MODELING OF REVENUE MOBILIZATION OF DISTRICT ASSEMBLIES

A CASE STUDY

BY

ABRAHAM BAMFO BOAKYE (Bsc)

A Thesis submitted to the Department of Mathematics,

Kwame Nkrumah University of Science and Technology

in partial fulfillment of the requirement for the degree

of

MASTER OF SCIENCE

College of Sciences

May 2009

DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.

ABRAHAM BAMFO BOAKYE 28-05-2009
PG. 72900-04

Student Name & ID	Signature	Date

Certified by:

PROF. I. K. DONTWI 28/05/09

Supervisor(s) Name	Signature	Date

Certified by:

Dr. Edward Prempeh 28-05-09

Head of Dept. Name	Signature	Date

ABSTRACT

The Act 462 of the Parliament of the Republic of Ghana, the Local Government Act empowers the District, Municipal and Metropolitan Assemblies to mobilize revenues within their boundaries, and to receive assistance from the Central Government and Donor Agencies for development.

The bulk of revenues to the Assemblies are from the Central Government and Donor Agencies. If the gap between the internally generated revenue and the external source of funding is not bridged, the development of most districts in the country would be dictated from outside.

This work was carried out in the Afigya-Sekyere District Assembly to investigate the trends of inflow of revenue into the District and to develop mathematical models which can be used to predict the inflow of revenues into the district in the years ahead. Regression analysis and differential equations were used to model the trend of inflow of revenue, both locally and external.

The trends of Ghana's GDP, both nominal and real were also modeled. Nominal GDP gave rise to logistic growth whilst the real GDP gave rise to almost exponential growth. The data on the real GDP of Ghana from 1993 to 2004 were analyzed using a model developed by Chukwu (1993). The model developed by Chukwu (1993), is used to determine whether or not there are oscillations or fluctuations in a country's GDP or GNP using available data. Chukwu's model can be extended to investigate the behaviour of the

economy of any district in Ghana provided there are data available. It was realized that there were no oscillations or fluctuations in the trend of the real GDP of Ghana from 1993 to 2004.

Two time dependent models for the inflow of revenue into the Assembly were identified: the exponential and the logistic models. The exponential model is expected to work when there is inflation in the country or more sources of revenue are being identified. Logistic model is expected to work for a longer period of time when the economy is be coming stable and almost all the potential sources of internally generated revenues have been identified.

DEDICATION

This thesis is dedicated primarily to the Almighty God, who gave me strength and guidance to complete this work. It is also dedicated to my wife and children.

TABLE OF CONTENTS

DECLARATION		ii
ABSTRACT		iii
DEDICATION		v
TABLE OF CONTENTS		vi
LIST OF TABLES		xi
LIST OF FIGURES		xii
ACKNNOWLEGWMENTS		xiv
ACRONYMS		xv
	CHAPTER 1	1
	INTRODUCTION	1
1.1	Local Government in Ghana	1
1.2	The Local Government Act, 1993	1
1.3	Statement of the Problem	2
1.4	Rationale of Study	3
1.5	Objectives of Study	3
1.6	The Ghanaian Local Government Structure	4
1.6.1	The Regional Coordinating Council	6
1.6.2	The District Assembly	7
1.6.3	Sub-District Political/ Administrative Structures	8
1.6.4	Sub-Metropolitan District Council	8
1.6.5	Urban Councils	9

1.6.6 Zonal Councils 10

1.6.7 Town! Area council 10

1.6.8 Unit Committees 11

1.6.9 Functions of the Assemblies 11

1.7 Local Government Revenue 12

1.7.1 Importance of Local Government Revenue in Service Delivery 12

1.7.2 Sources of Local Government Revenue 13

1.7.3 Locally Generated Revenue 13

1.7.4 Rates 13

1.7.5 Fees 14

1.7.6 Licenses 14

1.7.7 Trading Services 14

1.7.8 Importance of Locally Generated Revenue 15

1.7.9 Revenue from External Sources 15

1.7.10 Central Government Transfers 15

1.7.11 Grants-in-aid 16

1.7.12 Recurrent Expenditure Transfers 16

1.7.13 Ceded Revenue 16

1.7.14 District Assemblies Common Fund (DACF) 17

1.7.15 Challenges of DACF 18

1.7.16 Specialized Transfers 19

1.7.17 Special Payments 19

CHAPTER 2 20

REVIEW OF RELATED LITERATURE 20
2.1 Introduction 20
2.2 Mathematical Models and Modeling 20
2.3 Stages in Modeling Problems 21
2.4 Components of Mathematical Models 23
2.5 Types of Models 24
2.5.1 Graphical Models 24
2.5.2 Algebraic Models 24
2.6 Regression-Based Models 25
2.6.1 Linear Trend 26
2.6.2 Exponential Trend 26
2.6.3 Logistic Trend 27
2.6.4 Gompertz Trend 27
2.6.5 Autoregressive Models 28
2.7 Application of Regression Models 29
2.8 Limitations of Regression Models 31
2.9 Some Economic Growth Models 32
2.9.1 Domar Economic Growth Model 32
2.9.2 Solow Growth Model 33
2.9.3 The Simple Keynesian Model 34
2.9.4 Chukwu's Model 35
CHAPTER 3 36
APPLICATION OF MATHEMATICAL MODELS TO ECONOMICS 36

3.1	A Simplified National Economy	36
3.2	Continuous-Time Model of the Simple National Economy	37
3.3	National Economic Model Including Government Expenditure	39
3.4	The Four-Sector Economy Model	43
	CHAPTER 4	50
	DATA ANALYSIS AND DISCUSSION	50
4.1	Introduction	50
4.2	Data Collection	50
4.3	Locally Generated Revenue (LGR)	50
4.4	District Assembly Common Fund (DACF)	53
4.5	Total Grants	55
4.6	Total Revenue	57
4.7	The trend of Ghana's Gross Domestic Product from 1993-2006	60
4.7.1	Investigating of the Trend of Ghana's Real GDP Using Chukwu's Model	60
4.8	Discussion	65
4.8.1	Current Economic Situation in Ghana and Afigya-Sekyere District	69
4.8.2	The Way Forward	71
	CHAPTER 5	72
	CONCLUSIONS AND RECOMMENDATIONS	72
5.1	Conclusions	72
5.2	Recommendations	73
	REFERENCES	65
	APPENDICES	67

APPENDIX A 67

A.1 Graphs of Models of Revenue Mobilization in Afigya-Sekyre District and GDP of Ghana 67

APPENDIX B 75

B.1 Modeling of Ghana's Real GDP 75

LIST OF TABLES

Table 2.1 – Sub-Metropolitan District Councils under Respective Metropolitan Assemblies 9

Table 4.1 – Locally Generated Revenue in Ghana Cedis 51

Table 4.2 – Actual and Predicted LGRs, Exponential Model 52

Table 4.3 – Actual and Predicted LGRs, Quadratic Model 52

Table 4.4 – Actual and Predicted LGRs, Logistic Model 53

Table 4.5 – Actual DACF from 2002 to 2006 54

Table 4.6 – Actual and Predicted DACF from 2002 to 2006 55

Table 4.7 – Actual Total Grants from 1999 to 2005 55

Table 4.8 – Actual and Predicted Total Grants, Logistic Model 56

Table 4.9 – Actual and Predicted Total Grants, Exponential Model 57

Table 4.10 – Total Revenue from 1999 to 2005 58

Table 4.11 – Actual and Predicted Total Revenues, Logistic Model 59

Table 4.12 – Actual and Predicted Total Revenues, Exponential Model 59

Table 4.13 – Table of Values of $Y_t - Y_{t-1}$, $Y_{t-h} - Y_{t-(h+1)}$, Y_t and Y_{t-h} 53

Table B.1 – Ghana's GDP from 1993-2006 84

Table B.2 – Actual and Predicted Values of Nominal and Real GDPs of Ghana in Million Ghana Cedis from 1993 to 2004 86

LIST OF FIGURES

Figure 1.1 – Local Government Structure (ORGONOGRAM) 5

Figure 2.1 – Stages in Modeling 22

Figure 3.1 – A Simple Model of National Economy 36

Figure A.1 – ACTUAL LGR IN GHANA CEDIS 76

Figure A.2 – PREDICTED LGR IN GHANA CEDIS, EXPONENTIAL MODEL 76

Figure A.3 – ACTUAL LGR IN GHANA CEDIS, QUADRATIC MODEL 77

Figure A.4 – ACTUAL LGR IN GHANA CEDIS, LOGISTIC MODEL 77

Figure A.5 – ACTUAL DACF IN GHANA CEDIS 78

Figure A.6 – PREDICTED IN GHANA CEDIS 78

Figure A.7 – ACTUAL TOTAL GRANTS IN GHANA CEDIS 79

Figure A.8 – PREDICTED TOTAL GRANTS IN GHANA CEDIS, LOGISTIC MODEL 79

Figure A.9 – PREDICTED TOTAL GRANTS, IN GHANA CEDIS, EXPONENTIAL MODEL 80

Figure A.10 – ACTUAL TOTAL REVENUE IN GHANA CEDIS 80

Figure A.11 – PREDICTED TOTAL REVENUE IN GHANA CEDIS, LOGISTIC MODEL 81

Figure A.12 – PREDICTED TOTAL REVENUE IN GHANA CEDIS, EXPONENTIAL MODEL 81

Figure A.13 – GHANA'S REAL GDP FROM 1993 TO 2004 82

Figure A.14 – PREDICTED REAL GDP FROM 1993 TO 2004 82

Figure A.15 – ACTUAL NOMINAL GDP FROM 1993 TO 2004 83

Figure A.16 – PREDICTED NOMINAL GDP FROM 1993 TO 2004 83

ACKNOWLEDGEMENTS

Many people have contributed to the outcome of this thesis, both directly and indirectly, but a few stand in my mind for mention. First and foremost, I am extremely grateful to my Supervisor, Prof. I. K. Dontwi of the Department of Mathematics, KNUST, Kumasi for his patience, tolerance and devotion towards the success of this thesis. I am also grateful to my brother, Prof. J. K. Osei of the College of Agriculture and Consumer Services, University of Ghana, Legon, Accra for his support and encouragement.

I am equally thankful to Mr. C. A. Boadi of SIMMS Senior High School Fawoade, Kumasi, Mr Akwasi Oppong of Prempeh Senior High School, Kumasi and Mr. Owusu Agyemang Paul of UEW, Kumasi for typing and organizing the manuscript. Above all, I am grateful to the Almighty God for His guidance and wisdom given me in carrying out this work.

ABRAHAM BAMFO BOAKYE

ACRONYMS

DACF	District Assemblies Common Fund
GDP	Gross Domestic Product
GNP	Gross National Product
LGR	Locally Generated Revenue
NALAG	The National Association of Local Authorities of Ghana

CHAPTER 1

INTRODUCTION

1.1 Local Government in Ghana

Local Government in Ghana has gone through various forms since the introduction of native authorities by the colonial government in 1878. In 1988, the assembly system of Local Government was introduced, making the assembly the highest political, executing and administrative authority at the local level and giving the regions, coordinating, supervising and monitoring responsibilities (Institutes of Local Government Studies, 2007).

1.2 The Local Government Act, 1993

The Local Government Act, 1993 is an act to establish and regulate the local government system in accordance with the constitution and to provide for other connected purposes. Part I section 1, subsection 2 stipulates that "the President may by executive instrument:

a) Declare any area within Ghana to be a district;

b) Assign a name to the district."

According to Part I section 1, sub-section 3 "the President shall in the exercise of his powers under sub-section 2 (a) direct the Electoral Commission to make such recommendations as it considers appropriate for the purpose." Part I section 1 sub-section 4 takes into account, factors to be considered by the Electoral Commission before making recommendations under sub-section 3.

For the purpose of this work, Part I sub-section 4 (b) which mentions the geographical contiguity and economic viability of the area is considered. Part I subsection 5 explains "economic viability" as "the ability of an area to provide the basic infrastructure and other developmental needs from the monetary and other resources generated in the area".

District, Municipal, and Metropolitan Assemblies in Ghana are therefore responsible for mobilizing or generating funds within their boundaries to develop their districts and to improve the living standards of the people.

1.3 Statement of the Problem

District Assemblies which are responsible for accelerated development of the rural areas for economic take-off in Ghana have not lived up to expectation in most rural areas. This is because the income levels of the people in our rural communities are still generally low. Most districts in Ghana are characterized by lack of basic infrastructures such as good roads, good school buildings and equipment, modern water and sewerage system, hospitals and clinics, and many others to mention a few. These problems have come to stay because of the inability of the district assemblies to mobilize enough resources for such infrastructural development.

According to the New Local Government System (1996), District Assemblies have two main sources of revenue for development. They are traditional and external sources. Traditional here implies Locally Generated Revenue (LGR) which is derived from six main sources. These are: rate, lands, fees and fines, licenses, trading services and miscellaneous sources. The external sources of funding are from Central Government transfers which are District Assemblies Common Fund (DACF), ceded revenue, recurrent expenditure transfers and specialized transfers. It has been pointed out in the Medium Term Development Plan Document for Afigya-Sekyere District (1996 – 2000), that the external assistance forms the bulk of the district's revenue. The Medium Term Development Plan Document (2002 – 2004), for the same District has also confirmed this. According to both documents, if nothing is done to bring a balance between own sources of revenue and external assistance, in future, the District development policies would be dictated from outside. The total inflow of Locally Generated Revenue into Afigya-Sekyere District from 1999 to 2005 was about 430,000 Ghana cedis as compared with the total inflow of revenue from external sources which was about 2,100,000 Ghana cedis. This calls to question the districts effectiveness and efficiency in local revenue mobilization.

1.4 Rationale of the Study

The trends of inflow of Locally Generated Revenue for development in most districts in Ghana have not been encouraging. This calls for intervention. The study seeks to unearth the factors militating against the inflow of more revenues into most districts, and to use mathematical models to identify solutions.

1.5 Objectives of the Study

The main objectives of this study are:

1) To develop mathematical models using differential equation for improved revenue generation in Afigya-Sekyere District.
2) Use the models to predict the future trend of inflow of revenue into Afigya-Sekyere District and if possible, use it to predict the trend of inflow of revenue into other districts.
3) Use the model to determine the wealth of Afigya-Sekyere District and other districts that might want to do so. That is, have generic model applicable to all District Assemblies.
4) To determine whether or not there are oscillations or fluctuations in the trend of Ghana's real GDP growth.

1.6 The Ghanaian Local Government Structure

According to the New Local Government System (1996), and the Institute of Local Government Studies (2007), the Ghanaian Local Government System under Act 462 is made up of the Regional Coordinating Council, and a four-tier Metropolitan and three-tier Municipal/District Assemblies Urban/Town/Area/ZonalCouncils and Unit Committees. These units are shown in Figure 1.1 on page 5.

ORGONOGRAM

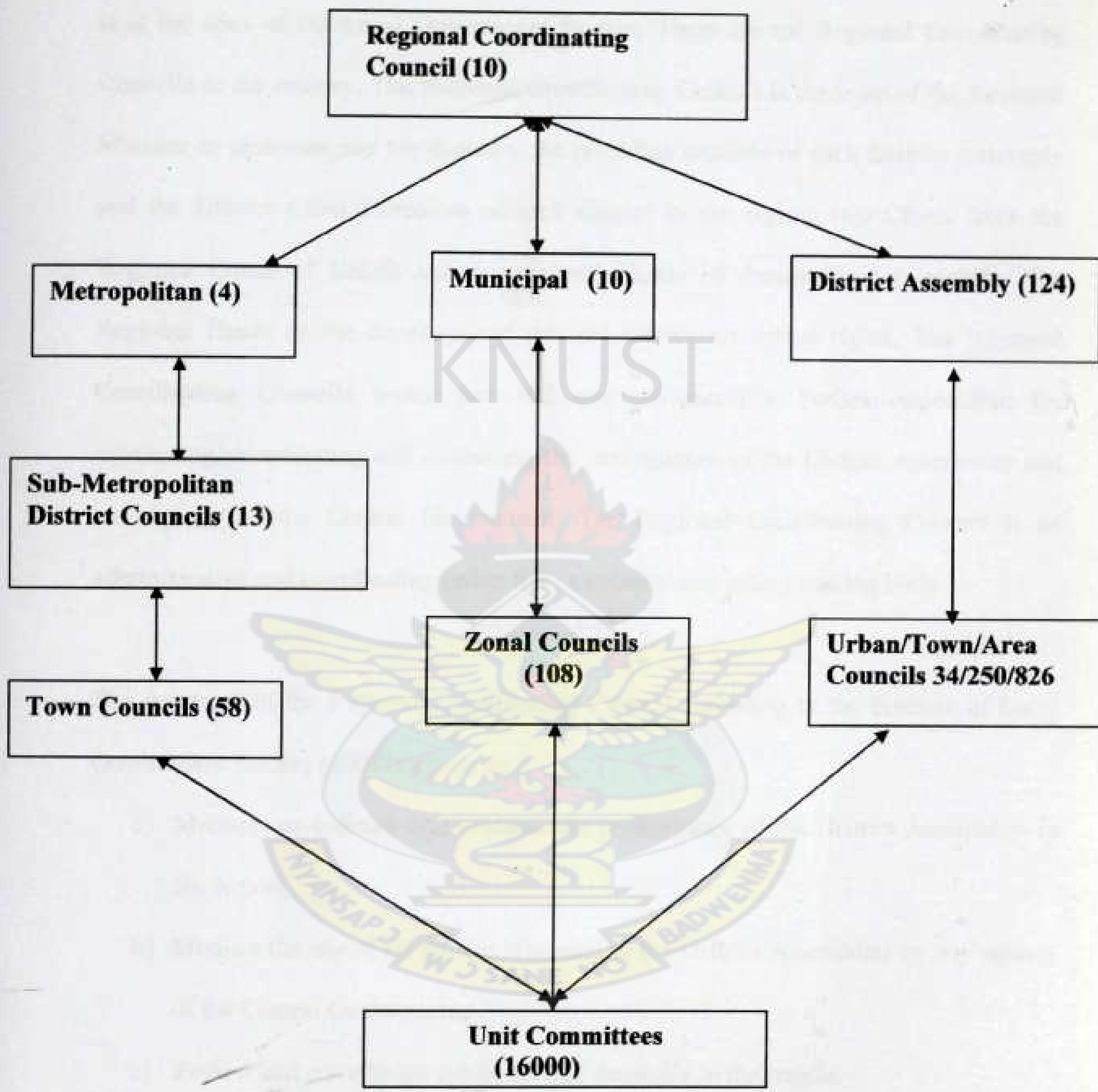


Figure 1.1 Local Government Structure

Source: Institute of Local Government Studies (2007).

1.6.1 The Regional Coordinating Council

To the Institute of Local Government Studies (2007), the Regional Coordinating Council is at the apex of the Local Government System. There are ten Regional Coordinating Councils in the country. The Regional Coordinating Council is made up of the Regional Minister as chairman and his deputies, the presiding member of each District Assembly and the District Chief Executive of each district in the region, two Chiefs from the Regional House of Chiefs and the regional Heads of decentralized ministries. The Regional Heads of the decentralized ministries have no voting rights. The Regional Coordinating Councils under Act 462 are non-executive bodies responsible for monitoring, coordinating and evaluating the performance of the District Assemblies and any agency of the Central Government. The Regional Coordinating Council is an administrative and coordinating rather than a political and policy making body.

The functions of the Regional Coordinating Council according to the Institute of Local Government Studies (2007) include to:

a) Monitor, co-ordinate and evaluate the performance of the District Assemblies in the region;
b) Monitor the use of all monies allocated to the District Assemblies by any agency of the Central Government;
c) Review and co-ordinate public service generally in the region;
d) Resolve any conflict between a District Assembly and any agency of the Central Government, public corporation, statutory body, non-governmental organization or individuals;

e) Provide security functions including managing conflicts in the region arising from chieftaincy, tribal and land disputes as well as religious factors;

f) Provide District Assemblies with information and data as is necessary to assist them in the formulation of district development plans;

g) Co-ordinate the plans and programmes of District Assemblies and harmonize them with national development policies and priorities;

h) Act on behalf of the Commission with respect to such national programmes and projects in the region as the Commission may direct;

i) Co-ordinate and monitor the programmes of all Departments under the Regional Coordinating Council and keeping it informed of all development;

j) Prepare annual report of the work of the Regional Coordinating Council within 3 months after the end of the financial year to the President and the Minister; and

k) Perform such other functions as may be assigned to it by or under any enactment.

1.6.2 The District Assembly

District Assemblies in Ghana are either Metropolitan, Municipal or District. This demarcation is based on population size. A Metropolitan Assembly is a Local Government unit whose population is above 250,000. A Municipal Assembly is a Local Government unit whose population is above 95,000 and a District Assembly has a population of between 75,000 and 95,000. Presently there are 4 Metropolitan Assemblies, 16 Municipal Assemblies and 124 District Assemblies.

The District Assembly consists of the District Chief Executive, who is the chief representative of central government in the district, elected and appointed Assembly members and the members of parliament representing constituencies in the District. Not more than 30% of the total membership of the Assembly is appointed by the president in consultation with traditional authorities and interest groups in the District. The rest are elected to represent electoral areas within the District. Member(s) of parliament, MP(s) are non-voting members.

1.6.3 Sub-District Political / Administrative Structures

The Sub-District Political/Administrative Structures are immediately below the Metropolitan, Municipal and District Assemblies. They perform functions assigned to them by the Instruments setting up the Assemblies. They are constituted by the Sub-Metropolitan District councils, urban /Town/Zonal/Area Councils, and Unit Committees (The New Local Government system, 1996).

1.6.4 Sub-Metropolitan District Council

These structures are immediately below the Metropolitan Assemblies. Between 1993 and 1996, 13 of these structures were established by law. Presently there are 29 of these structures established by law. These are shown on page 9

Table 2.1 Sub-Metropolitan District Councils under Respective Metropolitan Assemblies

Accra Metropolitan Assembly	Kumasi Metropolitan Assembly	Shama Ahanta East Metropolitan Assembly
1. La	1. Subin	1. Takoradi
2. Nungua		
3. Teshie	2. Bantama	2. Effia-Kuma
4. Osu-Klottey	3. Nhyiaeso	3.Secondi
5. Abosey Okai	4. Kwadaso	4. Essikadu-Ketan
6. South Ablekuma	5. Manshyia	5. Shama
7. North Ablekuma	6. Suame	
8. East Ayawaso	7. Old Tafo	
9. Central Ayawaso		
10. West Ayawaso	8. Asawase	
11. North Wugon		
12. North Okai Kwei	9. Oforikrom	
13. South Okai Kwei		
14. Ashiedu Keteke	10. Asokwa	

Source: NALAG Diary Desk, 2007

1.6.5 Urban Councils

To the New Local Government System (1996) and the Institute of Local Government Studies (2007), Urban Councils are created for settlements with population of 15,000.

Urban Councils consist of not less than 25 and not more than 30 members. Membership is made up of not more than 8 persons elected from among members of the relevant District Assemblies, not more than 12 representatives from Unit Committees in the area of authority of the urban Council and not more than 10 persons ordinarily resident in the urban area. 34 of such councils are created / established by law.

1.6.6 Zonal Councils

The Zonal Councils are in the "one town" Municipal Assemblies of Cape Coast, New Juaben, Tema, Techiman, Obuasi, Sunyani, Bolgatanga, Wa, Ho, and Bawku, for which the establishment of Town/Area councils will raise problems paralleling administrative structures. There are 108 of such councils. These are made up of not less than 15 and not more than 20 members. Membership is made up of not more than 5 persons elected from among the members of the relevant Municipal Assembly, not more than 10 representatives from Unit Committees and not more than 5 persons ordinarily resident in the zone.

1.6.7 Town/Area Councils

Town /Area Councils are created for settlement with population of more than 5000 but less than 15,000. Area Councils on the other hand are created for groups of villages and smaller towns which are geographically closest with population less or more than 5000. They are the rallying points in the development agenda of the assemblies. Their membership is made up of not more than 5 persons elected from among the members of the relevant Municipal Assembly not more than 10 representatives from the Unit Committees and not more than 5 persons ordinarily resident in the town area.

1.6.8 Unit Committees

To the New Local Government System (1996) and NALAG daily desk (2007), unit committees form the base structures of the New Local Government System. A unit is normally a settler or a group settlement with a population between 500-1,000 in the rural areas, and a higher population, 1,500 for the urban areas. Unit Committees being in close touch with the people, have the important roles of education, organization of communal labour, revenue raising and ensuring environmental cleanliness, registration of births and deaths, implementation and monitoring of self-help projects.

1.6.9 Functions of the Assemblies

These are deliberative, legislative and execution. Section 10 of the Local Government Act, 1993 Act 462 lists them as follows:

a) Be responsible for the overall development of the district and shall ensure the preparation and submission to the government for approval of the development plan and budget for the district;

b) Formulate programs and strategies for effective mobilization and utilization of human, physical, financial and other resources in the district;

c) Promote and support productive activity and social development in the district and remove any obstacles to initiative and development;

d) Initiate programs for the development of basic infrastructure and provide municipal works and services in the district;

e) Be responsible for the development, improvement and management of human settlements and the environment in the district;

f) In co-operation with the appropriate national and local security agencies, be responsible for the maintenance of security and public safety in the district;

g) Ensure ready access to the courts and public tribunals in the district for the promotion of justice;

h) Initiate, sponsor or carry out such studies as may be necessary for the discharge of any of the functions conferred by this law or any other enactment, and

i) Perform such other functions as may be referred to it by the government.

1.7 Local Government Revenue.

According to the Institute of Local Government Studies (2007), Local government revenue is income collected and received by a District, Municipal or Metropolitan Assembly.

1.7.1 Importance of Revenue In service Delivery

To the Institute of Local Government Studies (2007), the operations and functions of the Assemblies depend on the availability of revenue. Plans, sectoral activities and development activities in the district are financed from the revenue collected from different sources. The revenue is used for the following among others:

a) Finance development projects;

b) Pay allowances and other expenses for employees of the assembly;

c) Pay Assembly members' sitting allowances, ex-gratia awards and emoluments which are recurrent in nature, and

d) Finance maintenance costs.

1.7.2 Sources of Local Government Revenues

Revenue flows into every District, Municipal, or Metropolitan Assembly coffers through traditional and external sources.

1.7.3 Locally Generated Revenues (LGRs)

The New Local Government System (1996), considers LGRs (otherwise referred to as Traditional Local Government Sources of Revenue) as revenues that are derived from 6 main sources. These are: Rates, Lands, Fees, Licenses and Trading Services and Miscellaneous sources.

1.7.4 Rates

Included under rates are:

a) Basic Rate or Poll Tax which is levied on every adult between 18 and 70 years of age.

b) Property Rate which is levied on landed properties in the area of authority of the Assembly. It may also be levied on specified possessions of residents in the district.

c) Special Rates which are imposed by the assembly through fee-fixing resolutions to raise funds for specific projects. Such rates are project specific.

d) Rates payable by public corporations and organizations owing property in the area of jurisdiction of the District Assembly.

1.7.5 Fees

Section 34 of the Local Government Act (1993), gives the power to every District Assembly to charge fees for any service or facility provided by the Assembly or any license or permit issued by or on behalf of the Assembly. This can only be done within the guidelines set by the Minister for Local Government and Rural development. The sixth schedule of Act 462 lists some of the areas as Cattle Ponds, Slaughter Houses, Market Dues, Market Stalls/Stores, Lorry Park Dues, Chop Bars, and Corn Mills etc. Under Legislative Instrument 1530 of 1992, all crops other than cocoa, coffee and cotton are also levailable.

1.7.6 Licenses

The sixth schedule to Act 462 lists some of the items for which licenses are issued. These are Dog Licenses, Hawkers, Extension of Hours, Hotels and Restaurants, Beer and Wine sellers, Petroleum Installations, Palm Wine sellers, Self-employed Artisans, Fishing Tolls, etc.

1.7.7 Trading Services

District Assemblies may undertake trading activities from which revenue is earned.

1.7.8 Importance of Locally Generated Revenues

To the Institute of Local Government Studies (2007), revenue collection from internal source is important because:

a) It reduces pressure on Central Government and reliance on donations.

b) It sustains service delivery and autonomy of local Governments.

c) It is used for regulating businesses established in Assemblies.

1.7.9 Revenue from External Sources

The external sources of funding are from the Central Government directly or from external agencies of government which are directly transferred to the district or channeled through the Central Government and assistance received from donor agencies like Non-Governmental Organizations, World Bank, European Union and many others to mention a few.

1.7.10 Central Government Transfers

To the New Local Government System (1996), Central Government Transfers are revenues which are transferred from Central Government sources to the District Assemblies. The major ones are:

a) Grants-in-aid

b) Recurrent Expenditure Transfers

c) Ceded Revenue

d) District Assemblies Common Fund

e) Specialized Transfers

f) Special Payments.

1.7.11 Grants-In-Aid

Grants-in-aid, according to the Local Government System (1996), were administered by the Ministry of Finance and Economic Planning. Grants-in-aid were supposed to be project specific. Since the introduction of the present system of Local Government, Grants-in-aid to the District Assemblies have ceased even though it still remains on the statute books.

1.7.12 Recurrent Expenditure Transfers

Central Government has, since 1995, assumed full responsibilities for salaries and other remunerations, including pensions of the District Assemblies provided these fall within the approved manpower ceilings for the Assemblies. The Government also bears responsibility for the operational and administrative expenses of the Assemblies, referred to as "items 2-5". It is planned that once recurrent expenditure is fully decentralized, these payment will enable the Assemblies to rationalize their manpower and adopt other cost cutting measures in order to save money for development.

1.7.13 Ceded Revenue

Ceded revenue is derived from revenue sources listed in the sixth schedule of Act 462. Some of which are:

a) Entertainment Duty under Entertainment Duty Act ,1962 (Act 150)

b) Casino Revenue under the Casino Revenue Tax Decree ,1973 (NRCD 200)

c) Betting Tax under the Betting Tax Act 1965, (Act 268)

d) Gambling Tax under the Gambling Machines Decree, 1973 (NRC 1974)

e) Income Tax (Registration of Trade, Business, Profession or Vocation) Law, 1986 (PNDCL 156)

f) Income Tax payable by specified categories of self-employed persons in the informal sector

g) Daily Transport Tax under the Income Tax (Amendment) Law, 1987 (PNDCL 177)

h) Advertisement Tax under the Advertisement Tax Decree, 1976 (SMCD 50)

To the New Local Government System (1996), the ceded revenue is centrally collected by the Internal Revenue Service and the total ceded revenue collected for a year is transferred to the Ministry of Local Government which shares it among the District Assemblies using a Formula approved annually by the cabinet.

1.7.14 District Assemblies Common Fund (DACF)

The 1992 Constitution of the republic of Ghana provides for the establishment of a District Assemblies' Common Fund, which shall be:

a) Allowed annually by Parliament not less than 5 percent of the total revenues of Ghana and payable in quarterly installments for development;

b) Distributed among District Assemblies on the basis of a formula to be approved by Parliament; and

c) Administered by a District Assemblies' Common Fund Administrator.

The object of the DACF is to make available to the District Assemblies additional resources for development. Currently, the DACF is the main source of funding for Assemblies resource management.

1.7.15 Challenges of DACF

To the New Local Government System (2007), DACF like any other institution faces a number of challenges. Notable among them is the delays associated with its disbursement. This problem could be attributed to the system used in transferring the fund. The fund has to pass through a number of stages before it finally gets to the Assemblies. There is also the problem of delays in submitting reports or returns on the use of the fund by the District Assemblies. This in effect delays the Administrator's quarterly reports on the fund. Another challenge in administering the fund is the over reliance of assemblies on the DACF for every activity of their districts.

To the Institute of Local Government Studies (2007), again, the DACF was intended to supplement the efforts of District Assemblies in revenue generation. The common fund is intended exclusively for developmental projects. However, some assemblies have resorted to applying portions of the fund in recurrent expenditure. This defeats the purpose for which the fund was created.

1.7.16 Specialized Transfers

Specialized transfers include stool land royalties, timber royalties and development fund.

1.7.17 Special Payments

These are payments made by agencies and companies operating in areas of jurisdiction of the Assemblies sometimes gratuitously or as a mark of good co-operate citizenship.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1 Introduction

This chapter discusses the concept of mathematical model modeling and types of models. It discusses some models used to describe the relationship between two or more variables and some trend models. Studies that have been done elsewhere on trend models and some econometric models have also been included in this chapter.

2.2 Mathematical Models and Modeling

Mathematical model is defined as a mathematical problem whose solution allows us to describe or predict the behaviour of an associated physical system as it responds to a given set of inputs. The physical system is governed by a well-defined set of physical principles that then translated into corresponding mathematical statements. These statements often take the form of equations in which the "state" of the physical system plays the role of the unknown in the problem. The mathematical model is considered to be "well formulated" if the output or response of the physical system is uniquely determined by the input for the problem (DuChanteau and Zachmann, 1989).

Ledder (2005) defined a mathematical model as "a set of formulas and /or equations based on a quantitative description of real phenomena and created in the hope that the behaviour it predicts will resemble the real behaviour on which it is based."

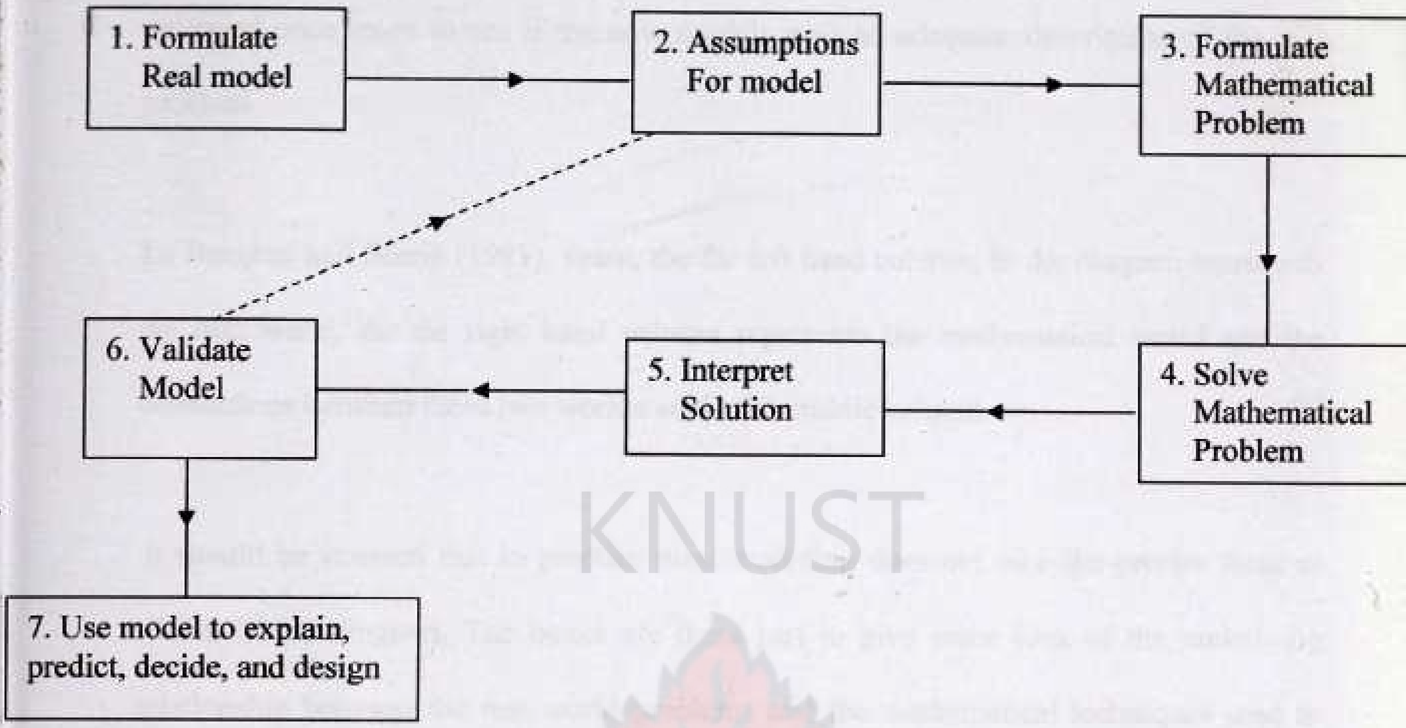
To him, a mathematical model could be as simple as a single formula relating two variables or as complicated as a set of equations describing the relationships between a set of unknowns. To him, again, the goal is not to produce a model that is "correct" but rather to elegantly describe and predict classes of observations. He further stated that "models do not represent reality–at best they represent an oversimplified characterization of reality. Typically, we do not need to use all details of a physical setting to create the model, which leaves additional features to compare with predictions made by the model. A successful mathematical model confirms the scientific understanding used to construct it and it makes predictions that can be tested to gain greater understanding. It is helpful to think of the process of mathematical description and model creation as being somewhat distinct. Conceptual model is an idealized characterization of a real-world situation. The mathematical model is a careful mathematical description of the conceptual model rather than the real situation.

Mathematical modeling is the art and science of constructing mathematical models and using them to gain insight into physical processes or to make predictions concerning physical processes. The science lies in constructing the mathematical model from the conceptual model, and the art lies in determining an appropriate conceptual model (Ledder, 2005).

2.3 Stages in Modeling Problems

The main stages in modeling, according to Burghes and Borrie (1981), are illustrated in the diagram on page 22.

Figure 2.1 Stages in Modeling



To Burghes and Borrie (1981), the problem may be to explain some observed data, or make some predictions, or take a decision. To achieve this we translate the real problem into a mathematical one by making a number of simplifying assumptions. Important variables must be identified, and the relationships between them postulated. The assumptions and relationships constitute the 'mathematical model', and generally lead to a mathematical problem of some sort, which is solved for the relevant variables using appropriate mathematical techniques. Attempts should be made to validate the model that is to check that the theoretical solution is in good agreement with the observations from the real situation. If there is good correlation, then the model can be used either to give a theoretical explanation for the observed phenomena, to predict further results, or to help in making decisions. On the other hand if the correlation between the theoretical and observed results is not adequate, we must return to the assumptions made in the model and decide which needs modification or what additions should be made. The cycle is then traversed once more to see if the new models give an adequate description of the real problem.

To Burghes and Borrie (1981), again, the far left hand column, in the diagram represents the real world, the far right hand column represents the mathematical world and the connections between these two worlds are in the middle column.

It should be stressed that in practice most modeling does not take the precise form as shown in the diagram. The boxes are there just to give some idea of the underlying relationship between the real world problems and the mathematical techniques used to find solutions to them. They further stressed that the important concepts to appreciate are the two translation stages, firstly from a real problem into a mathematical one through the model and secondly back from the mathematical solution to its interpretation in terms of the real problem.

2.4 Components of Mathematical Models

To Ledder (2005), mathematical quantities in models can be classified as variables, constants, parameters and input functions. To him, an independent variable is a quantity that takes on a range of values. Usually, independent variables are measures of time or position. The set of all possible values of the independent variables is the domain of the problem. A dependent variable is a quantity that changes during a given problem, depending on the value(s) of the independent variable(s). A constant is a quantity that has a single fixed value. A parameter is a quantity whose value is fixed throughout the domain of the model but can be varied to give a family of related problems. Parameters are there to serve the most important component of a mathematical model.

2.5 Types of Models

To Albright, Winston and Zappe (1999) "there are different types of models, and depending on the analyst's preferences and skills, each can be a valuable aid in solving a real problem". They described three types of models here: (1) graphical models, (2) algebraic models, and (3) spreadsheet models. For the purpose of this work, we discuss graphical and algebraic models.

2.5.1 Graphical Models

Albright et al. (1999) stated that "Graphical models are probably the most intuitive and least quantitative type of model. They attempt to portray graphically how different elements of a problem are related-what affects what".

2.5.2 Algebraic Models

Algebraic models are the opposite end of the spectrum. By means of algebraic equations and inequalities, they specify a set of relationships in a very precise way, and their preciseness and lack of ambiguity are very appealing to people with mathematical background. In addition, they can usually be stated concisely and with great generality (Albright et al. 1999).

A typical example of algebraic model is linear optimization model.

A linear optimization problem can be stated algebraically as follows:

$$\max \quad \sum_{j=1}^{n} p_j x_j \tag{2.1}$$

$$subject\ to \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i, \ \ 1 \le i \le m \tag{2.2}$$

$$0 \le x_j \le u_j, \ \ 1 \le j \le u \tag{2.3}$$

Here x_j is the amount of product j produced, u_j is the upper limit on the amount of product j that can be produced, p_j is the unit profit margin for product j, a_{ij} is the amount of resource i consumed by each unit of product j, b_i is the amount of resource i available, n is the number of products, and m is the number of scarce resources.

Albright et al. (1999) called this algebraic model the "product mix" problem because the model describes a company which can make several products, each of which contributes a certain amount to profit and consumes certain amount of several scarce resources.

2.6 Regression–Based Trend Models

Albright at al. (1999) stated that "many time series follow a long-term trend except for random variation. This trend can be upward or downward". To them, a straightforward way to model this trend is to estimate a regression equation for Y_t, using time t as the *single* explanatory variable. Five of such models are discussed below.

2.6.1 Linear Trend

A linear trend, according to Albright et al. (1999), means that the time series variable changes by a constant *amount* each time period. The relevant equation is

$$Y_t = a + bt + \varepsilon_t \tag{2.4}$$

where, a is the intercept, b is the slope, and ε_t is an error term the interpretation of b is that it represents the expected change in the series from one period to the next. If b is positive, the trend is upward; if b is negative, the trend is downward. The intercept term a is less important. It literally represents the expected value of the series at time $t = 0$.

2.6.2 Exponential Trend.

As observed by Albright et al. (1999), an exponential trend is appropriate when the time series changes by a constant percentage each period. The appropriate regression equation is

$$Y_t = ce^{bt}u_t \tag{2.5}$$

where c and b are constants, and u_t represents a *multiplicative* error term. By taking logarithms of both sides, and letting $a = In(c)$ and $\varepsilon_t = In(u_t)$, we obtain a linear equation that can be estimated by the usual linear regression method. We note howvever, that the response variable is now the logarithm of Y_t:

$$In(Y) = a + bt + \varepsilon_t \tag{2.6}$$

The coefficient b is approximately the percentage change per period. A more accurate percentage change is $e^b - 1$.

2.6.3 Logistic Trend.

Logistic growth model is one of the commonly used curves to analyze business and economic conditions. Logistic growth curve, which is also known as the Peal Real curve is governed by the model

$$Y_t = \frac{B}{1 + Ae^{-Bkt}} \tag{2.7}$$

where A, B, and K are positive constants. The curve rises steeply like exponential curve at first, then turns over and flattens out, approaching a horizontal asymptote. To Hoffmann, Bradley and Rosen (2005) the asymptotic line represents a saturation level for quantity represented by the logistic curve called carrying capacity of the quantity. The logistic model (2.7) cam also be expressed in the form

$$\frac{1}{Y_t} = \frac{1}{B} + \frac{A}{B}e^{-Bkt} \tag{2.8}$$

If the constant, B, is known a linear trend is obtained when $In\left(\frac{1}{Y_t} - \frac{1}{B}\right)$ is represented by Y, the response variable and t the independent variable. The logistic model is very popular in demographic studies and in many economic and business analyses.

2.6.4 Gompertz Trend

The equation for the Gompertz curve is

$$Y_t = CA^{B^t} \tag{2.9}$$

where $A, B,$ and C are constants. To Hoffmann et al. (2005) such curves are used by psychologists and others to describe such things as learning and growth within an organization.

Gompertz equation may be put in Logarithmic form

$$\log_e Y_t = \log_e C + B' \log_e A \tag{2.10}$$

A linear trend is obtained when $\log_e Y$ is represented as response variable and B' act as the independent variable.

2.6.5 Autoregressive Models

Autoregressive Model is a type of regression-based forecasting model; the independent variables are all previous values of the same time series. For example, if the series values are denoted $Y_1, Y_2, ..., Y_n$, then with a dependent a dependent variable Y_t, we might try to find an estimated regression equation relating Y_t to the most recent time series values Y_{t-1}, Y_{t-2}, and so on. With three most recent periods as independent variables, the estimated regression equation would be

$$\hat{Y}_1 = b_0 + b_1 Y_{t-1} + b_2 Y_{t-2} + b_3 Y_{t-3} \tag{2.11}$$

Regression models in which the independent variables are previous values of the time series are referred to as *autoregressive models.*

Another autoregressive model called Vector Autoregressive (VAR) model according to Kyreme (as cited in The Economy of Ghana, 2008) is a system of equations in which each in which each endogenous variable is a linear function of its past values of other endogenous variables in the system, exogenous variables that help determine the endogenous variables, and other non-random parts like constant terms or polynomial fuoctions of time. Even though VARs, over the past decade, have become popular among economists as forecasting tools, their use to test economic theories is controversial because empirical results are not necessarily stable and robust.

2.7 Application of Regression Models

Regression Models have been used and are being used to describe natural, physical and economic phenomena. Every day, managers make personal and professional decisions that are based on predictions of future events. To make these forecasts, they rely on the relationship between what is already known and what is to be estimated. If decision makers can determine how the known is related to the future event, they can aid the making process considerably.

Economic concepts and policy issues and forecasting problems are often with the relationships between two or more variables. The analysis is simplified by taking into account only two or more variables as in the case of demand and supply functions, income and expenditure functions, cost and production function and so on. These relationships are not exact functional relationships. Regression analysis attempts to measure the degree of relationships among the variables (Monga, 2000).

Hoffmann et al. (2005) have given problems about trends, some hypothetical and others factual. The factual ones are Population trend in United States of America (USA), Disposable Income and Consumption in the period of 1995-2000, Stock Market Average, Gasoline Prices, Gross Domestic Product figures for China for the period 1996-2001 and Spread of Aids. All the factual problems posed gave rise to linear trend models.

Kyeremeh (as cited in The Economy of Ghana, 2008) used vector autoregressive (VAR) models to study the effects of exchange-rate volatility & macroeconomic changes on economic growth in Ghana. The variables he used were cedi per dollar, exchange rate, Ghana's money supply, price level, per person real output, and Ghana's interest rate. His conclusions were that impulse responses suggest significantly dynamic interrelationship between the exchange rate and inflation. While inflation leads to the weakening of the cedi, a declining cedi in part may cause inflation. His co integration test suggested (a) a significant long-run relationship between the real output growth and the exchange rate; (b) a significant long-run relationship between price inflation and exchange rate; and (c) an insignificant long-run relationship between the real interest rate and exchange rate.

Quartey and Blankson (as cited in The Economy of Ghana, 2008) investigated household savings in Ghana. They used both classical linear regression technique and a logistic regression approach to compare the policy effects on savings under the two separate models. Their results were that higher levels of education (tertiary) would significantly increase the probability of savings in 1991/92 but cannot be said for 1998/9; the probability of savings increases as one attains tertiary education, but the marginal effect was not significant. Also, the probability of savings is also dependent on the type accommodation the household has; households living in rented or rent-free accommodation are likely to save more than more than those living in their own houses. Moreover, the marginal effects increased over two periods. They concluded that households living in rented accommodation are more likely to have financial savings perhaps to pay for rent advance or to put up their own houses than those living in their own hoses. Those living in their own houses may have used their savings to put up houses – a form of savings, particularly in a country where real interest rates are low and sometimes negative.

2.8 Limitations of Regression Models

The following are some of the limitations or using regression models.

1. Specific limited range- Regression relationship obtained from limited samples can be applied only to that specific range of data. For example relationship of level of consumption with salary clerks cannot be used to regress the level of consumption in relation to the salary of managers.
2. Cause and effects- In regression analysis, it is presumed that a change in one variable is cause by a change in the other variable. But regression and correlation can in no way determine cause and effect. There may be no cause and effect relation at all. For example, if we say there is a correlation between student's grade in college and their annual earnings 5 years after graduation, we are not saying that that one causes the other. Rather, both may be cause by other factors, such as, sociological background, parental attitudes, quality of teachers, effectiveness of the job-interviewing process, and economic status of parents-to name a few potential factors.
3. Using past trends to estimate future trends- We use historical data to estimate regression equations. But conditions can change and violate one or more of the assumptions on which our regression analysis depends. For example, a firm uses regression analysis to determine the relationship between the number of

employees and production volume. If the observations used in the analysis extend back for several years, the resulting regression line may be too steep because it may fail to recognize the effect of changing technology.

2.9 Some Economic Growth models

2.9.1 Domar Economic Growth Model

To Monga (2000) investment has a dual nature. It affects aggregate demand and promotes full employment. It also leads to expansion of capital stock and hence of supply output. Domar tries to determine how rapidly investment must grow to maintain full capacity output.

Let b stand for marginal propensity to consume, s marginal propensity to save and σ for the capital coefficient which represents the average as well as marginal productivity to capital.

If I, Y, S, and K stand for investment, income, savings and capital respectively, the multiplier $k = \frac{1}{1-b} = \frac{1}{s}$ and the production function is of the form $Y = \sigma K$, then we have

Increment to total demand = Increment to total supply

Since increment to total demand

$$\frac{dY}{dt} = k\frac{dI}{dt} = \left(\frac{1}{1-b}\right)\frac{dI}{dt} \tag{2.12}$$

and increment to total supply

$$\frac{dY}{dt} = \sigma \frac{dK}{dt} = \sigma I \tag{2.13}$$

$$\therefore \quad \left(\frac{1}{1-b}\right)\frac{dI}{dt} = \sigma I \text{ or } \frac{\mathrm{dI}}{\mathrm{I}} = \sigma sdt$$

solving we get

$$I = I_0 e^{\sigma st} \tag{2.14}$$

The major drawback of Domar model, according to Hardwick, Khan, and Langmead (1994) is its dependence on an inflexible production function where no substitution whatever is possible between labour and capital.

2.9.2 Solow Growth Model

Solow Growth Model, according to Monga (2000) assumes the following:

1. Capital, K, and labour, L, are combined in fixed proportions.
2. L grows exponentially, $L = L_0 e^{\lambda t}$ where λ is the growth of labour.
3. $\frac{K}{L} = V$ is the capital-labour ratio.
4. The production function is linearly homogeneous i.e.,

$$x = f(K,L) = Lf\left(\frac{K}{L}\right) = Lg(V).$$

Then, if a constant proportion of x is invested

$$\frac{dK}{dt} = sx = s.L_0 e^{\lambda t} g(V)$$

since $K = VL$ we have Solow model

$$\frac{dV}{dt} = sg(V) - \lambda V \tag{2.15}$$

2.9.3 The Simple Keynesian Model

Let C, I, G stand for consumption, investment, and government expenditure. Let the disposable income be $Y_d = Y - t$ where t is the level of net tax yield and let consumption be a linear function of disposable income:

$$C = a + bY_d = a + b(Y - t) \tag{2.16}$$

If we write the equation for the market equilibrium

$$\begin{aligned} Y &= C + I + G \\ &= \frac{a - bt + I + G}{1 - b} \end{aligned} \tag{2.17}$$

The equation (2.17) is called the Simple Keynesian model for a closed economy.

For an open economy the model becomes

$$Y = C + I + G + (X - M), \tag{2.18}$$

where $(X - M)$ is the net exports.

To Hardwick et al. (1994) the main limitations of the simple Keynesian model are that it fails to pay adequate attention to the effects of changes in wages and prices, and ignores the monetary sector of the economy.

2.9.4 Chukwu's Model

In 1993, Ethelbert N. Chukwu used the Keynesian model for an open economy to come out with the time dependent model

$$\dot{Y} - a_{-1}\dot{Y}(t-h) = -a_0 Y(t) + a_1 Y(t-h) + p(t) + q(t), \tag{2.19}$$

where Y is national income, t is time, and h is a delay in time before investments are made. $p(t)$ and $q(t)$ serve as control functions whilst $-a_{-1}, a_0$, and a_1 are parameters.

Chukwu's model has some advantages over the Keynesian model because

I. Chukwu's model has control functions but Keynesian model does not.

II. Chukwu's model is time dependent and can be used to investigate economic trends but Keynesian model does not have this property.

Chkwu's model can therefore be used to investigate the trend of growth of GDP of every nation.

CHAPTER 3

APPLICATION OF MATHEMATICAL MODELS TO ECONOMICS

3.1 A Simplified National Economy

Anthony and Biggs (1996) have considered a national economy under two simplifying assumptions:

(i) That the economy is closed, that is, there is no external trade;

(ii) There is no government that is; there are no taxes, welfare benefits and so on.

To them"in this situation we can identify four quantities which tell us something about the state of the economy":

Investment (I), production (Q), Income (Y) and consumption (C).

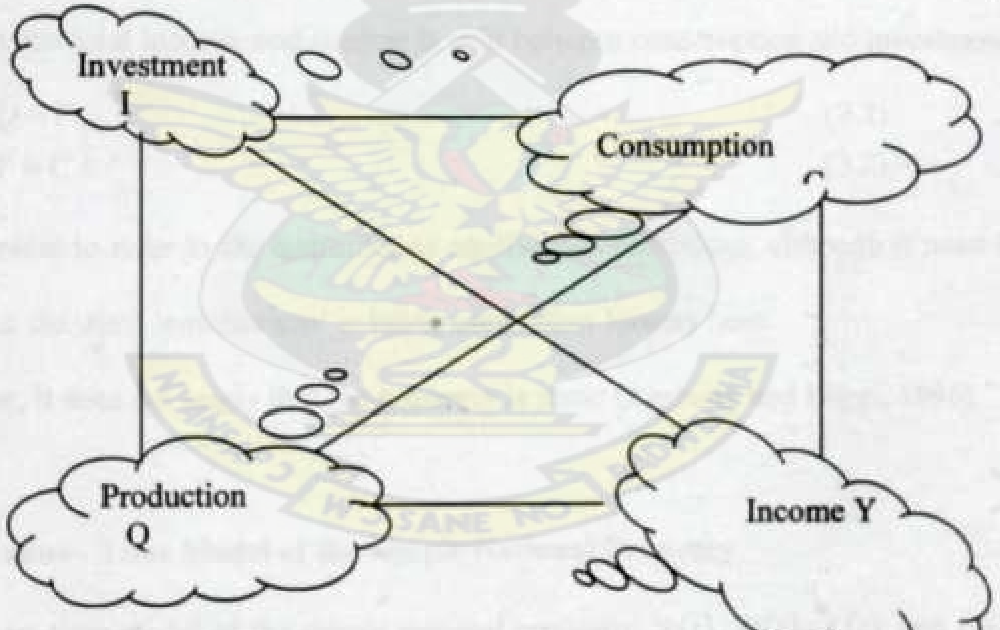


Figure 3.2 A simple model of national economy.

Anthony and Biggs (1996) further stated that "macroeconomists are concerned with the definitions of these terms and the relationships between them. We shall rely on the simple intuition that each of the four quantities affects the others, either directly or indirectly, as shown in Figure 3.1 above".

In general, it is dangerous to think between the variables as being one-way. However, there is a sense in which we can imagine an anticlockwise flow around the outer square: investment yields production which is converted into income some of which is required for consumption, leaving the rest for investment, which yields production and so on. there are simple relationships between the quantities I, Q, Y, C which express the fact that in very broad terms, the 'supply' and 'demand' are equal. Specifically production is balanced by the total income, and income is split between consumption and investment

$$Q = Y \tag{3.1}$$
$$Y = C + I \tag{3.2}$$

It is convenient to refer to the quantities as equilibrium conditions, although it must be stressed that the word 'equilibrium' is being used rather loosely here.
In particular, it does not imply that the economy is static (Anthony and Biggs, 1996)

3.2 Continuous - Time Model of the Simple National Economy

In continuous-time model of the simple national economy, $I(t), Q(t), Y(t)$ and $C(t)$ represent continuous differentiable functions of Investment, production, Income and consumption respectively.

To Anthony and Biggs (1996), suppose that the consumption $C(t)$ and income $Y(t)$ are linked by the equation.

$$C(t) = c + bY(t) \quad (b, c \text{ constants}). \tag{3.3}$$

This implies that the rate of change of C with respect to time is b times the rate of change of Y with respect to time;

$$\frac{dC}{dt} = b\frac{dY}{dt} \tag{3.4}$$

If we think of C as a function of Y, then,

$$\frac{dC}{dt} = b. \tag{3.5}$$

which explains why economists refer to the constant as the *marginal propensity to consume.*

Using the equilibrium condition $Y = C + I$ It follows that,

$$I = Y - C = Y - c - bY = -c + (1-b)Y = -c + sY,$$

where $s = 1 - b$ is equal to $\frac{dI}{dY}$ and is known as the *marginal propensity to invest*

In order to determine how the economy grows, we need more behavioral condition.

A very simple assumption is that production increases at a rate proportional to investment. That is,

$$\frac{dQ}{dt} = \rho I, \tag{3.6}$$

where ρ is a constant.

To determine the behaviour of the economy, the first step is to get an equation which involved only one of the basic functions in this case I. Using the results (3.4) and (3.5), we can argue as follows:

$$\frac{dI}{dt}=\frac{dI}{dY}\cdot\frac{dY}{dt}=\frac{sdY}{dt}=s\rho I \tag{3.7}$$

Equation (3.7) is a differential equation which involves I only and solving,

$$I(t)=Ae^{s\rho t}, \tag{3.8}$$

where A is a constant.

It is possible to make more complicated behavioural assumptions about how the economy grows, such assumptions lead to more complicated differential equations.

3.3 National Economic Model Including Government Expenditure

A simplified model for a national economy, according to Burghes and Borrie (1981), is described in terms of the following variables:

Y – *National output*
I – *Investment*
G – *Governmnt spending*
C – *consumption*

The demand, D, for goods and services is given by

$$D=C+I+G \tag{3.9}$$

It is assumed that,

$$C=(1-s)Y, \tag{3.10}$$

where s is savings coefficient.

Assume also that the output responds to excess demand so that

$$\frac{dY}{dt}=\ell(D-Y), \tag{3.11}$$

where ℓ is a constant; and investment responds according to

$$\frac{dI}{dt}=m\left(a\frac{dY}{dt}-I\right), \tag{3.12}$$

where $a\, and\, m$ are constants.

If G is taken as a constant, then from $(3.9)\, and\, (3.11)$

$$\frac{dY}{dt}=\ell(C+I+G-Y) \tag{3.13}$$

and that,

$$\frac{d^2Y}{dt^2}=\ell\left(\frac{dC}{dt}+\frac{dI}{dt}+\frac{dG}{dt}-\frac{dY}{dt}\right). \tag{3.14}$$

We note that,

$\frac{dG}{dt}=0$. Substituting $(3.10)\, \text{and}\, (3.12)$ into (3.14), we get

$$\frac{d^2Y}{dt^2}=\ell\left[(1-s)\frac{dY}{dt}+m\left(a\frac{dY}{dt}-I\right)-\frac{dY}{dt}\right] \tag{3.15}$$

$$=\ell\left(\frac{dY}{dt}-s\frac{dY}{dt}+ma\frac{dY}{dt}-mI-\frac{dY}{dt}\right)$$

$$=\ell\left[(ma-s)\frac{dY}{dt}-mI\right]$$

$$But\ I=\frac{1}{\ell}\frac{dY}{dt}-(1-s)Y-G+Y$$

$$=\frac{1}{\ell}\frac{dY}{dt}-(1-s)Y-G+Y$$

$$=\frac{1}{\ell}\left(\frac{dY}{dt}\right)+sY-G. \tag{3.16}$$

Substituting (3.16) into equation (3.15) gives

$$\frac{d^2Y}{dt^2}=\ell\left((ma-s)\frac{dY}{dt}-m(\frac{1}{\ell}\frac{dY}{dt}+sY-G)\right)$$

$$= \ell(ma-s)\frac{dY}{dt} - m\frac{dY}{dt} - mlsY + mlG$$

$$\Rightarrow \frac{d^2Y}{dt^2} - \left[\ell(ma-s)-m)\right]\frac{dY}{dt} + m\ell sY = m\ell G.$$

Let

$$\alpha = (m-\ell(ma-s), \; and \; \beta = mls,$$

then,

$$\frac{d^2Y}{dt^2} + \alpha\frac{dY}{dt} + \beta Y = m\ell G \qquad (3.17)$$

Equation (3.17) is a second order differential equation in Y. We let

$$Y = e^{Kt}.$$

The auxiliary Equation of (3.17) becomes

$$K^2 + \alpha K + \beta = 0. \qquad (3.18)$$

The solutions to the auxiliary Equation are:

$$K_1 = \frac{-\alpha + \sqrt{(\alpha^2 + 4\beta)}}{2}, \qquad (3.19a)$$

$$K_2 = \frac{-\alpha - \sqrt{(\alpha^2 - 4\beta)}}{2} \qquad (3.19b)$$

The solution of Equation (3.17) is of the form

$$p_c + p_b,$$

where p_c is the general solution of the associated homogeneous equation and p_b is a particular solution of equation (3.17) which will depend on the government expenditure, G. Now G is constant so the solution to equation (3.17) is therefore

$$Y = Ae^{K_1 t} + Be^{k_2 t} + m\ell G, \tag{3.20}$$

where A and B are constants.

For K_1 and k_2 to be real,

$$\alpha^2 - 4\beta \geq 0 \tag{3.21}$$

Assuming that A and B are positive constant, and that K_1 and K_2 are real, then

$Y \to m\ell G \text{ as } t \to \infty \quad \text{if } K_1 < 0 \text{ and } K_2 < 0$

$Y \to \infty \text{ as } t \to \infty$ if $K_1 < 0$ *and* $K_2 > 0$ or one of the Ks is positive and the other is negative. If $\alpha^2 - 4\beta < 0$ then K_1 and K_2 are complex and the solution becomes

$$Y(t) = e^{\gamma t}\left[E\cos\lambda t + F\sin\lambda t\right) + m\ell G \tag{3.22}$$

where E and F are constants,

$$\gamma = \frac{-\alpha}{2};$$

$$\lambda = \frac{\sqrt{(4\beta - \alpha^2)}}{2}$$

If $\gamma > 0, Y$ will oscillate and if $\gamma < 0, Y$ will still oscillate and will approach mlG as $t \to \infty$

3.4 The Four – Sector Economy Model.

The model below is developed by Chukwu, (1993).

Let Z denote aggregate demand, which is made up of consumption (C), investment (I), net exports (X), and government (G). These are all continuously differentiable functions which are related as follows:

$$Z = C + I + X + G. \tag{3.23}$$

We assume that:

$$C = C_0 + c(Y - T), \tag{3.24}$$

where Y is income, $C_0 > 0$ is autonomous consumption, that is does not depend on income $Y(t)$ and c, the marginal propensity to consume and $Y - T$ is the current after – tax income. Also, T is net taxes which is given by

$$T = T_0 + tY, \tag{3.25}$$

where $T_0 > 0$, is the level of non-income taxes and $0 < t < 1$.

The net export demand function

$$X = X_0 - mY - e_1 R;\ 0 < m < 1,\ e_1 > 0, \tag{3.26}$$

where m is the fraction of an increase o f income that is spent on other countries products and X_0 is autonomous exports and R is the real rate of interest. The real number e_1 is measured in dollars and can be estimated empirically. The total real government spending for goods and services and subsidies (adjusted for price level changes) is

$$G = G_0. \tag{3.27}$$

We assume investment outlays are split;

$$I(t) = I_0(t) + I_1(t),$$

where $I_0(t)$ is autonomous, that is, it does not depend on $Y(t)$, or $\dot{Y}(t)$, and $I_1(t)$ follows decision to invert B after a delay of h units of time:

$$I_1(t) = B(t-h).$$

We assume B is nonlinear and depends on future expectations of income growth:

$$B(t) = g(\dot{Y}(t-h)) .$$

That is decision to invest, B (t), is a nonlinear function of the current rate of change of income. Thus,

$$I_1(t) = B(t-h) = g(\dot{Y}(t-h)). \tag{3.28}$$

The function g is called Goodwin's investment function. It is nonlinear being linear only in its middle range and becoming completely inflexible at the lower and upper levels, we assume in the middle range

$$g(\dot{Y}(t-h) = a^{\cdot}_{-1}\dot{Y}(t-h), \tag{3.29}$$

where $a^{\cdot}_{-1}$ is the acceleration coefficient which has the dimension of time. With (3.23) and (3.28) aggregate demand reduces to

$$\begin{aligned} Z(t) &= C_0(t) + c\left(Y - T_0(t) - t_1 Y\right) + I_0(t) + g\left(\dot{Y}(t-h)\right) + X_0(t) - mY - e_1 R + G_0(t) \\ &= C_0(t) + I_0(t) + X_0(t) + G_0(t) + cY - cT_0 + ct_1 y - mY - e_1 R + g\left(\dot{Y}(t-h)\right) \\ &= \left(Z_0(t) + G_0(t) - cT_0(t)\right) + c\left((1-t_1) - m\right)Y + g\left(\dot{Y}(t-h)\right) - e_1 R \\ &= \left(Z_0(t) + G_0(t) - cT_o(t)\right) + \mu Y(t) + g\left(\dot{Y}(t-h)\right) - e_1 R, \end{aligned} \tag{3.30}$$

where

$$\mu = c(1-t_1) - m, 0 < c < 1, 0 < t_1 < 1. \qquad (3.30b)$$

and

$$Z_0 = C_0 + X_0 + I_0. \qquad (3.30c)$$

In equation (3), R is unknown. To obtain the value of R we now complete the theory of aggregate demand by providing the following four equations as a model of money demand and supply.

$$\frac{L}{P} = \frac{M}{P}. \qquad (3.31)$$

$$\frac{L}{P} = j + kY - rR, \qquad (3.32)$$

$$M = M_o, \qquad (3.33)$$

$$P = P_o. \qquad (3.34)$$

k is a fraction $(0 < k < 1)$ of income. R is the average real rate of interest while r is a real number. Here M is the nominal value of money supply which is controlled by the Central Bank, P is the price level. The real demand for money is denoted by $\left(\frac{L}{P}\right)$ and must be equal to the real value for money $\left(\frac{M}{P}\right)$ in equilibrium. The symbol j is autonomous real money demand. These are factors (other than Y or R) such as private individual expectation for the future which influence the demand for money.

Equations $(3.31) - (3.34)$ can be expressed as a single equation.

$$Y = \left(\frac{\frac{M_0}{P_0} - j}{k} \right) + \frac{rR}{k}, \tag{3.35}$$

so that

$$R = \frac{kY}{r} - \frac{\left[\left(\frac{M_o}{P_o} \right) - j \right]}{r}. \tag{3.36}$$

We use (3) in (3) to deduce that

$$Z(t) = (Z_0(t) + G_0(t) - cT_0 + \mu Y(t) + g(\dot{Y}(t-h)) + \frac{e_1}{r}\left[\left(\frac{M_o}{P_o} \right) - j - kY(t) \right]. \tag{3.37}$$

We note carefully that $Z_0 = C_0 + X_0 + I_0$ donates private spending while G_0 is government spending, T_0 non-income taxes and M_0 is nominal value of money supply are government policy instruments.

The parameters (j,k,r) and (μ, Z_0, h) can be estimated empirically.

On the supply side we denote total output or income by Y, and we postulate that

$$\frac{dY(t)}{dt} = -\lambda(Y(t) - Z(t)), \tag{3.38}$$

where λ is a constant. With equation (3.37) in (3.38) we obtain

$$\frac{1}{\lambda}\frac{dY(t)}{dt} = Z(t) - Y(t) = (Z_0(t) + G_0(t) - cT_0(t)) + \left(\mu - \frac{ke_1}{r} - 1 \right) Y(t) + g(\dot{Y}(t-h)) + \frac{e_1}{r}\left[\frac{M_0}{P_0} - j \right].$$

Thus

$$Y(t)-\lambda g(\dot{Y}(t-h))=-\lambda(1+\frac{e_1}{r}-\mu)Y(t)+\lambda\left[Z_0(t)-\frac{e_1 j}{r}\right]$$

$$+\lambda\left[G_0(t)-cT_0(t)+\frac{e_1}{r}\frac{M_0}{P_0}\right]. \qquad (3.39)$$

The government outlay can be split into public consumption G_{01} and public investment Go_2 that is into innovation

$$G_0(t)=\beta(Y(t-h))+v(t)\equiv G_{01}+G_{02}. \qquad (3.40)$$

The term $\beta(Y(t-h))$ is an upward drift of the consumption functions which is associated with the population and is dependent on the previous high income. The expression

$$\lambda\left[Go(t)-cTo(t)+\frac{e_1}{r}\frac{M_o}{P_o}\right]=\lambda\left[\beta Y(t-h)+v(t)-cTo+\frac{e_1}{r}\frac{M_0}{P_o}\right] \qquad (3.41)$$

represents government impact on the growth of $Y(t)$. Define as "solidarity" or "fraternity function" the expression,

$$\lambda\left[\frac{e_1 M_0}{rP_0}-cT_0(t)+v(t)\right]=q(t). \qquad (3.42)$$

It is the government control policy instrument.

The private sector contribution is the expression which is denoted by $p(t)$:

$$p(t)=\lambda\left[Z_0(t)-\frac{e_1}{r}j(t)\right]=\lambda\left[C_0(t)+I_0(t)+X_0(t)-\frac{e_1}{r}j(t)\right]. \qquad (3.43)$$

It is called *firm's initiative*, a function of autonomous consumption, investment, real money demand and autonomous net exports. Equation (3.4.2.17) becomes

$$\dot{Y}-\lambda g\left(\dot{Y}(t-h)\right)=-a_oY(t)+a_1Y(t-h)+p(t)+q(t), \tag{3.44}$$

where

$$a_o=\lambda\left(1+\frac{e,k}{r}-\mu\right),\ a_1=\lambda\beta.$$

Equation (3.44) is a nonlinear functional differential control system of neutral type. When g is linear and given by (3.29) equation (3.44) reduces to the linear system

$$\dot{Y}-a_{-1}\dot{Y}(t-h)=-a_0Y(t)+a_1Y(t-h)+p(t)+q(t), \tag{3.45}$$

where $a_{-1}=\dot{a}_{-1}\lambda$. The parameters a_{-1},a_0,a_1 can be estimated by known methods, such as matrix algebra. The public policy instrument, $q(t)$ as well as the private control function $p(t)$ may be taken as controls.

To Chukwu (1993), again, the system (3.44) which has a nonlinear investment function can generate limit cycles with expressions and contractions of different length. The limit cycle is the movement around an unstable, and possibly equilibrium. Thus the cycle is a natural endogenous outcome of our system: fluctuations of income Y can occur when Y satisfies (3.45). Indeed Chukwu (1993), has proved that if (3.45) is considered and we assume

(i) $p(t)+q(t)=0;$

(ii) $a_{-1},a_0,-a_1,h$ *are nonnegative real numbers such that* $a_{-1}\le 1,$

$$a_{-1}(a_0-2a_1)-a_1>\frac{1}{eh}.$$

Then every solution of (3.45) oscillates. By decreasing the value of a_1(and this can be done by decreasing β, the public consumption) the second inequality in (ii) can be destroyed. This will remove the oscillation.

To Chukwu (1993), it is unrealistic to assume that the parameters a_{-1}, a_0, a_1 will remain constant with changes in control policies of p and q. We consider the system

$$\dot{Y}(t) - g(\dot{Y}(t-h)) = L(t, Y_t, p_t)Y_t + B(t, Y_t, p_t)p_t + q(t). \tag{3.46}$$

The L and B are nonlinear functions. The symbol Y_t for $t \geq \sigma$ represents a function $Y_t : [-h, 0] \to E$ such that $Y_t(s) = Y(t+s), s \in [-h, 0]$. The function p_t is defined in a similar way.

In (3.45) $Y(t)$ is a real number, the value at time t of one nation's income. If we denote $x(t) = (Y_1(t), ..., Y_n(t))$ to the value of n nations GDP at time t and with corresponding private initiative $p = (p_1, ..., p_n)$ and solidarity function $q = (q_1, ..., q_n)$ we consider the n-dimensional system

$$\dot{x}(t) - A_{-1}\dot{x}(t-h) = A_0 x(t) + A_1 x(t-h) + p(t) + q(t), \tag{3.47}$$

which corresponds to (3.45) as a description of the growth of income of nations.

CHAPTER 4

DATA ANALYSIS AND DISCUSSION

4.1 Introduction

This chapter deals with the revenue situation of Afigya-Sekyere District Assembly. The chapter also deals with the models which give the picture of the approximate trend of inflow of revenue into the District. The trend of Ghana's Gross Domestic Product (GDP) from 1993 to 2006 has also been dealt with in this chapter.

4.2 Data Collection

Data on inflow of revenue into Afigya-Sekyere District Assembly were collected at the District Budget Office. As noted by Hoffmann et al. (2005), "a common procedure for associating a function with an observed physical phenomenon is to gather data, plot on a graph, and then find a function whose graph "best fits" the data in some meaningful way." With this in mind, the data collected from the Assembly were plotted on graphs and non-linear functional models were observed.

4.3 Locally Generated Revenue (LGR)

In table 4.1, there was a short-fall in revenue mobilization between 1999 and 2000; there was a general increase in the flow of Locally Generated Revenue throughout the period. The shape of the graph of Actual Locally Generated Revenue against time from 1999 to 2005 (Appendix A.1 Figure A.1 p. 76) may be assumed to obey three models: exponential, logistic and quadratic models. The tables on pages 51 and 52 show the actual and predicted values of the models. R = Revenue, t = time in years.

It is observed that if exponential model is assumed, (Appendix A.1 Figure A.2 p. 76) then, the equation for the model is given by

$$R = 9990.359622e^{0.3817211136t}; t \geq 1 \quad r^2 = 0.946 \tag{4.1}$$

If we assume quadratic model, (Appendix A.1 Figure A.3 p. 77) the equation for the model becomes

$$R = 5097.534881t^2 - 18391.14369t + 33978.02429; t \geq 1 \quad R_1^2 = 0.999 \tag{4.2}$$

If we try logistic model, (Appendix A.1 Figure A.4 p. 77) then, we have

$$R = \frac{1488237.912}{(1 + 189.4738065e^{-0.4430148171t})}; t \geq 1 \quad r^2 = 0.946 \tag{4.3}$$

The table below shows the trend of inflow of LGR into Afigya-Sekyere district from 1999 to 2005.

Table 4.1 Locally Generated Revenue in Ghana Cedis

YEAR	LGR IN GHANA CEDIS
1999	20588.23
2000	17522.70
2001	25569.50
2002	40037.60
2003	72021.89
2004	105173.84
2005	155635.27

Table 4.2 Actual and Predicted LGRs, Exponential Model.

YEAR	ACTUAL REVENUE	PREDICTED REVENUE	RESIDUALS
1999	20588.23	14633.91	5954.32
2000	17522.70	21435.81	-3913.11
2001	25569.50	31399.25	-5829.75
2002	40037.60	45993.73	-5956.13
2003	72021.89	67371.78	4650.11
2004	105173.84	98689.43	6484.41
2005	155635.27	144556.23	11079.04

Table 4.3 Actual and Predicted LGRs, Quadratic Model

YEAR	ACTUAL REVENUE	PREDICTED REVENUE	RESIDUALS
1999	20588.23	20684.42	-96.19
2000	17522.70	17585.88	-63.18
2001	25569.50	24682.41	887.09
2002	40037.60	41974.01	-1936.41
2003	72021.89	69460.68	2561.21
2004	105173.84	107142.42	-1968.58
2005	155635.27	155019.23	616.04

Table 4.4 Actual and Predicted LGRs, Logistic Model

YEAR	ACTUAL REVENUE	PREDICTED REVENUE	RESIDUALS
1999	20588.23	12132.97	8455.26
2000	17522.70	18810.35	-1287.65
2001	25569.50	29090.20	-3520.70
2002	40037.60	44816.66	-4779.06
2003	72021.89	68645.03	3376.86
2004	105173.84	104227.77	946.07
2005	155635.27	156225.31	-590.04

It is noted from the three tables above that quadratic model shows the best approximation. This is because the coefficient of variation in the case of quadratic model is the highest $\left(R_1^2 = 0.999\right)$ as compared with exponential and logistic models

4.4 District assemblies common fund (DACF)

Table 4.5 on page 54 shows the trend of flow of the District Assemblies common fund from year 2002 to year 2006.

Table 4.5 Actual DACF from 2002 to 2006

YEAR	DACF IN GHANA CEDIS
2002	281300
2003	421900
2004	465700
2005	528000
2006	579100

This table also shows an increasing flow of DACF.

The graph of the flow of Actual DACF from 2002 to 2006 (Appendix A.1 figure A.5 p. 78) shows a big rise of DACF from year 2002 to year 2003, but it showed almost linear relationship from year 2003 to 2006.The shape of the graph is assumed to obey logarithmic growth (Appendix A.1 figure A.6 p. 78) with the approximate equation

$$R = 284037.3561 + 178760.2497 Int; t \geq 1. \quad r^2 = 0.991 \tag{4.4}$$

Table 4.6 on page 55 depicts the actual and predicted values of DACF from 2002 to 2006.

Table 4.6 Actual and Predicted DACF from 2002 to 2006

YEAR	ACTUAL DACF	PREDICTED DACF	RESIDUALS
2002	281300	284037.36	-2737.36
2003	421900	407944.52	13955.48
2004	465700	480425.56	-14725.56
2005	528000	531851.68	-3851.68
2006	579100	571740.88	7359.12

4.5 Total Grants

The trend of inflow of total grants is as shown below.

Table 4.7 Actual Total Grants from 1999 to 2005

YEAR	TOTAL GRANTS IN GHANA CEDIS
1999	75413.79
2000	91389.07
2001	133642.80
2002	301666.12
2003	–
2004	650431.60
2005	785922.80

This trend shows a general progressive increase in the inflow of total grants from 1999 to 2005. The shape of the graph of total grants against time can be assumed to obey exponential growth or logistic growth (Appendix A.1 Figure A.7 p. 79)

If the graph is assumed to obey logistic growth, (Appendix A.1 Figure A.8 p.79) then, the equation for the growth is given by

$$R = \frac{989787.4909}{(1+47.05416538e^{-0.7462277789t})};t \geq 1 \quad r^2 = 0.974 \qquad (4.5)$$

If it is assumed to be exponential growth (Appendix A.1 Figure A.9 p.80) then, the Equation which describes the growth is given by

$$R = 44228.88742e^{0.428297101t};t \geq 1 \quad r^2 = 0.974 \qquad (4.6)$$

Table 4.8 Actual and Predicted Total Grants, Logistic Model

YEAR	ACTUAL TOTAL GRANTS	PREDICTED TOTAL GRANTS	RESIDUALS
1999	75413.79	42460.44	32953.35
2000	91389.07	85483.37	5905.70
2001	133642.80	164528.14	-30885.34
2002	301666.12	292983.41	8682.71
2003	–	465196.13	–
2004	650431.60	644942.75	5488.85
2005	785922.80	789603.67	-3680.87

Table 4.9 Actual and Predicted Total Grants, Exponential Model

YEAR	ACTUAL TOTAL GRANTS	PREDICTED TOTAL GRANTS	RESIDUALS
1999	75413.79	67875.51	7538.28
2000	91389.07	104164.60	-12775.3
2001	133642.80	159855.37	-26212.57
2002	301666.12	245320.75	56345.37
2003	–	376479.52	–
2004	650431.60	577761.26	72670.34
2005	785922.80	886656.67	-100733.87

The tables above (p.56 & 57) show the predicted and the actual total grants for the two models in Ghana cedis. The table which shows the logistic growth model seems to show a better approximation than that of the exponential growth although the have the same coefficient of determination, the logistic growth model agrees approximately with the projected revenue for the Assembly from 2006 to 2009.

4.6 Total Revenue

Table 4.10 on page 58 shows the total revenue situation of the Afigya-Sekyere district from year 1999 to year 2005. It is observed that the graph of total grant against time and that of the total revenue against time (Appendix A.1 Figure A.10 p.80) show the same general pattern of the inflow of total grants into the district.

Table 4.10 Total Revenue from 1999 to 2005

YEAR	TOTAL REVENUE IN GHANA CEDIS
1999	96020.12
2000	108911.77
2001	159212.29
2002	341703.72
2003	–
2004	755605.44
2005	941558.07

If we assume logistic growth equation for the flow of revenue, (Appendix A.1 Figure A.11 p.81) we obtain the equation.

$$R = \frac{1297803.703}{(1 + 44.26294494e^{-0.6828495111t})}; t \geq 1, r^2 = 0.975 \qquad (4.7)$$

If we try exponential growth model (Appendix A.1 Figure 12 p.81) we obtain the equation

$$R = 54455.40644e^{0.4195566997t}; t \geq 1, r^2 = 0.975 \qquad (4.8)$$

The tables on pages 58 and 59 show, the actual and the predicted total generated revenues for the district.

Table 4.11 Actual and Predicted Total Revenues, Logistic Model.

YEAR	ACTUAL TOTAL REVENUE	PREDICTED TOTAL RENENUE	RESIDUALS
1999	96020.12	55555.35	40464.77
2000	108911.77	105546.80	3364.97
2001	159212.29	193515.35	-34303.06
2002	341703.72	334247.32	7456.40
2003	–	528356.74	–
2004	755605.44	747717.10	7888.34
2005	941558.07	946161.41	-4603.34

Table 4.12 Actual and Predicted Total Revenues Exponential model

YEAR	ACTUALTOTAL RENENUE	PRDICTED TOTAL RENENUE	RESIDUALS
1999	96020.12	82843.16	13176.96
2000	108911.77	126029.52	-17117.75
2001	159212.29	191729.04	-32516.75
2002	341703.72	291677.90	50025.82
2003		443730.38	–
2004	755605.44	675048.21	80557.23
2005	941558.07	1026952.66	-85394.59

It appears that the logistic growth model seems to show a better approximation than the exponential growth model in terms of the locally generated revenue, the total grant and the total revenue flowing into the district

4.7 The Trend of Ghana's Gross Domestic Product from 1993 to 2006

The trend of Ghana's GDP from 1993 to 2006 is as shown in Appendix B 1 p. 84.

It is observed here that both the nominal and the real GDP have been growing progressively, however, between 1993 and year 2000, there were some fluctuations in terms of percentage real GDP but from year 2000 to year 2006, the GDP growth rate in terms of percentage has been increasing.

The graph of nominal GDP against time (Appendix A.1 FigureA.16 p 83) is assumed to obey logistic growth model. The equation of the model is

$$Y(t) = \frac{51005.37432}{(1+160.956495e^{-0.2845946128t})};t \geq 1 \quad Y(t) = income, t = time\ in\ years \tag{4.9}$$

The approximate model for the real GDP from the graph of GDP against time (Appendix A.1 Figure A.13 p.82) from 1993 to 2004 is

$$\begin{aligned} Y(t) = 367.0860196e^{0.042843816t} + 0.0200794045t^4 - 0.4835854529t^3 \\ + 3.9671537t^2 - 13.08610246t + 13.699504768 ;t \geq 1 \end{aligned} \tag{4.10}$$

4.7.1 Investigating the trend of Ghana's Real GDP Using Chukwu's Model

The model developed by Chukwu (1993), which describes the dynamics of the growth of wealth of nations (3. 45 p.48) is given by

$$\dot{Y} - a_{-1}\dot{Y}(t-h) = -a_0 Y(t) + a_1 Y(t-h) + p(t) + q(t)$$

Where $Y(t)$ is the GDP at time $t > 0$.

The values of $a_{-1}, a_0, and\, a_1$ can be estimated by using matrix algebra under the following assumptions:

1) That $p(t)+q(t)=0$ for all $t>0$;

2) The equation

$\dot{Y} - a_{-1}\dot{Y}(t-h) = -a_0 Y(t) + a_1 Y(t-h)$

be converted to difference equation, that is,

$$Y_t - Y_{t-1} - a_1(Y_{(t-h)} - Y_{t-(h+1)}) = -a_0 Y_{t-1} + a_1 Y_{t-(h+1)}; \qquad (4.11)$$

3) The real number, $h = \frac{108}{487}$ according to Ghana Data Profile (2007);

4) That $\dot{Y}(t-h)$ and $Y(t-h)$ be obtained by multiplying $\dot{Y}(t)$ and $Y(t)$ by $\left(1-\frac{108}{487}\right)$.

It has been pointed out by Chukwu (1993), that if

i. $p(t)+q(t)=0$ for all $t>0$;

ii. $a_{-1}, a_0\ and - a_1$ are nonnegative real numbers such that $a_{-1} \leq 1$;

iii. $a_{-1}(a_0 - 2a_1) - a_1 > \frac{1}{eh}$.

Then every solution of (3. 45 p.48) oscillates. Table 4.14 on page 62 shows the values of $Y_t - Y_{t-1}$, $Y_{t-h} - Y_{t-(h+1)}$, $Y_t\ and\ Y_{t-h}$ from 1993 to 2004.

Table 4.13 Table of Values of $Y_t - Y_{t-1}$, $Y_{t-h} - Y_{t-(h+1)}$, Y_t and Y_{t-h}

$Y_t - Y_{t-1}$	$Y_{t-h} - Y_{t-(h+1)}$	Y_t	Y_{t-h}
12.660	**9.85244**	**387.250**	**301.3711**
16.090	**12.52179**	**399.910**	**311.2236**
19.120	**14.87984**	**416.000**	**323.7454**
18.267	**14.21600**	**435.120**	**338.6252**
21.281	**16.56160**	**453.387**	**352.8412**
21.021	**16.35926**	**474.668**	**369.4028**
18.520	**14.4129**	**495.689**	**385.7621**
21.505	**16.73592**	**514.209**	**400.1750**
24.366	**18.96245**	**535.714**	**416.9109**
29.393	**22.87463**	**560.080**	**435.8733**
32.879	**22.58756**	**589.473**	**458.7480**

We let the values of $(Y_t - Y_{t-1}) = T$ be solution vector and the values of $(Y_{t-h} - Y_{t-(h+1)}, Y_t\ Y_{t-h}) = X$ be coefficient matrix. The systems of equations are as shown below:

$$9.85244a_{-1} - 387.250a_0 + 301.3711a_1 = 12.660$$
$$12.52179a_{-1} - 399.910a_0 + 311.2236a_1 = 16.090$$
$$14.87984a_{-1} - 416.000a_0 + 323.7454a_1 = 19.120$$
$$14.21600a_{-1} - 435.120a_0 + 338.6252a_1 = 18.267$$
$$16.56160a_{-1} - 453.387a_0 + 352.8412a_1 = 21.281$$
$$16.35926a_{-1} - 474.668a_0 + 369.4028a_1 = 21.021$$
$$14.41290a_{-1} - 495.689a_0 + 385.7621a_1 = 18.520$$
$$16.73592a_{-1} - 514.209a_0 + 400.1750a_1 = 21.505$$
$$18.96245a_{-1} - 535.714a_0 + 416.9109a_1 = 24.366$$
$$22.87463a_{-1} - 560.080a_0 + 435.8733a_1 = 29.393$$
$$22.58786a_{-1} - 589.473a_0 + 458.7480a_1 = 32.789$$

The coefficient matrix and the solution vector are as shown below:

$$X = \begin{pmatrix} 9.85244 & -387.250 & 301.3711 \\ 12.52179 & -399.910 & 3112.236 \\ 14.87984 & -416.000 & 323.7454 \\ 14.21600 & -435.120 & 338.6252 \\ 16.56160 & -453.387 & 352.8412 \\ 16.35926 & -474.668 & 369.4028 \\ 14.4129 & -495.689 & 385.7621 \\ 16.73592 & -514.209 & 400.1750 \\ 18.96245 & -535.714 & 416.9109 \\ 22.87463 & -560.080 & 435.8733 \\ 22.58756 & -589.473 & 458.7480 \end{pmatrix} \quad T = \begin{pmatrix} 12.660 \\ 16.090 \\ 19.120 \\ 18.267 \\ 21.281 \\ 21.021 \\ 18.520 \\ 21.505 \\ 24.366 \\ 29.393 \\ 32.879 \end{pmatrix}$$

The values of a_{-1}, a_0 *and* a_1 can be found using the formula

$$\begin{pmatrix} a_{-1} \\ a_0 \\ a_1 \end{pmatrix} = (X^T * X)^{-1} * X^T * T$$

$$(X^T * X) = \begin{pmatrix} 324465.1266 & -9027598.288 & 7025584.705 \\ -9027598.288 & 256142146.1 & -199338549 \\ 7025584.705 & -199338549 & 155132053.6 \end{pmatrix}$$

$$(X^T * X)^{-1} = \begin{pmatrix} 0.00015893 & 2.27081E\text{-}06 & -4.37547E\text{-}06 \\ 2.19108E\text{-}06 & 7949320.348 & 10214562.03 \\ -4.38215E\text{-}06 & 10214562.03 & 13125307.94 \end{pmatrix}$$

$$(X^T * X)^{-1} * X^T * T = \begin{pmatrix} 0.420317189 \\ -0.008153458 \\ 0.009390869 \end{pmatrix} = \begin{pmatrix} a_{-1} \\ a_0 \\ a_1 \end{pmatrix}$$

To Chukwu (1993), for every solution of (3. 45 p.48) to oscillate, the values of $a_{-1}, a_0 \; and -a_1$ should be nonnegative real numbers such that $|a_{-1}| \le 1$. We note that the values of a_0 and $-a_1$ are negative real numbers and this contradicts Chukwu's conditions. Hence every solution of (3. 45 p.48) will not oscillate. Another condition for oscillation is that

$$a_{-1}(a_0 - 2a_1) - a_1 > \frac{1}{eh}.$$

Now,

$$a_{-1}(a_0 - 2a_1) - a_1 = 0.420317189(\text{-}0.008153458\text{-}2(0.009390869))\text{-}0.009390869$$
$$= \text{-}0.020712185 < \frac{1}{\text{eh}} = 1.65883766$$

Hence no oscillation occurs in every solution of

$$\dot{Y} - a_{-1}\dot{Y}(t-h) = -a_0 Y(t) + a_1 Y(t-h)$$

To conclude, there are no oscillations in the trend of Ghana's real GDP from 1993 to 2004.

4.8 Discussion

As pointed out by Chukwu (1993), (3. 45 p.48) that the equation which describes the dynamics of the growth of wealth of nations is given by

$$\dot{Y} - a_{-1}\dot{Y}(t-h) = -a_0 Y(t) + a_1 Y(t-h) + p(t) + q(t)$$

This model tells us whether or not every solution of this equation will oscillate or fluctuate.

From the analysis of Ghana's real GDP from 1993 to 2004 using Chukwu's model, the values of $a_{-1}, a_0, and\, a_1$ are respectively 0.420317189, -0.008153458, and 0.009390869. The values of $a_0, and -a_1$ do not satisfy Chukwu's conditions for oscillation. So there are no oscillations or fluctuations in the trend of Ghana's real GDP from 1993 to 2004.

The fact that there are no oscillations or fluctuations in the economy does not mean that everything is well with our economy. To the African Peer Review Mechanism (2005), "the economy of Ghana remains relatively weak however; it is highly vulnerable to external shocks from the world economy and from sub-regional political instability due to factors originating in neighboring countries."

Chukwu's model can also be used to determine the behavior of the economy of every district in Ghana provided data are available. The GDP of every nation or district can change drastically should there be some technological advancements, innovations, discoveries or catastrophes. Chukwu's model, therefore, provides us information about the behavior of every economy and how best Governments and Private Sectors can come in to reduce or prevent fluctuations should there be any.

Comparing the graph of Ghana's nominal GDP with the flow of income into Afigya-Sekyere District Assembly, both graphs show similar pattern of growth. The graph of real GDP against time (Appendix A.1 Figure A.13 & A.14 p. 82) is almost exponential. This shows how the Economy of Ghana has been growing and improving over the years.

Beginning from year 1999, the inflow grants to Afigya- Sekyere District Assembly is likely to be of about 989,787.49 Ghana cedis by the year 2010. The maximum total grant for the District could be about 989787.49 Ghana cedis or about 100,000.00 Ghana cedis. Also, the maximum total revenue for the district could be about 1297803.703 Ghana cedis.

In terms of GDP growth, if the economy is to be growing as it is being observed, then, the maximum nominal GDP could be 51005.37432 million Ghana cedis.

If it is assumed that the inflow of LGRs into Afighya-Sekyere District should increase or grow from year to year, then, two models which are exponential growth model and logistic growth model could be used to predict the future outcome of the revenue. The two models were identified based on the data showing the trend of inflow of revenue with time into the District. The data which were available ranged from 1999 to 2005 and although from 1999 to 2005 is only 7 year period; the trend of inflow of revenue with time has suggested the two models.

If exponential growth model is to be used to predict the future outcome of locally generated revenues, then, the differential equation which leads to the formation of exponential growth model is

$$\frac{dY}{dt} = \lambda Y; \lambda > 0 \tag{4.12}$$

Y is the income to be generated during time t in years. The value of the constant, λ will depend on the rate of population growth, the rate of expansion of properties in the district and the level of production which will eventually yield income for the population and the depreciating value of our currency, the cedi.

The solution of equation (4.12) is

$$Y(t) = Ae^{\lambda t}; A > 0, \lambda > 0. \tag{4.13}$$

The exponential growth model is expected to work for a short time. This is because if the inflation rate continues to decline and the level of production by the District population does not increase, it will clearly show that the potential tax payers are therefore over-taxed and the inflow of revenue will decline.

If logistic growth model is to be used to predict the future outcome of locally generated revenues into the District, then, the differential equation whose solution leads to the formation of logistic growth function is

$$\frac{dY}{dt} = kY(A - Y); A > 0, k > 0. \tag{4.14}$$

Y is Locally Generated Revenue (LGR) to be generated during time t in years, k is a constant and A is the maximum revenue that the District can generate when the economy is relatively stable, almost all the potential tax payers are caught in the tax net, other sources of revenue have been identified and effectively mobilized and general considerations in revenue mobilizations have been met.

The solution of equation (4.14) is given by

$$Y(t) = \frac{A}{1+ce^{-\alpha t}}; c > 0, \alpha > 0 \qquad (4.15)$$

The speed with which $Y(t) \to A$ will depend on the values of the constants c and α. For $Y(t) \to A$ within the shortest possible time, the value of c should be such that

$$|c| < 1, \alpha \geq 1.$$

This will also depend on how quickly the Assembly should identify almost all the potential tax payers and other sources of revenue and the will power of the potential tax payers to pay tax. The logistic growth model is expected to work for a longer period of time because it reaches a certain equilibrium level, A. This is where general considerations of revenue mobilization would have been met, the potential tax payers have shown a strong will power to pay their taxes promptly and the economy is relatively stable. The model may break down should there be a high inflation rate, technological advancement and attraction of more investors into the district to invest so that the assembly can generate more revenues for development.

Locally Generated Revenues for the Assemblies in Ghana are likely to decline if the Assemblies lose contact with the people by the revenue collectors or fail to educate the people the need to pay tax. Overtaxing the people will also negatively affect the propensity of taxpayers to work, save, consume and invest.

The external sources of funding which form the bulk of the district's revenue and have been treated as Grants, the sum total in respective years since 1999 have also been obeying logistic growth model.

Total Revenue which is made up of Locally Generated Revenues and Revenue from external sources also obeys logistic growth model and the trend and pattern of flow of revenue into the district is similar to the nominal GDP of Ghana from 1993-2004.

Our real GDP is almost exponential and is increasing progressively which is an indication of a better economic growth.

4.8.1 Current Economic Situation in Ghana and Afigya-Sekyere District

According to African Peer Review Mechanism (2005), "Ghana's annual real Gross Domestic Product (GDP) rates exceeded 4% since year 2000." To the Peer Review Mechanism again, "the economy remains relatively weak; however, it is highly vulnerable to external shocks from the world economy and from sub-regional political instability due to factors originating in neighbouring countries."

The inflow of revenue into the country depends heavily on international donors such as the World Bank and the International Monetary Fund, European Union, African Development Bank and many others to mention a few. It is therefore not surprising that

Ghana became one of the highly indebted poor countries since year 2001. At present; our economy is regaining some stability because inflation rate has been consistently declining but recent inconsistent energy supply to our local industries, both formal and informal is likely to destabilize the economy some time to come.

Afigya-Sekyere District economy is mainly agrarian. In 2002 about 64% of the total workforce engaged in Agriculture. Industry and Services had 4% and 32% respectively of the total work force. As of now the total workforce engaged in Agriculture is 61%. The situation for Industry is 4% to 9% whereas Services and Commerce is 28% to 32% according to socio –economic survey of the district. Farming is basically on subsistence level and very little of the income generated by the farmers in the community can be saved for development. This may have contributed to the low development of the district. There are no heavy industries which can employ sizable number of the population in the District. Technological advancement and creativity are also lacking in the district so the District has to continue to enjoy grants from external sources for development.

Although the District's population is relatively poor in terms of income, the nominal inflow of revenues into the District Assembly increase from year to year and since inflation is continuing to decline from year to year in recent times, the District is therefore showing some remarkable improvement in local revenue mobilization. But it is still not the best since no comprehensive data on the sources of revenue for the Assembly have been collected and collated. For example, the sizes of the towns and villages in the District have been expanding, but no comprehensive data on the potential tax payers in the District are available. The only data on the income levels of the population of the District are 37.86 Ghana cedis as per capita income in 1996, and in 2002, the average household expenditure of 62.50 Ghana cedis annually. The 2006 Medium Term Plan document was silent about the levels of income of the population of the District. It is therefore very difficult, if not impossible to use these available data to develop a model to predict the future or long term income levels of the District.

4.8.2 The Way Forward

The income levels of the population in the District can be improved if the Assembly channels some of its resources into the youth. The Assembly should take it upon itself to help the youth to acquire formal education to the highest level. Those who cannot aspire to the highest level of formal education should assisted to acquire employable skills in the informal sector so that the Assembly will in the long run gain more revenue from such people for development. Most of the farmers should also be encouraged to go into commercial farming and this can be done if the Assembly channels some of its resources to the farmers through the extension services.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Two time dependent functional models have been identified for local revenue generation for development in Afigya-Sekyere District. One of them is exponential growth model which is of the form

$$Y(t) = Ae^{\lambda t}, A > 0$$

This model is expected to work for a short period when potential revenue sources are being identified and the population and private sector are growing at a higher rate. The model is likely to break down if the growth of the private sector becomes slower, inflation is declining and the people are seen to be burdened with high tax rates leading to low productivity, low consumption and low investment.

The other model is given by

$$Y(t) = \left(\frac{A}{1 + ce^{\alpha t}}\right).$$

The constant A>0 is the maximum revenue the District can generate when almost all the potential revenue sources have been identified in the district and there is minimal tax evasion. This model is also likely to break down when inflation rate becomes higher, and also there is a new technological advancement or a discovery of mineral deposit which will generate more revenue for the District. Also the speed with which $y(t) \to A$ will depend on the values of c and α .

Ghana's real GDP since 1993 has been growing at a rate exceeding 4% and analysis, using Chukwu's model indicates that there are no oscillations in the growth of our economy from 1993 to 2004.

5.2 Recommendations

The following recommendations, if implemented by the assemblies and the government will go a long way to improve the living standards of the population of every district.

(i) There should be statistical services department or its agency in every district to collect and collate comprehensive data on the total economy of every district from year to year.

(ii) Comprehensive data on all potential sources of revenue should be made available from year to year, before estimates on revenue mobilization and draft rates imposition and fee fixing resolution for every financial year are made by the Assemblies.

(iii) The plight of revenue collectors should be constantly addressed. If this is not done effective mobilization of revenue will be a mirage.

(iv) This project should be continued by some of the prospective post graduate Mathematics students. In addition to differential equation, statistical models should also be used to predict the rate of inflow of income into every district in Ghana. This will go a long way for effective planning of every district.

REFERENCES

1) African Peer Review Mechanism. Country Review Report and Programme of Action of the Republic of Ghana (2005), 47-51.

2) Albright, S. C., Wilson, W. L., and Zappe, C. (1999). Data Analysis & Decision Making with Microsoft Excel; Duxbury Press, Pacific Groove; 16-24.

3) Anthony, M. and Biggs, N., (1996) Mathematics for Economics and Finance, Methods and Modeling. Cambridge: Cambridge University Press; 343-344.

4) Burghes, D. N. and Borrie, M.N. (1981), Modeling with Differential Equations Susssex Horwood; 13-14, 117.

5) Chukwu, E. N. (1993) Mathematical Controllability Theory of the Growth of Wealth of Nations, Department of Mathematics, North Caroline State University, Raleigh, NC 27695-8205, USA; 87-90.

6) Duchanteau, P and Zachmann,D (2005) Applied Partial Differential Equations, New York: Dover Publications, Inc., Minneola, New York; 3.

7) Ghana Data Profile. World Development Indicators database, April 2007.

8) Hardwick, P. Khan, B. and LanGmead J. (1994). An Introduction to Mordern Economics (Fourth Edition). Longman Group Ltd, England; 351.

9) Hoffmann, L. D., Bradley G. L. and Rosen H. K. (2005). Applied Calculus for Business, Economics and Social and Life Sciences. New York: McGraw-Hill Companies, Inc., 1211 Avenue of Americas, New York, NY, 10020; 282, 295, 543, 602.

10) Ledder, G., (2005). Differential Equations, Modeling Approach. New York: McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY, 10020; 31, 37-38.

11) Institute of Local Government Studies *in collaboration with the* Ministry of Local Government, Rural Development and Environment. A Hand Book: The Role of the Assembly Member in Ghana's Local Government System (2007); 7-11, 14-15, 32-33.

12) Local Government Act, 1993, the Four Hundred and Sixty- Second Act of the Parliament of the Republic of Ghana. 7-8, 14-15.

13) Medium Term Development Plan for Afigya-Sekyere District (1996-2000). Submitted by Afigya-Sekyere District Assembly, Agona-Ashanti in May, 1996;

14) Medium Term Development Plan for Afigya-Sekyere District (2002-2007). Submitted by Afigya-Sekyere District Assembly, Agona-Ashanti in 2002; 18, 23-100.

15) Medium Term Development Plan for Afigya-Sekyere District (2007-2009). Submitted by Afigya-Sekyere District Assembly, Agona-Ashanti 2007; 106.

16) Monga, G. S. (2000) Mathematics and Statistics for Economics (Second Revised Edition). Vikas Publishing House PVT LTD, Noida; 322, 438-450.

17) The Economy of Ghana. Analytical Perspectives on Stability, Growth & Poverty (2008). James Currey Woeli Publishing Services; 97-98, 162-167.

18) The National Association of Local Authorities of Ghana (NALAG) Diary Desk (2007). 3-5.

19) The New Local Government System (1996). 9-17, 32, 49-55.

APPENDICES

APPENDIX A

A.1 Graphs of Models of Revenue Generation in Afigya-Sekyere District and GDP

Figure A.1

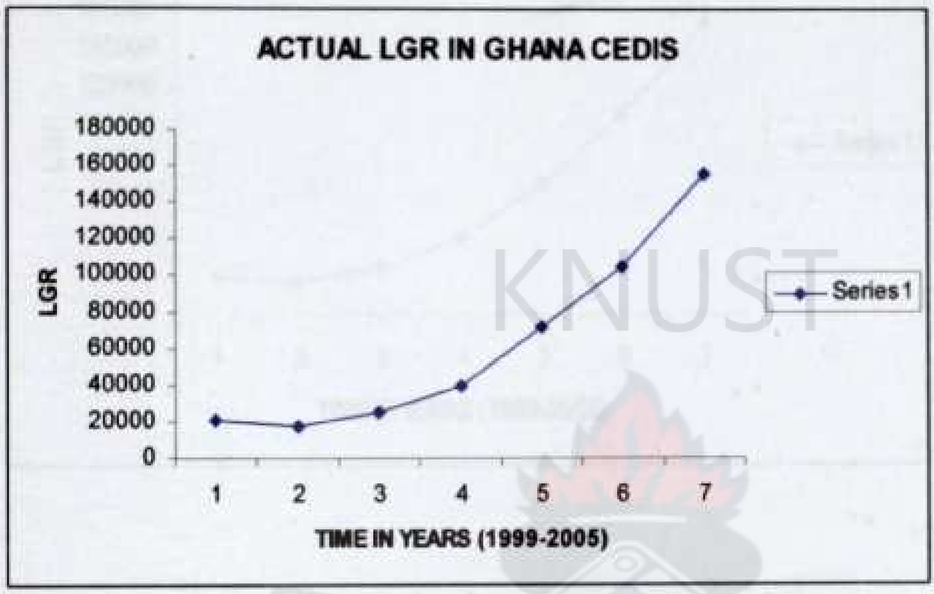


Figure A.2

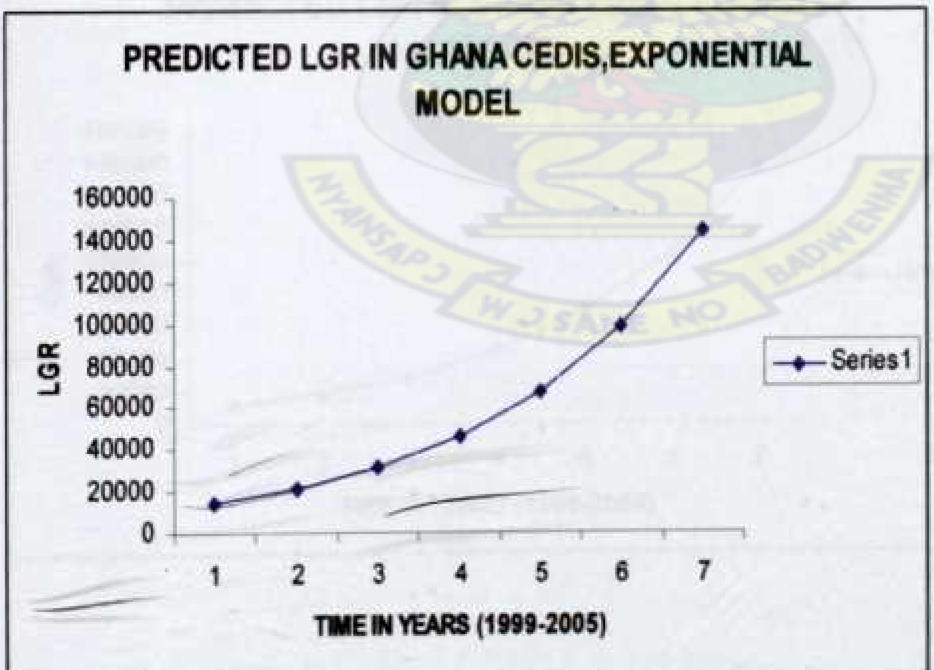


Figure A.3

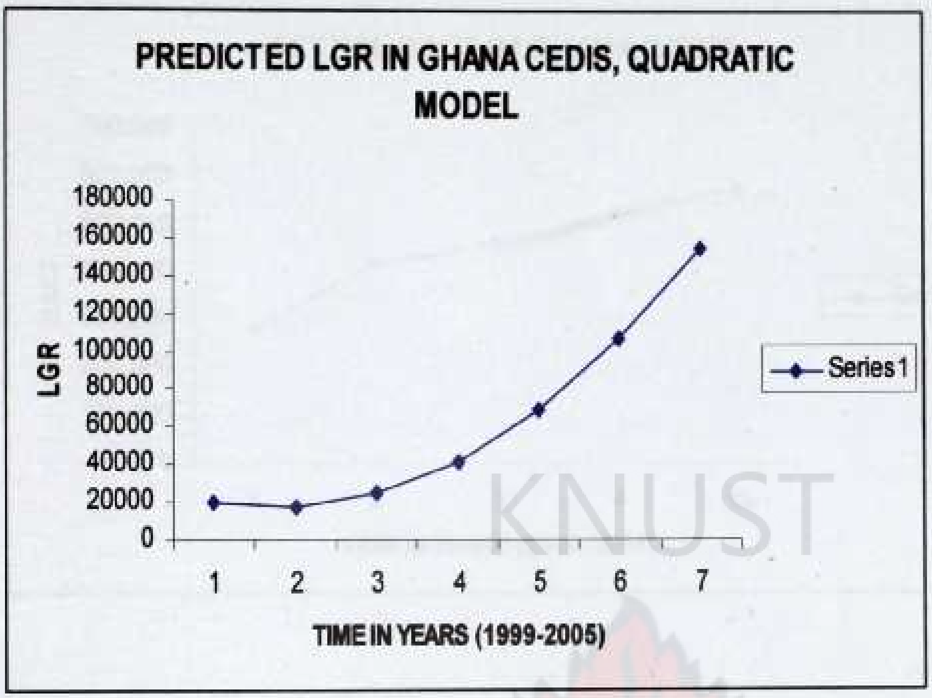


Figure A.4

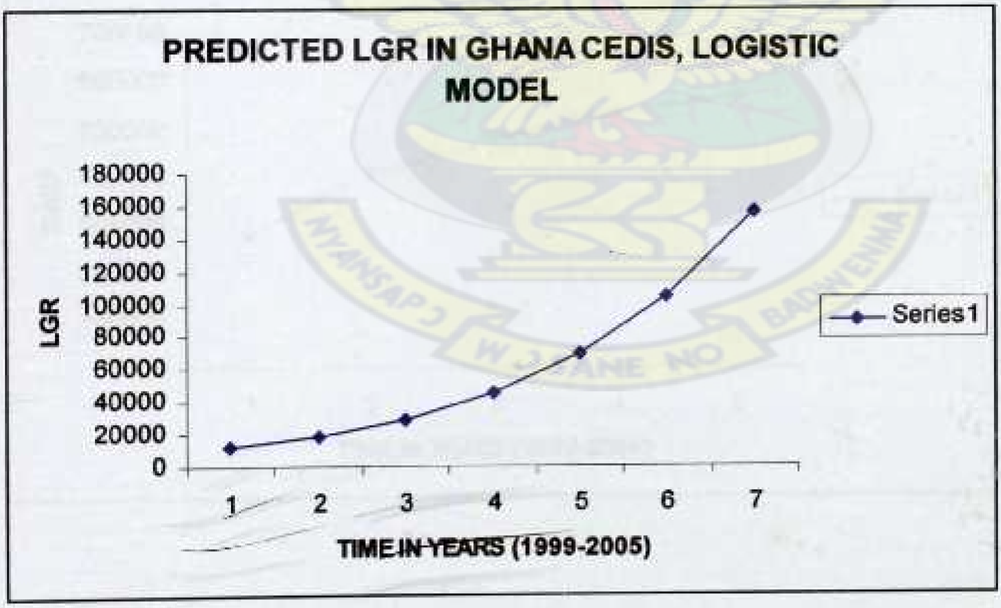


Figure A.5

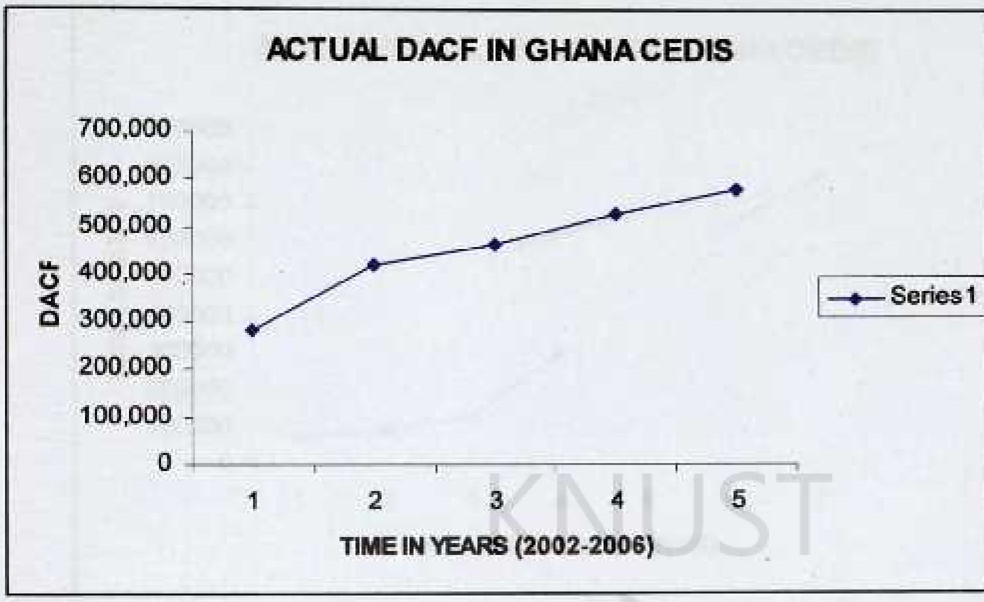


Figure A.6

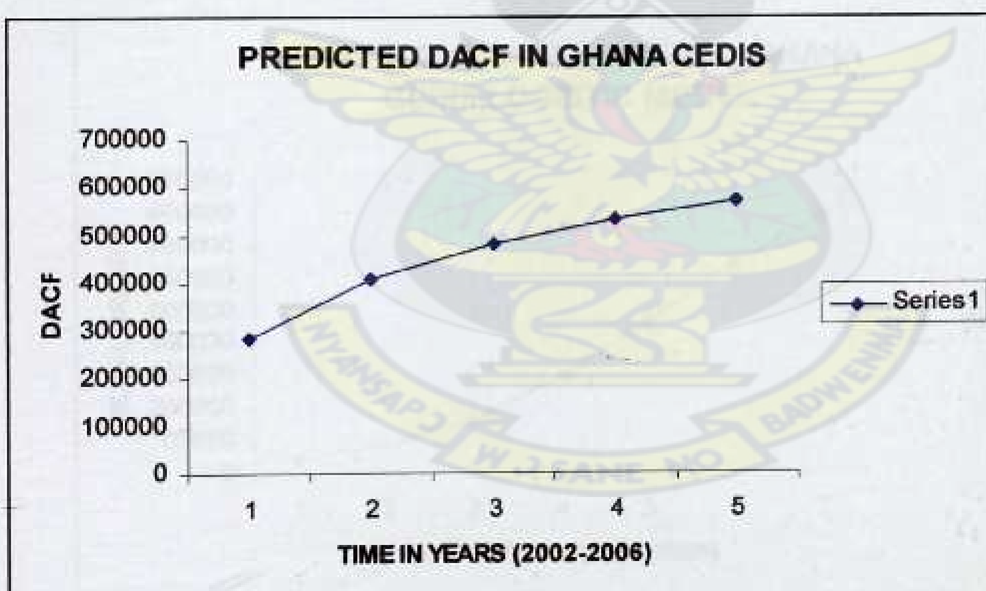


Figure A.7

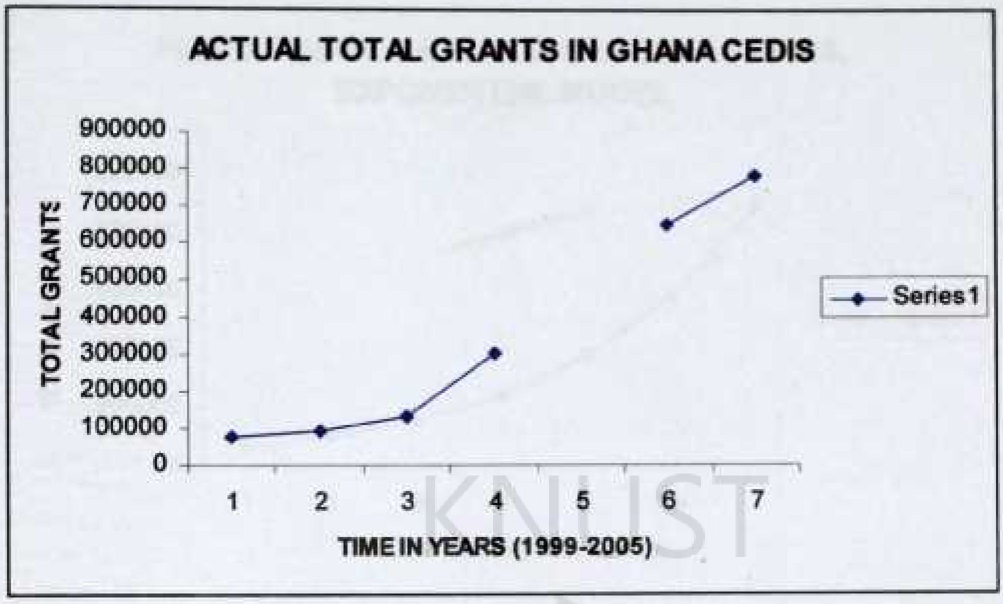


Figure A.8

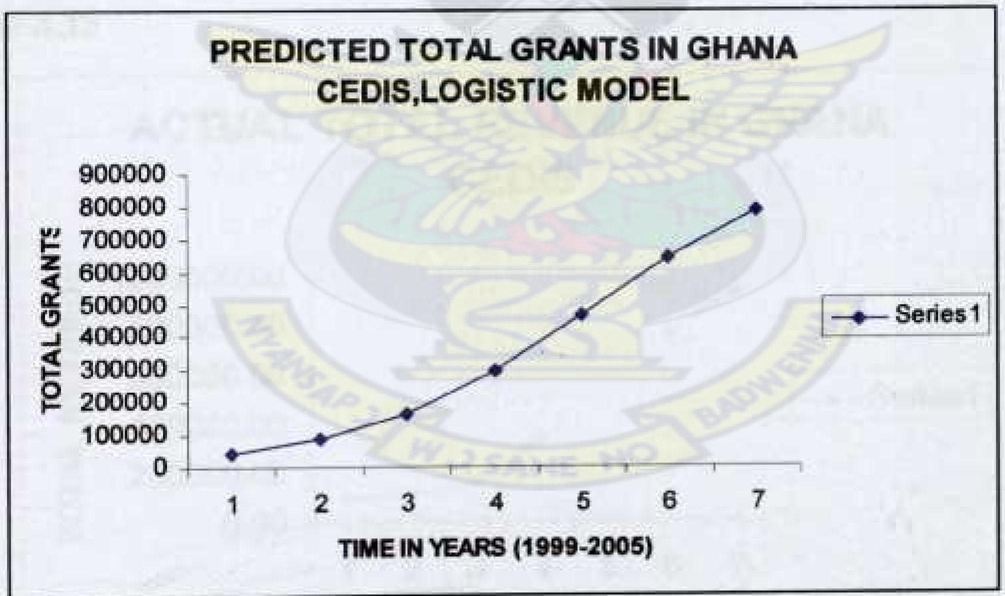


Figure A.9

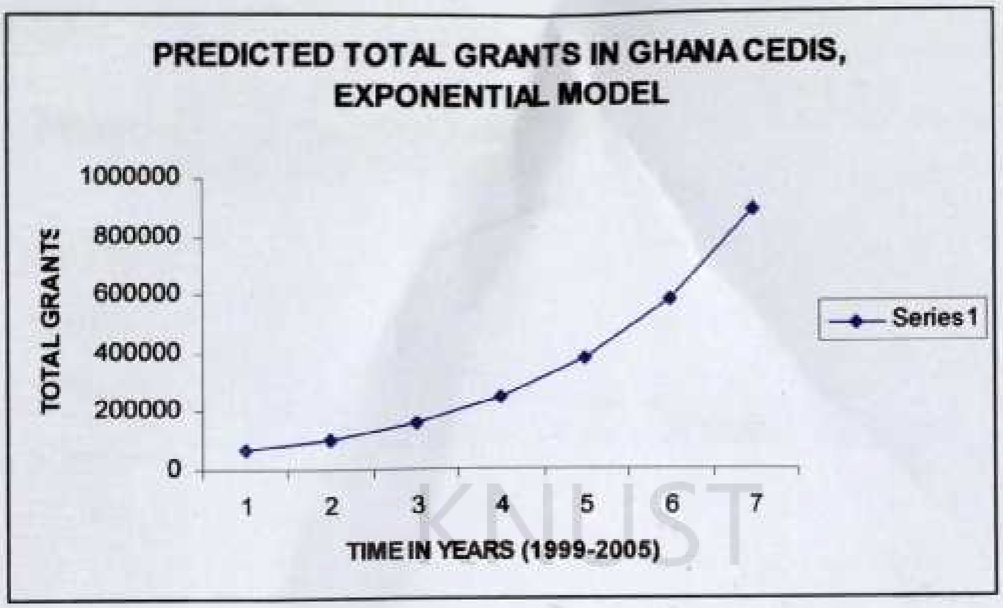


Figure A.10

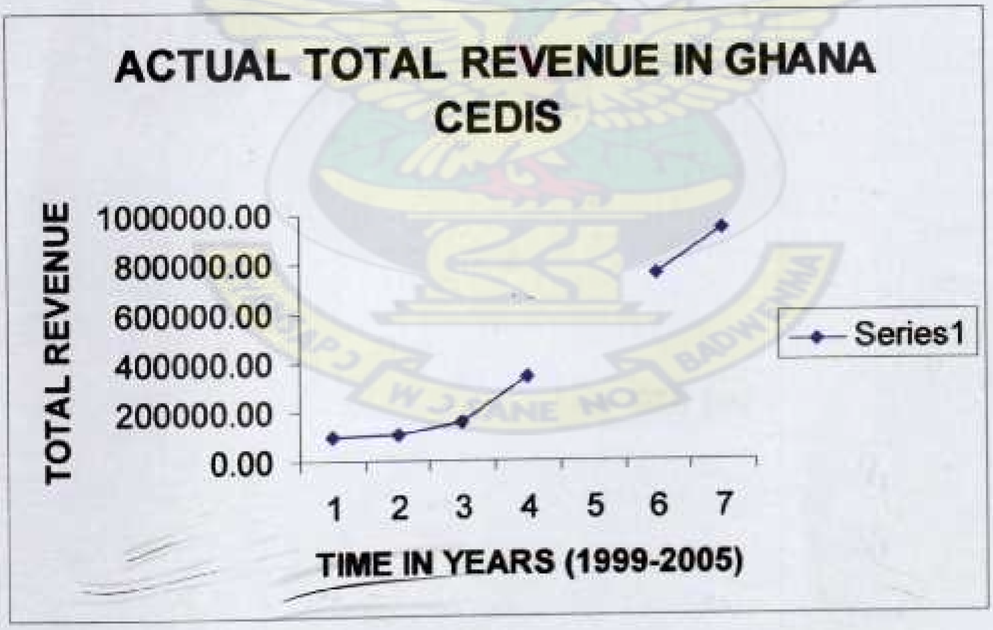


Figure A.11

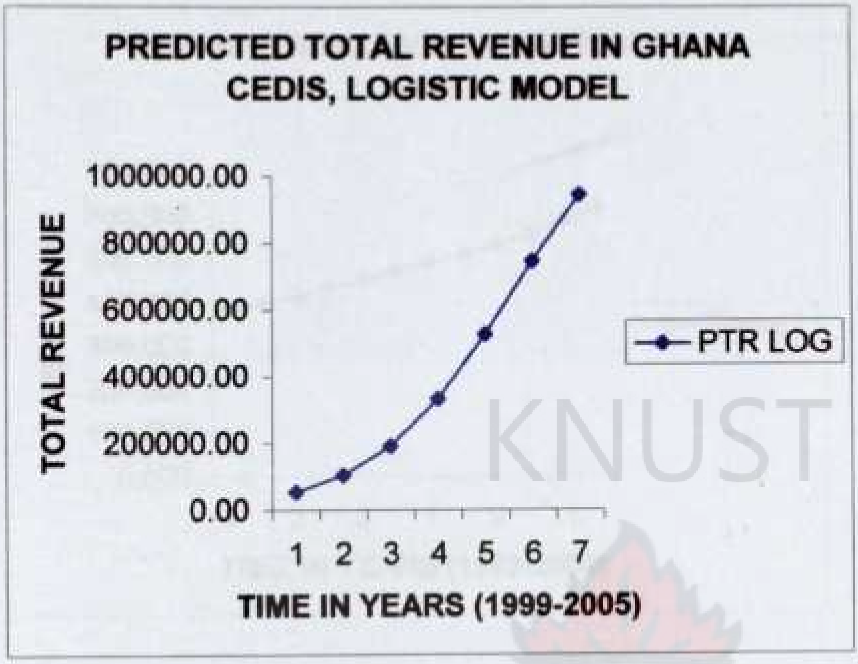


Figure A.12

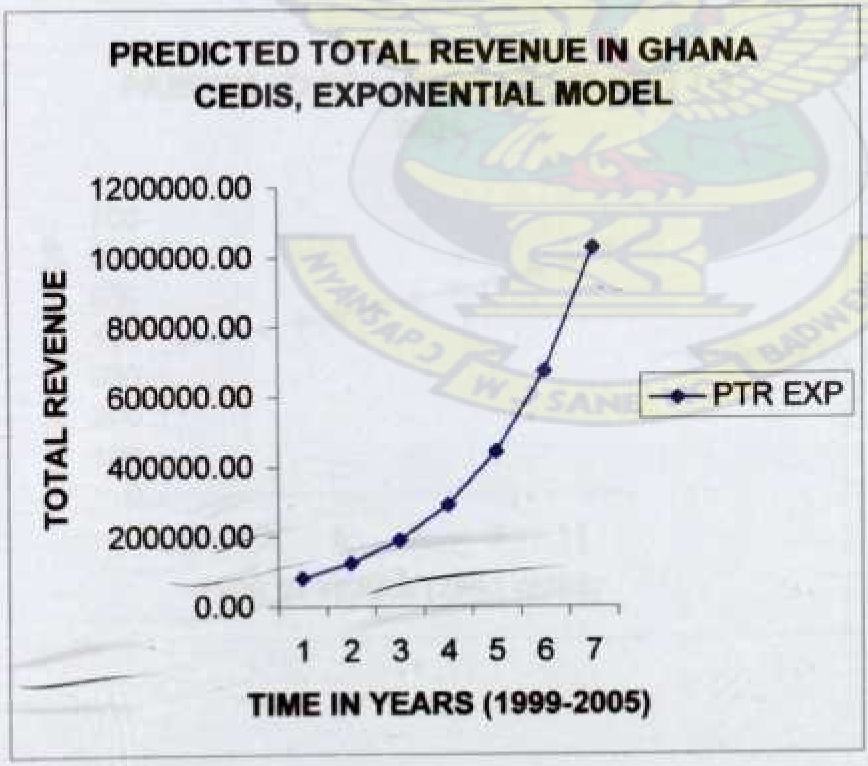


Figure A.13

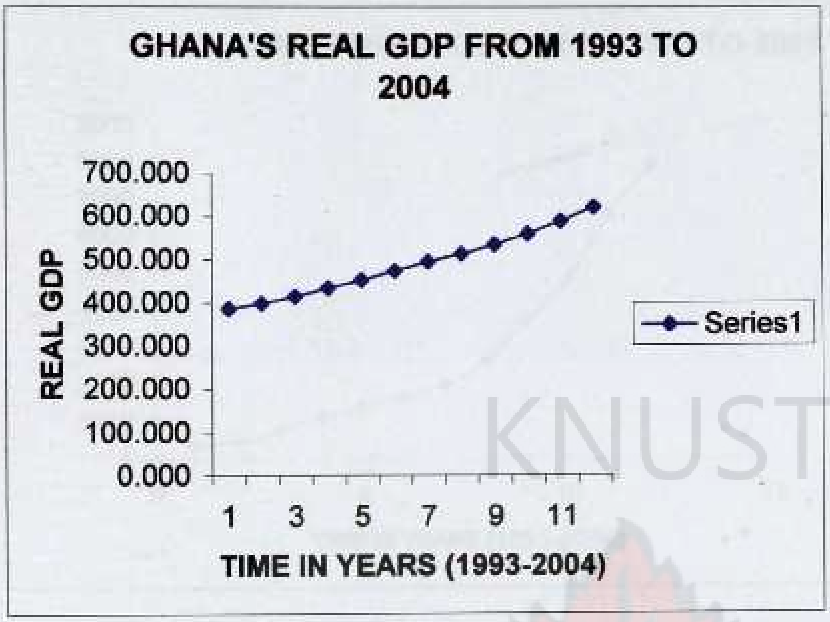


Figure A.14

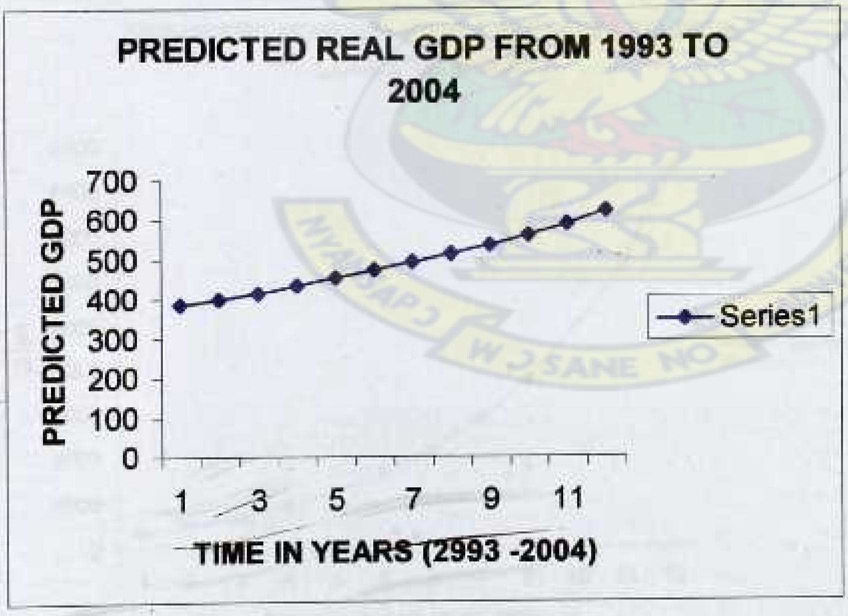


Figure A.15

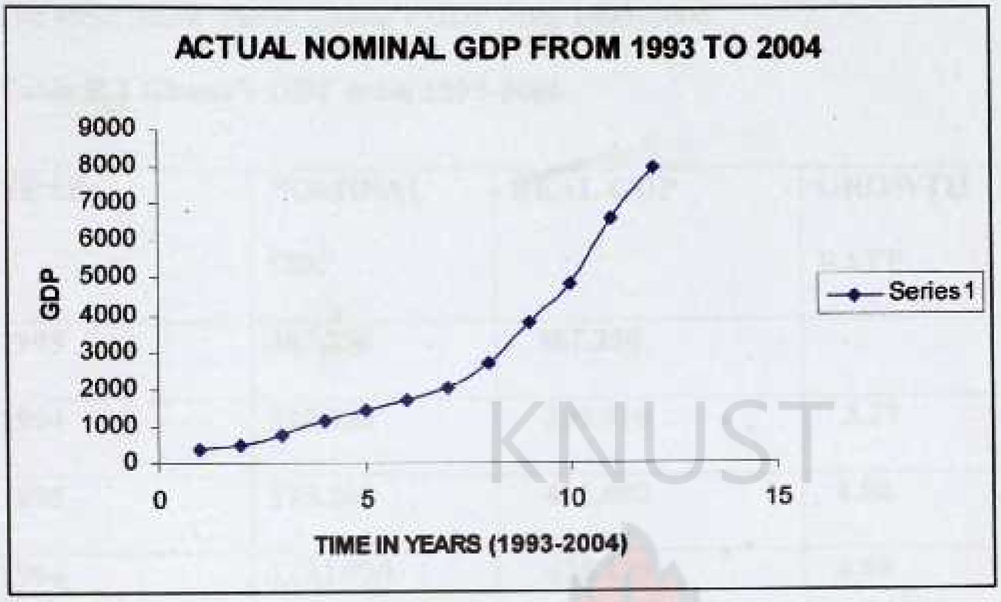
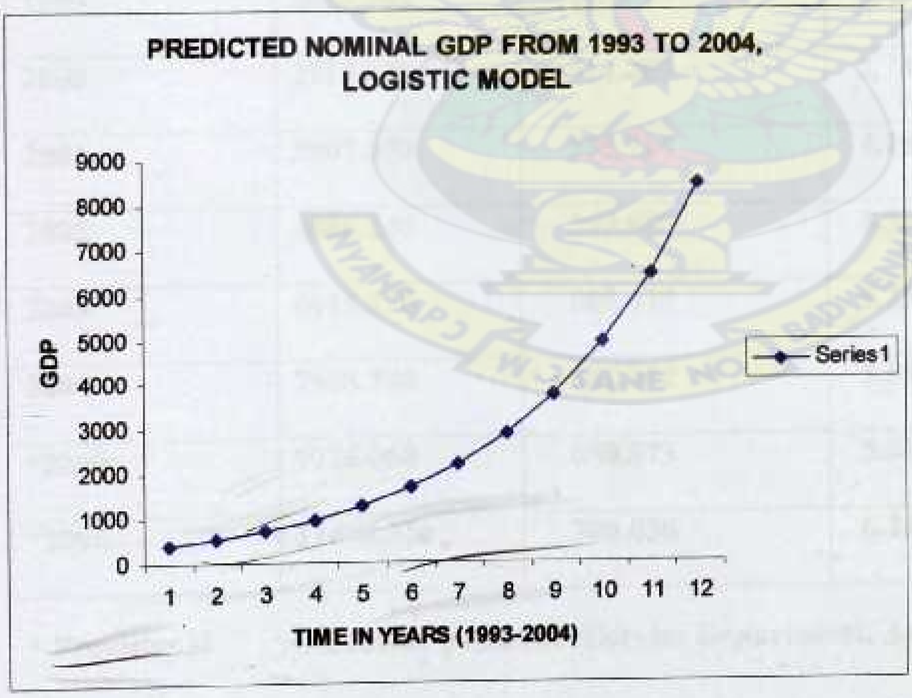


Figure A.16



APPENDIX B

B.1 Modeling of Ghana's Real GDP

The table below shows Ghana's GDP from 1993-2006.

Table B.1 Ghana's GDP from 1993-2006

YEAR	NOMINAL GDP	REAL GDP	GROWTH RATE
1993	387.250	387.250	
1994	520.520	399.910	3.27
1995	775.260	416.000	4.02
1996	1133.920	435.120	4.60
1997	1411.340	453.387	4.20
1998	1729.570	474.668	4.69
1999	2057.980	495.689	4.43
2000	2715.270	514.209	3.74
2001	3807.070	535.714	4.18
2002	4886.240	560.080	4.55
2003	6615.770	589.473	5.25
2004	7988.740	622.352	5.58
*2005	9726.060	658.873	5.87
*2006	11490.320	700.050	6.25

*** Provisional** **Source: Statistical Service Department, Accra.**

The model which approximately describes Ghana's real GDP from 1993 to 2004 (Table B.2 p.86) is given by

$$Y(t) = 367.0860196e^{0.0428843816t} + 0.0200794045t^4 - 0.4835854529t^3 + 3.9671537t^2 - 13.08610246t + 13.699504768 \qquad (2.2.1)$$

The model $Y(t)$, was obtained by displaying the values of the real GDP on a graph. From the shape of the curve, it was assumed that;

$$Y_1(t) = Ae^{\lambda t} \qquad (2.2.2)$$

Where A *and* λ are constants.

The real GDP values from 1993 -2004 were entered into TI-83 PLUS graphing calculator and it displayed the values of A *and* λ as 367.086019 and 0.042843816 respectively. The residuals, $E(t)$, obtained from the difference between actual and predicted values of the real GDP were also found to approximately obey the relationship

$$E(t) = 0.0200794045t^4 - 0.4835854529t^3 + 3.9671537t^2 - 13.08610246t + 13.699504768 \qquad (2.2.3)$$

The model which approximately describes the real GDP is therefore

$$Y(t) = Y_1(t) + E(t) = 367.0860196e^{0.0428843816t} + 0.0200794045t^4 - 0.4835854529t^3 + 3.9671537t^2 - 13.08610246t + 13.699504768; t \geq 1$$

The table on page 86 shows the actual and predicted values of nominal and predicted values of nominal and real GDPs

Table B.2 Actual and Predicted Values of Nominal and Real GDPs of Ghana in Million Ghana Cedis from 1993 to 2004.

YEAR	ACTUAL NOMINAL GDP	PREDICTED NOMINAL GDP	RESIDUALS	ACTUAL REAL GDP	PREDICTED REAL GDP	RESIDUALS
1993	387.250	417.7765	-30.5265	387.250	387.27357	-0.02357
1994	520.520	553.8115	-33.2915	399.910	399.78304	0.12696
1995	775.260	733.5169	41.7431	416.000	416.16561	-0.16561
1996	1133.920	970.4130	163.507	435.120	434.75668	0.36332
1997	1411.340	1281.8662	129.4738	453.387	454.37495	-0.98795
1998	1729.570	1689.9037	39.6663	474.668	474.32250	0.34550
1999	2057.980	2222.0218	-164.0418	495.689	494.38485	1.30415
2000	2715.270	2911.8003	-196.5303	514.209	514.83101	-0.62201
2001	3807.070	3799.0304	8.0396	535.714	536.41360	0.69960
2002	4886.240	4928.8949	-62.6549	560.080	560.36885	-0.28885
2003	6615.770	6349.5923	266.1777	589.473	588.41673	1.05627
2004	7988.740	8107.7328	-118.9928	622.352	622.76013	-0.40903