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FORENSIC ESTIMATION OF TIME SINCE DEATH: A COMPARATIVE STUDY OF POLYNOMIAL REGRESSION AND MODIFIED NEWTON'S MODELS

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the Msc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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DEDICATION

I dedicate this work to my wife, Matilda and lovely daughter Aseda.

ABSTRACT

Empirical studies have been made into the determination of Time Since Death (ATSD).This study attempts to briefly review most of the methods used to estimate TSD and concentrate on the development of a Polynomial Regression Model (PRM) and a Modified Newton's Model (MNM) that accurately predicts TSD and chose the most consistent, efficient and accurate estimator of TSD between the two. Such a model, when accurately developed, will add to the security services scientific ways of predicting TSD, as well as, help in civil and criminal cases. The study found out that the PRM best predicts TSD with an R^2 of 99.87% given the two secondary data obtained from the Journal of the Indian Academy of Forensic Medicine 2005 volume $27(3) : 170 - 176$ and the Acta Morphologica 2006, volume $3(3) : 52 - 53$ (Macedonia Association of Anatomists and Morphologists). The MAPE, MAD and SSE of the two models were also statistically tested and compared to that of the Actual Time Since Death (ATSD).The MNM relied on the use of an iterative method, to determine the number of times, time (denoted t in hours) changes over time for Rectal Temperature (RT) to coincide with the developed model of differential equation .The calculation embodied several parameters /conditions that affect the cooling rate of dead bodies.

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CONTENTS

LIST OF ABBREVIATION

ATSD ....Actual Time Since Death

PRM ....Polynomial Regression Model PRME

.... Polynomial Regression Model Estimates

MNM ....Modified Newton's Model

MNME .... Modified Newton's Model Estimate

....

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CHAPTER 1

INTRODUCTION

1.1 Review of chapter 1

Mathematical and Statistical models for determining the Time Since Death in homicide/murder, natural or an accidental death is just one tool within the reach of the forensic scientist. Proper application of such model to a real life situation can aid forensic investigator to answer the basic question asked by all, “When did death occur?”

Other measuring rod to estimate time of death at the disposal of the investigator are; Rigor Mortis the natural process occurring in death bodies through the contraction and relaxation of the body muscles. Livor mortis; the resultant colour of the dead body due to the pooling of blood as a result of gravity. Others are, forensic entomology, the study of insects at the crime scene, colour of the eye and DNA among others.

Estimation of the actual time of death has been a natural requirement in all human societies since ancient days. Connect ED: The California Centre for College and Career stated in her integrated Curriculum Unit on forensics crime scene investigation that forensic scientist have not only solved cold cases but also reopened “solved” cases. By reopening solved cases, some suspects can be freed/released and other suspects pursued. In other words estimating the time of death can serve as a screening process to free suspects who were not within victims reach at approximated time of death.

It is of interest to note that, unless death, defined in following paragraphs is supervised, by a physician/a medical practitioner, the exact time of death is estimated with an element of error.

1.2 Background Of Study

1.2.1 Brief History Of Temperature Based Models For Estimating TSD

Polynomial Regression Model (PRM) and Modified Newton's Models are based on temperature variations of the dead body. It follows the assumption of the Newton's Law of Cooling.

Several studies since the latter part of the 1800 revealed that,the temperature of dead human bodies initially shows a lag period and subsequently, reduce/increase towards the Ambient Temperature. "Significant contribution from the $19th$ century identified Post Mortem temperature drop among others. Rainy(1869) and Rosenthal (1872), mesured Rectal Temperature, instead of surface temperature. the Post-mortem Temperature plateau was already observed and described by this authors, who measured central core temperature 80 years before Shapiro (1965)" (Payne-James et al., 2003).

Rainy (1868),applied the Newton's Law of Cooling to calculate the length of TSD. This is the first known method for TSD that took into account a scientific principle for estimating the manner in which the body cools.

Rainy's formular for estimating TSD was developed based on temperature reading of 46 dead bodies. It required data on Ambient Temperature (AT) and two measurements of Rectal Temperature (RT) over a period of at least 1 hour to evaluate the actual temperature decrease. Rainy concluded that , if the RT is below 85^0F, then the maximum TSD will not exceed one and a half times the minimum TSD, Payne-James et al. (2003).

In 1962, Marshall and Hoare,established that, the basic assumption of the Newton's Law of Cooling was invalid when applied to the early stages of the cooling of a deceased human being. This was after they studied the cooling of 100 dead bodies. They suggested that, the cooling of the bodies are being influenced preliminary by a phenomenon whose effect decay with an exponential function. This exponential function synonymous to the temperature plateau is given as; Ce^{-pt} Marshall and Hoare (1962) .It is interesting to note that the Henssge Nomogram was also developed based on the Marshall and Hoare equation. Currently, the Marshal and Hoare formula is being used extensively to estimate TSD, due to its efficiency relative the initial Newton's formula.

1.2.2 Need to Study TSD

Several hundreds of thousand Murder/ Homicide occur throughout the world. "International homicide caused the deaths of almost half a million people (437,000) across the world in 2012. More than a third of those (36"%") occurred in the Americas, 31"%" in Africa and 28"%" in Asia, while Europe (5"%") and Oceania (0.3"%") (UNODC, 2014).

The huge number of murder cases gives credence to the need to develop a simple, efficient and easy to use mathematical model to estimate the time of death of murdered victims.

1.2.3 Heat loss in dead bodies

Process of heat loss in dead bodies: A dead body loses heat through the following ways;Vij (2011) , claimed the dead body loses heat mostly through conduction and convection. Conduction through the part of the body touching the ground or some other material. By convection, the body fluids of the dead body evaporates.

Radiation in dead bodies is due to the objects lying in the vicinity.

1.2.4 Measuring Dead Body

The average oral temperature of humans is 37.2^0C (98^0F). Temperature of the human body varies according to which part of the body it is taken. For instance it is averagely 36.4^0C (97.6^0F) underneath the armpit. An accurate way of measuring the temperature of a dead body is via the rectum where some intestinal organs could be reached. Claridge (2015), published that, by far the most accurate reading of a body temperature is the one that can be taken rectally. In taking the rectal temperature, care should be taken in order not to tamper with other evidence at the crime scene such as moving dead body and in murders associated with sex/rape.

1.2.5 Factors that affect cooling of dead body.

Several factors affect the rate of cooling of a deceased body. Cox (2009), described them as follows; body surface area to weight; (children cool more rapidly than adults under similar environmental conditions), likewise, thin, emaciated and dehydrated men cool more rapidly than women. The amount of clothing or blankets etc. that are on or wrapped around the body also affect the rate of post mortem cooling. A nude body will cool far more rapidly than a body clothed or wrapped in blankets.

Air movement also affects rate of cooling. Typically a body will cool more rapidly in moving air than still and more rapidly in a cold breeze than a warm breeze. Exposure to water also affects rate of cooling. A body cools much more rapidly in water than air of the same temperature.

1.2.6 Newton's Law of Cooling

Objects hotter than their ambient (surrounding) temperature reduces in temperature towards the ambient temperature through a law called Newton's law of cooling, named after Isaac Newton (1642 – 1727). The law states that the rate of the cooling of a body is determined by the difference between the temperature of the body and that of its environment.

Newton published the first known work on this law of cooling in 1701. In a paper filled, "Scala Graduum Colaris" published (in Latin) in the Philosophical Transactions of the Royal Society of London. Newton did not write any formula but expressed verbally his cooling law: The excess of the degree of the heat.... were in geometrical progression when the times are in an arithmetical progression (by “degree of heat”, Newton meant what we now call “temperature” so that “excess degrees of the heat” means “temperature different”)

He also wrote somewhere in the same article that; The heat which hot iron, in a determinate time, communicates to cold bodies near it, that is, the heat which the iron loses in a certain time is as the whole heat of the iron; and therefore (ideoque in Latin), if equal time of cooling be taken, the degree of heat will be in geometrical proportion (Besson, 2010). Newton’s study of cooling was done with the initial development of a measuring scale of temperature, using a linseed oil thermometer he developed.

In fact,Newton (1701) was mainly interested in defining a thermometric scale (scala graduum caloris) ... and more than half of his article is occupied by a list of values measured temperature and description of corresponding situations (calorum descriptiones et signa). The Newton’s law of cooling had been supported / confirmed as well as criticised by several scientist. The Russian and Swiss, G.W. Richman and Johann Heinrich Lambert (1755) respectively had confirmed the work of Newton whilst Scotsman George Martine (1738) and German I.C.P. Erxleben had criticised his work.

A much improved/accurate research on the measuring of temperature and on the law of cooling was published by the Frenchmen Pierre Dulong and Alexis Petit in 1817. They initially studied cooling process in a vacuum (due to radiation) using 2 mercury thermometers. They established that (p. 248) “the cooling of a body in a vacuum is given by the excess of its own emitted radiation over that of the surrounding bodies (Besson, 2010).

1.2.7 Polynomial Regression Model (PRM)

This is a form of linear regression in which the relationship between the independent variable *(x)* and the dependent variable *(y)* is modelled as an *nth* degree polynomial.They are usually fit using the method of least squares that minimizes the variables of the unbiased estimators of the coefficient, under the condition of the Gauss – Markov Theorem.

The first design of an experiment for polynomial regression appeared in an 1815 paper of Gergonne. In the twentieth century, polynomial regression played an important role in the development of regression analysis, with greater emphasis on issues of design and inference (wikepedia, t 25).

1.2.8 Mathematical Modelling

It is the process of writing differential equations to describe a physical situation. Quarteroni (2009), claimed that, mathematical modelling aims to describe the different aspects of the real world, their interactions and dynamics through mathematics. It offers new possibilities to manage the increasing complexity of technology, which is at the bases of modern industrial production. They can explore new solution in a very short period of time, thus, allowing the speed of innovation cycles to be increased, saving money and time.

It was further described as the abstractions of reality and a simplified representation of some real world entity. It can be in equations or computer codes. They are also characterised by assumptions about variables (things that change), parameters (things that do not change) and functional form (the relationship between the two; variables and parameters).

Davison (2003), defined statistical models as a probability distribution constructed to enable inferences to be drawn or decisions made from data. A mathematical and statistical models would be developed from the Newton's law of cooling (using differential equations) and the Polynomial Regression respectively to estimate the time since death(TSD) in this thesis.

1.2.9 Iterative Methods

Barrett et al. (1994), defined Iterative Methods as a wide range of techniques that use successive approximations to obtain more accurate solutions to a linear system at each step.

Better still, it is a mathematical procedure that generates a sequence of improving approximate solutions for a class of problems(Wikipedia, 2015).

1.2.10 Definition of terms

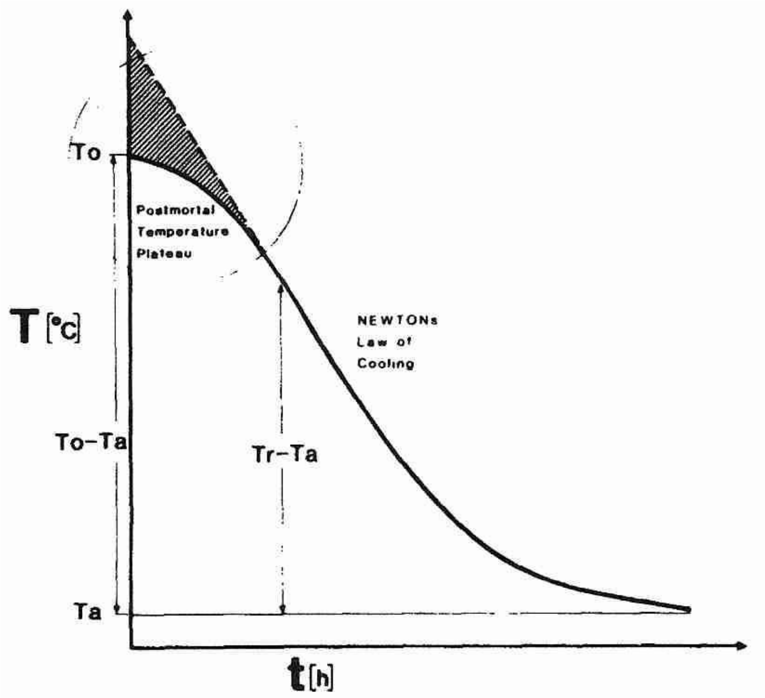
An attempt is made here to define a selection of terms used in this thesis.

Temperature Plateau

Post mortem interval/cooling does not begin immediately after death, it stays constant for a period before reducing or increasing as a result of the temperature difference between dead body and ambient temperature. (If the ambient temperature is hotter than the dead body, temperature of dead body will rise and vice versa).

This initial constant temperature was named as temperature plateau by Harry Rainy a professor of Medical Jurisprudence at the University of Glasgow. He noted that temperature may even rise after death. Rainy, (1968) defined the plateau as the period after death where the body does not cool at all and the body temperature may even rise a bit.

Figure 1.1: Temperature Plateau



(Source:Henbge and Madea (2004))

Death

The Canadian Blood Service and World Health Organisation defined death in the International Guideline for determination of death – phase 1 in May 30 – 31, 2012 (Montreal forum) as; The permanent loss of capacity for consciousness and loss of all brainstem functions. This may result from permanent cessation of circulation and/or after catastrophic brain injury.

In the context of death determination; Permanent: loss of function that will not resume spontaneously and will not be restored through intervention. Lack of capacity for consciousness means the lack of current or any future potential of awareness.

Post Mortem Interval

The time from death until finding the body (Tamiya, 2012).

Cold case

A cold case is a crime or an accident that has not yet been fully solved and it is not a subject of a recent criminal investigation, but for which new information could emerge from new witness' testimony, re-examined archives, retained material evidence, as well as fresh activities of the suspect. (Wikipedia, 2014)

Crime Scene Investigation (CSI)

It is the meeting point of science (e.g. fingerprint), logic and law (testifying in court). "Processing a crime scene" is a long, tedious process that involves purposeful documentation of the conditions at the scene and the collection of any physical evidence that could possibly illuminate what happened and point to who did it. There is no typical investigative approach (in short CSI involves all that goes on at the crime scene). At any given crime scene a CSI might collect dried blood from a windowpane, without letting his arm brush the glass in case there are any latent fingerprints there, lift hair off a victim's jacket using tweezers so he does not disturb fabric enough.

At the scene, police officers are typically the first to arrive at a crime scene. They arrest perpetrators if still at scene, and secures the scene. CSI unit documents the crime scene in detail and collects any physical evidence. Again a specialist (entomologist, forensic scientist, and forensic psychologists) may be called in if the evidence requires expect analysis. Detectives interview witnesses and consult with the CSI unit and follows the investigation of crime by following leads provided by witnesses and physical evidence.

The CSI begins with the arrival of CS Investigator at the scene and ensuring the place is secured. He/she walks through scene without touching anything. Second, she/he documents the scene by taking photographs and drawing sketches during a second walk-through. Thirdly, the CSI very carefully touch/collects all potential evidence and tagging. At this stage the collected evidence is handed over to the forensic scientist for laboratory work,(Julia, 2005). For the purpose of this research, taking of post mortem and ambient temperature among others fall under the CSI.

Forensic Investigation/Science.

This involves the laboratory analysis of the evidence collected at the crime scene. The term is loosely used for the analysis of all crimes, be it cybercrime, robbery, smuggling, drugs among others. For the purpose of this thesis, attention will be placed on death (murder/homicide, accident or natural death).

Julia (2005), defined what goes on in the laboratory after evidence had been brought in, as forensic science. Often, a piece of evidence passes through more than one department for analysis. Each department delivers a complete report of the evidence it analyses for the case and will be used to testify at the law court. The development of mathematical models, analysis of data and the validation of the results would be considered as a forensic investigation/ science in the thesis.

Homicide.

The UNODC (2014), defined homicide as the unlawful death, purposefully inflicted on a person by another person. It concerns itself only with those act in which the perpetrator intended to cause death or serious injury by his or her action. It therefore excludes death related to conflict, death caused when the perpetrators where reckless or negligent as well as killing that are usually considered justifiable according to penal law such as those by a law enforcement agents in their line of duty or in self-defence.

1.3 Problem Statement.

Accurate estimation/determinations of the time of death has several importance, be it in murder/accidental or natural death. Proper solution to a civil matter can be arrived at through the use of an accurate estimated time of death. Geberth (2006) put; "in civil matters, time may be what determines whether or not an insurance policy was in effect or void". For instance, if a client died hours before maturity of claims, it may not be paid.

In criminal cases, the time of death may serve as a screening process to either corroborate or refute suspects. Above all, it will satisfy the curiosity of the friends and loved ones of the dead. Sadly, security services in Ghana, Africa and parts of the world continue to rely on crude and unscientific methods to determine/estimate the time of death, especially in homicide/murder cases.

1.4 Main Objectives

Forensic estimation of the time of death using Polynomial Regression and Modified Newton's Models.

1.5 Specific Objective

The following are the specific objectives of the study.

- Determine a general solutions to estimate time of death analytically,using PRM and a MNM.
- Validate model using existing data
- Determine the best model for estimating TSD between the PRM and MNM.

1.6 Significance of the study.

It is imperative to conduct a research into the time of death so that an easy, reliable/efficient and quicker model to determine the time of death be realised. This in turn will help the security services, especially the police to at least add another scientific way of estimating TSD to the existing methods. Again, it would help in civil cases (help determine maturity of insurance claims etc), criminal cases (free and/or redefine suspect) and societal/family needs of satisfying curiosity of the time of death as well as determine who benefits from a will.

1.7 Methodology

The research will attempt to determine a Newton's mathematical model and a polynomial regression model to estimate the actual time since death analytically. A descriptive analysis of results obtained from the two models would be done.

The derived model would be validated using existing data.

1.8 Organisation of the Thesis

The thesis contains five main thematic chapters. A prelude to these chapters are the Abstract (summarize whole thesis) table of content, list of tables and figures, dedications and the acknowledgement. The initial chapter situates the thesis into perspective with the background of the thesis; objective of the study, organisation of the thesis, the limitation and definition of terms used. Chapter two contains the review of literature, chapter three comprises of the method adopted in the research. Chapter four gives an analysis of data, modelling data into a time of death estimator and the validation of results with existing data and discussions of findings. The last chapter summarizes findings, conclusions and recommendations.

1.9 Limitations of study

Various hospitals and health posts do not take adequate data on patients when they die, hence, it is difficult getting accurate data on the temperature of dead humans. Again most health posts lack tools for correct intestinal/rectal temperature reading as well as the weight of dead bodies.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

One of the most extensive areas of research has been the Time Since Death estimation/determination. The following are a review of some of the literature on this field of study.

2.2 Review of Some methods for estimating Time Since Death(TSD)

2.2.1 Livor mortis (Post Mortem lividity or hypostasis)

This is the resultant colour of a dead person's body due to pooling of blood and the work of gravity. At death,the heart stops beating and blood vessels stop moving, halting blood circulation. Blood flows down the inactive vessels into the lowest part of your body regardless of the body position.

It is observed as a dark purplish hue (shade of colour) onto the skin in which the blood had flown and pooled. Depending on the extent of time, this colour can/will become permanently fixed on the body. Lividity will be present after 2-8 hours of death. But can be removed easily with pressure on affected area. Hence, if colour disappears, a person probably died less than eight hours ago. If the colour remains person might have died beyond eight hours.

Environmental factors such as temperature, accessories and clothing on dead body can affect livor mortis.Environmental factors such as temperature, accessories and clothing on dead body can affect livor mortis. Extremely cold temperature slows down livor mortis and vice versa. Belts, wrist watches and clothing can also externally constrict blood passage slowing down livor mortis(Norman, 2012).

2.2.2 Rigor Mortis

Cox (2009), defined Rigor Mortis as the generalized stiffening of the skeletal and smooth muscles of the body following death. Norman (2012), also described it as the extremely stiffening of the body's muscles due to the build-up of calcium in muscle fibre and the inability to remove excess calcium.

Rigor mortis begins after two hours of death, starting from the head and slowly progressing to the feet. After 12 hours the dead body would be at its most rigor state. After 36 – 48 hours, the body would resume the soft dead body status. Process of rigor mortis depends on temperature, body weight, body temperature and sun exposure. It is faster in hotter bodies and slower in cooler bodies(Norman, 2012).

2.2.3 The colour of the eye

Cloudiness of the cornea will appear within a few hours to 24 hours, depending on the degree of closure of the eyelids, environmental temperature, humidity, and air movement. It is not recommended that you attempt to use cloudiness of the cornea for determination of post-mortem intervals due to the multitude of factors, which affect it (Cox, 2009). The transparency of the Cornea is very variable.Kumar et al. (2012) claimed that, the variability of the Cornea is less than other methods of estimating PMI. His analysis of 238 autopsis' cornea revealed that,transparent.

They concluded that changes of the Cornea occurs more in warm and moist weather than in warm and dry weather and cold and moist respectively and least in cold and dry weather. That changes of the Cornea occurs more in warm and moist weather than in warm and dry weather and cold and moist respectively and least in cold and dry weather.

2.2.4 Forensic Entomology

Forensic Entomology is the study of insects found at the crime scene. A pathologist is able to establish a more accurate time scale depending on which insect are found on the body and what they are at in their life cycle,Claridge (2015). Forensic entomology works in death scene investigation because insects are defined by certain characteristics and follows a predictable pattern of development (Warrington, 2010). Arthropods (invertebrate animals with joint limbs) are most important in forensic entomology because they eat dead vertebrate bodies, including man. One of the first groups of insects that arrive on a dead vertebrate is usually the blowflies (Dipteria, calliphoridae). The female blowflies oviposit (lay eggs) on dead body within few hours. (www.cienciaforense.com/pages/entomology/overview.htm)

Blowflies and insects go through four developmental stages called complete metamorphosis. A careful or correct calculation of these stages of development of insects as well as other creatures present at a crime scene can lead to the estimation of the time since death. The following table of approximation of the time of death,using insects present and stages of dead body is developed from the following article;Wikepedia (2014) and Carloye and Bambara (2006)

Table 2.1: Entomological Table

Approximate time of death	of Appearance body/stage	Insects present
Few minutes – 3 days	Fresh stage Just like sleeping. Last until body bloats	Blowflies and flesh flies arrive minutes after death. Begins to lay eggs/deposit maggot. Maggots begin to feed on body tissues. Ants begin arriving.

4 – 7 days	Bloated stage Body becomes visibly inflated, especially abdomen. Colour change and odour of putrefaction is noticeable.	Blowflies, houseflies, flesh fly, maggot, beetles, ants and other creatures feed on maggot.
8 – 18 days	Decay active stage Carcass deflates. Has wet appearance and a strong odour of putrefaction.	Ants, cockroaches, beetles and flies.
19 – 30 days	Advanced decay/post decay stage. Most flesh remove from carcass. Strong odour of decomposition begin to fade.	Adult skin beetle and mite arrives at carcass. Larval stages are absent. Instar calliphorid begin to leave carcass
31 days and over	Dry decay Bones, hair and portions of skin stench is no longer powerful	Centipedes, isopods, snails and cockroaches.

The above table gives a general decomposition model of a dead body. However, factors such as temperature (slow decomposition at cold periods and vice versa), drug present in body at death, clothing and access to dead body can affect rate of decomposition.

Goff (1993), postulated that insects are frequently the first organisms to arrive at a dead body. That, their activities begin a biological clock that will allow for an estimation of the postmortem interval (PMI).Five -5- stages of decomposition (fresh, bloated, decay post-decay and skeletal) are suggested as reference points in the decomposition process. Factors that may delay invasion of the remains by arthropods or alter developmental patterns, such as presence of drug and toxins, wrapping of body and climate were discussed by (Goff, 1993).

Goyal (2012), saw entomology as the application of knowledge of insects during investigation of crimes or other legal matters. His study of Insect evidence or presence revealed that insects of the form of blow flies in their different stages of development are found on fresh and decaying corpses. Beetles were found on skeletonised bodies.

2.2.5 Henssge Nomogram Method

The Nomogram method is based on a formula which follows the sigmoid shape of the cooling curve.The formula contains two exponential constant parts.The first represent the Post Mortem Plateau and the second constant shows the exponential drop of temperature after the Plateau according to the Newton's law of cooling, described by (Poposka et al., 2013).

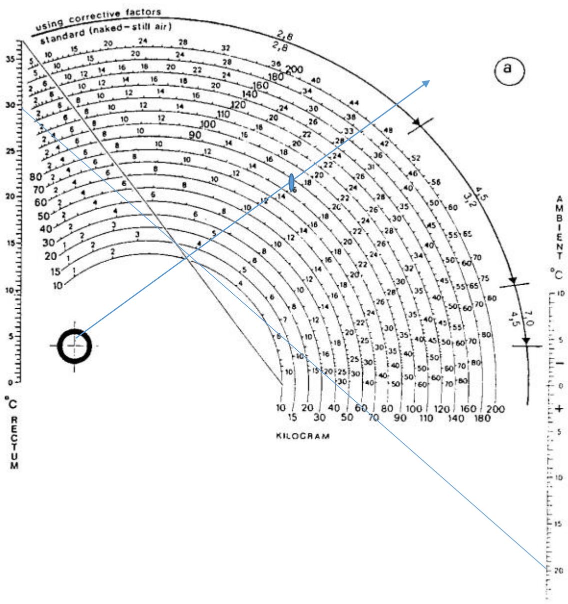
A widely used Nomogram for a forensic time of death determination is the one published by Henssge in 1981. He presented a simplified method to determine the Newton cooling coefficient, by finding out statistical figures of the deviation between calculated and real death times for cooling under standardized conditions (e.g. ambient temperature of below 23^0C and above 23^0C)

His Nomogram is in two folds; one for ambient temperature above 23^0C and one for ambient temperature below 23^0C. Based on conventional calculations such as those of; De Saram and Marshall as well as backed by a great volume of experimental data, Henssge has produced a method which can be carried out either by a simple computer program or a Nomogram. The results are given within different ranges of error, with a 95 per cent probability of the true time of death falling within this ranges. The ranges vary from 2.8 hours each side at a best estimate to 7 hours at worst(Saukko and Knight, 2004).

To read off the time of death, connect the rectal and ambient temperature on the Nomogram with a straight line, so that it crosses the diagonal line on the Nomogram. A second straight line is then drawn from the centre of the circle at the bottom left of the diagram to intersect the diagonal line and the initial line drawn on the Nomogram. The second line crosses the semi-circles of the body weight and that of the outermost semi-circle. The estimated time of death is read off .The outermost intersection gives the margin of error at 95%.

For instance, given a body temperature of 28.8^0C, ambient temperature of 21^0C and body weight of 83 kg,with an ATSD of 13 hours.Then, the estimated time of death becomes; $16+$ or -2.8 hours. The death occurred between 13.2 and 18.6 hours, before the rectal temperature measurement with a 95% reliability level.

Figure 2.1: Henssge Nomogram for ambient temperature up to 23^0C



(source:Poposka et al. (2013)

2.2.6 Algor Mortis

The Algor Mortis is the bases of all the temperature models for the estimation of the TSD. Extensive research has shown that, the human body temperature is averagely above that of the ambient (atmospheric) temperature. This temperature reduces in the body after death. This reduction in the body is called Algor mortis. Unlike forensic entomology, Algor Mortis is most effective during the first 24 hours of death. Its effectiveness fades as actual time since death increases.

2.2.7 Review Of Other Related Works

The behavior of the temperature within the external auditory canal (EAC) after death is investigated byRutty (2005). Results were compared to that of rectum. The EAC was then applied to previously published algorithm to estimate the TSD. Two experimental models were used to revisit and expand upon the question as to whether a temperature can be recorded from the EAC and used to estimate TSD.

Pellini (2011) studied the time since death and body Cooling using a revaluated Henssge Nomogram, which takes account of temperature and body weight ambient temperature, clothing worn andventilation. She assessed the reliability of Henssge Nomogram to estimate postmortem interval (PMI) accurately, using 46 dead bodies from a traffic accident with known PMI ranging from 2 to 44 hours. Rectal and ambient temperatures were measured/recorded along with variables such as clothing, sex, age. The body weight was also assessed on the basis of stature and muscle distribution. The Henssge Nomogram was then used to measure PMI and compared to the real one.17%of class had two PMI's coinciding,52%of cases, had real PMI fell within the range of calculated PMI whereas in31%of cases real PMI was outside calculated range. The Nomogram is more accurate when applied within ten hours after death.

Saram et al. (1956), studied or investigated the cooling rate of 41 executed prisoners under identical conditions. They realized that the body-room temperature difference has been found to have a definite bearing on the cooling rate. The influence of evaporation on the fall of temperature in a dead body was found as an important additional factor causing fall in temperature. They developed a mathematical formula to estimate the time of death in postmortem interval. It was suggested by saram that clothing did not appear to have significantly influenced the cooling rate. They also did less study into the effect weight of the body and of the humidity of the atmosphere.

However, recent studies postulated that clothing atmospheric temperature and body weight affect PMI significantly.

Kumar et al. (2014), in 2014 attempted to estimate time since death from a typically decomposed dead bodies. They noted that changes occurring in a specific sequence in after death helps in determining the time since death. Nevertheless, the specific sequence of noticeable changes of decomposition may not be found in all dead bodies. In such cases estimate time since death may be a difficult task. They used all other changes that occur after death to determine the time of death of two dead bodies with their decomposition occurring in an unusual pattern.

Olagunju and Abdulazeez (2013) focused their study on the determination of time of death models. They considered a mathematical model based on Newton's law of warming and cooling. The mathematical model was simulated with MATLAB program enabling a prediction of the time of occurrence.

Algorithms or supravital signs and early signs of death in the early postmortem period was used by Poposka et al. (2013),to estimate time since death. They noted that using several methos for determining the time since death increases significantly the preciseness and reliability upon estimation of the time since death. They performed five (5) analysis of parameters for estimating time since death. Supravital reactions (electrical and chemical excitability of muscles) and early signs of death (cooling of the body; algor mortis, postmortem lividity and rigor mortis) on 120 bodies with known time of death. Obtained results were then used to prepare a special table algorithm containing minimum and maximum values of the postmortem period for each tested parameters.

Brown and Marshall (1974),gave an explanation to the sigmoid shape of the cooling curve of the human corpse. They further discussed the equations for estimating the time of death from the body temperature. The reasons why an equation consisting of two exponential terms is adequate and the way such equation can be used in a real life problem were postulated by the authors.

Smart and Kaliszan (2010), also attempted to predict the time of death using Cosmol Multiphysics software: (an interactive environment for modeling and simulating scientific and engineering problems) to study the convective and conductive heat transfer from a dead human eye balls to the surrounding air. The study brings out a portion of a complete postmortem temperature decay curve. It is superimposed exactly upon the actual cooling curve development with a Comsol finite element method software. Time of death is then predicted.

Kaliszan et al. (2009), studied the estimation of the time of death using body cooling. They presented a review of the literature referring to the estimation of the time of death over nearly 200 years. Emphasis is put on the development of the methods taking advantage of the decrease in body temperature after death measured in various body sites.

Mall and Eisenmenger (2005),claims analysis of postmortem cooling provide the most accurate estimates. He noted that the main problem that initially prevented heat flow model from being used in determining time of death was solving complex heat flow equations. The study presented a three dimensional finite element-model of the human containing various tissue compartments with different thermal tissue properties. The initial temperature field is modeled homogeneously with a temperature gradient between body core and shell. Heat loss by conduction, convection and radiation as well as heat gain by supravital activity and irradiation from external sources can be simulated. The decrease rate of the supravital energy production was calibrated and the model successfully validated using the experimentally verified empirical model by Marshall and Hoare.

Ozawaemail et al. (2013), postulated that Rigor mortis is an important phenomenon to estimate the postmortem interval in forensic medicine, though it is affected by temperature. They measured stiffness of rat muscles using a liquid paraffin model to monitor the mechanical aspects of rigor mortis at five temperatures (37, 25, 10, 5 and 0^0C). At 37, 25 and 10^0C, the progression of stiffness was slower in cooler conditions. At 5 and 0^0C, the muscle stiffness increased immediately after the muscles were soaked in cooled liquid paraffin and then muscles gradually became rigid without going through a relaxed state. The phenomenon suggests that it is important to be careful when estimating the postmortem interval in cold seasons.

Poposka et al. (2011), attempted an estimation of the time since death through electric and chemical excitability of muscles. They examined the electric and chemical excitability of the muscles in 50 cases with a known time of death in order to determine their importance in the estimation of time since death. The electric excitability of the muscle was done with a device for electric simulation and the chemical excitability of the eye pupils was done by injecting miotic carbahol into the front eye chamber of the right eye and mydriatic adrenalin HCL into the front eye chamber of the left eye. By analysis and processing of obtained results, it was determined that electric excitability of the muscle is more important in determining the time since death within a postmortem period of up to 10 hours and chemical excitability for a postmortem period up to 12 hours.

Leinbach (2011), noted that the time of death can be an important information in cases involving insurance or criminal investigation. He further noted that Newton's Law of cooling using body temperature data obtained by a coroner was used to make a more accurate estimate towards the nineteen century. Again, they claim the Newton's Law does not really describe the cooling of a non-homogeneous human body. He discussed a more accurate model of the cooling process based on the theoretical work of Marshall and Hoare and the laboratory based statistical work of Claus Henssge. He used DERIVE 6 – 10 and the statistical work of Henssge to explore the double exponential cooling formula developed by Marshall and Hoare. The end result is a tool that can be used in the field by coroner's scene investigation to determine a 95% confidence interval for the time since death.

Mathura and Agrawala (2011), was of the view that determining time since death is an important goal in medico legal investigation. Since 1850's scientist have been working on different methods to determine PMI. Earlier methods were based on body cooling, rigor mortis, changes in the eye putrefaction, supravital reactions among others. They claimed those mentioned methods lack precision, hence, the focus is now shifted to biochemical methods. The biochemical methods are based on systematic pathophysiological changes and found to be more accurate since the effect of external conditions is less. They described the various methods used for estimating time since death.

Fitzgeralda and Oxenham (2009), developed a degree of decomposition index (DDI) by quantifying stages of decomposition for individual body elements. Two pigs were allowed to decompose undisturbed on the ground surface, one in full sun and the other in semi-shaded. The results of the regression modeling suggests that TSD accounts for the majority of variations in decomposition (using the DDI) while variation in Macro-environment (sun versus shade were not significant contributing factors). They concluded that quantifying decomposition is an effective method of estimating time since death (TSD), which negates variable environmental effects on the decomposition process. They claimed the implication for forensic investigations of recent deaths include the potential to provide an improved estimation of TSD at the time of body recovery.

Arikeri et al. (2013), Arikeri et al (2013) used and evaluated together physical and biochemical postmortem changes while estimating the time of death. Vitreous humor chemistry was used for postmortem analysis since serum values of many components are thought to be reflected in vitreous humor and stable for a prolonged PMI. A similar isolated component to vitreous humor is synovial fluid. They determined PMI by estimating sodium and potassium levels in joint fluid and to get hints for the reliability so as to establish reference value for synovial fluid. 82 cases of known interval of time since death synovial fluid was taken and analyzed for potassium levels on flame photometer. They found out that a positive relationship existed between time since death and levels of sodium and potassium. A definite equation was developed for computation of postmortem interval (PMI) with synovial potassium values.

Verica et al. (2007), postulated that postmortem cooling of the body is one of the pertinent parameters in estimation of time since death during the early postmortem period. She analyzed some of the existing methods, compared obtained results and determined which method gives precise results of the estimation of the time since death. Temperature of 50 cases of death with known time of death were taken and that of ambient temperature was taken as well. The weight of the body was also considered. They applied the Al-Alousi and Anderson method and the Henssge Nomogram method to estimate the time since death. She realized that the Henssge-Nomogram gives less discrepancy from the true time of death.

Henssge and Madea (2007), claimed the main principle of the determination of the time since death is the calculation of a measurable date along a time dependent curve back to the start point, that, the characteristics of the curve like slope as well as the start point are influence by internal and external antemortem and postmortem conditions. These influencing factors have to be taken into consideration quantitatively in order to improve precision of death time estimation. They concluded that, the amount of literature on estimating time since death has a reverse correlation with its importance in practice.

Madea et al. (1990), studied the precision of estimating the time since death by vitreous potassium – comparison of two different equations. The comparison was done on an independent sample of 100 cases. The noted that, the very flat slope in the Sturner's equation was the reason for a systematic over estimation of the time since death with much wider limits of confidence (95%) compared to the results using an own equation with a steeper slope of vitreous potassium.

Green and Wright (1985) also noted that one consistently used method of postmortem interval estimation is by means of body temperature measurements. The study concentrated on the extent of the errors to be encountered. Using this method among others and to device a more simple and accurate method. He device a method known as time dependent – Z equation (TDZE). This method required the measurement of two rectal temperatures. Results obtained using TDZE to estimate 67 coroner's cases compare favorably with results obtained by the use of other methods on the same data.

Shved et al. (1989), studied a diagnostic program for estimating the time of death using micro calculators MK-61 and B3-34 based on the process of dead body temperature. They noted that specific features of the process of changes in temperature measurements in diagnostical area of a dead body are characterized by a parameter which involves process sample obtained by temperature measurements in diagnostical area of a cadaver at the accident scene during 1hr. The programme provides for estimation of postmortem interval within the first 60 hrs after death with a deviation not exceeding $+/-3\%$ as compared to the real time of death.

Siddamsetty et al. (2014), claimed the estimation of the time since death (TSD) with a fair accuracy from postmortem changes still remains an important but difficult task to be performed by every autopsy surgeon. They estimated TSD from electrolyte analysis of postmortem vitreous humour collected from samples under semi-arid climate. Vitreous humour collected from 210 dead bodies were analyzed for sodium, potassium, calcium, chloride and glucose. The analyzed results showed a significant positive relationship between TSD and potassium, (r=0.841, p=0.000) a weak negative relationship between TSD and

Sodium $(r = 0.137, pc0.048)$ and glucose $(r = -0.241, p = 0.000)$ no significant relationship was established between TSD and calcium, and chloride.

Marshall (1965), in his article, the temperature methods of estimating the time of death reviewed the methods of estimating the time of death from the body temperature. He also made an assessment of the inaccuracy of those methods based on a formula containing a single exponential term.

Hiraiwa et al. (1980), used a simultation model of the approximate equations derived from the infinite cylinder to investigate the applicability of the theoretical curve to the actual one of rectal temperature. The rectum was found by the computer tomography to be near to the junction of the anterior three- quarters and the posterior quarter inside the body, this results were employed by the simulation. They found out, just as Akaishi et al did, that, fluctuation of less than 20^0C in ambient temperature was shown by the simulation to have little effect on the rectal temperature curve.

Oever (1976), reviewed the possibility and limitations in estimating the time of death. He noted that determining the exact moment of death in medicolegal cases is not possible as postmortem changes of the human body are variable and often misjudge. He reviewed the most reliable physical and biochemical methods of estimating the PMI and found out why they are neither popular nor practical in routine forensic medicine.it was suggested that for greater accuracy, in estimating the time of death further investigations should be carried out to find a suitable combination of some suitable combination of some physical and biochemical tests complementary to the data produced by each method or preventing large error range of each individual test.

Brown and Marshall (1974), derived an equation for determining time since death based ; Brown and Marshall (1974) double exponential cooling curve of human corpse. Five cases of known time of death were compared with the estimated time of death incorporating the technique of double exponential cooling model. Analysis of the errors between the actual postmortem period and the calculated time since death indicate that accuracy depends on the shape of the cooling curve.

Nokes et al. (1983), modified the equations suggested by Brown and Marshall (1974) which represented the cooling curve of human corpse in other to estimate the time of death without recourse to lengthy mathematical technique. They gave a comparison with the two other predictive procedures for estimating the time of death in the study.

Kuroda et al. (1982), measure the thermal properties (conductivity) of exorcised human skin by a transent hot wire method. Assuming theoretically that the human corpse is an infinite cylinder. The average value of 12 specimens were 0.30 kcal/m.h. The theoretical error due to the experimental condition such as the size of specimen or the heating probe was within about five percent and the deviation of three measure for each material was shifted.

Continuous postmortem temperature measurement was used to determine time of death by Simonsen et al. (1977). In 20 cases with known time of death's continuous postmortem measurements of the temperature fall in brain, calf, liver, axilla and rectum of the bodies were made and in addition to the ambient temperature was recorded. Observation was not done under standardized condition. The measurements of the brain temperatures have given the greatest accuracy in determining the time of death. The other sites of measurement permitted less reliable estimates of postmortem time, but none of them were found to be appropriate beyond 20 hours after the death. They noted that the temperature of the moment of death cannot be calculated. They found out that the maximum and minimum starting temperature ranges between 5^0C and 8^0C depending on the site of measurement. That irrespective of the site of measurement, postmortem interval is of the magnitude 1^0C per hour. They conclude that the determination of time of death will always be encumbered with great uncertainty. The most reliable estimate within the first 20 hours after death can be based upon the measurement of the brain temperature associated with an evolution of the development of the signs of death.

Nokes et al. (1986), determined postmortem period using the trachea temperature. The authors presented initial results involving the use of trachea temperature. The authors presented initial results involving the use of tracheas temperature as a means of determining the postmortem period. They developed a simple mathematical model based on the cooling curve of five corpses. They then noted that errors between actual and calculated postmortem period may be due to an initial temperature plateau or lack of knowledge of the body temperature at the time of death.

CHAPTER 3

METHODOLOGY

3.1 Introduction

The chapter presents a detailed description and explanation of the theories and the methods used in this study. An address of the Polynomial regression models (PRM) and the Modified Newton's Model (MNM) will be made. Both models are examples of temperature based models used in the estimation of TSD.

3.1.1 Polynomial Regression Model

Regression

It is a statistical technique to determine the linear relationship between two or more variables. They are primarily used for prediction and causal inference. In its simplest (bivariate) form, regression shows the relationship between one independent Variable (X) and a dependent Variable (Y) as in the formula below;

$$Y = \beta_0 + \beta_1 X + \epsilon \qquad (3.1)$$

Where,

- β_0 is the intercept parameter

- β_1 is the slope parameter (shows the magnitude and direction of that relationship)

- is the error term (explaining the amount of variation not predicted by the slope and intercept)

(Campbell and Campbell, 2008).

A Polynomial Regression Model(PRM)

This is a form of linear regression in which the relationship between the independent variable *(x)* and the dependent variable *(y)* is modelled as an *nth* degree polynomial.They are usually fit using the method of least squares that minimizes the variable of the unbiased estimators of the coefficient, under the condition of the Gauss – Markov Theorem(wikepedia, t 25).

In other words a Polynomial Regression Model can be said to be a form of a regression that fits a non-linear relationship between independent observed variables say, *(x)* and a resultant dependent variables *(y)*, where, ' X ' is expressed as a Vandermonde matrix of Kth degree.

PRM are considered as both Linear and multiple regression models.

3.1.2 Ordinary Least Square (OLS)

It is a procedure that computes the values of the β_i's that best fits an observation. It calculates those values by minimizing the Sum of Square Error (SSE) for all obervation (Campbell and Campbell, 2008).

The best β_i' estimate are those ones which will minimise the SSE '. The actual response less the predicted response gives the residuals denoted, ˆ

3.1.3 Assumptions under the Polynomial Regression Model

- Just as a linear regression assumes that, the relationship you are fitting a straight line to, is linear, polynomial regression assumes that you are fitting the appropriate kind of curve to your data. It assumes that data points are independent

- Y variable is normally distributed and homoscedastic for each value of x (McDonald, 2014).

- the behavior of a dependent variable y can be explained by a linear, or curvilinear, additive relationship between the dependent variable and a set of 'K' independent variables ($X_j, j = 1 \ldots K$)
- the relationship between the dependent variable y and any independent variable x_j are independent of each other and
- the error term (ϵ) are independent, normally distributed with mean zero and a constant variance /standard deviation. In other words, $\epsilon \backsim (0, \sigma^2)$

3.2 Building the Polynomial Regression Model

3.2.1 Order of the Model

The order *(K)* of polynomial models should be kept as low as possible. Chen (2014), advised that, higher-order polynomials *(K > 2)* should be avoided unless they can be justified for reasons outside the data. It is extensively known that the cooling/heating of the rectal temperature of a dead body follows a sigmoid *(S)* Shape, hence, cubic. Supported by Kamakar (2010), "The curve of the cooling (of a dead body) is sigmoid in pattern".

3.2.2 Model Building Strategy

There are several methods that aid in the selection of the degrees of a polynomial. An address of one is however considered.

Forward Selection

We start the model with a linear model of the form;

$$Y = \beta_0 + \beta_1 x + \epsilon \tag{3.2}$$

And go for a second order model of form,

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon \tag{3.3}$$

We then check for the significance of β_2 using say r^2 or test $Ho : \beta_2 = 0$ verses

$H1 : \beta_2 \neq 0$.

If it is significant, we go to the third order polynomial regression.

$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \epsilon$

We again do a test of significance for

$H_0 : \beta_3 = 0$

verses

$H_1 : \beta_3 \neq 0$.

We successively fit model of increasing order until the t – test for the highest order term is non-significant.

3.2.3 Modelling with Polynomial Regression

Given a general form for a polynomial of order three (3) as;

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \epsilon \tag{3.4}$$

Then, $E(y) = \bar{Y}$, gives;

$$\bar{Y} = \beta_0 + \beta_1 x_i + \beta_2 x_{2i} + \beta_3 x_{3i} \tag{3.5}$$

We pick the coefficients $(\beta_0, \beta_1, \beta_2 and \beta_3)'$ that best fits the curve of our data with a minimum error using the OLS method.

The general expression for any error, using the least square approach is;

$$SS_{res} = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2 \tag{3.6}$$

where; SS_{res}= Residual Sum of Squares $=\hat{e}^2$ $\hat{e}^2$ = square of the deviation of estimated fit $\bar{y}$ from observed data

$(y_i) = (y_i - \bar{y})^2$, But, from 3.5, $\bar{Y} = \beta_0 + \beta_1 x_i + \beta_2 x^2{}_i + \beta_3 x^3{}_i$

$$Therefore, SS_{res} = \sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2 \tag{3.7}$$

minimizing equation with respect to coefficients$\beta_i{}^0 s$ and equating them/each to zero;

$\frac{\partial SS_{res}}{\beta_0} = 0, \frac{\partial SS_{res}}{\beta_1} = 0, \frac{\partial SS_{res}}{\beta_2} = 0$ and $\frac{\partial SS_{res}}{\beta_3} = 0$

$$\frac{\partial SS_{res}}{\beta_0} = 2(\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2(-1) = 0 \tag{3.8}$$

$$\frac{\partial SS_{res}}{\beta_1} = 2(\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2(-x_i) = 0 \tag{3.9}$$

$$\frac{\partial SS_{res}}{\beta_2} = 2(\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2(-x_i^2) = 0 \tag{3.10}$$

$$\frac{\partial SS_{res}}{\beta_2} = 2(\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2(-x_i^3) = 0 \tag{3.11}$$

Simplifying further, and taking note that, $(\sum_{i=1}^{n} 1 = n)$ gives the following ;

$$\frac{\partial SS_{res}}{\beta_0} \Rightarrow \beta_0 + \beta_1 x_i \sum_{i=1}^{n} x_i + \beta_2 \sum_{i=1}^{n} x_i^2 + \beta_3 \sum_{i=1}^{n} x_i^3 = \sum_{i=1}^{n} y_i \tag{3.12}$$

$$\frac{\partial SS_{res}}{\beta_1} \Rightarrow \beta_0 \sum_{i=1}^{n} x_i + \beta_2 \sum_{i=1}^{n} x_i^2 + \beta_3 \sum_{i=1}^{n} x_i^3 + \beta_4 \sum_{i=1}^{n} x_i^4 = \sum_{i=1}^{n} x_i y_i \tag{3.13}$$

$$\frac{\partial SS_{res}}{\beta_1} \Rightarrow \beta_0 \sum_{i=1}^{n} x_i^2 + \beta_1 \sum_{i=1}^{n} x_i^3 + \beta_2 \sum_{i=1}^{n} x_i^4 + \beta_3 \sum_{i=1}^{n} x_i^5 = \sum_{i=1}^{n} x_i^2 y_i \tag{3.14}$$

$$\frac{\partial SS_{res}}{\beta_1} \Rightarrow \beta_0 \sum_{i=1}^{n} x_i^3 + \beta_1 \sum_{i=1}^{n} x_i^4 + \beta_2 \sum_{i=1}^{n} x_i^5 + \beta_3 \sum_{i=1}^{n} x_i^6 = \sum_{i=1}^{n} x_i^3 y_i \tag{3.15}$$

Equations; 3.12,3.13,3.14, and 3.15, are systems of simultaneous equations. We then write the equations in matrix form and use the Gaussian Elimination method to get the set of unknowns $\sum_{i=1}^{n} x_i \quad \sum_{i=1}^{n} x_i^2$ $(\beta_0,\beta_1,\beta_2,\beta_3)^0$

$n \quad \sum_{i=1}^{n} x_i^2 \quad \sum_{i=1}^{n} x_i^3 \quad \sum_{i=1}^{n} x_i^3$

$\sum_{i=1}^{n} x_i^4$

, X

$\sum_{i=1}^{n} x_{2i} \sum_{i=1}^{n} x_{3i} \sum_{i=1}^{n} x_{4i}$ $\quad \sum_{i=1}^{n} x_{5i}$ $\quad \beta_2$

$\sum_{i=1}^{n} x_i^6$ $\quad \beta_3$

$n\,3\ \sum^{n} x_{4i} \sum_{i=1}^{n} x_{5i} \sum_{i=1} x_i\ i=1$

$\sum_{i=1}^{n} y_i$

$\sum_{i=1}^{n} x_i y_i$

$B =$.

$\sum_{i=1}^{n} x_{2i} y_i$

$\sum_{i=1}^{n} x_{3i} y_i$

In other words, $AX = B$ becomes;

$n \quad \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_{2i} \sum_{i=1}^{n} x_{3i}$ $\quad \beta_0 \quad \beta_1$ $\quad \sum_{i=1}^{n} y_i$

$\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_{2i} \sum_{i=1}^{n} x_{i3} \sum_{i=1}^{n} x_{4i}$ $\quad \sum_{i=1}^{n} x_i y_i$

$\times \quad =$

$\sum_{i=1}^{n} x_{2i} \sum_{i=1}^{n} x_{3i} \sum_{i=1}^{n} x_{4i} \sum_{i=1}^{n} x_{5i}$ $\quad \beta_2$ $\quad \sum_{i=1}^{n} x_{2i} y_i$

$\sum_{i=1}^{n} x_{3i} \sum_{i=1}^{n} x_{4i} \sum_{i=1}^{n} x_{i5} \sum_{i=1}^{n} x_{6i}$ $\quad \beta_3$ $\quad \sum_{i=1}^{n} x_{3i} y_i$

$\hat{\beta}_0$

$\hat{\beta}_1$

Therefore, $X = B/A$ or $A^{-1}B$ $\hat{X} =$

$\sum_{i=1}^{n} x_i$ Let $A =$

β_0 β_1 $=$

, and

$i=1\ i$ $i=1\ i$ $i=1\ i$ $i=1\ i$ $i=1\ i$

The fitted Regression Model then becomes;

$$\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\beta}_2 x_{2i} + \hat{\beta}_3 x_{i3} \tag{3.16}$$

The Regression Function $E(Y) = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\beta}_2 x_i^2 + \hat{\beta}_3 x_i^3$ is a cubic response function, where;

- $\hat{\beta}_0$ Is the mean response when $X_i = 0, i.e., X_i = \bar{X}$.
- $\hat{\beta}_1$ Is called the linear response effect.
- $\hat{\beta}_2$ Is also called the quadratic response effect.
- $\hat{\beta}_3$ Is called the cubic response effect.

3.3 Polynomial Regression Model in Matrix Form

- Let X be an $n \times k$ matrix with observations on K independent variables for a total of n observations. As our model contains a constant term, the first columns in the X matrix will contain only ones
- Let Y be an $n \times 1$ vector of observations on the dependent variable.
- Let be an $n \times 1$ vector of disturbances or errors .

- Let β be a $k+1$ vector of unknown population parameters that we want to estimate.

Then the realization of this relationship in a general regression models can be written as a system of equations as follows;

$$\begin{bmatrix} Y_i \\ Y_{i+1} \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{21} & \cdots & X_{K1} \\ 1 & X_{21} & X_{22} & \cdots & X_{K2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & X_{1n} & X_{2n} & \cdots & X_{Kn} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} + \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

The general regression model can be written as $Y = X\beta + \epsilon$, in matrix. Therefore, the expected value of $Y = E(Y)$, gives; $\bar{Y} = X\hat{\beta}$ (Beck, 2001).

Using the Least Square Method in Matrix, we find the estimators $\hat{\beta}_i$, that minimizes the sum of squares residuals $(\sum_{i=1}^{n} \hat{e}_i^2)$, with a vector of residual of form; $e = y - \bar{y}$

Hence, $e = y - X\hat{\beta}$.

$$e^0 e = (y - X\hat{\beta})^0 (y - X\hat{\beta}) \tag{3.17}$$

, expanding,

$$e'e = y'y - \hat{\beta}' X'y - y'X\hat{\beta} + \hat{\beta}' X'X\hat{\beta} = y'y - 2\hat{\beta}' X'y + \hat{\beta}' X'X\hat{\beta} \tag{3.18}$$

Using the identity that the transpose of a scalar is a scalar $\hat{\beta}' X' y \Rightarrow$ $(\hat{\beta}' X' y)' = y' X\hat{\beta}$ and minimising $e'e$ with respect to $\hat{\beta}$

$$\frac{\partial e'e}{\partial \hat{\beta}} = 2X'y + 2X'X\hat{\beta} = 0 \tag{3.19}$$

$$\sum_{i=1}^{n} \hat{e}_i^2 = \begin{pmatrix} e_1 & e_2 & \cdots & e_n \end{pmatrix} \times \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix} = \begin{pmatrix} e_1^2 & e_2^2 & \cdots & e_n \end{pmatrix}$$

Hence; $X'X\hat{\beta} = X'y$

Multiply through the equation by $(X'X)^{-1}$ with the assumption that, the inverse of the matrix exists, gives; $\hat{\beta} = (X'X)^{-1}X'y$ The modelled Polynomial Regression (PRM) becomes

$$\hat{y} = X\hat{\beta} = X(X^0X)^{-1}X^0y \tag{3.20}$$

Replacing the R.H.S of 3.20 by the hat matrix (H), gives; $\hat{y} = HY$. Where H is idempotent ($H^2 = H \times H = H$) and symmetric ($H = H^T$).

3.3.1 Application of Data to the PRM

If we replace the X matrix by our observed rectal temperatures to form a Vandermonde matrix of degree 3, and solving the expression for $\hat{\beta}$, $[(X^0X)^{-1}X^0Y]$, we will get the following estimates of $\hat{\beta}_i, i = 0,1,2,3.$

	X	X_2	X_3
			942345.0
1	98.04	9611.842	859271.6
1	95.07	9038.305	
1	91.60	8390.560	768575.3
1	88.28	7793.358	
1	85.33	7281.209	687997.7
1	82.60	6822.760	
1	80.16	6425.626	621305.6
1	77.96	6077.762	
1	76.03	5780.561	
1	74.39	5533.872	
1	73.05	5336.302	760 563560.0
1	72.05	5191.202	
1	71.31	5085.116	

X=

y= 0, 2, 4, 6, 8, 10, 12

$$\begin{matrix} 515078.1 & 14 \\ 473822.3 & 16 \\ 439496.0 & 18 \\ 411664.7 & 20 \\ 389816.9 & 22 \\ 374026.1 & 24 \\ 362619.6 & 26 \end{matrix}$$

$$X'X = \begin{bmatrix} 13.000 & 1065.87 & 8.8368.48 & 7409579.00 \\ 1065.87 & 88368.48 & 7409579.00 & 628332032.41 \\ 88368.48 & 7409579.00 & 628332032.41 & 53877410000 \\ 7409579.00 & 628332032.41 & 53877410000 & 4669814000000 \end{bmatrix}$$

$$X'y = \begin{bmatrix} 182 \\ 14093 \\ 1098883 \\ 86330169 \end{bmatrix}, \quad \hat{\beta} = (X'X)^{-1}X'y = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 963.959713 \\ -30.894845 \\ 0.338438 \\ -0.001259 \end{bmatrix}$$

Hence, the 3rd degree Polynomial Regression Model from our data becomes;

$$\hat{y} = 963.959713 - 30.894845x + 0.338438X^2 - 0.001259X^3 \qquad (3.21)$$

3.3.2 Validity and Predictability of The Model

The following R summary and Anova outputs for the 3^{rd} degree Polynomial Regression Model aids in analysing and checking the validity and predictability of the Model.

Table 3.1: R Output

Coefficients	Estimates($\hat{\beta}$)	Standard Error	t-Value	p-Value
Intercept	963.959713	124.3	-6.915	0.0000289
X	-30.894845	4.46	5.471	0.0000695
X_2	0.338438	0.05323	6.358	0.000132
X_3	-0.001259	0.0002103	-5.986	0.000206
R_2	0.9987			
R^2 Adj.	0.9982			

Residual Standard Error	0.3267			

Table 3.2: Anova Table

	Degree of Fr.(DF)	Sum of Sq.	Mean Sq.	$F - Value$	$P - Value$
	3	727.04	242.346	2270.7	$2.846e - 13$
Residuals	9	0.96	0.107		

- The R^2 squared and Adjusted R^2 squared (i.e the percentage of variance in the dependent variable that can be explained by the predictors) are 99.87% and 99.82% respectively. This shows that the Polynomial Regression Model is an accurate fit or predictor of the Time since Death (TSD).

- The Standard Error of the estimate (cubic estimate) is very small, 0.0002103. Hence, the errors of the residual tends to be closer to the mean. This implies that, the model predicts well the dependent variable.

- Testing for a cubic relationship between the TSD and RT, we test the $H_0 : \beta_3 = 0$ against $H_1 : \beta_3 \neq 0$. Significant level ($\alpha = 0.05$), P-value of the coefficient of the X^3, is 0.000206. As the p-value $(0.000206) < \alpha(0.05)$, we reject the H0 in favour of H1 and conclude that, there is a cubic relationship between TSD and RT at 95% level of significance.

- The Mean Square Error (MSE) for our estimate is $(SSE/degreeoffreedom) = 0.107$. Hence, size of the errors in the regression is very small. In other words it shows that the regression fits the data very well.

- Again, the Mean Absolute Percentage Error (MAPE), calculated as

$$MAPE = \frac{100}{n}\sum_{i=1}^{n}|\frac{y_i - \hat{y}_i}{y_i}| \tag{3.22}$$

, gives; $2.7225 \simeq 2.72$ Since it lies less than 10% it shows that the PRM is an Excellent-Accurate forecaster of the TSD

Table 3.3: Summary Of Validity Of PRM

Calc. Stats	Value	Implication On PRM
R_2	99.87	Accurate Predictor Of TSD
SE	0.0002103	PRM is a good predictor of the independent variable.
		Small Error in the Regression. Fits data very well.
		Excellent and Accurate Forecaster of TSD.

The respective test statistics for calculating the Confidence Interval *(CI)* and Predictive intervals *(PI)* for the Regression Models are;

$$\hat{y} \pm t_{\frac{\alpha}{2}}, n-2.SE\sqrt{\frac{1}{n} + \frac{(X_A - \bar{X})^2}{\sum(X-\bar{X})^2}} \tag{3.23}$$

and

$$\hat{y} \pm t_{\frac{\alpha}{2}}, n-2.SE\sqrt{1 + \frac{1}{n} + \frac{(X_A - \bar{X})^2}{\sum(X-\bar{X})^2}} \tag{3.24}$$

Where, n is the total number of observation (13),$\hat{y}$ is the predicted value (estimated TSD), SE, is the standard deviation of the error term (0.3267), $\bar{X}$ is the mean of the RT (81.99),X is the actual observation and X_A is the actual time TSD we wanted estimate (10) .

Given a rectal temperature of 85.33^0F.

$\hat{TSD} = \hat{y} = 963.959713 - 30.894845(85.33) + 0.338438(85.33)^2 - 0.001259(85.33)^3$
=9.76 hours, which corresponds with the actual time of death of case 5.

The $t-value$ associated with $(1-\alpha)$ confidence level is,$t_{\frac{\alpha}{2}}, 13-2 = t_0.025, 11 =$ 2.201

n = 13, p = 3, s = 0.2374, $\hat{y}$ = 12.2542 and $t_{\frac{0.05}{2}}, (13-3) = t_{0.025}, 10 = 2.228$Hence, a 95% C.I and P.I are are as follows;

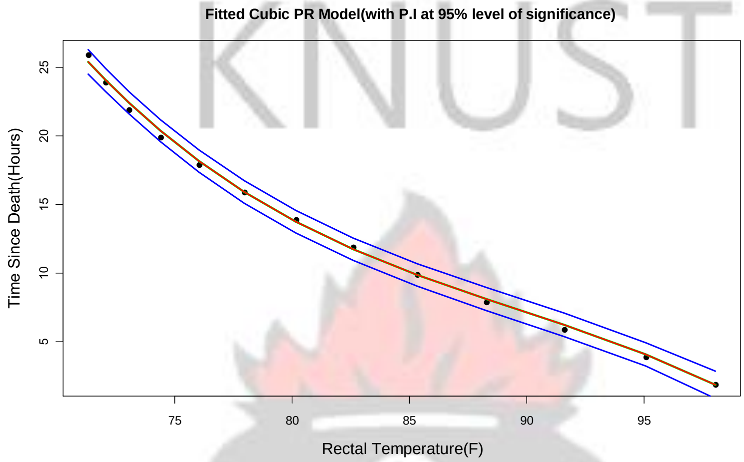
C.I at 95% level of significance for $T\hat{S}D$ = 9.76±2.201× 0.3267 × 0.27745627592 = 9.76 ± 0.19950956872

Hence, we are 95% confident that, the prediction of $T\hat{S}D$ will fall within

(9.5605,9.9595)

Also, the P.I at 95% level of significance for $T\hat{S}D$ = 9.76 ± 2.201× 0.3267 × 1.03786729761 = 9.76±0.74629581273 = (9.0137,10.5063). We are 95% confident that the estimate $T\hat{S}D$ for a RT of 85.33^0F would be contained in the interval (9.0137,10.5063)

Figure 3.1: A Plot of The Model of Estimates and P.I



3.3.3 Newton's Model

Mathematically the Newton's law of cooling is defined as follows

$$\frac{dT}{dt} = \propto (T - T_a) \tag{3.25}$$

Where;

- T, is the temperature of a cooling object, in our case a dead body .
- t, is the time in hours since the first measurement of the body
- T_a, is the ambient temperature.
- $\propto$, is the constant of proportionality between the ambient and body temperature

Solving the Above equation gives the following model

$\frac{dT}{dt} = -K(T - T_a)$,grouping like terms and itegrating;

$\frac{dT}{(T-T_a)} = Kdt$

$\int \frac{1}{T-T_a} dT = -K \int dt$

$ln(T - T_a) = -Kt + c$ where c is the constant of integration.
Taking the natural log of both sides and making t, the subject gives

$$T = T_a + c_1 e^{-Kt} \tag{3.26}$$

At initial time $t = 0$, $T(0) = T_0$

$$T(t) = T_a + T_0 - T_a e^{-Kt} \tag{3.27}$$

Which is the well Known Newton's Model. It assumes a constant Ambient Temperature and does not consider alot of parameters in its computation. This makes them largely inaccurate in estimating TSD. Marshall and Hoare (1962) established the fact that the basic assumption of Newton's law of cooling was invalid when applied to the very early stages of the cooling of deceased human body (Leinbach, 2011). Marshall and Hoare (1962) after graphing the general shape of the curve that describes the temperature of a cooling body added a "correcting factor" to the Newton's Law to represent the plateau. The said correction factor is exponential in form and decays with time. The expression
for this function is $Ce^{(-pt)}$ The differential equation for the deceased body then becomes;

$$\frac{dT}{dt} = -K(T - T_a) + Ce^{(-pt)} \tag{3.28}$$

Where K and P are cooling rate constants, all other parameter denote the same meaning as in the Newton's model above Writing the above equation in standard form as;

$$\frac{dT}{dt} + P(t)y = Q(t) \tag{3.29}$$

Where P(t)=K

Then the Integrating Factor (IF) becomes; $IF = e^{K\int(dt)} = e^{kt}$ multiplying through equation 3.28 by the IF gives;

$$\frac{dT}{dt}e^{kt} + K(T - T_a)e^{(kt)} = ce^{(-pt)}$$ (3.30) (3.31)

$$\frac{dT}{dt}e^{kt} + KTe^{kt} = KT_ae^{kt} + ce^{-pt} \tag{3.32}$$

But,simplifying the L.H.S, using the following product rule of differentiation

$$U(x)\frac{(dv(x))}{(dx)} + V(x)\frac{(du(x))}{dx} = \frac{d}{dx}(u(x)v(x)) \tag{3.33}$$

gives;

$$\frac{d}{dt}(Te^{kt}) = KT_ae^{Kt} + ce^{(-pt)}e^{kt} \tag{3.34}$$

Integrating both sides w.r.t. t

$$Te^{kt} = K\int T_ae^{kt}dt + C\int e^{((k-p)t)}dt \tag{3.35}$$

$$Te^{kt} = T_ae^{kt} + \frac{C}{(K-P)}e^{((K-P)t)} + A \tag{3.36}$$

Multiplying equation 3.36 by $\frac{1}{e^{kt}}$,gives;

$$T = T_ae^{kt}\frac{1}{e^{kt}} + \frac{C}{(K-P)}e^{kt}e^{-pt}\frac{1}{e^{kt}} + Ae^{-kt} \tag{3.37}$$

$$T(t) = T_a + \frac{C}{K-P}e^{-pt} + Ae^{-kt} \tag{3.38}$$

If the t = 0, $T(t)$ = 0 = T_0 ,then; $T_0 = T_a + \frac{C}{K-P} + A$. Hence,

$$A = T_0 - T_a - \frac{C}{K-P} \tag{3.39}$$

From 3.28, if t=0

$$C = K(T_0 - T_a) \quad (3.40)$$

put equation 3.40 into 3.39 and simplify

$$A = (T_0 - T_a)(1 - \frac{K}{K-P}) \quad (3.41)$$

Put equations 3.40 and 3.41 into equation 3.38 and simplify further

$$T(t) = T_a + \frac{K(T_0 - T_a)}{K-p}e^{-pt} + (T_0 - T_a)(1 - \frac{K}{K-P})e^{-kt} \quad (3.42)$$

$$T(t) = T_a + (T_0 - T_a)(\frac{K}{K-P})e^{-pt} + (1 - \frac{K}{K-P})e^{-kt}) \quad (3.43)$$

Which is the final Modified Newton's Model (MNM) for estimating time since death.

From Lynnerup (1993), The rate constant of the Newton's model (K) is given as;

K =$SizeFactor(SF)$ × 0.0006125- 0.05375

Where, $SF = 0.8 \times \frac{(Surfaceareaofbody(m^2)}{(Weightofbody(kg))}$

While the rate constant for the plateau (P) is given as 0.3 for clothed body and 0.4 for nude bodies

An Iterative method that iterates the number of times *time* (denoted$t$ in hours) changes over time for Rectal Temperature *(RT)* (T(t)) to coincide with the developed model of differential equation 3.43, embodying several parameters /conditions that affect the cooling of dead body is used to estimate the TSD.

A simple matlab code for executing this iteration can be seen in the appendix.

CHAPTER 4

Data Source, Presentation and Analysis of Results

4.0.4 Introduction

This chapter presents the sources of my data and the analyses of the estimated TSD generated using the Modified Newton's (MNM) and Polynomial Regression Models (PRM).

4.0.5 Data Source and Type

As this study seeks to model the time since death in homicides or in all deaths, using the MNM and the PRM, two secondary data were obtained from the Journal of the Indian Academy of Forensic Medicine 2005 volume $27(3) : 170-176$ and the Acta Morphologica 2006, volume 3(3) (Macedonia Association of of Anatomists and Morphologists).

The first data presents Rectal Temperature of 418 adult subjects aged between 18 and 60 years who died through accident at the Tropical region of Chandigarh (India) Singh et al. (2005). Subjects with known exact time of death, mode of death and other demographic profile were included in the study. This data was used to develop the PRM.

The second data, also presents an analysis of Rectal Temperature of 50 dead bodies with known time TSD, age, sex and body weight at the Institute of Forensic Medicine and Criminology in Skopje(Macedonia) (Poposka et al., 2006). This data was used to validate the PRM and MNM's

The following table presents the results obtained using the two models, PRM and the MNM's and the Actual or Known Time Since Death (KTSD).

Table 4.1: Estimated Results Of PRM and MNM

Case	Sex	$Ta^0C/^0F$	$RT^0C/^0F$	Weight(kg)	Clothed±	KTSD	PRME	MNME

1	M	17/34		65	+	4	4.04	2.76
2	F	22/62.6		65	+	4	4.62	4.48
3	M	19.3/66.74		78	+	4	3.80	4.28
4	F	22.5/72.5		75	-	4	4.28	4.85
5	M	22.4/72.32		65	-	4	2.76	2.96
6	M	24/75.2		75	-	5	6.16	7.93
7	M	21/69.8		80	-	5	1.91	2.06
8	M	22.4/72.32		80	-	6	6.36	7.7
9	F	22/62.6		58	-	6	6.98	5.82
10	M	21/ 69.8		75	-	6	5.08	5.41
11	M	16.5/61.7		60	+	6	5.95	5.47
12	M	20/68		78	-	6	6.16	6.47
13	M	24.5/76.10		70	+	6	4.97	6.86
14	F	21/69.8		57	-	7	7.19	7.15
15	M	16/60.8		80	+	7	8.64	8.79
16	M	21.3/70.34		80	+	7	6.93	8.42
17	M	24/75.2		78	+	7	6,16	8.85
18	M	24/75.2		82	+	7	5.19	7.44
19	M	24/75.2		84	+	7	5.30	7.69
20	M	22.5/72.5		70	+	7	6.16	7.76
21	M	30/86		75	+	7	3.42	7.17
22	M	21/69.8		72	-	8	7.19	7.86
23	M	21.2/70.16		55	-	8	6.78	6.66
24	M	21/69.8	34.9/94.82	85	-	8	6.57	7.60
25	M	24/75.2	34.4/93.92	50	-	9	7.19	8.16
26	M	19/66.2	35.1/95.18	78	+	10	7.60	8.61
27	M	23/73.4	34.7/94.46	80	+	10	8.32	12.14
28	M	15.7/60.26	35.9/96.62	75	+	12	12.92	12.38
29	M	24/75.2	33/91.4	80	+	13	9.75	16.23
30	M	21/69.8	36.5/97.7	83	-	13	10.71	13.64

			32.8/91.04
			32.2/89.96
			34/93.2
			33.2/91.76
			33/91.4
			34.1/93.38
			32/89.6
			30.6/87.08
			32.5/90.5
			33/91.4
			33.9/93.02
			33.8/92.84
			33/91.4
			33.4/95.72
			32/89.6
			32.4/90.32
			32.6/90.68
			32/89.6
			31.6/88.88
			30.9/87.62
			27.2/80.96
			29.6/85.28
			28.8/83.84

Table 4.2: Estimated Results Of PRM and MNM Continued

Case	Sex	$Ta^0C/^0F$	$RT^0C/^0F$	Weight(kg)	Clothed±	KTSD	PRME	MNME

31	M	16.5/61.7		80	-	13	11.62	11.14
32	M	18/64.4		75	-	14	11.27	11.23
33	F	20.6/69.08		54	-	14	12.92	13.42
34	F	20/68	28.1/82.58	45	-	14	12.47	11.42
35	M	23/73.4	28.4/83.2	75	+	15	13.23	21.55
36	M	24.4/75.92	27.2/80.96	76	-	15	15.11	33.06
37	M	23.6/74.48	27.5/81.5	75	-	15	18.90	51.95
38	M	20/68	27/80.6	80	+	15	17.97	23.35
39	M	21/69.8	25.9/78.62	73	+	16	19.15	27.07
40	M	23/73.4	24.1/75.38	95	+	17	20.41	58.26
41	M	19/66.2	24.5/76.1	80	+	19	21.77	25.91
42	M	22/71.6	24/75.2	95	+	19	20.94	43.85
43	M	22/71.6	23.5/74.3	85	+	19	21.77	45.04
44	M	17/62.6	23/73.4	75	+	20	22.63	21.53
45	M	17/62.6	23.3/73.94	70	+	20	23.22	21.30
46	M	17/62.6	23/73.4	76	+	20	23.22	22.19
47	F	20.6/69.08	22.7/72.86	65	-	21	24.6	34.56
48	M	21/69.8	22.5/72.5	73	+	22	26.11	49.23
49	M	20/68	22.5/72.5	80	+	24	27.15	41.49
50	M	21/69.8	22/71.6	60	-	24	25.44	39.65

			21.6/70.88 21.3/70.34 21.8/71.24

Source: *Poposkaet.all*2006

The following are the pictorial representation of the results in the table above.

Figure 4.1: Pictorial Presentation (Line Graph)of Results

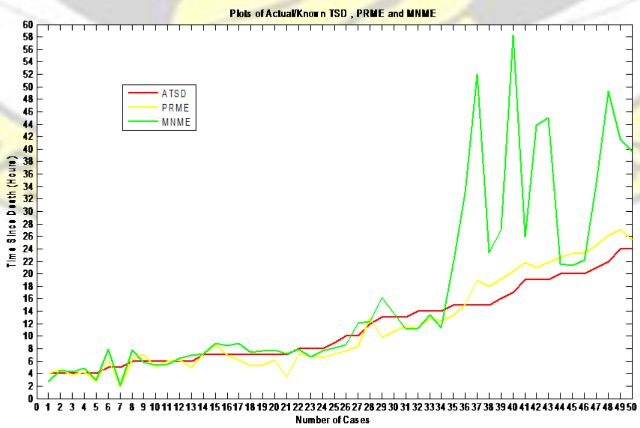
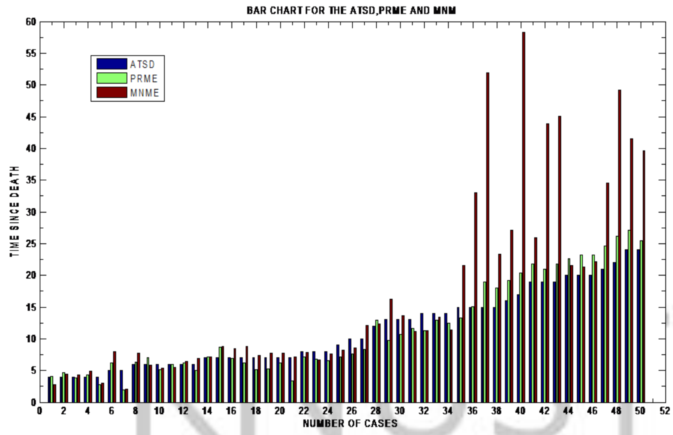


Figure 4.2: Bar Graph of Results



4.0.6 Analysis of Results

- Preliminary analysis of the result(s) churned out from the two models reveals that, both are good estimators of TSD. However, the existence of outliers in the results of the MNM makes it less accurate. Testing the Hypothesis at 95% significant level that the mean of the PRM and MNM are equal to that of KTSD, showed that, there is no difference between the estimates of PRM and ATSD. However,there existed a difference between estimates of MNM and ATSD except without the outliers.
- The parametric statistic of the population ,(ATSD) variance(σ^2)

$$\sum_{i=1}^{n} \frac{(X_i - \mu)^2}{N} = 35.60 \tag{4.1}$$

Hence, Standard Deviation is $\sqrt{\sigma^2} = 5.97$. and $\mu = 11.38$

Since σ^2 is known and $N > 30$ we use the Z test statistic of

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}} \tag{4.2}$$

and the Z critical is $Z_{0.025} = \pm 1.96$

Hypothesis For Model 1 (PRME)

For Model 1 (PRME)

$H_0 : \mu = 11.58$

$H_1 : \mu$ 6= 11.58

$$Z_1 = \frac{11.58 - 11.38}{\frac{5.97}{\sqrt{50}}} = 0.2369 \tag{4.3}$$

For Model 2 (MNME)

$H_0 : \mu = 16.62$

$H_1 : \mu$ 6= 11.62

$$Z_1 = \frac{16.62 - 11.38}{\frac{5.97}{\sqrt{50}}} = 6.2064 \tag{4.4}$$

Since Z_1 calculated for model 1 (0.2369) falls within the range of Z critical, we fail to reject the null hypothesis and conclude that, there is no difference between the mean of the ATSD and the estimates using the PRM. On the other hand,the Z calculated for model 2, (6.2064)falls beyond the range of the Z critical, hence, we reject the null hypothesis and state that there is enough evidence to conclude that, there are differences between the mean of the ATSD and the estimated TSD using the MNM. Though they are the same without the outliers

- The two models were further tested against the Mean Absolute Deviation (MAD) and the Sum of Squares Error (SSE)to determine which one is consistence and accurate in estimating TSD. The two tests provide an average of the absolute differences between the actual and estimated/forecast values.

MAD for Model 1

$$\sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{N} = \sum_{i=1}^{n} \frac{|ATSD - PRME|}{50} = 1.7498$$

SSE for Model 1

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (ATSD - PRME)^2 = 221.9887$$

MAD for Model 2

$$\sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{N} = \sum_{i=1}^{n} \frac{|ATSD - MNME|}{50} = 5.9728$$

SSE for Model 2

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(ATSD - MNME)^2 = 6531.009$$

Since the MAD (1.7498) and SSE (221.9887) respectively, for model 1 (PRM) are smaller than that of model 2 (MNME), we conclude that the PRM predicts TSD with accuracy and consistency than the MNM.

- Again, the Mean Absolute Percentage Error (MAPE)for the models 1 and 2 are;

  MAPE for Model 1

  $$MAPE = \frac{100}{n} \sum_{i=1}^{n} |\frac{ATSD - PRME}{ATSD}| = 61.247$$

  MAPE for Model 2

  $$MAPE = \frac{100}{n} \sum_{i=1}^{n} |\frac{ATSD - MNME}{ATSD}| = 152.4698$$

  Since, the MAPE for the estimates of the PRM are lower than that of the MNME, we can confidently claim that the Polynomial Regression Model is a better estimate of TSD.

- There was little to choose between the estimates of the two models for the first 15 hours since death(from case 1 through 34).Both models predicted exactly or with minimal errors, the TSD. The MNM begins to exhibit discrepancies as the TSD passes the 15 hour mark. This period is normally, marked by low temperature difference between the Rectal Temperature (RT) and the Ambient Temperature(AT). Those periods had a temperature difference range of (0.9 to 2.6). The highest estimate of error using the MNM occured at cases 37(with 36.95 hours more than KTSD) and 40(with 41.26 hours more than KTSD), where, the said difference between RT and AT was 0.9 respectively.

- The PRME on the other hand stayed within range, predicting TSD throughout the fifty (50) cases. little discrepancies occured at points where the cooling of the RT was extremely slowed down by certain parameters,(Case 21).

Table 4.3: Summary of Comparative validation of PRME and MNME

Calc.Stats	PRME	MNME	Implication On Estimates
MAD	1.7498	5.9728	PRM is an accurate predictor of TSD than MNM
SSE	221.9887	6531.009	PRM is a consistent predictor of TSD than MNM
MAPE	61.247	152.4698	PRM is a better predictor of TSD than MNM

CHAPTER 5

Conclusion And Recommendations

5.0.7 Conclusion

The study achieved all of its aims including determining the best model for estimating TSD between the PRM and the MNM. PRM estimated TSD better with little or no discrepancies. The MNM was also good in determining the TSD for the first 15 hours after death(Where $(RT - AT) > 2.6^0F$).

Further investigations revealed that, the MNM will produce estimates of TSD having larger / unacceptable errors, if the difference between the Rectal Temperature (RT) and Ambient Temperature (AT) is between $(0.1 and 5^0F)$. Hence,based on the analysis of the results, we conclude that the Polynomial Regression Model(PRM) is the best estimator of Time Since Death in all circumstances of death relative the Modified Newton's Model.

5.0.8 Recommendation

Further studies should be made into the Multiple Regression Model that considers the Ambient Temperature and the clothed state of the victim. This can further reduce the little discrepancies in the TSD results given by the PRM. Again, it will avoid the computational difficulties associated with the MNM and become an easy to apply approach in TSD investigation.

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APPENDIX A

5.1 Data One

Source:Journal of the Indian Academy of Forensic Medicine 2005 volume $27(3)$:

$170 - 176$

Figure 5.1: Data For Polynomial Regression Model

Figure 5.2: Data 1

Time since death (hrs)	Mean & SD Rectal temp (98.75±0.91°F)	Fall/2hours	Environmental temp. M=70.65±6.32°F Rectal temp	Fall/2hrs.	Correlations with rectal fall of temperature at various time interval with Body mass Index M=21.36±3.61 Rectal temp	Fall/2hrs.	body surface area M=16902.71±1907.71 cm² Rectal temp	Fall/2hrs.	Weight M=58.06±12.0 Kgs Rectal temp	Fall/2hrs.	Supine body length M=164.49±8.39cms Rectal temp	Fall/2hrs
2	98.04 ±1.55	0.71 ±1.08	0.60 ns	0.037 ns	0.124 ns	-0.051 ns	0.124 ns	-0.051 ns	0.121 ns	-0.048 ns	0.086 ns	-0.044 ns
4	95.07 ±2.08	3.68 ±1.80	0.225*	-0.216*	0.099 ns	-0.042 ns	0.099 ns	-0.042 ns	0.129 ns	-0.074 ns	0.012 ns	-0.032 ns
6	91.60 ±2.38	7.15 ±2.18	0.387***	-0.448***	0.143 ns	-0.095 ns	0.143 ns	-0.095 ns	0.180 ns	-0.133 ns	0.021 ns	-0.016 ns
8	88.28 ±2.67	10.47 ±2.54	0.570***	-0.623***	0.153 ns	-0.109 ns	0.153 ns	-0.109 ns	0.188 ns	-0.144 ns	0.025 ns	-0.007 ns
10	85.33 ±3.13	13.42 ±3.05	0.698***	-0.735***	0.135 ns	-0.094 ns	0.135 ns	-0.094 ns	0.165 ns	-0.126 ns	0.023 ns	-0.004 ns
12	82.60 ±3.67	16.15 ±3.64	0.778***	-0.800***	0.125 ns	-0.090 ns	0.125 ns	-0.090 ns	0.154 ns	-0.117 ns	0.024 ns	-0.001 ns
14	80.16 ±4.22	18.59 ±4.22	0.829***	-0.842***	0.108 ns	-0.076 ns	0.108 ns	-0.076 ns	0.132 ns	-0.100 ns	0.019 ns	-0.001 ns
16	77.96 ±4.72	20.78 ±4.76	0.864***	-0.868***	0.100 ns	-0.071 ns	0.100 ns	-0.071 ns	0.122 ns	-0.092 ns	0.018 ns	-0.001 ns
18	76.03 ±5.22	22.72 ±5.29	0.896***	-0.895***	0.104 ns	-0.077 ns	0.104 ns	-0.077 ns	0.127 ns	-0.100 ns	0.019 ns	-0.002 ns
20	74.39 ±5.62	24.35 ±5.70	0.912***	-0.909***	0.103 ns	-0.078 ns	0.103 ns	-0.078 ns	0.126 ns	-0.100 ns	0.022 ns	-0.007 ns
22	73.05 ±5.92	25.70 ±6.01	0.923***	-0.918***	0.102 ns	-0.079 ns	0.102 ns	-0.079 ns	0.126 ns	-0.101 ns	0.023 ns	-0.008 ns
24	72.05 ±6.06	26.69 ±6.16	0.930***	-0.923***	0.099 ns	-0.076 ns	0.099 ns	-0.076 ns	0.122 ns	-0.097 ns	0.021 ns	-0.007 ns
26	71.31 ±6.17	27.44 ±6.28	0.936***	-0.929***	0.101 ns	-0.078 ns	0.101 ns	-0.078 ns	0.124 ns	-0.099 ns	0.023 ns	0.009 n

APPENDIX B

5.1.1 Matlab Code For The Iteration Method

data = [*rectal,ambient,weight,clothed_state*]

data = [34.9 , 34 , 65 , 0.3]; *extimated_t* =

zeros(*size*(*data*,1)); *fori* = 1 : *size*(*data*,1) *t* =

0.001;

To = 98.96;

T = *data*(*i*,1); *Ta* = *data*(*i*,2);

W = *data*(*i*,3); *threshold* =

abs(*T* ∗ 0.0001); *BSA* =

0.1173 *?* (W^0.6466);

SF = 0.8 *?* *BSA* *?* 10000/*W*; *k* =

SF *?* 0.0006125 − 0.05375; *p* =

data(*i*,4); *loop* = 1; *whileloop*

$$tv = T_a + (T_0 - T_a)(\frac{K}{K-P}(e^{-pt} - e^{-kt}) + e^{-kt})$$

;

ans = *abs*(*T* − *tv*);

ifans ≤ *threshold*

extimated_t(*i*) = *t*;

loop = 0; else

t = *t* + 0.001;

end end

end *extimated_t*^{0}