OPTIMAL ADVERT PLACEMENT SLOT USING KNAPSACK MODEL.

A CASE STUDY: TELEVISION ADVERTISEMENT OF TV 3

By

Anthony Kwarteng, B. Ed Mathematics

A Thesis submitted to the Department of Mathematics,

Kwame Nkrumah University of Science and Technology

In partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Institute of Distance Learning

June, 2011

DECLARATION

I hereby declare that this submission is my own work towards the MSC and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

Anthony Kwarteng. PG 3014209 …………………… ………………….
Student Name & ID Signature Date

Certified by:

…………………………… ……………………… …………………….
Supervisor(s) Name Signature Date

Certified by:

……………………………… …………………………. …………………..
Head of Dept. Name Signature Date

Certified by:

……………………………..   ………………………….   …………………..

Dean of IDL.   Signature   Date

ABSTRACT

The study was about how to model the TV 3 adverts selection problems as 0 – 1 single knapsack problem so as to maximize the returns from their commercials.

Knapsack problem model is a general resource allocation model in which a single resource is assigned to a number of alternatives with the aim of maximizing the total return.

In this work, the researcher obtained the data on TV 3 adverts from the following zones:

A1: TV 3 News 360 (19:00 hours GMT),

A4: Music – Music (20 : 30 – 21:30 GMT) every Saturday.

A9: Mid Day Live (12 : 00 – 12 : 30 GMT)

Dynamic programming algorithm was used to solve the problem. To carry out the computations, the computer software, matlab was used to analyse the problem. The computational results showed that the optimal incomes of adverts from TV 3 News 360, Music – Music and Mid – Day Live programmes are Gh ¢30,005.00, Gh ¢15,696.00 and Gh ¢ 4,675.20 respective.

ACKNOWLEDGEMENT

In the course of my research work, a lot of people contributed immensely towards its success. The most prominent among them are Mr. K. F. Darkwah, a lecturer at KNUST Mathematics Dept. who supervised the work and Dr. S. K. Amponsah, the head of Mathematics Dept. KNUST, Kumasi.

I am also grateful to my wife, Janet Konadu, Madam Amma Achiaa (Maths Tutor, KNUST Senior High School), Madam Augustina Adu (Maths Tutor, St, Luis College of Education) and Mr. Collins Opoku (Archimedes),(Maths Tutor, Anglican Senior High School - Kumasi) for their support and encouragement.

I finally thank all those who directly or indirectly helped in making this study a success.

DEDICATION

This work is dedicated to my beloved wife, Janet Konadu and all my children.

TABLE OF CONTENT

CHAPTER 1

INTRODUCTION

CHAPTER 2

LITERATURE REVIEW

CHAPTER 3

METHODOLOGY

CHAPTER 4

DATA ANALYSIS AND RESULTS

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATION

CHAPTER 1

INTRODUCTION

1.1 Background Of The Study

One of the major problems confronting most of the business organizations is how to allocate their scarce resources (i.e capital, people ,time etc) across projects or other type of investments. Every business organization is a profit making entity, so the allocation of these resources should be done so as to maximize the total returns from a given investment. Generally, the objective is to select the particular subsets of projects which will be most effective and also be funded within the budget constraint.

Time is one of the greatest resources of Television and Radio stations. Broadcasting stations have to schedule programs interspersed with adverts or commercials. It is important to place adverts in such a way that the combination of adverts would have the largest exposure as possible so as to maximize the total returns from the adverts.

In order to achieve this goal, the broadcasting stations have to adopt appropriate scientific methods such as knapsack problem.

1.1.1 History of Broadcasting In Ghan

On the 31st July, 1935, the then governor of the Gold Coast, Sir Arnold Hodson together with an electrical engineer, F. A. W Byron, initiated the establishment of a wired radio distribution system in Accra.

It was began as a department of the Ministry of Information which was charged with the responsibility for the formulation of national mass communication policies and also for ensuring the effective use of mass media for the dissemination of information, and for economic and social development of the country.

In that same year, the Ministry established a Radio Broadcasting station in the country with approximately three hundred subscribers in Accra. Broadcasting began essentially as a relay service, re-broadcasting programmes from the BBC World Service. In 1936, the service began to expand and a re-diffusion station was opened in Cape Coast to cater for the people in the Central Region.

In 1940, a new broadcasting house with 1.3 KW transmitter was built in Accra which could broadcast to the neighbouring institutions.
Broadcasting in four of the major local languages: Twi, Fanti, Ga and Ewe started in the same year.

In 1952, a commission was appointed to advice the government on ways of improving and developing broadcasting in the country. The commission was charged to investigate the establishment and maintenance of a statutory corporation to assume direction and control of broadcasting services as was the case in Britain. Based on the commission's recommendations, the national service of the Gold Coast Broadcasting was established in 1954.

In 1956, locally produced programmes were increased. Also educational broadcast to schools and teacher training colleges started and. After the independent in 1957, the Gold Coast Broadcast System became the Ghana Broadcasting System or Radio Ghana. Mass communication was embraced as a way of changing society. Broadcasting in Ghana was thus to be a public service dedicated to the enlightenment of the people.

Since BBC was its main model, it adopted the public service model of the BBC from the onset.

In 1958, government of Ghana set another commission to advise it on the launching of an external service of the Radio Ghana. As a result of the commission's recommendations, the external service of radio Ghana was inaugurated in 1961. This was an unusually bold step for a newly independent country. The external service came about primarily because the Prime Minister, Dr. Kwame Nkruma, saw the broadcasting as an opportunity to propagate his pan African message to his follow African.

Four years later, GBC Television Service was launched. In 1997, GBC entered into agreement with WorldSpace to provide GBC with a channel on its Afristar satellite to enable it provide

a 24-hour, Direct Digital Broadcasting (DDB) service over a coverage

Chapter 12 of the 1992 Constitution guarantees the freedom and independence of the media. Article 2 explicitly prohibits censorship, while Article 3 pre-empts any licensing requirements for mass media. Journalists welcomed the liberal provisions of the constitution, hailing the 1990s as a new era of free expression in Ghana.

After the privatization of the airwave, the government gave approval to the allocation of frequencies to private television stations as well. As a result, two private channels, TV3 and Metro TV went on air in 1997.

1.1.2 Television Stations In Ghana

In Ghana, there is one state – run television station and several free-channels. The state-run television station, Ghana Television (GTV) is operated by the Ghana Broadcasting Corporation (GBC). It lays claim to covering 98% of Ghana's airwaves.

By law it is compelled to reach every nook and cranny of Ghana. GTV is presently available on 3 satellites:

1. Eutelsat W4 – through Multichoice Africa (popularly known as DSTV), GTV is available to DSTV subscribers in Ghana only.

2. Astra 2B at 28.2°E – [update] Ghana Broadcasting Corporation (GBC) now beams GTV and Uniiq FM to the world on Astra 2B.

3. Intelsat 903 at 34.5°W – home to several other free-to-air African TV stations, Intelsat 903 hosts GTV's satellite signal. GTV's signal on this satellite is beamed mainly on to Europe, fringes of North Africa and Asia.

TV3 Network Limited (TV3 Ghana) is one of the private, free-to-air television broadcasters in Ghana. It broadcasts its programmes via satellite.

Metro TV is another free-to-air television broadcaster in the country. It is a joint venture between Ghana Broadcasting Corporation and Media TV with 50/50 share holding. It has succeeded in broadcasting its network to all the regions in the country. Metro TV has struck a deal with South African's multichoice allowing it to broadcast throughout Africa.

A partnership between the government of Ghana and a private entity, Metro TV (Metropolitan Television) was the first Ghanaian TV station to go on satellite, a few years ago and has since been followed by many others. Metro TV is available on 2 satellites:

1. NSS 10 at 37.5°W – with a large footprint over Africa, the Middle East and Europe, NSS 10 brings Metro TV closer to billions of people across 3 continents.
2. Eutelsat W4 – Metro TV is the 2nd Ghanaian free-to-air TV service available on DSTV.

NSS 7 at 22.0°W – with a large footprint over West, East & Southern Africa, all the 3 next free-to-air Ghanaian TV stations can be accessed on this satellite. NSS7 also hosts 3 of the most popular Ghanaian FM radio stations viz Joy FM, Adom FM and Peace FM.

TV Africa is owned by a popular Ghanaian film maker Kwaw Paintsil Ansah, TV Africa started test transmission from Accra in 2002. The station was officially launched by the President of Ghana on 17th of May 2003. It is vailable on Nss7

Viasat 1 is a Ghanaian television channel owned by the Swedish listed media group, Modern Times Group (MTG). The channel was awarded a licence from the National Communications Authority of the Republic of Ghana in December 2007 and started broadcasting on December 12, 2008 on 6 p.m. Viasat 1 was MTG's first venture in Africa and became the fifth terrestrial television network in Ghana after GTV, Metro TV, TV3 and TV Africa. It broadcasts over Greater Accra, Central, Western and Ashanti region

Net 2 tv is another private television station in Ghana which is owned by Hon. Kennedy Ohene Agyapong, the New Patriotic Party member of Parliament for Assin North in the Central Region of Ghana. It was launched in 2007 in Ghana.

SMART TV is a new digital terrestrial television service launched in 2010 by Next Generation Broadcasting (NGB) in collaboration with Ghana Broadcasting Corporation (GBC).
The station offers both pay-TV service and free-to-air service of existing local stations on the Digital Terrestrial Television (DTT) network. The station currently airs in Accra and Kumasi and would later be extended to other regions in Ghana in the near future.

E.TV Ghana is a free – to – air commercial channel aimed at bringing premium television services to Ghana. The station was born out of a partnership between Global Media Alliance Ghana Ltd., and Sabido Investment (PTY) Ltd., of South Africa, the launch of e.tv Ghana stems from Sabido's success with e.TV in South Africa.

Other channels use either satellite or cable broadcasting. Multichoice provides its services through satellite, as well as Cable Gold whose service through cables serves part of Tema. V-Net TV and Fantazia TV also use cable broadcasting. Fontomfom TV telecasts in Kumasi. TV AGORO (TVA) is a pay-TV station which broadcasts wavelengths over Accra through bouquet of six premium channels comprising news channel CNN, music channel MCM, Cartoon Network for kids, Turner Classic Movies (TCM) and French channel CFITV. They also broadcast two religious channels on a 24-hour basis, being Trinity Broadcasting Network (TBN) and Catholic channel EWTN.

Currently, most of the channels in Ghana are on very high frequency (VHF). However, stations like CNN and others are broadcast on ultrahigh frequency (UHF) channels.

The trend in TV broadcast Ghana is towards the use of UHF channels.

1.1.3 Profile Of TV3 Ghana.

TV3 Network Limited (TV3 Ghana) is a private, free-to-air television broadcaster in Ghana which is owned by Malaysian media giant Media Prima. It was launched in Ghana in 1997.

TV3 Ghana airs and produces a variety of television programmes including acclaimed news bulletins, dramas and successful reality television and entertainment shows, some of which are apparently based on Malaysian programmes aired by television channels under Media Prima. For example: TV3 Ghana Mentor was based on the TV3 Malaysia's singing competition, also titled Mentor.[1] Media Prima claims that TV3 Ghana established itself as the most watched free-to-air television station in Ghana, having achieved 65% nationwide penetration at end-2006 and aiming to reach 90% by 2008.[2] Media Prima is also expected to list TV3 Network Ltd on the Ghana Stock Exchange by the first quarter of 2008. However, TV3 has experienced major competition in the likes of Metro TV which has succeeded not only in broadcasting its network to all the regions in the country but it also struck a deal with South Africa's Multichoice allowing it to be broadcast throughout Africa. Despite this, TV3 remains popular for its showing of Mexican telenovelas, Japanese series and music as compared to Metro's sports.

1.1.4 TV Broadcasting Programming or Scheduling

Broadcasting Scheduling is the act of organizing radio or television programmes in a daily, weekly or season-long schedule. Television scheduling strategies are adopted in such a way that the interest and satisfaction of the audiences would be reached and sustained. They are used to deliver programmes to audiences when they are most likely to want to watch them and deliver audiences to advertisers in the composition that makes their advertising most likely to be effective (Ellis, 2000).

1.1.5 Television Commercials Scheduling

As a business entity, every TV station has sources of income. One of the main sources of income for any private TV station is advertisement. The broadcasting is interspersed with advertising "breaks" normally three minutes long. In business, adverts are known as "spots". Typical spots last for 7, 15, 22, 30, 45, 60, 90, 120 seconds. In the advertising regulation, competing products should not be advertised within the same break. Hence, the products are scheduled into clash groups and products within the same group should not be advertised in the same break (Brown, 1969).

1.2 Statement of The Problem

The main aim of every entrepreneur is to allocate his or her scarce resources in order to maximize their returns. One of the greatest resources of Television stations is 'Time'. Hence, it should be well - managed so as to obtain a maximum returns.

As a private television station, the main sources of income for TV 3 Ghana are advertisements and sponsorships, which operate within the time constraint.

The administrators of the TV3, Ghana do face problems when selecting subsets of adverts or sponsors among the numerous adverts and sponsorships available that would earn them the optimal returns for the various programmes. The general practice is that, the selection is done by trial and error method and at the discretions of the administrators.

These approaches most times do not yield maximum results.

Television adverts selection problems can be modeled as the knapsack problems.

In this case, the adverts would be considered as the items (x), whilst the charges and duration for each advert are the values (v) and weights (w) respectively. The total time available for advertisements would be the capacity (c) of the knapsack. So Knapsack problem models could be used to solve advertising problems at ease for maximum returns. In addition, knapsack problem models can be applied to many real life applications or industrial situations such as budgeting, cargo loading and cutting stock etc.

1.3 Objectives

The objectives of the research are: (1) to model adverts scheduling as knapsack problem for TV3 station, (2) to maximize the total returns from TV 3 commercials.

1.4 Justification

Television stations play major roles in the political, educational and socio-economic development of the country. In order to discharge their functions well, the television stations need to generate enough revenue from the sponsored programmes and adverts to support their operations.

It is important to place adverts in such a way that the combination of adverts would maximize the total returns from the adverts. Without any adequate scientific methods of selecting from numerous adverts receive daily, the maximum returns from the adverts may not be achieved.

The adverts selecting problem can be modeled as knapsack problem and the appropriate algorithms could be applied as such, so that the maximum returns, is achieved.

When this happens, it would enable the TV 3 station to discharge its functions to the satisfaction of its cherished customers, which would justify its existence in the country. It may also lead to the expansion of the station, hence creating the employment for some people in the country.

In addition, knapsack problem can model many other managerial and industrial situations such as capital budgeting problem, cargo loading, cutting stock problems and routing of vehicles etc.

1.5 Methodology

The problem considered in the survey was the modeling of adverts selection problem as a single 0-1 knapsack problems that would yield optimal returns for television stations as well as the entire media. For instance, suppose the producer of TV 3 has n kinds of adverts (items), 1 through n. Each kind of advert has a charge (value) and duration (weight). The maximum time allotted for the adverts in the programme is C. The problem of the producer is to select subsets of adverts that would give him the maximum incomes taking into consideration the total time allotted for adverts.

The problem can be modeled as a single 0-1 knapsack problem as shown below:

Let x_i = the number of spots

v_i = value (charge) of the spots

w_i = weight (duration) of the spots

C = the maximum time allotted for the adverts.

$$Maximize\ Z = \sum_{i=1}^{n}(v_i x_i)$$

$$Subject\ to \sum_{i=1}^{n}(w_i x_i) \leq C$$

The data was collected from the office of TV 3 Ghana Ltd.

Dynamic programming algorithm was used to solve the problem.

For the computation, computer software, matlab was used to analysis the problem.

Resources for the study were : Library and internet.

1.6 Organization of Thesis

Chapter 1 deals with the background of the television industry in Ghana, Statement of Problem, the Objectives, Justification for the study, Methodology and thesis organization.

Chapter 2 provides the review of the relevant literature on the knapsack problem applications and the solution methods that have been proposed in the literature.

In chapter 3, the researcher outlined some algorithms for the solution methods such as the branch and bound, heuristic scheme, dynamic programming, simulation annealing and Genetic algorithm. Chapter 4 deals with the data collection and the results of analysis of the actual data from TV 3, Ghana. Chapter 5, provides the conclusion and recommendations of the work.

CHAPTER 2

LITERATURE REVIEW

Knapsack problems are notable of solving most of the industrial and managerial problems. In view of that, there have been many studies on it by both the theorists and practitioners in recent years. In this chapter, the researcher presents a review of the relevant literature on the knapsack problems and application

According to Zhong and Young, (2009) the integer programming tool and the Multiple Choice Knapsack problem should be used to provide optimal solutions to transportation programming problems in cases where alternatives versions of the projects are under consideration. In the study, they compared the optimization methods, which are used in the transportation programming process and also discussed the process of building and solving the optimization problems. They presented the concepts about the use of Multiple Choice Knapsack Problems (MCKP) and provided some examples of a real-world transportation programming at various budget levels.

Simoes et al., (2001) brought out an empirical study that compares the performance of the transposition-based Genetic Algorithm and the classical Genetic Algorithm for solving the 0-1 knapsack problem. The results obtained showed that the transposition is always superior to crossver.

According to Pendharker et al, (2005), the Information Technology Capital Budgeting, (ITCB) can be modeled as a 0 – 1 knapsack optimization problem. They further proposed two different simulation annealing (SA) heuristic solution procedures to solve the ITCB problems. They compared the performance of two SA heuristic procedures with the performance of two well-known ranking methods for capital budgeting. The results shown that the Information Technology (IT) investments chosen using the SA heuristics have higher after – tax profits than the IT investments selected using the two ranking methods.

Glickman and Allison, (1973) studied the problem of choosing among the technologies available for irrigation by tube wells to obtain an investment plan, which maximizes the net agricultural benefits from a proposed project in a developing country. They derived cost, benefit relationship, and incorporated it into a mathematical model that is solved by using a modification of the dynamic programming procedure for solving the knapsack problems. The study revealed that the optimal schedule favors small capacity wells, drilled by the indigenous methods with supplementary water distribution systems.

Lin and Yao, (2001) studied knapsack problem whereby all of the weight coefficients are fuzzy numbers. The assumption used was that each weight coefficient is imprecise because of the use of decimal truncation or rough estimation of the coefficients by the decision maker. The main purpose of the study was to extend the original knapsack problem into more generalized problem that could be useful in practical situations. The study showed that the fuzzy knapsack problem is an extension of the crisp knapsack problem. Moreover, the crisp knapsack problem is a special case of the fuzzy knapsack problem.

Akinc, (2006) addressed the formulation and solution of a variation of the classical binary knapsack problem, "fixed-charge knapsack problem". In this problem, the sub-sets of variables (activities) are associated with fixed costs. These costs may represent certain set-ups and/ or preparations required for the associated sub-set of activity to be scheduled. He discussed several potential real-world applications as well as problem extensions /generalization. The efficient solution of the problem depends on a standard branch- and – bound algorithm based on the following: (1) a non-negative polynomial algorithm to solve the LP relaxation, (2) various heuristic procedures to obtain good candidate solutions by adjusting the LP solutions, and (3) powerful rules to peg variables. From the computational experience, the suggested branch – and – bound algorithm shows excellent potential in the solution of a wide variety of large fixed-charge knapsack problems

Benisch et al, (2005) investigated the problem of choosing discriminatory prices for customers with probabilistic valuations and a seller with indistinguishable copies of a good. In the study, it revealed, that under certain assumptions, the problem could be reduced to the continuous knapsack problem (CKP). They presented a new fast epsilon-optimal algorithm for solving CKP instances with asymmetric concave reward functions. It was shown that the algorithm can be extended beyond the CKP setting to handle pricing problems with overlapping goods, rather than indistinguishable goods.

Bazgan et al., (2007) on their part presented an approach, based on dynamic programming for solving the 0-1 multi-objective knapsack problems. Their approach was based on the use of several complementary dominance relations to discard partial solutions that cannot lead to new non-dominated criterion vectors. They obtained an efficient method that performed better than the existing methods, both in terms of CPU time and size of the solved instances. Extensive numerical experiments on various types of instances were reported. They also compared their approach with other exact methods.

Eilon and Williamson, (1998) developed BARK (Budget Allocation by Ranking and Knapsack) to solve a particular problem in determining which projects should be selected, from a given array, for implementation subject to the budgetary constraints. This problem is often encountered in the public sector, where the values in financial terms, of competing projects are difficult or impossible to quantify, but where the projects may be ranked in terms of their perceived worth or benefit.

Argali at el, (2009) considered the allocation of a limited budget to a set of activities or investments in order to maximize return from investment. In a number of practical contexts, the return from investment in an activity is effectively modelled using an S-curve, where increasing returns to scale exist at small investment levels, and decreasing returns to scale occur at high investment levels.

Kalai at el, (2006) studied the robust Knapsack problem using a max-min criterion. The Knapsack problem is a classical combinatorial problem used to model many industrial situations.

Gerald et al, (2008) investigated a procurement problem where suppliers offer concave quantity discounts. The resulting continuous knapsack problem involves the minimization of a separable concave function.

Pisinger, (2007) gave a survey of upper bounds presented in the literature. In the study, the relative tightness of several bounds was shown. The technique for deriving the bounds included relaxation from upper planes, linearization, reformulation, Lagrangian decomposition, Lagrangian relaxation and semi definite programming. He also provided a short overview of heuristics, reduction techniques, branch-and-bound algorithms and approximation results, followed by an overview of the valid inequalities for the quadratic knapsack polytope. He concluded by an experimental study where the upper bounds presented were compared with respect to strength and computational effort

Martello et al, (1998) came out with a new algorithm for the optimal solution of the 0-1 Knapsack problem. The algorithm is very effective for large-size problems. It is based on the determination of an appropriate small subset of items and the solution of the corresponding “core problem”. They also derived a heuristic solution for the original problem that, with high probability, can be proved optimal. The algorithm incorporated a new method of computation of upper bound and coefficient implementations of reduction procedures.

Dantzig, (1957) proposed a greedy approximation algorithm to solve the unbounded knapsack problem. His version sorts the items in decreasing order of value per unit of weight. It then proceeds to insert them into the sack, starting with as many copies as possible of the first kind of item until there is no longer space in the sack for more. If there is an unlimited supply of each kind of item, and *m* is the maximum value of items that fit into the sack, then the greedy algorithm is guaranteed to achieve at least a value of m / 2. However, for the bounded problem, where the supply of each kind of item is limited, the algorithm may be far from optimal.

Ferreira, (1995) on his part, presented parallel algorithms for solving a knapsack problem of size n on PRAM and distributed memory machines. The algorithms were very effective because they achieved the optimal speedup with respect to the well-known solutions to the problem. In addition, they match the best current time/memory/ processors tradeoffs, while requiring less memory and / or processors. Since the PRAM is a theoretical model, and the aim was to obtain the practical algorithms for the knapsack problems, the study covered its solution in the distributed memory machine. For the first time in literature, work-efficient parallel algorithms on local memory-message passing architectures-are given. Time bounds for solving the problem on linear arrays, meshes, and hypercubes were proved.

Burkard et al, (1995) considered parametric knapsack problems where the cost respective weights are replaced by linear functions depending on a parameter, t. the aim is to find the smallest parameter, t such that the optimal solution value of the knapsack problem is equal to a pre-specified solution value. They developed pseudo-polynomial algorithms for the inverse parametric problems. In addition, with the help of the special properties of the parametric value function, they constructed search methods. Using computational experiments, the behaviour of these algorithms were investigated, and the favourable practical performance of different search methods exhibited.

Florios et al, (2009) solved instances of the Multi-Objective Multi-Constraints knapsack problem from the literature. The problem involved three objective functions and three constraints. They used both exact and approximate algorithms. Three branching heuristics and a more general purpose composite branching and construction heuristic were devised. The results from the exact algorithm showed that the branching heuristics greatly improved the performance of the Multi-Criteria Branch and Bound algorithm, which becomes faster than the adaptive

Pisinger,(1997) in his study, presented an algorithm for Knapsack Problem where the enumerated core size is minimal, and the computational effort for sorting and reduction is also limited according to the hierarchy. The algorithm was based on the dynamic programming approach in which the core size is extended by need. He presented computational experiments for several common occurring types of data instances. The tests indicated that his approach performed better than any known algorithm for Knapsack Problems, having very stable solution times.

Zhang et al, (2004) came out with a simple but useful method, the core of which is an efficient LP-based heuristic, for solving bi-objective 0-1 knapsack problems. The results of the computational experiments showed that the proposed method is able to generate a good approximation to the non-dominated set efficiently. In addition, they suggested three qualitative criteria to evaluate such an approximation. The method can be extended to other problems having properties similar to the knapsack problem.

Rinnooy et al, (1993) proposed a class of generalised greedy algorithms for the solution of the Multi-Knapsack Problem. In the algorithm, items are selected based on the decreasing ratios of their profit and a weighted sum of their requirement coefficient. The solution obtained depended on the choice of the weights. They gave the geographical representation of the method and exploited the relation to the dual of the linear programming relaxation of Multi-Knapsack. They also investigated the complexity of computing a set of weights that gives the maximum greedy solution value. Finally the heuristics were subjected to both a worst-case a probabilistic performance analysis.

Balev et al, (2008) presented a pre-processing procedure for the 0-1 Multidimensional Knapsack problem. They first generated non-increasing sequence of upper bound by solving LP relaxation, followed by creating a non-decreasing sequence of lower bound using dynamic programming. The comparison of the two sequences allowed either to prove that the best feasible solution obtained is optimal or to fix a subset of variables to their optimal value. They also obtained a heuristic solution. The computational experiments with a set of large-scale instances showed the efficiency of their reduction scheme. Particularly, it was shown that their approach allowed the reduction of the CPU time of leading commercial software.

Martello et al, (2000), gave the overview of the recent techniques for solving hard knapsack problems with special emphasis on the addition of cardinality constraints, dynamic programming, and rudimentary divisibility. They presented the computational results, which compared all the recent algorithms.

Taniguchi et al, (2008) introduced a kind of surrogated relaxation to derive upper and lower bounds quickly and proved that, with this pre-processing, the similar pegging tests can be applied to our problem. The branch-and-bound algorithm can be used to solve the reduced problem to optimality. They used the surrogated variables to evaluate the upper bound at each branch-and-bound node very quickly by solving a continuous Knapsack Problem. The study showed that the developed method finds upper and lower bounds of very high accuracy in a few seconds and solves large instances to optimality faster than the previously published algorithms.

Caprara et al, (2004) presented the two-dimensional Knapsack Problem with the aim of packing a maximum-profit subset of rectangles selected from a given set into another rectangle. In the study, they considered the natural relaxation of two-dimensional Knapsack problem given by the one-dimensional Knapsack Problem with item weights equal to the rectangle areas, proving the worst-case performance of the associated upper bound. They presented and compared computationally four exact algorithms based on the above relaxation and showed their effectiveness.

Pisinger, (1995), presented a new branch-and-bound algorithm for solving the exact solution of the 0-1 Knapsack Problem. The algorithm was used to solve an 'expanding core' which initially contains the break items, and it is expanded each time the branch-and-bound algorithm reaches the border of core. The computational experiments showed that most data instances are optimality solved without sorting or pre-processing a great majority of the items.

Yamada et al, (1998) came out with a branch-and-bound algorithm and a binary search algorithm to solve such problem to optimality. They implemented the algorithms and used the computational experiments to analyse the behaviour of the developed algorithms. The study revealed that the binary search algorithm solves Knapsack Sharing Problems with up to 20 000 variables in less than a minute in their computing environment.

Bortfeldt et al, (2001) presented a hybrid genetic algorithm for the container-loading problem with boxes of different sizes and a single container for loading. In the presentation. They generated stowage plans with several vertical layers each containing several boxes. They also represented the stowage plans by complex data structures closely related to the problem. The offspring were generated by means of some specific genetic operators that based on an integrated greedy heuristic. The process considers several practical constraints. Extensive test calculations including procedures from other authors vouch for the good performance of the Genetic Algorithm, above all for problems with strongly heterogeneous boxes.

Marcques et al, (2007) modelled an integer non-linear optimisation problem and for which some heuristic methods were designed. Finally, computational experiments were given to analyse the methods

Figuera et al, (2009) also investigated into a generic labelling algorithm for finding non-dominated outcomes of the multiple objective integer knapsack problems. The algorithm is used for solving the multiple objective shortest path problem on an underlying network. They presented the algorithms for constructing four network models. Every network consists of layers and network algorithm. When working forward layer by layer, they identified the set of all permanent non-dominated labels for each layer. The effectiveness of the algorithms is supported with numerical results obtained for the randomly generated problems for up to seven objectives. However, the exact algorithms in the literature solve the multiple objective binary knapsack problems with up to three objectives. It is realised that the approach can be applied to other classes of problems such as multiple constraints, binary variables, time depended objective functions etc.

Lokketangen et al, (1998) described a tabu search approach for solving general 0-1 mixed integer-programming problems that exploits the extreme point property of 0-1 solutions. They identified the specialised choice rules and aspiration criteria for the problems and expressed them as functions of integer infeasibility measures and objective function values. The first-level tabu search mechanisms were extended with advanced level strategies and learning. A critical event tabu search method that navigates both sides of the feasibility boundary has been shown that it is effective to solve the multidimensional knapsack problem.

Li and Curry, (2005) proved the merits of using surrogate constraint information vs. a Lagrangian relaxation scheme as choice rules for the problem class. They presented a constraint normalization method to strengthen the surrogate constraint information and improve the computational results. Additionally, they demonstrated the advantages of intensifying the search at critical solutions.

Hanafi et al (1998) on their part also described a approach to tabu search. They based on strategic oscillation and surrogate constraint information that provides a balance between intensification and diversification strategies. They provided new rules to the control the oscillation process for the 0-1 multidimensional knapsack problems. The results of the study showed that their method obtains solutions whose quality is at least as good as the best solutions obtained by previous methods, especially with large-scale instances. These encouraging results confirmed the efficiency of the tunnelling concept coupled with surrogate information when resource constraints are present.

Elsevier et al, (2003) presented the Utility Model for optimal Routing and Admission Control (RAC) of a data network supporting sessions requiring Quality of Service (QoS) guarantees. The model mapped the optimal RAC problem to a Multiple-choice Multi-dimension 0-1 Knapsack Problem (MMKP), a variation of the classical 0-1 Knapsack Problem. They also presented a design for an optimal RAC system based on the Utility Model. It was shown that the optimal RAC using the MMKP formulation (OptRAC) would provide 7–16% more revenue than the revenue provided by a traditional RAC system (TradRAC).

Elsevier et al, (2008) also introduced a utility model (UM) for resource allocation on computational grids. The allocation problem was formulated as a variant of the 0-1 multichoice multidimensional knapsack problem. The notion of task-option utility was introduced, and it was used to effect the allocation policies. They presented a variety of allocation policies, which were expressed as functions of metrics that are both intrinsic and external to the task and resources. The UM allocation strategy was shown to optimally allocate resources congruent with the chosen policies.

Okan et al, (2009) utilized admission control algorithms designed for revenue optimization with Quality of Service (QoS) guarantees to derive optimal pricing of multiple service classes in wireless cellular networks. A service provider typically adjusts pricing only periodically. Once a “global” optimal pricing is derived, it would stay static for a period, allowing users to be charged with the same rate while roaming. They utilized a hybrid partitioning-threshold admission control algorithm, to analyze a pricing scheme that correlates service demand with pricing, and to periodically determine optimal pricing under which, the system revenue is maximized while guaranteeing that QoS requirements of multiple service classes are satisfied.

Adlen et al (2007), proposed a new MAC protocol featuring a dynamic channel reservation by using dynamic TXOP limit parameter assignment, along with a fully distributed admission control algorithm. They generalized the model so thatsfied. Each AC active in the network may *a priori* assess the achievable QoS. The later model is used to design a fully distributed admission control algorithm that regulates the network load to protect the already admitted flows from new entering flows. Simulation results showed that compared to both EDCA and AEDCF (Adaptive EDCF), the protocol excels, in terms of network utilization and ability to guarantee the same QoS metrics’ performances to flows of the same AC (intra-class).

Pati et al, (2002) observed that with increasing use of mobile units for various applications, it has become imperative to provide services with quality of service (QoS) guarantees. They proposed techniques for bandwidth reservation and call admission control for wireless mobile networks. The proposed techniques utilize the available bandwidth efficiently to provide QoS guarantee with reduced new call rejection and handoff call blocking. They analyzed the performance of the scheme through extensive simulation studies. The results appeared promising.

Moraga et al, (2005) presented a solution approach, Meta-RaPS for the 0-1 Multidimensional Knapsack Problem. They implemented four different greedy priority rules within Meta-RaPS and compared them. Moreover, they implemented two simple local search techniques within Meta-RaPS' improvement stage. It was found that the Meta-RaPS approach performed better than many other solution methodologies in terms of differences from the optimal value and number of optimal solutions obtained. It is realized that Meta-RaPS approach is easy to understand, implement and achieves good results.

Ahmed et al., (1987) investigated into the problem of selecting a set of projects from a large number of available projects such that at least some specified levels of benefits of various types are realized at a minimum cost. The problem was formulated as 0-1 multidimensional knapsack problem. Because of the NP-completeness of the problems, polynomial bounded and efficient heuristic algorithms were proposed for its solutions. In the algorithm, the initial selection is done by prioritizing the projects according to a computed discarded index. Then the initial selection set is altered to reduce the total costs by using project exchange operations. According to the computational results, the proposed algorithm was quite effective in finding optimal or near the optimal solutions.

Carlo (1994) described a Lagrangean decomposition technique for solving multi-project planning problems with resource constraints and alternative modes of performing each activity in the projects. The decomposition can be useful in several ways: from one side, it provides bounds on the optimum, so that the quality of approximate solutions can be evaluated. Again, in the context of the branch- and- bound algorithms, it can be used for more fathoming of the tree nodes. Finally, in the modeling perspective, the Lagrangean optimal multipliers can provide insights to project managers as prices for assigning the resources to different projects.

The multidimensional 0-1 knapsack problem is considered as a knapsack with multiple resource constraints.

Freville et al., (2004) came out with an efficient preprocessing procedure for large-scale instances. The algorithm provides sharp lower and upper bounds on the optimal value. It is also a tighter equivalent representation by reducing the continuous feasible set and by eliminating constraints and variables. The scheme was proved to be very effective through a lot of computational experiments with test problems of the literature and large-scale randomly generated instances.

Abboud et al., (1997) presented an interactive procedure for the multi-objective multidimensional 0-1 knapsack problem that takes into consideration the incorporation of fuzzy goals of the decision maker. It is easy to use since it requires from the decision maker to handle only one parameter, the aspiration level of each objective. Additionally, it is fast and can treat such problem as 0-1 knapsack problem using already available software, the primal effective gradient method, which is meant for solving the large-scale cases

Ghorbani et al., (2009) studied a new multi-objective algorithm for project selection problem. They considered two objective functions to maximize total expected benefit of selected projects and minimized the summation of the absolute variation of allotted resources between each successive time- periods. A meta-heuristic multi-objective was proposed to obtain diverse locally non- dominated solutions. They compared the proposed algorithm, based on some prominent metrics with a well-known genetic algorithm, NSGA-II. the computational results showed that the proposed algorithm outperformed the NSGA-II.

Harper et al., (2001) presented a genetic algorithm as an aid for project assignment.

The assignmeent problem was about the allocation of the projects to students. If the students are to choos from a list of projects in advance, inevitably, most students would prefer the more popular projects. Hence such projects would be over-subscribed and assignment becomes a complex problem. The developed algorithm has compared well to an optimal integer programming approach. The main advantage of the genetic algorithm is that by its nature, the number of feasible project assignments can be produced, thus facilitating discussion on the merits of various allocations and supporting multi-objective decision making.

Gholamian et al., (2007) in their study, used a hybrid intelligent system instead of the mathematical model to solve the multi-objective problems. The main core of the system is fuzzy rule base, which maps decision space (Z) to the solution space (X). the system is designed on non-inferior region and gives a big picture of this region in the pattern of fuzzy rules. Since some of the solutions may be inferior, the specified feed forward neutral network is used to obtain non-inferior solutions in an exterior movement. They also provided the numerical examples of well-known NP-hard problems to clarify the accuracy of the developed system.

Aissi et al., (2007) studied into the approximation of min-max. (regret) versions of classical problems like shortest path, minimum spinning tree, and knapsack. For a constant number of scenarios, they established fully polynomial-time approximation schemes for the mini- max versions of these problems, using relationships between multi-objective and mini-max optimization. They used the dynamic programming and classical trimming techniques to construct a fully polynomial-time approximation scheme for min-max regret shortest path. Additionally, they established a fully polynomial-time approximation scheme for min-max regret spinning tree and proved that min-max regret knapsack is not at all approximable. For a non-constant number of scenarios, in which case, min-max and min-max regret versions of the polynomial-time solvable problems usually become strongly NP-hard, non-approximability result were provided for min-max (regret) versions of shortest path and spinning tree.

Lin et al., (2001) came out with an efficient linear search algorithm for solving the 0-1 knapsack problem. They included a net profit criterion in the linear search algorithm to generate a rescheduled candidate set. Four hard cases presented by Yang (1992) were tested and compared with the revised approach. The result showed that the proposed approach outperformed the previous works in terms of producing a small candidate set while retaining most of the information on optimal.

Again, Lin (2008) investigated the possibility of using genetic algorithms in solving the fuzzy knapsack problem without defining the membership functions for each imprecise weight coefficient. The proposed approach simulated a fuzzy number by distributing it into some partition points. The genetic algorithm was used to obtain the values in each partition point so that the final values represented the membership grade of the fuzzy number. The result of the study showed that the proposed approach performs better within the given bound of each imprecise weight coefficient than the fuzzy knapsack approach. The fuzzy genetic algorithm is different, but gave better results than the traditional fuzzy approach.

Balachandar et al., (2008) presented a heuristic to solve the 0-1 multi-constrained Knapsack problem, which is NP-hard. In the study, they exploited the dominance property of the constraints to reduce the search space to near optimal solutions of 0-1 multi-constrained knapsack problem. They also presented the space and computational complexity of solving 0-1 multi-constrained knapsack problems using that approach. The results from relative large size test problems showed that the heuristic can successfully be used for finding good solutions for highly constrained NP-hard problems.

Devyaterikova et al., (2009) presented discrete production planning problem, which may be formulated as the multidimensional knapsack problem. The resource quantities of the problem are supposed to be given as intervals. The approach to solve the problem based on using its relaxation set was suggested. Some L-class enumeration algorithms for the problem were described. They also presented the results of the computational experiments. The configuration of an optimization algorithm can made a significant difference to the efficiency of the solution process.

Realff et al., (1999) investigated the use of branch-and-bound for solution of the classical knapsack problem. It was shown that the best configuration of the algorithm could be the data dependent. So an 'intelligent' optimization system needs to configure itself automatically with the control knowledge appropriate to the problem the user is solving.

Gomes da Silva et al., (2007) studied the problem of inaccuracy of the solution generated by meta-heuristic approaches for combinatorial optimisation bi-criteria 0-1 knapsack problems. They proposed a hybrid approach that combines systematic and heuristic searches to reduce that inaccuracy in the context of a scatter search method. They also presented the comparisons with small and medium size instances solved by the exact methods. Large size instances were also considered and the quantity of the approximation was evaluated by taken into account the proximity to the upper frontier, devised by the linear relaxation, and the diversity of the solutions. They also compared the approach with other two well-known meta-heuristic approaches. The results showed the effectiveness of the proposed approach for small, medium and large size instances.

Yield management is an essential issue in the television advertisement. The major parts of the research in the revenue management focus on the airline or hotel industry. For television advertisements, the offers are decomposed into small television breaks or spot. In the advertising regulation, the competing products should not be advertised within the same break. Hence, the scheduling products into clash groups such that products within the same group are not advertised in the same break (Brown, 1969). Martin, (2004) proposed generic solution based on simulations and approximate.

Tsesmetzis et al, (2008) studied the problem of providers that receive multiple concurrent requests for services demonstrating different QoS properties. They introduced the " Selective Multiple Choice Knapsack Problem" that aims at identifying the services, which should be delivered in order to maximise the provider's profit, subject to maximum bandwidth constraints. The problem was solved by a proposed algorithm that had been empirically evaluated via numerous experiments.

Fubin et al, (2002) presented a simulated annealing (SA) algorithm for the 0-1 multidimensional knapsack problem. Problem-specific knowledge is incorporated in the algorithm description and evaluation of parameters. In order to look into the performance of finite-time implementation of SA computational results showed that SA performs much better than a genetic algorithm in terms of solution time, whilst requiring only a modest loss of solution quality.

Beasley, (2002) discussed the basic features of population heuristic and provided practical advice about their effective use for solving operations research problems including knapsack.

The Knapsack problem model is a general resource allocation model in which a single resource is assigned to a number of alternatives with the objective of maximizing the total return.

Owoloko et al., (2010) studied into the application of the knapsack problem model to the placement of advert slots in the media. The aim was to optimize the capital allocated for advert placements. The general practice is that funds are allocated by trial and error and at the discretions of persons. This approach most times do not yield maximum results, lesser audience are reached. But when the scientific Knapsack problem model was applied to industry data, a better result was achieved, wider audience and minimal cost was attained.

Oppong, (2009) considered the application of classical 0-1 knapsack problem with a single constraint to the selection of television advertisements at critical periods such as Prime Time News, new adjacencies and peak times. The television stations have to schedule programmes interspersed with adverts or commercials, which are the main sources of income for the broadcasting stations. The goal in scheduling commercials is to achieve wider audience satisfaction in order to maximise the total returns. The approach is flexible and can incorporate the use of knapsack for the profit maximisation in the television advert selection problem. The work was focused on the used of simple heuristic scheme (simple flip) for the solution knapsack problems. It was shown that the results from the heuristic method compares favourably with the well-known meta-heuristic methods such as Genetic Algorithm and Simulation Annealing.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter deals with the methodology of the study. It describes some types of knapsack problems, modelling of data, method for solving the problem and algorithm used to solve the problem.

3.2 TYPES OF KNAPSACKS PROBLEMS AND SOLUTIONS

Knapsack problems have known a large number of variations due to its wide range of applicability. Some of the variations are single and multiple-constrained knapsacks, knapsacks with disjunctive constraints, multidimensional knapsacks, multiple choice knapsacks, single and multiple objective knapsack, integer, linear, non-linear knapsacks, deterministic and stochastic knapsacks with convex or concave objective functions etc.

3.1.2 The Subset Sum Knapsack Problem

The problem is 0 – 1 problem as well as a decision problem with the values of items being the same as their respective weights.

The subset sum knapsack problem seeks to find from a given set of nonnegative integers, any subset of it that add up to exactly C, where C is the weight of the knapsack

The subset sum problem can be formulated as:

Let v_i = the value of the ith item

W_i = the weight of the ith item

C = the weight of the knapsack

But $v_i = w_i$

$$Maximize\ \ Z = \sum_{i=1}^{n} (w_i x_i) \ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

$$Subject\ to\ \sum_{i=1}^{n} (w_i x_i)\ \le C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

for $i = 1, 2, 3, \ldots, n$

Let $x_i = \{0, 1\}$ be the state of a piece of item being included in the knapsack

($x_i = 1$), if x_i item is included and ($x_i = 0$), if x_i item is excluded from the knapsack.

Equation 3.1 above gives the total value of items packed in the knapsack, whereas equation 3.2 gives the maximum weight (capacity), that the knapsack can take.

3.1.2 The Change-Making Problem

It is a bounded knapsack problem in which the value of each item is one,(i.e $v_i = 1$)

In the capacity constraint, the equality is imposed instead of the inequality. This can be formulated mathematically as:

Let w_i = the weight of ith item

b = the weight of the knapsack

$$Maximize, Z = \sum_{i=1}^{n}(x_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.3$$

,

$$Subject\ to\ \sum_{i=1}^{n}(w_i x_i) = b \ldots\ldots\ldots\ldots\ldots\ldots.3.4$$

For x_i ={ 0, 1 }, and i = 1, 2, 3, . . . , n

The performance function (i.e equation 3.3), that gives the total value of the items packed in the knapsack is the same as total number of items packed in the knapsack since $v_i = 1$.
The constraint function (i.e equation 3.4), prescribes the maximum weight that the knapsack can take.

The problem recalls the situation of a cashier having to assemble a given change b using the maximum or minimum number of coins in denomination, x_i, hence the name the change-making problem.

3.1.3 Multiple Knapsack Problems

It is 0 –1 Knapsack problem in which *m* number of containers with given capacities c_i,

(i= 1, 2, 3, . . . , *m*) are available.

In this type of problem, each solution variable, x_{ij}, is restricted to only binary values, $x_{ij} = 1$ if *item j* is selected for the container i, and 0 if otherwise.

The generalization arising when the item set is partitioned into subsets and the additional constraint is imposed that at most one item per subset is selected. That type of problem is called the Multiple-Choice Knapsack Problem. The multiple choice knapsack problem is defined as a knapsack problem with additional disjoint multiple choice constraint. The general description of the problem is given as: there is one knapsack with limited capacity. The objects to be packed in the knapsack are classified into multiple mutually exclusive classes. Within each class, there are several different items. The problem is to select some items from each class so as to minimize the total cost while the total size of the items do not exceed the limited capacity of the knapsack. This problem is of a generalized carryout problem and is NP-hard

The problem can be formulated mathematically as :

$$Maximize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n}(v\,x_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots.3.5$$

$$Subject\ to\ \sum_{j=1}^{n}\left(w_{ij}x_{ij}\right) \leq c_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.6$$

$$and\ \sum_{i=1}^{n}x_{ij} \leq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.7$$

for $x_j = \{0, 1\}$, $i = 1, 2, 3, \ldots, n$, and $j = 1, 2, 3, \ldots, n$.

The given performance function, 3.5 gives the total items packed in the knapsacks, whereas the constraints functions, 3.6 and 3.7 prescribe the maximum weights of the various knapsacks.

3.2.1 Data Modeling

The Single 0-1 Knapsack Problem

The single 0-1 knapsack problem, is an integer programming problem with single constraint. In this problem, each solution variable is restricted to only binary values. For instance, in the following, we have *n* kinds of items, 1 through *n*. Each kind of item has a value and a weight. We usually assume that all values and weights are nonnegative. The maximum weight that we can carry in the bag is C.

The most common formulation of the knapsack problem is the 0-1 knapsack problem, which restricts the inclusion of ith item number to zero or one. Mathematically the 0 – 1 knapsack problem can be formulated through the following integer linear programming:

Let v_i = the value of ith item

w_i = the weight of ith item

x_i = the number of pieces of ith item

C = the capacity of the knapsack

$$Maximize\ Z = \sum_{i=1}^{n} (v_i x_i) \dots\dots\dots\dots\dots\dots\dots\dots\dots 3.8$$

$$Subject\ to \sum_{i=1}^{n} (w_i x_i) \le C \dots\dots\dots\dots\dots\dots\dots\dots .. 3.9$$

for $i = 1, 2, 3, \ldots, n$

Let $x_i = \{0, \ 1\}$ be the state of a piece of item being included in the knapsack

($x_i = 1$), if x_i item is included and ($x_i = 0$), if x_i item is excluded from the knapsack.

Equation 3.1 above gives the total value of items packed in the knapsack, whereas equation 3.2 gives the maximum weight (capacity), that the knapsack can take.

3.2.2 Methods for solving Knapsack problems

The two basic methods for solving 0-1 knapsack problems are Branch - and – Bound and Dynamic programming methods. Moreover, meta-heuristics such as Simulated annealing, Genetic algorithm and Tabu search are used to solve large scale problems.

But for this study, dynamic programming method was used to solve the 0-1 knapsack problem.

3.2.3 Dynamic Programming Method

The dynamic programming is a paradigm of algorithm design in which an optimization problem is solved by a combination of caching subproblem solutions and appealing to the "principle of optimality. " There are three basic elements that characterize dynamic programming algorithm namely: substructure, table-structure and bottom-up computation.

a). Substructure

Decompose the given problem into smaller (and hopefully simpler) subproblems. Express the solution of the original problem in terms of solutions for smaller problems. Note that unlike divide-and-conquer problems, it is not usually sufficient to consider one decomposition, but many different ones.

b). Table Structure

After solving the subproblems, store the answers (results) to the subproblems in a table. This is done because (typically) subproblem solutions are reused many times, and we do not want to repeatedly solve the same problem over and over again.

c). Bottom-up Computation

Using table, combine solutions of smaller subproblems to solve larger subproblems, and eventually arrive at a solution to the complete problem. The idea of bottom-up computation is as follow:

i. Start with the smallest subproblems.

ii. Combining their solutions, obtain the solutions to subproblems of increasing size.

iii. Until one arrives at the solution of the original problem.

The dynamic programming approach can be used to solve knapsack problem provided certain integrality conditions of the coefficients hold. The first assumption is that the coefficients a_i, w are positive integers, a dynamic programming algorithm construct a table of dimension N* (W + 1).

The optimal solution may be found by backtracking through the table once the optimal value $P_N(W)$ is obtained. The complexity of this dynamic programming is O(NW).

here the number of items, i = 1, 2, . . ., n, are all positive integers.

3.2.4 Dynamic programming algorithm:

Let i = {1, 2, 3, …, n}

w_i = weight of the *i*th item

v_i = the value of the *ith* item

W = the maximum weight of the knapsack.

Step 1: Initialization

For $w = 0$ to W *and* $i = 1$ to n

Let B[0, *w*] = 0, and B[*i*, 0] = 0

Step 2: For $i = 1$ to n and $w = 0$ to W

Compute $B[i, w] = v_i + B[i\text{-}1, w\text{-}w_i]$; if $B[i, w] > B[i\text{-}1, w]$ and $w_i \leq w$,

then the item, i can be part of the solution in the table

else, go to step 3

Step 3: If $w_i \leq w$ but $B[i, w] \leq B[i\text{-}1, w]$,

compute $B[i, w] = B[i\text{-}1, w]$

Step 4: If $w_i > w$, compute $B[i, w] = [i\text{-}1, w]$

Repeat the process until all the data are considered.

Step 5: Select the maximum number from the solution table as the optimum solution.

Step 6: Select the knapsack items that gave the optimum solution:

Let i=n and k=W

if $B[i, k] \neq B[i–1, k]$ then, mark the ith item as in the knapsack

$i = i–1$, $k = k\text{-}w$, else

$i = i–1$

Example:

Let the number of items, $i = \{1, 2, 3, 4\}$, the maximum weight of the knapsack, $W = 5$kg. From the table below, find the optimum solution and the set of items that give the optimal solution.

Item, i	Weight, w_i	Value, v_i
1	2	3
2	3	4
3	4	5
4	5	6

Elements (weight, value) = {(2,3), (3,4), (4,5), (5,6)}.

From the above, n = 4,

Step 1: initialization,

W = 0 to W, *i.e* w= {0, 1, 2, 3, 4, 5}; i = {1, 2, 3, 4}

compute B[0, w] = 0; and B[i, 0] = 0, as shown in the table below

i\ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					
4	0					

Using the first item, (2, 3), compute for row 3 as shown in the below

$i = 1$, $v_i = 3$, $w_i = 2$, $w = 1$;

Since $w_i > w$ go to step 4

Step 4: compute B[i, w] = B[i-1, w]

B[i, w] = B[1, 1]

B**[**1, 1**] =** B[1-1, 1] = B[0, 1] = 0

Step 2:

For B[1, 2]; i =1, w = 2; since $w_i \leq w$,

Compute B[i, w] = v_i + B[i-1,w – w_i]

B[1, 2] = 3 + B[0,0] = 3

For B[1, 3]; i = 1, w = 3; since $w_i \leq w$,

B[1, 3] = 3 + B[1-1,3-2] = 3 + B[0, 1] = 3

For B[1, 4] = 3; and

B[1, 5] = 3

*i**w*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0					
3	0					
4	0					

Using the second item, (3, 4), compute the values for row 4 as shown in table below

w_i = 3, v_i = 4

For B[2, 1]; i = 2, w = 1, since $w_i > w$ .compute B[i, w] = B[i-1, w]

B[2, 1] = B[2-1, 1] = B[1, 1] = 0

For B[2, 2], $w_i > w$ then B[2, 2] = B[1, 2] = 3

For B[2,3], $w_i \leq w$; B[i, w] = v_i + B[i-1, w-w_i]

B[2, 3] = 4 + B[1, 0] = 4 + 0 = 4

B[2, 4] = 4 + B[1, 1] = 4 + 0 = 4

B[2, 5] = 4 + B[1, 2] = 4 + 3 = 7

*i**w*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0					
4	0					

Using item (4, 5), compute the values of row 5 as shown below

$w_i = 4$, $v_i = 5$

For B[3, 1], since $w_i > w$, *go to step 4*

compute B[i, w] = B[i-1, w]

B[3, 1] = B[2, 1] = 0

B[3, 2] = B[2, 2] = 3

B[3, 3] = B[2, 3] = 4

B[3, 4] , $w_i \leq w$, *compute* B[i, w] = v_i + B[i – 1, w - w_i]

B[3, 4] = 5 + B[2, 0] = 5 + 0 = 5

B[3, 5] = 5 + B[2, 1] = 5 + 0 = 5,

Since B[3, 5] < B[2, 5] , go step 3; (i.e. B[i, w] = B[i-1, w])

B[3, 5] = B[2, 5] = 7

i\ *w*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0					

Use the last item, (5, 6) to compute the entries in row 6 as shown below:

w_i = 5, v_i = 6

For B[4, 1], since $w_i > w$, *go to step 4*

Compute B[i, w] = B[i-1, w]

B[4, 1] = B[4-1, 1] = B[3, 1] = 0

B[4, 2] = B[3, 2] = 3

B[4, 3] = B[3, 3] = 4

B[4, 4] = B[3, 4] = 5

B[4, 5] = B[3, 5] = 7

i\ *w*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

Step 5: From the last table, the maximum number is 7. Therefore, the optimum solution is 7

Step 6: Finding the particular items to be included in the knapsack.

Let $i = n$ and $k = w$

if $B[i, k] \neq B[i-1, k]$ then, mark the ith item as in the knapsack

$i = i-1$, $k = k - w$, else

$i = i - 1$

From the completed solution table,

$i\backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

Let $i = 4$, k = 5, $v_i = 6$, $w_i = 5$

$B[i,k] = 7$ and $B[i\text{-}1, k] = 7$

B[i, k] ≠ B[i - 1], so item 4 should not be included in the knapsack. Consider, i=3, k = 5, v_i = 6 and w_i = 5

$i\backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

B[i, k] = 7 and B[i-1] = 7

B[i, k] ≠ B[i-1, k], so item 3 cannot be part of the knapsack

Consider, i = 2, k = 5, v_i = 4, and w_i = 3

i\ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

B[i, k] = 7 and B[i-1, k] = 3. Since B[i, k] ≠ B[i-1,k], then the item 2 should be included in the knapsack

$k - w_i = 2$

Also consider, i = 1, k = 2, v_i = 3 and w_i =2

i\ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

B[i, k] = 3, B[i-1,k] = 0, $k - w_i = 0$

Since B[i, k] ≠ B[i-1,k], then item 1 can be part of the knapsack.

Again, consider i = 0, and k = 0

i\ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

Therefore, the optimal solution is 7 which could be obtained by selecting items with number, i = {1, 2}

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 Data Collection

The researcher studied into the selection of adverts at TV 3, Ghana Ltd. As a private entity, TV 3 is mandated to generate revenue for the upkeep of the station. In view of that, TV 3 has various ways of generating income. These may include sponsorship of programmes, social and funeral announcements and advertisements. However, the research focused on advertisements, which are slotted in the programme schedules prepared quarterly to generate revenue to sustain the operations of TV 3.

for adverts, the rate structure and the programme schedule for the last quarter of 2010 was collected from the office of TV 3 Ghana Ltd. On the rate structure, the duration for the adverts have been zoned into ten zones i.e A1 - A10. Each of the zones has different rate attached. If a programme is placed under zone A1, then it is considered to be among the most patronised programmes on the channel. Therefore, the advert charges for such programmes are very high. Similarly, the advert charges for zone A10 programmes are very low as shown in the table 1 below

Table 1: TV 3 Rate Structure, 2010 (GH Cedis): Effective 1ST April, 2010

Duration Zone	15 Sec	20 Sec	30 Sec	45 Sec	60 Sec
A1	GH¢ 375.86	GH¢ 488.46	GH¢ 734.46	GH¢1084.29	GH¢ 1,505.49
A2	GH¢ 334.10	GH¢ 434.46	GH¢ 652.86	GH¢ 963.82	GH¢ 1,338.22
A3	GH¢ 257.00	GH¢ 344.20	GH¢ 502.20	GH¢ 741.40	GH¢ 1029.40
A4	GH¢ 197.70	GH¢ 264.80	GH¢ 386.30	GH¢ 570.40	GH¢ 791.90
A5	GH¢ 182.50	GH¢ 246.90	GH¢ 360.90	GH¢ 532.40	GH¢ 741.20
A6	GH¢ 173.40	GH¢ 228.20	GH¢ 342.30	GH¢ 513.00	GH¢ 703.20
A7	GH¢ 164.20	GH¢ 208.80	GH¢ 322.90	GH¢ 457.50	GH¢ 665.10
A8	GH¢ 152.10	GH¢ 205.30	GH¢ 304.20	GH¢ 456.30	GH¢ 646.50
A9	GH¢ 113.40	GH¢ 141.60	GH¢ 234.60	GH¢ 354.00	GH¢ 482.70
A10	GH¢ 98.70	GH¢ 120.10	GH¢ 200.30	GH¢ 308.40	GH¢ 416.10

The researcher obtained the data on TV 3 adverts from the following zones:

A1: TV 3 News 360 (19:00 hours GMT),

A4: Music – Music (20 : 30 – 21:30 GMT) every Saturday.

A9: Mid Day Live (12 : 00 – 12 : 30 GMT)

These data have been tabulated in tables 2, 3 and 4 below.

In table 2, column 1 represents the advert number, column 2 is the number of spots requested(x_i), column 3 is duration per spot, column 4 is advert charges per spot (v_i), column 5 is total duration for the spots ($w_i x_i$), column 6 is amount to be paid for each item ($v_i x_i$). i = {1, 2, 3, 4, 5, 6, …,78}

Table 2: The Requested Adverts For TV 3 News 360 At 7.00 – 8.30 pm

Total Time Available is 1200 seconds				**Zone : A1**	
Adverts No.	**No Of Spots Requested, x_i**	**Duration per spot in Second, w_i**	**Advert charges per spot(GH¢), v_i**	**Total duration for the spots (s), $w_i x_i$**	**Amount, (GH¢), $v_i x_i$**
1	4	30	734.46	120	2937.84
2	1	15	375.86	15	375.86
3	3	60	1505.49	180	4516.47
4	3	45	1084.29	135	3252.87
5	3	20	488.76	60	1466.28
6	4	30	734.46	120	2937.84
7	3	45	1084.29	135	3252.87
8	2	45	1084.29	90	2168.58
9	3	20	488.76	60	1466.28
10	4	60	1505.49	240	6021.96
11	2	45	1084.29	90	2168.58
12	3	60	1505.49	180	4516.47
13	2	20	488.76	40	977.52
14	5	15	375.86	75	1879.30
15	2	15	375.86	30	751.72
16	3	15	375.86	45	1127.58
17	1	60	1505..49	60	1505.49
18	1	20	488.76	20	488.76
19	2	30	734.46	60	1468.92
20	2	15	375.86	30	751.72
21	3	45	1084.29	135	3252.87
22	1	15	375.86	15	375.86
23	4	30	734.46	120	2937.84
24	2	45	1084.29	90	2168.58
25	4	15	375.86	60	1503.44
26	2	15	375.86	30	751.72
27	3	45	1084.29	135	3252.87
28	6	15	375.86	90	2255.16
Total	**78**			**2460**	**100123.47**

From table 3, column 1 represents the advert number, column 2 is the number of spots requested(x_i), column 3 is duration per spot, column 4 is advert charges per spot(v_i) column 5 is the total duration for the spots (w_ix_i) and column 6 amount to be paid for each item (v_ix_i). $i = \{1, 2, 3, 4, 5, \dots , 59\}$

Table 3: Requested Adverts For TV 3 Music-Music Adverts On Sat. 8.30-9.30 pm:

Total Time Available is 1200s				**ZONE : A 4**	
Advert No	**No of Spots Requested, x_i**	**Duration per Spot in Second, w_i**	**Charges per Spot (Gh ¢), v_i**	**Total Duration For The Spots, w_ix_i**	**Amount (Gh ¢), v_ix_i**
1	2	20	264.80	40	529.60
2	4	15	197.70	60	790.80
3	3	30	386.30	60	1158.90
4	2	15	197.70	30	395.40
5	1	15	197.70	15	197.70
6	1	45	570.40	45	570.40
7	3	20	268.80	60	806.40
8	2	15	197.70	30	395.40
9	4	15	197.70	60	790.80
10	2	30	386.30	60	772.60
11	2	60	791.90	120	1583.80
12	1	20	264.80	20	264.80
13	3	15	197.70	45	593.10
14	2	30	386.30	60	772.60
15	2	30	386.30	60	772.60
16	3	45	570.40	135	1711.20
17	3	20	264.80	60	794.40
18	1	15	197.70	15	197.70
19	2	30	386.30	60	772.60
20	1	15	197.70	15	197.70
21	3	45	570.40	135	1711.20
22	3	15	197.70	45	593.10
23	2	30	386.30	60	772.60
24	1	30	386.30	30	386.30
25	1	60	791.90	60	791.90
26	1	20	268.80	20	264.80
27	4	15	197.70	60	790.80
Total	**59**			**1460**	**29908.70**

From table 4, column 1 represents the advert number, column 2 is the number of spots Requested (x_i), column 3 is the duration per spot (w_i), column 4 is advert charges per spot (v_i), column 5 is total duration for the spots ($w_i x_i$) and column 6 is the amount to be paid for each item ($v_i x_i$). $i = \{1, 2, 3, 4, 5, \ldots, 36\}$.

Table 4: The Requested Adverts For Mid-Day Live At 12.00 – 12.30 pmTable 4 :

Total Time Available is 600 Seconds					**ZONE : A9**
Adverts No, *i*	**No of Spots Requested (s), x_i**	**Duration per Spot in Seconds,w_i**	**Advert charges per Spot, v_i**	**Total Duration For The Spots, $w_i x_i$**	**Amount(GH ¢), $v_i x_i$**
1	2	30	234.60	60	469.2
2	1	15	113.40	15	113.4
3	2	15	113.40	30	226.8
4	2	20	141.60	40	283.2
5	3	30	234.60	90	703.8
6	4	15	113.40	60	453.6
7	1	45	354.00	45	354
8	1	30	234.60	30	234.6
9	3	20	141.60	60	424.8
10	2	20	141.60	40	283.8
11	2	45	354.00	90	708
12	2	30	234.60	60	469.2
13	1	30	234.60	30	234.6
14	3	15	113.40	45	340.2
15	1	45	354.00	45	354
16	1	45	354.00	45	354
17	3	15	113.40	45	340.2
18	2	15	113.40	30	226.8
Total	**36**			**860**	**6754.2**

4.2 Model Formulation And Algorithm

From tables 2, 3 and 4, it could be seen that the total time (duration) demanded by the companies in each category exceeded the total available time for adverts. Hence, there is the need to select some subsets of adverts that would give maximum returns for the television station, so that the total time available for the adverts would not be exceeded.

The problem above can be formulated as a knapsack problem.

The following represent the general knapsack problem:

$$Maximize\ Z = \sum_{i=1}^{n} v_i x_i \ \ldots\ldots\ldots\ldots\ldots\ldots..\ 4.1$$

$$Subject\ to\ \sum_{i=1}^{n} w_i x_i \leq W \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 4.2$$

Where i = 1, 2, 3, . . ., n,

Let x_i = {0, 1} be the state of an advert been included in the knapsack or not. x_i = 1 if the *ith* advert is taken and x_i = 0 if the *ith* advert is not taken.

Each table gives the number, x_i of spots requested, the duration, w_i for each advert, the charges, v_i of the adverts and the total time available, W

x_i represents the column number two , w_i represents column three and v_i is column four in Tables 2, 3 and 4 above

Z represents the total income from the advertisement and W be the total time available.

Dynamic Programming Algorithm:

Dynamic programming algorithm was used to solve the above problem.

From Tables 2, 3 and 4,

Let x_i = column number two (i.e. the number of spots requested)

w_i = column number three (i.e. duration per spot in seconds)

v_i = column number four (i.e. advert charges per spot)

W = the maximum available time for adverts on each programme

Algorithm

Let $i = \{1, 2, 3, \ldots, n\}$

w_i = weight of the *i*th item

v_i = the value of the *ith* item

W = the maximum weight of the knapsack.

Step 1: Initialization

For $w = 0$ to W *and* $i = 1$ to n

Let $B[0, w] = 0$, and $B[i, 0] = 0$

Step 2: For $i = 1$ to n and $w = 0$ to W

Compute $B[i, w] = v_i + B[i-1, w-w_i]$; if $B[i, w] > B[i-1, w]$ and $w_i \leq w$, then the item, i can be part of the solution in the table.

else, go to step 3

Step 3: If $w_i \leq w$ but B[i, w] $\leq$ B[i-1, w],

compute B[i, w] = B[i-1, w]

Step 4: If $w_i > w$, compute B[i, w] = [i-1, w]

Repeat the process until all the data are considered.

Step 5: From the solution table, select the highest value to be the optimal solution.

Step 6: Select the knapsack items that gave the optimum solution:

Let $i = n$ and $k = W$

if $B[i, k] \neq B[i-1, k]$ then, mark the ith item as in the knapsack

$i = i - 1$ and $k = k - w_i$

else

$i = i - 1$

4.3 Computation Procedure

To carry out the computation, the computer software, matlab, was used to analyse each table. HP Laptop computer having the following specification: processor:- Intel (R), Core (TM) Duo CPU T2400 @ 1.83 GHz 1.83 GHz with Memory (RAM) 1015 MB was used to run each table.

4.4.1 Results for TV 3 News 360:

Let x_i represents the column two of table 2 above,

$i = \{1, 2, 3, 4, \ldots, 78\}$.

$x = \{0, 1\}^{78}$

($x_i = 1$), if *ith* item is included and ($x_i = 0$), if *ith* item is excluded from the knapsack

From the computer analysis, the optimal returns of table 2 is obtained when the following selections are made,

$x_i = \{0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1\}$

Advert No 1 = x_1, x_2, x_3, x_4; No 2 = x_5; No 3 = x_6, x_7, x_8; No 4 = x_9, x_{10}, x_{11}; No 5 = x_{12}, x_{13}, x_{14}; No 6 = $x_{15}, x_{16}, x_{17}\, x_{18}$; No 7 = x_{19}, x_{20}, x_{21}; No 8 = x_{22}, x_{23}; No 9 = x_{24}, x_{25}, x_{26}; No 10 = $x_{27}, x_{28}, x_{29}, x_{30}$: No 11 = x_{31}, x_{32},; No 12 = x_{33}, x_{34}, x_{35}; No 13 = x_{36}, x_{37}, No 14 = $x_{38}, x_{39}, x_{40}, x_{41}, x_{42}$; No15 = x_{43}, x_{44}; No 16 = x_{45}, x_{46}, x_{47}; No17 = x_{48}; No 18 = x_{49}; No 19 = x_{50}, x_{51}; No 20 = x_{52}, x_{53}; No 21= x_{54}, x_{55}, x_{56},; No 22 = x_{57}, No 23 = $x_{58}, x_{59}, x_{60}, x_{61}$,; No 24 = x_{62}, x_{63}; No 25 = $x_{64}, x_{65}, x_{66}, x_{67}$; No 26 = x_{68}, x_{69}; No 27 = x_{70}, x_{71}, x_{72}, No 28 = $x_{73}, x_{74}, x_{75}, x_{76}, x_{77}, x_{78}$.

The above results are tabulated in table 5 below:

Table 5: The Result Of TV 3 Adverts For News 360 At 7.00 – 8.00 pm.

	Total Time Available is 1200 seconds .					Zone : A1
Advert No	**No of Spots Requested, x_i**	**No of Spots used**	**Duration per spot in Second, w_i**	**Advert charges per spot(GH¢), v_i**	**Total duration for the spots (s), $w_i x_i$**	**Amount,(GH¢), $v_i x_i$**
1	4	0	0	0	0	0
2	1	1	15	375.86	15	375.86
3	3	3	60	1505.49	180	4516.47
4	3	0	0	0	0	0
5	3	0	0	0	0	0
6	4	4	30	734.46	120	2937.84
7	3	0	0	0	0	0
8	2	0	0	0	0	0
9	3	0	0	0	0	0
10	4	4	60	1505.49	240	6021.96
11	2	0	0	0	0	0
12	3	3	60	1505.49	180	4516.47
13	2	0	0	0	0	0
14	5	5	15	375.86	75	1879.30
15	2	2	15	375.86	30	751.72
16	3	3	15	375.86	45	1127.58
17	1	1	60	1505..49	60	1505.49
18	1	0	0	0	0	0
19	2	0	0	0	0	0
20	2	2	15	375.86	30	751.72
21	3	0	0	0	0	0
22	1	1	15	375.86	15	375.86
23	4	1	30	734.46	30	734.46
24	2	0	0	0	0	0
25	4	4	15	375.86	60	1503.44
26	2	2	15	375.86	30	751.72
27	3	0	0	0	0	0
28	6	6	15	375.86	90	2255.16
Total	**78**	**42**			**1200 seconds**	**GH¢ 30,005.00**

From column 7 of the table 5, the least amount charged for the selected adverts is Gh ¢ 375. 86 and the highest amount charged is Gh ¢ 4516.47.

The results revealed that adverts number 1, 4, 5, 7,8, 9, 11,13,18,19, 21,24,27 were used as shown in the table 5.

For advert number 23, only one advert was considered.

The optimal incomes of adverts from TV 3 News 360 is **GH¢ 30,005.00.**

4.4.2 Results For TV 3 Music – Music

From table 3, $i = \{1, 2, 3, 4, \ldots, 59\}$.

So $x = \{0, 1\}^{59}$

The computer analysis showed that the optimal returns of TV 3 advert for Music – Music could be obtained when the following selections are made:

x_i = {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

Advert No 1 = x_{1}, x_{2}, ; *No* 2 = x_{3}, x_{4} x_{5}, x_{6} ; *No* 3 = x_{7}, x_{8}, x_{9}; *No* 4 = x_{10}, x_{11} ; *No* 5 = x_{12},; *No* 6 = x_{13},; *No* 7 = x_{14}; x_{15}, x_{16},; No 8 = x_{17}, x_{18}; *No* 9 = x_{19}, x_{20}, x_{21} x_{22},; *No* 10 = x_{23}, x_{24} No 11 = x_{25}, x_{26},; No 12 = x_{27}; No 13 = x_{28}, x_{29}, x_{30} , No 14 = x_{31}, x_{32}; No15 = x_{33}, x_{34}; No,16 = x_{35}, x_{36}, x_{37}; No17 = x_{38}, x_{39}, x_{40}; No 18 = x_{41}; No 19 = x_{42}, x_{43}; No 20 = x_{44}; No 21= x_{4}, x_{46}, x_{47}; No 22 = x_{48}, x_{49}, x_{50}; ,No 23 = x_{51} x_{52},; No 24 = x_{53} ; No 25 = x_{54}; No 26 = x_{55}; No 27 = , x_{56}, x_{57} x_{58}, x_{59}.

The above results are tabulated in table 6 below:

Table 6: The Result Of TV 3 Adverts For Music - Music

Total Time Available Is 1200s Available.					ZONE : A 4	
Advert No	**No of Spots Requested, x_i**	**No of Spots used**	**Duration per Spot in Second, w_i**	**Charges per Spot (Gh ¢), v_i**	**Total Duration For The Spots, w_ix_i**	**Amount (Gh¢), v_ix_i,**
1	2	2	20	264.80	40	529.60
2	4	4	15	197.70	60	790.80
3	3	3	30	386.30	60	1158.90
4	2	2	15	197.70	30	395.40
5	1	1	15	197.70	15	197.70
6	1	1	45	570.40	45	570.40
7	3	3	20	268.80	60	806.40
8	2	2	15	197.70	30	395.40
9	4	4	15	197.70	60	790.80
10	2	2	30	386.30	60	772.60
11	2	2	60	791.90	120	1583.80
12	1	1	20	264.80	20	264.80
13	3	3	15	197.70	45	593.10
14	2	2	30	386.30	60	772.60
15	2	2	30	386.30	60	772.60
16	3	0	0	0	0	0
17	3	2	20	264.80	40	529.60
18	1	1	15	197.70	15	197.70
19	2	2	30	386.30	60	772.60
20	1	1	15	197.70	15	197.70
21	3	0	0	0	0	0
22	3	3	15	197.70	45	593.10
23	2	2	30	386.30	60	772.60
24	1	1	30	386.30	30	386.30
25	1	1	60	791.90	60	791.90
26	1	1	20	268.80	20	264.80
27	4	4	15	197.70	60	790.80
Total	**59**	**52**			**1200s**	**15,696.00**

From the above table 6, the least charge among the selected adverts is Gh ¢197.70 whilst the highest charge is Gh ¢1583.80.

The results showed that adverts with numbers, 16 and 21 were not used. Besides, for the advert number 17, two out of three spots requested were used.

The optimal returns of TV 3 adverts from Music-Music is Gh ¢ 15,696.00

4.4.3 Results of TV 3 Adverts for Mid-Day Live

From table 4, $i = \{1, 2, 3, 4, \ldots, 36\}$.

Let $x_i = \{0, 1\}^{36}$

From the computer analysis, the optimal returns of table 4 is obtained when the following

selections are made, $x_i = \{1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1,1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1,1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0\}$

Advert No. 1 = x_1, x_2, ; No. 2 = x_3,; No. 3 = x_4 x_5; No. 4 = x_6 x_7; No. 5 = x_8, x_9, x_{10}

No. 6 =, x_{11}, x_{12}, x_{13}, x_{14} ; No 7 = x_{15}; No. 8 = x_{16}; No. 9 = x_{17}, x_{18}, x_{19},; No. 10 = x_{20}, x_{21}

No. 11 = x_{22}, x_{23},; No. 12 = x_{24}, x_{25} ; No. 13 = x_{26}; , No. 14 = x_{27}, x_{28}, x_{29}; No.15 = x_{30};

No.,16 = x_{31}; No.17 = x_{32}, x_{33}, x_{34}; No. 18 = x_{35}, x_{36};

The above results are tabulated in table 7 below:

Table 7: The Results of TV 3 Adverts For Mid – Day Live

Total Time Available is 600s Available						**ZONE : A9**
Adverts No	**No of Spots Requested (s), x_i**	**No of Spots Used**	**Duration per Spot in Seconds, w_i**	**Advert charges per Spot, v_i**	**Total Duration For The Spots, $w_i x_i$**	**Amount (GH ¢), $v_i x_i$**
1	2	2	30	234.60	60	469.20
2	1	0	0	0	0	0
3	2	0	0	0	0	0
4	2	0	0	0	0	0
5	3	3	30	234.60	90	703.80
6	4	4	15	113.40	60	453.6
7	1	1	45	354.00	45	354.00
8	1	1	30	234.60	30	234.60
9	3	0	0	0	0	0
10	2	0	0	0	0	0
11	2	2	45	354.00	90	708.00
12	2	2	30	234.60	60	469.20
13	1	1	30	234.60	30	234.60
14	3	3	15	113.40	45	340.20
15	1	1	45	354.00	45	354.00
16	1	1	45	354.00	45	354.00
17	3	0	0	0	0	0
18	2	0	0	0	0	0
Total	**36**	**21**			**600s**	**4675.20**

From the column 7 of table 7 above, the least amount charged for the selected adverts is GH¢ 234.60, whilst the highest amount charged is GH¢708.00

It could also be seen that adverts with numbers; 2, 3, 4, 9, 10, 17 and 18 were not used.

The optimal returns for TV3 Mid-Day Life is **GH¢ 4675.20**

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Summary

Results from the analysis of data revealed that the optimal incomes of adverts from TV 3 News 360 is Gh ¢ 30,005.00. This would be obtained when the programme managers select the adverts with the advert numbers:

2, 3, 6, 10, 12, 14, 15, 16, 17, 20, 22, 23, 25, 26, 28 as shown in the table 5 above.

In the advert n*o* 23, the managers have to take only one out of the four adverts requested by the company.

Then also, the results showed that the optimal returns of adverts from TV 3 Music – Music programme is Gh ¢15, 696. 00 and would be obtained when the producers of TV 3 Music - Music programme choose the adverts with the advert numbers below:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27.

For the advert number 17, two out of 3 requested adverts were used.

The results also revealed that the producers of TV 3 Mid-Day Live would achieve the optimal returns from the numerous adverts if they select the adverts with the advert numbers below:

1, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16.

Hence, the optimal incomes is Gh ¢ 4675.20

5.2 Conclusion

From the study, the Television adverts selection problem was modelled as a single 0 - 1 knapsack problem as found in section 4.1 and 4.2 of the thesis.

Using knapsack problem in adverts selection, the optimum returns of TV 3 News 360, Music – Music and Mid – Day Live programmes are 30,005.00, 15,696.00 and 4,675.20 respective.

5.3 Recommendations

Since, the use of knapsack problem in television adverts selection problems gives the optimum returns , the researcher recommends that the customer relation managers or programme producers should incorporate the use of knapsack for profit maximization in the TV adverts selection.

The study considered the use of single 0-1 knapsack problem to solve adverts selection problems, the researcher suggests that there should be a future research on the application of multiple knapsack problem to solve Television adverts selection problem.

REFERENCES

1. . Abboud, N. J, Sakawa, M., Inuigushi, M. (1997). A fuzzy programming approach to multiobjective multidimensional 0 – 1 knapsack problem http://linghub.elsevier.com/retrieve/pii/S0165011495003622

2. Adlen, K., Abdelhamid, N., Abdelhak, G. and Mohamed N. (2007). A resource allocation protocol for QoS-sensitive services provisioning in 802. 11 networks. *Performance Evaluation. Vol. 64. Pp. 419 – 443.*

3. Alberto, C. and Michele, M. (2004). On the two dimensional Knapsack Operations Research Letters. http: // www.mathtu-dresden.de/~belov/publdownl/2D/on2Dkp.caprara_monaci.pdf

4. Arnaud, F. and Gérard, P. (2004). An efficient preprocessing procedure for multidimensional 0 – 1 knapsack problem http: // portal.acm.org/citation.cfm ?id=1460926 ..1k

5. Balachandar, S. R. and Kannan, K. (2008). A new polynomial time algorithm for 0 – 1 multiple knapsack problem based on dominant principle. Jorunal of heuristics. vol. 202(1). pp. 71 – 77.

6. Balev, S. (2008). The multidimensional knapsack problem http://www.optimization-onine.org/DB_FILE/2009/03/2258.PDF-31K

7. Beasley, J. E. (2002). Population heuristics.

http://linkinghub.elsevier.com/retrieve/pii/S0377221711002670 - -1k

8. Benisch, M. and Greenwald, A. (2005). Princing for customers with Probabilistic Valuation. Citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.179.5875.

9. Borteldt, A. and Hermann, G. (2001). A hybrid genetic algorithm for the container load problem. European Journal of Operational Research. Vol. 13(1). pp. 143 – 161.

10. Cai, Z. W. and Ong, H. L. (2004). Solving the biobjective zero - one knapsack problem by an efficient LP . www.faqs.org › Abstracts Index › Abstracts › Business, international - Cached

11. Carlo, V. (1994). Constrained multi – project planning problems, Lagrangean decomposition Approach. European Journal of Operational Research. vol. 78. Pp. 267- 275.

12. Caserta, M ., quinonez, R. and Marquez, U. (2008). A cross entropy algorithm for knapsack problem with setups. Computer and Operational Research, vol. 35. Pp. 241-252.

13. Dantzig, G. B. (1957). Discrete-Variable Extremum Problems. Journal of Operations Research. Vol. 5 (2). pp. 266 – 288.

14. Devyaterikova, M. V., Kolokolov, A. A. and Kolosov, A. P. (2009). L-class enumeration algorithms for a discrete production planning problem with interval resource quantities. European Journal of Operational Research. Vol. 13(1). Pp. 143 – 161.

15. Elsevier, B. V. (2009). Resource allocation on computational grids using a utility model and the knapsack problem. Future generation Computer Systems. Vol. 25. pp. 35 – 50

16. Eugénia, M. C., João, C., Jose′, F., Ernesto, M. and Jose′, L. S., (2009).

Solving bicriteria 0-1 knapsack problems using a labelling algorithm.

Computer and operations Research. vol. 36 (2). pp. 7.

17. Florios, K. (2009). Solving multiobjective, multiconstraint knapsack problems using mathematical programming and evolutionary .

lambda.qsensei.com/content/1mblgq - Cached

18. Gholamian, M. R. (2001). A hybrid intelligent system for multi-objective decision.

http://www.science/article/pii/S0950705106001729.

19. Halvard, A., Lars, M. H. and Arne, L. (2006). Adaptive memory search for multidemand multidimensional knapsack problems. Computer and Operations Research.

http: // www.portal.acm.org/citation.cfm?id=11410798 & picked=prox

20. Li, V. C. and Curry, G. L. (2005). Solving multidimensional knapsack problems with generalized upper bound constraints using critical event tabu search.

Computer and operational research. Vol. 32. pp. 825 - 848.

21. Lin, F. T. and Yao, J. S. (2001). A survey of DEA and knapsack formulation application in project selection.

European Journal of operation Research. vol. 135(1). pp. 9

22. Lokketangen, A. and Fred, G. (1998). Solving 0 – 1 mixed integer programming problems using tabu search. European Journal of operational research. vol. 106. Pp. 624-658.

23. Martello, S., Pisinger, D. and Paolo, T. (2000). New trends in exact algorithms for the 0 – 1 knapsack problem.

http: // citeseerx.istpsu/viewdoc/download?doi10.1.11.89068rep=rep|type=ps

24. Nazim, U. A. and Jatinder, N. D. (1987). An efficient heuristic algorithm for selecting projects.

Computer and Industrial Engineering Archives. vol. 12. pp. 153 – 158

25. Okan, Y. and Ing-Ray, C. (2009). Utilizing call admission control for pricing optimization of multiple service classes in wireless cellular networks.

Computer Communications. Vol. 32. Pp. 317-323.

26. . Oppong, E. O. (2010). Optimal Television Adverts Selection.

http://www.maxwellsci.com/print/rjit/v3-49-54.pdf - -1k

27. Owoloko, E. A. and Sagoe, E. T. (2010). Optimal advert placement slot – using the knapsack problem model. *American Journal of science and industrial research*

http://www.scihub.org/AJSIR. ISSN: 2153-649X

28. Pati, H. K., Mall, R. and Sengupta, I. (2002). An efficient bandwidth reservation and call admission control scheme for wireless mobile networks. *Computer Communications*. *Volume 25. Pp. 74-83*

29. Pendharkar, P. C. and Rodger, J. A. (2006). Information technology capital budgeting http://onlinelibrary.wiley.com/doi/10.1111/j.1475-3995.2006.... - -1k

30. Pisinger, D. (2001). Budgeting with bounded mmultiple choice constraints Journal of Heuristics. vol. 129. pp. 471- 480.

31. Pisinger, W. D., Rasmussen, A. B. and Sandvik, R. (2007). Solution of Large Quadratic Knapsack Problems Through Aggressive Reduction Informs Journal on Computing, vol. 19(2). Pp 280 - 290.

32. Realff , M. J., Kvam, P. H. and Taylor W. E. (1999). Combined analytical and empirical learning framework for branch and bound Citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.13

33. Reinaldo, J. M. (2005). Meta-RaPS approach for the 0-1 Multidimensional Knapsack Problem. http://port**al.**acm.org/citation.cfm?id=1651688 - -1k

34. Said, H. and Arnaud, F. (1998). An efficient tabu search approach for 0-1 multidimensional knapsack problem. European Journal of Operational Research. vol. 106. pp. 659 – 675.

35. Silvano, M., Pisinger D. and Paolo T. (2000). New trends in exact algorithm for the 0 – 1 knapsack problem.

http: // citeseerx.ist.psu/viewdoc/download?doi=10.1.11.89068rep=rep|&type=ps.

36. Simões, A. and Costa, E. (2001). An immune system-**based** genetic http://portal.acm.org/citati**on**.cfm?id=1144209 - -1k

37. Taniguchi, F. (2008). surrogate relaxation for deriving both upper and lower bounds efficiently. Taniguchi, F. (2008). surrogate relaxation for deriving both upper and lower bounds efficiently. http://imaman.oxfordjournals.org/content/19/3/227.full.pdf - -1k

38. Tao, Z. and Rhonda, Y. (2009). Multiple Choice Knapsack Problem.

Linkinghub.elsevier.com/retrieve/pii|S014971890900041x

39. Theodore, S. G. and Stephen, V. A. (1973). Investment planning for irrigation development projects.

linkinghub.elsevier.com/retrieve/pii/0038012173900505.

40. Umit, A. (2006). Approximate and exact algorithms for the fixed charge knapsack Problem. Http: // www. mathtu- dresden.del-belov/publ/down/kp_fixcharge pdf.

APPENDIX

Code of dynamic programming:

for w = 0 to W

B[0,w] = 0

for i = 1 to n

B[i,0] = 0

for i = 1 to n

for w = 0 to W

if wi <= w // item i can be part of the solution

if bi + B[i-1,w-wi] > B[i-1,w]

B[i,w] = bi + B[i-1,w- wi]

else

B[i,w] = B[i-1,w]

else B[i,w] = B[i-1,w] // wi > w

How To Actual Knapsack Items:

Let $i = n$ and $k = W$

if $B[i, k] \neq B[i-1, k]$ then, mark the *i*th item as in the knapsack

$i = i - 1$ and $k = k - w_i$

else

$i = i - 1$.