and Results.

Chapter Five contain conclusion and recommendations of the study.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

This chapter highlights the historical background of game theory, reviews the relevant contributions made so far on game theory applications as well as reviews on the application of Lemke-Howson Algorithm as a solution method.

2.1.1 Historical Background of Game Theory

Game theory is the study of how interdependent decision makers make choices. The decision makers as rational beings take into account the actions and inactions of their opponents in the decision making process. Game Theory is used primarily to provide insight into how to determine optimal decisions for participants in the decision making process.

The idea of game theory can be traced back to Antoine Cournot, (1838), in his study of 'duopoly'. Also, John Von Neumann, one of the greatest Mathematicians, who made an impact in formalizing the theory in the 20th century. In 1928, he published a paper that laid the foundation to the formulation of the theory 'two-person zero-sum game', where a person's gain equals an opponent loss. John Von Neumann later collaborated with Oskar Morgenstern, an economist, and they published a book entitled, "The Theory of Games and Economic Behaviour" (1944). This move, many believed to have given birth to 'game theory' as a field of study. In the 1950s, researchers like John Nash, Harold Kuhn, John Harsanyi, and others developed game theory further. John Nash on his part demonstrated that finite games always have an equilibrium point, at which all players choose actions which are best for them given their opponents choices,(*vonStengel,*2008).

The field of game theory has been used to capture and analyze the main features of situations that involve interactions between individuals. For instance, De Waegenaere et al (2005) applied game theory into banking. They developed a model that allows for reinvestment of intermediate returns. Thomson (2007) applied game theory in the airport situations to unravel the challenges associated with its operations.

2.1.2 Applications of Game Theory

Game theory since its evolution has been applied in diverse disciplines. These include economics, engineering, sports, telecommunication, finance, politics, and others. This section reviews some of the work done using game theory.

Rafael et al. (2012) carried out a research on the quality consumer service provided by Mobile telecommunication operators in the market. The study, primarily, aimed at determining whether consumers of mobile telecommunications are subject to abuses by the service providers. The researchers used simple game theoretical models where low-quality consumer service levels are part of an equilibrium strategy for the firms. It was found out that the low-quality consumer service is inefficient. It was also revealed that the inefficiency is due to demand-side market failure generated by incomplete information and that the efficiency may not be resolved through repeated interactions or competition. Rather, policy regulation should be tackled to ensure customer protection.

Zeng and Fan (2005) applied the 'Prisoner's Dilemma model of game theory to determine the levels of competition and cooperation between operators in the China's Telecommunication Industry. The researchers used the Nash Equilibrium Technique to determine the equilibrium point of the competition in the market. The study revealed that there was excess competition in the China's Telecom Industry (CTI) with its intended effect of price war between operators. The researchers concluded that if operators compete with each other by promoting the quality of service and product rather than cut down the price, they can get more profits. But without a common agreement and policy regulation, individual rationality will finally lead to lack of confidence and mistrust, and that only co-operation among the operators can bring about maximum profits.

Kjell and Jun (2011) studied impasse that exist between government and terrorists. That is, a defender-attacker game. Whiles government is doing everything possible to protect its assets by beefing up security; terrorists in their own ingenuity also find ways of countering government plan to stockpile resources. The researchers aimed at understanding a multiplicity of phenomenon such as how government allocates between defending its assets and attacking the terrorist's resources, how the terrorist allocates across defense and attack strategically, how government can deter the terrorist from attacking, how the terrorist can pacify the government, and when an interior-solution equilibrium exists where both players defend and attack. They employed two-stage-game model to analyze the phenomenon, where the government moves in the first stage and the terrorist moves in the second stage. They deduced subgame perfect Nash Equilibrium and used backward induction method to solve the game. They deduced four equilibrium solutions: The government attacks only, which deters the terrorist; both players defend and attack; the government defends but, does not attack and the terrorist attacks only; the terrorist attacks only and the government neither defend nor attack.

Sanjeev and Michael (2012) studied competition between two firms denoted by Red and Blue who used their resources to maximize products adoption by consumers located in a social network (Facebook, Google+, Twitter, etc). The researchers developed a gametheoretic framework for the study of competition between the firms who simultaneously allocated their resources on subsets of consumers, that is, to seed the network with initial adoptions. They used stochastic dynamics of local adoption to determine how the influence of each player's seed spreads through to create new adoption. They decomposed the dynamics into two parts: a switching function, which specifies the probability of a consumer switching from non-adoption to adoption as a function of the fraction of his neighbours who have adopted either of the two products Red and Blue; and a selection function, which specifies, conditional on switching, the probability that a consumer adopts (say) Red as function of the fraction of adopting neighbours who have adopted Red. The Price of Anarchy and Budget Multiplier were adopted by the researchers for solution. It was shown that network structure can interact in dramatic ways with the switching and selection functions at equilibrium.

Walid et al. (2009) considered how collaborative spectrum sensing between Secondary Users (SUs) and Primary Users (PUs) in cognitive radio network can be improved. They studied the impact of trade off that exists between collaborative spectrum sensing gains in terms of detection probability and cooperative cost in terms of false alarming probability on the topology and the dynamics of a network of (SUs). They explained the trade off as a way of reducing interference on (PUs) whiles maintaining good spectrum utilization. The researchers modeled the problem as a non-transferable coalitional game and used a distributed algorithm for coalition formation through simple merge and split rules. They observed that through the algorithm, the SUs can automatically collaborate and selforganize into disjoint independent coalitions, whiles maximizing their detection probability taking into consideration the cooperative costs.

They also studied the stability of the resulting network structure and showed that a maximum number of (SUs) per formed coalition exists for a proposed utility model. Simulations results obtained indicated that their proposed distribution algorithm reduced the average missing probability per SU up to 86.6% compared to non-cooperative case. Through the simulations they compared the performance of their proposed solution with respect to an optimal centralized solution that minimizes the average missing probability per SU relative to non-cooperative case, while maintaining a certain false alarm level. The results also indicated how their proposed algorithm automatically adapts the network topology to environmental change such as mobility.

Musa and Sunday (2008) employed the principles of game theory to determine optimal strategies for two competing banks to open their branches in a city within two business locations (location 1 and location 2). The MINIMAX (MAXMINI) Principle was used by the researchers to determine the optimal strategies for the banks. They established that the optimal strategy for both banks is to site their branches at the location 1. The utilities of the respective banks were calculated with the larger bank obtaining 60% and the smaller bank 40% of the business in the city.

Boah et al. (2014) applied the two-person constant-sum game of game theory to analyze the levels of patronage of two competing Radio Stations in Kumasi. The researchers identified the two stations common programmes and used them as the strategies of the stations. They interviewed a sample of hundred (100) regular listeners of both stations in the Ashanti Region to find their patronage of the ten (10) selected common progrommes, a utility matrix was constructed and analyzed. It was revealed that both stations had high patronage in diverse programmes and recommended that the two stations should not air similar programmes at the same time in order to get higher patronage.

Reinhardt and Dada (2005) used coalition symmetric game to research into how to fairly divide the cost savings realized by pooling critical resources. They made an assumption that by pooling demand, total cost can be reduced. They cited firms as an example that outsource their call centers to an independent provider. Since demand is pooled in this instance, variability can be reduced and utilization increased. The researchers used the Shapely Value as a solution concept. It was realized that the cost savings that must be divided among the players depend on the sum of each player's demand.

Andrea et al. (2010) provided a workable model to study multi-person environment where players can strategically transmit their private information to individuals who are connected to them in a communication network. They identified two forms of communications, which are private communication and public communication. They studied equilibrium information transmission within and across groups with different preferences. They also examined the equilibrium communication networks in a model where each player can communicate with another player at a small cost paid ex-ante. They studied the information aggregation problem of policy-makers who policy choices affect each other.

John and Flix (2012) studied the effects of social preferences on players cooperation. They investigated how social preferences and fairness concerns can affect the equilibrium behavior in both one-shot and infinitely repeated versions of the Prisoner's Delimma game which is appropriate for the study of strategic environments with extreme competitive incentives. The researchers analyzed how fairness concerns modify players incentives to cooperative in both versions of the game. They also analyzed the interaction between time and social preferences, and provided conditions under which observed cooperation in experimental settings can be explained using either social or time preferences alone or a combination of both types of preferences. Their findings showed that there is high level of cooperative behavior in infinitely repeated game. In effect, they showed how observed cooperative frequencies may be explained by time preferences alone or by a combination of both time and social preferences.

Bin et al. (2013) examined the effectiveness of individual-punishment mechanism at a constant fine-to-fee ratio in the context of potential punishment coordination problems that may occur in larger groups even when all free-riders can be identified. The researchers employed a within-person design of Punishment (P) verses Non-Punishment conditions to determine the effects of group size (small or large) in contribution to public good. Regression analysis was adopted to analyze such parameters as Marginal Per Capita Return (MPCR) and Marginal Group Returns (MGR). Their investigation revealed that the individual-punishment mechanism is robust when MPCR is held constant. It was also revealed that the efficiency gains from the punishment mechanism are significantly higher in the forty-participant than in the four participant treatment.

Ismail (2013) studied the effects of changes in divorce costs on marital dissolution in a two-period one-to-one matching model with nontransferable utilities under incomplete information. The researcher reviewed some literature on the effects of unilateral divorce laws on actual divorce rate in the USA and Europe and realized contradictory reports and thereby aims at determining whether stable matching theory, pioneered by Gale and Shapley can be used to offer some insights that other models could be missing. The Gale and Shapley's Deferred Acceptance Algorithm was used to determine stable equilibrium for both sexes. In conclusion, it was realized that divorce costs affect not only the individuals decisions to divorce and to marry, but also their decision of whom to marry. Consequently, the researcher established that the average probability of marital dissolution in the society is determined by three decision channels, which are: "The divorce channel", "The marital status channel" and "The marital composition channel".

Wei et al. (2013) studied and compared two pricing mechanisms for selling multiple heterogeneous advert positions in search engines (Yahoo!, Google, MSN, etc). They used the

Generalized Second-Price auction (GSP) which stipulates that the bidder winning the lowest position pays the larger value between the reserve price and the highest losing bid, and the GSP auction with Posted Reserve Price (PRP) a hybrid between the auction and a posted price mechanism, where the last winning position pays the reserve price regardless of how many positions sold. They established of Nash Equilibrium of both mechanisms. It is showed in the analysis that if advertiser's per-click value has an increasing generalized failure rate, the search engine revenue rate is quasi-concave and hence there exist an optimal reserve price under both mechanisms.

Arne et al. (2013) in their work investigated the dynamical properties of evolutionary multi-player games in finite populations. Properties such as fixation probability, fixation time and average abundance in mutation-selection equilibrium were investigated. The researchers were also interested in intrinsic stochastic effects induced by finite population size and nonlinearities in payoff induced by multi-player interaction. In the researchers analysis, they realized that the multi-player game can lead to substantial complications than the two-player game. They however suggested that more is still need to be done to understand the full complexity of such multi-player game interaction.

Jrme et al.(2013) highlighted two possible outcomes of gladiatorial combats game. One in which killing is the equilibrium strategy, which corresponds to the image portrayed in films of gladiatorial combats between slaves and the other, gladiatorial combats, which has instilled cooperation and identified survival of the defeated fighter as the equilibrium strategy, a situation which is a historical reality of professional gladiators. In effect, in the Ancient Greece, gladiatorial schools were setup to train novice gladiators how to handle weapons and put on good show. This was misconstrued by film actors over time as they incorporated killing into the game. The import derived from the study by the researchers is that stronger competitors in business should allow the weaker ones to survive in order to ensure a satisfactory supply flow of services.

Alex (2013) studied about the effects of entry by additional seller(s) or buyer(s) into a model of oligopoly market, in which there is bilateral relationship (cooperation) between sellers and buyers. It is assumed that the firms produced homogeneous goods. The researcher aimed at determining whether the conventional wisdom that characterized the Cournot oligopoly can be attributed to bilateral oligopoly. He constructed strategic supply and demand functions, intersections of which identify non-autarkic Nash Equilibrium in the game. These functions represented the aggregate supply and demand. First-Order differential technique was used to determine the utilities of both supply and demand. It was deduced that bilateral oligopoly is quasi-competitive and that the utilities of traders on the same side of the market to the entrant can increase so long as traders on the other side of the market sufficiently increase their market strategies.

Vincent and Ana (2013) adopted the Rubenstein's alternating-offer bargaining model with two-sided incomplete information to investigate the effects of having relative concerns on agreement and delay in reaching agreement. They indicated that with short time interval, the model has a unique limiting subgame perfect equilibrium (SPE) which approximates the Nash bargaining solution to the bargaining problem. The researchers also incorporated the Fehr and Schmidt's model of inequality aversion to examine what happens under complete information if both players have relative concerns. The conclusion drawn by the study is that an increase in relative concerns will reduce the delay in reaching an agreement. Arina (2013) studied the interaction of two rational players who are involved in a repeated play of normal form games drawn from a fixed family of games. The researcher assumed that reasoning resources are expensive and that players do not necessarily distinguish all games. He identified equilibrium payoffs that are consistent with evolutionary pressure that shapes the constraints on reasoning of agents in the long-run.

Joel and James(2013) developed a model to explicitly capture the constraints on implementation imposed by non-contractible opportunity for parties in contractual relationships to renegotiate. The model designed has three components: the method of incorporating costly renegotiation into the framework, the Renegotiation-Proofness Principle (RPP) with its validation conditions and the monotonicity theorem. The researchers observed in their analysis that the RPP is not applicable in some settings. However, the results of the monotonicity confirmed that the RPP's findings of negative consequences of renegotiation opportunities. They emphasized the need to incorporate institutional and technological constraints into mechanism-design analysis.

Alexandra (2013) used trust game experiment to classify subjects based on their behaviours. He aimed at identifying the correlation between an average proportion returned and amount sent by pairs of subjects in a gift exchange (reciprocity) game. The method used for analysis by the researcher was the Spearman's Correlation Method (). It was used to characterize subjects behavior on reciprocity effect. The results indicated high correlation between subjects actions, but rather attributable to a minority of the subjects.

William et al. (2013) used a laboratory experiment to investigate the effects of power asymmetries on conflict rates in a two-stage bargaining game model that is followed by conflict with a random outcome. The researchers used game-theoretic prediction techniques such as backwards induction reasoning to analyze the model. Some errors associated with expectations were observed and these were resolved by estimation of the Logit Quantal Response Model (LQRM) to ensure qualitative pattern. It was observed that demands and counterdemands are sensitive to relative power should a conflict arise, and that conflict does not occur in a significant proportion of cases. That conflict rates are not affected by power asymmetries, since the players adjust their demands base on the asymmetries.

Richard (2014) developed a game theoretic framework for voting called Condorcet Completion Method, where truthful revelation of preferences is an equilibrium strategy, leading to the election of a Condorcet Winner (a candidate who could win pairwise against each opponent), and devoid of any electoral fraud or manipulation. The researcher improved upon a previous research work by Potthof, (2013), which lacks strong non-manipulability properties. The method, in effect, ordered candidates in a set by first determining last-place candidate. He used mathematical induction algorithm to proof the non-manipulability of the electoral system.

Xiaojian and Ying-Ju (2013) constructed a stylized Principal-Agent Model to investigate the interactions among unawareness, reasoning and cognitive effort that characterized the principal-agent relationship. The researcher posited that in contractual relationship the contract proposer (Principal) does not always offer transparent and complete contract information to the contract follower (Agent). On the other hand, the agent as a sophisticated partner may suspect a fishy deal in the contract agreement and employ some defensive counteractions such as refusal of the contract or actively gathering information about the contract. The researchers used a number of solution concepts to account for various degrees of the unaware agent's sophistication. They used subgame-perfect solution which is an extension of Subgame Perfect Nash Equilibrium to update the agent's unawareness based on the principal contract offer. They also used justifiable solution method similar to that of game theory forward induction to evaluate the reasonability of the contract offer. They also used a trap-filtered solution method to synthesize all possible scenarios regarding how the agent perceives the principal contract offer. Though the researchers concentrated on a monopolistic principal's optimal contract design, they proposed a possible extension of the model to a multiple principals either homogeneous or heterogeneous competing in hiring an unaware agent.

Balzs and Caroline (2013) proposed a baseline population model to compare the growth rate of two populations. A population, in which suicide is genetically possible, and another, in which it is not. They assumed that individuals have equal potential of producing of a suicidal offspring who will unconditionally self-destruct regardless of her fertility, and the reproductive resources of those who commit suicide are inherited. The researchers used the limits principles of differentiation to determine steady-states of the competing populations. It was realized that the population where suicide is technologically feasible grow faster than the other.

Bernhard, et al. (2014) used an Agent-Based Model (ABM) to investigate the interdependent dynamics between individual agency and emergent socioeconomic structure, which leads to institutional change in a generic way. In this study, the researchers treated institutions as governed social structures with explicitly codified entry and exit conditions for agents as members. The agents (leaders) are perceived to be endowed with cognitive capabilities which feed their individual decisions. In their analysis, simulations were performed with payoff matrix for pairwise prisoners dilemma that is in line with non-degeneracy conditions. The results of the simulations indicated three scenarios of institutional change. These included static and ordered scenario, dynamic but highly fluctuating scenario and dynamic and complex scenario.

Reinhardt et al. (2008) constructed four new learning models: Impulse Balance Learning, Impulse Matching Learning, Active-Matching Learning and Payoff-Sampling Learning, which are based on behavourial reasoning of Payoff-Sampling Equilibrium (PSE), ActionSampling Equilibrium (ASE), and Impulse Balance Equilibrium (IBE). The authors tested the learning models together with Reinforcement Learning and the Self-Tuning Experience Weighting Attraction Learning (EWA) in an environment of twelve repeated (2 X 2) games. The results obtained were compared with an experimental data obtained in their previous work of Selten & Chmura, (2008). The experimental data comprised aggregate and individual behaviour in 12 completely mixed 2 X 2 games, Six (6) constant and Six (6) non-constant sum games. According to the study, participants were randomly matched and played for over 200 rounds. They concluded that the comparison of the six (6) models yielded the following order of predictive success from best to worst: Impulse Matching Learning, Action-Sampling Learning, Impulse Balance Learning, Payoff-Sampling Learning, Reinforcement Learning, Self-tuning EWA Learning.

Piotr et al. (2013) studied special case models of microbribery Swap Bribery and Shift Bribery Models and analyzed their complexities in some standard election systems including Plurality, K-approval, Borda, Copeland, and Maximin. The authors used their model and investigated whether bribery can influence voters preferences in voting. They used a polynomial time algorithm to compute optimal swap bribery that can transform voters preferences. It was revealed that Swap Bribery is NP-hard and there is the need to construct effective approximation algorithm for the model.

Ozan et al. (2011) studied the optimal strategies of a monopolist selling a divisible good (service) to consumers that are embedded in a social network. They considered a two-stage pricing-consumption game to model the interaction between the agents (consumers) and the monopolist. The first stage indicates the monopolist pricing function that maximizes his gains, whiles the second stage signifies the agent's consumption function that maximize his utility. In determining the monopolist optimal strategies, the researchers assumed three scenarios a setting where the monopolist can offer individualized price and derive an explicit characterization of the optimal price for each consumer as a function of her network position, a setting where the monopolist can offer a single uniform price for the goods and derive an algorithm polynomial in the number of agents to compute such a price, and a setting where the monopolist can offer the goods in two prices, full and discount. The researchers indicated in their findings that if the profit is nonnegative under any feasible price allocation, the algorithm guarantees at least 88% optimal profit.

Raul (2006) studied how social norms and emotions affect cooperation, coordination and punishment in a variety of games. The researcher adopted extensive form game with perfect information to model the phenomenon. The solution concept used by the researcher to solve the model is Subgame Perfect Nash Equilibrium (SPE). He observed that people feel badly when they deviate from the binding norm, and the less other players deviate the more badly the feel. He also observed that people get angry at transgressors and get pleasure from punishing them. He suggested that further experimental research should be conducted on social norm, emotion and reciprocity.

Raul and Marc (2012) explored the disapproval of allocation decisions that reveal behaviour patterns using experimental data from five (5) dictator games with a feedback stage, in which recipients have the possibilities of expressing their opinions about dictators choice. The researchers applied classification procedure to analyze the motives behind the recipients disapproval. They realized that subjects (humans) are heterogeneous in their disapproval patterns, distinguishing two main groups. That is, subjects who only disapprove choices that harm them and subjects who disapprove choices that are socially inefficient. Stefan and Jrgen (2013) investigated whether human subjects are willing to give up individual freedom in return for the benefits of improve coordination. They adopted an iterative public good game model and augment it two distinct contribution mechanisms a voluntary contribution mechanism (VCG) and an allocator contribution mechanism (AVG). Subjects are given the freedom to select one of two groups at the beginning of every period. In the VCG, subjects play a standard public good game by deciding how much of their endowment to keep for themselves and how much to invest for a public good. With the AVG, subjects have two treatment systems a coordinator treatment, where an allocator is randomly selected to set a uniform contribution for all group members including himself/herself, and a dictator treatment, where the allocator sets different contributions for himself/herself and all other group members. The researchers used prediction techniques to analyze the model. In effect, the MINIMAX principle was used to determine the payoff in the AVG and different payoff functions were used to predict the payoffs of two treatment systems of the AVG. The researchers in their analysis realized that the allocator groups achieved high contribution levels in both treatments than in the VCG.

Christiane and Christian (2013) reported experimental data on bidding behaviour from allpay auction format. They conducted first-price, sealed-bid, common value auctions with two and three subjects. They used two treatments the Recall and the NoRecall treatments. In the Recall treatment, subjects were allocated to groups of either two or three subjects and played ten consecutive but independent all-pay auctions for a prize of 100 ECU (experimental currency unit) in a partner matching. Full information about bids of group members in all previous rounds was provided. In the NoRecall treatment, subjects also played ten consecutive all-pay auctions for a prize of 100 ECU. But, in each round they were randomly reallocated to groups of either two or three subjects. They were informed about their group size but received no information about the outcome of the auction and the other subjects bids. The researchers used Nash Equilibrium as a solution method to analyze the model. In the end, they realized that subjects underbid in groups of two and overbid in groups of three.

Jane et al. (2013) adopted a two-person Prisoners Dilemma Model of an alliance between two firms to include the response of a rival firm, resulting in a version of three-player Prisoners Dilemma. They used Nash Equilibrium as a solution criterion for the model to analyze the impact on the stability of the alliance of the rivals competition, either with the alliance or with the individual partners. The researchers results indicated that the success or otherwise of an alliance is strongly influenced by the activity of a powerful third party in the same market sector and that for an alliance to be the Nash equilibrium, it is necessary for the allies to be under pressure from the third party and losing significant payoff as a result.

Todd and Jason (2013) carried out an experimental investigation to determine whether behaviour abnormalities are relevant in ultimatum games experiments. They auctioned scares participation rights to play the Proposer and Responder positions in the game. As a control mechanism, the researchers used two selection procedures called Auction and Random Treatments to select subjects for the game. In the Auction treatment, they conducted two multi-unit uniform price English clock auctions at the beginning of each round: one to determine four Proposers and their participation fee, and the other to determine four Responders and participation fee. In the Random treatment, they selected four subjects randomly from each of the Proposer and Responder groups at the beginning of each round. Forward induction and loss avoidance concepts were used as solution methods by the researchers to determine Nash Equilibrium. The researchers in their analysis observed strong coordination on offers by Proposers and also Responder behaviour consistent with the Nash Equilibrium.

Wakeel (2012) examined the challenges and prospects of power sector reform in Nigeria with focus on the market structure, market design and supply gap in the electricity generation within the context of power reform. The researcher adopted oligopolistic game theory based models of Cournot, Betrand and Supply Function Equilibrium to explain the complex interest groups in Nigeria energy sector and relate them to experiences in other countries. He concluded that competition on its own does not guarantee success, rather, a blend of competition with credible institutions. He also observed that when the rules in terms of market design, regulations and conduct regulatory agencies are strong, deregulation of electricity would be a success, and otherwise, where the institutions are very weak. The study also established that there a considerable supply gap in the electricity generation segment and this can discourage investors.

Oziegbe (2011) examined the application of Game Theory in solving of business decision problems in undeveloped countries. He used Nigeria as a case study. The researcher used the minimax maximin criterion to determine the saddle point or equilibrium point of the game. He observed that game theory is a good technique for analyzing manager's decision when dependent on the decision made by a competitor and that there is need for integration of game theory with the existing management techniques in organizations in Nigeria to enhance decision taking in strategic management. Ayo et al. (2012) examined the zoning/power formula with reference to the People Democratic Party (PDP) politics in Nigeria's fourth republic. The researchers applied multi-zero-sum model of game theory to model presidential primaries of the party held in 2011. They used simple majority formula to arrive at the winner. It is observed that the emergence of President Jonathan in 2011 presidential election is a dilemma because the 2015 election has to settle the contentious zoning formula of PDP to satisfy every zone, particularly the South East, to avoid an imploding consequence.

Oluyole et al (2013) utilized game theory to determine the cocoa production management system which maximizes the income of farmers under risks. They used data on cocoa production collected from a random sample of 200 farmers practicing three cocoa production management systems: Owner management system, Lease management system as well as Sharecropped management system. The games were constructed based on the income per hectare obtained from each of the three management systems. The researchers used Maximax and Maximin criteria of game theory for analysis. They observed in their analysis that the Maximax criterion showed that the Sharecropped management system was the best while the result of the Maximin showed that the Owner management system was the best for the farmers. The study therefore recommended for optimistic farmers to practice Sharecropped management system while Owner management system is recommended for

pessimistic farmers.

Adeoye et al. (2012) used game theory to determine vegetables and fruits which maximize net profit of farmers under risks, based on different characteristics of the farmers. They used data on vegetables production collected from a random sample of 60 farmers cultivating each of the selected vegetables and fruits from Oyo and Ondo state. The researchers constructed the game based on the net profit obtained from each of the vegetables and fruits. Maximax, Maximin, Regret, Utility and Laplace criteria of game theory were used in the analysis. They observed that the Maximax and Laplace criteria showed that the best vegetable and fruit to cultivate by farmers were Tomato and Pineapple, the results of the Maximin and Utility criteria indicated that Amaranthus and Pineapple were the best options and the regret criterion, Pepper and Plantain were the best options for the farmers. It is recommended for optimistic farmers to produce Tomato and Pineapple while Amaranthus is recommended for pessimistic farmers

Yusuf et al. (2014) used Two-Person Zero-Sum Game to determine the level of competition between the tanners and 'pomo' wholesalers in hides marketing competition in Nigeria. They used a sample size of forty three (43) respondents of which are thirteen (13) functional tanneries and the thirty (30) known 'pomo' Wholesalers in the study. The researchers adopted Nash Equilibrium technique as a solution method for the game. The result of their analysis revealed that market share of hides was 30% to 70% for tanners and ‘pomo’ wholesalers respectively and that there is relatively low competition between tanners and ‘pomo’ wholesalers in Nigeria. Finally, the researchers recommended that livestock production should be supported by government so that quality hides can be produced for tanning and ‘pomo’ consumption in the country.

Achugamonu et al. (2012) investigated the objective project optimization of MTN and GLOBACOM in advertising their products and other services they render within Nigeria. The researchers formulated linear programming model of game theory to determine an optimal resource allocation of both firms. They adopted the Simplex method and Duality Theory as their solution methods to obtain an optimum benefit for the two telecom firms. The result of their analysis indicated that for optimum performance, MTN should invest the X they have in advertisement only and nothing in service promotion and this will result in an optimal benefit value of 23X million. Similarly, GLOBACOM should invest her Y only in service promotion and nothing in advertisement which yields an optimal benefit value of 39Y million.

Hongxia and Tamer (2010) considered optimal nonlinear pricing policy design for a monopolistic network service provider in the face of a large population of users of different types. They used reverse Stackelberg (follower-leader) game to solve an incentive-design problem for which an -team optimal incentive (pricing) policy was obtained, which almost achieves

Pareto optimality for a monopolistic network service provider facing a large population of users, for the partially incomplete information game. A comparative study between games with information symmetry and asymmetry was conducted as well to evaluate the service provider's game preferences.

Kyung-Joon,et al (2009) investigated on how to improve network capacity by tuning the carrier sense threshold in Carrier Sense Multiple Access (CSMA) wireless networks. They presented a non-cooperative game-theoretic framework, which leads to a fully distributed algorithm for tuning the carrier sense threshold. They proved that the non-cooperative carrier sense game admits a unique Nash equilibrium (NE) under some technical conditions. They derived sufficient conditions that ensure the convergence of the synchronous and asynchronous update algorithms. Based on their analysis, the researchers proposed a fully distributed algorithm, entitled Non-Cooperative Carrier Sense Update Algorithm (NCUA) and then used simulation to show that NCUA outperforms standard CSMA with respect to the per-node through put by 10% to 50%.

Kyle (2014) researched on the potential of an international shale revolution, its impacts on the Organization of Petroleum Exporting Countries (OPEC) market shares and OPEC responses to the potential of shale developments. The researcher used time series regression model with data to study the dynamics of the Shale Oil in the USA and outside USA and a game theory model framework was used to analyze The OPEC responses to the potential of shale developments. In his analysis, he concluded that the prospective impacts of shale oil are far-reaching and carry significance concerning the future structure of oil markets, the effectiveness and viability of OPEC in the intermediate and long terms, and the dependency on OPEC oil for various segmented markets. With regards to the OPEC responds, the research concluded that in a perfectly collusive situation, OPEC has the capacity to hinder shale oil development, and, in an imperfectly collusive situation, OPEC's ability to effectively respond to prospective shale production is limited.

Harrison (2012) provided an analytical description of the nature of interaction between fiscal and monetary policy in Nigeria. He used game theoretical framework to model the phenomenon. His analysis revealed that misalignment of policy instruments and strategies, particularly fiscal dominance over monetary policy is the major factor responsible for the ineffectiveness of economic policies in Nigeria. The structure of the policy institutions and the credibility of the actors as reflected in their behaviour influence the nature of their interaction and that an optimal threshold that is able to synchronize the policy preferences of the economic agents could result into an optimal solution and improve the interaction between fiscal and monetary policy in Nigeria.

2.1.3 Historical Background of Lemke-Howson Algorithm

The study of bimatrix games via the best response polytopes can be traced back to the algorithm of Vorobev (1958) for finding all Nash equilibria of a bimatrix game. Important progress was made with the development of the algorithm by Lemke and Howson (1964). Shapley (1974) introduced facet labels, and described Nash equilibria of bimatrix games as completely labeled vertex pairs of the best response polytopes, which is crucial in the geometric approach of this algorithm. Of late, this algorithm has witnessed a lot of applications in finding Nash Equilibrium of non-degenerate bimatrix. Daphne et al. (1996) considered extensive two-person games with general payoffs, where the players have perfect recall. They first converted the extensive form game into strategic form game and then applied the Lemke's algorithm, a generalization of the LemkeHowson method to determine the Nash Equilibria of the game.

Kunpeng (2013) also investigated the use of the Lemke-Howson Algorithm to determine Nash Equilibrium of two-person bimatrix game. He realized that the method finds at least one Nash Equilibrium of non-degenerate bimatrix. Rahul and Von Stengel, B. (2006) studied about the computational problem of finding Nash equilibrium of a bimatrix game, a two-player game in strategic form. The researchers adopted the Lemke-Howson Algorithm as a solution method. Using polytope theory, they constructed the game using pairs of dual cyclic polytopes. They considered both cases of square and non-square matrix games.

Chapter 3

MODEL DEVELOPMENT

3.1 Introduction

In this chapter we provided explanation to some basic concepts of game theory, some basic solutions to two-person zero-sum games, and formulation of linear programming model of game theory. We also provided a brief description of The Lemke-Howson Algorithm as a solution method to two-person zero-sum games.

3.2 Definitions of Game Theory and Game

3.2.0.1 Game Theory

Game Theory is a mathematical concept that is used for analysis and resolution of conflict situations, in which parties have opposing interests. The concept of game theory provides a tool for formulating, analyzing and understanding different strategic scenarios. It attempts to address the functional relationship between selected strategies of individual players and their market outcomes which could be any quantifiable consequences (gain or loss).

3.2.0.2 Game

A game consists of players, actions, strategies, utilities, outcomes and equilibria. The players, actions and outcomes define the rules of the game.

Players: These are the decision makers of the game. In the context of this work, the players are the firms the MTN and the Vodafone Ghana.

Actions: These are all the possible moves that a player can make. In our context, there are the set of the selected service offers employed by both MTN and Vodafone-Ghana.

Strategy: It is one of the given possible actions of a player.

Utility: This consists of profit or expected gain a player receives after all players have selected strategies and the game has been played out. It is otherwise called payoff Outcome: This is the set of interesting results obtained from the values of actions/strategies after the game has been completed.

Equilibrium: It is a strategy combination that consists of the best response for each player in the game.

3.2.1 Assumptions in Game Theory

3.2.1.1 Rationality

In theory of game, every player is believed to be endowed with reasoning abilities that will enable him/her to compete favourably in competitions. A rational player always seeks to play in a manner that will maximize his/her payoff/utility. It is often assumed that the rationality of all players is common knowledge, Theodore and Von Stengel, (2001). In telecommunication industry, the players are usually electronic devices programmed to operate in a specific way, hence rationality is assured.

3.2.1.2 Maximization

The ultimate aim of every decision maker in any situation is to obtain a higher benefit. Players of a game, as rational beings, are in no exception. They always try their best to obtain a higher payoff/utility regardless of their opponents' actions, though, this may sound somehow selfish. However, in the business, especially, oligopoly market, every entity strives not to be disadvantaged.

3.2.1.3 Intelligence

Every decision maker for that matter a player has perfect knowledge of the game and his/her opponents. That is, he/she knows in full detail the rules of the game as well as the moves and payoffs of other player(s).

3.2.2 Classification of Games

Games are classified according to their properties. In this section we discuss some common types of games with their examples.

3.2.2.1 Cooperative and Non-cooperative Games

The Cooperative Game otherwise known as Coalitional Game is a type in which players enter into an agreement that allows them to plan mutual strategies and achieve higher benefits. It specifies what payoffs each potential group or coalition can obtain by the cooperation of its members. Examples are the bargaining problems, international relations. The Non-cooperative Game on the other hand is the type in which there is no negotiation or any mandatory agreement between players. With this type of game, players make choices out of their own interest. The Prisoner's Dilemma and the Battle of the Sexes are examples of Non-cooperative game. In effect, most game theoretic research works have been conducted using non-cooperative games in the telecommunication industry.

3.2.2.2 Games with complete and incomplete information

Games in which all players know the rules of the game are known as games with complete information. In these games, preferences or utility functions of players are common knowledge. The Prisoner's Dilemma is a typical example of a game of complete information. The payoffs for every combination of actions by players are a common knowledge.

Games in which at least one of the players does not know or is uncertain about another players preference or utility function are called games with incomplete information or pseudo games. For examples a sealed-bid art auction is a game of incomplete information. This is because the bidders payoff functions are not common knowledge, (*Feryal*&*Pinar,*2003).

3.2.2.3 Games with perfect and imperfect information

Games in which all players are fully informed about the moves of each other is said to have perfect information. Extensive games are those that can have perfect information.

On the other hand games in which players choose their moves independently and simultaneously are said to have imperfect information. Strategic form games are typically games with imperfect information. Game with imperfect information is a good model framework in telecommunications since service providers and/or users do not always know the exact actions of one another.

3.2.2.4 Strategic and Extensive form Games

The strategic/static/normal form of a game is defined by specifying for each player the set of strategies, and the payoff of each player for each strategy profile. In strategic form game each player chooses a strategy independently from and simultaneously with the other players. The players then receive their payoffs as given for the resulting strategy profile. The outcome of strategic form game can be either deterministic or may contain uncertainties. The solution for a strategic form game is Nash Equilibrium. For two players, the strategic form is conveniently represented in a table. The row of the table represents the strategies of player 1, and the column, the strategies of player 2. The strategic profile is a strategy pair, that is, a row and a column, with a corresponding cell of the table that contains two payoffs, one for player 1 and the other for player 2. Below is an example of a strategic form game.

Figure 3.1: An example of a strategic form game

A STRATEGIC FORM GAME

	PLAYER 2	
PLAYER 1	(1,3)	(1,3)
	(0,0)	(2,2)

The extensive form game, which is a detailed model of game tree, formalizes interactions where players can over time be informed about the actions of others. In an extensive game with perfect information, every player is at any point aware of the previous choices of all other players. One player moves at a time in the game, (*TheodoreandV onStengel,*2001). The basic structure of an extensive game is a tree. In the game, every branch point or node is associated with a player who makes move by choosing the next node. The connecting lines are labeled with the player's choices. The game starts at the initial node, called the root of the tree, and ends at a terminal node, called leaves, which establishes the outcome and determines the players' payoffs. Below is an example of a two-stage extensive game as
follows:

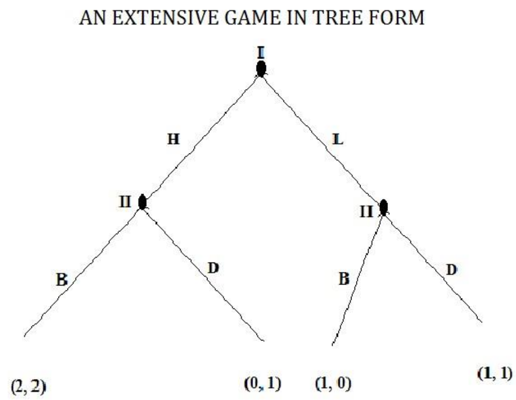
In a quality choice game, a service provider, player I, makes the first move, choosing either High (H) or Low (L) quality of service. Then the customer, player II, is informed about that choice. Player II can then decide separately between buy (B) and don't buy (D) in each case. The resulting payoffs are the same as in strategic form game. However, players in this game move in sequence rather than simultaneously.

Figure 3.2: An example of an extensive form game in matrix form

AN EXTENSIVE FORM GAME

		PLAYER II	
PLAYER I		B	D
	H	(2,2)	(0,1)
	L	(1,0)	(1,1)

Figure 3.3: An example of an extensive game in tree form



3.2.2.5 The Prisoner's Dilemma

In the Prisoner's Dilemma, two suspects of a crime are arrested and charged. Meanwhile, the police do not have sufficient information to convict the suspects, unless at least one of them confesses. Each suspect can either confess with a hope of a lighter sentence (defect) or refuse to talk (cooperate). As a result, the police interrogated the suspected criminals separately and offered them a deal. If they cooperate, then both will be convicted of minor offense and jailed for a month, if both defect, then they will be jailed for six months. If one defects and the other does not, then the defector will be released and the other is sentence for nine months. The prisoner's dilemma game is a strategic game. The players simultaneously choose their strategies, only one, and then the game is over. Below is a matrix representation of the suspects' possible actions and their corresponding jail sentences. A

Figure 3.4: Prisoners Dilemma

PLAYER 1	PLAYER 2		
		Cooperate(c)	Defect(d)
	Cooperate(C)	(-1,-1)	(-9,0)
	Defect(D)	(0,-9)	(-6,-6)

closed look at the outcomes of the game indicates that the outcome (D, d) is the equilibrium. The outcome (C, c), even though, has a higher payoff, the individual players are still liable to deviate.

3.2.3 Zero-sum Game

Zero-sum games are games whose utilities/payoffs are sum up to zero in every outcome. In games of zero-sum, a gain by one player is a lost by other player(s) in the competition.

A classic example of zero-sum game is the von Neumann Poker game, (von Neumann & Morgenstern 1944).

3.2.4 Two-Person Zero-Sum Games

These are games with only two players (who may be armies, firms, teams, and so on) in which one player wins what the other player losses. That is, a gain by one player is a lost to the other player. These games are characterized by: two players, two sets of strategies (one for each player), and a payoff table, (Hillier and Liebermann, 2001).

3.2.5 Nonzero-sum Games

Nonzero-sum games as the name suggest are games whose players utilities are not sum up to zero. Nonzero-sum game can be classified into two distinctive forms: The constant-sum game and the variable-sum game. If the utility of any combination of actions by the players in a game yields a constant sum, then the game is said to be constant-sum game. However, if the utility of any combination of actions by the players in a game yields different values, it is referred to as variable-sum game.

3.3 Solution Concepts

In game theory, every game is characterized by three sets of properties. These include a set of players, a set of strategies (finite or infinite) each for the players, and a possible outcome (payoffs/utilities). In effect, a game describes what actions the players can take and what the consequences of the actions are. This is implying that the solution to a game is the set of possible outcomes. In general, a solution of a game is an outcome from which no player wants to deviate unilaterally.

3.3.1 Nash Equilibrium

A Nash Equilibrium otherwise known as a Strategic Equilibrium is one of the most common solution method used in game theory. It consists of a profile (list) of strategies, one for each player, which has the property that no player can unilaterally change his/her strategy and gets a better payoff, Theodore and Von Stengel (2001). That is, if one player changes his/her strategy and the other players maintain their strategies, he/she will not gain anything better. For instance, in the battle of sexes if the Husband is playing Opera then the best response for the Wife is also play Opera. Thus, Opera is the best response for Wife against Opera. Similarly, Opera is the best response for Husband against Opera. This means that at (Opera, Opera), neither player would like to use a different strategy.

3.3.2 Pareto Optimality

This is a measure of efficiency. An outcome is said to be Pareto Optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player.

3.4 Pure Strategies and Mixed Strategies

In game theory, when a player makes a decision, he/she uses either a pure strategy or a mixed strategy. A pure strategy is the strategy deterministically chosen by a player from his/her strategies. A mixed strategy, on the other hand, is the strategy randomly selected from among pure strategies with assigned probabilities. That is, a mixed strategy is determined by the probabilities that it assigns to a player's pure strategies. It is only used when a player is indifferent between many pure strategies and also to keep an opponent in suspense.

3.4.1 Formulation of A Two-Person Zero-Sum Games

In general, a Two-Person Zero-Sum game in strategic form is of the form $(\{1,2\},X,Y,u_1,u_2)$. In this game, Player 1 denote a row player and Player 2 a column player; X, a nonempty set, denotes the set of strategies of Player 1; Y, a nonempty set, denotes the set of strategies of Player 2; u_1 denotes the payoff function of Player 1 and u_2 denotes the payoff function of Player 2. The payoff function of Player 2 is the negative of the payoff function of Player

1. The solution of the game contains in a table which is referred to as a Payoff Matrix.

Let N = {Player1,Player2}

$$X = \{x_1, x_2, \ldots, x_m\} \text{ and } Y = \{y_1, y_2, \ldots, y_n\}$$

$$u_1 = (x_i, y_j), u_2 = -u_1 = -(x_i, y_j), \forall x_i \in X \text{ and } \forall y_j \in Y$$

Figure 3.5: An example of a Two-Person Zero-Sum Game in Bimatrix Form

Player 1	Player 2		
	y_1	y_2	y_3
x_1	(3,-3)	(6,-6)	(1,-1)
x_2	(5,-5)	(-2,2)	(3,-3)
x_3	(4,-4)	(2,-2)	(-5,5)

Since the payoff function of player 2 is the negative of that of player 1, it implies that the payoff matrix can be reduced to a single matrix form for both players as follows:

Figure 3.6: Two-Person Zero-Sum Game Payoff Matrix in Single Form

Player 1	Player 2		
	y_1	y_2	y_3
x_1	3	6	1
x_2	5	-2	3
x_3	4	2	-5

In general, every zero-sum game is a constant-sum game:$(\{1,2\},X,Y,u_1,u_2)$, where$u_1(x_i,y_j)+ u_2(x_i,y_j) = C \forall x_i \in X, \forall y_j \in Y$ and C is a given constant.

3.4.2 Optimal Solution of A Zero-Sum Game With Pure Strategies: Nash Equilibrium

A finite Two-Person Zero-Sum Game in a strategic form:(X,Y,A), where $X = \{x_1,x_2,...,x_m\}$ and $Y = \{y_1,y_2,...,y_n\}$ are the respective pure strategies for the players 1 and 2 with a given payoff matrix

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}, \text{where } a_{ij} = (x_i, y_j)$$

In the game, player 1 is the row player whiles player 2 is the column player. The player 2 pays player 1 the entry in the chosen row and column. The entries of the matrix are winnings for the row player and losses for the column player.

Definition: Saddle Point of a Matrix

Given the matrix A, an element a_{ij} is said to be a saddle point if and only if it has the property that:

1. a_{ij} is the minimum of the *ith* row and

2. a_{ij} is the maximum of the *jth* column.

Theorem 1 *Given a matrix* $A = [a_{ij}]$.

Let

$$U_R = max_i min_j a_{ij}$$

$$U_C = min_j max_i a_{ij}$$

The matrix A has a saddle point if and only if $U_R = U_C$

Example

We consider the following payoff matrix A which denotes a gain for the row player.

$$
\begin{array}{lcc}
 & \text{Player 2} & \text{Row Minimum} \\
\text{Player 1} & \begin{pmatrix} 8 & 6 & 2 & 8 \\ 8 & 9 & 4 & 5 \\ 7 & 5 & 3 & 5 \\ 6 & 7 & 1 & 4 \end{pmatrix} & \begin{array}{l} 2 \\ 4 \leftarrow \text{Maximin} \\ 3 \\ 1 \end{array} \\
\text{Column Maximum} & \begin{array}{cccc} 8 & 9 & 4 & 8 \\ & & \uparrow & \\ & & \text{Minimax} & \end{array} &
\end{array}
$$

$U_R = max_i min_j a_{ij}\ min_j a_{ij} =$

$\{2,4,3,1\}^T$

$= max_i\{2,4,3,1\}^T$

$U_R = 4$

$U_C = min_j max_i a_{ij}\ max_i a_{ij} =$

$\{8,9,4,8\}$

$= min_j\{8,9,4,8\}$

$U_C = 4$

Since $U_R = U_C$ the game has a saddle point solution $a_{23} = 4$. Hence, the value of the game is 5.

3.4.3 Optimal Solution of A Zero-Sum Game With Mixed Strategies: Nash Equilibrium

In game theory, there are situations where a game does not possess a single saddle point. In such a case, a mixture of strategies are used. A mixed strategy is a probability distribution over the set of pure strategies of a player. A mixed strategy of Player 1 is a vector and consists of an **m**-tuple, $x = (x_1, x_2, \ldots, x_m)^T$ of probabilities, such that $\sum_{i=1}^{m} x_i = 1$. Each x_i denotes the probability of using a pure strategy Player 1. Similarly, a mixed strategy of Player 2 is a vector and consists of an **n**-tuple, $y = (y_1, y_2, ..., y_n)$ of probabilities, such that $\sum_{j=1}^{n} y_j = 1$. Each y_j denotes the probability of using a pure strategy Player 2. If player 1 uses the mixed strategy **x** and the player 2 chooses column **j** then the (average) payoff to Player 1 = $\sum_{i=1}^{m} x_i a_{ij}$. Also, if player 2 uses the mixed strategy **y** and the player

1 chooses row **i**, then the payoff to player 1 is $\sum_{i=1}^{m} a_{ij} y_j$. In effect, the objective for player 1 is to obtain at least an expected value V (the value of the game).

In mixed strategy game, each player plays all his/her strategies according to a predetermined set of probabilities, in order to obtain an optimal solution. Consider the following: Let $x_1, x_2, ..., x_m$ and $y_1 y x_2, ..., y_n$ denote the row and column probabilities used by the players 1 and 2 respectively to select their pure strategies, where:

$$\sum_{i=1}^{m} x_i = \sum_{j=1}^{n} y_j = 1$$

$$x_i \geq 0, x_i \geq 0 \forall i \text{ and} \forall j$$

Given the payoff matrix A as follows: where $a_{ij} = A(x_i, y_j)$

The concept of minimax criterion is extended to games that lacks saddle points and thus need mixed strategies. The criterion stipulates that player 1 selects the mixed strategy $\mathbf{x_i}$ that maximizes the minimum expected payoff to player 2. Equivalently, the player 2 also selects the mixed strategy $\mathbf{y_j}$ that minimizes the maximum expected loss to player 1. The

$$\begin{array}{cc} & \text{Player 2} \\ & \begin{array}{ccccc} & y_1 & y_2 & \cdots & y_n \end{array} \\ \text{Player 1} & \begin{array}{c} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_m \end{array} \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{array}$$

average payoff (the stable solution) of the game when both players use their optimal strategies is denoted by V.

Mathematically, the minimax criterion for a mixed strategy game is expressed as follows: Player 1 selects $x_i \geq 0, \sum_{i=1}^{m} x_i = 1$ that will result

$$max_{x_i}\{min(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, \ldots, \sum_{i=1}^{m} a_{in}x_i)\}$$

Let

$$V = \{min(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, \ldots, \sum_{i=1}^{m} a_{in}x_i)\}$$

Such that

$$\sum_{i=1}^{m} a_{ij}x_i \geq V, \quad \text{where } j = 1,2,\ldots,n \tag{3.1}$$

Player 2 selects $y_j \geq 0, \sum_{j=1}^{n} y_j = 1$ that will result

$$min_{y_j}\{max(\sum_{j=1}^{n} a_{1j}y_j, \sum_{j=1}^{n} a_{2j}y_j, \ldots, \sum_{j=1}^{n} a_{mj}y_j)\}$$

Also let

$$V = \{max(\sum_{j=1}^{n} a_{1j}y_j, \sum_{j=1}^{n} a_{2j}y_j, \ldots, \sum_{j=1}^{n} a_{mj}y_j)\}$$

Such that

$$\sum_{j=1}^{n} a_{ij}y_j \leq V, \quad \text{where } i = 1,2,\ldots,m \qquad (3.2)$$

The values of inequalities (3.1) & (3.2) respectively represent the maximin and minimax payoffs. Hence, minimax expected payoff ≥ maxmin expected payoff. Equality only occurs when $\mathbf{x_i}$ and $\mathbf{y_j}$ correspond to the optimal solution (the expected values of the game, V).

3.5 The Model

3.5.1 Linear Programming Formulation of Two-Person Zero-Sum Games

Game theory has a strong conceptual relationship with linear programming. In particular, every two-person zero-sum game can be expressed as a linear programming problem, Hamdy, (2007). In two-person zero-sum game, the players have opposing interests. One player objective is to maximize his gains and the other's is to minimize his losses. That is, in a two-person zero-sum game, a player either aims to maximize or minimize gains. Like linear programming, the objective of the game is a linear function of the decision variables. The linear programming, on the other hand, is a problem of maximizing (or minimizing) a linear function subject to a finite number of linear constraints on variables which are non-negative. It involves planning of activities to obtain an optimal result. The game of two-person zero-sum exhibits the primal-dual relationship as in linear programming. Also, when the players simultaneously use their optimal strategies the winner expected gain is equal to the looser expected loss.

3.5.2 The Primal-Dual Relationships

Weak Duality Property: if x is a feasible solution for the primal problem and y is a feasible solution for the dual, then: $yb \geq cx$, where cx and yb are respective maximizing and minimizing functions.

This property describes the relationship between any pair of solutions for the primal and dual problems where both solutions are feasible for their respective problems.

Strong Duality Property: if x^* is an optimal solution for the primal and y^* is an optimal solution for the dual, then $cx^* = y^*b$

The Minimax Theorem: If mixed strategies are allowed, the pair of mixed strategies that is optimal according to the minimax criterion provides a stable/equilibrium solution

(the value of the game, V), so that neither player can do better by unilaterally changing his/her strategy, (Hillier & Liebermann, 2001).

Consider the following: If Player 1's optimal strategies satisfy maximin criterion

$$\max_{x_i}\{\min(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, \ldots, \sum_{i=1}^{m} a_{in}x_i)\}$$

Subject to the constraints

$$x_1 + x_2 + \cdots + x_m = 1$$

and

$$x_i \geq o, \text{for } i = 1, \ldots, m$$

Let

$$V = \{min(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, \ldots, \sum_{i=1}^{m} a_{in}x_i)\}$$

The problem then becomes

$$\text{maximize } Z = V$$

subject to the constraints

$$\sum_{i=1}^{m} a_{ij}x_i \geq V, \text{where } j = 1, 2, \ldots, n$$

$$x_1 + x_2 + x_3 + \cdots + x_m = 1$$

and

$$x_i \geq 0, \text{for i} = 1, \ldots, m$$

Thus, the linear programming problem for Player 1 is

$$\text{maximize } Z = V$$

subject to the constraints

$$a_{11}x_1 + a_{21}x_2 + a_{31}x_3 + \cdots + a_{m1}x_m \geq V$$

$$a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + \cdots + a_{m2}x_m \geq V$$

...

$$a_{1n}x_1 + a_{2n}x_2 + a_{3n}x_3 + \cdots + a_{mn}x_m \geq V$$

$$x_1 + x_2 + x_3 + \cdots + x_m = 1$$

$$\forall x_i \geq 0, V \text{ is unrestricted}$$

The solution to the problem above gives the equilibrium mixed strategies ($x_1, x_2, dots, x_m$) for player 1 and the value of the game V.

Suppose $V > 0$, the constraints of the linear program becomes:

$$a_{11}\frac{x_1}{V} + a_{21}\frac{x_2}{V} + a_{31}\frac{x_3}{V} + \cdots + a_{m1}\frac{x_m}{V} \geq 1$$

$$a_{12}\frac{x_1}{V} + a_{22}\frac{x_2}{V} + a_{32}\frac{x_3}{V} + \cdots + a_{m2}\frac{x_m}{V} \geq 1$$

...

$$a_{1n}\frac{x_1}{V} + a_{2n}\frac{x_2}{V} + a_{3n}\frac{x_3}{V} + \cdots + a_{mn}\frac{x_m}{V} \geq 1$$

$$\frac{x_1}{V} + \frac{x_2}{V} + \frac{x_3}{V} + \cdots + \frac{x_m}{V} \geq \frac{1}{V}$$

and

$$\forall x_i \geq 0, i = 1,\ldots,m$$

Let

$$X_i = \frac{x_i}{V} \rightarrow i = 1, 2, \ldots, m$$

Since maximizing V is equivalent to minimizing $\frac{1}{V}$, it implies that

$$maxV = min\frac{1}{V} = min\{x_1 + x_2 + x_3 + \cdots + x_m\}$$

Hence the problem becomes

$$MinimizeZ = \{X_1 + X_2 + X_3 + \cdots + X_m\}$$

subject to the constraints

$$a_{11}X_1 + a_{21}X_2 + a_{31}X_3 + \ldots + a_{m1}X_m \geq 1$$

$$a_{12}X_1 + a_{22}X_2 + a_{32}X_3 + \ldots + a_{m2}X_m \geq 1$$

...

$$a_{1n}x_1 + a_{2n}x_2 + a_{3n}x_3 + ... + a_{mn}x_m \geq 1$$

and

$$x_i \geq 0, \text{for} \rightarrow i = 1,2,...,m$$

Player 2's optimal strategies $y_1, y_2 ..., y_n$ are determined by solving the minimax criterion below:

$$min_{y_j}\{max(\sum_{j=1}^{n} a_{1j}y_j, \sum_{j=1}^{n} a_{2j}y_j, ..., \sum_{j=1}^{n} a_{mj}y_j)\}$$

Subject to the constraints

$$y_1 + y_2 + \cdots + y_n = 1$$

$$y_j \geq 0, \text{for} \rightarrow j = 1,2,...,n$$

Following the same procedure as for Player 1, the above can also is expressed as a linear program as:

$$Maximize W = \{Y_1 + Y_2 + Y_3 + \cdots + Y_n\}$$

subject to the constraints

$$a_{11}y_1 + a_{21}y_2 + a_{31}y_3 + ... + a_{1n}y_n \leq 1$$

$$a_{12}y_1 + a_{22}y_2 + a_{32}y_3 + \ldots + a_{2n}y_n \leq 1$$

$$\ldots$$

$$a_{m1}y_1 + a_{m2}y_2 + a_{m3}y_3 + \ldots + a_{mn}x_n \leq 1$$

$$\forall y_j \geq 0, \text{for} \rightarrow j = 1,2,\ldots,n$$

and

$$w = \frac{1}{V}, Y_j = \frac{y_j}{V}, \text{for} \rightarrow j = 1, 2, \ldots, n$$

The linear programming problems for both Player 1 and Player 2 are dual to each other. This implies that the optimal mixed strategies for both players can be determined by solving one of the linear programming problems.

3.6 Solution Method: The Lemke-Howson Algorithm

This section introduces the solution method or algorithm that finds Nash Equilibria of Finite Two-Player Strategic Games. The information for this section is sourced from David, (2011) and Von Stengel (2002). The Lemke-Howson Algorithm is used in finding Nash Equilibria of two-person bimatrix games. It resembles the simplex algorithm, especially as it consists of iterated pivoting.

3.6.1 Similarities Between The Simplex and The Lemke-Howson Algorithms

- The Simplex Algorithm can take an exponential number of iterations so as the LemkeHowson Algorithm.

3.6.2 Differences Between The Simplex and The Lemke-Howson Algorithms

- In Simplex Algorithm the objective function dictates the choice of the next entering variable to enter the basis whiles in the Lemke-Howson Algorithm the complementary pivoting rule chooses the non-basic variable with duplicate labels to enter the basis.

Consider a two-person bimatrix game where the payoff matrices are A_{mxn} and B_{mxn}. A pair of strategies (x,y) is a Nash Equilibrium if and only if

$$\forall 1 \leq i \leq m, x_i > 0 \Rightarrow (A_{kj} y_j) = \max \sum_{j=m+1}^{m+n} (A_{ij} y_j)$$

$$\forall m+1 \leq j \leq m+n, y_j > 0 \Rightarrow (x_i^T B_{ik}) = \max \sum_{i=1}^{m} (x_i^T B_{ij})$$

Let $M = \{1,2,...,m$ and $N = \{m+1, m+2, ..., m+n$ respectively denote the actions of the Player 1 and Player 2. The matrix A_{mxn} represents the payoff for Player 1 and the matrix B_{mxn}

represents the payoff for Player 2. We think of Player 1 picking rows and Player 2 picking columns; so a mixed strategy for Player 1 is an **m**-element row vector that is stochastic (entries are non-negative and add up to 1) and similarly the mixed strategy for Player 2 is **n**-column vector. With the above notations, the payoff to Player 1 (resp. 2) under mixed strategy profile x^T, y is Ay(respx^TB). We used X to denote the set of all mixed strategies of Player 1 (the set of all stochastic m-element row vectors) and Y is defined similarly for Player 2.

Assumption 1: We assume that all entries of A and B are non-negative, that A has no all-zero columns and B has no all-zero rows.

The Concept of Lemke-Howson Algorithm

The basic idea of the Lemke-Howson Algorithm for finding all Nash Equilibria lies on guessing the support of equilibrium, and then solving a linear equation to determine the values of the non-zero variables. Here, we maintain a single guess as to what the supports should be, and in each iteration we change the guess only a little bit.

The Support of a Mixed Strategy

The support of a mixed strategy is the set of pure strategies that have positive probability. Thus

$$S(x) = \{i | x_i \geq 0\} \& S(y) = \{j | y_j \geq 0\}$$

Best Response

A best response to the mixed strategy y of player 2 is a mixed strategy x of player 1 that maximizes his expected payoff (Ay) . Similarly, a best response y of player 2 to x maximizes his expected payoff (x^TB). A Nash equilibrium is a pair (x, y) of mixed strategies that are best responses to each other.

Assumption 2: Non-degeneracy -A bimatrix game is non-degenerate if the number of pure best responses to any mixed strategy never exceeds the size of its support. The easiest description of the algorithm and the proof of Nash's theorem for two-person games rely on two polytopes. A polytope is the same as the feasible region for a Linear Program (LP): a system of linear equations and inequalities.

Let B_j denote the column of B corresponding to action j and let A^i denote the row of A corresponding to action i. The polytopes (P):

$$P_1 = \{x \in \mathrm{R}^M | (\forall i \in M : x_i \geq 0) \& (\forall j \in N : x^T B_j \leq 1)\}$$

$$P_2 = \{y \in \mathrm{R}^N | (\forall j \in N : y_j \geq 0) \& (\forall i \in M : A^i y \leq 1)\}$$

Here, the x and y are not restricted to be stochastic, only non-negative. For a non-zero non-negative x, we can normalize it to a stochastic vector normalize(x) as follows:

$$\text{nrml}(x) := \frac{x}{(\sum_i x_i)} = (\sum_i x_i)^{-1} x$$

$$\mathbf{nrml}(y) := \frac{y}{(\sum_j y_j)} = (\sum_j y_j)^{-1} y$$

The inequalities that define P_1 have the following meaning:

- If $x \in P$ meets $x_i \geq 0$ with equality then i is not in the support of x

- If $x \in P$ meets $x^T B_j \leq 1$ with equality then j is a best response to $nrml(x)$

Let assume that $x \in P$ has label k, where $k \in M \cup N = \{1,2,...,m + n\}$, if either $k \in M$ and $x_k = 0$, or $k \in N$ and $x^T B_k = 1$ Similarly, $y \in P_2$ has label k if either $k \in N$ and $y_k = 0$, or $k \in M$ and $A^k y = 1$. As a consequence of the Support Characterization, we

have the following.

Theorem 2 *Suppose that $x \in P_1$ and $y \in P_2$, and neither x nor y is the all-zero vector. Then x and y together have all labels from 1 to k if and only if (nrml(x),nrml(y)) is a Nash Equilibrium. All Nash Equilibria arises in this way.*

3.6.2.1 The Tableau Method (Lemke-Howson Algorithm)

In applying the tableau method to find Nash Equilibrium using the Lemke-Howson Algorithm, the following steps are used.

1. Preprocessing

2. Initialization of tableau

3. Repeated Pivoting

4. Recover Nash Equilibrium from final tableau

In the tableau method, we introduce slack variables, and use the terminologies basic and non-basic variables. For our purposes the basic variables and set of labels have opposite meanings since labels imply tight inequality and basic variables are not tight. Hence, "enter the basis"means the same as "label is removed" and "leave the basis" means that "label is added".

Step 1. Preprocessing

Here, we check to ensure all the entries are non-negatives and there is no pure Nash Equilibrium. Strictly dominated strategies are eliminated if any. The elimination reduces the size of the game, and therefore will reduce the amount of work involved with the pivoting later on. Hence, we apply the elimination before beginning. The linear Programming Problem Models for both players are then formulated using the polytopes constraints and the players respective payoff matrices.

Step 2. Initialization of Tableaux

For the purposes of solving the game we need two tableaux, one for each player. Let r_i be the slack variable in the constraints $A^iy \leq 1$ and let s_j be slack variable in the constraints $x^TB_j \leq 1$. We then obtain the system:

$$Ay + r = 1$$

$$B^T x + s = 1$$

$$x, y, r, s \geq 0$$

A binding inequality corresponds to a zero slack variable.

In the initial tableaux, the basis is $\{r_i | i \in M\} \cup \{s_j | j \in N\}$ we rewrite the equations so as to solve for them. In the payoff table, the entries are positive and there are no pure Nash

Example: A Bimatrix Payoff of Two-Player Game

P_1 \ P_2	4	5	6
1	1,2	3,1	0,0
2	0,1	0,3	2,1
3	2,0	1,0	1,3

Equilibria. All the entries are non-negatives and there is no pure Nash Equilibrium.

Table 3.1: Payoff Table for Player 1 (A)

	y_4	y_5	y_6
x_1	1	3	0
x_2	0	0	2
x_3	2	1	1

$$\max Z = \max \sum_{j=m+1}^{m+n} a_{ij} y_j, i = 1, 2, \ldots, m$$

subject to $A^i y \leq 1$ which are

$$y_4 + 3y_5 + 0y_6 \leq 1 \quad (3.3)$$

$$0y_4 + 0y_5 + 2y_6 \leq 1 \quad (3.4)$$

$$2y_4 + y_5 + y_6 \leq 1 \quad (3.5)$$

$$y_4 \geq 0, y_5 \geq 0, y_6 \geq 0$$

with strict inequality $y_j > 0$ for payoff calculation.

Table 3.2 represents Payoff Matrix for Player 2. All the entries are non-negatives and

Table 3.2: Payoff Matrix for Player 2 (B)

	y_4	y_5	y_6
x_1	2	1	0
x_2	1	3	1
x_3	0	0	3

there is no pure Nash Equilibrium.

$$\max W = \max \sum_{i=1}^{m} x_i{}^T b_{ij}, j = m+1, m+2, \ldots, m+n$$

subject to $x^T B_j \leq 1$ which are

$$2x_1 + x_2 + 0x_3 \leq 1 \quad (3.6) \quad x_1 + 3x_2 + 0x_3 \leq 1 \quad (3.7)$$

$$0x_1 + x_2 + 3x_3 \leq 1 \quad (3.8)$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

with strict inequality $x_i > 0$ for payoff calculation.

Adding slack variable r_i to (3.3),(3.4) and (3.5) convert the system to equalities:

$$y_4 + 3y_5 + 0y_6 + r_1 = 1 \quad (3.9) \quad 0y_4 + 0y_5 + 2y_6 + r_2 = 1$$

(3.10)

$$2y_4 + y_5 + y_6 + r_3 = 1 \qquad (3.11)$$

Adding the slack variable s_i to (3.6),(3.7) and (3.8) convert the system to equalities:

$$2x_1 + x_2 + 0x_3 + s_4 = 1 \qquad (3.12) \quad x_1 + 3x_2 + 0x_3 + s_5 = 1$$

$$(3.13) \quad 0x_1 + x_2 + 3x_3 + s_6 = 1 \quad (3.14)$$

Step 2: The Initial Tableaux

The Initial Tableaux are $r = 1 - Ay$

$$r_1 = 1 - y_4 - 3y_5 \; (3.15) \quad r_2 = 1 - 2y_6 \qquad (3.16)$$

$$r_3 = 1 - 2y_4 - y_5 - y_6 \qquad (3.17)$$

and $s = 1 - B^T x$,

$$s_4 = 1 - 2x_1 - x_2 \qquad (3.18) \quad s_5 = 1 - x_1 - 3x_2$$

$$(3.19) \quad s_6 = 1 - x_2 - 3x_3 \qquad (3.20)$$

Step 3. Pivoting

We then arbitrarily choose some x or y variable to bring in to the basis, corresponding to the arbitrary choice k_0 of label that we remove. Let's bring x_1 in. By considering the min-ratio rule (i.e. looking at the coefficients of x_1 in (3.12), (3.13) and (3.14)), it is s_4 that must leave the basis. Therefore, we solve (3.12) for x_1 obtaining a new equation (3.21) and then substitute the new equation into (3.13) and (3.14) and obtaining:

$$x_1 = \frac{1}{2} - \frac{1}{2}s_4 - \frac{1}{2}x_2 \tag{3.21}$$

$$s_5 = \frac{1}{2} + \frac{1}{2}s_4 - \frac{5}{2}x_2 \tag{3.22}$$

$$s_6 = 1 - x_2 - 3x_3 \tag{3.23}$$

The main feature of the Lemke-Howson Algorithm is that the variable which just left the basis determines the variable to enter the basis next. There are m x n complementary pairs of variables: $\{r_i,x_i\}$ for $i \in M$ and $\{s_j,y_j\}$ for $j \in N$. Each pair corresponds (in an inverse sense) to the labels we mentioned earlier, e.g., x_i is basic if and only if x does not have label i and s_j is basic if and only if x does not have label j

The m+n Complementary Conditions: (Orthogonality Condition) $r_ix_i = 0, i \in M$ and $s_jy_j = 0, j \in N$ indicate when to stop. Initially, all complementarity conditions are satisfied. We keep performing pivots until the complementarity conditions are again satisfied. Equivalently, we pivot until, between the two tableaux, in each complementary pair of variables, exactly one is basic and exactly one is non-basic.

In this case, since s_4 just left the basis, y_4 must be brought in. Examining (3.9), (3.10) and (3.11) tableau, we realized that r_3 is the winner of the min-ratio, and is therefore leaves the basis. We obtain the following as a result:

$$r_1 = \frac{1}{2} + \frac{1}{2}r_3 - \frac{5}{2}y_5 + \frac{1}{2}y_6$$

$$r_2 = 1 - 2y_6$$

$$y_4 = \frac{1}{2} - \frac{1}{2}r_3 - \frac{1}{2}y_5 - \frac{1}{2}y_6$$

since r_3 left, now x_3 enters the other tableau, and by the min-ratio rule s_6 leaves

$$x_1 = \frac{1}{2} - \frac{1}{2}s_4 - \frac{1}{2}x_2$$

$$s_5 = \frac{1}{2} + \frac{1}{2}s_4 - \frac{5}{2}x_2$$

$$x_3 = \frac{1}{3} - \frac{1}{3}x_2 - \frac{1}{3}s_6$$

since s_6 left, now y_6 enters and by the min-ratio rule r_2 leaves.

$$r_1 = \frac{3}{4} + \frac{1}{2}r_3 - \frac{5}{2}y_5 - \frac{1}{4}r_2$$

$$y_6 = \frac{1}{2} - \frac{1}{2}r_2$$

$$y_4 = \frac{1}{4} - \frac{1}{2}r_3 - \frac{1}{2}y_5 + \frac{1}{4}r_2$$

since r_2 left, now x_2 enters and by the min-ratio rule s_5 leaves.

$$x_1 = \frac{2}{5} - \frac{3}{5}s_4 + \frac{1}{5}s_5$$

$$x_2 = \frac{1}{5} + \frac{1}{5}s_4 - \frac{2}{5}s_5$$

$$x_3 = \frac{4}{15} - \frac{1}{15}s_4 - \frac{2}{15}s_5 - \frac{1}{3}s_6$$

since s_5 left, now y_5 enters and by the min-ratio rule r_1 leaves.

$$y_5 = \frac{3}{10} + \frac{1}{5}r_3 - \frac{2}{5}r_1 - \frac{1}{10}r_2$$

$$y_6 = \frac{1}{2} - \frac{1}{2}r_2$$

$$y_4 = \frac{1}{10} - \frac{3}{5}r_3 + \frac{1}{5}r_1 + \frac{3}{10}r_2$$

Step 4. Output

Since x_1 was the initial variable to enter the basis, and r_1 just left, the complementarity conditions are satisfied. That is, if x_i was the variable to enter, we stop when x_i or its complement leaves. In a tableau we obtain values for the basic variables by setting the non-basic variables to zero. Hence the variables values are:

$$r = (0,0,0), s = (0,0,0), x = (\frac{2}{5}, \frac{1}{5}, \frac{4}{15}), y = (\frac{1}{10}, \frac{3}{10}, \frac{1}{2}).$$

Therefore the Nash Equilibrium we found is

$$(\mathrm{nrml}(x), \mathrm{nrml}(y)) = ((\frac{6}{13}, \frac{3}{13}, \frac{4}{13}), (\frac{1}{9}, \frac{3}{9}, \frac{5}{9}))$$

The above problem is also solved on MATLAB by developing MATLAB codes which will be shown in appendix. The MATLAB codes developed were based on the pseudo-code below.

1. Enter player 1 and 2 matrices as A and B
2. Create tableaux for A as $r = 1 - Ay$ and B as $s = 1 - B^T x$
3. Choose a pivot by either choosing Player (A) or Player (B).
4. From step 3 select a basis thus if A then pick y otherwise pick x in the second column and on the first row. Divide that entire row by the coefficient of the basis. Find min (the 2nd column) and choose that row that recorded the minimum. This row then becomes the next column to be selected in the other matrix.
5. Repeat step 4 if a tie is found as the minimum value in the column then check if that number corresponding to the row has already been selected.
6. If yes choose the selected value and go to step 4 until all rows in both Player (A) and Player (B) variables have been used up.
7. Create a column vector from r and s by choosing all the first column in A and B respectively.
8. Normalize the vector.

MATLAB Results

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 2 \\ 2 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$(\mathrm{nrml}(x), \mathrm{nrml}(y)) = ((\frac{6}{13}, \frac{3}{13}, \frac{4}{13}), (\frac{1}{9}, \frac{3}{9}, \frac{5}{9}))$$

Chapter 4

DATA COLLECTION AND ANALYSIS

4.1 Introduction

This chapter presents the data used for the study and the results of analysis carried out. The data depicts customers preference between two competing mobile phone service providers based on the service offers available to them. The number of customers choosing a particular service offer from a service provider was recorded as a payoff for that service provider.

4.1.1 Data Collection

In this work, we made use of both primary and secondary data. The secondary data was obtained from the internet via websites of the two firms and confirmed at their respective outfits. The data consisted of the service offers employed by the two telecom companies to mobile phone users in the Ghanaian market. However, out of the various services they each offer to the public, we selected eight (8) services each for both firms. Our choice of these services was influenced by the following criteria:

- The services that were frequently advertised in the electronic media (the radio, television and the internet).
- The services that were targeting all groups/categories of customers of the respective firms.
- The level of awareness of the firms' services by the customers.

The service offers selected were regarded as strategies employed by the two firms to influence mobile phone users to patronize their products in the Ghanaian market.

The primary data for the study was obtained through a set of questionnaire that was prepared and administered to two hundred (200) respondents to indicate their preferred network based on the services employed by the two firms. The two hundred (200) respondents were sampled purposefully after putting them in a quota according to categories of customers. In all, forty (40) of respondents were tertiary students, sixty (60) formal sector workers and one hundred (100)informal sector workers. These techniques were adopted in order that we obtain varied views from the customers. We considered those customers who are subscribers to both firms. Each respondent was taken through on the nature of the questionnaire and how they are expected to respond. We also provided in-depth explanations on the selected services to the respondents to ensure more understanding. Table 4.1 and 4.2 shows the services selected for this research for MTN and Vodafone respectively.Table 4.3 contains the payoffs for both players.

Table 4.1: Service Offers employed by MTN

Service	MTN Zone	Family & Friends	Conference Calls	Extra Time	Free Beyond One	Mobile Money	Sunday Special	Pay4Me
Label	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8

Table 4.2: Service Offers employed by Vodafone

Service	Red Classic	Double Value Monthly	Supreme Value	Double Value Daily	Red Rush	Supreme Lite	Flat Rate	Red Hot
Label	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

Table 4.3: Payoff Table for MTN and Vodafone

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
x_1	124,76	56,144	61,139	68,132	117,83	75,125	98,102	125,75
x_2	126,74	62,138	63,137	72,128	115,85	89,111	98,102	122,78
x_3	129,71	78,122	68,132	69,131	95,105	83,117	92,108	111,89
x_4	112,88	65,135	67,133	63,137	102,98	78,122	97,103	111,89
x_5	118,82	72,128	128,72	74.126	108,92	85,115	100,100	119,81
x_6	48,52	137,63	122,78	131,69	156,44	144,56	150,50	157,43
x_7	124,76	94,106	79,121	88,112	72,128	101,99	114,86	137,63
x_8	94,106	74,126	73,127	74,126	98,102	86,114	95,105	97,103

The payoffs for MTN and Vodafone are shown in Table 4.4 and Table 4.5

The Table 4.4 contains the payoffs for MTN. All of its entries are non-negative and it has no pure Nash Equilibrium.

Table 4.4: Payoff Table for MTN (A)

	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}
x_1	124	56	61	68	117	75	98	125
x_2	126	62	63	72	115	89	98	122
x_3	129	78	68	69	95	83	92	111

x_4	112	65	67	63	102	78	97	111
x_5	118	72	128	74	108	85	100	119
x_6	148	137	122	131	156	144	157	150
x_7	124	94	79	88	72	101	114	137
x_8	94	74	73	74	98	86	95	97

Table 4.5 contains the payoffs for Vodafone-Ghana. All of its entries are non-negative and it has no pure Nash Equilibrium.

Table 4.5: Payoff Table for Vodafone-Ghana (B)

	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}
x_1	76	144	139	132	83	125	102	75
x_2	74	138	137	128	85	111	102	78
x_3	71	122	132	131	105	117	108	89
x_4	88	135	133	137	98	122	103	89
x_5	82	128	72	126	92	115	100	81
x_6	52	63	78	69	44	56	50	43
x_7	76	106	121	112	128	99	86	63
x_8	106	126	127	126	102	114	105	103

4.1.2 Model Formulation

For the purpose of solving the above game, we formulate the above constant-sum game as a zero-sum game by considering the payoffs of the individual players separately.First, we consider the payoff matrix table for the MTN and formulate our linear programming problem.

The following system of inequalities which defines the Polytope (P2) is generated from the MTN payoff table with MTN as the row player.

$$\max Z = max \sum_{j=m+1}^{m+n} a_{ij} y_j, i = 1, 2, \ldots, m.$$

subject to$A^i y \leq 1$which are:

$$124y_9 + 56y_{10} + 61y_{11} + 68y_{12} + 117y_{13} + 75y_{14} + 98y_{15} + 125y_{16} \leq 1$$

$$126y_9 + 62y_{10} + 63y_{11} + 72y_{12} + 115y_{13} + 89y_{14} + 98y_{15} + 122y_{16} \leq 1 \; 129y_9 +$$

$$78y_{10} + 68y_{11} + 69y_{12} + 95y_{13} + 83y_{14} + 92y_{15} + 111y_{16} \leq 1$$

$$112y_9 + 65y_{10} + 67y_{11} + 63y_{12} + 102y_{13} + 78y_{14} + 97y_{15} + 111y_{16} \leq 1$$

$$118y_9 + 72y_{10} + 128y_{11} + 74y_{12} + 108y_{13} + 85y_{14} + 100y_{15} + 119y_{16} \leq 1$$

$$148y_9 + 137y_{10} + 122y_{11} + 131y_{13} + 156y_{14} + 144y_{15} + 150y_{16} + 157y_{17} \leq 1$$

$$124y_9 + 94_y10 + 79y_{11} + 88y_{12} + 72y_{13} + 101y_{14} + 114y_{15} + 137y_{16} \leq 1$$

$$94y_9 + 74y_{10} + 73y_{11} + 74y_{12} + 98y_{13} + 86y_{14} + 95y_{15} + 97y_{16} + \leq$$

$$y_9 + y_{10} + y_{11} + y_{12} + y_{13} + y_{14} + y_{15} + y_{16} = 1$$

$$y_9 \geq 0, y_{10} \geq 0, y_{11} \geq 0, y_{12} \geq 0, y_{13} \geq 0, y_{14} \geq 0, y_{15} \geq 0, y_{16} \geq 0$$

with strictly inequality $y_j > 0$ for pay off calculation.

In the second case, we consider the payoff matrix of Vodafone and formulate the linear programming problem.The following system of inequalities which defines the Polytope (P1) is generated from the Vodafone payoff table with Vodafone as the column player.

$$\max W = max \sum_{i=1}^{m} x_i^T b_{ij}, j = m+1, m+2, \ldots, m+n.$$

subject to $x^T B_{ij} \leq 1$ which are:

$$76x_1 + 144x_2 + 139x_3 + 132x_4 + 83x_5 + 125x_6 + 102x_7 + 75x_8 \leq 1$$

$$74x_1 + 138x_2 + 137x_3 + 128x_4 + 85x_5 + 111x_6 + 102x_7 + 78x_8 \leq 1$$

$$71x_1 + 122x_2 + 132x_3 + 131x_4 + 105x_5 + 117x_6 + 108x_7 + 89x_8 \leq 1$$

$$88x_1 + 135x_2 + 133x_3 + 137x_4 + 98x_5 + 122x_6 + 103x_7 + 89x_8 \leq 1$$

$$82x_1 + 128x_2 + 72x_3 + 126x_4 + 92x_5 + 115x_6 + 100x_7 81x_8 \leq 1$$

$$52x_1 + 63x_2 + 78x_3 + 69x_4 + 44x_5 + 56x_6 + 50x_7 + 43x_8 \leq 1$$

$$76x_1 + 106x_2 + 121x_3 + 112x_4 + 128x_5 + 99x_6 + 86x_7 + 63x_8 \leq 1$$

$$106x_1 + 126x_2 + 127x_3 + 126x_4 + 102x_5 + 114x_6 + 105x_7 + 103x_8 \leq 1$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 1$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, x_7 \geq 0, x_8 \geq 0$$

with strictly inequality $x_i > 0$ for pay off calculation.

4.1.3 Computational Procedure

The systems of inequalities formulated were then solved on the MATLAB platform by invoking the Lemke-Howson Algorithm function. The computer used to perform this analysis was HP (WELCOME PC) with Processor: Intel(R) Celeron(R) CPU B815 @ 3.2 GHz, RAM: 4.00 GB and Type: 32-bit Operating System.

The systems were run five consecutive times in order to obtain credible results. The same results were obtained in each round.The input data were from figures 4.2, Figure 4.3 and Figure 4.4.

4.1.4 Results of Data Analysis

4.1.4.1 Results for MTN

The following results were obtained as the optimal solution for MTN using the data from figure 4.4.

$$\{x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = \frac{1}{7}, x_6 = \frac{6}{7}, x_7 = 0, x_8 = 0\}$$

The value of the game is $V = Z = 122.8571$

4.1.4.2 Results for Vodafone

The results obtained as optimal solution for Vodafone-Ghana using the data from figure 4.5 are as follows:

$$\{y_1 = 0, y_2 = 0, y_3 = \frac{19}{21}, y_4 = \frac{2}{21}, y_5 = 0, y_6 = 0, y_7 = 0, y_8 = 0\}$$

The value of the game is $V = W = 77.1429$

We summarized the results of the game of the two firms in Table 4.6 and 4.7 below:

Table 4.6: Nash Equilibrium Table for MTN

	0	0	19/21	2/21	0	0	0	0	$\sum_{j=m+1}^{m+n} a_{ij} y_j$
0	124	56	61	68	117	75	98	125	61.67
0	126	62	63	72	115	89	98	122	63.86
0	129	78	68	69	95	83	92	111	68.10
0	112	65	67	63	102	78	97	111	66.62
1/7	118	72	128	74	108	85	100	119	72.19
6/7	148	137	122	131	156	144	150	157	122.86
0	124	94	79	88	72	101	114	137	79.86
0	94	74	73	74	98	86	95	97	73.10
$\sum_{i=1}^{n} a_{ij} x_i$	143.71	127.71	122.86	122.86	149.14	135.57	142.86	151.57	

Table 4.7: Nash Equilibrium Table for Vodafone

	0	0	0	0	1/7	6/7	0	0	$\sum_{j=m+1}^{m+n} b_{ij} x_j$
0	76	74	71	88	82	52	76	106	56.29
0	144	138	122	135	128	63	106	126	72.29
19/21	139	137	132	133	72	78	121	127	77.14
2/21	132	128	131	137	126	69	112	126	77.14
0	83	85	105	98	92	44	128	102	50.86
0	125	111	117	122	115	56	99	114	64.43
0	102	102	108	103	100	50	86	105	57.14
0	75	78	89	89	81	43	63	103	48.43
$\sum_{i=1}^{m} b_{ij} y_i$	138.33	136.14	131.90	133.38	77.14	77.14	120.14	126.90	

4.1.5 Discussions of Results

4.1.5.1 MTN Results

The results obtained from figure 4.4 indicate that the optimal mixed strategies for MTN are:

$$\{x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = \frac{1}{7}, x_6 = \frac{6}{7}, x_7 = 0, x_8 = 0\}$$

and the value of the game V=Z=122.8571.This means that the MTN will have to adopt his mixed strategies with the following probabilities: MTN Free Beyond One = 0.1429 and MTN Mobile Money = 0.8571. The rest which include MTN Zone, MTN Family and Friends, MTN Conference Calls, MTN Extra Time, MTN Sunday Special and MTN Pay4Me have zero probabilities. This is to ensure an expected gain of 123 customers out of the 200 mobile phone users in the study from Tamale.

The interpretation of the results is that the MTN must pursue the strategy MTN Mobile Money more as it outperformed all the other strategies, followed by his strategy MTN Free Beyond One. The services with zero probabilities do not contribute to the value of the game.

However, the optimal mixed strategies for Vodafone-Ghana in response to MTN is the dual of MTN primal problem which is:

$$\{y_1 = 0, y_2 = 0, y_3 = \frac{19}{21}, y_4 = \frac{2}{21}, y_5 = 0, y_6 = 0, y_7 = 0, y_8 = 0\}$$

and the value of the game V=W= 122.8571. This means that the Vodafone-Ghana must adopt his mixed strategies with the following probabilities: Vodafone Supreme Value =

0.9048 and Vodafone Double Value Daily = 0.0952. The rest which include Vodafone Red Classic, Vodafone Double Value Monthly, Vodafone Red Rush, Vodafone Supreme Lite, Vodafone Flat Rate and Vodafone Red Hott have zero probabilities. This ensures an expected loss of 123 customers. The Vodafone-Ghana must pursue the strategy Vodafone Supreme Value more as it overshadowed all the other strategies, followed by his strategy Vodafone Double Value Monthly.

4.1.5.2 Vodafone Results

The results from obtained from table 4.5 indicate that the optimal mixed strategies for Vodafone is:

$$\{y_1 = 0, y_2 = 0, y_3 = \frac{19}{21}, y_4 = \frac{2}{21}, y_5 = 0, y_6 = 0, y_7 = 0, y_8 = 0\}$$

and the value of the game V = W = 77.1429. This means that the Vodafone-Ghana must adopt his mixed strategies with the following probabilities: Vodafone Supreme Value = 0.9048 and Vodafone Double Value Daily = 0.0952. The rest which include Vodafone Red Classic, Vodafone Double Value Daily, Vodafone Red Rush, Vodafone Supreme Lite, Vodafone Flat Rate and Vodafone Red Hott have zero probabilities. This ensures an expected gain of 77 customers.

The interpretation of the results is that the Vodafone-Ghana must pursue the strategy Vodafone Supreme Value more as it overshadowed all the other strategies, followed by his strategy Vodafone Double Value Daily. The services with zero probabilities do not contribute to the value of the game.

In the same vain, the optimal mixed strategies for MTN in response to Vodafone is the dual of the primal of Vodafones problem which is:

$$\{x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = \frac{1}{7}, x_6 = \frac{6}{7}, x_7 = 0, x_8 = 0\}$$

and the value of the game V = Z = 77.1429. This means that the MTN will have to adopt his mixed strategies with the following probabilities: MTN Free Beyond One = 0.1429 and MTN Mobile Money = 0.8571, since the rest have zero probabilities. This is to ensure an expected loss of 77 customers out of the 200 mobile phone used in the study from Tamale.The MTN must pursue the strategy MTN Mobile Money more as it outperformed all the other strategies, followed by his strategy MTN Free Beyond One.

Chapter 5

CONCLUSION AND

RECOMMENDATIONS

5.1 Conclusion

In this work, two linear programming models were formulated to find optimal strategies at equilibrium for the two most popular telecom companies operating in Ghana in section 4.2. The optimal strategies were found by the Lemke Howson Algorithm, the solution method used. We considered eight different service offers employed by each of these two companies to customers as a way of obtaining large shares of customers in our model.

The outcome of the work indicates that at equilibrium when MTN adopts his available strategies, Vodafone will respond by playing his strategies with probabilities as

$$y_1 = 0, y_2 = 0, y_3 = 0.9048, y_4 = 0.0952, y_5 = 0, y_6 = 0, y_7 = 0, y_8 = 0$$

and the value of the game

$$V = Z = 122.8571$$

.

On the other hand, at equilibrium when Vodafone adopts his strategies, MTN will respond by playing his strategies with the probabilities as

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0.1429, x_6 = 0.8571, x_7 = 0, x_8 = 0$$

and the value of the game

$$V = W = 77.1429$$

From the model formulated it was realized that at equilibrium, MTN will gain an expected payoff of 123 and Vodafone will lose an expected payoff of 123. This means that MTN will gain 123 customers and lose 77 customers whereas Vodafone will lose 123 customers and gain 77 customers. Therefore, the MTN gains 61% and Vodafone gains 39% of the 200 respondents used for the study.

5.1.1 Recommendations

1. MTN must pursue the strategy MTN Mobile Money more as it outperformed all the other strategies followed by his strategy MTN Free Beyond One.
2. The Vodafone-Ghana must pursue the strategy Vodafone Supreme Value more as it overshadowed all the other strategies followed by his strategy Vodafone Double Value

   Daily.
3. We also suggest that telecom companies should adopt game theory in analyzing their business strategies especially in relation to their competitors in the sector. Thus, there is the need for management of organizations to incorporate game theory into their existing management techniques to enhance decision making. This will, in effect,

assist managers in choosing best strategies towards improving acquisition and retention of their customers.

4. We recommend that researchers should turn their focus on game theory as it is applicable in every spectrum of life for decision making.

5. Further research work should be done in the telecom industry to involve all the operators in the sector using game theory models to enable them identify their optimal strategies and payoffs

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KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
COLLEGE OF PHYSICAL SCIENCES

DEPARTMENT OF MATHEMATICS

RESEARCH QUESTIONAIRE

I would be grateful if you could spare some minutes of your time to respond to this questionnaire which is for purely academic purpose. I am a final year MPhil. Mathematics student undertaking a research work on the topic: "*Mathematical Modelling of Consumers' Response to Service Offers by two Mobile Phone Service Providers: A case of MTN and Vodafone in Tamale Metropolis in the Northern Region of Ghana".*
The research is aimed at *determining how consumers react to promotions normally offered by the Service Providers.*

Please provide answer by ticking appropriately.

Respondents Personal Information

1. Sex: Male [ ] Female [ ]
2. Age bracket

   a) 18 – 25 [ ] b) 26 – 30 [ ] c) 31 – 35 [ ] d) 36 and above [ ]
3. Programme of study

   a) General [ ] b) Science/Mathematics [ ]
4. Religious affiliation

   a) Christianity [ ] b) Islam [ ] c) Traditional [ ] d) Others [ ]
5. Marital status

   a) Single [ ] b) Married [ ] c) Divorced [ ] d) Others [ ]

Information on Service Offers by MTN and Vodafone

The following are service offers by MTN and Vodafone which are paired.

Please for each box check your preference:

6. MTN Zone [ ] Vodafone Red Classic [ ]	**7.** MTN Zone [ ] Vodafone Double Value Monthly [ ]
8. MTN Zone [ ] Vodafone Supreme Lit [ ]	**9.** MTN Zone [ ] Vodafone Double Value Daily [ ]
10. MTN Zone [ ] Vodafone Red Rush [ ]	**11.** MTN Zone [ ] Vodafone Supreme Value [ ]
12. MTN Zone [ ] Vodafone Flat Rate [ ]	**13.** MTN Zone [ ] Vodafone Red Hott [ ]
14. MTN Family & Friends [ ] Vodafone Red Classic [ ]	**15.** MTN Family & Friends [ ] Vodafone Double Value Monthly [ ]
16. MTN Family & Friends [ ] Vodafone Supreme Lit [ ]	**17.** MTN Family & Friends [ ] Vodafone Double Value Daily [ ]
18. MTN Family & Friends [ ] Vodafone Red Rush [ ]	**19.** MTN Family & Friends [ ] Vodafone Supreme Value [ ]
20. MTN Family & Friends [ ] Vodafone Flat Rate [ ]	**21.** MTN Family & Friends [ ] Vodafone Red Hott [ ]

22. MTN Conference Calls [ ]	23. MTN Conference Calls [ ]

Vodafone Red Classic [ ]	Vodafone Double Value Monthly [ ]
24. MTN Conference Calls [ ] Vodafone Supreme Lit [ ]	25. MTN Conference Calls [ ] Vodafone Double Value Daily [ ]
26. MTN Conference Calls [ ] Vodafone Red Rush [ ]	27. MTN Conference Calls [ ] Vodafone Supreme Value [ ]
28. MTN Conference Calls [ ] Vodafone Flat Rate [ ]	29. MTN Conference Calls [ ] Vodafone Red Hott [ ]
30. MTN Extra Time [ ] Vodafone Red Classic [ ]	31. MTN Extra Time [ ] Vodafone Double Value Monthly [ ]
32. MTN Extra Time [ ] Vodafone Supreme Lit [ ]	33. MTN Extra Time [ ] Vodafone Double Value Daily [ ]
34. MTN Extra Time [ ] Vodafone Red Rush [ ]	35. MTN Extra Time [ ] Vodafone Supreme Value [ ]
36. MTN Extra Time [ ] Vodafone Flat Rate [ ]	37. MTN Extra Time [ ] Vodafone Red Hott [ ]
38. MTN Sunday Special [ ] Vodafone Red Classic [ ]	39. MTN Sunday Special [ ] Vodafone Double Value Monthly [ ]
40. MTN Sunday Special [ ] Vodafone Supreme Lit [ ]	41. MTN Sunday Special [ ] Vodafone Double Value Daily [ ]
42. MTN Sunday Special [ ] Vodafone Red Rush [ ]	43. MTN Sunday Special [ ] Vodafone Supreme Value [ ]

44. MTN Sunday Special [ ] Vodafone Flat Rate [ ]	45. MTN Sunday Special [ ] Vodafone Red Hott [ ]
46. MTN Mobile Money [ ] Vodafone Red Classic [ ]	47. MTN Mobile Money [ ] Vodafone Double Value Monthly [ ]
48. MTN Mobile Money [ ] Vodafone Supreme Lit [ ]	49. MTN Mobile Money [ ] Vodafone Double Value Daily [ ]
50. MTN Mobile Money [ ] Vodafone Red Rush [ ]	51. MTN Mobile Money [ ] Vodafone Supreme Value [ ]
52. MTN Mobile Money [ ] Vodafone Flat Rate [ ]	53. MTN Mobile Money [ ] Vodafone Red Hott [ ]
54. MTN Free Beyond One [ ] Vodafone Red Classic [ ]	55. MTN Free Beyond One [ ] Vodafone Double Value Monthly [ ]
56. MTN Free Beyond One [ ] Vodafone Supreme Lit [ ]	57. MTN Free Beyond One [ ] Vodafone Double Value Daily [ ]
58. MTN Free Beyond One [ ] Vodafone Red Rush [ ]	59. MTN Free Beyond One [ ] Vodafone Supreme Value [ ]
60. MTN Free Beyond One [ ] Vodafone Flat Rate [ ]	61. MTN Free Beyond One [ ] Vodafone Red Hott [ ]
62. MTNPay4Me [ ] Vodafone Red Classic [ ]	63. MTNPay4Me [ ] Vodafone Double Value Monthly [ ]
64. MTNPay4Me [ ] Vodafone Supreme Lit [ ]	65. MTNPay4Me [ ] Vodafone Double Value Daily [ ]

66. MTNPay4Me [ ] Vodafone Red Rush [ ]	67. MTNPay4Me [ ] Vodafone Supreme Value [ ]
68. MTNPay4Me [ ] Vodafone Flat Rate [ ]	69. MTNPay4Me [ ] Vodafone Red Hott [ ]

Thank You

APPENDIX 2: MATLAB CODE FOR LEMKE HOWSON ALGORITHM

```
function nashEqbm = LemkeHowson(A,B)
% if nargin  2  nargin  4
%        error('This function takes between two and four arguments');
%     end

% function nashEqbm = LEMKEHOWSON(varargin)
   %
   % This function computes a sample mixed strategy Nash equilibrium in a
   % bimatrix game.  This function implements the Lemke-Howson complementary
% pivoting algorithm for solving Bimatrix Games, a variant of the Lemke     %
algorithm for linear complementarity problems (LCPs).
   %
   % Syntax
   %   nashEqbm = LEMKEHOWSON(A, B)
   %   nashEqbm = LEMKEHOWSON(A, B, k0)
   %   nashEqbm = LEMKEHOWSON(A, B, k0, maxPivots)
   %
   % Parameters
   %   A an mn payoff matrix for the row player
   %   B an mn payoff matrix for the column player
   %   k0 an initial pivot in the set {1,...,m+n}
   %      (optional default = 1)
   %   maxPivots the maximum number of pivoting steps before termination
   %      (optional default = 500000);
   %
   % Return
   %   nashEqbm a 2x1 cell array where nashEqbm{1} and nashEqbm{2} are mixed
%   strategies for the row and column player, respectively.


   % A = input('enter player1 as a matrix.......');
   % B = input('enter player2 as a matrix.......');

   % A = [1 3 0;0 0 2;2 1 1] % this is the i-th row
   % B = [2 1 0;1 3 1;0 0 3] % this is the j-th column

A = [124 56 61 68 117 75 98 125;126 62 63 72 115 89 98 122;129 78 68 69      95
   83 92 111;112 65 67 63 102 78 97 111;118 72 128 74 108 85 100 119;
      148 137 122 131 156 144 150 157;124 94 79 88 72 101 114 137;
      94 74 73 74 98 86 95 97]; % this is the i-th row
B = [76 144 139 132 83 125 102 75;74 138 137 128 85 111 102 78;71 122 132
      131 105 117 108 89;88 135 133 137 98 122 103 89;82 128 72 126 92 115
```

```
        100 81;52 63 78 69 44 56 50 43;76 106 121 112 128 99 86 63;
        106 126 127 126 102 114 105 103]; % this is the j-th column


    if any(size(A) ~= size(B))
              error('Matrices must have same dimension');
    end

    [m,n] = size(A);
    size_ = [m,n];

    if nargin  2        k0 =
varargin{3};        if k0
1  k0  m+n
          error(['Initial pivot must be in {1,...,' num2str(n+m) '}']);
end     else        k0 = 1;
    end

    if nargin == 4       maxPivots
=  varargin{4};                    if
maxPivots  1
          error('Maximum  pivots  parameter  must  be  a  positive  integer!');
end
    else
          maxPivots    =    500000;
end

    %%  Scale  payoffs  to  be  strictly  positive
minVal  =  min(  min(min(A)),  min(min(B))  );
if minVal = 0
        A = A + ones(size(A))(1-minVal);
B = B + ones(size(A))(1-minVal);     end

    %% Build Tableaus
    Tab = cell(2,1);
    Tab{1} = [B',     eye(n), ones(n,1)];
    T1=Tab{1}
    Tab{2} = [eye(m), A,      ones(m,1)];
    T2=Tab{2}

    %%   Declare   row   labels
rowLabels       =       cell(2,1);
rowLabels{1} = m+1m+n;
    rowLabels{2} = 1m;
```

```
   %% Do complementary pivoting
k = k0;     if k0 = m        player = 1;
else        player = 2;
   end

   %        Pivoting        loop
numPiv = 0;    while numPiv
maxPivots           numPiv =
numPiv+1;

      % Use correct Tableau
      LP = Tab{player};
      [m_, ~] = size(LP);

      % Find pivot row (variable exiting)
max_ = 0;        ind = -1;        for i = 1m_
         t = LP(i,k)  LP(i, m+n+1);
if t   max_                        ind = i;
max_ = t;            end        end

      if max_  0
         Tab{player} = pivot(LP, ind, k);
         T3=Tab{player}
else            break;
      end

      % swap labels, set entering variable
temp       =       rowLabels{player}(ind);
rowLabels{player}(ind) = k;            k =
temp;

      % If the entering variable is the same
% as the starting pivot, break        if k ==
k0           break;
      end

         % update the tableau index
if player == 1               player = 2;
else            player = 1;        end

   end

   if numPiv == maxPivots
      error(['Maximum  pivot  steps  ('  num2str(maxPivots)  ')  reached!']);
end
```

```
    %% Extract the Nash equilibrium
    nashEqbm = cell(2,1);

    for player = 12

        x = zeros(size_(player), 1);
        rows = rowLabels{player};
        LP = Tab{player};

        for i = 1length(rows)            if player == 1 &&
rows(i) = size_(1)             x(rows(i)) = LP(i,m+n+1)
LP(i,rows(i));                 T4=x(rows(i))
            elseif player == 2 && rows(i)  size_(1);                x(rows(i)-
size_(1)) = LP(i,m+n+1)  LP(i,rows(i));
                T5=x(rows(i)-size_(1))
            end
end

        nashEqbm{player}       =       xsum(x);
format rat
        C=nashEqbm{player}
    end
end

function B = pivot(A,r,s)
% Pivots the tableau on the given row and column
        [m,~] = size(A);
    B = A;

    for  i  =  1    m
if     i     ==     r
continue;
else
            B(i,) = A(i,) - A(i,s)  A(r,s)  A(r,);
end     end
end
```