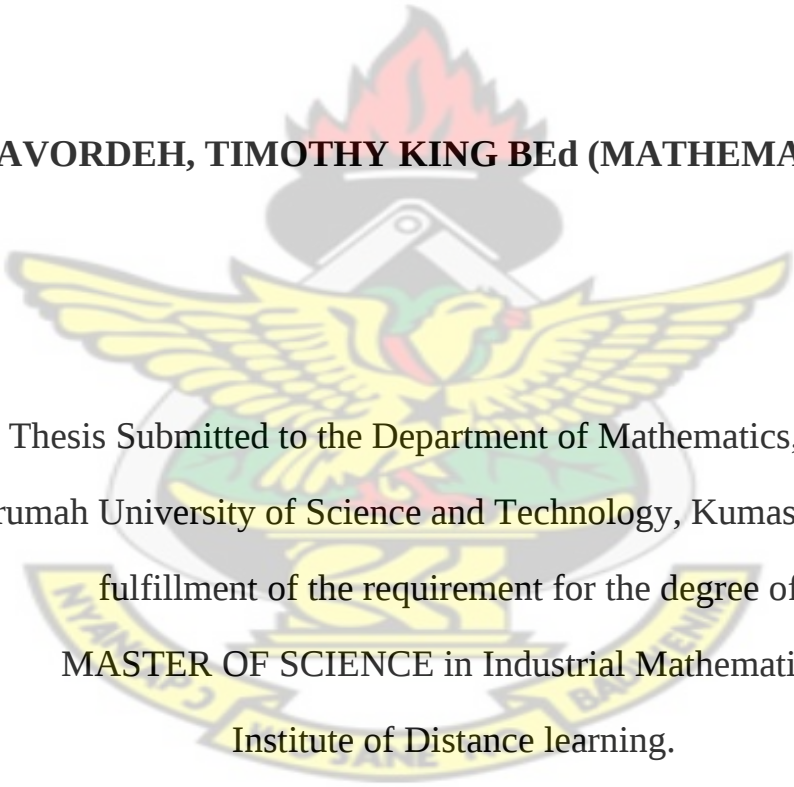


A MATHEMATICAL MODEL FOR THE SPREAD OF MALARIA

CASE STUDY: CHORKOR-ACCRA

KNUST
BY

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A Thesis Submitted to the Department of Mathematics, Kwame
Nkrumah University of Science and Technology, Kumasi, in partial
fulfillment of the requirement for the degree of
MASTER OF SCIENCE in Industrial Mathematics,
Institute of Distance learning.

MAY, 2011.

DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge; it contains no material previously published by another person(s) or material(s), which has been accepted for the award of any other degree of the University, except where the acknowledgement has been made in the text.

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ABSTRACT

Malaria is an infectious disease transmitted between humans through mosquito bites that kill about two million people a year. Many infectious diseases including malaria are preventable, yet they remain endemic in many communities like Chorkor in Accra due to lack of proper, adequate and timely control policies. The main goal of this thesis is to develop a mathematical model for the transmission of malaria. It has been shown that the model has unique disease-free and endemic equilibria.

A mathematical model for malaria is developed using ordinary differential equations. We analyze the existence and stability of disease-free and endemic malaria (malaria persisting in the population) equilibria. Key to our analysis is the definition of a reproductive number R_0 (the number of the new infections caused by one individual in an otherwise fully susceptible population, through the duration of the infectious period). The methods for controlling any infectious disease include a rapid reduction in both the infected and susceptible populations as well as a rapid reduction in the susceptible class if a cure is available. For diseases of malaria where there are no vaccines, it is still possible to reduce the susceptible group through a variety of control measures. The disease-free equilibrium is locally asymptotically stable, if $R_0 \leq 1$, and that the endemic equilibrium exist provided $R_0 > 1$.

Further simulation of the model clearly shows that, with proper combination of treatment and concerted effort aimed at prevention, malaria could be eliminated from our society. In fact, effective treatment offered to about fifty percent of the infected population together with about fifty percent prevention rate is all that is required to eliminate the diseases.

DEDICATION

This work is dedicated to God Almighty, to my Parents: the late Mr. Jonathan Kwami Kunugbe and Mrs. Cecilia Abla Kunugbe and also to my dearest brother Seth Kunugbe, as well as to all my mathematics teachers in Ghana and to Phil Tandoh for her support and tolerance throughout this program.

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ACKNOWLEDGEMENT

I wish to express my sincere gratitude to my supervisor Dr. E. Osei - Frimpong, for his guidance, patience, love encouragement and his overwhelming support provided me throughout this thesis.

I will also very much want to thank all my lecturers of the courses I offered during my first year of the MSc Industrial Mathematics program especially the late

Mr. Emmanuel Agyemang.

I appreciate the effort of Fordjour Richard and Ofori-Boateng Elvis for their friendly support and encouragement as well as suggestions offered me throughout the course. I also want to thank Mr. Sowah-Wilson Roberto sincerely and Mr. Barnes Benedict for reading through the entire script and doing the necessary corrections and I say God should bless you.

Finally, I am highly indebted to the James Amanquah's family for giving me the opportunity to pursue my lifelong dream. I am also grateful to my family and all friends who encouraged me in one way or the other to achieve this dream.

Above all, I thank God for His guidance and abundant favors upon me. To God is the glory.

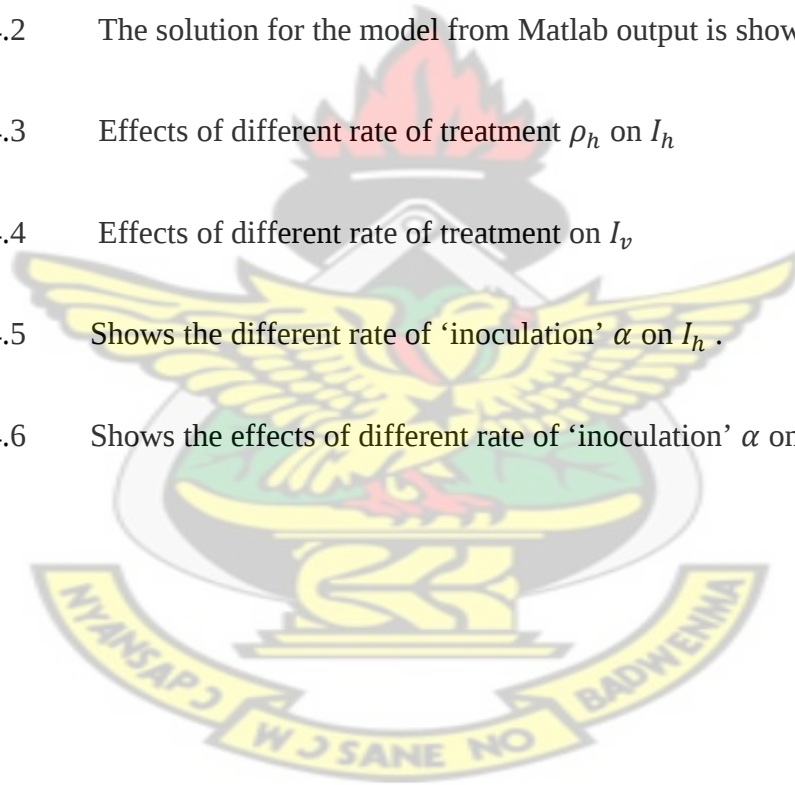
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