KWAME NKRUMAH UNIVERSITY OF SCIENCE
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DEPARTMENT OF MATHEMATICS

EFFICIENCY MEASUREMENT OF BASIC SCHOOLS USING DATA ENVELOPMENT ANALYSIS. CASE STUDY: THE EASTERN REGION, GHANA.

BY

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APPLIED MATHEMATICS

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DECLARATION

I hereby declare that this submission is my own work towards the award of the MPhil. Applied Mathematics degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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DEDICATION

This project is dedicated to the Almighty God for granting me guidance, strength, intelligence and grace to accomplish this project.

I also dedicate this work to my family. I owe you a debt of gratitude for your prayers, support and understanding during my period of studies.

My final dedication goes to my supervisor, Dr. E. Owusu-Ansah for his encouragement and support from beginning of this project to the end.

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ABSTRACT

Universally, education encompasses the transfer of knowledge, consciously or unconsciously from one generation to another in either formal or informal settings. In recent times, whenever the efficiency of Junior High School education is called into question, individuals use only the results of the BECE examinations to judge the schools without considering any other factors. The purpose of this study was to assess how efficient the junior high schools are in the Eastern region of Ghana considering both the performance of students and other factors in play. The Data Envelopment Analysis (DEA) method, which is a linear programming approach, is employed to address the problem of efficiency measurement for decision making units with several inputs and outputs. A case study based on the Eastern Region of Ghana was used with primary data sourced from 26 schools. The major findings revealed that out of the technical efficiencies of the 26 schools for the 2017/2018 academic year that were examined, eighteen schools in the region had less than one, indicating that the schools are producing below the production frontier and are therefore technically inefficient according to CCR model. This shows that in most of the schools, the resources available are being underutilised. For the BCC model, the technical efficiency of twelve out of the twenty six schools in the region were inefficient, this means more than half of the schools were efficient and were using their resources efficiently. It is recommended that, to improve academic performance of students, government and policy makers should pay more attention on how to lower the teacher-students ratio and the classroomstudents ratio in the various schools.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

Universally, education encompasses the transfer of knowledge, consciously or unconsciously from one generation to another in either formal or informal settings. Education involves the acquisition of knowledge, skills, beliefs, etc. It develops an individual's perception about life in general and shapes the person's point of view about concepts or ideas (Kasemsap, 2017). Junior High School is part of the First cycle schools in Ghana which also includes the lower and upper primary schools. This stage in a child's life is very important because it sets the foundation of his/her academic life. This study focuses on the efficiency of the junior high schools and covers the 2017/2018 academic year focused in the Eastern region of Ghana.

1.2 STATEMENT OF THE PROBLEM

In most cases, whenever the efficiency of the junior high schools is called into question, individuals use only the results of the BECE to judge the schools without considering any other factors. Such factors include the location, infrastructure, availability of teaching materials, qualification of teachers, among others. In our Ghanaian society, though minimal research has been undertaken, this is the case with some schools perceived as efficient because of the academic performance of their students. Such schools are most times having abundant resources at hand and it is difficult to determine whether they are fully utilizing it. It is imperative that these resources be considered before decisions concerning efficiency can be made. One needs to ensure whether all available resources have been utilized effectively and that the output, from the school, correlates with the use of resources. This will enable governmental bodies and private individuals who have keen interest in the academic development of students know the aspects of the school to improve, build upon and which aspect to shelve for a later time so as to obtain maximum efficiency.

1.3 OBJECTIVE OF THE STUDY

In order to ascertain the factors relevant in measuring efficiency at the junior high school level, this study shall pursue the following objectives:

1. To determine the efficiency of the various schools taking into consideration the performance of students in standardized test and the resources available to them.
2. To estimate the number of teacher to students ratio that can be employed in the schools for maximum efficiency.
3. To estimate the number of classroom to students ratio that can be employed in the schools for maximum efficiency.
4. To evaluate whether all resources in our various schools are being used efficiently.

1.4 METHODOLOGY

We will be considering the Data Envelopment Analysis (DEA) method, which is a linear programming approach, that is used to determine the efficiency of organisations using decision making units with several inputs and outputs. It is an effective quantifiable tool for measuring and evaluating performance (Cooper & Zhu, 2004). One significant aspect of DEA is that, it is not reliant on the prior specification of functional form.(Ghose, 2016). DEA inputs may also assume a variety of forms which admit of only ordinal measurements which will be useful in measuring the effectiveness of factors such as location, infrastructure, etc. DEA evaluates the achievements or outputs of a set of peer organisations called Decision Making Units (DMUs) which convert multiple inputs to multiple outputs (Cook & Zhu, 2005), this has been used in several studies which measures the efficiency and productivity of organisations.

Thus, the estimation of efficiency score for junior high schools in the region with the aid of DEA will provide adequate means in determining their effectiveness.

1.5 SIGNIFICANT OF THE STUDY

The researcher hopes to assess how efficient the junior high schools are in the Eastern region of Ghana considering both the performance of students and other factors in play. Also to determine the factors that causes a given amount of school resources to produce the highest academic achievement. The researcher again hopes to assess how some factors in the education sector is being underutilized .

1.6 LIMITATION OF THE STUDY

There is major challenge of data collection as educational data on both inputs and output are not readily available. Many important factors affecting technical efficiency of junior high schools are not observed or quantifiable, and ultimately are difficult to obtain. For example, teachers and students willingness to teach and learn are difficult to be measured. It was also costly and time consuming to move to the various schools in the region in search of data.

1.7 THESIS ORGANIZATION

The first chapter deals with the background of study, its significance, limitation and methodology. Chapter 2 contains the literature review and the contributions of other researchers regarding this study. In Chapter 3, the methodology is presented as mathematical treatment and logical presentation of formulation and models of solution. Chapter 4 deals with the data collection, analysis and result. Chapter 5 synthesis the whole study and presents the conclusion, summary and recommendation.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter contains the general overview of research done on the efficiency of the education sector both in Ghana and across the world using both data envelopment analysis and other types of methodologies.

2.2 EDUCATION IN GHANA

The Ghanaian education system is segmented into three main parts; the Basic Education, Secondary Education and the Tertiary Education. The Basic Education which is our focus for this study is free and compulsory and lasts up to 11 years. It is made up of the nursery/kindergarten level, the primary level and the junior high school and ends with the Basic Education Certificate Examination (BECE). English is used as the official language of instruction throughout the various stages. The local languages are mostly used within the first 3 years. The basic education helps to introduce the children to a wide variety of ideas and skills that will aid them to cope creatively with their environment.

The structure of the educational system is mostly 11 years of basic education with 3 years of secondary education and 4 years of tertiary education depending on the option chosen by the individual whether training college, university, Polytechnic, etc.

2.3 EMPIRICAL REVIEW

2.3.1 LITERATURE ON EDUCATION IN GHANA

According to Aminarh (2016), education is undoubtedly an effective and catalytic tool for economic development of a country. The study uses the stochastic frontier as its methodology to measure the technical efficiency and its determinants for forty sampled public primary schools in Ashanti Region of Ghana. Findings, based on the standardized test scores of the primary schools as reported by National Education Assessment (NEA) in 2014, revealed that pupil-teacher ratio and teacher's experience were associated with standardized test score of pupils. Results from the maximum likelihood estimate of the stochastic frontier showed that on average, public primary schools were 0.869 technically efficient; suggesting that about 13.1% of learning outcome could not be realized due to inefficiency. The study also showed that poverty and geographical location of a school were significant determinants of technical efficiencies of the public primary schools.

Ampiah (2008) discusses ways of promoting quality education considering the utilisation of input factors at classroom level. Using data from both rural and urban schools, the study assess the results of the Basic Education Certificate Examination in hope of finding the efficient schools and learning ways of improving the inefficient schools. The findings revealed that the selected schools were similar in how their input factors were utilized. It was also discovered that availability of textbooks and better and frequent usage of English contribute to the efficiencies of schools.

Inoue and Oketch (2008) accesses the cause of the implementation approach taken by Ghana and Malawi on equity and efficiency systems. The recently introduced Free Primary Education (FPE) policy as a means to realizing the 2015 Education for all and

Millennium Development Goals international targets is considered. The study follows the approach taken in implementing the FPE policy in the two countries.

2.3.2 LITERATURE ON THE EDUCATION SECTOR IN GENERAL

Kivenule (2015) studied differences between public and private secondary schools performances in Tanzania. The results obtained showed that private secondary schools were more efficient than public secondary schools due to certain factors in the study. The results also revealed that in private secondary schools, school managers were more active in the decision making process and that generally, private secondary schools were better equipped to perform than public secondary schools. Finally, the researcher has put forth that, the government has made more effort to steadiness education delivered by public secondary schools to its people.

Lassibille and Tan (2001) compared the efficiency of four school types which were: Government and Community schools in the public sector, and Christian and Wazazi schools in the private sector to determine whether growth in the private sector causes student learning to be efficient. Using data from 150 schools, the results reveal that both private schools under consideration were inefficient compared to the public schools under consideration when certain factors such as personal and family behaviour were considered and other factors such as endowment of schools resources were netted out. The recommendation given was for both sectors to communicate and build a strong relationships that could create networking opportunities.

Grosskopf and Taylor (2014) used an economic standpoint to explore certain policy issues that concern the provision of primary and secondary education in the United States. The findings of the study revealed that, current prices were unlikely to be equitable or efficient due to the supply of public goods in a broad sense and education to be specific and they considered basic efficiency for state-supported funding.

2.3.3 LITERATURE ON EDUCATION EFFICIENCY USING DATA ENVELOPMENT ANALYSIS

Barra and Zotti (2016) assess the technical efficiency of public university using data envelopment analysis. They focused mainly on teaching and research in the university. Using the Science and Technology (ST) sector and the Humanity and Social Science (HSS) sector as the study groups, the findings based on the data obtained from 2005 to 2009, showed that the HSS sector is less efficient in relation to quality of research than the ST sector. Whiles the ST sector achieves less efficiency in relation to teaching activities. They also provided confidence intervals using bootstrap method for efficiency scores and to obtain bias-corrected estimates.

Mizala and Farren (2002) assessed the technical efficiency of schools in Chile.
That is, they looked at certain schools ability to produce maximum outputs (academic achievement) given a specific amount of inputs (School resources). The methodologies used were stochastic production frontier and DEA. Each of these techniques has advantages and limitations, each of which were discussed in the paper. The results however was the same when a sample of 2000 schools is analyzed. The results obtained were informative for educational policy discussion in Chile.
Kempkes and Pohl (2010) analysed two methodologies; DEA and stochastic frontier analysis to determine the efficiency of 72 public German universities for a five year period from 1998-2003. This study considers the faculty composition of the schools which turns out to be an important factor in determining efficiency in the study. The results showed that the universities in the West were more efficient than those in the

East over the given duration, but those in the East were found to have outdone those in the West in terms of total factor productivity change.

Cunha and Rocha (2012) applied DEA techniques to assess and compare the efficiencies of public higher education institutions in Portugal. Three separate groups were analysed. These were public universities, public polytechnics and the several faculties of University of Porto.The findings revealed that a great part of the schools were operating inefficiently and that the schools resources were being underutilized.

Barbetta and Turati (2003) uses a sample size of 497 schools in the North-Western part of Italy in looking into the role of proprietary structure in efficiency. the schools were divided into public, private for profit and private non-profit schools. The study undergoes two stages, for the first stage, DEA and Stochastic Frontiers are used to estimate efficiency score and in stage two, they explained efficiency using proprietary structure. The results reveal that private for profit schools are less efficient than private non profit schools with the public schools been most effective.

2.4 DETERMINANTS OF SCHOOL'S EFFICIENCY

Some of the factors that determines the efficiency of a school is said to include the following:

- Socio-economic status of students:

  It has been established that when students have all their basic needs taken care of, they are able to perform better and increase the efficiency of their schools. That is, when they do not have to worry about certain needs, students are able to focus on their academic work and improve their performance. This is consistent with the study done by Alexander (2010).

- School type:

There are several types of schools including one gender schools and mixed schools, private and public schools, religious and non religious schools, specialized and general schools and many more. There are some advantages and disadvantages associated with each type. All these have an effect on efficiency whether positively or negatively as studied by Agasisti (2013); Alexander and Jaforullah (2004) and Alexander (2010).

- School location:

  The location of schools is said to be important when determining the efficiency of schools (Alexander and Jaforullah, 2004; Alexander, 2010). In a location where there are fewer to no distractions, students will be able to perform better as compared with a location with many distractions. Burney et al (2011) assessed the impact geographical region of a school has on its efficiency.

- Teachers characteristics:

  Here, we consider factors such as teacher qualifications, salary, motivation, experience etc. These all have an impact on determining school efficiency Rassouli-Currier (2007). That is, teachers who are highly motivated and experienced will be able to have an impact on the efficiency level of students. Alexander (2010) examined the proportion of teachers who have at least second-year university qualifications and discovered that it had a positive effect on school efficiency.

The literature reviewed indicates a gap in research in Ghana using quantifiable data obtained from a specified number of schools to determine the efficiencies of basic schools. Therefore, this study seeks to use quantifiable data to estimate the efficiencies of schools to give accurate descriptions when the need arises. This approach is novel in determining of the efficiency of basic schools within the context of Ghana as the data envelopment analysis technique is employed to determine the technical efficiencies. Consequently, we are able to incorporate several factors which enables us to have a clear view on which factors need to be increased or decreased. DEA is looked at in chapter three.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter looks at data envelopment analysis, its significance and some of the different types of models.

3.2 DATA ENVELOPMENT ANALYSIS

Data envelopment analysis (DEA) is a tool or method that is used to assist in identifying factors that lead to efficiency with given resources. That is, it provides the user clear information on the efficiencies of the decision making units, bringing to light factors responsible for the performance (Committee, 1997). DEA is a "data-oriented" approach to performance evaluation widely used by several organisations. It was invented by Charnes Abraham, Cooper W. William and Rhodes Edwardo in 1978 in a paper titled "Measuring the Efficiency of Decision Making Units". The model is called the CCR model. The next DEA model to be invented was proposed by Banker, Charnes and Cooper in 1984 and was named the BCC model. DEA evaluates the performance of units in the form of ratio of output and input to a process like efficiency and this helps in measuring productivity. It is a linear programming technique which identifies best practice within a sample and measures efficiency based on differences between observed and best practice units (Committee, 1997). DEA is typically used to measure technical efficiency of decision-making units (DMUs) which are homogeneous and also in the same line of work. For example, if we are to assess the performance of the education sector, all the schools should be in the same category, that is, we do not compare junior high schools with senior high schools. Also, DEA makes use of mathematical programming techniques which deals with multiple variables and constraints. This reduces the difficulties encountered when using techniques that allow for only a few variable options.

3.3 SIGNIFICANCE OF DEA

Data Envelopment Analysis is used mostly by non profit making organisations such as schools, courts, prisons, hospitals etc to determine their efficiencies. This is because it is able to calculate with factors that are difficult to quantify.

Also, DEA creates opportunities for collaboration between analysts and decisionmakers, this is because it is able to assess complex relation between several inputs and outputs which makes it possible for them to consider different scenarios that have or may occur and find the best cause of action.

DEA provides the observed efficiencies of individual agencies, which can be used to key out best performing agencies whose performance the inefficient agencies can target and work to achieve. That is, it identifies a point to serve as a benchmark to use in seeking improvements or for future examination. The benchmark is obtained by analysing the performance of several units. That is, DEA can help an organisation to be efficient by comparing it to similar organisations and providing benchmarks that inefficient organisations can imitate so as to find ways of improving its operations.

3.4 DEFINITIONS AND SYMBOLS

These are common terms and symbols that are often used in data envelopment analysis.

3.4.1 DECISION MAKING UNITS (DMU)

When studying DEA, the term DMU is used for the firms or units such as schools, prisons, hospitals, banks, courts, etc., that is being examined or considered in a DEA study. For the public sector, we mostly deal with non profit making organisations whereas the private sector is mostly made up of profit making organisations. DMU is important in DEA because it can be use in several applications as the tool used in the conversion of inputs into outputs and also used in performance assessments.

3.4.2 EFFICIENT FRONTIER

Data Envelopment Analysis is an analytical tool used in determining the efficient frontier of the production possibility set. The decision making units gets an efficiency score attached to them in the efficiency frontier. DEA is named so because the efficient frontier envelopes other data points. The “frontier line” refers to the line connecting the point of origin and the most efficient point. This is also called efficient frontier. It shows the most efficient unit that can be compared with other units serving as the benchmark.

3.4.3 TECHNICAL EFFICIENCY

Efficiency refers to the degree at which the expected output of a given resource matches the optimal output of the resource. The technical efficiency of an organisation is dependent on the organisation’s outputs divided by its inputs. It can also be reliant on its level of productivity. It is described as the conversion of inputs such raw materials and services into outputs. Technical efficiency computed and found to be one is said to be 100% efficient and that the organisation is operating at best practice.

3.4.4 SYMBOLS

The symbols here are notations that are used when calculating the technical efficiencies of organisations. The notations are adopted from Johnes (2004), and given as:

TE_k is given as the technical efficiency of organisation k with m inputs to obtain s outputs; that is, the performance of an entity considering its input and output. y_{rk} is given as the quantity of output r obtained by organisation k; that is, the total number of outputs produced by the organisation. x_{ik} is given as the quantity of input i used by organisation k; the total number of inputs used by the organisation. u_r is given as the weight of output r; v_i is given as the weight of input i;

n is given as the number of companies to be evaluated; s is given as the number of outputs; m is given as the number of inputs

These symbols are used in several models under DEA of which we will be considering three basic models in the next section.

3.5 MODELS UNDER DEA

DEA has gained worldwide recognition as a useful analytical tool for modelling operational processes and calculating the efficiency of organisations. Profit and non profit making organisations have benefited from the numerous studies done involving efficient frontier estimation. DEA now has several models that can use to determine the efficiency and evaluate performance. Most of its applications uses decision making variables in several forms to evaluate the performance of entities (Wen, 2015). Some frequently used DEA models are discussed below.

3.5.1 THE CCR MODEL

The CCR model calculates the relative efficiency for any decision making unit (DMU) using a weighted sum of outputs and a weighted sum of inputs. Here, all efficiency scores are confined between zero and one. The weight sum of outputs is divided by the weighted sum of inputs. This is given as

$$\text{Maximize} \qquad \mu = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \tag{3.1}$$

and subjected to the following constraints:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad j = 1, \cdots, n \tag{3.2}$$

$$u_r > 0 \qquad \forall \quad r = 1, \cdots, s \tag{3.3}$$

$$v_i > 0 \qquad \forall \quad i = 1, \cdots, m \tag{3.4}$$

We see from equation (3.2) that, the efficiency score of the weights applied to inputs and outputs of company k cannot be greater than one when applied to each company in the dataset. The model developed by Charnes and Rhodes (1978) works under the assumption of constant returns to scale (CRS model). The model is appropriate and can be used only when all firms operate at the optimal scale. Equation (3.3) tells us the weights of outputs are strictly positive whiles equation (3.4) shows the weights of inputs are strictly positive.

When the organisation is operating below the frontier line, it means its efficiency score is less than one. An efficiency score of one means the company is operating efficiency in the dataset. The DEA model have its variables in the form of input and output weights and the the most favourable weights to the unit under reference is given by the LP solution. Since the data sets contain the weights, the weights cannot be fixed ahead of time and the best weight may vary from one DMU to another DMU.

We consider two different ways to solve the linear programming problem. To begin with, we look at the output maximization DEA program, where the weighted sum of outputs is maximized and the weighted sum of input is constrained to 1. The other one is the input minimization DEA program, here we minimize weighted sum of inputs and the weighted sum of outputs is constrained to 1.

Disadvantages and Limitations of the CCR Models

One of the disadvantages of DEA is that it calculates the efficiency scores relative to best practice within a given study. When given the results of two or more studies, it is irrelevant to compare the results between them since there are unknown differences in their best practices. That is, a DEA study that uses information in a particular region cannot be compared with different studies if best practice is unknown.

Additionally, since DEA is not a statistical technique but a deterministic technique, the results obtained from a DEA analysis is particularly sensitive to measurement error. This implies, any mistake made when stating the inputs and outputs of an organisation can become an outlier and cause significant changes in the efficiency scores of neighbouring organisations and also distort the shape of the frontier.

Input and output variables and the sample size are important in a DEA study because DEA is sensitive to the choices considered. The number of organisations considered in the study have an impact on the results obtained. Discretionary power of the model reduces in small samples bringing into play the thumb rule.However, DEA is a very important tool for analysing the efficiency of public sector organisations and though its computations are sometimes difficult with large DMUs, DEA software have been made available to made the computation of efficiency scores easy.

Next, we consider two linear programming (LP) models, the Primal CCR and the Dual CCR model. Each of these models have two orientations, these are the input-oriented and output-oriented models. The input-oriented models are used to test if a DMU under evaluation can minimize its inputs while maintaining its output levels and the output-oriented models are done to check if a DMU under evaluation can maximize its outputs while maintaining its input levels. In the CCR model, efficiency values obtained by the input-oriented and output-oriented models are equal.

Primal CCR Model

The Primal model allows the DMU being measured to determine the sets of optimal weights for each of its factors so as to maximise its efficiency. A set of weights are selected so that the efficiency of any other unit with these weights will not exceed one, the value at which a unit is relatively efficient are considered as solutions.

The primal equation for CRS output-oriented model is given as

$$\text{Minimize} \quad \mu = \sum_{i=1}^{m} v_i x_{ik} \tag{3.5}$$

subject to

$$\begin{aligned} \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} &\leq 0 \qquad j = 1, \cdots, n \\ \sum_{r=1}^{s} u_r y_{rk} &= 1 \\ u_r &> 0 \quad \forall \quad r = 1, \cdots, s \\ v_i &> 0 \quad \forall \quad i = 1, \cdots, m \end{aligned} \tag{3.6}$$

The primal equation for CRS input-oriented model is given as

$$\text{Maximize} \quad \mu = \sum_{r=1}^{s} u_r y_{rk} \tag{3.7}$$

subject to

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad j = 1,\cdots,n$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1 \qquad (3.8)$$

$$u_r > 0 \qquad \forall \quad r = 1,\cdots,s$$

$$v_i > 0 \qquad \forall \quad i = 1,\cdots,m$$

Dual CCR Model

The Dual CCR model gives the same results (efficiency score) as the primal model and its considered as another way of solving the same problem. To determine the efficiency of the DMU, the model uses units already exist to create internally a hypothetical composite unit which will try to outdo the given unit. The unit is considered to be effective if they are not able to outperform it. In solving with linear programming, the dual problem is expressed with real variables φ, θ and a non-negative vector λ_j.

The dual form for CRS output-oriented model is given as

$$\text{Minimize} \qquad \varphi \qquad (3.9)$$

subject to

$$\sum_{j=1}^{n} \lambda_j y_{rj} \leq \phi y_{rk} \qquad r = 1,\cdots,s$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \geq x_{ik} \qquad i = 1,\cdots,m \qquad (3.10)$$

$$\lambda_j \geq 0 \qquad j = 1,\cdots,n$$

The efficiency score is given by $\frac{1}{\phi}$ with λ_j as the weights of inputs and outputs of a company.

The dual form for CRS input-oriented model is given as

$$\text{Maximize} \qquad \theta \qquad (3.11)$$

subject to

$$\begin{aligned} \sum_{j=1}^{n} \lambda_j y_{rj} &\leq y_{rk} & r = 1,\cdots,s \\ \sum_{j=1}^{n} \lambda_j x_{ij} &\geq \theta x_{ik} & i = 1,\cdots,m \qquad (3.12) \\ \lambda_j &\geq 0 & j = 1,\cdots,n \end{aligned}$$

The efficiency score is given by θ with λ_j as the weights of inputs and outputs of an organisation. We consider the next DEA model.

3.5.2 THE BCC MODEL

This model was proposed by Banker, Charnes and Cooper (1984) and was named the BCC model after them. It is the first adjustment done to the basic CCR model. This model works under the assumption of variable returns to scale (VRS model) and is used when firms are not operating at an optimal scale. The BCC model determines the pure technical efficiency of an organisation since it eliminates the component of scale efficiency that is found in the technical efficiency of CCR model. It has an added convexity constraint in the CCR model. We consider its primal and dual models as follows.

Primal BCC Model

We consider VRS to solve the linear programming problem because it includes a measure of returns to scale on the variables axis, c_k, for the firm k (Jean-Marc, 2012) giving us the following equations:

The primal model for BCC output-oriented model is given as

$$\text{Minimize} \quad \mu = \sum_{i=1}^{m} v_i x_{ik} - c_k \qquad (3.13)$$

subject to

$$\begin{aligned} \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - c_k &\leq 0 \quad j = 1,\cdots,n \\ \sum_{r=1}^{s} u_r y_{rk} &= 1 \\ u_r &> 0 \quad \forall \quad r = 1,\cdots,s \\ v_i &> 0 \quad \forall \quad i = 1,\cdots,m \end{aligned} \tag{3.14}$$

The primal equation for BCC input-oriented model is given as

$$\text{Maximize} \quad \mu = \sum_{r=1}^{s} u_r y_{rk} + c_k \tag{3.15}$$

subject to

$$\begin{aligned} \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - c_k &\leq 0 \quad j = 1,\cdots,n \\ \sum_{i=1}^{m} v_i x_{ik} &= 1 \\ u_r &> 0 \quad \forall \quad r = 1,\cdots,s \\ v_i &> 0 \quad \forall \quad i = 1,\cdots,m \end{aligned} \tag{3.16}$$

Dual BCC Model

The dual problem of the linear program is expressed with real variables φ_k, θ_k and a non-negative vector λ_j.

The dual form for BCC output-oriented model is given as

$$\text{Maximize} \quad \varphi_k \tag{3.17}$$

subject to

$$\begin{aligned} \sum_{j=1}^{n} \lambda_j y_{rj} &\leq \phi_k y_{rk} \quad r = 1,\cdots,s \; i = 1,\cdots \\ \sum_{j=1}^{n} \lambda_j x_{ij} &\geq x_{ik} \quad ,m \\ \sum_{j=1}^{n} \lambda_j &= 1 \\ \lambda_j &\geq 0 \end{aligned} \tag{3.18}$$

$j = 1,\cdots,n$

The efficiency score is given by $\frac{1}{\phi_k}$ with λ_j as the weights of inputs and outputs of a company.
The dual form for BCC input-oriented model is given as

$$\text{Minimize} \quad \theta_k \tag{3.19}$$

subject to

$$\begin{aligned} \sum_{j=1}^{n} \lambda_j y_{rj} &\leq y_{rk} \quad r = 1,\cdots,s \; i = 1,\cdots \\ \sum_{j=1}^{n} \lambda_j x_{ij} &\geq \theta_k x_{ik} \quad ,m \\ \sum_{j=1}^{n} \lambda_j &= 1 \end{aligned} \tag{3.20}$$

$$\lambda_j \geq 0 \qquad j = 1,\cdots,n$$

The efficiency score is given by θ_k with λ_j as the weights of inputs and outputs of a company.

3.5.3 SLACKS-BASED MEASURE OF EFFICIENCY (SBM) MODEL

Slacks-Based Measure of efficiency (SBM) is the second adjustment done to the basic CCR model. It was formulated by Tone in 1997. This model was developed when it became apparent that even though both the CCR and the BCC models are used to determine efficiency scores, neither is able to take into accounts the resulting amount of slack for inputs and outputs. Therefore, the SBM model is used to minimize the input and output slacks in measuring efficiency. This model serves as the basis for the definition of super efficiency. Efficiency is measured only by additional variables s_r and s_i.

Dual Model

Here, we look at the CCR and BCC dual models and adjust them for output and input slacks. Considering output slacks as s_r, and input slacks as s_i, we have the two models as follows.

CCR Dual Model

The dual model with inputs and output slacks for CCR output-oriented model is given as

$$\text{Maximize} \quad \mu = \phi + \epsilon \sum_{r=1}^{s} s_r + \epsilon \sum_{i=1}^{m} s_i \tag{3.21}$$

subject to

$$\begin{aligned}
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r &= \phi y_{rk} && r = 1,\cdots,s \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i &= x_{ik} && i = 1,\cdots,m \\
\lambda_j, s_r, s_i &\geq 0 && j = 1,\cdots,n; \quad r = 1,\cdots,s; \quad i = 1,\cdots,m
\end{aligned} \tag{3.22}$$

The efficiency score is given by $\frac{1}{\phi} = 1$ and the input and output slacks $s_i, s_r = 0$ $\forall i = 1,\cdots,m$ and $r = 1,\cdots,s$.

The dual form with input and output slacks for CRS input-oriented model is given as

$$\text{Minimize} \quad \mu = \theta - \epsilon \sum_{r=1}^{s} s_r - \epsilon \sum_{i=1}^{m} s_i \tag{3.23}$$

subject to

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r = y_{rk} \qquad r = 1,\cdots,s$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i = \theta x_{ik} \qquad i = 1,\cdots,m$$

$$\lambda_j, s_r, s_i \geq 0 \qquad j = 1,\cdots,n;\ r = 1,\cdots,s; \quad i = 1,\cdots,m$$

(3.24) The efficiency score is given by $\theta = 1$ and the input and output slacks $s_i, s_r = 0\ \forall i = 1,\cdots,m$ and $r = 1,\cdots,s$.

BCC Dual Model

The dual model with inputs and output slacks for BCC output-oriented model is given as

$$\text{Maximize} \quad \mu = \phi_k + \epsilon \sum_{r=1}^{s} s_r + \epsilon \sum_{i=1}^{m} s_i \qquad (3.25)$$

subject to

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r = \phi_k y_{rk} \qquad r = 1,\cdots,s$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i = x_{ik} \qquad i = 1,\cdots,m$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j, s_r, s_i \geq 0 \qquad j = 1,\cdots,n; \quad r = 1,\cdots,s; \quad i = 1,\cdots,m$$

(3.26)

The efficiency score is given by $\frac{1}{\phi_k} = 1$ and the input and output slacks $s_i, s_r = 0$ $\forall i = 1,\cdots,m$ and $r = 1,\cdots,s$.

The dual form with input and output slacks for CRS input-oriented model is given as

$$\text{Minimize} \quad \mu = \theta_k - \epsilon \sum_{r=1}^{s} s_r - \epsilon \sum_{i=1}^{m} s_i \qquad (3.27)$$

subject to

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r = y_{rk} \qquad r = 1,\cdots,s$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i = \theta_k x_{ik} \qquad i = 1,\cdots,m$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j, s_r, s_i \geq 0 \qquad j = 1,\cdots,n; \quad r = 1,\cdots,s; \quad i = 1,\cdots,m \tag{3.28}$$

The efficiency score is given by $\theta_k = 1$ and the input and output slacks $s_i, s_r = 0$ $\forall i = 1,\cdots,m$ and $r = 1,\cdots,s$.

3.6 EXAMPLE OF DEA ANALYSIS

Two inputs and one output case

We will evaluate the case of four branches of a manufacturing company where the stakeholders would like to know the impact the number of employees and the machinery available makes on the performance of the various branches. DEA will be used to analyse branches which are more efficient using the CCR model. That is, goods sold are unitized to 1 and the input values are normalized to values for getting 1 unit of goods sold.

Table 3.1: Illustrative example of four manufacturing shops

Store Branch	Number of Employees	Machinery available	Goods sold
1	8	6	1
2	4	8	1
3	16	2	1
4	8	4	1

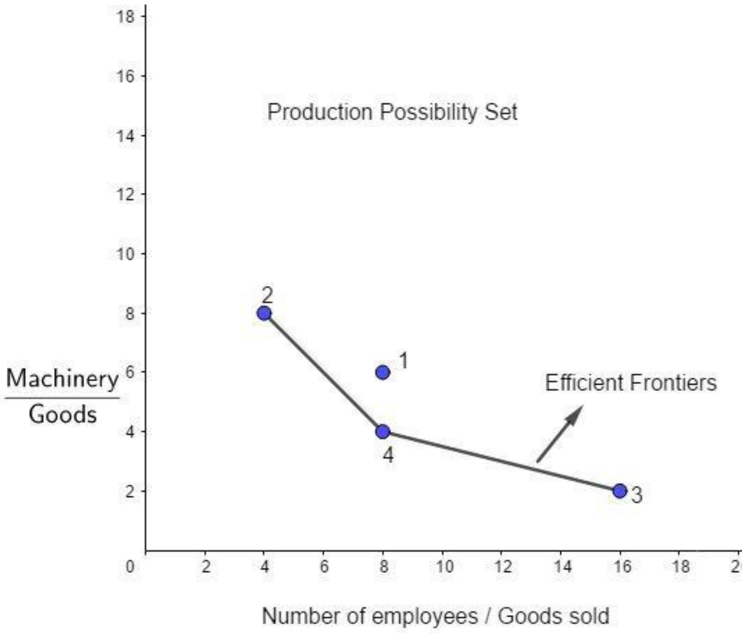

Figure 3.1: Two inputs and one output case

Interpretation

Stores on the efficient frontiers are assumed to be functioning efficiently these includes stores 2, 3 and 4. Store 1 is assumed to be inefficient since it lies above the efficient frontier. The graph shows that store 1 can decrease its input and still maintain its output level as compared with the efficient stores. The efficiency of store 1 can be calculated by referring to the frontier point. We draw a line from 0 to store 1 which crosses the frontier line at a specific point A. Then the efficiency of store 1 can be measured by

$$\frac{0A}{\text{store } 1} = \frac{7}{8} = 0.875$$

The reference set for store 1 then becomes stores 2 and 4 since point A lies on the line connecting them. This is shown clearly in the figure below.

Figure 3.2: Finding the efficiency of store 1

Improvements to store 1 can be done by decreasing the number of employees to seven and reducing machinery available to five. These numbers are the coordinates of the point A. To obtain the efficiency of store 4, store 1 has to maintain the number of employees whiles reducing machinery available to four.

As the size of the sample and its data increases, data envelopment analysis include mathematical formulae and computer packages that makes analysing the data simple.

Example of DEA Anaysis using the Models

Table 3.2: Multiple inputs and outputs case

DMU	Input 1	Input 2	Output 1	Output 2	Output 3
A	15	42	27	12	48
B	30	54	24	6	27
C	27	48	27	12	30
D	21	36	18	3	24

E	27	45	30	12	42

The primal equation for CCR output-oriented model for DMU A is given as

$$\text{Maximize} \quad 27v_1 + 12v_2 + 48v_3$$

subject to

$$15u_1 + 42u_2 = 1$$

$$27v_1 + 12v_2 + 48v_3 - 15u_1 - 42u_2 \leq 0$$

$$24v_1 + 6v_2 + 27v_3 - 30u_1 - 54u_2 \leq 0$$

$$27v_1 + 12v_2 + 30v_3 - 27u_1 - 48u_2 \leq 0$$

$$18v_1 + 3v_2 + 24v_3 - 21u_1 - 36u_2 \leq 0$$

$$30v_1 + 12v_2 + 42v_3 - 27u_1 - 45u_2 \leq 0$$

$$v_1, v_2, v_3, u_1, u_2 \geq 0$$

The primal equation for CCR input-oriented model for DMU A is given as

$$\text{Minimize} \quad 15u_1 + 42u_2$$

subject to

$$27v_1 + 12v_2 + 48v_3 = 1$$

$$15u_1 + 42u_2 - 27v_1 - 12v_2 - 48v_3 \leq 0$$

$$30u_1 + 54u_2 - 24v_1 - 6v_2 - 27v_3 \leq 0$$

$$27u_1 + 48u_2 - 27v_1 - 12v_2 - 30v_3 \leq 0$$

$$21u_1 + 36u_2 - 18v_1 - 3v_2 - 24v_3 \leq 0$$

$$27u_1 + 45u_2 - 30v_1 - 12v_2 - 42v_3 \leq 0$$

$$u_1, u_2, v_1, v_2, v_3 \geq 0$$

Analysis.

Using Excel Solver, It was discovered that the DMU A is efficient with efficiency score one. That is, the firm has minimized its inputs and is able to utilise its resources effectively.

	A	B	C	D	E	F	G	H
1								
2	DMU	Input 1	Input 2	Output 1	Output 2	Output 3		Weights
3	A	15	42	27	12	48		1
4	B	30	54	24	6	27		0
5	C	27	48	27	12	30		0
6	D	21	36	18	3	24		0
7	E	27	45	30	12	42		0
8								
9							Efficiency	1
10								
11						LHS		RHS
12			Input 1			15	<=	15
13			Input 2			42	<=	42
14			Output 1			27	>=	27
15			Output 2			12	>=	12
16			Output 3			48	>=	48
17			Total			1	=	1

Figure 3.3: Solver Solution for DMU A

On the other hand, DMU B was discovered to be inefficient and to obtain efficiency, DMU B had to reduce its inputs while maintaining its outputs considering DMU D as its benchmark.

	A	B	C	D	E	F	G	H
1								
2	DMU	Input 1	Input 2	Output 1	Output 2	Output 3		Weights
3	A	15	42	27	12	48		0
4	B	30	54	24	6	27		0
5	C	27	48	27	12	30		0
6	D	21	36	18	3	24		1
7	E	27	45	30	12	42		0
8								
9							Efficiency	1
10								
11						LHS		RHS
12			Input 1			20.2799	<=	20.3999
13			Input 2			36.71982	<=	36.71982
14			Output 1			19.07991	>=	16.31992
15			Output 2			4.07998	>=	4.07998
16			Output 3			26.87987	>=	18.35991
17			Total			1	=	1

Figure 3.4: Solver Solution for DMU B

We have DMU C to be inefficient with its efficient benchmark to be DMU A with DMU D and E to be efficient.

Using the DEA Frontier Opensolver, we obtain the following solutions.

DMU No.	DMU Name	Input-Oriented CRS Efficiency	Sum of lambdas	RTS	Optimal Lambdas with Benchmarks			
1	A	1.00000	1.000	Constant	1.000	A		
2	B	0.67105	0.816	Increasing	0.158	A	0.658	E
3	C	0.87500	1.000	Constant	1.000	A		
4	D	0.75184	0.604	Increasing	0.044	A	0.560	E
5	E	1.00000	1.000	Constant	1.000	E		

Figure 3.5: Opensolver Solution for all DMUs (CCR Model)

Here, the input-oriented CCR model shows us that DMU A and E are efficient with the rest B, C and D been inefficient. DMU A can be used as the efficiency benchmark for all the inefficient DMUs.

DMU No.	DMU Name	Input-Oriented VRS Efficiency	Optimal Lambdas with Benchmarks			
1	A	1.00000	1.000	A		
2	B	0.74074	0.667	A	0.333	D
3	C	0.87500	1.000	A		
4	D	1.00000	1.000	D		
5	E	1.00000	1.000	E		

Figure 3.6: Opensolver Solution for all DMUs (BCC Model)

The input-oriented BCC model shows DMU A, D and E to be efficient and shows B and C to be inefficient. DMU B can use DMU A or D as its efficiency benchmark whilst DMU C uses DMU A as theirs.

CHAPTER 4

ANALYSIS AND RESULTS

4.1 INTRODUCTION

This chapter deals with data collection and their analysis. The research will use primary data which will be sourced from 26 schools in the various districts in the Eastern Region of Ghana. We will be considering data for the 2017/2018 academic year.

Inputs

For DMU inputs, we can have a look at the total number of students in the various schools. Also the kind of resources available in the schools. These include library, computer lab and others. The classroom to students ratio, the teacher to student ratio, the male to female students ratio, the male to female teachers ratio and the number of graduate teachers.

Outputs

For DMU outputs, we can consider the standardized test scores of National Education Assessment (NEA) conducted by Ministry of Education for the 26 schools. Also, the standardized test scores of the Basic Education Certificate Examination for each of the schools. These include the total number of students who passed, the total number of girls who passed and the total number of students who passed with a single digit.

4.1.1 COLLECTION OF DATA

In a DEA study, we begin by selecting the DMUs. In this study, DMUs are the 26 schools in the Eastern region. Then we consider some of the inputs and outputs given above. For inputs, we use the number of students, the resources available, the number of classrooms and the number of teachers in the schools (N/T). For resources available, we looked at only libraries and computer labs (None - 0, library or computer lab - 1, library and computer lab - 2). For outputs, we considered the average score of the BECE for the various schools and the number of students who passed with single digit (S/D).This is presented in the table below.

Table 4.1: Schools with their inputs and outputs

DMU	Inputs				Outputs	
(Schools)	Students	Resources	Classrooms	Teachers	BECE Score	S/D
School 1	188	1	3	8	66.95	6
School 2	205	2	6	9	56.9	5
School 3	179	0	3	7	55.35	3
School 4	379	1	6	11	65.5	7
School 5	193	2	4	8	48.03	3
School 6	231	1	5	8	38.3	1
School 7	225	1	6	7	55.14	4
School 8	129	0	3	10	65.4	6
School 9	116	2	3	9	72.1	8
School 10	211	1	5	8	82.2	14
School 11	109	0	3	9	60.14	3
School 12	243	1	5	9	59.6	5
School 13	133	2	3	7	49.96	2
School 14	156	1	3	7	63.7	5
School 15	143	1	3	9	62.8	4
School 16	231	1	5	8	71.7	11
School 17	173	0	3	7	56.35	6
School 18	321	2	6	13	43.3	2
School 19	245	1	6	11	51.62	5
School 20	217	2	4	9	83.7	18
School 21	263	1	5	10	47.76	6
School 22	166	1	3	8	63.4	5
School 23	152	2	3	8	56.2	6
School 24	261	2	4	11	71.6	3

School 25	226	1	4	9	61.75	4
School 26	215	2	5	8	54.3	1

4.2 ANALYSIS OF DATA

Table 4.2: Schools with their efficiency scores using the input - oriented CCR model

DMU (Schools)	Input-Oriented CRS Efficiency	Sum of lambdas	Optimal Lambdas	Benchmarks
School 1	1	1	1	School 1
School 2	0.65	0.71	0.17	School 9
School 3	0.98	0.98	0.98	School 17
School 4	0.63	0.91	0.51	School 10
School 5	0.63	0.69	0.23	School 10
School 6	0.47	0.47	0.47	School 10
School 7	0.77	0.67	0.67	School 10
School 8	1	1	1	School 8
School 9	1	1	1	School 9
School 10	1	1	1	School 10
School 11	1	1	1	School 11
School 12	0.68	0.81	0.5	School 10
School 13	0.8	0.7	0.28	School 9
School 14	1	1	1	School 14
School 15	0.92	0.92	0.28	School 1
School 16	0.87	0.87	0.87	School 10
School 17	1	1	1	School 17
School 18	0.36	0.65	0.1	School 10
School 19	0.51	0.68	0.01	School 9
School 20	1	1	1	School 20
School 21	0.52	0.7	0.23	School 10
School 22	0.96	0.97	0.19	School 8
School 23	0.83	0.74	0.46	School 9

School 24	0.79	1.05	0.05	School 8
School 25	0.76	0.97	0.07	School 10
School 26	0.66	0.66	0.66	School 10

Table 4.3: Schools with their efficiency scores using the input - oriented BCC model

DMU (Schools)	Input-Oriented VRS Efficiency	Optimal Lambdas	Benchmarks
School 1	1.00	1.00	School 1
School 2	0.78	1.00	School 14
School 3	1.00	1.00	School 3
School 4	0.66	0.23	School 10
School 5	0.88	0.72	School 3
School 6	0.88	0.56	School 3
School 7	1.00	0.07	School 3
School 8	1.00	1.00	School 8
School 9	1.00	1.00	School 9
School 10	1.00	1.00	School 10
School 11	1.00	1.00	School 11
School 12	0.78	0.44	School 14
School 13	1.00	1.00	School 13
School 14	1.00	1.00	School 14
School 15	1.00	0.33	School 8
School 16	0.95	0.63	School 10
School 17	1.00	1.00	School 17
School 18	0.54	0.46	School 3
School 19	0.65	0.05	School 11
School 20	1.00	1.00	School 20
School 21	0.70	1.00	School 17
School 22	1.00	0.50	School 9
School 23	1.00	0.50	School 9
School 24	0.79	0.11	School 8

School 25	0.78	0.78	School 14
School 26	0.88	0.68	School 13

The technical efficiency scores related to the various schools in Table 4.1 are obtained from input-oriented CCR and BCC models. The results of the CCR and BCC models are illustrated in Table 4.2 and Table 4.3, respectively. These tables show the DMUs together with their efficiencies, sum of lambdas (weights) and their benchmarks.

Table 4.2 shows that, for the input-oriented CCR model, eight schools are efficient with efficiency scores of one. The remaining eighteen schools are technically less efficient. It is assumed that the inefficient schools can achieve efficiency at the same level as its efficient benchmark schools in DEA. Hence, it can be computed how much input quantity can be decreased and output quantity increased to improve the efficiency of inefficient schools. If we consider school 2, we discover its efficiency score to be 0.65 (65%) with the sum of its weights to be 0.71. That means that, it could be able to bring down its inputs by 35% and still produce the same output. To improve its efficiency, it requires an optimal weight of 0.17 with school 9 as its efficiency benchmark. The school with the lowest efficiency score (36%) was school eighteen with its efficiency benchmark school 10.

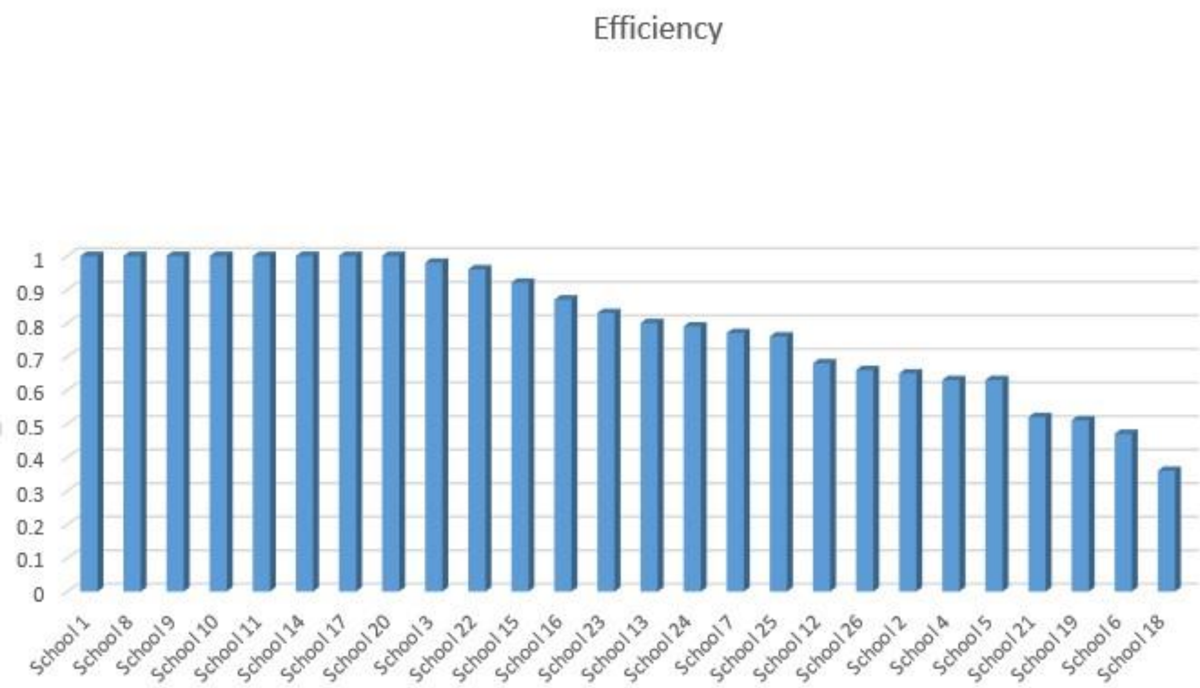

Figure 4.1: Schools with their efficiency scores using the input - oriented CCR model

Figure 4.1 shows that, for the input-oriented CCR model, eight schools are efficient. These are schools 1,8,9,10,11,14,17 and 20. The remaining eighteen schools are inefficient.

Table 4.3 shows that, for the input-oriented BCC model, fourteen schools are efficient with efficiency scores of one. The remaining twelve schools are technically less efficient. If we consider school 2, we discover it now has an efficiency score of 0.78 (78%). This is due to the fact that we are now under the assumption of variable returns to scale. To improve its efficiency, it required an optimal weight of 1.00 with school 14 as its efficiency benchmark. The school with the lowest efficiency score (54%) was still school eighteen with its efficiency benchmark school 3.

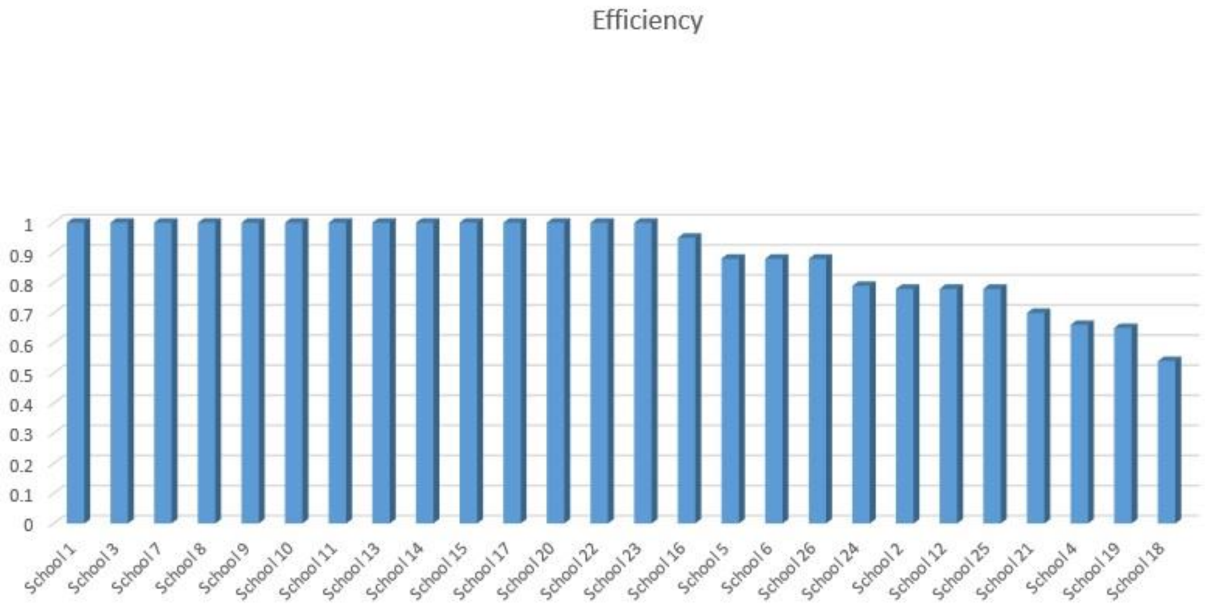

Figure 4.2: Schools with their efficiency scores using the input - oriented BCC model

From figure 4.2, we are shown that, for the input-oriented BCC model, fourteen schools are efficient. These are schools 1,3,7,8,9,10,11,13,14,15,17,20,22 and 23.

The remaining twelve schools are inefficient.

Table 4.4 and 3.6 shows the efficiency input and output target for both the CCR and the BCC model. It shows the optimal inputs and outputs values that will lead to efficiency in these schools.

Table 4.4: Schools with their efficiency input and output target (input - oriented CCR model)

DMU (Schools)		Efficient Input Inputs	Target		Efficient Output Outputs	Target
School 1	188.00	1.00	3.00	8.00	66.95	6.00
School 2	134.01	0.89	3.22	5.88	56.90	8.94
School 3	169.93	0.00	2.95	6.88	55.35	5.89
School 4	175.36	0.63	3.76	6.90	65.50	9.46
School 5	119.85	0.69	2.52	5.04	48.03	5.50
School 6	98.31	0.47	2.33	3.73	38.30	6.52
School 7	141.54	0.67	3.35	5.37	55.14	9.39

School 8	129.00	0.00	3.00	10.00	65.40	6.00
School 9	116.00	2.00	3.00	9.00	72.10	8.00
School 10	211.00	1.00	5.00	8.00	82.20	14.00
School 11	109.00	0.00	3.00	9.00	60.14	3.00
School 12	155.11	0.68	3.41	6.14	59.60	8.62
School 13	107.02	0.98	2.41	5.63	49.96	5.71
School 14	156.00	1.00	3.00	7.00	63.70	5.00
School 15	131.84	0.92	2.77	8.30	62.80	6.14
School 16	184.05	0.87	4.36	6.98	71.70	12.21
School 17	173.00	0.00	3.00	7.00	56.35	6.00
School 18	106.99	0.65	2.15	4.66	43.30	4.14
School 19	124.27	0.51	3.02	5.58	51.62	7.51
School 20	217.00	2.00	4.00	9.00	83.70	18.00
School 21	126.38	0.52	2.58	5.16	47.76	6.00
School 22	137.57	0.96	2.89	7.72	63.40	5.60
School 23	113.12	1.47	2.48	6.62	56.20	8.64
School 24	138.85	1.57	3.14	8.64	71.60	7.01
School 25	159.20	0.76	3.06	6.88	61.75	5.70
School 26	139.38	0.66	3.30	5.28	54.30	9.25

For example, under the CCR model, for school 2, it requires a student population of 134 with 1 resource, 3 classrooms and 6 teachers as its inputs. The outputs will require an average BECE score of 57% with the number of students obtaining single digits given as 9.

Table 4.5: Schools with their efficiency input and output target (input - oriented BCC model)

DMU (Schools)		Efficient Input Inputs	Target		Efficient Output Outputs	Target
School 1	188.00	1.00	3.00	8.00	66.95	6.00
School 2	156.00	1.00	3.00	7.00	63.70	5.00
School 3	179.00	0.00	3.00	7.00	55.35	3.00
School 4	174.65	0.66	3.47	7.23	65.50	7.44
School 5	168.88	0.42	3.00	7.00	54.28	3.00
School 6	158.88	0.88	3.00	7.00	52.99	2.56

School 7	153.97	1.00	3.00	7.00	55.14	4.00
School 8	129.00	0.00	3.00	10.00	65.40	6.00
School 9	116.00	2.00	3.00	9.00	72.10	8.00
School 10	211.00	1.00	5.00	8.00	82.20	14.00
School 11	109.00	0.00	3.00	9.00	60.14	3.00
School 12	165.48	0.44	3.00	7.00	59.60	5.56
School 13	133.00	2.00	3.00	7.00	49.96	2.00
School 14	156.00	1.00	3.00	7.00	63.70	5.00
School 15	129.83	1.00	3.00	9.00	67.24	7.00
School 16	196.75	0.63	4.25	7.63	72.51	11.00
School 17	173.00	0.00	3.00	7.00	56.35	6.00
School 18	154.23	1.08	3.00	7.00	52.45	2.46
School 19	158.17	0.65	3.00	7.10	59.16	5.00
School 20	217.00	2.00	4.00	9.00	83.70	18.00
School 21	173.00	0.00	3.00	7.00	56.35	6.00
School 22	144.50	1.00	3.00	8.00	64.23	7.00
School 23	144.50	1.00	3.00	8.00	64.23	7.00
School 24	142.16	1.58	3.17	8.70	71.60	8.83
School 25	159.78	0.78	3.00	7.00	62.07	5.22
School 26	140.26	1.68	3.00	7.00	54.30	2.95

Under the BCC model, for school 2, it requires a student population of 156 with 1 resource, 3 classrooms and 7 teachers as its inputs. The outputs will require an average BECE score of 64% with the number of students obtaining single digits given as 5.

To estimate the number of teacher to students and classroom to students ratio that can be employed in the various schools for maximum efficiency, we consider the ratios of the efficient input target for both the CCR and the BCC models.

Table 4.6: Teacher to Students Ratio and Classroom to Students Ratio for the CCR model

No.	DMU (Schools)	Teacher-Students ratio	Classroom-Students ratio
1.	School 1	1:24	1:63

2.	School 2	1:23	1:42
3.	School 3	1:25	1:58
4.	School 4	1:25	1:47
5.	School 5	1:24	1:48
6.	School 6	1:26	1:42
7.	School 7	1:26	1:42
8.	School 8	1:13	1:43
9.	School 9	1:13	1:39
10.	School 10	1:26	1:42
11.	School 11	1:12	1:36
12.	School 12	1:25	1:45
13.	School 13	1:19	1:44
14.	School 14	1:22	1:52
15.	School 15	1:16	1:48
16.	School 16	1:26	1:42
17.	School 17	1:25	1:58
18.	School 18	1:23	1:50
19.	School 18	1:22	1:41
20.	School 20	1:24	1:54
21.	School 21	1:24	1:49
22.	School 22	1:18	1:48
23.	School 23	1:17	1:46
24.	School 24	1:16	1:44
25.	School 25	1:23	1:52
26.	School 26	1:26	1:42

For the CCR model, Table 4.6 gives the specific ratios that can be employed for maximum efficiency in the various schools. For example, for school 2 to be efficient, there should be a teacher for every twenty three students and a classroom for every forty two students. For school 5, efficiency can be achieved by having a teacher for every twenty four students and a classroom for every forty eight students.

Table 4.7: Teacher to Students Ratio and Classroom to Students Ratio for the BCC model

No.	DMU (Schools)	Teacher-Students ratio	Classroom-Students ratio
1.	School 1	1:24	1:63
2.	School 2	1:22	1:52
3.	School 3	1:26	1:60
4.	School 4	1:24	1:50
5.	School 5	1:24	1:56
6.	School 6	1:23	1:53
7.	School 7	1:22	1:51
8.	School 8	1:13	1:43
9.	School 9	1:13	1:39
10.	School 10	1:26	1:42
11.	School 11	1:12	1:36
12.	School 12	1:24	1:55
13.	School 13	1:19	1:44
14.	School 14	1:22	1:52
15.	School 15	1:14	1:43
16.	School 16	1:26	1:46
17.	School 17	1:25	1:58
18.	School 18	1:22	1:51
19.	School 18	1:22	1:53
20.	School 20	1:24	1:54
21.	School 21	1:25	1:58
22.	School 22	1:18	1:48
23.	School 23	1:18	1:48
24.	School 24	1:16	1:45
25.	School 25	1:23	1:53
26.	School 26	1:20	1:47

For the BCC model, Table 4.7 gives the specific ratios that can be employed for maximum efficiency in the various schools. For example, for school 2 to be efficient, there should be a teacher for every twenty two students and a classroom for every fifty two students. For school 5, efficiency can be achieved by having a teacher for every twenty four students and a classroom for every fifty six students.

CHAPTER 5

CONCLUSION, SUMMARY AND RECOMMENDATIONS

5.1 SUMMARY AND CONCLUSION

This chapter presents summary of the finding, conclusions and recommendations based on the study.

5.1.1 SUMMARY.

The aim of this study was to assess how efficient the junior high schools are in the Eastern region of Ghana considering both the performance of students and other factors in play. The three research objectives that were addressed are: to determine the efficiency of the various schools taking into consideration the performance of students in standardized test and the resources available to them.; to estimate the number of students to teacher ratio that can be employed in the schools for maximum efficiency; to estimate the number of students to classroom ratio that can be employed in the schools for maximum efficiency; and to evaluate whether all resources in our various schools are used in the production process efficiently.

A case study based on the Eastern Region of Ghana was used with primary data sourced from 26 schools. The major findings revealed that out of the technical efficiencies of the 26 schools in the Eastern Region in 2018 that were examined, eight of the schools were efficient according to the CCR model. The BCC model showed that fourteen out of the 26 schools were efficient. For this purpose, we considered Data Envelopment Analysis, a non-parametric method which facilitates to examine different input-output components. For inputs; the number of students, the resources available, the number of classrooms and the number of teachers were considered. For outputs; the Basic Education Certificate Examination score and the number of students with single digits were used.

5.1.2 CONCLUSION.

One of the objectives of this study was to determine the technical efficiency of junior high schools taking into consideration academic achievements and resources of 26 schools in the Eastern Region of Ghana. Data envelopment analysis was used to obtain the following conclusions: The technical efficiency of eighteen out of the twenty six schools in the region is less than one, indicating that the schools are producing below the production frontier.This therefore makes the schools technically inefficient according to CCR model. This shows that in most of the schools, the resources available are being underutilised.

For the BCC model, the technical efficiency of twelve out of the twenty six schools in the region are inefficient, this means more than half of the schools were efficient and were using their resources efficiently. The study makes known that, the teacher-students ratio and the classroom-students ratio have significant effect on the academic performance of students in the various schools when we consider the efficiency input target for both models as the desired number of teachers and classrooms are given.

The mean technical efficiency estimate of 80% for the input - oriented CCR model suggests that the schools could increase their performance by 20% using the same level of inputs and existing methodologies. Whiles the mean technical efficiency estimate of 90% for the input - oriented BCC model suggests that the schools could increase their performance by 10%. This can be achieved by increasing the percentage score of the Basic Education Certificate Examination and the number of students who passed the examination with single digits.

5.2 RECOMMENDATIONS

Given the conclusions obtained from this study, the following recommendations should be noted. To improve academic performance of students, government and policy makers should pay more attention on how to lower the teacher-students ratio and the classroom-students ratio in the various schools. They should also provide adequate school structures and experienced teachers to the various schools.

The DEA models can also be used to analyse the efficiency differences over a period of time. To achieve this, they should be easier access to data from the various sources. The study recommends that future research should be done on the efficiency of basic schools considering the difference between the public and private sector. Future research should also include the significance of the physical environment on the performance of students in the various schools.

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