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DEPARTMENT OF MATHEMATICS

A SIMPLE MODEL OF THE ECONOMY OF GHANA

BY

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A SIMPLE MODEL OF THE ECONOMY OF GHANA

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BY

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DECLARATION

I, hereby declare that this submission is my own work towards the MSc. and that to the best of my knowledge no material previously published by another person nor material which has been accepted for the award of any degree of the University, except where due acknowledgement has been made in the text

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DEDICATION

This work is dedicated to my lovely wife and children.

ABSTRACT

The complexities presented by today's economies required a model approach to provide a clearer and a more reliable solution to today's economic problems of Ghana.

The research therefore approaches the above problem by constructing a simple economic model of the economy of Ghana.

This was done by using the Box-Jenkins method of Time Series Analysis of system identification with data from the Bank of Ghana, Kumasi Branch, and comprising quarterly series of the main economic components of the economy of Ghana such as; Inflation, Total money supply, Interest rate, Gross Domestic Product, Capital stock, Government Expenditure, Investment, Balance of Trade and Consumption.

As a result Autoregressive (AR) Models were constructed for each of the economic components.

A simple economic model of the economy of Ghana in the form of a difference equation model was constructed.

In addition, a homogeneous First order difference equation of the form $X_{k+1} = AX_k$ was found to be stable at a given equilibrium.

The non-homogeneous model of the economy of the form $X_{k+1} = AX_k + BU_k$ was found to be controllable.

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INTRODUCTION

1.1 BACKGROUND OF STUDY

Various economic policies have been put forward from independence to date, by the successive governments of Ghana, in an effort to raise the standard of living for the average Ghanaian. But these "good" intention has a lot of set back as a result of the imbalances in the World's economic order leaving the average Ghanaian worse off than before independence (A.K. Osei-Ofosu, 2003).

On the basis of today's levels of per capita income for grouping countries, Ghana would have been ranked as a middle-income country at the time of independence in 1957, a status that the country is aspiring to achieve by the year 2020 (Ibid 2003).

Ghana's economy depends on many indicators, Inflation, Interest Rate, Capital Stock, Investment, Total Money Supply, Government Expenditure, Gross Domestic Product, Consumption and Trade Balance. These above mentioned indicators have had their rise and fall in the performance of the Ghanaian economy before, and after independence.

A policy to rapidly develop social and economic infrastructure after independence in 1957 resulted in a quick draw down on the country's foreign exchange reserves, which saw growth in the economy (A. K. Osei-Fosu, 2003). This growth was throughout the 1960's and early 1970's.

However, the Ghanaian economy began a gradual down turn throughout the rest of the 1970, and early 1980. This was due to poor domestic policies, economic mismanagement, rapid increase in money supply, increased government expenditure, high inflation and interest rate, low investment and low GDP, as well as a decline in the per capita income.

Though various programmes and policies were put in place by the successive governments, from 1983 to 1991, to salvage the economy, expectations were not met. For instance, the Structural Adjustment Programme in 1983, saw real GDP, rose from (-7%) to over 8% in 1984, and maintain growth rate above 5% in 1985 and 1986. But between 1986 to 1990, it fell back to 4.8%. Inflation rose from 10% in 1985 to 37% by 1990. Balance of trade deficits increased from 2.7 billion dollars in 1986 to 3.5 billion dollars in 1990 (Ibid, 2003).

These declines in the economy were experienced from 1992 to 1995. Winning the election was a must for the government of the day, and so huge government expenditure was experienced, an increase in money supply, and a low drive towards investments. In effect the economic indicators did not perform any better.

Contemporary economies present challenges that require models, to be used to guide policy formulation. The challenges presented above indicate that domestic policies must be tailored along model equations which will help promote growth of the Ghanaian economy.

Constructing Autoregressive models for the various components of the economy was necessary. And a linear control discrete time model equation for the entire economy of Ghana that is stable and controllable, is required to guide the policy makers to achieve set goals, that promote economic growth and which will enable the average Ghanaian lead a descent life.

Included in these goals are;

1. the desire to reach full employment level, which is unemployment of about 5%,
2. an agenda of reaching optimal distribution of income so that the average person can live a decent life,
3. low level of inflation and
4. a satisfactory balance of payments.

1.2 STATEMENT OF THE PROBLEM

The path towards these goals has not been easy for Ghana, and one often asks whether any of these goals, could be achieved at all. In any case, it may be possible to reach the above goals absolutely only through an adroit use of economic research and policy.

However, modern economics is so constituted that at least some of these goals may to some extent be competitive. These are full employment, price level stability and satisfactory balance of payments.

Having more than one of these goals may sometimes involve having less of another. The concern of this research is to find out whether there is a way of obtaining a simple but reliable model of the Ghanaian economy which could be used to guide policy makers.

1.3 THE OBJECTIVES OF THE STUDY

Every economy has two kinds of variables namely; endogenous variables and exogenous variables. Endogenous variables include consumptions, national income and profit. Exogenous variables include government expenditure, net investments and indirect taxes.

The specific objectives of the study are as follows:

1. To obtain ARIMA models of the main economic indicators of the economy of Ghana.
2. To construct a homogeneous discrete time linear model of the economy of Ghana.
3. To construct a control theoretical economic linear model.
4. To determine the stability and controllability of the economic models.

1.4 METHODOLOGY

The data used for the study is a secondary data collected from the Bank of Ghana, Kumasi Branch, from 1996-2004 comprised quarterly performance on the major economic indicators of the economy of Ghana, namely Inflation, Total Money Supply, Interest Rate, Investment, Capital Stock, Government Expenditure, Gross Domestic Product, Consumption and Trade Balance.

The Box-Jenkins method of modelling time series data, implemented by SPSS package was used to identify AR(p) models .

MATLAB was also used to determine the eigenvalues, as well as stability and controllability of the difference economic models obtained.

1.5 STRUCTURE OF THE THESIS

The structure of the thesis is as follows;

Chapter 1 dealt with the Introduction, which comprises the Background of Study, the Problem Statement, the Objective of the Study and the Methodology.

Chapter 2 dealt with the review of the relevant literature; that is, presenting the available information while acknowledging and making reference of the work of originators of the ideas.

This chapter covered basic concepts on linear systems and spaces, discrete linear systems and concepts and theories as well as solutions to matrix equations, and time series concepts.

Chapter 3 dealt with the modelling and data analysis. The concepts of Chapter 2 would be applied in constructing and solving the various AR(p) models as well as the model equations in their matrix forms.

Chapter 4 summarized the results obtained in Chapter 3. It also presented conclusions of the study and then offered some recommendations based on the findings made.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

In Chapter 1, we considered the background and purpose of the research including problem statement leading to the objectives, methodology and structure of work.

In this chapter, literature will be reviewed based on its relevance to the study.

2.1 LINEAR SPACES

A set is an entity without internal mathematical structure and as such, of limited usefulness and interest. The sets that interest us are point sets on a line or plane and sets of ordered numbers. A linear space or vector is a set with an algebraic structure of real numbers. The operations of addition and scalar multiplication are extensions of the corresponding operations with real numbers or complex numbers.

The field $\mathbb{R}$ and the field $\mathbb{C}$ will refer to the set of real and complex numbers respectively due to their general algebraic structure. *(Halmos, 1958)*

Definition(Linear Spaces)

A linear space A over a field F is a set of vectors whose elements are to be called vectors and are defined as follows:

(i) An operation called vector addition which associates with each pair of vectors x and y in X a unique vector in X denoted by $x + y$, in such a way that

a) $x + y = y + x$

b) $x + (y + z) = (x + y) + z$

c) There is a unique vector $x \in X$, called the zero vector, such that

$$x + \hat{0} = x, \quad \text{for all } x \in X$$

d) For each vector $x \in X$, there is a unique vector in X, to be called $-x$, such that $x + (-x) = 0$

(ii) An operation called scalar multiplication, which associates with each scalar $c \in R$ and each $x \in X$, a unique vector in X, denoted by cx, in such away that

a) $1x = x$ for all $x \in X$

b) $(c_1 c_2)x = c_1(c_2 x)$, $c_1, c_2 \in F$

c) $c(x + y) = cx + cy$

d) $(c_1 + c_2)x = c_1 x + c_2 x$

The linear space importance to this discussion is the one whose elements are ordered sets of n real numbers, often called n-tuples or n-vectors.

This space is denoted by R^n and its elements by

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad 2.1$$

Addition operation in R^n is defined as

$$x + y = \begin{bmatrix} x_1 & + & y_1 \\ x_2 & + & y_2 \\ x_3 & + & y_3 \\ \vdots & \vdots & \vdots \\ x_n & + & y_n \end{bmatrix} \qquad 2.2$$

and multiplication by a scalar $c \in R$ as $cx = \begin{bmatrix} cx_1 \\ cx_2 \\ \vdots \\ cx_n \end{bmatrix}$ 2.3

(Hoffman, and Kunze, 1961)

2.1.1 LINEAR DEPENDENCE

A set { $x^1, x^2, \ldots, x^n$ } of vectors in X are linearly dependent if there exist scalars $c_1, c_2, \ldots, c_n$, in F not all of which are zero, such that $c_1x^1 + c_2x^2 + \ldots + c_nx^n = 0$.

Consider for example:

Let $x \in R^3$; the vector $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, is a linear combination of the vectors

$$x^1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \text{ and } x^2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix};$$

Since $x = x^1 + x^2$. Here $c_1 = 1$, $c_2 = 1$

The set $\{x, x^1, x^2\}$ is linearly dependent because $x - x^1 - x^2 = 0$.

(Rubio, 1971)

2.1.2 LINEAR INDEPENDENCE

Definition: A basis for a linear space X is a linearly independent set of vectors in X such that every vector in X can be written as a linear combination of elements in the set.

The set of vectors $\{e^1, e^2, \ldots, e^n\}$ in R^n, where all the entries in e^i are $zero$ except the ith entry, which is 1, is independent. A subset of this set is also independent; but a set formed by an arbitrary vector $x \in R^n$ and all the vectors $e^1, \ldots, e^n$, is linearly dependent. If it is assumed that x_i is to be the ith entry in the arbitrary vector; then $x - \sum_{i=1}^{n} x_i \ell^i = 0$.

If the vector $x = \sum_{i=1}^{n} c_i \alpha_i$ and the scalars c_i , $i = 1,\ldots,n$ are the coordinates of the vector x relative to the ordered basis $\{\alpha_i\}$.

If a vector x is assumed to have two sets of co-ordinates with respect to a basis

$\alpha = \{ x^1, x^2, \ldots, x^n\}$, then it can be written as

$$x = \sum_{i=1}^{n} a_i x^i = \sum_{i=1}^{n} b_i x^i$$

Therefore $\sum_{i=1}^{n} (a_i - b_i) x^i = 0$ 2.4

Since the vectors x^i are independent, because they form the basis of the linear space X. It follows that $a_i - b_i = 0,\ i = 1,\ 2, \ldots, n$ so that indeed the coordinates of a vector with respect to an ordered basis is unique. *(Rubio, 1971)*

2.1.3 LINEAR TRANSFORMATIONS AND MATRICES

Let X and Y be linear spaces over F, not necessarily distinct. A linear transformation from X into Y is a function $T: X \to Y$, such that

$$T\left(c_1 x^1 + c_2 x^2\right) = c_1 T\left(x^1\right) + c_2 T\left(x^2\right), \qquad 2.5$$

for all x^1 and x^2 in X and all scalars c_1 and c_2 in F. If $X = Y$, then a function T : X→ X, which satisfies (2.5) is said to be a linear operator, which under linear transformation maps a vector space unto itself. It must be noted that linear transformations are not necessarily onto.

(Cullen, 1972)

2.1.4 TRANSFORMATIONS BY MEANS OF MATRICES

Let a finite-dimensional linear space X have a basis $\alpha = \{x^1, \cdots, x^j, \cdots, x^n\}$.

Let T be a linear transformation from X into another finite-dimensional linear space Y, with basis $\beta = \{y^1, \cdots, y^i, \cdots, y^m\}$. Since $T(x^j) \in Y$, $j = 1, \ldots, n$, can be written as

$$T(x^j) = \sum_{i=1}^{m} a_{ij} y^i \qquad 2.6$$

where a_{ij} is the ith co-ordinate of $T(x^j)$ relative to the basis β. A vector $x \in X$ given by $x = \sum_{j=1}^{n} c_j x^j$ is transformed by T into

$$T(x) = \sum_{j=1}^{n} c_j T\left(x^j\right) = \sum_{j=1}^{n} c_j \sum_{i=1}^{m} a_{ij} y^i$$

$$= \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_{ij} c_j\right) y^i \qquad 2.7$$

It shall be noted that:

(i) the coordinates of $T(x)$ with respect to the bases β are completely specified, for any $x \in X$, for all coordinates of $x \in X$, and by the set of scalars $\{a_{ij}$, $i = 1,\ldots,m$, $j = 1,\ldots,n\}$. It this sense T is completely specified by this set of m.n scalars.

(ii) the set of scalars is independent on the bases α and β which have been chosen for X and Y respectively. While the transformation T itself does not depend on the bases chosen, the set of scalars a_{ij} described above will be written in array as follows:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad 2.8$$

relative to the bases α and β.

The scalars a_{ij} are called the entries of the matrix. The horizontal sub arrays such as a_{11}, $a_{12}, \ldots, a_{1n}$, are the rows of the matrix, while the vertical sub arrays such as;

$$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{m1} & a_{m2} \end{matrix}$$

are called the columns.

The matrix (2.8) has m rows and n columns, and so it is

an $m \times n$ matrix. The n-tuple in (2.8) can be considered as an $n \times 1$ matrix written as

$$\begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1n} \end{bmatrix}$$

It is however, customary to use the notation (2.8) (Halmos, 1958)

2.1.5 IDENTITY MATRIX (I_n)

An important operator is the identity operator I, which maps every vector in a vector space X into itself. Let X be n-dimensional vector and $\alpha = \{x^i, \cdots, x^n\}$ be a basis for X.

Then $I(x^j) = x^j$, j = 1,…,n ; so that the matrix of I with respect to any basis is the $n \times n$ identity matrix I_n. that is

$$I_n = \begin{bmatrix} I & & & & & \\ & I & & & & \\ & & I & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & I \end{bmatrix} \qquad 2.9$$

where all the entries not shown are zeros.

(Halmos, 1958)

2.1.6 THE ZERO MATRIX (O_n)

The zero operator, which maps every vector X into a vector O, has a matrix relative to any basis, the $n \times n$ zero matrix O_n, all of whose entries are zero. This matrix is sometimes denoted by O.

2.1.7 SUM OF MATRICES

The sum of two matrices of the same dimensions, $T_1 = (a_{ij})$ and $T_2 = (b_{ij})$ is a matrix with entries

$$a_{ij} + b_{ij}.$$

That is $$T_1 + T_2 = (a_{ij} + b_{ij}) \qquad 2.10$$

The matrix transformation $T_1 + T_2$ relative to the bases α and β is therefore $T_1 + T_2$.

Let T_1 and T_2 be two transformations which map a finite dimensional vector space X into a finite–dimensional vector space Y.

The sum of T_1 and T_2 as a transformation is denoted by $T_1 + T_2$, such that

$$(T_1 + T_2)x = T_1x + T_2x \qquad 2.11$$

Let X and Y have bases α and β as above. If the matrix of the operator T_1, relative to the bases α and β is $T_1 = a_{ij}$ and matrix operator T_2 is $T_2 = b_{ij}$,

$$T_1(x) = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_{ij}\, c_j\right) y^i \qquad 2.12$$

$$T_2(x) = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} b_{ij}\, c_j\right) y^i \qquad 2.13$$

where c_j, j = 1, …,n, are the coordinates of X relative to the basis α. Then,

$$(T_1 + T_2)x = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} (a_{ij} + b_{ij})c_j\right) y^i \qquad 2.14$$

Therefore the set of numbers $\{a_{ij} + b_{ij}, i = 1, \cdots, m;\ j = 1, \cdots, n\}$ completely specify the behavior of the operation $T_1 + T_2$ which leads to the definition under (2.14).

As an example, let $X = Y = R^3$, and T_1 and T_2 linear operators, defined by

$$T_1(x) = \begin{bmatrix} 3x_1 \\ x_1 + x_2 \\ x_3 \end{bmatrix}, \quad T_2(x) = \begin{bmatrix} x_2 \\ x_1 - x_2 \\ 3x_3 \end{bmatrix}, \ x \in R^3$$

Let $\alpha = \beta = \{x^1, x^2, x^3\}$, where

$$x^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ x^2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \ x^3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

The matrices T_1 and T_2 relative to these bases are:

$$T_1 = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

The matrix of the operator $T_1 + T_2$ relative to the same bases is

$$T_1 + T_2 = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 12 \end{bmatrix}.$$

(Birkhoff, 1965)

2.1.8 MULTIPLICATION OF MATRICES

Let an $m \times n$ matrix $T = a_{ij}$ be given. The $m \times n$ matrix cT is defined as

$$cT = (ca_{ij}) \qquad 2.15$$

where c is a scalar and $c \in F$, where F is a vector field. The matrix relative to the bases α and β of a transformation matrix cT, defined by

$$(cT)x = c(Tx) \qquad 2.16$$

is cT, where T is the matrix of T relative to the bases α and β.

Let $X, Y, and\ Z$ be finite-dimensional vector spaces of n, m, and r respectively.

Let $\alpha = \{ x^1, \ldots, x^n \}$, $\beta = \{ y^1, \ldots, y^m \}$ and $\gamma = \{z^1, \ldots, z^r\}$ be bases for X, Y ,Z in this order. Two transformations are defined, $T_1: X \longrightarrow Y$ and $T_2: Y \longrightarrow Z$, with matrices $T_1\ and\ T_2$ relative to the corresponding bases.

Consider the composite transformation T_1T_2 defined by

$$(T_1T_2)x = T_2(T_1x) \qquad 2.17$$

for all $x \in X$.

This transformation maps X into Z, and is well-defined and linear.

If $x = \sum_{j=1}^{n} c_j x^j$, then $T_1(x) = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_{ij} c_j \right) y^i$, where the scalars a_{ij} are elements of T_1

then,

$$T_2(T_1(x)) = \sum_{k=1}^{r}\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} a_{ij}c_j\right)b_{ki}\right]z^k \quad 2.18$$

where the scalars b_{ki} are the elements of T_2. Equation (2.18) can be arranged as

$$(T_2T_1(x))) = \sum_{k=1}^{r}\left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m} b_{ki}\,a_{ij}\right)c_j\right]z^k \quad 2.19$$

Thus the transformation T_1T_2 is completely defined by the set of scalars;

$\left\{\sum_{i=1}^{m} b_{ki}\,a_{ij},\ j=1,\cdots,n,\ k=1,\cdots,r\right\}$, by the rxn matrix $\left(\sum_{i=1}^{m} b_{ki}\,a_{ij}\right)$. This is the product of T_2 and T_1; that is:

$$T_1T_2 = \left(\sum_{i=1}^{m} b_{ki}\,a_{ij}\right) \quad 2.20$$

(Bellman, 1970)

2.1.9 PARTITION OF A MATRIX

Often matrices can be partitioned into sub matrices. For example, consider an $m \times n$ matrix A, with complex entries. Each of the m columns can be used to form a column vector with n complex entries. In this way vectors are constructed as $a^1 \ldots, a^m$ and it is written as

$$A = [\, a^1a^2\ldots a^m] \quad 2.21$$

For instant, if B is a $p \times n$ matrix, the product BA is equal to

$$BA = [\, Ba^1Ba^2\ldots Ba^m]. \quad 2.22$$

Similarly, matrices are partitioned as follows:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \qquad 2.23$$

The matrices $A_1, A_2, A_3,\ A_4$ can be of arbitrary dimensions provided that they cover the whole of the matrix A. The partition of matrix B is as follows:

$$B = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} . \qquad 2.24$$

Then supposed AB exists, it is given by;

$$AB = \begin{bmatrix} A_1B_1 + A_2B_3 & A_1B_2 + A_2B_4 \\ A_3B_1 + A_4B_3 & A_3B_2 + A_4B_4 \end{bmatrix} \qquad 2.25$$

Provided that the partitions of A and B are such that all the products which appear in the matrix AB are well defined. *(Hoffman and Kunze, 1958)*

2.1.10 EIGENVALUES AND EIGENVECTORS

Let T be an operator which maps X, an n-dimensional linear space over the field C, into itself. A number λ in C is an eigenvalue of T if there is at least one non-zero vector in X such that,

$$T(x) = \lambda x. \qquad 2.26$$

If λ is an eigenvalue of T, any vector $x \in X$ such that $T(x) = \lambda x$ is said to be an eigenvector of T. In other words, each eigenvector is associated with an eigenvalue and each eigenvalue is associated at least with one eigenvector. *(Rubio, 1971)*

2.1.11 REDUCTION (JORDAN CANONICAL FORM)

The canonical form for a matrix A when its characteristic roots are not all distinct, has a general result which is stated for any square matrix similar to the Jordan form as

$$\operatorname{diag}\left[J_{m1}(\lambda_1), J_{m2}(\lambda_1) = -J_{ms}(\lambda_1), J_{n1}(\lambda_2), J_{tv}(\lambda_q)\right] \qquad 2.27$$

Using the notation $J_k(\lambda)$ which is a kxk Jordan block is written as

$$J_k(\lambda) = \begin{bmatrix} \lambda & 1 & 0 & . & 0 & 0 \\ 0 & \lambda & 1 & . & 0 & 0 \\ . & . & . & . & . & . \\ . & 0 & 0 & . & \lambda & 1 \\ 0 & 0 & . & . & 0 & \lambda \end{bmatrix} \qquad 2.28$$

The distinct characteristic roots of A are $\lambda_1, \lambda_2, \ldots, \lambda q$ and the multiplicity of λ_i

is $m_1 + m_2 + \ldots + m_s$. If none of the characteristic roots of A is repeated so that $q = n$ then (2.28) reduces to the diagonal matrix and the Jordan block is simply

$$J_1(\lambda_i) = \lambda_i$$

There is just one linearly independent characteristic vector of A associated with each Jordan block, so the total number of such vectors is equal to the number of blocks in Jordan form.

An important case is stated here, that every symmetric matrix written as $(A^T = A)$ has a diagonal Jordan form and in addition a transforming matrix can be found which is orthogonal.

The Jordan form of (2.28) can be written as $A =$ dig [$J_2(1), J_1(1)$].

In this case A is derogatory, because it has two Jordan blocks associated with the root 1, and A is non derogatory if and only if there is only one Jordan block associated with each λ_i , (i = 1,2…, q) .

In addition if α_i is the largest Jordan block associated with λi, then the minimum polynomial of A is;

$$m(\lambda) = (\lambda - \lambda_1)^{\alpha_1} (\lambda - \lambda_2)^{\alpha_2} \cdots (\lambda - \lambda_q)^{\alpha_q} \qquad 2.29$$

which coincides with $k(\lambda)$ where $q = n$; and each αi being equal to unity. Since $J_k(\lambda)$ in (2.28) is triangular, its rank is k if $\lambda \neq 0$ and $k = 1$ if $\lambda = 0$.

In conclusion, it should be noted that although the Jordan form is of fundamental theoretical importance; it is only of little use in practical calculation. Being generally very difficult to compute. *(Barnett and Cameron, 1985)*

2.2.0 LINEAR SYSTEMS

A system is linear if when the response to some input , $u(t)$, is

$$y(t) = L(u),$$

where the response to $c_1u_1 + c_2u_2$ is $c_1L(u_1) + c_2L(u_2)$. Here L is some operator- differential, integral, probabilistic, etc, the c_i are constants; and u_j in general will be vectors.

(Chen, 1970)

2.2.1 DIFFERENCE EQUATION OF LINEAR SYSTEMS

This is another kind of linear systems whose domain is not in the closed interval of the real numbers, but a subset of the set of integers. The input and the output are defined only at discrete instants of time, designated as kT, where k is an integer and T an arbitrary positive real number.

The delay simply delays the sequence of numbers at its input by T seconds, so that if its input is

$w(k+1)T$, then the output is $w(k)T$. It follows that the input and output sequences are related by

$$y((k+2)T) = u(kt) - 2y((k+1)T) - 3y(kt) \quad 2.30$$

or

$$y((k+2)T) + 2y((k+1)T) + 3y(kt) = u(kt) \quad 2.31$$

By transformation, the equation above is

$x_1(kt) = y(kt)$, $x_2(kt) = y((k+1))$, so that

$$x_1((k+1)T) = x_2(kt) \quad 2.32$$

$$x_2((k+1)T) = -3x_1(kt) - 2x_2(kt) + u(kt)$$

$$y(kt) = x_1(kT)$$

Equation (2.30), gives a state variable at $(k+1)T$ in terms of the state variable at the previous time kT, and the values of the input at kT.

Equation (2.30) can be rewritten in the form,

$$x(k+1)T) = Ax(kT) + Bu(kT) \qquad 2.33$$

$$y(kT) = Cx(kT) \qquad 2.34$$

where x is a two dimensional state vector, u is a scalar input, y is a scalar output and

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \text{ from (2.30)}$$

In general, the difference linear system is define by the equations

$$x(k+1) = A(k)x(k) + B(k)u(k) \qquad 2.35$$

$$y(k) = C(k)x(k) + D(k)u(k), \quad k \in I \; . \qquad 2.36$$

The added extra term to the output represents direct transmission between input and output. Here x is an n-vector. A(k) is an nxn matrix, B(k) an nxr matrix, C(k) an mxn matrix, D(k) an mxr matrix. The input u(k) is an r-vector, the output y(k) an m-vector. All of these matrices and vectors are defined on a subset I of the set of integers of the type;

I = { p, p+1,p+2,…,p+q. }.

The matrices A and B are dependent on k, and as such, the systems are time-varying. All matrices and vectors have real components. *(Rubio,1971)*

2.2.2 THE HOMOGENEOUS SYSTEM

Consider the homogeneous system associated with the first equation of (2.35) and 2.36).

$$x(k+1) = A(k)x(k) \ , \ x(k_o) = x^o \ , \ k \in I \qquad 2.37$$

where the matrices A(k), $k \in I$ is nonsingular, for the uniqueness of solution of (2.37).

2.2.3 THE NONHOMOGENEOUS SYSTEM

Consider again equation (2.35) and (2.36) starting with (2.35)

$$x(k+1) = A(k)x(k) + B(k)u(k) \qquad 2.38$$

The input u is a sequence of numbers defined for $k \in I$. If $x(k_o)$ is specified. The aim, here, is to obtain an explicit expression for the solution of (2.38); with a preferred direction of time.

Let X(k) be a fundamental matrix of the homogeneous system associated with (2.38).

Let the solution of (2.38) be of the form;

$$x(k) = X(k)p(k) \qquad 2.39$$

which will be the solution if p satisfies equation (2.40) below.

$$x(k) = X(k+1)p(k+1) = A(k)X(k)p(k) + B(k)u(k).$$

since $\qquad X(k+1) = A(k)X(k)$

$$X(k+1)\{p(k+1) - p(k)\} = B(k)u(k)$$

Suppose now that X(k+1) is invertible then ,

$$p(k+1) = p(k) + X^{-1}(k+1)\, B(k)u(k) \qquad 2.40$$

If $x(k_o)$ is specified, the solution can be seen as

$$p(k) = X^{-1}(k_o)x(k_o) + \sum_{l=k_o}^{k-1} X^{-1}(l+1)B(l)u(l);$$

this implies at last that

$$p(k) = \phi(k,k_o)x(k_o) + \sum_{l=k}^{k-1} \phi(k,l+1)B(l)u(l) \qquad 2.41$$

(Rubio, 1971)

2.2.4 z-TRANSFORMATION

Definition: In a situation where the variables are measured or "sampled" only at discrete intervals of time, they produce what are referred to as Sampled-data or Discrete-time Systems.

The method of z-transform can be used also for obtaining explicit solutions. If $\hat{x}$, $\hat{u}$ are the z-transforms of x, u then, if $k_o = 0$

$$\hat{x}(z) = (zI_n - A)^{-1} zx^o + (zI_n - A)^{-1} B\hat{u}(z)$$

If the matrix A is diagonal with elements $c_{1},\ldots,c_{m}$; then,

$$A^k = \begin{bmatrix} c_1^k & & & & \\ & c_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots & \\ & & & & & c_m^k \end{bmatrix}$$

If A is in Jordan Canonical form, then,

$$A^k = \begin{bmatrix} A_1^k & & & & \\ & A_2^k & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots & \\ & & & & & A_q^k \end{bmatrix}$$

$$A^k{}_i = \begin{bmatrix} A_{i1}^k & & & & \\ & A_{i2}^k & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots & \\ & & & & & A_{is(i)}^k \end{bmatrix}$$

where, it can be recalled that,

$$A_{ij} = \begin{bmatrix} \lambda_i & & & & & \\ 1 & \lambda_i & & & & \\ & 1 & \ddots & & & \\ & & & & & \\ & & & & \lambda_i & \\ & & & & 1 & \lambda_i \end{bmatrix}_{n_{ij} x n_{ij}}$$

So that

$$A^k_{ij} = \begin{bmatrix} \lambda_i^k & 0 & \cdots & 0 \\ \binom{k}{1}\lambda_i^{k-1} & \lambda_i^k & \cdots & 0 \\ \binom{k}{2}\lambda_i^{k-2} & \binom{k}{1}\lambda_i^{k-1} & \cdots & \vdots \\ & & & 0 \\ \vdots & \vdots & \cdots & \lambda_i^k \end{bmatrix}$$

where $\binom{K}{1}, \binom{k}{2}, \ldots$ are the coefficients usually defined in connection with the binomial expansion.

If however, A is not in the Jordan Canonical form, we put z $=S^{-1}x$, where S is the matrix used in the transformation of A into Jordan canonical form, so that the matrix equation

$$x(k+1) = Ax(k) \; ; \; x(k_o) = I_n ,$$

becomes $$Z(k+1) = S^{-1}ASZ(k) \; , \; z(k_o) = S^{-1}$$

where the matrix $S^{-1}AS$ is in Jordan canonical form so that the transition matrix of the system is easily computed. It follows that

$$\Phi(k-k_o) = S(S^{-1}AS)^{k-k_o} S^{-1} \quad \text{where } k \in I .$$

(Barnett and Cameron, 1985)

2.2.5 DISCRETE –TIME SYSTEMS

A system in which the input and the output are defined only at discrete instants of time, instants which are designated kT, k being an integer and T an arbitrary positive real number.

The delay, simply delays the sequence of numbers at its input by T seconds, so that if its input is $w(k+1)T$, then its output is $w(k)T$.

Then the mathematical model in state variable form is

$$x(k+1) = A(kT)x(k) + B(kT)u(k) \qquad 2.42$$

where x(k), u(k) denote the values of the state and control vectors x(kT) and u(kT) respectively (k = 0, 1, 2, …).

To develop matrix methods for solution of (2.42). Consider first the situation when there is no control and A is a constant matrix, then (2.42) becomes

$$x(k+1) = Ax(k) , \quad x(k_o) = x_o \qquad 2.43$$

$$x(k_o+1) = Ax(k_o)$$

$$x(k_o+2) = Ax(k_o+1) = A^2x(k_o) , \text{ and so on}$$

Therefore, the solution of (2.43)

$$x(k) = A^{k-k_0}x_0 \qquad 2.44$$

The state transition matrix is defined by

$$\Phi(k, k_0) = A^{k-k_0} \qquad 2.45$$

And the solution of (2.45) can be written as

$$x(k) = \Phi(k, k_o)x_o \;;$$

with the following properties

$$\Phi(k+1,k_o) = A\Phi(k,k_o)$$

$\Phi(k,k) = I$, where I is the identity matrix,

$\Phi(k_o,k) = \Phi^{-1}(k,k_o)$, provided A is nonsingular 2.46

$\Phi(k,k_o) = \Phi(k,k_1)\Phi(k_1,k_o)$, $k \geq k_1 \geq k_o$ are the discrete analogue of the continuous time.

Now if A and B are still time invariant, then from (2.42),

$$\begin{aligned} x(k) &= A[Ax(k-2) + Bu(k-2)] + Bu(k-1) \\ &= A^2x(k-2) + ABu(k-2) + Bu(k-1) \\ &= A^2[Ax(k-3) + Bu(k-3)] + ABu(k-2) + Bu(k-1) \end{aligned}$$

.

.

.

$$= A^{k-ko}x^{o} + \sum_{i=ko}^{k-1} A^{k-i-1}Bu(i)$$

$$= \Phi(k,k_o)\left[x_0 + \sum_{i=ko}^{k-1}\phi(k_o, i+1)Bu(i)\right]$$

and assuming A is nonsingular then, (2.44) and (2.46) gives

$$\Phi(k,k_o) = \prod_{i=ko}^{k-1} A(iT) \qquad 2.47$$

(Cadzow, 1973)

2.2.6 MODEL FOR DISCRETE TIME CONTROL SYSTEM

Control is an attempt to compensate for disturbances that infect a system. Some are measurable and others are not measurable, and only manifest themselves as unexplained deviations from the target of the characteristic to be controlled. Consider the special case where unmeasured disturbances affect the output Y_t of a system, and suppose that feedback control is employed to bring the output as close as possible to the desired target value by adjustment applied to an input variable X_t.

Supposed that N_t represents the effect at the output of various unidentified disturbances within the system, which in the absence of control could cause the output to drift away from the desired target value or set point T.

(Kucera, 1979)

Then, despite adjustments that have been made to the process, an error:

$$\varepsilon_t = Y_t - T$$

$$\varepsilon_t = V(B)X_t + N_t - T$$

will occur between the output and its target value T, where B is the backward shift operator.

The object is to choose a control equation that the error ε will have the smallest possible mean square. The control equation expresses the adjustment

$$X_{t+1} = X_t - X_{t-1}$$

to be taken at time t, as a function of the present deviation ε_t. Previous deviations $\varepsilon_{t-1}, \varepsilon_{t-2}, \cdots$, and previous adjustments, $X_{t-1}, X_{t-2}, \ldots$

2.2.6.1 CONTROLLABILITY

Let Q be a matrix defined as $Q = [B \quad AB \quad \ldots \quad A^{n-1}B]$ has rank n. Then the set of all vectors in R^n which can be reached in n steps equals the whole of R^n, and in this case there will be n independent columns in Q. Therefore, the system

$$x(k+1) = Ax(k) + Bu(k) \qquad 2.49$$

is controllable if every state can be reached from the origin in a finite number of steps.

(Rubio, 1971)

Theorem 1

The system described by x(k+1) = Ax(k) + Bu(k) with A and B as constant matrices, is controllable if and only if the matrix Q has rank n.

The matrices A and B are considered constant in the sense that, the shift in the time origin changes neither the values of the state nor of the output, provided that the input is shifted accordingly. The system with such constant matrices is said to be time invariant or constant.

Thus if the matrix A is non-singular, let $x(k_o) = 0$, then the system

$$x(k_o+p) = \sum_{i=1}^{p} A^{p-1} Bu(k_0 + l - 1)$$

$$= Bu(k_o + p-1) + ABu(k_o + p-2) + \ldots + A^{p-2} Bu(k_o+1) + A^{p-1}Bu(k_o) \qquad 2.48$$

is controllable. Where the set of all vectors $x(k_o + p)$, and the input u takes all possible values at each k, $k = k_{o,}\ k_o+1, \ldots, k_o + p-1$; which is a subspace of R^n and is spanned by the columns of the matrices B, AB,…,A^{p-1}B.

(Rubio, 1971)

2.2.6.2 STABILITY

For a given finite dimensional system which in this case is the homogeneous difference system of the form

$$x(k+1) = A(k)x(k), \text{ as } k \to \infty; \qquad 2.50$$

the interval of definition of the matrix A is a set of type $[\ k_o, \infty\ ]$

If $\hat{x}$ is a point of equilibrium of x(k+1) = A(k)x(k), where the assumption is that A is a matrix function defined for all $k \geq k_o$,

if

$$A(k)\hat{x} = 0, \text{ for some } k \geq k_o \tag{2.51}$$

The origin is said to be a stable point of equilibrium if given t_0, a number $\delta(\varepsilon, t_0)$ can be found such that if, $\|x(t_0)\| < \delta(\varepsilon, t_0)$, then $\|x(t_0)\| < \varepsilon$ for $t \geq t_0$.

If the number $\delta(\varepsilon, t_0)$ does not depend on t_0, the origin, is said to be uniformly stable.

Definition: The origin is said to be an asymptotically stable point of equilibrium if it is stable and if every trajectory of this system, regardless of the initial condition, satisfies,

$$\lim_{t\to\infty}\|x(t)\| = 0$$

That is, given $x(t_o) = x^o$ and $\varepsilon > 0$, a number $N(x^o, t_o, \varepsilon)$ can be found such that $\|x(t)\| < \varepsilon$, for $t > N(x^0, t_0, \varepsilon)$. If the number $N(x^0, t_0, \varepsilon)$ does not depend on t_o, the origin is said to be uniformly asymptotically stable.

(Rubio, 1971)

Theorem 2:

The system $x(k+1) = Ax(k)$ 2.52

is asymptotically stable if and only if the eigenvalues of the matrix A have an absolute values less than unity.

Proof:

For all matrices $A(k)$, $k \in I$ are singular, then a transition matrix for the system

$$x(k+1) = A(k)x(k), \quad x(k_o) = x^o, \; k \in I \text{ is defined.}$$

If $x(k_o) = I_n$, in $x(k+1) = A(k)x(k)$, for $k \geq k_o$, defined on the set $\Phi(k,k_o)$ is given as

$$\Phi(k,k_o) = A(k-1)\ldots A(k_o), \; k > k_o, \; k \in I$$

then, the solution generated by $\Phi(k_o,k_o) = I_n$ from $x(k+1) = A(k)x(k)$, $x(k_o) = x^o$, $k \in I$ is

$x(k) = \Phi(k,k_0)x(k_o)$ for $k \geq k_o$, and has the properties:

$\Phi(k_o,k_o) = I$

$\Phi(k_2,k_1)\Phi(k_1,k_o) = \Phi(k_2,k_o)$ where $k_o \leq k_1 \leq k_2$; and the transition matrix $\Phi(k_1,k_o)$ is not invertible. Then if A is a diagonal matrix with elements $c_{1,\ldots,}c_{m,}$ then

$$A^k = \begin{bmatrix} c_1^k & & & & & \\ & c_2^k & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & c_m^k \end{bmatrix}, \text{ and}$$

$$A_{ij} = \begin{bmatrix} \lambda_i & & & & & \\ 1 & \lambda_i & & & & \\ & 1 & . & & & \\ & & & . & & \\ & & & & \lambda_i & \\ & & & & 1 & \lambda_i \end{bmatrix}_{n_{ij}\times n_{iij}}$$

So that

$$A_{ij}^k = \begin{bmatrix} \lambda_i^k & 0 & \cdots & 0 \\ \binom{k}{1}\lambda_i^{k-1} & \lambda_l^k & . \quad . \quad . & 0 \\ \binom{k}{2}\lambda_i^{k-2} & \binom{k}{1}\lambda_i^{k-1} & \cdots & 0 \\ & & & \vdots \\ \vdots & \vdots & & 0 \\ & & & \lambda_i^k \end{bmatrix}$$

Which holds when $K^n\lambda^k \to 0$ as $K \to \infty$, for a fixed n, such that $|\lambda| < 1$

Theorem 3

The origin is an asymptotically stable point of equilibrium for the system

$$x(k+1) = Ax(k),$$

if and only if, given any positive definite matrix G, there is a positive definite matrix F which is the unique solution of the equation

$$A^1FA^{-1}F = G \qquad 2.53$$

The equation (2.53) has a unique solution F if and only if, none of the eigenvalues of A, λ_1 , …, λ_n, and none of the sums $\lambda i + \lambda j$, $i \neq j$, $i,j = 1,\ldots,n$ are zero.

(Rubio, 1971)

2.3.0 TIME SERIES

A time series is a set of observation generated sequentially in time. If the set is continuous, the time series is said to be continuous. If the set is discrete, the time series is said to be discrete. Thus the observation from a discrete time series made at times $\tau_1, \tau_2, \tau_3, \ldots, \tau_n$ may be denoted by $z(\tau_1), z(\tau_2), z(\tau_3), \ldots, z(\tau_n)$

(Box et al, 1994)

2.3.1 COMPONENT OF TIME SERIES

1. **PERIODIC COMPONENT**

   If $Y_t = Y_{t+1} + e_t$, for all $t \epsilon N$, then the time series has a periodic component of period T.

2. **TREND COMPONENT**

   If $Y_t = y + \beta t + e_t$, then there exists a linear trend with the slope being β.

2.3.2 STATIONARY TIME SERIES

A time series is said to be strictly stationary if the joint distribution of $X_{t_1}, X_{t_2}, \ldots, X_{t_n}$ is the same as the joint distribution of $X_{t_{1+T}}, X_{t_{2+T}}, \ldots, X_{t_{n+T}}$ for all $t_{1+T}, t_{2+T}, \ldots, t_{n+T}$.

If there is a trend in the mean, differencing the time series data removes the trend to achieve stationarity.

Also if there is a trend in variance, transforming the time series data by $Y_t = \ln X_t$, where X_t is the original time series data, removes the trend to achieve stationarity.

(Box et al, 1994)

2.3.4 AUTOREGRESSIVE MODEL

A stochastic model can be used to represent certain practically occurring series. In this model the current value of the process is expressed as a finite linear aggregate of previous values of the process and a shock a_t.

Denote the values of a process at equally spaced times, t, t-1,t-2, … by z_t, z_{t-1}, z_{t-2},… . Also let $\overline{z_t}$, $\overline{z_{t-1}}$, …, be deviations from μ, for example $\overline{z_t} = z_t - \mu$, then,

$$\overline{z_t} = \phi_1 \overline{z_{t-1}} + \phi_2 \overline{z_{t-2}} + \ldots + \phi_p \overline{z_{t-p}} + a_t \qquad 2.54$$

and is called an autoregressive process of order p.

That is, given the linear model,

$$\overline{z} = \phi_1 \overline{x_1} + \phi_2 \overline{x_2} + \ldots + \phi_p \overline{x_p} + a \quad , \qquad 2.55$$

relating a dependence variable z to a set of independent variable $x_1, x_2, \ldots, x_p$, plus an error term α, is often referred to as a regression model and z is said to be regressed on x_1, x_2, …, x_p.

The variable z is said to be regressed on previous values of itself hence the model is autoregressive.

If the autoregressive operator of order p is represented by

$$\Phi(B) = 1 - \Phi_1 B - \Phi_2 B^2 - \ldots \Phi_p B^p \qquad 2.56$$

the autoregressive model may be written economically as $\Phi(B)\ \overline{z_t} = a_t$.

The model contains $p+2$ unknown parameters $\mu_1, \Phi_1, \Phi_2, \ldots, \Phi_p$, σ_a^2, which in practice have to be estimated from the data. The additional parameter σ_a^2 is the variance of the white noise process a_t .

2.3.5 AUTOREGRESSIVE PROCESSES

Consider the model

$$\overline{z_t} = \phi_1 \overline{z_{t-1}} + \phi_2 \overline{z_{t-2}} + \cdots + \phi_p \overline{z_{t-r}} + a_t \qquad 2.57$$

In which only the first p of the weights are nonzero. The symbols $\phi_1, \phi_2, \cdots, \phi_p$, are used for the finite set of weights parameters.

The process in 2.57 above is called an autoregressive process of order p, or more succinctly $AR(p)$ process. In particular, the autoregressive process of first order $(p=1)$ and of the second order $(p=2)$, are given respectively as

$$\overline{z_t} = \phi \overline{z_{t-1}} + a_t \qquad 2.58$$

$$\overline{z_t} = \phi_1 \overline{z_{t-1}} + \phi_2 \overline{z_{t-2}} + a_t \qquad 2.59$$

(Box et al, 1994)

2.3.6 STATIONARY CONDITIONS

A set of adjustable parameters $\phi_1, \phi_2, \cdots, \phi_p$ of an $AR(p)$ process.

$$\overline{z_t}\phi_1 \overline{z_{t-1}} + \cdots + \phi_p \overline{z_{t-p}} + a_t \tag{2.60}$$

or $$(1- \phi B - \cdots - \phi_p B^p)\overline{z_t} = \phi(B)\overline{z_t} = a_t \tag{2.61}$$

must satisfy certain conditions for the process to be stationary. For illustration, the first order autoregressive process $(1-\phi_1 B)^{-1}\overline{z_t} = a_t$ may be written $\overline{z_t} = (1-\phi_1 B)a_t = \sum_{j=0}^{\infty} \phi_1^j a_{t-j}$

Hence $\psi(B) = (1-\phi_1 B)^{-1} = \sum_{j=0}^{\infty} \phi_1^j B^j$ where $\phi(B)$ must converge for |B| ≤ 1.

This implies that the parameter ϕ_1 of an AR(1) process must satisfy the condition $|\phi_1| \le 1$ to ensure stationarity. Since the root of 1-ϕ_1B = 0 is B = ϕ_1^{-1}, this condition is equivalent to saying that the root of 1-$\phi_1 B$ =0 must lie outside the unit circle.

For the general AR(p) process $\overline{z_t} = \phi^{-1}(B)a_t$, is obtained as

Φ(B) = (1-G_1B)(1-G_2B)…(1-G_pB), where $G_1^{-1}, \cdots, G_p^{-1}$ are the roots of $\phi(B) = 0$ and expanding Φ^{-1}(B) in partial fractions yields

$$\overline{z_t} = \phi^{-1} B a_t = \sum_{i=1}^{p} \frac{K_i}{1-G_i B} a_t \ . \tag{2.62}$$

Hence , if $\psi(B) = \Phi^{-1}(B)$ is to be a convergent series for $|B| \leq 1$, that is if weights $\psi_i = \sum_{i=1}^{p} K_i G_i^j$ are to be absolutely summable so that the AR(p) will represent a stationary process, which gives $|G_i| < 1$, for i = 1,2,…,p.

(Box et al, 1994)

2.3.7 FIRST–ORDER AUTOREGRESSIVE PROCESS

The first-order autoregressive process is

$$\overline{z_t} = \phi_1 \overline{z_{t-1}} + a_t$$

$$= a_t + \phi_1 a_{t-1} + \phi_1^2 a_{t-2} \qquad 2.63$$

where it has been shown that ϕ_1 must satisfy the condition $-1 < \phi_1 < 1$, for the process to be stationary.

2.3.8 AUTOCORELATION FUNCTION

The autocorrelation function satisfies the first-order difference equation

$\rho_k = \phi_1 \rho_{k-1}$ k > 0, which , with $\rho_o = 1$, has the solution $\rho_k = \phi_1^k$, k≥0 .

This autocorrelation function decays exponentially to zero, when ϕ_1 is positive, but decays exponentially to zero, and oscillates in sign when ϕ_1 is negative.

In particular, $\rho_1 = \phi_1$

(Box et al, 1994)

2.3.9 VARIANCE

The variance of the process is $\sigma_z^2 = \dfrac{\sigma_a^2}{1-\rho_1\phi_1} = \dfrac{\sigma_a^2}{1-\phi_1^2}$.

on substituting, $\rho_1 = \phi_1$

(Box et al, 1994)

2.3.10 GENERAL FORM OF THE AUTOREGRESSIVE INTEGRATED MOVING AVERAGE PROCESS (ARIMA)

It is sometimes useful to consider a slight extension of the ARIMA model;

$$\phi(B)\nabla^d z_t = \theta(B)a_t$$

by adding a constant term θ_0 .

Thus the general form of the model to be used to describe ARIMA process is

$$\varphi(B) = \phi(B)\nabla^d z_t = \theta_0 + \theta(B)a_t$$

where $\phi(B) = 1-\phi_1 B - \phi_2 B^2 - \ldots - \phi_q B^q$.

$$\theta(B) = 1-\theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$$

It follows that:

1. $\phi(B)$ will be called autoregressive operator, which is assumed to be stationary; and that the roots of $\phi(B)$ =0, lie outside the unit circle.

2. $\varphi(B) = \phi(B)\nabla^d$, will be called the generalized autoregressive operator; it is a nonstationary operator with d of the roots of $\varphi(B) = 0$ equal to unity.

3. $\theta(B)$ will be called the moving average operator; it is assumed to be invertible, that is, the roots of θ(B) = 0 lie outside the unit circle.

(Box et al, 1994)

2.3.11 BACKSHIFT FORM OF THE MODEL

Direct use of the difference equation permits us to express the current value z_t of the process in terms of previous values of the z's and of current and previous values of the a's.

Thus if;

$$\varphi(B) = \phi(B)(1-B)^d = 1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_{p+q} B^{p+q} \quad .$$

If in π, $\theta_0 = 0, \pi$ it is written as,

$$z_t = \varphi_1 z_{t+1} + \cdots + \varphi_{p+q} z_{t-p-q} - \theta_1 a_{t-1} \cdots - \theta_q a_{t-q} + a_t$$

(Box et al, 1994)

CHAPTER 3

MODELLING AND DATA ANALYSIS

3.1.0 INTRODUCTION

In this chapter, the various components of the economy were analyzed using time series analysis and Box-Jenkins method of time series data using SPSS.

The main components of the economy include Inflation, Total Money Supply, Interest Rate, Gross Domestic Product, Capital Stock, Government Expenditure, Investment, Consumption and Balance of Trade.

A forty data point was used for each from 1996 to 2004, on a quarterly performance of the economy from the Bank of Ghana, Kumasi branch.

3.1.1 PRELIMINARY ANALYSIS

We present below a preliminary (descriptive) analysis of the data

Inflation

The graph in Fig. 3.1.1 below is the histogram on inflation values from 1996 to 2004. It has mean 30.28 and standard deviation 13.49 and it is positively skewed with mode 40.00.

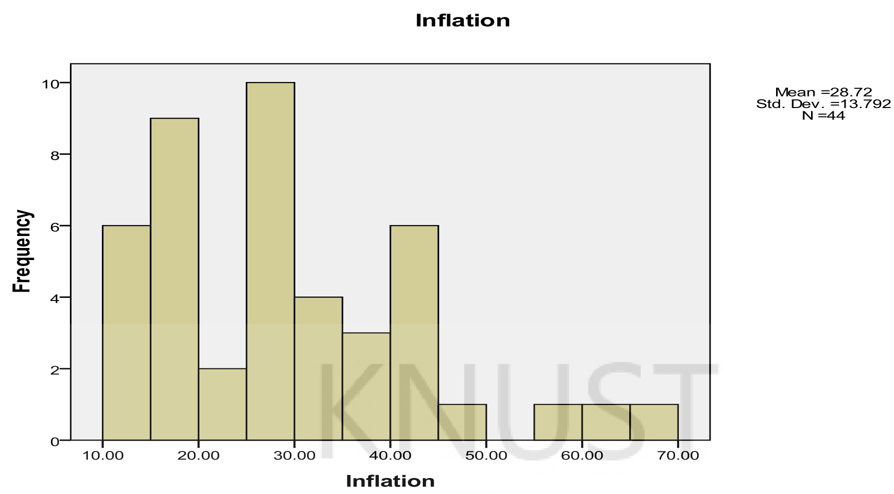

Fig. 3.1.1 Histogram on Inflation from 1996-2004

Interest Rate

The graph of Fig 3.1.2 as shown below represents the histogram of Interest Rate values from 1996 to 2004. It is approximately positively skewed and has mode 45.00, mean 28.89 and standard deviation 11.35.

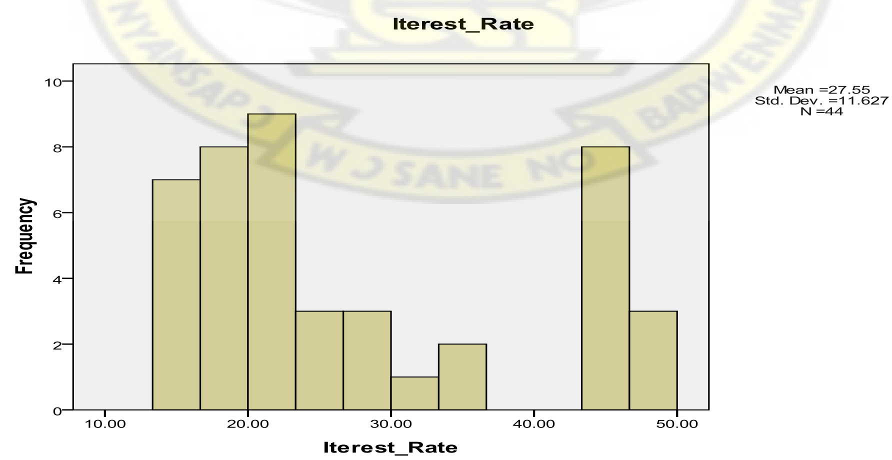

Fig. 3.1.2 Histogram on Interest Rate from 1996-2004

Total Money Supply

The graph in Fig. 3.1.3 below is the histogram on Total Money Supply values from 1996 to 2004. It has mean value of 191.65, its standard deviation value is 102.56, a modal value of 52.1 and it is approximately normally distributed.

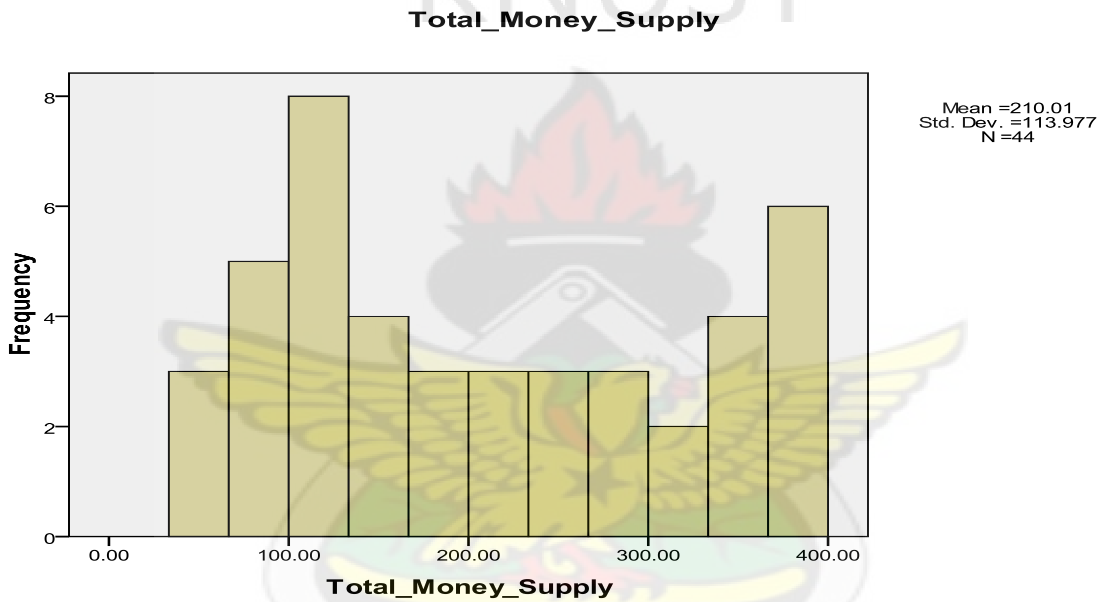

Fig. 3.1.3 Histogram on Total Money Supply from 1996-2004

Gross Domestic Product

The graph in Fig. 3.1.4 below is the histogram on Gross Domestic Product values from 1996 to 2004. It has deviated from the normal curve and positively skewed. Its modal value is 9631.90. The mean 5876.96 whiles its standard deviation has value equal to 1626.89.

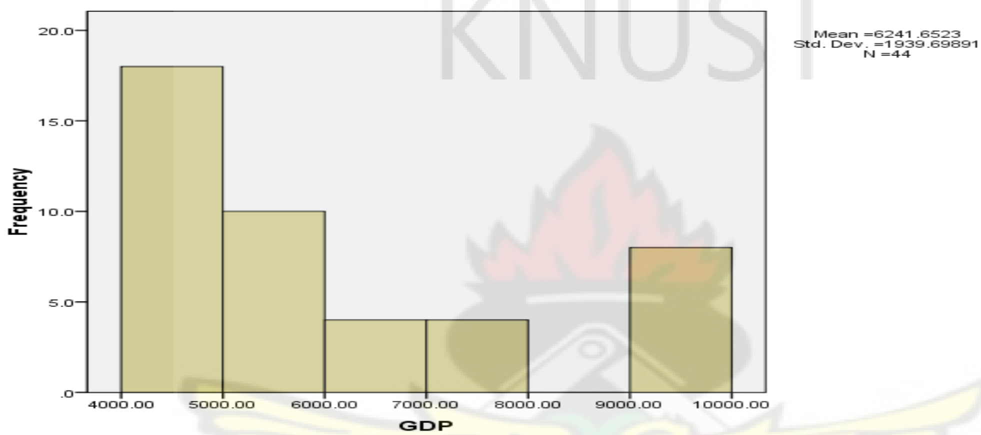

Fig. 3.1.4 Histogram on Gross Domestic Product from 1996-2004

Capital Stock

The graph in Fig. 3.1.5 below is the histogram of Capital Stock over the time interval 1996 to 2004. It is also positively skewed, with deviation and mean being 7516.29 and 8915.66 respectively.

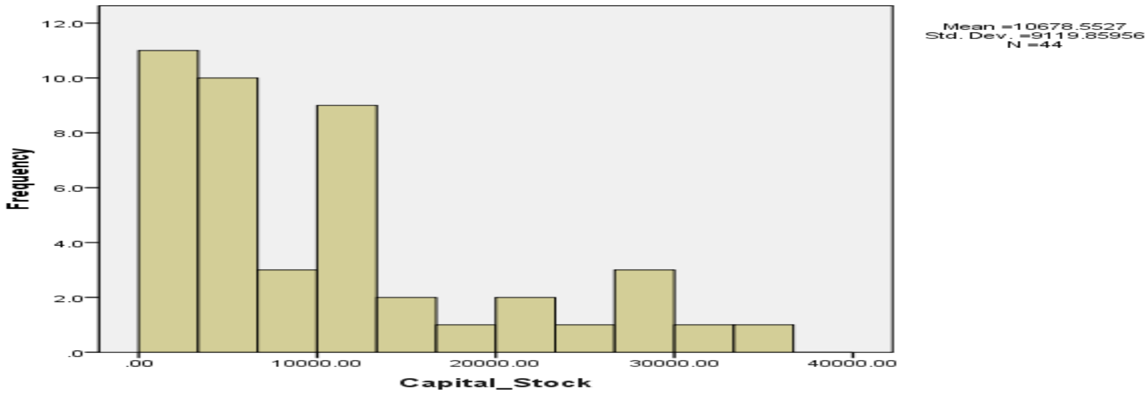

Fig. 3.1.5 Histogram on Capital Stock from 1996-2004

Government Expenditure

Fig. 3.1.6 below represents the histogram on Government Expenditure values. This has mode of 1384, mean 3526.48 and standard deviation 2550.72. This is observed to be positively skewed.

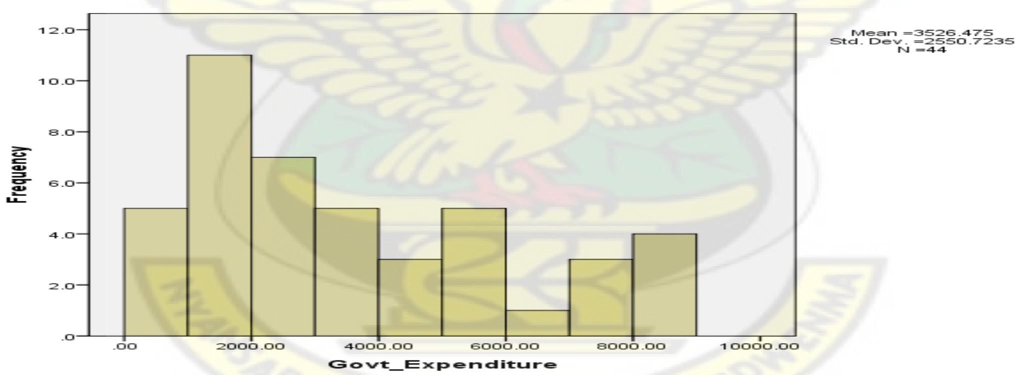

Fig. 3.1.6 Histogram on Government Expenditure from 1996-2004

Investment

Fig. 3.1.6 is the histogram representing the investment values in the economy, from 1996 to 2004. The graph is generally positively skewed. It has modal value of 97.40, mean 1982.26 and a standard deviation 2162.37

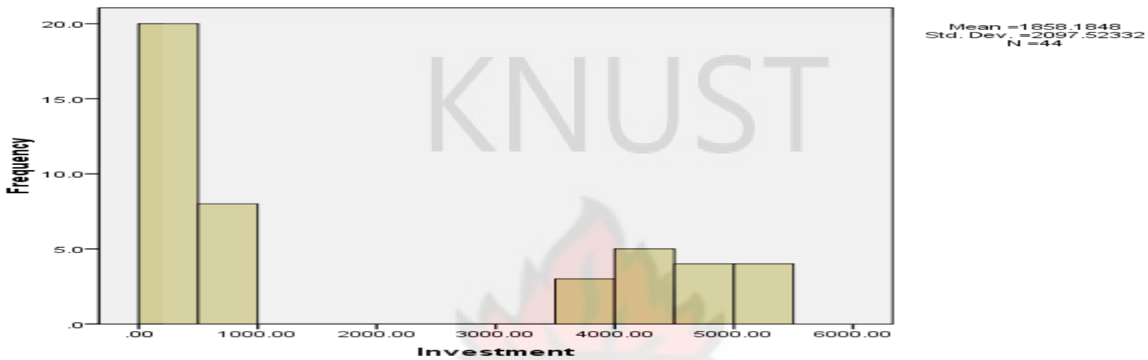

Fig. 3.1.7 Histogram on Investment from 1996-2004

Balance of Trade

Fig. 3.1.8 is the histogram representing the Balance of trade values during the period under consideration from 1996 to 2004. It is negatively skewed with mean and standard deviation being -942.82 and 1124.39 respectively. The mode is -2587.50.

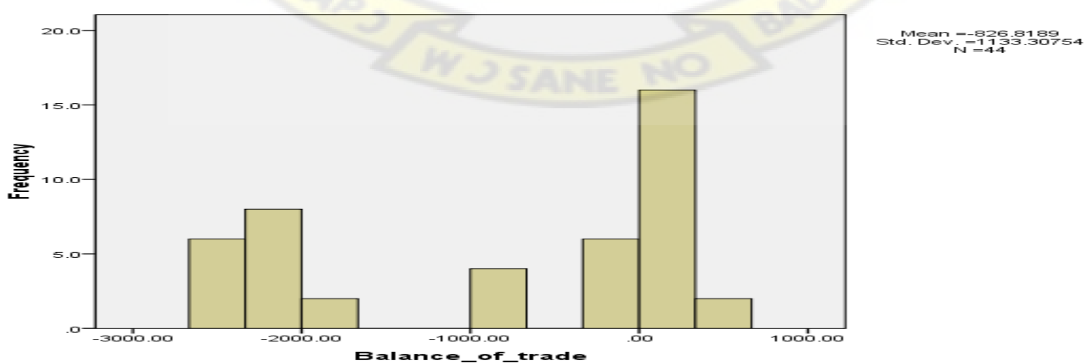

Fig. 3.1.8 Histogram on Balance of Trade from 1996-2004

Consumption

Fig. 3.1.9 is the histogram of the values of the Consumption component in the economy of Ghana from 1996 to 2004. It is a nearly normally distributed with mean, mode and standard deviation being 1775.98, -3631.0 and3732.50 respectively.

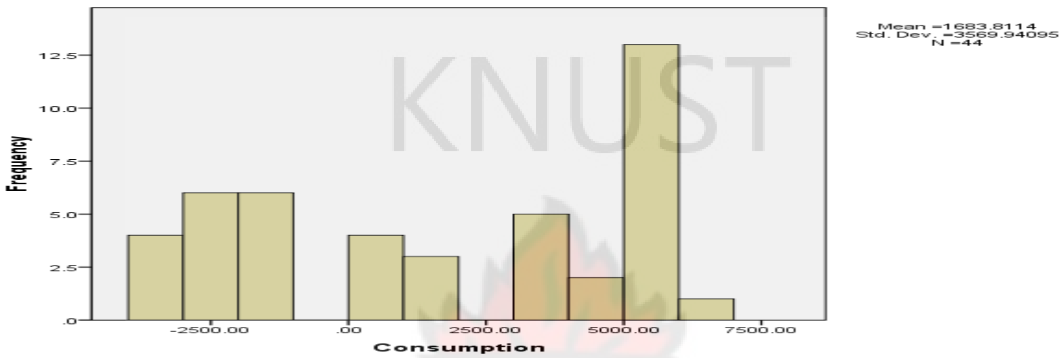

Fig. 3.1.9 Histogram on Consumption from 1996-2004

3.2 TIME SERIES ANALYSIS

Time series analysis was used to analyse the various components of the economy. Detailed analyses were done for all as follows:

3.2.1 INFLATION

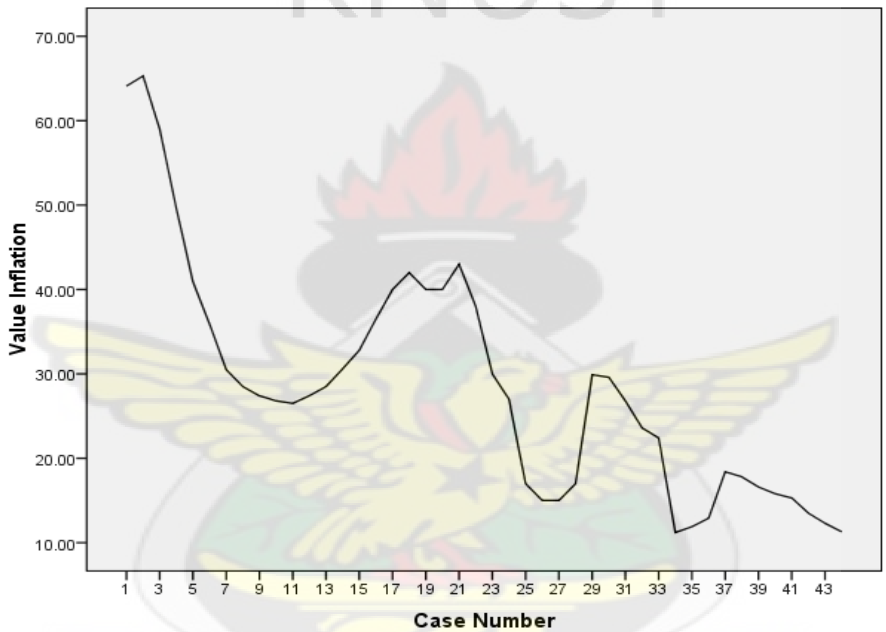

Fig 3.2.0 Trajectory of inflation from 1996 to 2004

Fig. 3.2.0 is the trajectory of the inflation values which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore, differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down quickly into a wave form whilst the partial autocorrelation function (PACF) truncates after lag 1 as exhibited in Figures 3.2.1 and 3.2.2 below.

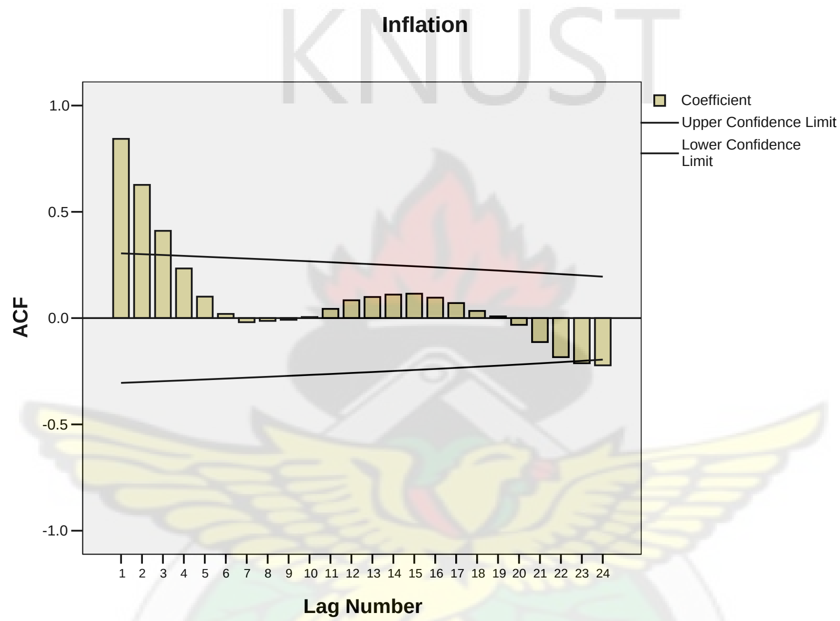

Fig 3.2.1 Autocorrelation function of inflation 1996-2004

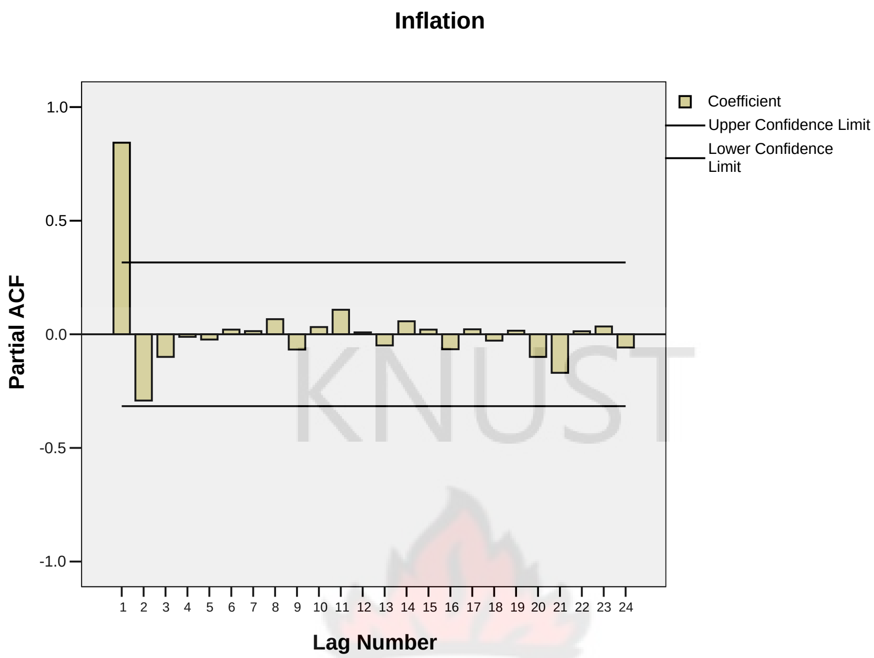

Fig 3.2.2 Partial autocorrelation function of inflation

The results from Fig.3.2.1 and Fig 3.2.2 above indicate an ARIMA(1,0,0) model. This model is now compared to two other practically likely models, i.e. ARIMA(1,1,0) and ARIMA(1,2,0) and the best is chosen based on the following tabular results.

Table 3.2.0 Comparison Table of likely models for inflation

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	23.331	244.315
ARIMA(1,1,0)	18.211	226.025
ARIMA(1,2,0)	23.563	230.016

From the results in Table 3.2.0 above, comparing both residual variance and AIC value we have ARIMA(1,1,0) to be our most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$.

Where Y_t is the observation at the present time, Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated as shown in the table below.

Parameter Estimates

		Estimates	Std Error	t	Approx Sig
Non-Seasonal Lags	AR1	.403	.149	2.696	.010
Constant		-1.190	1.125	-1.058	.297

Melard's algorithm was used for estimation.

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with

$k - p - q$ degrees of freedom. Where k = 24 (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is 35.172. Since the calculated $Q - statistic = 13.062 < 35.172$ the chosen model is adequate.

Thus our selected model for inflation is given by $Y_t = 1.403Y_{t-1} - 0.403Y_{t-2} - 1.19$

3.2.2 INTEREST RATE

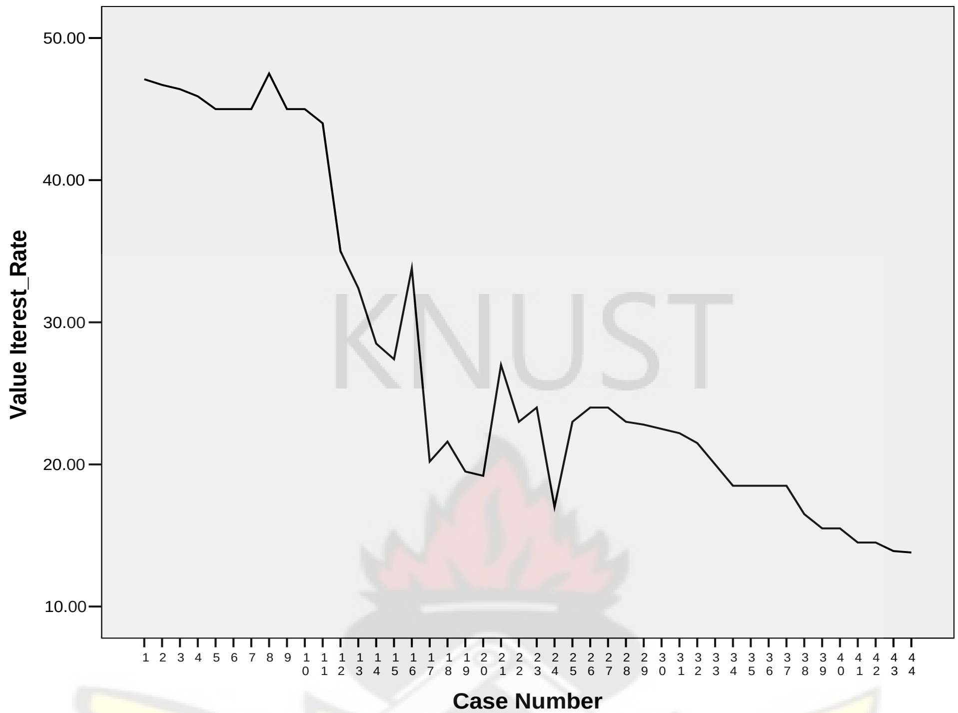

Fig 3.2.3 Trajectory of Interest Rate from 1996 to 2004

Fig. 3.2.3 is the trajectory of the Interest Rate values. This was analysed for any form of periodicity and seasonality but none appeared to exist. However, the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down into a wave form whilst the partial autocorrelation function (PACF) truncates after the first lag (lag 1) as exhibited in Figures 3.2.4 and 3.2.5 below.

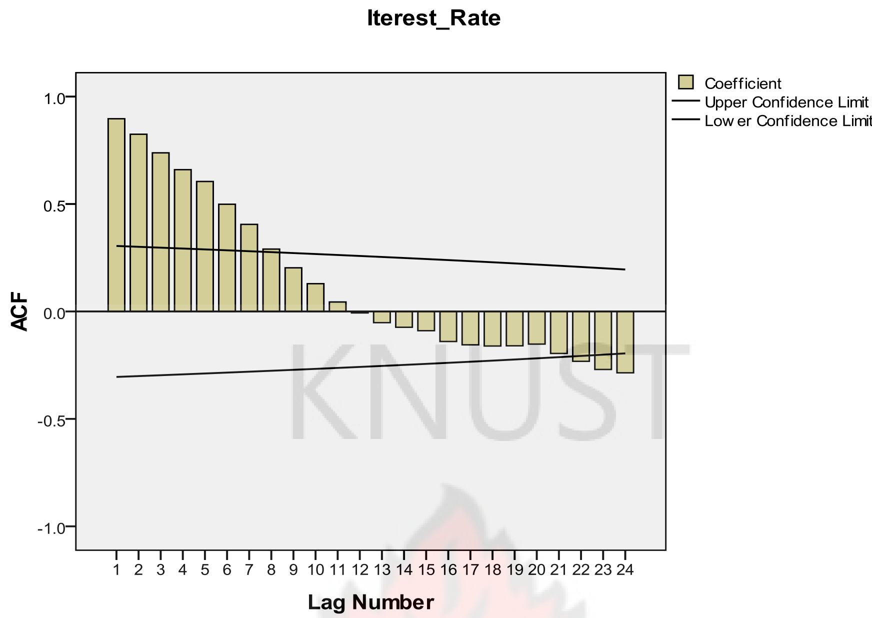

Fig 3.2.4 Autocorrelation function of Interest Rate

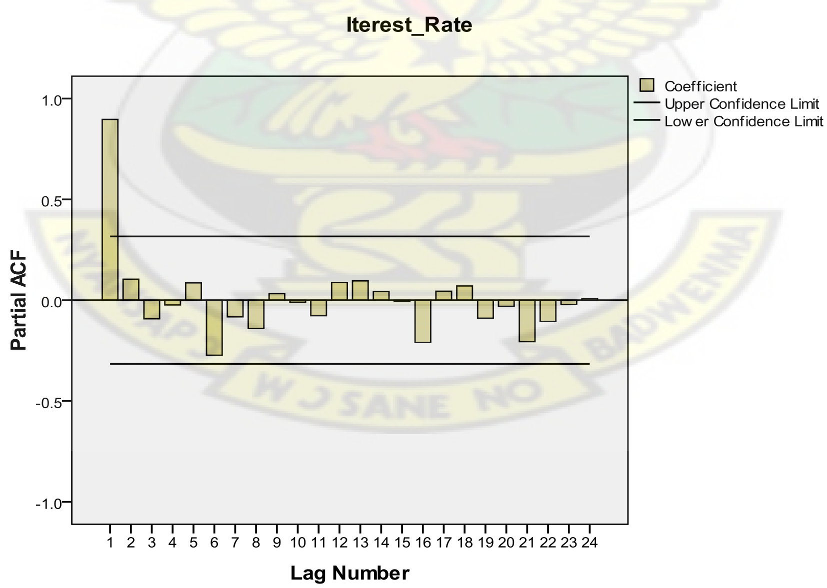

Fig 3.2.5 Partial autocorrelation function of Interest Rate

The results from Fig.3.2.4 and Fig 3.2.5 above indicate an ARIMA(1,0,0) model. This model is now compared to two other practically possible models, ARIMA(1,1,0) and ARIMA(1,2,0), and the best is chosen based on the following tabular results.

Table 3.2.1 Comparison Table of likely models for Interest Rate

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	23.331	244.315
ARIMA(1,1,0)	18.211	226.025
ARIMA(1,2,0)	23.563	230.016

From the results in Table 3.2.1 above, comparing both residual variance and AIC value we have ARIMA(1,1,0) as the statistically most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t, the white noise or error at the present time and α, parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

Parameter Estimates

		Estimates	Std Error	t	Approx Sig
Non-Seasonal Lags	AR1	.403	.149	2.696	.010
Constant		-1.190	1.125	-1.058	.297

Melard's algorithm was used for estimation.

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is $35.172.$ Since the calculated $Q - statistic = 13.062 < 35.172$ the chosen model is adequate.

Thus our selected model for Interest Rate is given by $Y_t = 1.403Y_{t-1} - 0.403Y_{t-2} - 1.19$

3.2.3 TOTAL MONEY SUPPLY

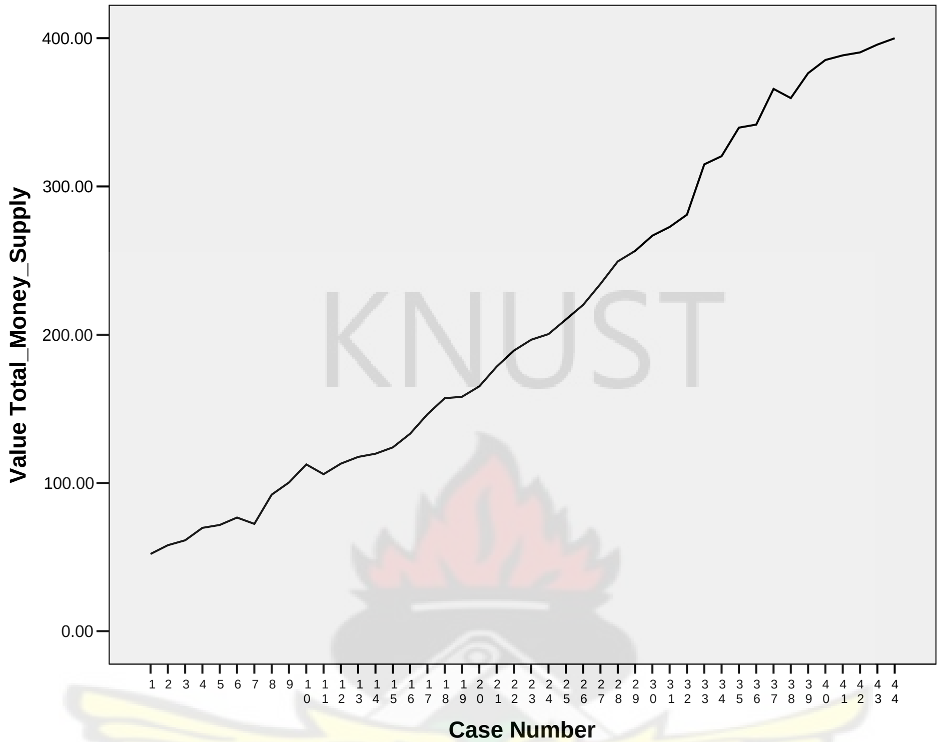

Fig 3.2.6 Trajectory of Total Money Supply from 1996 to 2004

Fig. 3.2.6 is the trajectory of the Total Money Supply values which was analysed for any form of periodicity and seasonality but none appeared to exist. However, the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down slowly whilst the partial autocorrelation function (PACF) truncates after the first lag as exhibited in Figures 3.2.7 and 3.2.8 below.

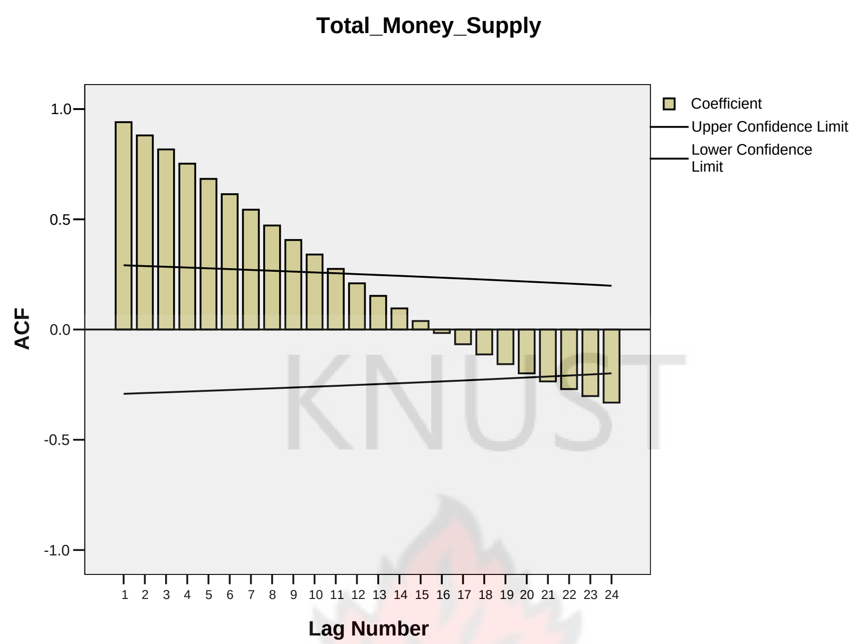

Fig 3.2.7 Autocorrelation function of Total Money Supply

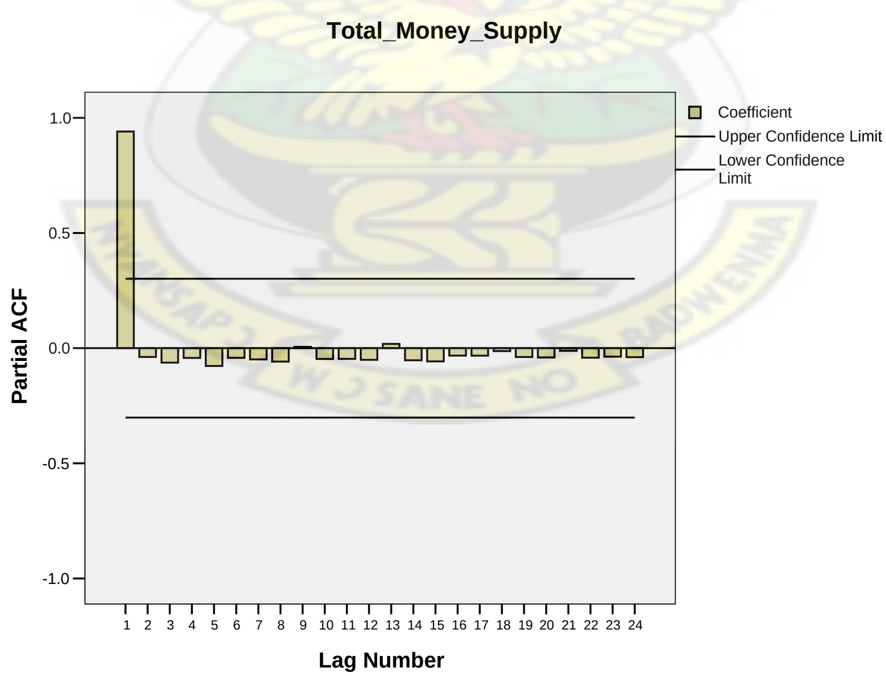

Fig 3.2.8 Partial autocorrelation function of Total Money Supply

The results from Fig.3.2.7 and Fig 3.2.8 above indicate an ARIMA(1,0,0) model. This model is now compared to two other practically possible models, ARIMA(1,1,0) and ARIMA(1,2,0), and the best is chosen based on the following tabular results.

Table 3.2.2 Comparison Table of likely models for Total Money Supply

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	125.947	345.220
ARIMA(1,1,0)	54.115	295.703
ARIMA(1,2,0)	69.843	300.199

From the results in Table 3.2.2 above, comparing both residual variance and AIC value gives ARIMA(1,1,0) as the most preferable since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

		Estimates	Std Error	T	Approx Sig
Non-Seasonal Lags	AR1	-.239	.152	-1.576	.123
Constant		8.116	.909	8.926	.000

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is $35.172.$ Since the calculated $Q - statistic = 20.33 < 35.172$ the chosen model is adequate.

Thus our selected model for Total Money Supply is given by

$$Y_t = 0.7461Y_{t-1} + 0.239Y_{t-2} + 8.116$$

3.2.4 GROSS DOMESTIC PRODUCT (GDP)

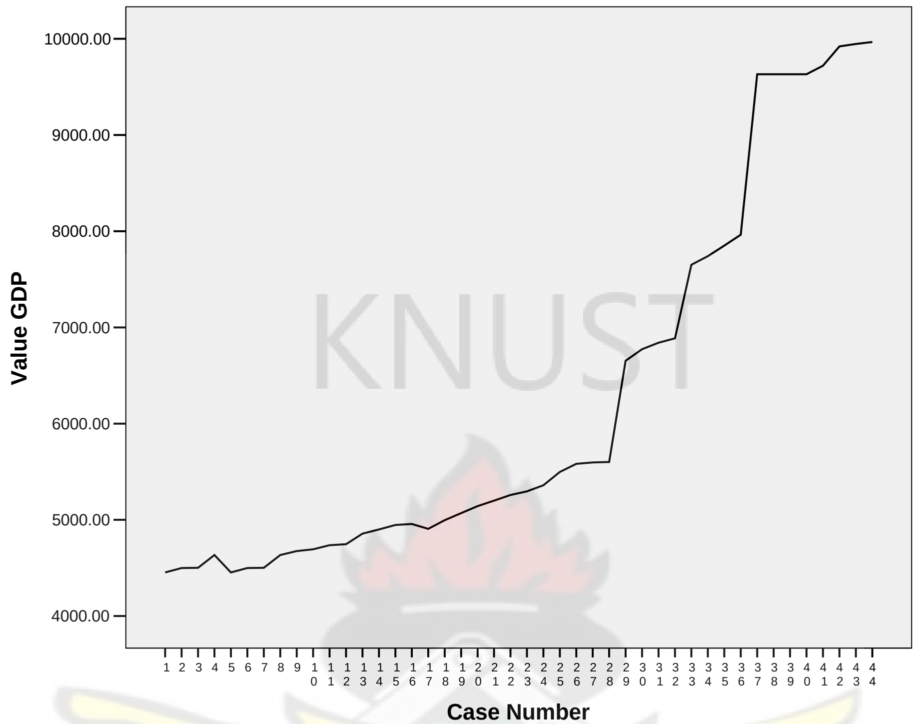

Fig 3.2.9 Trajectory of Gross Domestic Product (GDP) from 1996 to 2004

Fig. 3.2.9 is the trajectory of the Gross Domestic Product values which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down slowly into a wave form whilst the partial autocorrelation function (PACF) truncates after as exhibited in Figures 3.2.10 and 3.2.11 below.

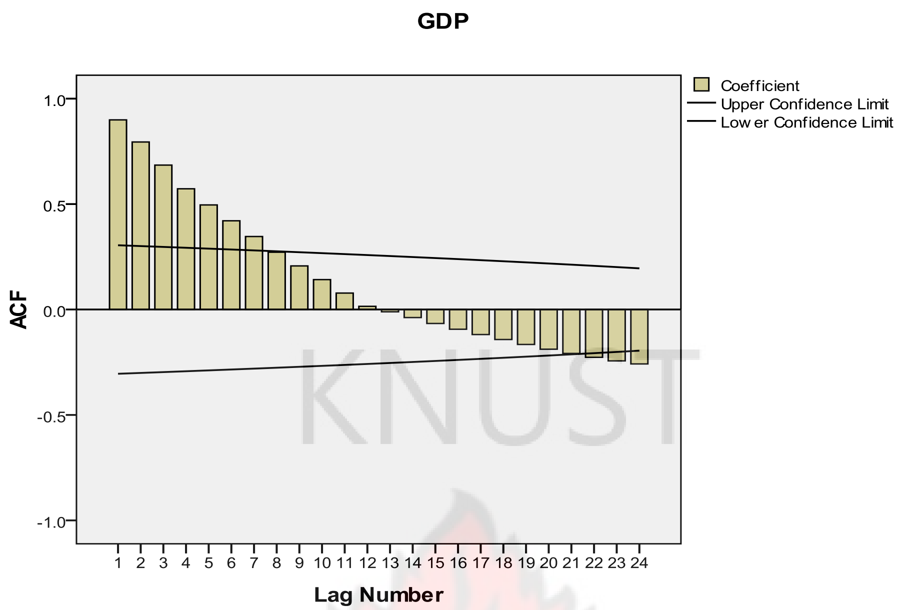

Fig 3.2.10 Autocorrelation function of Gross Domestic Product

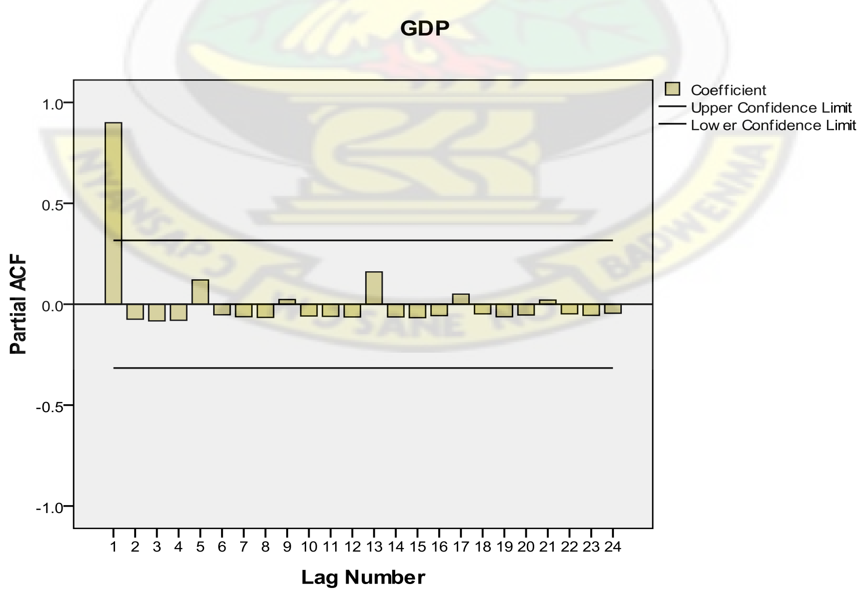

Fig 3.2.11 Partial autocorrelation function of Gross Domestic Product

The results from Fig.3.2.10 and Fig 3.2.11 above indicate an ARIMA(1,0,0) model. This model is now compared to two other practically possible models, ARIMA(1,1,0) and ARIMA(1,2,0), and the best is chosen based on the following tabular results.

Table 3.2.3 Comparison Table of likely models of Gross Domestic Product

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	125973.5	589.123
ARIMA(1,1,0)	107581.8	564.534
ARIMA(1,2,0)	174814.8	568.814

From the results in Table 3.2.3 above, comparing both residual variance and AIC value gives ARIMA(1,1,0) as the most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t, the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

		Estimates	Std Error	T	Approx Sig
Non-Seasonal Lags	AR1	-.051	.156	-.326	.746
Constant		128.450	45.438	2.827	.007

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared with the critical value of χ^2_{23} which is $35.172.$ Since the calculated $Q - statistic = 19.558 < 35.172$ the chosen model is adequate.

Thus our selected model for Gross Domestic Product is given by

$$Y_t = 0.949Y_{t-1} + 0.051Y_{t-2} + 128.45$$

3.2.5 CAPITAL STOCK

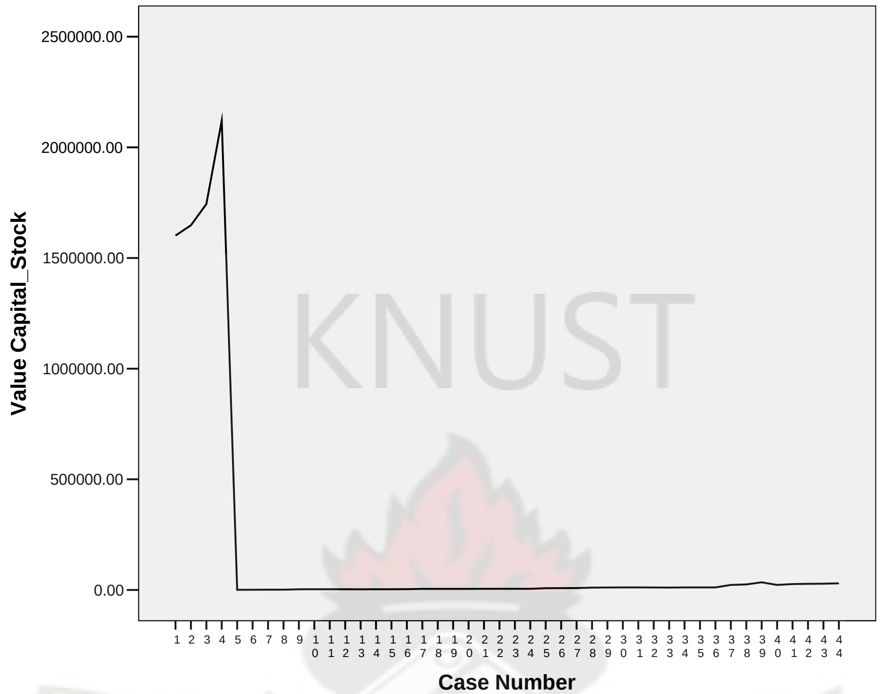

Fig 3.2.12 Trajectory of Capital Stock from 1996 to 2004

Fig. 3.2.12 is the trajectory of the Capital Stock values which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

<u>Model Identification</u>

The autocorrelation function (ACF) dies down very quickly to zero whilst the partial autocorrelation function (PACF) truncates after the first lag as exhibited in Figures 3.2.13 and 3.2.14 below.

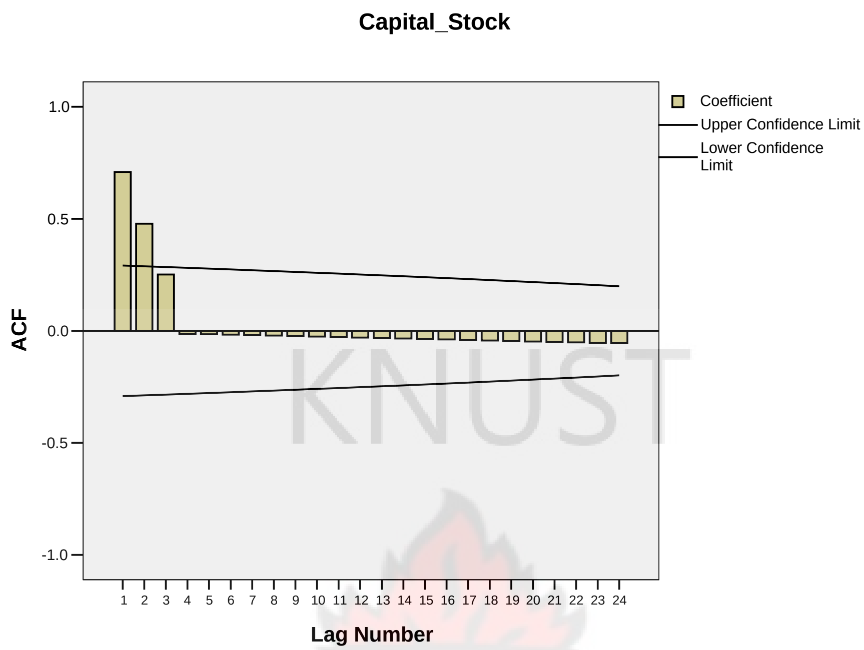

Fig 3.2.13 Autocorrelation function of Capital Stock

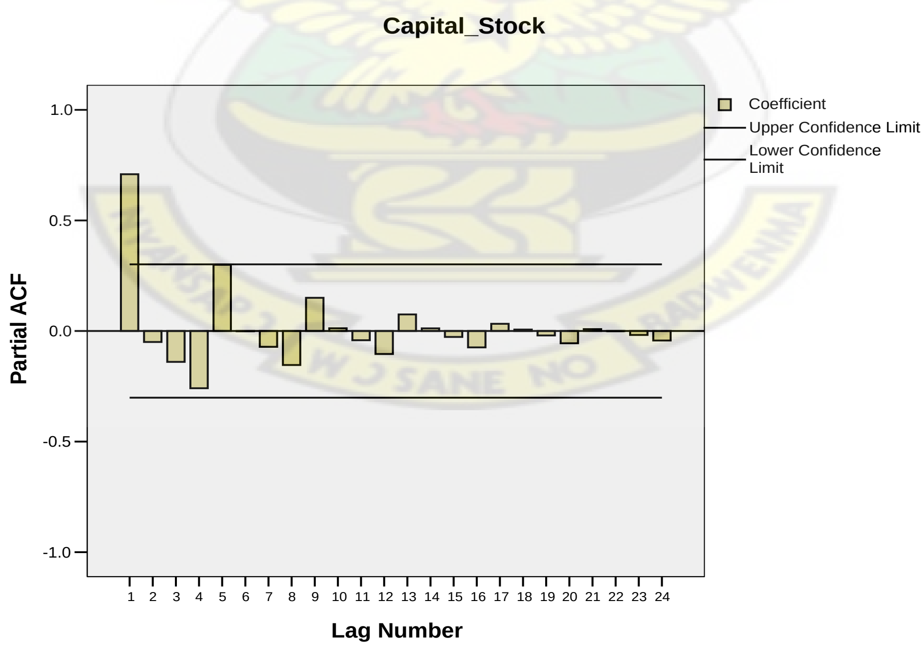

Fig 3.2.14 Partial autocorrelation function of Capital Stock

The results from Fig.3.2.13 and Fig 3.2.14 above indicate an ARIMA(1,0,0) model. This model is now compared to two other possible models, ARIMA(1,1,0) and ARIMA(1,2,0), and the best is chosen based on the following tabular results.

Table 3.2.4 Comparison Table of likely models for Capital Stock

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	2×10^7	779.842
ARIMA(1,1,0)	2×10^7	760.982
ARIMA(1,2,0)	2×10^7	753.984

From the results in Table 3.2.4 above, comparing both residual variance and AIC value gives ARIMA(1,2,0) as the statistically most preferable model since its AIC values are least.

The identified model is of the form $Y_t = \alpha Y_{t-1} + Y_{t-2} - \alpha Y_{t-3} + e_t$, where Y_t is the observation at the present time Y_{t-1}, Y_{t-3} and Y_{t-2} are respectively observations at time lag 1 and time lag 2 and time lag 3, e_t the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

		Estimates	Std Error	T	Approx Sig
Non-Seasonal Lags	AR1	-.693	.131	-4.122	.000
Constant		-102.395	45.438	43731.854	-.035

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with

$k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is 35.172. Since the calculated $Q - statistic = 4.72 < 35.172$ the chosen model is adequate.

Thus our selected model for Capital Stock is given by

$$Y_t = -0.693Y_{t-1} + Y_{t-2} + 0.693Y_{t-3} - 102.395$$

3.2.6 GOVERNMENT EXPENDITURE

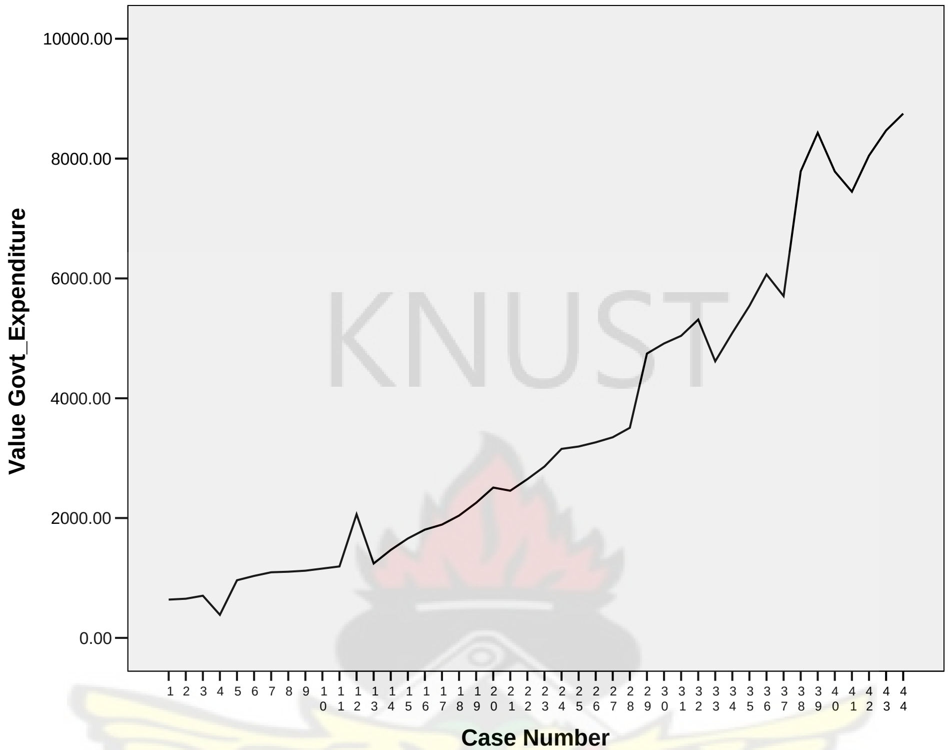

Fig 3.2.15 Trajectory of Government Expenditure from 1996 to 2004

Fig. 3.2.15 is the trajectory of the Government Expenditure values, which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down slowly into a wave form whilst the partial autocorrelation function (PACF) truncates after lag 1 as exhibited in Figures 4.2.1 and 4.2.2 below.

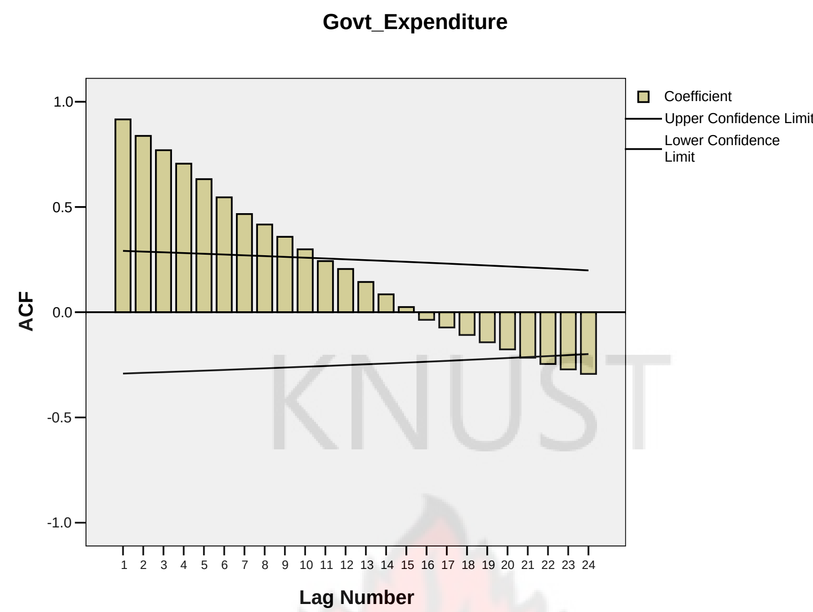

Fig 3.2.15 Autocorrelation function of Government Expenditure

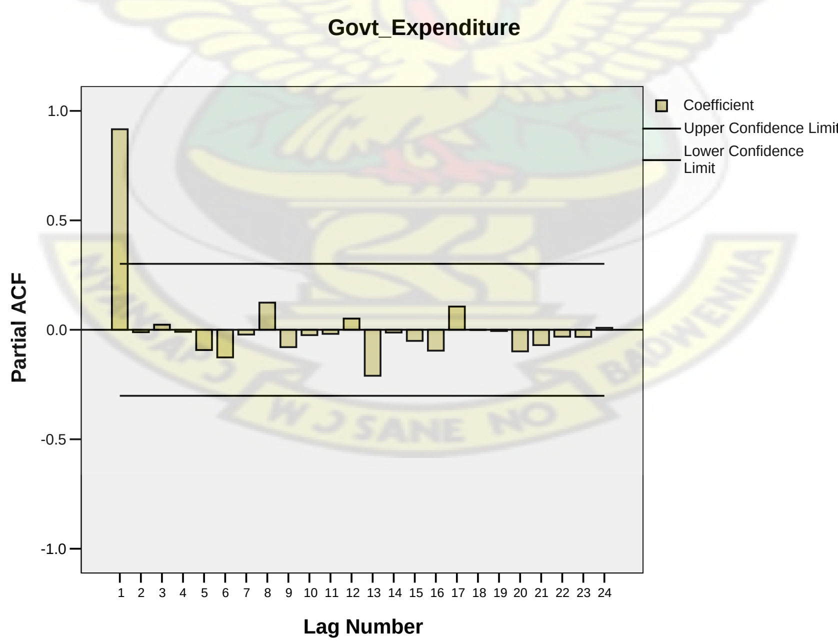

Fig 3.2.16 Partial autocorrelation function of Government Expenditure

The results from Fig.3.2.1 and Fig 3.2.2 above indicate an ARIMA(1,0,0) model. This model is now compared to two other possible models ARIMA(1,1,0) and ARIMA(1,2,0) and the best is chosen based on the following tabular results.

Table 3.2.5 Comparison Table of likely models for Government Expenditure

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	281004.3	621.151
ARIMA(1,1,0)	237479.3	595.455
ARIMA(1,2,0)	438451.3	603.789

From the results in Table 3.2.5 above, comparing both residual variance and AIC value gives ARIMA(1,1,0) to be the most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t, the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

	Estimates	Std Error	T	Approx Sig
Non-Seasonal Lags AR1	-.149	.154	-.968	.338
Constant	188.877	63.924	2.955	.005

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared with the critical value of χ^2_{23} which is 35.172. Since the calculated $Q - statistic = 12.505 < 35.172$ the chosen model is adequate.

Thus our selected model for Government Expenditure is given by

$$Y_t = 0.851Y_{t-1} - 0.149Y_{t-2} + 188.877$$

3.2.7 INVESTMENT

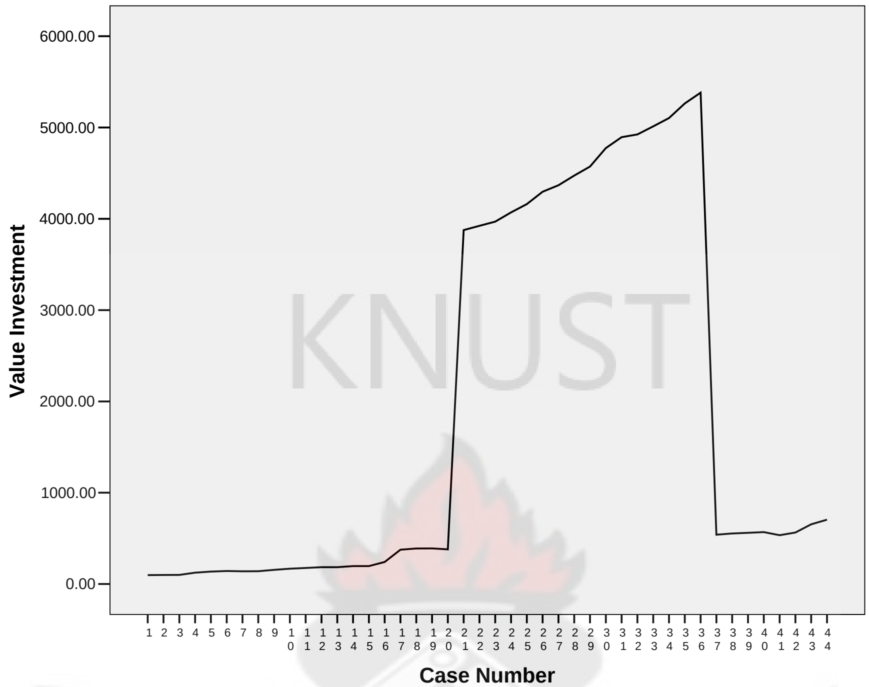

Fig 3.2.17 Trajectory of Investment from 1996 to 2004

Fig. 3.2.17 is the trajectory of the Investment values, which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down into a wave form whilst the partial autocorrelation function (PACF) truncates after the first lag as exhibited in Figures 3.2.18 and 3.2.19 below.

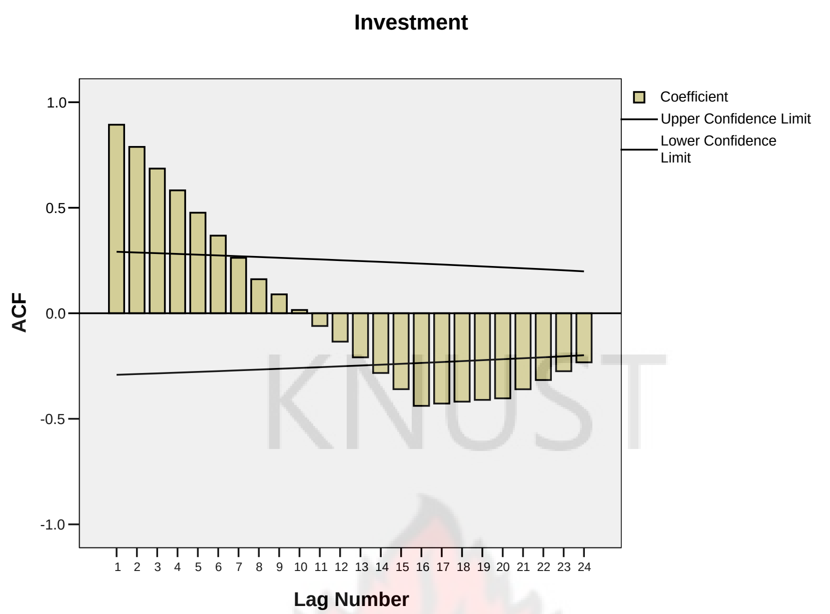

Fig 3.2.18 Autocorrelation function of Investment

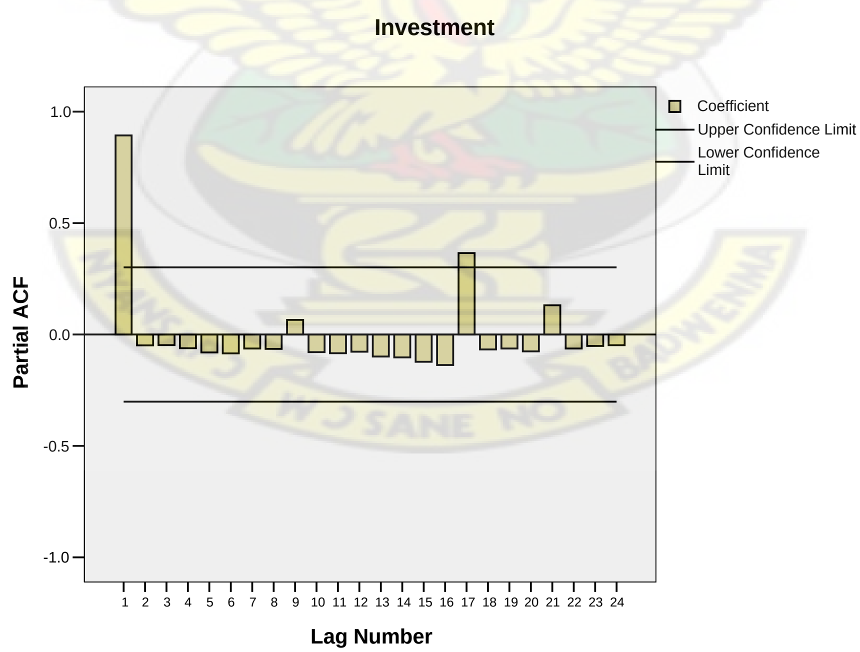

Fig 3.2.19 Partial autocorrelation function of Investment

The results from Fig.3.2.18 and Fig 3.2.19 above indicate an ARIMA(1,0,0) model. This model is now compared to two other possible models, ARIMA(1,1,0) and ARIMA(1,2,0), and the best is chosen based on the following tabular results.

Table 3.2.6 Comparison Table of likely models for Investment

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	908000.8	665.818
ARIMA(1,1,0)	9069290.7	650.266
ARIMA(1,2,0)	1514257.0	650.851

From the results in Table 3.2.6 above, comparing both residual variance and AIC value produce ARIMA(1,1,0) as the most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t, the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

		Estimates	Std Error	T	Approx Sig
Non-Seasonal Lags	AR1	-.010	.156	-.063	.950
Constant		14.112	141.295	.100	.921

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is $35.172.$ Since the calculated $Q - statistic = 15.546 < 35.172$ the chosen model is adequate.

Thus our selected model for Investment is given by $Y_t = 0.99Y_{t-1} + 0.01Y_{t-2} + 14.112$

3.2.7 BALANCE OF TRADE

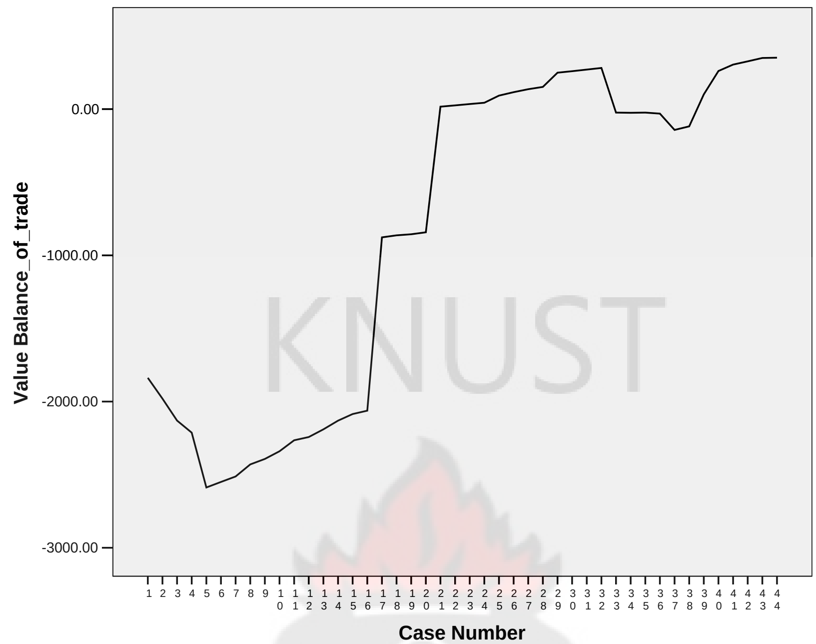

Fig 3.2.20 Trajectory of Balance of Trade from 1996 to 2004

Fig. 3.2.20 is the trajectory of the Balance of Trade values which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down slowly into a wave form whilst the partial autocorrelation function (PACF) truncates after the first lag as exhibited in Figures 3.2.21 and 3.2.22 below.

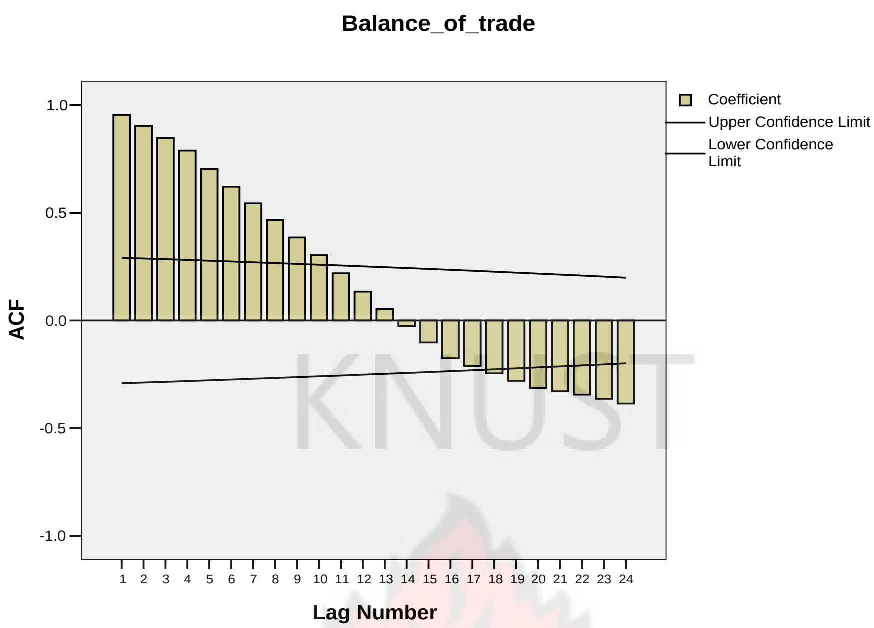

Fig 3.2.21 Autocorrelation function of Balance of Trade

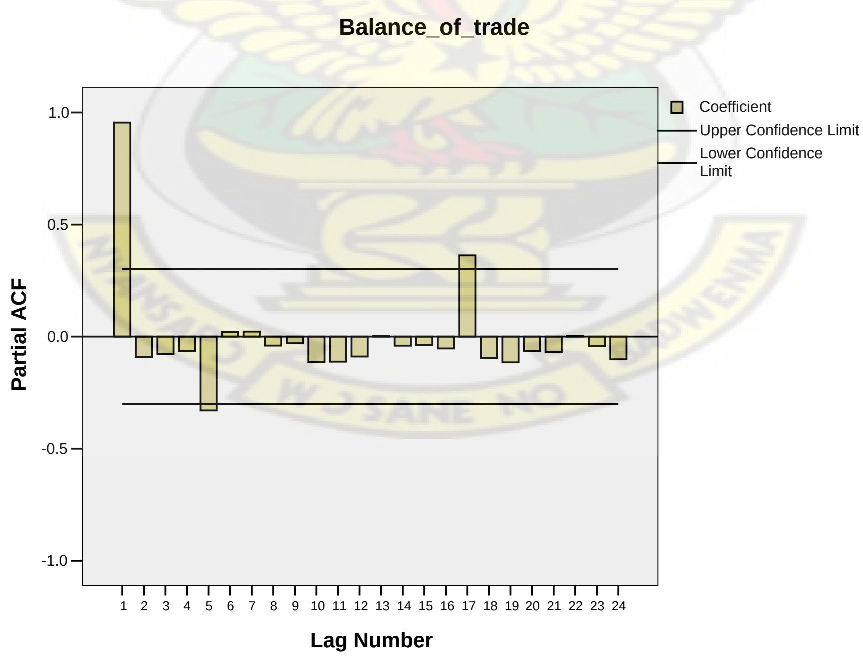

Fig 3.2.22 Partial autocorrelation function of Balance of Trade

The results from Fig.3.2.21 and Fig 3.2.22 above indicate an ARIMA(1,0,0) model. This model is now compared to two other practically possible models ARIMA(1,1,0) and ARIMA(1,2,0) and the best is chosen based on the following tabular results.

Table 3.2.7 Comparison Table of likely models for Balance of Trade

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	67968.425	563.456
ARIMA(1,1,0)	65994.489	545.474
ARIMA(1,2,0)	98368.612	546.962

From the results in Table 3.2.7 above, comparing both residual variance and AIC value providuce ARIMA(1,1,0) as the most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t, the white noise or error at the present time and α, AR parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

		Estimates	Std Error	T	Approx Sig
Non-Seasonal Lags	AR1	.026	.156	.168	.867
Constant		50.758	38.230	1.328	.192

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is $35.172.$ Since the calculated $Q - statistic = 24.544 < 35.172$ the chosen model is adequate.

Thus our selected model for Balance of Trade is given by

$$Y_t = 1.026Y_{t-1} - 0.026Y_{t-2} + 50.758$$

3.2.8 CONSUMPTION

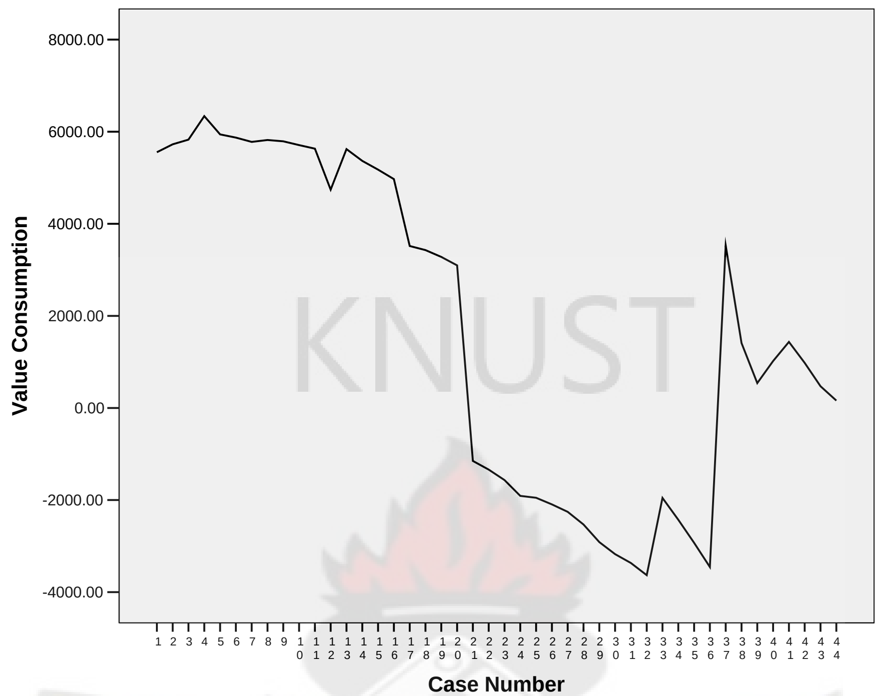

Fig 3.2.23 Trajectory of Consumption from 1996 to 2004

Fig. 3.2.23 is the trajectory of the Balance of Trade values which was analysed for any form of periodicity and seasonality but none appeared to exist. However the data appears to be non-stationary in mean therefore differencing is required to make it stationary.

Model Identification

The autocorrelation function (ACF) dies down into a wave form whilst the partial autocorrelation function (PACF) truncates after lag 1 as exhibited in Figures 4.2.1 and 4.2.2 below.

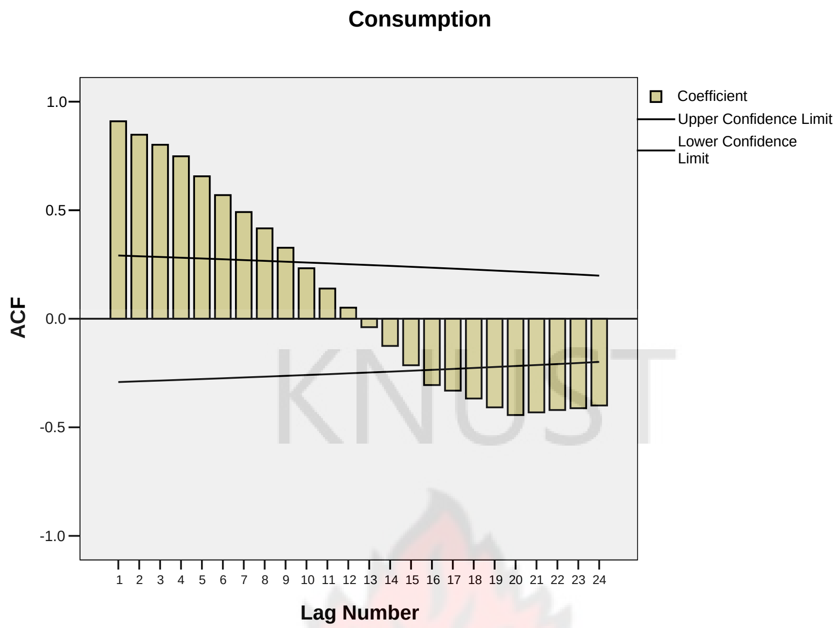

Fig 3.2.24 Autocorrelation function of Consumption

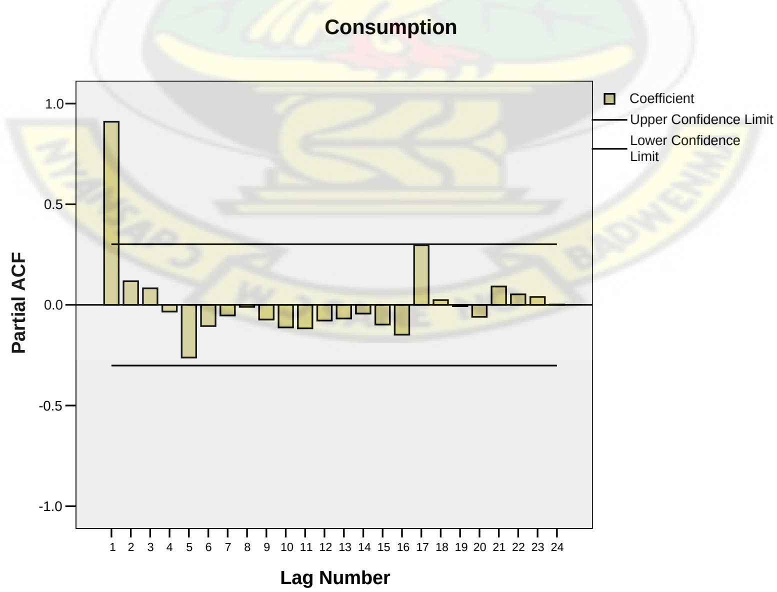

Fig 3.2.25 Partial autocorrelation function of Consumption

The results from Fig.3.2.24 and Fig 3.2.25 above indicate an ARIMA(1,0,0) model. This model is now compared to two other possible models, ARIMA(1,1,0) and ARIMA(1,2,0), and the best is chosen based on the following tabular results.

Table 3.2.8 Comparison Table of likely models for Consumption

MODEL	Residual Variance	Akaike's Information Criterion(AIC) Value
ARIMA(1,0,0)	2077066	699.49
ARIMA(1,1,0)	2076258	680.015
ARIMA(1,2,0)	3788279	685.760

From the results in Table 3.2.8 above, comparing both residual variance and AIC value gives ARIMA(1,1,0) as the most preferable model since its residual values are least.

The identified model is of the form $Y_t = (1+\alpha)Y_{t-1} - \alpha Y_{t-2} + e_t$, where Y_t is the observation at the present time Y_{t-1} and Y_{t-2} are respectively observations at time lag 1 and time lag 2, e_t, the white noise or error at the present time and α, parameter to be estimated.

After the model has been identified the parameters of the model were estimated and shown in the table below.

		Estimates	Std Error	t	Approx Sig
Non-Seasonal Lags	AR1	-.199	.153	-1.300	.201
Constant		-125.852	175.679	-.716	.478

Diagnostic Testing

On the adequacy of the model, Q-statistic was estimated to be $\chi^2 - distributed$ with $k - p - q$ degrees of freedom. Where $k = 24$ (maximum lag) used for Q, p is the order of the AR process and q is the order of the MA process. The $Q - statistic$ is compared to the critical value of χ^2_{23} which is $35.172.$ Since the calculated $Q - statistic = 17.55 < 35.172$ the chosen model is adequate.

Thus our selected model for Consumption is given by $Y_t = 0.801Y_{t-1} - 0.199Y_{t-2} - 125.852.$

3.3 MODELLING

The economy of Ghana could be likened to the theoretical four sector economy. Such as

$$Y = I + C + G \qquad 3.1$$

an open economy, that is one involving an international trade; equation (3.1) above becomes;

$$Y = C + I + G + (X - M) \qquad 3.2$$

Where the macro economic variables are defined as

Y = National Income

C = Consumption

I = Investment

G = Government Expenditure

X = Export expenditure

M=Imports

On the other hand expenditure on imports is a leakage from the system; some part of consumption, investment and government expenditure is likely to be spent on foreign produced goods, and these expenditure (M) needs to be deducted so that equilibrium condition requires the equality of planned expenditure on domestically produced goods and domestic national output.

Consider also a simple consumption function

$$C = a + c(Y - T) \qquad 3.3$$

Where a is autonomous consumption and c is the propensity to consume.

As regards imports, assume that they are positively related to real disposable income so that

$$M = b + m(Y - T) \text{ , } 0 < m < 1 \qquad 3.4$$

That is a rise in real disposable income that will cause an increase in planned import expenditure, which is directly related to consumption function.

m is the marginal propensity to import, which is the proportion of an increase in real disposable income that is spent on imports and b is autonomous import.

Substituting (3.3) and (3.4) into (3.2) gives

$$Y = a + c(Y - T) + I + G + X - (b + m(Y - T)$$

$$Y = a - b + I + G + X + c(Y - T) - m(Y - T)$$

$$Y - c(Y - T) + m(Y - T) = a - b + I + G + X$$

$$Y - cY + cT + mY - mT = a - b + I + G + X$$

$$Y - cY + mY = a - b + I + G + X + mT - cT$$

$$Y(1 - c + m) = a - b + I + G + X + mT - cT$$

$$Y = (a - b + I + G + X + mT - cT) \left(\frac{1}{1 - c + m}\right) \qquad 3.5$$

Autonomous elements are $a - b + I + G + X + mT - cT$, and Multiplier $\frac{1}{1-c+m}$

Trade balance or Balance of trade is defined as

$$B_T = X - M \text{ , where } M = b - mY \qquad 3.6$$

$$B_T = X - b - mY \qquad 3.7$$

For a given foreign interest rates, one expects net financial flows into the domestic economy (F) to be an increasing function of the domestic interest rate.

That is; $F = F(r)$ 3.8

$F' > 0$. That is to say that the first derivative is positive.

From (3.5), (3.6) and (3.7) gives

$$B_T = X - b - mY + F(r) \qquad 3.9$$

A linear system of difference equation was used to model the simplified version of the United States of America's (U.S.A.) economy of 1969 and is given as follows:

$$
\begin{aligned}
C(k) &= \alpha_0 + \alpha_1 Y(k) + \alpha_2 C(k-1) + u_1(k) \\
I(k) &= \beta_0 + \beta_1 P(k) + \beta_2 K(k-1) + u_2(k) \\
W(k) &= \alpha_0 + \gamma_1 Y(k) + \gamma_2(t) + u_3(k) \\
Y(k) &= C(k) + I(k) + G(k) \\
P(k) &= Y(k) - W(k) \\
K(k) &= K(k-1) + I(k)
\end{aligned}
\qquad 3.10
$$

Where C = consumption

W = wage income

K = net capital stock

G = government expenditure u_i, i= 1,2,3,… ; are interventions

Y = net product income, I = investments, t = time, P = non-wage income and

β_i γ_i, α_i are controls.

(USA Treasury Department, 1969)

We now attempt to simplify the above system using linear models of a four sector economy.

3.4 ASSUMPTIONS MADE REGARDING THE SIMPLIFIED MODEL

1. The dependent variable is obtained base on the previous year's policy instruments. That is to say that the independent variables lagged one time unit the dependent.
2. $\beta_2 K(k\text{-}1)$ = the capital stock cannot exist independent of the Investment and income.
3. In fact, Investment and capital stock are directly proportional and so $\beta_2 K(k\text{-}1)$ is equal to $I(k\text{-}1)$.
4. The consumption and investment in Ghana's economy depends largely on the Government's expenditure.
5. All the autonomous factors namely;

a) α_o = consumption

b) β_o = Investment

c) γ_{o} = wages

are considered as having a negligible influence on the Economy of Ghana, and so are ignored.

3.5 MODEL SIMPLIFICAION

$$C(k) = \alpha_1 Y(k) + \alpha_2 C(k-1) \qquad 3.11$$
$$I(k) = \beta_1 P(k) + \beta_2 K(k-1) \qquad 3.12$$
$$W(k) = \gamma_1 Y(k) \qquad 3.13$$
$$Y(k) = C(k-1) + I(k-1) + G(k-1) \qquad 3.14$$
$$P(k) = Y(k) - W(k) \qquad 3.15$$
$$K(k) = K(k-1) + I(k-1) \qquad 3.16$$

Substituting (3.14) into (3.11) for Y(k) gives

$$C(k) = \alpha_1[C(k-1) + I(k-1) + G(k-1)] + \alpha_2 C(k-1)$$

$$C(k) = \alpha_1 C(k-1) + \alpha_1 I(k-1) + \alpha_1 G(k-1) + \alpha_2 C(k-1)$$

$$C(k) = (\alpha_1 + \alpha_2) C(k-1) + \alpha_1 I(k-1) + \alpha_1 G(k-1) \qquad 3.17$$

Equation (3.17) is now reduced to

$$C(k) = a_{11} C(k-1) + a_{12} I(k-1) + b_1 G(k-1) \qquad 3.18$$

Where $a_{11} = \alpha_1 + \alpha_2$, $a_{12} = \alpha_1$ and $b_1 = \alpha_1$

Substituting (3.15) into (3.12) gives

$$I(k) = \beta_1[(Y(k) - W(k)] + \beta_2 K(k-1), \qquad 3.19$$

Substituting (3.13) into (3.19) gives

$$I(k) = \beta_1[Y(k) - \gamma_1 Y(k)] + \beta_2 K(k-1)$$

$$I(k) = \beta_1(1 - \gamma_1)Y(k) + \beta_2 K(k-1) \qquad 3.20$$

Using equation (3.14), substituting for Y(k) in (3.20), gives

$$I(k) = \beta_1 (1 - \gamma_1) [C(k-1) + I(k-1) + G(k-1)] + \beta_2 K(k-1), \qquad 3.21$$

$$I(k) = \beta_1 (1 - \gamma_1)C(k-1) + \beta_1(1 - \gamma_1)I(k-1) + \beta_1(1 - \gamma_1) G(k-1) + \beta_2 K(k-1) \qquad 3.22$$

But $K(k) = K(k-1) + I(k-1)$ and $\beta_2 K(k-1) = I(k-1)$,

Since investment is part of capital stock. (3.22) can then be written as

$$I(k) = \beta_1(1 - \gamma_1)C(k-1) + \beta_1(1 - \gamma_1)I(k-1) + \beta_1(1 - \gamma_1)G(k-1) + I(k-1)$$

and simplifying gives

$$I(k) = \beta_1(1 - \gamma_1)C(k-1) + [\beta_1(1 - \gamma_1) + 1)]I(k-1) + \beta_1(1 - \gamma_1)G(k-1)$$

3.23

Equation (3.23) is now reduced to

$$I(k) = a_{21}C(k-1) + a_{22}I(k-1) + b_2 G(k-1) \qquad 3.24$$

where $a_{21} = \beta_1(1-\gamma_1)$, $a_{22} = \beta_1(1-\gamma_1)+1$ and $b_2 = \beta_1(1-\gamma_1)$

The two equations (3.18) and (3.24) are now resolved into matrix form as

$$\begin{pmatrix} C(k \\ I(k) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} C(k-1) \\ I(k-1) \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} G(k-1) \qquad 3.25$$

Equations (3.17) and (3.23) are the linear equations of the Economic Model whiles the matrix form of the Economic Model is equation (3.25)

G(k-1) = government expenditure , a control variable

C(k-1) = consumption , a state variable

I(k-1) = investment , a state variable

Equation (3.25) is then compared to the non-homogenous state matrix equation

$$X(k) = Ax(k-1) + Bu(k-1),$$

where A = an n x n matrix, B = an n x 1 matrix, X(k-1) = a state vector

u(k-1) = a state control vector.

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $x(k-1) = \begin{pmatrix} C(k-1) \\ I(k-1) \end{pmatrix}$, u(k-1) = G(k- 1) and

$X(k) = \begin{pmatrix} C(k) \\ I(k) \end{pmatrix}$

3.6 DATA BASED MODEL

3.6.1 Homogeneous (Free) System Model

The homogeneous (free) system model of the form

$$\begin{pmatrix} C(k \\ I(k) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} C(k-1) \\ I(k-1) \end{pmatrix}$$

where $a_{11} = 0.870,\ a_{12} = -0.122,\ a_{21} = -0.068,\ and\ a_{22} = 0.793$, which were obtained using the data (refer to Appendix) is given in a matrix equation as

$$\begin{pmatrix} C(k) \\ I(k) \end{pmatrix} = \begin{pmatrix} 0.870 & -0.122 \\ -0.068 & 0.793 \end{pmatrix} \begin{pmatrix} C(k-1) \\ I(k-1) \end{pmatrix}$$

The homogeneous model equation above has the matrix $H = \begin{pmatrix} 0.870 & -0.122 \\ -0.068 & 0.793 \end{pmatrix}$ and the characteristic value equation

$$|H - \lambda I| = \begin{vmatrix} 0.870 - \lambda & -0.122 \\ -0.068 & 0.793 - \lambda \end{vmatrix} = \lambda^2 - 1.663\lambda + 0.681614 = 0$$

The above equations give eigenvalues $\lambda_1 = 0.93038, \quad \lambda_2 = 0.732615.$

The $|\lambda| < 1$ for all eigenvalues suggests that the homogeneous model system is stable.

3.6.2 The Non-homogeneous (Controlled) System Model

The non-homogeneous (controlled) system model which has the system matrix

$a_{11} = 0.197,\ \ a_{12} = -0.984,\ a_{21} = 0.405,\ \ a_{22} = 1.399$ with the control matrix

$b_1 = -0.505,\ and\ b_2 = 0.355.$

The system equation is given in the matrix notation as:

$$\begin{pmatrix} C(k) \\ I(k) \end{pmatrix} = \begin{pmatrix} 0.197 & -0.984 \\ 0.405 & 1.399 \end{pmatrix} \begin{pmatrix} C(k-1) \\ C(k-1) \end{pmatrix} + \begin{pmatrix} -0.505 \\ 0.355 \end{pmatrix} G(k-1)$$

The control parameters are the government expenditure and taxation, whiles the state variables are investment and consumption.

The controllable matrix of the system model has the matrix

$A = \begin{pmatrix} 0.197 & -0.984 \\ 0.355 & 0.29212 \end{pmatrix}$, $B = \begin{pmatrix} -0.505 \\ 0.355 \end{pmatrix}$ and $AB = \begin{pmatrix} -0.448805 \\ 0.29212 \end{pmatrix}$ is given in the form

$Q = (B \quad AB)$, that is $Q = \begin{pmatrix} -0.505 & -0.448805 \\ 0.355 & 0.29212 \end{pmatrix}$

The rank of Q is 2 and the model is controllable.

Table 3.3 Summary of Models of the components of the economy of Ghana

COMPONENT OF THE ECONOMY	MODEL
Inflation	$Y_t = 1.403Y_{t-1} - 0.403Y_{t-2} - 1.19$
Interest Rate	$Y_t = 1.403Y_{t-1} - 0.403Y_{t-2} - 1.19$
Total Money Supply	$Y_t = 0.7461Y_{t-1} + 0.239Y_{t-2} + 8.116$
GDP	$Y_t = 0.949Y_{t-1} + 0.051Y_{t-2} + 128.45$
Capital Stock	$Y_t = -0.693Y_{t-1} + Y_{t-2} + 0.693Y_{t-3} - 102.395$
Government Expenditure	$Y_t = 0.851Y_{t-1} - 0.149Y_{t-2} + 188.877$
Investment	$Y_t = 0.99Y_{t-1} + 0.01Y_{t-2} + 14.112$
Balance Of Trade	$Y_t = 1.026Y_{t-1} - 0.026Y_{t-2} + 50.758$
Consumption	$Y_t = 0.801Y_{t-1} - 0.199Y_{t-2} - 125.852$

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.0 INTRODUCTION

Since the model economy like any other economy be it developing or developed seeks stability and control, it is necessary that the economic model equation satisfies these basic requirements of a standard economic model.

The main components of the research model are consumption, investment and government expenditure. For any three sector economy the national income depends on investment, consumption and government expenditure. Since investment is the main drive of an economy, the economic growth of a typical developing country like Ghana will therefore require such a model for a trial.

4.1 CONCLUSION

The summary of findings as regards the work done is as follows;

1. The following table is the ARIMA models of the various components of the economy.

Table 4.0 Summary of Models of the components of the economy of Ghana

COMPONENT OF THE ECONOMY	**MODEL**
Inflation	$Y_t = 1.403Y_{t-1} - 0.403Y_{t-2} - 1.19$
Interest Rate	$Y_t = 1.403Y_{t-1} - 0.403Y_{t-2} - 1.19$
Total Money Supply	$Y_t = 0.7461Y_{t-1} + 0.239Y_{t-2} + 8.116$
GDP	$Y_t = 0.949Y_{t-1} + 0.051Y_{t-2} + 128.45$
Capital Stock	$Y_t = -0.693Y_{t-1} + Y_{t-2} + 0.693Y_{t-3} - 102.395$
Government Expenditure	$Y_t = 0.851Y_{t-1} - 0.149Y_{t-2} + 188.877$
Investment	$Y_t = 0.99Y_{t-1} + 0.01Y_{t-2} + 14.112$
Balance Of Trade	$Y_t = 1.026Y_{t-1} - 0.026Y_{t-2} + 50.758$
Consumption	$Y_t = 0.801Y_{t-1} - 0.199Y_{t-2} - 125.852$

2. Based on time series analysis the following series were obtained.

   i. The homogenous (free) system model is given as

$$\begin{pmatrix} C(k) \\ I(k) \end{pmatrix} = \begin{pmatrix} 0.870 & -0.122 \\ -0.068 & 0.793 \end{pmatrix} \begin{pmatrix} C(k-1) \\ I(k-1) \end{pmatrix}$$

   ii. The non-homogeneous (controlled) system model is given as

$$\begin{pmatrix} C(k) \\ I(k) \end{pmatrix} = \begin{pmatrix} 0.197 & -0.984 \\ 0.405 & 1.399 \end{pmatrix} \begin{pmatrix} C(k-1) \\ C(k-1) \end{pmatrix} + \begin{pmatrix} -0.505 \\ 0.355 \end{pmatrix} G(k-1)$$

iii. The controllability matrix model system was found to be of rank 2 thereby implying that the system is controllable.

4.2 RECOMMENDATION

1. Stakeholders should make use of mathematical models of the economy such as the one obtained, even if simple, for the purposes or a guide to policy making, especially in the area of using government expenditure as a major tool for managing the economy.
2. For further studies or work, it is recommended, that, a non-linear system model which will be more reliable should be formulated, using preferably updated data. Also an optimal control model could be studied even in the linear case by incorporating an objective function such as the welfare integral

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APPENDIX

DATA FOR THE MODEL

	1996				
PARAMETER	Q1	Q2	Q3	Q4	TOTAL
INFLATION	64.10	65.30	59.00	49.60	238.00
INTEREST RATE	47.10	46.70	46.40	45.90	186.10
TOTAL MONEY SUPPLY	52.10	58.00	61.30	69.70	241.10
GDP	4452.86	4498.69	4501.60	4633.90	18087.05
CAPITAL STOCK	1601400.00	1647900.00	1744000.00	2120400.00	7113700.00
GOVT. EXPENDITURE	638.80	654.20	703.60	834.60	2831.20
INVESTMENT	97.40	98.60	100.10	124.00	420.10
BALANCE OF TRADE	-1837.50	-1980.00	-2130.00	-2212.50	-8160.00

	1997				
parameter	Q1	Q2	Q3	Q4	TOTAL
INFLATION	40.90	35.90	30.50	28.50	135.80
INTEREST RATE	45.00	45.00	45.00	47.50	182.50
TOTAL MONEY SUPPLY	71.63	76.64	72.36	92.07	312.7
GDP	4452.86	4498.69	4501.62	4633.90	18087.07
CAPITAL STOCK	813.96	1197.84	1448.67	1504.70	4965.17
GOVT. EXPENDITURE	962.50	1034.20	1095.60	1103.90	4196.20
INVESTMENT	136.40	142.80	139.20	140.10	558.50
BALANCE OF TRADE	-2587.50	-2550.00	-2512.50	-2430.00	-10080.00

	1998				
parameter	Q1	Q2	Q3	Q4	TOTAL
INFLATION	27.40	26.80	26.50	27.42	108.12
INTEREST RATE	45.00	45.00	44.00	35.00	169.00
TOTAL MONEY SUPPLY	100.30	112.50	105.90	113.00	431.70
GDP	4675.25	4693.88	4735.90	4746.70	18851.73
CAPITAL STOCK	3035.56	3226.53	3227.82	3232.85	12722.76
GOVT. EXPENDITURE	1121.70	1156.80	1193.20	2063.40	5535.10
INVESTMENT	154.80	168.10	175.90	184.20	683.00
BALANCE OF TRADE	-2392.50	-2340.00	-2265.00	-2242.50	-9240.00

parameter	1999				
	Q1	Q2	Q3	Q4	TOTAL
INFLATION	28.50	30.60	32.80	36.50	128.40
INTEREST RATE	32.40	28.50	27.40	33.80	122.10
TOTAL MONEY SUPPLY	117.53	119.71	123.96	133.20	494.40
GDP	4856.61	4899.62	4946.84	4956.92	19659.99
CAPITAL STOCK	3146.29	3239.29	3334.72	3579.96	13300.26
GOVT. EXPENDITURE	1241.20	1468.41	1659.46	1806.90	6175.97
INVESTMENT	184.30	196.42	196.51	241.28	818.51
BALANCE OF TRADE	-2190.20	-2130.41	-2085.56	-2062.50	-8468.67

parameter	2000				
	Q1	Q2	Q3	Q4	TOTAL
INFLATION	40.00	42.00	40.00	40.00	162.00
INTEREST RATE	20.20	21.60	19.50	19.20	80.50
TOTAL MONEY SUPPLY	146.34	157.13	158.13	165.12	626.72
GDP	4906.55	4996.41	5069.63	5142.19	20114.78
CAPITAL STOCK	4972.20	4983.80	5012.34	5214.80	20183.14
GOVT. EXPENDITURE	1891.00	2041.63	2254.77	2508.49	8695.89
INVESTMENT	375.36	389.16	390.12	379.96	1534.60
BALANCE OF TRADE	-876.83	-863.42	-855.89	-842.30	-3438.44

parameter	2001				
	Q1	Q2	Q3	Q4	TOTAL
INFLATION	43.00	38.00	30.00	27.00	138.00
INTEREST RATE	27.00	23.00	24.00	17.00	91.00
TOTAL MONEY SUPPLY	178.43	189.26	196.59	200.42	764.70
GDP	5199.76	5258.36	5296.23	5359.12	21113.47
CAPITAL STOCK	5267.38	5384.46	5399.26	5492.90	21544.00
GOVT. EXPENDITURE	2456.80	2648.92	2859.66	3154.70	11120.08
INVESTMENT	3876.84	3922.88	3968.88	4070.22	15838.82
BALANCE OF TRADE	16.50	25.50	34.50	43.50	120.00

	2002				
parameter	Q1	Q2	Q3	Q4	TOTAL
INFLATION	17.00	15.00	15.00	17.00	64.00
INTEREST RATE	23.00	24.00	24.00	23.00	94.00
TOTAL MONEY SUPPLY	210.23	220.14	234.26	249.44	914.07
GDP	5497.63	5581.96	5596.40	5600.80	22276.79
CAPITAL STOCK	8003.50	8165.80	8534.00	10513.60	35216.90
GOVT. EXPENDITURE	3194.00	3263.50	3348.60	3506.29	13312.39
INVESTMENT	4161.24	4296.39	4367.65	4473.48	17298.76
BALANCE OF TRADE	92.23	115.92	136.16	151.80	496.11

.	2003				
parameter	Q1	Q2	Q3	Q4	TOTAL
INFLATION	29.90	29.60	26.80	23.60	109.90
INTEREST RATE	22.80	22.50	22.20	21.50	89.00
TOTAL MONEY SUPPLY	256.49	266.82	272.69	280.93	1076.93
GDP	6653.14	6773.36	6840.43	6886.73	27153.66
CAPITAL STOCK	11194.90	11673.00	11473.40	11375.90	45717.20
GOVT. EXPENDITURE	4745.30	4913.64	5042.20	5312.55	20013.69
INVESTMENT	4572.40	4774.60	48994.42	4924.28	63265.70
BALANCE OF TRADE	249.32	259.44	270.48	281.62	1060.86

	2004				
parameter	Q1	Q2	Q3	Q4	TOTAL
INFLATION	22.40	11.20	11.90	12.90	58.40
INTEREST RATE	20.00	18.50	18.50	18.50	75.50
TOTAL MONEY SUPPLY	314.90	320.44	339.63	341.72	1316.69
GDP	7650.91	7740.82	7850.63	7963.80	31206.16
CAPITAL STOCK	10957.80	11502.70	11521.50	11628.40	45610.40
GOVT. EXPENDITURE	4615.62	5090.40	5540.61	6066.59	21313.22
INVESTMENT	5012.32	5103.48	5263.41	5381.90	20761.11
BALANCE OF TRADE	-24.05	-25.41	-23.64	-30.98	-104.08