KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

A TIME OPTIMAL VEHICLE ROUTING MODEL FOR DISASTER RELIEF DELIVERY

By

Kofi Agyei Adane

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Declaration

I hereby declare that this submission is my own work towards the award of the M.Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

Kofi Agyei Adane	....	....
Student	Signature	Date

Certified by:

Professor F. T. Oduro	....	....
Supervisor	Signature	Date

Certified by:

Professor A. O. Adebanji	....	....
Head of Department	Signature	Date

Dedication

This work is dedicated to my parents, for their heart-warming affection showered upon me since the day of my birth.

Abstract

In linear programming, the transportation problem is one of the most important and successful applications of quantitative analysis for solving business problems in products distribution. It has also proved to be a handful tool in the optimal distribution of relief items at minimized times during disasters. This thesis presents a solution for transportation problem using computing modeling for the optimal distribution of relieve items in some selected districts of the Greater Accra Region of Ghana which has been known over the years to be susceptible to flooding. The considered areas included Ada Foah, Ada West, Lekma, Prampram and Tema. This thesis aim to minimize the time for sending relieve items from the National Disaster Management Organization (NADMO) office to disaster sites. The intent is that, the needs of each arrival area are met and every shipping vehicle operates within its capacity. These vehicles, nine in number, which served as sources of relieve items included the $\frac{1}{2}$ ton pickup, 1 ton pickup, 4 wheeler truck, 6 wheeler truck, 10 wheeler truck, 12 wheeler truck, 14 wheeler truck, 18 wheeler truck and the 22 wheeler truck. The underlying transportation tableau is derived and according to its governing mathematical model, the Vogel's approximation method is used to search the basic feasible solution and the Modified Distribution Method (MODI) is also employed in optimizing the basic feasible solution.

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Chapter 1

Introduction

1.1 General Overview

Nobody wishes unfortunate and certain unforeseen circumstances to befall them. As humans we often than not envisage the good life, as defined by the individual, and do not wish to have to face some of harsh realities that goes on around us. A disaster can be defined as an unexpected natural or man-made catastrophe that is of substantial extent, and causes significant physical damage or destruction, loss of life or sometimes permanent change to natural environment. The United Nations defines a disaster as a serious disruption of the functioning of a community or a society.

Disasters involve widespread human, material, economic or environmental impacts, which exceed the ability of the affected community or society to cope using its own resources. It mostly falls on the central government and other entities such as Non Governmental Organizations (NGO's).

1.2 Background

The Worldwatch Institute asserts and warns that natural disasters are becoming more and more frequent. Wherever disasters hit, havoc is wreaked. Yearly, thousands of people are affected by storms, hurricanes, droughts, earthquakes, floods, etc. And whether they are displaced, lose their homes, their work or relatives, the impacts of natural disasters on the populations are dramatic and colossal (Rigolet, 2015). In developing countries, the growing incidence of natural disasters is highly correlated to the increasing vulnerability of households and communities as previous socioeconomic vulnerabilities may aggravate the impact of natural disaster, making the process of recovery very difficult (Vatsa et al., 2000). Thus, the impact of such events could result in an immediate increase in poverty and deprevation (Carter et al., 2007). In the aftermath of a disaster, the affected who have limited financial reserves, may become more disadvantaged with some even falling into poverty (Rodriguez-Oreggia et al., 2013). The American National Red Cross, some time past, used a ’Need not loss’ criterion in providing relief assistance. That is, individuals and families ought to be assisted to the extent that they would have a certain standard of living irrespective of what they had lost. One result of this scheme was that some victims of the disaster ended up living better than they had before impact, though that scheme left those who had substantially more prior to the disaster at a loss (Quarantelli, 1999). But overall the general intent of the scheme was good

1.2.1 Disaster Management

The Red Cross and Red Crescent societies define disaster management as the organisation and management of resources and responsibilities for dealing with all humanitarian aspects of emergencies, in particular preparedness, response and recovery in order to lessen the impact of disasters. Some of the disasters include fires and floods, earthquake and tornado, epidemics and starvation, heat and cold, rats and locusts (Stanley Jaya Kumar, 2000).

Disasters in Ghana have been grouped into 6 main parts by the national disaster management organization (NADMO). They are;

TYPES OF HAZARDS/DISASTERS	COVERAGE AREA
Hydro-meteorological Disasters	Flood, Windstorm, Rainstorm, Drought and Tidal-waves
Pest and Insect Infestation Disasters	Armyworm, Anthrax, Blackfly, Locust, Larger Grain Borer etc.
Geological/Nuclear Radiological Disasters	Earthquakes, Tsunamis, Gas Emissions and Landslide etc
Fires and Lightning Disasters	Bush/Wild fires, Domestic and Industrial fires and Lightening
Disease Epidemics Disasters	Cholera, Yellow Fever, Cerebro-Spinal Meningitis(CSM), Pandemic Influenza etc.
Man-Made Disasters	Social conflicts, Collapse of Building, Vehicular and Aviation Accidents, Lake/Boat Accidents, Marine and Railway Disasters, etc.

The national disaster management organization (NADMO) has tackled major disasters since its inception in 1997. Like, the Outbreak of Cerebro-Spinal Minigitis (CSM), which affected the three Northern Regions of Ghana and claimed about 1,356 lives in 1997;
Cholera Outbreaks mostly occurring yearly in Greater Accra and Central Regions, which cost tens of thousands of cedis to tackle every time;
Armyworm Invasion in the three Northern Regions, Ejura in the Ashanti Region and Dahwenya in the Greater Accra Region. Relief provided included seeds, chemicals, protective clothing etc.;

Northern Floods in 1999 that swept through the Upper West, Upper East, Northern and the Northern parts of the Brong Ahafo and Volta Regions affected over three hundred thousand (300,000) persons. And other secondary disasters of water-borne and water related diseases to contend with (NADMO disaster profile, 2018).

In recent years, Ghana has been plagued with yearly floods across the length and breadth of the country, especially in the rainy season. Also fire disasters have also been a major national issue with the loss of so many lives and properties. The afore mentioned examples, as well as, other forms all require immediate response.

Disaster response is a very important stage in disaster management. It is made up of a number of elements, like; warning/ evacuation, search and rescue, providing immediate assistance, damage assessment, immediate restoration or construction of infrastructure,etc. The singular aim of emergency response is to provide immediate assistance in order to maintain life, improve health and provide emotional support and relief of the affected population. Such assistance ranges from the provision of specific but limited aid needed at that instance, such as medical assistance, temporary shelter, food, and the initial repairs to damaged infrastructure.

The primary focus of disaster response is to keep people safe, prevent further damage to life and property, and meet the basic needs of the affected until more permanent and sustainable solutions are found. It is the duty of the government or governments in whose territory the disaster occurred to provide response and address the situation. Additionally, Humanitarian organizations are often strongly present in disaster response stage, particularly in countries where the government lacks the resources to respond adequately to the needs.

1.3 Problem Statement

The effectiveness and usefulness of disaster response mainly depends on its timeliness. In a quest to provide effective and quick relief to areas and people hit with disasters, responders need to take into consideration how fast to take relief items to destination points. That means determining which vehicle can carry the required weight of materials and also travel to the destination sights as soon as possible.

1.4 Research Objectives

The main objective of this research is to find the optimal time that a car will travel from the supply source to the disaster location. Under this objective we answer the following questions;

1. What is the optimal speed of the vehicle used in transport?

2. What is the carrying capacity of the supply source in respect of the commodities?

3. Clearly state the ideal vehicle type based on its carrying capacity and speed

1.5 Methodology

The research uses the transportation problem technique, specifically, the optimal vehicle routing model in the delivery of disaster relief items. The goal is to minimize the time a vehicle takes in carrying commodities to disaster locations.

1.6 Scope of the Study

This research considers flooding disasters. Floods occur country wide. But the size of this research is limited to Five (5) districts/metropolis in the Greater Accra region; namely, Ada Foah district, Ada West district, Ningo Prampram district, Lekma (Ledzokuku-Krowor) municipal and Tema municipal, where there are frequent occurrence of floods.

Disaster management does not only entail the provision of relief alone. As a matter of fact, a proper and effective disaster management takes stock of the actual extent of damage done and the exact number of people affected. It is always prudent for Disaster management organizations, MMDA?s (Metropolitan, Municipal, and District Assemblies), and governments to have an idea of disaster prone areas in their catchment, so as to give special attention to these areas and also figure out ways to effectively dispatch assistance to these areas.

Below are maps of the considered areas, highlighting their overall land size, flood and non flood areas, total population and population likely to be affected by the floods, etc.

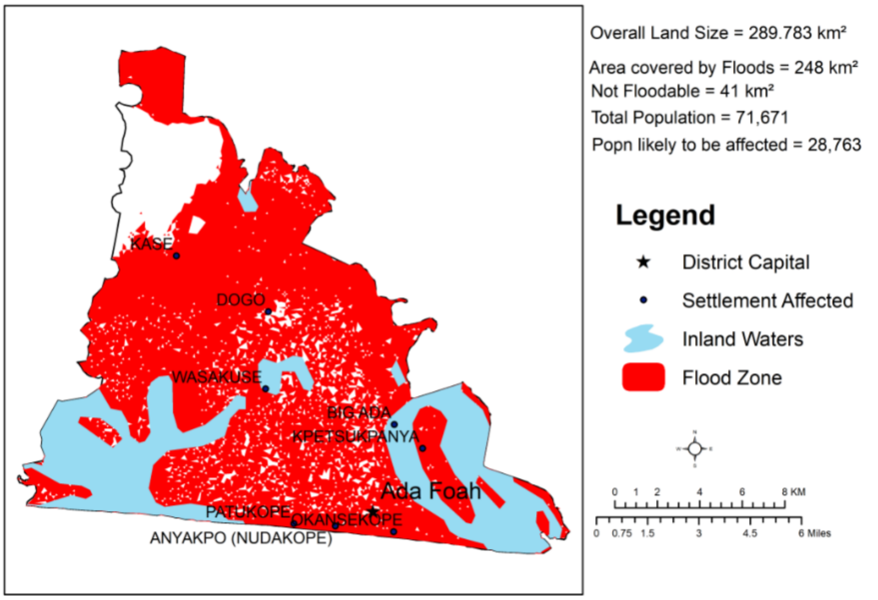

Figure 1.1: Map of Ada Foah

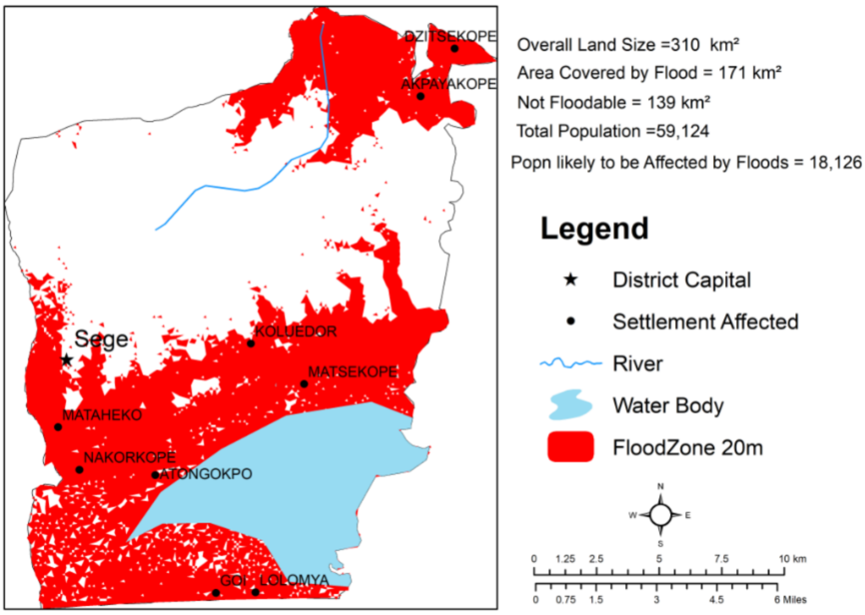

Figure 1.2: Map of Ada West

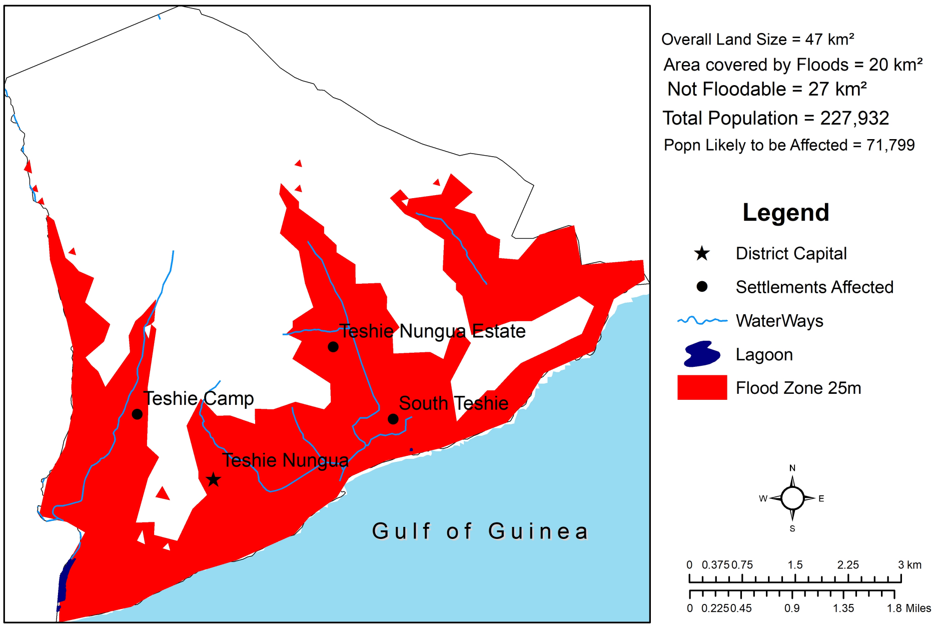

Figure 1.3: Map of LEKMA

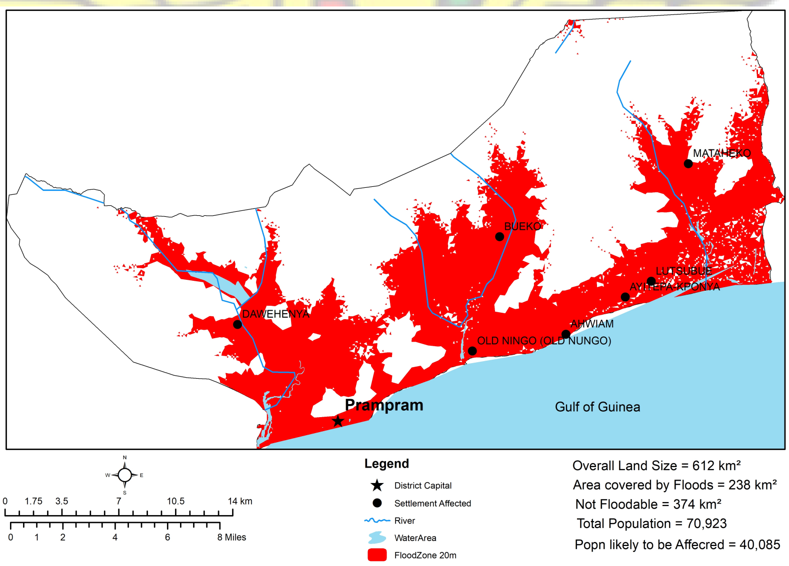

Figure 1.4: Map of Prampram

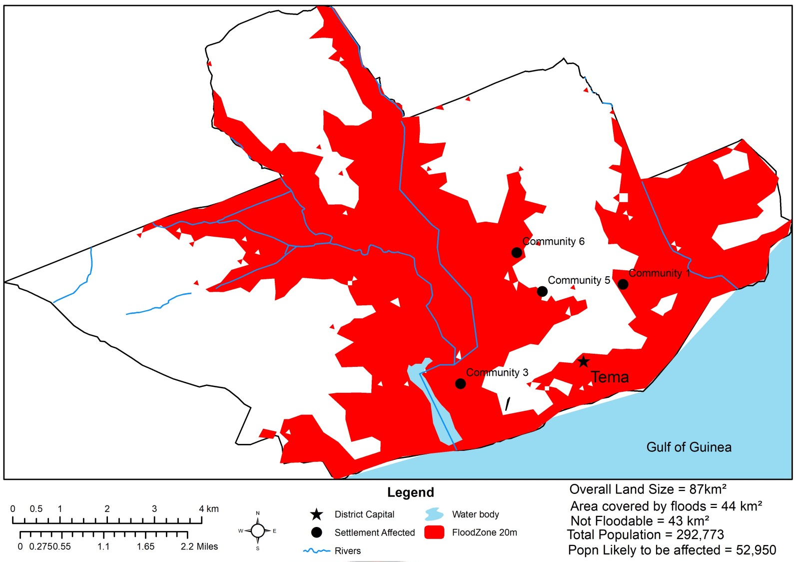

Figure 1.5: Map of Tema

1.7 Assumptions of the Study

In order to achieve the set objectives, the following assumptions were made;

1. The warehouse of the goods or supplies becomes the car type.

2. The speed of the car is not affected by the load it carries.

In justification to the second assumption, the times taken by the vehicles to reach these relief items were taken when the cars were loaded with relief items at full capacity. Thus, the differences in the speed of the cars were as a result of differences in their original speeds independent of the loads they took.

1.8 Significance of the Study

Disasters are accompanied by the loss of life and property, and other essential and important resources. And in the bid to curb or curtail its effects, the government in charge of that particular area, owe it to the citizens to respond to swiftly and with an immediate plan of action. The foremost action of disaster management is to provide timely and immediate response in the form of reliefs and other intervention services to the disaster hit areas and people. The whole concept of disaster management thus hinges on timeliness and thus innovative ways should be adopted in ensuring that relief is received on time.

1.9 Organization of the Study

This work seeks to optimize the time taken for relief to be delivered to a disaster zone,

1. Chapter one talks about the general introduction to disaster and disaster management, and identifies the problem, as well as the general structure of the study.

2. Chapter two covers some definitions and related work on disaster management, and its theories.

3. Chapter three examines the presentation of model formulation, relationships and solutions to the optimization Equations derived.

4. Chapter four comprises analysis of data collected as well as discusses the results obtained from the case study.

5. Chapter five contains the conclusions and recommendations necessary for this research.

Chapter 2

Literature Review

2.1 Overview

This chapter contains the review of the literature related to the study and the theoretical frame work related to the important aspect of the study. This mainly comprises the theory that has been reviewed under the following heading: Background of routing problems, Disaster management, supply chain management and Emergency response logistics.

2.2 Background of Routing Problems

The conveyance problem is classical in optimization routing of goods. The transportation problem was formalized by the French mathematician Monge (1781). In the 1920s A.N Tolstol was one of the first to study the transportation problem mathematically. In 1930, in the National commissariat of Transportation of the soviet union he published a paper ‘methods of finding the minimal kilometrage in Cargo- transportation in space.Major advances were made in space in Economics the eld during World war 2 by the Soviet/Russian Mathematician and Leonid Kantoronch consequently the problem as it is sometimes known as the Monge-Kantorovich transportation problem. The problem as it is now stated is sometimes known as the Monge-Kantorovich transportation problem. Kantorovich (1942), published a paper on continuous version of the problem and later with Gavurian, and applied study of the capacitated transportation problem Kantorovich and Gavurin (1949).Major advances were made in the field during World War II by the So- viet/Russian mathematician and economist Leonid Kantorovich. Transportation Problem (TP) is centered on supply and demand of commodities transported from several sources to the different destinations. The origin of transportation was first presented by Hitchcock (1941) also presented a study entitled (The Distribution of a Product from Several sources to numerous localities). This presentation is considered to be the first important contribution to the solution of transportation problems. Koopmans (1949) presented an independent study, not related to Hitchcock and called (Optimum Utilization of the Transportation System). These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem received this name because many of its applications involve determining how to optimally transport goods. During the preparedness phase, plans are set up in case a disaster occurs (e.g. preplanning of logistics operations, stockpiling of relief items, establishing communication plans, training of relief personnel). The response phase requires an immediate dispatch of personnel, equipment and other items to the disaster area. During the recovery phase, efforts are made to restore the affected areas to their previous state by reconstructing houses and public facilities Nikbakhsh et al. (2011). In the mitigation phase, 7 measures to prevent hazards from turning into disasters or to reduce their negative impacts are set (e.g. construction of flood levees, strengthening of existing buildings, land-use planning, insurances) Bullock et al. (2004). Hence, this phase requires high long-term planning and investment Wilson (2009).

2.3 Disaster Management

Disaster management is defined as “the body of policy and administrative decisions and operational activities which pertain to the various stages of a disaster at all levels” (United Nations, 1992, p. 28). The collection of internationally agreed glossary of basic terms related to disaster is a result of a joint effort of many national institutions, non-governmental organizations, and inter-governmental organizations (United Nations, 1992).This section will look at the disaster types and phases. In order to lessen the harmful impacts of disasters, the design of preventive measures and recovery plans is necessary. Related to time, disaster relief operations can be separated into four phases:

1. before a disaster strikes (preparation phase)
2. shortly after (immediate response phase)
3. in the aftermath (reconstruction phase)
4. and afterwards (mitigation phase)

These steps build the so-called Disaster Management Cycle Long (1997). Each phase requires different resources and skills. For instance, the first two phases mainly focus on strategic planning and preparation, whereas the last stage requires actual project management Kovács and Spens (2007). During the preparedness phase, plans are set up in case a disaster occurs (e.g. preplanning of logistics operations, stockpiling of relief items, establishing communication plans, training of relief personnel). The response phase requires an immediate dispatch of personnel, equipment and other items to the disaster area. During this stage there is a need for quick response to disaster affected zones. During the recovery phase, efforts are made to restore the affected areas to their previous state by reconstructing houses and public facilities Nikbakhsh et al. (2011). In the mitigation phase,there are seven measures to prevent hazards from turning into disasters or to reduce their negative impacts are set (e.g. construction of flood levees, strengthening of existing buildings, land-use planning, insurances) Bullock et al. (2004). Hence, this phase requires high long-term planning and investment Wilson (2009). Appendix 2 lists tasks and activities executed in each particular phase of the Disaster Management Cycle. Particular disaster phases vary from research to research. For example, some researchers prefer a four-phase approach - mitigation, preparedness, response and recovery - based on the Comprehensive Emergency Management concept proposed in the 1978 report of the National Governors' Association Emergency Preparedness Project. Differently, Lee and

Zbinden (2003), as well as Kovács and Spens (2007), separate the cycle into three phases:

1. preparedness (preparation),
2. during operation (immediate response),
3. post operation (reconstruction).

Also, there is a two-phase method: disaster mitigation (assessment, prevention, preparedness) including "measures taken in advance of a disaster aimed at decreasing or eliminating its impact on society and environment" (United Nations, 1992, p. 53), and disaster response (relief, rehabilitation, reconstruction) which is "a sum of decisions and actions taken during and after disaster, including immediate relief, rehabilitation, and reconstruction" (United Nations, 1992, p. 29).

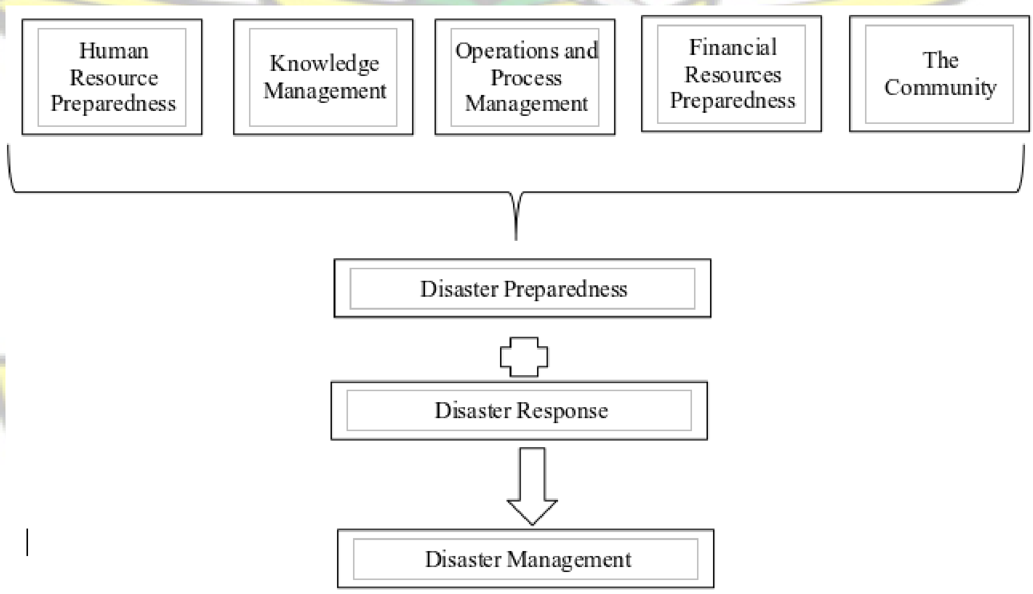

Figure 2.1: Establishing effective disaster management (adapted from Samii et al. (2002))

2.4 Supply Chain Management

A term often used interchangeably with logistics supply chain management (SCM) refers to the planning, procurement, manufacture and distribution of goods in, military and humanitarian sectors. In the humanitarian field, SCM is narrower in scope than logistics as the latter term is taken to cover a number of other functions as well as SCM. The distribution of relief supplies in the first few days after a major disaster is critical to human welfare. Designing and controlling a logistics distribution network involve different levels of decision-making, which are not independent of each other but exhibit interactions Bramel and Simchi-Levi (1997) claim that, in logistics management practice, the tendency to use decision rules that were adequate in the past, or that seem to be intuitively good, is still often observed. However, it proved to be worthwhile using scientific approaches to certificate a good performance of the supply chain or to detect opportunities for improving it. Many times this leads to a more effective performance of the supply chain while maintaining or even improving the customer service level. To ensure an efficient performance of the supply chain, decisions having a significant impact on each other must be coordinated. For instance, companies believe that capacity is expensive (see Bradley and Arntzen (1999)). This has a twofold consequence. Firstly, the purchase of production equipment is made by top managers,while the production schedules and the inventory levels are decided at lower levels in the company. Geoffrion and Powers (1995) summarize some of the main reasons for the increasing role of optimization techniques in the design of distribution systems. The most crucial one is the development of the capabilities of computers that allow for the investigation of richer and more realistic models than could be analyzed before. In these extended models, additional important issues, for e.g. scenario analysis, can be included. This development in computer technology is accompanied by new advances in algorithms,(Nemhauser, 1994)

2.5 Emergency Response and Logistics

The term "logistics" refers to the procurement, distribution, maintenance, and replacement of materiel and personnel. After a disaster occurred, a high demand for various relief items exists. The Pan American Health Organization and the World Health Organization (2011) released a minimum list of required relief commodities for disaster management. Those products are listed in Table 2.1

Table 2.1: Minimum list of required disaster relief items (Pan American Health Organization & World Health Organization, 2011

Disaster Relief Item
Food
Water and Sanitary Items
Medicine (general pharmaceutical products and/or specific pharmaceutical products in possible cases of epidemics)
Health kits and supplies for supporting health-care processes
Field hospitals
Clothing and blankets
Items associated with infants and children (e.g. instant milk, diapers, toys)
Shelters and temporary housing facilities (e.g. tents)
Electrical power generating equipment
Fuel (e.g. coal, gas, or oil)
Field kitchen equipment and utensils
Cleaning supplies
Agricultural commodities and livestock
Specialized equipment for handling hazardous materials
Communication equipment
Firefighting equipment
Debris-removal equipment and vehicles
Construction equipment and vehicles

The Asian tsunami of December 2004 was one of the first and biggest demonstrations of the social change consequences due to increasing urbanization and social marginalization (Perry, 2007). We mostly see on TV documentaries that most vulnerable populations were the first affected by this disaster: Perry states that one-third of the affected population was living under the poverty line. This vulnerability has been increased by diverse factors; socially and economically they were the poorest people; geographically, mainly fishermen, they were living on the coast; politically, they were powerless and therefore unable to change their situation; culturally, traditions are an important part of their way of living and solutions were not appropriate to this tragedy; finally, psychologically, those populations were not able to prepare, fight and cope. In the 12 affected countries over 300,000 people disappeared and more than one million were displaced (Couldrey and Morris, 2005).

Chapter 3

Methodology

3.1 Introduction

This chapter discuss the proposed solution methodology and approach for a time optimal vehicle routing model for disaster relief delivery in the selected districts in the Greater Accra Region of Ghana. This is a divergent disaster relief supply network which the solution of the transportation problem can be used to solve. The transportation problem seeks to minimise the time of vehicle f, carrying commodities of type g from supply source h to i destinations which is the disaster location. Every transportation problem is an embodiment of linear programming. Linear programming model will be used to solve the problem.

3.2 Linear Programming

Linear programming is a powerful mathematical model designed to solve problems involving allocation. These are some of the terminologies in linear programming.

1. The Decision Variables refer to the economic or physical quantities which are under the control of the decision maker and their values are used to find the solution of the model. In general it is denoted by $x_1, x_2, \ldots, x_n$

2. Objective function is a function of decision variables expressing the objective of the decision maker. It denotes the function you want to minimise or maximise. Objectives functions is represented as:
   Maximise or Minimise $Z = \sum_{j=1}^{n} c_j x_j$
   Where c_j is the coefficient of the objective function corresponding to the ith variable

and x_j is the jth decision variable.

3. Constraints are limitations on resources, which are to be shared among the various decision variables. Constraints are expressed as linear equation or linear inequalities in terms of the decision variables. There are so many types of constraints that are equality constraints, inequality constraints and integer constraints. Constraints come as a result of limited resources, physical laws etc. In general constraints in linear programming(LP) is denoted as $\sum_{j=1}^{m} a_{ij}x_j \leq b_i \qquad \text{for } i = 1, \ldots, m$
   Where x_j is the *jth* decision variable,
   a_{ij} is the coefficient of x_j in constraint i,
   b_i is the right hand side coefficient on constraint i.

4. Non-negativity Restriction shows that all decision variables must be greater than or equal to zero.It is denoted by $x \geq 0$ where x_i is the *ith* decision variable.

The general linear programming (LP) problem is given as

$$\text{Maximise or Minimise} \qquad Z = \sum_{j=1} c_j x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, i = 1, \ldots, m$$

$$x_j \geq 0, \qquad j = 1, \ldots, n$$

3.2.1 Assumptions of Linear Programming

There are four assumptions in linear programming and all the assumptions are implicit in the model formulation given. These are:

Proportionality Assumption Proportionality assumption refers to the individual variables in the objective function and the constraints. We assume that their proportional to their values that is if the value of a variable is doubled, we double the contribution of the variable to the objective function as well as each constraint in which the variable appears. The contribution per unit of the variable is constant. For example, suppose the variable x_j is the number of units of product j produced and c_j is the cost per unit of product j.If doubling the amount of product j produced doubles its cost, the per unit cost is constant nd the proportionality assumption is satisfied.

Additivity Assumption Additivity means that the total value of the objective function and each constraint function is obtained by summing up the individual contributions from each variable

Divisibility Assumption For divisibility assumption each decision variables are allowed assume fraction within a specified range by the constraint. That is variables are not restricted to integers

Certainty Assumption Each linear coefficient of the objective function and constraints is known

3.2.2 Examples of Linear Programming

Transport Problem

There are i warehouse, $w_1, \ldots, w_n$, that supply a certain commodity, and there are j shops, $M_1, \ldots, M_m$, to which this commodity must be shipped. Warehouse w_i possesses an amount s_i of the commodity $(i = 1, 2, \ldots, n)$, and market M_j must receive the amount u_j of the commodity $(j = 1, \ldots, m)$. Let c_{ij} be the cost of transporting one unit of the commodity from warehouse w_i to market M_j. The problem is to meet the market requirements at minimum transportation cost.

Let x_{ij} be the quantity of the commodity shipped from warehouse w_i to market

M_j. The total transportation cost is

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij}$$

The amount sent from the warehouse w_i is $\sum_{j=1}^{m} X_i j$ and since the amount available at the warehouse w_i is S_i we must have

$$\sum x_{ij} \leq S_i, \qquad \text{for} i = 1, \ldots, n$$

The amount sent to the market M_j is $\sum_{i=1}^{n} x_{ij}$ and since the amount required there u_j

$$\sum_{i=1}^{n} x_{ij} \geq u_j, \qquad \text{for} j = 1, \ldots, m$$

$$x_{ij}, \qquad i = 1, \ldots, n \text{ and } j = 1, \ldots, m$$

The Investment Problem

We have a sum p unit of money which may be invested in various business areas, each of them producing a certain benefit. Let $c_j, 1 \leq j \leq n$, be the sum invested in the j-th activity. The problem above can be modelled using linear programming, that is:

$$\text{Maximise} \sum_{j=1}^{n} c_j x_j$$

Subject to

$$\sum_{j=1}^{n} x_j \leq p$$

3.3 Methods of Solution

These are some of the important definition and concepts that are used in solving linear programming problem.

1. Solution is a set of values of decision variables which satisfies all constraint
2. Feasible solution is any non-negative solution which satisfies all the constraints.
3. Feasible Region is the collection all feasible solution
4. Infeasible problem a linear programming problem is said to be infeasible if there is no solution that satisfies all the constraints.
5. An optimal solution is a feasible solution that has the most favourable value of the objective function.
6. Optimal feasible solution is any feasible solution which minimises or maximises the objective function.
7. Corner point is a point in the feasible region that occurs at the intersection of two boundary lines.

3.3.1 The Graphical Method

The graphical method is use to solve linear programming problems involving two decision variables. Steps for Graphical method of solving a linear programming problem involves

1. Formulate the linear programming problem from the given question.
2. Change the inequality sign in the constraints into equality sign. Use the constraints equation to graph the feasible region and find the corner points of the feasible region. The coordinate of the corner points can be obtained

by either inspection or by solving the two equations of the lines intersecting at that point simultaneously.

3. Put the values obtained as corn points in the objective function.

4. Determine the optimal solution from step 3. If the problem is maximization type it correspond to the largest value obtained from the objective function whiles minimization correspond to the smallest value obtained from the objective function.

Let us consider one example on graphical method

$$\text{Maximise} Z = 3x_1 + 5_2$$
$$\text{Subject to}$$
$$x_1 \leq 14$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$
$$x_1 \geq 0, x_2 \geq 0$$

To solve this, we change all the inequality signs in the constraints into equality signs. Now we have

$$x_1 = 4, (4, 0)$$
$$x_2 = 5, (0, 6)$$
$$3x_1 + 2x_2 = 18, (6, 9). \quad \text{We plot } (6, 0) \text{ and } (0, 9).$$

With these points above, we plot them and find the optimal value.

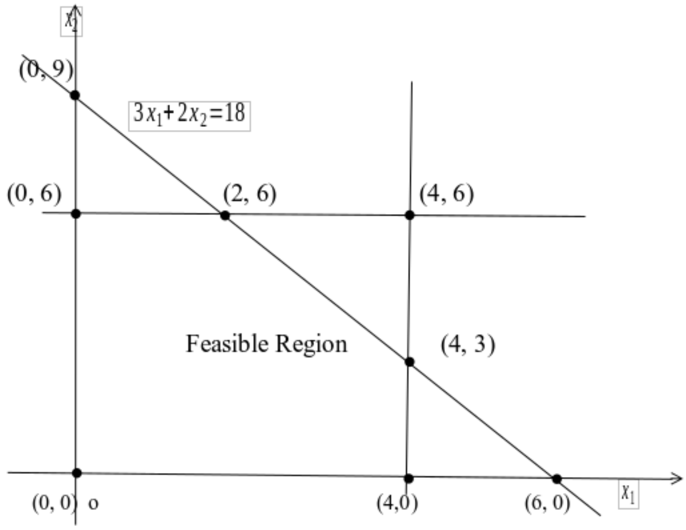

Figure 3.1: Diagram to show feasible region

The shaded region is called the feasible region. There are five corner point feasible solution that is (0, 0), (0, 6), (2, 4) and (4, 0). The points that lie outside the feasible region are called corner point infeasible solutions. They are (6, 0), (4, 6) and (0, 9). The optimal solution is $x_1 = 2$ and $x_2 = 6$. The optimal value is $z = 3(2) + 5(6) = 26$

3.3.2 Simplex Method

Most real world linear programming problems have more than two variables and thus too complex for the graphical solution. It is better to use solution methods that are adaptable to computers. One of the methods used to solve linear programming with large variables is simplex method. The simplex method was developed by George Dantzig in 1947. The underlying concepts are geometric, but the solution algorithm is an algebraic procedure. The simplex method is an algorithm which provides us with systematic way examining the corner points of the feasible region to determine the optimal value of the objective function.

General Simplex Formulation

We start by stating a general linear programming problem involving n decision variables and m constraints as

$$\text{Maximise} z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$.$$
$$.$$
$$.$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

We can only use the simplex method when the inequality signs in the constraints are replaced by equation signs. After adding a slack variable, the corresponding systems of constraint equations is

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + s_1 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + s_2 = b_2$$
$$.$$
$$.$$
$$.$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + s_m = b_m$$

Where $x_i \geq 0, s_i \geq 0$, s_i are slack variables

S_i always have coefficient of zero in the objective function, that is $S_i = 0$

$$z = c_i x_i + 0s_i, \text{ for } i = 1, 2, \ldots, m$$

In matrix notation, the standard form of the linear programming problem is given by an $m \times n$ matrix

$$z = Cx$$
$$Ax = b$$

Where

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad C = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{pmatrix}$$

Z is the objective function, subject to the constant $Ax = b$. We note the following things about the standard linear programming.

1. The objective function is the same. The slack variable included in the objective function takes a coefficient of zero
2. The m constraints of the new system are represented by m equations and there n+m unknown variables.
3. All variables are nonnegative that is both the decision variable and slack variables
4. The values at the right side are nonnegative

The Steps of the Simplex Algorithm

1. Add slack variables to remove the inequality in the set of constraints to an equation.

2. Create the initial simplex tableau. In this step we arrange the various matrices and vectors involved in the matrix form in the simplex tableau. The tableau contains all the information about the current basic variables and their corresponding values, the optimality status of the solution. An initial simplex has this form

    - The top row of an initial tableau represents both $n + m$decision variables and slack variables $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$.
    - The middle row shows the coefficients of the constraints variables.
    - The last row is the z equation, showing the objective coefficients.
    - The column labelled "RHS" shows the values on the values of the constraints.

Basic variables	Coefficient						RHS	Ratio
	x1	…	xn	xn+1	…	xn+m		
xn+1	a11	…	a1n	1	…	0	b1	
.	.		.	.		.	.	
.	.		.	.		.	.	
.	.		.	.		.	.	
xn+m	am1	…	amn	0	…	1	bm	
z	-c1	…	-cn	0	…	0	0	

Figure 3.2:

3. Find an initial basic feasible solution (bfs). If there is no initial basic feasible solution then the model is infeasible, so stop. The easiest choice of an initial feasible is to assume that the entire decision variables are not basic hence the basic solutions consist of slack variables.

4. Find the entry column by locating the most negative entry in the bottom row. Since this variable will become basic, one of the existing basic variable

will have to become nonbasic .The entry column is called the pivot column.

5. Locate the departing row. This done by computing the ratios entries in the right hand column with their corresponding positive entries. The smallest nonnegative ratio $\frac{b_i}{a_{ij}}$ is taking as the departing row. The intersection of pivot column and pivot row determines the pivot element.

6. The entry in the departing row and entering column is called the pivot. The pivot is converted to one by using elementary row operation and all entries to the top and down of the pivot in the same column goes to zero by using elementary row operation.

7. If all entries in the last row are zero or positive, stop. If not go to step 3 and start till the last row is zero or positive

8. The optimal value of the final tableau is given by the entry in the lower - right column.

Optimality Test

The Optimality criteria is met when all coefficient in the objective row are non-negative. The optimal solution can be determined from the final tableau. The solution is made up of basic decision variables in the extreme left column and it corresponding values appear in the extreme right column. Any decision variable which falls out in the basic set has a value of zero.

3.3.3 Special Cases in the Simplex Method

Certain situations may occur which do not comply with the assumptions so far made in implementing the simplex method. Some of these situations are discussed below.

Tie Breaking

The most negative coefficient in the bottom row that is the z- equation always determines the entering variable (pivot column). In a situation where the most negative entry in the bottom row is more than one, then there is a tie. Normally tie is broken by making an arbitrary selection of the entering among those that qualify.
The same approach is used for leaving basic variable.

Unbounded Solution

Unboundedness happens when the entire coefficient in the pivot column is either zero or negative. In this case it will not be possible for us to select a coefficient from the pivot column. Hence there would be no way of computing a leaving variable.

3.4 Transportation Problem

The general transportation problem is a special type of linear programming where the objective is to determine an optimal approach for dispensing a commodity from a group of supply centres, such as factories, to various receiving centres, such as warehouses, in such a way as to minimise total distribution costs and to maximise profit. The supply centre is called source and the distribution centre is called destination. The source (origin) of a transportation problem is the location from which freights are dispatched. The destination of a transportation problem is the location to which freights are transported.
Each source is able to supply a fixed number of units of the product, usually called the capacity, and each destination has a fixed demand, often called the requirement. The transportation model can be used to determine how to allocate the supplies from the various factories to the warehouses that demand those goods, in such a way that total shipping cost is minimised.

3.4.1 Mathematical Formulation of Transportation Problem

Suppose a factory has m warehouses and n receiving centres. One item is to be shipped from warehouses to the receiving centres. Each warehouse has a limit it can supply and each receiving has a given level of demand. The cost of transporting one unit of the commodity from each warehouse to each receiving centre is known. These costs are assumed to be linear.

A transportation problem is specified by the following information:

1. A set of m supply points from which a good is shipped. Supply point i can supply at most s_i units, where $i = 1, 2, \ldots, m$
2. A set of n demand points to which the good is shipped. Demand point j must receive at least d_j unit of the shipped good, where $j = 1, 2, \ldots, n$
3. Each unit produced at supply point i and shipped to demand point j incur a variable cost of c_{ij} where $i = 1, 2, \ldots, m$ and $j = 1, 2 \ldots, n$

Decision Variable

Decision variable is the number of units shipped from the supply point to the demand point. It is represented by x_{ij}, where i is the supply point and j is the demand point.

Objective Function The objective is function is given by

$$\text{Minimise} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

c_{ij} is the cost of transporting one unit of the commodity from source i to destination j.

Constraints

The constraints are the conditions that force supply and demand needs to be satisfied. Every node in transportation has a constraint. Let s_i denote source capacity and d_j denote the number of unit being received by destination. A supply constraint says that the sum of all shipments from a source cannot exceed the available supply. That is

$$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad \text{for } i = 1, 2, \ldots, m$$

The demand constraint says that the quantity of goods shipped to a destination must be at least as large as the demand. That is

$$\sum_{i=1}^{m} x_{ij} \geq d_j \qquad \text{for } j = 1, 2, \ldots, n$$

The transportation problem is

$$\text{Minimise} \qquad \sum_{i=i}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad \text{for } i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} x_{ij} \geq d_j \qquad \text{for } j = 1, 2, \ldots, n$$
$$x_{ij} \geq 0$$

The number of constraints of the transportation problem is $m + n$ while the number of variables is mn.

3.4.2 Types of Transportation Problem

Balanced Transportation Problem

Balance transportation problem is a transportation problem when total supply at the source is equal to the total requirements at the destination. That is

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

In a balance transportation model, each of the constraints takes an equal sign. That is

$$\sum_{j=1}^{n} x_{ij} = s_i, \qquad \text{for } i = i, 2, \ldots, m$$
$$\sum_{i=1}^{m} x_{ij} = d_j, \qquad \text{for } j = i, 2, \ldots, n$$

A balance transportation model has $n + m - 1$ independent constraint equation.

Unbalanced Transportation Problem

An unbalance transportation problem is when the total supply is not equal to the total demand.

$$\sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j$$

There two cases of unbalance transportation problem

1. When the total supply is more than total demand

$$\sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j$$

   When the total supply is more than the total demand, a dummy demand point is created to absorb the excess supply. Since shipments to the dummy demand point are not real shipments, they are given a cost value of zero.

Shipments to the dummy demand point indicate unused supply capacity.

2. When the total supply is less than total demand

$$\sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j$$

If total supply is less than total demand, the actual problem has no feasible solution. If the demand is more than the supply, dummy supply point equal to the amount of excess demand is created to absorb the excess demand.

3.4.3 Transportation Problem Tableau

The model of a transportation problem can be represented in a tabular form with all the relevant parameters. A typical transportation problem takes the form of a standard form. The information needed to use the model consist of the following

- A list of origins and each one's supply quantity per period.
- A list of destinations and each one's demand per period.
- The unit cost of shipping commodities from source to destination.

		Cost per Unit Distributed				
		Destination				
		1	**2**	⋯	***n***	**Supply**
Source	1	c_{11}	c_{12}	⋯	c_{1n}	s_1
	2	c_{21}	c_{22}	⋯	c_{2n}	s_2
	⋮	⋯	⋯	⋯	⋯	⋮
	m	c_{m1}	c_{m2}	⋯	c_{mn}	s_m
Demand		d_1	d_2	⋯	d_n	

Figure 3.3: The Transportation problem Tableau

If $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$ then this is a balance transportation problem.From the above transportation tableau, the column to the extreme left represent the source (i). The centre columns depict the destination points (j). The column to the extreme right represents the supply (s_i) and the bottom row represents the demand (d_j). Each cell represents one route. The unit cost of shipment from source i to destination j, c_{ij}, is depicted in the right corner of the cell (i, j). The number of units shipped from the source i, to destination j, is depicted in the centre of the cell (i, j).

3.4.4 Network Representation of the Transportation Problem

The transportation problem is represented graphically as a network with m origin nodes, n destination nodes, and a set m.n directed arc. This is shown in

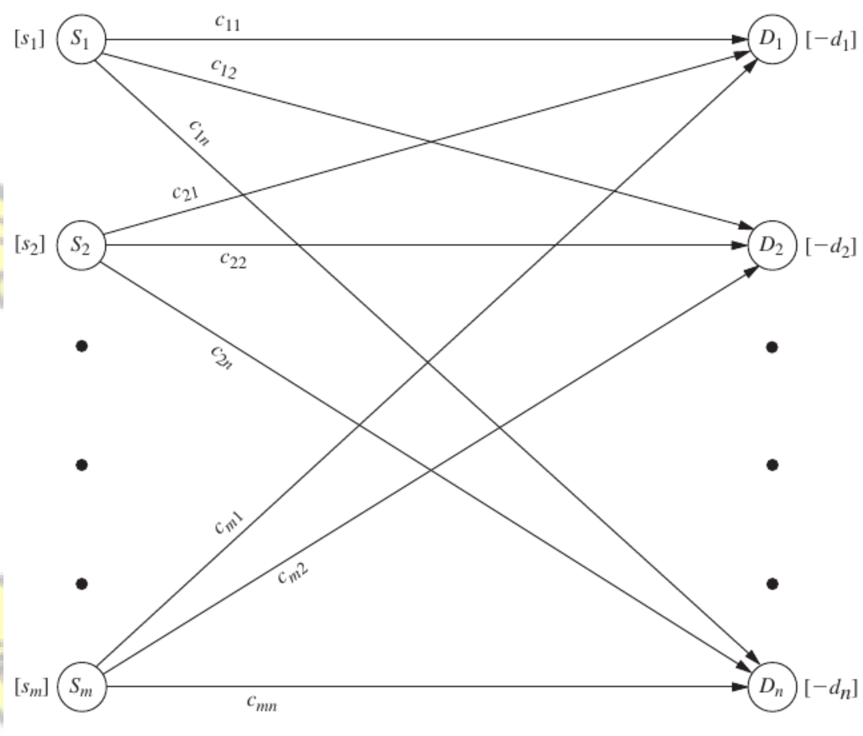

Figure 3.4:

In the diagram each node represents a source m or destination n. The route between a source and destination is represented by a directed arc moving from the source to the destination.

3.4.5 Finding the Initial Basic Feasible Solution for Transportation Problem

In general, any initial feasible solution of a transportation problem with m source and n destinations should have $m + n - 1$ non zero basic variables. There are three ways of obtaining initial basic feasible solution for a transportation problem which are presented in the next subsections.

Least Cost Method

The steps for least cost method are given below

1. Identify the cell with the least cost in the entire tableau
2. Make a maximum allocation to the cell with the smallest unit cost
3. If there is a tie then choose arbitrarily. Cross out row or column that is exhausted (maximum demand or capacity) by this
4. Repeat the above steps till all units have been allocated.

The North-West Corner Method

The steps for the north - west corner method are given below

1. Allocate maximum amount to the upper left corner of the transportation tableau.
2. Continue to the next cell of the row, depending on the remaining supply for that row and the demand requirement for the next column. Continue till the row (or column) total is exhausted.
3. Cross out exhausted row (or column)
4. Repeat 1 to 3 till the entire demand and supply is exhausted

The Vogel's Approximate Method(VAM)

The steps for the Vogel's Approximation method are given below

1. Subtract the smallest from the second smallest cost of each row. After subtraction you write the answers at extreme right of the tableau. In case of tie between smallest costs, the difference should be taken as zero.

2. Subtract the smallest from the second smallest cost of each column. After subtraction you write the answers at the bottom of the tableau. In case of tie between the smallest costs, the difference should be taken as zero.

3. Choose the row/column with the largest difference. Select the variable with the least cost cell corresponding to the row/column.

4. Check the demand and supply values for the least cost cell. If the demand value is less than the supply value, then the column relating to the demand value leaves the system. If the supply value is less than the demand value, then the row relating to the supply value leaves the system.

5. Repeat step 3 and 4 till entire demand and supply is exhaust. That is until one uncrossed cell remains.

3.4.6 Optimality Test

A basic feasible solution of a transportation problem is optimal if and only if $c_{ij} - u_i - v_j$, where
c_{ij} is the shipping cost per unit of occupied cell
u_i is the row index
v_j is the column index
If all the cost index values obtained for all the currently empty cells are nonnegative, then the current solution is optimal. If there are negative values then the solution has to be improved.

For optimality test, you have to compute the values of u_i, v_j and $c_{ij} - u_i - v_j$ (opportunity cost). For optimality test ui gives m row and vj gives n column, therefore the non-zero basic variables becomes $m + n$. We assign zero to ui to make it redundant and also change it from $m + n$ to $m + n - 1$. There are two ways of checking for optimality, these are

Stepping-Stone Method

The steps involved in checking for optimality using the Stepping-Stone method include

1. First choose unoccupied cell and create a closed path, which starts from the unoccupied cell and ends at the same cell.

2. Once the path is created, assign positive (+) or negative (−) sign alternatively on each occupied cell in the closed loop. Begin with positive (+) sign for the unoccupied cell.

3. Add cells with positive signs and subtract cells with negative signs. Compute for the unoccupied cell cost

4. Repeat these steps again until all the unoccupied cells get evaluated.

5. If all the unoccupied cells have zero or positive values, then solution is optimal

6. In case, one of the values of the empty cells becomes negative, then the solution is not optimal. Then, select that unoccupied cell which has the most negative change and assign as many units as possible. Subtract the unit that added to the unoccupied cell from the other cells with a negative sign in a loop, to balance the demand and supply requirements.

Modified Distribution Method (MODI)

Another method for evaluating empty cells is the modified distribution method (MODI). It involves solving row index (u_i) and column index (v_j) numbers that can be used for cell evaluation. The modi is made up of the following steps:

1. Determine an initial basic feasible solution using the three methods

2. Determine the values of the index row (u_i) and index column (v_j), using $u_i + v_j = c_{ij}$. Begin by assigning zero to the first row (u_i). Do this using only completed cells.

3. Obtain cell evaluations for unused cells using the relationship $\Delta_{ij} = c_{ij} - (u_i + v_j)$, where Δ_{ij} is the opportunity cost, c_i is the cell cost, is the row index and v_j is the column index.

4. Check the sign of each opportunity cost (Δ_{ij}). If the opportunity cost for all empty cell is either positive or zero, then the given solution is optimal. If only one unoccupied cell has negative cost, then the given solution is not optimal and it can be improved.

5. Choose the unused cell with the smallest negative cell to be added in the next solution

6. Draw a closed path starting from the unoccupied cell and assign positive (+) and negative (−) sign alternatively. The cell with negative (−) signs are called donors and the ones with positive (+) signs are called recipient.

7. Determine the maximum number of units that should be moved to the unoccupied cell. In the loop, choose the smallest value with a negative position. Add it to the cells in the loop with positive sign and subtract it from the cells with negative (−) sign in the loop.

3.4.7 Degeneracy

If the basic feasible solution of a transportation problem with m sources and n destinations is less than $m+n-1$ positive variables (occupied cells). It always follows that whenever the number of occupied cells is less than $m+n-1$, then the transportation problem is a degenerate one. Degeneracy occurs at two stages

1. At the initial solution

2. During the optimality test stage

Degeneracy is resolved by introducing an artificial quantity (Δ). The quantity is assigned to the unoccupied cell which has the minimum transportation cost, so that the total number of occupied cells will be $m+n-1$. The value of Δ is infinitesimal; it has no effect on the supply and demand constraints. In case of tie, choose arbitrarily.

Chapter 4

Results and Analysis

4.1 Introduction

This chapter entails description of the data used for the analysis, formulation of the model; which comprises of detailed description of the decision variables, objective functions and constraints. Some essential intermediate results ao the Vogel's approximation method is present alongside the final solution. Finally the results obtained are discussed.

4.2 Data Description

Prior to the objective of the research, the main task is to optimise the time taken to convey relief items from the NADMO head office to the affected disaster areas including; Ada Foah, Ada West, Lekma , Prampram and Tema. The table below gives the quantities of relief items demanded by each of the affected areas.

Table 4.1: Table of Demand of Relief Items by Affected Areas

District	Demand (In Tons)
Ada Foah	235.5
Ada West	150.5
Lekma	588
Prampram	326
Tema	433.5

The break down of the demand of relief items for the given districts are given in the Tables 4.2, 4.3, 4.4, 4.5 and 4.6.

For the Ada Foah District, the initial supply table was given as Table 4.2

Table 4.2: Ada Foah District

Vehicle Type	Number of Vehicle	Capacity	Total Capacity
4 Wheeler Truck	2	1.5	3
10 Wheeler Truck	1	15	15
6 Wheeler Truck	1	9	9
12 Wheeler truck	2	21	42
14 Wheeler Truck	2	23	46
18 Wheeler Truck	2	27	54
22 Wheeler Truck	2	33	66
1/2 Ton Pickup	1	1/2	1/2
Total			235.5

The breakdown for Ada West was also given as with the respective number of vehicles sent was given as Table 4.3

Table 4.3: Ada West District

Vehicle Type	Number of Vehicle	Capacity	Total Capacity
4 Wheeler Truck	1	1.5	1.5
10 Wheeler Truck	1	15	15
6 Wheeler Truck	1	9	9
12 Wheeler truck	2	21	42
14 Wheeler Truck	1	23	23
18 Wheeler Truck	1	27	27
22 Wheeler Truck	1	33	33
Total			150.5

Also, the breakdown of vehicles dispatched to Lekma was also given as Table 4.4

Similarly, the breakdown of vehicles dispatched to the Prampram district was given as Table 4.5

The vehicles dispatched to Tema was broken down as Table 4.6

Table 4.4: Lekma District

Vehicle Type	Number of Vehicle	Capacity	Total Capacity
4 Wheeler Truck	4	1.5	6
10 Wheeler Truck	4	15	60
6 Wheeler Truck	5	9	45
12 Wheeler truck	4	21	84
14 Wheeler Truck	4	23	92
18 Wheeler Truck	5	27	135
22 Wheeler Truck	5	33	165
1 Ton Pickup	1	1	1
Total			588

Table 4.5: Prampram District

Vehicle Type	Number of Vehicle	Capacity	
4 Wheeler Truck	1	1.5	1.5
10 Wheeler Truck	1	15	15
6 Wheeler Truck	2	9	18
12 Wheeler truck	2	21	42
14 Wheeler Truck	3	23	69
18 Wheeler Truck	3	27	81
22 Wheeler Truck	3	33	99
1/2 Ton Pickup	1	1/2	1/2
Total			326

Every good model is as good as it's assumptions, and with respect to this fact, we made an assumption in Chapter One that, the warehouse of the goods or supplies becomes the type of vehicle to be used to convey the relief items. With respect to that we break the supply down into the capacity of these vehicles and also show the number of vehicles available in the Table 4.7.

Table 4.6: Tema District

Vehicle Type	Number of Vehicle	Capacity	Total Capacity
4 Wheeler Truck	3	1.5	4.5
10 Wheeler Truck	3	15	45
6 Wheeler Truck	3	9	27
12 Wheeler truck	4	21	84
14 Wheeler Truck	4	23	92
18 Wheeler Truck	3	27	81
22 Wheeler Truck	3	33	99
1 Ton Pickup	1	1	1
Total			433.5

Table 4.7: Table of supply with respect to given vehicles

Vehicle Type	Capacity(In Tons)	Number Of Vehicles	Source Total Capacity
4 Wheeler Truck	1.5	11	16.5
6 Wheeler Truck	9	12	108
10 Wheeler Truck	15	10	150
12 Wheeler Truck	21	14	294
14 Wheeler Truck	23	14	322
18 Wheeler Truck	27	14	378
22 Wheeler Truck	33	14	462
1 Ton Pickup	1	2	2
1/2 Ton Pickup	1/2	2	1

4.3 Model Formulation

4.3.1 Decision Variables

The decision variables under consideration are the vehicles types used in conveying the relief items to the disaster sites. Since each vehicle type has a unique amount of relief items it can convey, the maximum amount associated with each vehicle type is used in the actual computations. The vehicle types considered are shown in Table 4.8;

Table 4.8: Vehicle Types

i	Vehicle Type, X_i
1	$\frac{1}{2}TonPickup$
2	1 Ton Pickup
3	4 Wheeler Truck
4	6 Wheeler Truck
5	10 Wheeler Truck
6	12 Wheeler Truck
7	14 Wheeler Truck
8	18 Wheeler Truck
9	22 Wheeler Truck

4.3.2 Objective Function

In order to achieve the primary objective of this thesis which is to find the optimal/minimal time needed to deliver the largest amount of relief items using a specific vehicle, the objective function is stated as minimization problem. Where we minimize the sum-product of time and carrying capacity of vehicle i to destination j. This is stated mathematically as;

$$\text{Minimise} \qquad \sum_{i=1}^{5}\sum_{j=1}^{9} t_{ij}x_{ij} \tag{4.1}$$

where

t_{ij} is the time taken for vehicle i to take relief items to destination j

x_{ij} is the quantity of relief items (in tonnes) from vehicle i to destination j

The total minimal time is computed as

$$T = \sum_{i=1}^{5}\sum_{j=1}^{9} w_{ij}t_{ij} \tag{4.2}$$

where

$$w_{ij} = \begin{cases} 1, & \text{Allocation is made from source } i \text{ to destination } j \\ 0, & \text{otherwise} \end{cases}$$

4.3.3 Constraints

Resources are not infinite in nature, as such there are limitations/constraints on the resources used delivering the relief items and the relief items itself. The fundamental constraints on any disaster delivery system is the the quantity of relief items to send the various destinations and vehicle type to convey this relief items. The objective function is subjected to the following constraints;

$$\sum_{j=1}^{5} x_{ij} \leq s_i \qquad \text{for } i = 1, 2, \ldots, 9$$

$$\sum_{i=1}^{9} x_{ij} \geq d_j \qquad \text{for } j = 1, 2, \ldots, 5$$

where

s_i is the maximum relief item that can be supplied by the i^{th} vehicle

d_j is the minimum relief item demanded by the j^{th} destination.

4.4 Transportation Problem

The times taken by these vehicles are presented in the Table 4.9:

The entries represented with $(-)$ implies that it is impossible for the trucks to move to those destinations. Henceforth in the computation, these times will be represented in such a way that the trucks move to these areas at infinite times. The problem can be modelled as the transportation problem since it can be represented in the form of the transportation tableau. Thus the transportation tableau for the given problem is represented as shown in Table 4.10

As discussed earlier in the previous chapter, solving the transportation problem

Table 4.9: Table of time taken by vehicles to reach disaster areas

Vehicle	Ada Foah	Ada West	Lekma	Prampram	Tema
1/2 Ton pickup	95.4	-	-	46.8	-
1 Ton Pickup	-	-	25.0	-	37.0
4 Wheeler Truck	116.6	79.2	27.4	57.2	40.7
6 Wheeler Truck	127.2	86.4	30.0	62.4	44.4
10 Wheeler Truck	148.4	100.8	35.0	72.8	51.8
12 Wheeler Truck	159.0	108.0	37.5	78	55.5
14 Wheeler Truck	169.6	115.2	40	83.2	59.2
18 Wheeler Truck	190.8	129.6	45.0	93.6	66.6
22 Wheeler Truck	212.0	144.0	50	104.0	74

Table 4.10: Transportation Tableau for the given data

Source/Destination	Ada Foah	Ada West	Lekma	Prampram	Tema	Supply
1/2 Ton Pickup	95.4	-	-	46.8	-	1
1 Ton Pickup	-	-	25.0	-	37.0	2
4 Wheeler Truck	116.6	79.2	27.4	57.2	40.7	16.5
6 Wheeler Truck	127.2	86.4	30.0	62.4	44.4	108
10 Wheeler Truck	148.4	100.8	35.0	72.8	51.8	150
12 Wheeler Truck	159.0	108.0	37.5	78.0	55.5	294
14 Wheeler Truck	169.6	115.2	40.0	83.2	59.2	322
18 Wheeler Truck	190.8	129.6	45.0	93.6	66.6	378
22 Wheeler Truck	212.0	144.0	50	104.0	74.0	462
Demand	235.5	150.5	588	326	433.5	1733.5

involves two steps of finding the:

1. basic feasible solution

2. optimal feasible solution

Thus, in finding the basic feasible solution, we use the vogel's approximation method and also the Modified Distribution method in finding the optimal feasible solution. The MATLAB programming language was used in all of our computations because of its ability to handle matrices very fast and accurately. Unfortunately, MATLAB had no inbuilt function to help solve the Transportation problem. Luckily, there was a way around this. MATLAB has a large community of programmers which aid in the exchange of code thus making it of high interest to its users. The code presented by was employed to help solve the transportation problem given our current data.

4.5 Basic Feasible Solution

The initial step to solving the Transportation problem is to finding the basic feasible solution which in our case we are going to employ the services of the Vogel's Approximation method to help us find this. Thus, the objective function will be given as

$$\text{Minimise} \quad \sum_{i=1}^{5}\sum_{j=1}^{9} t_{ij}x_{ij} \tag{4.3}$$

Subject to

$$\sum_{j=1}^{5} x_{ij} \leq s_i \qquad \text{for } i = 1, 2, \ldots, 9$$

$$\sum_{i=1}^{9} x_{ij} \geq d_j \qquad \text{for } j = 1, 2, \ldots, 5$$

We check whether the formulated transportation problem is balanced or not. For a balanced transportation problem

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Hence

$$\sum_{j=1}^{5} d_j = \quad 235.5 + 150.5 + 588 + 326 + 433.5 = 1733.5$$

$$\sum_{i=1}^{11} s_i = \quad 1 + 2 + 16.5 + 108 + 150 + 294 + 322 + 378 + 462 = 1733.5$$

Since the two computed values are the same it implies the formulated transportation problem is balanced. Applying the Vogel's Approximation Method on the given data in MATLAB yielded the following results as shown in Table 4.11.

Table 4.11: A Tableau Representation of the basic feasible Solution

Source/Destination	Ada Foah	Ada West	Lekma	Prampram	Tema	Supply
$\frac{1}{2}$ Ton Pickup	95.4 0	Inf 0	Inf 0	46.8 1	Inf 0	1
1 Ton Pickup	Inf 0	Inf 0	25.0 0	Inf 0	37.0 2	2
4 Wheeler Truck	116.6 16.5	79.2 0	27.4 0	57.2 0	40.7 0	16.5
6 Wheeler Truck	127.2 108.0	86.4 0	30.0 0	62.4 0	44.4 0	108
10 Wheeler Truck	148.4 111.0	100.8 39.0	35.0 0	72.8 0	51.8 0	150
12 Wheeler Truck	159.0 0	108.0 111.5	37.5 0	78.0 182.5	55.5 0	294
14 Wheeler Truck	169.6 0	115.2 0	40.0 0	83.2 142.5	59.2 179.5	322
18 Wheeler Truck	190.8 0	129.6 0	45.0 126.0	93.6 0	66.6 252.0	378
22 Wheeler Truck	212.0 0	144.0 0	50 462.0	104.0 0	74.0 0	462
Demand	235.5	150.5	588	326	433.5	1733.5

Thus the basic feasible solution of the product of the time and allocation

$$\sum_{i=1}^{5}\sum_{j=1}^{9} t_{ij}x_{ij} = 1(46.8) + 2(37) + 16.5(116.6) + 108.0(127.2) + 111.0(148.4) + 39(100.8) + 182.5(78) + 142.5(83.2) + 179.5(59.2) + 126.0(45) + 252(66.6) + 462(50) + 111.5(108.0) = 130500$$

Thus from the above, the optimal time thus becomes

$$T = \sum_{i=1}^{9}\sum_{j=1}^{5} ct_{ij}$$

$$= 46.8 + 37 + 116.6 + 127.2 + 148.4 + 100.8 + 108.0$$

$$+ 78 + 59.2 + 83.2 + 45 + 50 + 66.6$$

$$= 1066.8 \text{ (Minutes)}$$

where

$$c = \begin{cases} 1, & \text{Allocation is made from source } i \text{ to destination } j \\ 0, & \text{otherwise} \end{cases}$$

4.6 Optimal Feasible Solution

To check for optimality, we apply the idea of the Modified distribution method on the basic feasible solution presented in the previous table. Similarly, we supply the basic feasible solution to the Modified Distribution algorithm yielding Table 4.12

Table 4.12: Optimal Feasible Solution's Tableau

Source/Destination	Ada Foah	Ada West	Lekma	Prampram	Tema	Supply
1/2 Ton Pickup	95.4 1	Inf 0	Inf 0	46.8 0	Inf 0	1
1 Ton Pickup	Inf 0	Inf 0	25.0 0	Inf 0	37.0 2	2
4 Wheeler Truck	116.6 16.5	79.2 0	27.4 0	57.2 0	40.7 0	16.5
6 Wheeler Truck	127.2 108.0	86.4 0	30.0 0	62.4 0	44.4 0	108
10 Wheeler Truck	148.4 111.0	100.8 39.0	35.0 0	72.8 0	51.8 0	150
12 Wheeler Truck	159.0 0	108.0 111.5	37.5 0	78.0 182.5	55.5 0	294
14 Wheeler Truck	169.6 0	115.2 0	40.0 0	83.2 142.5	59.2 179.5	322
18 Wheeler Truck	190.8 0	129.6 0	45.0 126.0	93.6 0	66.6 252.0	378
22 Wheeler Truck	212.0 0	144.0 0	50 462.0	104.0 0	74.0 0	462
Demand	235.5	150.5	588	326	433.5	1733.5

4.7 Discussion

Given the optimal feasible solution in the previous table, then minimum value associated to minimising the time and optimising allocation would be given by

$$
\begin{aligned}
\sum_{i=1}^{5}\sum_{j=1}^{9} t_{ij}x_{ij} =& 1(95.4) + 2(37) + 16.5(116.6) + 108.0(127.2) + 111.0(148.4) + 39(100.8) \\
&+ 111.5(108.0) + 78(182.5) + 83.2(142.5) + 179.5(59.2) + 126(45) \\
&+ 252(66.6) + 50(462.0) \\
=& 130470
\end{aligned}
$$

As part of our assumptions, we can clearly see that the given times are not unit times, that is they do not give the times taken for the vehicle to reach their destinations given that they are empty. Therefore to introduce the concept of unitary time, we must then divide each product of time and allocation by the given time. This also affirms that the time taken is independent of the allocation, thus the minimum times associated to these travels will just be the sum of times for those with allocations. Thus, our optimal feasible time will be

$$T = \sum_{i=1}^{9} \sum_{j=1}^{5} w_{ij} t_{ij}$$

where

T is the minimum time

$$w_{ij} = \begin{cases} 1, \text{Allocation is made from source } i \text{ to destination } j \\ 0, \text{otherwise} \end{cases}$$

Hence from the above, the minimum time thus becomes

$$\begin{aligned} T &= \sum_{i=1}^{9} \sum_{j=1}^{5} w_{ij} t_{ij} \\ &= 95.4 + 37 + 116.6 + 127.2 + 148.4 + 108.0 + 78 + 83.2 \\ &+ 59.2 + 45 + 66.6 + 50 + 100.8 \\ &= 1115.4 \text{ (Minutes)} \end{aligned}$$

Next, a breakdown of the minimum time and allocation per each destination is presented.

Ada Foah: The optimal allocation for the Ada Foah District is represented in the first column of the optimal feasible solution's tableau. The allocation is simplified in Table 4.13

Table 4.13: Allocation and Time taken Table for Ada Foah

Source	Allocation(Tons)	Time Taken
1/2 Ton Pickup	1	95.4
4 Wheeler Truck	16.5	116.5
6 Wheeler Truck	108.0	127.2
10 Wheeler Truck	111.0	148.4

Hence the optimal time from the NADMO office to the Ada Foah District, is the sum of the time taken column in Table 4.13,thus,

$$95.4 + 116.6 + 127.2 + 148.4 = 487.60 \text{ (Minutes)}$$

Now with the Allocation, we can see that two $\frac{1}{2}$ Ton Pickups must be assigned with 1 ton of relief items to this district, eleven 4 wheeler truck must be assigned to this district. In addition, twelve 6 wheeler trucks and eleven 10 wheeler trucks must be assigned to this district.

Ada West: Also,the optimal allocation for the Ada West District is represented in the second column of the optimal feasible solution's tableau.The allocation is simplified in Table 4.14

Table 4.14: Allocation and Time taken Table for Ada West

Source	Allocation(Tons)	Time Taken
10 Wheeler Truck	39	100.8
12 Wheeler Truck	111.5	108.0

Hence the minimum time from the NADMO office to the Ada West District, is the sum of the time taken column in Table 4.14, thus,

$$100.8 + 108.0 = 208.8 \text{ (Minutes)}$$

Now with the Allocation, we can see that three 10 wheeler truck must be assigned with 1 ton of relief items to this district, five 12 wheeler truck must be assigned to this district.

Lekma: The optimal allocation of relief items and the optimised times for these allocation to reach the Lekma district which is represented by the third column in the Optimal feasible solution tableau is given in Table 4.15.

Table 4.15: Allocation and Time taken Table for Lekma

Source	Allocation(Tons)	Time Taken
18 Wheeler Truck	126.0	45.0
22 Wheeler Truck	462.0	50.0

The associated minimum time is calculated as

$$45.0 + 50.0 = 95.0 \text{ (Minutes)}$$

Thus, four 18 wheeler trucks and thirteen 22 wheeler trucks must be assigned to this district in order to meet their demand in optimal time.

Prampram: In addition, for Prampram District which is represented in the fourth column of the optimal feasible solution's tableau, the allocation is be simplified in Table 4.16

Table 4.16: Allocation and Time taken Table for Prampram

Source	Allocation(Tons)	Time Taken
12 Wheeler Truck	182.5	78.0
14 Wheeler Truck	142.5	83.2

Thus the optimized time for relief items to reach this district is calculated as

$$78 + 83.2 = 161.2 \text{ (Minutes)}$$

With this, the implication is that eight 12 wheeler trucks and six 14 wheeler trucks must be assigned to the Prampram district in other to minimise time and also meet their demand of relief items.

Tema: Lastly, for the fifth column of the optimal feasible solution's tableau which represents the Tema district. The allocation is simplified in Table 4.17

Table 4.17: Allocation and Time taken Table for Tema

Source	Allocation(Tons)	Time Taken
1 Ton pickup	2	37.0
14 Wheeler Truck	179.5	59.2
18 Wheeler Truck	252.0	66.6

Hence the optimal time for relief items to reach the Tema District is

$$37.0 + 59.2 + 66.6 = 162.8 \text{ (Minutes)}$$

Breaking down the optimal allocation, two 1 ton pick ups, eight 14 wheeler trucks and nine 18 wheeler trucks must be assigned relief items to district in order to minimize time and also meet their demand of relief items.

Chapter 5

Conclusion and Recommendations

5.1 Overview

This chapter talks about the summary of results in chapter 4 and conclusions drawn from these results, and finally recommendations.

5.2 Summary of Results

Table 4.7 shows the overall number of the different vehicle types available at the NADMO head office and their respective carrying capacities. Table 4.9 shows the average time by these tracks to different disaster zones and also highlights those vehicles that are unable to travel to some of the destinations, represented by a single dash (-). To determine the time optimality, the problem was solved using the transportation problem, where the Vogel's approximation method was first used to find the basic feasible solution (Table 4.11). Then by the application of the Modified distribution method on the basic feasible solution, we obtained our optimal solution (Table 4.12).

5.3 Conclusion

Comparison of the two Tables (4.11 and 4.12) showed no change or difference in the time required for a loaded vehicle to reach its destination, but rather a massive deficit in the number of vehicles required to meet the optimal time for relief delivery. Below are tables illustrating these deficits.

NOTE: Vehicle type column- shows vehicle types needed, No. Available column- shows number of that vehicle type available to the assembly, and Deficit column-shows the number of vehicle type deficit of the assembly.

Table 5.1: Vehicle Deficit for Ada Foah

Vehicle type	No. Required	No. available	Deficit
1/2 Ton Pickup	2	1	-1
4 Wheeler Truck	11	2	-9
6 Wheeler Truck	12	1	-11
10 Wheeler Truck	16	10	-6

Table 5.2: Vehicle Deficit for Ada West

Vehicle type	No. Required	No. available	Deficit
10 Wheeler Truck	3	1	-2
12 Wheeler Truck	5	2	-3

Table 5.3: Vehicle Deficit for LEKMA

Vehicle type	No. Required	No. available	Deficit
18 Wheeler Truck	4	5	+1
12 Wheeler Truck	13	5	-8

Table 5.4: Vehicle Deficit for Prampram

Vehicle type	No. Required	No. available	Deficit
12 Wheeler Truck	8	2	-6
14 Wheeler Truck	6	3	-3

Table 5.5: Vehicle Deficit for Tema

Vehicle type	No. Required	No. available	Deficit
1 Ton Truck	2	1	-1
14 Wheeler Truck	8	4	-4
18 Wheeler Truck	9	3	-6

5.4 Recommendation

Based on the findings, there is no need to buy new vehicles, but we recommend that there be reshuffle of vehicles for the recommended types to be sent to the appropriate destination, to ensure fast delivery of relief in the event of any disaster (specifically floods).

Appendix

Example of simplex method

$$\text{Maximize} \qquad Z = 3x_1 + 5x_2$$

Subject to

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

We will change all the inequality signs into equality signs by introducing slack variables. The augmented form will be

$$\text{Maximize} \qquad Z = 3x_1 + 5x_2$$

Subject to

$$x_1 + x_3 = 4$$

$$2x_2 + x_4 = 12$$

$$3x_1 + 2x_2 + x_5 = 18$$

$$x_j \geq 0, \text{ for } j = 1, \ldots, 5$$

x_3, x_4, x_5 are called slack variables.

	Coefficient						
Basic variables	x1	x2	x3	x4	x5	RHS	Ratio
x3	1	0	1	0	0	4	
x4	0	2	0	1	0	12 →	$\frac{12}{2} = 6$ ← Departing
x5	3	2	0	0	1	18 →	$\frac{18}{2} = 9$
Z	-3	-5 ↑ entering	0	0	0	0	

The largest negative value in z determines the entering basic variable and the minimum ratio determines departing basic variable. The minimum ratio is 6, which means that x_4 which is the pivot row will leave the next simplex tableau and x_2 which is the pivot column will enter the next simplex tableau. Since x_2 is taking the position of x_4, then we need to reproduce the first tableau pattern of coefficients in the $x_4(0, 0, 1, 0)$ for x_2 in the next tableau. Divide the pivot row by the pivot number. Make sure the numbers in the same column with the pivot number are the same. Add or subtract the new row to/from the various rows depending on their sign. This is done to make all the numbers in the same column with the pivot number to be zero. We continue with the elementary row

	Coefficient						
Basic variables	x1	x2	x3	x4	x5	RHS	Ratio
x3	1	0	1	0	0	4	$\frac{4}{1} = 4$
x2	0	1	0	$\frac{1}{2}$	0	6	
x5	3	0	0	-1	1	6	$\frac{6}{3} = 2$ Departing
Z	-3 ↑ entering	0	0	$\frac{5}{2}$	0	30	

operation until optimality is reached (that is when there is non-negative values in the z values). x_1 will enter the next simplex tableau and x_5 will leave the tableau. Since there is negative number in the z values we continue with the elementary row operation till all the negative values becomes non-negative values. In this tableau, there are no negative elements in the bottom row. We have therefore

Basic variables	Coefficient					RHS	Ratio
	x1	x2	x3	x4	x5		
x3	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x2	0	1	0	$\frac{1}{2}$	0	6	
x1	1	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	2	
Z	0	0	0	$\frac{3}{2}$	1	36	

determined the optimal solution to be $(x_1, x_2, x_3, x_4, x_5) = (2, 6, 2, 0, 0)$

$$z = 3x_1 + 5x_2$$

$$z = 3(2) = 5(6) = 36$$

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