APPLICATION OF GAME THEORY IN FINANCIAL MANAGEMENT
(A CASE STUDY OF OAK FINANCIAL SERVICES)

BY

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DECLARATION

I hereby declare that the submission of this compilation is the true findings of my own researched work presented towards an award of a second degree in the Industrial Mathematics and that, to the best of my knowledge, it contains no material previously published by another person nor submitted to any other University or institution for the award of degree except where due acknowledgement has been made in text .However, references from the work of others have been clearly stated.

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Dedication

I dedicate to my parent, Wife and my darling boy kwame Nimo Asiedu

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Any limitations in this book, however are exclusively mine, but the good comments must be shared among those named above.

Abstract

A computational study of Game Theory applied to investment decisions in optimal portfolio selection Problem is considered. Emphasis will be placed on investment decision problem, which is modeled as Game Theory Problem. Data from Oak financial Service for 2012 is examined.

The decision – maker has to select at least one option from all possible options in which he can invest. The problem here is to decide what action or a combination of actions to take among the various possible options with the given rates of return.

The solution to game theory application in financial investment planning is effective in giving optimal solution as compared with personal discretion means of investment by an investor. From the concept of investment using game theory, the solution to this problem consists of many feasible options investment opportunities where an investor can invest where the limit of the investment amount is not violated.

According to the developed model, the value of the game from the various investment options was 5.3 percent growth rate in mutual fund and bonds. The solution shown gave remarkably better results than the independent model normally used by the institution. We therefore recommend that our model should be adopted by the institution for its investment planning.

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CHAPTER 1

1.0 INTRODUCTION

Finance is concerned with how the savings of investors are allocated through financial markets and intermediaries to firms, which use them to fund their activities. Finance can be broadly divided into two fields. The first is asset pricing, which is concerned with the decisions of investors. The second is corporate finance, which is concerned with the decisions of firms. Traditional neoclassical economics did not attach much importance to either kind of finance. It was more concerned with the production, pricing and allocation of inputs and outputs and the operation of the markets for these. Models assumed certainty and in this context financial decisions are relatively straightforward. However, even with this simple methodology important concepts such as the time value of money and discounting were developed. Finance developed as a field in its own right with the introduction of uncertainty into asset pricing and the recognition that classical analysis failed to explain many aspects of corporate finance.

We shall give an overview of how a first generation of game theory models tackled those problems, and discuss the successes and failures. We shall discuss some of the main issues in finance and how game theory has played a major role in overcoming it.

1.1 BACKGROUND OF STUDY

1.1.1 The Main Issues in Finance

1.1.2 Asset Pricing

The focus of Keynesian macroeconomics on uncertainty and the operation of financial markets lead to the development of frameworks for analyzing risk. Keynes (1936) and Hicks (1939) took account of risk by adding a risk premium to the interest rate. However, there was no systematic theory underlying this risk premium. The key theoretical development which eventually leads to such a theory was von Neumann and Morgenstern's (1947) axiomatic approach to choice under uncertainty. Their notion of expected utility, developed originally for use in game theory, underlies the vast majority of theories of asset pricing.

1.1.3 Portfolio Theory and the Capital Asset Pricing Model

Markowitz (1952; 1959) utilized a special case of von Neumann and Morgenstern's expected utility to develop a theory of portfolio choice. He considered the case where investors are only concerned with the mean and variance of the payoffs of the portfolios they are choosing. This is a special case of expected utility provided the investor's utility of consumption is quadratic and/or asset returns are multinormally distributed. Markowitz's main result was to show that diversifying holdings is optimal and the benefit that can be obtained depends on the covariances of asset returns. Tobin's (1958) work on liquidity preference helped to establish the mean-variance framework as the standard approach to portfolio choice problems. Subsequent authors developed portfolio theory considerably (see Constantinides and Malliaris (1995)).

It was not until sometime after Markowitz's original contribution that his framework of individual portfolio choice was used as the basis for an equilibrium theory, namely the Capital Asset pricing Model (CAPM). Brennan (1989) has argued that the reason for the delay was the boldness of the assumption that all investors have the same beliefs about the means and variances of all assets. Sharpe (1964) and Lintner (1965) showed that in equilibrium

$$Er_i = r_f + \beta_i(Er_M - r_F),$$

Where

Er_i is the expected return on asset i,

r_f is the return on the risk free asset,

Er_M is the expected return on the market portfolio (i.e. a value weighted portfolio of all the assets in the market) and

β_i = cov (r_i r_M) / var (r_M).

Black (1972) demonstrated that the same relationship held even if no risk free asset existed provided r_F was replaced by the expected return on a portfolio or asset with b = 0. The model formalizes the risk premium of Keynes and Hicks and shows that it depends on the covariance of returns with other assets.

Despite being based on the very strong assumptions of mean-variance preferences and homogeneity of investor beliefs, the CAPM was an extremely important development in finance. It not only provided key theoretical insights concerning the pricing of stocks but also lead to a great deal of empirical work testing whether these predictions held in practice. Early tests such as Fama and Macbeth (1973) provided some support for the model. Subsequent tests using more sophisticated econometric techniques have not been so encouraging. Ferson (1995) contains a review of these tests. The CAPM is only one of many asset-pricing models that has been developed.

Other models include the Arbitrage Pricing Theory (APT) of Ross (1977a) and the representative agent asset-pricing model of Lucas (1978). However, the CAPM was the most important not only because it was useful in its own right for such things as deriving discount rates for capital budgeting but also because it allowed investigators to easily adjusts for risk when considering a variety of topics. We turn next to one of the most important hypotheses that resulted from this ability to adjust for risk.

1.1.4 Market Efficiency

In models with competitive markets, symmetric information and no frictions like transaction costs, the only variations in returns across assets are due to differences in risk. All information that is available to investors becomes reflected in stock prices and no investor can earn higher returns except by bearing more risk. In the CAPM, for example, it is only differences in b's that cause differences in returns. The idea that the differences in returns are due to differences in risk came to be known as the Efficient Markets Hypothesis. During the 1960's a considerable amount of research was undertaken to see whether U.S. stock markets were in fact efficient. In a well-known survey, Fama (1970) argued that the balance of the evidence suggested markets were efficient. In a follow up piece, Fama (1991) continued to argue that by and large markets were efficient despite the documentation of many anomalies during the intervening period.

Standard tests of market efficiency involve a joint test of market efficiency and the equilibrium asset-pricing model that is used in the analysis. Hence a rejection of the joint hypothesis can either be a rejection of market efficiency or the asset-pricing model used or both. Hawawini and Keim (1995) surveyed these “anomalies.” Basu (1977) discovered one of the first. He pointed out that price to earnings (P/E) ratios provided more explanatory power than b’s. Firms with low P/E ratios (value stocks) tend to outperform stocks with high P/E ratios (growth stocks).

Banz (1981) showed that there was a significant relationship between the market value of common equity and returns (the size effect). Stattman (1980) and others have demonstrated the significant predictive ability of price per share to book value Per Share (P/B) ratios for returns. In an influential paper, Fama and French (1993) have documented that firm size and the ratio of book to market equity are important factors in explaining average stock returns. In addition to these cross-sectional effects there are also a number of significant time series anomalies.Perhaps the best known of these is the January effect.

Rozeff and Kinney (1976) found that returns on an equal weighted index of NYSE stocks were much higher in January than in the other months of the year. Keim (1983) demonstrated that the size effect was concentrated in January. Cross (1973) and French (1980) pointed out that the returns on S&P composite index are negative on Mondays. Numerous other studies have confirmed this weekend effect in a wide variety of circumstances.

These anomalies are difficult to reconcile with models of asset pricing such as the CAPM. Most of them are little understood. Attempts have been made to explain the January effect by tax loss selling at the end of the year. Even this is problematic because in countries such as the U.K. and Australia where the tax year does not end in December there is still a January effect. It would seem that the simple frameworks most asset pricing models adopt are not sufficient to capture the richness of the processes underlying stock price formation.

Instead of trying to reconcile these anomalies with asset pricing theories based on rational behavior, a number of authors have sought to explain them using behavioural theories based on foundations taken from the psychology literature. For example, Dreman (1982) argues that the P/E effect can be explained by investors' tendency to make extreme forecasts. High (low) P/E ratio stocks correspond to a forecast of high (low) growth by the market. If investors predict too high (low) growth, high P/E stocks will underperform (overperform). De Bondt and Thaler (1995) surveys behavioural explanations for this and other anomalies.

1.1.5 Continuous Time Models

Perhaps the most significant advance in asset pricing theory since the early models were formulated was the extension of the paradigm to allow for continuous trading. This approach was developed in a series of papers by Merton (1969; 1971; 1973a) and culminated in his development of the inter-temporal capital asset pricing model (ICAPM). The assumptions of expected utility maximization, symmetric information and frictionless markets are maintained. By analyzing both the consumption and portfolio decisions of an investor through time and assuming prices per share are generated by Ito processes, greater realism and tractability compared to the mean variance approach is achieved. In particular, it is not necessary to assume quadratic utility or normally distributed returns. Other important contributions that were developed using this framework were Breeden's (1979) Consumption CAPM and Cox, Ingersoll and Ross's (1985) modeling of the term structure of interest rates.

The relationship between continuous time models and the Arrow–Debreu general equilibrium model was considered by Harrison and Kreps (1979) and Duffie and Huang (1985). Repeated trading allows markets to be made effectively complete even though there are only a few securities. One of the most important uses of continuous time techniques is for the pricing of derivative securities such as options. This was pioneered by Merton (1973b) and Black and Scholes (1973) and lead to the development of a large literature that is surveyed in Ross (1992). Not only has this work provided great theoretical insight but it has also proved to be empirically implementable and of great practical use.

1.1.6 Corporate Finance

The second important area considered by finance is concerned with the financial decisions made by firms. These include the choice between debt and equity and the amount to pay out in dividends. The seminal work in this area was Modigliani and Miller (1958) and Miller and Modigliani (1961). They showed that with perfect markets (i.e., no frictions and symmetric information) and no taxes the total value of a firm is independent of its debt/equity ratio. Similarly they demonstrated that the value of the firm is independent of the level of dividends. In their framework it is the investment decisions of the firm that are important in determining its total value.

The importance of the Modigliani and Miller theorems was not as a description of reality. Instead it was to stress the importance of taxes and capital market imperfections in determining corporate financial policies. Incorporating tax deductibility of interest, but not dividends and bankruptcy costs, leads to the trade-off theory of capital structure. Some debt is desirable because of the tax shield arising from interest deductibility but the costs of bankruptcy and financial distress limit the amount that should be used. With regard to dividend policy, incorporating the fact that capital gains are taxed less at the personal level than dividends into the Modigliani and Miller framework gives the result that all payouts should be made by repurchasing shares rather than by paying dividends.

The trade-off theory of capital structure does not provide a satisfactory explanation of what firms do in practice. The tax advantage of debt relative to the magnitude of expected bankruptcy costs would seem to be such that firms should use more debt than is actually observed. Attempt to explain this, such as Miller (1977) that incorporate personal as well as corporate taxes into the theory of capital structure, have not been successful. In the Miller model there is a personal tax advantage to equity because capital gains are only taxed on realization and a corporate tax advantage to debt because interest is tax deductible. In equilibrium, people with personal tax rates above the corporate tax rate hold equity while those with rates below hold debt. This prediction is not consistent with what occurred in the United State in the late 1980's and early 1990's when there were no personal tax rates above the corporate rate. The Miller model suggests that there should have been a very large increase in the amount of debt used by corporations but there was only a small change.

The tax-augmented theory of dividends also does not provide a good explanation of what actually happens. Firms have paid out a substantial amount of their earnings as dividends for many decades. Attempts to explain the puzzle using tax based theories such as the clientele model have not been found convincing. They are difficult to reconcile with the fact that many people in high tax brackets hold large amounts of dividend paying stocks and on the margin pay significant taxes on the dividends.

Within the Modigliani and Miller framework other corporate financial decisions also do not create value except through tax effects and reductions in frictions such as transaction costs. Although theoretical insights are provided, the theories are not consistent with what is observed in practice. As with the asset pricing models discussed above this is perhaps not surprising given their simplicity. In particular, the assumptions of perfect information and perfect markets are very strong.

1.1.7. The Game Theory Approach

The inability of standard finance theories to provide satisfactory explanations for observed phenomena lead to a search for theories using new methodologies. This was particularly true in corporate finance where the existing models were so clearly unsatisfactory. Game theory has provided a methodology that has lead to insights into many previously unexplained phenomena by allowing asymmetric information and strategic interaction to be incorporated into the analysis. We start with a discussion of the use of game theory in corporate finance where to date it has been most successfully applied. We subsequently consider its role in asset pricing.

Game theory is a branch of applied mathematics concerned with how agents interact with one another through the choices they make. This interaction, or game, is both independent – made by autonomous agents – and interdependent – the outcome relies on the combination of choices made by the agents (Kelly 2003). These are strategy games where the knowledge of all possible outcomes is readily available to the player at the time of choice making, as opposed to games of chance where the outcome can be determined in whole or in part by a randomized factor.

While game theory typically applies practically to economics and mathematics, it has found wide use in a variety of fields such as politics, evolutionary biology, psychology and finance. Games as entertainment can also learn from the discoveries provided by game theory, especially as more Massively Multiplayer Online games (MMOs) create large social and economic systems and need to balance choices and interactions among the player population. Tapping into the patterns of choice among humans can aid developers in creating more meaningful player-driven game play, regardless if that game play is cooperative or competitive in nature.

1.1.8 Zero-Sum Games

Game theory already applies to many aspects of real-life activities. Zero-sum games are games where the total net wins cancels out the total net losses. An example of this would be a two-player game where one player always wins and one player always loses, or else they both tie. This game play can be seen in any one-versus-one fighting game such as Soul Caliber (Namco 1998) or Mortal Kombat (Midway 1992). The main characteristics of zero-sum games are that "the interests of the two players are always strictly opposed and competitive, with no possibility of, or benefit in, cooperation" (Kelly 2003). Cooperation, however, can be found in non-zero sum games. The total outcome for these games does not equal zero, allowing multiple players to win or lose with variations on each. Racing games often use this system where there are first, second, and third place winners. The possibility for more than one player to win allows the opportunity for cooperative strategies to form.

1.1.9 Non-Zero Sum Games

The most famous non-zero sum game is the Prisoner's Dilemma. Considered the standard model for the evolution of cooperation, the Prisoner's Dilemma presents the situation of two prisoners in separate rooms given the choice to betray their comrade in exchange for a lower sentence, or stay silent to serve minimal time for their crime. The complexity arises when we take into account both prisoners' choices. If both of them stay silent, they each serve six months for their crime. If one of them stays silent and the other betrays, then the betrayer can go free and the silent prisoner serves ten years. However, if they both betray one another, they both serve five years (Elizabeth, 2006).

A rational player, when taking into account another player's choice, will choose to betray out of defense against serving ten years and the possibility of going free (Workman 2004). However, if each player is rational each chooses to betray, giving them a net loss when the combined cooperative strategy would be a better overall outcome. In game theory, a Nash Equilibrium is the optimal solution for both players where neither can gain anything by changing their and only their decisions. The option to betray is the Nash equilibrium for the Prisoner's Dilemma.

While the Prisoner's Dilemma is the formal name for this decision matrix and refers to the original wording of the game, its mathematical representation applies across a range of possible situations. The terms cooperate and defect replace stay silent or betray and each outcome translates to a numerical pay-off matrix.

A true Prisoner's Dilemma must meet certain conditions in order to be accurate to the mathematical game. The players must be aware of all possible choices (cooperate or defect) and the consequences of each. Player may not communicate with one another and may not know the choice the other player makes until after both choices are made.

‘‘Interactive Decision Theory’’ would perhaps be a more descriptive name for the discipline usually called Game Theory. This discipline concerns the behaviour of decision makers (players) whose decisions affect each other. As in non-interactive (one-person) decision theory, the analysis is from a rational, rather than a psychological or sociological viewpoint.

The term ‘‘Game Theory’’ stems from the formal resemblance of interactive decision problems (games) to parlor games such as chess, bridge, poker, monopoly, diplomacy or battleship. The term also underscores the rational, ‘‘cold,’’ calculating nature of the analysis. The major applications of game theory are to economics, political science (on both the national and international levels), tactical and strategic military problems, evolutionary biology, and, most recently, computer science. There are also important connections with accounting, statistics, the foundations of mathematics, social psychology, and branches of philosophy such as epistemology and ethics. Game theory is a sort of umbrella or ‘‘unified field’’ theory for the rational side of social science, where ‘‘social’’ is interpreted broadly, to include human as well as non-human players (computers, animals, plants).

In the earliest years, game theory was preoccupied with strictly competitive games, more commonly known as two-person zero-sum games. In these games, there is no point in cooperation or joint action of any kind: if one outcome is preferred to another by one player, then the preference is necessarily reversed for the other. This is the case for most two-person parlor games, such as chess or two-sided poker; but it seems inappropriate for most economic or political applications. Nevertheless, the study of the strictly competitive case has, over the years, turned out remarkably fruitful; many of the concepts and results generated in connection with this case are in fact much more widely applicable, and have become cornerstones of the more general theory.

1.2 PROBLEM STATEMENT

The problem with Finance is how the savings of investors are allocated through financial markets and intermediaries to firms, which use them to fund their activities. Finance can be broadly divided into two fields, namely; asset pricing, which is concerned with the decisions of investors and corporate finance, which is concerned with the decisions of firms. Traditional neoclassical economics did not attach much importance to either kind of finance. It was more concerned with the production, pricing and allocation of inputs and outputs and the operation of the markets for these. Models assumed certainty and in this context financial decisions are relatively straightforward. However, even with this simple methodology important concepts such as the time value of money and discounting were developed.

Our study focuses on the use of game theoretic approach for managing financial investments of investors.

1.3 OBJECTIVES

The objectives of this study are :

(i) To model investment of investors as a game theoretic mathematical problem that maximizes the returns from their investments.

(ii) To determine which investment options will give the investor an optimum investment yield.

(iii) To determine the optimality in the investment policies.

1.4 METHODOLOGY

Game theory bears a strong relationship to Linear Programming (LP), since every finite two person zero sum game can be expressed as a LP and conversely every LP can be expressed as a game.

If the problem has no saddle point, dominance is unsuccessful to reduce the game and the method of matrices also fails, then LP offers the best method of solution. So far several authors namely Bansal (1980), Martin (2002), Stephen (2000), Thedor (2001) and many other authors proposed different types of theoretical discussion of game problems with their strategies also.

In this study, we will discuss some methods and definitions of LP and game theory with some relevant theorems and propositions. We shall also give a discussion of simplex method and Minimax-Maximin method for solving game problems. A short discussion of rectangular 2×2 game would also be given.

1.5 JUSTIFICATION

In decision analysis, the decision-maker has to select at least and at most one option from all possible options. This certainly limits its scope and its applications..

The mathematical game theoretic models and techniques considered in decision analysis are concerned with prescriptive theories of choice (action). This answers the question of exactly how a decision maker should behave when faced with a choice between those actions which have outcomes governed by chance, or the actions of competitors, hence the reason for the study.

1.6 Significance of the study

The findings of the study will provide well–researched information, which can be useful to researchers for academic purposes in the area of investment management. To the investors, the study hopes to provide them with useful information like the recommended options of investments to obtain optimum return from their investment. To the investment company, the recommendations of the study may enable them to design investment management policies to improve the smooth running of the firm, thereby satisfying customers and generally minimizing risk.

1.7 SCOPE OF THE STUDY

The scope of the study will be limited to the impact of investment management in the capital market of an organization. The study will be carried out at Oak Financial Services.

1.8 ORGANIZATION OF THE THESIS

In chapter one, we presented a background study of finance and game theoretic models.

In chapter two, related work in game theoretic model in financial management would be discussed.

In chapter three, the game theoretic optimization procedures and methods that would be applied in solving our problem will be introduced and explained.

Chapter four will provide a computational study of the algorithm applied to our financial investment management instances.

Chapter five will conclude this thesis with additional comments and recommend

CHAPTER TWO

LITERATURE REVIEW

The thorniest issue in finance has been what Black (1976) termed “the dividend puzzle.” Firms have historically paid out about a half of their earnings as dividends.
Many of these dividends were received by investors in high tax brackets who, on the margin, paid substantial amounts of taxes on them. In addition, in a classic study Lintner (1956) demonstrated through game theoretical model that managers “smooth” dividends are less variable than earnings. This finding was confirmed by Fama and Babiak (1968) and numerous other authors.

In their original article on dividends, Miller and Modigliani (1961) had suggested that dividends might convey significant information about a firm’s prospects. However, it was not until game theoretic methods were applied that any progress was made in understanding this issue. Bhattacharya’s (1979) model of dividends as a signal was one of the first approaches in finance to use these game theoretic tools.

Bhattacharya assumes that managers have superior information about the profitability of their firm’s investment. They can signal this to the capital market by “committing” to a sufficiently high level of dividends. If it turns out the project is profitable these dividends can be paid from earnings without a problem. If the project is unprofitable then the firm has to resort to outside finance and incur deadweight transaction costs. The firm will therefore only find it worthwhile to commit to a high dividend level if in fact its prospects are good. Subsequent authors like Miller and Rock (1985) and John and Williams (1985) developed a game theoretic models which did not require assumptions such as committing to a certain level of dividends and where the deadweight costs required to make the signal credible were plausible.

One of the problems with signaling models of dividends is that they typically suggest that dividends will be paid to signal new information. Unless new information is continually arriving there is no need to keep paying them. But in that case the level of dividends should be varying to reflect the new information. This feature of dividend signaling models is difficult to reconcile with smoothing. In an important piece, Kumar (1988) developed a 'coarse signaling' theory that is consistent with the fact that firms smooth dividends. Firms within a range of productivity all pay the same level of dividends. It is only when they move outside this range that they will alter their dividend level.

Another problem in many dividend signaling models (including Kumar (1988)) is that they do not explain why firms use dividends rather than share repurchases. In most models the two are essentially equivalent except for the way that they are taxed since both involve transferring cash from the firm to the owners. Dividends are typically treated as ordinary income and taxed at high rates whereas repurchases involve price appreciation being taxed at low capital gains rates. Building on work by Ofer and Thakor (1987) and Barclay and Smith (1988), Brennan and Thakor (1990) suggested that repurchases have a disadvantage in that informed investors are able to bid for undervalued stocks and avoid overvalued ones. There is thus an adverse selection problem. Dividends do not suffer from this problem because they are pro rata. Some progress on understanding the dividend puzzle has been made in recent years. This is one of the finance applications of game theory that has been somewhat successful.

The trade-off theory of capital structure has been a textbook staple for many years. Even though it had provided a better explanation of firms' choices than the initial dividend models, the theory is not entirely satisfactory because the empirical magnitudes of bankruptcy costs and interest tax shields do not seem to match observed capital structures. The use of game theoretic techniques in this field has also allowed it to move ahead significantly (Harris and Raviv,1991).

The first contributions in a game theoretic vein were signaling models. Ross (1977) develops a model where managers signal the prospects of the firm to the capital markets by choosing an appropriate level of debt. The reason this acts as a signal is that bankruptcy is costly. A high debt firm with good prospects will only incur these costs occasionally while a similarly levered firm with poor prospects will incur them often.

Leland and Pyle (1977) considered a situation where entrepreneurs use their retained share of ownership in a firm to signal its value. Owners of high value firms retain a high share of the firm to signal their type. Their high retention means they don't get to diversify as much as they would if there was symmetric information and it is this that makes it unattractive for low value firms to mimic them.

Two subsequent papers based on asymmetric information which have been very influential are Myers (1984) and Myers and Majluf (1984). If managers are better informed about the prospects of the firm than the capital markets they will be unwilling to issue equity to finance investment projects if the equity is undervalued. Instead they will have a preference for using equity when it is overvalued. Thus equity is regarded as a bad signal. Myers (1984) used this kind of reasoning to develop the "pecking order" theory of financing. Instead of using equity to finance investment projects it will be better to use less information sensitive sources of funds. Retained earnings are the most preferred, with debt coming next and finally equity. The results of these research and the subsequent literature such as Stein (1992) and Nyborg (1995) are consistent with a number of stylized facts concerning the effect of issuing different types of security on stock price and the financing choices of firms. However, in order to derive those strong assumptions such as overwhelming bankruptcy aversion of managers are often necessary.

Dybvig and Zender (1991) stressed they often assume suboptimal managerial incentive schemes. The authors showed that if managerial incentive schemes are chosen optimally the Modigliani and Miller irrelevance results can hold even with asymmetric information.

A second influential strand of the literature on capital structure that has used game theoretic concepts is concerned with agency costs. Jensen and Meckling (1976) pointed to two kinds of agency problems in corporations. One is between equity holders and bondholders and the other is between equity holders and managers. The first arises because the owners of a levered firm have an incentive to take risks; they receive the surplus when returns are high but the bondholders bear the cost when default occurs.

Diamond (1989) showed how reputation considerations can ameliorate this risk shifting incentive when there is a long time horizon. The second conflict arises when equity holders cannot fully control the actions of managers. This means that managers have an incentive to pursue their own interests rather than those of the equity holders.

The agency perspective has also lead to a series of important papers by Hart and Moore and others on financial contracts. These used game theoretic techniques to shed light on the role of incomplete contracting possibilities in determining financial contracts and in particular debt. Hart and Moore (1989) considered an entrepreneur who wishes to raise funds to undertake a project. Both the entrepreneur and the outside investor can observe the project payoffs at each date, but they cannot write explicit contracts based on these payoffs because third parties such as courts cannot observe them. The focus of their analysis is the problem of providing an incentive for the entrepreneur to repay the borrowed funds. Among other things, it is shown that the optimal contract is a debt contract and incentives to repay are provided by the ability of the creditor to seize the entrepreneur's assets.

The Modigliani and Miller (1958) theory of capital structure is such that the product market decisions of firms are separated from financial market decisions. Essentially this is achieved by assuming there is perfect competition in product markets. In an oligopolistic industry where there are strategic interactions between firms in the product market, financial decisions are also likely to play an important role. Allen (1986), Brander and Lewis (1986) and Maksimovic (1986) considered various different aspects of these interactions between financing and product markets.

Allen (1986) considered a duopoly model where a bankrupt firm is at a strategic disadvantage in choosing its investment because the bankruptcy process forces it to delay its decision. The bankrupt firm becomes a follower in a Stackelberg investment game instead of a simultaneous mover in a Nash-Cournot game.

Brander and Lewis (1986) and Maksimovic (1986) analyzed the role of debt as a pre-commitment device in oligopoly models. By taking on a large amount of debt a firm effectively pre-commits to a higher level of output.

Titman (1984) and Maksimovic and Titman (1993) considered the interaction between financial decisions and customers' decisions. Titman (1984) looked at the effect of an increased probability of bankruptcy on product price because, for example, of the difficulties of obtaining spare parts and service should the firm cease to exist.

Maksimovic and Titman (1993) considered the relationship between capital structure and a firm's reputational incentives to maintain high product quality.

A significant component of the trade-off theory is the bankruptcy costs that limit the use of debt. An important issue concerns the nature of these bankruptcy costs. Haugen and Senbet (1978) argued that the extent of bankruptcy costs was limited because firms could simply renegotiate the terms of the debt and avoid bankruptcy and its associated costs. The literature on strategic behavior around and within bankruptcy that this contribution lead to used game theoretic techniques extensively (see Webb (1987), Giammarino (1988), Brown (1989) and for a survey Senbet and Seward (1995)). It was shown that Haugen and Senbet's argument depended on the absence of frictions. With asymmetric information or other frictions bankruptcy costs could occur in equilibrium.

The concept of the market for corporate control was developed verbally by Manne (1965). The author argued that in order for resources to be used efficiently, it is necessary that firms be run by the most able and competent managers. The author also suggested that the way in which modern capitalist economies achieve this is through the market for corporate control. There are several ways in which this operates including tender offers, mergers and proxy fights.

The paper that provided a formal model of the takeover process and renewed interest in the area was Grossman and Hart (1980). They pointed out that the tender offer mechanism involved a free rider problem. If a firm makes a bid for a target in order to replace its management and run it more efficiently then each of the target's shareholders has an incentive to hold out and say no to the bid. The reason is that they will then be able to benefit from the improvements implemented by the new management. They will only be willing to tender if the offer price fully reflects the value under the new management. Hence a bidding firm cannot make a profit from tendering for the target. In fact if there are costs of acquiring information in preparation for the bid or other bidding costs the firm will make a loss. The free rider problem thus appears to exclude the possibility of takeovers. The authors solution to this dilemma was that a firm's corporate charter should allow acquirors to obtain benefits unavailable to other shareholders after the acquisition. They term this process "dilution."

Another solution to the free rider problem, pointed out by Shleifer and Vishny (1986), is for bidders to be shareholders in the target before making any formal tender offer. In this way they can benefit from the price appreciation in the "toehold" of shares they already own even if they pay full price for the remaining shares they need to acquire. The empirical evidence is not consistent with this argument, however. Bradley, Desai and Kim (1988) find that the majority of bidders own no shares prior to the tender offer.

A second puzzle that the empirical literature has documented is the fact that bidding in takeover contests occurs through several large jumps rather than many small ones. For example, Jennings and Mazzeo (1993) found out through a study that the majority of the initial bid premiums were over 20% of the market value of the target 10 days before the offer. This evidence conflicts with the standard solution of the English auction model that suggests there should be many small bid increments.

Fishman (1988) argued that the reason for the large initial premium is to deter potential competitors. In his model, observing a bid alerts the market to the potential desirability of the target. If the initial bid is low a second bidder will find it worthwhile to spend the cost to investigate the target. This second firm may then bid for the target and push out the first bidder or force a higher price to be paid. By starting with a sufficiently high bid the initial bidder can reduce the likelihood of this competition.

Much of the theoretical literature has attempted to explain why the defensive measures that many targets adopt may be optimal for their shareholders. Typically the defensive measures are designed to ensure that the bidder that values the company the most ends up buying it. For example, Shleifer and Vishny (1986) developed a model where the payment of greenmail to a bidder, signals to other interested parties that no "white knight" is waiting to buy the firm. This puts the firm in play and can lead to a higher price being paid for it than initially would have been the case.

In 1963 the U.S. Securities and Exchange Commission undertook a study of IPOs and found that the initial short run return on these stocks was significantly positive.
Logue (1973), Ibbotson (1975) and numerous subsequent academic studies have found a similar result. In a survey of the literature on IPOs, Ibbotson and Ritter (1995) give a figure of 15.3% for the average increase in the stock price during the first day of trading based on data from 1960-1992. The large short run return on IPOs was for many years one of the most glaring challenges to market efficiency. The standard symmetric information models that existed in the 1960s and 1970s were not at all consistent with this observation.

The first research to provide an appealing explanation of this phenomenon was Rock (1986). In the authors model, the under pricing occurs because of adverse selection. There are two groups of buyers for the shares; one is informed about the true value of the stock while the other is uninformed. The informed group will only buy when the offering price is at or below the true value. This implies that the uninformed will receive a high allocation of overpriced stocks since they will be the only people in the market when the offering price is above the true value. Rock suggested that in order to induce the uninformed to participate they must be compensated for the overpriced stock they ended up buying. Under-pricing on average is one way of doing this.

The theories that have been put forward to explain long run underperformance are behavioral.Miller (1977) argued that there will be a wide range of opinion concerning IPOs and the initial price will reflect the most optimistic opinion. As information is revealed through time, the most optimistic investors will gradually adjust their beliefs and the price of the stock will fall.

Shiller (1990) argued the market for IPOs is subject to an 'impresario' effect. Investment banks will try to create the appearance of excess demand and this will lead to a high price initially but subsequently to underperformance.

Ritter (1991) and Loughran and Ritter (1995) suggested that there are swings of investor sentiment in the IPO market and firms use the "window of opportunity" created by overpricing to issue equity. Although IPOs represent a relatively small part of financing activity they have received a great deal of attention in the academic literature. The reason perhaps is the extent to which under-pricing and overpricing represent a violation of market efficiency. It is interesting to note that while game theoretic techniques have provided many explanations of under-pricing they have not been utilized to explain overpricing. Instead the explanations presented have relied on eliminating the assumption of rational behaviour by investors.

An area that has been significantly changed by game theoretic models is intermediation. Traditionally, banks and other intermediaries were regarded as ways of reducing transaction costs (Gurley and Shaw, 1960). Models of banking were not very rich. The field was dramatically changed by the modeling techniques introduced in Diamond and Dybvig (1983). The study considered a simple model where a bank provides insurance to depositors against liquidity shocks. At the intermediate date customers find out whether they require liquidity then or at the final date. There is a cost to liquidating long term assets at the intermediate date. A deposit contract is used where customers who withdraw first get the promised amount until resources are exhausted after which nothing is received (i.e., the first come first served constraint). These assumptions result in two self-fulfilling equilibria. In the good equilibrium everybody believes only those who have liquidity needs at the intermediate date will withdraw their funds and this outcome is optimal for both types of depositor. In the bad equilibrium everybody believes everybody else will withdraw. Given the first come first served assumption and that liquidating long term assets is costly, it is optimal for early and late consumers to withdraw and there is a run on the bank. The authors argued the bad equilibrium can be eliminated by deposit insurance. In addition to being important as a theory of runs, the paper was also important in terms of the way in which liquidity needs were introduced and a similar approach has been adopted in the investigation of many topics.

Jacklin and Bhattacharya (1988) compared what happens with bank deposits to what happens when securities are held directly so runs are not possible. In their model some depositors receive a signal about the risky investment. They showed that either bank deposits or directly held securities can be optimal depending on the characteristics of the risky investment. The comparison of bank-based and stock market-based financial systems has become a widely considered topic in recent years (see Thakor (1996) and Allen and Gale (1999)).

Diamond (1984) presented a model of delegated monitoring where banks have an incentive to monitor borrowers because otherwise they will be unable to pay off depositors. A full account of the recent literature on banking is contained in Bhattacharya and Thakor (1993).

Kyle (1985) developed a model with a single risk neutral market maker, a group of noise traders who buy or sell for exogenous reasons such as liquidity needs and a risk neutral informed trader. The market maker selects efficient prices and the noise traders simply submit orders. The informed trader chooses a quantity to maximize his expected profit.

Glosten and Milgrom (1985) presented a risk neutral market maker, noise traders and informed traders model. The main difference between this model and that of Kyle is that the quantities traded are fixed and the focus is on the setting of bid and ask prices rather than the quantity choice of the informed trader. The market maker sets the bid ask spread to take into account the possibility that the trader may be informed and have a better estimate of the true value of the security. As orders are received, the bid and ask prices change to reflect the possibility that the trader is informed. Also, the model is competitive in the sense that the market maker is constrained to make zero expected profits.

A number of other asset-pricing topics in addition to market microstructure have been influenced by game theory. These include market manipulation models (Cherian and Jarrow, 1995).

Abel and Mailath (1994) presented risk neutral investors model subscribe to securities paid from a new project's revenues. They note that it is possible that all investors subscribe to the new securities even though all investors' expected return is negative. This could not happen if it was common knowledge that all investors' expected return was negative.

Allen, et al., (1993) considered a rational expectations equilibrium of a dynamic asset trading economy with a finite horizon, asymmetric information and short sales constraints. They note that an asset may trade at a positive price, even though every trader knows that the asset is worthless. Even though each trader knows that the asset is worthless, he attaches positive probability to some other trader assigning positive expected value to the asset in some future contingency. It is worth holding the asset for that reason. Again, this could not occur if it were common knowledge that the asset was worthless.

Kraus and Smith (1989) described a model where the arrival of information about others' information (not new information about fundamentals) drives the market.

Kraus and Smith (1998) considered a model where multiple self-fulfilling equilibria arise because of uncertainty about other investors' beliefs. They term this "endogenous sunspots". It is shown that such sunspots can produce "pseudo-bubbles" where asset prices are higher than in the equilibrium with common knowledge.

Shin (1996) compared the performance of decentralized markets with dealership markets. While both perform the same in a complete information environment, the author noted that the decentralized market performs worse in the presence of higher order uncertainty about endowments. The intuition is that a decentralized market requires co-ordination that is sensitive to a lack of common knowledge, whereas the dealership requires less coordination.

A more developed literature has been concerned with informational cascades. An early example was Welch (1992). A group of potential investors must decide whether to invest in an Initial Public Offering (IPO) sequentially. Each investor has some private information about the IPO. Suppose that the first few investors happen to observe bad signals and choose not to invest. Later investors, even if they observed good signals, would ignore their own private information and not invest on the basis of the (public) information implicit in others' decisions not to invest. But now even if the majority of late moving investors has good information, their good information is never revealed to the market. Thus inefficiencies arise in the aggregation of private information because the investors' actions provide only a coarse signal of their private information. This type of phenomenon has been analyzed more generally by Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992).

Finance applications are surveyed in Devenow and Welch (1996). One major weakness of the informational cascade argument is that it relies on action sets being too coarse to reveal private information (see Lee (1993)). There are some contexts where this assumption is natural: for example, investors' decisions whether to subscribe to initial public offerings at a fixed offer price (although even then the volume demanded might reveal information continuously). But once prices are endogenized, the (continuum) set of possible prices will tend to reveal prices. Two natural reasons why informational cascades might nonetheless occur in markets with endogenous price formation have been introduced in the literature. If investors face transaction costs, they may tend not to trade on the basis of small pieces of information (Lee (1997)). In this case, market crashes might occur when a large number of investors, who have observed bad news but not acted on it, observe a (small) public signal that pushes them into trading despite transaction costs.

Avery and Zemsky (1996) exploited the fact that although prices may provide rich signals about private information, if private information is rich enough (and, in particular, multi-dimensional), the market will not be able to infer private information from prices.

For some purposes, it does not matter if differences in beliefs are explained by different information or differences in priors. For example, Lintner (1969) derived a CAPM with heterogeneous beliefs and – assuming, as he did, that investors do not learn from prices – the origin of their differences in beliefs did not matter. It is only once it is assumed that individuals learn from others' actions (or prices that depend on others' actions) that the distinction becomes important. Thus the distinction began to be emphasized in finance exactly when game theoretic and information theoretic issues were introduced. Most importantly, "no trade" theorems, such as that of Milgrom and Stokey (1982), established that differences in beliefs based on differences in information alone could not lead to trade.

But while the distinction is undoubtedly crucial, this does not justify a claim that heterogeneous prior beliefs are inconsistent with rationality (see Morris (1995) for a review of attempts to justify this claim; see also Gul (1998) and Aumann (1998)). In any case, there is undoubtedly a significant middle ground between the extreme assumptions that (1) participants in financial markets are irrational; and (2) all differences in beliefs are explained by differences in information. We will briefly review some work in finance within this middle ground.

Harrison and Kreps (1978) considered a dynamic model where traders were risk neutral, had heterogeneous prior beliefs (not explained by differences in information) about the dividend process of a risky asset, and were short sales constrained in that asset. They observed that the price of an asset would typically be more than any trader's fundamental value of the asset (the discounted expected dividend) because of the option value of being able to sell the asset to some other trader with a higher valuation in the future.

Morris (1996) presented a model where although traders start out with heterogeneous prior beliefs, they are able to learn the true dividend process through time; a re-sale premium nonetheless arises from reflecting the divergence of opinion before learning has occurred. Thus this model provides an explanation of the opening market overvaluation of initial public offerings: lack of learning opportunities implies greater heterogeneity of beliefs implies higher prices.

Harris and Raviv (1993) presented a model where traders disagree about the likelihood of alternative public signals conditional on payoff relevant events. They present a simple model incorporating this feature that naturally explains the positive autocorrelation of trading volume and the correlation between absolute price changes and volume as well a number of other features of financial market data.

Managing water resources systems usually involves conflicts. Behaviors of stakeholders, who might be willing to contribute to improvements and reach a win–win situation, sometimes result in worse conditions for all parties. Game theory can identify and interpret the behaviors of parties to water resource problems and describe how interactions of different parties who give priority to their own objectives, rather than system's objective, result in a system's evolution. Outcomes predicted by game theory often differ from results suggested by optimization methods which assume all parties are willing to act towards the best system-wide outcome. This study reviews applicability of game theory to water resources management and conflict resolution through a series of non-cooperative water resource games. Kaveh (2009) illustrates the dynamic structure of water resource problems and the importance of considering the game's evolution path while studying such problems.

Interest in water resources conflict resolution has increased over the last decades (Dinar, 2004) and various quantitative and qualitative methods have been proposed for conflict resolution in water resources management, including, but not limited to

Interactive Computer-Assisted Negotiation Support system (ICANS) (Thiessen and Loucks, 1992; Thiessen et al., 1998), Graph Model for Conflict Resolution (GMCR) (Kilgour et al., 1996; Hipel et al., 1997), Shared Vision Modeling (Lund and Palmer, 1997), Adjusted Winner (AW) mechanism (Massoud, 2000), Alternative Dispute Resolution (ADR) (Wolf, 2000), Multivariate Analysis Biplot (Losa et al., 2001), and Fuzzy Cognitive Maps (Giordano et al., 2005). Wolf (2002) presented some significant papers and case studies on the prevention and resolution of conflict (using descriptive methods) over water resources.

Many researchers have attempted water conflict resolution studies in a game-theoretic framework. Carraro et al. (2005), Parrachino et al. (2006), and Zara et al. (2006) reviewed game theoretic water conflict resolution studies. Game theory applications in water resources literature cover a range of water resource problems, locations, solution methods, analysis types, and classifications. It may be possible to place some studies in more than one category. However, the main aspect of the study was considered for categorization. The authors applied game theory for (1) water or cost/benefit allocation among users; (2) groundwater management; (3) water allocation among trans-boundary users; (4) water quality management; and (5) other types of water resources management problems.

Carraro et al., (2005) believed that many natural resource management issues have the characteristics of a Prisoner's Dilemma game: players' dominant strategy is not cooperative, and the resulting equilibrium is not Pareto-optimal. Similarly, most papers dealing with sharing natural resources problems have made the same assumption about the game to be the Prisoner's Dilemma. However, all common resource problems might not be Prisoner's Dilemmas (Sandler, 1992).

The conditions of a natural resource sharing problem might favour the possibility of cooperation (Taylor, 1987). Water resource games are not necessarily rival (there might be multiple users and usage by one user does not prevent simultaneous usage by other users). Thus, coordination among the parties might be beneficial to all and can create externalities. However, some water resources games can be treated as anti-coordination games in which the available resource is rival (the resource can only be consumed by one user), sharing the resource comes at a cost to users, and the resource is not excludable (it is not possible to prevent a player who does not pay for the resource from enjoying its benefits).

Identifying the structure of water resource games is essential as the results can be misleading if wrong assumptions are made in conflict modeling. For instance, characteristics of an anti-coordination water game cannot be captured if the conflict is modeled as Prisoner's Dilemma. Bardhan (1993) believes that the literature usually jumps to the case of Prisoner's Dilemma in case of free-riders. Sometimes, the player might not be able to reach his objective on his own. Under that condition (Stag-Hunt game) a player cooperates when the other player also cooperates and defects when the other one defects. In some common resource examples consequences of defection might be so bad that a player prefers not to defect if the other player defects (Chicken game) (Bardhan, 1993). Here, two non-Prisoner's Dilemma water resource games, useful for understanding water conflicts, are introduced to support the fact that not all water resources games are Prisoner's Dilemmas.

Cloud computing is a newly emerging paradigm in which a client pays as it uses computing resources owned by a cloud provider. Since multiple clients share the cloud's resources, they could potentially interfere with each others' tasks.

Current pricing and resource allocation mechanisms are quite preliminary (e.g., fixed pricing in Amazon EC2/S3) and do not take into account the conflict of interests between multiple clients using the cloud simultaneously. This can lead to clients being overpriced, depending upon their allocated resources. Further, these mechanisms do not allow the provider to optimize its resource utilization.

Virajith et al., (2003), took the first step towards modeling the complex client-client and client-provider interactions in a cloud by using game theory. The authors defined a new class of games called Cloud Resource Allocation Games (CRAGs). CRAGs solve the resource allocation problem in clouds using game-theoretic mechanisms, ensuring that clients are charged (near) optimal prices for their resource usage and that resources of the cloud are used near their optimal capacity. Tyhe authors presented the conditions for reaching various stable equilibria in CRAGs and provide algorithms that ensure close to optimal performance. The authors further provided results of several experiments performed using traces from PlanetLab and the Parallel Workload Archives which show that the new mechanisms result in as much as 15% to 88% increase in performance compared to existing resource allocation mechanisms like Round-Robin.

Wireless technologies and devices are becoming increasingly ubiquitous in our daily life. On the one hand, wireless resources are natural and fixed, and on the other hand, wireless technologies and devices are increasing day-by-day resulting in spectrum scarcity. As a consequence, efficient use of limited wireless resources is a central and fundamental step in wireless systems. As the demand increases, the management of limited wireless resources is crucial to allocate the resources optimally. Moreover, optimal allocation of limited wireless resources results in dissemination of information to large areas both reliably and quickly. Recently, game theory has emerged as an efficient tool for optimal allocation of wireless resources, which has social optimal points called Nash equilibrium.

Danda et al., (2009) outlined that the Nash equilibrium does not necessarily produce optimal outcome, therefore the optimal point is also referred as social optimal point. The authors first presented the game theory and its application in resource allocation at different layers of the protocol stack of the wireless network model. Furthermore, they showed that the static assignment of spectrum bands by governmental bodies such as the FCC (Federal Communications Commission) in the United States is inefficient since the licensed systems do not always fully use their frequency bands. The secondary unlicensed (cognitive radio) users can identify the idle spectrum and use it opportunistically. Therefore, in order to access the licensed spectrum opportunistically and optimally, dynamic spectrum access functionality is important in next generation (XG) wireless systems, which was the subject matter of their study. In particular, they presented different game theoretic approaches for dynamic spectrum access.

Game Theory (GT) is a mathematical method that describes the phenomenon of conflict and cooperation between intelligent rational decision-makers. In particular, the theory has been proven very useful in the design of wireless sensor networks (WSNs). Hai-Yan et al., (2011) presented a survey on the recent developments and findings of GT, its applications in WSNs, and provided the community a general view of this vibrant research area. The authors first introduced the typical formulation of GT in the WSN application domain. The roles of GT were described that included routing protocol design, topology control, power control and energy saving, packet forwarding, data collection, spectrum allocation, bandwidth allocation, quality of service control, coverage optimization, WSN security, and other sensor management tasks. Then, three variations of game theory were described, namely, the cooperative, non-cooperative, and repeated schemes. Finally, existing problems and future trends were identified for researchers and engineers in the field.

The problem of mapping tasks onto a computational grid with the aim to minimize the power consumption and the make span subject to the constraints of deadlines and architectural requirements is considered by Samee (2008). To solve this problem, the author proposed a solution from cooperative game theory based on the concept of Nash Bargaining Solution. The proposed game theoretical technique was compared against several traditional techniques. The experimental results showed that when the deadline constraints are tight, the proposed technique achieves superior performance and reports competitive performance relative to the optimal solution.

Predictable allocations of security resources such as police officers, canine units, or checkpoints are vulnerable to exploitation by attackers. Recent work has applied game-theoretic methods to find optimal randomized security policies, including a fielded application at the Los Angeles International Airport (LAX). This approach has promising applications in many similar domains, including police patrolling for subway and bus systems, randomized baggage screening, and scheduling for the Federal Air Marshal Service (FAMS) on commercial flights. However, the existing methods scale poorly when the security policy requires coordination of many resources, which is central to many of these potential applications. Christopher et al., (2009) developed new models and algorithms that scale to much more complex instances of security games. The key idea was to use a compact model of security games, which allows exponential improvements in both memory and runtime relative to the best known algorithms for solving general Stackelberg games. The authors developed even faster algorithms for security games under payoff restrictions that are natural in many security domains. Finally, they introduced additional realistic scheduling constraints while retaining comparable performance improvements. The empirical evaluation comprises both random data and realistic instances of the FAMS and LAX problems. The author's new methods scale to problems several orders of magnitude larger than the fastest known algorithm.

Omar and Wessam (2011) developed an optimal solution for resource allocation between secondary users in a cognitive radio network (CRN). The authors assume a CRN that contains a set of primary users (PUs) coexisting with secondary users (SUs) in an underlay spectrum sharing paradigm. PUs use licensed bands of the spectrum while SUs try either to use unoccupied bands or coexist with PUs in the same band without harmfully affecting primary transmissions. The authors proposed an algorithm based on the VCG (Vickrey–Clarke–Groves) model in a non cooperative game for spectrum allocation between secondary transmissions that guarantee a required minimum data rate for both PUs and SUs, assuming a fixed value of the bit error rate. It aimed to find the optimal and fair assignment of secondary transmissions to spectrum bands that maximizes their sum data rate. Simulation results showed that the proposed solution maximizes the sum data rate depending on the transmit power of primary transmissions and the data rate required for secondary transmissions. Using Jain's fairness index, they also showed that their proposition is almost 98% fair.

Corruption is an important social and ethical problem; fight with it requires changes in values, norms and behavioral patterns of the society. This is usually a long and difficult process. Decades should pass to change deep values of a society. In the mean time, it is possible to combat corruption by changing incentive structures in the economy. If deep causes of the problem are analyzed carefully, a new system of governance can be established, such that, even most opportunist individuals do not find getting involved in corrupt practices profitable.

Bayar (2003) presented a thesis which examined characteristics of the system providing a fertile environment for corruption and to figure out factors stimulating corrupt transactions using game theoretical models. The first two models examined corruption as a kind of transaction between the briber and the bribee. In the models, it is shown that intermediaries sector occur from the profit maximization behavior of agents. This sector, by establishing long term, trust based relationships with bureaucrats, decreases risks occurring from the fact that the two parties involved in a corrupt transaction do not know each other perfectly. This sector, by reducing the likelihood of detection, serves corrupt transactions, and in return for the service it provided, takes commission, so gets benefit. Third model examines a strange type of corruption, a case of (spurious) middlemen obtaining bribe from the public service bureaucrats give, by pretending that he has influence on the acceptance or speed of it. The model tries to detect the characteristics of the environment making such a deception process persistent.

Intan (2011) studied food allocation in bird broods from the perspective of cooperative game theory. We want to explore whether or not food distribution data fit into the known solution concepts of cooperative game theory. A first issue to be handled is the fact that in the bird brood data we only see the solutions, while the starting position, the game, is not immediately clear. As such we need to reconstruct the game from the solutions given. A second issue is that there are many different solution concepts (e.g. Shapley value, nucleolus, etc) and we want to analyze which of these fits best. Most interesting is to specifically address the properties that lead to these solutions, because these would be most useful in finding a motivation for the specific solution concept found in nature.

Cognitive radio technology, a revolutionary communication paradigm that can utilize the existing wireless spectrum resources more efficiently, has been receiving a growing attention in recent years. As network users need to adapt their operating parameters to the dynamic environment, who may pursue different goals, traditional spectrum sharing approaches based on a fully cooperative, static, and centralized network environment are no longer applicable. Instead, game theory has been recognized as an important tool in studying, modeling, and analyzing the cognitive interaction process. Beibei et al., (2010) presented the most fundamental concepts of game theory, and explain in detail how these concepts can be leveraged in designing spectrum sharing protocols, with an emphasis on state-of-the-art research contributions in cognitive radio networking. Research challenges and future directions in game theoretic modeling approaches were also outlined. The author's study provided a comprehensive treatment of game theory with important applications in cognitive radio networks, and will aid the design of efficient, self-enforcing, and distributed spectrum sharing schemes in future wireless networks.

Christian (2008) presented a game theoretic thesis which investigated if (World Trade Organisation) members' shifting behaviour in the Dispute Settlement Mechanism may be explained using a game theoretic approach. To do this, the author uses a three step process: First, a thorough quantitative analysis of how the DSM works and how the members behave in it is carried out. Second, with the result from the first part in mind, a game theoretic model pointing out different possible strategies that may be pursued in the DSM is constructed. And third, the implications of the model are compared with the findings in the first part in order to evaluate whether the model could be used to explain the members' different behaviour in the DSM or not. The main result is that many aspects of the member states' strategies could be explained whit the constructed model but as a few commonly used strategies seem completely irrational from the model's perspective it is far from a perfect explanation.

Economic aspects of service systems are interesting and of great importance to real-life applications. Traditional studies of service systems are mainly conducted using stochastic queueing networks. Game theory, a well-established tool that is used to model the interactions between individuals, can be used together with stochastic queueing networks to study the economic aspects of service systems. CHOI (2010) presented a thesis which used both stochastic queueing networks and game theory to study economic problems related to service systems with multiple servers, namely the problem of finding the optimal pricing strategy, and the study and control of the economic behavior of independently-operating service providers. Pricing decisions are important in a service system as they affect, apart from the profit, the demand of customers and thus waiting times. The model studied here considered the optimal pricing scheme in a two-stage tandem queueing system with different types of customers. It was assumed that the demand of each type of customers has a negative linear relationship with the price of the service. The authors analysis gave explicitly the optimal pricing scheme which maximizes the total profits while maintaining the second-stage expected sojourn time under a given level. Further discussion was given for the case where the constraint is imposed on the total waiting time of the two stages instead of only the second stage. The economic behavior of service providers in a competitive environment is another important and interesting research problem concerning economic aspects of service systems. The focus of the authors study was the role and impact of service capacity in capturing larger market share and maximizing long-run expected profits in a multiple-server setting. They first focus on the analytical results of the case of a common-queue service system. The problem is formulated as a multiple- player strategic game. Equilibrium solutions are analyzed when the queueing system is stable. The equilibrium service capacity in a multiple-server separate- queue environment is analyzed similarly and compared with the common-queue case. The analysis shows that in the case of multiple servers the separate queue allocation scheme creates more competition incentives for servers and induces higher service capacities. In particular, a high compensation level tends to favour the separate queue allocation when there are not severe diseconomies associated with increasing service capacity.

When in 1950 John Nash was getting his PhD degree on theory of non-cooperative games, no one could foresee that he was more than several decades ahead of his times. One could also not predict that almost half a century later his equilibria will be celebrated and Nash himself, along with other notable economists, will be given the Nobel Prize. Nowadays, no one undermines the relevance of Game Theory as science which influences other sciences, starting with mathematics and economy, and ending with philosophy and biology. The forefathers of GT are generally agreed to be John von Neumann and Oskar Morgenstern and their Theory of Games and Economic Behavior'. Since then, GT has developed into an extensive science branch that attempts to uniformly and unambiguously explain the behavior of humans, social groups, corporations, and governments, as well as animals and other living creatures, collectively called players. A player has their own interest in mind when behaving in a particular way. GT tries to explain that behavior and anticipates the best possible solutions. Tomasz (2012) presented a thesis which focused on the vital aspects of GT and the behavior of players in (none) coalition and (non) cooperative circumstances. The focuse of the thesis was the art of implementing game theories in reality, particularly in the small business milieu. Finally, the expectations based on the knowledge presented in the previous chapters are juxtaposed with real life cases. For that, an interview with small business representatives has been done and the results are presented in form of reflections and tips of how a small business should act.

The importance of information superiority has been emphasized as a critical capability that future joint forces must be able to achieve. No longer simply a future concept, it is being officially defined and incorporated in doctrinal publications like Joint Publication 3-13, Information Operations. Unfortunately, our ability to effectively measure its contribution relative to other battlefield systems remains limited. John (2008) studied a model that focused on exploring the limits of the contributions that information superiority can make, examining the sensitivity of information superiority to varying information quality and comparing those contributions with other contributing factors to battlefield results. Furthermore, an effort is made to identify some of the risks associated with using information superiority as a force multiplier. A simple decision model was developed based on the concepts of a two-person zero sum game to explore these questions. In the model, one side is provided varying degrees of an information advantage, while also varying degrees of information quality to the information advantage. Additionally, a variety of scenarios were considered involving varied levels of opposing side force levels. Experimental design techniques were employed to efficiently explore the model output space while allowing for sufficient replications of the model at each design point in order to provide a sufficient data set for analysis.

Multi-hop wireless network are promising techniques in the field of wireless communication. The dynamic topology of the network and the independent selfish participants of the network make it difficult to be modeled by traditional tools. Game theory is one of the most powerful tools for such problems. However, most current works have certain limitations. There has not been a widely accepted solution for the problem yet. Miao (2007) presented a thesis on bandwidth sharing in wireless networks. The author assumed the nodes are rational, selfish, but not malicious, independent agents in the game. In their model, nodes are trying to send their data to the gateway. Some nodes may require others to forward their packets to successfully connect to the gateway. However, nodes are selfish and do not wish to help others. Therefore it is possible that some nodes may refuse the requirement. In that case, the unpleasant nodes may punish the others by slowing down their traffic, in which case both parties will suffer. Therefore it is non-trivial to find out the equilibrium for these nodes after the bargaining process. What is the proper distribution of resources among these nodes? We propose a solution based on the game theory. The author's solution fulfilled the goal of fairness and social-welfare maximization.

In recent years, China has been flexing its military power and strengthening its claim to the resource-rich Spratly and Paracel Islands in the South China Sea. These islands are also being claimed by five other countries: Brunei, Malaysia, the Philippines, Taiwan, and Vietnam. Currently China claims the entire South China Sea as its territorial waters. The U.S. has great interest in this issue because its trade routes with the Asia-Pacific region go through the South China Sea. Throughout history, Vietnam and China have had a contentious relationship. Like China, Vietnam is currently modernizing its military and strengthening its claims to the South China Sea. Of the claimants to the South China Sea, Vietnam seems to be the only country that is willing to challenge Chinese assertiveness in the region. Since the normalization of relations between the United States and Vietnam, the two former enemies have become important trading partners. The United States and Vietnam are conducting yearly high-level military visits; however, the U.S. wants to take this relationship to the next level. Ngan (2012) presented a thesis which applied game theory to analyze whether the U.S. can influence Vietnam to open a more formal military relationship to counterbalance the assertiveness of China in the South China Sea. This thesis concluded that, from the game theoretic Strategic Moves perspective, the U.S. currently cannot apply threats, promises, or a combination thereof to compel or coerce Vietnam toward a more formal military alliance to counterbalance the assertiveness of China in the South China Sea.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

Game theory is a mathematical theory that deals with the general features of competitive situations like parlour games, military battles, political campaigns, advertising and marketing campaigns by competing business firms and so forth. It is a distinct and interdisciplinary approach to the study of human behaviour. The disciplines most involved in game theory are mathematics, economics and the other social and behavioural sciences. Game theory (like computational theory and so many other contributions) was founded by the great mathematician John von Neumann (1947). The concepts of game theory provide a language to formulate structure, analyze, and understand strategic scenarios.

Game theory bears a strong relationship to Linear Programming (LP), since every finite two person zero sum game can be expressed as a LP and conversely every LP can be expressed as a game.

If the problem has no saddle point, dominance is unsuccessful to reduce the game and the method of matrices also fails, then LP offers the best method of solution. So far several authors namely Bansal (1980), Martin (2002), Stephen (2000), Thedor (2001) and many other authors proposed different types of theoretical discussion of game problems with their strategies also.

In this chapter of the study, we will discuss some methods and definitions of LP and game theory with some relevant theorems and propositions. We shall also give a discussion of simplex method and Minimax-Maximin method for solving game problems. A short discussion of rectangular 2×2 game would also be given.

3.1 LINEAR PROGRAMMING AND GAME THEORY

Consider the standard L P problems as follows,

$$\text{Maximize } Z = C^T \boldsymbol{x} \quad (3.1)$$

$$\text{Subject to } Ax\ (\leq, =, \geq)\ \boldsymbol{b} \quad (3.2)$$

$$\boldsymbol{x} \geq 0 \quad (3.3)$$

Where A is an m × n matrix and $x=(x_1, x_2, x_3, \ldots\ldots x_n)$, $b=(b_1, b_2, b_3, \ldots\ldots b_m)^T$ are column vectors. We shall consider any number of rows and columns, $b \neq 0$ and the system of linear equations are given in equation 3.2. We shall also denote the i[th] column of A by $A^{(i)}$.

Objective function

The linear function $z = \sum_{j=1}^{n} C_j X_j = c_1x_1 + c_2x_2 + \ldots \ldots \ldots + c_nx_n$ which is to be maximized (or minimized) is called objective function of the general linear programming problem (GLPP).

Constraints

The set of equations or inequalities is called the constraints of the general linear programming problem. **Ax** (≤, =, **≥) b** is the set of constraints in the GLPP.

Solution of GLPP

An n-tuple (x_1, x_2, … … , x_n) of real numbers which satisfies the constrains of a GLPP is called the solution of GLPP.

Feasible Solution

Any solution to a GLPP which also satisfies the nonnegative restrictions of the problem is called a feasible solution to the GLPP. Or, A feasible solution to the LP problem is a vector x = (x_1, x_2, … … , x_n) which satisfies the conditions $\sum_{j=1}^{n} C_j X_j$ (≤, =, ≥) $\mathbf{b_i}$ i = 1, … … , m and j = 1, … … , n; $x_i \geq 0$.

Matrix Form

Suppose we have found the optimal solution to (3.1). Let BV_i be the basic variable for row *i* of the optimal tableau. Also define $BV = \{BV_1, BV_2, \dots \dots, BV_m\}$ to be the set of basic variables in the optimal tableau, and define the $m \times 1$ vector as,

$$x_{BV} = \begin{bmatrix} X_{BV1} \\ \cdot \\ \cdot \\ X_{BVm} \end{bmatrix}$$

We also define *NBV* = the set of nonbasic variables in the optimal tableau

$x_{NBV} = (n - m) \times 1$ vector listing the nonbasic variables (in any desired order)

Using our knowledge of matrix algebra, we can express the optimal tableau *n* terms of *BV* and the original LP (3.1). Recall that c_1, c_2, … … c_n are the objective function coefficients for the variables x_1, x_2, ... , x_n (some of these may be slack, excess or artificial variables).

Here, *CBV* is the $1 \times m$ row vector $[C_{BV1} \quad C_{BV2} \quad . \; . \quad C_{BVM}]$

Thus the elements of C*BV* are the objective function coefficients for the optimal tableau's basic variables. C*NBV* is the $1 \times (n - m)$ row vector whose elements are the coefficients of the no basic variables (in the order of *NBV*). The $m \times m$ matrix *B* is the matrix whose j^{th} column is the column for BV_j in (3.1). a_j is the column (in the constraints) for the variable x_j in (3.1). *N* is the $m \times (n - m)$ matrix whose columns are the columns for the non-basic variables (in the *NBV* order) in (3.1). The $m \times 1$ column vector *b* is the right-hand side of the constraints in (3.1).

Game

A game is a formal description of a strategic situation (Davis, 1983).

Strategy

A strategy of a player P is a complete enumeration of all the actions that he will take for every contingency that might arise (Sasieni, 1966).

Pay-off

The pay-off is a connecting link between the sets of strategies open to all the players. Suppose that at the end of a play of a game, a player p_i (i=1,2,……….,n) is expected to obtain an amount v_i, called the pay-off to the player p_i. .

Pay-off matrix

A pay-off matrix is the table that represents the pay-off from player II to player I for all possible actions by players (McKinsey, 1952).

Fair game

A game is said to be fair game if the value of the game is zero.

Pure strategy

A pure strategy for player I (or player II) is the decision to play the same row (or column) on every move of the game (Sasieni, 1966).

Consider the matrix game A=(a_{ij}) for two players. If both players employ pure strategies, the outcome of each move is exactly the same and the game is completely predictable. For example, if player I always chooses the ith row and player II always chooses the jth column, then on every play of the game player I receive (a_{ij}) units from player II.

Mixed strategy

A mixed strategy is an active randomization, with given probabilities that determine the player's decision. As a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies.

Suppose player I does not want to play each row on each play of the game with probability 1 or 0, as was the case with pure strategies. Instead, suppose he decides to play row i with probability x_i with i=1, 2,…..,m, where more than one x_i is greater than zero, and $\sum_i^m x_i$ **= 1.** This decision is denoted by

$$X=[x_1 \quad x_2 \quad . \ . \quad x_p \ . \ \ . \ \ x_m]$$

is called a mixed strategy for player I (Thomas, 1969). In like manner, if player II decides to play column j with probability y_j with j=1, 2,……,n where more than one y_i is greater than zero, and $\sum_{i=1}^m y_i$ **= 1**

Then

$$Y= [y_1 \quad y_2 \quad . \ . \quad y_p \ . \ \ . \ \ y_m]$$

Player

A player is an agent who makes decisions in a game.

Strategic form

A game in strategic form, also called normal form, is a compact representation of a game in which players simultaneously choose their strategies. The resulting payoffs are presented in a table with a cell for each strategy combination.

Two-person zero-sum game

A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player's gain is the other player's loss, so their interests are diametrically opposed (Harvey, 1956).

Saddle point

A saddle point of a payoff matrix is that position in the payoff matrix where the maximum of row minima coincides with the minimum of the column maxima .The payoff at the saddle point is called the value of the game and is obviously equal to the maximin and minimax values of the game.

Theorem 3.1

If mixed strategies are allowed, the pair of mixed strategies that is optimal according to the minimax criterion proves a stable solution with V= V= V , so that neither player can do better by unilaterally changing her or his strategy (Meyerson, 1991).

Theorem 3.2

In a finite matrix game, the set of optimal strategies for each player is convex and closed (Kambo, 1991).

Theorem 3.3

Let v be the value of an m × n matrix game. Then if Y= $[y_1 \quad y_2 \quad \cdot\cdot \quad y_p \cdot \cdot \; y_m]$ is an optimal strategy for player II with y_i > 0, every optimal strategy x for player I must have the property

$$\sum_{i,j=1}^{m} a_{ij}\, x_i = \text{V}$$

Similarly, if the optimal strategy x has x_i > 0, then the optimal strategy y must be that

$$\sum_{i,j=1}^{n} a_{ij}\, y_i = \text{V}$$

Proposition 3.1: The set S= {x | **A**x = b, x ≥ 0 } is convex.

Proposition 3.2: x ≥ 0 is a basic nonnegative solution of (3.2) if and only if x is a vertex of (3.1).

Proposition 3.3: If the system of equations (3.2) has a nonnegative solution, then it has a basic nonnegative solution.

Proposition 3.4: S has only a finite number of vertices (Marcus, 1969).

3.2. Simplex Method

The Simplex method is an iterative procedure for solving linear programming problems expressed in standard form. In addition to the standard form, the Simplex method requires that the constraint equations be expressed as an economical system from which a basic feasible solution can be readily obtained. If the standard faint of LP is not in canonical form, one has to reduce it to a variable. Then we remove these artificial variables by applying two- phase method or Big-M method. The Simplex method is developed by George B. Dantzig (1947). The Simplex method has a wide range of applications including financial, agriculture, industry, transportation and other problems in economics and management science.

3.3. Minimax-Maximin pure strategies

Since each player knows that the other rational and same objective that is, to maximize the pay off from the other player, each might decide to us the conservative minimax criterion to select an action. That is, player I examines each row in the payoff matrix and selects the minimum element in each row, say p_{ij} with i=1, 2 ………,m. Then he selects the maximum of these minimum elements, say p_{rs}.

Mathematically, $V= p_{rs} = \max[\min(p_{ij})]$

The element p_{rs} is called the maximin value of the game, and the decision to play row r is called the maximin pure strategy. Likewise, player II examines each column in the payoff matrix to the column with the smallest maximum loss.

Let,

$$V = p_{tu} = \min[\max(p_{ij})]$$

Then p_{tu} is called the minimax value of the game and the decision to play column u is called minimax pure strategy. It can be shown that, the minimax value v represents a lower bound on a quantity called the value of the game, and also v represents a upper bound on the value of the game.

RECTANGULAR 2×2 GAME

In this section, we present a short discussion about 2×2 particular game problems (Stanley, 1954).

First, consider a 2×2 game with the payoff matrix.

Let x_i be the probability player II plays row I with i =1, 2, and let y_j be the probability player I plays column j with j=1, 2.

Since

$$\begin{array}{cc} & \text{Player II} \\ \text{Player I} & \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \end{array} \quad \ldots\ldots(3.4)$$

$\sum_{I=1}^{2} x_i = 1$ and $\sum_{I=1}^{2} y_i = 1$

So we can write, $x_2=1- x_1$ and $y_2= 1 - y_1$.

The saddle point is necessarily the value of the game. If a saddle point does not exist, then we have to follow the following procedure.

Let, the optimal strategy of player I is $\bar{y} = \begin{pmatrix} y*1 \\ y*2 \end{pmatrix}$

The optimal strategy of player II is $\bar{x} = \begin{pmatrix} x*1 \\ x*2 \end{pmatrix}$

$$y^*_1= \frac{P_{22}- P_{21}}{P_{11}+P_{22}-P_{12}-P_{21}} \quad \ldots\ldots \ldots \ldots \ldots \ldots (3.5)$$

$$y^*_2=1-y^*_1 \quad \ldots\ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.6)$$

$$x^*_1= \frac{P_{22}- P_{21}}{P_{11}+P_{22}-P_{12}-P_{21}} \quad \ldots\ldots \ldots \ldots \ldots \ldots (3.7)$$

$$x^*_2=1-x^*_1 \quad \ldots\ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.8)$$

These will be optimal minimax strategies for player I and player II respectively.

Finally the value of the game is

$$V= y^*_1x^*_1\, P_{11}+ y^*_1\, (1-x^*_1)\, P_{12}+ (1-y^*_1)\, x^*_1\, P_{21}+ (1-y^*_1)(1-x^*_2)\, P_{21} \ldots (3.9)$$

SOLVING GAME PROBLEMS REDUCING INTO L P

Here we discuss generalized m × n game problems for converting it into LP to find the two players strategies with the help of LP method.

In many applications, one needs to compute basic solutions of a system of linear equations. For example, in dealing with many linear programming problems, especially degenerate and cycling problems (Beale, 1955), it is often more convenient to locate the extreme points by applying the usual simplex method.

An m × n Game

Any Game with mixed strategies can be solved by transforming the problem to a linear programming problem. Let the value of game is v. Initially, player I acts as maximize and player II acts as minimize. But after transforming some steps when we convert the LP then inverse the value of the game. For this objective function also changes.

First consider, the optimal mixed strategy for player II,

Expected payoff for player II $=\sum_i^m \sum_j^n p_{ij} y_j\, x_i$ and the player II strategy (x_1, x_2, ... , x_m) is optimal if $\sum_i^m \sum_j^n p_{ij} y_j\, x_i \;\leq\; v$ for all opposing strategies i.e. player I is (y_1,y_2,.........,y_n).After some necessary calculations we get the following two forms of player II and player I respectively.

Player II :

Maximize, $\frac{1}{v} = x_1+x_2+\ldots\ldots\ldots\ldots\ldots\ldots.+x_m$

Subject to,

$p_{11}x_1 + p_{12}x_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + p_{1n}x_n \leq 1$

$p_{21}x_1 + p_{22}x_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + p_{2n}x_n \leq 1$ ....(3.10)

$p_{m1}x_1 + p_{m2}x_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + p_{mn}x_n \leq 1$

$x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + x_n = 1$

and $x_j \geq 0$,for j=1 , 2, ………n.

Player I:

Minimize, $\frac{1}{V} = y_1+y_2+\ldots\ldots\ldots\ldots\ldots+y_m$

Subject to,

$p_{11}y_1 + p_{21} y_2 + \ldots\ldots\ldots\ldots\ldots + p_{m1}y_m \geq 1$

$p_{12}y_1 + p_{22}y_2 + \ldots\ldots\ldots\ldots\ldots + p_{m2}y_m \geq 1$ ....(3.11)

$P_{1n}y_1 + p_{2n}y_2 + \ldots\ldots\ldots\ldots\ldots + p_{mn}y_m \geq 1$

$y_1 + y_2 + \ldots\ldots\ldots\ldots\ldots + y_m = 1$

and $y_i \geq 0$,for i=1 , 2, ……………m.

We can solve equation (3.10) and equation (3.11) by suitable L P method such as usual simplex method or Big M simplex method or Primal-dual simplex method. In this paper we will develop a computer technique incorporate with usual Simplex method.

All game problems can be solved by our procedure. Here we consider a real life problem, which illustrates the implementation and advantage of the above procedure.

Algorithm

Here we first discuss the algorithm of the game by Minimax-Maximin,2×2 strategies and for the modified matrix of the game problems.

Step (1): If the pay-off matrix is 2×2 then find the game value.

Sub step (I): Search the maximum element from each row of the payoff matrix of equation (3.4).

Sub step (II): Search the minimum element from each column of the payoff matrix of equation (3.4).

Sub step (III): If they coincide then the value of the game is V= Maximin element=Minimax element. Then Stop .If we fail to get such value, go to Sub step (IV).

Sub step (IV): Find the mixed strategies for player I using (3.5) and (3.6).

Sub step (V): Find the mixed strategies for player II using (3.7) and (3.8).

Sub step (VI): Finally, we get value of the game by (3.9).

Otherwise go to Step (2) for m , n >2.

Step (2): Search the minimum element from each row of the reduced payoff matrix and then find the maximum element of these minimum elements.

Step (3): Search the maximum element from each column of the reduced payoff matrix and then find the minimum element of these maximum elements

.**Step (4):** For the player I if the Maximin less than zero then find k which is equal to addition of one and absolute value of Maximin.

Step (5): For the player II if the Minimax less than zero then find k which is equal to addition of one and absolute value of Minimax.

Step (6): If Maximin and Minimax both are greater than zero then $k \geq 0$.

Step (7): Finally to get the modified payoff matrix adding k with each payoff elements of the given payoff matrix.

Step (8): Then to find the mixed strategies with game value of the two players, follow the algorithm below.

Algorithm for player I and player II

Here, we present a computational procedure incorporated with simplex method in terms of some steps for finding their strategies with the game value from the modified matrix for $m \times n$ game problems.

Step (1): First, take the modified payoff matrix for the player II and player I and the value of k.

Step (2): We will get equations (3.10) and (3.11)for the player II and player I respectively.

Step (3): We take input for player II from the equation (3.10).

Step (4): Define the types of constraints. If all are of "≤" type goes to step (6).

Step (5): We follow the following sub-step.

Sub-step (I): Express the problem in standard form.

Sub-step (II): Start with an initial basic feasible solution in canonical form and set up the initial table.

Step (2): We will get equations (3.10) and (3.11)for the player II and player I respectively.

Step (3): We take input for player II from the equation (3.10).

Step (4): Define the types of constraints. If all are of "≤" type goes to step (6).

Step (5): We follow the following sub-step.

Sub-step (I): Express the problem in standard form.

Sub-step (II): Start with an initial basic feasible solution in canonical form and set up the initial table.

Sub-step (III): Use the inner product rule to find the relative profit factors $\bar{C}_j$ as follows

$\bar{C}_j$ = C_j - Z_j- (inner product of $C_B$and the column corresponding to X_j in the canonical system).

Sub-step (IV): If all $\bar{C}_j \leq 0$, the current basic feasible solution is optimal and stop. Otherwise select the non-basic variable with most positive $\bar{C}_j$ to enter the basis.

Sub-step (V): Choose the pivot operation to get the table and basic feasible solution.

Sub-step (VI): Go to Sub-step (III).

Step (6): At first express the problem in standard form by introducing slack and surplus variables. Then express the problem in canonical form by introducing artificial variables if necessary and form the initial basic feasible solution. Go to Sub-step (III).

Step (7): If any $\bar{C}_j$ corresponding to non-basic variable is zero, the problem has alternative solution, take this column and go to Sub-step (V).

Step (8): Finally, we find all the strategies for player II is in corresponding their right hand side (RHS) and strategies of player I is in corresponding the $\bar{C}_j = C_j - Z_j$ of the slack variables.

Step (9): Calculate the value of the object functions for each feasible solution.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter, we shall consider a computational study of Game Theory applied to investment decisions in optimal portfolio selection Problem. Emphasis will be placed on investment decision problem, which is modelled as Game Theory Problem. Data from Oak financial Service for 2012 shall be examined.

4.1 Data Collection and Analysis

Oak financial Service, a micro-financial institution operates a financial investment services in the stock market and also offer a consultancy advice to investors on the type of investment investors should invest their finances into. The following set of data for 2012 financial year from the treasury department are shown in Table 4.1

Table 4.1 Investment Options for the 2012 Financial Year

Actions	States of Nature (Events) in Percentages			
	Growth (G)	Medium Growth (MG)	No Change (N)	Low (L)
Bonds	28.5	27.5	20.0	13.5
Stocks	19.5	17.5	14.5	11.5
Deposits	17.5	14.5	12.0	10.5
Mutual Fund	29.5	22.5	18.5	19.5

The decision – maker has to select at least one option from all possible options in which he can invest his investment. The problem here is to decide what action or a combination of actions to take among the various possible options with the given rates of return as shown in the Table 4.1. Comparing the above problem with the game theory problem, the above can be formulated as if the investor is playing a game against nature.

To solve the above problem, we first check whether the game has a Saddle point.

Thus, at Saddle point,

Min{Column Maximum} = Max{Row Minimum}

From the above, the game has no Saddle point.

From the States of Nature point of view, the player is a maximizing player, while from the action point of view the player is a minimizing player. We then write their inequalities and apply Linear Programming approach in solving them.

Let y_j, where j = 1,2,3, and 4 be the probabilities with which States of Nature plays his strategies and x_i where i = 1,2,3, and 4 with which action lays his strategies. Then the inequalities of States of Nature are:

Maximize Z = $\sum_{j=1}^{n} Y_j$

Subject to $\sum_{i,j=1}^{n} R_{ij} Y_j$

$Y_j \geq 0$

Thus,

Maximize Z = $Y_1 + Y_2 + Y_3 + Y_4$

Subject to

$28.5Y_1 + 27.5Y_2 + 20.0Y_3 + 13.5Y_4 \leq 1$

$19.5Y_1 + 17.5Y_2 + 14.5Y_3 + 11.5Y_4 \leq 1$

$17.5Y_1 + 14.5Y_2 + 12.0Y_3 + 10.5Y_4 \leq 1$

$29.5Y_1 + 22.5Y_2 + 18.5Y_3 + 19.5Y_4 \leq 1$

$Y_1 \geq 0, Y_2 \geq 0, Y_3 \geq 0, Y_4 \geq 0$

Also the inequalities of the Action Player are:

Minimize Z = $\sum_{i=1}^{n} X_i$

Subject to $\sum_{i,j=1}^{n} R_{ij}X_i$

$X_i \geq 0$

Thus,

Minimize Z = $X_1 + X_2 + X_3 + X_4$

Subject to

$28.5X_1 + 19.5X_2 + 17.5X_3 + 29.5X_4 \geq 1$

$27.5X_1 + 17.5X_2 + 14.5X_3 + 22.5X_4 \geq 1$

$20.0X_1 + 14.5X_2 + 12.0X_3 + 18.5X_4 \geq 1$

$13.5X_1 + 11.5X_2 + 10.5X_3 + 19.5X_4 \geq 1$

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, X_4 \geq 0$

4.2 RESULTS

The various feasible solutions of iteration generated by the Quantitative Method (QM) Optimization Software are shown in Table 4.2.

Table 4.2 Values showing the various Iterations

Cj	Basic Variables	1 Y1	1 Y2	1 Y3	1 Y4	0 slack 1	0 slack 2	0 slack 3	0 slack 4	Quantity
Iteration 1										
0	slack 1	28.5	27.5	20	13.5	1	0	0	0	1
0	slack 2	19.5	17.5	14.5	11.5	0	1	0	0	1
0	slack 3	17.5	14.5	12	10.5	0	0	1	0	1
0	slack 4	29.5	22.5	18.5	19.5	0	0	0	1	1
	zj	0	0	0	0	0	0	0	0	0
	cj-zj	1	1	1	1	0	0	0	0	
Iteration 2										
0	slack 1	0	5.7627	2.1271	-5.339	1	0	0	-0.9661	0.0339
0	slack 2	0	2.6271	2.2712	-1.3898	0	1	0	-0.661	0.339
0	slack 3	0	1.1525	1.0254	-1.0678	0	0	1	-0.5932	0.4068
1	Y1	1	0.7627	0.6271	0.661	0	0	0	0.0339	0.0339
	zj	1	0.7627	0.6271	0.661	0	0	0	0.0339	0.0339
	cj-zj	0	0.2373	0.3729	0.339	0	0	0	-0.0339	
Iteration 3										
1	Y3	0	2.7092	1	-2.51	0.4701	0	0	-0.4542	0.0159
0	slack 2	0	-3.5259	0	4.3108	-1.0677	1	0	0.3705	0.3028
0	slack 3	0	-1.6255	0	1.506	-0.4821	0	1	-0.1275	0.3904
1	Y1	1	-0.9363	0	2.2351	-0.2948	0	0	0.3187	0.0239
	zj	1	1.7729	1	-0.2749	0.1753	0	0	-0.1355	0.0398
	cj-zj	0	-0.7729	0	1.2749	-0.1753	0	0	0.1355	
Iteration 4										
1	Y3	1.123	1.6578	1	0	0.139	0	0	-0.0963	0.0428
0	slack 2	-1.9287	-1.7201	0	0	-0.4991	1	0	-0.2442	0.2567
0	slack 3	-0.6738	-0.9947	0	0	-0.2834	0	1	-0.3422	0.3743
1	Y4	0.4474	-0.4189	0	1	-0.1319	0	0	0.1426	0.0107
	zj	1.5704	1.2389	1	1	0.0071	0	0	0.0463	0.0535
	cj-zj	-0.5704	-0.2389	0	0	-0.0071	0	0	-0.0463	

The various range of values associated with the options of investments and their reduced cost are shown in Table 4.3.

Table 4.3 Values showing the various Ranging of investment options

Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
Y1	0	0.5704	1	-Infinity	1.57
Y2	0	0.2388	1	-Infinity	1.24
Y3	0.04	0	1	0.95	1.48
Y4	0.01	0	1	0.68	1.05
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Constraint 1	7.1301	0	1	0.69	1.08
Constraint 2	0	0.2566	1	0.74	Infinity
Constraint 3	0	0.3743	1	0.63	Infinity
Constraint 4	4.6345	0	1	0.93	1.44

The Solution Lists are also shown in Table 4.5.

Table 4.5 Values showing the various feasible solution lists of investment options

Variable	Status	Value
Y1	NONBasic	0
Y2	NONBasic	0
Y3	Basic	4.278
Y4	Basic	1.0695
slack 1	NONBasic	0
slack 2	Basic	0.2566
slack 3	Basic	0.3743
slack 4	NONBasic	0
Optimal Value (Z)		5.3475

4.3 CONCLUSIONS

The solution to game theory application in financial investment planning is effective in giving optimal solution as compared with personal discretion means of investment by an investor. From the concept of investment using game theory, the solution to this problem consists of many feasible options investment opportunities where an investor can invest where the limit of the investment amount is not violated.

We therefore recommend that our model should be adopted by the institution for its investment planning.

CHAPTER FIVE

RECOMMENDATIONS AND CONCLUSIONS

5.0 INTRODUCTION

The inability of standard finance theories to provide satisfactory explanations for observed phenomena lead to a search for theories using new methodologies. This was particularly true in corporate finance where the existing models were so clearly unsatisfactory. Game theory has provided a methodology that has lead to insights into many previously unexplained phenomena by allowing asymmetric information and strategic interaction to be incorporated into the analysis.

Game theory as a branch of applied mathematics is concerned with how agents interact with one another through the choices they make. This interaction, or game, is both independent – made by autonomous agents – and interdependent – the outcome relies on the combination of choices made by the agents. These are strategy games where the knowledge of all possible outcomes is readily available to the player at the time of choice making, as opposed to games of chance where the outcome can be determined in whole or in part by a randomized factor.

Thus, studying game theory applications in financial decision making problem can never be considered as an abstract research with no real importance.

5.1 RECOMMENDATIONS

The use of mathematical models has proved to be efficient in the computation of optimum results and gives a systematic and transparent solution as compared with an arbitrary method. Operation has become one of the key competitive advantages with optimization-based approaches being expected to play an important role. Using optimization-based approaches to model industrial problem gives a better result. Management will benefit from the proposed approach for investors who would be investing in the stock market in order to optimize returns and minimize risk. We therefore recommend that our model should be adopted by the institution for its investment planning.

5.2 CONCLUSIONS

In this thesis we have studied the concept of game theory in investment decision making problem. Our solutions to the data instance from an investment institution in Ghana by applying our developed model gave optimal value of the game from various combinations of investment options.

According to the developed model, the value of the game from the various investment options was 5.2435 percent growth rate in mutual fund and bonds. The solution shown gave remarkably better results than the independent model normally used by the institution.

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