KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

TOPIC

PORTFOLIO INVESTMENT OPTIMIZATION

PORTFOLIO SELECTION PROBLEM

A CASE STUDY OF FIVE HYOTHETICAL COMPANIES ON THE GHANA STOCK EXCHANGE

BY:

SAMUEL NYAMEKYW BENYAH

THIS THESIS IS PRESENTED TO THE KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF MASTER OF SCIENCE IN INDUSTRIAL MATHEMATICS.

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SUPERVISOR :

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DECLARATION

I declare that with the exception of certain documentary and resources which I cited and duly acknowledged, this submission is the output of my own research and that it has neither in whole or in part been presented for another degree elsewhere.

Furthermore, I also declare that neither my supervisor nor any other person but the author alone is responsible for whatever errors and omissions that might appear in the work.

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Dr. S. K Amponsah		21/05/12
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Last but not the least; I devote this study to my Mother, Elizabeth Wallace and my deceased father, Isaac Kwesi Benyah (late), for being my source of inspiration.

May God bless us all.

DEDICATION

I dedicate this thesis to God for given me the grace to come this far. To my father Isaac Kwesi Benyah (Late), my mother Elizabeth Wallace and family who through thick and thin have supported me in diverse ways and stood by me through my education.

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ABSTRACT

Portfolio investment and Optimization involves efficient portfolio selection strategy that maximizes returns and minimizes risk. The key objective of portfolio managers in asset management and investment diversification application is to optimize portfolio investment by either maximizing return or minimizing risk. Optimization models play a critical role in determining portfolio in these risk minimization strategies for investors. This thesis rides at the back of the traditional Harry Markowitz Mean Variance optimization approach which has only one objective, which fails to meet the demand of investors who have multiple investment objectives. The objective of this study is presents a multi-objective approach to securities on portfolio optimization problems. We carry out the operations research of portfolio selection model where the main objective is to minimize risk and maximize the individual's return. We compare the Mean Absolute Deviation (MAD) performance of this model to that of the Markowitz's quadratic programming model. Using the risk aversion utility function we use the optimal portfolio strategy to analyze the various risk tolerance. Detailed analysis based on a financial toolkit utility function is used to access the risk levels for the computed returns on the optimization. Chapter three outlines the methodology of the various models including our MAD model study. Data consisting of five shares securities over twelve months is used to examine whether these various formulations provide similar portfolios or not.

The Linear Programming of Mean Absolute Deviation (MAD) model would be used to help in the selection of the right portfolio for an investor through the portfolio utility risk aversion function. In conclusion the MAD model would be suitable for historical data at least twelve months or more. Using the risk utility it is recommended that for large historical data investors can use the MAD model in the selection of a portfolio and a minimal risk level for their clients. For short-term investments the Harry Markowitz model would be a suitable model for this resource allocation and investment decisions.

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CHAPTER 1

1.0 INTRODUCTION

Businesses and individuals are concerned with returns on investments and the risk factors arising because of the market volatility. Prices of goods and services in a competitive era determine the prices of shares and stocks on the trading markets. Private businesses such as banks, financial institutions and other share holders have to make critical decisions on the value of their portfolio investments in the future.

The time value of money is very important for various authorities such as business, project managers, and individuals to put together what is termed efficient frontiers to overcome the market volatility which is mostly affected by inflation and rate fluctuations.

Consider a scenario where we have to select the times of various traffic lights in a section of a city so as to obtain an optimum control of traffic flow. If we know the transportation needs of all the people in a city, before suggesting an implementation of "optimum traffic flow, we need a selection criterion by which we can compare patterns of traffic-light durations.

One way of optimizing this is to assign as cost to each decision the total amount of time taken to make all the trips within this section. An alternative goal may be to minimize the maximum waiting time that is spent at stop lights in each trip.

It is important to note that mathematical models are not meant to replace management ~~tools but~~ are technological improvement that facilitates the decision making process provided by Financial and Project Managers. They are new tools or techniques to be used side-by-side with other managerial techniques for improving performance of a portfolio.

Let's take for instance a SSNIT contributor whose pension fund portfolio is made up of a mixture of domestic and local assets such as shares, bonds and estate property. We want to know how risky it is to hold that portfolio.
Mathematical tools can be used to aid in the decision of this concerned pensioner who is a 28-year old man who wants to use the portfolio to purchase a pension at the age of 60. The expected return of this portfolio as he hopes, must appreciate very well in order to purchase a better quality of life at retirement than her current banking savings level.

To assist the man make an informed decision, certain mathematical and financial analysis must be done to alleviate his fear of risk so that, it will deliver low output and therefore make him settle for a significantly lower quality of life. How do we determine this risk for him to invest his wealth at this age in the selected portfolio?

I agree with the author Barry du Toit (2004) in his book "Risk, theory, reflection: Limitations of the stochastic model of uncertainty in financial risk analysis" which states, the world is a complex place with very powerful forces evolving and interacting in complex ways".

There had been periods of stability during the industrialized age that had increased the national and personal wealth and quality of life of individuals and organizations.

However, since the future is uncertain as it was in the recent world economic downturn of many economies, we need to be careful in extrapolating from a model to factor in all these risk. As it is with an organization so the quality or standard of life may radically be altered by all sorts of economic developments, as it is with the value of the assets that are invested for the future.
The challenge here is to find an appropriate model that is used for underestimating the risk and employ all the intelligent guesses we can about the future, and build those into our investment plans.

Risk analysis and Management decisions incorporate these uncertain predictions for the future for which we have reasonable grounds, including future uncertainty which we can model, as well as cater for the uncertainties which we cannot model at all. In fact we do this sort of thing in many areas of life all the time, such as in the study of medicine and health, due to this and a number of other factors. Everyone has a de facto asset allocation scheme, even if everything they have is in a bank current account (or indeed in an overdraft). So we don't have to arrive at the correct asset allocation scheme, just a better one than most people currently have, and one based on plausible judgments. We have the tools already available for this. At the same time, we also need to be sure that we are not blinded by the dominance of some of our more successful risk technologies.

The argument is simply that we do not plan for the future as well as we can because our understanding of financial risk has been distorted by the phenomenal success of a particular model of risk. The areas where that model is most appropriate have prospered, and the areas where it is least useful have either been neglected, or else have simply been approached using the conventional methods, regardless of relevance.

In the following chapters we consider the developments of portfolio selection model; firstly, Markowitz's idea of portfolio diversification, and secondly, the Konno and Yamazaki mean absolute deviation portfolio model.

Definitions of Terminology used in the Thesis.

- **a.** Portfolio: the lists of investments such as shares, bills, notes, bonds, individual stocks, ~~mutual funds~~ etc, held by a person or business.
- **b.** Risk: this is the possibility of losing an asset either stocks, bonds, mutual funds, etc, an undesirable attribute of a random outcome of a given financial investment.
- **c.** Investment Optimization means;
    - i. Minimization of risk for a specific expected return

ii. Maximization of the expected wealth for a specific risk
iii. Minimization of risk and maximization of return using a specific risk aversion factor.
iv. Minimization of risk regardless of the expected return
v. Maximization of the expected return regardless of the risk
vi. Minimization of expected return regardless of the risk.

The mathematical problem of portfolio optimization was first introduced in the fifties by a Nobel Prize winner Professor Harry Markowitz. This model called Mean-Variance model has been the foundation for many investment in the financial industries.

In this study we are going to consider the selection of assets and equities those investors rely on to maximize returns or minimize risk on their portfolios. In view of this, various expected returns models were introduced beginning with Markowitz Mean-Variance optimization theory used by many financial industries and researchers today. Historical data of five listed companies from the Ghana Stock exchange sites would be used in the study. The main reason for this is that, Ghana Stock Exchange (GSE) is the principal stock exchange of Ghana. The exchange was incorporated in July 1989 with trading commencing in 1990. It currently has around 30 listed companies and 2 corporate bonds. All types of securities can be listed. Criteria for listing include capital adequacy, profitability, spread of shares, years of existence and management efficiency. The GSE is located in Accra.

In 1993, the GSE was the sixth best index performing emerging stock market, with a capital appreciation of 116%. In 1994 it was the best index performing stock market among all emerging markets, gaining 124.3% in its index level. 1995's index growth was a disappointing 6.3%, partly because of high inflation and interest rates. Growth of the index for 1997 was 42%, and at the end of 1998 it was 868.35 (see the 1998 Review for more information). As of October 2006 the market capitalization of the Ghana Stock Exchange was about 111,500bil cedis ($11.5 billion). As of December 31 2007, the GSE's market capitalization was 131,633.22bil cedis. In 2007, the index appreciated by 31.84% (see the "Publications" section on the GSE's website for more information).

The manufacturing and brewing sectors currently dominate the exchange. A distant third is the banking sector while other listed companies fall into the insurance, mining and petroleum sectors. Most of the listed companies on the GSE are Ghanaian but there are some multinationals. The Analysis of this study is based on these companies for which and investor would make a choice to produce a least risk and the highest return.

1.1. BACKGROUND TO THE STUDY

Investments of selected assets and / or equities such as stocks and bonds are key for an investor. The Financial Analyst must understand that the investor is concerned about the diversification of assets such that it yields the highest returns and a measureable level of risk.

To help Financial Analyst address the apprehension of their clients, Markowitz (1952) Theory, said expected return would be measured by mean and the risk in collection of the investments called herewith portfolio investment is measured by variance (or standard deviation).

The sum of the individual correlation of risk of a properly constructed portfolio from equities such as shares in leading markets is less than the risk from the individual investments. All these analysis must be considered to address the selection of assets and its investments in a reliable stock market.

In summary, the complexity of a portfolio investment and it optimization is of prime importance to an investor and that the investor is guided by a critical selection model that models risk and returns with a changing markets rates trend to take away the fears of losing assets.

1.2. PROBLEM STATEMENT

For the Ghanaian economy to continuously keep on growing, financial institutions and individuals need to invest in reliable ventures. This would help individuals plan for the future and for companies also to pay higher dividends and improve on shareholder capital. The concern is not that Ghanaians or individuals don't want to invest, but what amount to invest and into what company is the problem. This thesis seeks to address the selection problem for these investors.

Portfolio optimization plays a critical role in the determination of portfolio strategies for these investors. Investors hope to achieve from portfolio optimization by maximizing portfolio returns and minimize portfolio risk. An optimal portfolio is determined by an investor's risk-return preference called aversion.

One of the primary responsibilities also by financial advisors is to help their clients in developing a confident investment portfolio in which the returns are maximized and risk is minimized. You will need to develop an algorithm that will provide good estimates for investments such as stock return and use these estimates to build an optimal portfolio.

Markowitz foundation Model and other similar developments are used in this thesis to provide answers. In these models reward and risk are measured by expected return and variance of a portfolio respectively. Expected return is calculated based on historical performance of an asset, and variance is a measure of the dispersion of returns.

The key to achieving investment objectives is to provide an optimal portfolio strategy which shows investors how much to invest in each security in a given portfolio. Therefore, the decision variable of portfolio optimization problems is the percentage of security weights for these investments.

In summary, this project makes extensive use linear programming and the operations research method called Optimization and other statistical measures such as mean, variance, Covariance and standard deviation to help give confidence to invest and solve the selection problem. To aid our computation, mathematical tools such as MATHLAB, LINGO other excel tools would be employed to do analyses of data collected for a period of one year on the Ghanaian stock market from the internet.

1.3. OBJECTIVE OF THE STUDY

This study employs the effectiveness of optimization techniques to solve a portfolio selection problem. Optimization technique was employed in the selection of stocks in the Ghana Stock market. The objectives of applying this technique are:

To compare Portfolio Optimization Models that jointly selects the appropriate investments among risky assets and minimizing the risk of investment;

To identify the best portfolio optimization model that captures returns and the performance of risk (as a result of uncertainty);

Predict the investment portfolio selection criterion that minimizes risk and maximizes return using specified risk aversion factor;

Empirical validation of the model through risk aversion utility function using market data to enable financial decision.

1.4. METHODOLOGY

The financial decision considered is a portfolio selection of common stocks. It is assumed that the investor has a budget or wealth and seeks to form a portfolio from risking assets in the face of future temporal predictive uncertainty.

In addition, economic conditions of the future are uncertain which plays a vital role in the financial decision that an investor makes so as to optimize portfolio.
In this thesis we compare three different models which either uses linear programming formulation, statistical or numerical analysis models. These models each would generate returns and risk as a base line for the risk aversion financial index toolkit which

is used to determine the investor's level of acceptable risk for which he/she would invest with confidence and eliminate the fears of the selection problem.

The thesis puts together practical scenarios for an investor and tries to use the tools, variables or parameters discussed above to generate an efficient frontier for investors.

1.4.1 THESIS REPORT ORGANIZATION

The study of portfolio optimization to solve the selection problem is outlined as follows:

Chapter 1 opens the study by summarizing given a brief discussion on portfolio investment and the challenges that investors go through in making a selection and rebalancing their portfolios over a time horizon. It also discusses the journey of a financial model and the modern direction of this all important investment application.
Chapter 2 focuses on the literature reviews of previous models and research direction which introduces the background knowledge and the terminology of later chapters. Mean-risk portfolio optimization model and various risk measures would be reviewed and discussed.

In this chapter we consider the formulation of a Linear programming model of determining the model using the random variables to make up the portfolio optimization linear or quadratic functions used for the formulation.

Chapter 3 is a ~~numerical and model parameters~~ discussion on the return and risk of portfolio investments to help in the financial decision making process.

Chapter 4 is based both on the statistical characteristics of the historical data set and scenario generation of real world market data on Ghana stock investment. The computations and test were described and empirical results are discussed. The numerical analysis of the various models from the previous chapter is presented and comparison of the result between different risks levels of return incremental done. The model is solved by the Math Lab, Excel solver and other modern tools.

Chapter 5 represents the discussion of results application of the optimization model as well as discussions on the limitations of the model.

I.5 JUSTIFICATION

I. The future is uncertain and every investor would want to consider the risk before any investment is made. Therefore the mathematical model to assist in making the best decision.

II. It is very important to analyze the risk associated with every investment to assist in assets diversification.

1.6 LIMITATIONS

i. This research does not include transaction cost, hedging which may later be considered to give the investor more confidence to invest in diversified assets.

CHAPTER 2

LITERATURE REVIEW OF THE SUBJECT AREA

2.0 INTRODUCTION

Optimization tools have become the pivot around which most Financial Analysts, Project Managers and organizations use to make decisions on investments. The objective of every investment is to minimize risk or maximize profit.

However, the decision or selection criteria to make are the thorn of every financial Advisor or Investor. Over the years Investors and other people have had to answer which selection criteria to obtain the portfolio that can attain a specific rate of return with a minimum risk.

Markowitz (1952) developed a model called the Mean –Variance model (MV) which uses the mean as the return and the variance as the risk on the investments.

Markowitz's work on portfolio diversification is key in the history of the development of this discipline, perhaps the foundational point. In his paper "Portfolio Selection", he emphasized that investors are concerned with risk and the return or the benefits of their securities invested. Investment analyst would study the volatility of the market and the various opportunities that offer the least risk and help the investor formulate a portfolio from these securities.

Markowitz in his theory suggested that investors would consider the overall risk-return benefits instead of considering only compiling portfolios from securities that had attractive risk-reward characteristics. It was based on these suggestions that he proposed the random variables standard variation, variance, mean and correlation. This led to the ability to calculate the expected return and volatility of any portfolio constructed with these securities.

Although Markowitz published his ideas in 1952, in what was pretty much their mature form, they only came to influence financial decision-making in the 1970's. Rather remarkably, the modern edifice of mathematical modeling, risk analysis and financial engineering only really began in that decade, although many of the important ideas had been around for years, and had in fact been taken to relatively advanced levels in some other statistical and economic sciences.

Many reasons have been put forward for the timing of the rise of modern financial engineering, including the volatility in global markets in the 1970's, the rise of inflation in stagnating economies, the growth in computational power and the spread of access to computational technology, and so on. For whatever reason the time was right, and Markowitz's ideas provided the necessary conceptual framework for other models to emerge. The reason why Markowitz's model has not been used extensively is summarized in the preceding chapters and also be found in (Elton, Gridber, Padberg, 1976; Konno and Yamazaki, 1991).

Sharpe (1971) and stone (1973) tried to reformulate the portfolio problem into linear programming model. Konno and Yamazaki (1991) proposed a new portfolio optimization modle as alternative to Markowitz's Mean-Variance model. The employed the L1 mean absolute deviation as a risk measure instead of the variance introduce by Markowitz in order to over the computational difficulty.

2.1 Markowitz's Mean – Variance Model (MV) Review

Markowitz's "Portfolio Selection" paper in 1952 included the first significant approach aimed at optimizing multi-asset portfolios. The Markowitz Model looks at the mean and variance of returns to find the minimum point in a feasible problem space. This helps minimize the deviation of the portfolio while meeting other simple constraints. Computations of the Markowitz Model deal with covariance matrices that highlight the relationship between all individual assets and can get very large with increasing numbers of assets. As a result, computational complexity associated with Markowitz is often so high that it prevents real-time information processing. For example, a portfolio with 100 assets can have a 100 X 100 covariance matrix, making it easy to see how such a model can become computationally complex. Over the years, many new techniques have tried to address the shortcomings of the Markowitz Model by way of simplifying the covariance matrix and decreasing the computational complexity. Some of these models, such as CAPM, have received much attention and success.

One new idea however, which avoids dealing with covariance matrices altogether, is Mean Absolute Deviation. By avoiding covariance matrices, MAD stays away from the computational difficulties associated with quadratic models. Since MAD only deals with mean absolute deviation of each stock it is considered a linear model and hence is much simpler to solve.

Tobin (1958) expanded on Markowitz work and added a risk-free asset to the analysis in order to leverage or de-leverage, as appropriate portfolios on the "efficient frontier" leading to the super efficient portfolio and capital market line. With leverage, portfolios on the capital market line could outperform portfolios on the efficient frontier. Sharpe (1964) then prepared a capital asset pricing model that noted that all the investors should hold the market portfolio, whether leveraged or de- leveraged, with positions on the risk-free assets.

Minimize

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\sigma_{ij}X_iX_j\right)$$

Subject to **2.1**

$$\sum_{j=1}^{n} r_j X_j \geq \rho M_o$$

$$0 \leq X_j \leq U_j \quad j=1\ldots n$$

Where;

σ_{ij} = covariance between assets i and j,

X_j = the amount invested in asset j,

r_j = the expected return of asset j per period

ρ = a variable showing the minimal rate of return by an investor

M_o = total amount of fund to be invested into all the aassets

U_j = maximum amount of money which can be invested into asset j.

2.2 Konno and Yamazaki (1991) proposed the Mean Absolute Deviation (MAD)

Konno and Yamazaki (1991) were the first researchers to present the Mean Absolute Deviation (MAD) portfolio optimization model. The MAD model was designed to retain the advantages of the Mean-Variance (Markowitz's) model while removing some of its shortcomings, thereby making the MAD model more suitable for use by working brokers. The optimum MAD model portfolio is the one that minimizes the return's average absolute deviation subject to the restriction of a given mean return.

The MAD model was formulated with linear functions that are solved using linear programming techniques, thus avoiding the difficulties presented by quadratic programming. The model's construction process consists basically of creating a non-linear formulation that approximates a linear one. This non-linear form is the absolute deviation from the portfolio's mean return, described as follows:

Konno and Yamazaki (1991) introduced the L_1 risk function called the mean absolute deviation;

Minimize

$$W(x) = E\left|\sum R_j X_j - E\left[\sum_{j=1}^{n} R_j X_j\right]\right|$$

Subject to **2.2**

$$\sum_{j=1}^{n} E[R_j]X_j \geq \rho M_o$$

$$\sum_{j=1}^{n} X_j = M_o$$

$$0 \leq X_j \leq U_j \quad j=1\ldots n$$

Konno and Yamazaki assumed that the expected variable value of the random variable can be approximated by the average from the data as indicated below

$$r_j = E[R_j] = \frac{1}{T}\sum_{t=1}^{n} r_{jt}$$

Where r_{jt} is the realization of random variable R_j during the period t (where t=1...T).

The above W(x) can be approximated by; $1/_T \sum_{t=1}^{T} \left|\sum_{j=1}^{n} (r_{jt} - r_j) X_j\right|$

If we denote $a_{jt} = (r_{jt} - r_j)$ for the same conditions above for t, model 2.1 can be expressed as below;

Minimize $\frac{1}{T}\left[\sum_{t=1}^{T} Y_t\right] \quad t = 1\ldots T$

Subject to

$$Y_t + \sum_{j=1}^{n} a_{jt} X_j \geq 0 \quad t = 1\ldots T$$

$$Y_t - \sum_{j=1}^{n} a_{jt} X_j \geq 0 \quad t = 1\ldots T$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq X_j \leq U_j \quad j=1\ldots n$$

Konno and Yamazaki's concluded that there were some advantages over the traditional Markowitz's model as follows;

I. The model does not make use of the covariance as in the traditional model

II. Solving this model is much easier than the Markowitz's quadratic model

III. The maximum number of assets invested in can be projected to **2T+2**($U_j = \infty$) as against Markowitz's model that can only be **n** assets.

IV. T can serve as a control variable to indicate the number of assets.

Feinstein and Thapa (1993) revisited the MAD portfolio optimization model and came up with a new model that is equivalent to Konno and Yamazaki's model in terms of number of assets to T+2 by subtracting a non-negative surplus $2V_t$ **and** $2W_t$ to reduce the linear form of the MAD model to and equality formulation as below;

$$Y_t + \sum_{j=1}^{n} a_{jt} X_j = 2V_t \qquad t = 1 \ldots T$$

$$Y_t - \sum_{j=1}^{n} a_{jt} X_j = 2W_t \qquad t = 1 \ldots T$$

This according the Feinstein and Thapa's (1993) reduced the transaction cost of the investments. The model therefore became

Minimize

$$\sum_{t=1}^{T} (V_t + W_t)$$

Subject to

$$V_t - W_t - \sum_{j=1}^{n} a_{jt} x_j$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq X_j \leq U_j \quad j=1\ldots n;\ V_t \geq 0\,,\ W_t \geq 0$$

Chang (2005) reformulated Feinstein and Thapa's optimization model of T+2 assets and introduced a continuous variable d_t to handle the challenges in the number of iterations involved when the number of assets for the investor.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

Investors look out for investments that have minimum risk but yield the highest profit. For many financial analyst and project evaluation managers, the decision to invest into any portfolio is of highest importance. Many model evolved from Henry Markowitz model is used in analyzing the feasibility and the final returns for and investment in a volatile environment.

In this chapter the LP of MAD model is considered to facilitate in the selection of an optimized five stock investment from the stock market in Ghana.

Many constraints are often taken into consideration when models are built to select an optimal portfolio. However, to make a model for portfolio selection feasible, it is very rare that all of the existing constraints are modeled. Moreover, in mathematical models, numerous assumptions are often made about the market and the assets even though these assumptions do not always hold. Nevertheless, they are pretty good approximations of reality, and therefore still allow accurate and feasible modeling.

The most common assumptions that apply to many mathematical models representing financial problems include the following:

a. Market price is not affected by the actions performed. In general, this is not true since increase in demand leads to increase in price and vice versa. However, if the trading happens in small quantities, the effects on the market will be insignificant.

b. The liquidity assumption states that, at any time, an individual can buy or sell infinite amounts of any asset in a market. This might be true for the market of foreign currency exchange since these assets exist in (almost) infinite amounts, but this assumption is never true if an investor is trying to buy an "infinite" amount of stocks of a small company.

c. Short-selling assumes that an investor can sell assets he/she does not hold, which will be represented by negative amounts in the portfolio distribution.

d. Fractional amounts of assets could be bought.

e. There is no transaction cost for trading assets.

Numerous models have been developed using variety of computational techniques to solve the problem of optimal portfolio selection. Return-based strategies, methods involving stochastic processes, and intelligent systems techniques have been used under different assumptions to efficiently solve the selection of portfolio problem. Return based strategies, as the name suggests, rely mostly on the return of the assets and aim at maximizing the return of the portfolio whiles minimizing the risk on these investments. These are the simplest models of the financial investments and could usually be solved by using linear programming techniques.

Methods and procedures involving stochastic processes centre on predicting the behavior of assets rather than finding the optimal diversification of wealth among the various strategies. Predicting the values of the return and the risk of securities is a very important to enable diversification assets. Various intelligent systems and techniques are used such as linear programming. Modern techniques and tools such as genetic algorithms, rule-based expert systems, neural networks and others are also employed where ~~the~~ objective functions and other and constraints show some complexity of the problem.

From the preceding chapters we had made certain assumptions about the investor and the market for the modeling the optimal portfolio and its selection.

The last two assumptions are usually used to simplify the optimization part of the problem. We can allow dynamic change of the portfolio or we can select a portfolio and do not change it until the end of the time allocated to achieve a specified goal. If we decide to select a static portfolio, we also assume that there is no transaction cost associated with this portfolio as the fee will be paid only once. Other assumptions might be made by imposing (or non-imposing) certain constraints on the problem. After making general assumptions about the problem, we can define a multi-criteria decision making setting for portfolio selection. The set of alternatives is defined as the set of all possible portfolios in a given market.

Many business investors periodically adopt an asset allocation policy that specifies target percentages of value for each of several asset classes. Typically a policy is set by a fund's board after evaluating the implications of a set of alternative policies. The staff is then instructed to implement the policy, usually by maintaining the actual allocation to each asset class within a specified range around the policy target level. Such asset allocation (or asset/liability) studies are usually conducted every one to three years or sooner when market conditions change radically.

Most asset allocation studies include at least some analyses that utilize standard mean/variance optimization procedures and incorporate at least some of the aspects of equilibrium asset pricing theory based on mean/variance assumptions (typically, a standard version of the Capital Asset Pricing Model, possibly augmented by assumptions about asset mispricing.)

In a complete asset allocation study a fund's staff (often with the help of consultants) typically:

Selects desired asset classes and representative benchmark indices;

Chooses a representative historic period and obtains returns for the asset classes;

Computes historic asset average returns, standard deviations and correlations;

Estimates future expected returns, standard deviations. Historic data are typically used, with possible modifications, for standard deviations. Expected returns are often based more on current market conditions and/or typical relationships in capital markets;

Finds several mean/variance efficient asset mixes for alternative levels of risk tolerance;

Projects future outcomes for the selected asset mixes, often over many years;

Presents to the board relevant summary measures of future outcomes for each of the selected asset mixes, then;

Asks the board to choose one of the candidate asset mixes to be the asset allocation policy, based on their views concerning the relevant measures of future outcomes.

The focus of this thesis is on the key analytic tools employed in steps 4 and 5. In step 5 analysts typically utilize a technique termed portfolio optimization. To provide reasonable inputs for such optimization, analysts often rely on informal methods but in some cases utilize a technique termed reverse portfolio optimization. For expository purposes I begin with a discussion of portfolio optimization methods, then turn to reverse optimization procedures. In each case I review the standard analytic approach based on mean/variance assumptions and then describe a more general procedure that assumes investors seek to maximize expected utility. Importantly, mean-variance procedures are special cases of the more general expected utility Formulation.

The prescription to select a portfolio that maximizes an investor's expected utility is hardly new. Nor are applications in the area of asset allocation. Particularly relevant in this respect is the recent work by [Cremers, Kritzman and Page 2005] and [Adler and Kritzman 2007] in which a "full-scale optimization" numerical search algorithm is used to find an asset allocation that maximizes expected utility under a variety of assumptions about investor preferences.

This thesis adds to the existing literature in three ways. First, it presents a new optimization algorithm for efficiently maximizing expected utility in an asset allocation setting. Second it provides a straightforward reverse optimization procedure that adjusts a set of possible future asset returns to incorporate information contained in current asset market values. Finally, it shows that traditional mean/variance procedures for both optimization and Maximization of the minimum return or MAD optimization are obtained if the new procedures are utilized with the assumption that all investors have quadratic utility and that this can be linearized.

Much of modern investment theory and practice builds on Markowitz' assumption that in many cases an investor can be concerned solely with the mean and variance of the probability distribution of his or her portfolio return over a specified future period. Given this, only portfolios that provide the maximum mean (expected return) for given variance of return (or standard deviation of return) warrant consideration. A representative set of such mean/variance efficient portfolios of asset classes can then be considered in an asset allocation study, with the one chosen that best meets the board's preferences in terms of the range of relevant future outcomes over one or more future periods.

A focus on only the mean and variance of portfolio return can be justified in one of three ways. First, if all relevant probability distributions have the same form, mean and variance may be sufficient statistics to identify the full distribution of returns for a portfolio. Second, if an investor wishes to maximize the expected utility of portfolio return and considers utility a quadratic function of portfolio return, only mean/variance efficient portfolios need be considered. Third, it may be that over the range of probability distributions to be evaluated, a quadratic approximation to an investor's true utility function may provide asset allocations that provide expected utility adequately close to that associated with a fully optimal allocation, as argued in [Levy and Markowitz 1979].

Asset allocation studies often explicitly assume that all security and portfolio returns are distributed normally over a single period (for example, a year). If this were the case, the focus on mean/variance analysis would be appropriate, no matter what the form of the investor's utility function. But there is increasing agreement that at least some return distributions are not normally distribute, even over relatively short periods and that explicit attention needs to be given to "tail risk" arising from greater probabilities of extreme outcomes than those associated with normal distributions. Furthermore, there is increasing interest in investment vehicles such as hedge funds that may be intentionally designed to have non-normal distributions and substantial downside tail risk. For these reasons, in at least some cases the first justification for mean/variance analysis as a reasonable approximation to reality may be insufficient.

The second justification may also not always suffice. Quadratic utility functions are characterized by a "satiation level" of return beyond which the investor prefers less return to more –an implausible characterization of the preferences of most investors. To be sure, such functions have a great analytic advantage and may serve as reasonable approximations for some investors' true utility functions. Nonetheless, many investors' preferences may be better represented with a different type of utility function. If this is the case, it can be taken into account not only in choosing an optimal portfolio but also when making predictions about tradeoffs available in the capital markets.

While it is entirely possible that in a given setting mean/variance analyses may provide a sufficient approximation to produce an adequate asset allocations, it would seem prudent to at least conduct an alternative analysis utilizing detailed estimates of possible future returns and the best possible representation of an investor's preferences to evaluate the efficacy of the traditional approach. To facilitate this I present more general approaches to optimization and reverse optimization.

3.1 STATISTICAL ANALYSIS OF PORTFOLIO PARAMETERS

3.1.1 Central Tendency (Mean / average/ expected value), $\overline{R_p}$

$$\overline{R_p} = w_1\bar{r}_1 + w_2\bar{r}_2 + \cdots + w_n\bar{r}_N \quad 3.1$$

$$\overline{R_p} = \sum_{i=1}^{N}(w_i\bar{r}_i) \qquad 3.2$$

Where i = Assets

P= Portfolio

N= number of assets in the portfolio

w_i = weight, proportion of amount invested in asset i

$$w_i = \frac{\text{amount invested in asset i}}{\text{Total value of investment}} \qquad 3.3$$

Example: An Investment and Financial Services want to invest a total of $100; Oil =50%; Gold=50%.

The average returns ($\bar{r}$), are 15% and 20% respectively

Answer

$$\overline{R_p} = \frac{1}{2} \times \overline{r_{TOil}} + \frac{1}{2} \times \overline{r_{AGold}}$$

$$= \frac{1}{2} \times 15\% + \frac{1}{2} \times 20\%$$

Total Return for portfolio p, $\overline{R_p} = 7.5\% + 10\% = 17.5\%$

The total annual return formula can be summarized;

$$R_i = \left(\prod_{i=1}^{n}(1+r)\right)^{1/n} - 1 \qquad 3.1.1.1$$

Where R_i the ith daily return and Greek letter pi (Π) is denotes a repeated product.

3.1.2 CO-VARIANCE

$$Cov(X,Y) = \frac{(X_1 - \bar{X})\,(Y_1 - \bar{Y})(X_2 - \bar{X})((Y_2 - \bar{Y}) + \cdots + (X_n - \bar{X})(Y_n - \bar{Y})}{(N-1)}$$

Example	Oil%	Gold%
I=1	1	6
I=2	2	2
I=3	3	4
Average =	2	4

$$Cov(Oil, Gold) = \frac{(1\%-2\%)(6\%-4\%)+(2\%-2\%)(2\%-4\%)+(3\%-2\%)(4\%-4\%)}{3-1}$$

$$= -0.02\%$$

CO-VARIANCE $= \frac{\sum(X-\bar{X})(Y-\bar{Y})}{n-1}$

3.1.2.1

Where x = the value of one variable

$\bar{x}$ = the average value of that variable

y = the value of a second variable

$\bar{y}$ = the average value of the second variable

n = the number of data values (must be the same for each variable)

3.1.3 Risk Analysis tools called the Sharpe Ration and the financial Utility toolkit

To assist determine an efficient frontier (a set of efficient portfolios) is the Sharpe ration derived by economist William Sharpe which measures the return-to-risk ratio of a portfolio.

This is largely dependent on the level of risk with which the investor is comfortable. Choosing a level of risk is very much a judgment call for the investor, but mathematical methods exist to aid in this choice. Two common mathematical tools are the *Sharpe ratio* and *utility functions.*

The Sharpe ratio is defined as follows:

$$S = \frac{R_x - r_x}{\sigma_p} \qquad 3.2.1$$

Where S is the Sharpe ratio

r_x= expected annualized return of portfolio X_i

R_x= is the annualized return of an investment such as a bond, stock or MFund.

σ_p= is the standard deviation of X_i

Based on this phenomenon the following model is proposed for the study on portfolio optimization. We will use the excel solver to analyze the statistical method of the Markowitz Mean Variance model.

3.3 PORTFOLIO OPTIMIZATION MODELS.

Portfolio Optimization model is formulated as an appropriate optimized risk measure subject to operating constraints, and parametric constraints that a desirable performance measure (such as expected portfolio return) meets a pre-defined target level.

3.3.1 Markowitz (1952)

Lets n be the number of securities in the portfolio

X_i a random variable representing the return on the i[th] security, the values of the

I. expected return, μ_i = E [(X_i) 3.3.1

II. Variance, δ_i^2=E [$(X_i$ - $\mu_i^2)$] 3.3.2

III. For each pair of the security, the autocorrelation is given by:

$$P(i,j) = \frac{E[(X_i - \mu_i)(X_i - \mu_j)]}{\delta_i \delta_j} \qquad 3.3.3$$

Now if we put weights, w, on the amount an Investor invests in a certain portfolio, r_p, the expected return of the entire portfolio, r_i the expected return of the i^{th} stock, w_i the weight of the i^{th} stock, i.e. the fraction of your money that is invested in this stock ;

I. Then the return on the entire, n, portfolio is given by:

$$r_p = \sum_{i=1}^{n} w_i \; r_i \qquad 3.3.4$$

Where $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$ 3.3.5

From equation 3.3.1, expected returns on the entire portfolio is given by Maximization of the Investments equation;

$$Z = r_p = \mu_p = E[\sum_{i=1}^{n} w_i \mu_i] = \sum_{i=1}^{n} w_i \mu_i \qquad 3.3.6$$

II. The variance of the portfolio is given by

$$Variance,\ \delta^2 = E[\sum_{i=1}^{n} w_i \mu_i - E(\sum_{i=1}^{n} w_j \mu_j)]^2$$

$$\text{Variance, } \delta^2 = E[\sum_{i=1}^{n} w_i \mu_i - E(\sum_{i=1}^{n} w_j \mu_j)][\sum_{i=1}^{n} w_i \mu_i - E(\sum_{i=1}^{n} w_j \mu_j)]$$

$$\text{Variance, } \delta^2 = E\{[\sum_{i=1}^{n} w_i \;(X_i - \mu_i)][\sum_{j=1}^{n} w_j \;(X_j - \mu_j)]\}$$

$$\text{Variance, } \delta^2 = E[\sum_{i=1}^{n} w_i w_j (X_i - \mu_i)\;(X_j - \mu_j)]$$

$$\delta^2 = \sum_{i=1}^{n} w_i w_j E[(X_i - \mu_i)\;(X_j - \mu_j)]$$

From equation 3.3.2 the above can be written as:

$$\delta_i\delta_j\rho_{i,j=}\ E[(X_i-\mu_i)\ (X_j-\mu_j)]$$

Therefore

$$\delta^2 = \sum_{i=1}^{n} w_i\, w_j\ \underbrace{[\ \delta_i\delta_j\rho_{i,j}\ ]}Q_{i,j}$$

$$\delta^2 = \sum_{i=1}^{n} w_i\, w_j\ Q_{i,j}$$

3.3.7

Where $Q_{i,j}$ is the covariance of the $X_j\ and\ \mu_j$.

Based on Markowitz's Model of Mean–Variance, the MV Model, an Investor defines an efficient frontier as follows:

THE OPTIMIZATION PROBLEM

i. The least risk for a given amount of return.

Minimize $\sum_{i=0\cdot}^{n} \sum_{j=1}^{n} w_i\, w_j\ Q_{i,j}$ 3.3.8

Subject to:

$$\sum_{i=1}^{n} w_i\mu_i \geq \mu^*\ ;\ 0 \leq w_i \leq 1;\ \sum_{i=1}^{n} w_i = 1$$

ii. The most return for a given amount of risk, or

Maximize $\sum_{i=1}^{n} w_i\mu_i$ 3.3.9

Subject to

$$\sum_{i=1}^{n} \cdot \sum_{j=1}^{n} w_i w_j \ Q_{i,j} \ \leq \delta_*^2 \ ; w_i \geq 1; 1 \leq i \leq n$$

iii. Keep the expected return large and the variance small

Maximize $\quad Z = \sum_{i=0}^{n} \cdot \sum_{j=1}^{n} w_i w_j \ Q_{i,j} \ - \sum_{i=1}^{n} w_i \mu_i$

Subject to $\quad w_i \ \geq 0; \ w_i \ = 1$

Where K>0 is a risk tolerance parameter for all the weightings.

3.3.3 Proposed Mean Absolute Deviation (MAD) Model

This model is the piecewise linear risk model proposed by Konno (1990) and Konno and Yamazaki (1991) is equivalent to the MV model if the returns is mean multivariate normally distributed. Thus under this assumption, the minimization of the sum of the absolute deviations of the portfolio return about the mean is equivalent to the minimization of the variance.

Also the piecewise LP model, MAD, has a faster rate of convergence than the quadratic programming model (to be considered in the next chapter) of the MV model.

The other aim of the MAD model proposed by Konno and Konno and Yamazaki is overcome the limit of the Capital Assets Pricing model (CAPM) for pricing securities.

Konno and Yamazaki (1991) developed a linear model called MAD model where risk is measured by the absolute deviations instead Markowitz definition of risk as the variance. The model is equivalent to Markowitz's model if the returns are multivariate, normally distributed and that the MAD does not require specific type of return distributions. According to Konno and Yamazaki the multivariate and normally distributed form of the MV model is equivalent to minimizing the sum of the absolute deviations from the averages associated with the X_j choices as follows:

Minimize $$E([\sum_{j=1}^{n} R_j X_j - E(\sum_{j=1}^{n} R_j X_j)]) \qquad 3.3.3.1$$

Subject to; $$\sum_{j=1}^{n} X_j \; E(R_j) \geq \rho \qquad 3.3.3.2$$

$$\sum_{j=1}^{n} X_j = 1 \qquad 3.3.3.3$$

$$0 \leq X_j \leq 1 \qquad 3.3.3.4$$

The drawback in the input parameter estimates 2n+n(n-1)/2 of the MV model is eliminated.

The MAD model can be formulated as a LP model as, Linear MAD, as follows

Minimize $$Z = \left(\frac{1}{T}\right) \sum_{k=1}^{T} y_k \qquad 3.3.3.5$$

Subject to; $$y_k + \sum_{j=1}^{n} \left(r_{j\,k} - E(R_j)\right) X_j \geq 0 \,; k = 1, \ldots, T \qquad 3.3.3.6$$

$$y_k - \sum_{j=1}^{n} \left(r_{j\,k} - E(R_j)\right) X_j \geq 0 \,; k = 1, \ldots, T \qquad 3.3.3.7$$

Where the T= period of the investment

$r_{j\,k}$= return of the j^{th}asset at time k.

X_j= weight or proportion of the investments

In the preceding chapters we discuss the Linear Programming method of the Henry Markowitz model of Portfolio selection. We explore the LP of the MAD model in this direction.

3.4 PORTFOLIO MODEL FOR THE CASE STUDY

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{ij} X_i X_j$ **3.4.1**

Subject to:

$\sum \bar{r}_i x_j \geq \alpha * \beta$ **3.4.2**

$\sum_{j=1}^{n} X_j = \beta$ **3.4.3**

$0 \leq U_J \leq U_{J;}$ j=1, 2,.., n **3.4.5**

With I and j securities over T periods

$$\delta_{ij} = \frac{1}{T} \sum_{t=1}^{T} (X_{it} - \bar{r}_i)(X_{jt} - \bar{r}_j) \qquad \textbf{3.4.6}$$

X_{jt} = return of security j at period t

$\bar{r}_i$ = average return in security i over the entire period T

X_j = portfolio allocation of security j that is not larger than the upper bound, U_j

α = the minimum return demanded by a particular investor

β = the total budget that is invested in the varios securities

This classical model is always valid given two important assumptions: (a) that the expected return is multivariate normally distributed; (b) that the investor is risk averter and prefers lower risk.

If constraint 3 is excluded and short selling is permitted a different solution will be obtained by using perhaps all securities, either with positive or negative weights.

However, the minimum variance portfolio is inefficient because of its complexity in solving the Non-linear objective quadratic function form which is almost always the case it is normally difficult to find the optimal solution when the no of securities is perhaps large. For instances, if we have 300 securities, then we may need to compute a Variance-covariance matrix of (n*(n+1))/2= 44,850 combinations.

Not is the computation cumbersome but the implementation of optimal solution is somehow very impossible. In reality we would settle for a minimal or a sub optimal solution .for about 500 securities; there might be 200 of them that takes positive values. The investor is forced to allocate part of his budget into large number of small blocks of shares resulting in transaction cost. This may not be profitable to split the budget into many small blocks of shares.

Experts even argue that even integer Quadratic Programming with more 50 securities might be difficult to solve.

The following present scenarios for solving these portfolios for making it easy for selection by Investors. Using the Sharpe ratio model the risk selection is made to optimize the solution for Investment clients.

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

The Markowitz model has under a many of re-formulation to linearize the quadratic programming form that will facilitate the computation and the selection of a portfolio. The models discussed above were formulated to see which of the models has low risk and yet the most profitable return. We compare the results from these formulations to determine which one yield the highest return and lower risk for the preceding sections.

4.1 PROBLEM / SCENARIO FORMULATION

4.1.1 Scenario 1

Using a one year annual closing prices of five stocks on the Ghana stock exchange market, the following analyses will help us to develop the Markowitz quadratic programming model and the Mean Absolute model.

Based on the equation 3.1.1, 3.1.2, 3.1.1.3 the following statistical measurements are obtained;

TABLE 4.0 **QUARDARTIC MODEL MATRIXES**

Covariance Matrix	-	-	-	-	-
	A	**B**	**C**	**D**	**E**
A	0.00007027	0.00000119	0.00000050	0.00000010	0.00000002
B	0.00000119	0.00037033	0.00000350	0.00000284	0.00000063
C	0.00000050	0.00000350	0.00014177	0.00000087	0.00000107
D	0.00000010	0.00000284	0.00000087	0.00009136	-0.00000001
E	0.00000002	0.00000063	0.00000107	-0.00000001	0.00000243

TABLE 4.1 ESTIMATION OF CORRELATION FOR THE MV MODEL

Correlation Matrix					
	A	**B**	**C**	**D**	**E**
A	1	0.0073849	0.005008476	0.001263944	0.00179701
B	0.0073849	1	0.015278315	0.015455219	0.02089729
C	0.00500848	0.01527831	1	0.007607126	0.057547734
D	0.00126394	0.01545522	0.007607126	1	-0.000570101
E	0.00179701	0.02089729	0.057547734	-0.000570101	1

TABLE 4.2 **RETURN AND RISK ANALYSIS**

Expected Cumulative Return (annualized)				
A	**B**	**C**	**D**	**E**
13.33%	13.65%	-9.16%	-5.79%	-1.14%
Standard Deviation (annualized)				
A	**B**	**C**	**D**	**E**
0.008399211	0.01928224	0.01193025	0.00957704	0.001562638

aily return for a one year stock investment is calculated using the following formula:

$$\text{Daily Return of an investment} = \left(\frac{\text{Stock Price today} - \text{Stock Price Yesterday}}{\text{Stock Price Yesterday}}\right)$$

The tables above indicate the various returns for the five stocks on the daily closing balances of the stock markets. Based on the risk (standard deviation an investor will make a choice of the investment that maximizes the investments.

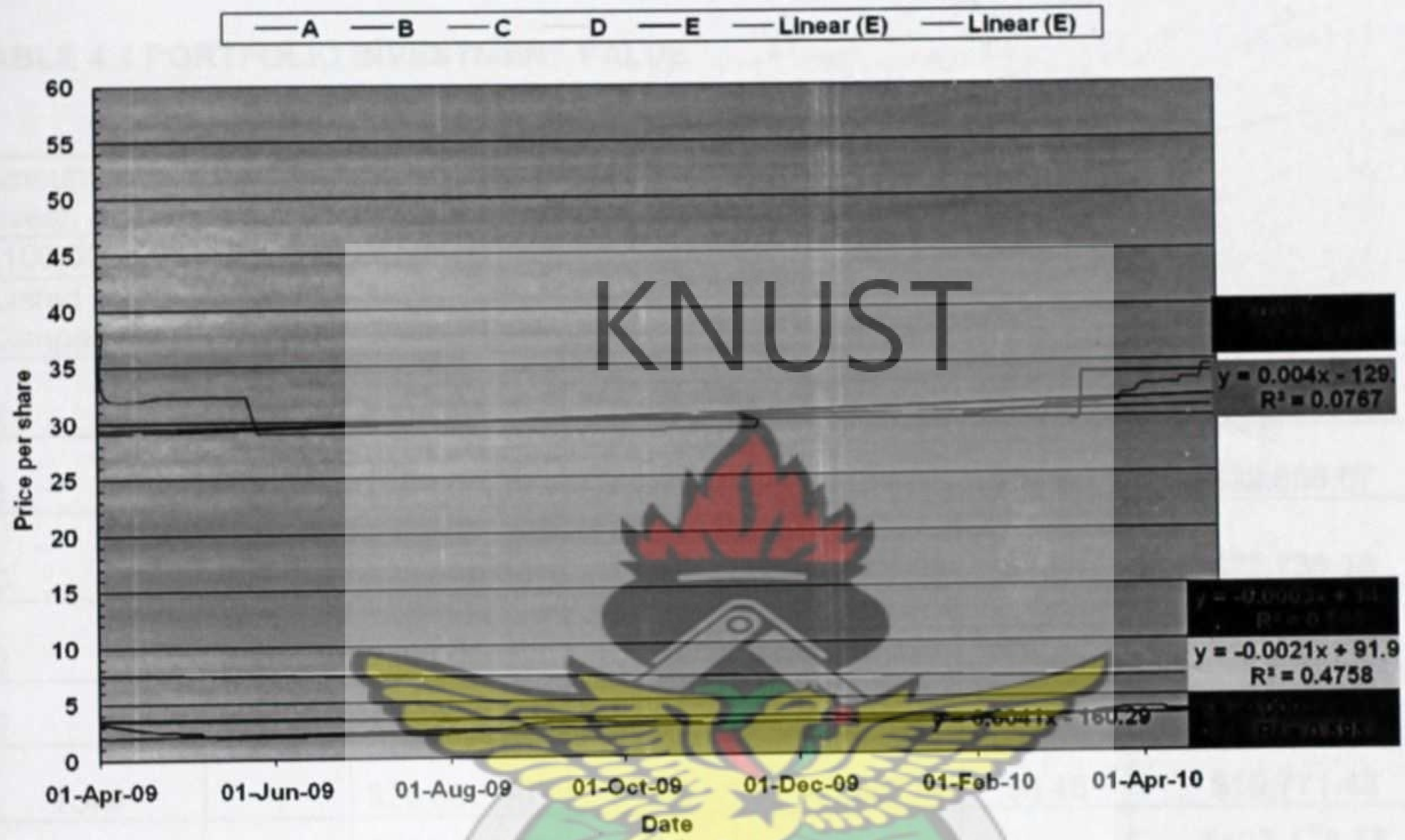


FIGURE 4.0 **PLOT OF DAILY SHARE PRICES FOR THE FIVE COMPANIES**

The value of the correlation indicates how good a linear model approximates the trend in stock prices (the closer the value of the correlation coefficient to 1, the better the linear fit). Stocks that fluctuate a great deal and have lower correlations could be considered "risky" investments.

The highest annualized return of 13.65% or 13.33% at the end of the investment period, client has a choice between selection of either Portfolio A and B to yield the greatest return. The distinctive factor is the risk which is determined by the annualized standard deviation of the portfolios. The line of best fit for B would produce the highest slope, which shows a high rate of return and the lowest risk compared with A. Besides, stock

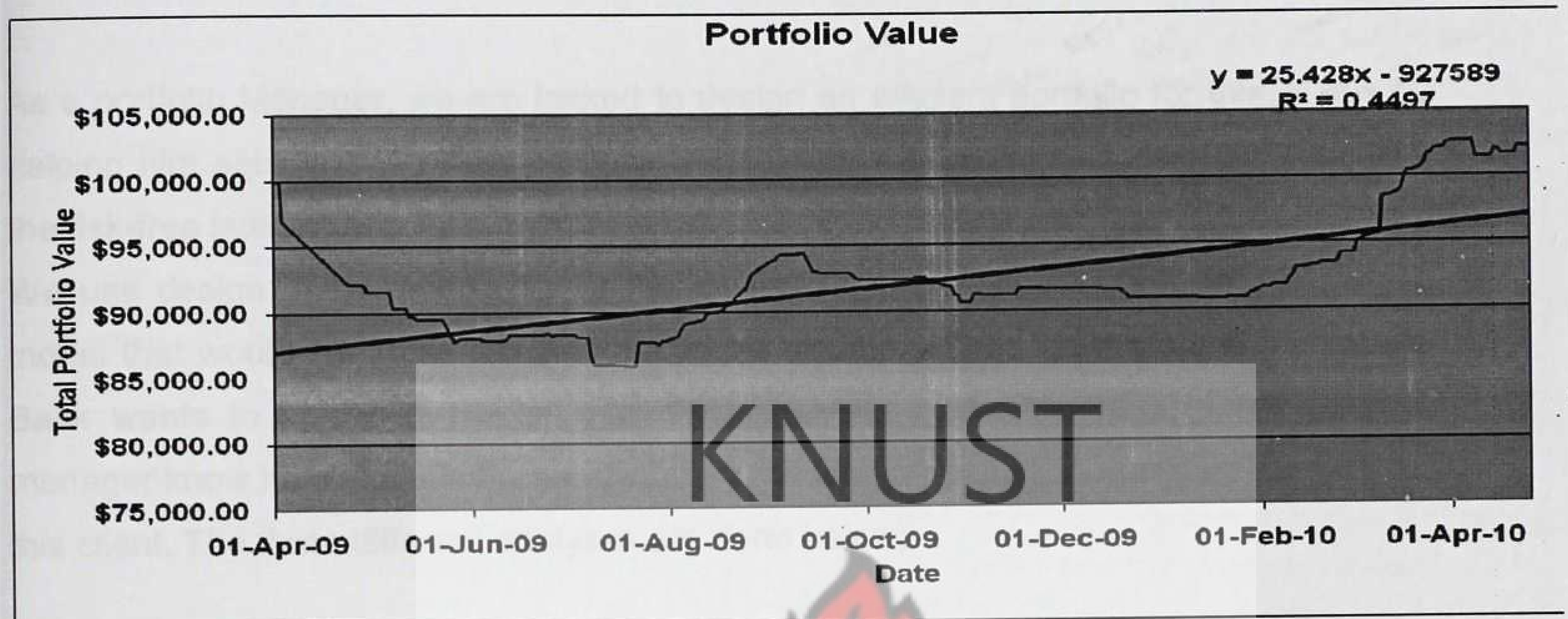
A also appears to fluctuate in price to a greater extent than the other companies. Therefore B and the other companies may also represent the riskiest investment despite the high rate of return.

TABLE 4.4 PORTFOLIO INVESTMENT VALUE

Amount to invest:						
$100,000.00						
Listed stock Companies	Weight					
A	0.2	Amount invested	Initial price	# of shares	Final price	Value by end Investment
B	0.2	$20,000.00	$30.00	666.6666667	$34.00	$22,666.67
C	0.2	$20,000.00	$3.15	6349.206349	$3.58	$22,730.16
D	0.2	$20,000.00	$38.00	526.3157895	$34.52	$18,168.42
E	0.2	$20,000.00	$7.60	2631.578947	$7.16	$18,842.11
Total	1	$20,000.00	$3.50	5714.285714	$3.46	$19,771.43
	Initial	$100,000.00				$102,178.78
Result Summary					Final	$100,000.00
					Return	$102,178.78
					Return	2.18%

Based on the table 4.4 above, an investment of $100,000 yields a return of 2.18%. Under what risk would the investor go for the even weights (0.2) of the investments in all five stocks. This will make an efficient frontier for a risk-averse client.

Figure 4.2 VOLATILITY LEVEL FOR COMPANY A



The investment of portfolio amount $20,000 in Company B, the company with the highest expected return.

The above shows the graph exactly following the zigzag in the price of company A's stock. Company A's investment portfolio has a high return at the end of the period. But, the stock price appears to be somehow volatile; there are fluctuations in the trend of the prices indicated by a increases/drops in stock prices. A client selecting to invest all into this company obviously faces a risky investment.

4.1.2 Scenario 2 - Developing and Comparing Models

A Bank has a policy to retain its clients through proper allocation of investments securities or assets that yield higher returns at a low risk.

Company A is an environmental sanitation company and a corporate client who is concerned with a good investment to help in its work in the improvement of the Ghana's environment. The Client plans to invest at most 100,000 USD, and stands to demand a monthly return of at least 3 % return on its investments (or 3,000 USD) and wishes that no share will receive more that 75% (or 75,000 USD) of his budget.

As a portfolio Manager, we are tasked to design an efficient portfolio for this clients by helping him selecting the best portfolio to minimize the customer's risk, by neglecting the risk-free interest rate.

We use design three models to fix the customers preference to help in selecting a model that would minimize risk and maximize returns. Based on the stock market the Bank wants to invest in five shares. The analyses below would help the portfolio manager know how much is invested in any of these companies in the listed markets for this client. The three different analyses are done below;

4.1.1 QUADRATIC PROGRAMMING (QP) FORMULATION

TABLE 4.1 ESTIMATE OF VARIANCE – COVARIANCE FOR SCENARIO 2

The variance-covariance over the 5 shares representing companies A, B, C, D, and E

σ	σ_A	σ_B	σ_C	σ_D	σ_E
σ_A	0.001467	0.0002617	0.00052	0.000174	0.000432
σ_B		0.001116	-0.000011	0.000590	0.000486
σ_C			0.001058	-0.000023	-0.001706
σ_D				0.001085	0.000552
σ_E					0.001003

From the above model and variance-covariance we formulate the following:

Minimize

$$Z = 0.001467X_A^2 + 0.00116X_B^2 + 0.001069X_C^2 + 0.00109X_D^2 + 0.001004X_E^2 + 0.000262X_AX_B + \cdots + 0.000553X_DX_E \qquad \textbf{4.1.1}$$

Subject to

1. Budget constraint

$$X_A + X_B + X_C + X_D + X_E \geq 0.075 \qquad \textbf{4.1.2}$$

iii. Return demand constraints

$$0.0207X_A + 0.0316X_B + 0.0323X_C + 0.0207X_D + 0.0207X_E \geq 0.03 \qquad \textbf{4.1.3}$$

iv. Lower and Upper Limits of all shares

$$0 \leq X_A, X_B, X_C, X_D, X_E \leq 0.75 \qquad \textbf{4.1.4}$$

Using modern tools a potential investor with the above investments produces the following output for the QP model as:

$$X_A =, X_B = 0.016416, X_C = 0.33148, X_D = 0.18841, X_E = 0.31595,$$

$$Expected\ return, r = 3.412\%, \qquad risk, \sigma = 1.847\%$$

4.1.2. MAXIMZATION OF THE MINIMUM RETURN (MaxMin)

Market analysts, researchers, and other traders regard risk as far away from symmetry or normally distributed. Very often a small loss may make one sad so is a little profit.

The alternative to the formulation above is to maximize the minimum return demanded by the investor. When the shares/data is normally distributed or skewed, the Maximization of the Minimum return called in this study as MaxMin formulation might be more appropriate method, compared with the classical method above. The MaxMin formulation might also be preferable, if the number of decision variable including integer variables of the portfolio optimization is large.

Using the same scenario and variables above and the minimum return from the optimal portfolio as Z ≥ 0;

The objective function then is to maximize the minimum return as given below

Maximize (Max) Z

Formulation of the case study above generates the following from January - December;

January;

$$0.054X_A + 0.03X_B + 0.064X_C + 0.038X_D + 0.05X_E - Z \geq 0$$

$$0.054X_A + 0.03X_B + 0.064X_C + 0.038X_D + 0.05X_E \geq Z \qquad 4.1.2.1$$

...

...

December;

$$0.05X_A + 0.017X_B + 0.032X_C + 0.025X_D + 0.040X_E \geq Z \qquad 4.1.2.2$$

Budget constraints

$$X_A + +X_B + X_C + X_D + X_E \leq 100,000 \qquad 4.1.2.3$$

Return demanded constraints as

$$0.020X_A + 0.0316X_B + 0.0323X_C + 0.0337X_D + 0.0376X_E \geq 3,000 \quad \textbf{4.1.2.4}$$

Lower and upper bounds constraints

$$0 \leq X_A, X_B, X_C, X_D, X_E \leq 75,000 \quad \textbf{4.1.2.5}$$

The optimal solution to this formulation is

$Z = 98.5$, $X_c = 45{,}959.6$, $X_E = 54{,}040.4$, $expected\ return, r = 0.5164\%$.

This optimal solution is more than Zoom Lion's return expectation of 0.3%.

4.1.3. MEAN ABSOLUTE DEVIATION (MAD) FORMULATION

Market analysts regard sigma, δ as the main risk measure. An alternative to simply the Markowitz classical model is to use the absolute deviation as a as a risk measure proposed by Konno & Yamazaki.

According to Konno & Yamazaki, if the return is a multivariate normally distributed, the minimum of the absolute deviation provides similar results with the classical Markowitz formulation.

The Mean Absolute Deviation (MAD) of the risk or volatility is defined by

$$|\sigma| = \frac{1}{T}\sum_{t=1}^{T}\left|\sum(X_{it} - \bar{r}_i)X_i\right| \quad \mathbf{4.1.3.1}$$

This replaces the classical Markowitz variance-covariance formulation and will be minimized the linear form.

Using Konno & Yamazaki MAD model and with the variables as above we form the following:

$$12Y_t \geq 0 \; variables \,, t = 1,2,\ldots,12$$

Where Y_t *is linear mappings of the non – linear form of* $|\sum(X_{it} - \bar{r}_i)X_i|$

This is defined for every month for the previous example of 5 shares A, B, C, D, and E.

Thus objective function is to minimize the Mean Absolute deviation i.e.

$$\text{Min} \quad = \quad \frac{1}{12}\{Y_1 + Y_2 + \cdots + Y_{12}\} \qquad 4.1.3.2$$

Budget Constraints equality formulation

$$A + B + C + D + E = 100{,}000 \qquad 4.1.3.3$$

Return Demand Formulation

$$0.020A + 0.0316B + 0.0323C + 0.0337D + 0.0376E \geq 3{,}000 \qquad 4.1.3.4$$

Mapping Y_t with shares A, B, C, D, and E for all the months starting with January:

$$Y_t = \pm \sum_{i=1}^{5} \left|\sum (X_{it} - \bar{r}_i)X_i\right| \qquad 4.1.3.5$$

January

$Y_1 = -\{0.333A + 0.0004B + 0.0083C + 0.0043D + 0.0114E\}$ »

$$Y_1 + 0.333A + 0.0004B + 0.0083C + 0.0043D + 0.0114E \geq 0 \qquad 4.1.3.6$$

And

$Y_1 = +\{0.333A + 0.0004B + 0.0083C + 0.0043D + 0.0114E\}$ »

$Y_1 - 0.333A - 0.0004B - 0.0083C - 0.0043D - 0.0114E \geq 0$ **4.1.3.7**

$Y_2 = -\{0.0243A + 0.0234B + 0.0203C + 0.0283D + 0.02948E\}$ »

$Y_2 + 0.0243A + 0.0234B + 0.0203C + 0.0283D + 0.02948E \geq 0$ **4.1.3.8**

$Y_2 - 0.0243A - 0.0234B - 0.0203C - 0.0283D - 0.0294E \geq 0$ **4.1.3.9**

…

…

…

$Y_{12} + 0.0313A - 0.0486B - 0.0037C - 0.0087D + 0.0024E \geq 0$ 4.1.3.10

$Y_{12} - 0.0313A + 0.0486B + 0.0037C + 0.0087D - 0.0024E \geq 0$ 4.1.3.11

Lower and Upper bounds for the shares

$0 \ll A, B, C, D, E \leq 75{,}000$

The optimal solution for this formulation yields the B=8,568.9, C=36,295.2, D=1,600.2, E=53,535.7, Min=1,286.7 and a slack for the return demanded 0.510%.

The MAD model has many advantages over the other models:

i. No variance-co-variance matrix estimation

ii. Faster optimization as compared with the non-linear formulation QP model

iii. This model can be re-formulated as an Integer LP to take into consideration other cost or decision variables.

iv. Does very were with a higher population of historical data

4.2 RISK AVERSION

This measures an investor's satisfaction with a particular investment which can be evaluated through a utility function; the various models above can be tested with this utility function to determine which investment does well by varying the risk.
The Financial Tool kit tool for selection of a optimal portfolio at a given level of risk is represented below:

$$U(k) = r_k - 0.005 * A * {\sigma^2}_k \qquad 4.2.1$$

U (k) = the utility of the portfolio k.

r_k = the expected return,

σ_k = the standard deviation for the investment,

This return-risk curve is used to establish the increment in return that an investor will need in order to make an increment in risk worthwhile.

A is a positive constant between 2.0 And 4.0 that measures an individual's aversion to risk. The graph displace the scenario that a value of A greater than or equal to 4 (above the smooth line) would represent a very risk-averse investor, whereas a value of A close to 0(below the dotted line) means an investor is very tolerant of risk.

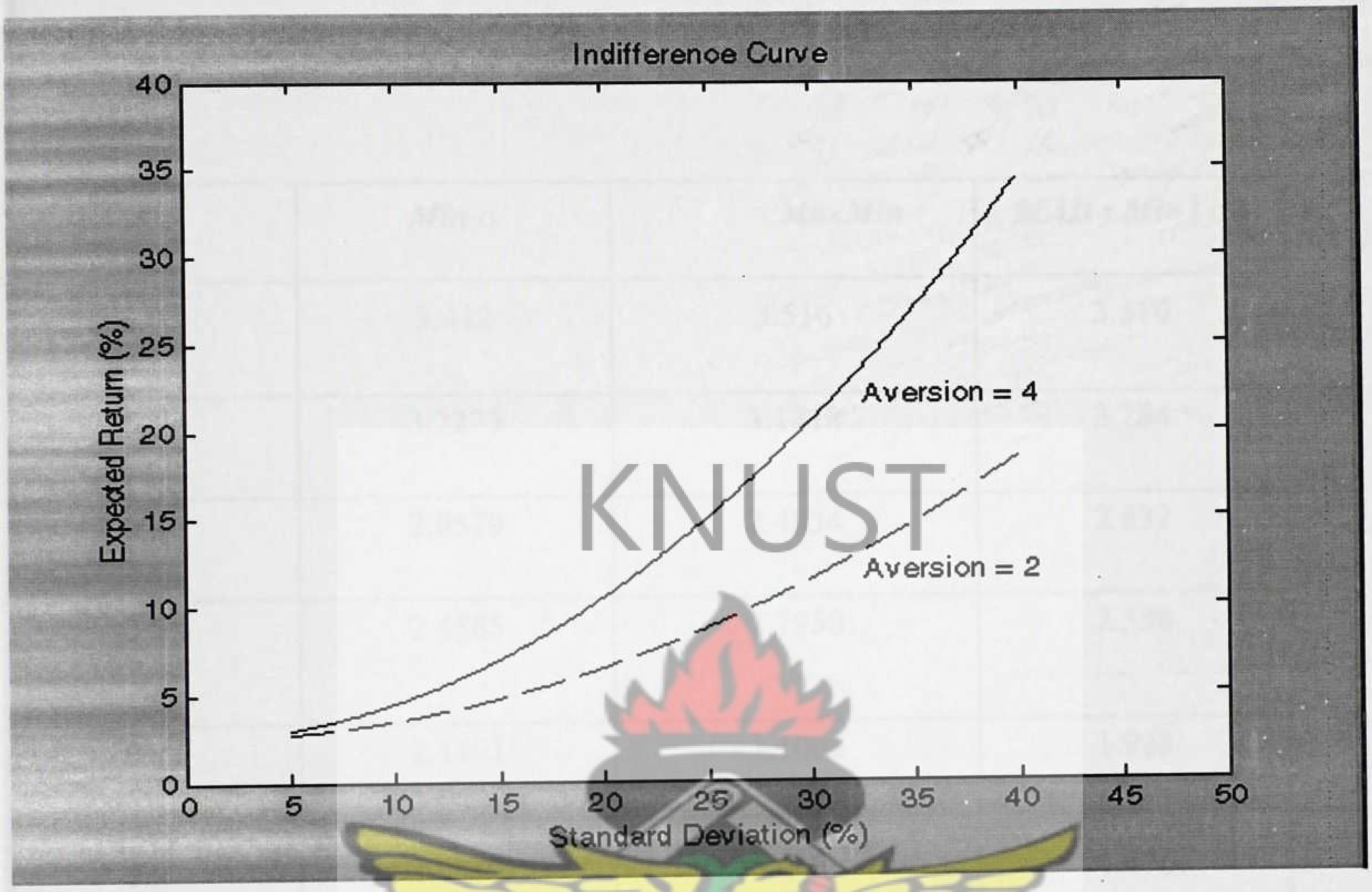


Figure 4.3 CUSTOMER'S INDIFFERENCE OR RISK AVERSION CURVE

The higher the value of U, the more satisfactory an investor is with their chosen portfolio. A Higher expected return tend to increase utility, whereas higher risk results in lower utility.

Using the above utility function and the risk levels below we form the following on the three types of models solutions discussed.

TABLE 4.2 CUSTOMER RISK/RETURN INDEX ANALYSIS

A	*Min σ*	*MaxMin*	*MAD : Min | σ |*
0.0	3.412	3.516	3.510
0.1	3.2273	3.1718	3.284
0.3	2.8579	2.4834	2.832
0.5	2.4885	1.7950	2.380
0.7	2.1191	1.1066	1.928
0.9	1.7497	0.4182	1.476
1.0	1.5650	0.0740	1.250
1.5	0.6415	-1.6470	0.120

The table above enables us to make a choice for the portfolios to achieve an efficient frontier at various levels of risk aversion.

4.2 COMPARISON OF THE PROTFOLIO MODELS TO AID IN SELECTION

After the formulation the results of these three models are compared to help company A to select an efficient portfolio to make an efficient frontier.

TABLE 4.3 MODELS COMPARISON TO AID IN BEST SELECTION

Model	Weights	12 Months	3 Months	6 Months
		Risk, δ and Return, r	Risk, δ and Return, r	Risk, δ and Return, r
QP	$X_B = 0.0164$ $X_C = 0.331$ $X_D = 0.188$ $X_B = 0.315$	$\delta = 1.84$ $r = 3.41$	$\delta = 0.90$ $r = -0.30$	$\delta = 0.75$ $r = -0.66$
MaxMin	$X_C = 0.4595$ $X_E = 0.5404$	$\delta = 2.29$ $r = 3.52$	$\delta = 1.72$ $r = 0.079$	$\delta = 1.55$ $r = -0.69$
MAD	$X_B = 0.0164$ $X_C = 0.3314$ $X_D = 0.188$ $X_B = 0.315$	$\delta = 2.26$ $r = 3.51$	$\delta = 1.49$ $r = 0.04$	$\delta = 1.34$ $r = -0.65$

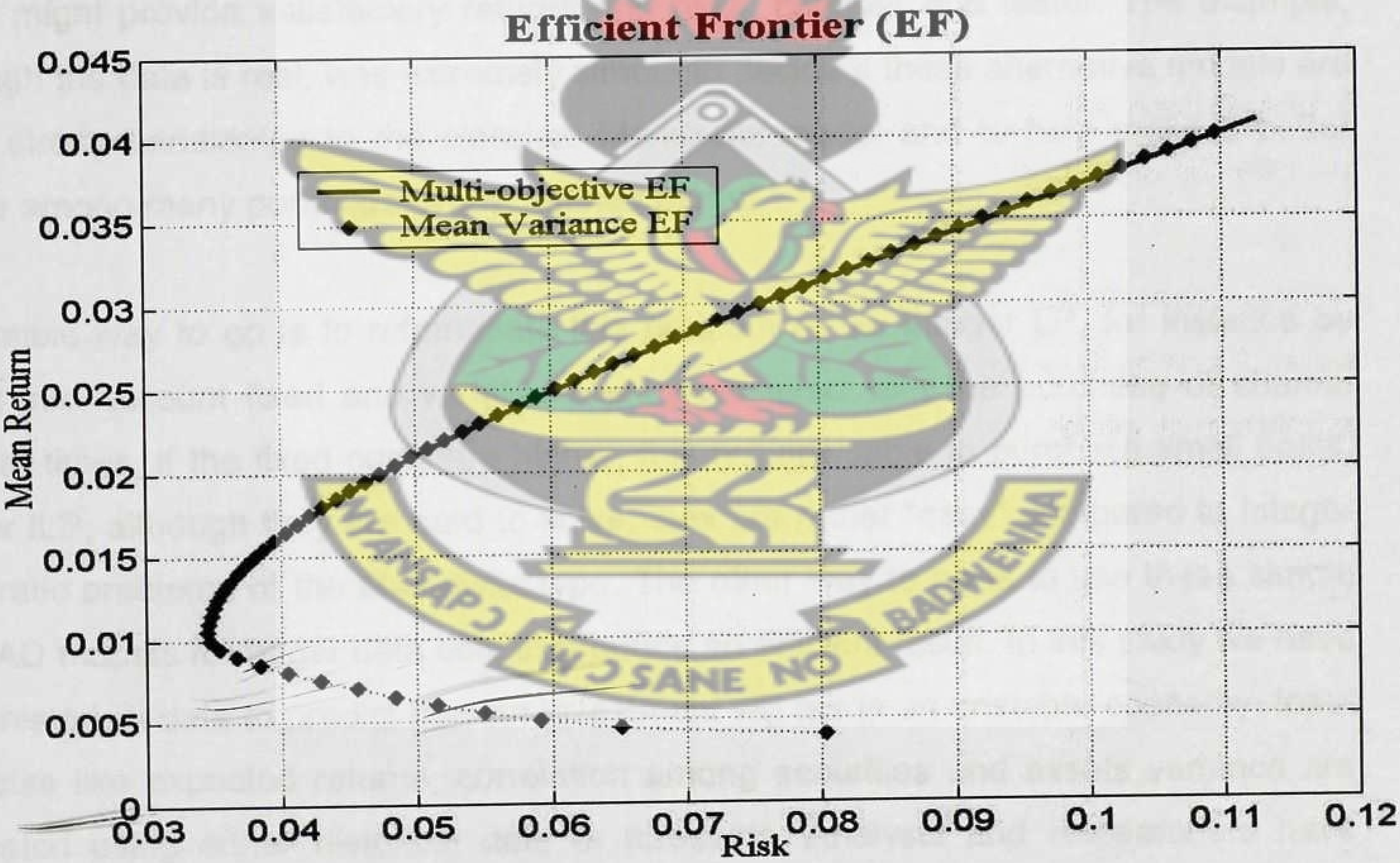
The tale above summarizes the three models over the 5 share for a twelve month period data and projected for the next three and six months.

The MaxMin has the highest return; min δ of the QP has the lowest risk and return while the MAD model lies between the two models.

However, the min δ of the QP model performs the worse during the next 3 and 6 months periods. Though the MAD model has a marginal difference compared with the other models, yet this provides the lowest losses during the 6 months.

The curve below a representation of most efficient portfolio selection frontier used to represent most of portfolio selection from many optimization books. Risk has been a very dictating factor when it comes to selection of an efficient portfolio to maximize return to achieve the hopes of an investor.

Figure 4.4 Optimal Risk/ Return for Every Scenario



Optimal portfolios define a line in the risk/return plane called efficient frontier. Anything below it is not efficient and anything above it is not possible with the given set of constraints. An efficient frontier must lie on the tangent of this curve to indicate level of risk an investor will allow for their investments.

CHAPTER 5

SUMMARY AND CONCLUSION

5.1 Summary of Results

The capacity of computers and their speed though has increased considerably over recent years, optimal solutions of portfolios on very large quadratic problems might be hard to achieve, especially if one wants integer solutions of the formulation discussed in these pages.

Konno and Yamazaki's alternative linear approximations, as those presented in this paper might provide satisfactory returns and risks, cheaper and faster. The example, although the data is real, was extremely simple to decide if these alternative models are to be strong candidates to the classical Markowitz model and to help make a better choice among many portfolios over a defined period.

A possible way to go is to reformulate this problem as an Integer LP, for instance by taking into account fixed and variable costs associated with the purchase of shares. Most at times, if the fixed costs are higher, it is not advisable to purchase small posts. Higher ILP, although they are hard to solve, they are rather "easy" compared to Integer Quadratic problems of the Markowitz type. The other way to go is to use these simple LP MAD models for larger data sets to produce an efficient result. In this study we have used historical data to predict how an investment will fair in an unstable economy. Input variables like expected returns, correlation among securities and assets variance are estimated using either historical data or forecasts. Analysts and Researchers have found that estimation errors in the input parameters overwhelm the theoretical benefits of the mean-variance paradigm.

In this study the MAD model by Konno and Yamazaki was compared with other models and from the results concluded that for small data, the quadratic programming model of the Markowitz model produces the best selection of the various securities to make an efficient portfolio at a certain level of risk. For large historical data set, the MAD model produces the best selection of assets to for a portfolio.

Based on the comparison of models, it has been established that the MAD model is better when selecting a portfolio's initial return and risks to help use the financial toolkit make a better selection for optimal portfolio solution and efficient frontier.

5.2 **Portfolio Optimization in financial decision making**

The objective of every Investor in portfolio optimization is to manage risk through diversification and obtain an optimal risk-return tradeoff.

Many Portfolio theories assume that for a given level of risk, investors prefer higher returns to lower returns on their investment.

Furthermore, for a given level of expected return, investors prefer lower risk to greater risk. Not only that but also, one can assume that investors would go for an efficient portfolio, that is, a portfolio in which there is no other portfolio that offers a greater return with the same or lower risk.

In many financial analyses, expected return is commonly employed as a measure of return, and variance or standard deviation of return is commonly employed as a measure of risk. In practice, mean-variance efficient portfolios have been found to be quite unstable.

During the last several years, alternative methods have emerged for optimizing uncertain financial decisions concerning risk. Modern portfolio management systems integrates multi-objective optimization and interactive frontier decision-making techniques that improves financial asset allocation or diversification by introducing more flexible, robust, and realistic assumptions and providing more sophisticated portfolio analysis.

This study concentrated on the mathematical model presented by the Nobel Prize winner Harry Markowitz and later by Konno and Yamazaki called MAD model to facilitate Portfolio selection.

5.3 Conclusion and recommended research areas.

Portfolio optimization management arises in every organization and in situations where resources need to be optimized under certain risk conditions.

Investors or Financial Analyst are challenged with developing the best frontiers for their clients to maximize returns for an appreciable level of risk.

As the complexity of the models of the portfolio becomes large and the speed of computation decreases, investors need to develop the model such that clients can receive an immediate investments diversifications report.

The main difference between the mean variance and MAD approach is their problem formulations and their computation burden. The MAD approach puts two optimization objectives (minimizing risk and maximizing expected return) into one objective function where as the mean variance approach has only one objective of minimizing risk. The MV model and method places the expected value as a constraint in the formulation, which forces the optimization model to provide the minimal risk for each specified level of expected return.

There are two comparative advantages for the MAD formulation over the mean variance formulation. First, since the mean variance approach assumes that the investor's sole objective is to minimize risk, it may not be a good fit for investors who are extremely risk seeking. The multi-objective formulation is applicable for investors of any risk tolerance. Second, the mean variance method requires investors to place an expected value constraint, but there are times when investors do not want to place any constraints on their investment or do not know what kind of return to expect from his investment. The multi-objective optimization provides the entire picture of optimal risk-return trade off.

Another key difference between these two methods lies in their approach to producing efficient frontiers. The efficient frontier of the multi-objective optimization is determined by the risk aversion index of the risk because different values of the risk determine different values of risk and expected return. The efficient frontier of the mean variance method is generated by varying the proportion of two optimal portfolios because the Two Fund Separation Theorem guarantees that any optimized portfolio can be duplicated by a combination of two optimal portfolios.

Hence, in the event of generating the efficient frontier for the mean variance optimization, there is the need to use the minimum variance portfolio to replicate a secondary portfolio with the given expected return. As a result, using the mean variance method to generate the efficient frontier can be numerically more cumbersome than the multi-objective approach.

Finally, the traditional single-objective approach, such as the mean variance method, solves the problem by having one of the optimization objectives in the objective function and fixes the other objective as a constraint. Consequently, investors have to choose the optimal solution based on given expected return or risk.

The multi-objective optimization provides an alternative solution to the portfolio optimization problem, generating the same optimal solution as the mean variance method. It can be applied to investors of any risk tolerance, including those who are extremely risk-seeking and risk-averse. The risk-aversion index measures how much an investor weights risk over expected return. Given any specified value of risk-aversion index, the multi-objective optimization provides investors with optimal asset allocation strategy that can simultaneously maximize expected return and minimize risk.

To alleviate the fears of clients, It is recommended that hedging must be consider in future direction of this research based on the Harry Markowitz mean variance or Konno and Yamazaki's MAD models.

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APPENDIX

Table 1

	Daily Closing Prices					Daily Return				
Date	A	B	C	D	E	A	B	C	D	E
01-Apr-09	30	3.15	38	7.6	3.5					
02-Apr-09	30	3.15	38	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
03-Apr-09	30	3.15	32.3	7.6	3.5	0.00000	0.00000	-0.15000	0.00000	0.0000
23-Apr-09	30	2.4	32.3	7.6	3.5	0.00000	-0.23810	0.00000	0.00000	0.0000
24-Apr-09	30	2.39	32.3	7.6	3.5	0.00000	-0.00417	0.00000	0.00000	0.0000
27-Apr-09	30	2.39	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
28-Apr-09	30	2.39	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
29-Apr-09	30	2.3	32.3	7.6	3.5	0.00000	-0.03766	0.00000	0.00000	0.0000
30-Apr-09	30	2.3	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
04-May-09	30	2.3	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
05-May-09	30	2.3	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
06-May-09	30	2.3	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
07-May-09	30	2.1	32.3	7.6	3.5	0.00000	-0.08696	0.00000	0.00000	0.0000
08-May-09	30	2.1	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
11-May-09	30	2.1	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
12-May-09	30	2	32.3	7.6	3.5	0.00000	-0.04762	0.00000	0.00000	0.0000
13-May-09	30	2	32.3	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
14-May-09	30	1.97	32.29	7.6	3.5	0.00000	-0.01500	-0.00031	0.00000	0.0000
15-May-09	30	1.97	32.29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
18-May-09	30	1.97	32.29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
19-May-09	30	1.97	32.29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
20-May-09	30	1.97	32.29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
21-May-09	30	1.97	32.29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
22-May-09	30	1.97	32.29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
26-May-09	30	1.97	29	7.6	3.5	0.00000	0.00000	-0.10189	0.00000	0.0000
27-May-09	30	1.99	29	7.6	3.5	0.00000	0.01015	0.00000	0.00000	0.0000
28-May-09	30	2	29	7.6	3.5	0.00000	0.00503	0.00000	0.00000	0.0000
29-May-09	30	2	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
01-Jun-09	30	2	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
02-Jun-09	30	2	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
03-Jun-09	30	2	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
04-Jun-09	30	1.98	29	7.6	3.5	0.00000	-0.01000	0.00000	0.00000	0.0000
05-Jun-09	30	1.98	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
08-Jun-09	30	1.98	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
09-Jun-09	30	1.98	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
10-Jun-09	30	1.98	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
11-Jun-09	30	2	29	7.6	3.5	0.00000	0.01010	0.00000	0.00000	0.0000
12-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.02500	0.00000	0.00000	0.0000

15-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
16-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
17-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
18-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
19-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
22-Jun-09	30	2.05	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
23-Jun-09	30	2.06	29	7.6	3.5	0.00000	0.00488	0.00000	0.00000	0.0000
24-Jun-09	30	2.06	29	7.6	3.5	0.00000	0.00000	0.00000	0.00000	0.0000
25-Jun-09	30	2.05	29	7.6	3.49	0.00000	-0.00485	0.00000	0.00000	0.00286
26-Jun-09	30	2.06	29	7.6	3.49	0.00000	0.00488	0.00000	0.00000	0.0000
29-Jun-09	30	2.06	29	7.6	3.49	0.00000	0.00000	0.00000	0.00000	0.0000
30-Jun-09	30	2.06	29	7.6	3.49	0.00000	0.00000	0.00000	0.00000	0.0000
02-Jul-09	30	2.06	29	7.6	3.49	0.00000	0.00000	0.00000	0.00000	0.0000
03-Jul-09	30	2.06	29	7.6	3.45	0.00000	0.00000	0.00000	0.00000	0.01146
06-Jul-09	30	2.06	29	7.6	3.45	0.00000	0.00000	0.00000	0.00000	0.0000
07-Jul-09	30	2.06	29	7.1	3.45	0.00000	0.00000	0.00000	-0.06579	0.0000
08-Jul-09	30	2.06	29	6.8	3.45	0.00000	0.00000	0.00000	-0.04225	0.0000
09-Jul-09	30	2.06	29	6.8	3.45	0.00000	0.00000	0.00000	0.00000	0.0000
10-Jul-09	30	2.06	29	6.8	3.45	0.00000	0.00000	0.00000	0.00000	0.0000
13-Jul-09	30	2.06	29	6.8	3.45	0.00000	0.00000	0.00000	0.00000	0.0000
14-Jul-09	30	2.06	29	6.8	3.45	0.00000	0.00000	0.00000	0.00000	0.0000
15-Jul-09	30	2.06	29	6.8	3.44	0.00000	0.00000	0.00000	0.00000	0.0029
16-Jul-09	30	2.06	29	6.8	3.44	0.00000	0.00000	0.00000	0.00000	0.0000
17-Jul-09	30	2.06	29	6.8	3.44	0.00000	0.00000	0.00000	0.00000	0.0000
20-Jul-09	30	2.06	29	6.8	3.43	0.00000	0.00000	0.00000	0.00000	0.0029
21-Jul-09	30	2.06	29	6.8	3.43	0.00000	0.00000	0.00000	0.00000	0.0000
22-Jul-09	30	2.06	29	7.5	3.43	0.00000	0.00000	0.00000	0.10294	0.0000
23-Jul-09	30	2.06	29	7.5	3.43	0.00000	0.00000	0.00000	0.00000	0.0000
24-Jul-09	30	2.06	29	7.5	3.43	0.00000	0.00000	0.00000	0.00000	0.0000
27-Jul-09	30	2.06	29	7.5	3.43	0.00000	0.00000	0.00000	0.00000	0.0000
28-Jul-09	30	2.06	29	7.5	3.4	0.00000	0.00000	0.00000	0.00000	0.00875
29-Jul-09	30	2.1	29	7.5	3.4	0.00000	0.01942	0.00000	0.00000	0.0000
30-Jul-09	30	2.1	29	7.5	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
31-Jul-09	30	2.11	29	7.5	3.4	0.00000	0.00476	0.00000	0.00000	0.0000
03-Aug-09	30	2.16	29	7.5	3.4	0.00000	0.02370	0.00000	0.00000	0.0000
04-Aug-09	30	2.21	29	7.5	3.42	0.00000	0.02315	0.00000	0.00000	0.00588
05-Aug-09	30	2.28	29	7.5	3.42	0.00000	0.03167	0.00000	0.00000	0.0000
06-Aug-09	30	2.3	29	7.5	3.42	0.00000	0.00877	0.00000	0.00000	0.0000
07-Aug-09	30	2.3	29	7.5	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
10-Aug-09	30	2.32	29	7.5	3.42	0.00000	0.00870	0.00000	0.00000	0.0000
11-Aug-09	30	2.34	29	7.5	3.42	0.00000	0.00862	0.00000	0.00000	0.0000
12-Aug-09	30	2.4	29	7.5	3.42	0.00000	0.02564	0.00000	0.00000	0.0000
13-Aug-09	30	2.41	29	7.5	3.42	0.00000	0.00417	0.00000	0.00000	0.0000
14-Aug-09	30	2.42	29	7.5	3.42	0.00000	0.00415	0.00000	0.00000	0.0000

17-Aug-09	30	2.45	29	7.5	3.42	0.00000	0.01240	0.00000	0.00000	0.0000(
18-Aug-09	30	2.5	29	7.5	3.42	0.00000	0.02041	0.00000	0.00000	0.0000(
19-Aug-09	30	2.51	29	7.5	3.42	0.00000	0.00400	0.00000	0.00000	0.0000(
20-Aug-09	30	2.55	29	7.5	3.42	0.00000	0.01594	0.00000	0.00000	0.0000(
21-Aug-09	30	2.6	29	7.5	3.42	0.00000	0.01961	0.00000	0.00000	0.0000(
24-Aug-09	30	2.7	29	7.5	3.42	0.00000	0.03846	0.00000	0.00000	0.0000(
25-Aug-09	30	2.8	29	7.5	3.42	0.00000	0.03704	0.00000	0.00000	0.0000(
26-Aug-09	30	2.86	29	7.5	3.42	0.00000	0.02143	0.00000	0.00000	0.0000(
27-Aug-09	30	2.9	29	7.5	3.42	0.00000	0.01399	0.00000	0.00000	0.0000(
28-Aug-09	30	2.93	29	7.5	3.42	0.00000	0.01034	0.00000	0.00000	0.0000(
31-Aug-09	30	3.01	29	7.5	3.42	0.00000	0.02730	0.00000	0.00000	0.0000(
01-Sep-09	30	3.02	29	7.5	3.42	0.00000	0.00332	0.00000	0.00000	0.0000(
02-Sep-09	30	3.05	29	7.5	3.42	0.00000	0.00993	0.00000	0.00000	0.0000(
03-Sep-09	30	3.09	29	7.5	3.42	0.00000	0.01311	0.00000	0.00000	0.0000(
04-Sep-09	30	3.11	29	7.5	3.42	0.00000	0.00647	0.00000	0.00000	0.0000(
07-Sep-09	30	3.11	29	7.5	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
08-Sep-09	30	3.11	29	7.5	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
09-Sep-09	30	3.11	29	7.5	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
10-Sep-09	30	3.13	29	7.5	3.42	0.00000	0.00643	0.00000	0.00000	0.0000(
11-Sep-09	30	3.13	29	7.5	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
14-Sep-09	30	3.12	29	7	3.42	0.00000	-0.00319	0.00000	-0.06667	0.0000(
15-Sep-09	30	3.12	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
16-Sep-09	30	3.1	29	7	3.42	0.00000	-0.00641	0.00000	0.00000	0.0000(
17-Sep-09	30	3.1	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
18-Sep-09	30	3.1	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
22-Sep-09	30	3.09	29	7	3.42	0.00000	-0.00323	0.00000	0.00000	0.0000(
23-Sep-09	30	3.09	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
24-Sep-09	30	3.09	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
25-Sep-09	30	3.09	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
28-Sep-09	30	3.02	29	7	3.42	0.00000	-0.02265	0.00000	0.00000	0.0000(
29-Sep-09	30	3.02	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
30-Sep-09	30	3.02	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
01-Oct-09	30	3.02	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
02-Oct-09	30	3.02	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
05-Oct-09	30	3.02	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
06-Oct-09	30	3.02	29	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
07-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00034	0.00000	0.0000(
08-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
09-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
12-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
13-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
14-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
15-Oct-09	30	3.02	29.01	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
16-Oct-09	30	3.02	29.2	7	3.42	0.00000	0.00000	0.00655	0.00000	0.0000(
19-Oct-09	30	3.02	29.2	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000(
20-Oct-09	30	2.95	29.2	7	3.42	0.00000	-0.02318	0.00000	0.00000	0.0000(
21-Oct-09	30	2.95	29.25	7	3.42	0.00000	0.00000	0.00171	0.00000	0.0000(
22-Oct-09	30	2.94	29.25	7	3.42	0.00000	-0.00339	0.00000	0.00000	0.0000(

23-Oct-09	30	2.94	29.25	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
26-Oct-09	30	2.9	29.25	7	3.42	0.00000	-0.01361	0.00000	0.00000	0.0000
27-Oct-09	30	2.9	29.25	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
28-Oct-09	30	2.8	29.25	7	3.42	0.00000	-0.03448	0.00000	0.00000	0.0000
29-Oct-09	30	2.8	29.25	7	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
30-Oct-09	30	2.8	29.25	6.8	3.42	0.00000	0.00000	0.00000	-0.02857	0.0000
02-Nov-09	30	2.8	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
03-Nov-09	30	2.9	29.25	6.8	3.42	0.00000	0.03571	0.00000	0.00000	0.0000
04-Nov-09	30	2.9	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
05-Nov-09	30	2.9	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
06-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	-0.00345	0.00000	0.00000	0.0000
09-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
10-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
11-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
12-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
13-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
16-Nov-09	30	2.89	29.25	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
17-Nov-09	30	2.9	30	6.8	3.42	0.00000	0.00346	0.02564	0.00000	0.0000
18-Nov-09	30	2.9	30	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
19-Nov-09	30	2.9	30	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
20-Nov-09	30	2.9	30	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
23-Nov-09	30	2.9	30	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
24-Nov-09	30	2.9	30	6.8	3.42	0.00000	0.00000	0.00000	0.00000	0.0000
25-Nov-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.00292
26-Nov-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
30-Nov-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
01-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
02-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
03-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
07-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
08-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
09-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
10-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
11-Dec-09	30	2.9	30	6.8	3.41	0.00000	0.00000	0.00000	0.00000	0.0000
14-Dec-09	30	2.9	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.00293
15-Dec-09	30	2.9	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
16-Dec-09	30	2.9	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
17-Dec-09	30	2.9	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
18-Dec-09	30	2.9	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
21-Dec-09	30	2.8	30	6.8	3.4	0.00000	-0.03448	0.00000	0.00000	0.0000
22-Dec-09	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
23-Dec-09	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
24-Dec-09	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
29-Dec-09	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
30-Dec-09	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
31-Dec-09	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
04-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000

05-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
06-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
07-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
08-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
11-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
12-Jan-10	30	2.78	30	6.8	3.4	0.00000	-0.00714	0.00000	0.00000	0.0000(
13-Jan-10	30	2.78	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
14-Jan-10	30	2.78	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
15-Jan-10	30	2.78	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
18-Jan-10	30	2.78	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
19-Jan-10	30	2.78	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
20-Jan-10	30	2.78	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
21-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00719	0.00000	0.00000	0.0000(
22-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
25-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
26-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
27-Jan-10	30	2.8	30	6.8	3.4	0.00000	0.00000	0.00000	0.00000	0.0000(
28-Jan-10	30	2.84	30.2	6.8	3.4	0.00000	0.01429	0.00667	0.00000	0.0000(
29-Jan-10	30	2.85	30.2	6.8	3.4	0.00000	0.00352	0.00000	0.00000	0.0000(
01-Feb-10	30	2.9	30.2	6.8	3.4	0.00000	0.01754	0.00000	0.00000	0.0000(
02-Feb-10	30	2.9	30.2	6.8	3.39	0.00000	0.00000	0.00000	0.00000	0.0029
03-Feb-10	30	2.9	30.31	6.8	3.39	0.00000	0.00000	0.00364	0.00000	0.0000(
04-Feb-10	30	2.92	30.35	6.8	3.39	0.00000	0.00690	0.00132	0.00000	0.0000(
05-Feb-10	30	2.92	30.35	6.8	3.39	0.00000	0.00000	0.00000	0.00000	0.0000(
08-Feb-10	30	2.92	30.39	6.8	3.39	0.00000	0.00000	0.00132	0.00000	0.0000(
09-Feb-10	30	3	30.39	6.8	3.39	0.00000	0.02740	0.00000	0.00000	0.0000(
10-Feb-10	30	3	30.45	6.8	3.39	0.00000	0.00000	0.00197	0.00000	0.0000(
11-Feb-10	30	3.1	30.66	6.8	3.39	0.00000	0.03333	0.00690	0.00000	0.0000(
12-Feb-10	30	3.1	30.66	6.8	3.39	0.00000	0.00000	0.00000	0.00000	0.0000(
15-Feb-10	30	3.11	30.66	6.8	3.39	0.00000	0.00323	0.00000	0.00000	0.0000(
16-Feb-10	30	3.12	30.66	6.8	3.39	0.00000	0.00322	0.00000	0.00000	0.0000(
17-Feb-10	30	3.12	30.7	6.8	3.39	0.00000	0.00000	0.00130	0.00000	0.0000(
18-Feb-10	30	3.12	30.7	6.8	3.39	0.00000	0.00000	0.00000	0.00000	0.0000(
19-Feb-10	30	3.15	30.7	6.8	3.39	0.00000	0.00962	0.00000	0.00000	0.0000(
22-Feb-10	30	3.15	30.7	6.8	3.39	0.00000	0.00000	0.00000	0.00000	0.0000(
23-Feb-10	30	3.15	30.7	6.8	3.39	0.00000	0.00000	0.00000	0.00000	0.0000(
24-Feb-10	30	3.15	30.85	6.8	3.39	0.00000	0.00000	0.00489	0.00000	0.0000(
25-Feb-10	30	3.16	31	7	3.39	0.00000	0.00317	0.00486	0.02941	0.0000(
26-Feb-10	30	3.16	31.05	7	3.39	0.00000	0.00000	0.00161	0.00000	0.0000(
01-Mar-10	30	3.16	31	7	3.39	0.00000	0.00000	-0.00161	0.00000	0.0000(
02-Mar-10	30	3.3	31.05	7	3.39	0.00000	0.04430	0.00161	0.00000	0.0000(
03-Mar-10	30	3.3	31.05	7	3.39	0.00000	0.00000	0.00000	0.00000	0.0000(
04-Mar-10	30	3.32	31.05	7.01	3.39	0.00000	0.00606	0.00000	0.00143	0.0000(
05-Mar-10	30	3.34	31.05	7.01	3.39	0.00000	0.00602	0.00000	0.00000	0.0000(
09-Mar-10	30	3.37	31.2	7.01	3.39	0.00000	0.00898	0.00483	0.00000	0.0000(
10-Mar-10	34	3.38	31.22	7.01	3.39	0.13333	0.00297	0.00064	0.00000	0.0000(
11-Mar-10	34	3.4	31.23	7.01	3.39	0.00000	0.00592	0.00032	0.00000	0.0000(

12-Mar-10	34	3.4	31.23	7.01	3.39	0.00000	0.00000	0.00000	0.00000	0.0000
15-Mar-10	34	3.45	31.23	7.01	3.39	0.00000	0.01471	0.00000	0.00000	0.0000
16-Mar-10	34	3.5	31.23	7.01	3.39	0.00000	0.01449	0.00000	0.00000	0.0000
17-Mar-10	34	3.6	31.3	7.1	3.39	0.00000	0.02857	0.00224	0.01284	0.0000
18-Mar-10	34	3.65	31.33	7.1	3.39	0.00000	0.01389	0.00096	0.00000	0.0000
19-Mar-10	34	3.65	31.33	7.1	3.39	0.00000	0.00000	0.00000	0.00000	0.0000
22-Mar-10	34	3.69	31.37	7.1	3.39	0.00000	0.01096	0.00128	0.00000	0.0000
23-Mar-10	34	3.75	31.37	7.1	3.39	0.00000	0.01626	0.00000	0.00000	0.0000
24-Mar-10	34	3.8	32	7.1	3.39	0.00000	0.01333	0.02008	0.00000	0.0000
25-Mar-10	34	3.8	32.08	7.1	3.39	0.00000	0.00000	0.00250	0.00000	0.0000
26-Mar-10	34	3.85	32.08	7.1	3.39	0.00000	0.01316	0.00000	0.00000	0.0000
29-Mar-10	34	3.85	32.25	7.13	3.39	0.00000	0.00000	0.00530	0.00423	0.0000
30-Mar-10	34	3.85	32.64	7.15	3.39	0.00000	0.00000	0.01209	0.00281	0.0000
31-Mar-10	34	3.85	32.75	7.16	3.39	0.00000	0.00000	0.00337	0.00140	0.0000
01-Apr-10	34	3.85	32.85	7.16	3.39	0.00000	0.00000	0.00305	0.00000	0.0000
06-Apr-10	34	3.85	32.85	7.16	3.39	0.00000	0.00000	0.00000	0.00000	0.0000
07-Apr-10	34	3.85	32.9	7.16	3.4	0.00000	0.00000	0.00152	0.00000	0.00295
08-Apr-10	34	3.65	32.9	7.16	3.4	0.00000	-0.05195	0.00000	0.00000	0.0000
12-Apr-10	34	3.65	32.9	7.16	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
13-Apr-10	34	3.65	32.9	7.16	3.4	0.00000	0.00000	0.00000	0.00000	0.0000
14-Apr-10	34	3.65	33.4	7.16	3.46	0.00000	0.00000	0.01520	0.00000	0.01765
15-Apr-10	34	3.64	33.4	7.16	3.46	0.00000	-0.00274	0.00000	0.00000	0.0000
16-Apr-10	34	3.59	33.4	7.16	3.46	0.00000	-0.01374	0.00000	0.00000	0.0000
19-Apr-10	34	3.59	33.4	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
20-Apr-10	34	3.59	33.4	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
21-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	-0.00279	0.03353	0.00000	0.0000
22-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
23-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
24-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
25-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
26-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
27-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
28-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
29-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000
30-Apr-10	34	3.58	34.52	7.16	3.46	0.00000	0.00000	0.00000	0.00000	0.0000

Table 2

(Daily return + 1)				
-	-	-	-	-
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	0.85000	1.00000	1.00000
1.00000	0.76190	1.00000	1.00000	1.00000
1.00000	0.99583	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	0.96234	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
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1.00000	1.00488	1.00000	1.00000	1.00000
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1.00000	1.02564	1.00000	1.00000	1.00000
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1.00000	1.01240	1.00000	1.00000	1.00000
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1.00000	0.99661	1.00000	1.00000	1.00000

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1.00000	1.00352	1.00000	1.00000	1.00000
1.00000	1.01754	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	0.99706
1.00000	1.00000	1.00364	1.00000	1.00000
1.00000	1.00690	1.00132	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00132	1.00000	1.00000
1.00000	1.02740	1.00000	1.00000	1.00000
1.00000	1.00000	1.00197	1.00000	1.00000
1.00000	1.03333	1.00690	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00323	1.00000	1.00000	1.00000
1.00000	1.00322	1.00000	1.00000	1.00000
1.00000	1.00000	1.00130	1.00000	1.00000
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1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00489	1.00000	1.00000
1.00000	1.00317	1.00486	1.02941	1.00000
1.00000	1.00000	1.00161	1.00000	1.00000
1.00000	1.00000	0.99839	1.00000	1.00000
1.00000	1.04430	1.00161	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00606	1.00000	1.00143	1.00000

1.00000	1.00602	1.00000	1.00000	1.00000
1.00000	1.00898	1.00483	1.00000	1.00000
1.13333	1.00297	1.00064	1.00000	1.00000
1.00000	1.00592	1.00032	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.01471	1.00000	1.00000	1.00000
1.00000	1.01449	1.00000	1.00000	1.00000
1.00000	1.02857	1.00224	1.01284	1.00000
1.00000	1.01389	1.00096	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.01096	1.00128	1.00000	1.00000
1.00000	1.01626	1.00000	1.00000	1.00000
1.00000	1.01333	1.02008	1.00000	1.00000
1.00000	1.00000	1.00250	1.00000	1.00000
1.00000	1.01316	1.00000	1.00000	1.00000
1.00000	1.00000	1.00530	1.00423	1.00000
1.00000	1.00000	1.01209	1.00281	1.00000
1.00000	1.00000	1.00337	1.00140	1.00000
1.00000	1.00000	1.00305	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00152	1.00000	1.00295
1.00000	0.94805	1.00000	1.00000	1.00000
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1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.01520	1.00000	1.01765
1.00000	0.99726	1.00000	1.00000	1.00000
1.00000	0.98626	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	0.99721	1.03353	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
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1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000

Table 3 Historic monthly returns for five selected Company shares over one year for 3 Models

Months	X_A	X_B	X_C	X_D	X_E
Jan	0.054	0.032	0.064	0.038	0.049
Feb.	0.045	0.060	0.060	0.060	0.070
Mar	(0.030)	(0.040)	0.050	(0.040)	(0.040)
Apr	(0.020)	0.050	0.010	0.050	0.050
May	0.040	0.050	0.050	0.070	0.050
June	0.050	0.030	0.040	(0.040)	0.040
July	0.060	0.060	0.020	0.060	0.060
Aug	0.040	0.050	0.050	0.030	0.030
Sep	(0.040)	0.030	(0.060)	0.040	0.050
Oct	(0.040)	0.040	0.050	0.060	0.020
Nov	0.050	0.040	0.040	0.060	0.050
Dec	0.050	(0.017)	0.032	0.030	0.040
Mean	**0.021**	**0.032**	**0.032**	**0.034**	**0.038**

Table 4 Absolute deviation per month

Months	A	B	C	D	E
Jan	0.0333	0.0004	0.0083	0.0043	0.0114
Feb.	0.0243	0.0234	0.0203	0.0283	0.0294
Mar	(0.0507)	(0.0676)	0.0123	(0.0707)	(0.0766)
Apr	(0.0387)	0.0204	(0.0087)	0.0163	0.0134
May	0.0223	0.0154	0.0173	0.0313	0.0114
June	0.0263	0.0024	0.0003	(0.0767)	(0.0006)
July	0.0343	0.0314	0.0013	0.0283	0.0174
Aug	0.0153	0.0164	0.0113	0.0003	(0.0126)
Sep	(0.0597)	(0.0066)	(0.0747)	0.0013	0.0144
Oct	(0.0637)	0.0084	0.0113	0.0223	(0.0176)
Nov	0.0253	0.0044	0.0043	0.0233	0.0074
Dec	0.0313	(0.0486)	(0.0037)	(0.0087)	0.0024

Table 5 Real returns over the next 6-months

Months	X_A	X_B	X_C	X_D	X_E
Jan	(0.028)	(0.023)	0.018	(0.021)	0.013
Feb.	(0.011)	0.018	0.009	(0.034)	0.007
Mar	0.016	0.011	(0.016)	(0.011)	(0.024)
Apr	0.023	(0.026)	(0.021)	0.018	(0.018)
May	(0.033)	(0.015)	(0.011)	(0.023)	0.005
June	0.019	0.021	(0.019)	0.015	(0.025)