OPTIMAL LOCATION OF FIRE STATION USING ABSOLUTE CENTER MODEL. CASE STUDY: NEW JUABEN MUNICIPALITY

BY

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CHAPTER ONE

Introduction

1.1 Background of Study

For the years, the location of semi-obnoxious (also known as semi-desirable) facility has been a widely studied topic by researchers in location theory. A facility is said to be semi-desirable when it gives service to certain customers in the neighbourhood but, on the other hand, is felt as obnoxious to its environment. For example stadia, airports, train stations and fire stations are examples of semi-obnoxious facilities. Since they are useful and necessary for the community, but they are a source of negative effects such as noise. New Juaben Municipality is one of municipalities of the Eastern Region and has estimated population of 152,858 people kilometers with a population density of 1,507.

The Municipality shares boundaries on the north with East Akim Municipality on the south with Akuapem North, Yilo Krobo District on the East. Suhum Kraboa Coalter District on the west. It lies between latitude 60^0N and 70^0N.

The new Juaben Municipality with Koforidua as capital is co-terminus with Eastern Regional Capital. Koforidua is located at the junction of the major truck roads in the Eastern Region. Farming is the main agricultural activities of most inhabitants in the Municipality. The major factory in the municipality is the Intravenous Infusion Limited at Koforidua that produces intravenous fluids for distribution throughout the country. The municipality has one fire station at Asokore serving the Municipality and East Akim Municipality as East Akim has no fire station. Most fire outbreaks in Ghana could be linked to misuse of electrical gadgets, wrongful electrical connection, careless usage of candles, wrongful disposal of live cigarette butts and many other factors and behaviours.

In U.K the great fire of London in 1666 set in motion changes which laid the foundation for organized firefighting. The only equipment available to fight in 1666 which burnt for five days was two-quart (2.28 litres) hand syringes and a similar slightly large syringe (Louisa et al., 2006) . In the wake of the fire, the city council established the first fire insurance company “THE FIRE OFFICE” in 1677 which employed small teams of Thames Watermen as firefighters and provided them with uniforms and arm badges showing the company to which they belonged. The first organized municipal fire brigade in the world however, was established in Edinburgh, Scotland, when the Edinburgh fire engine Establishment was formed in 1824. It was led by James Braidwood. In 1832, London fire Engine Establishment was also formed.

1.2 Problem Statement

It is a fact that cities or towns in Ghana do not have well located fire stations hence minor incidents which would easily be managed results is massive loss of property and even lives. Again roads are not properly layed out that access to places of fire outbreaks is simply not possible. The belief is that fire station should be located in such a way that allows firefighters to respond in a timely manner to emergencies. Facts that influence this decision are as follows:

i. The risk of fire is not the same in all areas; industrial parts lf the municipality is more vulnerable to fire outbreaks.

ii. Population is not spread equally around the municipality, and as a result there are parts of the municipality that are more populated than others.

It has been shown that frequency of incidents is higher in the most populated areas.

It is against this background that this study is being undertaken to develop a decision support system that will help authorities of New Juaben Municipality to strategically locate fire station.

1.3 Objectives of the Study

The objectives of the study are as follows:

i. To model the location of fire station as Absolute Center Problem

ii. Determine the optimal location and service coverage distance.

1.4 Methodology

The problem is to develop a decision support system to optimally locate fire station in the New Juaben Municipality. The p-center heuristics was used in the study. The study was descriptive and analytical in nature and therefore, made use of quantitative and qualitative data collection tools whereas the analyses of data involved the use of mathematical procedure. Data were obtained from statistical department of the New Juaben Municipality, formal and informal interviews with fire service management and some operational men. Map of the Municipality was obtained from planning department of the municipality. Floyd Warshalls' algorithm was used to compute Euclidean distance between all pairs of nodes.

Other information were obtained from the internet and department of mathematical library of KNUST (Kwame Nkrumah University of Science and Technology)

1.5 Justification

Optimal location of fire station in the country would boost foreign and local investor confidence in their economic activities. Plants and animal species which could be pushed to extinction as a result of wild bush fire will be reduced. Degradation of ecosystem, increased soil erosion, reduced water quality and increased soil salinity resulting from fire outbreaks will also be also be addressed thereby increasing productivity of the country.

Damages, injuries loss of property and even death that, both human beings and animals suffer will be a large extent prevented. The findings of the study can be implemented by authorities of New Juaben Municipality.

Future researchers can replicate the study at other parts of the country using the work done as reference material.

1.6 Organization of the study

Chapter one talks about the profile of the area of study, fire history, causes of fire outbreaks, objectives, justification and methodology of the study. Chapter two is primarily about review of some location problem models.

Chapter three considers three location models, strategies involved in choosing a site, network-based algorithms, absolute center problem, determination of upper envelope, local center and absolute center.

Chapter four is about result of location vertex or node center, local center and discussion. Chapter five considers conclusions and recommendations.

CHAPTER TWO

Literature Review

2.1 Introduction

This chapter introduces some of the methods and models that other researchers have applied in solving location problems.

2.2 Review of Location Problem Models

Hongzhong et al. (2005), first surveyed general facility location problems and identified models used to address common emergency situations, such as house fires and regular health care needs. The authors then analyzed the characteristics of large-scale emergencies and proposed a general facility location model that is sited for large-scale emergencies. This general facility location model could be cast as a covering model, a P-median model or a P-center model, each suited for different needs in large-scale emergencies. Illustrative examples were given to show how the proposed model could be used to optimize the location of facilities for medical supplies to address large-scale emergencies.

The associated FORTRAN computer programme could be utilized to determine the travel time from a source of fire to a smoke detector. The difference in travel time from an isolated fire source to two or more detectors could be used to isolate those airways in which the source of fire is located. This model also has application in mine emergency stage. To determine the optimum location fire detectors, the mine network was divided into zones each of which was associated with a difference in calculated smoke arrival time between a pair of detectors.

Church and ReVelle (1974) and White and Case (1974), developed a maximal covering location problem model that did not require full coverage to all demand points. Instead, the model sought the maximal coverage with a given number of facilities.

The maximal covering location problem and different variants of it had been extensively used to solve various emergency service location problems. A notable example was the work of Eaton et al. (1985), that used the maximal covering location problem to plan the emergency medical service in Austin Texas. The solution gave a reduced average emergency responses time with increased calls for service.

Schilling et al., (2005), generalized the maximal covering location problem model to locate emergency fire fighting servers and depots in the city of Baltimore. In the authors' model, known as FLEET (facility location and Equipment Emplacement Technique) two different types of servers needed to be located simultaneously. A demand point was regarded 'covered' only if both servers were located within a specified distance.

Hodder and Dincer (1986), consider the location of capacitated facilities globally under exchange rate uncertainty. The model incorporates the financing aspects of plant construction by endogenously deciding how much of each plants' total cost to borrow from each country; the per-period cost of this financing is a random variable since the exchange rate are uncertain.

In addition, cost and per-unit profit are uncertain. The model maximizes a mean-variance expression concerning the total profit. This objective is quadratic and involves a large-variance-covariance matrix, each off diagonal term of which requires a bilinear term in the objective function. Therefore, the author proposes an approximation scheme that effectively diagonalizes the variance-covariance matrix so that the objective function contains only squared terms and no bilinear terms.

The resulting model is solved using an off-the-shelf quadratic programming solver for small problems and using a gradient search method for larger ones. No discussion is provided concerning the form of uncertainty (discrete or continuous) or the probability distributions governing it, but in theory any approach could be used as long as the random parameters can be expressed adequately in the form needed for the approximation.

Berman and Odoni (1982), studied a single-facility location problem in which travel times are stochastic and the facility (e.g. Ambulance) may be relocated at a cost as conditions change. Travel times are scenario-based, and scenario transitions occur according to a discrete-time Markov process. The objective is to choose a facility location for each scenario to minimize expected transportation and relocation costs. The authors show that Hakimi property applies to this problem and that the problem on a tree is equivalent to the deterministic problem; any scenario can be used to determine the optimal location since I-median on a tree is independent of the edge of lengths. They then present a heuristic for the problem on a general network that involves iteratively fixing the location in all but one scenario and solving what amounts to I-median problem. They discuss simple bounds on the optimal objective value of the multi-facility problem. Berman and LeBlanc, (1984), introduce a heuristic for this problem that loops through the scenarios, performs local exchanges within each, and then performs exchanges to link the scenarios in an effort to reduce relocations costs.

Carson and Batta, (1990), present a case study of a similar problem in which a single ambulance is to be relocated on the Amherst campus of SUNY buffalo as the population moves about the campus throughout the day (from class-room buildings to dining halls to dormitories, etc.). Given the difficulties inherent in identifying probability distributions and estimating allocation costs in practice, Carson and Batta simply divide the day into four unequal time periods and solve I-median problem in each. Relocation costs are not explicitly considered, but the decision to use four time periods was arrived at in consideration of the trade off between frequent relocation and increased response times.

Ghosh and McLafferty, (1982), introduce a model for locating multiple stores so as maximize market share in a competitive environment with demand uncertainty (actual, uncertainty as to which stores competitor plans to close, but in this setting they amount to the same thing). The authors discuss a model from the marketing literature for estimating market share given fixed store locations.
The location model itself is formulated as a multi objective model, with each objective representing the market-share-maximization objective in a given scenario.

Ultimately, the objectives are combined into a weighted sum to be minimized. If the weights represent scenario probabilities, the objective is equivalent to minimizing the expected cost; otherwise, the weights can be adjusted systematically to find nondominated solutions (solutions for which no objective can be improved without degrading another objective). For a given set of weights, the problem is solved using an exchange heuristic. On a small sample problem, three noninferior solutions were found, and the authors provide some discussion as to how to choose among them.

Benedict, (1983), Eaton et al., (1986), and Hogan and ReVelle, (1986), developed covering maximal location problem models for emergency service that had a secondly "back up-coverage" objective. The models ensured that a second (back up) facility could be available to service a demand area in case that the first facility was unavailable to provide services. Based on a hypercube queuing model, Javis (1977) developed a descriptive model for operation characteristics of an EMS system with a given configuration of resources and a location model for determining the placement of ambulances to minimize average response time or other geographically based variables.

Marianov and ReVelle, (1996), created a realistic location model for emergence systems based on results from queuing theory. In their model the travel times or distances along arcs of network were considered as random variables. The goal was to place limited numbers of emergency vehicles, such as ambulances, in away as to maximize the call for service.

Carbone, (1974), formulated a deterministic p-median model with the objective of minimizing the distance travelled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node was uncertain. The author further extended the deterministic p-median node to a chance constrained model. The model sought to minimize distance and user costs, and maximize demand and utilization.

Paluzzi, (2004), discussed and tested p-median based heuristic location model for placing emergency service facilities for the city of Carbondale. The goal of this model was to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. The results were compared with results from other approaches and the comparison validated the usefulness and effectiveness of the p-median based location model.

Doeksen and Oehrtman, (1976), used a general transportation model based on alternative objective functions to obtain optimal fire stations for the rural fire system. The different objectives used to obtain the optimal sites include: minimizing responses time to fire, minimizing total mileage for fighting rural or country fires and minimizing protection per dollars' worth of burnable property.

Plane and Hendricks, (1977), used the maximum covering distance concept to develop a hierarchical objective function for the set covering formulation of the fire station location problem. The objective function permitted the simultaneous minimization of the number of fire stations and minimization of the existing fire station within the minimum total number of stations.

Badri et al. (1998), underlined the need for a multi objective model in determining the fire station location. The authors used a multiple criteria modeling approach via integer goal programming in everlasting potential sites in 31 sub-areas in the state of Dubai. Their model determined the location of fire stations and the areas they are supposed to serve. It considered eleven (11) strategic objectives that incorporated travel times and travel distances from stations to demand sites, and also other cost-related objectives and criteria-technical and political in nature.

Church (2002), exhaustively reviewed the existing work linking GIS location science and asserted that GIS could support a wide range of spatial queries that aid location studies. He explored the integration of a heuristic algorithm into GIS for spatial optimization of fire station locations. This novel approach to solving optimization problem led to a paradigm shift in solving spatial analytical problems of a similar nature in the disciplines of transportation, networking and infrastructure design.

Tzeng and Chen (1999), used a fuzzy multi objective approach to determine the optimal number and sites of fire stations in Taipeis' international airport. A genetic algorithm was then executed to weigh against the brute-force enumeration method. The results proved that the genetic algorithm was suitable for solving such location problems. Nevertheless, its efficiency still remained to be verified by large-scale problems.

Talwar (2002), utilized a p-center model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands from accidents of tourist activities such as skiing, hiking and climbing at the north and south of Alphine mountain ranges. One of the models' aims was to minimize the maximum (worst) response times and the author use effective heuristics to solve the problem.

ReVelle and Hogan et al., (1989), formulated a model that sought to minimize a population which had a service available within a desired travel time with a stated reliability, given that only P servers were to be located. The authors computed the number Pi of servers needed for reliable coverage of node i, and maximized the population in nodes i, with pi or more servers.

De Palma et al (1989), study a multi firm competitive facility location with random consumer utilities. A consumers' utility for firm i is expressed as a constant a_i (the mean utility for the firm) minus the distance from the consumer to the firms' nearest facility minus a random error term. After choosing its maximum–utility firm, each consumer will choose the nearest facility within that firm.

Firm i will open m_i facilities to maximize its expected sales (market share). The authors proved that if the m_i–median solution is unique for all i and if the consumers' tastes are sufficiently diverse, then there exists a unique location equilibrium, and in that equilibrium, firm i locates its facilities at the m_i–median solution. The problem therefore reduces to solving a separate PMP for each firm.

MirHassani et al(2000), formulate a study chain network design problem as a stochastic program with fixed recourse; the SP has binary first- stage variables and continuous second – stage variables. The objective function coefficient are deterministic; uncertainty is present only in the right- hand sides of the recourse constraints, which may represent for example, demands or capacities. The authors focus especially on parallel implementation issues for their proposed Benders decomposition algorithm.

Tsiakis et al. (2001), consider a multiproduct, multiechelon supply chain under scenario- based demand uncertainty. The goal is to choose middle- echelon facility locations and capacities, transportation links and flows to minimize expected costs. Transportation cost are piecewise linear concave. The model is formulated as a large –scale MIP and solved using CPLEX.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter introduces a number of Locations models (p-median, center-of gravity etc) formulated and used in solving location problem. The chapter also discusses the methodology that would be used to in finding the optimal location where a fire station would be located in the Koforidua Township to ensure optimal response time for incident responders in the service coverage area.

3.2 Spatial Representation of Location

In support of decision processes that involve facility siting, location models are generally used.

To formulate a location model, it is necessary to identify where the demand is located and where facilities can be sited.

The problem of siting p facilities in some universe so as to satisfy a given set of criteria poses the following:

i) The universe to be considered;

ii) The assumptions to simplify the problem without distorting the solution radically; and

iii) The objectives to be optimized.

These result in the emergence of many different formulations to the fundamental location problem. As one would expect, the more accurately a model reflects the 'real life' situations the more complex the problem becomes.

Three different universes will be addressed;

i) Planar;

ii) Network; and

iii) Discrete.

The whole essence of the siting problem is to locate several facilities to optimize a certain set of objectives. The objectives function could be any of the following:

i) Minimise the maximum Euclidean distance;

ii) Minimise average travel time or cost;

iii) Minimise maximum travel time or cost;

iv) Minimise net; and

v) Minimise response time.

3.3 The Universe to be considered

The first universe to be considered is that of the entire plane, entitled the Planar location problem. Here the set of points making up the entire plane is the set of feasible solutions. For this basic formulation, the planar model assumes direct distance metric e.g Euclidean. On the other hand in a network problem, potential customers will normally travel the arcs or edges of the network, road or rail. This prompts the formulation of the network location model, where the facilities may be positioned on a vertex or an edge of network. Distances are then reformulated to be the shortest path linking facilities and customers. There is also the discrete problem of siting a facility on vertices of a network.

(Francis et al., 1983)

3.3.1 Planar Location Models

A planar location model involves the location of p new facilities $p \in N$ within a feasible plane, so as to minimize some cost of the distance from each new facility to the other new facilities and any existing facility within the plane.

Assumptions:

Before any formulation of the above can be established a set of assumptions must be made:

i) Any point in the plane can be a member of the feasible solution.

ii) Each facility can be approximated by a point, i.e. it has no area.

iii) A subset of the earths' surface can be approximated by a plane.

The above assumptions immediately raise several questions about accuracy. Assumption (i) does not allow for the occurrence of infeasible area within the plane, such as property owned by other organizations, natural barriers are inaccessible sites. In these cases the model assumes that a site close to the optimal may be chosen with no loss of satisfaction. Assumption (ii) states that the feasible plane is infinitely bigger than the area taken by a facility. This is obviously unrealistic and may affect the results if the feasible area is on a very local scale and the potential facilities on large site area. Assumption(iii) assumes that the feasible set is small enough so that the spherical curve of the sphere does not alter the shortest distance.

3.3.2 Network Location Models

A network is a system of interconnecting routes which allows movement from one centre to others. It is made up of nodes (vertices) which may be population centres and links (edges) which are routes or services which connect them. In the network location model, the distance metric is measured along a road or rail system, or a set of flight or shipping routes. It may therefore be preferred for placement of the facility to occur on the edges or nodes of the network.

Assumptions

To adopt this model, the set of assumptions made above must first be modified as:

(i) Each facility can be approximated by a point i.e. it has no area.

(ii) Network distances between points are defined as shortest path distances which can be computed using Djikstra algorithm or Floyd Warshall algorithm.

(iii) Any point in the network can be a member of the feasible solution.

These assumptions are similar to those of the planar model and will result in similar formulations. However, if the assumption that all the facilities provide the same kind of service and that a customer will only have to travel to the closest facility is introduced, a subset of the minimax or minisum formulation is addressed.

3.3.3 Discrete Location Models

Planar and Network location models have some limitations, in that:

i) Every point in the plane or network is a candidate solution;

ii) Fixed costs for siting individual facilities at a particular point are ignored or assumed to be independent of the location chosen and so do not affect the optimal solution.

These limitations are confronted when the solution set is reduced to that of a finite number of candidate solutions. Each candidate can be assigned an individual location cost which in turn can be incorporated into the objective function. (Moon I.D and Chandhry S.S, 1994) and (Mirchandani P.B and Francis R.L, 1990)

3.4 Strategies Involved in Choosing a Site

Location simplify refers to a place where something happens or exist.

Many factors, both quantitative and qualitative have to be considered in selecting a location. Some of these factors are more important than others so people can use weightings to make the decision process more objective. Three of the main location strategies are the location break-even analysis, factor rating and centre-of-gravity methods.

3.4.1 The Location Break-Even Analysis

The location break-even analysis is the use of cost-volume analysis to make economic comparison of location alternatives. By identifying fixed and variable costs and graphing them for each location we can determine which one provides the lowest cost. Location break-even analysis can be done mathematically or graphically. The graphic approach has the advantage of providing the range of volume over which each location is preferable. There are three steps in location break-even analysis.

These are:

i) Determine the fixed and variable costs for each location.

ii) Plot the cost for each location, with cost on the vertical axis of the graph and production on volume the horizontal axis.

iii) Select the location that has the lowest total cost for the expected production volume.

3.4.2 The Factor Rating Method

The factor rating is popular because a wide variety of factors, from education to labour skill to recreation, can be objectively included. The factor rating method has six steps:

i) Develop a list of relevant factors.

ii) Assign a weight to each factor to reflect its relative importance in the companys' objectives.

iii) Develop a scale for each (e.g. 1 to 10 or 1 to 100)

iv) Assign a score to each location for each factor using the scale in step (iii)

v) Multiply the score by the weights for each factor and total the score for each location.

vi) Make a recommendation based on the maximum point score, considering the results of quantitative approaches as well.

When a decision is sensitive to mirror changes, further analysis of either the weighting or the points assigned may be appropriate. Alternatively, management may conclude that these intangible factors are not the proper criteria on which to base a location decision. Managers therefore place primary weight on the more quantitative aspects of the decision. (Amponsah, 2006)

3.4.3 Centre-of-Gravity Method

The centre-of-gravity method is a mathematical technique used for finding the location of a distribution centre which minimizes distribution costs. This method takes into account the location of markets, the volume of goods shipped to those markets, and shipping costs in finding the best location for a distribution centre.

The first step in the centre-of-gravity method is to place the location on a co-ordinate system. The origin of the co-ordinate system in the scale is arbitrary, just as long as the relative distances are correctly represented. This can be done by placing a grid over an ordinary map of the location in question. The centre-of-gravity is determined by equations (3.1) and (3.2)

$$C_x = \frac{\sum d_{ix} W_i}{\sum W_i} \quad \text{.........................(3.1)}$$

$$C_y = \frac{\sum d_{iy} W_i}{\sum W_i} \quad \text{.........................(3.2)}$$

Where

$C_x = x -$ Coordinate of the centre-of-gravity

$C_y = y -$ Coordinate of the centre-of-gravity

$d_{ix} = x -$ Coordinate of location ***i***

$d_{iy} = y -$ Coordinate of location ***i***

$W_i =$ Volume of goods to and from location i

Once the x and y-coordinates have been obtained, the new location is placed on the previously described map to determine the actual position on the map. If that particular location does not fall directly on a city, simply locate the nearest city and place new distribution centre there. (Louisa et al., 2006)

3.5 Network-Based Algorithms

3.5.1 Shortest Path Problems

Shortest path problems are the most fundamental and most commonly encountered problems in the study of transportation and communication networks (Salhi S, 1998). There are many types of shortest path problems. For example, we may be interested in determining the shortest path from one specified node in the network to another specified node or we may need to find the shortest paths from a specified node to all other nodes. Shortest path between all pairs of nodes in a network are required in some problems while sometimes one wishes to find the shortest path from one given node to another given node that passes through certain specified intermediate nodes. In some application, one requires not only the shortest path but also the second and the third shortest paths. There are instances when the actual shortest path is not required but only the shortest distance. We shall consider two most important shortest-path problems:

i) How to determine (a shortest path) from a specific node S to another specific node T,

ii) How to determine distance (a path) from every node to every other node in the network.

3.5.1.1 Floyd-Warshall Algorithm

The Floyd-Warshall algorithm obtains a matrix of shortest path distance within $O\ \ n^3$ computations. The algorithm is based on inductive arguments developed by an application of a dynamic programming technique.

Let represent the length of the shortest path from node i to node j subject to the condition that this path uses the nodes 1, 2, …, k – 1 as internal nodes. Clearly, $d^{k+1}\ \ i, j$ represent the actual shortest path distances from the node i to j. The algorithm first computes $d^2\ \ i, j$ for all node pairs i and j using $d^1\ \ i, j$, it then computes $d^2\ \ i, j$ for all node pairs i and j. It repeats this process until obtains $d^{k+1}\ \ i, j$ using $d^k\ \ i, j\ = \min\ \ d^k\ \ i, k\ \ , d^k\ \ k, j$. The Floyd-Warshall algorithm remains of interest because it handles negative weight edges correctly.

3.5.1.2 Dijkstras' Algorithm

The Dijkstras' algorithm finds the shortest path from a source s to all other nodes in the network with nonnegative lengths. It maintains a distance label d(i) with each node i, which is an upper bound on the shortest path length from the source node s to any other node j. At any intermediate step, the algorithm divides the nodes of the network under consideration into two groups: those which it designates as permanently labeled (or permanent), and those which it designates as temporarily labeled (or temporal). The distance label to any permanent node represents the shortest distance from the source node to that node. The fundamental idea of the algorithm is to find out from source node s and permanently labeled nodes in the order of their distances from the node s.

Initially, node s is assigned a permanent label of zero (0) and each other node j a temporary label equal to infinity. At each iteration, the label of a node i is its shortest distance from the source node along a path whose internal nodes (i.e. node i other than s or node i itself) are all permanent labeled. The algorithm selects a node i within the minimum temporary label (breaking ties arbitrarily), makes it permanent and reaches out from that node (i.e. it scans all the edges coming out from the node i to update the distances label of adjacent nodes). The algorithm terminates when it has designated all nodes permanent.

Dijkstras' algorithm can be expressed as a set of steps.

Step 1: Assign the permanent label O to the starting vertex.

Step 2: Assign temporary labels to all the vertices that are connected directly to the most recently permanent labeled vertex

Step 3: Choose the vertex with the smallest temporary label and assign a permanent label to that vertex.

Step 4: Repeat steps 2 and 3 until all vertices have permanent labels.

Step 5: Find the shortest path by tracing back through the network.

3.6 Absolute Center Problem

The center problem was first proposed by Sylvester (1857) more than one hundred years ago.

The problem asked for the center of a circle that had the smallest radius to cover all desired destinations. The k-center model and its extensions had been applied in the context of locating facilities such as EMS centers, hospitals, fire station and other public facilities.

For a point x on the network G, let m(x) denote max d(x, n_i) where d(x, n_i), is the cost or distance of the 'shortest' path between x and 'farthest' demand node n_i. The general absolute center problem is

i) Formulated as $\min\left[m(x)\right] = \min\left[\max d(x, n_i)\right]$ subject to $x \in G$

The above formulation is applied in finding the vertex and local centers.

ii) The vertex center (or node center) $x_n \in N$ is a node such that for every node $y \in N, m(x_n) \le m(y)$.

The local center of an edge (p, q) is a point x, on (p, q) such that for every point y on.

iii) (p, q), $m(x_i) \le m(y)$. The absolute center x_a is a point on G such that for every point y on G, (y may be on an edge of G), $m(x_a) \le m(y)$ (Mirchandi P. B, and Francis R. L, 1990)

To find a node center, we compute the matrix of the shortest paths costs (travel times, distances) for all pairs of nodes using the Floyd-Warshalls' or Dijkstras' algorithm, and then choose a node such that the maximum entry in its row in the matrix is smallest among the maximum entries of the rows.

For example figure 3.1 shows a network of an urbanized area with nodes n_1, n_2, n_3, n_4 *and* n_5 representing points where demand for services is generated.

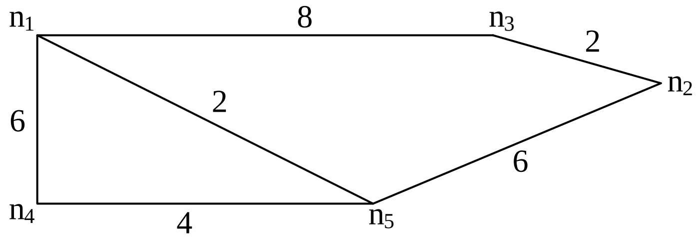


Fig. 3.1: Example of network showing demand nodes and distance

By using the Floyd-Warshalls' algorithm we obtain a matrix of the shortest paths of the network of figure 3.1.

The algorithm computes d(p, q) for all node pairs p and q are shown in table 3.1

Table 3.1: Matrix of shortest path distance for pairs of nodes for fig.3.1

	n_1	n_2	n_3	n_4	n_5	Row max
n_1	0	8	8	6	2	8
n_2	8	0	2	10	6	10
n_3	8	2	0	12	8	12
n_4	6	10	12	0	4	12
n_5	2	6	8	4	0	8

From table 3.1, the smallest among the entries in all rows occurs at either n_1 or n_5, with $m(n_1) = m(n_5) = 8$ and therefore n_1 or n_5 may be taken as the node center.

3.7 Finding the Absolute Centre

The absolute centre minimizes the cost (distance travel time). We look for the path of minimum cost (Euclidean distance) by finding the shortest path among all pairs of vertices using Floyd-Warshalls' or Dijkstras' algorithm. A vertex is a designated point in a network and an edge is a direct distance or arc between two vertices, p and q denoted by c (p, q) which is the edge cost or edge distance.

A shortest path is the total distance between two vertices which may not be direct but passing through other vertices. Thus a shortest path may not be a direct distance or cost between two vertices. This is denoted by d(p, q) and is described as the minimum path cost;

$$d(p,q) = \min \sum_{i=1}^{i-1} c(n_i, n_{i+1})$$

Consider the edge (p, q) with a point *x* on it as shown in figure 3.2

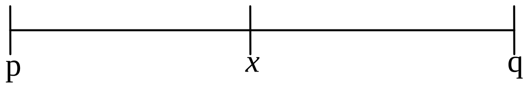


Fig 3.2: Movement of x along edge (p, q)

$$d(x,p) + d(x,q) = c(p,q) \qquad \Rightarrow d(x,p) = c(p,q) - d(x,q)$$

For an undirected graph (a two-way road) with non-negative weight (cost), put

$m(x) = \max d(x, p)$

If x is on an edge or a node we require $m(x^1) \leq m(x)$, where $x_{pq} = x^1$, the distance (cost) of the point *x* on edge (p, q) away from p.

To calculate $m(x^1)$

i. Evaluate all vertices and find the vertex center value and its cost.
ii. Evaluate all edges to find the local center with minimum cost.
iii. Compare the two costs, i.e., the minimum vertex center cost and the minimum edge cost, the lowest of the two costs is the solution, $m(x^1)$

The local center for each edge can be found as shown. Consider an edge (p, q) with a point *x* on it. Assuming we want to move from *x* to n_i where n_i is any node or vertex on the network G, we find the minimum cost by moving to n_i through p or q.

p and q are demand points and n_i is the farthest desired destination.

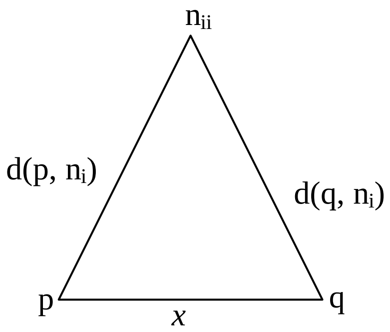


Fig. 3.3 Distance of x to n_i through p and q

$x + d(p, n_i) = c(p,q) - x + d(q, n_i)$ is an edge and its cost is $c \;\; p,q$. From the fig. 3.3 $d(p,x) = x$ and $d(q,x) = c(p,q) - x$ hence, $d(p,x) + d(x,q) = c(p,q)$

The movement from x to n_i (any of the nodes or vertices on the network G) can be done in two directions i.e. through p or q given rise to respectively the equations below;

$y_1 = x + d(p, n_i)$.......................(3.1)

$y_2 = c(p,q) - x + d(q, n_i)$...........................(3.2)

Where y_1, is the distance from x to n_i through p and y_2 is the distance from x to n_i through q. As x moves along the edge (p, q) there will be a point when the two distances or costs would be equal. At this point $y_1 = y_2$ and the kink/maximum/pareto could be found. Solving for the path of equal cost we have:

$$x + d(p, n_i) = c(p,q) - x + d(q, n_i)$$

$$x = \frac{c(p,q) + d(q, n_i) - d(p, n_i)}{2}$$

Where x can be denoted by x_m being the minimum cost. The equations (1) and (2) involving y_1 and y_2 are therefore used to draw graphs for the edge (p, q) from which the local centre can be determined. As n_i assumes all the nodes on the network, a number of equations will be generated under equations (1) and (2). These equations would then be sketched on the same axes in a given range obtained from solving the kink point for the paths of equal distance for each pair of equations.

An upper envelope is then obtained by tracing all paths of lines beyond which there are no higher points for the *x*-value in the given range on the graph. These graphs are indicated by thicks lines. The local center $x_{pq} = x_i$ is the point that minimizes the upper envelope. The absolute center at termination of the process is the point x_a (node centre x_n or local center $x_{pq} = x_i$) that assigned the least value to m(x)

Using figure 3.1 we would evaluate all edges in the given network to illustrate how the absolute centre can be found on a given network as follows:

3.8 Location on edge (n_1n_3)

Consider

$m(x) = y_1 = x + d(p, n_i)$..............(3.3)

$y_2 = c(p,q) - x + d(q, n_i)$………(3.4)

Choosing n_3 as the origin, we let p = n_3 ad q = n_1 such that $0 \le x \le c\ (p,q)$

Putting $n_i = n_1$, ie $i = 1$ then Table 3.1 we $d(p, n_i) = d(n_3, n_1) = 8$

$d(q, n_i) = d(n_i, n_i) = 0$ and $c(p,q) = c(n_3, n_1) = 8$

Thus from (i) and (ii) $y_1 = x + 8$ and $y_2 = 8 - x$. Solving for the path of equal distance or cost, we have $x + 8 = 8 - x, x = 0$. That is, the kink point for the two equations being on left endpoint of the interval. By sketching, the equation $y_1 = x + 8$ falls outside the range and hence rejected.

Putting $n_i = n_3$, *i.e.* $i = 3$ then $d(p, n_i) = d(n_3, n_3) = 0$ and $d(q, n_i) = d(n_1, n_3) = 8$.

Thus $y_1 = x$, $y_2 = 16 - x$ and solving for the path of equal distance or cost, we have $x = 16 - x,\ x = 8$ which is the kink point. It is at the right end point of the interval.

By sketching, the equation $y_2 = 16 - x$ falls outside the range and hence rejected. In both instances above, we can accept and sketch the two equations below

$y_1 = x \quad 0 \le x \le 8$.................(3.5)

$y_2 = 16 - x \quad 0 \le x \le 8$....................(3.6)

Putting $n_i = n_2$, *i.e.* $i = 2$ then $d(p, n_i) = d(n_3, n_2) = 2$ and $d(q, n_i) = d(n_1, n_2) = 8$.

The resulting equations $y_1 = x + 2$ and $y_2 = 16 - x$ when solved for the path of equal distance or cost, we have $x + 2 = 16 - x \Rightarrow x = 7$ which is the kink point. The following equations are then sketched in the given ranges

$y_1 = x + 2 \quad 0 \le x \le 7$.................(3.7)

$y_2 = 16 - x \quad 7 \le x \le 8$....................(3.8)

Putting $n_i = n_4$, *i.e.* $i = 4$ then $d(p, n_i) = d(n_3, n_4) = 12$ and $d(q, n_i) = d(n_1, n_4) = 6$

The resulting equations, $y_1 = x + 12$ and $y_2 = 14 - x$ when solved for the path of equal distance or cost, we have $x + 12 = 14 - x,\ x = 1$ which is the kink point.

The following equations are then sketched within the given ranges.

$y_1 = x + 12 \quad 0 \le x \le 1$.................(3.9)

$y_2 = 14 - x \quad 0 \le x \le 8$....................(3.10)

Putting $n_i = n_5$, *i.e.* $i = 5$ then $d(p, n_i) = d(n_3, n_5) = 8$ and $d(q, n_i) = d(n_1, n_5) = 2$

The resulting equations, $y_1 = x + 8$ and $y_2 = 10 - x$ when solved for the path of equal distance or cost, we have $x + 8 = 10 - x \Rightarrow x = 1$ which is the kink point.

The following equations are then sketched within the given ranges.

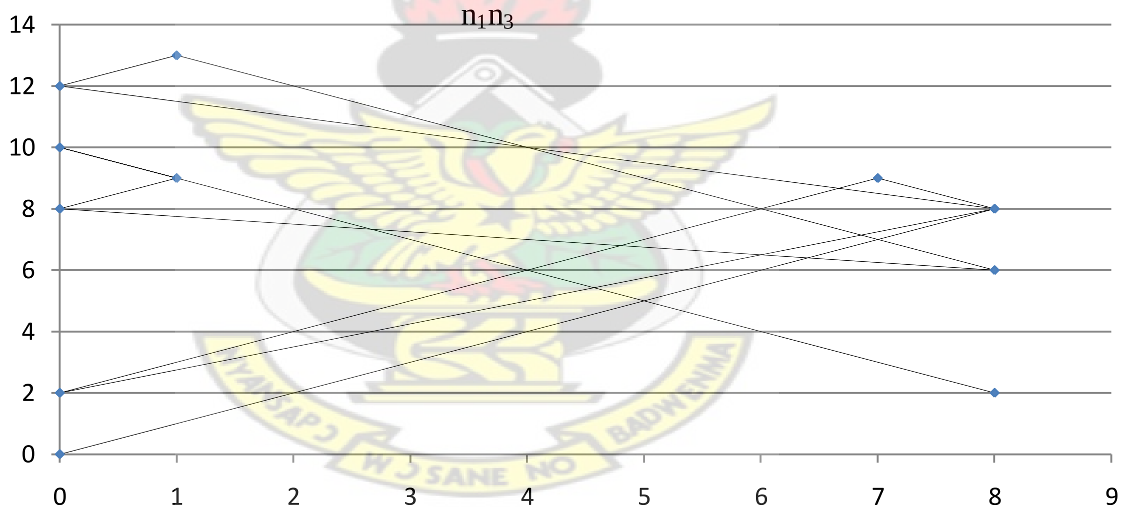
$y_1 = x + 8 \quad 0 \le x \le 1$.................(3.11)

$y_2 = 10 - x \quad 0 \le x \le 8$....................(3.12)

The eight equations are then sketched on the same axes as shown in fig. 3.4. The minimum cost or distance of the path can be found from the graph using the upper envelope

$x_{n_1 n_3} = 8 \;\; and \;\; m(x_l) = 8$

The thick line represents the upper envelope of the graph and the minimum point on it is the local center



$X_{n_3} = 8 \qquad m(x_l) = 8$

Fig. 3.4 Graph showing upper envelope and local center for edge n_1n_3

Thus the minimum cost on edge (n_1, n_3) i.e. $x_{n_1 n_3}$, is selected by considering the point corresponding to the maximum cost for all nodes. In the example above, the minimum cost/distance for edge (n_1, n_3) is given as $x_{n_1, n_3} = 8 \;\; and \;\; m(x_1) = 8$ units.

3.9 Construction of the Upper Envelope

After sketching all the equations resulting from the location on edge (n_1,n_3) on the same axes as shown in fig. 3.4 there is the need to construct an 'upper envelope' which gives the minimum cost/distance of a shortest path from *x* to a farthest node on the given edge. To construct the upper envelope, we trace all paths of lines beyond which there are no higher points for the same x-value in the given range. This path is indicated by a thick line as shown in the figure 3.4

3.10 Local center

For each edge (p, q), the local center is found by plotting $d(x,n_i)$ for each node $n_i \in N$ where $0 \le x \le c\ (p,q)$.

The local center $x_{pq} = x_l$ is the point that minimizes the upper envelope. The absolute center x_a is the minimum point among the local centers. This occurs on the edge $n_5 n_2$ with $d(x_a, n_5) = 2$ and $d(x_a, n_2) = 4$, $m(x_a) = 6$ which implies, the maximum distance from point *x* to the farthest node is 6 units that is both nodes n_5 and n_2 hence the optimum location of the facility is on edge (n_2,n_5) which is 2 units from node n_5 and 4 units from node n_2.

Finding a single absolute center of a network is more involving. In practice, where a network has a large number of nodes, there would be equally a large number of edges to be enumerated for their respective local centers.

Table 3.2 local centers and corresponding cost or distance for figure 3.1

Edge	Edge distance	Local centre X_l	Cost $m(x_l)$
(n_1, n_3)	8	At n_3	8 units
(n_2, n_5)	6	2 units from n_3	6 units
(n_1, n_5)	2	At n_1 or n_5	8 units
(n_1, n_4)	6	At n_1	8 units
(n_5, n_4)	4	At n_5	8 units
(n_3, n_2)	2	At n_2	10 units

The computation of the equations and graphs for the locations on the other edges in the network are shown in appendix A

Fortunately, as indicated in the propositions (i) and (ii) below, many edges do not need to be explicitly enumerated for their respective local centers.

Preposition (i)

For the set of all points x on a fixed edge (p,q) of G, the maximum distance function $m(x)$ is piecewise linear whose slope is always $+1 or -1$.

Preposition (ii)

For an edge (p,q), the local center satisfies the equation,

$m(x_l) \geq \dfrac{m(p)+m(q)-c(p,q)}{2}$ where $c(p,q)$ denotes the cost of edge (p,q).

Proof

Consider any point on the edge p,q. Let $x:0\le x\le c\ \ p,q$ denote the point p such that $x=0$ and the point $x=c\ \ p,q$ denote q. We take $d\ \ x,p$ to be x and $d\ \ x,q$ to be $c\ \ p,q\ -x$. The cost $d\ \ x,p$ of a shortest path between x and the farthest demand node p is piecewise linear with a slope $+1 or -1$ at each point of x. Its value at $x=0$ is $m\ \ p$ and its value at $x=c\ \ p,q$ is $m\ \ q$ where $m\ \ p$ and $m\ \ q$ are nodes centers for nodes p and q. Hence,

$m\ \ x\ \ge m\ \ p\ \ -x\ \left[For\, all\, x:0\le x\le c\ \ p,q\ \right]$......................(3.13)

$m\ \ x\ \ge m\ \ q\ -\ c\ \ p,q\ -x\ \left[For\, all\, x:0\le x\le c\ \ p,q\ \right]$......................(3.14)

By adding the two inequalities (i) and (ii), we obtain

$$m\ \ x_l\ \ge \frac{m\ \ p\ +m\ \ q\ -c\ \ p,q}{2}$$

Where x_l simultaneously satisfies the two inequalities above.

From these preposition and from observation that, by definition the maximum distance associated with the node center must be greater than or equal to the corresponding distance from the absolute i.e. $m\ \ x_n\ \ge m\ \ x_a$, we can derive the following test:

If for edge $p,q\ ,m\ \ x_{\mathrm{n}}\ \le \dfrac{m\ \ p\ +m\ \ q\ -c\ \ p,q}{2}$ then the local center x_l of p,q cannot improve on $m\ \ x_n$ and therefore need not be found. This test which takes advantage of the fact that it is very simple to find the local center x_l often leads to considerable reduction in the computation effort required to obtain the absolute center. With respect to the five-node, six-edged network in fig. 3.1, we found easily that the node center is at nodes n_1 and n_5 and that $m(x_n) = m(n_1) = m(n_5) = 8$

On application of the test to the six edges of the network, we obtain

Edge (n_1, n_3) $: \frac{m(n_1) + m(n_3) - c(n_1, n_3)}{2} = \frac{8+12-8}{2} = 6 < 8$

Table 3.3: Results of edges whose local centers are to be determined

Edge	For edge (p,q): $\frac{m(p)+m(q)-c(p,q)}{2}$	m$(X_n) = 8 \leq \frac{m(p)+m(q)-c(p,q)}{2}$
(n_1, n_3)	6	6< 8
(n_2, n_3)	10	$10 > 8$
(n_2, n_5)	6	6<8
(n_1, n_5)	7	7<8
(n_1, n_4)	7	7<8
(n_4, n_5)	8	8=8

The results of the test above clearly suggest that the local center needs to be found for only edges. Edges (n_1, n_3), (n_2, n_5), (n_1, n_5) *and* (n_1, n_4). This makes significant savings in the computational effort and time.

3.11 Summary

Planar, network and discrete location models which may be used to represent location problems and their respective assumptions have been discussed.

A detailed explanation of p-center problem a heuristic method which is the means of locating a fire station at the New Juaben Municipality has been provided.

The next chapter is data collection and analyses.

CHAPTER FOUR

DATA COLLECTION AND ANALYSES

4.1 Introduction

The chapter provides New Juaben municipal map (Appendix B) and selected demand areas specifying the road distances between them. Data was obtained from municipal planning office and municipal town planning department and would be analyzed using the center-problem to identify where a fire station has to be optimally located in the municipality.

Locations considered are:

A – Koforidua

B – Effiduase

C – Baako Krom

D – Koforidua Ada

E – Affian

F – Nyamekrom

G – Asokore

H – Agyeso

I – Adweso

J – Oyoko

K – Kwakyekrom

L – Mile 50

M – Wawase

N – Jumapo

O – Kentenkiren

P – Begrey

Q – Agricultural station

R – Poposo

S – Suhyen

T – Akwadum

Table 4.1: Selected edges specifying the road distance between them.

NO.	EDGE CONSIDERED	DISTANCE (METRES)
1	(A, B)	2200
2	(A, C)	4000
3	(A, D)	2300
4	(A, R)	5500
5	(A, H)	3800
6	(A, I)	4100
7	(B, G)	1100
8	(B, L)	7200
9	(B, M)	4800
10	(C, B)	5000
11	(C, F)	750
12	(C, E)	2300
13	(C, G)	5800
14	(D, R)	5000
15	(E, F)	1800
16	(F, G)	6600
17	(G, M)	4600
18	(G, J)	3200
19	(G, S)	8400
20	(H, I)	1500
21	(I, K)	1200
22	(I, L)	900
23	(J, N)	3700
24	(J, S)	5200
25	(K, O)	2500
26	(M, P)	3000
27	(M, Q)	3850
28	(M, T)	4750
29	(N, S)	1700

Developed network of data of table 4.1 having capital letters as nodes or vertices and the figures as distances between pairs of nodes

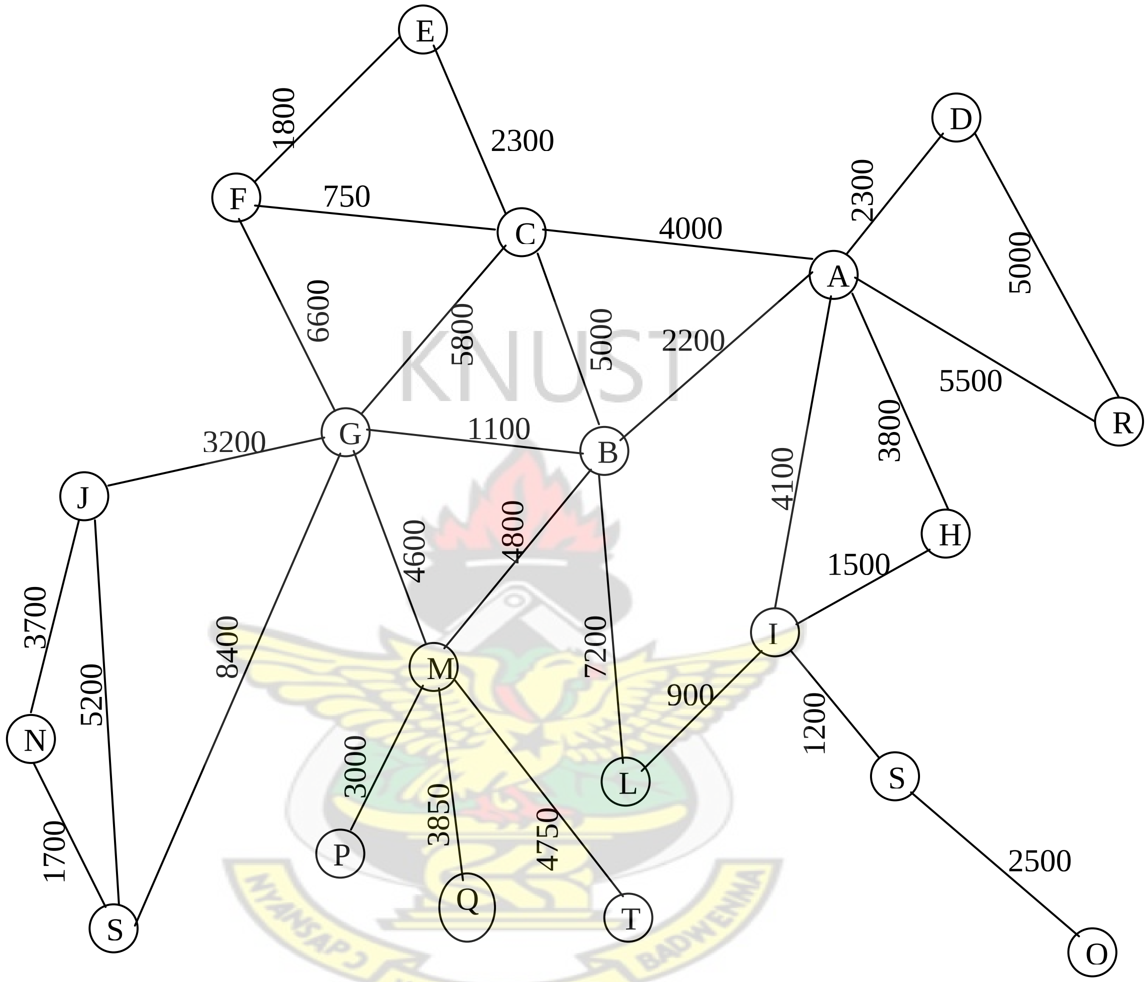


Fig. 4.1: Developed Network for selected demand destinations of New Juaben Municipality.

4.2 All Pairs Shortest Path for the Data Collected

From the network in figure 4.1 the minimum distance matrix d(i, j), that is the matrix of the shortest path using the Floyd-Warshalls' algorithm was obtained and is shown in Table 4.2

Table 4.2 : Matrix of shortest path distance for all pairs of nodes from fig 4.1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	Row max
A	-	2200	4000	2300	6300	4750	3300	3800	4100	6500	5300	5000	7000	10200	7800	10000	10850	5500	11700	11750	11750
B	2200	-	5000	4500	7300	5750	1100	6000	6300	4300	7500	7200	4800	8000	10000	7800	8650	7700	9500	9550	10000
C	4000	5000	-	6300	2300	750	5800	7800	8100	9000	9300	9000	9800	12700	11800	12800	13650	9550	14200	14550	14550
D	2300	4500	6300	-	8600	6300	5600	6100	6400	8000	7600	7300	9300	12500	10000	12300	13150	5000	14000	14050	14050
E	6300	7300	2300	8600	-	1800	8100	10100	10400	11300	11600	11300	12100	15000	14100	15100	15950	11800	16500	16850	16850
F	4800	5800	800	7100	1800	-	6600	8550	8900	9800	10100	9800	10600	13500	12600	14200	15050	10300	15000	15950	15950
G	3300	1100	5800	5600	8100	6600	-	7100	7400	3200	8600	8300	4600	6900	11100	7600	8450	8800	8400	9350	11100
H	3800	6000	7800	6100	10100	8550	7100	-	1500	10300	2700	2400	10800	14000	5200	13800	14560	9300	15500	15550	15550
I	4100	6300	8100	6400	10400	8850	7400	1500	-	10600	1200	900	11100	14300	3700	14100	14950	9600	18300	15850	18300
J	6500	4300	9000	8800	11300	9800	3200	10300	10600	-	11800	11500	7800	3700	14300	10800	11650	10020	5200	12550	14300
K	5300	7500	9300	7600	11600	10050	8600	2700	1200	11800	-	2100	12300	15500	2500	15300	16150	10800	17000	17050	17050
L	5000	7200	9000	7300	11300	9750	8300	2400	900	11500	2100	-	12000	15200	4600	15000	15850	10500	16700	16750	16750
M	7000	4800	9800	9300	12100	11200	4600	10800	11100	7800	12300	12000	-	11500	14800	3000	3850	12500	13000	4750	14800
N	10200	8000	12700	12500	15000	13500	6900	14000	14300	3700	15500	15200	11500	-	18000	14500	15350	15700	1700	16250	18000
O	7800	10000	11800	10100	14100	12550	11100	5200	3700	14300	2500	4600	14800	18000	-	17800	18650	13300	19500	19550	19550
P	10000	7800	12800	12300	15100	14200	7600	13800	14100	10800	15300	15000	3000	14500	17800	-	6850	15500	16000	7750	17800
Q	10850	8650	13650	13150	15950	15050	8450	14650	14950	11650	16150	15850	3850	15350	18650	6850	-	16350	16850	8600	18650
R	5500	7700	9550	5000	11800	10250	8800	9300	9600	10020	10800	10500	12500	15700	13300	15500	16350	-	17200	17250	17250
S	11700	9500	14200	14000	16500	15000	8400	15500	18300	5200	17000	16700	13000	1700	19500	16000	16850	17200	-	17750	19500
T	11750	9550	14550	14050	16850	15950	9350	15550	15850	12550	17050	16750	4750	16250	19550	7750	8600	17250	17750	-	19550

4.3 Results

4.3.1 Locating the Vertex/Node Center

Row 1 represents demand nodes of the network and Row 2 represents row maximum from table 4.2

(a) Table 4.3 Vertex/Node Center from table 4.2

NODE	A	B	C	D	E	F	G	H	I	J
ROW MAX	11750	10000	14550	14050	16850	15950	11100	15550	18300	14300

NODE	K	L	M	N	O	P	Q	R	S	T
ROW MAX	17050	16750	14800	18000	19550	17800	18650	17250	19500	19550

The node or vertex center (x_n) is chosen as the smallest among the maximum entries of all rows in the matrix. From Table 4.2 the row with the minimum among the maximum entries occurs at node/vertex B with a maximum distance (cost) of 10000 metres. Thus the node/vertex centre for the network in figure 4.1 is B, hence $m(B) = 10000$.

4.3.2 Locating the Local Centers

$$Edge\ (A,B) = \frac{m(A) + m(B) - c(A,B)}{2} = \frac{11750 + 10000 - 2200}{2} = 9775 < 10,000$$

Table 4.4 test for edges whose local centers are to be determined for the developed network.

Edge	For edge (p,q): $\frac{m(p)+m(q)-c(p,q)}{2}$	$m(X_n) = 10000 \leq \frac{m(p)+m(q)-c(p,q)}{2}$
(A,B)	9775	$9775 < 10000$
(A,C)	11150	$11150 > 10000$

(A,D)	11750	11750> 10000
(A,R)	11750	11750> 10000
(A,H)	11750	11750> 10000
(A,I)	12975	12975> 10000
(B,G)	10000	10000= 10000
(B,L)	9425	9425< 10000
(B,M)	10000	10000= 10000
(C,B)	9775	9775< 10000
(C,F)	148750	148750> 10000
(C,E)	14550	14550> 10000
(C,G)	9925	9925< 10000
(D,R)	13150	13150> 10000
(E,F)	15500	15500> 10000
(F,G)	1675	1675> 10000
(G,M)	10650	10650> 10000
(G,J)	11100	11100> 10000
(G,S)	11100	11100> 10000
(H,I)	16175	16175> 10000
(I,K)	17075	17075> 10000
(I,L)	17075	17075> 10000
(J,N)	14300	14300> 10000
(J,S)	14300	14300> 10000
(K,O)	17050	17050> 10000
(M,P)	15925	15925> 10000

(M,Q)	15925	15925> 10000
(M,T)	15925	15925> 10000
(N,S)	17900	17900> 10000

From table 4.4 edges whose local centers are to be determined are (A,B), (B,L), (C,B) and (C,G)

Location on edge (A, B)

Let $A = P$, $B = q$ such that $0 \le x \le c\ \ p,q$ and $c\ \ p,q\ = c\ \ A,B\ = 2200$

Putting $n_i = A,$ then $d\ \ p,n_i\ = d\ \ A,A\ = 0, d\ \ q,n_i\ = d\ \ B,A\ = 2200$

$y_1 = x$ and $y_2 = 4400 - x$ when solved

$x = 4400 - x, x = 2200$ Kink point

$y_1 = x \quad 0 \le x \le 2200$.................. 4.1

Putting $n_i = B,$ then $d\ \ p,n_i\ = d\ \ A,B\ = 2200, d\ \ q,n_i\ = d\ \ B,B\ = 0$

$y_1 = x +$ and $y_2 = 2200 - x$ when solved

$x + 2200 = 2200 - x, x = 0$ Kink point

$y_2 = 2200 - x \quad 0 \le x \le 2200$.................. 4.2

Putting $n_i = C,$ then $d\ \ p,n_i\ = d\ \ A,C\ = 4000, d\ \ q,n_i\ = d\ \ B,C\ = 5000$

$y_1 = x + 4000$ and $y_2 = 7200 - x$ when solved

$x + 4000 = 7200 - x, \Rightarrow x = 1600$ Kink point

$y_1 = x + 4000 \quad 0 \le x \le 1600$.................. 4.3

$$y_2 = 7200 - x \quad 1600 \le x \le 2200 .... \; 4.4$$

Putting $n_i = D$, then $d(p, n_i) = d(A, D) = 2300, d(q, n_i) = d(B, D) = 4500$

$y_1 = x + 2300$ and $y_2 = 6700 - x$ when solved

$$x + 2300 = 6700 - x, \Rightarrow x = 2200 \quad \text{Kink point}$$

$$y_1 = x + 2300 \quad 0 \le x \le 2200 .... \; 4.5$$

Putting $n_i = E$, then $d(p, n_i) = d(A, E) = 6300, d(q, n_i) = d(B, E) = 7300$

$y_1 = x + 6300$ and $y_2 = 9500 - x$ when solved

$$x + 6300 = 9500 - x, \Rightarrow x = 1600 \quad \text{Kink point}$$

$$y_1 = x + 6300 \quad 0 \le x \le 1600 .... \; 4.6$$

$$y_2 = 9500 - x \quad 1600 \le x \le 2200 .... \; 4.7$$

Putting $n_i = F$, then $d(p, n_i) = d(A, F) = 4750, d(q, n_i) = d(B, F) = 5750$

$y_1 = x + 4750$ and $y_2 = 6950 - x$ when solved

$$x + 4750 = 6950 - x, \Rightarrow x = 1100 \quad \text{Kink point}$$

$$y_1 = x + 4750 \quad 0 \le x \le 1100 .... \; 4.8$$

$$y_2 = 6950 - x \quad 1100 \le x \le 2200 .... \; 4.9$$

Putting $n_i = G$, then $d(p, n_i) = d(A, G) = 3300, d(q, n_i) = d(B, G) = 1100$

$y_1 = x + 3300$ and $y_2 = 5500 - x$ when solved

$$x + 3300 = 5500 - x, \Rightarrow x = 1100 \quad \text{Kink point}$$

$y_1 = x + 3300 \quad 0 \le x \le 1100$.................. 4.10

$y_2 = 5500 - x \quad 1100 \le x \le 2200$.................. 4.11

Putting $n_i = H$, then $d \;\; p, n_i \;\; = d \;\; A, H \;\; = 3800, d \;\; q, n_i \;\; = d \;\; B, H \;\; = 6000$

$y_1 = x + 3800$ and $y_2 = 8200 - x$ when solved

$x + 3800 = 8200 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 3800 \quad 0 \le x \le 2200$.................. 4.12

Putting $n_i = I$, then $d \;\; p, n_i \;\; = d \;\; A, I \;\; = 4100, d \;\; q, n_i \;\; = d \;\; B, I \;\; = 6300$

$y_1 = x + 4100$ and $y_2 = 8500 - x$ when solved

$x + 4100 = 8500 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 4100 \quad 0 \le x \le 2200$.................. 4.13

Putting $n_i = J$, then $d \;\; p, n_i \;\; = d \;\; A, J \;\; = 6500, d \;\; q, n_i \;\; = d \;\; B, J \;\; = 4300$

$y_1 = x + 6500$ and $y_2 = 6500 - x$ when solved

$x + 6500 = 6500 - x, \Rightarrow x = 0$ Kink point

$y_2 = 6500 - x \quad 0 \le x \le 2200$.................. 4.14

Putting $n_i = K$, then $d \;\; p, n_i \;\; = d \;\; A, K \;\; = 5300, d \;\; q, n_i \;\; = d \;\; B, K \;\; = 7500$

$y_1 = x + 5300$ and $y_2 = 9700 - x$ when solved

$x + 5300 = 9700 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 5300 \quad 0 \le x \le 2200$.................. 4.15

Putting $n_i = L$, then $d(p, n_i) = d(A, L) = 0, d(q, n_i) = d(B, L) = 2200$

$$y_1 = x + 5000 \text{ and } y_2 = 9400 - x \text{ when solved}$$

$$x + 5000 = 9400 - x, \Rightarrow x = 2200 \quad \text{Kink point}$$

$$y_1 = x + 5000 \quad 0 \le x \le 2200 .... \; 4.16$$

Putting $n_i = M$, then $d(p, n_i) = d(A, M) = 7000, d(q, n_i) = d(B, M) = 4800$

$$y_1 = x + 7000 \text{ and } y_2 = 7000 - x \text{ when solved}$$

$$x + 7000 = 7000 - x, \Rightarrow x = 0 \quad \text{Kink point}$$

$$y_1 = 7000 - x \quad 0 \le x \le 2200 .... \; 4.17$$

Putting $n_i = N$, then $d(p, n_i) = d(A, N) = 10200, d(q, n_i) = d(B, N) = 8000$

$$y_1 = x + 10200 \text{ and } y_2 = 12400 - x \text{ when solved}$$

$$x + 10200 = 12400 - x, \Rightarrow x = 1100 \quad \text{Kink point}$$

$$y_1 = x + 10200 \quad 0 \le x \le 1100 .... \; 4.18$$

$$y_2 = 12400 - x \quad 0 \le x \le 2200 .... \; 4.19$$

Putting $n_i = O$, then $d(p, n_i) = d(A, O) = 7800, d(q, n_i) = d(B, O) = 2200$

$$y_1 = x + 7800 \text{ and } y_2 = 12200 - x \text{ when solved}$$

$$x + 7800 = 1200 - x, \Rightarrow x = 2200 \quad \text{Kink point}$$

$$y_1 = x + 7800 \quad 0 \le x \le 2200 .... \; 4.20$$

Putting $n_i = P$, then $d(p, n_i) = d(A, P) = 10000, d(q, n_i) = d(B, P) = 7800$

$y_1 = x + 10000$ and $y_2 = 10000 - x$ when solved

$x + 10000 = 10000 - x, \Rightarrow x = 0$ Kink point

$y_2 = 10000 - x \quad 0 \le x \le 2200$.................. 4.21

Putting $n_i = Q$, then $d(p, n_i) = d(A, Q) = 10850, d(q, n_i) = d(B, Q) = 8650$

$y_1 = x + 10850$ and $y_2 = 10850 - x$ when solved

$x + 10850 = 10850 - x, \Rightarrow x = 0$ Kink point

$y_2 = 10850 - x \quad 0 \le x \le 2200$.................. 4.22

Putting $n_i = R$, then $d(p, n_i) = d(A, R) = 0, d(q, n_i) = d(B, R) = 7700$

$y_1 = x + 5500$ and $y_2 = 9900 - x$ when solved

$x + 5500 = 9900 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 5500 \quad 0 \le x \le 2200$.................. 4.23

Putting $n_i = S$, then $d(p, n_i) = d(A, S) = 11700, d(q, n_i) = d(B, S) = 9500$

$y_1 = x + 11700$ and $y_2 = 11700 - x$ when solved

$x + 11700 = 11700 - x, \Rightarrow x = 0$ Kink point

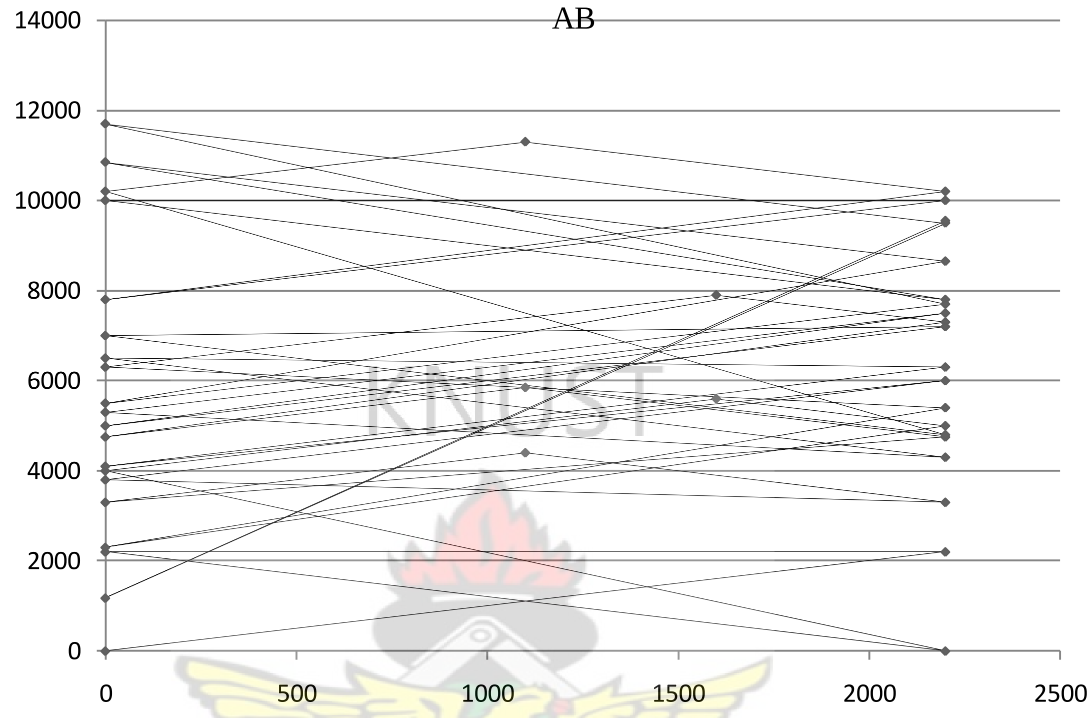
$y_2 = 11700 - x \quad 0 \le x \le 2200$.................. 4.24

Putting $n_i = T$, then $d(p, n_i) = d(A, T) = 11750, d(q, n_i) = d(B, T) = 9550$

$y_1 = x + 11750$ and $y_2 = 11750 - x$ when solved

$x + 11750 = 11750 - x, \Rightarrow x = 0$ Kink point

$y_2 = 11750 - x \quad 0 \le x \le 2200$................. 4.25



$X_B = 2200, m(x_l) = 10200$

Fig. 4.2 Graph showing upper envelope and local center for edge AB

Computations for equations for the locations on edges (B,L), (C,B) and (C,G) and graphs to determine their local centers are shown in appendices D and E respectively.

(b) Local Center

Table 4.5 below has column one as edge number, column two is edge name, column three is location of edge center and column four is the least point of the upper envelope of each of the edges

Table 4.5 Shows local centers and cost for edges (A, B), (B,L), (C, B), and (C,G)

No.	Edge	Local Center (x_l)	cost $m(x_l)$
1	(A, B)	At node B	10200
2	(B, L)	At node B	10000
3	(C, B)	At node B	10000
4	(C, G)	At node G	11100

The least of the local centers of table 4.5 is 10000 and occurred at node B

4.4 Discussion

From table 4.3 the node or vertex center, $m(x_n)$ is 10000 metres. The least of the local centers, $m(x_l)$ from table 4.4 is 10000 metres. The least of the local centers is compared with the vertex or node center and the minimum is taken as the absolute center. By inspection the minimum of the node center and the least of the local centers is 10000. Hence the absolute center, $m(x_a)$ is 10000 metres and occurred at node *B* of the network.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The main objective of the study was to use the Absolute center-heuristic method to optimally locate a fire station in the New Juaben Municipality. The following findings were realized

1. The optimal location of the fire station for selected demand destinations of New Juaben Municipality was found to be at Effiduase (node B of the network).
2. The optimal service coverage distance was found to be 10000 metre radius from node B.

5.2 Recommendation

From the results obtained, the following recommendations are made:

1. No fire station should be sited without the appropriate scientific technique.
2. Studies be carried out to find the optimal locations and service coverage areas for other fire stations.

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APPENDIX A

Computations of the equations for the locations

n_1, n_4 , n_1, n_5 , n_4, n_5 , n_5, n_2 *and* n_3, n_2

LOCATION ON EDGE n_1, n_4

Choosing n_1 as the origin, let $p = n$ and $q = n_4$ such that $0 \le x \le c(p,q)$

Putting $n_i = n_1, i.e.\ i = 1$ then $d(p, n_i) = d(n_1, n_1) = 0, d(q, n_i) = d(n_4, n_1) = 6$ and

$c(p,q) = c(n_1, n_4) = 6,$ The resulting equations $y_1 = x$ and $y_2 = 12 - x$ when solved

$x = 12 - x \Rightarrow x = 6$ (Kink point)

$y_1 = x \quad 0 \le x \le 6$.......................$(i)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d(p, n_i) = d(n_1, n_2) = 8, d(q, n_i) = d(n_4, n_1) = 10$

$y_1 = x + 8$ and $y_2 = 16 - x$ when solved

$x + 8 = 16 - x \Rightarrow x = 4$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 4$.......................$(ii)$

$y_2 = 16 - x \quad 4 \le x \le 6$.......................$(iii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d(p, n_i) = d(n_1, n_3) = 8, d(q, n_i) = d(n_4, n_3) = 12$

$y_1 = x + 8$ and $y_2 = 18 - x$ when solved

$x + 8 = 18 - x \Rightarrow x = 5$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 5$.......................$(iv)$

$y_2 = 18 - x \quad 5 \le x \le 6$.......................$(v)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d(p, n_i) = d(n_1, n_4) = 6, d(q, n_i) = d(n_4, n_4) = 0$

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x+6=6-x \Rightarrow x=0$ (Kink point)

$y_2=6-x \quad 0 \le x \le 6$.......................$(vi)$

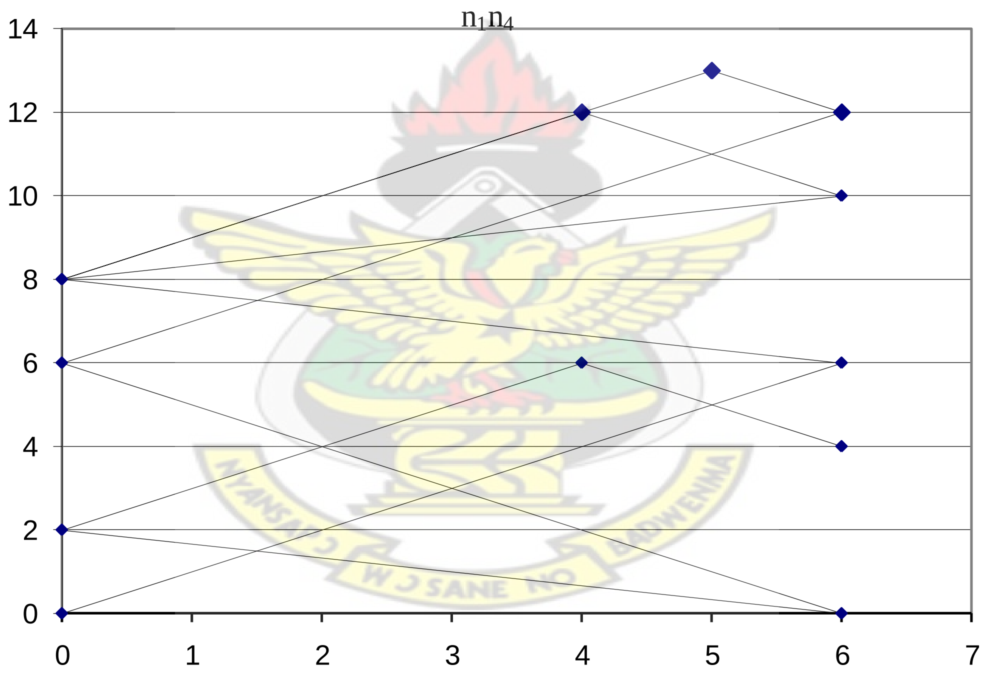
Putting $n_i = n_5, i.e.\ i=5$ then $d\ \ p,n_i\ \ =d\ \ n_1,n_5\ \ =8, d\ \ q,n_i\ \ =d\ \ n_4,n_5\ \ =4$

$y_1 = x+2$ and $y_2 = 10-x$ when solved

$x+2=10-x \Rightarrow x=4$ (Kink point)

$y_1 = x+2 \quad 0 \le x \le 4$.......................$(vii)$

$y_2 = 10-x \quad 4 \le x \le 6$.......................$(viii)$



$X_{n_1 n_4} = 0 \qquad m(x_l) = 8$

LOCATING ON EDGE (n_1, n_5)

Choosing $n_1 = p$ and $q = n_5$ such that $c(p,q) = c(n_1,n_5) = 2$

Putting $n_i = n_1, i.e.\ i = 1$ then $d(p,n_i) = d(n_1,n_1) = 0, d(q,n_i) = d(n_5,n_1) = 2$

$y_1 = x$ and $y_2 = 4 - x$ when solved

$x = 4 - x \Rightarrow x = 2$ (Kink point)

$y_1 = x \quad 0 \le x \le 2$.......................$(i)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d(p,n_i) = d(n_1,n_2) = 8, d(q,n_i) = d(n_5,n_2) = 2$

$y_1 = x + 8$ and $y_2 = 8 - x$ when solved

$x + 8 = 8 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 8 - x \quad 0 \le x \le 2$.......................$(ii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d(p,n_i) = d(n_1,n_3) = 8, d(q,n_i) = d(n_5,n_3) = 8$

$y_1 = x + 8$ and $y_2 = 10 - x$ when solved

$x + 8 = 10 - x \Rightarrow x = 1$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 1$.......................$(iv)$

$y_2 = 10 - x \quad 1 \le x \le 2$.......................$(v)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d(p,n_i) = d(n_1,n_4) = 6, d(q,n_i) = d(n_5,n_4) = 4$

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x + 6 = 6 - x \Rightarrow x = 0$ (Kink point)

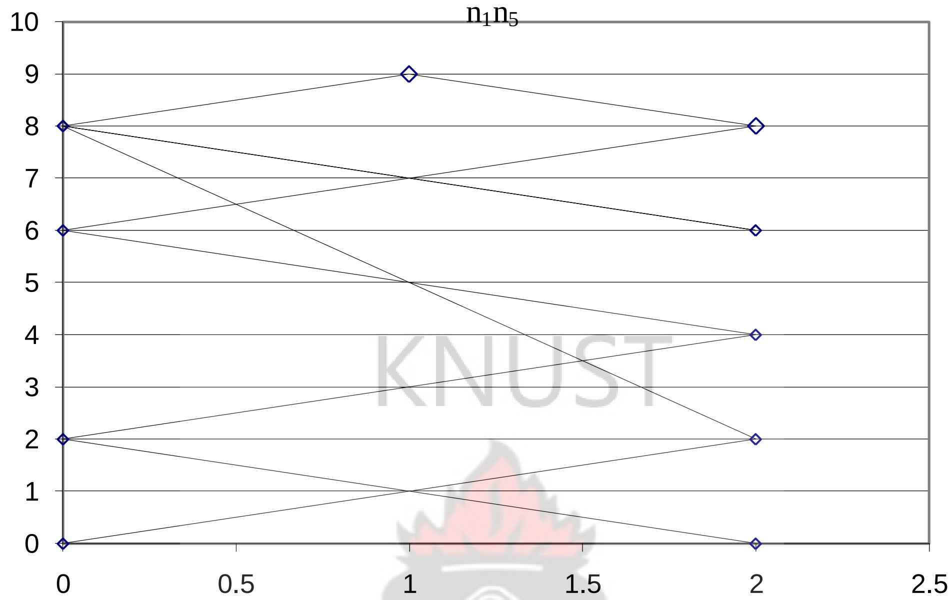
$y_2 = 6 - x \quad 0 \le x \le 2$.......................$(v)$

Putting $n_i = n_5, i.e.\ i = 5$ then $d(p,n_i) = d(n_1,n_5) = 2, d(q,n_i) = d(n_5,n_5) = 0$

$y_1 = x + 2$ and $y_2 = 2 - x$ when solved

$x+2=2-x \Rightarrow x=0$ (Kink point)

$y_2 = 2-x \quad 0 \le x \le 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(vi)$



$X_{n_1n_5} = 0 \text{ or } 2 \qquad m(x_l) = 8$

LOCATING ON EDGE n_4, n_5

Choosing $n_4 = p$ and $q = n_5$ such that $c(p,q) = c(n_4,n_5) = 4$

Putting $n_i = n_1, i.e.\ i=1$ then $d(p,n_i) = d(n_4,n_1) = 6, d(q,n_i) = d(n_5,n_1) = 2$

$y_1 = x+6$ and $y_2 = 6-x$ when solved

$x+6=6-x \Rightarrow x=0$ (Kink point)

$y_1 = 6-x \quad 0 \le x \le 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$

Putting $n_i = n_2, i.e.\ i=2$ then $d(p,n_i) = d(n_4,n_2) = 10, d(q,n_i) = d(n_5,n_2) = 6$

$y_1 = x+10$ and $y_2 = 10-x$ when solved

$x+10=10-x \Rightarrow x=0$ (Kink point)

$y_2 = 10 - x \quad 0 \le x \le 4$.......................$(ii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d(p, n_i) = d(n_4, n_3) = 12, d(q, n_i) = d(n_5, n_3) = 8$

$y_1 = x + 12$ and $y_2 = 12 - x$ when solved

$x + 12 = 12 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 12 - x \quad 0 \le x \le 4$.......................$(iii)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d(p, n_i) = d(n_4, n_4) = 0, d(q, n_i) = d(n_5, n_4) = 4$

$y_1 = x$ and $y_2 = 8 - x$ when solved

$x = 8 - x \Rightarrow x = 4$ (Kink point)

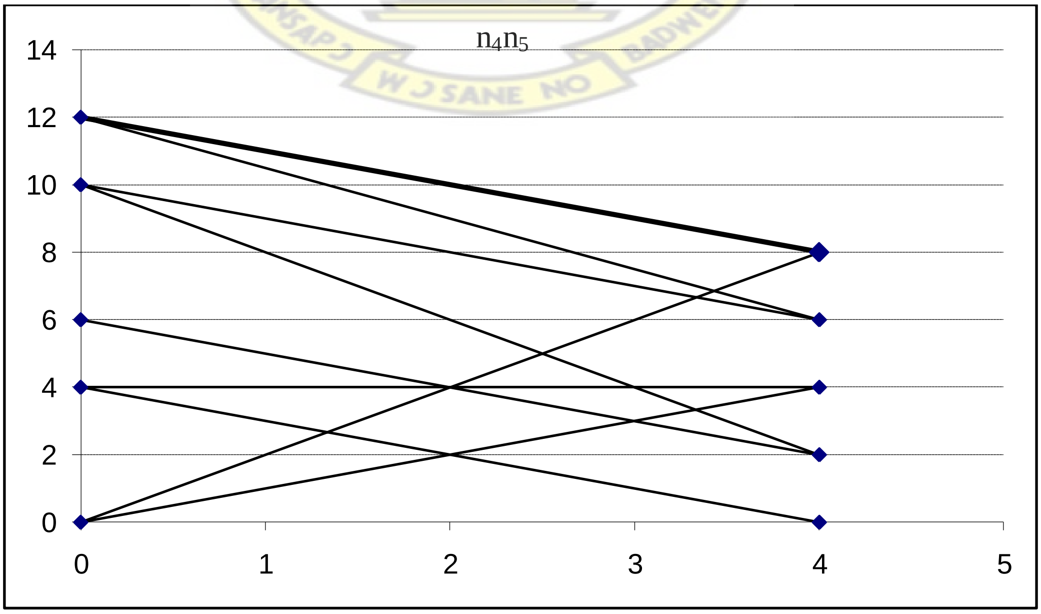
$y_1 = x \quad 0 \le x \le 4$.......................$(iv)$

Putting $n_i = n_5, i.e.\ i = 5$ then $d(p, n_i) = d(n_4, n_5) = 4, d(q, n_i) = d(n_5, n_5) = 0$

$y_1 = x + 4$ and $y_2 = 4 - x$ when solved

$x + 4 = 4 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 4 - x \quad 0 \le x \le 4$.......................$(v)$



$X_{n_4 n_5} = 4 \qquad m(x_l) = 8$

LOCATING ON EDGE n_5, n_2

Choosing $p = n_5$ and $q = n_2$ such that $c\ \ p,q\ \ = c\ \ n_5, n_2\ \ = 6$

Putting $n_i = n_1, i.e.\ i = 1$ then $d\ \ p,n_i\ \ = d\ \ n_5,n_1\ \ = 2, d\ \ q,n_i\ \ = d\ \ n_2,n_1\ \ = 8$

$y_1 = x + 2$ and $y_2 = 14 - x$ when solved

$x + 2 = 14 - x \Rightarrow x = 6$ (Kink point)

$y_1 = x + 2\ \ 0 \le x \le 6$.......................$(i)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d\ \ p,n_i\ \ = d\ \ n_5,n_2\ \ = 6, d\ \ q,n_i\ \ = d\ \ n_2,n_2\ \ = 0$

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x + 6 = 6 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 6 - x\ \ 0 \le x \le 6$.......................$(ii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d\ \ p,n_i\ \ = d\ \ n_5,n_3\ \ = 8, d\ \ q,n_i\ \ = d\ \ n_2,n_3\ \ = 2$

$y_1 = x + 8$ and $y_2 = 8 - x$ when solved

$x + 8 = 8 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 8 - x\ \ 0 \le x \le 6$.......................$(iii)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d\ \ p,n_i\ \ = d\ \ n_5,n_4\ \ = 4, d\ \ q,n_i\ \ = d\ \ n_2,n_4\ \ = 10$

$y_1 = x + 4$ and $y_2 = 16 - x$ when solved

$x + 4 = 16 - x \Rightarrow x = 6$ (Kink point)

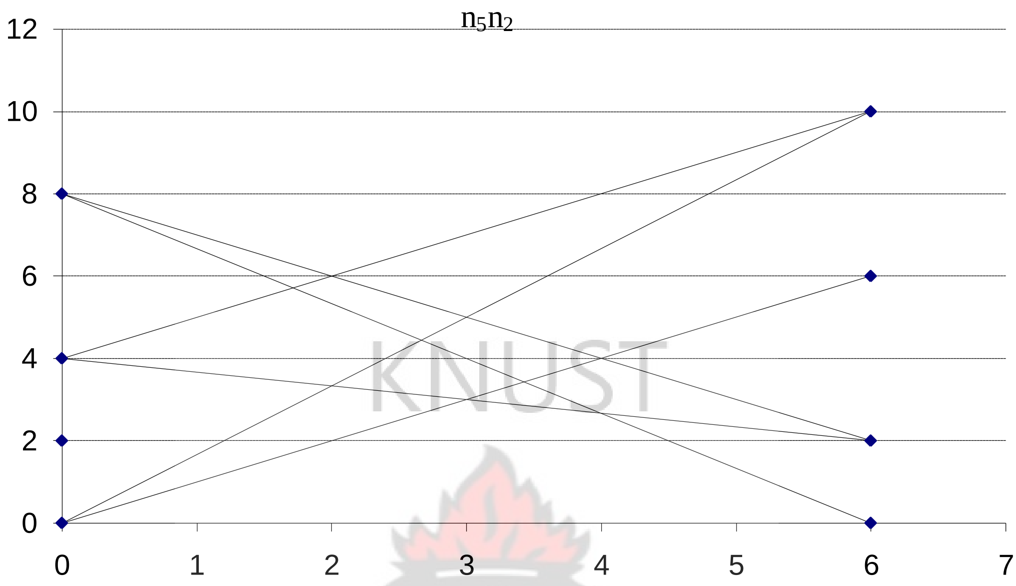
$y_1 = x + 4\ \ 0 \le x \le 6$.......................$(iv)$

Putting $n_i = n_5, i.e.\ i = 5$ then $d\ \ p,n_i\ \ = d\ \ n_5,n_5\ \ = 0, d\ \ q,n_i\ \ = d\ \ n_2,n_5\ \ = 6$

$y_1 = x$ and $y_2 = 12 - x$ when solved

$x = 12 - x \Rightarrow x = 6$ (Kink point)

$y_1 = x \quad 0 \le x \le 6$.......................$(v)$



$$X_{n_5 n_2} = 2 \qquad m(x_l) = 6$$

LOCATING ON EDGE n_3, n_2

Choosing $p = n_3$ and $q = n_2$ such that $c\left(p,q\right) = c\left(n_3,n_2\right) = 2$

Putting $n_i = n_1, i.e.\ i = 1$ then $d\left(p,n_i\right) = d\left(n_3,n_1\right) = 8, d\left(q,n_i\right) = d\left(n_2,n_1\right) = 8$

$y_1 = x + 8$ and $y_2 = 10 - x$ when solved

$x + 8 = 10 - x \Rightarrow x = 1$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 1$.......................$(i)$

$y_2 = 10 - x \quad 1 \le x \le 2$.......................$(ii)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d\left(p,n_i\right) = d\left(n_3,n_2\right) = 2, d\left(q,n_i\right) = d\left(n_2,n_2\right) = 0$

$y_1 = x + 2$ and $y_2 = 2 - x$ when solved

$x+2=2-x \Rightarrow x=0$ (Kink point)

$y_2 = 2-x \quad 0 \le x \le 2$.......................$(iii)$

Putting $n_i = n_3, i.e.\ i=3$ then $d(p,n_i) = d(n_3,n_3) = 0, d(q,n_i) = d(n_2,n_3) = 2$

$y_1 = x$ and $y_2 = 4-x$ when solved

$x = 4-x \Rightarrow x=2$ (Kink point)

$y_1 = x \quad 0 \le x \le 2$.......................$(iv)$

Putting $n_i = n_4, i.e.\ i=4$ then $d(p,n_i) = d(n_3,n_4) = 12, d(q,n_i) = d(n_2,n_4) = 10$

$y_1 = x+12$ and $y_2 = 12-x$ when solved

$x+12 = 12-x \Rightarrow x=0$ (Kink point)

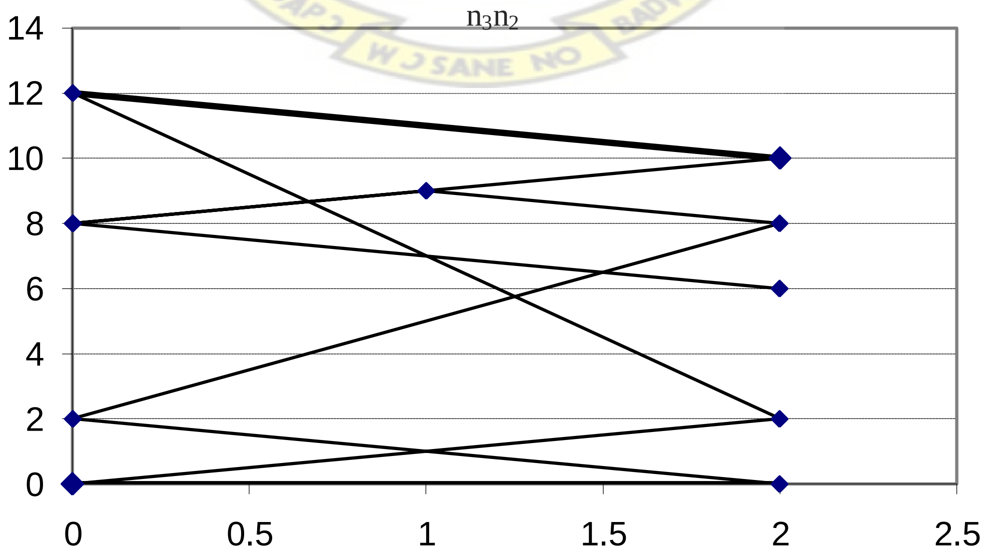
$y_2 = 12-x \quad 0 \le x \le 2$.......................$(v)$

Putting $n_i = n_5, i.e.\ i=5$ then $d(p,n_i) = d(n_3,n_5) = 8, d(q,n_i) = d(n_2,n_5) = 6$

$y_1 = x+8$ and $y_2 = 8-x$ when solved

$x+8 = 8-x \Rightarrow x=0$ (Kink point)

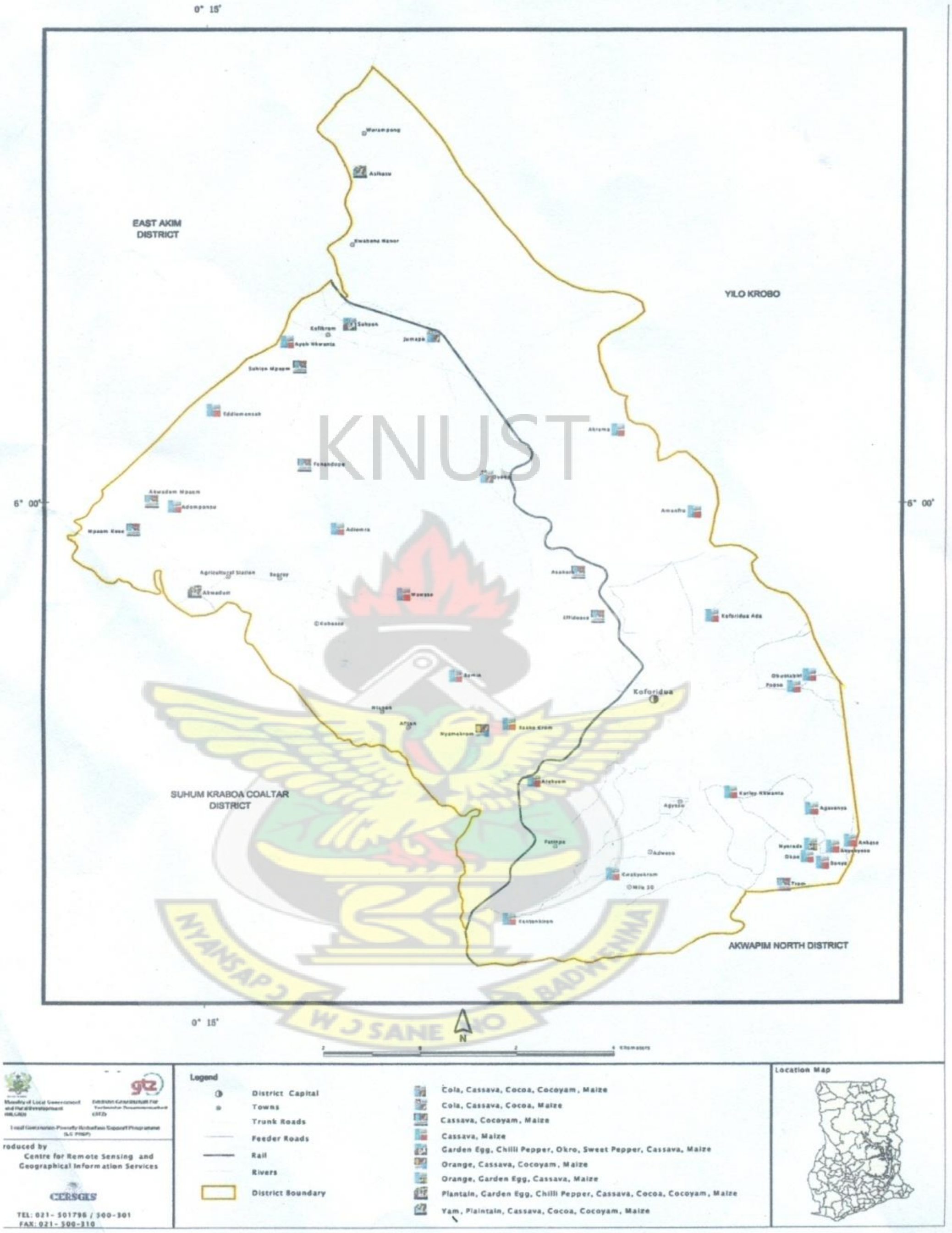
$y_2 = 8-x \quad 0 \le x \le 2$.......................$(vi)$



$X_{n_3 n_2} = 2 \qquad m(x_l) = 10$

APPENDIX B

New Juaben Municipality Map, having capital letters as selected demand nodes



OPTIMAL LOCATION OF FIRE STATION USING ABSOLUTE CENTER MODEL. CASE STUDY: NEW JUABEN MUNICIPALITY

BY

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CHAPTER ONE

Introduction

1.1 Background of Study

For the years, the location of semi-obnoxious (also known as semi-desirable) facility has been a widely studied topic by researchers in location theory. A facility is said to be semi-desirable when it gives service to certain customers in the neighbourhood but, on the other hand, is felt as obnoxious to its environment. For example stadia, airports, train stations and fire stations are examples of semi-obnoxious facilities. Since they are useful and necessary for the community, but they are a source of negative effects such as noise. New Juaben Municipality is one of municipalities of the Eastern Region and has estimated population of 152,858 people kilometers with a population density of 1,507.

The Municipality shares boundaries on the north with East Akim Municipality on the south with Akuapem North, Yilo Krobo District on the East. Suhum Kraboa Coalter District on the west. It lies between latitude 60^0N and 70^0N.

The new Juaben Municipality with Koforidua as capital is co-terminus with Eastern Regional Capital. Koforidua is located at the junction of the major truck roads in the Eastern Region. Farming is the main agricultural activities of most inhabitants in the Municipality. The major factory in the municipality is the Intravenous Infusion Limited at Koforidua that produces intravenous fluids for distribution throughout the country. The municipality has one fire station at Asokore serving the Municipality and East Akim Municipality as East Akim has no fire station. Most fire outbreaks in Ghana could be linked to misuse of electrical gadgets, wrongful electrical connection, careless usage of candles, wrongful disposal of live cigarette butts and many other factors and behaviours.

In U.K the great fire of London in 1666 set in motion changes which laid the foundation for organized firefighting. The only equipment available to fight in 1666 which burnt for five days was two-quart (2.28 litres) hand syringes and a similar slightly large syringe (Louisa et al., 2006) . In the wake of the fire, the city council established the first fire insurance company "THE FIRE OFFICE" in 1677 which employed small teams of Thames Watermen as firefighters and provided them with uniforms and arm badges showing the company to which they belonged. The first organized municipal fire brigade in the world however, was established in Edinburgh, Scotland, when the Edinburgh fire engine Establishment was formed in 1824. It was led by James Braidwood. In 1832, London fire Engine Establishment was also formed.

1.2 Problem Statement

It is a fact that cities or towns in Ghana do not have well located fire stations hence minor incidents which would easily be managed results is massive loss of property and even lives. Again roads are not properly layed out that access to places of fire outbreaks is simply not possible. The belief is that fire station should be located in such a way that allows firefighters to respond in a timely manner to emergencies. Facts that influence this decision are as follows:

i. The risk of fire is not the same in all areas; industrial parts lf the municipality is more vulnerable to fire outbreaks.

ii. Population is not spread equally around the municipality, and as a result there are parts of the municipality that are more populated than others.

It has been shown that frequency of incidents is higher in the most populated areas.

It is against this background that this study is being undertaken to develop a decision support system that will help authorities of New Juaben Municipality to strategically locate fire station.

1.3 Objectives of the Study

The objectives of the study are as follows:

i. To model the location of fire station as Absolute Center Problem

ii. Determine the optimal location and service coverage distance.

1.4 Methodology

The problem is to develop a decision support system to optimally locate fire station in the New Juaben Municipality. The p-center heuristics was used in the study. The study was descriptive and analytical in nature and therefore, made use of quantitative and qualitative data collection tools whereas the analyses of data involved the use of mathematical procedure. Data were obtained from statistical department of the New Juaben Municipality, formal and informal interviews with fire service management and some operational men. Map of the Municipality was obtained from planning department of the municipality. Floyd Warshalls' algorithm was used to compute Euclidean distance between all pairs of nodes.

Other information were obtained from the internet and department of mathematical library of KNUST (Kwame Nkrumah University of Science and Technology)

1.5 Justification

Optimal location of fire station in the country would boost foreign and local investor confidence in their economic activities. Plants and animal species which could be pushed to extinction as a result of wild bush fire will be reduced. Degradation of ecosystem, increased soil erosion, reduced water quality and increased soil salinity resulting from fire outbreaks will also be also be addressed thereby increasing productivity of the country.

Damages, injuries loss of property and even death that, both human beings and animals suffer will be a large extent prevented. The findings of the study can be implemented by authorities of New Juaben Municipality.

Future researchers can replicate the study at other parts of the country using the work done as reference material.

1.6 Organization of the study

Chapter one talks about the profile of the area of study, fire history, causes of fire outbreaks, objectives, justification and methodology of the study. Chapter two is primarily about review of some location problem models.

Chapter three considers three location models, strategies involved in choosing a site, network-based algorithms, absolute center problem, determination of upper envelope, local center and absolute center.

Chapter four is about result of location vertex or node center, local center and discussion. Chapter five considers conclusions and recommendations.

CHAPTER TWO

Literature Review

2.1 Introduction

This chapter introduces some of the methods and models that other researchers have applied in solving location problems.

2.2 Review of Location Problem Models

Hongzhong et al. (2005), first surveyed general facility location problems and identified models used to address common emergency situations, such as house fires and regular health care needs. The authors then analyzed the characteristics of large-scale emergencies and proposed a general facility location model that is sited for large-scale emergencies. This general facility location model could be cast as a covering model, a P-median model or a P-center model, each suited for different needs in large-scale emergencies. Illustrative examples were given to show how the proposed model could be used to optimize the location of facilities for medical supplies to address large-scale emergencies.

The associated FORTRAN computer programme could be utilized to determine the travel time from a source of fire to a smoke detector. The difference in travel time from an isolated fire source to two or more detectors could be used to isolate those airways in which the source of fire is located. This model also has application in mine emergency stage. To determine the optimum location fire detectors, the mine network was divided into zones each of which was associated with a difference in calculated smoke arrival time between a pair of detectors.

Church and ReVelle (1974) and White and Case (1974), developed a maximal covering location problem model that did not require full coverage to all demand points. Instead, the model sought the maximal coverage with a given number of facilities.

The maximal covering location problem and different variants of it had been extensively used to solve various emergency service location problems. A notable example was the work of Eaton et al. (1985), that used the maximal covering location problem to plan the emergency medical service in Austin Texas. The solution gave a reduced average emergency responses time with increased calls for service.

Schilling et al., (2005), generalized the maximal covering location problem model to locate emergency fire fighting servers and depots in the city of Baltimore. In the authors' model, known as FLEET (facility location and Equipment Emplacement Technique) two different types of servers needed to be located simultaneously. A demand point was regarded 'covered' only if both servers were located within a specified distance.

Hodder and Dincer (1986), consider the location of capacitated facilities globally under exchange rate uncertainty. The model incorporates the financing aspects of plant construction by endogenously deciding how much of each plants' total cost to borrow from each country; the per-period cost of this financing is a random variable since the exchange rate are uncertain.

In addition, cost and per-unit profit are uncertain. The model maximizes a mean-variance expression concerning the total profit. This objective is quadratic and involves a large-variance-covariance matrix, each off diagonal term of which requires a bilinear term in the objective function. Therefore, the author proposes an approximation scheme that effectively diagonalizes the variance-covariance matrix so that the objective function contains only squared terms and no bilinear terms.

The resulting model is solved using an off-the-shelf quadratic programming solver for small problems and using a gradient search method for larger ones. No discussion is provided concerning the form of uncertainty (discrete or continuous) or the probability distributions governing it, but in theory any approach could be used as long as the random parameters can be expressed adequately in the form needed for the approximation.

Berman and Odoni (1982), studied a single-facility location problem in which travel times are stochastic and the facility (e.g. Ambulance) may be relocated at a cost as conditions change. Travel times are scenario-based, and scenario transitions occur according to a discrete-time Markov process. The objective is to choose a facility location for each scenario to minimize expected transportation and relocation costs. The authors show that Hakimi property applies to this problem and that the problem on a tree is equivalent to the deterministic problem; any scenario can be used to determine the optimal location since I-median on a tree is independent of the edge of lengths. They then present a heuristic for the problem on a general network that involves iteratively fixing the location in all but one scenario and solving what amounts to I-median problem. They discuss simple bounds on the optimal objective value of the multi-facility problem. Berman and LeBlanc, (1984), introduce a heuristic for this problem that loops through the scenarios, performs local exchanges within each, and then performs exchanges to link the scenarios in an effort to reduce relocations costs.

Carson and Batta, (1990), present a case study of a similar problem in which a single ambulance is to be relocated on the Amherst campus of SUNY buffalo as the population moves about the campus throughout the day (from class-room buildings to dining halls to dormitories, etc.). Given the difficulties inherent in identifying probability distributions and estimating allocation costs in practice, Carson and Batta simply divide the day into four unequal time periods and solve I-median problem in each. Relocation costs are not explicitly considered, but the decision to use four time periods was arrived at in consideration of the trade off between frequent relocation and increased response times.

Ghosh and McLafferty, (1982), introduce a model for locating multiple stores so as maximize market share in a competitive environment with demand uncertainty (actual, uncertainty as to which stores competitor plans to close, but in this setting they amount to the same thing). The authors discuss a model from the marketing literature for estimating market share given fixed store locations.

The location model itself is formulated as a multi objective model, with each objective representing the market-share-maximization objective in a given scenario.

Ultimately, the objectives are combined into a weighted sum to be minimized. If the weights represent scenario probabilities, the objective is equivalent to minimizing the expected cost; otherwise, the weights can be adjusted systematically to find nondominated solutions (solutions for which no objective can be improved without degrading another objective). For a given set of weights, the problem is solved using an exchange heuristic. On a small sample problem, three noninferior solutions were found, and the authors provide some discussion as to how to choose among them.

Benedict, (1983), Eaton et al., (1986), and Hogan and ReVelle, (1986), developed covering maximal location problem models for emergency service that had a secondly “back up-coverage” objective. The models ensured that a second (back up) facility could be available to service a demand area in case that the first facility was unavailable to provide services. Based on a hypercube queuing model, Javis (1977) developed a descriptive model for operation characteristics of an EMS system with a given configuration of resources and a location model for determining the placement of ambulances to minimize average response time or other geographically based variables.

Marianov and ReVelle, (1996), created a realistic location model for emergence systems based on results from queuing theory. In their model the travel times or distances along arcs of network were considered as random variables. The goal was to place limited numbers of emergency vehicles, such as ambulances, in away as to maximize the call for service.

Carbone, (1974), formulated a deterministic p-median model with the objective of minimizing the distance travelled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node was uncertain. The author further extended the deterministic p-median node to a chance constrained model. The model sought to minimize distance and user costs, and maximize demand and utilization.

Paluzzi, (2004), discussed and tested p-median based heuristic location model for placing emergency service facilities for the city of Carbondale. The goal of this model was to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. The results were compared with results from other approaches and the comparison validated the usefulness and effectiveness of the p-median based location model.

Doeksen and Oehrtman, (1976), used a general transportation model based on alternative objective functions to obtain optimal fire stations for the rural fire system. The different objectives used to obtain the optimal sites include: minimizing responses time to fire, minimizing total mileage for fighting rural or country fires and minimizing protection per dollars' worth of burnable property.

Plane and Hendricks, (1977), used the maximum covering distance concept to develop a hierarchical objective function for the set covering formulation of the fire station location problem. The objective function permitted the simultaneous minimization of the number of fire stations and minimization of the existing fire station within the minimum total number of stations.

Badri et al. (1998), underlined the need for a multi objective model in determining the fire station location. The authors used a multiple criteria modeling approach via integer goal programming in everlasting potential sites in 31 sub-areas in the state of Dubai. Their model determined the location of fire stations and the areas they are supposed to serve. It considered eleven (11) strategic objectives that incorporated travel times and travel distances from stations to demand sites, and also other cost-related objectives and criteria-technical and political in nature.

Church (2002), exhaustively reviewed the existing work linking GIS location science and asserted that GIS could support a wide range of spatial queries that aid location studies. He explored the integration of a heuristic algorithm into GIS for spatial optimization of fire station locations. This novel approach to solving optimization problem led to a paradigm shift in solving spatial analytical problems of a similar nature in the disciplines of transportation, networking and infrastructure design.

Tzeng and Chen (1999), used a fuzzy multi objective approach to determine the optimal number and sites of fire stations in Taipeis' international airport. A genetic algorithm was then executed to weigh against the brute-force enumeration method. The results proved that the genetic algorithm was suitable for solving such location problems. Nevertheless, its efficiency still remained to be verified by large-scale problems.

Talwar (2002), utilized a p-center model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands from accidents of tourist activities such as skiing, hiking and climbing at the north and south of Alphine mountain ranges. One of the models' aims was to minimize the maximum (worst) response times and the author use effective heuristics to solve the problem.

ReVelle and Hogan et al., (1989), formulated a model that sought to minimize a population which had a service available within a desired travel time with a stated reliability, given that only P servers were to be located. The authors computed the number Pi of servers needed for reliable coverage of node i, and maximized the population in nodes i, with pi or more servers.

De Palma et al (1989), study a multi firm competitive facility location with random consumer utilities. A consumers' utility for firm i is expressed as a constant a_i (the mean utility for the firm) minus the distance from the consumer to the firms' nearest facility minus a random error term. After choosing its maximum–utility firm, each consumer will choose the nearest facility within that firm.

Firm i will open m_i facilities to maximize its expected sales (market share). The authors proved that if the m_i–median solution is unique for all i and if the consumers' tastes are sufficiently diverse, then there exists a unique location equilibrium, and in that equilibrium, firm i locates its facilities at the m_i–median solution. The problem therefore reduces to solving a separate PMP for each firm.

MirHassani et al(2000), formulate a study chain network design problem as a stochastic program with fixed recourse; the SP has binary first- stage variables and continuous second – stage variables. The objective function coefficient are deterministic; uncertainty is present only in the right- hand sides of the recourse constraints, which may represent for example, demands or capacities. The authors focus especially on parallel implementation issues for their proposed Benders decomposition algorithm.

Tsiakis et al. (2001), consider a multiproduct, multiechelon supply chain under scenario- based demand uncertainty. The goal is to choose middle- echelon facility locations and capacities, transportation links and flows to minimize expected costs. Transportation cost are piecewise linear concave. The model is formulated as a large –scale MIP and solved using CPLEX.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter introduces a number of Locations models (p-median, center-of gravity etc) formulated and used in solving location problem. The chapter also discusses the methodology that would be used to in finding the optimal location where a fire station would be located in the Koforidua Township to ensure optimal response time for incident responders in the service coverage area.

3.2 Spatial Representation of Location

In support of decision processes that involve facility siting, location models are generally used.

To formulate a location model, it is necessary to identify where the demand is located and where facilities can be sited.

The problem of siting p facilities in some universe so as to satisfy a given set of criteria poses the following:

i) The universe to be considered;

ii) The assumptions to simplify the problem without distorting the solution radically; and

iii) The objectives to be optimized.

These result in the emergence of many different formulations to the fundamental location problem. As one would expect, the more accurately a model reflects the 'real life' situations the more complex the problem becomes.

Three different universes will be addressed;

i) Planar;

ii) Network; and

iii) Discrete.

The whole essence of the siting problem is to locate several facilities to optimize a certain set of objectives. The objectives function could be any of the following:

i) Minimise the maximum Euclidean distance;

ii) Minimise average travel time or cost;

iii) Minimise maximum travel time or cost;

iv) Minimise net; and

v) Minimise response time.

3.3 The Universe to be considered

The first universe to be considered is that of the entire plane, entitled the Planar location problem. Here the set of points making up the entire plane is the set of feasible solutions. For this basic formulation, the planar model assumes direct distance metric e.g Euclidean. On the other hand in a network problem, potential customers will normally travel the arcs or edges of the network, road or rail. This prompts the formulation of the network location model, where the facilities may be positioned on a vertex or an edge of network. Distances are then reformulated to be the shortest path linking facilities and customers. There is also the discrete problem of siting a facility on vertices of a network.

(Francis et al., 1983)

3.3.1 Planar Location Models

A planar location model involves the location of p new facilities $p \in N$ within a feasible plane, so as to minimize some cost of the distance from each new facility to the other new facilities and any existing facility within the plane.

Assumptions:

Before any formulation of the above can be established a set of assumptions must be made:

- i) Any point in the plane can be a member of the feasible solution.
- ii) Each facility can be approximated by a point, i.e. it has no area.
- iii) A subset of the earths' surface can be approximated by a plane.

The above assumptions immediately raise several questions about accuracy. Assumption (i) does not allow for the occurrence of infeasible area within the plane, such as property owned by other organizations, natural barriers are inaccessible sites. In these cases the model assumes that a site close to the optimal may be chosen with no loss of satisfaction. Assumption (ii) states that the feasible plane is infinitely bigger than the area taken by a facility. This is obviously unrealistic and may affect the results if the feasible area is on a very local scale and the potential facilities on large site area. Assumption(iii) assumes that the feasible set is small enough so that the spherical curve of the sphere does not alter the shortest distance.

3.3.2 Network Location Models

A network is a system of interconnecting routes which allows movement from one centre to others. It is made up of nodes (vertices) which may be population centres and links (edges) which are routes or services which connect them. In the network location model, the distance metric is measured along a road or rail system, or a set of flight or shipping routes. It may therefore be preferred for placement of the facility to occur on the edges or nodes of the network.

Assumptions

To adopt this model, the set of assumptions made above must first be modified as:

(i) Each facility can be approximated by a point i.e. it has no area.

(ii) Network distances between points are defined as shortest path distances which can be computed using Djikstra algorithm or Floyd Warshall algorithm.

(iii) Any point in the network can be a member of the feasible solution.

These assumptions are similar to those of the planar model and will result in similar formulations. However, if the assumption that all the facilities provide the same kind of service and that a customer will only have to travel to the closest facility is introduced, a subset of the minimax or minisum formulation is addressed.

3.3.3 Discrete Location Models

Planar and Network location models have some limitations, in that:

i) Every point in the plane or network is a candidate solution;

ii) Fixed costs for siting individual facilities at a particular point are ignored or assumed to be independent of the location chosen and so do not affect the optimal solution.

These limitations are confronted when the solution set is reduced to that of a finite number of candidate solutions. Each candidate can be assigned an individual location cost which in turn can be incorporated into the objective function. (Moon I.D and Chandhry S.S, 1994) and (Mirchandani P.B and Francis R.L, 1990)

3.4 Strategies Involved in Choosing a Site

Location simplify refers to a place where something happens or exist.

Many factors, both quantitative and qualitative have to be considered in selecting a location. Some of these factors are more important than others so people can use weightings to make the decision process more objective. Three of the main location strategies are the location break-even analysis, factor rating and centre-of-gravity methods.

3.4.1 The Location Break-Even Analysis

The location break-even analysis is the use of cost-volume analysis to make economic comparison of location alternatives. By identifying fixed and variable costs and graphing them for each location we can determine which one provides the lowest cost. Location break-even analysis can be done mathematically or graphically. The graphic approach has the advantage of providing the range of volume over which each location is preferable. There are three steps in location break-even analysis.

These are:

i) Determine the fixed and variable costs for each location.

ii) Plot the cost for each location, with cost on the vertical axis of the graph and production on volume the horizontal axis.

iii) Select the location that has the lowest total cost for the expected production volume.

3.4.2 The Factor Rating Method

The factor rating is popular because a wide variety of factors, from education to labour skill to recreation, can be objectively included. The factor rating method has six steps:

i) Develop a list of relevant factors.

ii) Assign a weight to each factor to reflect its relative importance in the companys' objectives.

iii) Develop a scale for each (e.g. 1 to 10 or 1 to 100)

iv) Assign a score to each location for each factor using the scale in step (iii)

v) Multiply the score by the weights for each factor and total the score for each location.

vi) Make a recommendation based on the maximum point score, considering the results of quantitative approaches as well.

When a decision is sensitive to mirror changes, further analysis of either the weighting or the points assigned may be appropriate. Alternatively, management may conclude that these intangible factors are not the proper criteria on which to base a location decision. Managers therefore place primary weight on the more quantitative aspects of the decision. (Amponsah, 2006)

3.4.3 Centre-of-Gravity Method

The centre-of-gravity method is a mathematical technique used for finding the location of a distribution centre which minimizes distribution costs. This method takes into account the location of markets, the volume of goods shipped to those markets, and shipping costs in finding the best location for a distribution centre.

The first step in the centre-of-gravity method is to place the location on a co-ordinate system. The origin of the co-ordinate system in the scale is arbitrary, just as long as the relative distances are correctly represented. This can be done by placing a grid over an ordinary map of the location in question. The centre-of-gravity is determined by equations (3.1) and (3.2)

$$C_x = \frac{\sum d_{ix} W_i}{\sum W_i} \qquad (3.1)$$

$$C_y = \frac{\sum d_{iy} W_i}{\sum W_i} \qquad (3.2)$$

Where

$C_x = x -$ Coordinate of the centre-of-gravity

$C_y = y -$ Coordinate of the centre-of-gravity

$d_{ix} = x -$ Coordinate of location ***i***

$d_{iy} = y -$ Coordinate of location ***i***

$W_i =$ Volume of goods to and from location i

Once the x and y-coordinates have been obtained, the new location is placed on the previously described map to determine the actual position on the map. If that particular location does not fall directly on a city, simply locate the nearest city and place new distribution centre there. (Louisa et al., 2006)

3.5 Network-Based Algorithms

3.5.1 Shortest Path Problems

Shortest path problems are the most fundamental and most commonly encountered problems in the study of transportation and communication networks (Salhi S, 1998). There are many types of shortest path problems. For example, we may be interested in determining the shortest path from one specified node in the network to another specified node or we may need to find the shortest paths from a specified node to all other nodes. Shortest path between all pairs of nodes in a network are required in some problems while sometimes one wishes to find the shortest path from one given node to another given node that passes through certain specified intermediate nodes. In some application, one requires not only the shortest path but also the second and the third shortest paths. There are instances when the actual shortest path is not required but only the shortest distance. We shall consider two most important shortest-path problems:

i) How to determine (a shortest path) from a specific node S to another specific node T,

ii) How to determine distance (a path) from every node to every other node in the network.

3.5.1.1 Floyd-Warshall Algorithm

The Floyd-Warshall algorithm obtains a matrix of shortest path distance within $O(n^3)$ computations. The algorithm is based on inductive arguments developed by an application of a dynamic programming technique.

Let represent the length of the shortest path from node i to node j subject to the condition that this path uses the nodes 1, 2, …, k – 1 as internal nodes. Clearly, $d^{k+1}(i, j)$ represent the actual shortest path distances from the node i to j. The algorithm first computes $d^2(i, j)$ for all node pairs i and j using $d^1(i, j)$, it then computes $d^2(i, j)$ for all node pairs i and j. It repeats this process until obtains $d^{k+1}(i, j)$ using $d^k(i, j) = \min(d^k(i,k), d^k(k, j))$. The Floyd-Warshall algorithm remains of interest because it handles negative weight edges correctly.

3.5.1.2 Dijkstras' Algorithm

The Dijkstras' algorithm finds the shortest path from a source s to all other nodes in the network with nonnegative lengths. It maintains a distance label d(i) with each node i, which is an upper bound on the shortest path length from the source node s to any other node j. At any intermediate step, the algorithm divides the nodes of the network under consideration into two groups: those which it designates as permanently labeled (or permanent), and those which it designates as temporarily labeled (or temporal). The distance label to any permanent node represents the shortest distance from the source node to that node. The fundamental idea of the algorithm is to find out from source node s and permanently labeled nodes in the order of their distances from the node s.

Initially, node s is assigned a permanent label of zero (0) and each other node j a temporary label equal to infinity. At each iteration, the label of a node i is its shortest distance from the source node along a path whose internal nodes (i.e. node i other than s or node i itself) are all permanent labeled. The algorithm selects a node i within the minimum temporary label (breaking ties arbitrarily), makes it permanent and reaches out from that node (i.e. it scans all the edges coming out from the node i to update the distances label of adjacent nodes). The algorithm terminates when it has designated all nodes permanent.

Dijkstras' algorithm can be expressed as a set of steps.

Step 1: Assign the permanent label O to the starting vertex.

Step 2: Assign temporary labels to all the vertices that are connected directly to the most recently permanent labeled vertex

Step 3: Choose the vertex with the smallest temporary label and assign a permanent label to that vertex.

Step 4: Repeat steps 2 and 3 until all vertices have permanent labels.

Step 5: Find the shortest path by tracing back through the network.

3.6 Absolute Center Problem

The center problem was first proposed by Sylvester (1857) more than one hundred years ago.

The problem asked for the center of a circle that had the smallest radius to cover all desired destinations. The k-center model and its extensions had been applied in the context of locating facilities such as EMS centers, hospitals, fire station and other public facilities.

For a point x on the network G, let m(x) denote max d(x, n_i) where d(x, n_i), is the cost or distance of the 'shortest' path between x and 'farthest' demand node n_i. The general absolute center problem is

i) Formulated as $\min[m(x)] = \min[\max d(x, n_i)]$ subject to $x \in G$

The above formulation is applied in finding the vertex and local centers.

ii) The vertex center (or node center) $x_n \in N$ is a node such that for every node $y \in N, m(x_n) \leq m(y)$.

The local center of an edge (p, q) is a point x, on (p, q) such that for every point y on.

iii) (p, q), $m(x_i) \leq m(y)$. The absolute center x_a is a point on G such that for every point y on G, (y may be on an edge of G), $m(x_a) \leq m(y)$ (Mirchandi P. B, and Francis R. L, 1990)

To find a node center, we compute the matrix of the shortest paths costs (travel times, distances) for all pairs of nodes using the Floyd-Warshalls' or Dijkstras' algorithm, and then choose a node such that the maximum entry in its row in the matrix is smallest among the maximum entries of the rows.

For example figure 3.1 shows a network of an urbanized area with nodes n_1, n_2, n_3, n_4 *and* n_5 representing points where demand for services is generated.

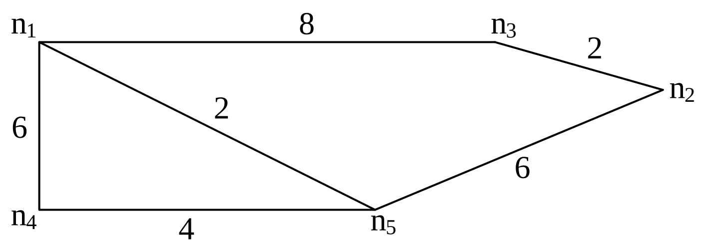


Fig. 3.1: Example of network showing demand nodes and distance

By using the Floyd-Warshalls' algorithm we obtain a matrix of the shortest paths of the network of figure 3.1.

The algorithm computes d(p, q) for all node pairs p and q are shown in table 3.1

Table 3.1: Matrix of shortest path distance for pairs of nodes for fig.3.1

	n_1	n_2	n_3	n_4	n_5	Row max
n_1	0	8	8	6	2	8
n_2	8	0	2	10	6	10
n_3	8	2	0	12	8	12
n_4	6	10	12	0	4	12
n_5	2	6	8	4	0	8

From table 3.1, the smallest among the entries in all rows occurs at either n_1 or n_5, with $m(n_1) = m(n_5) = 8$ and therefore n_1 or n_5 may be taken as the node center.

3.7 Finding the Absolute Centre

The absolute centre minimizes the cost (distance travel time). We look for the path of minimum cost (Euclidean distance) by finding the shortest path among all pairs of vertices using Floyd-Warshalls' or Dijkstras' algorithm. A vertex is a designated point in a network and an edge is a direct distance or arc between two vertices, p and q denoted by c (p, q) which is the edge cost or edge distance.

A shortest path is the total distance between two vertices which may not be direct but passing through other vertices. Thus a shortest path may not be a direct distance or cost between two vertices. This is denoted by d(p, q) and is described as the minimum path cost;

$$d(p,q) = \min \sum_{i=1}^{i-1} c(n_i, n_{i+1})$$

Consider the edge (p, q) with a point x on it as shown in figure 3.2

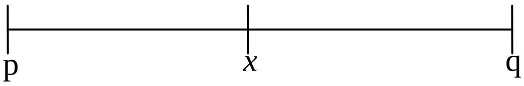


Fig 3.2: Movement of x along edge (p, q)

$$d(x,p) + d(x,q) = c(p,q) \quad \Rightarrow d(x,p) = c(p,q) - d(x,q)$$

For an undirected graph (a two-way road) with non-negative weight (cost), put

$m(x) = \max d(x,p)$

If x is on an edge or a node we require $m(x^1) \leq m(x)$, where $x_{pq} = x^1$, the distance (cost) of the point x on edge (p, q) away from p.

To calculate $m(x^1)$

i. Evaluate all vertices and find the vertex center value and its cost.

ii. Evaluate all edges to find the local center with minimum cost.

iii. Compare the two costs, i.e., the minimum vertex center cost and the minimum edge cost, the lowest of the two costs is the solution, $m(x^1)$

The local center for each edge can be found as shown. Consider an edge (p, q) with a point x on it. Assuming we want to move from x to n_i where n_i is any node or vertex on the network G, we find the minimum cost by moving to n_i through p or q.

p and q are demand points and $n_i$is the farthest desired destination.

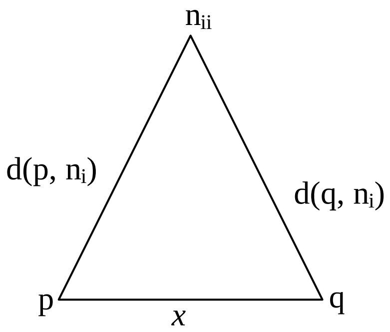


Fig. 3.3 Distance of x to n_i through p and q

$x + d(p, n_i) = c(p,q) - x + d(q, n_i)$ is an edge and its cost is $c \;\; p,q$. From the fig. 3.3

$d(p, x) = x$ and $d(q, x) = c(p,q) - x$ hence, $d(p, x) + d(x, q) = c(p,q)$

The movement from x to n_i (any of the nodes or vertices on the network G) can be done in two directions i.e. through p or q given rise to respectively the equations below;

$y_1 = x + d(p, n_i)$.......................(3.1)

$y_2 = c(p,q) - x + d(q, n_i)$...........................(3.2)

Where y_1, is the distance from x to n_i through p and y_2 is the distance from x to n_i through q. As x moves along the edge (p, q) there will be a point when the two distances or costs would be equal. At this point $y_1 = y_2$ and the kink/maximum/pareto could be found. Solving for the path of equal cost we have:

$$x + d(p, n_i) = c(p,q) - x + d(q, n_i)$$

$$x = \frac{c(p,q) + d(q, n_i) - d(p, n_i)}{2}$$

Where x can be denoted by x_m being the minimum cost. The equations (1) and (2) involving y_1 and y_2 are therefore used to draw graphs for the edge (p, q) from which the local centre can be determined. As n_i assumes all the nodes on the network, a number of equations will be generated under equations (1) and (2). These equations would then be sketched on the same axes in a given range obtained from solving the kink point for the paths of equal distance for each pair of equations.

An upper envelope is then obtained by tracing all paths of lines beyond which there are no higher points for the *x*-value in the given range on the graph. These graphs are indicated by thicks lines. The local center $x_{pq} = x_i$ is the point that minimizes the upper envelope. The absolute center at termination of the process is the point x_a (node centre x_n or local center $x_{pq} = x_i$) that assigned the least value to m(x)

Using figure 3.1 we would evaluate all edges in the given network to illustrate how the absolute centre can be found on a given network as follows:

3.8 Location on edge (n_1n_3)

Consider

$m(x) = y_1 = x + d(p, n_i)$..............(3.3)

$y_2 = c(p,q) - x + d(q, n_i)$………(3.4)

Choosing n_3 as the origin, we let p = n_3 ad q = n_1 such that $0 \leq x \leq c\ (p,q)$

Putting $n_i = n_1$, ie $i = 1$ then Table 3.1 we $d(p, n_i) = d(n_3, n_1) = 8$

$d(q, n_i) = d(n_i, n_i) = 0$ and $c(p,q) = c(n_3, n_1) = 8$

Thus from (i) and (ii) $y_1 = x + 8$ and $y_2 = 8 - x$. Solving for the path of equal distance or cost, we have $x + 8 = 8 - x, x = 0$. That is, the kink point for the two equations being on left endpoint of the interval. By sketching, the equation $y_1 = x + 8$ falls outside the range and hence rejected.

Putting $n_i = n_3$, *i.e.* $i = 3$ then $d(p, n_i) = d(n_3, n_3) = 0$ and $d(q, n_i) = d(n_1, n_3) = 8$.

Thus $y_1 = x, y_2 = 16 - x$ and solving for the path of equal distance or cost, we have $x = 16 - x,\ x = 8$ which is the kink point. It is at the right end point of the interval.

By sketching, the equation $y_2 = 16 - x$ falls outside the range and hence rejected. In both instances above, we can accept and sketch the two equations below

$y_1 = x \quad 0 \le x \le 8$.................(3.5)

$y_2 = 16 - x \quad 0 \le x \le 8$....................(3.6)

Putting $n_i = n_2$, *i.e.* $i = 2$ then $d(p, n_i) = d(n_3, n_2) = 2$ and $d(q, n_i) = d(n_1, n_2) = 8$.

The resulting equations $y_1 = x + 2$ and $y_2 = 16 - x$ when solved for the path of equal distance or cost, we have $x + 2 = 16 - x \Rightarrow x = 7$ which is the kink point. The following equations are then sketched in the given ranges

$y_1 = x + 2 \quad 0 \le x \le 7$.................(3.7)

$y_2 = 16 - x \quad 7 \le x \le 8$....................(3.8)

Putting $n_i = n_4$, *i.e.* $i = 4$ then $d(p, n_i) = d(n_3, n_4) = 12$ and $d(q, n_i) = d(n_1, n_4) = 6$

The resulting equations, $y_1 = x + 12$ and $y_2 = 14 - x$ when solved for the path of equal distance or cost, we have $x + 12 = 14 - x,\ x = 1$ which is the kink point.

The following equations are then sketched within the given ranges.

$y_1 = x + 12 \quad 0 \le x \le 1$.................(3.9)

$y_2 = 14 - x \quad 0 \le x \le 8$....................(3.10)

Putting $n_i = n_5$, *i.e.* $i = 5$ then $d(p, n_i) = d(n_3, n_5) = 8$ and $d(q, n_i) = d(n_1, n_5) = 2$

The resulting equations, $y_1 = x + 8$ and $y_2 = 10 - x$ when solved for the path of equal distance or cost, we have $x + 8 = 10 - x \Rightarrow x = 1$ which is the kink point.

The following equations are then sketched within the given ranges.

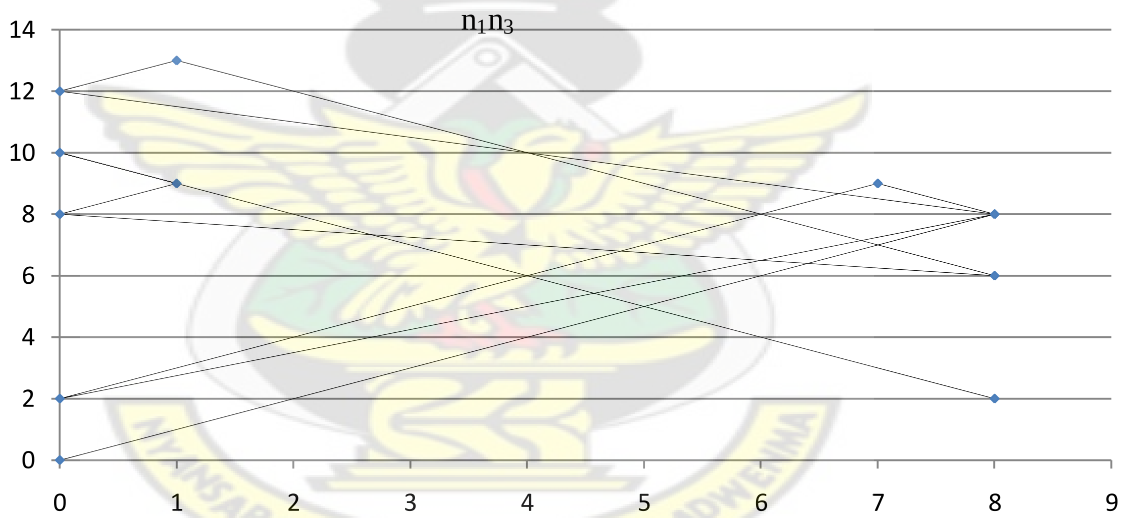
$y_1 = x + 8 \quad 0 \le x \le 1$.................(3.11)

$y_2 = 10 - x \quad 0 \le x \le 8$....................(3.12)

The eight equations are then sketched on the same axes as shown in fig. 3.4. The minimum cost or distance of the path can be found from the graph using the upper envelope

$x_{n_1 n_3} = 8 \;\; and \;\; m(x_l) = 8$

The thick line represents the upper envelope of the graph and the minimum point on it is the local center



$X_{n_3} = 8 \qquad m(x_l) = 8$

Fig. 3.4 Graph showing upper envelope and local center for edge n_1n_3

Thus the minimum cost on edge (n_1, n_3) i.e. $x_{n_1 n_3}$, is selected by considering the point corresponding to the maximum cost for all nodes. In the example above, the minimum cost/distance for edge (n_1, n_3) is given as $x_{n_1, n_3} = 8 \;\; and \;\; m(x_1) = 8$ units.

3.9 Construction of the Upper Envelope

After sketching all the equations resulting from the location on edge (n_1,n_3) on the same axes as shown in fig. 3.4 there is the need to construct an 'upper envelope' which gives the minimum cost/distance of a shortest path from *x* to a farthest node on the given edge. To construct the upper envelope, we trace all paths of lines beyond which there are no higher points for the same x-value in the given range. This path is indicated by a thick line as shown in the figure 3.4

3.10 Local center

For each edge (p, q), the local center is found by plotting $d(x,n_i)$ for each node $n_i \in N$ where $0 \le x \le c\ \ (p,q)$.

The local center $x_{pq} = x_l$ is the point that minimizes the upper envelope. The absolute center x_a is the minimum point among the local centers. This occurs on the edge $n_5 n_2$ with $d(x_a,n_5) = 2$ and $d\ \ x_a,n_2\ \ = 4$, $m(x_a) = 6$ which implies, the maximum distance from point *x* to the farthest node is 6 units that is both nodes n_5 and n_2 hence the optimum location of the facility is on edge (n_2,n_5) which is 2 units from node n_5 and 4 units from node n_2.

Finding a single absolute center of a network is more involving. In practice, where a network has a large number of nodes, there would be equally a large number of edges to be enumerated for their respective local centers.

Table 3.2 local centers and corresponding cost or distance for figure 3.1

Edge	Edge distance	Local centre X_l	Cost $m(x_l)$
(n_1, n_3)	8	At n_3	8 units
(n_2, n_5)	6	2 units from n_3	6 units
(n_1, n_5)	2	At n_1 or n_5	8 units
(n_1, n_4)	6	At n_1	8 units
(n_5, n_4)	4	At n_5	8 units
(n_3, n_2)	2	At n_2	10 units

The computation of the equations and graphs for the locations on the other edges in the network are shown in appendix A

Fortunately, as indicated in the propositions (i) and (ii) below, many edges do not need to be explicitly enumerated for their respective local centers.

Preposition (i)

For the set of all points x on a fixed edge (p,q) of G, the maximum distance function $m(x)$ is piecewise linear whose slope is always $+1 or -1$.

Preposition (ii)

For an edge (p,q), the local center satisfies the equation,

$m(x_l) \geq \dfrac{m(p)+m(q)-c(p,q)}{2}$ where $c(p,q)$ denotes the cost of edge (p,q).

Proof

Consider any point on the edge (p,q). Let $x: 0 \le x \le c(p,q)$ denote the point p such that $x = 0$ and the point $x = c(p,q)$ denote q. We take $d(x,p)$ to be x and $d(x,q)$ to be $c(p,q) - x$. The cost $d(x,p)$ of a shortest path between x and the farthest demand node p is piecewise linear with a slope $+1 or -1$ at each point of x. Its value at $x = 0$ is $m(p)$ and its value at $x = c(p,q)$ is $m(q)$ where $m(p)$ and $m(q)$ are nodes centers for nodes p and q. Hence,

$m(x) \ge m(p) - x \left[For\, all\, x: 0 \le x \le c(p,q)\right]$......................(3.13)

$m(x) \ge m(q) - (c(p,q) - x) \left[For\, all\, x: 0 \le x \le c(p,q)\right]$......................(3.14)

By adding the two inequalities (i) and (ii), we obtain

$$m(x_l) \ge \frac{m(p) + m(q) - c(p,q)}{2}$$

Where x_l simultaneously satisfies the two inequalities above.

From these preposition and from observation that, by definition the maximum distance associated with the node center must be greater than or equal to the corresponding distance from the absolute i.e. $m(x_n) \ge m(x_a)$, we can derive the following test:

If for edge (p,q), $m(x_{\mathrm{n}}) \le \dfrac{m(p) + m(q) - c(p,q)}{2}$ then the local center x_l of (p,q) cannot improve on $m(x_n)$ and therefore need not be found. This test which takes advantage of the fact that it is very simple to find the local center x_l often leads to considerable reduction in the computation effort required to obtain the absolute center. With respect to the five-node, six-edged network in fig. 3.1, we found easily that the node center is at nodes n_1 and n_5 and that $m(x_n) = m(n_1) = m(n_5) = 8$

On application of the test to the six edges of the network, we obtain

Edge (n_1, n_3) $: \dfrac{m(n_1) + m(n_3) - c(n_1, n_3)}{2} = \dfrac{8+12-8}{2} = 6 < 8$

Table 3.3: Results of edges whose local centers are to be determined

Edge	For edge (p,q): $\frac{m(p)+m(q)-c(p,q)}{2}$	m$(X_n) = 8 \leq \frac{m(p)+m(q)-c(p,q)}{2}$
$(n_{1,}n_3)$	6	6< 8
$(n_{2,}n_3)$	10	$10 > 8$
$(n_{2,}n_5)$	6	6<8
$(n_{1,}n_5)$	7	7<8
$(n_{1,}n_4)$	7	7<8
$(n_{4,}n_5)$	8	8=8

The results of the test above clearly suggest that the local center needs to be found for only edges. Edges (n_1, n_3), (n_2, n_5), (n_1, n_5) *and* (n_1, n_4). This makes significant savings in the computational effort and time.

3.11 Summary

Planar, network and discrete location models which may be used to represent location problems and their respective assumptions have been discussed.

A detailed explanation of p-center problem a heuristic method which is the means of locating a fire station at the New Juaben Municipality has been provided.

The next chapter is data collection and analyses.

CHAPTER FOUR

DATA COLLECTION AND ANALYSES

4.1 Introduction

The chapter provides New Juaben municipal map (Appendix B) and selected demand areas specifying the road distances between them. Data was obtained from municipal planning office and municipal town planning department and would be analyzed using the center-problem to identify where a fire station has to be optimally located in the municipality.

Locations considered are:

A – Koforidua

B – Effiduase

C – Baako Krom

D – Koforidua Ada

E – Affian

F – Nyamekrom

G – Asokore

H – Agyeso

I – Adweso

J – Oyoko

K – Kwakyekrom

L – Mile 50

M – Wawase

N – Jumapo

O – Kentenkiren

P – Begrey

Q – Agricultural station

R – Poposo

S – Suhyen

T – Akwadum

Table 4.1: Selected edges specifying the road distance between them.

NO.	EDGE CONSIDERED	DISTANCE (METRES)
1	(A, B)	2200
2	(A, C)	4000
3	(A, D)	2300
4	(A, R)	5500
5	(A, H)	3800
6	(A, I)	4100
7	(B, G)	1100
8	(B, L)	7200
9	(B, M)	4800
10	(C, B)	5000
11	(C, F)	750
12	(C, E)	2300
13	(C, G)	5800
14	(D, R)	5000
15	(E, F)	1800
16	(F, G)	6600
17	(G, M)	4600
18	(G, J)	3200
19	(G, S)	8400
20	(H, I)	1500
21	(I, K)	1200
22	(I, L)	900
23	(J, N)	3700
24	(J, S)	5200
25	(K, O)	2500
26	(M, P)	3000
27	(M, Q)	3850
28	(M, T)	4750
29	(N, S)	1700

Developed network of data of table 4.1 having capital letters as nodes or vertices and the figures as distances between pairs of nodes

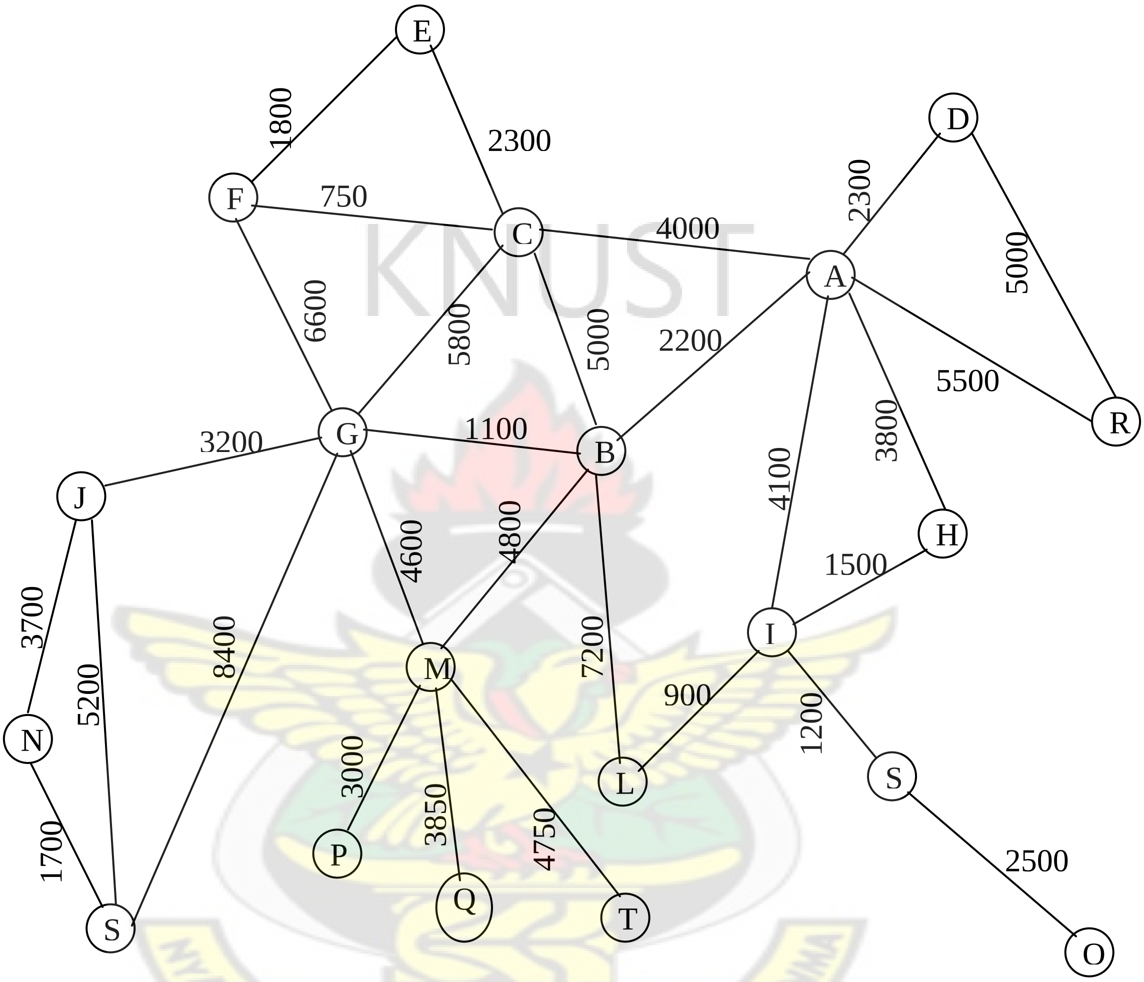


Fig. 4.1: Developed Network for selected demand destinations of New Juaben Municipality.

4.2 All Pairs Shortest Path for the Data Collected

From the network in figure 4.1 the minimum distance matrix d(i, j), that is the matrix of the shortest path using the Floyd-Warshalls' algorithm was obtained and is shown in Table 4.2

Table 4.2 : Matrix of shortest path distance for all pairs of nodes from fig 4.1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	Row max
A	-	2200	4000	2300	6300	4750	3300	3800	4100	6500	5300	5000	7000	10200	7800	10000	10850	5500	11700	11750	11750
B	2200	-	5000	4500	7300	5750	1100	6000	6300	4300	7500	7200	4800	8000	10000	7800	8650	7700	9500	9550	10000
C	4000	5000	-	6300	2300	750	5800	7800	8100	9000	9300	9000	9800	12700	11800	12800	13650	9550	14200	14550	14550
D	2300	4500	6300	-	8600	6300	5600	6100	6400	8000	7600	7300	9300	12500	10000	12300	13150	5000	14000	14050	14050
E	6300	7300	2300	8600	-	1800	8100	10100	10400	11300	11600	11300	12100	15000	14100	15100	15950	11800	16500	16850	16850
F	4800	5800	800	7100	1800	-	6600	8550	8900	9800	10100	9800	10600	13500	12600	14200	15050	10300	15000	15950	15950
G	3300	1100	5800	5600	8100	6600	-	7100	7400	3200	8600	8300	4600	6900	11100	7600	8450	8800	8400	9350	11100
H	3800	6000	7800	6100	10100	8550	7100	-	1500	10300	2700	2400	10800	14000	5200	13800	14560	9300	15500	15550	15550
I	4100	6300	8100	6400	10400	8850	7400	1500	-	10600	1200	900	11100	14300	3700	14100	14950	9600	18300	15850	18300
J	6500	4300	9000	8800	11300	9800	3200	10300	10600	-	11800	11500	7800	3700	14300	10800	11650	10020	5200	12550	14300
K	5300	7500	9300	7600	11600	10050	8600	2700	1200	11800	-	2100	12300	15500	2500	15300	16150	10800	17000	17050	17050
L	5000	7200	9000	7300	11300	9750	8300	2400	900	11500	2100	-	12000	15200	4600	15000	15850	10500	16700	16750	16750
M	7000	4800	9800	9300	12100	11200	4600	10800	11100	7800	12300	12000	-	11500	14800	3000	3850	12500	13000	4750	14800
N	10200	8000	12700	12500	15000	13500	6900	14000	14300	3700	15500	15200	11500	-	18000	14500	15350	15700	1700	16250	18000
O	7800	10000	11800	10100	14100	12550	11100	5200	3700	14300	2500	4600	14800	18000	-	17800	18650	13300	19500	19550	19550
P	10000	7800	12800	12300	15100	14200	7600	13800	14100	10800	15300	15000	3000	14500	17800	-	6850	15500	16000	7750	17800
Q	10850	8650	13650	13150	15950	15050	8450	14650	14950	11650	16150	15850	3850	15350	18650	6850	-	16350	16850	8600	18650
R	5500	7700	9550	5000	11800	10250	8800	9300	9600	10020	10800	10500	12500	15700	13300	15500	16350	-	17200	17250	17250
S	11700	9500	14200	14000	16500	15000	8400	15500	18300	5200	17000	16700	13000	1700	19500	16000	16850	17200	-	17750	19500
T	11750	9550	14550	14050	16850	15950	9350	15550	15850	12550	17050	16750	4750	16250	19550	7750	8600	17250	17750	-	19550

4.3 Results

4.3.1 Locating the Vertex/Node Center

Row 1 represents demand nodes of the network and Row 2 represents row maximum from table 4.2

(a) Table 4.3 Vertex/Node Center from table 4.2

NODE	A	B	C	D	E	F	G	H	I	J
ROW MAX	11750	10000	14550	14050	16850	15950	11100	15550	18300	14300

NODE	K	L	M	N	O	P	Q	R	S	T
ROW MAX	17050	16750	14800	18000	19550	17800	18650	17250	19500	19550

The node or vertex center (x_n) is chosen as the smallest among the maximum entries of all rows in the matrix. From Table 4.2 the row with the minimum among the maximum entries occurs at node/vertex B with a maximum distance (cost) of 10000 metres. Thus the node/vertex centre for the network in figure 4.1 is B, hence $m(B) = 10000$.

4.3.2 Locating the Local Centers

$$Edge\ (A,B) = \frac{m(A) + m(B) - c(A,B)}{2} = \frac{11750 + 10000 - 2200}{2} = 9775 < 10,000$$

Table 4.4 test for edges whose local centers are to be determined for the developed network.

Edge	For edge (p,q): $\frac{m(p)+m(q)-c(p,q)}{2}$	$m(X_n) = 10000 \leq \frac{m(p)+m(q)-c(p,q)}{2}$
(A,B)	9775	9775< 10000
(A,C)	11150	11150> 10000

(A,D)	11750	11750> 10000
(A,R)	11750	11750> 10000
(A,H)	11750	11750> 10000
(A,I)	12975	12975> 10000
(B,G)	10000	10000= 10000
(B,L)	9425	9425< 10000
(B,M)	10000	10000= 10000
(C,B)	9775	9775< 10000
(C,F)	148750	148750> 10000
(C,E)	14550	14550> 10000
(C,G)	9925	9925< 10000
(D,R)	13150	13150> 10000
(E,F)	15500	15500> 10000
(F,G)	1675	1675> 10000
(G,M)	10650	10650> 10000
(G,J)	11100	11100> 10000
(G,S)	11100	11100> 10000
(H,I)	16175	16175> 10000
(I,K)	17075	17075> 10000
(I,L)	17075	17075> 10000
(J,N)	14300	14300> 10000
(J,S)	14300	14300> 10000
(K,O)	17050	17050> 10000
(M,P)	15925	15925> 10000

(M,Q)	15925	15925> 10000
(M,T)	15925	15925> 10000
(N,S)	17900	17900> 10000

From table 4.4 edges whose local centers are to be determined are (A,B), (B,L), (C,B) and (C,G)

Location on edge (A, B)

Let $A = P$, $B = q$ such that $0 \le x \le c(p,q)$ and $c(p,q) = c(A,B) = 2200$

Putting $n_i = A,$ then $d(p,n_i) = d(A,A) = 0, d(q,n_i) = d(B,A) = 2200$

$y_1 = x$ and $y_2 = 4400 - x$ when solved

$x = 4400 - x, x = 2200$ Kink point

$y_1 = x \quad 0 \le x \le 2200$.................. 4.1

Putting $n_i = B,$ then $d(p,n_i) = d(A,B) = 2200, d(q,n_i) = d(B,B) = 0$

$y_1 = x +$ and $y_2 = 2200 - x$ when solved

$x + 2200 = 2200 - x, x = 0$ Kink point

$y_2 = 2200 - x \quad 0 \le x \le 2200$.................. 4.2

Putting $n_i = C,$ then $d(p,n_i) = d(A,C) = 4000, d(q,n_i) = d(B,C) = 5000$

$y_1 = x + 4000$ and $y_2 = 7200 - x$ when solved

$x + 4000 = 7200 - x, \Rightarrow x = 1600$ Kink point

$y_1 = x + 4000 \quad 0 \le x \le 1600$.................. 4.3

$y_2 = 7200 - x \quad 1600 \le x \le 2200$.................. 4.4

Putting $n_i = D,$ then $d\ \ p,n_i \ = d\ \ A,D \ = 2300, d\ \ q,n_i \ = d\ \ B,D \ = 4500$

$y_1 = x + 2300$ and $y_2 = 6700 - x$ when solved

$x + 2300 = 6700 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 2300 \quad 0 \le x \le 2200$.................. 4.5

Putting $n_i = E,$ then $d\ \ p,n_i \ = d\ \ A,E \ = 6300, d\ \ q,n_i \ = d\ \ B,E \ = 7300$

$y_1 = x + 6300$ and $y_2 = 9500 - x$ when solved

$x + 6300 = 9500 - x, \Rightarrow x = 1600$ Kink point

$y_1 = x + 6300 \quad 0 \le x \le 1600$.................. 4.6

$y_2 = 9500 - x \quad 1600 \le x \le 2200$.................. 4.7

Putting $n_i = F,$ then $d\ \ p,n_i \ = d\ \ A,F \ = 4750, d\ \ q,n_i \ = d\ \ B,F \ = 5750$

$y_1 = x + 4750$ and $y_2 = 6950 - x$ when solved

$x + 4750 = 6950 - x, \Rightarrow x = 1100$ Kink point

$y_1 = x + 4750 \quad 0 \le x \le 1100$.................. 4.8

$y_2 = 6950 - x \quad 1100 \le x \le 2200$.................. 4.9

Putting $n_i = G,$ then $d\ \ p,n_i \ = d\ \ A,G \ = 3300, d\ \ q,n_i \ = d\ \ B,G \ = 1100$

$y_1 = x + 3300$ and $y_2 = 5500 - x$ when solved

$x + 3300 = 5500 - x, \Rightarrow x = 1100$ Kink point

$y_1 = x + 3300 \quad 0 \le x \le 1100$.................. 4.10

$y_2 = 5500 - x \quad 1100 \le x \le 2200$.................. 4.11

Putting $n_i = H$, then $d(p, n_i) = d(A, H) = 3800, d(q, n_i) = d(B, H) = 6000$

$y_1 = x + 3800$ and $y_2 = 8200 - x$ when solved

$x + 3800 = 8200 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 3800 \quad 0 \le x \le 2200$.................. 4.12

Putting $n_i = I$, then $d(p, n_i) = d(A, I) = 4100, d(q, n_i) = d(B, I) = 6300$

$y_1 = x + 4100$ and $y_2 = 8500 - x$ when solved

$x + 4100 = 8500 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 4100 \quad 0 \le x \le 2200$.................. 4.13

Putting $n_i = J$, then $d(p, n_i) = d(A, J) = 6500, d(q, n_i) = d(B, J) = 4300$

$y_1 = x + 6500$ and $y_2 = 6500 - x$ when solved

$x + 6500 = 6500 - x, \Rightarrow x = 0$ Kink point

$y_2 = 6500 - x \quad 0 \le x \le 2200$.................. 4.14

Putting $n_i = K$, then $d(p, n_i) = d(A, K) = 5300, d(q, n_i) = d(B, K) = 7500$

$y_1 = x + 5300$ and $y_2 = 9700 - x$ when solved

$x + 5300 = 9700 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 5300 \quad 0 \le x \le 2200$.................. 4.15

Putting $n_i = L$, then $d \; p, n_i \; = d \; A, L \; = 0, d \; q, n_i \; = d \; B, L \; = 2200$

$y_1 = x + 5000$ and $y_2 = 9400 - x$ when solved

$x + 5000 = 9400 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 5000 \quad 0 \le x \le 2200$.................. 4.16

Putting $n_i = M$, then $d \; p, n_i \; = d \; A, M \; = 7000, d \; q, n_i \; = d \; B, M \; = 4800$

$y_1 = x + 7000$ and $y_2 = 7000 - x$ when solved

$x + 7000 = 7000 - x, \Rightarrow x = 0$ Kink point

$y_1 = 7000 - x \quad 0 \le x \le 2200$.................. 4.17

Putting $n_i = N$, then $d \; p, n_i \; = d \; A, N \; = 10200, d \; q, n_i \; = d \; B, N \; = 8000$

$y_1 = x + 10200$ and $y_2 = 12400 - x$ when solved

$x + 10200 = 12400 - x, \Rightarrow x = 1100$ Kink point

$y_1 = x + 10200 \quad 0 \le x \le 1100$.................. 4.18

$y_2 = 12400 - x \quad 0 \le x \le 2200$.................. 4.19

Putting $n_i = O$, then $d \; p, n_i \; = d \; A, O \; = 7800, d \; q, n_i \; = d \; B, O \; = 2200$

$y_1 = x + 7800$ and $y_2 = 12200 - x$ when solved

$x + 7800 = 1200 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 7800 \quad 0 \le x \le 2200$.................. 4.20

Putting $n_i = P,$ then $d \;\; p, n_i \;\; = d \;\; A, P \;\; = 10000, d \;\; q, n_i \;\; = d \;\; B, P \;\; = 7800$

$y_1 = x + 10000$ and $y_2 = 10000 - x$ when solved

$x + 10000 = 10000 - x, \Rightarrow x = 0$ Kink point

$y_2 = 10000 - x \quad 0 \le x \le 2200$.................. 4.21

Putting $n_i = Q,$ then $d \;\; p, n_i \;\; = d \;\; A, Q \;\; = 10850, d \;\; q, n_i \;\; = d \;\; B, Q \;\; = 8650$

$y_1 = x + 10850$ and $y_2 = 10850 - x$ when solved

$x + 10850 = 10850 - x, \Rightarrow x = 0$ Kink point

$y_2 = 10850 - x \quad 0 \le x \le 2200$.................. 4.22

Putting $n_i = R,$ then $d \;\; p, n_i \;\; = d \;\; A, R \;\; = 0, d \;\; q, n_i \;\; = d \;\; B, R \;\; = 7700$

$y_1 = x + 5500$ and $y_2 = 9900 - x$ when solved

$x + 5500 = 9900 - x, \Rightarrow x = 2200$ Kink point

$y_1 = x + 5500 \quad 0 \le x \le 2200$.................. 4.23

Putting $n_i = S,$ then $d \;\; p, n_i \;\; = d \;\; A, S \;\; = 11700, d \;\; q, n_i \;\; = d \;\; B, S \;\; = 9500$

$y_1 = x + 11700$ and $y_2 = 11700 - x$ when solved

$x + 11700 = 11700 - x, \Rightarrow x = 0$ Kink point

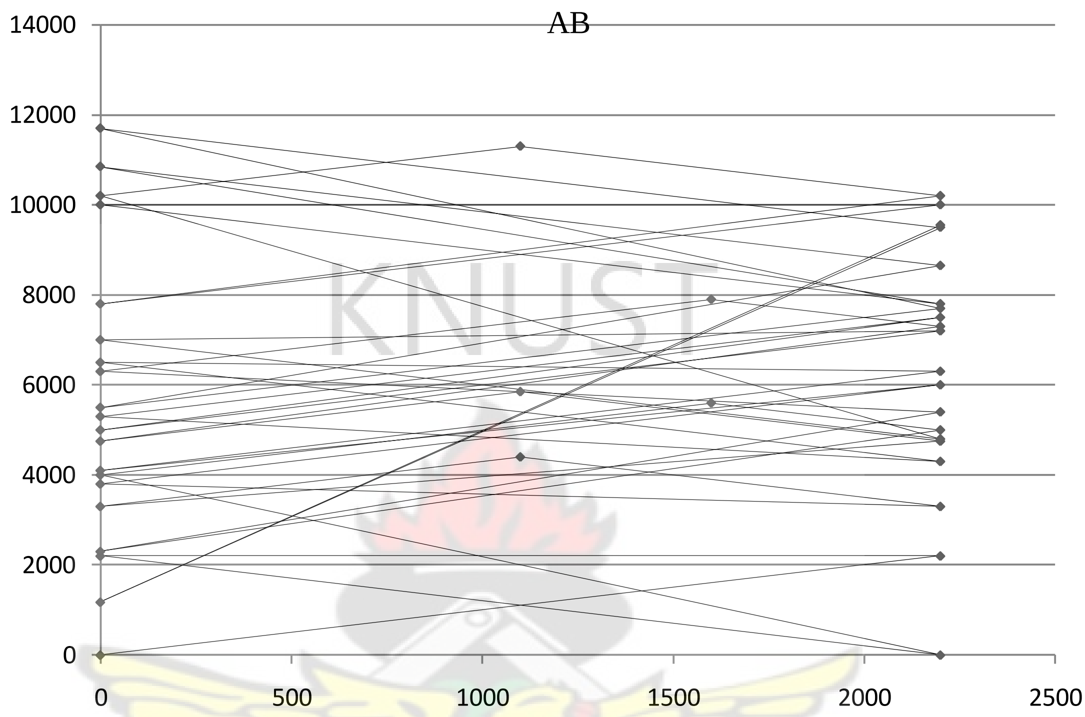
$y_2 = 11700 - x \quad 0 \le x \le 2200$.................. 4.24

Putting $n_i = T,$ then $d \;\; p, n_i \;\; = d \;\; A, T \;\; = 11750, d \;\; q, n_i \;\; = d \;\; B, T \;\; = 9550$

$y_1 = x + 11750$ and $y_2 = 11750 - x$ when solved

$x+11750=11750-x, \Rightarrow x=0$ Kink point

$y_2 = 11750 - x \quad 0 \le x \le 2200$.................. 4.25



$X_B = 2200, m(x_l) = 10200$

Fig. 4.2 Graph showing upper envelope and local center for edge AB

Computations for equations for the locations on edges (B,L), (C,B) and (C,G) and graphs to determine their local centers are shown in appendices D and E respectively.

(b) Local Center

Table 4.5 below has column one as edge number, column two is edge name, column three is location of edge center and column four is the least point of the upper envelope of each of the edges

Table 4.5 Shows local centers and cost for edges (A, B), (B,L), (C, B), and (C,G)

No.	Edge	Local Center (x_l)	cost $m(x_l)$
1	(A, B)	At node B	10200
2	(B, L)	At node B	10000
3	(C, B)	At node B	10000
4	(C, G)	At node G	11100

The least of the local centers of table 4.5 is 10000 and occurred at node B

4.4 Discussion

From table 4.3 the node or vertex center, $m(x_n)$ is 10000 metres. The least of the local centers, $m(x_l)$ from table 4.4 is 10000 metres. The least of the local centers is compared with the vertex or node center and the minimum is taken as the absolute center. By inspection the minimum of the node center and the least of the local centers is 10000. Hence the absolute center, $m(x_a)$ is 10000 metres and occurred at node *B* of the network.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The main objective of the study was to use the Absolute center-heuristic method to optimally locate a fire station in the New Juaben Municipality. The following findings were realized

1. The optimal location of the fire station for selected demand destinations of New Juaben Municipality was found to be at Effiduase (node B of the network).
2. The optimal service coverage distance was found to be 10000 metre radius from node B.

5.2 Recommendation

From the results obtained, the following recommendations are made:

1. No fire station should be sited without the appropriate scientific technique.
2. Studies be carried out to find the optimal locations and service coverage areas for other fire stations.

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APPENDIX A

Computations of the equations for the locations

(n_1, n_4) , (n_1, n_5) , (n_4, n_5) , (n_5, n_2) *and* (n_3, n_2)

LOCATION ON EDGE (n_1, n_4)

Choosing n_1 as the origin, let $p = n$ and $q = n_4$ such that $0 \le x \le c(p,q)$

Putting $n_i = n_1, i.e.\ i = 1$ then $d(p, n_i) = d(n_1, n_1) = 0, d(q, n_i) = d(n_4, n_1) = 6$ and

$c(p,q) = c(n_1, n_4) = 6,$ The resulting equations $y_1 = x$ and $y_2 = 12 - x$ when solved

$x = 12 - x \Rightarrow x = 6$ (Kink point)

$y_1 = x \quad 0 \le x \le 6$.......................$(i)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d(p, n_i) = d(n_1, n_2) = 8, d(q, n_i) = d(n_4, n_1) = 10$

$y_1 = x + 8$ and $y_2 = 16 - x$ when solved

$x + 8 = 16 - x \Rightarrow x = 4$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 4$.......................$(ii)$

$y_2 = 16 - x \quad 4 \le x \le 6$.......................$(iii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d(p, n_i) = d(n_1, n_3) = 8, d(q, n_i) = d(n_4, n_3) = 12$

$y_1 = x + 8$ and $y_2 = 18 - x$ when solved

$x + 8 = 18 - x \Rightarrow x = 5$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 5$.......................$(iv)$

$y_2 = 18 - x \quad 5 \le x \le 6$.......................$(v)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d(p, n_i) = d(n_1, n_4) = 6, d(q, n_i) = d(n_4, n_4) = 0$

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x+6=6-x \Rightarrow x=0$ (Kink point)

$y_2=6-x \quad 0\le x\le 6$.......................$(vi)$

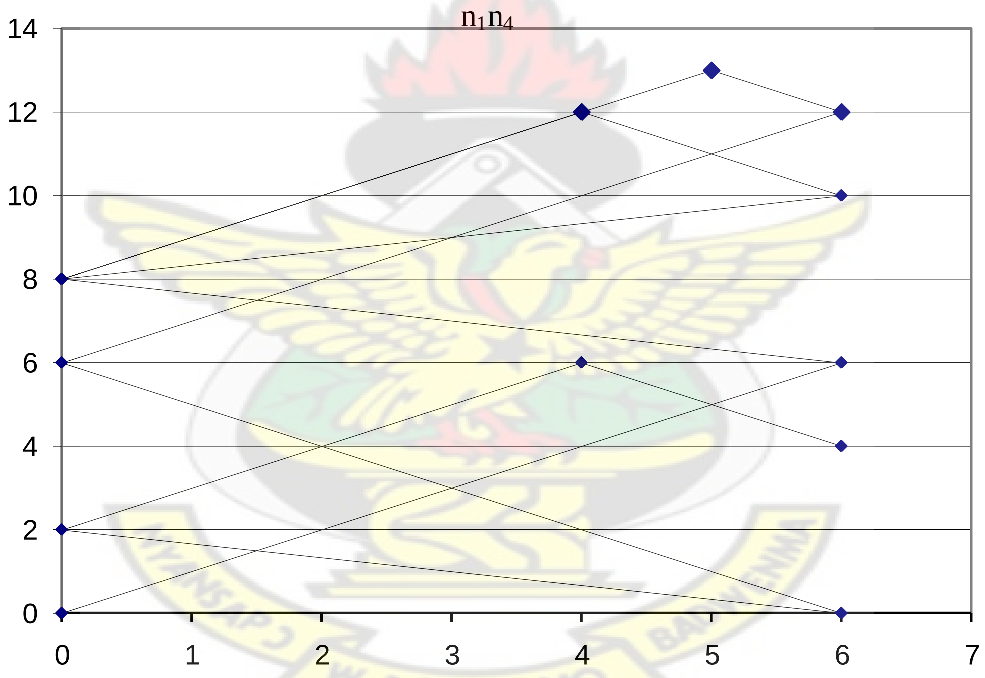
Putting $n_i=n_5, i.e.\ i=5$ then $d\ \ p,n_i\ \ =d\ \ n_1,n_5\ \ =8, d\ \ q,n_i\ \ =d\ \ n_4,n_5\ \ =4$

$y_1=x+2$ and $y_2=10-x$ when solved

$x+2=10-x \Rightarrow x=4$ (Kink point)

$y_1=x+2 \quad 0\le x\le 4$.......................$(vii)$

$y_2=10-x \quad 4\le x\le 6$.......................$(viii)$



$X_{n_1 n_4}=0 \qquad m(x_l)=8$

LOCATING ON EDGE (n_1, n_5)

Choosing $n_1 = p$ and $q = n_5$ such that $c(p,q) = c(n_1,n_5) = 2$

Putting $n_i = n_1, i.e.\ i = 1$ then $d(p,n_i) = d(n_1,n_1) = 0, d(q,n_i) = d(n_5,n_1) = 2$

$y_1 = x$ and $y_2 = 4 - x$ when solved

$x = 4 - x \Rightarrow x = 2$ (Kink point)

$y_1 = x \quad 0 \le x \le 2$.......................$(i)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d(p,n_i) = d(n_1,n_2) = 8, d(q,n_i) = d(n_5,n_2) = 2$

$y_1 = x + 8$ and $y_2 = 8 - x$ when solved

$x + 8 = 8 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 8 - x \quad 0 \le x \le 2$.......................$(ii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d(p,n_i) = d(n_1,n_3) = 8, d(q,n_i) = d(n_5,n_3) = 8$

$y_1 = x + 8$ and $y_2 = 10 - x$ when solved

$x + 8 = 10 - x \Rightarrow x = 1$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 1$.......................$(iv)$

$y_2 = 10 - x \quad 1 \le x \le 2$.......................$(v)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d(p,n_i) = d(n_1,n_4) = 6, d(q,n_i) = d(n_5,n_4) = 4$

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x + 6 = 6 - x \Rightarrow x = 0$ (Kink point)

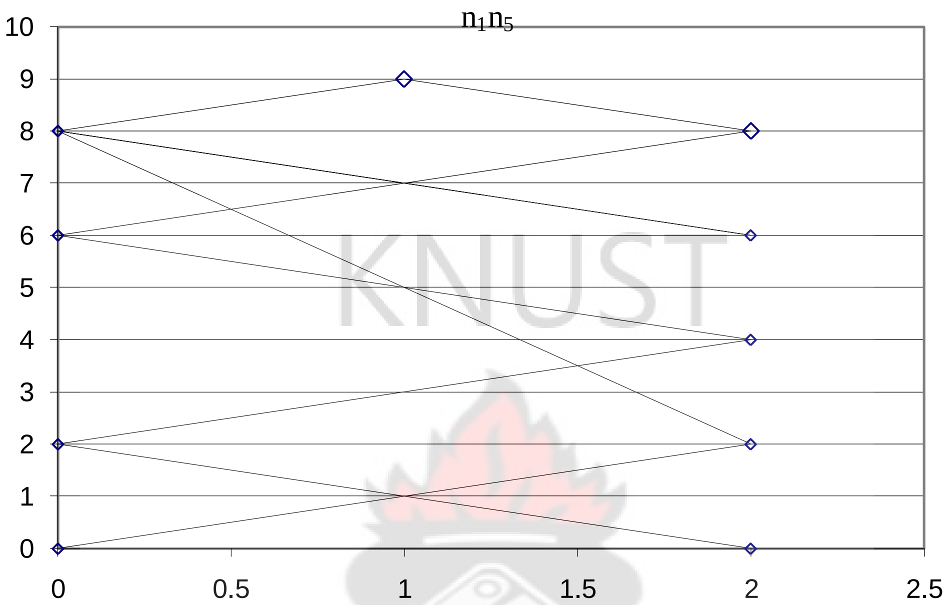
$y_2 = 6 - x \quad 0 \le x \le 2$.......................$(v)$

Putting $n_i = n_5, i.e.\ i = 5$ then $d(p,n_i) = d(n_1,n_5) = 2, d(q,n_i) = d(n_5,n_5) = 0$

$y_1 = x + 2$ and $y_2 = 2 - x$ when solved

$x+2=2-x \Rightarrow x=0$ (Kink point)

$y_2 = 2-x \quad 0 \le x \le 2$......................$(vi)$



$X_{n_1 n_5} = 0 \ or \ 2 \qquad m(x_l) = 8$

LOCATING ON EDGE (n_4, n_5)

Choosing $n_4 = p$ and $q = n_5$ such that $c(p,q) = c(n_4,n_5) = 4$

Putting $n_i = n_1$, i.e. $i=1$ then $d(p,n_i) = d(n_4,n_1) = 6, d(q,n_i) = d(n_5,n_1) = 2$

$y_1 = x+6$ and $y_2 = 6-x$ when solved

$x+6=6-x \Rightarrow x=0$ (Kink point)

$y_1 = 6-x \quad 0 \le x \le 4$......................$(i)$

Putting $n_i = n_2$, i.e. $i=2$ then $d(p,n_i) = d(n_4,n_2) = 10, d(q,n_i) = d(n_5,n_2) = 6$

$y_1 = x+10$ and $y_2 = 10-x$ when solved

$x+10=10-x \Rightarrow x=0$ (Kink point)

$y_2 = 10 - x \quad 0 \le x \le 4$.......................$(ii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d \quad p, n_i \quad = d \quad n_4, n_3 \quad = 12, d \quad q, n_i \quad = d \quad n_5, n_3 \quad = 8$

$y_1 = x + 12$ and $y_2 = 12 - x$ when solved

$x + 12 = 12 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 12 - x \quad 0 \le x \le 4$.......................$(iii)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d \quad p, n_i \quad = d \quad n_4, n_4 \quad = 0, d \quad q, n_i \quad = d \quad n_5, n_4 \quad = 4$

$y_1 = x$ and $y_2 = 8 - x$ when solved

$x = 8 - x \Rightarrow x = 4$ (Kink point)

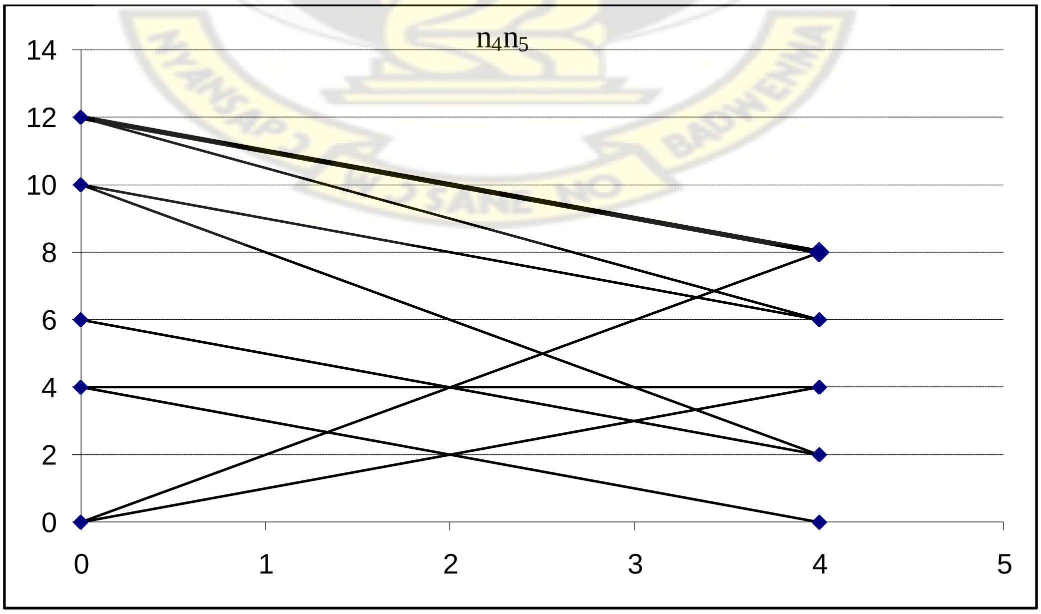
$y_1 = x \quad 0 \le x \le 4$.......................$(iv)$

Putting $n_i = n_5, i.e.\ i = 5$ then $d \quad p, n_i \quad = d \quad n_4, n_5 \quad = 4, d \quad q, n_i \quad = d \quad n_5, n_5 \quad = 0$

$y_1 = x + 4$ and $y_2 = 4 - x$ when solved

$x + 4 = 4 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 4 - x \quad 0 \le x \le 4$.......................$(v)$



$X_{n_4 n_5} = 4 \qquad m(x_l) = 8$

LOCATING ON EDGE n_5, n_2

Choosing $p = n_5$ and $q = n_2$ such that $c(p,q) = c(n_5, n_2) = 6$

Putting $n_i = n_1, i.e.\ i = 1$ then $d(p,n_i) = d(n_5,n_1) = 2, d(q,n_i) = d(n_2,n_1) = 8$

$y_1 = x + 2$ and $y_2 = 14 - x$ when solved

$x + 2 = 14 - x \Rightarrow x = 6$ (Kink point)

$y_1 = x + 2 \quad 0 \le x \le 6$.......................$(i)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d(p,n_i) = d(n_5,n_2) = 6, d(q,n_i) = d(n_2,n_2) = 0$

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x + 6 = 6 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 6 - x \quad 0 \le x \le 6$.......................$(ii)$

Putting $n_i = n_3, i.e.\ i = 3$ then $d(p,n_i) = d(n_5,n_3) = 8, d(q,n_i) = d(n_2,n_3) = 2$

$y_1 = x + 8$ and $y_2 = 8 - x$ when solved

$x + 8 = 8 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 8 - x \quad 0 \le x \le 6$.......................$(iii)$

Putting $n_i = n_4, i.e.\ i = 4$ then $d(p,n_i) = d(n_5,n_4) = 4, d(q,n_i) = d(n_2,n_4) = 10$

$y_1 = x + 4$ and $y_2 = 16 - x$ when solved

$x + 4 = 16 - x \Rightarrow x = 6$ (Kink point)

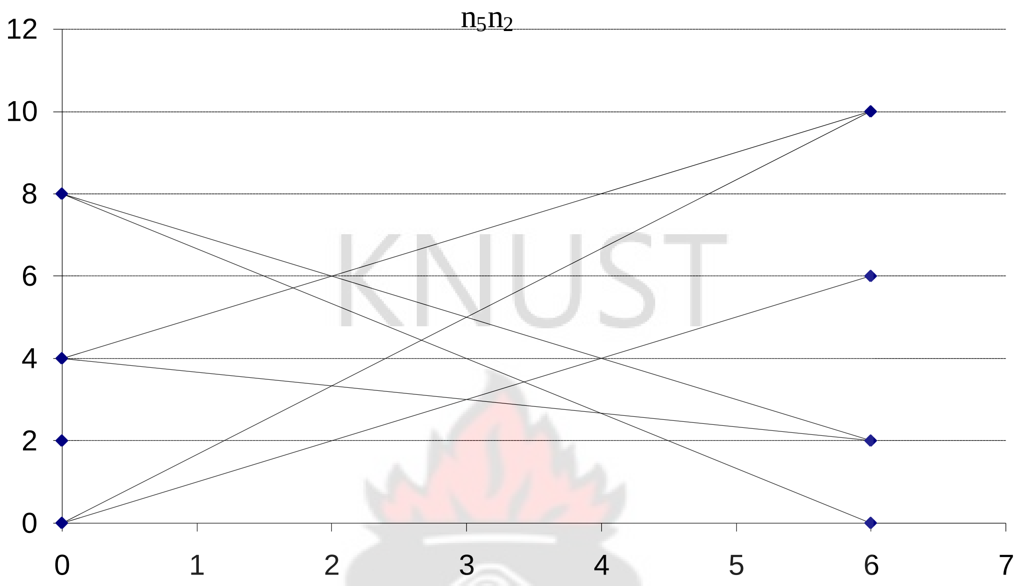
$y_1 = x + 4 \quad 0 \le x \le 6$.......................$(iv)$

Putting $n_i = n_5, i.e.\ i = 5$ then $d(p,n_i) = d(n_5,n_5) = 0, d(q,n_i) = d(n_2,n_5) = 6$

$y_1 = x$ and $y_2 = 12 - x$ when solved

$x = 12 - x \Rightarrow x = 6$ (Kink point)

$y_1 = x \quad 0 \le x \le 6$.......................$(v)$



$X_{n_5 n_2} = 2 \qquad m(x_l) = 6$

LOCATING ON EDGE n_3, n_2

Choosing $p = n_3$ and $q = n_2$ such that $c(p,q) = c(n_3, n_2) = 2$

Putting $n_i = n_1, i.e.\ i = 1$ then $d(p, n_i) = d(n_3, n_1) = 8, d(q, n_i) = d(n_2, n_1) = 8$

$y_1 = x + 8$ and $y_2 = 10 - x$ when solved

$x + 8 = 10 - x \Rightarrow x = 1$ (Kink point)

$y_1 = x + 8 \quad 0 \le x \le 1$.......................$(i)$

$y_2 = 10 - x \quad 1 \le x \le 2$......................$(ii)$

Putting $n_i = n_2, i.e.\ i = 2$ then $d(p, n_i) = d(n_3, n_2) = 2, d(q, n_i) = d(n_2, n_2) = 0$

$y_1 = x + 2$ and $y_2 = 2 - x$ when solved

$x+2=2-x \Rightarrow x=0$ (Kink point)

$y_2 = 2-x \quad 0 \le x \le 2$.......................$(iii)$

Putting $n_i = n_3, i.e.\ i=3$ then $d\ \ p,n_i\ \ = d\ \ n_3,n_3\ \ = 0, d\ \ q,n_i\ \ = d\ \ n_2,n_3\ \ = 2$

$y_1 = x$ and $y_2 = 4-x$ when solved

$x = 4-x \Rightarrow x=2$ (Kink point)

$y_1 = x \quad 0 \le x \le 2$.......................$(iv)$

Putting $n_i = n_4, i.e.\ i=4$ then $d\ \ p,n_i\ \ = d\ \ n_3,n_4\ \ = 12, d\ \ q,n_i\ \ = d\ \ n_2,n_4\ \ = 10$

$y_1 = x+12$ and $y_2 = 12-x$ when solved

$x+12 = 12-x \Rightarrow x=0$ (Kink point)

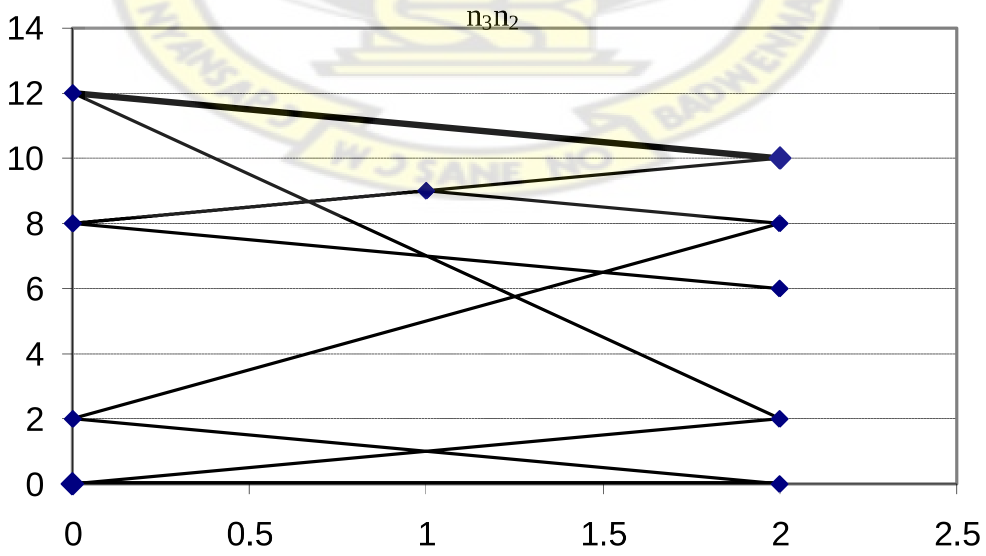
$y_2 = 12-x \quad 0 \le x \le 2$.......................$(v)$

Putting $n_i = n_5, i.e.\ i=5$ then $d\ \ p,n_i\ \ = d\ \ n_3,n_5\ \ = 8, d\ \ q,n_i\ \ = d\ \ n_2,n_5\ \ = 6$

$y_1 = x+8$ and $y_2 = 8-x$ when solved

$x+8 = 8-x \Rightarrow x=0$ (Kink point)

$y_2 = 8-x \quad 0 \le x \le 2$.......................$(vi)$



$X_{n_3 n_2} = 2 \qquad m(x_l) = 10$

APPENDIX B

New Juaben Municipality Map, having capital letters as selected demand nodes