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APPLICATION OF QUEUING SYSTEMS: A CASE STUDY OF THE OUTPATIENT DEPARTMENT OF KWAHU GOVERNMENT HOSPITAL, ATIBIE

BY

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DECLARATION

I hereby declare that this thesis is the result of my own original work and that no part of it has been presented for another degree in this university or elsewhere.

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ABSTRACT

This research work was undertaken to investigate the queue situation at the outpatient department of Kwahu Government Hospital, Atibie. Queuing theory is thus applied in the evaluation of queue situations at the four sections of the outpatient department of the hospital namely: records section, health insurance section, history section and consulting rooms.

Data was taken on the number of patients entering the system and the service time of patients. The data was then used to measure the performance of the system.

The research shows which section of the outpatient department is busies and where patients spend more time. It concludes with ways by which management of the hospital can reduce waiting time of patients thereby achieving most of its objectives.

ACKNOWLEGEMENT

My foremost appreciation goes to the almighty God for the strength He has given me to complete this work.

I recognize my Supervisor, Dr. F.T Oduro , Senior Lecturer, Department of Mathematics, Kwame Nkrumah University of Science and Technology for his relentless effort in supervising my work till completion.

I am grateful to the management of Kwahu Government Hospital, Atibie for allowing me to use their department for my work.

DEDICATION

I dedicate this work to my family especially my mother

TABLE OF CONTENTS

LIST OF TABLES

FIGURES

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

Queuing theory is the mathematical study of waiting lines. It is also a methodology that helps match random demand to fixed capacity .In computer science, queuing theory is the study of queues as a technique for managing processes and objects in a computer. A queue can be studied in terms of the source of each queued item, how frequently items arrive on the queue, how long they can or should wait, whether some items should jump ahead in a queue, how multiple queues might be formed and managed, and the rules by which items are enqueued and dequeued.

Queues are common in everyday experiences. Queues are formed because resources are limited. In fact it makes economic sense to have queues. For how many buses and trains would be needed to avoid queues? Queues help facilities or business to provide services in an orderly fashion.

Queuing theory is considered as a branch of operation research because the results are often used when making business decisions about the resources needed to provide services. The theory also seeks to determine how best to design and operate a system usually under conditions requiring the allocation of scare resources.

Queuing theory enables mathematical analysis of several related process including arriving at the queue, waiting in the queue and being served by the server(s). The theory permits the derivation and calculation of several performances such as the average number of arrivals entering a system per unit time, the average number of customers present in the queuing system, the average number of customers waiting in line and the average number of customers in service. The theory also calculates the average time a customer spends in a system, the average time a customer spends in a line, and the average time a customer spends in a service.

Queuing theory is applicable in hospitals, banks, post offices, barbering shops, restaurants etc. It is also applicable to intelligent transportation systems, cell centres, telecommunications and traffic flow. In banks, queuing theory is used to determine the expected number of customers present in the bank, the fraction of time that a particular teller is ideas and how many customers on the average can a teller serve per hour. It can also be used to determine the cost involved in hiring a teller. In applying queuing theory on our roads, we can determine the traffic flow in a particular day of the week. We can also determine what time and when the traffic usually occurs. In our hospitals where queuing theory is mostly applied, it could be used to determine the number of nurses or doctors needed at particular time. The theory can also be used to determine the number of beds needed at a particular time.

1.2 STATEMENT OF THE PROBLEM

In 2003, the government of Ghana replaced the existing "cash and carry" health financing system with the National Heath Insurance Scheme.

The "cash and carry'' system which was a system that made it compulsory for everybody to pay money immediately before and after treatment in our hospital prevented many Ghanaians who are poor from seeking medical care resulting in needless deaths. With the introduction of the National health insurance scheme, there has been a remarkable increase in attendance at government hospitals. These hospitals unfortunately have not had a commensurate increase in capacity in terms of personal, space and equipment to provide satisfactory services to meet the demands of these patients.

Atibie Government Hospital established in 1954 can be found in the Kwahu South district of Ghana. The estimated population of the district is about one hundred and seventy one thousand, four hundred and thirteen (171, 413) (Kwahu South Assembly Report, 2007).

The hospital seeks to (1) increase assess to health (2) provide quality health care in all departments and (3) improve efficiency in health service delivering.

Since the introduction of the national health insurance scheme in 2005, patients visiting it's out patience department of the hospital has increased. Sixty-six percent (66%) of this number was covered by the National health insurance scheme. Within this period, the hospital had five doctors and twenty five general nurses catering for the patients who visited the outpatients department of the hospital.

1.3 OBJECTIVES

The primary aim of this study is to investigate the queue situation at the outpatient department of Atibie Government Hospital. Specifically this study seeks to

(a) Construct queuing models for various sections of the department

(b) Determine performance parameters such as the average waiting time of a patient, average queue length, probability that that a customer will wait etc.

(c) Study the day to day patterns of the queues.

1.4 METHODOLOGY

Data for this study was obtained from the Atibie Government Hospital. The data is purely primary data. The data was taken between 8:00am and 12:00pm for five days. A stop watch was used in taking the data.

1.5 RELEVANCE OF THE STUDY

This study hopes to help the management of the hospital minimize the time that customers (patients) have to wait in a queue and maximize the utilization of it servers or resources (doctors, nurses, hospital beds etc.).

When a patient is waiting in a queue, he may decide to forgo the service because he does not wish to wait any longer. Most research assigns costs to patient waiting time to each server. Thus this study would help management improve customer satisfaction by reducing waiting times and adjusting staffing. It would also help them reduce cost and thus improve profit. This study would also serve as a reference for future academic work.

1.6 ORGANIZATION OF THE WORK

This study is organized into as follows: Chapter one looks at the background of the Study, problem statement, objectives, the methodology used and the relevance of the study. Chapter two looks at the supporting literature of the study. Chapter three covers the methodology in detailed. In this Chapter, the sources and the method of collection of data would be given. Chapter four presents the results and the analysis of the data obtained from the study. Chapter Five will be the concluding Chapter. Discussions of results, conclusions and recommendations would be presented.

CHAPTER TWO

LITERATURE REVIEW

2.1 A HISTORICAL PERSPECTIVE

Agner Kranup Erlang, a Danish engineer who worked for the Copenhagen telephone exchange published the first paper on queuing theory (1909). In his paper, he lays the foundation for the place in queuing theory.

A probabilistic approach to the analysis of a queuing system was initiated by Kendall (1951, 1953) when he demonstrated that imbedded markov chains can be identified in the queue length process in systems M/G/I and A/B/C/D/E

2.2 BASICS OF QUEUING THEORY

In the application of queuing theory, an input and output process should be specified. Some examples of an input and output processes are given in Table 2.1.

Table 2.1: Examples of input and output processes.

Situation	Input process	Output process
Hospital	Arrival of the patient at the registration centre	Assessment,triage provision of services, discharge
Bank	Customer arrives at the bank	The tellers serve the customers

2.2.1THE INPUT PROCESS

The input process is usually called the arrival process. We assume that not more than one arrival can occur at a given instant. If that happens, we say bulk arrivals are allowed.

There are two situations in which the arrival process may depend on the number of customers present. The first occurs when the arrivals are drawn from a small population called finite source models. The second occurs when the rate at which customers arrive at a facility decreases when it is too crowded. When a facility is too crowded, balking or reneging occurs. If a customer arrives but fails to enter the system, we say the customer has balked. When a patient is waiting in a queue, he may decide to forgo the service because he does not wish to wait any longer. This phenomenon is known as reneging.

2.2.2 THE OUTPUT PROCESS

To describe the output process of a queuing system, we usually specify the service time distribution (service mechanism). The service mechanism is the way that customers receive service once they are selected from the front of a queue. The service mechanism is also called a server.

Servers are usually arranged in parallel or in series. Servers are in parallel if all the servers provide the same type of service and a customer need only pass through one server to complete service. Servers are in series if a customer must pass through several servers before completing service.

2.2.3 QUEUE DISCIPLINE

A queue discipline is a priority rule for determining the order which the customers are served. Queue discipline also determines the manner in which resources are divided among customers. The selected rules can have an effect on the system's overall performance. The number of customers in line, the average waiting time and the efficiency of the service facility among others are affected by the choice of priority rules. Examples of queue discipline include the FCFS (First come, first serve), LCLS (Last come last serve), SIRO (Service in random order), and priority queuing discipline.

The most common queue discipline is the FCFS, in which customers are served in the order of their arrival. This means that the customer that has been waiting the longest is served first. In the LCLS, discipline, the customer with the shortest time is served first. If a customer to enter service is chosen randomly from customers waiting for service, it is known as SIRO queuing discipline. Priority queuing discipline places arriving customers in different queues. These queues are given priority level and within each priority level customers enter service on an FCFS basis. Priority disciplines are often used in emergency rooms to determine the order in which customers receive treatment.

There are two major practical problems associated in using any rule: One is ensuring that customers know and follow the rule. The other is ensuring that a system exists to enable the employees to manage the line.

2.3 TYPES OF QUEUING SYSTEMS

There are basically four types of queuing systems and different combinations of the same can be used for very complex networks.

2.3.1 SINGLE CHANNEL- SINGLE PHASE SYSTE

In this system there is a single queue of customers waiting for service and only one phase of service is involved. An example is flu vaccination camp where a nurse practitioner is the server who does all the paper work and vaccination.

2.3.2 SINGLE CHANNEL-MULTIPLE PHASE SYSTEM

In this case there is still a single queue but the service involves multiple phases. For example patients first arrive at the registration centre, get the registration done and then again wait in a queue to be seen by the doctor. There is a queue formation or waiting at each phase of the system.

2.3.3 MULTIPLE CHANNEL-SINGLE PHASE SYSTEM

In this type of queuing system, customers form multiple queues waiting for the service which involves only one phase. Customers have the liberty to switch from one line to the other. An example is customers waiting at the pharmacy store.

2.3.4 MULTIPLE CHANNEL-MULTIPLE PHASE SYSTEM

This type of system has numerous queues and a complex network of multiple phases of service involved. This type of service is typically seen in a hospital setting, outpatients clinics etc. For example in a hospital setting, patients first forms queue for registration, then for assessment, diagnostics, review, treatment, prescription and finally exit from the system.

2.4 MEASURES OF PERFORMANCE OF QUEUING SYSTEMS

There are various ways used to determine whether a queuing system is performing. This enables policy makers to analyze the results, identify the need and carry out then necessary interventions or modifications. It is like the feedback loop of any planning cycle. The salient performance measures of queuing systems are:

(i).Time customers spend waiting.

(ii).The average number of customers in line.

(iii).The utilization rate of servers.

(iv).Cost of operating the system.

The manager would use these measures to determine the optimum number of servers and the capacity for the queuing system.

2.5 POISSON PROCESS AND EXPONENTIAL DISTRIBUTION

A useful queuing model both (a) represents a real- life system with sufficient accuracy and (b) analytically tractable. A queuing model based on the Poisson process and exponentially distributions often meet these requirements

2.5.1 POISSON PROCESS

A Poisson process named after the French mathematician Simeon-Dennis Poisson(1781-1840) is a stochastic process (function of two variables), which is used in modelling random events such as customer arrival, a request for action from the a web server. In the Poisson probability distribution, the observer records the number of events in a time interval.

The Poisson process is a continuous-time process. Continuous-time markov processes are examples of Poisson process. Types of Poisson processes are the homogeneous, non-homogeneous and spatial processes. Orderliness and Memorylessness (would be explained in detailed in the section) are general characteristics of the Poisson process.

ORDERLINESS: This roughly means

$$\lim_{\Delta t \to 0} P(N(t+\Delta t) - N(t) > 1 \setminus N(t+\Delta t) - N(t) \geq 1) = 0$$

This implies that arrivals don't occur simultaneously.

MEMORYLESSNESS: The number of arrivals occurring in any bounded interval of time t is independent of the number of arrivals occurring before time t.

Poisson distribution probabilities can be calculated using the formula

$$p(x) = \frac{\mu^x e^{-\mu}}{x!} \quad , x \geq 0$$

Where x is the number of successes and μ is the mean of the distribution given by $\mu = np$.

The variance of the Poisson distribution is also the same as its mean. That is $\sigma^2 = np$

2.5.2 EXPONENTIAL DISTRIBUTION

The exponential distribution occurs naturally when describing the lengths of inter-arrivals time in a homogeneous Poisson processes. The exponential distribution describes the time for a continuous process to change state. In queuing theory, the inter-arrival times are often modelled as exponentially variables.

The length of a process that can be thought of as a sequence of several independent tasks is better modelled by a variable following the Erlang distribution.

A random variable X has an exponential distribution with parameter λ if the density function of x is given by

$f(x) = \lambda e^{-\lambda t}$, $(t \geq 0)$

Then $E(X) = \frac{1}{\lambda}$ and Var $T = \frac{1}{\lambda^2}$

λ is the expected mean

Memorylessness is an important property of the exponential distribution

MEMORYLESSNESS

If a random variable T is exponentially distributed, then its conditional probability obeys

$P(T > s = t \backslash T > s) = P(T > t)$ For s, t ≥ 0

This says that the conditional probability that we need to wait, for example more than another 10 seconds before the first arrival, given that the first arrival has not yet happen after 30 seconds is no different from the initial probably that we need to wait more than 10 seconds for the first arrival. To summarize a memorylessness of a probably distribution of the waiting time T until the first arrival means , $P(T > 40 \backslash T > 30) = P(T > 10)$.

The memoryless property of the exponential distribution is important because it implies that if we want to know the probability distribution of the time until the next arrival, then it does not matter how long it has been since the last arrival. Suppose inter-arrival times are exponentially distributed with $\lambda = 4$, then the memoryless property implies that no matter how long it has been since the last arrival, the probability distribution governing the time until the next arrival has the density function $4\lambda^{-4t}$. This means that to predict future arrival patterns, we need not keep track of how long it has been since the last arrival. This observation can be appreciably simplifying analysis of a queuing theory.

2.6 THE ERLANG DISTRIBUTION

If inter-arrival times do not appear to be exponential, they are often modelled by an Erlang(1909) distribution. An Erlang distribution is a continuous random variable (T) whose density function is specified by two parameters: a rate parameter R and a shape parameter K (K must be a positive integer).

Given values of R and K, the Erlang distribution has the following density function

$$f(t) = \frac{R(Rt)^{k-1} e^{-Rt}}{(k-1)!}, (t \geq 0)$$

Using integration by parts, E (T) $= \frac{K}{R}$ and varT $= \frac{K}{R^2}$

2.7 BIRTH- DEATH PROCESSES

Most queuing systems with exponential inter-arrival times and exponential services times are often modelled as birth-death processes. A birth- death process is a continuous stochastic process for which the systems state at any time is a nonnegative integer. If a birth- death process is in a state j at time t, then the motion of the process is govern by the following laws:

(a) With probability $\lambda_j \Delta t + 0(\Delta t)$, a birth occurs between time t and time $t + \Delta t$. A birth increases the system state by 1 to j+1. The variable λ_j is called the birth rate in state j. In most queuing systems the birth is simply an arrival.

(b) With probability $\mu_j \Delta t + 0(\Delta t)$, a death occurs between time t and time $t + \Delta t$. A death decreases the system state by 1, to j-1.

The variable μ_j is the death rate in state j. In most queuing systems, a death is a service completion. Note that $\lambda_0 = 0$ must hold or a negative state could occur

(c) Birth and death are independent of each other.

2.8 MATHEMATICAL MODELLING OF QUEUING SYSTEMS

There two approaches to consider when analyzing a queue mathematically:

(a) <u>ANALYTICAL MODEL</u>-behaviour of all elements in the queue is described by systems of (differential) equations. In applying analytical model in queuing systems requires some assumptions to be made. These assumptions include (i) both inter-arrival times and service times are exponential (ii) steady state of the system does exist.

(b) SIMULATION MODEL-this takes the form of a set of assumptions about the operation of the system expressed as mathematical or logical relations between the objects of interest in the system. The simulation process involves executing or running the model through time usually on a computer.

In both the analytical and simulation model there are four major elements of any queuing situation, which must be considered. These include arrivals, services, number of servers and queue discipline.

2.9 A SINGLE-SERVER QUEUING MODEL

2.9.1 KENDALL'S NOTATION FOR CLASSIFICATION OF QUEUE TYPES

There is a standard notation for classifying queuing systems into different types. This was proposed by Kendall (1951, 1953). Systems are described by the notation: A/B/C/D/E

Where: A, B, C, D and E are represented Table 2.2

Table 2.2: Description of Kendall's notations

A	Distribution of inter-arrival times of customers
B	Distribution of service times
C	Number of servers
D	Maximum total number of customers which can be accommodate in the system
E	Queue discipline

A and B can take any of the following types:

Table 2.3: Description of Kendall's notation

M	Exponential distribution
D	Deterministic inter-arrival times
E_K	Erlang distribution(k=shape parameter
G	General distribution

2.9.2 EXAMPLES OF QUEUING MODELS

M/M/1-This would describe a queue with Poisson arrivals, Exponential service time distribution, single server, infinite number of waiting positions

M/ E_z / 2/K-This would describe a queue with Poisson arrivals Erlangian of order-2 service time distribution, 2 servers, maximum number k in queue

G/M/2 – This would describe queues with generalized arrivals, exponential service time distribution, 2 servers, and infinite number of waiting positions.

2.10 MATHEMATICAL MODEL OF M/M/1 SYSTEM

2.10.1 MODEL DESCRIPTION

1. Arrivals are random, and come from the Poisson probability distribution.
2. Each service time is also assumed to be a random variable following the exponential distribution.
3. Service times are assumed to be independent of each other and independent of the arrival process.
4. There is one single server in the queue.
5. The queue discipline is FCFS, and there is no limit on the size of the line.

2.10.2 QUEUING SYSTEM NOTATION

As queuing theory is a mathematical theory, there are several mathematical notations used in it. Few of the basic notations used in queuing model calculations are enumerated in Table 2.4

Table 2.4: Basic mathematical notations of queuing models

Notation	Description
λ	Arrival rate
μ	Service rate
L	Average number of customers in the service
W_q	Average time customers spend in the system
w	Average time customers spend in the system
W	Average time customers spend in the system
$\frac{1}{\mu}$	Service time
ρ	Service utilization
p_n	Probability of n units in system

2.10.3 BASIC NUMERIC CHARACTERISTICS OF M/M/1 QUEUE

$\rho = \frac{\lambda}{\mu}$ is defined as utilization of the system, we assume that $\rho < 1$

1. Probability that there are n arrivals in the system is given by

$$p_n = P^N (1-\rho)$$

2. Probability of the system being empty (expected idle time of the system) is given by

$$p_0 = (1-\rho)$$

3. The expected number of customers in the system is given by

$$L = \frac{\lambda}{\mu - \lambda}$$

4. The expected number of customers in the queue is:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

5. The average waiting time (in the queue) of an arrival is:

$$W_q = \frac{L_q}{\lambda} = \frac{L}{\mu}$$

6. The average time an arrival spend s in the system (both waiting and in service) is:

$$W = W_q + \frac{1}{\mu}$$

2.10.4 EXAMPLE

Analyze system of queue in an office. Population of customers is very large so that we can consider it is an infinite one. Customers for the service arrive randomly following a Poisson process. The office can process customers at an average rate of five patients an hour (one at 12 minutes). The service process is also Poisson. Customers are served at an average of four per hour (one at 15 minutes). The office operates 12 hours a day.

First, all measures assume the process has been operating long enough for the probabilities resulting from the physical characteristics of the problem to have made themselves felt; that is the system is in equilibrium.

Second, utilization of the system of the system is $\rho = \frac{\lambda}{\mu} = \frac{4}{5} = 0.8 < 1$, thus we count relations as follows:

(1) Probability of the system being empty (expected idle time of the system):

$p_0 = 1 - \rho = 1 - 0.8 = 0.2$

This means that on the average, the office will be idle 20 percent of the time and busy 80 percent of the time.

(2) The expected number in the system (both in waiting line and being serviced) is:

$$L = \frac{\lambda}{\mu - \lambda} = \frac{4}{5-4} = 4$$

This means that there will be an average of four persons in line and being served.

(3) The expected number in the waiting line is:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{4 \times 4}{5(5-4)} = \frac{16}{5} = 3.2$$

Thus, there will be an average of 3.2 people in waiting line.

(4) The average waiting time (in the queue) of an arrival is:

$$W_q = \frac{L_q}{\lambda} = \frac{3.2}{4} = 0.8$$

(5) The average time an arrival spends in the system (both waiting and in service) is:

$$W = W_q + \frac{1}{\mu} = 0.8 + \frac{1}{5} = 1$$

Thus, the average time spent in the system both in waiting and obtaining service is one hour. If we assume a 12 hour workday, there will be an average of 48 customers arriving per day, and the expected total lost time of the customers waiting will be $T = \lambda \times 12 \times W_q = 4 \times 12 \times 0.8 = 38.4$.

There would be cost associated with these 38.4 hours. Assume the cost to the society is 10 dollars for each hour lost by a customer waiting. The average cost per pay from waiting is 38.4*10=384 dollars

Suppose that we could in some fashion increase the service rate from five to six per hour and thereby decrease the average time spent in service from 12 minutes to 12 minutes. What could be the effect of this change?

With $\mu = 6$, the expected number in the queue is

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \frac{4^2}{6(6-4)} = 1.33$$

The average wait for the patient is now: $W_q = \frac{L_q}{\lambda} = \frac{1.33}{4} = 0.33$

Before the change, each customer spent an average of 12 minutes being served and 48 minutes waiting. After the change, each patient will spend an average of 10 minutes waiting. This can be verified by computing the total time in the system:

$$W = W_q + \frac{1}{\mu} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \text{(of an hour)}$$

Each day there would 48 customers and each will save half an hour in total. At a cost of 10 dollars per hour, the daily cost of savings is:

$$48 \times \frac{1}{2} \times 20 = 480 \text{ Dollars}$$

2.12 MULTIPLE-SERVER QUEUING MODEL

As explained in Section 2.4, multiple-server, or multiple-channel model involves more than one server.

2.11.1 BASIC PROPERTIES OF MULTIPLE-SERVER MODEL

We shall suppose that these conditions are satisfied:

(i). A Poisson arrival rate.

(ii). An exponential service time.

(iii). There are S servers in the system.

(iv). The processing order is FCFS

(v). The calling population is infinite.

(vi). There is no upper limit to the queue length.

(vii). All the servers are homogeneneous-e.g have the same mean rate.

2.11.2 MULTI-SERVER FORMULAS

The following formulae are used to measure the performance of the multi-server systems:

(i). System utilization is given by

$$\rho = \frac{\lambda}{S\mu}$$

(ii). Probability of n units in the system, where $n \leq S$ is:

$$P_n = P_0 \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}$$

(iii). Probability of n units the system, where n>S

$$P_n = P_0 \frac{\left(\frac{\lambda}{\mu}\right)^n}{S!\left(S^{n-s}\right)}$$

(iv). Probability of zero units in the system:

$$P_0 = \left[\sum_{n=0}^{S-1}\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{S!\left(1-\frac{\lambda}{S\mu}\right)}\right]^{-1}$$

(v). Probability that an arrival will have to wait for service

$$P_w = \left(\frac{\lambda}{\mu}\right)^s \frac{P_0}{S!\left(1 - \frac{\lambda}{S\mu}\right)}$$

(vi). Average number in line is;

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(S-1)!(S\mu-\lambda)^2} P_0$$

(vii). Average number in the system is:

$$L = L_q + \frac{\lambda}{\mu}$$

(viii). Average time spent in line is:

$$W_q = \frac{L_q}{\lambda}$$

(ix). Average time spent in the system is:

$$W = W_q + \frac{1}{\mu}$$

2.11.3 EXAMPLE

Management of a shop plans to open during Sundays. The shop will have three cash registers with an expected service time of 1 minute; projected mean arrival rate is expected to be 1.2 customers per minute.

We shall compute basic performance measures using formulas for multiple-server model

First, $\lambda = 1.2; \mu = 1; S = 3$

Thus the system utilization is

$$\rho = \frac{1.2}{3\times 1} = 0.4$$

Using formula 4 we calculate

$$P_0 = \frac{1}{\left[\frac{\left(\frac{1.2}{1}\right)^0}{0!} + \frac{\left(\frac{1.2}{1}\right)^1}{1!} + \frac{\left(\frac{1.2}{1}\right)^2}{2!}\right] + \frac{\frac{\left(\frac{1.2}{1}\right)^3}{0!}}{3!\left(1-\frac{1.2}{3\times 1}\right)}} = 0.294$$

Using p_0 we can easy determine other measures:

$$L_q = \frac{1.2\times 1\left(\frac{1.2}{1}\right)^3}{(3-1)(3\times 1-1.2)^2} P_0 = 0.32\times P_0 = 0.32\times 0.294 = 0.094$$ Customers in the line

$$L = 0.094 + \frac{1.2}{1} = 1.294$$ Customers in the system

$$W_q = \frac{0.094}{1.2} = 0.078$$ is the average time a customer spends in the line.

$$W = 0.078 + \frac{1}{1} = 1.078$$ is the average time a customer spends in the system.

When will a customer have to wait? The probability of this event must be P_w calculated for S=3:

$$P_w = \left(\frac{1.2}{1}\right)^3 \frac{0.294}{3!\left(1-\frac{1.2}{3\times 1}\right)} = 0.141$$

We can conclude that, practically no queue exists in the system, the probability that a customer will wait is very small.

2.13 STEADY-STATE PROBABILITIES

Let P be the transition probability matrix for an ergodic markov chain with states 1, 2.....S (with ijth element P_{ij}). After a large number of periods have elapsed, the probability π_j may be found by solving the following set of linear equations

$$\pi_j = \sum_{k=1}^{k=s} \pi_k p_{kj} (j = 1,2,......s)$$

$$\pi_1 + \pi_2 + ... + \pi_s = 1$$

2.14 SYSTEM UTILIZATION/TRAFFIC INTENSITY OF THE QUEUING SYSTEM

Let λ be the arrival rate of a customer and

μ be the service rate of a server

Then we define the system utilization of the server as $\rho = \frac{\lambda}{\mu}$

For a steady state probability to exist, we assume that $0 \leq \rho \leq 1$

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

This chapter covers the sources and method of collecting the data. Methods are formulated to achieve the set of objectives in chapter one.

3.2 DATA COLLECTION

Data was collected from the outpatient department of the Kwehu Government Hospital-Atibie in the Eastern Region of Ghana.

For a patient to see a doctor, his/her folder would have to go through four stages. Meaning a patient joins four queues before he/she sees the doctor. These stages include: Records section (arrival point), health insurance section, history section and consulting rooms.

When a patient is not covered by insurance, he/she joins three queues. Data was taken from all the four sections of the outpatients department from Monday to Friday. Data was taken between 8:00am and 12:00pm on each day. This was so because; the outpatient department of the hospital is open to patients from 8:00am to 12:00pm. After this period, patients are either referred to the casualty section of the hospital or are asked to go home and come the following day.

Data was taken on the following: (1) the number of arrivals entering the system (8:00am to 12:00pm.) (2) The service time of each patient observed.

3.3 FORMULATION OF PLOBLEM

In this section, formulae are derived (sometimes quoted). These formulae would be used in the analysis of the data and help determine the set of objectives.

3.3.1 Mean arrival rate, λ

Let n be the number of patients who entered the system between 8:00 am to 12:00pm

h be the number of hours between 8:00am and 12:00pm

Hence the mean arrival rate is given by

$$\lambda = \frac{n}{h} \qquad (3.1)$$

The mean arrival rate λ will have units of arrivals per hour

3.3.2 Mean service rate, μ

The method for determining the mean service time of patients at the various stages of the outpatient department differs. In the records and insurance sections of the outpatients department, folders of patients are sorted out/worked on in batches. The service times at these two sections was taken within an hour. Within this period, there were several batches of folders been sorted out/worked on. Thus, at every point when folders were sorted out/worked on, the service time was taken.

In the history section and consulting rooms however, each patient was attended to one at a time. Hence the service time of each patient was taken whenever he/she was attended to.

3.3.2.1 Mean service rate at the Records and Insurance sections

Let $S_1, S_1 \text{------} S_n$ be the mean service times when the folders of patients are sorted out/worked on in batches.

Let n be the number of folders of patients being sorted out/worked on.

w be the start service time for sorting out/working on patient's folder.

v be the finish service time for sorting out/working on patient's folder.

N be the total number of batches of folders been sorted out.

Hence $S_1, S_1 \ldots\ldots S_n$ would be

$$S_1 = \frac{v_1 - w_1}{n_1}$$

$$S_2 = \frac{v_2 - w_2}{n_1} \quad (3.2)$$

$$S_n = \frac{v_n - w_n}{n_n}$$

Hence the average service rate, μ at these stages is given by

$$\mu = \frac{1}{N}\sum_{i=1}^{n} S_i \quad (3.3)$$

3.3.2.2 Mean service rate at the History section and consulting rooms.

Let $o_1, o_2 \ldots\ldots o_n$ be the observed service times of patients.

Let w_p be the start service time of patients at any of these sections.

v_p be the finished service time of patients at any of these sections.

n be the number of patients observed by a server and

N be the number of servers.

Thus the observed service time of patients would be

$$O_i = v_{pi} - w_{pi} \quad \text{(Where i=1, 2......n)}$$

and the mean service rate for a server would be

$$\mu_i = \frac{1}{n_i}\sum_{i=1}^{n} o_i$$

Thus the mean service rate for all the servers is given by

$$u = \frac{1}{N}\sum_{i=1}^{n} u_i \qquad (3.4)$$

The mean service rate μ will also have units of arrivals per hour.

3.3.3 MEASURING THE PERFORMANCE OF THE SYSTEM

Having calculated λ *and* μ (from the four sections of the outpatients department), these two variables are then used to measure the performance of the system. In measuring the performance of the system the following are determined:

(i) The average time each customer spends in the queue.

(ii)The average queue length.

(iii) The average time each customer spends in the system.

(iv)The average number of customers in the system.

(v)The probability that the service facility will be idle

(vi)The utilization factor of the system, and

(vii)The probability of the number of patients in the system.

In measuring the performance of the system, two types of queues are considered. These are: (a) a single queue with a server and (b) a single queue with multiple servers. Also, in order to determine the amount of time that a patient spends in the system, the queuing formula $L = \lambda W$ is used.

3.3.4 THE QUEUING FORMULA, $L = \lambda W$

This formula is known as the Little's formula.

Where L is defined as the average number of customers present in the queuing system.

λ is defined as the mean arrival rate.

W is defined as the expected time a customer spends in the queuing system

Note: $W = w_s + w_q$

Where w_s is defined as the average time a customer spends in service and w_q is defined as the average time a customer spends in the queue.

For any queuing system in which a steady state distribution exists, the following relations hold.

$$L = \lambda W$$

$$L_q = \lambda w_q \qquad (3.5)$$

$$L_s = \lambda w_s$$

Note: L_q *and* L_s are defined as the average number of customers waiting in line and the average number of customers waiting in service respectively.

3.3.5 SINGLE-SERVER QUEUING FORMULAE

The following formulas are used in measuring the performance of a single queue with a single server:

The utilization factor of the system, ρ is given by

$$\rho = \frac{\lambda}{\mu} \qquad (0 \le \rho \le 1) \qquad (3.6)$$

Probability that there are n arrivals in the system is given by

$$p_n = P^N (1-\rho) \qquad (0 \le \rho \le 1) \qquad (3.7)$$

Probability of the system being empty (expected idle time of the system) is given by

$$p_0 = (1-\rho) \qquad (0 \le \rho \le 1) \qquad (3.8)$$

The expected number of customers in the system is given by

$$L = \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho} \qquad (3.9)$$

The expected number of customers in the queue is:

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho} \qquad (3.10)$$

The average waiting time (in the queue) of an arrival is:

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)} \qquad (3.11)$$

The average time an arrival spend s in the system (both waiting and in service) is:

$$W = \frac{L}{\lambda} = \frac{\rho}{\lambda(1-\rho)} = \frac{1}{\mu-\lambda} \qquad (3.12)$$

3.3.6 MULTI-SERVER FORMULAE

The following formulas are used in measuring the performance of a queue with multiple servers.

The System utilization is given by

$$\rho = \frac{\lambda}{S\mu} \qquad (3.14)$$

Probability of n units in the system, where n $\le S$ is:

$$P_n = P_0 \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} \qquad (3.15)$$

Probability of n units the system, where n>S

$$P_n = P_0 \frac{\left(\frac{\lambda}{\mu}\right)^n}{S!\left(S^{n-s}\right)} \qquad (3.16)$$

Probability of zero units in the system:

$$P_0 = \left[\sum_{n=0}^{S-1}\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{S!\left(1-\frac{\lambda}{S\mu}\right)}\right]^{-1} \qquad (3.17)$$

Probability that an arrival will have to wait for service

$$P_w = \left(\frac{\lambda}{\mu}\right)^s \frac{P_0}{S!\left(1-\frac{\lambda}{S\mu}\right)} \qquad (3.18)$$

Average number in line is;

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(S-1)!(S\mu-\lambda)^2} P_0 \qquad (3.19)$$

Average number in the system is:

$$L = L_q + \frac{\lambda}{\mu} \qquad (3.20)$$

Average time spent in line is:

$$W_q = \frac{L_q}{\lambda} \qquad (3.21)$$

Average time spent in the system is:

$$W = W_q + \frac{1}{\mu} \qquad (3.22)$$

3.3.7 LIMITATIONS OF THE STUDY

There were several limitations to this study. Most of these limitations are the basic assumptions for the application of queuing models. Some of the limitations are:

(i) Both inter-arrival and service times are exponential.

(ii) Arrivals are served on first come first served basis.

(iii) Service times are independent from one another.

(iv) The system assumes a steady or equilibrium state.

CHAPTER FOUR

DATA ANALYSIS AND MODELLING

4.1 INTRODUCTION

In this chapter, data analysis of the work is presented. These results will be summarised using tables and then briefly discussed. Use will be made of the formulae in chapters two and three.

4.2 PRESENTATION AND PRELIMINARY ANALYSIS DATA

This section deals with the analysis of data collected from the four sections (records, insurance, history and consulting rooms) of the outpatients department of the hospital on Monday, Tuesday, Wednesday, Thursday and Friday. In this section, mean arrival rates and service rates are calculated from the data presented. The mean arrival rates and mean service rates are then used to measure the performance of the system as stated in chapter three.

4.2.1 Presentation of Data Collected on Monday

The number of patients who entered the system between 8:00am to 12:00pm is 160.

From (equation 3.1), Mean arrival rate (λ) = $\frac{160}{4}$ = 40 patients per hour.

Table 4.1: Service times of patient at records section on Monday

NO. OF FOLDERS	SERVICE TIME(minutes)
19	22
8	12
10	13

From Table 4.1, Mean service rate (μ) =47.2 patients per hour (This was calculated using equations 3.2 and 3.3).

Number of servers=1

Thus the system utilization is $\rho = \frac{40}{47.2} = 0.85$

$$P_0 = 1 - 0.85 = 0.15$$

$$L_q = \frac{(0.85)^2}{1 - 0.85} = 4.82$$

$$W_q = \frac{4.82}{40} = 0.1205 hours (7.23 Minutes)$$

$$L = \frac{40}{47.2 - 40} = 5.6$$

$$W = \frac{5.6}{40} = 0.14 hours (8.4 Minutes)$$

The result shows that the server will be busy 85 percent of the time and idle 15 percent of the time. More so, the an average number of patients in the queue is 4.82, average waiting time of patients in the queue is 7.23 minutes, the average number of patients in the system is 5.6 and the average waiting time of patients in the system is 8.4 minutes.

Table 4.2: Service times of patients at health insurance section on Monday

NO. OF FOLDERS	SERVICE TIMES
9	8
16	14
8	8
16	11

From Table 4.2, the mean service rate (μ) =71.43 patients per hour (This was calculated using equations 3.2 and 3.3).

Number of servers=1

Thus the system utilization is $\rho = \frac{40}{71.43} = 0.56$

$$P_0 = 1 - 0.56 = 0.44$$

$$L_q = \frac{(0.56)^2}{1 - 0.56} = 0.71$$

$$W_q = \frac{0.71}{40} = 0.02 hours (1.2 Minutes)$$

$$L = \frac{40}{71.43 - 40} = 1.27$$

$$W = \frac{1.27}{40} = 0.032 hours (1.92 Minutes)$$

The results show that the server will be busy 56 percent of the time and idle 44 percent of the time. The average number of patients in the queue is 0.71, the average time a patient spends in the queue is 1.2 minutes, the average number of patients in the system is 1.27 and the average waiting time of patients in the system is 1.92 minutes.

Table 4.3: Service times of patients at history section on Monday

CUSTOMER NO.	SERVICE TIME(minutes)			
	NURSE 1	NURSE 2	NURSE 3	NURSE 4
1	6	4	5	3
2	3	6	5	4
3	2	6	5	3
4	3	6	5	2
5	6	7	10	3
6	8	9	5	3
7	5	7	5	5
8	4	3	5	4
9	5	2	5	3
10				5
11				4

From Table 4.3, the mean service rate μ (per server per minute) =4.76

Hence μ=12.61 patients per hour. (This value was calculated using equation 3.4).

Number of servers=4

Thus the system utilization, $\rho = \frac{40}{4 \times 12.61} = 0.79$

$$P_0 = \frac{1}{1 + \frac{\left(\frac{40}{12.61}\right)^1}{1} + \frac{\left(\frac{40}{12.61}\right)^2}{2} + \frac{\left(\frac{40}{12.61}\right)^3}{6} + \frac{\left(\frac{40}{12.61}\right)^4}{24(1-0.79)}} = 0.03$$

The probability that a server is idle is $p_0 + \frac{3}{4}p_1 + \frac{2}{4}p_2 + \frac{1}{4}p_3$

From (equation 3.15) $p_1 = 0.10$ $p_2 = 0.15$ and $p_3 = 0.16$

Hence $p_0 + \frac{3}{4}p_1 + \frac{2}{4}p_2 + \frac{1}{4}p_3 = 0.03 + \frac{3}{4}\times 0.10 + \frac{2}{4}\times 0.15 + \frac{1}{4}\times 0.16 = 0.22$

From (3.19) $$L_q = \frac{40 \times 12.61 \times \left(\frac{40}{12.61}\right)^4}{3!(4 \times 12.61 - 40)^2} \times 0.03 = 2.43$$

$$W_q = \frac{2.34}{40} = 0.06 hours (3.60 Minutes)$$

$$L = 2.43 + \frac{40}{12.61} = 5.60$$

$$W = \frac{5.60}{40} = 0.14 hours (8.4 Minutes)$$

The results show that the system utilization of the servers is 79 and the probability that a server will be idle is 22 percent. The average number of patients in a queue is 2.43; the average time a patient spends in the queue is 3.60 minutes, the average number of patients in the system is 5.60 and the average time a patient spends in the system is 8.4 minutes.

Table 4.4: Service times of patients at the consulting rooms on Monday

CUSTOMER NO.	SERVICE TIME(minutes)			
	DOCTOR 1	DOCTOR 2	DOCTOR 3	DOCTOR 4
1	4	4	5	5
2	4	4	5	3
3	3	8	5	3
4	3	5	5	5
5	2	3	4	6
6	3	7	6	2
7	5	4	10	3
8	2	2	5	3
9	3	1	6	
10		1	4	
11		3		
12		7		
13		4		
14		4		

From Table 4.4, the mean service time μ (per server per minute) =4.17

Hence μ=14.39 patients per hour (This value was calculated using equation 3.4).

Number of servers=4

Thus the system utilization $\rho = \dfrac{40}{4\times 14.39} = 0.69$

$$P_0 = \frac{1}{1 + \frac{\left(\frac{40}{14.39}\right)^1}{1} + \frac{\left(\frac{40}{14.39}\right)^2}{2} + \frac{\left(\frac{40}{14.39}\right)^3}{6} + \frac{\left(\frac{40}{14.39}\right)^4}{24(1-0.69)}} = 0.05$$

The probability that a server is idle is $p_0 + \frac{3}{4}p_1 + \frac{2}{4}p_2 + \frac{1}{4}p_3$

From (equation 3.15) $p_1 = 0.14$ $p_2 = 0.19$ and $p_3 = 0.18$

Hence $p_0 + \frac{3}{4}p_1 + \frac{2}{4}p_2 + \frac{1}{4}p_3 = 0.05 + \frac{3}{4}\times 0.14 + \frac{2}{4}\times 0.19 + \frac{1}{4}\times 0.18 = 0.30$

$$L_q = \frac{40\times 14.39\times\left(\frac{40}{14.39}\right)^4}{3!(4\times 14.39-40)^2}\times 0.05 = 0.93$$

$$W_q = \frac{0.93}{40} = 0.023\,\textit{hours}\,(1.4\text{minutes})$$

$$L = 0.93 + \frac{40}{14.39} = 3.71$$

$$W = \frac{3.71}{40} = 0.093 hours (5.58 minutes)$$

The results show that the capacity utilization is 69 percent and the probability that a server is idle 0.30. The average number of patients in the queue is 0.93, the average time a patient spends in a queue is 1.4 minutes, the average number of patients in the system is 3.71 and the average time a patient spends in the system is 5.58 minutes.

4.2.2 Presentation of Data collected on Tuesday.

The number of patients who entered the system between 8:00am to 12:00pm is 165.

From (equation 3.1), Mean arrival rate $\mu = \frac{165}{4} = 41.25$ patients per hour.

Table 4.5: Service times of patients at the records section on Tuesday

NO. OF FOLDERS	SERVICE TIME
45	60
18	25

From Table 4.5, the mean service rate (μ) =44.44 patients per hour (This was calculated using equations 3.2 and 3.3).

Number of servers=1

Thus the system utilization (ρ) $= \frac{41.25}{44.44} = 0.93$

$$P_0 = 1 - 0.93 = 0.07$$

$$L_q = \frac{(0.93)^2}{1-0.93} = 12.36$$

$$W_q = \frac{12.36}{41.25} = 0.3 hours (18 \text{ Minutes})$$

$$L = \frac{41.25}{44.44 - 41.25} = 12.93$$

$$W = \frac{12.93}{41.25} = 0.31 hours \text{ (18.60minutes)}$$

The results show that the server will be busy 93 percent of the time and idle 7 percent of the time. The average number of patients in the queue is 12.36; the average time a patient spends in the queue is 18 minutes, the average number of patients in the system is 12.93 and the average time a patient spends in the system is 18.60 minutes.

Table 4.6: Service times of patients at health insurance section on Tuesday

NO. OF FOLDERS	SERVICE TIME(minutes)
43	55
18	22

From Table 4.6, the mean service rate (μ) =47.62 patients per hour (This was calculated using equations 3.2 and 3.3).

Number of servers=1

Thus the system utilization $\rho = \frac{41.25}{47.62} = 0.87$

$$P_0 = 1 - 0.87 = 0.13$$

$$L_q = \frac{(0.87)^2}{1 - 0.87} = 5.82$$

$$W_q = \frac{5.82}{41.25} = 0.14 hours \text{ (8.4minutes)}$$

$$L = \frac{41.25}{47.62 - 41.25} = 6.48$$

$$W = \frac{6.48}{41.25} = 0.15 hours \text{ (9minutes)}$$

The results show that the server is busy 87 percent of the time and idle 13 percent of the time. The average number of patients in the queue is 5.82, the average time a patient spends in the queue is 8.4 minutes, the average number of patients in the system is 6.48 and the average time a patient spends in the system is 9 minutes.

Table 4.7: Service times of patients at history section on Tuesday

CUSTOMER NO.	SERVICE TIME(minutes)			
	NURSE 1	NURSE 2	NURSE 3	NURSE 4
1	6	5	3	5
2	3	5	5	5
3	3	5	5	5
4	2	5	4	5
5	3	5	5	5
6	3	5		5
7	4	4		5
8	4	5		5
9	4	4		
10	4	6		
11	8			

From the table, the mean service rate μ (per server per minute) =4.6

Hence μ=13.04 patients per hour (This value was calculated using equation 3.4).

Number of servers=4

Thus the system utilization $W = \frac{3.18}{40} = 0.08 hours$

$$P_0 = \frac{1}{1+\frac{\left(\frac{41.5}{13.04}\right)^1}{1}+\frac{\left(\frac{41.5}{13.04}\right)^2}{2}+\frac{\left(\frac{41.5}{13.04}\right)^3}{6}+\frac{\left(\frac{41.5}{13.04}\right)^4}{24(1-0.8)}} = 0.028$$

The probability that a server is idle is $p_0+\frac{3}{4}p_1+\frac{2}{4}p_2+\frac{1}{4}p_3$

From (equation 3.15) $p_1 = 0.09$ $p_2 = 0.14$ and $p_3 = 0.15$

Hence $p_0 + \frac{3}{4}p_1 + \frac{2}{4}p_2 + \frac{1}{4}p_3 = 0.028 + \frac{3}{4}\times 0.09 + \frac{2}{4}\times 0.14 + \frac{1}{4}\times 0.15 = 0.20$

From (17) $$L_q = \frac{41.5\times 13.04\left(\frac{41.5}{13.04}\right)^4}{3!(4\times 13.04 - 41.5)^2}\times 0.028 = 2.28$$

$$W_q = \frac{2.28}{41.5} = 0.055 hours \text{ (3.3 minutes)}$$

$$L = 2.28 + \frac{41.5}{13.04} = 5.4$$

$$W = \frac{5.4}{41.5} = 0.13 hours \text{ (7.8minutes)}$$

The results show that the system utilization is 80 percent and the probability that a server is idle is 20 percent. The average number of patients in the queue is 2.28, the average time a patient spends in the queue is 3.3 minutes, the average number of patients in the system is 5.4 and the average time a patient spends in the system is 7.8 minutes.

Table 4.8: Service times of patients at the consulting rooms on Tuesday

CUSTOMER NO.	SERVICE TIMES(minutes)		
	DOCTOR 1	DOCTOR 2	DOCTOR 3
1	4	5	2
2	5	5	4
3	3	8	1
4	4	4	1
5	4	6	2
6	2	4	2
7	6	10	4
8	9	8	4
9	2	5	5
10	3		10
11	3		2
12	9		4
13			3
14			3
15			2
16			4
17			8

From Table 4.8, the mean service rate μ (per server per minute) =4.39

Hence μ =13.68 patients per hour (This value was calculated using equation 3.4).

Number of servers=3

Thus the system utilization $\rho = \frac{41.5}{3*13.68} = 1.01$

The system utilization $\rho > 1$ means that the mean arrival rate λ of patients is greater than the mean service rate μ of the three doctors.

4.2.3 Presentation of Data Collected on Wednesday.

The number of patients who entered the system between 8:00am to 12:00pm is 150.

From (equation 3.1), Arrival rate $\lambda = \frac{150}{4} = 37.5$ patients per hour

Table 4.9: Service times of patients at the records section on Wednesday

NO. OF FOLDERS	SERVICE TIME(minutes)
21	30
8	12

From Table 4.9, the mean service rate (μ) =41.4 patients per hour (This was calculated using equation 3.2 and 3.3)

Number of servers=1

Thus the system utilization $\rho = \frac{37.5}{41.4} = 0.91$

$$P_o = 1 - 0.91 = 0.09$$

$$L_q = \frac{(0.91)^2}{1-0.91} = 9.2$$

$$W_q = \frac{9.2}{37.5} = 0.25 hours \text{ (15minutes)}$$

$$L = \frac{37.5}{41.4 - 37.5} = 9.62$$

$$W = \frac{9.62}{37.5} = 0.26 hours \text{ (15.6minutes)}$$

The results show that at the records section on Wednesday, the server is busy 91 percent of the time and idle 9 percent of the time. The number of patients in the queue is 9.2, the average time a patient spends in a queue is 15 minutes, the average number of patients in the system is 9.62 and the average time a patient spends in the system is 15.6 minutes.

Table 4.10: service times of patients at the health insurance section on Wednesday

NO. OF FOLDERS	SERVICE TIME(minutes)
18	30
7	12

From Table 4.10, the mean service rate (μ) =45.45 patients per hour (This was calculated using equations 3.2 and 3.3)

Number of server=1

Thus the system utilization $\rho = \frac{37.5}{45.45} = 0.83$

$$P_0 = 1 - 0.83 = 0.17$$

$$L_q = \frac{(0.83)^2}{1-0.83} = 4.06$$

$$W_q = \frac{4.06}{37.5} = 0.108 hours \text{ (6.5minutes)}$$

$$L = \frac{37.5}{45.45-37.5} = 4.72$$

$$W = \frac{4.72}{37.5} = 0.13 hours \text{ (7.8minutes)}$$

The results show that at the insurance on Wednesday, the server is busy 83 percent of the time and idle 17 percent of the time. The average number of patients in the queue is 4.06, the average time a patient spends in the queue is 6.5 minutes, the average number of patients in the system is 4.72 and the average time a patient spends in the system is 7.8 minutes.

Table 4.11: Service times of patients at the history section on Wednesday

CUSTOMER NO.	SERVICE TIME(minutes)			
	NURSE 1	NURSE 2	NURSE 3	NURSE 4
1	5	6	5	7
2	1	3	3	5
3	5	5	2	3
4	9	5	5	9
5	2	3		6
6	3	4		5
7	2	4		6
8	4	5		6
9		4		
10		4		
11		3		
12		4		

From Table 4.11, the mean service rate μ (per server per minute) =4.5

Hence μ=13.33 patients per hour. (This value was calculated using equation 3.4)

Number of servers=4

Thus the system utilization $\rho = \frac{37.5}{4\times 13.33} = 0.7$

$$p_0 = \frac{1}{1+\frac{\left(\frac{37.5}{13.33}\right)^1}{1}+\frac{\left(\frac{37.5}{13.33}\right)^2}{2}+\frac{\left(\frac{37.5}{13.33}\right)^3}{6}+\frac{\left(\frac{37.5}{13.33}\right)^4}{24(1-0.70)}}=0.05$$

The probability that a server is idle is $p_0+\frac{3}{4}p_1+\frac{2}{4}p_2+\frac{1}{4}p_3$

From (equation 3.15) p_1 =0.14 p_2 = 0.20 and p_3 =0.18

Hence $p_0+\frac{3}{4}p_1+\frac{2}{4}p_2+\frac{1}{4}p_3=0.05+\frac{3}{4}\times 0.14+\frac{2}{4}\times 0.20+\frac{1}{4}\times 0.18=0.30$

$$L_q = \frac{37.5\times 13.33\times\left(\frac{37.5}{13.33}\right)^4}{6(4\times 13.33-37.5)^2}\times 0.05 = 1.04$$

$$W_q = \frac{1.04}{37.5} = 0.027\textit{hours}\ (1.66\text{minutes})$$

$$L = 1.04 + \frac{37.5}{13.33} = 3.85$$

$$W = \frac{3.85}{37.5} = 0.103 hours\,(6.16 minutes)$$

The results show that on Wednesday at the history section, the system utilization of the servers is 70 percent; the probability that a server is idle is 30 percent. More so the average number of patients in the queue is 1.04, the average time a patient spends in a queue is 1.66 minutes, the number of patients in the system is 3.85 and the average time a patient spends in the system is 6.16 minutes.

Table 4.12: Service times of patients at the consulting room on Wednesday

CUSTOMER NO.	SERVICE TIME(minutes)
	DOCTOR 1
1	5
2	3
3	2
4	4
5	3
6	2
7	3
8	5
9	2
10	2
11	2

From Table 4.11, mean service rate μ (per server per minute) =3

Hence $\mu = 20$ patients per hour (This value was calculated using equations 3.4)

Number of servers=1

Thus the system utilization $\rho = \frac{37.5}{20} = 1.88$

The system utilization $\rho > 1$ means that the mean arrival rate λ of patients is greater than the mean service rate μ of the doctor.

4.2.4 Presentation of Data on Thursday

The number of patients who entered the system between 8:00am to 12:00pm is 160

From (equation 3.1), mean arrival rate $\lambda = \frac{160}{4} = 40$ patients per hour

Table 4.13: Service times of patients at the records section on Thursday

NO. OF FOLDERS	SERVICE TIME(minutes)
8	16
4	10
10	15
7	10

From Table 4.13, the mean service rate (μ) =42.55 patients per hour.(This value was calculated using equations 3.2 and 3.3)

Number of servers=1

Thus the system utilization $\rho = \frac{40}{42.55} = 0.94$

$$P_0 = 1 - 0.94 = 0.06$$

$$L_q = \frac{(0.94)^2}{1-0.94} = 14.73$$

$$W_q = \frac{14.73}{40} = 0.37 (22.2\text{minutes})$$

$$L = \frac{40}{42.55-40} = 15.69$$

$$W = \frac{15.69}{40} = 0.39\textit{hours} (23.54\text{minutes})$$

The results show that on Thursday at the records section, the server is busy 94 percent of the time and idle 6 percent of the time. The average number of patients in the queue is 14.73, the average time a patient spends in the queue is 22.2 minutes, the average number of patients in the system is 15.69 and the average time a patient spends in the system is 23.54 minutes.

Table 4.14: Service times of patients at the health insurance section on Thursday

NO. OF FOLDERS	SERVICE TIME(minute)
7	5
12	19
10	11
7	6

From Table 4.14, the mean service rate (μ) =52.63patients per hour. (This value was calculated using equations 3.2 and 3.3)

Number of servers=1

Thus the system utilization $\rho = \dfrac{40}{52.63} = 0.76$

$$P_0 = 1 - 0.76 = 0.24$$

$$L_q = \frac{(0.76)^2}{1-0.76} = 2.4$$

$$W_q = \frac{2.4}{40} = 0.06\textit{hours}\ (3.6\text{minutes})$$

$$L = \frac{40}{52.63-40} = 3.18$$

$$W = \frac{3.18}{40} = 0.08\textit{hours}\ (4.8\text{minutes})$$

The results show that on Thursday at the insurance section, the server is busy 76 percent of the time and idle 14 percent of the time. More so, the average number of patients in the queue is 2.4, the average time a patient spends in the queue is 3.6 minutes, the average number of patients in the system is 3.18 and the average time a patient spends in the system is 4.8 minutes.

Table 4.15: Service times of patients at the history section on Thursday

CUSTOMER NO.	SERVICE TIME(minutes)			
	NURSE 1	NURSE 2	NURSE 3	NURSE 4
1	10	11	5	6
2	7	4	5	5
3	10	6	5	3
4	5	10	2	6
5	5	4	5	5
6	5	14	4	
7	5	10	2	
8	4		3	
9	7		5	
10	6		7	
11			2	
12			6	
13				
14				

From Table 4.15, the mean service rate, μ (per server per minute) =5.85

Hence μ=10.26 patients per hour. (This value was calculated using equation 3.4)

Number of servers=4

Thus the system utilization $\rho = \dfrac{40}{4\times 10.26} = 0.98$

$$P_0 = \frac{1}{1+\dfrac{\left(\dfrac{40}{10.26}\right)^1}{1}+\dfrac{\left(\dfrac{40}{10.26}\right)^2}{2}+\dfrac{\left(\dfrac{40}{10.26}\right)^3}{6}+\dfrac{\left(\dfrac{40}{10.26}\right)^4}{24(1-0.98)}} = 0.002$$

The probability that a server is idle is $p_0+\frac{3}{4}p_1+\frac{2}{4}p_2+\frac{1}{4}p_3$

From (equation 3.15) $p_1=0.01$ $p_2= 0.02$ and $p_3=0.02$

Hence $p_0+\frac{3}{4}p_1+\frac{2}{4}p_2+\frac{1}{4}p_3=0.002+\frac{3}{4}\times 0.01+\frac{2}{4}\times 0.02+\frac{1}{4}\times 0.02=0.02$

From (17) $$L_q=\frac{40\times 10.26\times\left(\dfrac{40}{10.26}\right)^4}{6(4\times 10.26-40)^2}\times 0.002=29.22$$

$$W_q = \frac{29.22}{40} = 0.73\text{Hours (43.8minutes)}$$

$$L = 29.22 + \frac{40}{10.26} = 33.12$$

$$W = \frac{33.12}{40} = 0.83\,hours\text{ (49.8minutes)}$$

The results show that on Thursday, at the history section the system utilization of a server is 98 percent and the probability that a server is idle is 2 percent. The average number of patients in the queue is 29.22; the average time a patient spends in the queue is 43.8 minutes, the average number of patients in the system is 33.12 and the average time a patient spends in the system is 49.8 minutes.

Table 4.16: Service times of patients at the consulting rooms on Thursday

CUSTOMER NO.	SERVICE TIME(minutes)		
	DOCTOR 1	DOCTOR 2	DOCTOR 3
1	3	3	3
2	5	4	9
3	3	1	4
4	5	7	5
5	3	9	5
6	4	2	2
7	6	2	3
8		3	3
9		2	3
10		1	6
11		4	
12		4	
13		2	
14		4	

From Table 4.16, the mean service rate μ, (per server per minute) =3.87

Hence μ=15.5 patients per hour (This value was calculated using equation 3.4)

Thus the system utilization $\rho = \frac{40}{3 \times 15.5} = 0.86$

$$P_0 = \frac{1}{1 + \frac{\left(\frac{40}{15.5}\right)^1}{1} + \frac{\left(\frac{40}{15.5}\right)^2}{2} + \frac{\left(\frac{40}{15.5}\right)^3}{6} + \frac{\left(\frac{40}{15.5}\right)^4}{24(1-0.86)}} = 0.044$$

The probability that a server is idle is $p_0 + \frac{2}{3}p_1 + \frac{1}{3}p_2$

From (equation 3.15) $p_1 = 0.11$ and $p_2 = 0.15$

Hence $p_0 + \frac{2}{3}p_1 + \frac{1}{3}p_2 = 0.044 + \frac{2}{3} \times 0.11 + \frac{1}{3} \times 0.15 = 0.17$

$$L_q = \frac{40 \times 15.5\left(\frac{40}{15.5}\right)^4}{6(4 \times 15.5 - 40)^2} \times 0.044 = 0.42$$

$$W_q = \frac{0.42}{40} = 0.011 hours\ (0.6\text{minutes})$$

$$L = 0.42 + \frac{40}{15.5} = 3.00$$

$$W = \frac{3.00}{40} = 0.08 hours\ (4.8\text{minutes})$$

The result show that on Thursday at the consulting rooms, the system utilization of a server is 86 percent and the probability that a server is idle is 17 percent. The average number of patients in the queue is 0.42, the average waiting time of a patient in the queue is 0.6 minutes, the average number of patients in the system is 3.00 and the average time a patient spends in the system is 4.8 minutes.

4.2.4 Presentation of Data Collected on Friday

The number of patients who entered the system between 8:00am to 12:00pm is 146

From (equation 3.1), Arrival rate (λ) $= \frac{146}{4} = 36.5$ patients per hour

Table 4.17: Service times of patients at the records section on Friday

NO. OF FOLDERS	SERVICE TIME(minutes)
10	14
12	18

From Table 4.17, the mean service rate μ=41.38 patients per hour (This value was calculated using equations 3.2 and 3.3)

Number of servers=1

Thus the system utilization $\rho = \frac{36.5}{41.38} = 0.88$

$$P_0 = 1 - 0.88 = 0.12$$

$$L_q = \frac{(0.88)^2}{1-0.88} = 6.5$$

$$W_q = \frac{6.5}{36.5} = 0.18 \text{ Hours (10.8minutes)}$$

$$L = \frac{36.5}{41.38 - 36.5} = 7.48$$

$$W = \frac{7.48}{36.5} = 0.20 \textit{hours} \text{ (12.30minutes)}$$

The results show that on Friday at records section, the server is busy 88 percent of the time and idle 12 percent of the time. The average number of patients in the queue is 6.5; the average time a patient spends in the queue is 10.8 minutes, the average number of patients in the system is 7.48 and the average time a patient spends in the system is 12.30 minutes.

Table 4.18: Service times of patients at the health insurance section on Friday

NO. OF FOLDERS	SERVICE TIME(minutes)
10	12
12	15

From Table 4.18, mean service $\mu = 48.8$ patients per hour. (This value was calculated using equations 3.2 and 3.3)

Number of servers=1

Thus the system utilization $\rho = \frac{36.5}{48.8} = 0.75$

$$P_0 = 1 - 0.75 = 0.25$$

$$L_q = \frac{(0.75)^2}{1-0.75} = 2.25$$

$$W_q = \frac{2.25}{36.5} = 0.06 hours\,(3.6\text{minutes})$$

$$L = \frac{36.5}{48.8-36.5} = 2.97$$

$$W = \frac{2.96}{36.5} = 0.081 hours\,(4.86\text{minutes})$$

The results show that on Friday at the insurance section, the server is busy 75 percent of the time and idle 25 percent of the time. More so, the average number of patients in the queue is 2.25, the average time a patient spends in the queue is 3.6 minutes, the average number of people in the system is 2.97 and the average time a patient spends in the queue is 4.86 minutes.

Table 4.19: Service times of patients at the history section on Friday

CUSTOMER NO.	SERVICE TIME(minutes)		
	NURSE 1	NURSE 2	NURSE 3
1	5	5	5
2	5	4	1
3	5	4	5
4	5	8	8
5	5	5	5
6	5	4	4
7	5	4	5
8	4	3	5
9	5	5	5
10	5	6	5
11	5		
12	5		

From Table 4.19, the mean service rate μ (per server per minute) =4.84

Hence μ=12.40 patients per hour (This value was calculated using equation 3.4)

Number of servers=3

Thus the system utilization $\rho = \dfrac{36.5}{3\times 12.40} = 0.98$

From (15) $$P_0 = \frac{1}{1+\frac{\left(\frac{36.5}{12.40}\right)^1}{1}+\frac{\left(\frac{36.5}{12.40}\right)^2}{2}+\frac{\left(\frac{36.5}{12.40}\right)^3}{6(1-0.98)}} = 0.0041$$

The probability that a server is idle is $p_0 + \frac{2}{3}p_1 + \frac{1}{3}p_2$

From (3.15) p_1 =0.01 and p_2 = 0.02

Hence $p_0 + \frac{2}{3}p_1 + \frac{1}{3}p_2 = 0.0041 + \frac{2}{3}\times 0.01 + \frac{1}{3}\times 0.02 = 0.02$

$$L_q = \frac{36.5\times 12.40\times\left(\frac{36.5}{12.40}\right)^3}{2(3\times 12.40-36.5)^2}\times 0.0041 = 48.29$$

$$W_q = \frac{48.29}{36.5} = 1.32\,\textit{hours}\ (79.2\text{minutes})$$

$$L = 48.29 + \frac{36.5}{12.40} = 51.23$$

$$W = \frac{51.23}{36.5} = 1.40 \textit{hours} \text{ (84minutes)}$$

The result show on Friday at the history section, the system utilization of the servers is 98 percent and the probability that a server is busy is 2 percent. The average number of patients in the is 48.29, the average time a patient spends in the queue is 79.2 minutes, the average number of patients in the system is 51.23 and the average time a patient spends in the system is 84 minutes.

Table 4.20: Service times of patients at the consulting rooms on Friday

CUSTOMER NO.	SERVICE TIME(minutes)		
	DOCTOR 1	DOCTOR 2	DOCTOR 3
1	6	7	2
2	6	2	4
3	4	3	5
4	4	3	3
5	4	2	3
6	4	8	3
7	6	4	2
8	3	4	7
9	2		
10	4		
11	11		

From the table 4.20, the mean service rate μ (per server per minute) =4.37

Thus μ=13.73 patients per hour. (This value was calculated using equation 3.4)

Number of servers=3

Hence the system utilization $\rho = \frac{36.5}{3 \times 13.73} = 0.89$

$$P_0 = \frac{1}{1 + \frac{\left(\frac{36.5}{13.73}\right)^1}{1} + \frac{\left(\frac{36.5}{13.73}\right)^2}{2} + \frac{\left(\frac{36.5}{13.73}\right)^3}{6(1-0.89)}} = 0.028$$

The probability that a server is idle is $p_0 + \frac{2}{3}p_1 + \frac{1}{3}p_2$

From (equation 3.15) $p_1 = 0.07$ and $p_2 = 0.10$

Hence $p_0 + \frac{2}{3}p_1 + \frac{1}{3}p_2 = 0.028 + \frac{2}{3}\times 0.07 + \frac{1}{3}\times 0.10 = 0.11$

From (17) $$L_q = \frac{36.5\times 13.73\times\left(\frac{36.5}{13.73}\right)^3}{2(3\times 13.73 - 36.5)^2}\times 0.028 = 5.99$$

$$W_q = \frac{5.99}{36.5} = 0.16\,hour\ (9.6\text{minutes})$$

$$L = 5.99 + \frac{36.5}{13.73} = 8.65$$

$$W = \frac{8.65}{36.5} = 0.24\,hours\ (14.4\text{minutes})$$

The results show that on Friday at the consulting rooms, the system utilization of the servers is 89 percent and the probability that a server is idle is 11 percent. More so, the average number of patients in queue is 5.99, the average time a patient spends in the queue is 9.6 minutes, the average number of patients in the system is 8.65 and the average time a patient spends in the system is 14.4 minutes.

CHAPTER FIVE

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter, the discussions of key findings of the study are presented. These results are then summarized in a conclusion and pertinent recommendations made.

5.2 DISCUSSION OF RESULTS

Comparing the mean arrival rate of patients per hour on the days that the study was undertaken(see Figure 5.1), it was observed that Tuesday had the highest arrival rate(41.5 patients per hour) and Friday had the least arrival rate(36.5 patients per hour). Thus, from 8:00am to 12:00pm on Tuesday and Friday, 166 patients and 146 patients respectively would have entered the system.

Inferring from figure 5.2, it can be said that the insurance section has the least mean service rate followed by the records section, consulting rooms and history section respectively.

Analysis of the system utilization of the servers at the records section, insurance section, history section and consulting rooms (see Figure 5.3) shows that the server at the records section is the busiest on Monday. On Tuesday and Wednesday, the servers at the consulting rooms are the busiest whiles on Thursday and Friday, the servers at the history section are the busiest.

The system utilization of the servers at the consulting rooms on Tuesday and Wednesday is greater than one (steady-state probability does not exist). What this means is that, the mean arrival rate of the patients is greater than the mean service rate of the servers. On Tuesday for example, the number of patients in the system at the consulting rooms increases by 0.42 patients every hour whiles on Wednesday, the number of patients in the system at the consulting rooms increases by 17.5 patients every hour. Thus from 8:00am to 12:00pm, the number of patients in the system at the consulting rooms on these two days would have increased by 1.68 patients 70 patients respectively. This situation (especially on Wednesday) would make the system congested and crowded since the queue length at the consulting rooms would be too long.

Suppose the number of servers in the consulting rooms on Tuesday is increased from three to four, the system utilization of the servers becomes 75.84 percent (the arrival rate of servers is less than the service rate of servers). The number of patients in the queue becomes 1.83, the average time a patient spends in the queue would be 2.65 minutes, the average number of patients in the system would be 4.86 and the average time a patient spends in the system would be 7 minutes. Suppose the mean service rate on Tuesday at the consulting rooms is instead reduced from 4.39minutes to 3.39minutes, the system utilization of the servers becomes 78.15 percent. The queue length then becomes 2.10, the average time a patient spends in the queue becomes 3.04 minutes, the average number of patient spends in the system becomes 4.4 and the average time a patient spends in the system would be 6.36 minutes.

On Wednesday, suppose we add another server to the server in the consulting room, the system utilization of the server becomes 94 percent. The average number of patients in the queue becomes 12.72, the average time a patient spends in the queue becomes 20.35 minutes, the average number of patients in the system becomes 14.60 and average time a patient spends in the system becomes 23.36 minutes. Suppose the mean service rate is rather reduced from 3 minutes to 1.5 minutes, the system utilization of the servers becomes 93.40. The queue length becomes 14.73, the average time a patient spends in the queue becomes 23.56 minutes, the average number of patients in the system becomes 15 and the average time a patient spends in the system becomes 24 minutes.

As shown on Tuesday and Wednesday, increasing the number of servers and reducing the mean service rate of servers are methods used to make a system reach a steady state. Reducing the mean service rate of servers to make a system reach a steady state is cost effective as compared to increasing the number of servers to make a system reach a steady state. In a hospital setting, since proper diagnosis of patients is required (especially at the history section and consulting rooms), the average service rate would be the best guide to determine whether to increase the number of servers or reduce the mean service rate to make the system reach a steady state. On the average, a patient spends four minutes at the consulting rooms from Monday to Friday. thus it would therefore not be prudent to reduce the mean service rate(as was done) to make the system reach a steady state since doing that would affect the proper diagnosis of patients. So increasing the number of servers for the system to reach a steady state would be the best option.

Analysis of the system utilization of the servers also shows that as the busy time of the servers becomes high(less idle), the queue length becomes long thus making patients spend more time in the queue. For instance, on Friday the history section and the consulting rooms have three servers each but the system utilization of the servers are 98 percent and 89 percent respectively. At the history section, the queue length and the average time a patient spends in the queue is 48.29 and 72.2 minutes respectively whiles at the consulting rooms, the queue length and the average time a patients spends in the queue is 5.99 and 9.6 respectively.

On the average the number of minutes a patient spends in the system before seeing a doctor is as follows: 18.72 minutes on Monday, 35.4 minutes on Tuesday, 29.56 minutes on Wednesday, 78.18 minutes on Thursday and 101.16 minutes on Friday. The above results show that a patient spends more time in the system on Friday than the other days.

The total time a patient spends in the system is equal to the addition of the average times a patient spends at each of the four sections of the outpatients department. Thus on the average, the total time a patient spends in the system are as follows: 24.3 minutes on Monday, 82.89 minutes on Thursday and 115.56 minutes on Friday.

On Tuesday and Wednesday, the total time a patient spends in the system is difficult to calculate since steady state probability distribution does not exist in the consulting rooms on both days. However, suppose we use the assumptions in paragraph four and five of this section, then total time a patient spends in the system on these days (Tuesday and Wednesday) are 42.4 minutes and 52.92 respectively(when the servers are increased).

Relating the arrival rate of patients to the total time patients spend in the system, we see that though patients spent more in the system on Friday than the other days, the mean arrival rate on this day was the least. The reason for this situation was that the mean arrival rate on this day was very high at the various sections as compared to the other days. More so, the number of servers at the history section and consulting rooms were three each as compared to four servers on the other days.

It is important to note that the queue lengths at the history section and the consulting rooms may in actual sense be higher than the ones calculated from the results. The first reason for this situation could be that during the study, most of the servers did not start attending to the patients at the same time. The second reason is that, where as some servers attended to ten patients in an hour, others attended to five. Moreover after patients had been attended to at the history section, they had to wait for some time before been attended to by a doctor.

Fig 5.1:A Graph of the arrival rate from monday to Friday

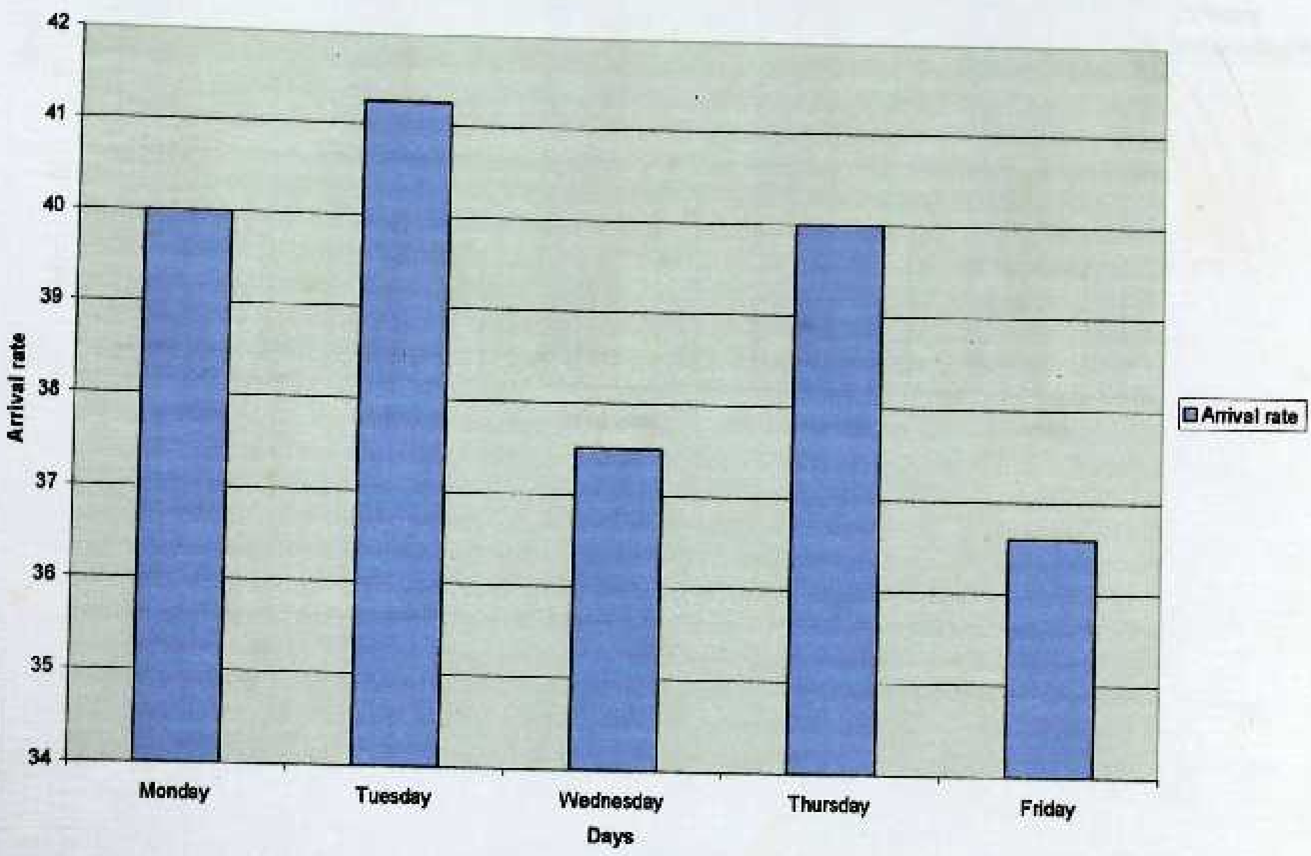


Fig 5.2: A graph of the mean service rate of a server at the various sections from Monday to Friday

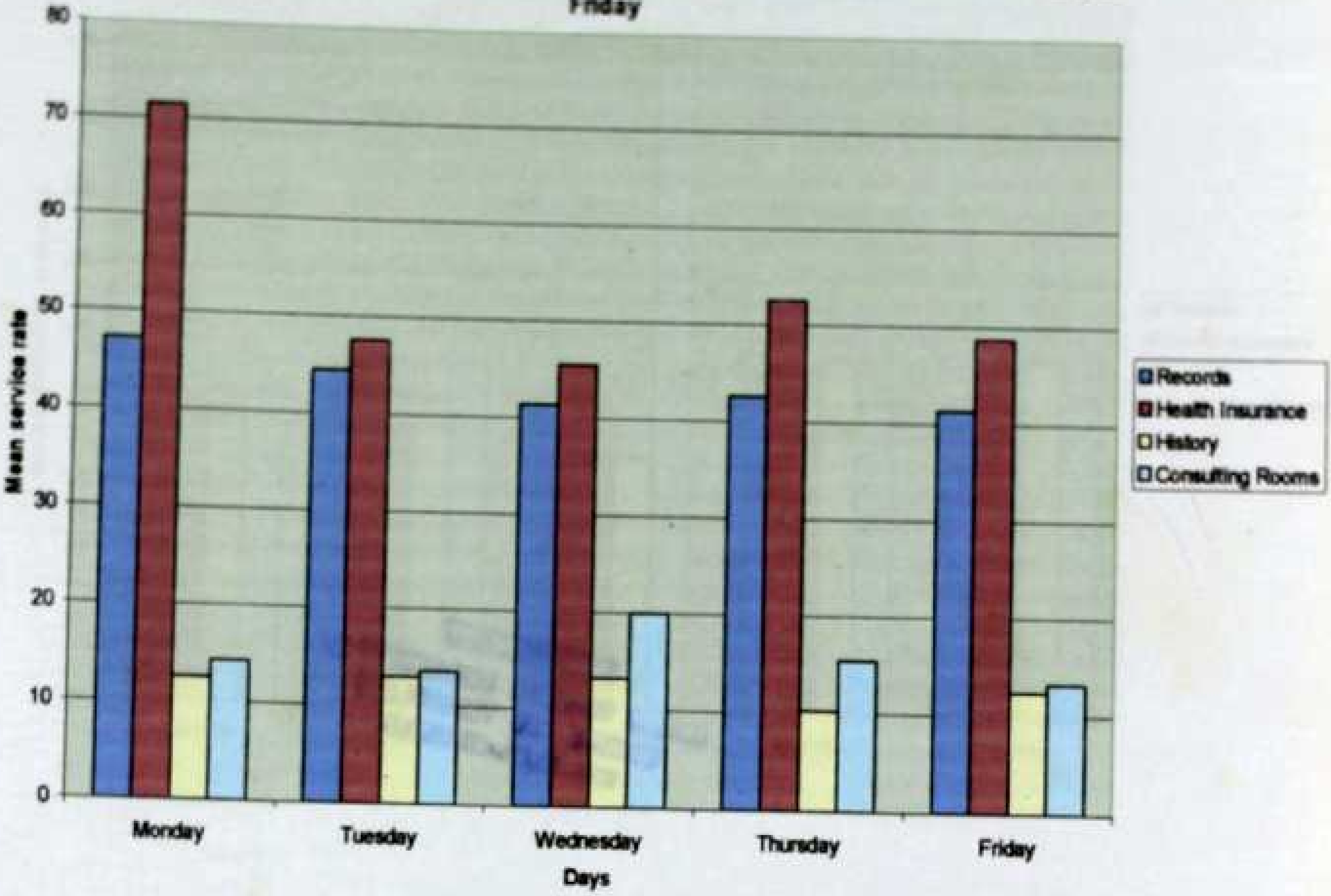
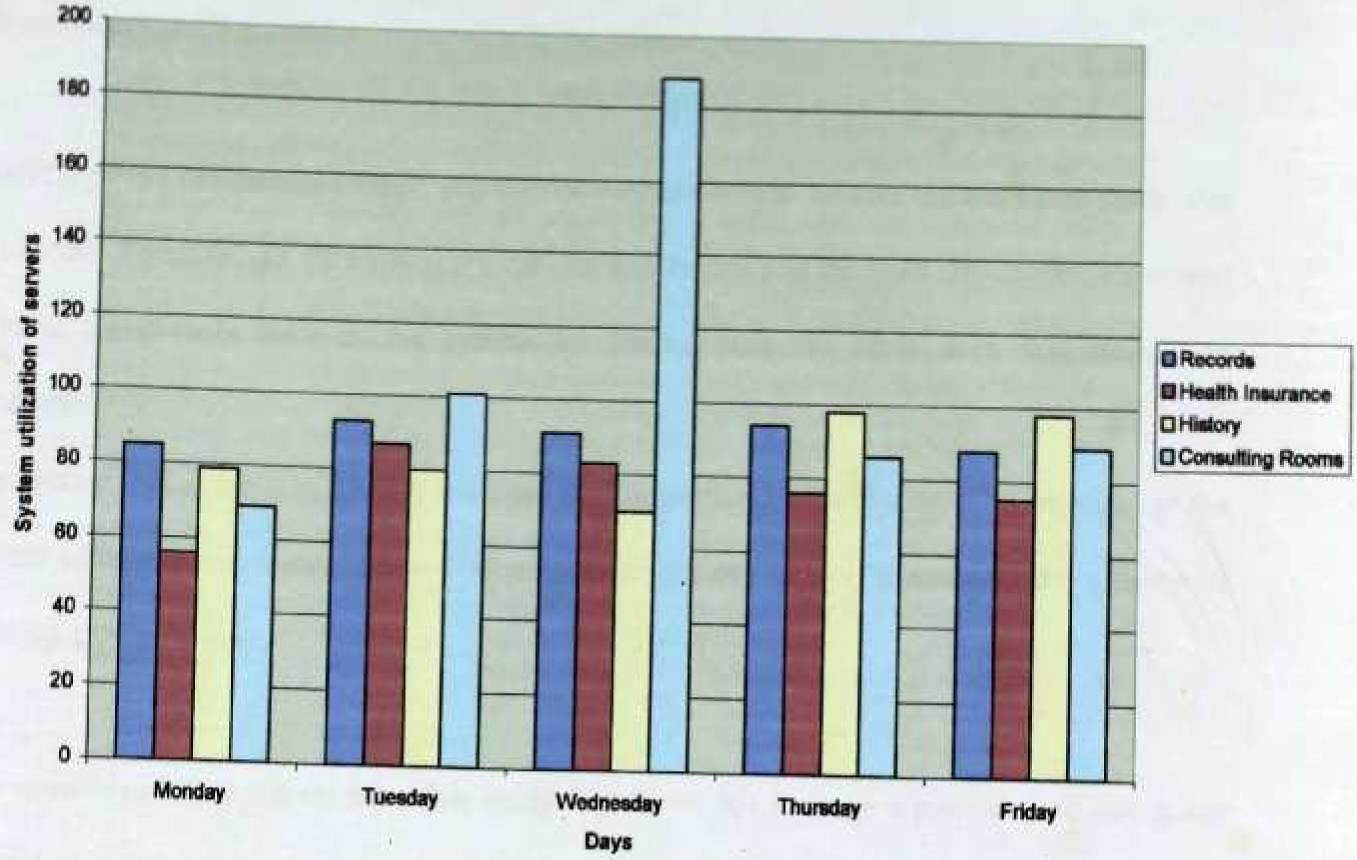


Fig. 5.3:A graph of the system utilization of servers at the various section from Monday to Friday



5.3 CONCLUSION

The management of hospital facilities such as the outpatient department is very complex and demanding because of long queues and congestions in the system. The most common objectives of hospitals have included the reduction of patient's time in the system, improvement on customer service, better resource utilization and reduction of operating cost. To achieve these objectives, we make use of queuing theory.

The primary objective of the study was to look at the queue situation at the outpatient department of Kwahu Government Hospital-Atibie. In looking at the queue situation at the outpatient department of the hospital the study looked at the arrival rate of patients, service rate of servers and the system utilization of servers. These parameters were then used to determine the measures of performance of the system such as waiting time of patients in the queue/system and the number of patients in the queue/system.

Analysis of the results shows that, there exist variable arrival rate on the days that study was conducted. Tuesday had the highest arrival rate and Friday had the least arrival rate. However, patients spent more time in the system on Friday than the other days that study was conducted.

The system utilization was used to measure the congestion of the system. For instance, as the system utilization increases, (near 100 percent or greater than 100 percent) the amount of queuing also increases.

The main conclusion drawn from this study is that, on the average a patient need not spend more than two hours at the outpatient department of this hospital. However, servers who do not start work on the same time could make patients spend more time than expected.

5.4 RECOMMENDATIONS

Hospital settings are confronted with the problems of long queues and congestions. When these problems are not well managed, any hospital setting would not achieve it broad objective of providing assessable and quality healthcare to its patients. In the view this research, when the following suggestions are adhered to by management of the hospital it would help them achieve their objectives.

When utilization is high, waiting times could be minimized by giving priority to patients who require shorter service times. Moreover patients with ailment which require regular check ups should be given specific days to attend hospitals.

Since system utilization measures the performance of any system, it is important to have a fixed target utilization rate for all the sections at the outpatients department. This fixed utilization rate should be the utilization rate, which would make patients spend less time in the system. Hence having system utilization above this fixed utilization rate means patients would spend more time in the system.

Since management of the hospital has a fair idea of the mean arrival rate of patients to the outpatient department of the hospital, there should be a known average service rate at the various sections. Servers especially at the history section and consulting rooms should take the start and finish times of patients when attending to them. This known average service rate would serve as check to servers.

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