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KUMASI, GHANA

INSTITUTE OF MINING & MINERAL ENGINEERING

DEPARTMENT OF MINERAL PROCESSING & EXTRACTIVE

METALLURGY / MATERIAL SCIENCE & ENGINEERING

TOPIC

MATHEMATICAL MODEL FOR AIR QUALITY IN THE INDUSTRIAL AREA (ATONSU) OF KUMASI

By

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DECLARATION

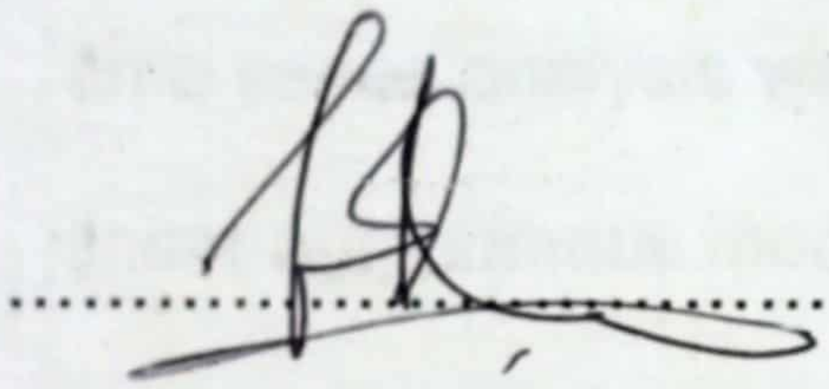
I HEREBY DECLARE THAT EXCEPT FOR REFERENCE TO OTHER PEOPLES' WORKS, WHICH HAVE BEEN DULY ACKNOWLEDGED, THIS THESIS SUBMITTED TO THE BOARD OF POSTGRADUATE STUDIES (KNUST) IS THE RESULT OF MY OWN INVESTIGATION AND HAS NOT BEEN PRESENTED FOR ANY OTHER DEGREE



.....

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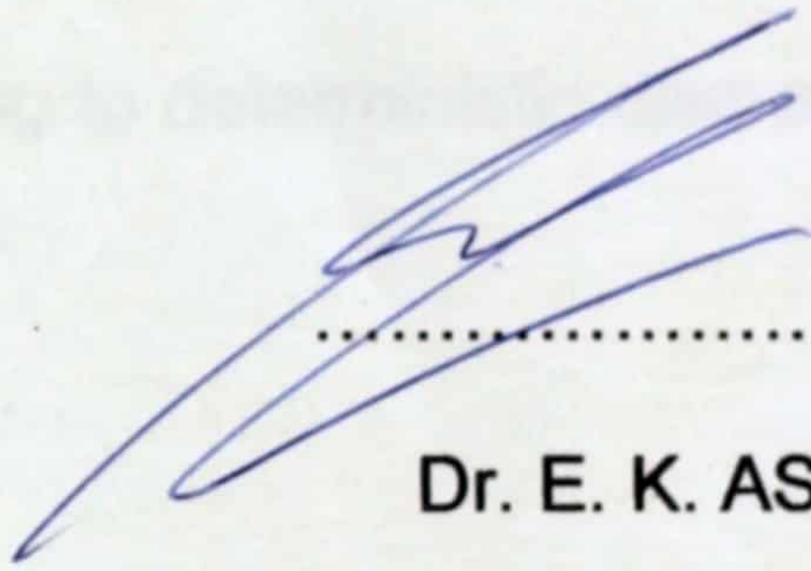
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ABSTRACT

Air pollution, an aspect of environmental degradation, poses a serious threat to the society, causing health hazards to living things and destruction of property.

This study aims at modeling the concentrations of SO_2, NO_2 and CO as well as some aerodynamic quantities namely air temperature, wind speed and direction to determine their steady state dynamics.

A statistical method of system identification based on autoregressive time series analysis was used leading to deterministic discrete - time linear autonomous models.

The dynamics of the concentrations of SO_2, NO_2, as well as the temperature of the air were found to be of the first order and unstable with steady state values generally not less than 0.0114ppm, and 0.0110ppm and 0.9246ppm respectively, indicating a need for immediate and adequate measures to avert the associated devastating consequences in the near future.

ACKNOWLEDGEMENT

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CHAPTER ZERO

INTRODUCTION

0.1 BACKGROUND TO THE STUDY

0.1.1 Air

Air is one of the essential gifts to man. It is a life supporting system without which life cannot exist. It forms a life blanket of the earth, the essential ingredient for all living things. Air covers every part of the 200 million square miles of the earth's surface.

In the course of a day, a single person breathes between 30 and 35 pounds of air in a constant automatic response to extract life-giving oxygen [1]. The supply of air in the atmosphere is estimated to be 6 quadrillion tonnes [1] and it is constantly in movement about the earth's surface in both northerly and easterly directions.

0.1.2 Components Of The Air

The air is a mixture of gases and is composed of nitrogen (78 percent) oxygen (21 percent) and the various other gases: argon, carbon dioxide, helium, hydrogen, krypton, neon and xenon and water vapour [2].

The various components of air play various roles for the benefit of man. For instance, the oxygen component is needed for respiration. It also supports burning. The carbon dioxide component is required by green plants for photosynthesis. Nitrogen for fixation of nitrates to improve soil fertility by the nitrogen - fixing bacteria called azotobacter.

0.1.3 Air Quality

The air is truly the closest man can come to possession of an effective infinite and free resource. Over a long period of time, the atmosphere was in fact a free good.

Until recently the earth's living systems evolved fairly in a balance. The atmosphere; hydrosphere and lithosphere were unaffected by humans' activity [3].

0.1.4 Air Pollution

Recent research [4] has given a rough picture of the physical and chemical transformation that takes place in the atmosphere.

Industrialization has brought prosperity to man since the output of goods and services per man-hour has increased dramatically over

the levels attained in earlier ages. This has occurred since machine work has replaced human toil. Then the energy consumption per man hour and also per capita has steadily increased Predominantly, this energy has been generated by means of the combustion of fossil fuels such as coal and oil. Hereby waste products are released making the air unhealthy and degrading air quality in the neighbourhood of the industries [5].

Since the development of agriculture and technology, increasing human impact on the atmosphere has occurred with its associated health hazards [1]. The combination of industrial and domestic discharges and thermal inversion has led to human fatalities and widespread diseases in several air pollution catastrophes. In less forms; as eye-stinging and lung – irritating conditions accompany thermal inversions in urban areas as man's activities greatly aggravate a situation of natural air pollution potential. Thus, increasing industrialization and urbanization have created growing demands to use the atmosphere, whether consciously or not, as a waste disposal medium [6].

Conditions of deteriorating air quality have grown more severe as urbanization and mass consumption have become the standard way of life throughout the large part of the world. Economy and ultimately, mine wastes are the underlying factor behind the emergence of serious air pollution problems. The decline in air quality signals the end of free air resources [4].

Although the use of the air largely remains free to each individual disposing waste into the atmosphere, the social costs of poor air quality in the form of dangers to human health, agricultural damages; property destruction, and many other (including presently unknown) effects accrued to society as a whole [3].

Foul cities' air is observed since hundreds of years ago but not until the latest decades did effective measures against this problem become instituted. Often the stacks of the factories were built higher in order to disperse the pollutants over larger volumes of air, thus lowering the ambient concentration of pollutants [1].

0.2 Nature and Sources of Pollutants

0.2.1 Primary Pollutants

Particulate matter suspended in the air is one of the most noticeable of air pollutants and it consists of chemically stable substances such as dust, soot, ash and smoke. These pollutants are called primary pollutants indicating that they do not change form after altering the atmosphere. They are dispersed and diffused by natural air movements but remain structurally unaltered within the air [7].

0.2.2 Secondary Pollutants

Although the general decline in the level of primary pollutants is an encouraging development in itself, the switch to liquid and gas fuels has not led to overall decline in the level of air pollution.

Instead, new gaseous pollutants, of more complex nature in terms of their structure, stability, identifiability and effects have emerged with the basic changes in both fuel and transportation technologies.

These new pollutants called secondary pollutants cause considerably more difficult control problems and pose perhaps even greater health and economic dangers than primary pollutants [7].

0.2.3 Sulphur Dioxide

SO_2 is a colourless corrosive gas, which is directly damaging to plant, animals and property. To various degrees, fossil fuels all contain sulphur. When these fuels are burned, the sulphur produces sulphur dioxide and sulphur trioxide. Both of these gases have serious health effects living things. In addition, sulphur trioxide reacts with water vapour or dissolves in water droplets in the atmosphere to form sulphuric acid. At times in certain areas, it has literally rained sulphuric acid, which corrodes, deteriorates and weakens structures and results in considerable property damage [8]. It is also the result of the acid rain that many aquatic life forms also die.

Studies [8] have shown that sulphur dioxide in the atmosphere is taken into plants through the respiratory process and, in combination with water naturally contained within the plant, becomes toxic to plant cells. Basic cereal crops such as wheat and barley are severely

attacked by sulphur dioxide. Damage by sulphur dioxide can be extensive enough to totally destroy the entire market value of a crop. It also affects trees, and there have been recent instances of damages to forested areas resulting from power plant discharges of sulphur dioxide [9]. SO_2 damage is second to only smoke as causes of air polluted-related health damage.

0.2.4 Nitrogen Oxides

The oxides of nitrogen are usually represented by the general term NO_x because of their interconvertibility [8]. Nitrogen dioxide is the main result of the incomplete burning of gas and oil fuels and the motor vehicle is the major source. Other sources are power plant and industries. The oxides of nitrogen make up the other gaseous component that contributes to smog formation, which has visibility problems and breathing difficulty as two major results.

In addition, organic substances in the atmosphere combine photochemically with nitrogen oxides to create ozone, a bluish gas that has devastating effect on vegetation, causes rubber to deteriorate, and is responsible for a significant amount of other

property damage. In this sense, it leads to the formation of a single or atomic oxygen (O) by splitting either molecular oxygen (O_2) or Nitrogen dioxide (NO_2). The single oxygen then reacts with another O_2 to form ozone (O_3).

The photochemical equation may be summarized as follows:

$$NO_{2(g)} \xrightarrow{Sunlight} NO_{(g)} + \tfrac{1}{2}O_{2(g)}$$

$$\tfrac{1}{2}O_{2(g)} + O_{2(g)} \xrightarrow{Sunlight} O_{3(g)}$$

$$NO_{2(g)} \xrightarrow{Sunlight} NO_{(g)} + \tfrac{1}{2}O_{2(g)}$$

Leading to the photostationary state:

$$NO_{2(g} + O_{2(g)} \qquad NO + O_{3(g)}$$

The NO combines with organic compounds generally represented by RO_2 to produce more ozone as follows:

$$RO_{2(g)} + NO_{(g)} \xrightarrow{Sunlight} RO_{(g)} + NO_{2(g)}$$

$$NO_{2(g)} + O_{2\,(g)} \qquad NO_{(g)} + O_{3(g)}$$

The O_3 formed in the stratosphere provides valuable shield for the biosphere by absorbing incoming harmful ultraviolet radiation. In ambient air, however, O_3 is a strong oxidizing agent with a devastating effect [8].

0.2.5 Carbon Monoxide

This is a colourless and odourless gas produced by the inefficient combustion of carbon fuel. It can pose a serious health problem. It is highly toxic at significant levels of concentration and can cause decreased human efficiency in low but chronic doses [1]. Although motor vehicles are the major sources of carbon monoxide emissions other economic activities [6] are also responsible for significant contribution of this pollutant.

0.2.6 Effects of Poor Air Quality

As alarming as these are, evidence [1] suggests that more widespread and ultimately more damaging effects may accompany persistent human exposure to poor air quality over long time periods. The connection between air quality and respiratory diseases has received increasing attention [1]. Although medical evidence [1] has established that a general relation exists between air quality and human health; an important problem is to quantify this for a multitude of other factors that determine human health.

Empirical research [6] efforts have attempted to isolate the relation between air quality and such specific diseases as bronchitis, asthma, lung cancer and emphysema in addition to general mortality and morbidity rates. The overall weight of this evidence leaves little doubt that widespread and adverse human effects are associated with poor air quality.

Other studies [6] have pointed to an association of non-respiratory cancers with air quality indices and / or urban versus rural residency. Various infant mortality rates, total mortality rates, pneumonia and pulmonary tuberculosis diseases also show increases with higher levels of air pollution [1].

Although pollution prevention is the best way to protect the atmosphere, it is very difficult to achieve. Everyday tonnes of pollutants are liberated into the atmosphere. However it is possible to minimize the extent of atmospheric pollution. One way of doing this is to develop a model for system of pollutants [10].

Finally, several studies [1] indicate that work absences are significantly higher when atmospheric conditions reflect poor air quality than when the air is relatively clean.

0.3 STATEMENT OF THE PROBLEM

The aim of this research is to determine the extent or level of pollution in terms of SO_2, NO_2 and CO concentrations as well as the effect of wind speed and direction on the distribution of these pollutants. This will enable policy makers to formulate the required policy for all stakeholders to observe and therefore contribute their quota towards the realization of a pollution free environment.

0.4 OBJECTIVES

i. To set up a mathematical model to assess air pollutant dynamics for SO_2, NO_2 and CO, and air temperature.

ii. To determine the growth rates of the SO_2, NO_2, CO and air temperature.

0.5 JUSTIFICATION

Atmospheric pollution is a threat to socio-economic development and could adversely affect the survival of the entire population.

There is therefore an urgent need to:

i. Understand the impact of the atmospheric pollution.

ii. Assess the various parameters such as source, rate of emission and growth of certain pollutants for effective control measures to be taken.

0.6 METHODOLOGY

i. A statistical method of system identification using autoregressive time series analysis.

ii. First order difference equations dynamics modeling.

0.7 STRUCTURE OF THESIS

The thesis is structured such that Chapter zero gives a background to the study.

In Chapters one and two the concepts of time series (Autoregressive models) and dynamical models (First order difference equations) are reviewed.

Chapter three deals with the process of modeling time series data using autoregressive modeling transformed into deterministic difference equation models and solved for a general solution. The results obtained will also be discussed in Chapter Three.

A summary of the results, conclusion and recommendation is dealt with in Chapter Four.

CHAPTER ONE

1.0 TIME SERIES

Time series is the term used to describe some statistical variable or numerical data measured over a uniform set of time points. Put mathematically, a time dependent sequence x_1, x_2, ... x_p where 1, 2, ...,p denotes time steps is called time series.

Thus $X_p = F(p)$

Where X_p is the level of the quantity and F(p) is the function of time step p.

1.1 TIME SERIES CYCLE

Normally time series data exhibits a general pattern which broadly speaking repeats and it is called Ti me Series Cycle.

1.2 MATHEMATICAL MODELING

Mathematical modelling is the use of a simplified mathematical representation of a real system, process, or theory.

Mathematical models are developed in order to enhance our ability to understand, predict and possibly control the beha:iour of the system being modeled.

Mathematical models are symbolic and help to express ideas and problems clearly. They may use equations such as

$$F = ma \qquad (1.2.0)$$

Where F = force, M = mass and a = acceleration

Mathematical models may also use diagrams and groups such as eiectronic computers, circuit diagrams, or stock – market index performance charts.

1.3 TIME SERIES MODELS

The components of the atmosphere must be kept fairly constant. They are used to help control current and future environmental activities. To use time series effectively the data has to be organized and analyzed.

In order to explain the movements of time series data, models can be constructed. These models describe how the various components combine to form the individual data values. For example

$$X = F + L \qquad (1.3.0)$$

Where

X = extent of environmental pollution

F = fixed or natural phenomenon

L = amount of pollutants liberated

1.4 TIME SERIES ANALYSIS

Time Series Analysis is the evaluation and extraction of the components of a model that "break down" a particular series into understandable and explainable portions and enables;

(a) Trends to be identified

(b) Extraneous factors to be eliminated

(c) Forecast to be made

The understanding, description and use of these processes is known as Time Series Analysis.

1.5 STANDARD TIME SERIES MODEL

Depending on the nature, complexity and extent of the analysis required, there are various types of time series models including

- Time Series Additive Model

  $X = t + s + r$

- Time Series Multiplication Model

  $X = t \times s \times r$

where

x = a given time series value

t = trend component

s = seasonal component

and r = residual component

Thus given a set of data, every single given (x) value can be expressed as the sum or product of the components. It is the evaluation and interpretation that is the main aim of the overall analysis.

1.6 DESCRIPTION OF TIME SERIES COMPONENTS

a) *Trend*: It enables the underlying, long-term tendency of the data to be highlighted.

b) *Seasonality*: It describes the short-term cyclic fluctuations in the data about the trend

c) *Residual variation*: These include other factors not explained by (a) and (b) above.

This variation normally consists of 2 components.

(i) Random factors: These are disturbances due to unpredictable influences, such as weather conditions.

(ii) Long term cyclic factors. These can be (if they exist) due to underlying economic causes outside the scope of the immediate environment [11]

1.7 AUTOREGRESSIVE INTEGRAL TIME SERIES MODELS

Autoregressive models differ from other regression models with respect to the assumption of the independence of error term.

Autoregressive models are time series forecasting tools.

Deterministic models are models in which no reference is made to the source or nature of the underlying randomness in the series.

Essentially it involves extrapolation techniques [12]

1.7.1 SIMPLE RANDOM MODEL

Simple Random Model consists of

(1) Overall arithmetic mean μ

(2) Error component e_t

It is given by

$$Y_t = \mu + e \quad (1, 0, 0) \qquad (1.7.1)$$

That is, there is no autoregressive AR portion (Y_t does not depend on Y_{t-1}). So it is a completely random series. It is classified as AR (0, 0).

In completely random time series, the series or autocorrelation would be zero. Each succeeding value would not be related to its previous value. This is also called a normal distribution random series or often called white noise process.

1.7.2 FIRST ORDER AR MODEL

A time series of the form

$$Y_t = \mu + \alpha_1 Y_{t-1} + e_t$$

is classified as AR(1, 0) and is an autoregressive model of first order.

1.7.3 AUTOREGRESSIVE MODEL

A model of the form

$$Y_t = \rho Y_{t-1} + \rho_2 Y_{t-2} + \ldots\ldots\ldots + \rho_p Y_{t-p} + e_t$$

is called an Autoregressive Model of the order p and is denoted by AR(p,0)

1.8 A STATIONARY TIME SERIES

A stationary time series is a time series whose basic statistical properties such as the mean or variance remain constant over time [12]

1.8.1 STATIONARITY OF AUTOREGRESSION MODEL

If for a joint probability distribution function $F(X_1, X_2, \ldots\ldots, X_k)$ and any arbitrary shift, τ, along the time axis

$$F(x_1, \ldots\ldots, x_k,) = F(x_{1+r}, \ldots\ldots, x_{k+r}) \qquad (1.8.1a)$$

Where x_1, $x_k, \ldots\ldots, x_k$ are in a given time series then the time series is said to be stationary.

In particular, assuming that the process starts in the infinite past and all X_k have bounded means and variances, then an AR (p) process is stationary if all roots z_o of the polynomial (known as the characteristic polynomial):

$$\theta_p(z) = 1 - \theta_1 z - \theta_2 z^2 - \ldots\ldots\ldots - \theta_p z^p \qquad (1.8.1b)$$

has a modulus

$$|Z| = \sqrt{Z_1^2 + Z_2^2} \quad > \quad 1 \qquad (1.8.1c)$$

where $z_1 = \mathrm{Re}(z)$ and $z_2 = \mathrm{Im}(z)$

1.8.1.1 Example

For the AR (1) process

$$x_k = \rho x_{k-1} + e_k, \qquad (1.8.1d)$$

$$1 - \rho z = 0$$

is its characteristic polynomial and it has a solution of

$$Z = 1/\rho.$$

This means that the condition for stationarity is

$$|Z| = \left|1/\rho\right| > 1$$

which is equivalent to $|\rho| < 1$.

1.9 AUTOREGRESSIVE EXOGENOUS MODEL (ARX)

Knowing the variable $Y_{k-1}, \ldots, Y_{k-p}$ to be exogenous, it is possible to introduce them into the AR (p) model as shown below

$$X_k = \theta_1 X_{k-1} + \ldots + \theta_p X_{k-p} + \beta_1 Y_{k-1} + \ldots + \beta_p Y_{k-p} + \mu + e_k \qquad (1.9.0)$$

This is called an autoregressive exogenous model of order p and denoted by ARX (p).

1.9.1 Example

(i) An ARX (1) process is given by

$$X_k = \theta_1 X_{k-1} + \beta_1 Y_{k-1} + \mu + e_k \qquad (1.9.1)$$

1.9.2 Example

(ii) An ARX (2) process is given by

$$X_k = \theta_1 X_{k-1} + \theta_2 X_{k-2} + \beta_1 Y_{k-1} + \beta_2 Y_{k-2} + \mu + e_k \qquad (1.9.2)$$

1.10 VECTOR AUTOREGRESSIVE MODEL (VAR)

A Vector autoregressive model of order P, denoted by VAR (p) for a series of N-vector variables $X_k = (X_{1k}, \ldots, X_{NK})$ may be defined as

$$X_k = A_1 X_{k-1} + \ldots\ldots + A_p X_{k-p} + \mu + e_k \qquad (1.10.0)$$

where

$$A = \begin{pmatrix} a_{11,i} & a_{12,i}\ldots\ldots & a_{1N,i} \\ a_{21,i} & a_{22,i}\ldots\ldots & a_{2N,i} \\ . & . & \\ . & . & \\ a_{N1,i} & a_{N2,i}\ldots\ldots & a_{NNi} \end{pmatrix} \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_{2\ N,I} \\ . \\ . \\ \mu_N \end{pmatrix} \quad \mu_k = \begin{pmatrix} e_{1k} \\ e_{2k} \\ . \\ . \\ e_{2N} \end{pmatrix}$$

The A_i are (n x n) coefficient matrices and e_k is a vector white noise process such that $E(e_k) = 0$ and the same (non singular) covariance $\sum_v = E(e_k, e_k^1)$ for all k. Moreover, e_k and e_s are uncorrelated for $k \neq s$.

In practice, the parameters A_i and μ will be unknown. Before the process (1.10.0) can be used for forecasting and analysis purposes they have to be estimated from the available data [13]. The stationarity of the VAR process will be discussed before considering the estimation of parameters in (1.10.0) since it will be useful in deriving the asymptotic properties of the parameter estimators.

1.10.1 Example

(1) A VAR (1) process is given by

$$X_k = A_1 X_{k-1} + \mu + e_k \qquad (1.10.1)$$

Specifically, if N = 2 we have

$$X_k = \begin{bmatrix} X_{1k} \\ X_{2k} \end{bmatrix}, \quad A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \quad e_k = \begin{bmatrix} e_{1k} \\ e_{2k} \end{bmatrix}$$

1.10.2 Example

(2) A V AR (2) process is given by

$$X_k = A_1 X_{k-1} + A_2 X_{k-2} + \mu + e_k \qquad (1.10.2)$$

1.10.3 Remark

The VAR model may be regarded as a matrix form of a simultaneous system of (AR) equations.

1.11 VECTOR AUTOREGRESSIVE EXOGENOUS MODEL (VARX)

If a set of vector variables U_k is known a priori to be exogenous, they can easily be substituted in a Vector Autoregressive model of order p, VAR (p) as

$$X_k = A_1X_{k-1} + \ldots + A_pX_{k-p} + BU_k + e_k \qquad (1.11.0)$$

with B as the coefficient matrix of the exogenous variables and the intercept terms are also included in U_k. More generally lagged

exogenous variables may enter the model in addition to the current variables U_k as

$$X_k = A_1X_{k-1} + \ldots + A_pX_{k-p} + B_1U_{k-1} + \ldots + B_pU_{k-p} \qquad (1.11.1)$$

Where

B is an n x m matrix

U_k is an Exogenous Vector

Models of the type (1.11.0) are called vector autoregressive exogenous models of order p, denoted VARX (p).

1.11.1 Example

(i) A VARX (1) model is given by

$$X_k = A_1X_{k-1} + B_1U_{k-1} + \mu + e_k \qquad (1.11.2)$$

1.11.2 Example

(ii) A VARX (2) model is given by

$$X_k = A_1X_{k-1} + A_2X_{k-2} + B_1U_{k-1} + B_2U_{k-2} + \mu + e_k \qquad (1.11.3)$$

1.12 STATIONARITY OF VAR MODELS

For an N - dimensional random vectors x_{k-1}, y_k, y_{k+1} a vector stochastic process, VAR (P) process (1.10.0) is stationary if it has bounded means and covariance matrices and the polynomial defined by the determinant:

$$\det (1- A_1Z – A_2Z^2 - \dots - A_pZ^p) = 0 \qquad 1.12.0)$$

has all its roots outside the complex unit circle (i.e $|z| > 1$)

1.12.1 Example

Consider a bivariate VAR (1) process

$$\begin{bmatrix} X_{1k} \\ X_{2k} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_{1k-1} \\ X_{2k-1} \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} e_{1k} \\ e_{2k} \end{bmatrix} \qquad (1.12.1a)$$

The determinant in (1.12.0) becomes

$$det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} Z \right) = 1 - (\theta_{11} + \theta_{22})Z + (\theta_{11}\theta_{22} - \theta_{21}\theta_{12})Z^2 \qquad (1.12.1b)$$

and it is a polynomial of degree two. The process will be stationary if this polynomial has no roots in or on the unit circle.

Suppose, for example, that

$$A = \begin{pmatrix} 0.004 & 0.534 \\ 0.218 & 0.261 \end{pmatrix}$$

the polynomial in (1.8.1b) becomes

$$1 - (0.004 + 0.261)\,Z + [(0.004)(0.261) - (0.218)(0.534)]Z^2$$

$$= 1 - 0.265Z - 0.116022Z^2$$

with the roots as $z = -1.7371$ and $z = 4.0215$ each of which is greater than one in absolute value. Therefore, the corresponding process is stationary.

CHAPTER TWO

DYNAMIC SYSTEM MODEL

2.1 INTRODUCTION

A system is one or more sets of elements and the various operations and the relations involving them. It can be defined as an organized set of ideas or principles usually intended to explain the arrangement or working of a systematic whole or as a regularly interacting or independent group of items forming a unified whole. It gives a method of representation or notation of numbers.

2.2 DYNAMICAL SYSTEM

Dynamic as a term, is used to describe the time – changing pattern producing process in which the characteristics of the problem at two different times are interrelated. When the process involves a system then we have dynamical system.

There are several examples of such systems. A typical example is environmental system like changes in the extent of pollution.

In order to handle dynamic systems appropriate mathematical language and framework are set up. In this direction the two types of equations used are

- Differential equations
- Difference equations.

They offer the appropriate structure used to link variables to each other giving an indication of the behavior of the system in terms of occurance as continuous – time or discrete – time.

When a dynamic behavior establishes a relation between a current value and the derivatives of a dynamic variable then it is a continuous time – behaviour and such behaviour is described by differential equations.

On the other hand if the dynamic behavior establishes a relation between a current value of a variable at one time and those at an adjacent time then it is described by difference equations.

With the advent of technology and its associated increasing growth rate it has become inevitable that analytical procedures for solving difference and differential equations also be pursued vigorously.

When dealing with events or consequences (eg discharge of pollutants into the atmosphere which occur at discrete - time periods such as hourly, daily, monthly or yearly periods it is convenient to use difference equations [14].

Mathematical modelling technique has been employed as an essential part in the bid to achieve the said goal. However, selecting a model depends on the following factors:

1. The depth of understanding needed at a particular stage of the problem study.

2. The use to which the model will be put.

Differential and difference equations indicate only special case of more general dynamic model understanding since they are defined in terms of a single dynamics variable. Notwithstanding this, they are capable of providing an important first step in developing techniques for the

mathematical analysis of dynamic phenomena through problem formulation or theoretical development [14].

These two types of equations enable the dynamic situation and the theories behind them to be understood.

The following give qualitative treatment of systems of first order difference equations.

2.3 DIFFERENCE EQUATIONS

Suppose that

1. there is a defined sequence of points, representing discrete equally spaced time points, indexed by k.
2. there is a value x(k) a real number associated with each point. An equation relating the value of x(k), at point k, to the value at previous points can be established. Such an equation is called a difference equation.

Below is an example:

$$X(k) = ax(k-1) + b \qquad (2.3.0a)$$

Where a and b are coefficients that may or may not depend on k.

2.3.1 ORDER OF DIFFERENCE EQUATION

The difference between the highest and lowest indices that appear in the equation is called the order of a difference equation.

2.3.1.1 Example

First – Order Difference Equation

If the rule defines the k^{th} member of the sequence in terms of the $(k-1)^{th}$ member (and possibly also the number **K** itself) then it is a first-order difference equation. An example is the following:

i.e $$x(k) = a_1 x(k-1) + b_0 \qquad (2.3.0b)$$

2.3.1.2 Example

Second – Order Difference Equation

A rule which defines the k^{th} member of the sequence in terms of the (k-2)th member (and possibly also the (k -1)th member or the number k itself) is called a second order difference equation.

$$X(k) = a_1 x(k-1) + a_2 x(k-2) + b \qquad (2.3.0c)$$

2.3.1.3 Example (In the general case)

N^{th} – Order Difference Equation

Generally, the n^{th} order difference equation which defines the kth member of the sequence in terms of the (k-n)th member is called the nth – order difference equation.

$$X(k) = a_1x(k-1) + a_2x(k-2) + \dots + a_{n-1}x(k-n+1) + a_nx(k-n) + b \qquad (2.3.0d)$$

2.3.2 LINEARITY OF DIFFERENCE EQUATION

If a difference equation has a form

$$a_nx(k-n) + a_{n-1}x(k-n+1) + \dots + a_1x(k-1) + a_ox(k) = g(k) \qquad (2.3.2)$$

for some given functions g(k) and a_i, $i = 0,1,2, \dots n$ then it is described as a linear difference equation

The unknown function x appears linearly in the equation. The equation is said to have constant coefficients or to be autonomous if these coefficients do not depend on k, The function g(k) is variously called the forcing term or driving term.

if the function g(k) = 0 then equation (2.3.0d) is described as homogeneous otherwise it is nonhomogeneous.

2.4 GENERAL SOLUTION TO LINEAR AUTONOMOUS EQUATION OF FIRST ORDER

Since higher order difference equations can be converted to a system of first order difference equations we will consider only the solution of first order difference equations.

Suppose that x(k) is the level of a pollutant (eg NO_2) in the atmosphere at anytime k and x(k –1) is the level of the pollutant in the atmosphere in the previous day, then the two may be related by a linear autonomous first order difference equation as

$$X(k) = ax(k-1) + b \qquad (2.4.1)$$

Where *a* and *b* are parameters to be determined.

A function x(k) that reduces the equation to an identity is the solution of the difference equation.

(i) Consider the homogeneous equation of the equation ((2.4.1)

$$X(k) = aX(k-1) \qquad (2.4.2)$$

The characteristic *equation corresponding* to this homogeneous equation is

$$\lambda - a = 0$$

Which has the single solution $\lambda = a$.

We therefore expect that the solution of the form

$$X(k) = Ca^k$$

Consider the non-homogeneous equation (2.4.1)

The general solution to this equation implies we determine values recursively. Thus we specify, arbitrarily, the value of x at an initial point k_0 say $k_0 = 0$, and specify $x(0) = C$. This leads immediately to the following successive values:

$X(0) = c$

$$X(1) = ax(0) + b = ac + b$$

$$X(2) = ax(1) + b = a(ac + b) + b = a^2c + (a + 1)\, b$$

.

.

.

$$x(k) = a^kc + (a^{k-1} + a^{k-2} + \ldots + a + 1)\, b$$

If $a = 1$, then the expression reduces to

$$X(k) = c + kb$$

For $a \neq 1$, the expression is simplified as

$$x(k) = a^k c + \left(\frac{1 - a^k}{1 - a}\right) b$$

Hence, the desired solution in closed – form is

$$x(k) = \begin{cases} c + kb, & a = 1 \\ a^k c + \left(\frac{1 - a^k}{1 - a}\right) b & a \neq 1 \end{cases}$$

2.4.1 Example

Solve the difference equation

$$X(k) = 4X(k-1) + 6$$

Solution

In the equation above, $a = 4$, $b = 6$

$$x(k) = 4^k c + \left(\frac{4^k - 1}{4 - 1}\right) 6$$

$$x(k) = 4^k c + \left(\frac{4^k - 1}{3}\right) 6$$

$$= 4^k c + 2(4^k - 1)$$

$$= (c + 2)\, 4^k - 2$$

2.5 LINEAR STATE EQUATIONS

Here we combine the concept of dynamics, which is represented by a system of ordinary difference equations with the machinery of linear algebra to digest dynamic systems [16]. The study here will centre on the first – order equations of a linear Discrete – Time Autonomous System.

2.5.1 DISCRETE – TIME LINEAR AUTONOMOUS DYNAMICAL SYSTEM

2.5.1.1 Systems Of First – Order Equations

In discrete time an nth – order dynamic system is defined in terms of n variables $x_1(k)$, $x_2(k)$, …, $x_n(k)$ that are each functions of the index k. a system of n first – order difference equations of the following form:

$$X_1(k+1) = f_1(x_1(k), x_2(k), \ldots, x_n(k), k)$$

$$X_2(k+1) = f_2(x_1(k), x_2(k), \ldots, x_n(k), k) \qquad (2.5.1)$$

.

.

.

$$X_n(k+1) = f_n(x_1(k), x_2(k), \ldots, x_n(k), k)$$

relate these n variables.

The functions f_i, i = 1, 2, …, n define the system and the variable $x_i(k)$, i = 1, 2, …, n are referred to as state variable.

The variable $x_i(k)$,

i = 1, 2, …, n are regarded as the unknowns whose values are determined by the system of equations.

The n values $x_1(0), x_2(0), \ldots, x_n(0)$ are referred to as initial conditions if the system is defined for k = 0, 1, 2, ...

2.5.2 LINEARITY OF DISCRETE – TIME DYNAMIC SYSTEM

A discrete – time system is linear if it has the following form

$$\begin{aligned} X_1(k+1) &= a_{11}x_1(k) + a_{12}x_2(k) + \ldots + a_{1n}x_n(k) + w_1(k) \\ X_2(k+1) &= a_{21}x_1(k) + a_{22}x_2(k) + \ldots + a_{2n}x_n(k) + w_1(k) \qquad (2.5.2) \\ &\vdots \\ X_n(k+1) &= a_{n1}x_1(k) + a_{n2}x_2(k) + \ldots + a_{nn}x_n(k) + w_n(k) \end{aligned}$$

where the variable $x_1(k)$, $x_2(k)$, ..., $x_n(k)$ are the state variables of the system, and they may take on arbitrary real values. The fixed parameters or coefficients of the system are values a_{ij}, i = 1, 2, ... n, j = 1, 2, ..., n which are the values of $w_i(k)$, i = 1, 2, .., n parameters called driving or forcing terms.If the parameter a_i is independent of k, the system is said to have constant coefficient or to be autonomous [17].

The system (2.5.2) can be written in a matrix form as

$$X(k+1) = AX(k) + w(k) \qquad (2.5.3)$$

where X(k) is the n × 1 state vector and w(k) is the forcing vector.

i.e.

$$x(k) = \begin{pmatrix} x_1(k) \\ x(k) \\ . \\ . \\ x_n(k) \end{pmatrix} \qquad W(k) = \begin{pmatrix} w_1(k) \\ w_2(k) \\ . \\ . \\ w_n(k) \end{pmatrix} \qquad (2.5.4)$$

The matrix A is the square n × n matrix that consists of the coefficients $a_{ij}(k)$. It is referred to as the system matrix.

2.5.3 LINEAR DISCRETE - TIME SYSTEM WITH INPUTS

In most applications the forcing or driving terms in a system are derived from a single or perhaps a few, specific inputs to the system. Often the source or input and the resulting forcing terms are simply structurally connected.

For instance, all the equations in the system may be affected by a single input source or an input may enter only a few equations. It is useful to explicitly display the particular structural relation in the formulation of the system equation. The definition of a linear system is expanded slightly to account for this additional structure [17].

A discrete – time linear system with inputs has the following form.

$$X_1(k+1) = a_{11}(k)\, x_1(k) + a_{12}(k)\, x_2(k) + \ldots + a_{1n}(k)\, x_n(k) + b_{11}(k)U_1(k) +$$

$$\ldots + b_{1m}(k)\, U_m(k)$$

$$X_2(k+1) = a_{21}(k)\, x_1(k) + a_{22}(k)\, x_2(k) + \ldots + a_{2n}(k)\, x_n(k) + b_{21}(k)U_1(k) +$$

$$\ldots + b_{2m}(k)\, U_m(k)$$

.

.

$$X_n(k+1) = a_{n1}(k)\, x_1(k) + a_{n2}(k)\, x_2(k) + \ldots + a_{nn}(k)\, x_n(k) + b_{n1}(k)U_1(k) + \ldots +$$

$$b_{nm}(k)\, U_m(k) \qquad (2.5.5)$$

Where the variables $u_1(k)$, $u_2(k)$, … , $u_m(k)$ are control variables of the system.

In matrix notation the system takes the form

$$X(k+1) = A(k)\, x(k) + B(k)\, u(k) \qquad (2.5.6)$$

Where B(k) is an n x m distributive matrix and u(k) is an m x 1 control vector.

NB: The vector x(k) in discrete – time system is called state vector because it is a complete description of the system at the time k at least as far as determining future behaviour.

The state space is referred to as the n-dimensional space in which the state vector is defined.

2.5.3.1 Conversion to state form

Ordinary difference equations can be easily converted to equivalent systems of first order equations.

Consider the linear difference equation

$$x_1(k+1) = a_{n-1}(k)\, x_1(k + n - 1) + \ldots\ldots + a_0(k)\, x(k) + U(k)$$

where k = 0, 1, 2 . . .

To construct a suitable system representation, define n state variables as n successive values of x(k). in particular, let

$$X_1(K) = x(k)$$

$$X_2(K) = x(k + 1)$$

$$X_3(K) = x(k + 2)$$

.

.

$$X_n(K) = x(k + n - 1)$$

With these definitions, it follows that

$$X_1(k + 1) = x_2(k)$$
$$X_2(k + 1) = x_3(k)$$

.

.

$$X_{n-}(k + 1) = x_n(k)$$

The value of $x_n(k + 1)$ can be found from the original difference equation as

$$x_n(k+1) = a_0(k)\, x_1(k) + a_1(k)\, x_2(k) + \ldots + a_{n-1}(k)\, x_n(k) + U(k)$$

Defining the state vector x(k) having component $x_1(k), x_2(k), \ldots, x_n(k)$ as above produces the linear system

$$x(k+1) = x(k) + u(k)$$

$$a_0(k)\ a_1(k)\ a_2(k) \ldots a_{n-2}(k)\ a_{n-1}(k)$$

$$\begin{pmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ . & . & . & & . & . \\ . & . & . & & . & . \\ 0 & 0 & 0 & & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ 1 \end{pmatrix}$$

$$= A(k)\, x(k) + B(k)\, u(k)$$

2.5.3.1.1 Example

Consider linear difference equation

$$X(k + 2) + 2x(K + 1) + 3\, x(k) = u(k)$$

Let $X_1(k) = x(k)$

$X_2(k) = x(k + 1)$

Then,

$X_1(k + 1) = x_2(k)$

$X_2(K + 1) = -2x_2(k) - 3x_1(k) + u(k)$

2.5.3.2 HOMOGENEOUS DISCRETE - TIME SYSTEM

If there is no forcing term in a linear system then the system is described as a homogeneous linear system [17].

In discrete – time the state vector representation for such a system is given by

$$x(k +1) = Ax(k) \qquad (2.5.7)$$

Otherwise it is nonhomogeneous.

2.5.4 SOLUTION TO LINEAR DISCRETE – TIME SYSTEM

2.5.4.1 A Homogeneous Case

Here solution to system of constant coefficient (ie time invariant) will be treated. Also unforced (homogeneous) case will be looked at first.

(a) Suppose that no input is applied to the system (2.5.8) then we set

$$u(0) = u(1) = u(2) = \ldots = u(k) = 0$$

then the state evolves according to the difference equation

$$x(k + 1) = Ax(k)$$

If the initial state vector is x(0), then the response of the system can be readily deduced from the recursive set of linear equations

$$x(1) = Ax(0)$$

$$x(2) = Ax(1) = A^2x(0)$$

.

.

$$x(k) = Ax(k - 1) = A^kx(0)$$

From these systems of equations the following facts can be deduced:

(i) If $x(0) = 0$, then $x(k) = 0$ for all k

(ii) $X(j) = A^j x(0)$

(iii) Let k and j index arbitrary values of time, then for $k \geq j$

$$X(k) = A^{k-j}x(j)$$

2.5.4.2 A Nonhomogeneous Case

Suppose there is an input applied to the system, so that the state difference equation is

$$X(k + 1) = Ax(k) + Bu(k)$$

If the initial state is x(0) and the input sequence is

[u(0), u(1), u(2), ..., U(k)] the resultant state sequence is generated as follows:

$$X(1) = Ax(0) + Bu(0)$$

$$X(2) = Ax(1) + Bu(1) = A^2x(0) + ABu(0) + Bu(1)$$

$$X(k) = Ax(k-1) + Bu(k-1)$$

$$= A^k x(0) + A^{k-1}BU(0) + \ldots + ABU(k-2) + BU(k-1)$$

These can be simplified as

$$X(k) = A^k x(0) + \sum_{j=0}^{k-1} A^{k-j-1} BU(j)$$

2.5.5 SOLUTIONS IN TERMS OF EIGENVALUES AND EIGENVECTORS

2.5.5.1 System Eigenvectors

In this section it is shown that an eigenvector of the system matrix defines an independent first-order subsystem of the total system. It follows that if a complete set of n linearly independent eigenvectors can be found, then the overall system can be decomposed into n decoupled first - order systems.

Consider the homogeneous discrete-time system

$$X(k + 1) = Ax(k) \qquad (2.5.8)$$

where x(k) is an n-dimensional state vector and A is an n × n matrix.

Suppose the vector v_i is an eigenvector of A with associated eigenvalue λ_i. [18]. That is,

$$Av_i = \lambda_i v_i$$

Suppose also that the initial state vector is set equal to a scalar multiple of this eigenvector; that is, $x(0) = \alpha v_i$. Then from (2.5.8) and (2.3.2) it follows that

$$X(1) = Ax(0) = \alpha Av_i = \lambda_i \alpha v_i = \lambda_i x(0)$$

Therefore, in this special case, the next state is just λ_i, times the initial state. Also, it is easy to see that all successive states are also various scalar multiples of the initial state.

From the above observation it means that once the state is equal to a multiple of an eigenvector it remains a multiple of the eigenvector and this makes it easy to achieve the state and can be characterized at any particular time. This is done by specifying the corresponding multiplying factor once this situation has been achieved.

Thus,

$$X(k) = z_i\,(k)v_I$$

where $z_i(k)$ is the appropriate scalar multiplying coefficient.

The multiplying coefficient itself satisfies a first-order difference equation. To see this , note that the system equation yields

$$X(k + 1) = Ax(k) = \lambda_i z_i(k)v_i$$

which, by definition of $z_i(k + 1)$, shows that $z_i(k + 1) = \lambda_i z_i(k)$.

the following theorem summarizes these results

Theorem

If the state vector of a linear homogeneous constant coefficient system is initially aligned with an eigenvector, it continues to be aligned in subsequent time periods. The coefficient, $z_i(k)$, determining the multiple of the eigenvector at a time k, satisfies the first-order equation.

$$z_i(k + 1) = \lambda_i z_i(k) \qquad (2.5.9)$$

2.5.5.2 Example:

Consider the second-order difference equation

$$2y(k +2) = 3y(k + 1) - y(k)$$

By defining

$$x_1(k) = y(k)$$

$$x_2(k) = y(k + 1)$$

The original equation can be converted to the system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

One eigenvalue and associated eigenvector for this system is

$$\lambda_1 = \frac{1}{2}, \qquad v_1 = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

with x_1 and x_2 initiating the state vector in the proportions 1 to ½, the state vector will continue to have these same properties for all k. A multiplicative factor equal to $\lambda_1 = ½$ reduces the state vector at each step [19].

With reference to the original difference equation, the system state is equal to this eigenvector if two successive y(k)'s are in the proportions 1 to ½. If that condition holds for any k it will hold for all future k's. That

means that each successive pair of y (k)'s will be in the proportions 1 to ½, and hence, that for every k, y (k + 1) = (½) y (k). Therefore, in this case, the eigenvectors translate immediately into a simple relation among successive values of the variable that defines the difference equation.

2.5.6 DIAGONALIZATION OF A SYSTEM

Again consider the homogeneous system

$$X(k + 1) = Ax(k) \qquad (2.5.10)$$

Suppose that the system matrix A has a complete set of n linearly independent eigenvectors $v_1, v_2, \ldots, v_n$ with corresponding eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. The eigenvalues may or may not be distinct. We shall show how these n eigenvectors can be used to define n separate first-order systems. Although this procedure is perhaps the most important conceptually, it is sometimes of direct computational benefit.

Let an arbitrary value of the state x(k) be specified. Since there are n eigenvectors, this state can be expressed as a linear combination of the eigenvectors in the form

$$X(k) = z_1(k)v_1 + z_2(k)v_2 + \ldots + z_n(k)v_n$$

where $Z_i(k)$, i = 1, 2, …, n are scalars. Using the fact $Av_i = \lambda_i v_i$, multiplication of (2.4.7)) by the matrix A yields.

$$X(k + 1) = Ax(k)$$

$$= \lambda_1 z_1(k)v_1 + \lambda_2 z_2(k)v_2 + \ldots + \lambda_n z_n(k)v_n$$

Therefore, as x(k+1) is expressed as a linear combination of eigenvectors in the form

$$X(k + 1) = z_1(k + 1)v_1 + z_2(k + 1)v_2 + \ldots + z_n(k + 1)v_n$$

It can be seen that the scalar coefficient z_i satisfies the first-order equations

$$\begin{aligned} z_1(k + 1) &= \lambda_1 z_1(k) \\ z_2(k + 1) &= \lambda_2 z_2(k) \\ &\vdots \\ z_n(k + 1) &= \lambda_n z_n(k) \end{aligned} \qquad (2.5.11)$$

making it possible to look at the state vector at each time instant as consisting of a linear combination of the n eigenvectors. As time

progresses, the weighting coefficients change (each independently of the others) so that the relative weights may change. As a result, the system can be looked at as n separate first-order systems, each governing the coefficient of one eigenvector.

2.5.6.1 Change Of Variable

We can transform the analysis above directly into a convenient manipulative technique by formally introducing change of variables.

Suppose M is the modal matrix of **A**. That is, **M** is the **n** × **n** matrix whose n columns are the eigenvectors of **A**. For a given x (k), we define the vector z(k) by

$$\mathbf{X}(k) = \mathbf{M}z(k)$$

This is, of course, just the vector representation of the earlier equation (2.5.7) with the components of the vector z(k) equal to the earlier $z_i(k)$'s. If this change variable is substituted in the system equation, we have.

$$\mathbf{M}z(k + 1) = \mathbf{AM}z(k)$$

Or, equivalently,

$$z(k + 1) = M^{-1}AMz(k)$$

This gives the definition of a new system which bears a relation with the original system by a change of variable.

The matrix $\mathbf{M^{-1}AM}$ is the new system matrix that corresponds to the system that governs the z_i (k)'s expressed earlier by (2.5.11).

Therefore, we may write that $\mathbf{M^{-1}AM} = \Lambda$ is the diagonal matrix with the eigenvalues of **A** on the diagonal. The modal matrix **M** defines a new coordinate system in which **A** is represented by the diagonal matrix Λ.

The detailed form of (2.5.2) becomes

$$\begin{pmatrix} z_1(k+1) \\ z_2(k+1) \\ \\ . \\ \\ z_n(k+1) \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & 0 & \lambda_n \end{pmatrix} \begin{pmatrix} z_1(k) \\ z_2(k) \\ \\ \\ \\ z_n(k) \end{pmatrix}$$

The A_i are (n x n) coefficient matrices and

which explicitly displays the diagonal form obtained by the change of variables.

2.5.7 MULTIPLE EIGENVALUES

It may not be impossible to find a complete set of eigenvalues if the system matrix **A** has repeated roots of its characteristic polynomial corresponding to the presence of multiple eigenvalues making it impossible to diagonalize the matrix **A** and for that matter reduction of the system to a set of uncoupled first-order systems also becomes impossible such a system has characteristic response which is somewhat different than that system which can be diagonalised [19].

2.6 DOMINANT MODEL [EIGENVALUE(S)]

For a linear constant coefficients system it is often the behaviour of only one or two of its eigenvalues and corresponding eigenvectors that largely determines the long-term behaviour. Hence the special interest attached to these dominant eigenvalues and eigenvectors.

2.6.1 DISCRETE- TIME SYSTEM

Consider the discrete-time system

$$X(k + 1) = Ax(k) \qquad (2.6.0)$$

Suppose the matrix A has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ with $|\lambda_1| > |\lambda_2| \geq \ldots \geq |\lambda_n|$. The eigenvalue λ_1 of greatest magnitude is the dominant eigenvalue. Therefore in the system matrix **A**, the eigenvalue with the greatest magnitudes are described as the dominant eigenvalues. For simplicity, let us assume that there is a complete set of eigenvectors, and that there is only one eigenvalue of greatest magnitude. In situations where there were two greatest of equal magnitude, then both would be considered dominant.

Any initial condition vector can be expressed as a linear combination of all eigenvectors in the form

$$X(0) = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n \qquad (2.6.1)$$

Correspondingly, the solution to (2.4.5) at an arbitrary time k > 0 is

$$X(k) = \sum_{i=1}^{n} \alpha_i \lambda_i^k v_i \qquad (2.6.2)$$

Since λ_1^k grows faster than λ_i^k for i = 2, 3, …, n, it follows that for large k

$$\left|\alpha_1 \lambda_1^k\right| >> \left|\alpha_i \lambda_i^k\right| \qquad \text{for i = 2, 3, …, n}$$

as long as $\alpha_1 \neq 0$. this is to say that if the value of k is large, then the coefficient of the first eigenvector in the expansion (2.6.2) is large relative to the other coefficients. Hence, for large values of k, the state vector x(k) is essentially aligned with the eigenvector v_1 [20].

If $\alpha_1 = 0$, the first coefficient in (2.6.2) is zero for all values of k, and the state vector will not be in line with v_1 and from the theoretical point of view, in this case the eigenvalue of next greatest magnitude would

from the practical point of view the dominant eigenvalue almost always takes hold – at least ultimately – for a slight perturbation at any step introduces a small $\alpha_1 \neq 0$ that grows faster than the other term.

A similar analysis applies to a nonhomogeneous system such as

$$X(k + 1) = Ax(k) + b$$

For a real dominant eigenvalue(s) greater than one, this indicates that there is long-term growth of the system.

The solution to the system

$$X(k) = Ax(k-1) + BU(k-1)$$

Expressing this in terms of the dominant eigenvalue(s), we have

$$X_p = \alpha_i \Lambda_i^p V_i + \left(\lambda_i^p - 1\right)\left(\lambda_i - 1\right)^{-1} B \qquad (2.6.3)$$

where

$$A_i = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & 0 & \lambda_n \end{pmatrix}$$

α_i – Scalar Multiple of V_i

V_i – The eigenvectors corresponding to λ_i

λ_i – dominant eigenvalues

2.6.2 EQUILIBRIUM POINT OF LINEAR DISCRETE – TIME SYSTEM

2.6.2.1 Definition

If a vector X(k) has a property that once the system state vector X(k) is equal to $\bar{x}$ it remains equal to $\bar{x}$ for all future time then the vector X(k) is described as an equilibrium point of the system.

Thus, it is a point where the state vector is in equilibrium and does not move. This helps in determining the stability of the system.

For a homogenous system, the origin (the point $\bar{x} = 0$) is the only equilibrium point. Once the state is equal to zero it will remain unchanged.

Other equilibrium points may also occur in equilibrium. An equilibrium point must satisfy the condition

$$\bar{x} = A\bar{x} \tag{2.6.4}$$

and this condition is identical to the statement that x is an eigenvector of **A** with corresponding eigenvalue equal to unity. Therefore if unity is an eigenvalue of **A**, any corresponding eigenvector is an equilibrium point. If unity is not an eigenvalue of **A**, then the only equilibrium point of the system is the origin.

For a system which is nonhomogeneous, an equilibrium point of such

a system must satisfy the equation

$$\bar{x} = A\bar{x} + b \qquad (2.6.5)$$

If unity is not an eigenvalue of **A**, then the matrix **I – A** is nonsingular, with

$$\bar{x} = [I - A]^{-1} b \qquad (2.6.6)$$

as its unique solution and also unity is an eigenvalue, then depending on whether or not (2.6.5) represents a consistent set of equations, there may be no equilibrium point or infinity of such points. In most cases of interest, there is a unique equilibrium point given by (2.6.6).

2.6.3 STABILITY

The term stability is used to describe the property of an object that is in equilibrium, which makes the object resist or oppose any attempt to

move it. That is, any movement sets up an opposing system to resist the movement. It shows how easy or difficult it is for the system to lose balance. In the setting of dynamic systems, stability is defined with respect to a given equilibrium point. An equilibrium point is stable if when the state vector is removed slightly away from that point, it tends to return to it, or at least does not keep moving further away.

Knowledge of stability helps in our everyday life. For instance in areas such as

- Scientific industry where objects like vehicles, buildings, household items (eg. iron) are designed.

- Biological systems where terrestrial or land plants develop roots to make them stable. A very interesting thing is that many forest trees have developed thick roots (buttress roots) at a height of 1 – 2m from the trunk to the ground as an adaptation to give them stability.

- Trees have broad base and they become narrow at the highest height.

Objects may be in (i) stable equilibrium (ii), unstable equilibrium or (iii) neutral equilibrium depending on whether or not the line of action occurs inside or outside the base are after they are slightly displaced.

Here we consider stability of equilibrium points corresponding to linear constant coefficients systems of the form

$$X(k + 1) = Ax(k) + b \qquad (2.6.7)$$

$$X(t) = Ax(t) + b \qquad (2.6.8)$$

In order to make the discussion precise, the following definition is introduced.

2.6.3.1 Definition

If for any initial condition the state vector tends to x as time increases then the equilibrium point x of a linear constant coefficient system (2.6.7) or (2.6.8) is described as asymptotically stable. The point is unstable if for some initial condition the corresponding state vector tends towards infinity.

It is important to note that stability issues for (2.5.7) is tied directly to the corresponding equations. Suppose, for example, that x is an equilibrium point of (2.5.7). Then we have

that x is an equilibrium point of (2.6.0). Then we have

$$x(k+1)-\bar{x} = Ax(k)-A\bar{x}+b-b$$

And thus,

$$x(k+1)-\bar{x} = A(x(k)-\bar{x})$$

It is clear that the condition for x (t) to tend to $\bar{x}$ in (2.6.7) is identical to that for z(k) to tend to 0 in the homogeneous system

$$Z(k+1) = Az(k)$$

Therefore, in the case of the linear system (2.6.7) and (2.6.8),

asymptotic stability or instability is explicitly independent on the equilibrium point, but instead on the properties of the homogeneous equation.

In other words, the complete solution to, say, (2.6.7), consists of a constant x (a particular solution) and a solution to the homogeneous equation. Asymptotic stability holds if every homogeneous solution tends to zero.

It is the eigenvalues of the matrix **A** that determines the character of the solution to the homogeneous equation. The discrete - time case is considered below.

2.6.4 STABILITY CONDITION OF A LINEAR DISCRETE-TIME SYSTEM

Consider the system

$$X(k + 1) = Ax(k) \qquad (2.6.8)$$

In order to obtain conditions for asymptotic stability assume initially that ***A*** can be diagonalized. Then there is a matrix **M** such that

$$\boldsymbol{A} = \mathbf{M}\boldsymbol{A}\mathbf{M}^{-1}$$

where

$$\boldsymbol{A}_i = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & 0 & \lambda_n \end{pmatrix}$$

and the λ_i's are the eigenvalues of **A**. Moreover,

$$\boldsymbol{A}^K = \boldsymbol{M}\boldsymbol{A}^K\boldsymbol{M}^{-1} = \boldsymbol{M}\begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & 0 & \lambda_n \end{pmatrix}\boldsymbol{M}^{-1}$$

The requirements of asymptotic stability and that the matrix $\boldsymbol{A}^k$ tend toward the zero matrix as k increases are equivalent, since otherwise some initial conditions could be found that had a corresponding

solution not tending toward zero. This requirement, in turn, is equivalent to requiring that all the terms tend toward zero.

All terms will tend toward zero if and only if $|\lambda_j| < 1$ for every j. Thus, for asymptotic stability a necessary and sufficient condition is that all eigenvalues of **A** lie inside the unit circle of the complex plane.

Deducing a partial converse result concerning instability is easy. If the magnitude of any eigenvalue is greater than one, then there would be the corresponding eigenvector a vector X(0), which leads to a solution that increases geometrically toward infinity. Thus, an eigenvalue with a magnitude greater than one is sufficient to indicate that instability exists. With the background knowledge of the fact that the homogenous system determines stability all the above may be summarized and stated in terms of a theorem as below:

Theorem

A necessary and sufficient condition for an equilibrium point of the system (2.5.0) to be asymptotically stable is that the eigenvalues of **A**

all have magnitude less than one (that is, the eigenvalues must all lie inside the unit circle in the complex plane).

If at least one eigenvalue has magnitude greater than one, the equilibrium point is unstable.

Because the stability of the equilibrium point of a linear discrete-time system depends only on the structure of the system matrix **A**, it is common to refer to stability of the system itself, or stability of the matrix **A**. Thus, we say, "the system is asymptotically stable " or "the matrix **A** is asymptotically stable" when, in the context of a discrete-time linear system, the matrix **A** has all of its eigenvalues inside the unit circle.

These have been illustrated in the figures 2.6.1 (a), (b) and (c)

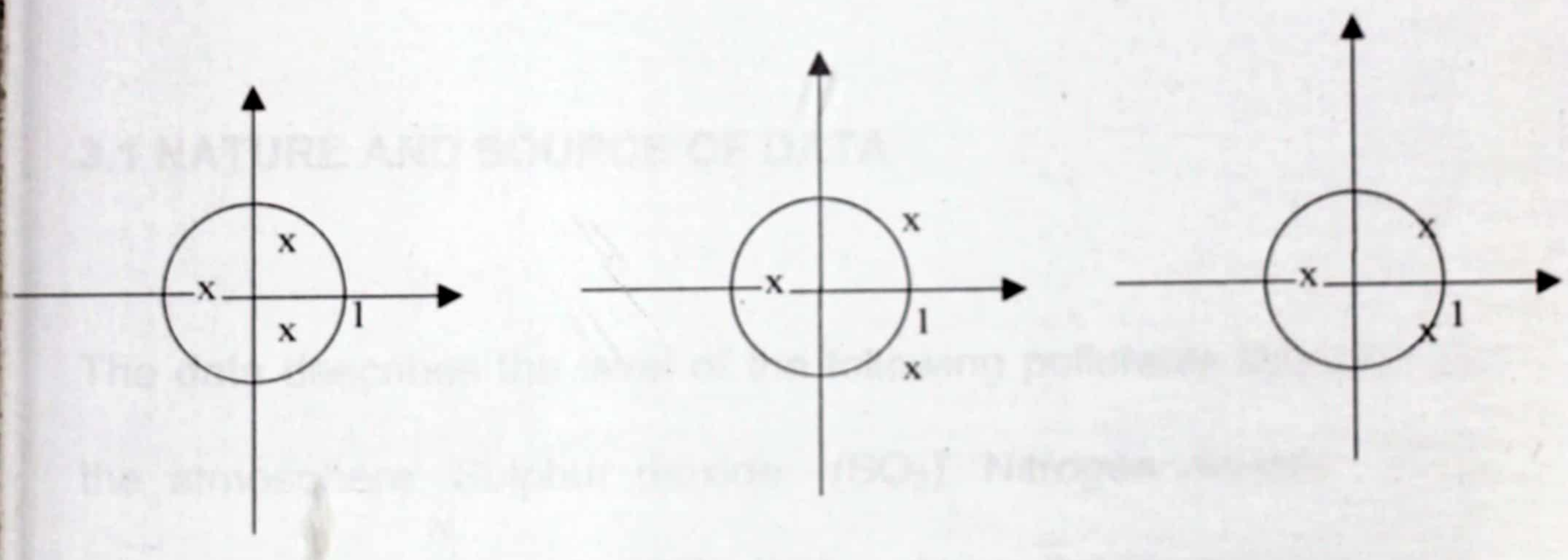


Figure 1. Eigenvalues and stability in discrete-time system.

(a) Asymptotically stable. (b) Unstable. (c) Marginally stable.

CHAPTER THREE

MODELLING AND DATA ANALYSIS

3.0 INTRODUCTION

This chapter involves the identification of the model for the various pollutants, air temperatures and wind direction and speed dynamics recorded using the methods described in chapter two with the help of computer program called Statistical Package for Social Science (SPSS). The specific quantities used to determine the best models were: probability, residual variance and Q – statistics.

3.1 NATURE AND SOURCE OF DATA

The data describes the level of the following pollutants liberated into the atmosphere: Sulphur dioxide (SO_2), Nitrogen dioxide (NO_2), Carbon monoxide dioxide (CO). It also shows the temperature of the air and the direction and speed of the wind.

This data was obtained printed from a machine called Environmental Monitoring Station (EMS) installed by the Environmental Protection Agency on top of a building (at a height of about 20 meters) in the premises of the Ghana brewery limited near Atonsu, a suburb of Kumasi.

The instrument captures and records these quantities at an interval of one hour. Therefore there were twenty-four values for each quantity for each day. The data for each day had been equally grouped into four under the headings: Morning, Afternoon, Evening and Night and their averages provided.

The data describes levels of these quantities in the period from 30th May 2001 to 30th May 2002.

3.3 IDENTIFICATION OF THE MODELS

3.3.1 SO_2 model

The averages of the levels of the quantities recorded were run on the SPSS using autoregressive model. The results obtained are displayed in the tables below.

The model selected as the best has been marked with the asterisk.

Table 3.3.3 Models For SO_2 (Evening)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	0.00497173	0.0	0.42260998	0.0
*** 110**	0.00354621	0.0	-0.26871725	0.0
101	0.00469247	0.0	-0.35591778 0.31254002	0.0
111	0.00499316	0.0 0.0	0.289140039 0.439113068	0.0
200	0.00478233	0.0	0.36628163 0.99862325	0.0
210	0.00578273	0.0	0.40112470 -0.67348675	0.0

Table 3.3.4 Models For SO_2 (Night)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	0.00264166	0.02588733	0.05515387	0.0
*** 110**	0.00230392	0.0	-0.37523006	0.0
101	0.00260586	0.0	-0.88381890	0.0
111	0.00287031	0.00001275 0.0	0.12804213 0.52644352	0.0
200	0.00255349	0.00733765 0.0	0.06538088 -0.94586350	0.0
210	0.00367602	0.00741622	0.51901241 0.90022785	0.0

3.3.2 OBSERVATIONS

From the tables above, the Autoregressive model AR(1,1) was identified as the best in each period of the day when the recorded concentrations of SO_2 for morning, afternoon, evening and night were ran with the SPSS. This model was found to be the least in terms of probability of least error, residual variance and the Q – value.

The AR (1,1) model is defined as (as explained in chapter one)

$$W_k = \alpha w_{k-1} + \mu + \varepsilon_k \ldots\ldots \qquad (3.3.1)$$

where

α_1 = parameter to be determined

$W_k = X_k - X_{k-1}$

μ = Mean of pollutant level

ε_k = error term or white noise.

When (3.3.1) is integrated the following Autoregressive model of the form

$$X_k = \alpha_1 X_{k-1} + \mu k + c + \varepsilon_k \qquad (3.3.2)$$

Is obtained.

Where μ and c are a constant and the concentration of pollutants in the air respectively.

The table 3.3.5 below shows the parameters of the AR(1,1) model estimated using the SPSS software:

Table 3.3.5: Parameters of SO_2

Period	α	μ
Morning	- 0.446	0.0046
Afternoon	- 0.464	0.0100
Evening	- 0.269	0.0114
Night	-0.375	0.0071

3.3.3 Remark: In all cases c, the integration constant was found to be zero to three decimal places.

Therefore equation (3.3.2) becomes

$$X_k = -0.446X_{k-1} + (0.0046)k + \varepsilon_k \text{ for (morning)}$$

$$X_k = -0.464X_{k-1} + (0.0100)k + \varepsilon_k \text{ for (Afternoon)}$$

$$X_k = -0.269X_{k-1} + (0.0114)k + \varepsilon_k \text{ for (Evening)}$$

$$X_k = -0.375X_{k-1} + (0.0071)k + \varepsilon_k \text{ for (Night)}$$

3.3.4 DETERMINISTIC SO_2 CONCENTRATION DYNAMICS MODEL

The AR (1,1) model defined by the equation

$$X_k = \alpha_1 X_{k-1} + \mu k + \varepsilon_k \qquad (3.3.4)$$

is a stochastic model that leads to a deterministic dynamics difference equation since the error term, ε_k is zero. This deterministic dynamics difference equation is given by

$$X_k = \alpha_1 X_{k-1} + \mu k$$

$$X_k = -0.446X_{k-1} + (0.0046)k \text{ for (morning)}$$

$$X_k = -0.464X_{k-1} + (0.0100)k \text{ for (Afternoon)}$$

$$X_k = -0.269X_{k-1} + (0.0114)k \text{ for (Evening)}$$

$$X_k = -0.375X_{k-1} + (0.0071)k \text{ for (Night)}$$

The resulting deterministic dynamics difference equations are all first order difference equation with constant coefficients. The reason being that confidence level (probability) for these models were found to be zero to two decimal places. (See the tables 3.3.1, 3.3.2, 3.3.3 and 3.3.4).

The general solution to the deterministic equation for SO_2 concentration dynamics is therefore given by

$$X_k = \alpha^k (X_0) + a + bk \qquad (3.3.5)$$

Where X_0 = initial concentration of SO_2 during the various periods.

So we have the following:

$X_k = -0.446^k (X_0) + 0.0010 + 0.0032k$ for morning

$X_k = -0.464^k (X_0) + 0.0022 + 0.0068k$ for afternoon

$X_k = -0.269^k (X_0) + 0.0019 + 0.0090k$ for evening

$X_k = -0.375^k (X_0) + 0.0023 + 0.0052k$ for night

The dynamics of SO_2 concentration has been graphed as shown below.

FIG. 2: A Graph of SO_2 Concentration (ppm) in the Morning

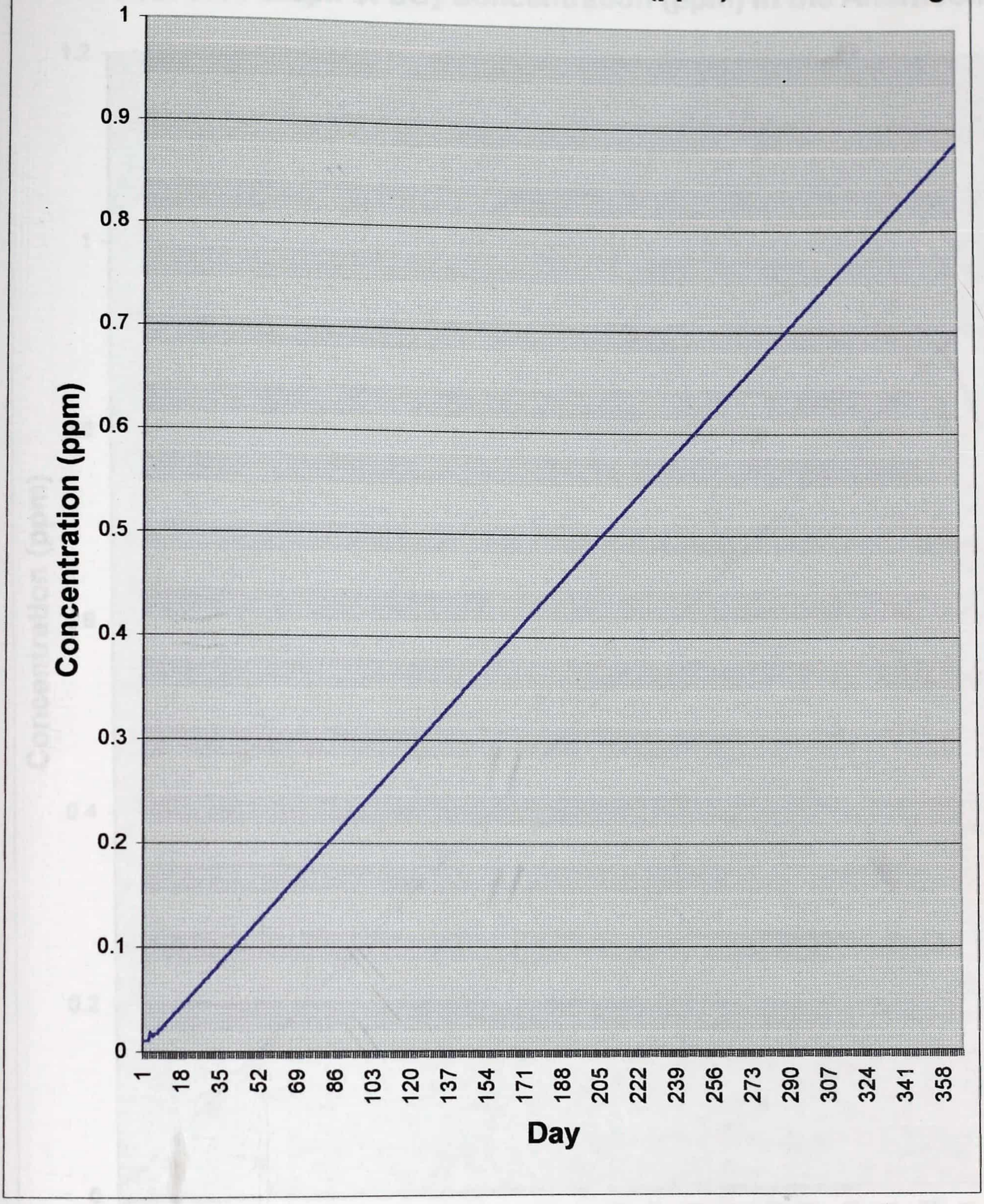


FIG. 3: A Graph of SO_2 Concentration (ppm) in the Afternoon

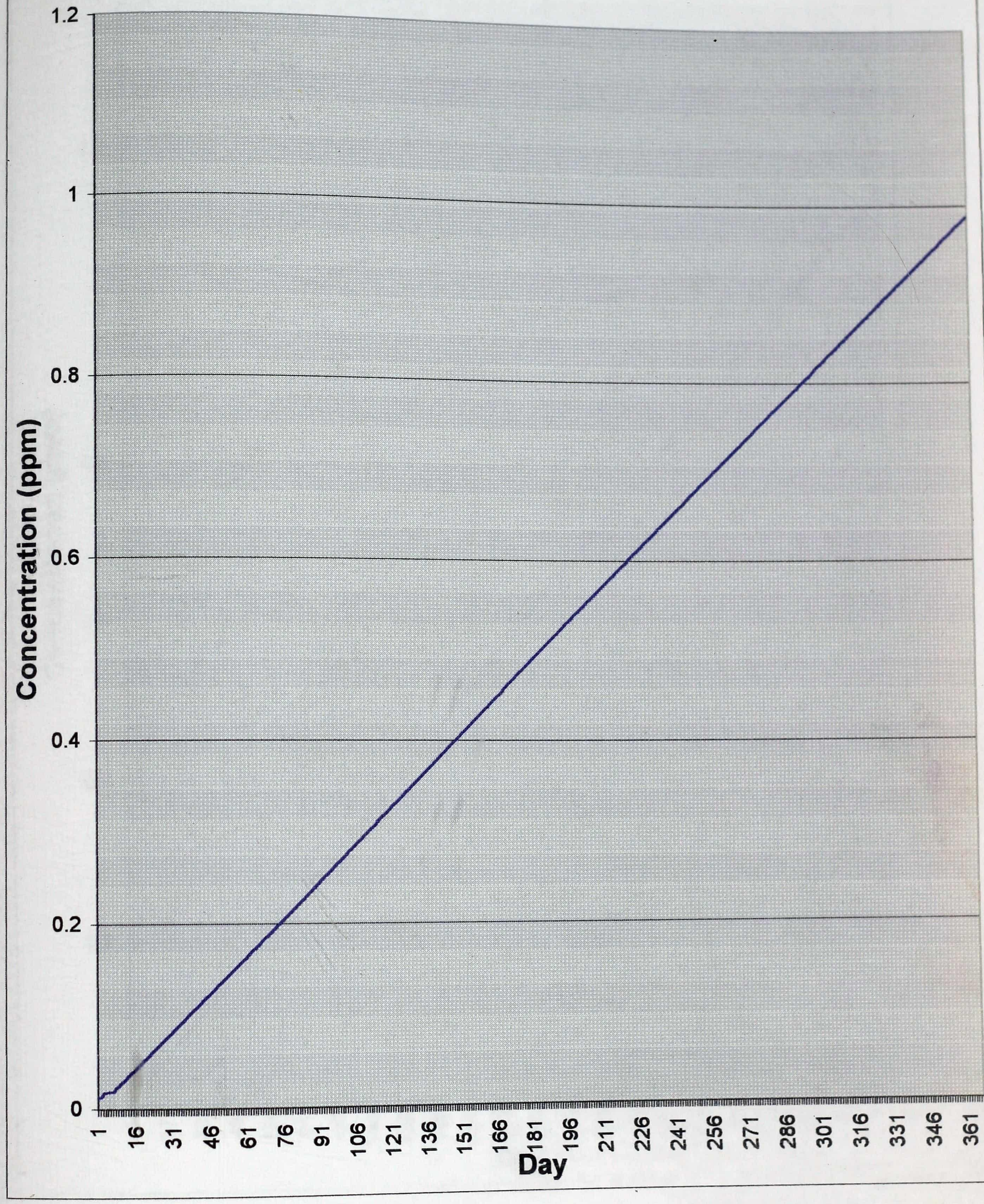


FIG. 4: A Graph of SO_2 Concentration (ppm) in the Evening

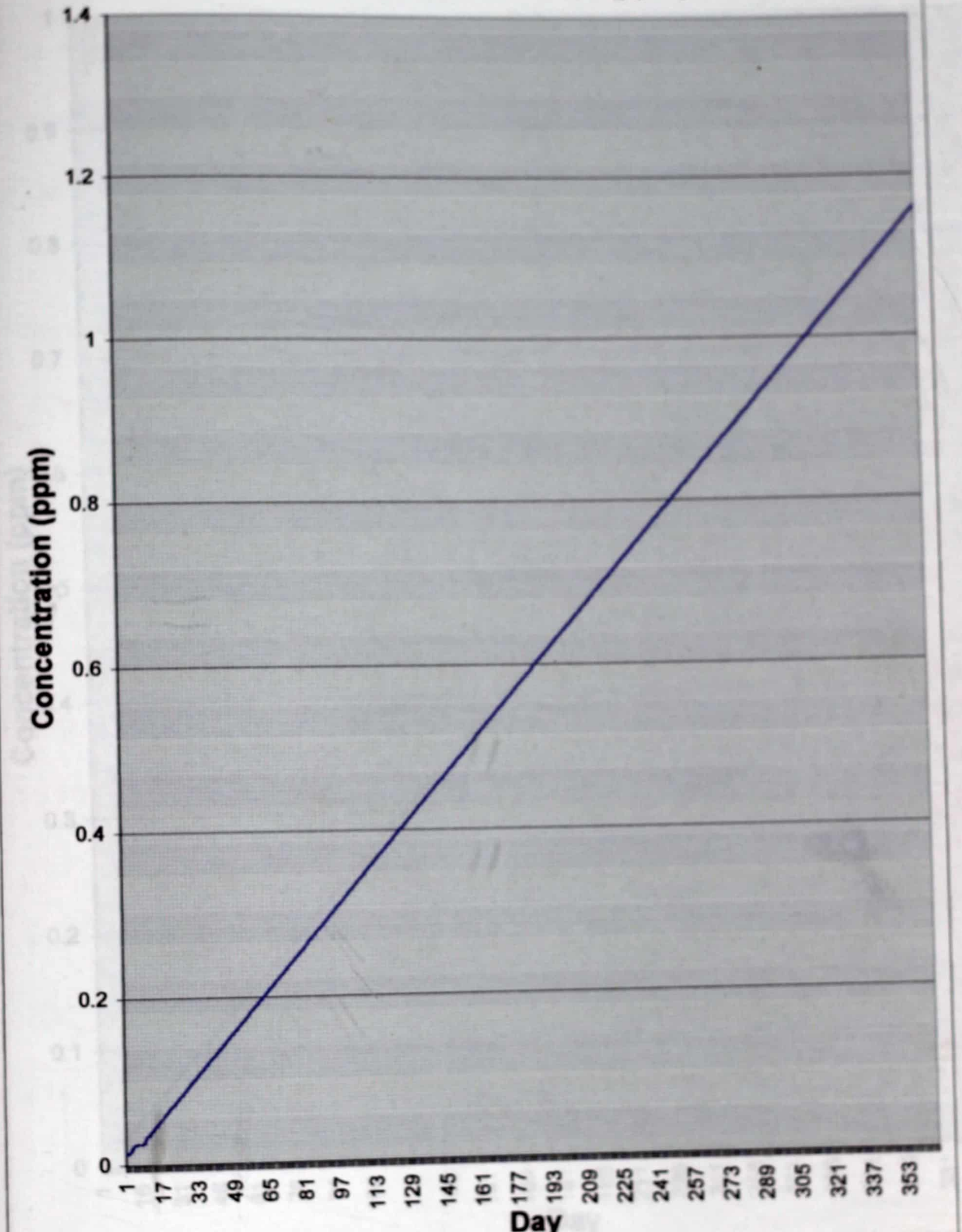
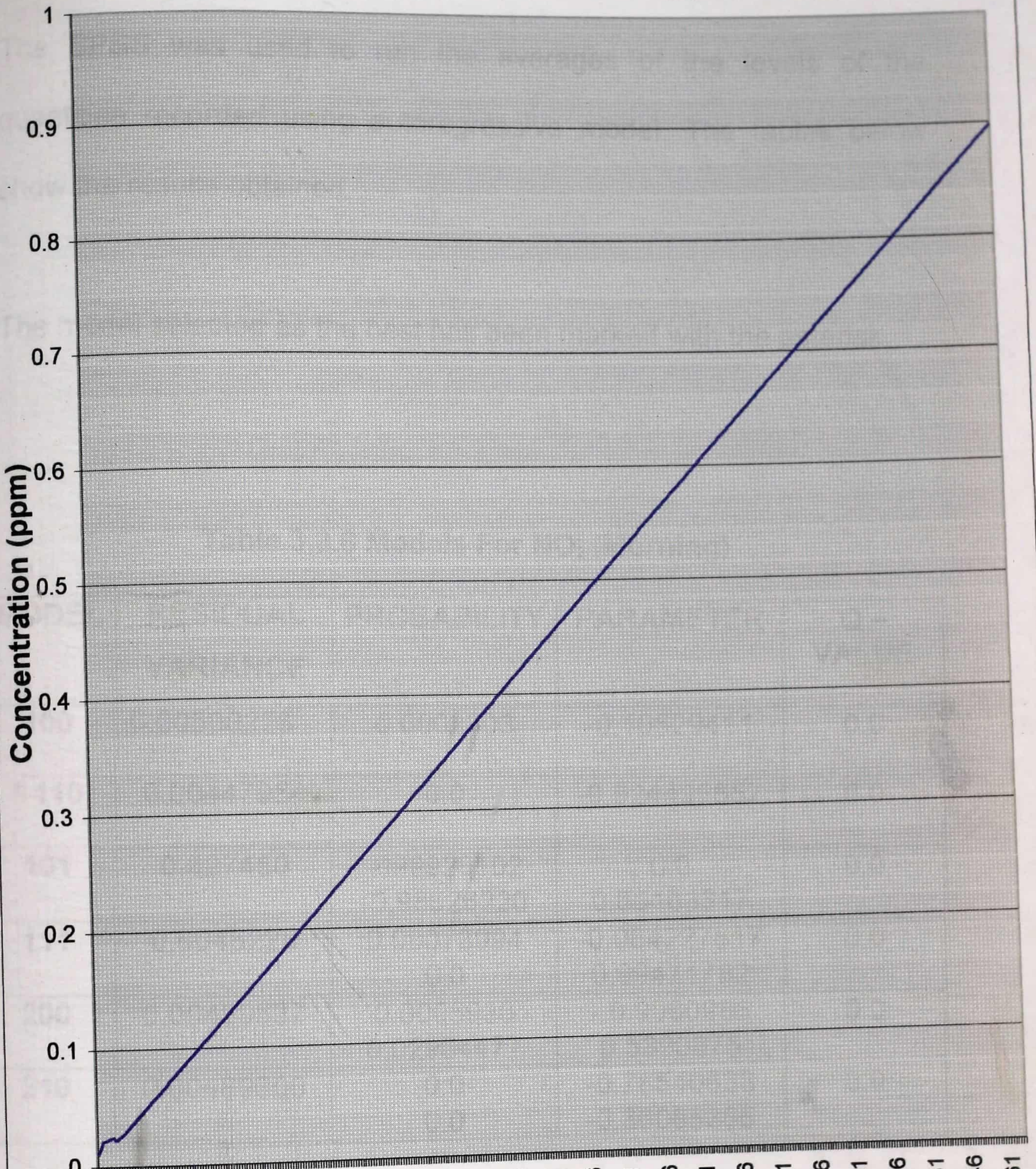


FIG. 5: A Graph of SO_2 Concentration (ppm) in the Night



3.3.5 NO_2 model

The SPSS was used to run the averages of the levels of the quantities recorded using autoregressive model. The tables below show the results obtained.

The model selected as the best has been marked with the asterisk.

Table 3.3.6 Models For NO_2 (Morning)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	0.00500226	0.0002303	-0.10529411	0.0
***110**	0.00447954	0.0	-0.524495657	0.0
101	0.027450	-099926102 -0.98626220	0.0 0.66403317	0.0
111	0.0045228	0.00073994 0.0	-0.084227817 0.99471782	0.0
200	0.00499537	0.0005930 0.02964477	-.0.9960986 0.5390875	0.0
210	0.00489800	0.0 0.0	-0.78540833 -0.36085385	0.0

Table 3.3.7 Models For NO_2 (Afternoon)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	0.0027606	0.37587764	0.2193490	0.0
***110**	0.00264302	0.0	-0.42732420	0.0
101	0.00224803	-0.8226188629 -089201545	0.0 0.0	0.0
111	0.00228754	0.16150049 0.0	0.03521618 0.99619612	0.0
200	0.00224404	0.31674109 0.0	0.02463182 -0.12110496	0.0
210	0.00331251	0.0 0.0	-0.55613930 -0.30204959	0.0

Table 3.3.8 Models For NO_2 (Evening)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	0.00229391	0.005986615	0.6801792	0.0
***110**	0.00218717	0.0	0.0 -0.41634739	0.0
101	0.00248726	0.0 0.0	- 0.72105646 - 0.88256223	0.0
111	0.00233710	0.00039954 0.0	0.9150962 0.98108639	0.0
200	0.00222549	0.00258751 0.00017152	0.07445500 -0.09298518	0.0
210	0.00315509	0.0 0.0	-0.55680736 -0.33598092	0.0

Table 3.3.9 Models For NO_2 (Night)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	0.27111360	0.27111360	-0.02726171	0.0
* 110	0.00262714	0.0	-0.47068731	0.0 0.0
101	0.00307829	0.0 0.0	0.65262994	0.0
111	0.00224990	0.9241116 0.0	0.00243946 0.9949046	0.0
200	0.00224990	0.22848617 0.0	-0.02973764 0.63403102	0.0
210	0.00340801	0.0 0.0	-5.8680704 -0.24709199	0.0

3.3.6 OBSERVATIONS

It can be inferred from the tables above that the integrated Autoregressive model (AR (1,1)) is the best in each period of the day when the recorded concentrations of NO_2 for morning, afternoon, evening and night were ran with the SPSS because it has the least probability of least error, residual variance and Q – statistics values.

The AR (1,1) model is defined as

$$W_k = \alpha w_{k-1} + \mu + \varepsilon_k \ldots\ldots \quad (3.3.6)$$

where

α_1 = parameter to be determined

$W_k = X_k - X_{k-1}$

μ = Mean of pollutant level

ε_k = error term or white noise.

When (3.3.6) is integrated the following Autoregressive model of the form

$$X_k = \alpha_1 X_{k-1} + \mu k + c + \varepsilon_k \qquad (3.3.7)$$

Is obtained

Where c and X_k are constant and the concentration of pollutants in the air respectively.

The table 3.3.10 below shows the parameters of the AR(1,1) model estimated by the SPSS:.

Table 3.3.10: Parameters NO_2

Period	α_1	μ
Morning	-0.524	0.0068
Afternoon	- 0.427	0.0100
Evening	- 0.416	0.0111
Night	-0.471	0.0075

3.3.7 Remark: In all cases c, the integration constant was found to be zero to two decimal places.

Therefore equation (3.3.2) becomes

$$X_k = -0.524X_{k-1} + (0.0068)k + \varepsilon_k \text{ for (morning)}$$

$$X_k = -0.427X_{k-1} + (0.0100)k + \varepsilon_k \text{ for (Afternoon)}$$

$$X_k = -0.416X_{k-1} + (0.0111)k + \varepsilon_k \text{ for (Evening)}$$

$$X_k = -0.471X_{k-1} + (0.0075)k + \varepsilon_k \text{ for (Night)}$$

3.3.7 DETERMINISTIC NO_2 CONCENTRATION DYNAMICS MODEL

The AR (1,1) model defined by the equation

$$X_k = \alpha_1 X_{k-1} + \mu k + \varepsilon_k \qquad (3.3.2)$$

is a stochastic model that leads to a deterministic dynamics difference equation since the error term, ε_k is zero. This deterministic dynamics difference equation is given by

$$X_k = \alpha_1 X_{k-1} + \mu k \qquad (3.3.8)$$

$$X_k = -0.524X_{k-1} + (0.0068)k \text{ for (morning)}$$

$$X_k = -0.427X_{k-1} + (0.0100)k \text{ for (Afternoon)}$$

$$X_k = -0.416X_{k-1} + (0.0111)k \text{ for (Evening)}$$

$$X_k = -0.471X_{k-1} + (0.0075)k \text{ for (Night)}$$

These resulting deterministic dynamics difference equations are all first order difference equations with constant coefficients because the confidence level (probability) for these models was found to be zero to two decimal places. (See table's 3.3.6, 3.37, 3.3.8 and 3.3.9).

The general solution to the deterministic equation for NO_2 concentration dynamics is therefore given by

$$X_k = \alpha^k (X_0) + a + bk \qquad (3.3.5)$$

Where X_0 = initial concentration of NO_2 during the various periods.

So we have the following:

$X_k = -0.524^k (X_0) + 0.0015 + 0.0045k$ for morning

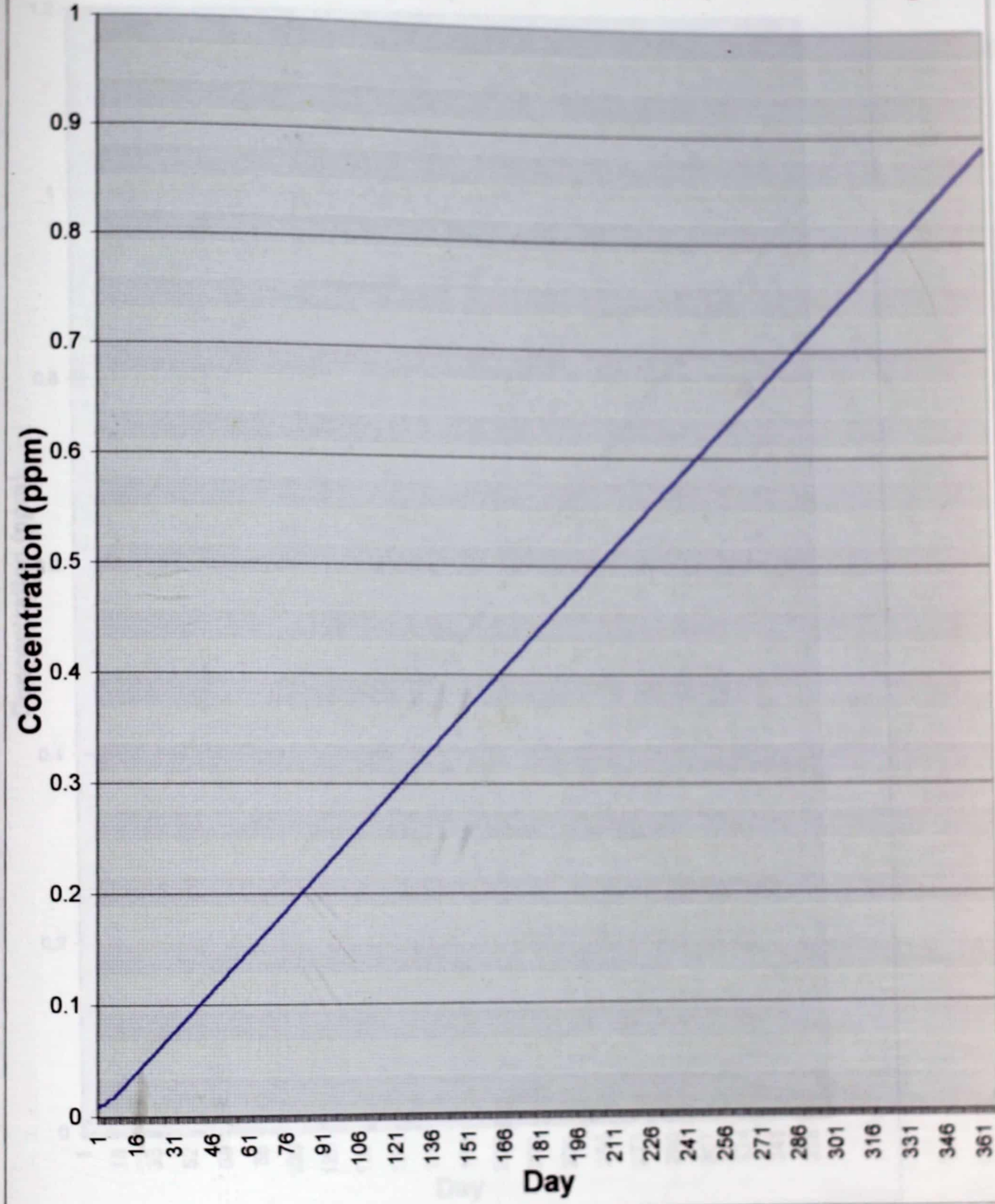
$X_k = -0.427^k (X_0) + 0.0021 + 0.0070k$ for afternoon

$X_k = -0.416^k (X_0) + 0.0024 + 0.0078k$ for evening

$X_k = -0.471^k (X_0) + 0.0022 + 0.0051k$ for night

These NO_2 concentration dynamics have been illustrated graphically as shown below.

FIG. 6: A Graph of NO_2 Concentration (ppm) in the Morning



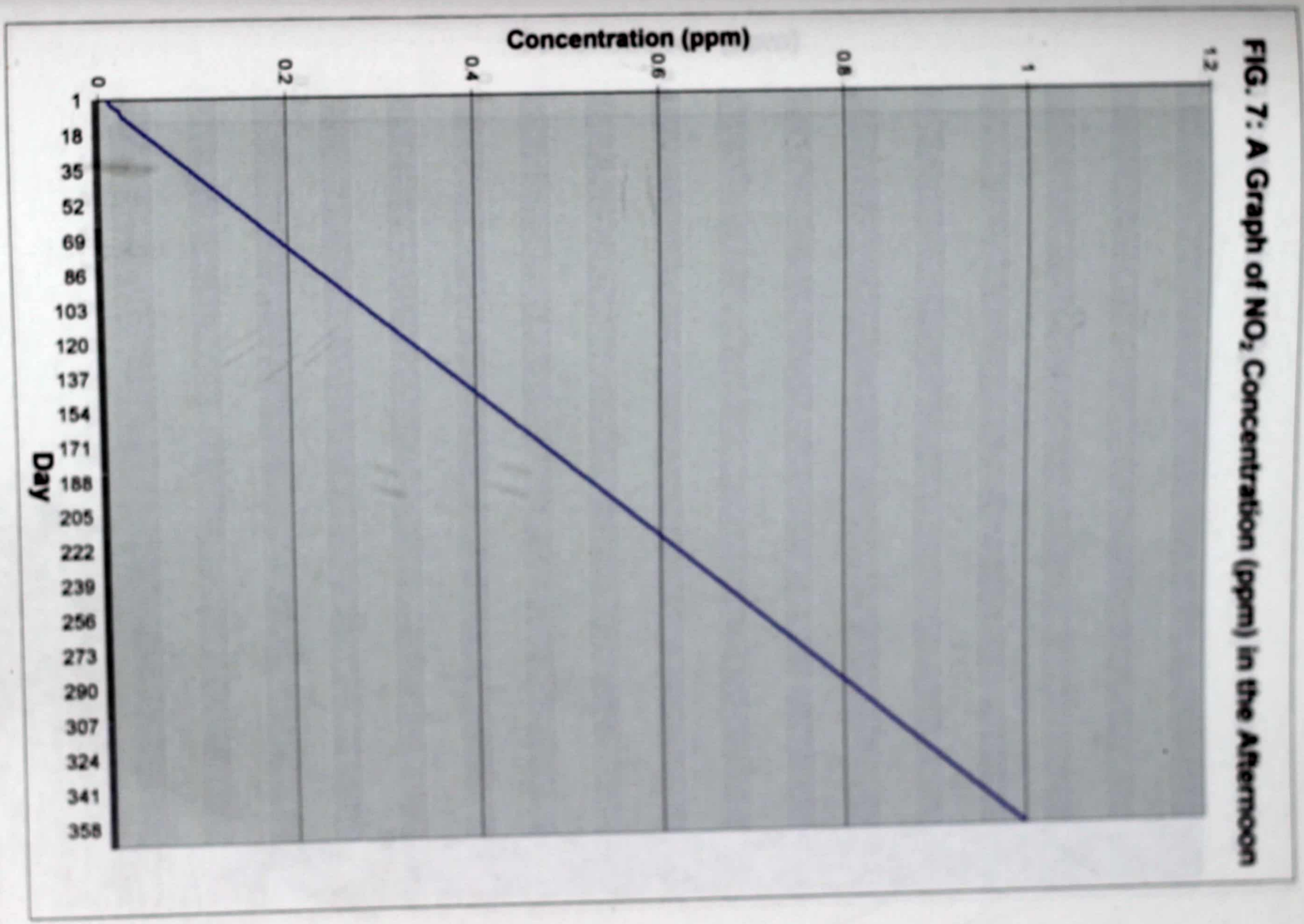


FIG. 7: A Graph of NO_2 Concentration (ppm) in the Afternoon

FIG. 8: A Graph of NO_2 Concentration (ppm) in the Evening

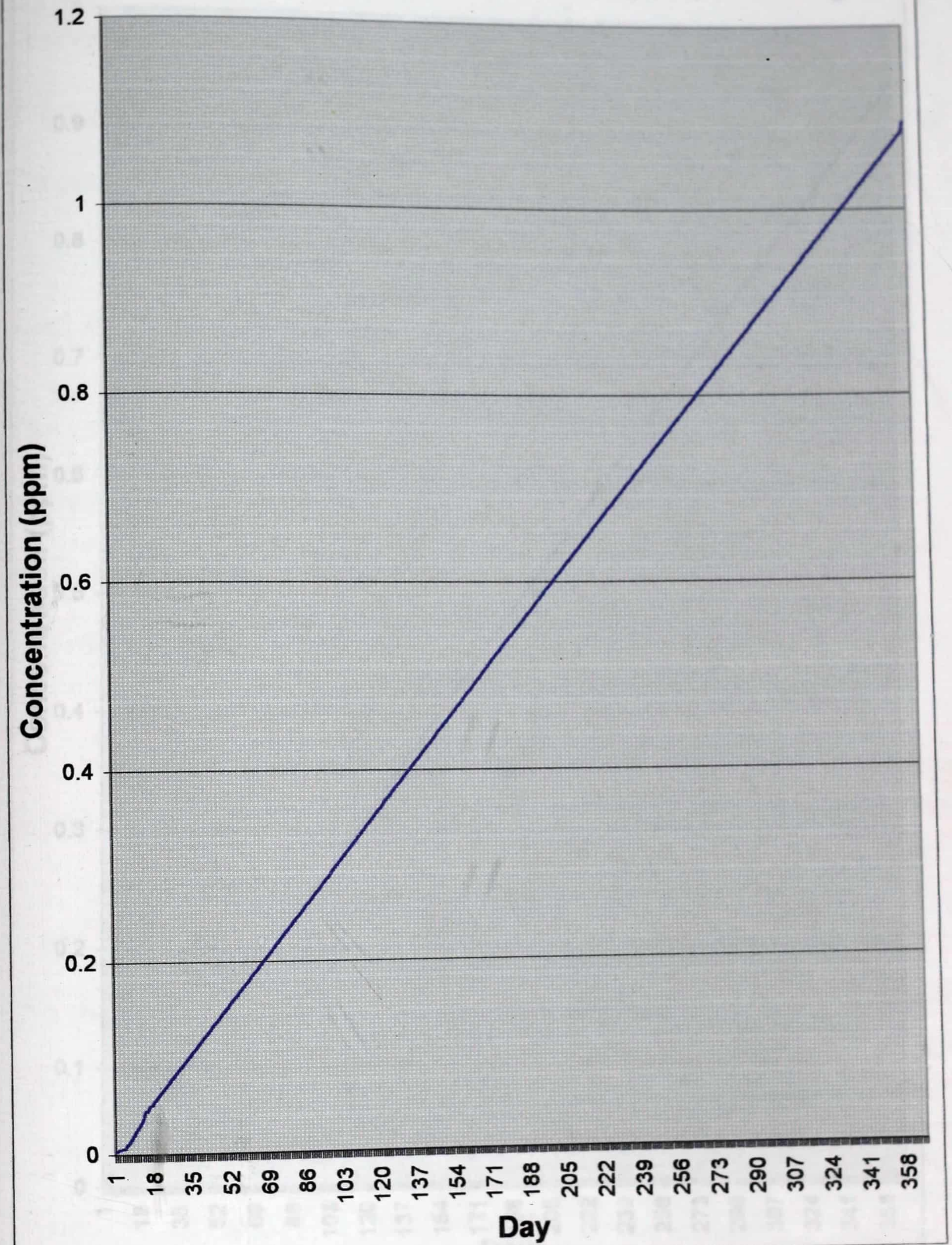
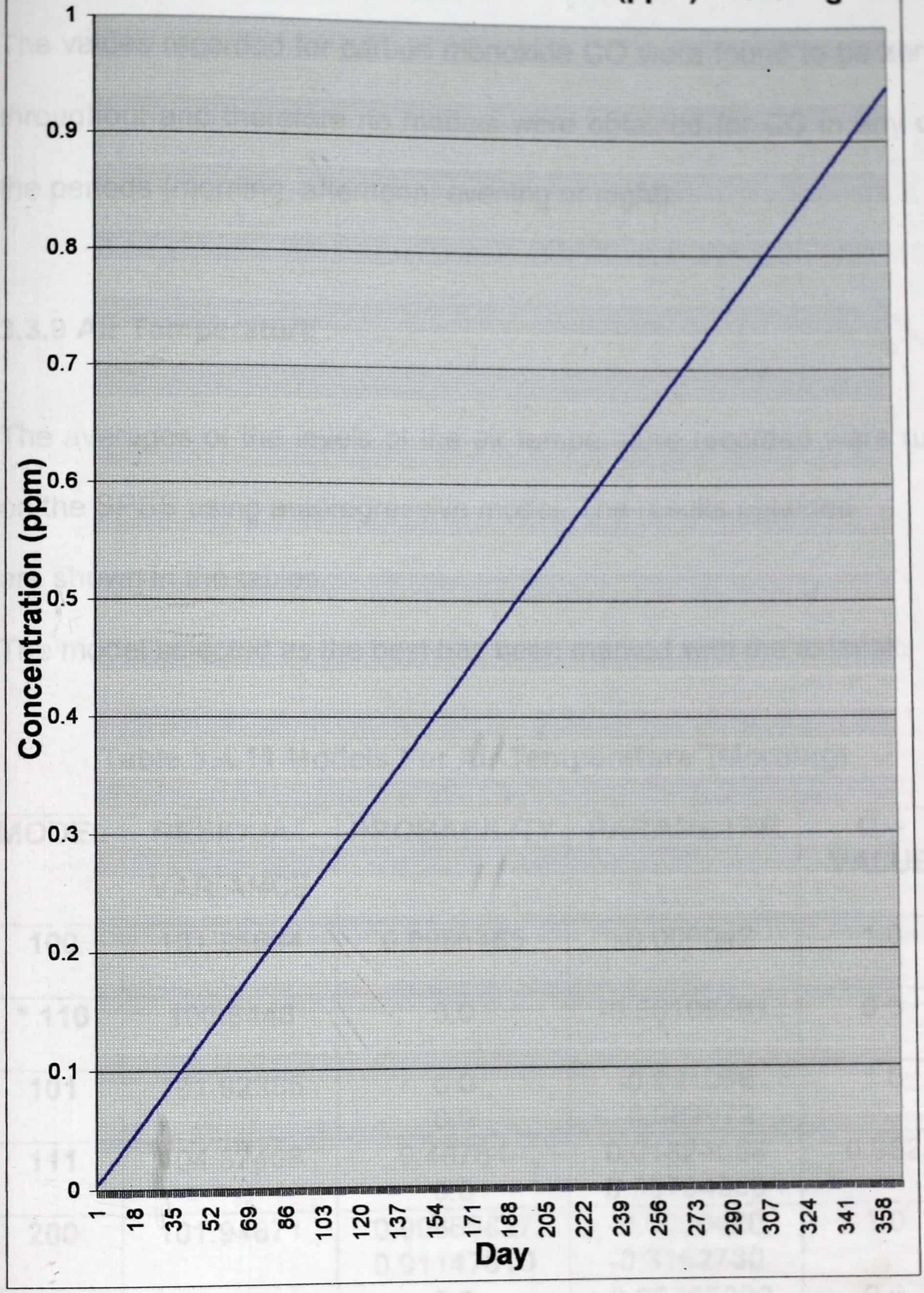


FIG. 9: A Graph of NO_2 Concentration (ppm) in the Night



3.3.8 CO Model

The values recorded for carbon monoxide CO were found to be zero throughout and therefore no models were obtained for CO in any of the periods (morning, afternoon, evening or night).

3.3.9 Air Temperature

The averages of the levels of the air temperature recorded were run on the SPSS using autoregressive model. The results obtained are shown in the tables.

The model selected as the best has been marked with the asterisk.

Table 3.4.11 Models For Air Temperature (Morning)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	101.88694	0.9986465	-0.000042	1.0
*** 110**	100.6446	0.0	-0.56109461	0.0
101	101.92335	0.0 0.0	-0.991066 -0.989072	1.0
111	104.82408	0.48784 0.0	0.01824058 0.95794353	0.982
200	101.94871	0.99865827 0.91147610	-0.0000420 -0.3162730	1.0
210	135.85822	0.0 0.0	-6.66767822 -0.33203819	0.0

Table 3.4.12 Models For Air temperature (Afternoon)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	25.760939	0.49003954	0.17098	1.00
* 110	27.863011	0.0	-050267806	0.0
101	25.767008	0.31095270 0.33633901	0.627752 0.607568	1.0
111	26.047932	0.41870892 0.0	0.02215145 0.95861546	0.990
200	25.763504	0.50001601 0.0	0.1670110 0.022644	1.0
210	33.95730	0.0	- 0.66468952 -0.6804622	0.0

Table 3.4.13 Models For Air temperature (Evening)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	3.4425017	0.0	0.413733	0.0
* 110	3.0461947	0.0 0.0	-.505198021	0.0
101	3.1854244	0.0	0.821430	0.0
111	3.422810	0.11696464 0.0	-0.6189627 0.58774709	0.0
200	3.1437980	0.0 0.0	0.291455 0.295449	0.0
210	3.6167450	0.0000000 0.0033531	0.546402 0.295449	0.0

Table 3.4.14 Models For Air temperature (Night)

MODEL	RESIDUAL VARIANCE	PROBABILITY	PARAMETER	Q – VALUE
100	101.88694	0.99864650	-0.0000042	0.858
*** 110**	101.64458	0.0	-0.50109461	0.0
101	101.91335	0.0 0.0	-0.991066 0.924615	1.0
111	171.08345	0.0 0.0	0.48812619 0.91654814	0.0
200	148.58466	0.0 0.0	0.383584 -0.193927	0.0
210	195.50461	0.0 0.0	-0.28505923 -0.35174732	0.0

3.4.0 OBSERVATIONS

The tables above indicate that the integrated Autoregressive model (AR(1,1)) is the best in each period of the day when the recorded concentrations of air temperature for morning, afternoon, evening and night were ran with the SPSS because it has the least probability of least error, residual variance and Q – statistics values.

The AR (1,1) model is defined as

$$W_k = \alpha w_{k-1} + \mu + \varepsilon_k \ldots\ldots \qquad (3.3.6)$$

where

α_1 = parameter to be determined

$W_k = X_k - X_{k-1}$

μ = Mean of air temperature

ε_k = error term or white noise.

When (3.3.1) is integrated the following Autoregressive model of the form

$$X_k = \alpha_1 X_{k-1} + \mu k + c + \varepsilon_k \qquad (3.3.2)$$

Is obtained

Where c and X_k are constant and the temperature of the air respectively.

The table 3.3.15 below shows the parameters of the AR(1,1) model estimated by the SPSS:.

Table 3.4.15 Parameters of Air Temperature

Period	α_1	μ
Morning	-0.561	0.8951
Afternoon	- 0.503	0.9132
Evening	- 0.505	0.9225
Night	-0.501	0.9111

3.4.1 Remark: In all cases c, the integration constant was found to be zero to two decimal places.

Therefore equation (3.3.2) becomes

$$X_k = -0.5261_{k-1} + (0.8951)k + \varepsilon_k \text{ for (morning)}$$

$$X_k = -0.503X_{k-1} + (0.9132)k + \varepsilon_k \text{ for (Afternoon)}$$

$$X_k = -0.505X_{k-1} + (0.9225)k + \varepsilon_k \text{ for (Evening)}$$

$$X_k = -0.501X_{k-1} + (0.9111)k + \varepsilon_k \text{ for (Night)}$$

3.4.2 DETERMINISTIC AIR TEMPERATURE DYNAMICS MODEL

The AR (1,1) model defined by the equation

$$X_k = \alpha_1 X_{k-1} + \mu k + \varepsilon_k \qquad (3.3.2)$$

is a stochastic model that leads to a deterministic dynamics difference equation since the error term, ε_k is zero. This deterministic dynamics difference equation is given by

$$X_k = \alpha_1 X_{k-1} + \mu k$$

$$X_k = -0.5261_{k-1} + (0.8951)k \text{ for (morning)}$$

$$X_k = -0.503X_{k-1} + (0.9132)k \text{ for (Afternoon)}$$

$$X_k = -0.505X_{k-1} + (0.9225)k \text{ for (Evening)}$$

$$X_k = -0.501X_{k-1} + (0.9111)k \text{ for (Night)}$$

These resulting deterministic dynamics difference equations are all first order difference equations with constant coefficients because the confidence level (probability) for these models were found to be zero to two decimal places. (See the table's 3.4.11, 3.4.12, 3.4.13 and 3.4.14).

Therefore the general solution to the deterministic equation for air temperature dynamics is given by

$$X_k = \alpha^k (X_0) + a + bk \qquad (3.3.5)$$

Where X_0 = initial air temperature during the various periods.

So we have the following:

$X_k = -0.561^k (X_0) + 0.2051 + 0.5734$ for morning

$X_k = -0.503^k (X_0) + 0.2033 + 0.6076k$ for afternoon

$X_k = -0.505^k (X_0) + 0.2057 + 0.6130k$ for evening

$X_k = -0.501^k (X_0) + 0.2032 + 0.6060k$ for night

Graphing these air temperature dynamics, we have:

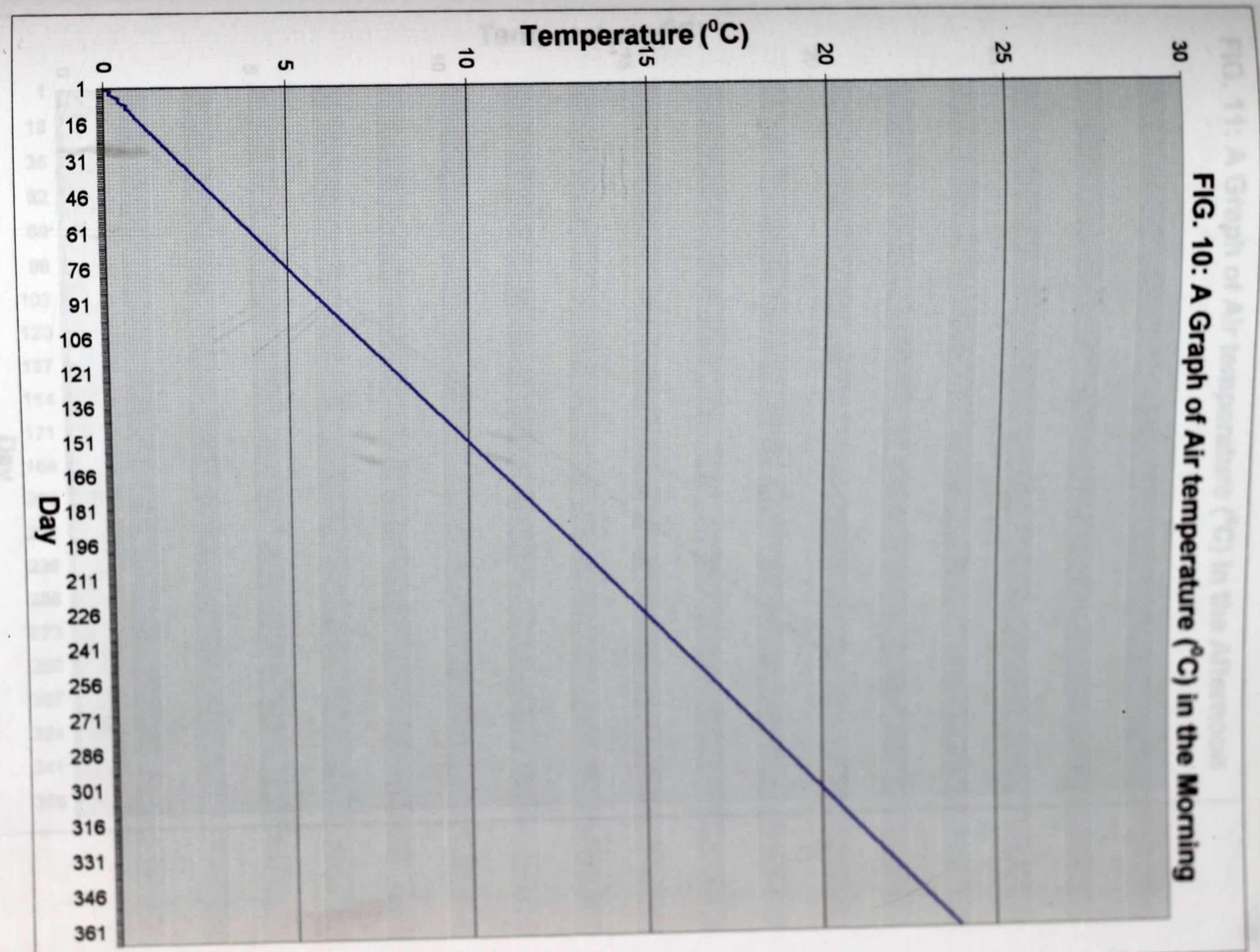


FIG. 10: A Graph of Air temperature (^{0}C) in the Morning

FIG. 11: A Graph of Air temperature (^{0}C) in the Afternoon

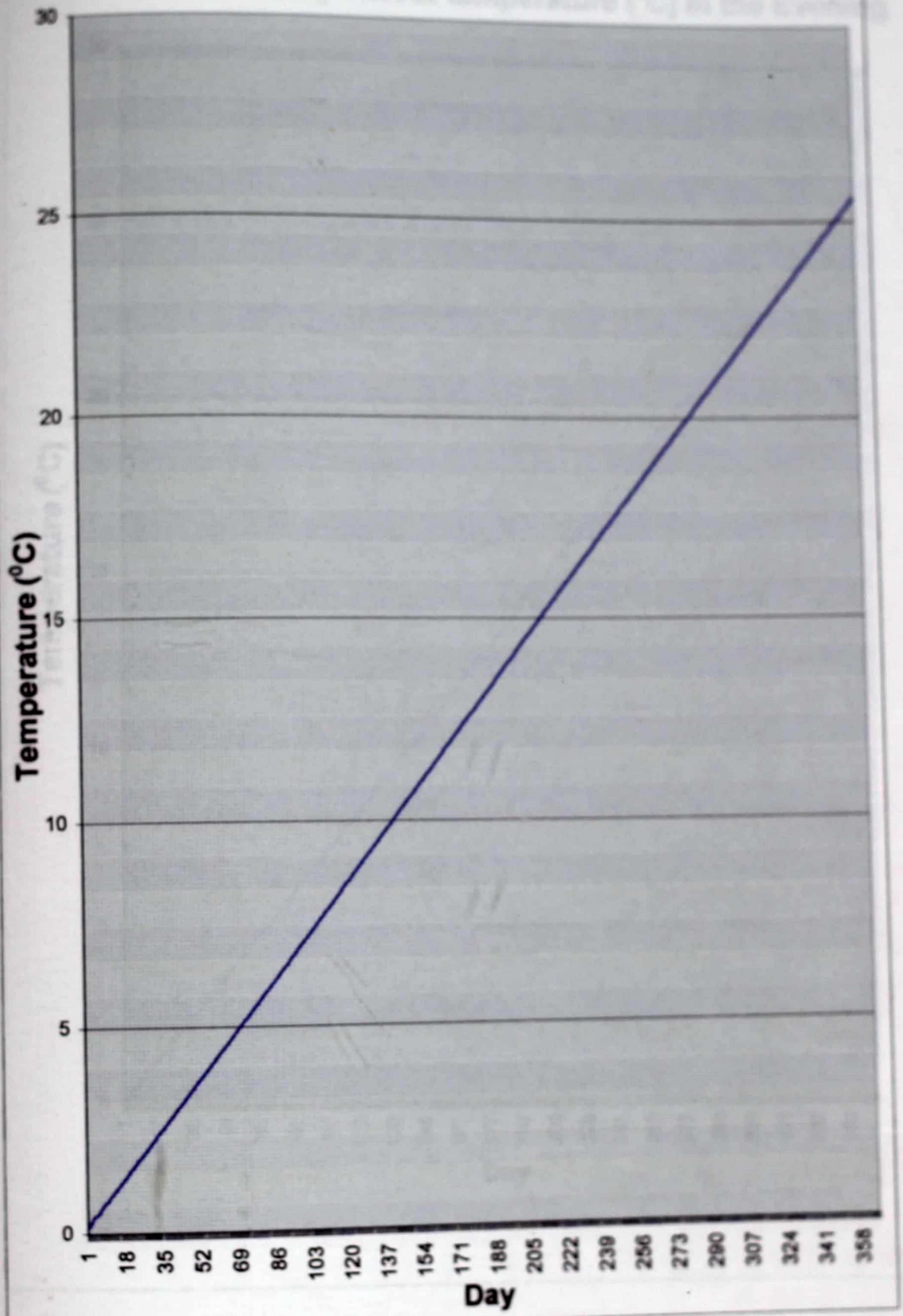


FIG. 12: A Graph of Air temperature (^{0}C) in the Evening

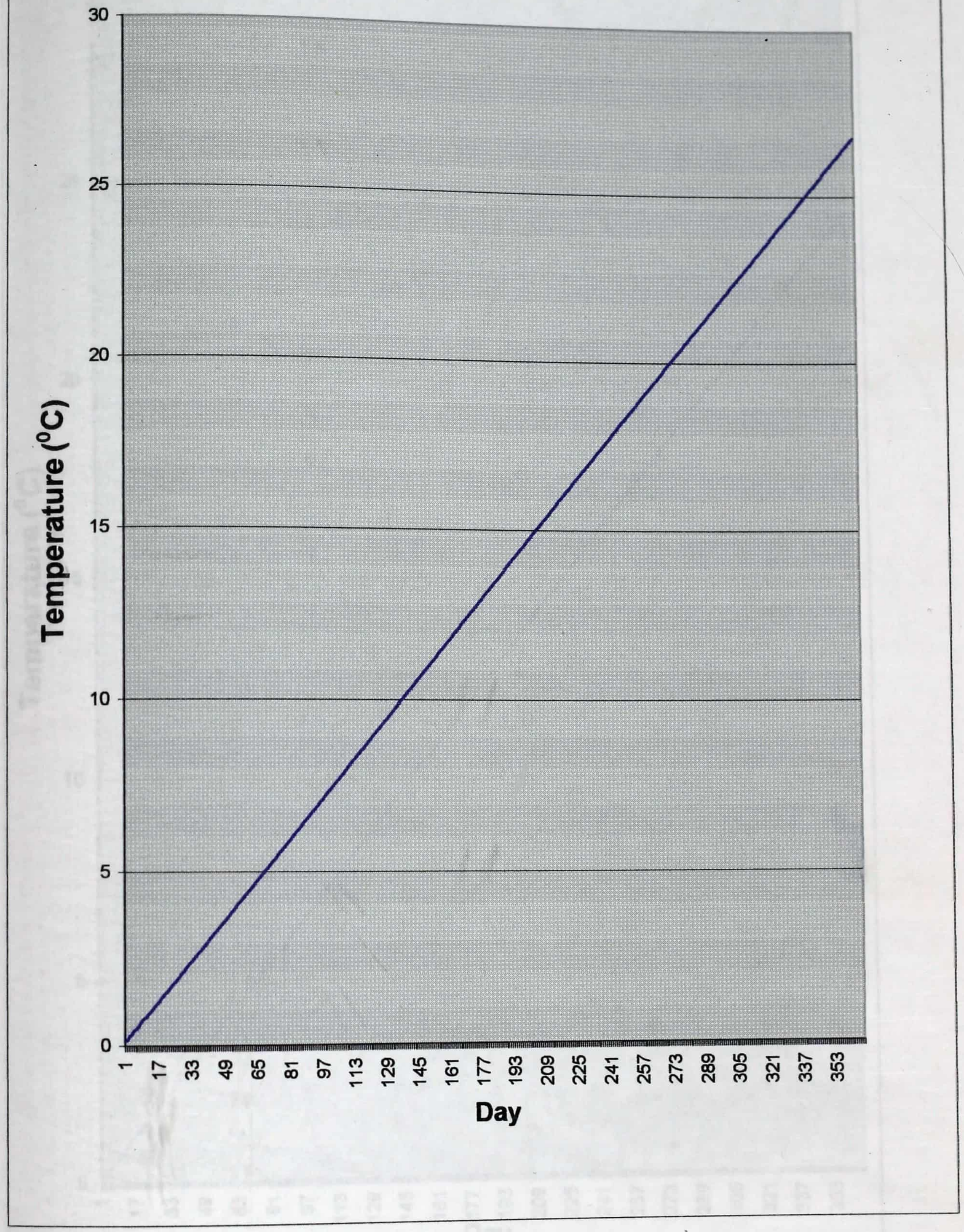
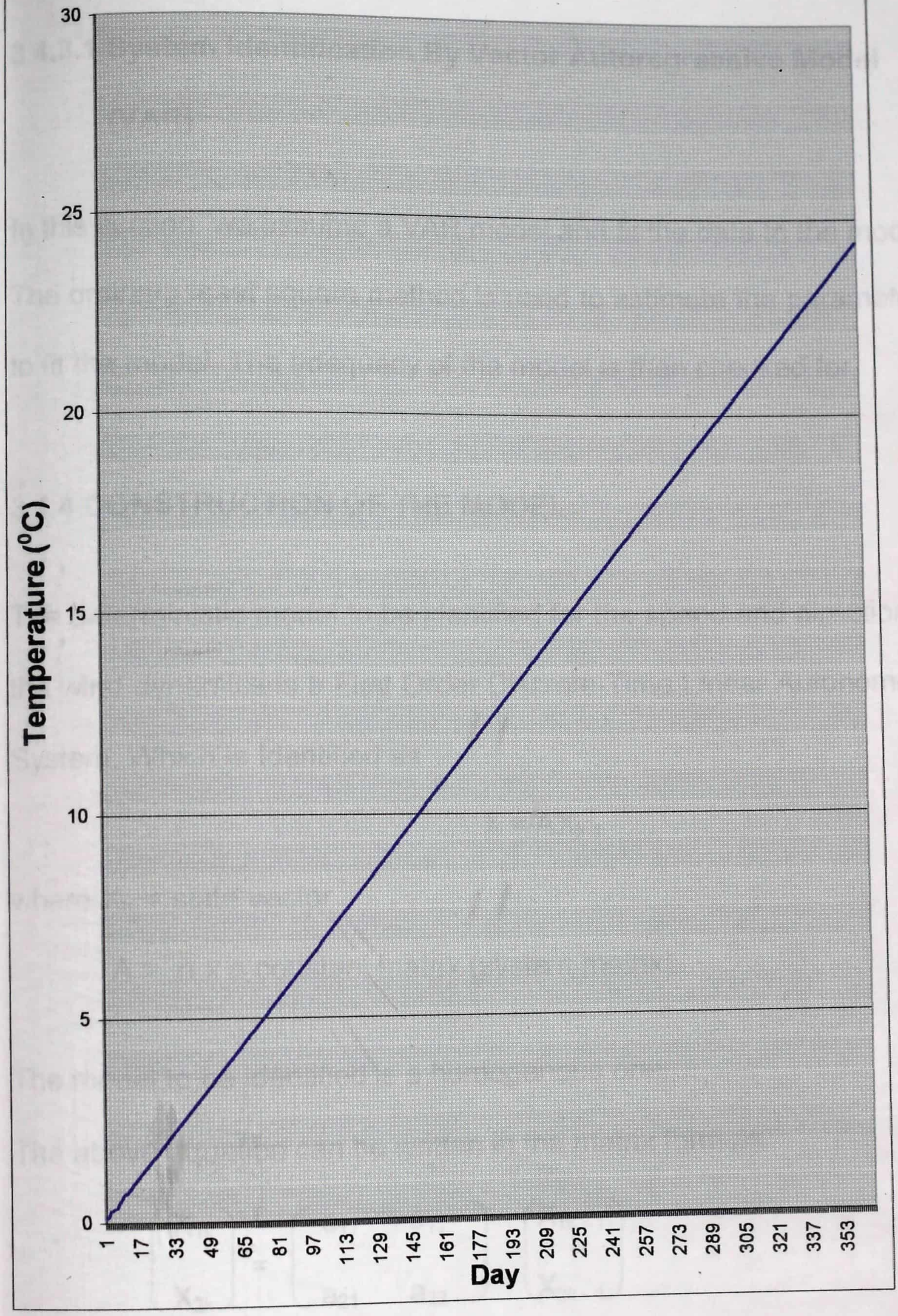


FIG. 13: A Graph of Air temperature (^{0}C) in the night



3.4.3 WIND MODEL [IN TERMS OF SPEED AND DIRECTION]

3.4.3.1 System Identification By Vector Autoregressive Model (VAR)

In this section, we assume a VAR model and fit the data to the model. The ordinary least square method is used to estimate the parameters to fit the model. The adequacy of the model is then checked for.

3.4.4 CONSTRUCTION OF THE MODEL

The deterministic model to be identified for the speed and direction of the wind dynamics is a First Order Discrete-Time Linear Autonomous System, Which Is Identified as

$$X = AX_{k-1}$$

where X_K = state vector

A = n x n constant matrix (system matrix).

The model to be identified is a homogenous one.

The above equation can be written in the matrix form as

$$\begin{pmatrix} X_{1k} \\ X_{2k} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} X_{1k-1} \\ X_{2k-1} \end{pmatrix}$$

In the form of equation, this can be written as

$$X_{1k} = a_{11} X_{1k-1} + a_{12} X_{2k-1}$$

$$X_{2k} = a_{21} X_{1k-1} + a_{22} X_{2k-1}$$

3.4.5 ESTIMATION OF PARAMETERS OF HOMOGENOUS SYSTEM

The parameters of the unforced system above are estimated using linear regression model.

The estimated parameters with their respective probabilities are shown in tables 3.4.5, 3.4.6, 3.4.7 and 3.4.8.

TABLE 3.4.16 Parameters of Wind Speed And Direction (Morning)

	Parameter	Probability
a_{11}	-0.0206*	0.0000
a_{12}	-0.0718	0.4561
a_{21}	-0.1814	0.7348
a_{22}	-0.2751*	0.0000

TABLE 3.3.11 AFTERNOON

	Parameter	Probability
a_{11}	-0.4769*	0.0000
a_{12}	-0.4421	0.0800
a_{21}	-0.2048	0.1527
a_{22}	-0.4149*	0.0000

TABLE 3.3.12 EVENING

	Parameter	Probability
a_{11}	-0.4817*	0.0000
a_{12}	-0.6164	0.2112
a_{21}	-0.2048	0.1482
a_{22}	-0.4986*	0.0000

TABLE 3. 3.13 NIGHT

	Parameter	Probability
a_{11}	-0.5053*	0.0000
a_{12}	-0.4316	0.0724
a_{21}	-0.2048	0.6971
a_{22}	-0.4986*	0 0000

The parameters with probability greater than $\alpha = 0.05$ are taken as zeros. From the above, with the exception of the parameters with asterisk, all the other parameters are zeros because they have probabilities greater than 0.05.

The matrices are:

The matrices are:

$$A_{MORNING} = \begin{pmatrix} -0.0206 & 0 \\ 0 & -0.2751 \end{pmatrix}$$

$$A_{AFTERNOON} = \begin{pmatrix} -0.4769 & 0 \\ 0 & -0.4149 \end{pmatrix}$$

$$A_{EVENING} = \begin{pmatrix} -0.5053 & 0 \\ 0 & -0.4984 \end{pmatrix}$$

$$A_{NIGHT} = \begin{pmatrix} -0.4769 & 0 \\ 0 & -0.4149 \end{pmatrix}$$

3.4.6 FEATURES OF CONCENTRATION DYNAMICS OF POLLUTANTS

- The level of air pollution in terms of SO_2 in the Atonsu area of Kumasi in the Ashanti Region is found to be growing linearly at mean growth rates of 0.0046, 0.100, 0.0114 and 0.0071 in the order of Morning, Afternoon, Evening and Night respectively.

- The level of air pollution in terms of and NO_2 in the Atonsu area of Kumasi in the Ashanti Region is found to be growing linearly at at mean growth rates of 0.0068, 0.0100, 0.0111 and -0.0075 for Morning, Afternoon, Evening and Night respectively.

3.4.7 STABILITY OF THE SYSTEM (HOMOGENEOUS)

The stability of the system depends on the eigenvalues of the matrix **A** above. The matrices are diagonal matrices it implies that the eigenvalues are the diagonals elements and the magnitudes of their values are less than one which is indicative of stability in terms of wind speed and direction.

$$X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1}$$

$$X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1}$$

3.4.8 INTERPRETATION OF RESULTS

3.4.8.1 SO_2

The results show that the average concentration of SO_2 in the morning is 0.13ppm this value increased to 0.28ppm in the afternoon and reached its peak in the evening at a value of 0.32ppm. It then dropped to 0.20ppm in the night.

The observed increasing order may be due to the order of increasing activities associated with emission of the gas. The major source of this gas emission are the industries such as the wood industries, power stations and domestic emissions such as burning plants, protein – base foods.

The drop in SO_2 concentration in the night is a reflection of the decline in these SO_2 emission activities in the night.

The peak hourly average concentrations in a 24-hour period are quite significant according to the ambient air quality guidelines by the EPA, Ghana, which indicates an average of 0.3375ppm as the maximum permissible SO_2 level for the industrial area.

The values recorded for these periods are 38.52%, 82.96%, 94.81% and 59.26% for morning, afternoon, evening and night respectively of the EPA, Ghana guidelines and are generally very significant.

Similarly they are significant based on the WHO standard of 0.1875ppm (a value described as health guideline) and the department of environment of USA standard of < 0.06ppm (described as a very good quality air and >0.399ppm as very poor air quality).

Moreover the growth rates of the SO_2 emission in the respective periods signals alarming situation.

Fig.2, 3, 4 and 5 clearly show the high rates of growth of SO_2 in the atmosphere during the morning, afternoon, evening and night respectively. There is therefore the existence of potential health hazards such as respiratory irritants, acid rains which in turn has a negative effect on the forest, aquatic habitats, corrosion of property, etc.

3.4.8.2 NO_2

The mean concentration of NO_2 recorded also increased from morning (0.19ppm) through the afternoon (0.28ppm) to its peak in the evening (0.31ppm). It also dropped to 0.21ppm in the night, a trend similar to that observed in the case of SO_2.

The maximum permissible NO_2 level for the industrial area set by EPA, Ghana, indicates an average of 0.15ppm. When compared to standards, the values recorded for these periods are 126.6667%, 186.6667%, 206.6667% and 140.0% (for morning, afternoon, evening and night respectively.

On the account of WHO NO_2 concentration should be < 0.0750ppm, a value described as health guideline, while the department of environment of USA has < 0.0360ppm and >0.2149ppm described as a very good quality air and very poor quality air respectively).

These observations indicate that values recorded in this research work are significant and are very alarming.

The growth rates of the NO_2 emission during these periods indicate an unbounded growth and for that matter capable of exerting its health and other negative effects if the necessary steps are not taken to minimize the growth rate.

The high growth of NO_2 in the atmosphere during the morning, afternoon, evening and night respectively are shown in figs. 6, 7, 8 and 9 respectively.

The observed high growth rate may be due to clustering of vehicular traffic, industries and power plants in the area.

The steps to minimize the growth rate of NO_2 are very Important because the gas contributes to the formation of low-level ozone (O_3). The effect is that it is poisonous even at lower concentrations and brings about respiratory irritants as well as acid rains.

3.4.8.3 CO

The levels of CO recorded were zero throughout. This may be due to its high instability and its subsequent oxidation to CO_2 at lower level before reaching the censor, which is cited at a height of about 20m.

The ferrous ion in haemoglobin has higher affinity for CO than O_2. Therefore haemoglobin will readily combine with any CO instead of O_2 to form a relatively more stable compound called carboxyhaemoglobin which is a respiratory poison.

3.4.8.4 AIR TEMPERATURE

The trend exhibited by the air temperature did not differ from those observed for the concentrations of SO_2 and NO_2 all rising from morning through the afternoon to the evening and dropping in the night. The means for air temperature were 25.09^0C, 25.05^0C, 25.83^0C and 25.51^0C in the order of morning, afternoon, evening and night. However the values remained fairly constant, fluctuating around 25^0C The growth rate, however, indicate the possibility higher values if the necessary steps are not taken.

The patterns of growth in the morning, afternoon, evening and night have been displayed graphically in figs.10, 11, 12 and 13 respectively.

3.4.8.5 EFFECT OF WIND SPEED AND DIRECTION ON THE MOVEMENT (DISTRIBUTION) OF THE GASES.

The system of equations 3.3.4 is simplified with all the estimated parameters and are dealt with separately to bring out their interpretations below.

3.4.8.5.1 morning

The first equation in the system of difference equation

$X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1}$ with $a_{11} = -0.0206$ and $a_{12} = 0$ becomes

$X_{1k} = -0.0206\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.0206\ X_{1k-1}$ (Easterly Direction),

And from $X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1}$

with $a_{21} = -0.2751$ and $a_{22} = 0$

we have $X_{1k} = -0.2751\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.2751\ X_{1k-1}$ (Northerly Direction).

In these equations the left hand side X_{1k} and X_{2k} are the levels of the SO_2 and NO_2 in the air in the kth period (morning).

It can be seen that the spread of the gases in the easterly direction is not affected significantly by the speed and direction of the wind.

However it is affected by speed and direction of the wind northerly.

3.4.8.5.2 Afternoon

The system of difference equation

$X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1}$ with a_{11} =-0.4769 and a_{12} = 0 becomes

$X_{1k} = -0.4769\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.4769\ X_{1k-1}$ (Easterly direction),

And from $X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1}$

with a_{21} = -0.4149 and a_{22} = 0

we have $X_{1k} = -0.4149\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.4149\ X_{1k-1}$ (Northerly Direction).

In these equations the left hand side X_{1k} and X_{2k} are the levels of the pollutants SO_2 and NO_2 in the air in the kth period (afternoon).

It is observed that the speed and direction of the wind affect the spread of the gases in both the easterly and northerly directions significantly.

3.4.8.5.3 Evening

The system of difference equation

$X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1}$ with a_{11} =-0.4877 and a_{12} = 0

becomes

$X_{1k} = -0.4877\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.4877\ X_{1k-1}$ (Easterly direction),

And form $X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1}$

with a_{21} = -0.4986 and a_{22} = 0

we have $X_{1k} = -0.4986\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.4986\ X_{1k-1}$ (Northerly Direction).

In these equations the left hand side X_{1k} and X_{2k} are the levels of the pollutants SO_2 and NO_2 in the air in the kth period (afternoon).

This indicates that the effect of the wind speed and direction is quite significant on the spread of the gases in both the easterly and northerly directions.

3.4.8.5.4 Night

The system of difference equation

$X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1}$ with $a_{11} = -0.5053$ and $a_{12} = 0$

becomes

$X_{1k} = -0.5053\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.5053\ X_{1k-1}$ (Easterly Direction),

And form $X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1}$

with $a_{21} = -0.4984$ and $a_{22} = 0$

we have $X_{1k} = -0.4984\ X_{1k-1} + 0X_{2k-1}$

So $X_{1k} = -0.4984\ X_{1k-1}$ (Northerly Direction).

In these equations the left hand side X_{1k} and X_{2k} are the levels of the pollutants SO_2 and NO_2 in the air in the kth period (afternoon).

This equation shows that the effect of the wind speed and direction on the spread of the gases in both the easterly and northerly directions is quite significant.

The direction and speed of the wind have been shown generally to affect the movement of the gases in both easterly and northerly directions during the afternoon, evening and night.

That is, there is no preferred wind direction except for the morning where the dominant direction is northerly. This observation compares favourably with [6] who noted that skies are generally clear in the night, allowing rapid radiant heat loss.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 CONCLUSION

From the study and results obtained, it can be concluded that:

- The level of pollution in terms of SO_2 is, at the moment, very alarming. Moreover, the growth rate indicates an possible increase in future.

- NO_2 concentration during the study period is well beyond acceptable limit. In addition to this, the growth rate suggests a further threatening situation.

- CO concentration during the study period is not sensored by virtue of the height at which the EMS sensor device is installed. Thus the CO is oxidised to CO_2 at the level of emission, which is generally close to the ground.

- Air temperature is fairly constant on the average.

- There is no preferred wind direction except for the morning where the dominant direction is northerly.

4.2 RECOMMENDATIONS

The tendency for the pollutants SO_2 and NO_2 as well as the air temperature to increase is very high. Therefore all activities that enhance this should be controlled well in order to avert a health-threatening situation.

This may be achieved by:

- Relocating some of the factories or industries to ensure that the gases liberated by these industries into the atmosphere are dispersed and hence prevent concentration of these gases at a given area. This measure will in turn prevent their harmful effect on human beings, animals and property.

- Over - aged vehicles as well as unmaintained vehicles should not be declared roadworthy and therefore should not be allowed to ply the roads. Also the use of vehicle engines which have catalytic converters, should be encouraged.

- Bypasses should be built around the industrial area to prevent the concentration of vehicles in the industrial area.

- Social amenities such as schools, hospitals, markets, entertainment centers, etc should be provided around the industrial area to prevent people oscillating between the industrial area and the central business district, which is in turn linked with heavy vehicular traffic and its associated pollution.

- Chimneys should be built high enough to ensure that the pollutants emitted are dispersed widely and well diluted. This will ensure that the ground level concentrations are not harmful to human and animal health and also to property.

- The sensor device, EMS should be installed at a lower the height to enable it capture the levels or concentrations of CO to enable the state of CO to be known and hence the necessary precaution to be taken.

- The sensor device should be adjusted to record values to three decimal places and beyond. This will ensure that the truncation error is further reduced and that more sensitive effects (e. g CO pollution) are captured.

- Law enforcement agencies should enforce environmental laws to ensure that all stakeholders do their best to conserve the environment.

- Education of the public on the need to conserve the environment should be intensified.

- Further studies on the control theory of these pollutants should be carried out.

REFERENCES

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APPENDICES

APPENDIX 1: DAILY AVERAGE SO_2 CONCENTRATION (ppm)

APPENDIX 1A: MORNING

0.0	0.1	0.1	0.2	0.1	0.1	0.0	0.1	0.1
0.0	0.2	0.1	0.1	0.1	0.2	0.1	0.1	0.2
0.1	0.1	0.2	0.2	0.2	0.1	0.2	0.0	0.2
0.1	0.2	0.1	0.1	0.1	0.2	0.1	0.1	0.2
0.1	0.2	0.1	0.0	0.2	0.1	0.1	0.1	0.1
0.1	0.1	0.2	0.2	0.1	0.2	0.1	0.2	0.1
0.1	0.0	0.1	0.1	0.2	0.1	0.2	0.1	0.2
0.1	0.2	0.2	0.2	0.2	0.1	0.2	0.1	0.2
0.1	0.1	0.2	0.1	0.1	0.2	0.1	0.1	0.1
0.2	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.1
0.2	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.1
0.2	0.2	0.2	0.1	0.2	0.1	0.2	0.1	0.2
0.1	0.1	0.2	0.2	0.2	0.0	0.1	0.2	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.1	0.1
0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.2	0.2
0.0	0.2	0.0	0.1	0.1	0.1	0.1	0.2	0.1
0.2	0.1	0.1	0.1	0.2	0.1	0.0	0.1	0.2
0.1	0.2	0.1	0.1	.0.0	0.0	0.1	0.1	0.1
0.2	0.1	0.2	0.1	0.1	0.0	0.2	0.1	0.1
0.1	0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.1
0.1	0.2	0.1	0.1	0.2	0.1	0.2	0.2	0.2
0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.1	0.2
0.1	0.2	0.2	0.1	0.1	0.2	0.1	0.2	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.1
0.1	0.2	0.1	0.1	0.2	0.1	0.1	0.1	0.1
0.0	0.1	0.2	0.1	0.1	0.0	0.2	0.2	0.0
0.1	0.2	0.1	0.1	0.0	0.1	0.1	0.2	0.1
0.1	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1
0.1	0.1	0.2	0.1	0.1	0.0	0.2	0.2	0.2
0.1	0.0	0.2	0.2	0.1	0.2	0.1	0.1	0.1

0.0	0.1	0.1	0.1	0.2	0.2	0.1	0.0	0.2
0.0	0.2	0.1	0.1	0.2	0.2	0.2	0.2	0.1
0.2	0.1	0.1	0.1	0.1	0.0	0.1	0.1	0.2
0.1	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2
0.2	0.2	0.1	0.2	0.1	0.1	0.2	0.1	0.1
0.2	0.1	0.2	0.1	0.2	0.2	0.1	0.1	0.2
0.1	0.1	0.2	0.1	0.2	0.1	0.1	0.2	0.1
0.2	0.1	0.0	0.1	0.1	0.2	0.2	0.2	0.2
0.1	0.2	0.1	0.1	0.2	0.2	0.2	0.1	0.2
0.1	0.2	0.1	0.2	0.0	0.1	0.2	0.2	
0.0	0.2	0.2	0.1	0.1	0.1	0.2	0.2	

APPENDIX 1B: AFTERNOON

0.2	0.3	0.3	0.3	0.2	0.3	0.2	0.3	0.2
0.2	0.3	0.3	0.2	0.3	0.3	0.3	0.2	0.3
0.4	0.3	0.3	0.2	0.3	0.3	0.2	0.2	0.3
0.2	0.3	0.3	0.4	0.3	0.3	0.2	0.2	0.2
0.3	0.3	0.2	0.2	0.3	0.3	0.2	0.2	0.2
0.3	0.2	0.3	0.3	0.3	0.2	0.2	0.3	0.2
0.3	0.2	0.3	0.3	0.2	0.3	0.3	0.3	0.2
0.2	0.2	0.2	0.3	0.2	0.3	0.2	0.3	0.2
0.2	0.3	0.3	0.2	0.2	0.2	0.3	0.2	0.2
0.2	0.2	0.3	0.2	0.3	0.3	0.3	0.2	0.4
0.3	0.2	0. 3	0.2	0.2	0.3	0.2	0.1	0.2
0.2	0.3	0.2	0.3	0.2	0. 3	0.2	0.3	0.3
0.2	0.3	0.2	0.2	0.3	0.2	0.3	0.2	0.3
0.3	0.2	0.2	0.2	0.3	0.2	0.1	0.2	0.3
0.3	0.2	0.3	0.3	0.2	0.2	0.2	0.3	0.3
0.4	0.2	0.3	0.3	0.2	0.3	0.3	0.2	0.2
0.2	0.2	0.3	0.4	0.2	0.3	0.3	0.1	0.3
0.3	0.3	0.2	0.2	0.2	0.3	0.2	0.2	0.4
0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.2	0.2
0.2	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.3
0.2	0.2	0.3	0.2	0.3	0.3	0.3	0.2	0.2

0.3	0. 3	0.2	0.2	0.2	0.3	0.3	0.2	0.3
0.2	0.2	0.3	0.3	0. 3	0.2	0.3	0.2	0.3
0.2	0.2	0.3	0.2	0.2	0.3	0.2	0.2	0.3
0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.3	03
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3
0.3	0.4	0.2	0.2	0.2	0.2	0.2	0.3	0.3
0.3	0.2	0.3	0.3	0.4	0.3	0.1	0.2	0.3
0.2	0.3	0.2	0.3	0.2	0.2	0.2	0.3	0.2
0.2	0.3	0.2	0.2	0.3	03	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.3
0.3	0.2	0.2	0.2	0.2	0.3	0.2	0.2	0. 3
0.3	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.3	0.4	0.2	0.2	0.2	0.2
0.2	0.3	0.2	0.2	0.2	0.2	03	0.3	0.3
0.2	0.2	0.3	0.2	0.3	0.3	0.3	0.2	0.2
0.2	0.3	0.3	0.2	0.2	0.2	0.3	0.2	0.2
0.3	0.2	0.3	0.2	0.2	0.3	0.2	0.3	
0.3	0.2	0.2	0.3	0.2	0.2	0.2	0.2	
0.3	0.3	0.3	0.2	0.3	0.3	0.2	0.3	
0.2	0.3	0.3	0.3	0.3	0.2	0.3	0.2	

APPENDIX 1C: EVENING

0.3	0.3	0.4	0.4	0.3	0.3	0.2	0.3	0.3
0.2	0.3	0.3	0.3	0.2	0.4	0.3	0.2	0.4
0.4	0.4	0.2	0.3	0.4	0.3	0.2	0.1	0.2
0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.2	0.3
0.3	0.3	0.2	0.2	0.2	0.3	0.3	0.2	0.3
0.3	0.3	0.3	0.4	0.4	0.4	0.2	0.4	0.3
0.4	0.3	0.3	0.3	0.2	0.4	0.2	0.2	0.4
0.3	0.4	0. 4	0.3	0.3	0.2	0.2	0.3	0.3
0.5	0.2	0.2	0.2	0.3	0.3	0.2	0.3	03
0.3	0.4	0.3	0.4	0.4	0.3	0.2	0.2	0.3
0.4	0.2	0.4	0.3	0.2	0.4	0.3	0.4	0.3
0.3	0.2	0.4	0.3	0.3	0.3	0.2	0.3	0.2
0.2	0.3	0.2	0.2	0.3	0.4	0.2	0.3	0.4

0.3	0.2	0.3	0.3	0.2	0.3	0.3	0.2	0.2
0.4	0.2	0.3	0.2	0.3	0.3	0.2	0.3	0.3
0.4	0.3	0.4	0.3	0.4	0.4	0.2	0.2	0. 3
0.2	0.3	0.2	0.2	0.3	0.4	0.3	0.2	0.2
0.3	0.3	0.3	0.3	0.4	0.3	0.2	0.3	0.2
0.2	0.4	0.3	0.3	0.2	0.3	0.2	0.4	0.3
0.2	0.3	0.4	0.3	0.3	0.4	0.2	0.3	0.3
0.2	0. 3	0.4	0.3	0.3	0.3	0.3	0.3	0.4
0.2	0.3	0.3	0.3	0.4	0.4	0.2	0.2	0.4
0.3	0.4	0.3	0.4	0.4	0.3	03	0.4	0.2
0.3	0.2	0.4	0.3	0.4	0.3	0.3	0.3	0.1
0.4	0.3	0.4	0.3	0.3	0.3	0.3	0.4	0.4
0.3	0.3	0.4	0.4	0.3	0.3	0.4	0.2	0.3
0.3	0.4	0.2	0.3	0.4	0.4	0.2	0.3	0.2
0.3	0.2	0.3	0.2	0.3	0.3	0.3	0.3	0.2
0.3	0.3	0.3	0.3	0.4	0.3	0.4	0.2	0.2
0.4	0.3	0.4	0.3	0.3	0.4	0.3	0.2	0.3
0.3	0.4	0.2	0.4	0.3	0.3	0.2	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.2	0.3	0.3	0.4
0.3	0.2	0.4	0.2	0.4	0.2	0.4	0.2	0.4
0.3	0.3	0.2	0.3	0.3	0.3	0.4	0.2	0.2
0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.3	0.4
0.4	0.4	0.3	0.3	0.4	0.2	0.1	0.3	0.2
0.3	0.2	0.4	0.3	0.3	0.4	0.3	0.4	0.4
0.4	0.3	0.2	0.3	0.3	0.3	0.2	0.1	0.3
0.3	0.3	0.3	0.2	0.3	0.4	0.3	03	
0.4	0.4	0.2	0.4	0.4	0. 3	0.2	0.3	
0.3	0.3	0.2	0.3	0.3	0.3	0.2	0.2	

APPENDIX 1D: NIGHT

0.2	0.1	0.2	0.3	0.2	0.1	0.3	0.2	0.3
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.1	0.2
0.1	0.1	0.3	0.4	0.3	0.2	0.3	0.2	0.2
0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.3	0.1	0.2	0.2	0.1	0.3	0.2	0.3	0.2

0.1	0.1	0.1	0.3	0.2	0.3	0.3	0.2	0.3
0.2	0.1	0.3	0.2	0.2	0.3	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.1	0.2	0.1	0.3	0.2
0.2	0.1	0.2	0.2	0.3	0.1	0.2	0.2	0.2
0.1	0.2	0.3	0.2	0.3	0.2	0.3	0.4	0.2
0.1	0.1	0.2	0.1	0.3	0.2	0.2	0.2	0.1
0.2	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.1	0.2	0.2	0.1	0.2	0.1	0.2	0.3	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.1	0.2
0.2	0.1	0.2	0.2	0.3	0.4	0.2	0.2	0.3
0.1	0.2	0.2	0.2	0.2	0.4	0.1	0.1	0.3
0.2	0.1	0.1	0.2	0.2	0.1	0.2	0.2	0.2
0.1	0.2	0.1	0.2	0.3	0.2	0.2	0.2	0.2
0.2	0.1	0.4	0.1	0.2	0.1	0.2	0.1	0.2
0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.1	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.1	0.2	0.1	0.2	0.2	0.2	0.3	0.2
0.1	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2
0.2	0.1	0.1	0.3	0.2	0.1	0.1	0.1	0.3
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.1	0.2	0.3	0.3	0.2	0.2	0.2	0.2	0.1
0.2	0.2	0.2	0.1	0.1	0.3	0.3	0.1	0.3
0.1	0.2	0.3	0.2	0.2	0.2	0.1	0.2	0.2
0.2	0.2	0.1	0.2	0.2	0.1	0.2	0.2	0.1
0.1	0.3	0.2	0.1	0.3	0.2	0.2	0.2	0.2
0.2	0.2	0.4	0.2	0.2	0.2	0.1	0.1	0.2
0.1	0.1	0.2	0.1	0.3	0.2	0.2	0.2	0.1
0.2	0.2	0.2	0.2	0.1	0.2	0.3	0.2	0.2
0.1	0.1	0.1	0.1	0.2	0.1	0.2	0.3	0.2
0.2	0.1	0.2	0.3	0.2	0.2	0.1	0.2	0.4
01	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.3	0.1	0.2	0.1	0.2	0.2	0.1
0.3	0.1	0.2	0.2	0.1	0.2	0.1	0.3	0.3
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
0.2	0.2	0.2	0.2	0.2	0.1	0.3	0.1	
0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
0.2	0.3	0.2	0.2	0.2				

APPENDIX 2: DAILY AVERAGE NO_2 CONCENTRATION (ppm)

APPENDIX 2A: MORNING

0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.1
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.2
0.2	0.2	0.1	0.1	0.3	0.2	0.3	0.2	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.1	0.3	0.2	0.2	0.2	0.2	0.1
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2
0.3	0.2	0.2	0.2	0.2	0.3	0.2	0.2	0.2
0.2	0.2	0.1	0.2	0.3	0.2	0.3	0.1	0.2
0.2	0.2	0.2	0.3	0.3	0.2	0.1	0.2	0.3
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.1	0.3	0.2	0.2	0.2	0.1	0.2	0.2	0.2
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.3	0.2
0.2	0.2	0.2	0.1	0.1	0.2	0.2	0.2	0.1
0.2	0.2	0.2	0.2	0.3	0.2	0.2	0.2	0.3
0.2	0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.2
0.1	0.1	0.1	0.2	0.3	0.3	0.2	0.2	0.2
0.2	0.1	0.1	0.2	0.3	0.2	0.2	0.2	0.3
0.1	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.1	0.2	0.2	0.1	0 2	0.2	0.2
0.1	0.2	0.1	0.2	0.2	0.2	0.2	0.1	0.2
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.1	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.1
0.2	0.1	0.2	0.3	0.2	0.1	0.2	0.2	0.0
0.2	0.2	0.2	0.2	0.2	0.3	0.2	0.2	0.1
0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.2	0.2
0.2	0.2	0.2	0.2	0.3	0.2	0.1	0.2	0.3
0.2	0.2	0.3	0.2	0.2	0.1	0.1	0.2	0.2
0.1	0.2	0.2	0.2	0.1	0.1	0.2	0.2	0.2
0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.1
0.1	0.2	0.1	0.1	0.2	0.2	0.2	0.2	0.1

0.2	0.1	0.2	0.1	0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.3	0.2	0.2	0.3	0.2
0.1	0.2	0.1	0.2	0.2	0.0	0.2	0.2	0.2
0.2	0.1	0.2	0.2	0.1	0.2	0.2	0.1	0.2
0.2	0.1	0.2	0.1	0.1	0.2	0.2	0.1	
0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	
0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.3	

APPENDIX 2B: AFTERNOON

0.3	0.1	0.2	0.2	0.3	0.2	0.3	0.3	0.3
0.3	0.2	0.2	0.2	0.3	0.2	0.1	0.2	0.1
0.3	0.3	0.1	0.3	0.1	0.2	0.2	0.1	0.2
0.3	0.1	0.1	0.3	0.3	0.2	0.3	0.2	0.3
0.3	0.3	0.3	0.2	0.2	0.1	0.3	0.2	0.2
0.2	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.3
0.2	0.3	0. 3	0.3	0.2	0.3	0.2	0.2	0.3
0.1	0.3	0.2	0.3	0.3	0.2	0.2	0.2	0.3
0.3	0.2	0.1	0.2	0.3	0.3	0.2	0.2	03
0.3	0.4	0.2	0.2	0.2	0.3	0.2	0.2	0.3
0.3	0.2	0.1	0.2	0.1	0.2	0.3	0.3	0.3
0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.3	0.3
0.2	0.3	0.2	0.2	0.2	0.2	0.2	0.3	0.2
0.3	0.2	0.1	0.3	0.2	0.1	0.2	0.2	0.2
0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3
0.2	0.3	0.3	0.3	0.1	0.2	0.1	0.2	0. 3
0.2	0.1	0.1	0.2	0.3	0.2	0.2	0.2	0.2
0.2	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2
0.2	0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.3
0.1	0.3	0.2	0.3	0.2	0.3	0.2	0.2	0.2
0.2	0.2	0.1	0.3	0.3	0.3	0.2	0.3	0.2
0.2	0.1	0.2	0.2	0.1	0.3	0.2	0.2	0.1
0.2	0.2	0.2	0.3	0.3	0.2	03	0.2	0.2
0.1	0.1	0.3	0.2	0.2	0.2	0.3	0.3	0.1
0.1	0.2	0.2	0.3	0.2	0.2	0.3	0.2	0.0
0.1	0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.3

0.3	0.2	0.2	0.3	0.3	0.2	0.2	0.3	0.2
0.3	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.2
0.2	0.2	0.3	0.2	0.3	0.2	0.1	0.2	0.3
0.2	0.2	0.2	0.3	0.2	0.2	0.3	0.2	0.2
0.2	0.1	0.2	0.2	0.2	0.3	0.3	0:2	0.3
0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.2	0.3
0.3	0.3	0.3	0.2	0.2	0.2	03	0.2	0.2
0.3	0.2	0.3	0.3	0.3	0.1	0.1	0.2	0.3
0.3	0.1	0.3	0.2	0.2	0.2	0.2	0.2	0.1
0.2	0.2	0.3	0.3	0.3	0.3	0.2	0.3	0.2
0.2	0.2	0.1	0.2	0.3	0.2	0.2	0.3	0.3
0.2	0.1	0.2	0.3	0.2	0.3	0.1	0.1	0.2
0.2	0.2	0.3	0.2	03	0.2	0.3	03	
0.1	0.2	0.3	0.2	0.3	0.2	0.2	0.3	
0.3	0.2	0.2	0.2	0.3	0.2	0.2	0.2	

APPENDIX 2C: EVENING

0.4	0.3	0.4	0.4	0.3	0.2	0.2	0.3	0.3
0.3	0.2	0.3	0.3	0.2	0.2	0.3	0.2	0.4
0.4	0.4	0.2	0.3	0.4	0.2	0.2	0.1	0.2
0.4	0.4	0.3	0.3	0.3	0.2	0.3	0.2	0.3
0.3	0.3	0.2	0.2	0.2	0.1	0.3	0.2	0.3
0.2	0.3	0.3	0.4	0.3	0.2	0.2	0.4	0.3
0.4	0.3	0.3	0.3	0.2	0.3	0.2	0.2	0.4
0.3	0.4	0. 4	0.3	0.3	0.2	0.2	0.3	0.3
0.3	0.2	0.2	0.2	0.3	0.3	0.2	0.3	03
0.3	0.4	0.3	0.4	0.2	0.3	0.2	0.2	0.3
0.3	0.4	0.3	0.4	0.2	0.3	0.2	0.2	0.3
0.3	0.2	0.4	0.3	0.3	0.2	0.3	0.4	0.3
0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.3	0.2
0.2	0.3	0.2	0.2	0.2	0.4	0.2	0.3	0.4
0.3	0.2	0.2	0.3	0.2	0.3	0.3	0.2	0.2
0.4	0.2	0.3	0.2	0.3	0.2	0.2	0.3	0.3
0.4	0.4	0.3	0.3	0.4	0.2	0.2	0.2	0.3
0.2	0.2	0.2	0.2	0.3	0.4	0.3	0.2	0.2
0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.3	0.2
0.2	0.4	0.3	0.3	0.2	0.2	0.2	0.4	0.3

0.2	0.3	0.2	0.3	0.2	0.4	0.2	0.3	0.3
0.2	0. 3	0.1	0.3	0.3	0.3	0.3	0.3	0.4
0.2	0.2	0.2	0.3	0.4	0.2	0.2	0.2	0.4
0.2	0.4	0.3	0.4	0.3	0.3	03	0.4	0.2
0.3	0.2	0.3	0.3	0.3	0.2	0.3	0.3	0.1
0.4	0.3	0.4	0.3	0.2	0.3	0.3	0.4	0.4
0.4	0.3	0.2	0.2	0.3	0.3	0.4	0.2	0.3
0.3	0.2	0.2	0.3	0.2	0.4	0.2	0.3	0.2
0.3	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.2
0.2	0.2	0.3	0.2	0.4	0.3	0.4	0.2	0.2
0.2	0.2	0.2	0.3	0.3	0.4	0.3	0.2	0.3
0.2	0.2	0.2	0.4	0.3	0.3	0.2	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.2	0.3	0.3	0.4
0.3	0.2	0.4	0.2	0.4	0.2	0.4	0.2	0.2
0.3	0.3	0.2	0.3	0.3	0.3	0.4	0.2	0.3
0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.3	0.2
0.4	0.2	0.3	0.3	0.2	0.2	0.1	0.3	0.2
0.3	0.2	0.4	0.3	0.2	0.4	0.3	0.4	0.2
0.2	0.2	0.2	0.3	0.2	0.3	0.2	0.1	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	
0.2	0.3	0.3	0.2	04	0.4	0.3	03	
0.4	0.2	0.2	0.4	0.2	0. 3	0.2	0.3	

APPENDIX 2D: NIGHT

0.2	0.1	0.2	0.2	0.3	0.2	0.2	0.4	0.2
0.2	0.2	0.1	0.2	0.3	0.2	0.3	0.2	0.2
0.1	0.1	0.3	0.4	0.4	0.2	0.2	0.2	0.2
0.3	0.2	0.2	0.2	0.3	0.2	0.2	0.3	0.2
0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.2	0.3
0.1	0.1	0.1	0.3	0.2	0.3	0.2	0.2	0.2
0.2	0.1	0.1	0.2	0.2	0.3	0.1	0.3	0.2
0.2	0.2	0.1	0.2	0.1	0.2	0.2	0.2	0.2
0.2	0.1	0.1	0.2	0.3	0.3	0.3	0.4	0.2
0.4	0.2	0.2	0.2	0.3	0.2	0.2	0.2	0.3
0.1	0.2	0.1	0.1	0.3	0.2	0.2	0.2	0.2
0.3	0.2	0.1	0.2	0.2	0.2	0.3	0.3	0.2

0.1	0.2	0.1	0.1	0.2	0.1	0.2	0.3	0.2
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.3
0.2	0.2	0.2	0.2	0.3	0.4	0.2	0.2	0.3
0.1	0.2	0.2	0.2	0.3	0.2	0.2	0.2	0.2
0.4	0.1	0.1	0.2	0.3	0.3	0.3	0.2	0.2
0.1	0.2	0.1	0.2	0.3	0.2	0.2	0.2	0.2
0.2	0.2	0.4	0.1	0.2	0.1	0.2	0.1	0.2
0.1	0.2	0.1	0.2	0.3	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3
0.2	0.1	0.2	0.4	0.2	0.2	0.2	0.2	0.1
0.1	0.2	0.2	0.2	0.3	0.2	0.1	0.3	0.3
0.2	0.1	0.2	0.3	0.2	0.1	0.2	0.2	0.1
0.2	0.2	0.1	0.2	0.2	0.3	0.2	0.2	0.2
0.2	0.2	0.3	0.3	0.2	0.4	0.3	0.1	0.3
0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.2	0.2
0.1	0.2	0.3	0.2	0.3	0.1	0.2	0.2	0.4
0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0.1	0.2	0.2	0.2	0.3	0.3	0.1	0.1	0.2
0.2	0.2	0.4	0.2	0.2	0.2	0.2	0.2	0.2
0.1	0.2	0.2	0.1	0.3	0.2	0.3	0.2	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2
0.1	0.2	0.1	0.1	0.4	0.4	0.2	0.2	0.2
0.2	0.1	0.2	0.3	0.2	0.2	0.2	0.3	0.2
0.2	0.2	0.2	0.2	0.3	0.2	0.2	0.2	0.2
0.2	0.2	0.3	0.4	0.2	0.1	0.1	0.1	0.3
0.1	0.1	0.2	0.2	0.3	0.2	0.2	0.4	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.2	
0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.3	
0.2	0.1	0.1	0.2	0.2	0.2	0.2	0.3	

APPENDIX 3: DAILY AVERAGE AIR TEMPERATURES (^{0}C)

APPENDIX 3A: MORNING

22.4	28.2	23.2	23.3	20.7	23.4	22.2	22.9
23.5	25.2	23	23.1	23.6	24.4	22.6	24.4
21.3	25.5	25.3	23.2	25.2	20.7	23.3	23.1
21.0	25.7	22.2	22.2	21.1	23.6	23.2	25.4
24.1	22.9	24.5	22.2	24.5	25.2	22.6	21.2
21.6	23.9	23.4	26.6	23	21.1	23.0	22.9
23.3	24.2	25.3	24.1	23.1	24.5	22.5	23.1
23.7	23.4	23.5	23.6	24.7	23.6	24.8	23.2
22.2	25.7	23.2	28.2	23.2	23.1	23.1	22.4
21.2	24.9	23.5	25.2	24.5	24.7	21.6	24.4
23.2	25.2	21.3	24.9	23.3	23.2	23.3	20.7
22.8	22.9	22.1	25.7	21.6	23.7	23.1	24.5
22.2	24.9	24.1	22.9	24.6	25.3	23.4	25.2
22.7	23.1	21.6	23.9	23.4	25.1	23.5	21.1
22.8	23.1	23.3	24.0	25.3	24.5	22.2	24.5
23.0	25.4	23.6	23.4	21.5	23.4	26.6	23.7
23.4	22.9	22.2	25.7	24.2	25.3	24.1	22.1
23.7	24.5	21.2	24.9	23.4	23.5	23.6	24.5
22.2	23.9	22.5	25.2	25.7	23.2	28.2	24.6
22.7	23.2	22.8	22.9	24.5	23.5	25.2	24.5
23.1	23.3	22.2	24.9	25.2	21.3	24.9	23.6
22.5	25.4	22.7	23.1	22.9	22.1	25.7	22
23.3	22.7	22.8	23.5	24.9	24.1	22.9	23.5
23.0	25.6	23.4	25.4	23.1	21.6	23.9	22.7
21.6	22.2	23.4	22.9	23.3	23.3	24.0	24.4
22.5	24.1	23.7	24.5	25.4	23.6	23.4	23.1
23.3	24.1	22.2	23.2	22.9	22.2	25.9	26.2
22.5	23.5	22.7	25.7	24.5	21.2	24.9	21.2
22.2	24.6	23.1	23.3	23.9	24.5	25.2	22.9
22.6	24.4	23.7	25.4	23.2	22.8	22.9	23.0
23.3	23.1	23.3	22.7	23.3	23.3	25.9	23.2
23.2	23.6	22.8	25.6	25.4	22.7	23.1	24.1
22.6	20.4	21.6	22.2	26.1	22.8	24.2	24.4
23.1	22.9	22.7	24.6	25.6	23.2	25.4	20.7
22.5	23.5	23.3	24.1	22.2	23.4	22.9	25.4
23.2	23.2	24.6	24.6	24.1	23.7	24.5	25.2
23.1	23.4	22.2	24.6	24.4	22.2	23.2	23.9
23.2	24.4	22.6	24.4	23.5	22.7	25.7	24.5
23.3	20.7	23.3	23.1	24.6	23.1	23.3	24.1

22.9	23.6	23.2	22.8	24.4	23.3	25.4	23.3
22.9	25.2	22.6	21.4	23.1	23.3	22.7	25.6
22.2	21.1	23.5	22.9	23.6	21.7	25.6	24.2
22.2	24.5	22.5	245	20.4	21.6	22.2	23.6
26.6	23.0	23.2	23.2	22.9	22.4	24.6	25.2
24.1	23.1	23.1	22.4	23.5	25.4	21.5	
24.6	24.7	23.2	24.4	23.2	24.6	24.6	

APPENDIX 3B: AFTERNOON

26.3	22.5	23.5	23.3	24.1	22.2	23.4	22.9
26.8	23.2	23.2	24.6	24.6	24.1	28.1	24.5
27.3	23.1	23.4	22.2	24.6	24.4	22.2	23.2
26.6	23.2	24.4	22.6	24.4	23.5	22.7	25.7
26.8	23.3	20.7	28.2	23.1	26.7	23.1	23.3
26.7	22.9	23.6	23.2	22.5	27.0	23.0	25.4
25.1	28.1	25.2	22.6	21.4	28.1	23.3	22.7
21.6	22.2	21.1	23.5	28.8	23.2	21.7	25.6
24.0	22.2	24.5	22.5	245	20.4	21.6	22.2
24.3	26.6	23.2	23.2	23.2	22.9	22.5	24.6
24.0	24.1	23.1	23.1	22.4	23.5	25.4	21.5
22.4	24.6	24.7	23.2	24.4	23.2	24.6	24.6
26.1	27.4	23.2	23.3	20.7	23.4	22.2	22.9
28.2	25.2	23.7	23.1	23.6	24.4	22.6	24.4
24.6	25.5	25.3	23.6	25.2	20.7	23.3	23.1
23.3	25.7	22.2	22.2	21.1	23.6	23.2	25.4
24.2	22.9	23.3	22.2	24.5	25.2	22.6	21.2
26.6	23.9	23.4	28.9	23.2	21.1	28.7	22.9
27.1	28.8	25.3	24.1	23.1	24.5	22.5	23.5
23.7	23.4	23.5	23.6	24.7	23.0	24.8	23.2
22.2	25.7	23.2	28.2	23.2	23.1	23.1	22.4
21.2	24.9	28.2	25.2	24.5	24.7	21.6	24.4
23.1	25.2	21.3	24.9	23.3	26.8	23.3	20.7
26.6	22.9	22.1	25.7	21.6	23.7	23.1	24.5
22.2	24.9	24.1	22.9	24.8	25.3	27.6	25.2
27.8	23.1	21.6	23.9	23.4	25.1	26.3	21.1
20.6	23.4	23.3	24.4	25.3	24.5	22.2	24.5
23.4	25.4	23.6	27.5	21.5	23.4	26.6	23.4
23.4	22.9	27.8	25.7	24.2	25.3	24.1	22.1
23.7	24.5	21.2	24.9	23.4	23.5	23.6	24.5

22.2	23.9	22.5	25.2	25.7	23.2	28.2	24.6
22.7	23.2	22.8	22.9	27.4	27.5	25.2	24.5
23.1	27.7	22.2	24.9	25.2	21.3	24.9	23.6
23.8	25.4	22.7	23.1	22.9	22.1	25.7	22.0
23.3	22.7	22.8	23.4	24.9	24.1	22.9	23.5
23.4	25.6	23.2	25.4	23.1	21.6	28.0	22.7
21.6	22.2	23.4	22.9	23.2	23.3	24.2	24.4
22.4	24.1	23.7	24.5	25.4	23.6	23.4	23.1
23.3	24.1	22.2	23.2	22.9	22.2	25.9	26.3
22.5	23.5	22.7	25.7	24.5	21.2	24.9	21.2
22.2	24.6	23.1	27.6	23.9	24.5	25.2	22.9
22.6	26.1	23.0	25.4	23.2	22.8	22.9	23.4
27.4	27.7	23.3	22.7	23.3	23.3	25.9	23.2
23.2	23.8	28.7	25.6	25.4	22.7	23.1	24.1
22.6	20.4	21.6	22.2	26.0	26.5	24.2	
28.1	22.9	22.8	24.6	25.6	23.3	25.4	

APPENDIX 3C: EVENING

29.3	22.5	24.1	23.7	24.5	25.4	23.6	23.4
29.5	26.8	24.1	22.2	23.2	22.9	22.2	22.9
30.5	22.5	23.5	22.7	25.7	24.5	21.2	28.8
31.4	29.7	24.6	23.1	27.6	23.9	24.5	25.2
29.0	22.6	26.3	23.4	25.4	25.9	27.3	22.9
31.5	27.3	27.7	23.3	22.7	23.3	23.3	25.9
31.9	28.6	23.2	28.7	25.6	25.4	22.7	23.1
29.5	22.6	20.4	21.6	22.2	26.4	26.5	24.2
28.8	28.1	23.4	22.7	24.6	25.6	22.8	25.4
30.1	28.9	23.5	23.3	27.2	22.2	23.4	22.9
28.4	23.2	23.2	24.6	24.6	24.1	28.7	24.5
28.3	26.7	23.4	22.2	24.6	24.4	22.2	25.9
26.6	23.2	24.4	25.0	24.4	23.5	22.7	25.7
26.8	26.9	24.6	28.2	23.1	26.7	23.1	23.4
27.2	28.7	23.6	23.2	22.4	27.4	23.6	25.4
26.5	28.4	25.2	22.6	21.4	28.1	23.3	26.3
21.6	22.2	24.3	23.1	28.8	23.7	24.4	25.6
24.6	26.4	24.5	22.5	245	20.4	21.6	24.5
24.3	26.6	23.3	23.2	23.2	22.9	22	24.6
24.7	24.1	23.1	23.1	22.4	26.5	25.4	25.7
24.4	24.6	25.4	23.2	24.4	23.2	24.6	28.2
26.1	27.4	23.2	23.3	20.7	23.4	22.2	26.9
28.0	24.2	23.2	23.1	26.9	24.4	22.6	25.6

25.6	25.5	25.3	24.1	25.2	20.7	23.3	24.4
23.3	25.7	22.2	22.2	21.1	23.6	23.2	26.1
27.9	22.9	24.4	22.2	24.5	25.2	22.6	22.5
26.6	23.9	23.4	28.9	23.1	21.1	28.7	23.7
27.1	28.8	26.4	24.1	23.1	26.1	22.5	25.1
29.6	23.4	23.5	23.6	24.7	23.3	24.8	27.4
22.2	25.7	23.2	28.2	23.2	23.1	23.1	22.4
28.5	24.9	28.3	25.2	27.5	24.7	23.6	23.6
23.4	25.2	21.3	24.9	23.3	26.8	23.3	20.7
26.6	22.9	22.1	25.7	21.6	23.5	23.1	25.8
29.7	24.9	24.1	22.9	24.3	25.3	27.6	25.6
30.4	23.1	21.6	23.9	23.4	25.1	26.3	24.3
21.0	23.2	23.3	24.2	25.3	24.5	22.2	25.8
23.8	25.4	23.6	27.5	21.5	23.4	26.6	29.9
23.4	22.9	27.8	25.7	24.2	25.3	24.1	24.2
23.7	24.5	21.2	24.9	23.4	23.5	27.2	23.6
22.2	23.9	22.5	25.2	25.7	23.2	28.2	27.3
22.7	23.2	22.8	27.3	27.4	27.5	25.2	26.6
23.1	27.7	26.6	24.9	25.2	22.5	24.9	24.1
29.1	25.4	22.7	23.1	22.9	22.1	25.7	23.8
28.4	22.7	22.8	23.4	24.9	24.1	22.9	25.4
24.8	25.6	23.4	25.4	23.1	21.6	28.2	
21.6	22.2	23.4	22.9	23.1	23.3	24.4	

APPENDIX 3D: NIGHT

25.8	23.3	24.1	22.2	23.2	22.9	22.2	24.3
22.9	22.5	23.5	22.7	27.4	24.5	21.2	27.3
24.9	22.2	24.6	23.1	27.6	23.9	24.5	25.2
28.4	22.6	26.1	23.0	25.4	23.2	22.8	22.9
24.1	27.2	27.7	23.3	22.7	23.3	23.3	25.9
25.1	23.2	23.6	28.7	25.6	25.4	28.4	25.6
25.8	22.6	20.4	21.6	28.4	26.5	26.5	24.2
22.2	28.1	26.9	22.8	24.6	25.6	23.2	25.4
21.7	22.5	23.5	23.3	24.1	22.2	23.4	22.9
24.1	23.2	23.2	24.6	24.6	24.1	28.4	27.4
24.3	23.1	23.4	22.2	24.6	24.4	22.2	23.2
24.8	23.2	24.4	22.6	23.9	25.7	22.7	25.7
23.4	23.3	20.7	28.2	23.1	29.3	23.1	23.3
24.8	22.9	23.6	23.2	22.6	27.1	23.6	25.4
25.6	28.1	25.2	22.6	21.4	28.1	23.3	24.1
23.3	22.2	23.6	24.1	28.8	23.8	29.1	25.6

22.1	22.2	27.1	22.5	24.5	20.4	21.6	22.2
23.5	26.6	23.6	23.2	23.2	22.9	22.4	24.6
23.1	28.1	23.1	23.1	22.4	23.5	25.4	22.3
23.1	24.6	24.7	23.2	24.4	23.2	24.6	24.6
22.2	27.4	23.2	23.3	20.7	23.4	22.2	22.9
23.4	25.2	23.1	23.1	25.3	28.7	22.6	26.0
22.2	25.5	25.3	23	25.2	20.7	23.3	23.1
24.5	25.7	22.2	22.2	24.8	28.1	23.2	27.6
27.2	22.9	25.3	22.2	29.6	25.2	22.6	21.2
26.8	23.9	23.4	28.9	22.4	21.1	28.7	22.9
25.6	28.8	25.3	24.1	23.1	24.5	22.5	26.4
24.3	23.4	23.5	25.6	24.7	23.7	24.8	23.2
25.5	25.7	23.2	28.2	23.2	22.1	23.1	25.4
23.9	24.9	28.1	25.2	24.5	24.7	21.6	24.4
28.2	25.2	21.3	24.9	23.3	26.8	23.3	21.4
26.6	22.9	22.1	25.7	21.6	23.1	23.1	24.5
22.2	24.9	24.8	22.9	24.4	25.3	27.6	26.4
27.8	23.1	21.6	23.9	23.4	25.3	26.3	24.0
20.9	23.6	23.3	24.4	25.3	24.5	24.4	24.5
23.1	25.4	23.6	27.5	21.5	27.5	26.6	21.6
23.4	29.1	27.8	25.7	24.2	22.9	24.1	22.1
23.7	24.5	21.2	24.9	24.7	23.5	23.6	25.4
22.2	23.9	22.5	25.2	25.7	23.2	28.2	27.3
22.7	23.2	22.8	22.9	27.4	27.5	25.2	25.2
23.1	27.7	22.2	24.9	25.2	21.3	24.9	23.6
23.4	25.4	22.7	23.1	22.9	22.1	23.6	24.8
23.3	22.7	28.6	23.2	27.3	24.1	22.9	29.1
23.6	25.6	23.7	25.4	23.1	21.6	28.7	23.7
21.6	22.2	23.4	22.9	23.3	23.3	24.2	
22.4	27.4	23.7	24.5	26.9	23.6	23.4	

APPENDIX 4: DAILY AVERAGE SPEED OF THE WIND (m/s)

APPENDIX 4A: MORNING

0.6	0.7	0.7	1.1	1.2	0.6	0.7	0.6	0.7
0.5	0.6	0.3	1.2	1.3	0.6	0.6	0.5	0.3
0.4	0.6	0.4	0.6	1.2	0.7	0.6	0.4	0.4
0.7	0.7	0.7	0.5	0.6	0.3	0.7	0.7	0.7
1.1	0.3	0.6	0.4	0.7	0.4	0.3	1.1	0.6
1.2	0.4	0.6	0.7	0.3	0.7	0.4	1.2	0.6
0.6	0.7	0.7	1.1	0.4	0.6	0.7	0.6	0.7
0.5	0.6	0.3	1.2	0.7	0.7	0.6	0.5	0.3
0.4	0.7	0.4	1.3	0.6	0.3	0.6	0.4	0.4
0.7	0.3	0.7	1.2	0.6	0.4	0.5	0.7	0.7
1.1	0.4	0.5	0.6	0.7	0.7	0.4	1.1	0.6
1.2	0.7	0.6	0.7	0.3	0.6	0.7	1.2	0.7
1.3	0.6	0.5	0.3	0.4	0.6	1.1	1.3	0.3
1.2	0.6	0.4	0.4	0.7	0.7	1.2	1.2	0.4
0.6	0.7	0.7	0.7	0.6	0.3	0.6	0.6	0.7
0.7	0.3	1.1	0.6	0.7	0.4	0.5	0.7	0.6
0.3	0.4	1.2	0.6	0.3	0.6	0.4	0.3	0.6
0.4	0.7	0.6	0.7	0.4	0.5	0.7	0.4	0.7
0.7	0.6	0.5	0.3	0.7	0.4	1.1	0.7	0.3
0.6	0.6	0.4	0.4	0.6	0.7	1.2	0.6	0.4
0.6	0.5	0.7	0.7	0.6	1.1	1.3	0.6	0.7
0.7	0.4	1.1	0.6	0.7	1.2	1.2	0.7	0.6
0.3	0.7	1.2	0.7	0.3	0.6	0.6	0.3	0.6
0.4	1.1	1.3	0.3	0.4	0.5	0.7	0.4	0.5
0.7	1.2	1.2	0.4	0.7	0.4	0.3	0.7	0.4
0.6	0.6	0.6	0.7	0.6	0.7	0.4	0.6	0.7
0.7	0.5	0.7	0.6	0.6	1.1	0.7	0.7	1.1
0.3	0.4	0.3	0.6	0.5	1.2	0.6	0.3	1.2
0.4	0.7	0.4	0.7	0.4	1.3	0.6	0.4	0.6
0.7	1.1	0.7	0.3	0.7	1.2	0.7	0.7	0.5
0.6	1.2	0.6	0.4	1.1	0.6	0.3	0.6	0.4
0.6	1.3	0.6	0.7	1.2	0.7	0.4	0.5	0.7
0.7	1.2	0.7	0.6	0.6	0.3	0.7	0.4	1.1
0.3	0.6	0.3	0.7	0.5	0.4	0.6	0.7	1.2
0.4	0.7	0.4	0.6	0.4	0.7	0.7	1.1	1.3
0.7	0.3	0.7	0.5	0.7	0.6	0.3	1.2	1.2

0.6	0.4	0.6	0.4	1.1	0.6	0.4	0.6	0.6
0.6	0.7	0.7	0.7	1.2	0.7	0.7	0.5	0.7
0.5	0.6	0.3	1.1	1.3	0.3	0.6	0.4	
0.4	0.6	0.4	1.2	1.2	0.4	0.6	0.7	
0.7	0.7	0.7	0.6	0.6	0.7	0.7	1.1	

APPENDIX 4B: AFTERNOON

0.6	1.1	0.7	1.4	1.2	0.8	0.6	0.7	0.6
1.2	1.1	0.6	1.3	0.8	0.8	1.4	0.6	1.2
1.1	1.2	1.4	0.9	0.6	1.2	1.4	1.4	1.1
1.1	0.8	1.4	0.7	1.2	1.4	1.3	1.4	1.1
1.2	0.8	1.3	0.6	1.1	1.4	0.9	1.3	1.2
0.8	1.2	0.9	1.4	1.1	1.3	0.7	0.9	0.8
0.6	1.4	0.7	1.4	1.2	0.9	0.6	0.7	0.6
1.2	1.4	0.6	1.3	0.8	0.7	1.4	0.6	1.2
1.1	1.3	1.4	0.9	0.8	0.6	1.3	0.6	1.1
1.1	0.9	1.3	0.7	1.2	1.4	0.9	1.2	1.1
1.2	0.7	0.9	0.5	1.4	1.4	0.7	1.1	1.2
0.8	0.6	0.7	0.6	1.4	1.3	0.6	1.1	0.8
0.8	1.4	0.6	1.2	1.3	0.9	1.4	1.2	0.8
1.2	1.4	1.4	1.1	0.9	0.7	1.4	0.8	1.2
1.4	1.3	1.4	1.1	0.7	0.6	1.3	0.6	1.4
1.4	0.9	1.3	1.2	0.6	1.4	0.9	1.2	1.4
1.3	0.7	0.9	0.8	1.4	1.3	0.6	1.1	1.3
0.9	0.6	0.7	0.6	1.4	0.9	1.2	1.1	0.9
0.7	1.4	0.6	1.2	1.3	0.7	1.1	1.2	0.7
0.6	1.3	0.6	1.1	0.9	0.6	1.1	0.8	0.6
1.4	0.9	1.2	1.1	0.7	1.4	1.2	0.8	1.4
1.4	0.7	1.1	1.2	0.6	1.4	0.8	1.2	1.4
1.3	0.6	1.1	0.8	1.4	1.3	0.6	1.4	1.3
0.9	1.4	1.2	0.8	1.3	0.9	1.2	1.4	0.9
0.7	1.4	0.8	1.2	0.9	0.7	1.1	1.3	0.7
0.6	1.3	0.6	1.4	0.7	0.6	1.1	0.9	0.6
1.4	0.9	1.2	1.4	0.6	0.6	1.2	0.7	1.4
1.3	0.6	1.1	1.3	1.4	1.2	0.8	0.6	1.3
0.9	1.2	1.1	0.9	1.4	1.1	0.8	1.4	0.9
0.7	1.1	1.2	0.7	1.3	1.1	1.2	1.4	0.7
0.6	1.1	0.8	0.6	0.9	1.2	1.4	1.3	0.6

1.4	1.2	0.8	1.4	0.7	0.8	1.4	0.9	1.2
1.4	0.8	1.2	1.4	0.6	0.6	1.3	0.7	1.1
1.3	0.6	1.4	1.3	0.5	1.2	0.9	0.6	1.1
0.9	1.2	1.4	0.9	0.6	1.1	0.7	1.4	1.2
0.7	1.1	1.3	0.7	1.2	1.1	0.6	1.3	0.8
0.6	1.1	0.9	0.6	1.1	1.2	1.4	0.9	0.6
0.6	1.2	0.7	1.4	1.1	0.8	1.4	0.7	1.2
1.2	0.8	0.6	1.3	1.2	0.8	1.3	0.6	1.1
1.1	0.8	1.4	0.9	0.8	1.2	0.9	1.4	
1.1	1.2	1.4	0.7	0.6	1.4	0.7	1.4	
1.2	1.4	1.3	0.6	1.2	1.4	0.6	1.3	

APPENDIX 4C: EVENING

0.3	0.6	1.3	1.0	0.8	0.9	1.2	1.0	0.8
1.2	1.0	0.9	1.0	0.9	1.0	1.1	0.8	0.3
1.1	0.9	1.0	0.8	0.6	1.0	0.8	0.3	1.2
0.8	0.8	1.0	0.3	0.3	0.8	0.9	1.3	1.1
0.9	0.4	0.8	1.2	1.2	0.3	0.6	0.9	0.8
0.6	0.9	0.3	1.1	1.1	1.2	0.3	1.0.	0.9
0.3	1.0	1.3	0.8	0.8	1.1	1.2	1.0.	0.6
1.2	1.5	0.9	0.9	0.9	0.8	1.1	0.8	0.3
1.1	1.3	1.0	0.6	0.6	0.9	0.8	0.3	1.2
0.8	0.9	1.0	0.3	1.0	0.6	0.9	0.9	1.1
0.9	1.0	0.8	1.2	1.5	0.3	0.6	1.0	0.8
0.6	1.0	0.3	1.1	1.3	1.2	1.0	1.0	0.9
1.0	0.8	0.9	0.8	0.9	1.1	1.5	0.8	0.6
1.5	0.3	1.0	0.9	1.0	0.8	1.3	0.3	1.0
1.3	1.3	1.0	0.6	1.0	0.9	0.9	1.3	1.5
0.9	19	0.8	1.0	0.8	0.6	1.0	0.9	1.3
1.0	0.5	0.3	1.5	0.3	1.0	1.0	1.0	0.9
1.0	1.0	1.3	1.3	1.3	1.5	0.8	1.0	1.0
0.8	0.8	0.9	0.9	0.9	1.3	0.3	0.8	1.0
0.3	0.3	1.0	1.0	1.0	0.9	1.3	0.3	0.8
1.3	0.9	1.0	1.0	1.0	1.0	0.9	0.3	0.3
0.9	1.0	0.8	0.8	0.8	1.0	1.0	1.2	1.3
1.0	1.0	0.3	0.3	0.3	0.8	1.0	1.1	0.9
1.0	0.8	0.3	1.3	0.9	0.3	0.8	0.8	1.0
0.8	0.3	1.2	0.9	1.0	1.3	0.3	0.9	1.0

0.3	1.3	1.1	1.0	1.0	0.9	0.9	0.6	0.8
0.9	0.9	0.8	1.0	0.8	1.0	1.0	0.3	0.3
1.0	0.7	0.9	0.8	0.3	1.0	1.0	1.2	0.9
1.0	1.0	0.6	0.3	1.3	0.8	0.8	1.1	1.0
0.8	0.3	0.3	0.9	0.9	0.3	0.3	0.8	1.0
0.3	1.2	1.2	1.0	1.0	0.9	1.3	0.9	0.8
1.3	1.1	1.1	1.0	1.0	1.0	0.9	0.6	0.3
0.9	0.8	0.8	0.8	0.8	1.0	1.0	1.0	1.3
1.0	0.9	0.9	0.3	0.3	0.8	1.0	1.5	0.9
1.0	0.6	0.6	1.3	0.3	0.3	0.8	1.3	1.0
0.8	0.3	1.0	0.9	1.2	1.3	0.3	0.9	1.0
0.3	1.2	1.5	1.0	1.1	0.9	0.3	1.0	0.8
0.3	1.1	1.3	1.0	0.8	1.0	1.2	1.0	0.3
1.2	0.8	0.9	0.8	0.9	1.0	1.1	0.8	
1.1	0.9	1.0	0.3	0.6	0.8	0.8	0.3	
0.8	0.6	1.0	0.3	0.3	0.3	0.9	1.3	

APPENDIX 4D: NIGHT

1.0	0.9	0.8	0.9	0.6	0.4	1.0	0.7	0.8
0.5	0.3	1.0	0.8	0.4	0.7	1.3	1.0	0.9
0.9	0.6	1.3	0.9	1.0	1.0	0.9	1.3	0.8
0.3	0.4	0.9	0.8	0.5	1.3	0.8	0.9	1.0
0.6	0.7	0.8	1.0	0.9	0.9	0.9	0.8	1.3
0.4	1.0	0.9	1.3	0.3	0.8	0.8	0.9	0.9
1.0	1.3	0.8	0.9	0.6	0.9	1.0	0.8	0.8
0.5	0.9	1.0	0.8	0.4	0.8	0.9	1.0	0.9
0.9	0.8	0.9	0.9	0.7	1.0	0.8	1.3	0.8
0.3	0.9	0.8	0.8	1.0.	1.3	0.9	0.9	1.0
0.6	0.8	0.9	1.0	1.3	0.9	0.8	0.8	0.8
0.4	1.0	0.8	1.0	0.9	0.8	1.0	0.9	1.0
0.7	1.3	1.0	0.5	0.8	0.9	1.3	0.8	0.5
1.0	0.9	1.3	0.9	0.9	0.8	0.9	1.0	0.9
1.3	0.8	0.9	0.3	0.8	1.0	0.8	0.9	0.3
0.9	0.9	0.8	0.6	1.0	0.9	0.9	0.8	0.6
0.8	0.8	0.9	0.4	1.3	0.8	1.0	0.9	0.4
0.9	1.0	0.8	1.0	0.9	0.9	0.5	0.8	1.0
0.8	0.9	1.0	0.5	0.8	0.8	0 9	1.0	0.5
1.0	0.8	1.0	0.9	0.9	1.0	0.3	0.5	0.9
1.3	0.9	0.5	0.3	0.8	1.3	0.6	0.9	0.3

0.9	0.8	0.9	0.6	1.0	0.9	0.4	0.3	0.6
0.8	1.0	0.3	0.4	0.9	0.8	1.0	0.6	0.4
0.9	1.3	0.6	0.7	0.8	0.9	0.5	0.4	0.7
0.8	0.9	0.4	1.0	0.9	0.8	0.9	1.0	1.0
1.0	0.8	1.0	1.3	0.8	1.0	0.3	0.5	1.3
0.9	0.9	0.5	0.9	1.0	1.0	0.6	0.9	0.9
0.8	1.0	0.9	0.8	1.3	0.5	0.4	0.3	0.8
0.9	0.5	0.3	0.9	0.9	0.9	0.7	0.6	0.9
0.8	0.9	0.6	0.8	0.8	0.3	1.0	0.4	0.8
1.0	0.3	0.4	1.0	0.9	0.6	1.3	0.7	1.0
1.3	0.6	0.7	1.3	0.8	0.4	0.9	1.0	1.3
0.9	0.4	1.0	0.9	1.0	1.0	0.8	1.3	0.9
0.8	1.0	1.3	0.8	0.8	0.5	0.9	0.9	0.8
0.9	0.5	0.9	0.9	1.0	0.9	0.8	0.8	0.9
0.8	0.9	0.8	0.8	0.5	0.3	1.0	0.9	0.8
1.0	0.3	0.9	1.0	0.9	0.6	1.3	0.8	1.0
1.0	0.6	0.8	0.9	0.3	0.4	0.9	1.0	0.9
0.5	0.4	1.0	0.8	0.6	0.7	0.8	1.3	
0.9	0.7	1.3	0.9	0.4	1.0	0.9	0.9	
0.3	1.0	0.9	0.8	1.0	1.3	0.8	0.8	

APPENDIX 5: DAILY AVERAGE DIRECTION OF THE WIND (^{0})

APPENDIX 5A: MORNING

68.8	55.7	69.9	131.7	133.5	83.4	116	62.3	110.7
100.9	83.3	83.6	124.5	93.4	87.9	47.6	83.9	96.7
73.8	88.9	110.7	266.1	89.8	86.9	87.2	72.2	90.7
66.5	108.7	87.4	270.1	85.8	66.2	132.7	82.5	104.5
66.5	92.7	99.4	67.8	55.7	69.9	131.7	133.5	83.4
81.2	81.4	122.9	100.5	83.3	83.6	124.5	90.3	87.9
112.3	50.3	138.5	73.8	88.9	110.7	267.4	89.8	50.3
112.1	60.8	168.7	66.5	108.7	87.4	270.9	85.8	60.8
101.7	108.8	61.4	66.5	92.7	99.4	68.8	55.7	108.8
27.6	81.4	96.3	81.2	81.4	122.9	100.9	83.3	100.4
89.2	70.4	100.4	112.3	50.3	138.5	73.8	88.9	70.4
47.0	93.8	150.3	112.1	60.8	168.7	66.5	108.7	87.6
117.6	89.2	133.9	101.7	108.8	62.4	66.5	92.7	89.2

52.6	76.9	145.5	27.6	81.4	129.4	81.2	81.4	106.7
87.5	66.3	74.3	89.2	70.4	100.4	112.3	50.3	82.2
61.2	98.7	126.4	48.6	93.5	150.3	112.1	60.8	70.5
94.0	108.3	177.9	116.8	88.9	133.9	101.7	108.8	103.5
70.3	103.7	89.2	52.6	76.9	145.5	27.6	81.4	88.3
89.2	112.8	118.3	87.5	66.3	74.3	89.2	70.4	76.9
89.7	116.7	94.8	61.2	98.7	126.4	47.5	93.8	66.3
94.5	114.1	138.1	94.4	108.3	177.9	117.6	89.2	98.1
109.1	102.9	81.1	70.3	103.7	94.35	52.6	76.9	105.3
81.4	77.3	133.9	89.2	112.8	118.3	87.5	66.3	126.2
76.6	89.6	145.5	89.7	116.7	75.3	61.2	98.7	83.5
131.5	95.1	74.3	94.5	114.1	138.1	98.7	108.3	64.8
144.3	123.6	126.4	109	102.9	81.1	70.3	103.7	76.3
106.7	124.7	177.9	81.4	77.3	133.9	89.2	112.8	81.9
77.8	110.7	89.2	76.6	89.6	145.5	89.7	116.7	100.0
48.1	89.2	118.3	131.5	95.1	74.3	94.5	114.1	113.4
96.1	91.8	75.3	144.3	124.5	126.4	102.6	102.9	110.7
78.6	126.2	113.6	106.7	124.7	187.0	81.4	77.3	110.7
99.2	83.5	81.1	78.7	110.7	89.2	76.6	89.6	96.7
37.4	64.8	66.5	48.5	89.2	118.3	131.5	95.1	90.7
100.9	73.3	153.4	96.1	91.8	75.3	144.3	123.6	104.5
74.5	84.4	249.3	78.6	126.2	138.1	106.7	124.7	83.4
139.8	80.6	117.7	101.2	83.5	81.1	77.8	110.7	87.7
98.2	113.7	150.3	93.4	64.8	66.5	48.1	89.2	86.6
62.3	110.7	68.5	100.9	73.3	153.4	96.1	91.8	66. 2
83.9	96.7	80.8	74.5	84.4	149.3	78.6	126.2	105.3
72.2	90.7	85.9	139.8	80.6	117.7	99.2	83.5	
82.5	104.5	79.6	98.2	113.7	150.3	37.4	64.8	
133.5	83.4	116.1	62.3	110.7	68.5	100.9	73.3	

APPENDIX 5B: AFTERNOON

108.1	100.6	117.6	161	121.3	92.8	71.3	106.5	78.6
96.8	142.2	144.8	94.3	124.3	65.8	105.5	133.4	127.8
125.2	118.8	125.5	13.1	83.2	99.1	115.2	113.1	174.5
127.2	91.9	143.4	28.2	59.6	72.3	110.6	151.5	86.4
86.9	87.9	101.2	108.1	117.6	100.6	161.3	121.3	92.8
50.4	136.9	99.1	96.8	144.8	142.2	94.3	102.9	65.8
83.4	113.7	94.7	125.2	125.5	118.8	13.1	80.6	99.1
124.6	125	127.2	127.6	143.4	100.2	38.4	59.6	72.3

153.4	76.5	112.1	90.5	101.2	104	108.1	117.6	100.6
141.3	90.6	143.3	50.4	99.1	136.9	96.8	144.8	142.2
127.6	130.2	134.2	83.4	94.7	113.7	125.2	125.5	118.8
106.2	88.5	125.8	124.6	127.2	127.6	127.8	143.4	91.2
116.2	91.9	59.4	153.4	113.9	76.5	86.9	101.2	104.8
88.6	128.5	94.9	141.3	143.3	90.6	50.4	99.1	136.9
83.0	146.7	148.2	127.6	134.2	130.2	83.4	94.7	113.7
98.7	127	65.7	106.2	125.8	89.4	124.6	127.2	125.3
112.2	188.5	90.2	116.2	59.4	91.9	153.4	112.1	76.5
131.4	71.5	108.4	86.7	94.9	128.5	141.3	143.3	90.6
100.7	64.3	131.5	83	148.2	146.7	127.6	134.2	130.2
63.4	61.7	101.1	98.7	65	124	106.2	125.8	88.5
104.7	139.8	152.8	112.2	90.2	188.5	116.2	59.4	91.9
155.4	89.5	90.7	131.4	108.4	69.1	88.6	94.9	128.5
117.4	91.9	105.8	100.7	131.5	64.3	83.8	148.2	146.7
111.1	127.4	89.2	63.4	101.1	61.7	98.7	65.9	127.1
125.8	146.7	207.1	104.7	152.8	139.8	112.2	90.2	188.5
101.2	127.7	111.5	157.9	90.6	89.2	131.4	108.4	71.5
150.7	52.9	113.7	117.7	105.8	91.9	100.7	131.5	64.3
135.7	69.1	67.1	111.1	90.5	128.5	63.4	101.1	61.7
114.4	64.3	127.3	125.8	207.1	146.7	104.7	152.8	139.8
103.5	61.7	136.4	101.2	111.5	129.3	155.4	90.8	89.3
111.2	139.8	77.9	150.7	113.7	52.9	117.7	105.8	91.9
63.5	88.5	111.4	135.7	67.1	69.1	111.1	89.2	127.4
156.1	104.8	95.3	113.5	127.3	64.3	125.8	207.1	146.7
84.7	116.8	85.6	103.5	136.4	61.7	101.2	111.5	127.0
130.4	87.9	106.4	111.2	77.9	139.8	150.7	113.7	52.9
76.4	109.2	128.2	63.5	111.4	83.1	135.7	67.1	69.1
67.4	90.6	176.3	156.1	96.9	104.8	114.4	127.3	64.3
106.5	34.5	78.6	84.7	79.5	116.8	103.5	136.4	61.7
133.4	86.5	127.8	130.4	106.4	87.9	111.2	77.9	
113.1	91.8	174.5	76.5	128.1	109.2	63.5	111.4	
151.5	120.4	86.4	67.4	176.3	90.6	156.1	95.3	

APPENDIX 5C: EVENING

131.9	97.1	70.3	108.9	129	70.2	90.4	81.6	81.7
191.8	74.8	79.5	101.4	119.4	41.1	66.3	99.7	120.4
189.4	80.1	70.2	246.2	75.5	133.8	93.7	127.1	75.3
55.6	136.9	72.5	29.4	35.8	66.5	107.3	155.8	160.6
70.9	63.9	71.2	131.9	97.1	70.3	108.9	131.7	70.2

34.3	37.9	68.6	191.8	74.8	79.5	152.6	119.4	41.1
66.2	108.1	96.4	189.4	80.1	70.2	246.2	74.2	140.5
160.4	89.8	109.8	55.6	136.9	72.7	30.1	35.8	100.1
128.8	143.3	71.5	70.9	63.9	75.8	131.9	97.1	72.6
124.1	83.1	38.6	34.3	37.9	68.6	191.8	74.8	111.8
72.6	108.6	121.8	66.2	108.1	96.4	189.4	80.1	102.9
164.8	151.4	65.8	160.4	89.8	109.8	55.6	136.9	98.6
102.2	82.6	90.5	128.8	143.3	71.5	70.9	63.9	85.4
110.6	52.1	84.7	124.1	83.1	38.6	34.3	37.9	99.4
85.4	30.8	63.6	72.6	108.6	121.8	66.2	108.1	96.4
99.4	155.1	74.5	164.8	151	65.8	160.4	89.8	109.8
117.2	104.0	163.1	102.5	82.6	90.4	128.8	143.3	71.5
161.5	89.9	47.9	110.6	60.4	84.7	124.1	83.1	38.6
82.6	127.5	83.4	85.4	30.8	63.6	72.6	108.6	121.8
70.9	70.3	167	99.4	155.1	74.5	164.8	151	65.8
119.9	80.5	64.3	117.2	104	165	102.3	82.6	90.5
125.3	168.8	65.8	161.5	89.9	47.9	110.6	52.1	84.7
106.8	52.1	90.5	82.6	127.5	83.5	85.4	30.8	63.6
48.2	70.3	84.7	70.9	70.3	167.7	99.4	155.1	80.3
73.5	91.5	63.6	119.9	80.5	64.3	117.2	104.4	167.3
219.9	83.6	64.5	125.3	168.8	65.8	161.5	89.9	64.3
100.8	106.2	163.1	106.8	52.1	90.5	82.6	127.5	91.8
96.1	90.1	47.9	48.2	74.4	84.7	70.9	70.3	86.5
71.3	72.7	83.3	73.5	91.5	63.6	119.9	80.5	50.3
87.0	44.8	167	219.9	83.6	64.5	125.3	168.8	53.4
88.5	64.4	64.3	100.8	106.2	163.1	106.8	52.1	149.5
111.9	143.6	71.6	96.4	90.1	47.9	48.2	70.3	93.3
64.5	84.3	86.5	71.3	72.7	83.3	73.5	91.5	96.7
132.4	68.2	57.2	87.6	44.8	167.3	219.9	83.6	51.6
80.7	96.7	59.5	88.5	64.4	64.3	100.8	103.8	105.2
79.5	51.6	149.3	111.9	143.6	71.6	96.3	90.1	81.7
127.1	75.3	92.8	79.5	51.6	149.3	111.9	143.6	70.4
155.8	160.6	121.1	51.6	95.1	193.3	64.5	84.3	99.1
131.7	70.2	90.4	81.6	81.7	39.4	132.4	68.2	
119.4	41.1	66.4	99.7	120.4	115.9	80.7	96.7	
74.2	133.8	69.1	127.1	75.3	92.8	79.5	51.6	

APPENDIX 5D: NIGHT

265.1	64.1	101.9	96.7	124.7	123.7	98.3	61.6	140.2
119.8	92.7	73.8	45.3	100.1	86.9	77.2	100.9	110.3
82.7	87.1	73.8	87.4	73.1	71.2	79.1	172.6	67.7
99.6	96	86.1	24.8	117.6	80.1	78.7	60.8	110.2
55.3	67.5	102.6	265.1	64.1	101.9	100	124	123.8
146.4	86.5	68.7	119.8	92.6	73.8	45.3	100.1	86.9
43.5	60.9	69.6	82.7	87.1	73.5	87.4	70.7	82.7
113.3	77.4	191.9	99.6	96.5	86.1	26.7	117.6	99.6
85.5	42.4	75.7	55.3	67.5	102.6	265.1	64.1	55.3
77.3	53.6	87.5	146.4	86.5	68.7	119.8	92.3	75.9
104.9	73.4	130.9	43.4	60.9	69.3	82.7	87.1	61.2
101.6	78.1	79.9	113.3	75.8	191.9	99.6	96.4	73.1
97.9	75.9	85.2	85.5	42.3	78.9	55.3	67.5	79.9
80.7	61.2	96.3	77.3	53.4	86.4	146.4	86.5	85.2
49.4	73.1	86.8	104.9	73.4	130.9	43.4	60.9	96.3
61.6	90.1	72.4	101.6	78.1	79.9	113.3	77.4	100.4
72.2	105.4	85.3	97.9	75.9	85.2	85.5	42.3	45.3
36.2	76.5	95.3	80.7	61.2	96.3	77.3	53.5	73.7
53.1	86.5	121.7	49.9	73.1	86.8	104.9	73.4	86.1
41.7	82.6	59.8	61.3	90.1	72.4	101.6	78.1	102.6
77.1	71.4	150.4	72.2	105.4	85.3	97.9	75.9	128.7
81.9	100.3	79.9	36.2	76.9	95.3	80.7	61.2	88.8
48.2	80.8	85.2	53.1	86.5	121.7	49.7	73.1	32.5
127.7	68.4	96.3	41.7	82.6	59.8	61.3	90.1	127.7
95.3	45.6	86.3	77.1	71.4	150.4	72.2	105.4	95.3
81.6	111.1	72.4	81.9	100.3	80.2	36.2	76.5	81.4
128.8	103.6	85.3	24.5	80.8	85.2	53.1	86.5	98.8
155.6	97.2	95.3	127.7	68.4	96.3	41.7	82.6	105.6
60.4	128.7	121.7	95.3	45.6	86.3	77.1	71.4	87.2
67.5	88.8	59.8	81.6	115.4	72.4	81.9	100.3	115.3
115.3	67.3	150.4	128.8	103.6	85.3	48.2	80.8	129.4
119.1	75.3	84.4	155.6	97.2	95.3	127.7	68.4	94.6
94.6	77.5	89.5	60.4	128.7	122.4	95.3	45.6	67.3
67.3	101.8	97.4	67.5	88.8	59.8	81.6	111.1	51.1
84.8	85.7	77.2	115.3	67.3	150.4	128.8	103.6	98.3
103.5	93.7	80.5	122.5	76.4	84.4	155.6	97.2	80.5
54.5	64.8	133.8	94.6	77.5	89.5	60.4	128.7	133.8
61.6	140.2	105.7	67.3	101.8	97.1	67.5	88.8	105.7
100.9	110.3	90.7	84.2	85.7	77.2	115.3	67.3	
172.6	67.7	90.9	103.5	93.5	80.5	119.1	75.6	
60.8	110.2	51.1	54.5	64.8	133.8	94.6	77.5	