KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

APPLICATION OF QUEUING THEORY IN THE EFFECTIVE MANAGEMENT OF TIME IN MONEY DEPOSIT BANKS-A STUDY OF GHANA COMMERCIAL BANK IN OBUASI MUNICIPAL.

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the Msc. Industrial Mathematics degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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DEDICATION

This work is dedicated to my parents and all my family members especially my brothers Nazif Mohamed, Majid Dimah, Shariff Dimah, and my sister Zaharah Awudu.

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LIST OF ABBREVIATION

GCB... Ghana Commercial Bank

KNUST....... Kwame Nkrumah University of Science And Technology

ATM.. Automated Teller Machine

adb...Agricultural Development Bank

FIFO.. First - In - First - Out

LIFO... Last - In - First - Out

SPT.. Shortest Processing Time First

FCFS.. First Come First Served

ABSTRACT

Waiting for services is a phenomenon in Ghana . We wait to eat in restaurants, we queue to board buses, we line up for service in post office and in banks either to deposit or withdraw money. The waiting phenomenon is not an experience limited to human beings only but objects too. Jobs wait to be processed on a machine, planes circle in stack before getting permission to land at an airport and cars stop at traffic lights waiting for their turn. Waiting cannot be eliminated entirely without incurring inordinate expenses and the goal is to reduce its adverse impact to a tolerable levels.

The objective of this study is to use already available systems to identify and also make known the effects and ramifications of keeping customers waiting in the queue and also the cost banks had to bear if idle facilities are not put into good use, with special emphasis on Obuasi branch of GCB. The queuing characteristics at the bank were analyzed using a Multi-server single-queue Model to achieve this major objective. Data for this study was collected by direct observation with the help of research assistants, a stop watch to record the number of hours/minutes spent by each customer at the bank. The data collected from the bank showed that 28th of June, from an hour of 9:30am-10:30am recorded the highest number of customers in the waiting line(113) whiles the least number of customers (17) in the waiting line was recorded on 18th of July hour of 9:00am-10:00am. customers had to wait an average of 0.0165 hours in the queue and 0.0183 hours in the system before leaving the bank.

We finally suggested that, queuing theory is worth studying; the findings of it can be used by managers of banks to determine and install the optimum service facilities or in other words put in place the appropriate technologies to help deal with long queues in their banks.

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Chapter 1

Introduction

In Ghana forming queues for something has become generally a phenomenon of what we do and we never seems to renege on that. A waakyi(A local rice combine with beans) seller will tell people who were there for her services to form a queue,in restaurants we have to queue for service, National service secretariat(NSS) queues must be form, mentioning; National Health Insurance Service unit(NHIS), bus terminals, bank counters, hospitals, traffic jams,drawing water from pips etc. Usually what bring this non-stopping queuing issues has been the fact that, the service facilities may not be enough ie people there are more than the service facilities operating or it may also come as results of inefficiencies of tellers(provider of the services) or also when there are lapses in timing for services facility to operate. When the number of customers demanding service be more than service facilities queue will definitely be form. In any case when the service facilities are more than the total number of customers its suppose to serve some service facilities will be come empty or idle, this usually happens at low peak days in banks or any organization.

Simply put, Queue; is define as customers or people lining up for service from service facilities and it has two important component; First, the drawing population ie the population of which the institution or organization draw their customers from, it can be the whole village, town, city, garage or farm etc. The population can be classified as finite example provision of services to your staff or repairing cars in a garage; where there are limit or limited in take and infinite example serving customers in a bank or provision of service in a bus terminal; where customers continue to troupe-in none stop. Second component, service system comprising of service facilities and service providers(tellers). Banking in Ghana has become very competitive couple with the challenges currently the have to endure through. In terms of financial sector, banks are the biggest so to say the least and their operations and what the offer to a country is undeniable very important. They contribute very heavily to the countries economy playing an intermediary between the nations deficit and surplus economy ie when the economy is struggling the provide financial support, more so when the country has reserves the play a roll of keeping it save. Hence, it has become an area where people or customers will like to transact businesses, tapping into their service provision system and go on testing the competencies of services they provide.But the inability of Ghanaian banks to match the number of customers to the service facilities they have, has cause a lot of time wasting and delays of customers in the banking hall of which GCB Obuasi branch can not be exempted. This not-do-a way canker of delay and time wasting has its own ramifications which can results in customers switching banks or stay entirely from businesses of bank and the consequences are detrimental to the operations of banks, because customers are they most valuable assets of every serious bank.

Another problem to be considered is the inefficiency of some tellers or service personal, they at times causes unnecessary delayers instead to focus and be serious on task at hand; moving to and fro, charting on phone or with their co-workers without regard and respect for customer's time, some intentionally (Service personal) do that to frustrate customers. Customers in general prefer to spend lessor time in transacting or doing business in their respective banks or from their service providers, because time wasting is not what the do cherish and this is a problem yet to be address by banks or being address but could not find a way through it. But board and managers of banks including staff prerogative is to attract and retain customer without letting go, at the same time maximized profit. For a bank to make the require profit, effective and efficient service delivery couple with good time management should be adhere to.

One major still not-resolve problem in Ghanaian Banks is congestion and overcrowding that continue to perpetuate at the banking premises; of which the repercussion has been customers switching banks hence looking for banks that understand their per-lite. Irrespective of improvement made by the banking sector, engaging in Internet Banking, usage of Automated Teller Machine(ATM), operating Mobile Banking etc in a way of subduing over-the-counter problems in their banks still the problem persist, which means it has not yielded the anticipated results probably due to technological(Computerization) breakdown, inefficiencies and irresponsible activities of managers and staff. Hence, the longer queues continue to exist without lessening. The fear of continuing keeping customers in the waiting line can bring discomfort, dissatisfaction and economic cost to them. Elsewhere This wasted time could be maximize or utilize ie opportunity cost of time spent in queuing.

In waiting line system, management decision of which service to bring on board or offer must be intone to customer satisfaction.To state also, provision of cheaper services may not cost much or may alleviate cost incursion for a small period space of time but the consequences will be felt in a long run; customer dissatisfaction in addition to them being bitter in transacting future business with the said banks. A high provision of service to customers ie increasing the number of working tellers or employing more technologies will be expensive and bitter to taste in terms of cost incur by the banks, that is temporal but when properly Mann and strategies it will bring relief in future operations of the bank. To properly take stalks of improvements in services, management of banks weighs the cost of providing services to customers against the brought-out costs of delays customers have to go through in the waiting line. The aim of the analysis is to find a middle ground between reducing economic effects(cost) to the system and giving quick, satisfying and fast service delivery.This study seeks to help the management of GCB-Obuasi branch make their service delivery efficient, effective, satisfying and time-bound to their value customers when they are in their banking hall to deposit or transact businesses.

1.1 Background of the Study

The problems people had to pass through when receiving services in an area where queuing is massive results in customers becoming disappointed. These problems sometimes result into insults and when prolong leads to infighting and it always happens.In the operations of banks, it may not be prudent to have customers waiting but simple put in reality one cannot totally avoid queues. The professional treat to this systematic problem is to find a well leverage or balance service facilities for the day to day transactions of the bank. This is reasonable because, bank which is a place where avalanche of business transaction operate of which queues to the effect cannot be entirely do a way with.

A bank is often noted as an institution which provides basic services such as accepting deposits, managing monies of their customers, make available transparent information of customers accounts and provision of financial help or assistance when customers are in need. There are financial institutions that are not in the status of bank also provide similarly banking services like astute banks without meeting the legal description of a bank, in the mids of banks struggling and folding up in Ghana recently, managers must do more including meeting the legal financial requirements set by the regulator(Bank of Ghana) which may help them to stay in the business of banking. Banks are very relevant in the financial sector industries in Ghana and their roles can not be underestimate. Not only should operations of banks in Ghana be free from challenges but be able to rub-shoulders to modernity of advance technology and at same time taking into account internal and external factors. History of banks in Ghana dated back in 1886 and 1917 of which Bank of British West Africa, now name Standard Chartered Bank and Dominion, Colonial and Overseas now Barclay Banks of Ghana were the first two banks established courtesy GCB (2007) Circular to Shareholders. The circular reiterate that, there was only one indigenous bank in the country on the eve of independent namely Bank of Gold Coast now called Ghana Commercial

Bank(GCB) dated in history 1953.

The bank of Gold Coast which bears the new name GCB was passed under a law of ordinance by the established constitutionally body Legislative Assembly in 1952, which was open operationally for business in May 20, 1953 courtesy GCB (2007) Circular to Shareholders. It was largely own by the then Government of Ghana ie the Government of Ghana was the sole shareholder at incorporation and the bank's objective was to provide financial support or assistant to small Ghanaian traders, farmers and corporate business.The Bank in no means, is the largest player in financial sector industry in Ghana currently, providing it's services to a broader client-ale. In 1950, the bank started with only one branch, currently having a total branches of 214 , more than l05 ATMs and a number of agencies. Government of Ghana shares as December, 2016 stood at 21.4 percent, individual and institution 78.6 percent courtesy Ghana Commercial Bank limited-Wikipedia (2016). The bank arguably is the largest employer in terms of human resources in the financial sector industry, with a staff strength as at December, 2016 stood 1,532 employees; Ghana Commercial Bank Limited-Wikipedia (2016), excluding the absorption of staff of the collapse banks. This shows the strength of the bank which started with only 27 staff. The bank has also tap into the expertise of various professionals in the field to help cement the operational objectives of the bank.

The growth of the bank has gone far and beyond of which its branches and networks can be found in every district in the country. In 1996 was the first time Ghana Commercial Bank(GCB) name was listed on the notice of Ghana Stock Exchange Market (Wikipedia, 2016). The bank's performance, innovative product, profitability and corporate social responsibility are some of the hall marks of its growth or astute achievement.The Board and Managers of the bank has also done a tremendous work in advancing the bank's Internet banking, Royal banking, information technology system, mobile banking, smart pay system and international remittance like money gram and western union. Currently has absorb both UT Bank and Capital Bank in its operations, that indicate the potency of the Bank.

The necessity of regulating institution was naturally needed to sanitize operations of banks and put them back on track. After independence the Bank of Ghana (BOG) was established (1957), to perform the role of regulating and monitoring the banking institutions in Ghana. The banking sector is getting more competitive probably due to upsurge in regulatory supervision, global banking and customers also becoming wise or a ware of their right. The demand of Customers are soring each day as they are searching for higher quality, low price, value for their savings and a very good service provision system. They also require an improve working ethics and value-addition from their partnering or banks they have chosen to do business with (Olaniyi, 2004). To deliver Service to customers, banks must see it to be personal unless otherwise either customers are provided service immediately or had to wait in the waiting line when the servers are busy or may decide to leave.

Queues are formed where facilities needed for fast services delivery are not enough, which by way cannot satisfy ever increasing demand of their services at a particular period of time. More so, customers have always express dislike for waiting line or queues (Olaniyi, 2004). The challenge of continuing keeping customers waiting in the queue can become a cost to them, because they say time is money. According to Elegalam (1978),customers wouldn't like or are not prepared to wast more of their precious time in the waiting queue, which has always become a time-cost to them. This time wasted would have beneficially utilized doing something different without call for alarm. Immense improvement has been made in Ghana's banking sector since the period of financial sector reforms. But nonetheless, Banks in Ghana are now in serious turmoil, struggling to stay on their feet and some collapsing or folding up which has become a stressing time for both financial sectors and their customers. Managers of banks and the regulator needs to put their expertise into high front to get these problems solve before more banks fold up, which has a die consequences not only for banks but customers as well. what banks need to do more in this situation is also to find a way to retain their value customers rather than loosing them. Because every bank's prosperity depends on availability of their value customers and what customer hate is to come to bank and waste more of their precious time there, instead of using this time for things that will add value to their life or bring them prosperity. when banks do not put measures to ease this situation of queuing, which most customers complain a lot and it continues, massive switch of banks may occur or they may even stop saving in banks or any form of transaction they have with the banks may halt and the consequences are there for all to see.

1.2 Brief History of Ghana Commercial Bank Obuasi Branch

The bank which is located at a suburb call Toy in Obuasi town, is opposite to both the main post office and metro mass transient terminal; was established in 1953 with only 10 staff (Manager of GCB Obuasi branch).It is the biggest bank in the town and it's competitors being Agricultural Development Bank(adb),Standard Charted Bank, Allied Bank, Golden Pride all in the same vicinity. Is only Odotobire Rual Bank which is further from them. The bank has expanded tremendously with current staff number 15, teller box 4: one queue having three tellers attending to them ie those depositing or withdrawing from the bank. The 4th Teller attend to those who's families live in Abroad and have send money true moneygram or western-union money transfer for cashing. There is another queue that lead to a table where people go there to check their balance or get help, either for corrections upon making mistakes when filling a form or may have problem of understanding certain things in the bank's operations.The vision of the bank is to become a strong leader in the financial sector without reneging on its market value and her mission is to be a first class banking in the country at the same time providing banking solutions for her customers and valued for all stakeholders.

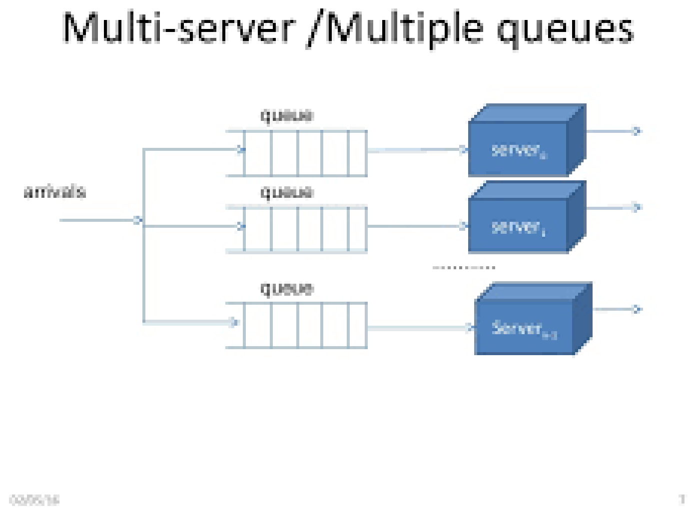

Figure 1.1: Multi-server/multi-queue

The setup of the bank has been the type, multiple queuing and multiple server (M/M/S:∞/FIFO) but the researcher concentration will be on where the customers deposit or save money, which has one queue comprising those withdrawing monies and depositors leading to three servers/tellers. Hence single-queue and multiple-server.

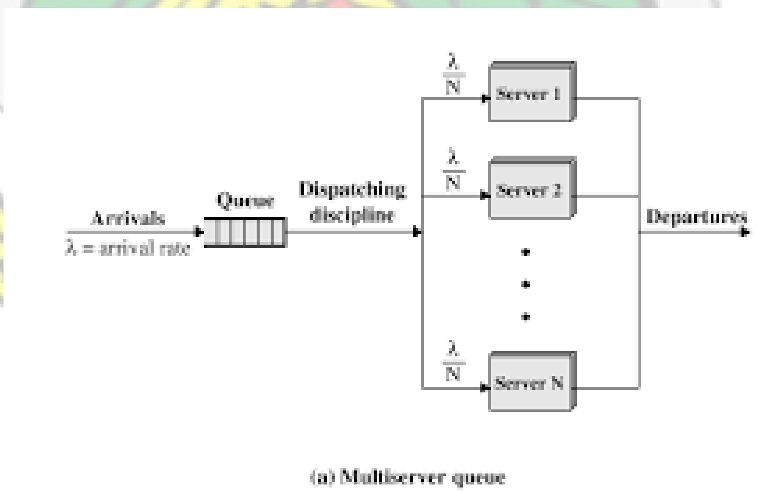

Figure 1.2: Single queue/Multi-server

1.3 Statement of the Problem

It is undesirable to have more people waiting in the queue which result to time the had to wast, discomfort and money constraints customers had to stress through; however queues seems to be part of our lives and activities we engage in. In Ghana Commercial Bank, it has become a routine of always citing long queues in the banking halls of which the queue can exist for many minutes or hours causing precious loss of time, makes banks transaction more tedious and thwart productivity hence the current struggles. Per important contributions or roles banks play in countries economic development, a tilt in performance may seriously have an impact probably negative on the economy. Continues queuing in banking halls usually end up with customers losing much of their precious time, become frustrated, confused and results in chaos. The inherent traditional method of banking in Ghana has made financial transaction tedious and time consuming. When an informal survey couple with analysis carried out at the branch of Ghana Commercial Bank Obuasi. Some customers of the bank complain bitterly about the frustration and value of time wasted or gone through before and during services. Base on these challenges it has become expedient to use mathematical modeling to check problems of queuing system at the branch. Since this bank provide services for a very vast-coverage area for both far and near customers in Adansi

1.4 Objectives of the Study

Setting good objective is relevant hence, the objective that necessitated this study is to analyze queuing systems at Obuasi branch of Ghana Commercial Bank in other to understand the effect and behavior of the disturbing processes for an informed and cogent decisions to be taken by relevant management body and those concerns. They following specific objectives are what the thesis is yenning to achieve

I. To investigate the applicability of queuing model in the management of time in obuasi branch of Ghana commercial bank.

II. Use the performance measures or systems to check customers' time they had to spend in the bank and effects when facilities are idle.

III. To examine, help and explain the operating characteristics of queuing system, using Little's equation for calculating or computing the numerical values generated for effective decision making in the Bank.

IV. To discuss and make recommendations based on the analysis gone through by the researcher.

1.5 Research Methodology

An M/M/S:∞/ and queuing analytical model tool will be used in the course of the study. Primary data will be collected at Obuasi branch of Ghana Commercial Bank for the analysis. A stopwatch will be used to monitor the time spend in serving a customer and arrival time of each customer as well as duration of time customers spend either in queue or system. The data will be collected during two different sessions that is peak days (end of the month) and non-peak days. In addition, other sources of information for this study are the Internet, KNUST library, past research works, articles and journals for relevant literature.First come first serve(FCFS) will be ideal queuing discipline used.A research assistance will be needed or used to aid fast gathering of data for onward computations.

1.6 Significant of the Study

Queuing systems primarily involve the provision of services and how fast these services are provided to ease discomfort. These systems also involve the arrival and departure of customers at service centers in search of efficient and consistent services. Queuing system extend beyond waiting lines in banking halls and the usual phenomenon of delay caused by busy servers. The systematic study of queuing system may be useful in contributing towards other areas in the society such as:

1. Analysis of reducing spending too much time in banking halls and other areas
2. Recommending to institutions after the research to peeve up their operational management.
3. Serves as statistical science that has too much to offer either in managerial role or set up response team in many fields of human activity
4. Reductions of queues in banking halls help alleviate stress and discomfort of cause ensuring ease of doing business and these attract more customers to join their services.

1.7 Limitation of the Study

Data collection was restricted to customers who arrived when the researcher was available on that day, of which the collections are going to be done randomly (an hour a day). A maximum of seven(7) days will be selected sparsely to collect the data, so all those who came before or after the period were not captured. In fact, data collection or gathering should have been done continuously whole month probably more but not to limit it to the seven working days time period; this is due to time and financial constraints. The recorded data was for those who came to the bank at that particular period the researcher was doing the collection. This imply, the results may be strictly related to a particular case study and its generalization may be restricted.

1.8 Organization of the Study

Organization of this thesis are categories into five chapters. Chapter one, talks about introduction of the study.This includes the study background, statement of problem, objectives, research methodology, limitations and organization of the study. Chapter two presents pertinent literature in the area on queuing systems and also reviewing of models. Research methodology discusses in Chapter Three. Collection of data and its analysis treated in Chapter Four.Lastly Chapter five ends the thesis by providing brief conclusion and a way forward.

Chapter 2

Literature Review

This chapter titled Literature review dwells on discussing the views and ideals of other scholars as it relates to the topic under study. The study reviews the following sections: Concept of Queuing Theory, Queuing Theory, Queues Application, Elements of Queue, Effects of Queue, Queuing Management, Operating Characteristics of Queuing System and Modeling of Queues.These topics reviewed, contributed to the conclusion and recommendation of the study.

2.1 Concept of Queuing Theory

According to Taylor (1994) waiting in queues to receive services is one of the natural phenomenon people had to stress through in every day of a persons life. For a person who has gone to shopping or to watch movies has experienced unease of waiting in queues to make purchases of a ticket. Not only do people spend a vital portion of their time waiting in lines, but products line up in production plants, machinery queues in line to be served, Aeroplans wait to take off and land, and so on. Since time is a valuable resource or asset, the reduction of waiting time becomes an important topic of analysis.

In the view of,Anderson *et al.* (1991), they developed quantitative models to help management of organization understand and make better and rightful decisions concerning the operation of waiting lines. In an area of science definition, a waiting line is referred to as a queue, and the body of knowledge describing and giving proper explanatory tutorial of waiting line is known as queuing theory. Before, during and after the beginning of civilization issues of queues exit. History books have explained, indicating citizens of communities queuing with discomfort waiting for share of feed from their kings and Queens, slaves were had to be chain in queues waiting for an audience with their kings or masters, the needy or prisoners queuing for a bowl of food etc. Though queuing is date back to history, studying of queues became new. In the early 1900s A. K. Erlang work on queuing theory became the first research work on queues which resulted in the study of queuing theory to date.Cutting-edge technology telephones was the research work he spent many of his years investigating (Brockmeyer *et al.*, 1948). The researcher(A. K Erlang); a Danish born mathematician in 1909 was credited with pioneering of probability and telephone conversation. In the latter of his works, he wants on to revealed that studying of a telephone system generally follows Poisson input, exponential holding (service) times, constant holding times, multiple and a single channel (servers).

In the view of Talia (2006), the study of queuing systems aid in quantifying or aggregating the phenomenon of queues using representation and realistic performance measures, such as the mean number of people in the waiting line or system, expected time spend in the queue or system, Rho (average facility utilization), and systems idle. The outcome of analyzing queues can go a long way of aiding managers of bank operationalize their cost optimality, where we seek realistic optimization of the two costs: the cost of providing service to customers and the cost of wasted time in the queue. It is difficulty to be able to quantify the stress level and amount of time wasted by customers in the system, especially when the attitude of tellers and customer are integral part of the operations.

According to Okeke (1996), queue is simple describe as a collection of objects awaiting service. It is usual to think of objects in the queue so do human beings. In studying queuing systems, we need to broaden our thought of the objects involved to include for example cars awaiting routine service, or stopping to buy fuel at a filling station, machines awaiting repairs, files pile up in the office to be gone through, in fact any discrete object which needs one type of function or the other to be performed on it. Queues are in fact about delays. Whether the actual queue is observed or not, queues are form because it is not at all impossible to organize the supplies to be exactly equal to the demand.

2.2 Queues

According to Morse (1958). The number of waiting lines and their respective lengths are the two basic aspects of the 'Queue'. The number of waiting lines is essentially a function of the configuration of the service facilities.That is, one waiting line for each different point of entry into the service system. Therefore, queue can either be single where there is only one waiting line or multiple where you have more than one waiting line .

In the view of Lee (1966), the length or size of the queue is influenced by such factors as;

a. Physical space, (Example, limited space at a banking hall or gas stations).

b. Legal restrictions example, city ordinance against forming queues on specified city streets.

c. Attitude of the customers example, long lines discourage some customers from joining the

d. The relationship of the capacity of the input source to the capacity of the service facilities.

The length of the queue can be limited or unlimited. The queue is limited (truncated) when there is a finite beyond which it cannot increase, example, the queue at a gas station). The queue is unlimited when there is no finite on its size example, the number of mail orders for development of photos,Lee (1966).

2.3 Queuing Theory

Wikipedia describes queuing system as,"the mathematical study of waiting lines (queues)". The expanded systems have actually help in using mathematical study and analysis of various related processes in looking at areas like customers arriving in the queue, being in the queue and departing the queue. It reiterated that, The prop-ponded theories were used in calculations, derivatives and analysis of systems performance such as mean waiting time in the system or queue, mean number of waiting customers and the probability of having system operating in certain states; idle, busy, availability of service facilities or waiting in a certain period to be served". Morse (1958), on his view on queuing theory stated that, it is important for one to study queuing system which essentially helps to establish the use of mathematical modeling to evaluate the effectiveness and efficiencies of queues. It is base on this ground that, solutions of queuing management can be optimize. Consistent with Wikipedia writers, queuing discipline ie the servers or tellers style of serving customers base on who to be pulled out of the waiting line for serving. Conventionally, there are three(3) types of service disciplines widely used in queue theory. They include First In First Out, Last In First Out and Processor Sharing. In First In First Out, the first person or object leading the queue will be provided service first and also be the first to exit the queue.In First In Last Out, the person or object first in the queue will be the last person or object to exit the queue, they may be several issues that may lead to that.Lastly, processor Sharing type of discipline serves all persons or objects in the waiting line equally or randomly.

Queuing theory help establish a steady state system of performance measures which of course is a good plane for short and long term strategic decision making. Crowley *et al.* (1995) came with a system of analyzing performance measures during the design and initialization of electromechanical devices.Flow ratio analysis was the steps description of the process which hitches on Jackson waiting line networks and give an elaborate approximation for labor and resource needs before the design of a more detailed simulation model. In addition to explanation of queuing theory Vasumathi and Dhanavanthan (2010) used the theory in various field of business situations namely; banks, transportation, hospitals, restaurants and matters involving customers. Generally, customers in every where expect a good level of services from the institutions they are engage in, whereas the institution provides services and taking account of customers waiting time at the same time maintaining efficient and effective require service delivery. Models of queues are widely used in restaurants, manufacturing unit, hospitals etc to address issues of time wasting. Also to detect unit arrive at regular or irregular period of time at a given point entry. Queues repercussions are not only a worry to customers but the institutions too, both safer the disadvantages. Waiting in the queue has a serious ramification on customers discomfort and uneasiness. The precious time customers spend waiting in the queue not only affects their daily activities but also their' expectations of leaving the queue early for other things the want to engage in. Taylor (1994); Obamiro (2006).

2.4 The Elements of Queue

According to Bose (2002), the most important things to consider when dealing with queuing situation in an institution are the customer who are there for service and the service facilities. The sources of Customers ie where an institution draw its customers from can be finite or infinite as stated early.When customers are arriving for service at the center either the customer is serve immediately when the servers are free or join the waiting line for are while when the servers are busy and will pull or call a waiting customer from the line if any. When there is zero customers in the queue ie when no one is in the queue, the service facility becomes empty until some one than come for services. In queuing analysis how customers arrive for service is denoted as inter-arrival time which is between successive customers and it follows Poisson distribution.Services rendered or received follow exponential distributions which is described as the ratio of arrival per time customer spend while receiving services . Generally, customers inter-arrival and time of receiving service could be probabilistic in a case of how customers arrive or receive service in restaurant or deterministic as in the arrival of application of job interviews.

In the view of Egbo (2001), he state elements of queue as;

a) Arrivals

b) Queue

c) Service

d) Exit/Departure

2.4.1 Arrivals

This describes people or objects coming into the system to receive service. The arrival pattern of these people or objects affects ways of operating queues. The arrival may be systematic ie orderly or it may be random or systematic-random. The arrival rate is a measure of time an object or persons takes to join a queue.

2.4.2 Queue

This represents waiting line and time period customer spend in waiting before he/she receive services. As the objects or customers arrive, there is the likelihood that they will wait or queue for their turn depending on the state of the service facility.

2.4.3 Service

This describes receiving attention or providing customers what they are there for. The purpose of coming to the system is to receive service or be served and the time spent before or being served is mostly what customers dislike especially when it took so long. The service rate describes the time spent in actually receiving the service attention. Like the arrival rate, the service rate may be random or systematic or both.

2.4.4 Exit/Departure

After receiving service or served, the object or customer will have to leave the system. Exiting or leaving the system is also referred to as departure. This departure completes the cycle and of cause customer do most appreciate and satisfy when the depart from the system early.

2.5 Effects of Queuing

Today when running business either within the country or outside country there is the need to prepare for competition because the world is now a global village where your customers can be snatch by other businesses hence the need to protect your customers base by upping your service delivery. Financial sector which is the pillar for business and economic empowerment of countries cannot afford to disappoint its value customers. Managers of banks had to deal with tough and challenging problems when managing their banks and its up them to think through the problems including issues of queues to fine a strategic solutions in solving them. Banks are undeniably they major player in the financial sector development, mediating between the reserves and broke economic sectors of a country, which is a place where many customers will like to transact business. A common problem usually one will notice in Ghanaian banks is congestion it seems not to be stopping. Customers are always dissatisfy when the see this kind of congestions and long queue, when it continues what run into their mind is to change banks. According to Sokefun (2001), Customers are satisfy when services deliver to them are of high quality, quick and efficient service delivering is what customers are yenning for. It is therefore important for managers of bank and staff to improve.The length of queues and congestions most at time happens at high peak period usually when the weekend is approaching where most people need money for their expenses. Often too longevity of waiting lines could be as a result of system malfunctioning ie computers and counting machine used by these tellers breaks down and also tellers inefficiencies of slowness,skipping responsibilities, non- focusing,leaving teller boxes empty without recourse to customers time etc. In the view of Zeithaml (2000), one major problem to note is measuring the relationship between when customers are satisfy and when banks are making improvement in profit . There are certain variables which influence performance such as price charges, ease of doing business, how transactions operate, openness/transparent dissemination of information, customer care etc. There for these indicators should be incorporated in the operational ethics of the bank or any organization because they play a middle ground between satisfaction and result. Efficient Services banks provide to their value customers also come with some challenges this includes legal issues, problems of recovery of loans , lending rate very high, poor customer relation, poor managerial skills, political instability or interference, transaction services charges, higher risks and low profitability. These have in turn inhibited quality of service delivering. often times, remedies needed to be found to curb these measures that pull done businesses in other to be able to propel customers to patronize services of banks. The reason being that a customer who was once disappointed by a bank will with hold his/her commitment to the said bank. Of course customers are the most potent advertise stakes, one can use to trend his/her business vise a vi customers can turn the spoils by informing other customers either creating rumors or in truth to get customers turn or stop transacting businesses with the bank.Which will become a bitter pill for the bank to swallow Banks (2000) concluded that time wasting and queuing challenges are the most common and normal issues in our life. We queue and wast time at the banks, postal office, NHIS offices, in bus terminal or in a traffic jam but also in more technical areas, such as machines queuing in factories either for repairs or services, computer network jamming up and telecommunication congestions. These technologies help speed up business transactions and re-engineering purposes in administrative tasks. "Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems".Customers are satisfied when they see a fast moving queue because they know the will be wasting little time to have their services rendered to them. They however become unsatisfied when they see a long, slow-pacing queue (Katz and Martin, 1989).

2.6 Queue Management

This approach deals with the structuring of customers, queues and servers visa-vi how they could be well coordinated towards the goal of rendering effective service at the least cost. When queue management is practice well it has the tendency of addressing issues of queues which will intend help lessening customers waiting time. Increasing productivity by training existing staff on good customer management or employing more staff is the way to achieve the operation management approach to satisfy customer's demands (Katz and Martin, 1989).

According to AL-Jumaily *et al.* (2011) their paper spotlight on bank's queuing system, the varying waiting line algorithms used by most banks in in providing services to their value customers and the period of time in average the spend or wast. The objective of their research work was to build automatic queuing system for the organization and structuring of queues in banks,to which the system should be able to analyze the status of queue and decide which customer to serve.According to the testing results, the new queuing architecture model can help switch between different scheduling algorithms and also factor in average customer waiting time. The clear design of their work, reasons with the modeling of average customer waiting time which to them is the point of reference. In addition to the processes of switching to the scheduling algorithm that gives the best average waiting time.

GoshaKinnis (2007) Suggested that a Queue Management System such as Queue Administration will help improve the satisfaction of a customer in the barber's shop as well as they barbers in question. The tool used in their study; Queue Administration, is a database driven in an on-line application which help to manage the different waiting list of a barbershop, in other to provide better functionality, efficiency and to maximize use of all the information collected. They classified Queue Administration into three interface, namely, the administrative, employee and customer interface. In a restaurant, when customers are waiting in a long hours and the service is not forth coming, a skill manager will at times instruct his/her service workers to move from customer to customer service table to drop free snacks or explain to them reasons of delaying or anything that will get customers calm down especial when their services are delaying which of cause is a good practice. Many patrons will definitely do appreciate that and this will have way of subsiding stress level. Services of that nature isn't what they are after, but it definitely calm nerves perhaps that is what one may think.

Many customers wonder why banks won't open more branches, employ many staff in other to solve problems of long queues and have customers stop complaining about their poor services. (Olusola and Okolie, 2013). More so, when capacity management skills of both managers and staff are build it plays a significant role in the aspect of designing service operations, especially when queuing model is seen as a solution to the problems of queues in the banking system. Anichebe (2013) suggested that in terms of performance criteria, it make sense to use three(3) servers or service facility in serving customers rather than the two(2) or four(4) servers. He reiterated that it far play reasons in managing better both cost and waiting line perfectly. The suggesting then is to use three(3) server facilities for serving customers this will in a way reduce long queuing problems. Ogbadu and Usman (2012) studied and researched on Imperativeness of Customer Relationship Management in Banking Industry. What the find out is that, relationship exist between managing customers and their loyalty as well as profit made by banks.The suggestion then is Managers of bank needs to improve or have their staff train on management relationships program to improve on their human relationship.

2.7 Queuing Theory Application

There have been many ways of applying queuing theory , some written done in Management of Science, literature of Probability as well as Operational Research. Areas applied were repairing of machine, telecommunications, networking, at the harbor when ships are being load or unload, planning of patients in hospital or clinics, inventory control etc. Queuing theory can also be used or applied in both assessment measures and security evaluation.In fact the use of queuing theory model in banks has increase which has help some of the banks to reduce the long queues found in their halls, productivity has gone up and customers satisfy. In an area of computer simulation models, queuing theory has also been used to help businesses strive.

According to Moss (1987), queuing theory can help play a role in the following; numerical strength of pharmacy staff, prescription process, dispensing process, and an outpatient time he/she has to wait. He then used mathematical tool for the computation and search. In machine interference, service and repair models, queuing theory ideas has been used. More so counting or listing leaflet of papers there has been application of queuing theory.When using queuing theory applications managers of banks have to Analysis, Simulate, compute numerals generated and make use of different approximation procedures for the study of queues. Now,some basic results are review and at same time providing some recent issues of queuing theory.We look for the books and papers that concerns queuing system and list them, example models of finite source and their applications.These relevant materials(papers and books) were gathered and collected from different sources including database and other reliable sources. Some researchers both past and present have done a pretty good work in queuing theory and has been used in such areas as machine repair system. Choudhury and Ke (2012) studied and incorporated an unreliable retrial queue with delaying repair and general retrial times under Bernoulli vacation schedule. The processes of system of queues and how is apply in service delivery or provisions were recently suggested by Bakari *et al.* (2014), they designed and applied modeling of multi-server repair problem with switching failure and reboot delay under related profit analysis. Liu and Ke (2014)also studied and applied multi-server machine interference with modified Bernoulli vacations.

Yue *et al.* (2013) did a perfect work on performance analysis and optimization of a machine repair problem with warm spares and two heterogeneous repairmen. Machine repair with mixed spares, balking and reneging was duly worked on presented by Maheshwari and Ali (2013). Trivedi (2002) applied and explained Probability and Statistics with Reliability queuing and Computer Science. Ke and Wang (2007) developed the reliability analysis of balking and reneging in repairable system with warm standbys. Jain *et al.* (2002) also developed diffusion process for multi-repairmen machining system with spares and balking. Age repair policies for the machine repair problem has been discussed by Arulmozhi (2002). Armstrong (2002) obtained age repair policies for the machine repair problem. Ciny (2001) developed Markovian approximation for manufacturing systems of unreliable mechanism tandem. Queuing theory and customer satisfaction, a review of terminology, trends and applications to pharmacy hospital management was discussed by Nosek and Wilson (2001). Shawkey (2000) dealt the machine interference model.

Zeltyn and Mandelbaum (2004) They used customer desperations and analytical queuing models to explain nonlinear relationships between customers waiting time and them being abandon. They cited that, in the context of call-center outsourcing, the common use of service level agreements based on delay thresholds at the upper-tail of the distribution (e.g. 95% of the customers wait less than 2 minutes) was in consistent with customers waiting behavior and non-linear effects

2.8 Operating Features of Queuing System

Behavior of queuing system are denoted by these variabilities; rate of customers arrival,time spend waiting in the queue , time spend when being served, emptiness of the system, amount of time in total a value customer spend in the system etc. The distributions of such variables need to be know, including their computational values, probabilities and performance measures of which the variables be less or more than a set targeted numerical value. For example indicate the probability of time spend waiting is ten(10) minutes less or more and it ramification. The operational features of waiting line system refer to the values (i.e. performance measures, probabilities, mean etc) of which varieties of variables so needed to either assess the performance of an old(existing) queuing system or to create a new one, (Tanner, 1995). Example, the emergence of operating features came as a result of different interaction among the various elements or performance measures of waiting line system (arrival rate, service rate, Rho, idle servers,etc). The computation of numerical values of various operating features can be calculated either analytically (that is by using mathematical equations relating to specific queuing models) or by simulation from an early collated system of queues which vis a vi helps to make finite decision on targeted areas.

2.9 Modeling Queues

The mathematical description or representation of queuing system is refer to as queuing model. Which makes precise assumptions concerning performance variabilities of customers arrival and service processes,type of servers and how many they are, queue behavior, organization and type of queue discipline to use. Many of these models were researched, developed and applied to various field of study ie queuing models have been widely used to improve various areas of troubling waiting line situation. Formulas are develop for onward calculations or computations of different performance measures results or values which of course help to design a new model system or improve on an existing models as well as many other service systems

In a study by Brown (2011),he stated that managers who manages institutions or organization, if used researched works of queuing models properly can go along way of helping them to comprehend and control or stir the effects of rework, but reasons of over-site the research model tool which should be a guide to them in managing waiting line problems is overlook partly because of over complexity environment of accuracy and/or the need for definitive assumptions. One of the objective of his researched work was to broaden/widening scope of having those in need of research models to understand or comprehend the accuracy of simple queuing in a system of variables. He further proposed queuing model that, combines G/G/1 modeling techniques for rework with efficient time processing techniques for machine availability and the precision of this model was tested under different levels of external arrival variability, rework and availability of machine.The tested results indicated that, the performance of the model was best under exponential patterns of arrival and can even be better under high rework conditions. Also in terms of using this tool for allocation of job to a particular worker and/or machine, generalizations are made with respect to that which was then base on a known rework rate with the ultimate aim of minimizing or reducing queuing time.

Nosek and Wilson (2001) stated that mathematical models and performance measures are often utilize by queuing theory researchers to assess and hopefully improve service flow rendered to customers who found themselves in long queues. Queuing theory has been applied in so many areas of field of study and very well be utilized by many service industry operators. Mentioning some areas of its application include early stages of staff planning, customer waiting time, behavior of queues, working environment, productivity etc. Essential, there are six(6) major components when it comes to queuing system or waiting line and they are:

populations, arrival, queues itself, queue discipline, service mechanism and the departure or exit.

Gurumurthi (2004) searched or considered heterogeneous servers and systems of queue with multiple classes of customers of which these customers were able to be served by more than one service facility. More so the server also have the capacity to serve or process more than one customer at the same time. He provided a unique framework used for the analysis and modeling of systems under arbitrary customer and server flexibility necessary for formulating a rich control policies which includes; customer selection and a priority scheme for customers or servers to be specific for server. The modeling formulated was also used to generate several insights into the effect of system configuration and control policies. In particular, the model examines the relationship between flexibility, control policies and throughput under varying assumptions for system parameters.

Rouba and Ward (2009) used heavy traffic limits and computer simulation to study the performance of alternative real time delay estimators in the overloaded GI/GI/ s+ GI multi server queuing model, allowing customer abandonment. They developed delay estimates which were used at call centers for delay announcements and other related service systems. The characterized system performance by expected average squared error in steady state. In addition they established approximations for performance measures with a non-exponential abandonment-time distribution to obtain new delay estimators that effectively cope with non-exponential abandonment-time distributions.

Udayabhanu *et al.* (2010) established that congestion and overcrowding in a waiting line system has a serious ramifications, so that it is never optimal to operate at 100% utilization levels. They developed an expression for the optimal utilization level for an M/D/1 queue, and demonstrate its similarity to the EOQ model of the inventory literature. The formulated model can also be used to achieve an optimal mean arrival rate, or to necessary adjust the available capacity so that the desired system utilization level is attained.

Chapter 3

Research Methodology

To make sure that the study are in line with the objectives set, proper care has to be taken in drawing the research design. The chapter highlights queue discipline and customer behavior, types of queues and their structure in banks, queuing system notation, arrival distribution, service distribution,server utilization, Little's law, Kendall's Notation,and illustrative examples.

This study focuses on M/M/s :(1 /FCFS) queuing system (infinite capacity of customers). This system has a single-queue and two or more service facilities. The assumption here is that, both inter sequencing arrival and service time are exponentially distributed and only one arrival can occur during a given time space.

3.1 Queue Discipline And Customer Behavior

Queue discipline; This is where customers/objects are logical arrange in a queue to the effect that when service facility is free customer/object is chosen for serving. Some types of queuing discipline are enumerated below:

I First-in-first-out (FIFO)- Services start from the first person in the queue and continues in that order.

II Last-in-first-out (LIFO)- Services start from the last person in the queue and continues in that order.

III Services administer in random order (SIRO)- Services are rendered in a random manner without notice on who come first or last.

IV Shortest processing time first (SPT)- Customers with least problem or whose problem can be solved fast are served.

V Service according to priority (PR)- Services are provided in order of importance or prestige of customer on the basis of their service requirements.

At times customers in the queue exhibit certain behavior when the see that the queue isn't moving as fast as the expect ie behavior or actions that some customers freely exhibit before and during queuing for example:

- Balk: quickly leave when there are many people in the queue to be served.
- Renege: Join the queue for a while before leaving.
- Jockey: Jumping from one queue to the order looking for one moving fast.

3.2 Types Of Queues And Their Structure In The Banks

Four major types of queuing system can be mention; 1. simple queue , 2. circular queue, 3. priority queue and 4. de-queue (double ended queue).The most frequently used type of queue in the banking hall is the simple queue. Simple queue describe simple operations of waiting line in which insertion occurs at the rear of the list and deletion occurs at the front of the list.

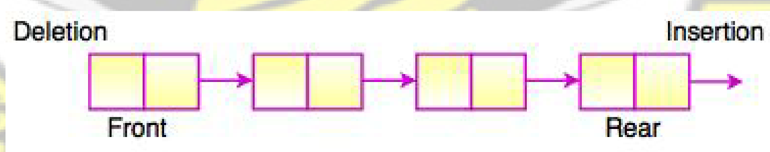

Figure 3.1: Simple queue

In addition to that, the servers (tellers) also determine how long customers wait in the hall. In Obuasi branch they following queuing formats are what they operate on; 1. One queue and One server 2. One queue and Many servers. 3. Many queues and Many servers.

3.2.1 Single Queue And Single Server (Teller)

In this type of queuing system, there is only one queue and only one server.If server becomes idle a customer moves to the teller to be served immediately, if not an arriving customer will have to wait or joins a queue for a while when the services facility is busy. After a customer has been served complete by the service facility and he/she feels satisfy, the customer then leaves the system. More so when there are some customers in the waiting line, one is immediately dispatched to the server for servicing. This type of queue is practice at the areas where customers follow that queue to make enquirers, check balance, open new accounts and boarding a bus.

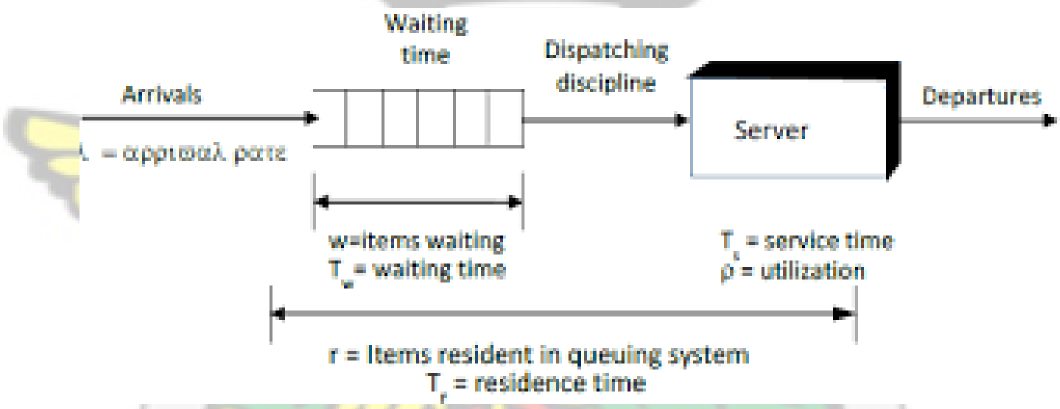

Figure 3.2: Queuing system structure for a single server queue

3.2.2 Single Queue And Multiple Server(Tellers)

This is a system where by there is more than one service facility (server) providing same services and drawn from a single waiting line.Some Ghanaian banks do practice this type of queue.

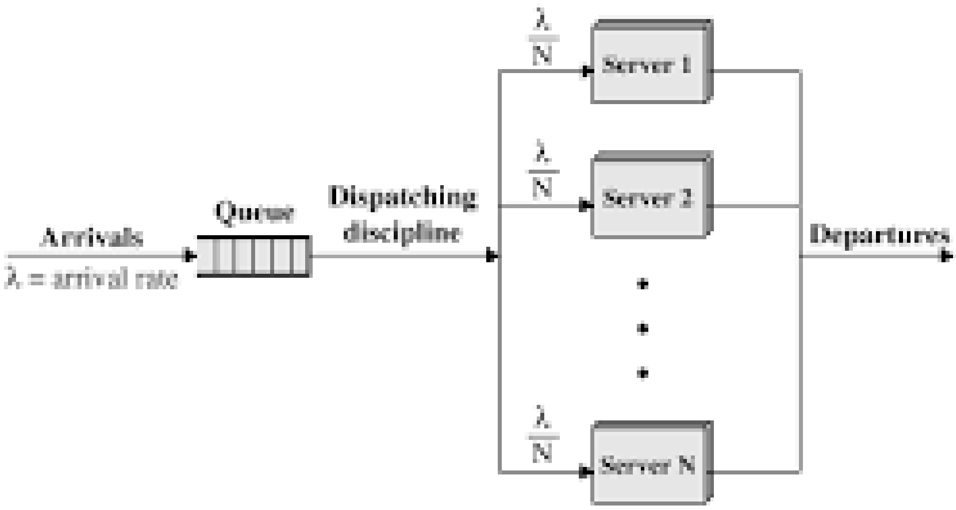

Figure 3.3: Single queue/Multi server

3.2.3 Multiple Queue And Multiple Servers(Tellers)

This can also be called Single Stag Queue in Parallel. It is similar to that of Single Queue-Server, only that there are many servers (tellers)performing the same task with each having a queue to be served.

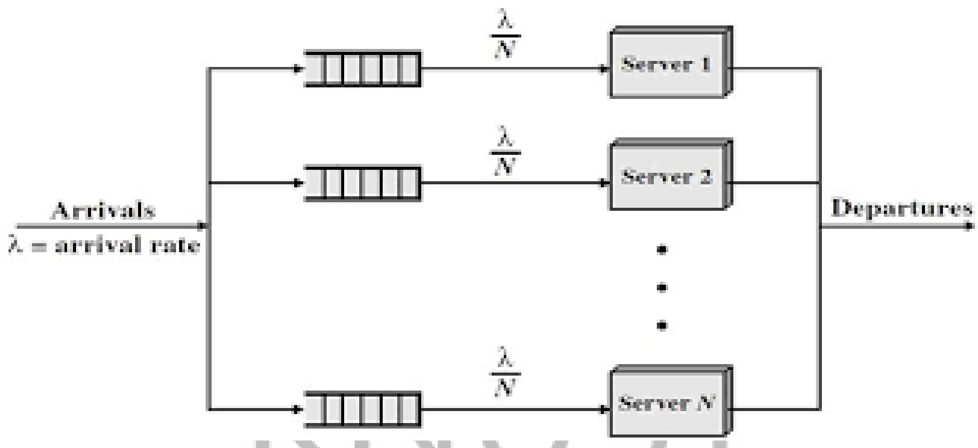

Figure 3.4: Multi server queue

3.3 Queuing System Notations

Queuing theory is a mathematical theory with its own standard notations. Few of the primary performance measures used in queuing theory that are relevant in this study are enumerated below;

λ: Average (mean) arrival rate i.e. how customers/patients join the queue/system.

μ: Average (mean) service rate i.e. the rate or how services are provided to customers/patients.

$\frac{\lambda}{\mu}$: Number of people expected in the waiting line, given by ratio of arrival rat to service rate, which in obedience to probability theory lies between zero and one

$\frac{1}{\lambda}$: Expected inter-arrival time.

$\frac{1}{\mu}$: Expected inter-service time.

ρ : Rho(Utilization factor) i.e. $\rho = \frac{\lambda}{s\mu}$ to which 's' is denote as the number of service facilities. It describes the fraction of the system's service capacity ($s\mu$) that is being utilized in the average by arriving customers (λ) (Hiller and Lieberman, 2001).

Lq : The number of customers in the queue or length of the queue. i.e

Lq $= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{p^2}{1-p^2}$

Ls : Number of customers in the system or length of the system.ie Ls $= \frac{\lambda}{\mu-\lambda}$ or

$\frac{p}{1-p}$

Wq : Average time customers spent in the queue. i.e Wq $=\frac{\lambda}{\mu-\lambda}$

Ws : Average time customers spent in the system. ie Ws $= \frac{1}{\mu-\lambda}$ or $\frac{1}{\lambda(1-p)}$

Po : probability of zero customers in the system. i.e Po $=1-\frac{\lambda}{\mu}$ or $1-p$

Pn : probability of exactly n units or customers in the system. i.e P(x = n) : $1-\frac{\lambda}{\mu}(\frac{\lambda}{\mu})^n = (1-p)p^n$

Probability of more than n units in the system $= p^n$

3.4 Arrival Process

In queuing system customers/patients coming into or joining the queue is the first procedure in queuing structure. Customers may join the queue individual or in batches and also at what time interval did they join. The sources of customers drawn from the center can be finite or infinite.These informations are term as Arrival process.

structure of arrival process and queuing system is shown below.

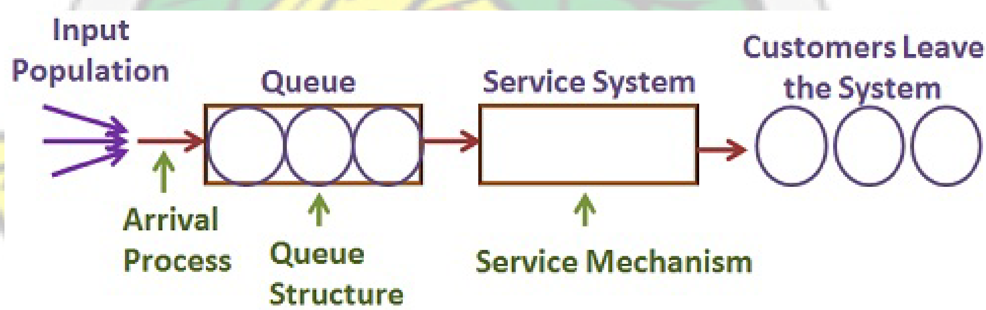

Figure 3.5: Structure of arrival process and queuing system

Arrival process from imput customers/population can be classified as follows.

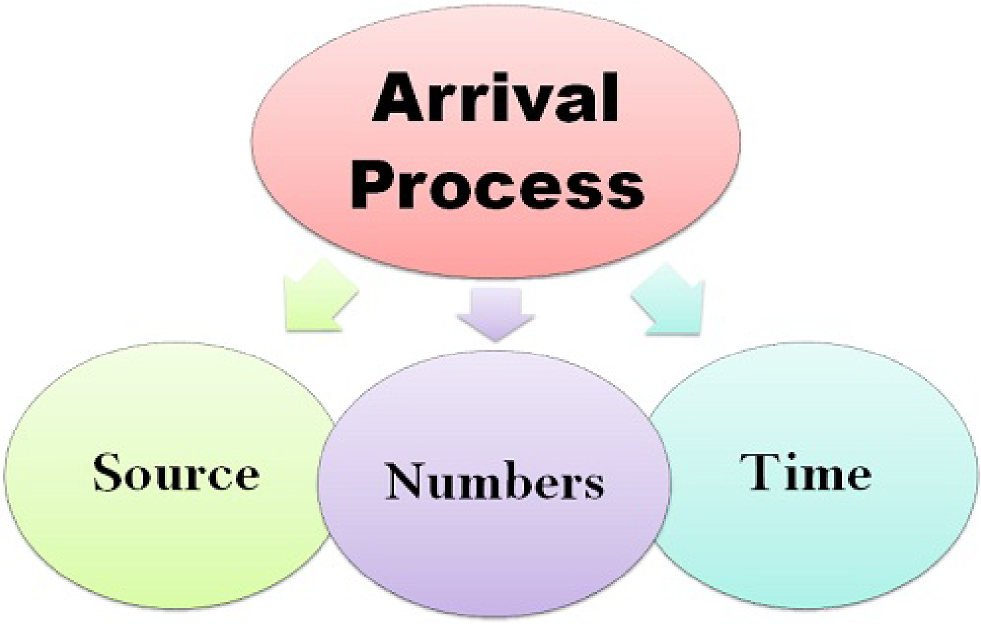

Figure 3.6: Arrival process

According to the Source

Sources here refers to where institutions/banks gets their customers from. The sources from which institutions/banks draw their customers from can be finite or infinite. Finite means population source is exhaustive or is limited example working staff, repairing cars, machines in a factory etc. Infinite, the sources of population is not exhaustive ie not limited example people in the city, town, village etc.

According to Numbers

Number here referring to customer quantity who arrived in the system for services. Customers arriving for service can either happen individually or in groups for example, a customer can individually walk to barbering shop for hair cut or a student singularly walk to library for his/her personal studies. In groups, a family may decide for today to all go to restaurant for a feast or friends, families arriving in bus terminal to board a bus etc. Inter arrival time and how sequential these arrivals happens is of important to researcher of queuing theory.

According to time

The arriving time interval customers do arrive in the queuing system is of very relevant in the study of queuing theory. Customers coming into the system may come in at a known time interval or may arrive randomly. For customers who arrive at a regular/known time space interval, their procession of arrival is term as deterministic models. Moreover, many are times where arrivals of customers do not follow a particular sequential time order ie when and how customers arrive in the system can't be tolled or known hence, this unknown per unit time arrival of customers is term as probability distribution. What it means is, customers arriving in the system may follow any pattern ie it can be deterministic or probabilistic. But all in all it assumed that, how customers arrive in the system must follow a particular order and that is Poisson Distribution. All the necessary information related to how many people arriving in the system at a different time interval is very well handle and taken care of while gathering and designing the queue structure. Given an average arrival rate (λ). Let Ai be the arrival time interval which exist between the arrivals of the (i-1)th and ith customers, we shall represent the mean (or expected) inter-arrival time by E(A) and (λ) = 1/ E(A) is the arrival frequency.

Basic Queuing Theory Relation

Assume that, the arrival time interval distribution, period of service distributions, number of tellers, capacity of the system and system queuing discipline were provided, these enable describing and computing queuing system parameters and state of performance measures. Assume customers who arrive at the queuing system are numbered from

n^{th} arriving customer called customer -n.

Let t_n represent period of time when n^{th} customer arrives in the system and hence t_n-t_{n-1} an inter-arrival time.

Let P_n represents service period for the n^{th} customer.

Let B_n represent period when the n^{th} customer leaves/exists.

Let L represent a possible length(number) of serving customers. We denote L(h) to be number of people/customers in the system at period h, L_n be the number of client/customers in the system just after n^{th} customer leaves. The waiting time T is the period that a customer stay(spend) in a queue waiting to receive service. We also denote T_n as the time spend waiting by n^{th} customer and T(h) total period taken to serve all the customers in the waiting queue at period h (the total remains of work to do at period (h)).

Let λ represents the expected number of customers in a given unit of period(time) (average)

$$\lambda = \frac{\text{number of customers arriving}}{\text{total time involved}}$$

λh denote average number of customers arriving at a period h

μ average number of customers a service facility can handle for a period.

$$\mu = \frac{\text{number of customers a server can handle}}{\text{total time involved}}$$

3.4.1 Poisson Distribution

Poisson distribution;is a discrete/divisive probability distribution that expresses the probability of a given number of events occurring in a fixed interval pattern of time or quad. The distribution was theorist, birthed and pro-pondered by a French mathematician SimAon Denis Poisson and hence named after him. In the field of study, it applications are mostly used in probability theory and statistics for solving several discrete and matters that follow fixed interval patterns of arrival. Especially, when the occurring event constant rate is known and are independently of the time since the last event occur. Poisson distribution is very relevant and can be used for number of events which occur particularly in a specified time interval such as distance, area or volume. For example a person can keep trace of number of mails he/she receives in average each day per properly using analysis of Poisson distribution. Of course per the analysis each receipt of mail in future must/will not depend on the previous mail received ie the sources of mail receive today and in future are independently of one another. Then the assumption here is that, mails receive each day follows/obeys a Poisson distribution. Some examples which follow Poisson's distribution includes number of phone calls a call center receive per minute/hour. In radioactive source, we can also talk of decay process per minute/hour a substance gradually stages through. In conclusive Poisson distribution is popular for fashioning models that has do with number of times an event occur in a pattern of time interval or space of which both occurring events are independently of each other with the parameter λt . This parameter λt is the average number of arrivals in time t which is also the variance of the distribution. If n denotes the number of arrivals within a time interval t, then the probability function p(x) is represented mathematically as,

$$p(x)=\frac{e^{-\lambda}\lambda^{x}}{x!} \text{ where } x = 0,1,2,3...$$

- λ represent number of event occur in average per interval
- 'e' denote number 2.71828...(Euler's numbers) based on the natural logarithms
- X! is X*(X-1)*(X-2)*...*2*1 is the factorial of X

This distribution or procession arrival is refer to as Poisson input. Let t denote a period(time) variable. Let say the system starts at t=0. Customers arriving at a specific period which happens randomly, the 1st at t_1, the 2nd at t_2, etc. The random variable t_n represents the clock period at which the nth customer arrives, and the values t_n where n = 1,2,3,.. are called points of occurrence.

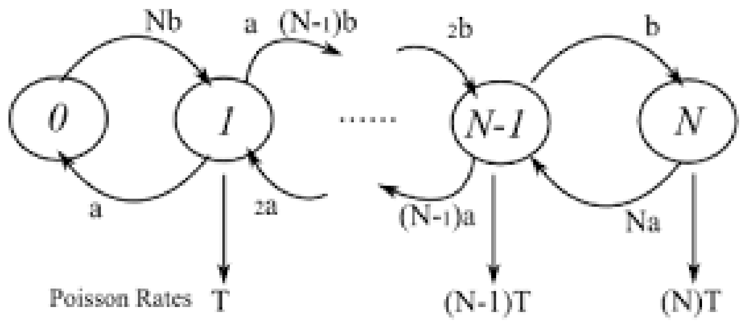

Figure 3.7: Exponential distributed inter-arrival times

Let t_0 of which n≥1 for T_n represents interval of time between the (n-1)th and nth arrival. For ordered of random variables sequentially (T_n,n≥1) is refer to as procession of an inter arrival stated in Fig 3.7 above

The processes of collection time arrival in a random order that explains some systems of evolution over time period is refer to as Stochastic process. For this step system (X(t_n,n≥0)), is said to be processes of counting only if X(t_n) denote total number of arrival customers occurred up to time (0,t_n). The system of counting must term up :

a. X(t_n≥0) and X(0)= 0

b. X(t_n) denote valued integer .

c. If $T_{n-1} \leq$ t, then X(t_{n-1})≤ X(t_n) where t_n represent time period until the next arrival.

d. For $t_{n-1} \leq t_n$,X(t_{n-1}) ≤ X(t_n) represent arrivals number which happens to occurred in the interval (t_{n-1},t_n).

The processes of counting can only be said to have independent increments if arrivals number that occur in time disjoint intervals are independent. Example X(t_n) and X($t_n + t_{n-1}$) ≤ X(t_n) - X(t_n) not depending on each other.

processes of counting (X(t_n), n $\geq$ 0) can only be said to follow Poisson system processes when having randomly selection of time, T arrival interval in an ordered time interval (t_{n-1},t_n), T = t = t_n - t_{n-1} having n points from the probability with a rate of arrival λ, $\lambda \geq 0$, if

a. X(0) = 0 i.e. when a customer arriving joins idle waiting line.

b. processes of the system has increment which is independent .

c. The arriving number of customers in an interval of any length 't' is Poisson distributed with an average λt. Which represents all t_{n-1}, t $\geq$ 0.

Of which probability of no arriving customer in the interval [0, t] is,
p(no customer arrival in[0,t]) = p(0) = $e^{-\lambda t}$.
More so, p(no customer arrival in[0,t])= p(arrival which happens next occur after 't')
= p(the successive time between two arrival exceed 't'). The indication here is that, probability density function of time inter-arrival is indicated as,
$e^{-\lambda t}$ for t $\geq$ 0
Which referred as negative exponential distribution with a given parameter or exponential distribution simple put. The distribution of standard deviation and average inter-arrival time are both 1/(λ) of which, (λ) is rate of arrival.
i.e. E(t) = $\frac{1}{\lambda}$
To add, the average time 't' of inter-arrival will then become reciprocal of rate of arrival.

3.5 Service Distribution

The service part of queuing system is very relevant when it comes to ameliorating queuing problems.The number of servers/service facilities available determine how efficient, effective and fast a system can operate but also not manage well and determine the efficient number needed can bring untold cost to the operator. It can also be said that it is the supply side of the system with its number of server specifications (denoted by 's'). The service facilities are the service point or entry point of service such as cashiers, staff, receptionist, machine, ticket server etc.In a bank, tellers are the servers or service point. Traffic intersections, the lightening; red, yellow and green are the service point for traffics. Also in restaurant, people come round your table asking type of menu needed for serving or drop menu on your table, they are the servers or service points. System service facilities can be categories into these forms

- Single server: here the point of entry for service is only one.
- Parallel servers: the point of entry for services are many.
- Tandem queue: many service facilities available and is up to a customer where to queue for his/her services to be provided.

Efficacy of servers in serving customers will determine system performance. The quicker of servers to be able to serve a frustrating and depressing customer, the better system performance to minimize queuing delays.
Let S_i be time receiving service by ith customer, we then denote customer service mean time by E(S) and $\mu = 1/(E(S))$ the service rate of a server.

3.5.1 Exponential Distribution

The exponential distribution which can also be referred as negative exponential distribution is a system distribution which explains time occurrence between events in a point of Poisson process ie is a distribution where there is a continuously and independent events occurrence at an average constant rate. In statistics and probability theory queuing models are the most commonly used which are of course based on exponential service time and inter arrival time assumptions .

The distribution parameter μ is represented as$\mu e^{-\mu t}$ for 't' $\geq$ 0. If 'T' denote random variable which then represents time inter-service exponential distribution, we further that p(T $\leq$ t) = 1-$e^{\mu t}$ and p(T $\geq$ t) = $e^{-\mu t}$

The cumulative distribution function(cdf) of 'T' given as

F(t) = P(T $\leq$ t) = 1- P(T$\geq$ t)

= 1 - $e^{-\mu t}$ Where 0 $\leq$ t$\leq$ ∞

Inter-arrival time density function is

f(t) =$\frac{d}{dt}Ft = \{^{\mu e^{-\mu t}}_{0}$

0$\leq$ t $\leq$ ∞

t$\leq$ 0

Average inter customer arrival time indicated as

E(t) = $\int_0^\infty$t f(t)dt = $\int_0^\infty$ μt$e^{-\mu t}$dt

Using integration by parts.

$= \mu[\frac{-te^{-\mu t}}{\mu}|_{\mu}^{\infty} + \frac{1}{\mu}\int_0^\infty e^{-\mu t}dt]_0^\infty$

$= \mu[\frac{-te^{-\mu t}}{\mu}|_0^\infty + \frac{1}{\mu}[\frac{e^{-\mu t}}{\mu}]_0^\infty]$

$= \mu[0 + \frac{1}{\mu} * \frac{1}{\mu}]$

$= \mu[\frac{1}{\mu^2}] = \frac{1}{\mu}$

Exponential distribution posses an exciting characteristics that is, its average is match to its measure of deviation E(T) = $\frac{1}{\mu}$

3.5.2 Properties of Poisson And Exponential Distribution

Poisson and Exponential distributions both are similarly some how related but whiles Poisson distribution deals with a fixed occurrence of number of events within a space of time, Exponential distribution had to relate itself to the time between occurrence of ordered/successive events with a continuous flowing time. There are three vital features of Poisson and exponential distributions namely;

i. Memoryless Property

This property states that, the occurrence of two events in terms of time, now and previous are expected to remain the same no matter how long the previous event occur .

$$\Pr(x \leq T + t | x > T) = \Pr(x \leq t)$$

This seems difficult to realize in "real world". For example, traffic light has three sets of colors; red, yellow and green each has a fixed time to lighten/shown. If it takes ten(10) seconds for a red light to go off and enable yellow light to be on and yellow light go off and enable green light to be on and the cycle continuous without each lighten on and off depending on how long one of them occur ie the probability of traffic light turning yellow in the next ten(10) seconds does not depend how long red light has been on. For situation of such nature is term as memoryless.

ii. Additive Property

The property of Additive is when 'n' independent Poisson process with parameter λi are sum for i = 1, 2,... n. That is a Poisson process with parameter $\lambda 1 + \lambda 2 + ... + \lambda n$.

iii. Decomposition Property

Suppose that n(t) is a Poisson process with expected rate λ and that each arrival is marked with probability p, independent of all other arrivals. Let $n_1(t)$ and $n_2(t)$ respectively represent the number of marked and unmarked arrivals in [0, t]. Then $n_1(t)$ and $n_2(t)$ are two independent Poisson process with respective rates λp and $\lambda(1 - p)$ (Bastani, 2007).

3.6 Server(Teller) Utilization Factor (p)

If the queuing system consists of a unity host hence, the utilization or the state of steadiness ρ is the fraction of period where the service facility is busy that is occupied to the arrival. In a case where customers are drawn from is infinite and there isn't limit on the arrival of customers in the sole server(teller) queue, the utilization factor is define mathematically as: $\rho = \frac{\text{arrival rate}}{\text{service rate}} = \frac{\lambda}{\mu}$ The steady state of queuing theory with many service facility is the average fractional distribution of servers which are active. Referring to the above mention case since the number of working facilities is the product; sμ is the overall services distribution this implies $\rho = \frac{\lambda}{s\mu}$ and rho can be used to formulate issues of static behavior mentioned previously. For ρ (rho) to be stable it must fall between the range of zero to one. $0 \leq \rho \leq 1$. If Rho or utilization computation is greater than one('1'), what it means is number of customers arriving is much more than the rate they are being serve.Simply analysis pounded on indicate that, queuing length will continue to grow unabated. But when Rho is less than one('1'), waiting line will in an average reach a steady state. In any case, if Rho or utilization exceeds the target, the system then is not stable and will require new service facility. The indication here is that, in an average number of customers arriving in the queue by a given time period must be less than rate of being serve or provision of service. Other than that measures must be put in place which include additional server indicated early to ameliorate situations of growth in queue length .

3.7 Little's Law

In Mathematical probability theory which has queuing theory as a discipline or subject matter area of study comes with Little's; Results, Theorem, Lemma, Law or formula initiated or theorist by John Little. For the expression, he simply put it as; how long in an average 'L' number of customers stay waiting in a stationary system equal to the product of average arrival rate λ within a long effective space of time and average time customers spend waiting 'W' in the system. State algebraically as

$$L= \lambda W$$

Consider a system which start from t=0 to t=∞ ($0 \leq t \leq \infty$) and as time progresses quantities of various values of interest which has a connection between average queue length of customers in the system, customer waiting time etc are invariably look at and computed. Let

L(t)= Number of customers in the system at time t

λ(t) = Number of customers who arrived in the interval [0, t]

W_j = Time spent in the system by the jth arriving customer our intuitive notion of the "typical" number of customers in the system observed up to time t is

$$L_t = \frac{1}{t} \int_0^\infty L(T)dT$$

Where time 't' represents average time period. Of course, values of L_t changes when invariably time' t' changes, but in many cases of interest,L_t stays at a steady-state L as t increases, represented as,

$$L = \lim_{\longrightarrow\infty} L_t$$

In this case, 'L' which is the average steady-state time of L(T) is stated as

$$\lambda_t = \frac{\propto(t)}{t}$$

when average arrival time rate is measure over the interval [0,t], the arrival steady-state rate is express as

$$\lambda = \lim_{\longrightarrow\infty} \lambda_t$$

Let preempt limit exists, the average waiting time of customers or delays measure to time t is similarly express as

$$W_t = \frac{\sum_{j=0}^{\propto(t)} W_j}{\propto(t)}$$

To also indicate average time each customer spend in the system waiting for service measure to time t. The average steady-state time spend or delay by customer is also express as

$$W = \lim_{\longrightarrow\infty} W_j$$

Again, preempting limit exists, it came to bear that variables L, λ, and W were similarly related by a simple expression or formula which makes it possible to fine one when the other two are given/provided.The result is term as Little's Theorem,(L=(λ)W).

The expression made by Little's Theorem says, systems that are over crowded(large L) are naturally associated with long customer queues and delays (large W) and vice versa. For example, during the end of the month salary workers troop to their various banks for their salaries and other transaction coursing bank halls to be over crowded (large L). Of course , this will definitely be associated with long queue hours of waiting time in the banking hall (large W). We can also look at a small fast-food restaurant (small W) which ideally needs a smaller space room for it business (small L) than a fully large restaurant for the same customer arrival rate. More so average number of customers in the queue at time 't' can be express as

$$L_q = \lambda W_q$$

and the average customers number receiving service at time 't'

$$L_s = \lambda W_s$$

3.8 Kendall's Notation

Kendall's notational system (or sometimes Kendall notation) is the measure system notations for explaining and classifying a queuing node. D. G. Kendall describes queuing framework using three component namely A/S/c in 1953 where the letter 'A' represents arrival time that exist between customers joining the queue, 'S' denote job size and 'c' representing servers or service facilities number at the point of serving. Since then it has expanded into A/S/c/K/N/D. For all queues that are form have the following characterization; arrival rate, service rate and queuing discipline. System of queues are usually explain in shorten form by using what were stated early. The general D. G Kendall's notation expanded is shown below:

[A/B/s]:d/e/f

Where,

A = Probability of arrival rate distribution

B = Probability distribution of the departures

s = Number of servers (channels)

d = The capacity of the queue(s)

e = The size of the calling population

f = Queue ranking rule (Ordering of the queue)

Some special notations for various probability distributions have been developed, explaining arrivals and systems departure. For example probability distribution of arrival or departure in Poisson process, Erlang probability distribution, General distribution, Independent distribution etc. Thus, we can cite example where the system has arrivals and departures to be Poisson process distribution as [M/M/1]:infinity/infinity/FCFS having a single server, queue length that is infinite, infinite calling population and FCFS being queuing discipline. For mathematical studies, this system of queue is the simplest and expressed/referred as M/M/1 queue. Banking queues possess Markovian Memoryless properties, therefore in this study the focus will be on M/M/1 and M/M/s in determining service facilities needed for each time interval. As stated earlier about the three main queuing systems , that is Single Server with Single Queue, Multiple Server with Single Queue, and Multiple Single Server with Single Queue in Parallel using queuing models M/M/1 and M/M/s. In these models, arrival of customers follows Poisson process or distribution which in implies that arrivals are independent of each other. Probability of arrival in any interval of time does not depend on the starting point of inception or on the specific history of arrivals preceding it, but depends only on length being property of Stationarity and Lack of Memory. Both assumptions are said to be Markovian, hence using the two "M,s" for the notation used in the models.

3.8.1 M/M/1 Model

This type of model M/M/1 represents a system where it first 'M' denote arrival which follows Poisson distribution, the second 'M' represents system service time follows exponential distribution and '1' denote a sole/single service facility . The naming of the model/system is written in Kendall's notation. Per description, the M/M/1 model is the basic queuing model or the very simplest model to use, it is a fanciful object of study as expressions of closed-form can be sourced for many metrics of interest using this model. An M/M/s model which is more or less complex and having more than one service facility is an extension of M/M/1 model . The model which is a stochastic process have a state space set as 0,1,2,3,... where in the system, the values obtain correspond to number of customers including currently receiving services.

- According to Poisson process arrival rate of customers occur at λ and then the process is shift from state i to i + 1.

- Service systems follows exponential distribution with rate of parameter as μ in the model of M/M/1 queue, where the average/mean service time is formulated as $1/\mu$.

- An operated single server provide services to customers one at a time and after the other following the discipline of first-come-first-serve order. When a customer receives his/her service and satisfied, the customer must leave the system for the turn of others.This progressively reduces the length of the queue.

- The drawing population which forms the queue is of infinite in size,therefore, there is no limit of customers the queue can contain in the state space 0,1,2,3,....

Continuous time Markov chain poses same similarity as in a birth-death process("birth" refers to customers arriving to the queue while "death" refers to customers departing/leaving the queue after service). The diagram below is the state space diagram.

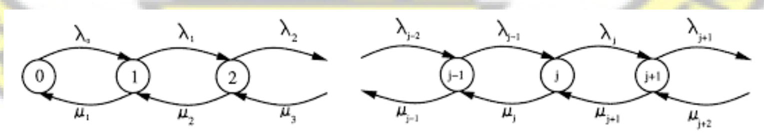

Figure 3.8: The state space diagram

It continues that for equilibrium, the number expected to depart from stage j is of equal to the expected number entering into stage j. Thus, Average rate in (arrivals)= Average rate out (departures).

Assuming there is a settling down of the system to a steady state in stage j, a fraction p_j of the system time is spend in stage j. For j$\geq$ 1 , a customer in the system only can enter or leave stage j by passing to stage p_{j+1} or p_{j-1}, more so j$\geq$ 1, Use expressions below to derived the expected number of people in the queue/system, denoting:

P_o

P_1

P_2 ...

P_n

and also

L_s; average number of people in the system

L_q; average number of people in the queue

W_s; average waiting time in the system

W_q; average waiting time in the queue.

p_j = p_{j-1}(t) * probability of one arrival and no service + p_{j+1} * probability of no arrival and one service + p_j(t) * probability of no arrival and no service.

where

- λh is probability of one arrival.
- μh is probability of one service.
- 1-λh is probability of no arrival.
- 1-μh is probability of no service.

$$p_j(t+h) = p_{j-1}(t) * \lambda h(1-\mu h) + p_{j+1}(t) * \mu h(1-\lambda h) + p_j(t) * (1-\lambda h)\ (1-\mu h)$$

simplify by leaving out the highest or second order terms.

$$p_j(t+h) = p_{j-1}(t)\lambda h + p_{j+1}(t)\mu h + p_j(t)(1-\lambda h-\mu h).$$

Divide through by "h"

$\frac{p_j(t+h)-p_j(t)}{h}$ = $p_{j-1}(t)\lambda + p_{j+1}(t)\mu - p_j(t)(\lambda+\mu)$

At steady state

$\frac{p_j(t+h)-p_j(t)}{h} = 0$

Therefore:

$\lambda p_{j-1} + \mu p_{j+1} = (\lambda+\mu)p_j$ ... (eqn 1)

If j=0 ie probability that no person in the queue.

$p_0(t+h) = p_1(t)$ * probability of no arrival and one service + $p_0(t)$ * probability of no arrival and no service.

$p_0(t+h) = p_1(t)(1-\lambda h)\mu h + p_0(t)(1-\lambda h)1$

where probability of no service is one

$p_0(t+h) = p_1(t)\mu h + p_0(t)(1-\lambda h)$

leaving highest order out and divide through by "h".

$\frac{p_0(t+h)-p_0(t)}{h} = p_1(t)\mu - p_0(t)\lambda$

At steady state

$\frac{p_0(t+h)-p_0(t)}{h} = 0$

$p_1(t)\mu = p_0(t)\lambda$ ... (eqn 2)

$p_1 = \frac{\lambda}{\mu} p_0$

put j=0 in (eqn 1)

$\lambda p_0 + \mu p_2 = (\lambda + \mu) p_1$

$\lambda p_0 + \mu p_2 = \lambda p_1 + \mu p_1$

where $p_1(t)\mu = p_0(t)\lambda$

$\mu p_2 = \lambda p_1$

$p_2 = \frac{\lambda}{\mu} p_1$ but

$p_1 = \frac{\lambda}{\mu} p_0$

Therefore

$p_2 = (\frac{\lambda}{\mu})(\frac{\lambda}{\mu}) p_0$

or

$p_2 = (\frac{\lambda}{\mu})^2 p_0$

Generally, $p = \frac{\lambda}{\mu}$ being the expected number of people in the system

$p_1 = p p_0$

$p_2 = PP_1 = P^2P_0$

$P_3 = PP_2 = P^3P_0$

$P_n = P^n p_0$

Here the sum of steady state is equal to "1" ie

$p_0 + p_1 + p_2 \ldots + \infty = 1$ or

$p_0 + pp_0 + p^2p_0 + p^3p_0 \ldots + \infty = 1$

$p_0[1 + p + p^2 + p^3 \ldots + \infty] = 1$

$p_0[\frac{1}{1-p}] = 1$

Therefore $p_0 = 1 - p$

To find average length in the system (Ls), we set

$p_j = p^j p_0$

$\sum_{j=0}^{\infty} jp_j = \sum jp^j p_0$

$p_0 p \sum jp^{j-1}$

$p_0 p \sum \frac{d}{dp} p^j$

$p_0 p \frac{d}{dp} \sum p^j$

$p_0 p \frac{d}{dp}[1 + p + p^2 ...+ \infty]$

Applying geometric series

$p_0 p \frac{d}{dp} \frac{1}{1-p}$

$p_0 p \frac{1}{(1-p)^2}$

but $p_0 = 1 - p$

$Ls = \frac{p}{1-p}$ Ls can also express the following

Ls = average length of queue (Lq) + expected number of people $(\frac{\lambda}{\mu})$

$Ls = Lq + \frac{\lambda}{\mu}$

Where length of the queue can be express as;

$Lq = Ls - \frac{\lambda}{\mu}$

Applying Little's equation

$Ls = \lambda$ * average waiting time in the system

$Ls = \lambda Ws$ where

$Ws = \frac{Ls}{\lambda}$ being average waiting time in the system

Lq(average length of the queue)= λ * average waiting time in the queue

Lq = λWq where

Wq = $\frac{Lq}{\lambda}$ being average waiting time in the queue.

3.8.2 Illustrative Example

A bank consisting of only on window, a solitary employee performs all the service required and the window remains continuously open from 9am to 3pm. It has been discovered that an average number of clients are 48 during the day and the average service time is 5mins / person. Then the following can be known, the utilization factor, average number of clients in the system, average waiting time etc. Average number of client (arrival) = 48 Average service time per person = 5 minutes Time (9am to 3pm) = 6 hours
Implies that the mean arrival rate $\lambda = \frac{48}{6} = 8$ client/hour
and mean service that $\mu = \frac{60}{5} = 12$ clients/hour
The utilization factor p = $\frac{\lambda}{\mu} = \frac{8}{12} = 0.67$
It implies $\lambda = 8$ clients/hour and $\mu = 12$ clients/hour

- Number of customers in average found in the system

$Ls = \frac{p}{1-p}$ where $p = \frac{\lambda}{\mu} = 0.67$

$Ls = \frac{0.67}{1-0.67} = 2.03$ or 2 client

- Number of customers in average found in the queue

$Lq = Ls - \frac{\lambda}{\mu}$

$Lq = 2.03 - 0.67 = 1.36$ client

- Average waiting time in the system

$Ws = \frac{Ls}{\lambda} = \frac{2.03}{8} = 0.25$/hour

- Average waiting time in the queue.

$Wq = \frac{Lq}{\lambda} = \frac{1.36}{8} = 0.17$

Probability of zero, one and two persons in the queue is

$p_0 = 1 - p$ where $p = \frac{\lambda}{\mu}$

$p_1 = (\frac{\lambda}{\mu})^1 p_0$

$p_2 = (\frac{\lambda}{\mu})^2 p_0$ ...

$p_n = (\frac{\lambda}{\mu})^n p_0$

$P_n = (1 - \frac{\lambda}{\mu})(\frac{\lambda}{\mu})^n$

Where n = (0,1,2 ...)

$p_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{8}{12} = 0.33$

$p_1 = (\frac{\lambda}{\mu})^1 p_0 = (\frac{8}{12})^1(0.33) = (0.67)^1(0.33) = 0.2211$

$p_2 = (\frac{\lambda}{\mu})^2 p_0 = (\frac{8}{12})^2(0.33) = (0.67)^2(0.33) = 0.15$

3.8.3 M/M/s Model

M/M/s queue model have similar description/characteristics as classic M/M/1 model of queue but the differences have been the number of service facilities in each of them, whiles M/M/s deals with many servers/service facilities when modeling, M/M/1 has to deal with only one service facility for same modeling process. In the system, the number of customers found in the queue at time 't' x(t) of M/M/s can be modeled as a Markov Continuous Chain.

ρ(Rho) which is refer as service utilization factor has a condition stability label as $\rho = \frac{\lambda}{s\mu}$, which indicate the average proportion time for how busy each server is.The total service rate should surpass the arrival rate, which is $s\mu > \lambda$, and if $s\mu \leq \lambda$ the queue eventually grows infinitely large.

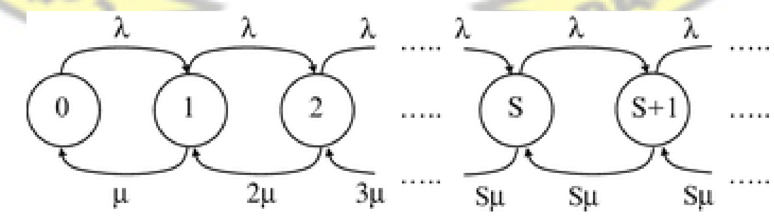

Figure 3.9: State diagram for multiple queuing

Thus, Expected rate in (arrivals) = Expected rate out (departures) principle, thus $\lambda_s = \lambda$. Servers/service facilities will be idle only when the number of customers joining or in the queue is less than the operational servers/service facilities denoted as s < c, here the service rate is represented as μ_s = sμ. In a case where the number of people "s" is more than number of servers "c" s > c. What it means is the service facilities will be busy and can be express as μ_s = cμ

The general expression.

$$p_s = p^s p_0 = \frac{\lambda^s}{\mu^s} P_0$$

When s<c ie the number of people "s" being less than number of servers "c p_s can be express as

$$p_s = (\frac{\lambda^s}{\mu, 2\mu, ... s\mu}) p_0 = (\frac{\lambda}{\mu})^s \frac{1}{s!} p_0$$

When s> c ie the number of people "s" being more than number of servers "c" p_s can be express as

Let say they are 4 servers and infinite queue length.

$$p_s = (\frac{\lambda^s}{\mu, 2\mu, 3\mu, 4\mu, 4\mu, 4\mu ... \infty}) p_0 = (\frac{\lambda^s}{\mu^s c! c^{s-c}}) p_0$$

Probability that no one is in the queue ie p_0.

$$p_0 + p = 1$$

$$p_0 + p^1 p_0 + p^2 p_0 + p^3 p_0 + ... + \infty = 1$$

$$p_0 [1 + p^1 + p^2 + p^3 + ... + \infty] = 1$$

$\sum_{s=0}^{c-1}\frac{p^s}{s!}p_0 + \sum_{s=c}^{\infty}\frac{p^s}{c!cs-c}p_0 = 1$

$p_0[\sum_{n=0}^{c-1}\frac{p^n}{s!} + \sum_{s=c}^{\infty}\frac{p^c}{c!}\frac{p^{s-c}}{c^{s-c}}] = 1$

$p_0[\sum_{s=0}^{c-1}\frac{p^s}{s!} + \frac{p^c}{c!}(1 + \frac{p}{c} + (\frac{p}{c})^2 + ...\infty)] = 1$

with geometric series

$(1 + \frac{p}{c} + (\frac{p}{c})^2 + ...\infty) = \frac{1}{1-\frac{p}{c}}$

$p_0 = [\sum_{s=0}^{c-1}\frac{p^s}{s!} + \frac{p^c}{c!}\frac{1}{1-\frac{p}{c}}] = 1$

$p_0 = \frac{1}{\sum_{s=0}^{c-1}\frac{p^s}{s!}+\frac{p^c}{c!}\frac{1}{1-\frac{p}{c}}}$

$\therefore p_s = pp_0 = (\frac{\lambda^s}{\mu^s c!c^{s-c}})\ (\frac{1}{\sum_{s=0}^{c-1}\frac{p^s}{s!}+\frac{p^c}{c!}\frac{1}{1-\frac{p}{c}}})$

Where s = 1,2,3 ...∞

The average length of the queue can be express as (Lq):

$= \sum_{s=c}^{\infty}(s - c)p_s = \sum_{s=0}^{\infty}sp_c + s$

$\sum_{s=0}^{\infty}\frac{sp^{c+s}}{c!c^s}p_0 = \sum\frac{p^{c+1}p_0}{c!c}sp^{s-1}$

$\frac{p^{c+1}p_0}{c!c}\sum\frac{d}{d(\frac{p}{c})}(\frac{p}{c})^s$

$\frac{p^{c+1}p_0}{c!c}\frac{d}{d(\frac{p}{c})}\sum(\frac{p}{c})^s$

using infinite geometry series in the above

$$\frac{p^{c+1}p_0}{c!c}\frac{d}{d(\frac{p}{c})}\frac{1}{1-(\frac{p}{c})}$$

differentiate the above

$$\text{Lq} = \frac{p^{c+1}p_0}{c!c}\frac{1}{1-(\frac{p}{c})^2} = \frac{p^{c+1}p_0}{(c-1)!(c-p)^2}$$

3.8.4 Illustrative Example

In a certain bank there are three(3) tellers. The bank was open from 9am to 3pm. It has been discovered that an average number of clients are 48 during the day and the average service time is 5mins/person.The the following can be known, the utilization factor. Number of client(arrival) = 48. Service time person = 5 minutes. Time range(9am to 3pm) = 6 hours. Number of servers/Tellers (c) = 3

- Mean arrival rate $\lambda = \frac{48}{6} = 8$ clients/hour

- Mean service rate $\mu = \frac{60}{5} = 12$ clients/hour

- The utilization factor p $= \frac{\lambda}{c\mu} = \frac{8}{3(12)} = 0.22$/hour or 22%

- probability of no one in the system(p_0) $= \frac{1}{[\sum_0^{c-1}\frac{p^s}{s!}+\frac{p^c}{c!}\frac{1}{(1-\frac{p}{c})}]}$

  $$= \frac{1}{1+\frac{0.4444}{2}+\frac{0.2962}{6}\frac{1}{1-0.22}} = 0.7779$$

- Average length of the queue (Lq) $= \frac{p^{c+1}p_0}{(c-1)!(c-p)^2} = \frac{(0.1975)(0.7779)}{(2)(3-0.6667^2)} = 0.030$

- Average length of the system (Ls)= Lq + p = 0.030 + 0.6667 = 0.6967

- Average waiting time in the queue (Wq) $= \frac{Lq}{\lambda} = \frac{0.030}{8} = 0.00375$min

Chapter 4

Data Collection And Analysis

This chapter looks at data collection and analysis during the period of the study. Calculation of various performance parameters discussed in chapter three are presented in this chapter. Tables would be used to show the data collected and the findings used to answer the research objectives that were formulated in Chapter one.

The queuing model used in the analysis is M/M/s which involves a single-line with multiple servers in the system.

Assumptions

1 The calling population size is infinite. This assumption implies that input sources are unlimited.

2 The arrival rate of customers follows or is characterize by Poisson distribution.

3 No Balking in the queue. Means we preempt that, customers who join the queue stays.

4 No reneging in the queue. The assumption here is that customers stay in the queue till services are provided.

5 The queuing discipline used here is first come, first served (FCFS). Which means the first customer in the waiting line receive the first service.

6 Customers continue to join the queue at any given time without restrictions ie the queue length is infinite.

7 The system service time is characterize by exponential distribution.

8 The system service rate is assume to be greater than its arrival rate (i.e., N> 2).

4.1 Source Of Data Collection

Data for the study was duly collected from primary source, which was gathered from GCB Obuasi at different days and time respectively. Data was collated randomly without a particular order. An hour each day and seven working days selected randomly. The concentration was on both period where we have many customers visiting the bank (peak days) and period where we have few customers visiting the bank (off peak days) which involve customers arrival and service time respectively. The method of Data collection; customers were observed as they come into the bank to withdraw or deposit monies. As we observed, we also recorded with the help of materials such as notepad, wrist watch and pen; information picked up also include 1. Arrival time of each customer 2. Number of customers 3. Waiting time of each customer in the queue 4. Service time of each customer, with the help of one research assistance. One(1) hour or sixty(60) minute during the days of 28th and 29th of June, 2018. 2nd, 9th, 16th, 18th and 25th in the month of July, 2018 were used to collect the data or say spent at the bank collecting the needed data for onward computations. Samples of data collated were recorded and placed accordingly. Summary of data collected are indicated below.

Table 4.1: **Multiple Channel Queue Situation in Obuasi Branch of Ghana Commercial Bank**

DATE	TIME RANGE	NO. OF TELLER	NO. OF ARRIVALS	NO. SERVED
28TH JUNE	9:30am-10:30am	3	113	38
29TH JUNE	1:00pm-2:00pm	3	98	33
2ND JULY	8:30am-9:30am	3	103	36
9TH JULY	2:15pm-3:15pm	3	100	36
16TH JULY	11:00am-12:00pm	3	33	17
18TH JULY	9:00am-10:00am	3	17	12
25TH JULY	8:30am-9:30am	3	88	30

The table above summarizes all data collected.The collection of data was done within an hour or sixty minute. During the peak days, that is on the 28th, 29th of June and 2nd of July saw many customers arrived at the banking hall. Due to the payment of salaries and other transactions around these time of the month, a lot of customers visited the bank to collect their salaries and redeposit them in their saves account, also parents/guidance who are salaried workers also deposit monies into schools accounts as their wards school fees. Consequently, pressure is exerted on the network servers culminating the delay of services due to the unstable nature of the network. Customers therefore, complain bitterly about the service operations of the bank during these peak periods. On the other hand, there is less pressure on the network during off peak days due to minimal arrival of customers into the banking hall. Hence network problems were rarely experienced during these periods.

4.2 Method of Analysis

The method used for the analysis is multi-server method of modeling queues and the type of discipline; First-come First-served denoted as (M/M/S): (∞/FCFS). Here the performance analysis for the study was computed, these includes arrival time of each customer, server utilization, time waited in the queue, time spent during service etc using relevant tools. The tabular representation of the computed performance analysis values were done. In chapter five analyses of the results were done with the help of queuing theory analysis model.

4.3 Parameters From Data

For the analysis, an hour(01:00:00) not minute will be taking as 01:00, in order to have simplified average computations. The rate of customers arriving at the bank fluctuate throughout the day and there might be differences in arrival from day to day or hour to hour but the assumption is that they are independent and similarly distributed.

4.3.1 From Table 4.1,Computation On The 28th Of June Data

In the single channel queue situation, there is only one service point, while in this case, more than one server is available. The number of customers entering the bank within 9:30am-10-30am (Arrival rate) =113

The number of customers served within 9:30am - 10:30am (service rate) = 38

The number of tellers = 3

Total time spent in hours = 1 hr

Mean arrival rate $(\lambda) = \frac{\text{arrival rate}}{\text{total time in hours}} = \frac{113}{1} = 113$ customers/hour

Mean service rate $(\mu) = \frac{\text{service rate}}{\text{total time in hours}} = \frac{38}{1} = 38$ customers/hour

Table 4.2: **Summary for mean arrival rate and mean service rate for data collected**

DATE	N0. OF SERVERS	TOTAL TIME	ARRIVAL RATE	SERVICE RATE
28TH JUNE	3	1	113	38
29TH JUNE	3	1	98	33
2ND JULY	3	1	103	36
9TH JULY	3	1	100	36
16TH JULY	3	1	33	17
18TH JULY	3	1	17	12
25TH JULY	3	1	88	30

4.4 Results(Performance Measure)

Computation for the data of 28th June, 2018. The computations and outcomes for other days follow similarly, calculations will be done manually or using queuing solver.

- Rho(Utilization factor) for 28th June is given by. $\rho = \frac{\lambda}{3\mu} = \frac{113}{3*38} = 0.9912 \approx$ 99.12%

- The probability of system being idle(ie there are no customers waiting in the queue).

$$p_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+4.4214+4.3826(113.6364)} = 0.00199 \approx 0.20\%$$

- The length of the queue (L_q)

$$L_q = \frac{p^{c+1} p_0}{(c-1)!(c-p)^2} = \frac{(78.1951)(0.00199)}{(2)(0.0007)} = 111.1487 \approx 111 \text{ customers}$$

- Length of system (L_s)

$$= L_q + \frac{\lambda}{\mu} = 111.1487 + 2.9737 = 114.1224 \approx 114 \text{ customers}$$

- Average time customer spent in the queue before receiving service(W_q).

$$W_q = \frac{L_q}{\lambda}\ \frac{111.1487}{113} = 0.9836 \text{ hours or 59 minutes}$$

- Average time customer spent in the system (W_s).

$$W_s = W_q + \frac{1}{\lambda} = 0.9836 + \frac{1}{113} = 1.5146 \text{ hours or 1hr, 31 minutes}$$

The results show that the server would be busy 0.9912 or 99.12% of the time and idle 0.00199 or 0.20% . Also, total number of customers in the queue is 111 and total number customer in the system on average is 114. Moreover, time customer had to spent in the queue is 0.9836 hours or 59 minutes and time spent in the system is 1.5146 hours or 1hr, 31 minutes.

4.4.1 Computation On 29th of June Data

The number of customers entering the bank between 1:00pm-2:00pm (Arrival rate) = 98

The number of customers served between 1:00pm-2:00pm(Service rate) = 33

The number of tellers = 3

Total time spent in hours = 1hr

Mean arrival rate(λ)= $\frac{\text{arrival rate}}{\text{total time in hours}} = \frac{98}{1}$= 98 customers/hr

Mean service rate (μ)= $\frac{\text{service rate}}{\text{total time in hours}} = \frac{33}{1} = 33$

Performance Measures

Results for the performance measures are shown below for the data of 29th June, 2018

- Utilization factor for 29th June, is $\rho = \frac{\lambda}{3\mu} = \frac{98}{3*33} = 0.9899 \approx 98.99\%$

- Probability that there is no one found in the system(Idle)

$$P_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+4.4096+4.3650(99.0099)} = 0.0023 \approx 0.23\%$$

- Length of the queue(Lq)

$$= \frac{p^{c+1} p_0}{(c-1)!(c-p)^2} = \frac{(77.7765)(0.0023)}{(2)(0.0009)} = 99.3889 \approx 99 \text{ customers}$$

- Length of the system(Ls)

$$= Lq + \frac{\lambda}{\mu} = 99.3889 + 2.9697 = 102.3586 \approx 102 \text{ customers.}$$

- Average time a customer spend in the queue waiting for his/her turn(Wq)

$$= \frac{Lq}{\lambda} = \frac{99.3889}{98} = 1.0142 \text{ hr}$$

- Average time a customer spend waiting in the system(Ws)

$$= Wq + \frac{1}{\lambda} = 1.0142 + 0.0102 = 1.0244 \text{ hr}$$

The results in the above shows that on 29th June per the hour observed, the server would be busy 0.9899 or 98.99% of the time and zero number of customers or no any customer present in the system 0.0023 or 0.23% approximately. Also the average number of customer present or waiting in the queue that afternoon is 99 customers and those in the queue plus those receiving service add up to 102 customers. To add, the period spend in the waiting line before he/she the customer receive services is 1.0142 hr and the total duration he/ she has to spend before leaving the bank is 1.0244 hr .

4.4.2 Computation On 2nd of July Data

The results collated and computed are shown below for 2nd of July, 2018.

The number of customers entering the bank between an hour 8:30am-9:30am(Arrival rate) = 103

The number of customers served between 8:30am-9:30am(Service rate)= 36

The number of tellers = 3

Total time spent in hours = 1hr

Mean arrival rate(λ) = $\frac{\text{arrival rate}}{\text{total time in hours}} = \frac{103}{1} = 103$ customers/hr

Mean service rate(μ) = $\frac{\text{service rate}}{\text{total time in hours}} = \frac{36}{1} = 36$customers/hr

Performance Measures

The results for the performance measures are shown below for the data of 2nd July, 2018.

- Utilization factor for 2nd July, is

$$\rho = \frac{\lambda}{3\mu} = \frac{103}{3*36} = 0.9537 \approx 95.37\%$$

- The probability that the system is empty or no one is in the bank(Idle)

$$P_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+4.09298+3.9035(21.5983)} = 0.0112 \approx 1.12\%$$

- Length of the queue(Lq)

$$= \frac{p^{c+1} p_0}{(c-1)!(c-p)^2} = \frac{(67.0099)(0.0112)}{(2)(0.0193)} = 19.4433 \approx 19 \text{ customers}$$

- Length of the system(Ls)

$$Lq + \frac{\lambda}{\mu} = 19.4433 + 2.8611 = 22.3044 \approx 22 \text{ customers}$$

- Amount of time customer spent in the queue(Averagely)before service is provided(Wq)

$$\frac{Lq}{\lambda} = \frac{19.4433}{103} = 0.1888\text{hr or } 11 \text{ minute}$$

- Amount of time customer spent in the system in an average(Ws)

$$Wq + \frac{1}{\lambda} = 0.1888 + 0.0097 = 0.1985\text{hr or } 12 \text{ minute.}$$

The permutation above indicate that, the server was busy 0.9537 or 95.37% and can only be empty or idle 0.0112 or 1.12%.It further indicate that, the length of the queue or average number of customers waiting in the queue to be served as at 1:00pm-2:00pm is 19 customers and those waiting in the queue plus those receiving service sum up to 22 customers. Moreover, a customer at that period in the bank had to spend 0.1888hr or 11 minute before he/she gets to a teller for service and to leave the bank he/she had to spend or wait for 0.1985hr or 12 minute before departing, unless he/she had something different to transact at the bank, a part from depositing money.

4.4.3 Computation On 9th of July Data

On 9th of July the researcher visited the bank, took some data and computed it as follows.

The number of customers arriving at the bank between an hour of 2:15pm-3:15pm(Arrival rate) = 100

Those who were served between the hour 2:15pm-3:15pm(Service rate)= 36 customers

The number of active servers for the period = 3

Total time spent in the bank to collect the data = 1hr

Mean arrival rate(λ) = $\frac{\text{arrival rate}}{\text{total time in hours}} = \frac{100}{1} = 100$ customers/hr

Mean service rate(μ) = $\frac{\text{service rate}}{\text{total time in hours}} = \frac{36}{1} = 36$ customers/hr

Performance Measures

The results for the performance measures are shown below for the data of 9th July, 2018.

- Utilization factor for 9th July is given by

$$\rho = \frac{\lambda}{3\mu} = \frac{100}{3*36} = 0.9259 \approx 92.59\%$$

- The probability that the system is empty(Idle).

$$P_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+3.8580+3.5722(13.4953)} = 0.0188 \approx 1.88\%$$

- Length of the queue(Lq)

$$= \frac{p^{c+1} p_0}{(c-1)!(c-p)^2} = \frac{(59.5374)(0.0188)}{(2)(0.0494)} = 11.3289 \approx 11 \text{ customers}$$

- Length of the system(Ls)

$$\text{Lq} + \frac{\lambda}{\mu} = 11.3289 + 2.7778 = 14.1067 \approx 14 \text{ customers}$$

- Average time a customer had to wait in the queue(Wq)

$$\frac{Lq}{\lambda} = \frac{11.3289}{100} = 0.1133\text{hr or 7 minutes}$$

- Average time a customer had to wait in the system(Ws)

$$\text{Wq} + \frac{1}{\lambda} = 0.1133 + 0.01 = 0.1233\text{hr or 7 minutes}$$

From the above computations, the serve was busy 0.9259 or 92.59% and can only be empty or idle 0.0188 or 1.88%.The number of customers in the queue is 11. The number of customers in the system, those being served inclusive is 14 customers. The time a customer is expected to spend before he/she receive service is 0.1133 hours or 7 minutes and in the system, a customer is expected to spend 0.0.1233 hours or 7 minutes.

4.4.4 Computation On 16th of July Data

The number of customers entering the bank within 11:00am-12:00 noon(Arrival rate) = 33

The number of customers served within 11:00am-12:00 noon(Service rate) = 17

The number of tellers = 3

Total time spend in hours = 1hr

Mean arrival rate(λ) = $\frac{\text{arrival rate}}{\text{total time in hours}} = \frac{33}{1} = 33$

Mean service rate(μ) = $\frac{\text{service rate}}{\text{total time in hours}} = \frac{17}{1} = 17$

Performance Measures

The results for the performance measures are shown below for the data of 16th July, 2018.

- Utilization factor for 16th July is given by:

$$\rho = \frac{\lambda}{3\mu} = \frac{33}{3{*}17} = 0.6471 \approx 64.71\%$$

- The probability that no one is in the queue(Empty)

$$\mathrm{P}_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+1.8841+1.2191(2.8337)} = 0.1578 \approx 15.78\%$$

- Total customers in average waiting in the queue(Lq)

$$= \frac{p^{c+1} p_0}{(c-1)!(c-p)^2} = \frac{(14.1991)(0.1578)}{(2)(1.1211)} = 0.9993 \approx 1 \text{ customers}$$

- Average number of customers waiting in the system(Ls)

$$Lq + \frac{\lambda}{\mu} = 0.9993 + 1.9412 = 2.9405 \approx 3 \text{ customers}$$

- Average waiting time customer had to spend in the queue(Wq)

$$\frac{Lq}{\lambda} = \frac{0.9993}{33} = 0.0303\text{hr}$$

- Average waiting time customer had to spend in the system(Ws)

$$Wq + \frac{1}{\lambda} = 0.0303 = 0.0303 = 0.0606\text{hr}$$

The permutation above indicate that, the server was busy 0.6471 or 64.71% and empty or idle 0.1578 or 15.78%. It further indicate that,the average number of customer waiting in the queue to be served as at 11:00am-12:00noon is 0.9993 or 1 customer and those waiting in the queue plus those receiving service sum up to 2.9405 or 3 customers. Moreover, a customer at that period in the bank had to spend 0.0303 hours before he/she gets to a teller for service and to leave the bank he/she had to spend 0.0606 hours before departing the premises, unless the customer had something deferent to transact in the bank.

4.4.5 Computation On 18th of July Data

On 18th of July the researcher visited the bank again, took some data and computed it as follows:
The number of customers arriving at the bank between an hour of 9:00am-10:00am(Arrival rate) = 17
Those who were served within the hour 9;00am-10:00am(Service rate) = 12

The number of active servers within the period = 3

Total time spend in the bank to collect the data = 1hr

Mean arrival rate(λ) = $\frac{\text{arrival rate}}{\text{total time in hours}} = \frac{17}{1} = 17$ customers/hr

Mean service rate(μ) = $\frac{\text{service rate}}{\text{total time in hours}} = \frac{12}{1} = 12$ customers/hr

Performance Measures

The results for the performance measures are shown below for the data 18th July, 2018.

- Utilization factor for 18th July is:

$$\rho = \frac{\lambda}{3\mu} = \frac{17}{3*12} = 0.4722 \approx 47.22\%$$

- The probability that the system is idle(Empty)

$$\text{P}_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+1.0035+0.4739(1.8947)} = 0.3447 \approx 34.47\%$$

- Total customers in average waiting in the queue(Lq)

$$\frac{p^{c+1}p_0}{(c-1)!(c-p)^2} = \frac{(4.0278)(0.3447)}{(2)(2.5069)} = 0.2769 \approx 0 \text{ customers}$$

- Average number of customers waiting in the system(Ls)

$$\text{Lq} + \frac{\lambda}{\mu} = 0.2769 + 1.4167 = 1.6936 \approx 2 \text{ customers}$$

- Average waiting time customer had to spend in the queue(Wq)

$$\frac{Lq}{\lambda} = \frac{0.2769}{17} = 0.0163 \text{ hours}$$

- Average waiting time customer had to spend in the system(Ws)

$$\text{Wq} + \frac{1}{\lambda} = 0.0163 + 0.0588 = 0.0751 \text{ hours}$$

From the above results, the server was busy 0.4722 or 47.22% and probability that, the system is empty or idle is 0.3447 or 34.47%. There is no customer in the queue and the number of customers in the system is 2. The time a customer is expected to wait before he/she receive service is 0.0163 hours and in the system, a customer is expected to spend 0.0751 hours.

4.4.6 Computation On 25th July Data

The results collated and computed are shown below for 25th of July, 2018.

The number of customers entering the bank within the hour 8:30am-9:30am(Arrival rate) = 88

The number of customers served within 8:30am-9:30am(Service rate) = 30

The number of tellers = 3

Total time spent in hour = 1hr

Mean arrival rate(λ) = $\frac{\text{arrival rate}}{\text{total time in hours}} = \frac{88}{1} = 88$ customers/hr

Mean service rate(μ) = $\frac{\text{service rate}}{\text{total time in hours}} = \frac{30}{1} = 30$

Performance Measures

The results for the performance measures are shown below for the data of 25th July, 2018.

- Utilization factor for 25th July,is given as

$$\rho = \frac{\lambda}{3\mu} = \frac{88}{3*30} = 0.9778 \approx 97.78\%$$

- The probability that, the system is empty or no one is in the bank(Idle)

$$\mathrm{P}_0 = \frac{1}{[\sum_0^{c-1} \frac{p^s}{s!} + \frac{p^c}{c!} \frac{1}{(1-\frac{p}{c})}]} = \frac{1}{1+4.3022+4.2066(45.0450)} = 0.0051 \approx 0.51\%$$

- Total customers waiting in the queue(Lq)

$$= \frac{p^{c+1} p_0}{(c-1)!(c-p)^2} = \frac{(74.0365)(0.0051)}{(2)(0.0044)} = 42.9075 \approx 43 \text{ customers}$$

- Total customers waiting in the system.(Ls)

$$\mathrm{Lq} + \frac{\lambda}{\mu} = 42.9075 + 2.9333 = 45.8408 \approx 46 \text{ customers}$$

- Amount of time customer spend in the queue before he/she receives services(Wq)

$$\frac{Lq}{\lambda} = \frac{42.9075}{88} = 0.4876\text{hr or } 29 \text{ minute}$$

- Average waiting time a customer spend in the system(Ws)

$$\mathrm{Wq} + \frac{1}{\lambda} = 0.4876 + 0.0114 = 0.499\text{hr or } 30 \text{ minute}$$

The results shows that on 25th July per the hour observed, the server would be busy 0.9778 or 97.78% of the time and idle or no customer present in the system 0.0051 or 0.51% approximately. Also the average number of customers present or waiting in the queue that morning is 43 customers and those in the queue plus those receiving service 46 customers. Moreover, a customer is expected to spend 0.4876hr or 29 minute in the waiting line and in the system customer spent 0.499hr or 30 minute.

4.4.7 Results Summary

From Table 4.1 data was collected,computed and analyzed using queuing analytical model.The results below summarizes all what was being down above showing the dates in which data was collected, inputs, intermediate calculation and performance measures.

Table 4.3: **Collated Data, Calculations And Results, Ghana Commercial Bank- Obuasi Branch**

DATE	28TH JUNE	29TH JUNE	2ND JULY	9TH JULY	16TH JULY	18TH JULY	25TH JULY
Total Time(t)hrs	1	1	1	1	1	1	1
No. of Arrivals	113	98	103	100	33	17	88
No. Served	38	33	36	36	17	12	30
No. of Servers	3	3	3	3	3	3	3
Model Type	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3
Mean Arrival Time	0.0088	0.0102	0.0097	0.01	0.0303	0.0588	0.0114
Mean Service Time	0.0263	0.0303	0.0278	0.0278	0.0588	0.0833	0.0333
Server Utilization	0.9912	0.9899	0.9537	0.9259	0.6471	0.4722	0.9778
System Empty	0.0020	0.0023	0.0112	0.0188	0.1578	0.3447	0.0051
Lq	111.1487	99.3889	19.4433	11.3289	0.9993	0.2769	42.9075
Ls	114.1224	102.3586	22.3044	14.1067	2.9405	1.6936	45.8408
Wq(hrs)	0.9836	1.0142	0.1888	0.1133	0.0303	0.0163	0.4876
Ws(hrs)	1.5146	1.0244	0.1985	0.1233	0.0606	0.0751	0.49896

4.4.8 Results For Average Data

The total number of customers observed was 552, out of that 202 was served, a total of 7 hours was spent observing and 3 tellers were there doing the serving.Table 4.4 shows the intermediate calculations and performances measure.

Table 4.4: **Average Data Calculation And Their Results**

INPUTS	Values
Total Time Involved	7hours
Total Number of Customers Arrived	552
Total Number of Customers Served	202
Total Number of Servers	3
Model Type	M/M/3
Intermediate	
Mean Arrival Time	0.0018
Mean Service Time	0.0056
Performance Measure	
Rho(Average Server Utilization)	0.9109
Probability of Empty Queue(p_0)	0.0233
Average Length of the Queue(Lq)	9.0987
Average Length of the System(Ls)	11.8314
Average Waiting Time in the Queue(Wq)	0.0165
Average Waiting Time in the System(Ws)	0.0183

The results in Table 4.4 shows that, tellers would be busy $0.9109 \approx 91.09\%$ of the time and system idle $0.0233 \approx 2.33\%$. Also, total customers in average waiting in the queue is 9 and total customers in average waiting in the system is 12. Moreover, the average time a customer spends in the queue is 0.0165 hours and average time a customer spend in the system is 0.0183 hours. The data analysis is as follows:

• Utilization factor helps to determines how the tellers or servers are busy as shown in Table 4.5 .

Table 4.5: **Rho(Utilization Factors For Each Day)**

DATE	AVERAGE SERVER UTILIZATION(Rho)
28TH JUNE	0.9912
29TH JUNE	0.9899
2ND JULY	0.9537
9TH JULY	0.9259
16TH JULY	0.6471
18TH JULY	0.4722
25TH JULY	0.9778

From Table 4.5, the busiest of all the days is 28th of June. Its utilization factor is 99.12% followed by 29th June with a utilization factor of 98.99%. 18th of July recorded the least of 47.22%. On the average, the utilization factor for all the days was 91.09% from Table 4.4. The days with higher utilization factor recorded a high turnout of customers and vice versa.

- Probability of the system being empty or idle(Idle Facility which come with cost)

Table 4.6: **Probability of System Being Empty(Idle)**

DATE	PROBABILITY OF SYSTEM EMPTY(IDLE)
28TH JUNE	0.0020
29TH JUNE	0.0023
2ND JULY	0.0112
9TH JULY	0.0188
16TH JULY	0.1578
18TH JULY	0.3447
25TH JULY	0.0051

From Table 4.6, System being empty or idle implies that, no customer will be in the queue or system to be served. The faster customers are served the likely-Hood the system will become empty. On the average, the probability of the system idle is 0.0233 from Table 4.4.The highest probability for which the system will be empty is 0.3447 which occurred on the 18th July whiles on the 28th June recorded a least probability of 0.0020. It was on that day, servers were busy for a long period.

• Average number of customer in the queue(Lq).This determine number of people in the queue excluding those being served.

Table 4.7: **Average Number of Customers in the Queue(Lq)**

DATE	AVERAGE LENGTH OF THE QUEUE(Lq)
28TH JUNE	111.1487
29TH JUNE	99.3889
2ND JULY	19.4433
9TH JULY	11.3289
16TH JULY	0.9993
18TH JULY	0.2769
25TH JULY	42.905

The average length of the queue(Lq) from Table 4.4 is 9.0987 $\approx$ 9 customers. June, 28th recorded the highest number of customers turn up 111.1487 approximately 111 customers were on the waiting line and followed by 29th of June with 99.3889 approximately 99 customers as compared to 16th and 18th July with least number of customers with 0.9993 $\approx$ 1 customer and 0.2769 $\approx$ 0 customer respectively as shown in table 4.7. Managers of the bank could take a perspective of this and suggest a way forward to curb long queues and at the same time prevent the system being empty.

• Average number of customers in the system(Ls).This helps to determine number of people in the queue including those being served.

Table 4.8: **Average Number of Customers in the System(Ls)**

DATE	AVERAGE LENGTH OF THE SYSTEM
28TH JUNE	114.1224
29TH JUNE	102.3586
2ND JULY	22.3044
9TH JULY	14.1067
16TH JULY	2.9405
18TH JULY	1.6936
25TH JULY	45.8408

The average number of customers in the system from Table 4.4 is 11.8314, which means that on average 12 customers will be found banking. 28th June recorded the highest number of customers in the system being $114.1224 \approx 114$ customers, followed by 29th of June with $102.3586 \approx 102$ customers as compared to 16th July and 18th July with least number of customers with $2.9405 \approx 3$ customers and $1.6936 \approx 2$ customer respectively as shown in Table 4.8.

- Average waiting time in the queue(Wq).This help to determine time spend in the queue by customers.

Table 4.9: **Average Waiting Time in the Queue(Wq)**

DATE	AVERAGE WAITING TIME IN THE QUEUE(Wq)/hr
28TH JUNE	0.9836
29TH JUNE	1.0142
2ND JULY	0.1888
9TH JULY	0.1133
16TH JULY	0.0303
18TH JULY	0.0163
25TH JULY	0.4876

The average time customer spend in the waiting line from Table 4.4 is 0.0165 hours. which imply that on the average a customer will spend or wait that amount of time in the banking hall before being served.29th of June recorded the highest waiting time spent in the queue with 1.0142 hours and followed by 28th June with 0.9836 hours as compared to 16th July and 18th July recorded the least time spend in the queue before being served with 0.0303 and 0.0163 hours respectively as shown in Table 4.9

- Average waiting time in the system(Ws).This help to determine time spend in the system by customers.

Table 4.10: **Average Waiting Time in the System(Ws)**

DATE	AVERAGE WAITING TIME IN THE SYSTEM(Ws)/hr
28TH JUNE	1.5146
29TH JUNE	1.0244
2ND JULY	0.1985
9TH JULY	0.1233
16TH JULY	0.0606
18TH JULY	0.0751
25TH JULY	0.499

The average time a customer spent in the system from Table 4.4 is 0.0183 hours.June, 28th recorded the highest time a customer will spend in the system with 1.5146 hours and followed by 29th of June with 1.0244 as compared to 16th and 18th of July recorded the least time spent in the system with 0.0606 and 0.0751 hours respectively as shown in Table 4.10.

4.5 Performance Measure Analysis For Utilization Factor

From a managerial perspective, utilization is often seen as a measure of productivity and therefore it is considered desirable for it to be high. The entire utilization factor for the data is stable. Utilization factor determine how busy or engaged they tellers are or working; the waiting time, which is how long customers wait before being served. These two items moved hand in hand that is, as utilization factor decreases, so do the waiting time also decreases and as it increases waiting time also increases. The average utilization factor for all the hours or days is $0.9109 \approx 91.09\%$ from Table 4.4. The highest utilization factor ie the day tellers were very busy from table 4.5 is $0.9912 \approx 99.12\%$ which happens on the 28th of June, followed by $0.9899 \approx 98.99\%$ on the 29th of June and 0.9778 $\approx 97.78\%$ also on the 25th of July all happen during the peak days. We also noticed that, these higher utilization factors also recorded a higher waiting time during these periods; 1.5146 hours, 1.0244 hours and 0.499 hours respectively from Table 4.10. Considering the data collected and through observation at the bank's all, the average arriving rates are greater than the average service rate, it was also noticed that the closer the average service rate to the average arriving rate, the smaller utilization factor becomes and the least waiting time. As tellers were executing their duties, it was found out that on the 18th of July, 2018 was the day tellers were at their best when it recorded the lowest utilization factor of $0.4722 \approx 47.22\%$ with its arrival rate of 17 customers and out of that 12 customers were served. On the 28th of June, 2018 recorded the highest capacity utilization factor of $0.9912 \approx 99.12\%$ with an arrival rate of 113 customers and out of that 38 customers were served for the period where all tellers were available per the hour the data was collected. This implies that the tellers were on their best on that day and managed to serve many customers as possible and when the number of tellers decreases, the higher the utilization factor gets closer to one and the signs are not good for both the bank and customers.If proper measures are not put in place, the probability of customers leaving to join other banks will be high.

4.6 Probability of the System Being Idle(Empty)

The probability of system being empty is characterized by how busy the tellers are engage in their duties,Table 4.6 shows the highest and lowest probability of the system being empty or idle. The utilization factor and the system being empty or idle share some relationship. The smaller the probability of the system being empty, the greater or increase capacity of utilization factor. When the system approaches one i.e. the higher probability of the system being empty, the smaller the utilization factor and the number of customers in the entire queuing system approaches zero. The highest probability for system emptiness/ idle occurred on the 18th of July which is 0.3447 and on the 28th of June recorded the lowest probability of the system being empty/idle which is 0.0020 from Table 4.6. This means, managers of the bank on 18th of July could have signed one of the tellers to transact other business rather than receiving customers who are there to deposit or save money. July 28th, adding another teller to the already existing tellers would have gone a long way to ease long queue and teller-fatigue hence reducing customer frustration and discomfort.

4.7 Performance Measures Analysis for Highest And Lowest of Customers Arrivals

Considering the highest arrival of customers 113, that was on the 28th of June and the lowest arrival of customers 17 that was on the 18th of July, for the peak and off peak days respectfully. We observe that there is a vast difference between their utilization factors; $0.9912 \approx 99.12\%$ and $0.4722 \approx 47.22\%$ and probability of system being empty or idle is 0.0020 and 0.3447 respectively. This means that, there will be 114.1224 or 114 and 1.6936 or 2 average number of customers in the system and would take 1.5146 and 0.0751 hour(s) average time to be in the system respectively.

4.8 Projection Using 28th June, 2018 Data

Now we consider one of the days in which the bank recorded the highest arrival rat, highest utilization factor and highest waiting time of customers.The researcher will like to use this day or date to study the performance analysis assuming there is one-teller, two-tellers, three-tellers, four-tellers and five-tellers then compare their results to make policy recommendation. Taking an average arrival rat, $\lambda = 113$ and service rate, $\mu = 38$ on 28th June for this analysis. The Table 4.11 presents the results for considering one to five tellers at a given time using the inputs obtained on the 28th June, 2018

Table 4.11: **The table Shows Results of Types of Models From One to Five Tellers**

TYPES OF MODEL	M/M1	M/M/2	M/M/3	M/M/4	M/M/5
Performance Measures					
Rho(Average Utilization Factor)	2.9737	1.4868	0.9912	0.7434	0.5947
Zero customers in the queue(Empty)	-1.9737	-0.1957	0.0020	0.0553	0.1106
Average Length of the Queue(Lq)	-4.4804	-5.4278	111.1487	2.0347	0.7761
Average Length of the System(Ls)	-1.5067	-2.4541	114.1224	5.0080	3.7498
Waiting Time in the Queue(Wq)	-0.0396	-0.0480	0.9836	0.0180	0.0069
Waiting Time in the System(Ws)	-0.0133	-0.0392	1.5146	0.0269	0.0157

From one teller to the five tellers. The inappropriateness of M/M/1 and M/M/2 model for solving customers- waiting time problems become apparent as it shows negative figures for all performance criteria in Obuasi branch of Ghana Commercial Bank per the hours collected. However, M/M/3, M/M/4, and M/M/5 models were compared and it is seen that;

- Using a four-teller system is better than a three-teller system in all the computing results shown. For instance, assuming during that morning, there were four tellers serving the customers, there would have been 5.0080 $\approx$ 5 customers waiting in the system instead of $114.1224 \approx 114$ customers and a customer would have to spend 0.0269 hours in the system instead of 1.5146 hours.

- A five-teller system has a high probability of being idle with $0.1106 \approx 11.06\%$ than four-teller, three-teller and two-teller system.

4.9 Discussion

This study is to design a system that will optimize the performance of tellers and reduce the time spent by customers when depositing or withdrawing money in the bank, by determining the following objectives;
Objective I: To Investigate the Applicability of Queuing Model in the Management of Time in Obuasi branch of GCB. Based on the analysis of this study and the theoretical frame work on queuing theory, queuing model is applicable in the management of time and cost in service industries such as commercial banks , hospitals, hotels, educational institutions, supermarkets, motor mechanic workshops, transport companies and in manufacturing firms where jobs are processed in sequence. However, it is observed from the study that most of these service industries including Obuasi branch of GCB do not apply queuing model in reality. One of the major problems associated with the non application of queuing model by commercial banks is the long queue we always see in the banks today.
Objective II: To Design A System that Minimizes the Cost of Customers Waiting and the Cost of idle Facilities. The study of queues deals with quantifying the phenomena of waiting in lines using descriptive measures of performance such as average queue lengths, average time spend in the queue, and rho or facility utilization. Examples above shows or demonstrates how these measures can be used to design a service facility. Customers and services facilities are the key actors when it comes to queuing situation. Customers are generated from a source on arrival at a service facility, services may commence immediately or customers might wait in a queue if server is busy. When a facility completes serving a cus-

tomer, it automatically "pulls" a waiting customer if any from the waiting line. If the queue is empty, the facility becomes idle until a new customer arrives. The problem associated with this situation, is the cost of customers waiting and the idle facilities. To minimize these costs, queuing model is designed as shown in model formation in the preceding chapters of this study. In designing the system, the components of a queuing system, queue discipline, queue characteristics and queue assumptions are applied as indicated in the research methodology.

Objective III: To Examine and Explain the Operating Characteristics of Queuing System and Provide Equations For Calculating their Numerical Values for Effective Decision Making. A convenient queuing theory notation for properly summarizing the characteristics of the queuing situation in Obuasi branch of GCB and the equations for calculating their numerical values is given by the following format: For Multiple channel queue:

1. The probability that customers wait for service is denoted by "P".
2. Average length of the queue is denoted by Lq.
3. Average system process time is denoted by Ws.
4. Average time a customer spend in the queue is denoted by Wq.

The equations for calculating these systems are contained in the analysis.

Chapter 5

Conclusion And Recommendation

5.1 Conclusion

The main objective of this thesis was to apply queuing theory in the management of time in GCB Obuasi with respect to depositing/withdrawing monies. The thesis therefore aims at designing a system that will optimize a state measure of performance such as the time/costs of customers waiting and cost of idle facilities. The result of queuing analysis can be used in the context of a cost optimization model, where we seek the minimization of the sum of two costs; the costs of offering the service and the cost of waiting. The main challenge in implementing queuing models is the difficulties of obtaining reliable estimates of the customers involved, particularly when human behavior is an integral part of the operation. It is determined that customers spend more time at the banking hall of GCB Obuasi branch on peak days that is 1.5146 hours or 1hr 31 minutes and off peak days customers spend a minimum of 0.0606 hours or 4 minutes. On the average a customer spends 0.0183 hours in the system . For the highest arrival rate(113), average time a customer spent in the system for three servers is 1.5146 hours and four servers is 0.0269 hours. An increased in service unit to five servers would reduce the average waiting time in the system drastically to 0.0157 hours. There should be more tellers available to help ease build up queues since the banking hall has extra teller space to avoid long queues at that period. Loss of goodwill and withdrawal of accounts on the part of customers because of long queues compensate for the cost of extra hand to be hired.

5.2 Recommendation

Sequel to the findings of this study, we recommend the following:

1. Management of any organization should know that when a customer feels frustrated because of long queue there is always some cost involved, either he/she begin to think of closing his/her account or switch banks. Which, when not solve becomes detrimental to any serious bank who means business, given the current turbulence in the financial sector in Ghana hence supervision as to what each teller does including management should be monitored.

2. Management should also be aware that in trying to increase the service facilities, cost are also involved.The greater the service facility the quicker the queue will disappear and the service capacity stays idle. For that, management should know the peak-period and off-peak period in other to be able to adjust their services accordingly ie either employing more tellers or not and also when to install new counting machine.

3. One area that also needs improvement is the Automated Teller Machine(ATM). The frequent breakdown of it especially during peak period needs to be checked and rectified, this can go a long way to augment tellers work hence, work hand in hand with tellers in reducing long queues in the bank.

In general, queuing theory is worth studying to enable the business executives determine and install the optimum service facilities so that the overall service cost is minimized.

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Appendices

DATA SOURCE: GHANA COMMERCIAL BANK-OBUASI BRANCH

QUEUING SYSTEM: MULTIPLE SERVER

DATE: 28TH JUNE, 2018

TIME: 9:30AM-10:30AM

A 1.0 Samples of Data Collected

NO. OF CUST.	ARRIVAL IN MINUTES	TELLER 1	TELLER 2	TELLER 3
1	0:00:56	0:09:00		
2	0:02:23		0:14:01	
3	0:06:12			0:10:02
4	0:07:56	0:20:08		
5	0:09:05			0:10:04
6	0:10:29		0:23:00	
7	0:12:34	0:2:09		
8	0:12:34			0:03:02
9	0:12:34		0:04:22	
10	0:12:34			0:03:58
11	0:12:42		0:05:00	
12	0:13:55		0:11:02	
13	0:14:42	0:16:01		
14	0:14:51			0:12:00
15	0:15:42	0:08:09		
16	0:18:25		0:08:40	
17	0:18:43			0:06:20
18	0:18:47	0:02:56		
19	0:18:58	0:04:03		
20	0:21:46			0:06:43
21	0:24:19		0:11:00	
22	0:24:37	0:00:59		
23	0:25:43			0:20:03
24	0:28:07		0:13:12	
25	0:28:15	0:16:48		
26	0:31:02			0:25:00
27	0:31:03	0:21:05		
28	0:31:40			0:19:23
29	0:33:55		0:23:13	
30	0:34:17	0:13:14		
31	0:35:41		0:07:20	
32	0:36:31			0:14:09
33	0:36:56	0:18:05		
34	0:38:00		0:17:12	
35	0:38:00		0:20:00	
36	0:38:00			0:16:29
37	0:38:00	0:15:00		
38	0:39:16			0:19:43
39	0:40:30		0:06:06	

NO. OF CUST	ARRIVAL IN MINUTE	TELLER 1	TELLER 2	TELLER 3
40	0:41:22	0:15:26		
41	0:41:22		0:23:00	
42	0:41:22			0:12:11
43	0:41:49		0:17:13	
44	0:43:11	0:18:09		
45	0:43:18			0:21:00
46	0:45:01		0:19:16	
47	0:45:10	0:15:20		
48	0:45:21		0:18:00	
49	0:45:20			0:13:12
50	0:46:00		0:17:15	
51	0;46:02	0:20:09		
52	0:46:02			0:21:45
53	0:47:55		0:17:37	
54	0:49:00	0:26:00		
55	0:49:01			00:05:14
56	0:49:43		0:03:34	
57	0:50:57	0:08:13		
58	0:50:56			0:12:07
59	0:51:32		0:09:14	
60	0:52:12	0:09:07		
61	0:52:47	0:20:12		
62	0:54:10		0:15:23	
63	0:54:37			0:17:00
64	0:54:50	0:13:23		
65	0:55:15			0:19:00
66	0:58:02		0:21:00	
67	0:58:49			0:02:56

DATE: 2ND JULY, 2018

TIME: 8:30AM-9:30AM

A.2.0 Sample of Data Collected

NO. OF CUST.	ARRIVAL IN MINUTES	TELLER 1	TELLER 2	TELLER 3
1	0:00:57	0:10:04		
2	0:00:57		0:07:	
3	0:00:57			0:12:00
4	0:03:05	0:16:01		
5	0:04:10			0:13:12
6	0:04:10		0:04:56	
7	0:05:56	0:18:20		
8	0:06:12		0:20:00	
9	0:06:20			0:15:13
10	0:07:00	0:15:00		
11	0:07:00	0:14:27		
12	0:07:06			0:07:27
13	0:07:58	0:17:02		
14	0:08:30			0:09:16
15	0:11:12	0:04:00		
16	0:11:17		0:10:20	
17	0:11:45			0:10:12
18	0:15:03			0:13:56
19	0:15:03		0:27:06	
20	0:15:10	0:23:12		
21	0:15:16			0:15:20
22	0:16:54			0:15:02

NO. OF CUST.	ARRIVAL IN MINUTES	TELLER 1	TELLER 2	TELLER 3
23	0:16:55	0:20:00		
24	0:16:55		0:07:32	
25	0:18:01			0:09:12
26	0:18:48			0:10:29
27	0:19:14	0:12:00		
28	0:20:12		0:03:20	
29	0:20:20			0:05:11
30	0:21:10	0:05:19		
31	0:21:30		0:09:50	
32	0:21:50			0:13:00
33	0:21:56	0:20:00		
34	0:24:01		0:18:03	
35	0:24:01			0:15:09
36	0:24:01	0:15:23		
37	0:24:30		0:14:15	
38	0:25:02			0:10:20
39	0:25:02	0:13:00		
40	0:25:05		0:16:20	
41	0:25:10			0:17:30
42	0:25:10	0:20:00		
43	0:25:10		0:18:20	
44	0:25:17			0:17:14
45	0:25:17		0:21:14	
46	0:25:43	0:21:36		
47	0:26:43		0:25:24	
48	0:26:45			0:19:19
49	0:28:57	0:12:28		
50	0:29:00		0:19:56	
51	0:29:00			0:20:17
52	0:29:00	0:08:55		
53	0:29:00		0:15:20	
54	0:31:23			0:21:09
55	0:31:31	0:13:45		
56	0:31:50		0:18:14	
57	0:33:19	0:16:48		
58	0:34:20			0:17:00
59	0:34:40		0:07:47	
60	0:34:52			0:16:37
61	0:35:03		0:21:00	

NO. OF CUST.	ARRIVAL IN MINUTES	TELLER 1	TELLER 2	TELLER 3
62	0:35:06	0:14:22		
63	0:35:22		0:17:23	
64	0:37:55			0:18:14
65	0:38:01	0:22:09		
66	0:38:01		0:16:00	
67	0:38:01			0:17:34
68	0:38:01	0:05:40		
69	0:38:20		0:20:07	
70	0:38:36			0:19:20
71	0:38:40	0:09:32		
72	0:38:40		0:34:08	
73	0:38:40	0:03:02		
74	0:38:40		0:08:00	
75	0:38:57			0:12:00
76	0:49:01			0:14:07
77	0:49:01		0:07:00	
78	0:49:01	0:10:11		
79	0:49:01	0:08:40		
80	0:49:01			0:20:00
81	0:49:01		0:13:00	
82	0:49:01	0:19:00		
83	0:49:44		0:21:09	
84	0:50:38			0:11:00
85	0:50:38	0:12:12		
86	0:51:53		0:07:08	
87	0:53:54			0:09:20
88	0:56:15	0:10:56		
89	0:56:20		0:19:20	
90	0:56:37			0:12:00
91	0:56:57	0:22:00		
92	0:55:01		0:14:45	
93	0:55:29			0:23:00
94	0:57:45	0:08:10		
95	0:58:04		0:06:34	
96	0:58:11			0:05:34
97	0:58:12	0:06:45		
98	0:58:12		0:15:00	
99	0:58:12			0:23:00
100	0:57:03	0:13:45		

NO. OF CUST.	ARRIVAL IN MINUTES	TELLER 1	TELLER 2	TELLER 3
101	0:57:10	0:10:20		
102	0:57:20		0:09:56	
103	0:59:00			0:20:34