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THE EFFECTS OF REDIALS AND RETRIALS ON THE GSM NETWORK

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DECLARATION

I hereby declare that, except for specific references which have been duly acknowledged, this work is the result of my own field research and it has not been submitted either in part or whole for any other degree elsewhere.

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ABSTRACT

The scarcity of spectral resources limits the number of channels that can be used on a telecommunication network. Consequently, a user requesting a service particularly a call may be blocked because all channels may be occupied.

Initial planning and design of GSM network may not necessarily be accurate because all parameters including the traffic which determines the blocking of the network are based on estimations. Such networks may not meet the industry standard especially when it comes to blocking which is a measure of the Grade of Service (GoS) of the network. A user or the system may experience blocking at one time or the other. Both users and the network may attempt to get a connection. These reattempts constitute redials on the part of the user and retrials on the part of the network and they introduce a phenomenon which cannot be neglected in the analysis of a network. All of these retrials and redials are registered as separate calls. As a result, during the busy periods when blocking is observed in a cell, counters register a much larger volume than the effective call attempts.

This research seeks to study of the effect of these reattempts on the GSM network. We seek to find how the blocking probability, redial probability, number of redials and the number of retrials affect the GoS of the network. Four analytical models namely Generalized Redial/Retrial Model (GRRM), Uniform Redial Model (URM), were applied to measured load to extract the excess load which comprise of redials and retrials. When the extraction is complete, the actual load that is supposed to be on the network would be obtained after which an efficient and a better network can be designed.

The findings in this study revealed that network traffic is inflated with redials and retrials generated by networks subscribers and the network respectively. All the

models when applied produced actual loads that were less than the measured loads. GRRM and SHM proved to be better models to use on highly congested networks while URM and IRM proved to be useful for all congestion conditions. It was also observed that an increase in the blocking probability and the redial probability introduced more load onto the network as the redials and retrials increase.

The study has also shown that the network blocking probability may not always meet the industry standard and the number of channels that are used for network operations may often be inadequate.

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LIST OF SYMBOLS

A_C	Traffic intensity in a cell
C	Number of channels
C_E	Number of effective call attempts
C_F	Number of effective fresh call attempts
C_H	Number of effective hand-off call attempts
C_{HF}	Number of effective hand-off feedback call attempts
C_T	Total number of call attempts
f	Hand-off to total attempt count ratio
m	Maximum redial count
n	Maximum retrial count
N_R^{GRRM}	Mean number of retrials/redials per effective call in GRRM
N_R^{IRM}	Mean number of retrials/redials per effective callin IRM
N_R^{URM}	Mean number of retrials/redials per effective call in URM
N_F^{SHM}	Mean number of retrials/redials per effective fresh call in SHM
N_H^{SHM}	Mean number of retrials/redials per effective hand-off call in SHM
P_b	Probability of redialling after the retrials are all blocked
α	Redial probability
β	Blocking probability
α_i	i^{th} redial probability
α_H	Hand-off feedback probability

ABBREVIATION LIST

GoS: Grade of Service

GSM: Global System for Mobile communication

OMC: Operations and Maintenance Centre

PCS: Personal Communication Services

CTMC: Continuous Time Markov Chain

IMTS: Improve Mobile Telephone Service

1G: First Generation

2G: Second Generation (Analogue Cellular)

3G: Third Generation (Digital Cellular)

GPRS: General Packet Radio Service

EDGE: Enhanced Data Rate for GSM Evolution

SMS: Short Message Service

ITU: International Telecommunication Union

PLMN: Public Land Mobile Network

MS: Mobile Station

BTS: Base Transceiver Station

MSC: Mobile Switching Centre

BSC: Base Station Controller

PSTN: Public Switched Telephone Network

CDMA: Code Division Multiple Access

EV-DO: Evolution Data Optimized

DECT: Digital Enhanced Cordless Telecommunication

IMT-2000: International Mobile Telecommunication - 2000

TDMA: Time Division Multiple Access

iDEN: IntegratedDigital Enhanced Network

SMSS: Switching and Management Subsystem

BSS: Base Station Subsystem

OMSS: Operations and Maintenance Subsystem

FCA: Fixed Channel Allocation

DCA: Dynamic Channel Allocation

FDD: Frequency Division Duplexing

RF: Radio Frequency

BCCH: Broadcast Channel

MAHO: Mobile Assisted Hand-off

BHT: Busy Hour Traffic

TCNS: Traffic Communication Network Statistics

TCH: Number of Channels

CHAPTER ONE
INTRODUCTION

1.1 Motivation

Wires and cables have been the original medium for distance communication and it continues to provide communication to majority of people even though wireless communication has evolved and taken a centre stage in telecommunications. In cellular radio systems, the concept of trunking allows the users to share the somewhat small number of channels in a cell by providing on-demand transmission access to each user from a pool of available channels [1]. Therefore, wireless telephony systems rely greatly on teletraffic or trunking theory to accommodate a large number of users in a limited spectrum having a target to provide a certain optimal Grade of Service (GoS) [2]. Network engineers make use of the probability theory application to the solution of problems concerning planning, performance and assessment on the operation and maintenance of telecommunication systems. This is described as Teletraffic theory [3] or Traffic Engineering.

GSM cellular networks rely on the efficient use of the spectrum therefore the service area are divided into smaller cells, ideally with no gaps or overlaps. Each cell is served by its own base station and a set of frequencies. The goal behind this concept was to increase capacity so the network can provide service to a large number of subscribers while maintaining optimal system performance to the satisfaction of the subscriber. The operation of this network is such that a portable handset can be used in any one cell and moved through a couple of cells during a particular connection or transmission [4].The performance of cellular networks is the most important issue that network operators are concerned with. One of the performance indicators of a cellular network is the call blocking probability. The blocking probability is the statistical probability that a telephone connection cannot be established due to insufficient transmission resources in the network. This quantity is used to determine the GoS of cellular network.

The GoS of Global System for Mobile Communication (GSM) is set to 2% and it indicates that for optimum operation of the system, it should operate at 2% blocking. However this is not always the case because the initial dimensioning of cellular mobile networks is based on rough estimations of traffic [5] and thus, the initial capacities may not provide the required GoS figure. Consequently, users may be blocked while in the process of making calls. Most users retry till they get connection while others give up. Due to these repeated reattempts, the allowable blocking rate in the network may increase. This situation unfortunately introduces unnecessary traffic which exaggerates the load on the network. There is therefore the need to monitor the GoS of the network once it is deployed.

1.2 Objectives

The cellular network system has succeeded in increasing capacity on the network at the expense of scarce resources. Users making a call or the system retrying a connection get blocked because of the large subscriber base and insufficient channel resources. All these attempts exaggerate the network traffic especially in cells where the blocking probability is high.

This thesis aims to study the effect of reattempts on the GoS of the GSM network using a theoretical approach. The theoretical models are validated using real traffic data obtained from field installation of a telecommunication operator. The work seeks to investigate the effects of the number of redials, number of retrials, redial probability and call blocking probability on the efficiency of the network.

1.3 Organisation of the Thesis

This thesis is organized as follows: in Chapter 2, related works are discussed and an accurate description of the retrial and redial models is given. Chapter 3 illustrates the thesis methodology of proceedings. Chapter 4 is devoted to the analysis and discussion of results from the models when applied to real data collected from the Operations and Maintenance Centre (OMC) of a telecommunication company in Ghana. Calculations are displayed graphically to demonstrate the effect of the changing system parameters on the blocking probability and eventually the GoS. The effective load is also tabulated against the measured load to see the extent of the excess and also to see the effective number of channels needed for channel allocation. Conclusions and recommendations are given in Chapter 5.

CHAPTER TWO
LITERATURE REVIEW

2.1 Introduction

This chapter discusses previous researches conducted to investigate the effect of network system parameters especially reattempts on network efficiency. It also gives an overview of the Global System for Mobile communication (GSM) network, its attributes and capabilities. We also look at the Erlang-B model: a model used for calculating the blocking probability of a network. Repeated attempts and the redial and retrial models used in this study are also introduced here.

2.2 Related Works

Blocking is experienced in a network when a user wanting to access the network is prevented from doing so because all channels are in use. It can be experienced when making a new call or during a hand-off process where all channels in the destination cell are in use. Sidi *et al* [**6**], provided analytic expressions for the two kinds of blocking mentioned earlier for two types of users: a slowly moving mobile user and a fast mobile user. The study proposed an approximation for the moderate mobility (neither slow nor fast) using a multi-dimensional continuous-time Markov chain. In [**7**], the moderate mobility was studied with regard to cell size. It was shown that, the channel holding times in a cell get smaller as the cell residence times get smaller and the number of hand-off calls and the hand-off call blocking probabilities increase. In [**8**], a model for studying the effect of redials in a Personal Communication Services (PCS) network was studied. The probability of redial, the redial rate and the portable mobility is varied and their effect was studied on the following parameters: probability of forced termination, probability of a new call blocking, call completion probability, expected number of redials before a new call is accepted or rejected and calls accepted per simulated second. It was observed in the study that:

- Redialling when done at a low rate and with a high probability helps to reduce the call incompletion probability.
- Redialling when done with a high probability can cause an increase in the probability of force termination.
- The study indicated that redialling may improve the call incompletion probability. However from the user's point of view, frequent redialling may indicate poor quality of service.
- There could be a case where the telecommunication network could get congested to the extent that the network functions can even breakdown especially at an area where a lot of people have gathered due to an event.

Generalized gamma distribution was used for the cell residence times and it was concluded that the blocking probability distribution approaches the Erlang-B distribution as the size of the cell gets bigger. In [**9**], a model for the estimation of blocking probabilities in cellular mobile telephony with customer retrials was proposed. This model is based on Continuous Time Markov Chain (CTMC). The reason for the proposal of the model was that in previous work, it was assumed that the call request time sequence is described by means of a Poisson process and the time for which a call requires service within a cell (the call *dwell time*) is exponentially distributed. This assumption did not allow the description of the increase in call request rate due to repeated attempts (redials and retrials) by users whose call request was refused due to the lack of available resources**.** It was observed that blocking probabilities for systems without reattempts were lower than the blocking probabilities for systems with reattempts.

Another model was also proposed to evaluate the impact of customer retrial on capacity re-dimensioning of mobile cellular networks [**10**].This work was based on [**11**].The study considered a two dimensional Markov chain with 2 parameters: persistent intensity (redials) and retrial intensity (retrials). It was observed that these parameters when increased cause the apparent blocking probability to take on high values than the real blocking probabilities. Thus the parameters control the blocking probability which can be evaluated from the measurements. They also contribute to the overall blocking probability of the system and that under overload conditions; repeated attempts increase the blocking probability.

All researches discussed here proposed models for the study of the effects of reattempts on the telecommunications network. None of these models has been validated to investigate its authenticity and possible implementation by network operators. This thesis seek to validate models proposed in [**2**],[**18**],[**35**],[**36**].

2.3 **The GSM Network Evolution**

The latter part of the 1940s saw the first radiotelephone service introduced in the US, and it was meant to serve as a connection between mobile users in cars and the public fixed network. In the 1960s, Bell Systems launched a system called Improved Mobile Telephone Service (IMTS), which brought much innovation like direct dialling and higher bandwidth. Later, they launched the first analog cellular systems based on IMTS and were developed in the late 1960s and early 1970s [**12**]. In these systems, the radio signals were analog and were known as first generation (1G). The systems were called "cellular" because coverage areas were split into smaller areas called cells; each of which is served by low power radio transceivers. Towards the end of the 1980s, radio signals were digitized [**12**] and this led to the advent of second generation (2G) technology. The introduction of this technology provided better quality of service and increased capacity at low cost to the subscribers. GSM, GPRS and EDGE are examples of 2G technologies. 2G systems introduced data services such as SMS. Technological advancement saw the advent of the 3G mobile technology. The attributes of the 3G system are as follows [**13**]:

- Global standards to allow for low cost and worldwide roaming.
- Higher QoS for voice.
- Support for advance services such as multimedia, bandwidth of demand and high speed data.
- Flexibility for evolution allowing for backward compatibility and to cope with future market discontinuity.
- Multi-environment capabilities like vehicular and fixed wireless Access.
- Compatibility of services with fixed networks.

This system is based on the ITU family of standards under the IMT-2000 [**14**]. The mobile environment in this system provides services including video calls, wide-area wireless voice telephone, and broadband wireless data.

Each cell in cellular network is served by a base station which transmits at low power. By the use of multiple networks (in the form of cells), a single network can handle a large number of subscribers with very little spectrum. The size of the cells depends on the radio signal propagation, local morphology and the user density in the cell. A group of channels representing a portion of the limited radio spectrum is allocated to the cells [**1**][**15**][**16**]. As the need for more channels increases, the number of cells can be increased as long as the interference between transmitters is kept minimal as possible [**2**].

The essential feature of a cellular network is that the final link between the subscriber and the fixed network should be radio. This however comes with a number of consequences [**17**] such as:

- Radio spectrum is a finite resource and the amount of spectrum available for mobile communication is strictly limited.
- The radio environment is subject to multipath propagation, fading and interference and it is therefore not an ideal transmission medium.
- The subscriber is able to move and this movement must be accommodated by the system.

Considering the effect of these consequences, the following should be noted that since the radio spectrum is limited, appropriate strategies should be employed to accommodate the increasing number of subscribers and that interference should be kept below a certain level as possible. Mobility on the part of the subscriber should also not be impeded.

The most important limiting factor in the performance of the cellular radio systems is interference. The sources of interference include [**18**] another mobile station in the same cell, a connection in the neighbouring cell, other base stations operating in the same frequency and the leakage of energy into the cellular frequency band from any other sources.

Cellular networks offer a number of advantages over alternative solutions. The advantages include: increased capacity, reduced power usage, larger coverage area and reduced interference from other signals.

2.4 Structure of a Cellular Network

The cellular network consists of both land and radio based sections. Such network is often referred to as Public land mobile network (PLMN). The land-based section of the network is the Public switched telephone network (PSTN) whilst the radio based section is composed of the following entities [**19**]:

Cell: the transmission radius produced by a base station transceiver

Mobile Station (MS): A wireless device used to communicate over the cellular network.

Base Transceivers Station (BTS): A transmitter/receiver used to transmit/receive signals over the radio interface section of the network.

Mobile Switching Centre (MSC): The heart of the network which sets up and maintains calls made over the network. This unit provides connection to the PSTN.

Base Station Controller (BSC): Controls communication between a group of BTSs and a single MSC.

Figure 2.1 illustrates the structure of a cellular network

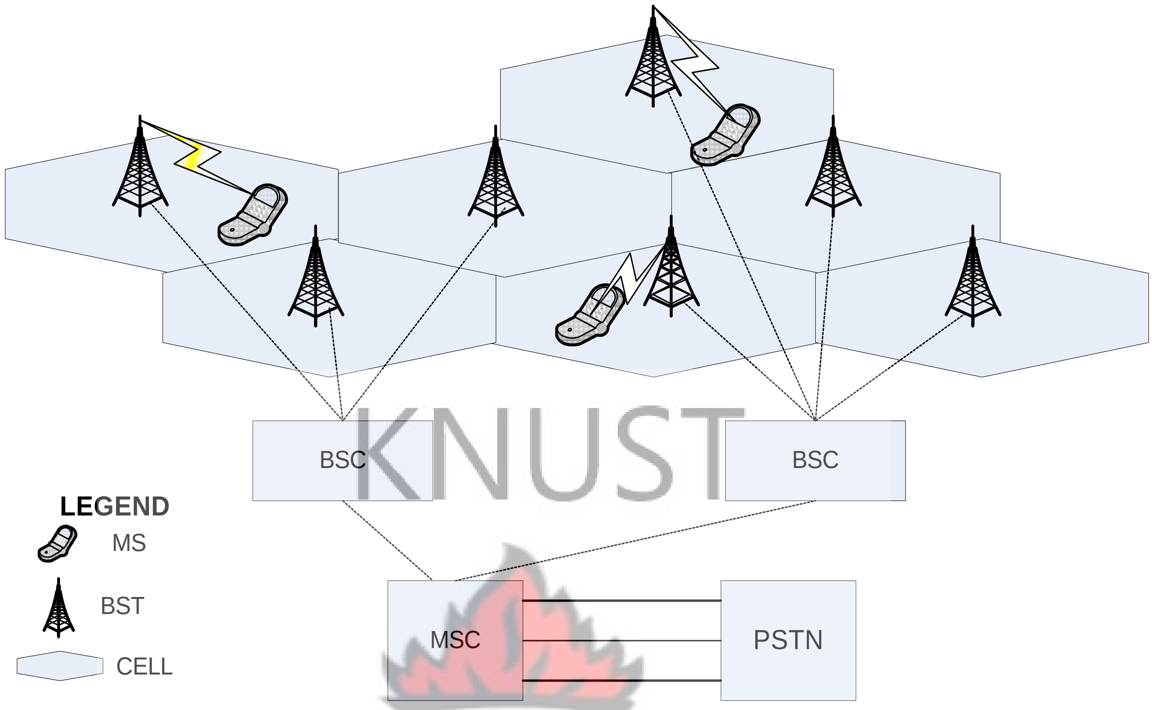

Figure 2.1 A Cellular Network

2.5 The GSM Network

There are a number of different digital cellular technologies which include: GSM, GPRS, CDMA, EV-DO, EDGE or EGSM, DECT, Digital AMPS (IS-136/TDMA), and iDEN [**20**].

GSM is the most popular of all the digital cellular technologies [**9**]. This mobile communication standard was developed to address compatibility issues. The objectives of the GSM are [**21**] a broad offering of speech and data services, compatibility with wired networks, automatic roaming and hand-off, efficient frequency reuse, support for different types of mobile terminal equipment, digital transmission and supplier independence.

There are different specifications for the various flavours of GSM. Those of the GSM-900 networks are listed below [**22**]

- *Access Method:* GSM is a digital technology based on the TDMA technique. This means that several different calls may share the same carrier and each call is assigned a time slot. This choice was found to improve capacity.
- *Modulation:* the modulation scheme for GSM is the GMSK. This facilitates the use of narrow bandwidth and allows for coherent and non-coherent detection capabilities.
- *Frequency Band:* In GSM, the uplink (from the mobile station to the base station) frequency ranges from 890 to 915 MHz and the downlink (base station to mobile station) frequency ranges from 935 to 960 MHz [**23**]
- *Bandwidth:*The bandwidth for GSM as observed from the frequency band is 25 MHz. This provides 124 carriers and each had a bandwidth of 200 kHz with 8 users per carrier. This means there is about 1000 actual speech or data channels for transmission. This number can be further doubled with the introduction of a half rate speech coder.
- *Carrier Spacing:* Spacing for each of the carrier is 200 kHz.

2.5.1 Architecture of the GSM Network

The GSM network has two key components [**24**]: the fixed installed infrastructure and the mobile subscribers who use the services provided by the network and communicate over the air interface. The fixed section of the network is subdivided into three major systems: the SMSS, BSS, and the OMSS. The functions of the various systems are found in details in [**24**][**25**].

2.5.2 Capacity and Channel Allocation

In the GSM network, each base station is allocated a different carrier frequency and each cell has a usable bandwidth associated with the carrier. Capacity can be defined as the number of active users that the network can support at a given time. Because only a finite part of the radio spectrum is allocated to the cellular radio, the number of carrier frequency is limited and hence capacity. Thus capacity can only be increased by reusing the carrier frequency [**17**]. The remarkable expansion of the wireless/mobile users' population together with the requirements of bandwidth for multimedia applications requires efficient reuse of the limited radio spectrum allocated to wireless/mobile communications. From a cost-of-service point of view, it is also important to efficiently use the radio spectrum. The number of base stations when reduced would proportionately reduce the cost-of-service and this can be achieved by more efficient reuse of the radio spectrum.

For efficient utilization of the radio spectrum, a frequency reuse scheme that is consistent with the objectives of increasing capacity and minimizing interference is required [**26**]. To achieve these objectives a variety of channel assignment strategies have been developed. It can be used either fixed or dynamic [**27**]. The performance of the system can be affected by the choice of a channel assignment strategy [**26**][**27**] as such care should be taken when choosing the various strategies.

Channel allocation schemes can be divided into a number of different categories based on assessment basis. Channel allocation strategies in GSM include FCA, Channel Borrowing and DCA. [**26**] and [**27**] give detailed information on these strategies.

2.5.3 Frequency Reuse

Communication in a cellular network is full duplex, where communication is attained by sending and receiving messages on two different frequencies - FDD. The reason for the cellular topology of the network is to enable frequency reuse. Cells, a certain distance apart, can reuse the same frequencies, which ensure the efficient usage of limited radio resources [**28**]. Frequencies, reused in the system accommodate more users. The group of cells in which frequencies cannot be reused is termed as a cluster. The minimum distance which allows the same frequency to be reused will depend on many factors [**17**] such as: Number of co-channel in the vicinity of the centre cell, geographic terrain, antenna height and transmitted power within each cell

The basic prohibiting factor in radio spectrum or frequency reuse is interference caused by the environment or other mobiles. Interference can be reduced by deploying efficient radio subsystems and by making use of channel assignment techniques [**26**] as discussed in the previous section.

2.5.4 Hand-off

A mobile station in a cell would use a certain frequency/channel in that cell. When a call is in progress and a mobile station moves from one cell to another, the call is expected to be terminated because there is a change in frequency. But this does not happen because technically the call can be handed over to another channel in a different cell.

The parameters that are needed to determine whether a hand-off is necessary are [**29**] the signal strength of the base station with which the communication is being made, the signal strengths of the surrounding base stations and the availability of channels.

The mobile station monitors the strength of the signal of the base station while the cellular network has knowledge of the status of the availability of the channels and makes decisions about when the hand-off is to take place and to which channel of which cell.

Due to the TDMA technique that the GSM network uses, the transmitter transmits for one out of eight slots, and similarly the receiver receives for only one in eight slots. In effect the RF section of the MS could be said to be idle for six out of the total eight slots. This is however not the case because during the slots that are not communication with the BTS scan the other radio channels looking for beacon frequencies that may be stronger or more suitable for transmission and reception. In addition to this, when the MS communicates with a particular BTS, one of the responses it make is to send out a list of radio channels of the beacon frequencies of neighbouring BTSs via the BCCH. The MS scans these and reports back the quality of the link to the BTS. In this way the MS assists in the hand-off decision and as a result this form of GSM hand-off is termed MAHO.

The network has knowledge of the quality of the link between the MS and the BTS as well as the strength of local BTSs as reported back by the MS. It also knows the availability of channels in the nearby cells. As a result it has all the information it needs to be able to make a decision about whether it needs to hand the mobile over from one BTS to another. If the network decides that it is necessary for the mobile to hand over, it assigns a new channel and time slot to the MS. It informs the BTS and the MS of the change. The MS then retunes during the period it is not transmitting or receiving, i.e. in an idle period. This process is called hand-off (or as handover as other literatures may put it) because the call in progress is literally handed over to an idle channel in the new cell by the old channel in the previous cell.

The possible types of hand-off are [**17**]

- Intra-cell hand-off – this occurs between traffic channels within the same cell
- Inter-cell hand-off – this occurs between traffic channels on different cells
- Inter-MSC hand-off – this occurs between cells belonging to different MSCs

The time for a hand-off should be cautiously selected to ensure user satisfaction as blocking a new session setup is considered to be less harmful than blocking a hand-off attempt [**30**] as it irritates the user.

2.5.5 Grade of Service (GoS)

The probability of a call being blocked or queued for some period of time due to limited system resources is termed the blocking probability. This measure is used to determine the GoS of the system. This value is checked during the busy hour of the day. GoS is expressed as a decimal fraction. For example, 3% blocking indicates that there is a probability (P) that three (3) in every one hundred (100) call attempts will encounter a blockage condition (i.e., will fail to gain access to the system) during the busy hour of the day.

Once the network is operational, continuous monitoring of the *GoS* is required. This is necessary because although the network might be correctly dimensioned, there are overload and failure situations not considered in the dimensioning where short term (minutes, hours) network traffic management actions have to be taken. In situations considered in the dimensioning, traffic forecast errors or approximations errors made in the dimensioning models may lead to a GoS different from the one expected. GoS monitoring is needed to detect these problems and to produce feedback for traffic characterisation and network design. Depending on the problems detected, network reconfigurations, changes of the routing patterns or adjustment of traffic control parameters can be made in medium term (weeks, months). The urgency of a long term planning of network extensions may also be assessed [**31**].

To achieve and maintain a given GoS, the operator must ensure that sufficient telecommunications circuits or routes are available to meet a specific level of demand. It should also be kept in mind that too many circuits will create a situation where the operator is providing excess capacity which may never be used, or at the very least may be severely underutilized. This adds costs which must be borne by other parts of the network. GoS is normally set at 2% which is the industry standard.

2.6 The Erlang-B Model

The Erlang-*B* model which was designed and analyzed by A.K. Erlang [**32**] is represented by *M/M/N/N* where the first *M* stands for Poisson arrival process; the second *M* represents independent and identical exponential service times. The number of agents *N* is the same as the number of trunk lines *N* which implies that waiting is not allowed. The Erlang-*B* model is a "blocked calls lost" model, in which when servers are unavailable; the service requestor is denied service and must retry the request. When all telephone trunks are exhausted, the caller receives a busy signal. Thus the caller must hang up or redial repeatedly until a server becomes available. Erlang-B calculates the blocked call probability (loss probability for a given traffic load and a number of servers).

The Erlang-B model is based upon the following assumptions [**1**]

- Call requests are memoryless, implying that all users, including those that are blocked, may request a channel any time.
- All free channels are available for servicing call requests until they are all occupied.
- The channel occupancy time (the probability of a user occupying a channel) is exponentially distributed. Thus, longer calls are less likely to happen.
- There are a finite number of channels available to service the call requests.
- Traffic requests (call arrivals) are described by a Poisson process, which implies exponentially distributed call inter-arrival times.
- Call inter-arrival times are independent of each other.

The probability that a call will be blocked is derived as follows:

Let

λ = the arrival rate and

μ= the service rate and the

A= offered load in Erlangs

C= number of trunk lines

Then offered load, A is derived from equation (1) as:

$$A = \frac{\lambda}{\mu} \tag{1}$$

Theprobability that a call will be blocked$P_B(C/A)$ can be derived using results from (ITU-D Study Group 2) [**2**][**31**] as follows:

$$P_B(C/A) = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C}\frac{C^k}{k!}} \quad (2)$$

From equation (2), given *A* and *C*, the blocking probability can be determined. The reverse problem is to determine *C* for a given call load, *A* and a prescribed blocking probability which is done using tables or calculators which are readily available.

2.7 Repeated Attempts

The traffic designer must be able to use some traffic value against which he or she can design his or her network to determine the number of trunk circuits required. The traffic level is an average of all traffic taken over several days and over the busiest period. The period is usually one (1) hour (60 minutes) and the average traffic over that hour is called the 'Busy Hour Traffic', BHT [17].

When a particular user requests service and all the radio channels are occupied, access to the system is denied and the user is blocked. This condition leads to congestion mostly at BHT. When a call is blocked, it may be reattempted by the initiator. Repeated calls can have quite a negative impact on the system performance and should therefore not be neglected in network design and planning [**9**][**30**][**33**][**34**].

Redials are calls attempts that are generated by the user when an attempt to initiate a connection fails and he or she retries [**6**]. Retrials are calls attempts that are generated by the system when an attempt to initiate a connection fails and it retries. These attempts are normally termed as automatic retrials.

Retrials are deterministic [**2**] normally set at three to six times by the network operator while the redials are stochastic and it normally depends on the user behaviour. An impatient subscriber will retrial infinitely until a connection is established. These call attempts contribute to the traffic load and thus should be filtered out so that the actual load on the network can be obtained.

Reattempts may also be introduced by the hand-off process. During the hand-off process, it is possible that there may not be available channels in the new cell which would trigger some retrials in the system because the system would want to maintain the connection for the call in progress. Redials are however absent in hand-off because by the time the user redials it will be registered as a new call in the new cell.

2.8 Retrial and Redial Models

Different models have been designed in [**2**],[**18**],[**35**],[**36**] to accommodate fresh calls and both fresh and hand-off calls. The models are outlined in the next two sections.

2.8.1 Generalized Retrial/Redial Model (GRRM)

In [**2**], [**35**], three different models (the general model and two other models which are a modification on the former) were designed to accommodate the retrial and the redials based on the assumptions that were made. The flow chart of the model is shown in Figure 2.2:

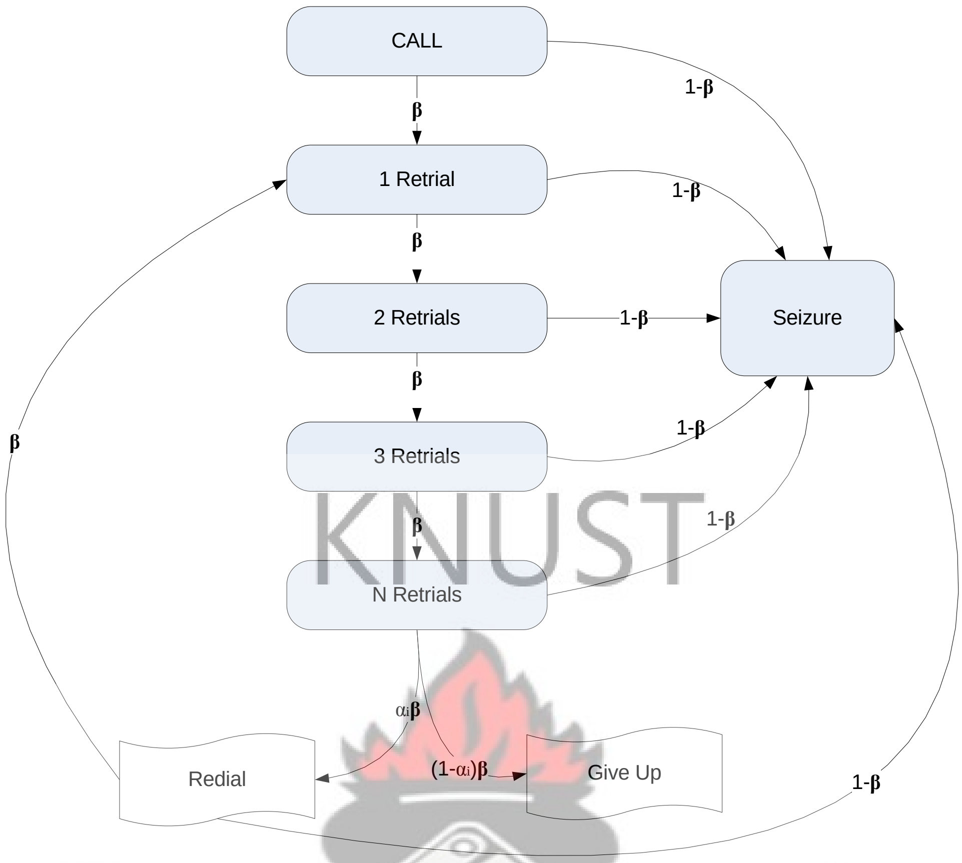

Figure 2.2 The Generalized Redial/Retrial Model

The process starts at the ***Call setup*** state and stops either in the ***Seizure*** or the ***Give up*** state. The intermediate states represent the retrials or redials. An invariant blocking ratio,β of the number of total blocked attempts to the total attempts is assumed to be acting through the retrials. Each call attempt, as well as the redials bring along a maximum of $n$retrials. When the i^{th}call attempt fails, including all the associated n automatic retrials, the caller either redials with a probability α_i (thereby reinitiating the entire call procedure with retrials), or quits with a probability $1-\alpha_i$. This model assumes at most $m$redial per person and ignores the extremely impatient customers as outliers.

Let N_R^{GRRM} denote the expected number of retrials and redials per original fresh call. Based on the transitions depicted in Figure 2.2, N_R^{GRRM} is expressed as in equation (3)

$$N_R^{GRRM} = \sum_{i=0}^{m} \left\{ \left\{ \left(\sum_{j=0}^{n-1} [i(n+1)+j]\, \beta^{(i+1)n+j}(1-\beta) \right. \right. \right.$$

$$\left. \left. \left. + [(i+i)n+i]\beta^{(i+1)n+1}\left(1-\alpha_{(i+1)}\beta\right) \right) \right\} \xi_i \right\} \qquad (3)$$

where

N_R^{GRRM} = mean number of redials/retrials per effective fresh calls in *GRRM*

m = maximum redial count

n = maximum retrial count

i = i^{th} redial

j = j^{th} retrial

β = blocking probability

α = redial probability

$$\xi_i = \begin{cases} 1 & if i = 0, \\ \prod_{k=1}^{i} \alpha_k \; if i = 1, \dots, m. \end{cases} \qquad (4)$$

Thus for every call attempt initiated by a caller, N_R^{GRRM} additional retrials and redials take place on the average. Because of the difficulty in finding an appropriate redial probability distribution, *GRRM* can be simplified by assuming a constant redial probability. We look at a uniform redial probability distribution, where $\alpha_i = \alpha, i = 1,2, \dots, m,$ we have $N_R^{GRRM} = N_R^{URM}$ where *URM* stands for Uniform Redial Model.

Given that

N_R^{URM}= mean number of redials/retrials per effective fresh calls in *URM*, equation (3) becomes

$$N_R^{URM} = \sum_{j=0}^{m}\left\{\left[\sum_{i=0}^{n-1}(j(n+1)+i)\beta^{(jn+1)}(1-\beta)\right] + \big((j+1)n+j\big)\beta^{(j+i)n}(1-\alpha\beta)\right\}(\alpha\beta)^j$$

$$= \frac{1-P_b^{m+1}}{1-P_b}\left[P_b + \frac{\beta(1-\beta^n)}{1-\beta}\right] - (m+1)(n+1)P_b^{m+1}, \qquad (5)$$

where $P_b = \alpha\beta^{n+1}$ is the probability of redialling after the original call and all its retrials are blocked. *URM* can be further simplified by assuming users redial till they get a channel or give up redialling by omitting *m*. When we omit *m*, we obtain the Infinite Redial Model (*IRM*) expressed by equation (6) as:

$$N_R^{IRM} = \lim_{m\to\infty} N_R^{URM} = \frac{\alpha\beta^{n+1}}{1-\alpha\beta^{n+1}} + \frac{\beta(1-\beta^n)}{(1-\beta)(1-\alpha\beta^{n+1})} \qquad (6)$$

where

N_R^{IRM} = mean number of redials/retrials per effective fresh calls in *IRM*

Now including the original call, the following scaling gives the expected effective load(C_E) as expressed in equation (7):

$$C_E = \frac{C_T}{N_R^{Model}+1}, where Model = GRRM, URM or IRM \qquad (7)$$

where C_T is the total measured load, which contains the retrials and redials. The effective load can then be calculated and the replanning activity can now be done more accurately.

2.8.2 Separated Hand-off Model (SHM)

Hand-off calls are not subject to redials and thus it is necessary to model hand-off calls separately from fresh calls in the analysis of the retrial and redial phenomenon in GSM networks [**36**].

When a hand-off call is dropped, the caller may generate a new call to continue with the conversation which is always the case because the loss of a hand-off request or a retrial is less desirable than the loss of a new session [**30**]. The new call is considered as a fresh call in the new cell and not a redial of the hand-off call. These types of calls are recorded by the fresh call counters. Thus, the hand-off call counters do not register the fresh calls generated after the dropped hand-off calls. The crucial point in this operation is that the user might generate a new fresh call in another cell after the call drops. The dropping of the calls might be due to variations in power level measurements by the mobile stations that occur due to user mobility [**2**].

In view of the above mentioned observations, the GRRM is revised to include the hand-off calls in the Separated Hand-off Model (SHM) [**2**],[**18**], [**36**]. The SHM is shown in Figure 2.3.

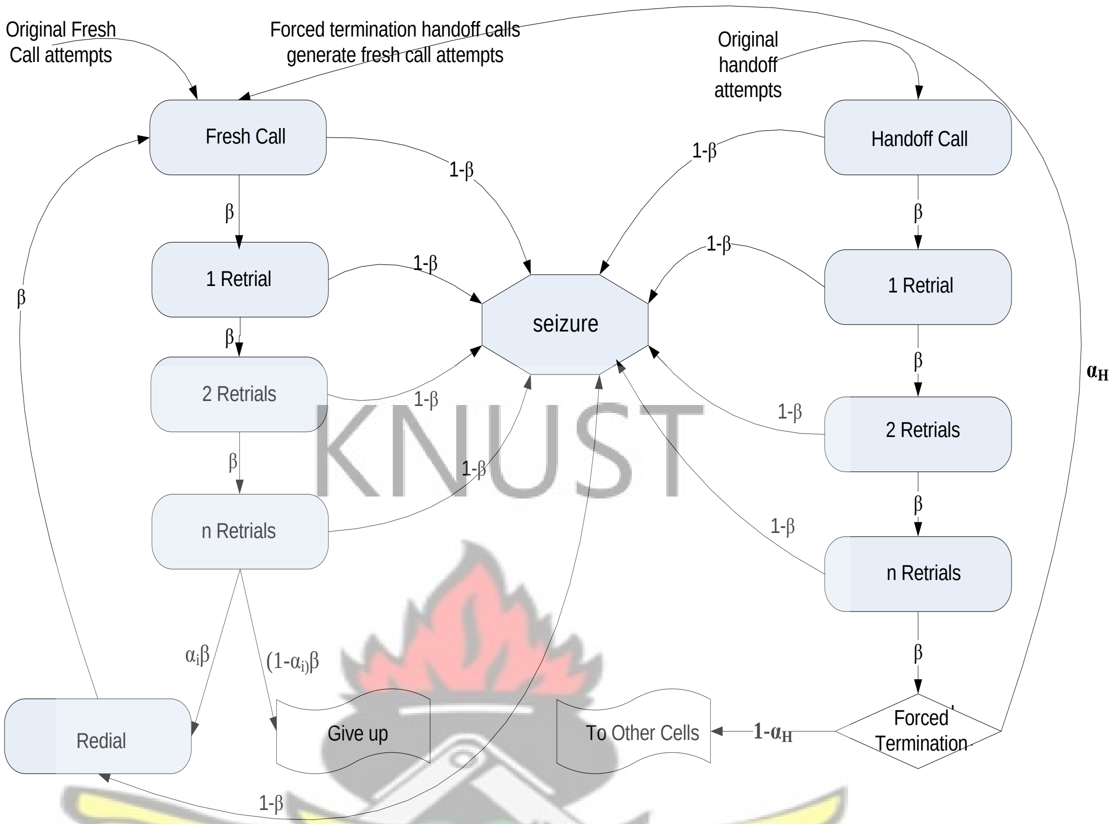

Figure 2.3 The Separated Hand-off Model

The connection setup starts from the ***Fresh Call*** state and the hand-off starts from the ***Hand-off Call*** state. The fresh call portion is the same as discussed with the GRRM. Hand-off calls are merely retried up to a specific limit defined by the GSM operator. The retrial scheme is identical for both the fresh calls and hand-off calls, and this is represented by the maximum retrial count, n in the model. From the figure, the fresh call attempts that are a result of hand-off forced termination are shown by a feedback loop. The hand-off feedback probability, α_H is a small value in practice because users might have crossed over to a neighbouring cell by the time they redial. The mean number of retrials and redials generated by the fresh calls (N_F^{SHM}) can be calculated by equation (8):

$$N_F^{SHM} = \sum_{i=0}^{m} \left\{ \left\{ \sum_{j=0}^{n-1} [i(n+1)+j]\, \beta^{(i+1)n+j}(1-\beta) + [(i+1)n+i]\beta^{(i+1)n+1}\left(1-\alpha_{(i+1)}\beta\right) \right\} \xi_i \right\} \quad (8)$$

where ξ_i is defined in equation (4). The mean number of retrials by the hand-off calls (N_H^{SHM}) can be calculated as shown in equation (9):

$$N_H^{SHM} = \sum_{i=0}^{n-1} i\beta^i(1-\beta) + n\beta^n = \frac{\beta(1-\beta^n)}{1-\beta} \quad (9)$$

Now assuming that, the total load is c_T, the handoff feedback probability is α_H and f percent of the total load c_T, is the hand-off call and the retrial load. If we denote the effective fresh call load by c_F (c_F includes the hand-off feedback call load), and the effective hand-off call load by c_H and the hand-off feedback load by c_{HF}, then

$$C_F = \frac{(1-f)C_T}{N_F^{SHM}} \quad (10)$$

$$C_H = \frac{fC_T}{N_H^{SHM}} \quad (11)$$

$$C_{HF} = \beta^{n+1}\alpha_H C_H \quad (12)$$

If the cell capacity was replanned adequately to handle the attempts for a given GoS, then the hand-off feedback calls and their retrials and redials would not be generated. Thus, the hand-off feedback load should be deducted from the expected effective load. The expected effective load c_E can thus be calculated with equation (13) as:

$$C_E = C_F - C_{HF} + C_H \quad (13)$$

2.9 Conclusion

This chapter gave a review of all literature encountered in this study. Equations for the calculation of the mean number of redials and retrials for the various models we seek to validate were also reviewed here. The next chapter gives us a step by step approach to how the study was conducted.

CHAPTER THREE
METHODOLOGY

3.1 Introduction

The planning and design stage of cellular networks may involve estimations of voice traffic, data, frequency allocations, number of channels and so on. These estimations may not necessarily reflect the actual condition on the network after deployment. This situation may also result in congestion on the network which leads to the blocking of some of the calls. Contrary to the initial dimensioning when the engineer works with estimations, the engineer is now equipped with the measured call attempt data which gives information about traffic on the network after the network is deployed. This data helps the engineer to monitor the performance of the network and do some replanning if need be.

Data from the network in the form of call attempts is collected for processing, filtering and analysis. During the filtering stage, the actual redial probability of callers is needed so we conducted a survey to obtain those probabilities.

3.2 Research Statement

Previous researches on the effect of redials and retrials were limited to theoretical simulation work. This could be explained by the fact that gathering data on a real network is challenging because of competition among operators. Consequently, little research has been conducted with field data.

The proposed study debunk from the previous studies by validating a theoretical model with measured traffic data by evaluating the effects of user and system reattempts on the Grade of service.

3.3 Data collection, Filtering and Analysis method

This section gives a brief description of how data was collected, processed and analysed.

3.3.1 Data Collection

For the study, we considered a 900 MHz GSM network of one of the telecommunications companies in Ghana with an industry standard GoS of 2% blocking. Traffic data was gathered from the OMC of the network and was processed by the TCNS software into an excel spreadsheet.

The data consisted of call attempt data per cell for 60 minute periods. Out of the data collected only the BHT was considered. The ten highest figures were selected from the BHT to make up our data sets. It is assumed that since the individual call attempts are not available, the data is made up of the original calls and reattempted calls [**10**], all as independent call attempts.

The Erlang B calculator was used to determine the blocking probability (beta) of the data sets based on the BHT and number of channels.

For the application of GRRM and SHM models, the exact redial distribution of callers was needed. A survey was conducted among some 50 students of the final year class of the Construction Technology and Management students in the College of Architecture and Planning. After the redial distribution was derived out of the student sample, the exact redial probabilities were then calculated based on the number of students that had redialled a particular number of times. It is actually a fraction of the number of students that had redialled i times to the total number of students that had redialled $i + 1$. Based on this, the choice of any sample size whether small or large would not influence the results that would be obtained. It is assumed that this is the same reason that prompted the choice of 45 students that was used in [2],[18] and [35].

For URM and IRM we use a uniform redial probability with $\alpha \in \{0.3, 0.6, 0.9\}$.

3.3.2 Data Filtering

Data was filtered by applying GRRM, URM, IRM and SHM developed in [**2**], [**18**],[**35**],[**36**]to obtain the effective traffic load without the reattempts. The models were programmed with the MATLAB software and were applied to the measured load. Matlab codes for the various models are displayed in Appendix A. Results obtained are displayed in Appendix B.

3.3.3 Data Analysis

With the help of the MATLAB software appropriate graphs and tables were obtained from the results. Graphs for the various models were compared and conclusions drawn. The GoS of the system was obtained for both the measured load and effective load, comparisons were made and conclusions drawn. The required TCH for the calculated effective load were also computed and compared to the TCH already on the network.

3.4 Conclusion

This chapter outlined the various steps taken to gather data for our study. After the data was collected, appropriate software was employed to code the equations used for extracting the excess load from the measured load. The next chapter discusses results obtained.

CHAPTER FOUR

RESULT AND DISCUSSION

4.1 Introduction

This chapter gives the results and discussion of the work done. In all figures, the number of retrials is plotted against the effective load based on a particular redial probability and/or redial. We consider results for the four different models: GRRM, URM, IRM and SHM. All values for the measured and effective loads are in erlangs.

4.2 Real Traffic Measurement

Measured load obtained from a GSM operator in Ghana is displayed in Table 4.1. The real traffic measurements are applied to all models to obtain results. The Measured Load *(C_T)*, the Number of Channels *(TCH)* and the Blocking Probability *(β)* are displayed in the table. The measurement duration was set to 60 minutes.

Table 4.1 Real traffic measurements over 60 minutes

Data Set	Measured Load (C_T)	Number of channels (TCH)	Blocking Probability (β)
1	**123.75**	**25**	**0.80**
2	**75.38**	**41**	**0.470**
3	69.35	41	0.427
4	62.56	44	0.326
5	60.45	44	0.305
6	56.28	41	0.307
7	54.88	27	0.524
8	**45.78**	**44**	**0.133**
9	39.99	25	0.407
10	39.23	25	0.397

For the analysis of each of the models, three values of β:0.80, 0.470 and 0.133were selected: representing high, medium and low blocking. The choice of these values was necessary because we wanted to study the effect of the redials and retrials at every level of blocking. The selected values are highlighted as shown in Table 4.1.After which all the data sets would be used when a comparisonof the models is made.

4.3 Generalized Redial/Retrial Model (GRRM)

For GRRM, we consider the exact redial distribution of the students as discussed in Section 3.2.1.Figure 4.1shows the redial distribution of the students.

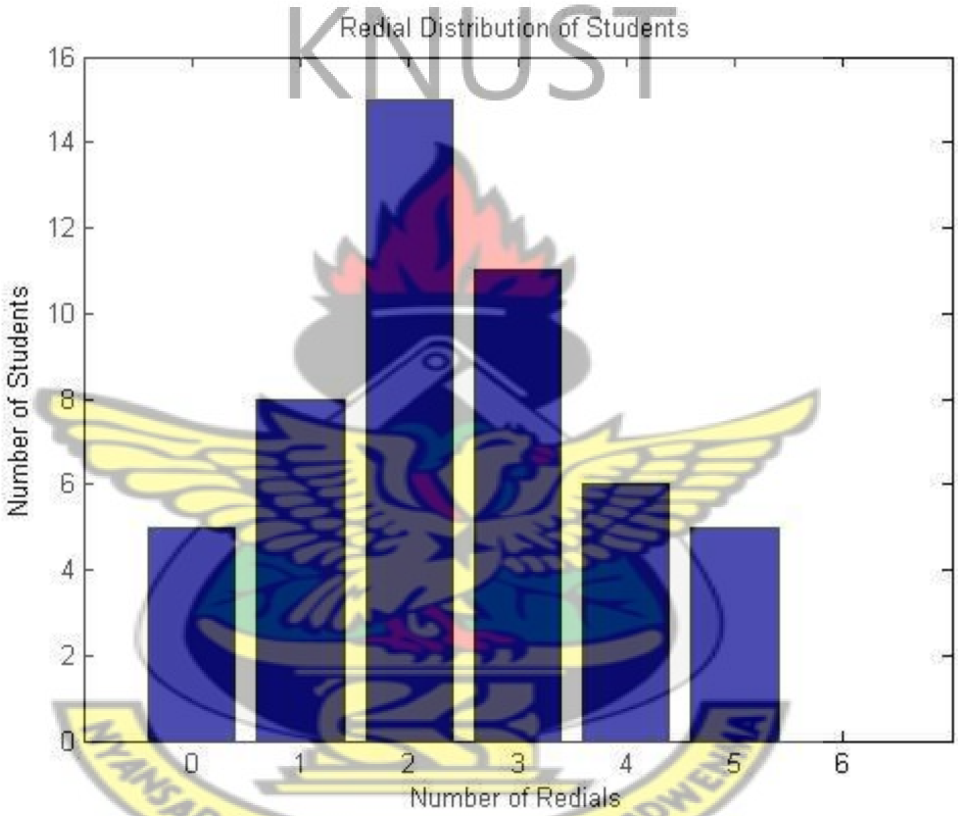

Figure 4.1 Redial Distribution of Students

Based on the conditional probability that the customer has already redialled and failed $i-1$ times redial probabilities as shown in Table 4.2 was obtained for the students.

From Figure 4.1, we observe that 5 students do not redial. This means 45 out of the 50 redial at least once. Out of the 45, 37 redial for the second time. 22 out of the 37 redial the third time and 11 redial four times leaving 5 to redial the fifth time. From this analysis we calculate the conditional probabilities of redialling listed in Table 4.2.

Table 4.2 Exact Redial Probability of students

Redial probability	Value
α_0	0.1
α_1	0.9
α_2	0.8222
α_3	0.5946
α_4	0.5
α_5	0.45

For this model, we consider redials and retrials. From the histogram the mean number of redials was 4. Therefore the maximum redial count, m, was set to 4 resulting in the choice of the first five redial probabilities (α_0, α_1, α_2, α_3, α_4) of the students. This m is used for all the other models. We set n = 6 and analyse the results obtained when we applied this model to the measured load.

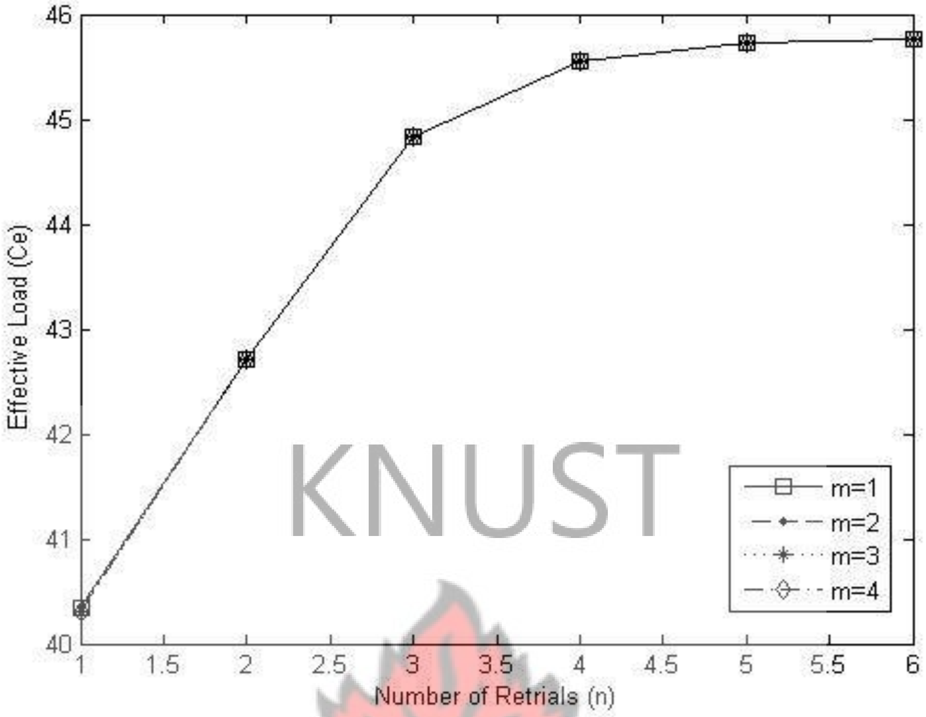

Figure 4.2 Ce against n for β = 0.133 when m = 4

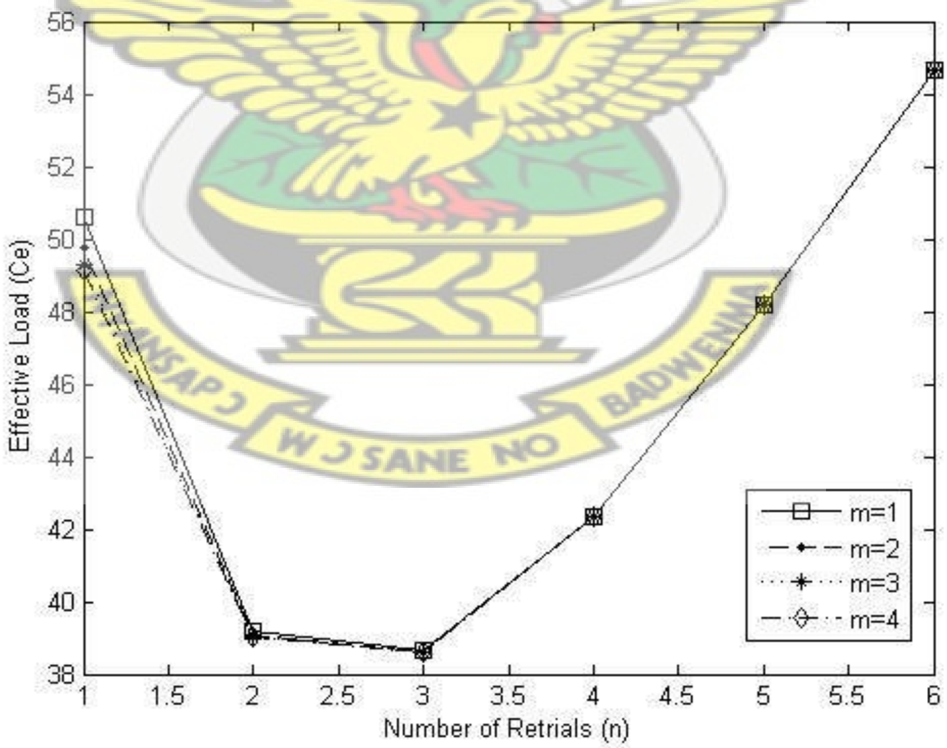

Figure 4.3 Ce against n for β = 0.470 when m = 4

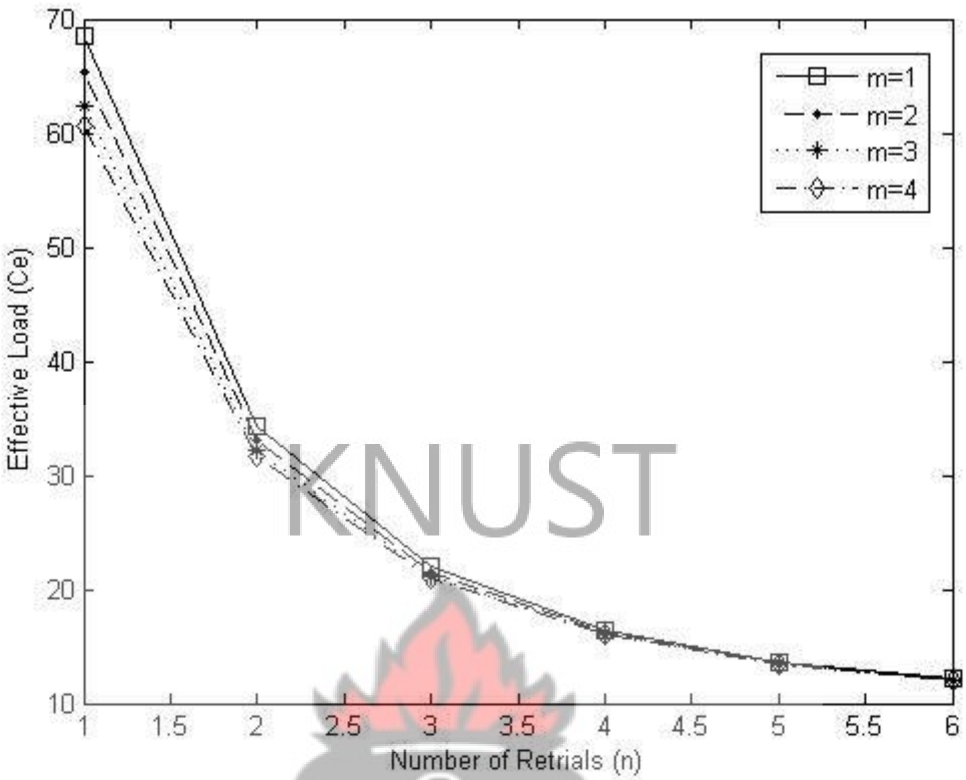

Figure 4.4 Ce against n for β = 0.80 when m = 4

Figure 4.2 shows the effective loads obtained for β = 0.133 when m = 4. The graphs overlap indicating that a change in the redials do not affect the expected number of redials and retrials, $N_R^{GRRM}$and thus the effective load,C_E. Figures 4.3 and 4.4 displays graphs for β = 0.470 and β = 0.80 respectively. In these graphs, an increase in m produces an increase in N_R^{GRRM}which causes a decrease in C_E.

In the case ofthe retrials, n, Figure 4.2 shows an increase inC_E as n increases. This observation is in contrast to what Figure 4.4 displays where an increase in n produces a corresponding decrease inC_E. A condition that is to be desired because from equation (7), we observe that the expected number of redials and retrials for this model,N_R^{GRRM} is inversely proportional to the effective load, C_E and thus an increase in N_R^{GRRM} means a decrease in C_E and vice versa. Figure 4.3 however shows a decreasing sequence for C_E from n = 1 to n = 3 and increases from n =3 to n = 6.

β = 0.80 produces optimistic results for this model than the other values of β. An increase in the redials and retrials produces an increase in the expected number of redials and retrials which eventually decrease the effective load on the network as observed in **[**10**]** and **[**11**]**. GRRM has proven to be a better model to use on highly congested networks.

4.4 Uniform Redial Model (URM)

We consider redials, retrials and redial probability. We consider a maximum redial count of 4, a maximum retrial count of 6 and a uniform redial probability, α of 0.3, 0.6 and 0.9.

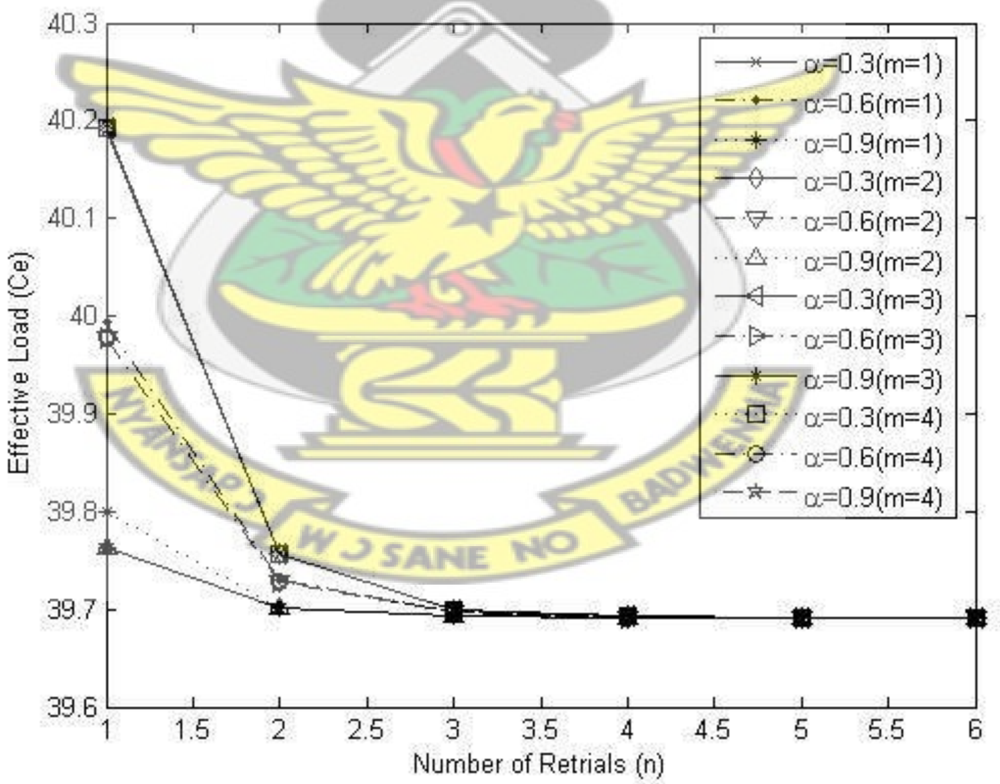

Figure 4.5 Ce against n for β = 0.133 when m = 4 and α=0.3, 0.6 and 0.9

All graphs obtained from this model followed a decreasing sequence. C_E decreases as redials and retrials increase for all values of α as shown in Figures 4.5, 4.6 and 4.7. For all values of β, C_E decreases from m = 1 to m = 4 as α increases except for β = 0.133 at α = 0.3 where the graphs for m = 1 and m = 4 overlap. The reason being that redialling at a probability of 0.3 on a 13.3% blocking on a network do not introduce significant load on the network thus the change in C_E is almost negligible. Redialling at 0.6 and 0.9 however introduce more redials and retrials which eventually increase the expected number of redials and retrials, N_R^{URM} causing a slight decrease in C_E as seen in Figure 4.5.

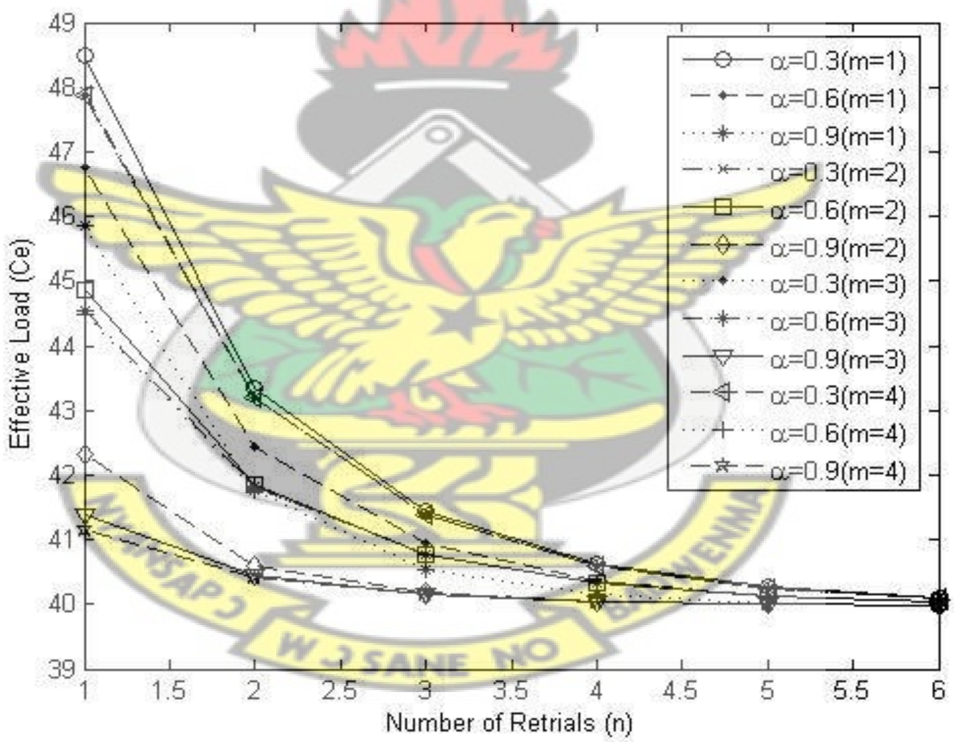

Figure 4.6 Ce against n for β = 0.470 when m = 4 and α=0.3, 0.6 and 0.9

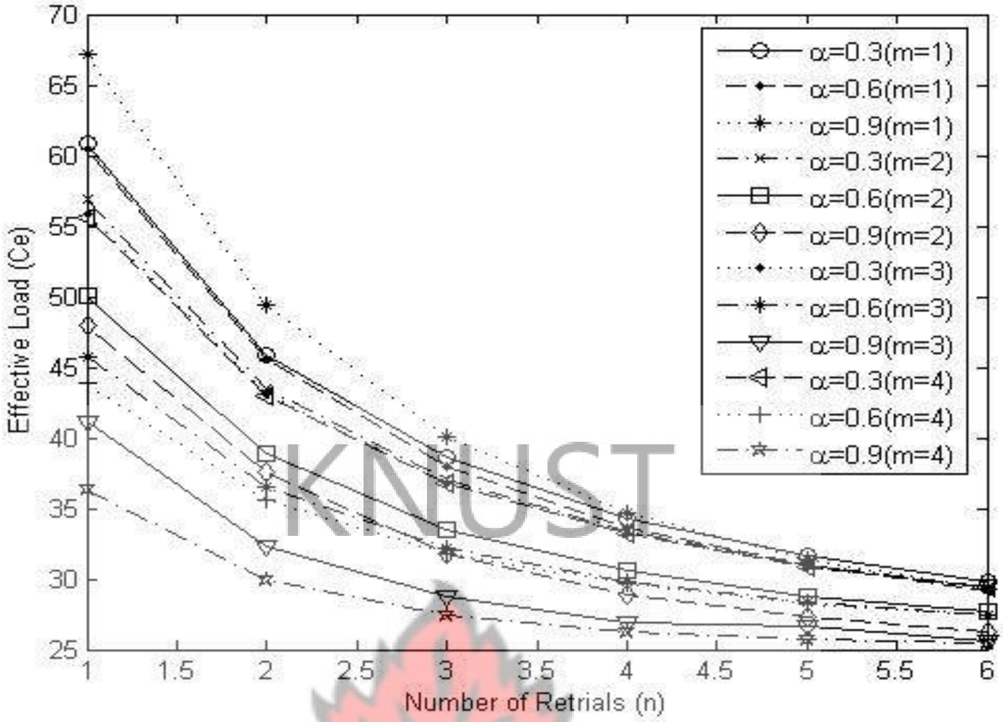

Figure 4.7 Ceagainst n for β = 0.80 when m = 4 and α=0.3, 0.6 and 0.9

Figures 4.6 and 4.7 however exhibit curves slightly different from that of Figure 4.5. It is observed that the curves of Figures 4.6 and 4.7 do not overlap. The curves in Figure 4.6 depicts that the difference in C_E is small compared to that of Figure 4.7. This is because the value of β is higher in Figure 4.7 than that of Figure 4.6.For all the figures, the difference in the effective loads for m=1 and m=4 increases as α increases.

It was observed that the higher α and β the more redials are introduced onto the network as discussed in [**8**] where it was observed that redialling at a higher probability cause blocking on the network.

For the retrials, a study of Figures 4.5, 4.6 and 4.7 reveals that C_E decreases as n increases.However graphs for β = 0.133 converge at n=5 and remains constant. This model gives optimistic results for any value of β and can also be used when the effect of redials, retrials and the redial probability are being studied.

4.5 Infinite Redial Model (IRM)

This model omits the redials as it is assumed that the user redials infinitely. We therefore consider the effect of α and nonC_E.Values of α and n are same as ones used in URM. From Figures 4.8, 4.9 and 4.10 it is shown that C_E decreases as α increases for all values α: an indication that the expected number of redials and retrials for this model, N_R^{IRM} increases as the redials and retrials increase.

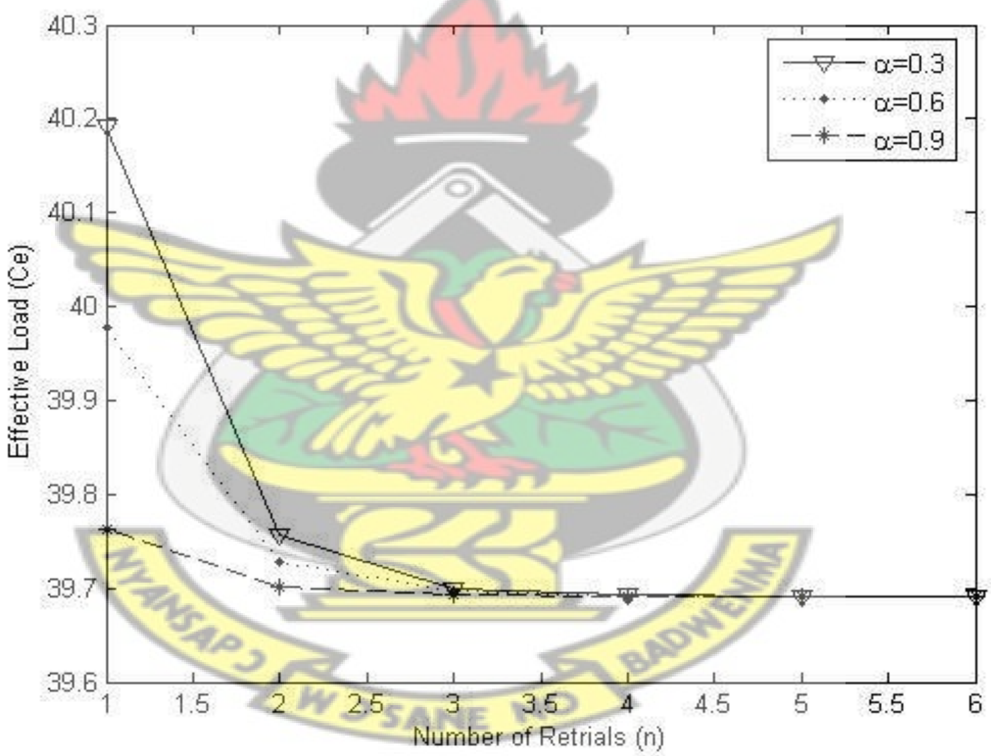

Figure 4.8 Ce against n for β = 0.133

The effective load decreases as the redial probability increases. Figures 4.9 and 4.10 give a better picture of the changes in C_E. It is observed that the higher the redial probability, the higher the deviation of the effective load from the measured load.

This means the redials and retrials form a significant portion of the measured load.IRM gives optimistic values for all values of β even though for β = 0.133 (Figure 4.8) the curves for all values of α converge at n=5 and remains constant.

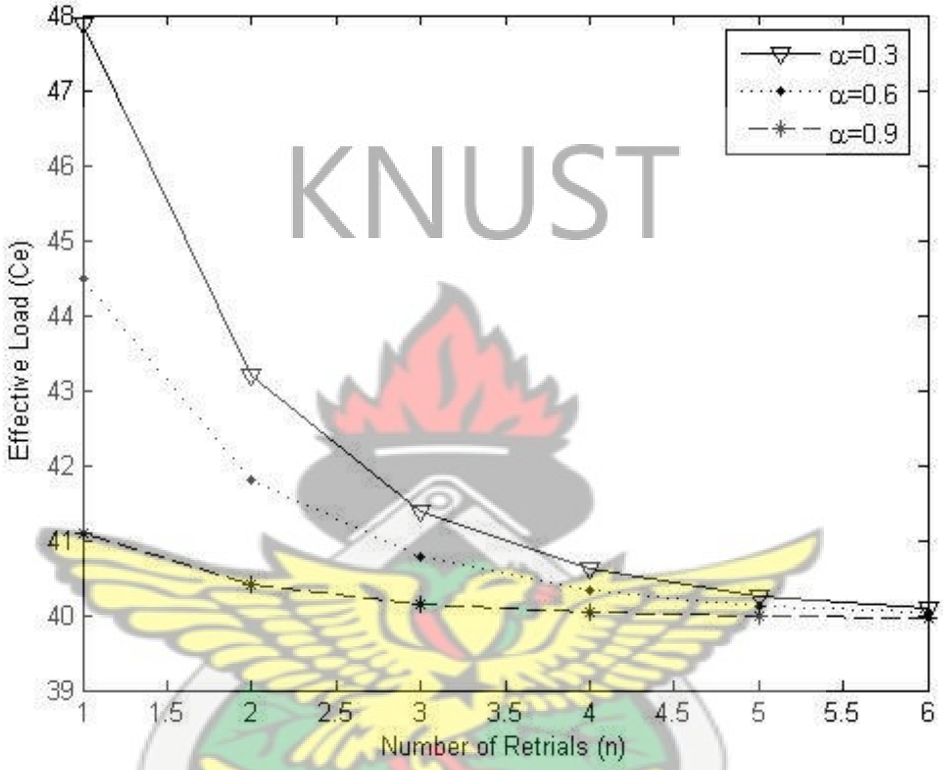

Figure 4.9 Ce against n for β = 0.470

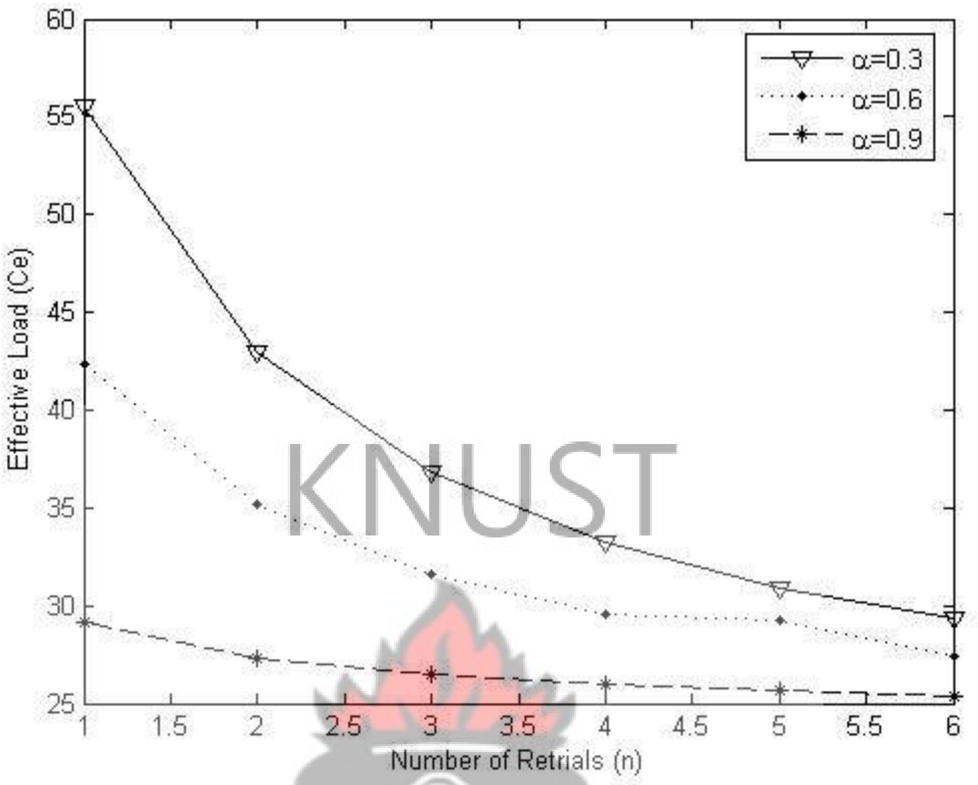

Figure 4.10 Ce against n for β = 0.80

4.6 Separated Hand-off Model(SHM)

As mentioned earlier, this model considers fresh calls with the hand-off calls. The mobile station measures the power level gained from several different cells. It is therefore likely for the mobile station to camp on a cell other than the one to which the hand-off is attempted after the call drops. Moreover, the dropped attempt can result in a fresh call attempt depending upon the user. Thus, it is possible to conclude that the hand-off feedback probability, α_H is not large in practice [**2**]. We therefore set α_H to 0.2 and f to 0.45 and all other parameters as was set for GRRM.

The effect of redials and retrials is shown in Figures 4.11, 4.12 and 4.13.

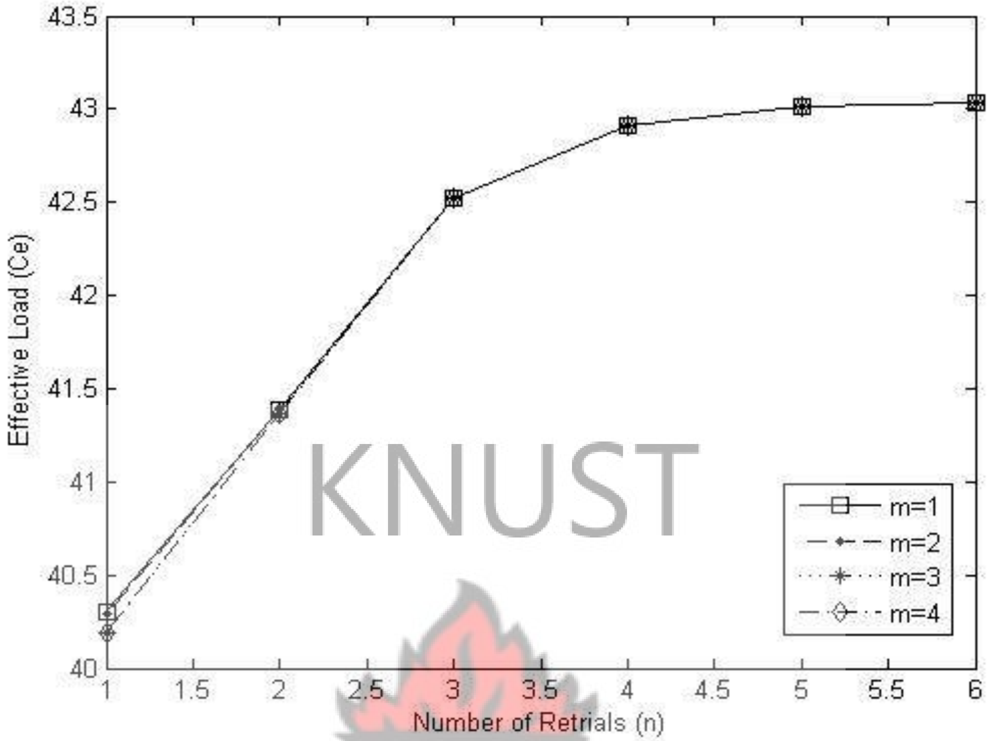

Figure 4.11 Ceagainst n for β = 0.133 when m = 4

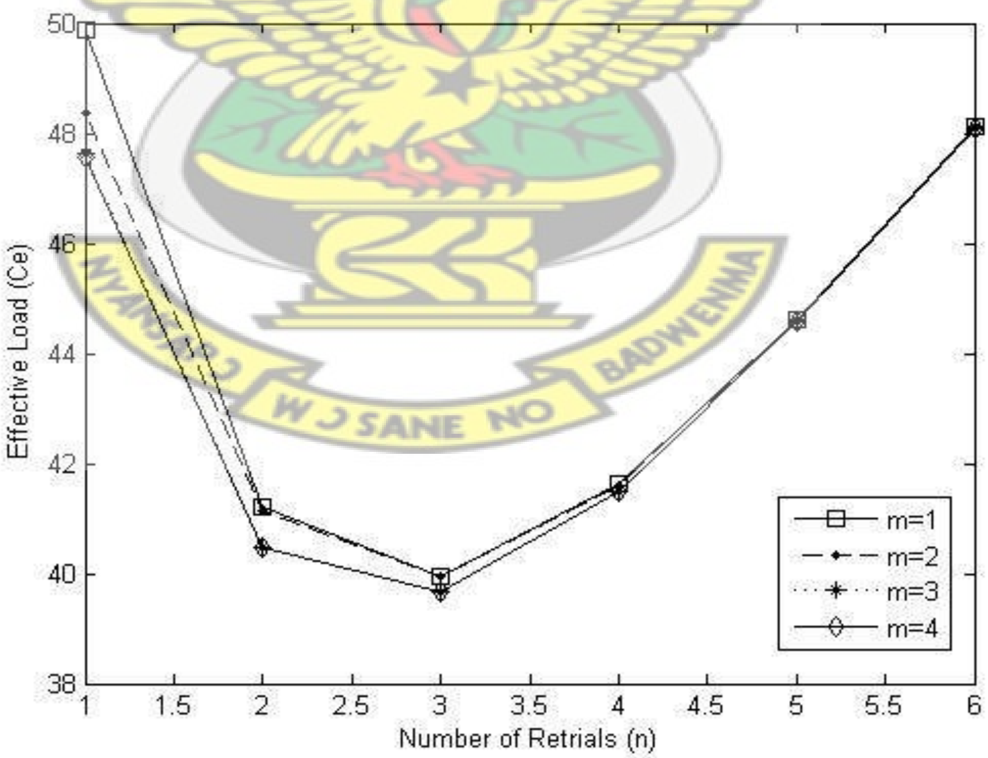

Figure 4.12 Ce against n for β = 0.470 when m = 4

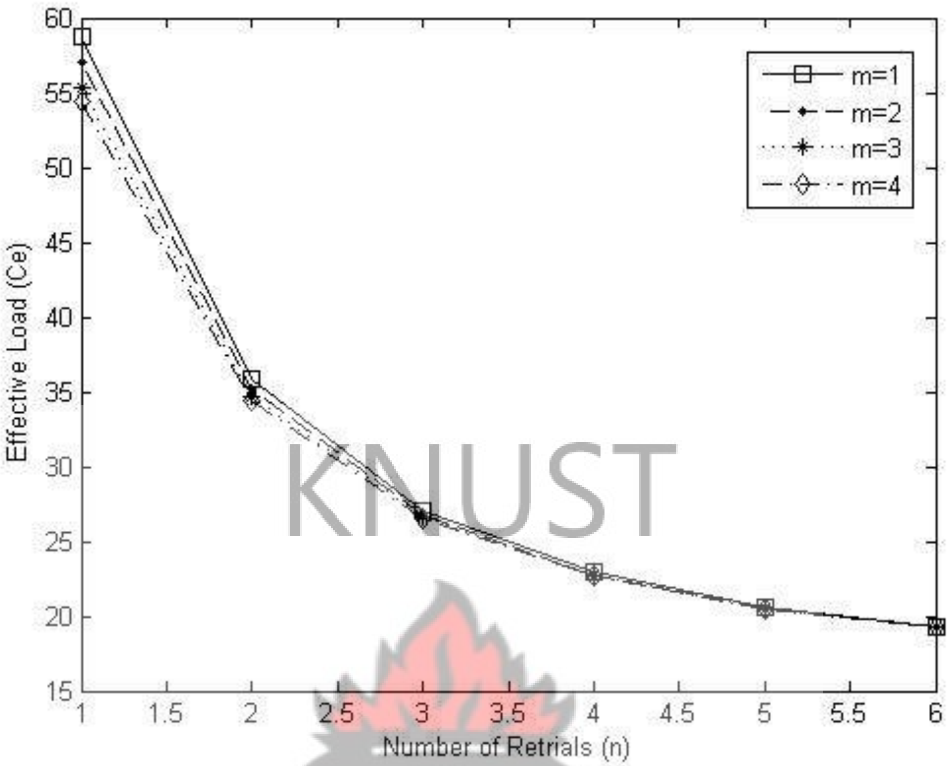

Figure 4.13 Ce against n for β = 0.80 when m = 4

SHM produces similar results like GRRM: optimistic results for higher values of β. The difference in C_E is accounted for by the introduction of the hand-off load. Both GRRM and SHM can be used when the exact redial probability of the callers can be determined.

4.7 Comparison of URM and IRM

Application of URM and IRM on the measured loads produced effective loads and blocking probabilities that were less than the actual measured loads and their corresponding blocking probabilities. However the blocking probabilities did not meet the 2% industry standard.Table 4.3 lists the effective loads and blocking probabilities for bothmodels for n = 5, α = 0.6 and m =2 in the case of URM. It is observed that $C_E$for URM and IRM are in agreement with the exception of C_T =

123.75, 75.38, 62.56 and 54.88 where they vary slightly. The blocking probabilities however remain the same for all values of C_E for both URM and IRM.

Table 4.3 Comparison of C_E and β for URM and IRM when n = 5, α = 0.6 and (m = 2 in the case of URM)

Measured Load (C_T)	Number of Channels (TCH)	Blocking Probability (β)	Effective Load (C_E) For URM	β for URM	Effective Load (C_E) For IRM	β for IRM
123.75	25	0.80	28.8148	0.211	28.2673	0.211
75.38	41	0.470	40.1256	0.103	40.1255	0.103
69.35	41	0.427	39.8345	0.100	39.8345	0.100
62.56	44	0.326	42.1856	0.089	42.1857	0.089
60.45	44	0.305	42.0263	0.087	42.0263	0.087
56.28	41	0.307	39.0151	0.089	39.0151	0.089
54.88	27	0.524	26.3442	0.120	26.3438	0.120
45.78	44	0.133	39.6913	0.061	39.6913	0.061
39.99	25	0.407	23.7574	0.118	23.7574	0.118
39.23	25	0.397	23.6929	0.117	23.6929	0.117

Table 4.4 lists the effective loads and blocking probabilities for URM and IRM respectively for n = 5, α = 0.9 and m =2 in the case of URM. It is observed that results for URM and IRM are in agreement with the exception of C_T = 123.75, 75.38, 69.35 and 54.88 which vary slightly. The effective loads produced by Table 4.4 are lower those produced in Table 4.3. An indication that the higher the redial probability, the more redials and its associated retrials are introduced onto the network and the expected number of redials and retrials increase thereby reducing the effective load.

From the two tables, we observed that blocking probabilities for systems without reattempts were lower than the blocking probabilities for systems with reattempts: an observation made in [**9**]. URM and IRM can be used for all blocking conditions.

Table 4.4 Comparison of C_E and β for URM and IRM when n = 5, α = 0.9 and (m = 2 in the case of URM)

Measured Load (C_T)	Number of Channels (TCH)	Blocking Probability (β)	Effective Load (C_E) For URM	β for URM	Effective Load (C_E) For IRM	β for IRM
123.75	25	0.80	27.2469	0.190	25.6293	0.157
75.38	41	0.470	39.9953	0.102	39.9949	0.102
69.35	41	0.427	39.7619	0.099	39.7618	0.099
62.56	44	0.326	42.1705	0.089	42.1705	0.089
60.45	44	0.305	42.0161	0.087	42.0161	0.087
56.28	41	0.307	39.0053	0.089	39.0053	0.089
54.88	27	0.524	26.1796	0.123	26.1781	0.123
45.78	44	0.133	39.6913	0.061	39.6913	0.061
39.99	25	0.407	23.7249	0.118	23.7249	0.118
39.23	25	0.397	23.6650	0.117	23.6650	0.117

4.8 Comparison of GRRM and SHM

From Sections4.2 and 4.4, we observed that GRRM and SHM work well with high values of β. For a comparison of GRRM and SHM, we select the 1st data set from Table 4.1since it is the one with the highest blocking probability.

Table 4.5 illustrates the effective loads for the two models. A difference of 7.0336 erlangs is observed. This is accounted for by the introduction of the hand-off load. The blocking probability for GRRM meets the industry standard while that for SHM does not even though it is below the system blocking probability.

Table 4.5 Comparison of C_E and β for GRRM and SHM when n = 5 and m = 2

Measured Load (C_T)	Number of Channels (TCH)	Blocking Probability (β)	Effective Load (C_E) For GRRM	β for GRRM	Effective Load (C_E) For SHM	β for SHM
123.75	25	0.80	13.5162	0.02	20.5498	0.059

4.9 Required TCH for 2% (0.02) blocking

From Sections 4.7 and 4.8, we observe that the effective loads obtained after extracting the excess load produced blocking probabilities higher than 2% (0.02), which is the industry standard. It therefore became necessary that we investigate and find out the number of channels that can accommodate the calculated effective loads to obtain the 2% blocking probability.

The required TCH for obtaining 2% blocking for all four models are listed in Tables 4.6 and 4.7.

Table 4.6 Required TCH for 2% (0.02) blocking with C_E from URM and IRM

Measured Load		URM		IRM	
C_T	TCH	C_E	TCH	C_E	TCH
123.75	25	27.2469	36	25.6293	35
75.38	41	39.9953	50	39.9949	50
69.35	41	39.7619	50	39.7618	50
62.56	44	42.1705	53	42.1705	53
60.45	44	42.0161	52	42.0161	52
56.28	41	39.0053	49	39.0053	49
54.88	27	26.1796	35	26.1781	35
45.78	44	39.6913	50	39.6913	50
39.99	25	23.7249	32	23.7249	32
39.23	25	23.6650	32	23.6650	32

The required TCH for URM and IRM are in close agreement while that for GRRM and SHM vary. It was observed that the TCH provided by the network operators are not adequate for the effective load as can be seen from the tables. The network needs to be replanned and channels increased so they can accommodate the effective load.

Table 4.7 Required TCH for 2% blocking with C_E from GRRM and SHM

Measured Load		GRRM		SHM (α_H= 0.2, f =0.45)	
C_T	TCH	C_E	TCH	C_E	TCH
123.75	25	13.5162	21	20.5498	29

4.10 Conclusion

This chapter gave the results obtained after the extraction of the excess load. The results for each of the models were compared to find out which of the models worked best for specific network conditions. We also looked at how the redial probability, number of redials and number of retrials affect the Gos of the network. Based on the expected load obtained the required number of channels was computed to know whether the channels were enough to carry the network traffic. The next chapter gives us the import of the results obtained.

CHAPTER FIVE

CONCLUSION

GSM Network operators often design and plan network based on estimations. Such estimations may not necessarily reflect the actual situation when the network is deployed. There is therefore the need to monitor such networks during operation to know whether they are operating optimally or not. Blocking introduces the retrial phenomenon on the network and this inflates the traffic on the network. This thesis studied the effect of this phenomenon on the GSM network.

When blocking is experienced in the process of making a call, it displeases the caller and it is irritating when it is during a hand-off process because the caller is in the middle of a conversation. Blocking when not monitored could congest a network and even lead to overload conditions thereby causing network breakdown. Calls are blocked when channels are insufficient to carry the traffic on the network. This is why it is necessary to monitor the network after deployment.

One way of monitoring the network is by checking the GoS to know whether it meets the industry standard. The GoS is also a measure of the blocking and can be obtained if the load on the network and the number of channels used on the network are known. Previous researches have proven that measured load is inflated with redials and retrials thus the measured load should be refined to obtain the actual load so that correct GoS figures could be obtained for proper analysis of the network. The effect of redials and retrials could be disastrous and therefore cannot be neglected. Many models have been designed to study the effect of these redials and retrials on cellular networks of which GSM is an example.

Our research has mainly investigated the effect redial and retrial using four analytical models, GRRM, URM, IRM and SHM designed in [**2**],[**18**],[**35**],[**36**] to refine measured load obtained from a GSM operator in Ghana.

The GRRM has proven to be a better model to use when the blocking probability is high while URM and IRM have been capable of handling all blocking conditions. The results have indicated:

- the effective load was less than the measured load. This is an indication that there was some amount of excess traffic on the network. The degree of excess was however dependent on the value of the blocking probability. The percentage of excess load was about 76.9%, 46.8% and 13.3% for high, medium and low blocking respectively.
- the higher the blocking probability, the higher the difference between the measured load and the effective load because more redials and retrials are introduced onto the network.
- redialling at a higher rate increased the redials and retrials thereby increasing the load on the network.
- and, the actual GoS figures obtained after refining the measured load did not reflect the industry standard. We obtained 17.4%, 10.2% and 6.1% as opposed to the 2%.

In view of the above, it is necessary to factor the effects of the redial and retrial phenomenon when designing a GSM network as their effect is very significant .Network replanning and redesigning should be done for networks so GoS figures would reflect industry standard and for customer satisfaction.

Further studies may be conducted in more than one GSM technology based network and the findings compared. Investigations may also be conducted with other parameters of the GSM network and their effects compared. The University through the National Communications Authority (NCA) could collaborate with the telecommunication companies for researches to be conducted to improve operations. This could also help students get access to data and information for research as it is difficult to come by these because of security reasons.

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APPENDICE

APPENDIX A

Matlab Codes

GRRM

```
function tel=ngrrm (m,beta,alpha)
ct=; %(ASSIGN RELEVANT VALUE)
n=1;
while n<=6
    outer=0;
for i=0:m
inner=0;
for j=0:n-1
    first=(i*(n+1)+j)*(beta^((i+1)*n+j))*(1-beta);
    second=((i+1)*n+i)*(beta^((i+1)*n+i))*(1-alpha (i+1)*beta);
    final=first+second;
    inner=inner+final;
end
if i==0
    eps=1;
else
    eps=1;
    for k=1:i
        eps=eps*alpha (k);
    end
end
out=inner*eps;
```

```
outer=outer+out;
end
tel=outer;
tel
ce (n)=ct/(1+tel);
ce
v (n)=n;
n=n+1;
end
```

URM

```
m = ; % (ASSIGN RELEVANT VALUE)
 n = ; % (ASSIGN RELEVANT VALUE)
 beta = ; % (ASSIGN RELEVANT VALUE)
 alpha = ; % (ASSIGN RELEVANT VALUE)
 Ct = ; %(ASSIGN RELEVANT VALUE)
 Nurm_r = 0;
 sum_j = 0;
 inner_sum = 0;
for j = 0:m
   sum_j = 0;
   inner_sum = 0;
   for i = 0:n-1
        sum_j = sum_j + (j*(n+1) + i)*(beta^(j*n+i))*(1 - beta);
   end
   inner_sum = sum_j + ((j+1)*n + j)*(beta^((j+1)*n)*(1 - alpha*beta));
```

```
    Nurm_r = Nurm_r + inner_sum*((alpha*beta)^j);
end
Nurm_r
Ce=Ct/(Nurm_r+1);
Ce
```

IRM

```
beta = ; % ((ASSIGN RELEVANT VALUE)
alpha = ; % (ASSIGN RELEVANT VALUE)
n = ; % (ASSIGN RELEVANT VALUE)
Ct=; % (ASSIGN RELEVANT VALUE)
 Nirm_r = (alpha*beta^(n+1))/(1 - alpha*beta^(n+1))...
       + (beta*(1-beta^n))/((1-beta)*(1-alpha*beta^(n+1)));
Nirm_r
Ce=Ct/(1+Nirm_r)
Ce
```

SHM

For fresh calls code refer to GRRM Code. Below is code for hand-off calls and other values

```
beta = ; % (ASSIGN RELEVANT VALUE)
 alpha = ; % (ASSIGN RELEVANT VALUE)
 n = ; % (ASSIGN RELEVANT VALUE)
 Nshm_h = beta*(1-beta^n)/(1-beta);


Nshm_f = ;% (ASSIGN RELEVANT VALUE)
```

```
beta = ;%(ASSIGN RELEVANT VALUE)

n=; %(ASSIGN RELEVANT VALUE)

Ct=;% (ASSIGN RELEVANT VALUE)

alpha_h=; % (ASSIGN RELEVANT VALUE)

f=0.45; % (ASSIGN RELEVANT VALUE)

Nshm_h = beta*(1-beta^n)/(1-beta);

Nshm_h

Cf = (1-f)*Ct/(Nshm_f+1);

Ch = f*Ct/(Nshm_h+1);

Chf = beta^(n+1)*alpha_h*Ch;

Ce = Cf+Ch-Chf;

Cf

Ch

Chf

Ce
```


APPENDIX B

Results from the various models

GRRM Results for a load of 45.78 for m=1

n	N_R^{GRRM}	C_E
1	0.1349	40.3377
2	0.0720	42.7057
3	0.0212	44.8279
4	0.0050	45.5528
5	0.0010	45.7328
6	1.9749×10^{-4}	45.7710

GRRM Results for a load of 45.78 for m=2

n	N_R^{GRRM}	C_E
1	0.1349	40.3377
2	0.0720	42.7057
3	0.0212	44.8279
4	0.0050	45.5528
5	0.0010	45.7328
6	1.9749×10^{-4}	45.7710

GRRM Results for a load of 75.38 for m=1

n	N_R^{GRRM}	C_E
1	0.4893	50.6145
2	0.9230	39.2002
3	0.9512	38.6329
4	0.7793	42.3649
5	0.5648	48.1726
6	0.3789	54.6665

GRRM Results for a load of 75.38 for m=2

n	N_R^{GRRM}	C_E
1	0.5154	49.7414
2	0.9298	39.0604
3	0.9524	38.6087
4	0.7795	42.3605
5	0.5648	48.1718
6	0.3789	54.6663

GRRM Results for a load of 123.75 for m=1

n	N_R^{GRRM}	C_E
1	0.8046	68.5744
2	2.6001	34.3737
3	4.6377	21.9503
4	6.5225	16.4507
5	8.0679	13.6471
6	9.2086	12.1221

GRRM Results for a load of 123.75 for m=2

n	N_R^{GRRM}	C_E
1	0.8919	65.4091
2	2.7376	33.1098
3	4.7772	21.4203
4	6.6393	16.1991
5	8.1557	13.5162
6	9.2720	12.0494

IRM Results for a load of 45.78 for α=0.3

n	N_R^{IRM}	C_E
1	0.1390	40.1916
2	0.1515	39.7568
3	0.1531	39.7000
4	0.1534	39.6924
5	0.1534	39.6914
6	0.1534	39.6913

IRM Results for a load of 45.78 for α=0.6

n	N_R^{IRM}	C_E
1	0.1452	39.9772
2	0.1523	39.7287
3	0.1533	39.6962
4	0.1534	39.6919
5	0.1534	39.6913
6	0.1534	39.6913

IRM Results for a load of 45.78 for α=0.9

n	N_R^{IRM}	C_E
1	0.1513	39.7627
2	0.1531	39.7006
3	0.1534	39.6925
4	0.1534	39.6914
5	0.1534	39.6913
6	0.1534	39.6913

IRM Results for a load of 75.38 for α=0.3

n	N_R^{IRM}	C_E
1	0.5743	47.8807
2	0.7453	43.1913
3	0.8214	41.3861
4	0.8563	40.6078
5	0.8725	40.2561
6	0.8801	40.0938

IRM Results for a load of 75.38 for α=0.6

n	N_R^{IRM}	C_E
1	0.6946	44.4824
2	0.8028	41.8028
3	0.8489	40.7712
4	0.8692	40.3265
5	0.8786	40.1255
6	0.8830	40.0328

IRM Results for a load of 75.38 for α=0.9

n	N_R^{IRM}	C_E
1	0.8348	41.0842
2	0.8652	40.4142
3	0.8772	40.1564
4	0.8824	40.0452
5	0.8847	39.9949
6	0.8858	39.9717

IRM Results for a load of 123.75 for α=0.3

n	N_R^{IRM}	C_E
1	1.2277	55.5500
2	1.8828	42.9270
3	2.3656	36.7695
4	2.7281	33.1940
5	3.0042	30.9052
6	3.2167	29.3475

IRM Results for a load of 123.75 for α=0.6

n	N_R^{IRM}	C_E
1	1.9221	42.3500
2	2.5219	35.1369
3	2.9139	31.6183
4	3.1843	29.5751
5	3.3779	29.2673
6	3.5202	27.3771

IRM Results for a load of 123.75 for α=0.9

n	N_R^{IRM}	C_E
1	3.2453	29.1500
2	3.5252	27.3467
3	3.6756	26.4671
4	3.7676	25.9563
5	3.8285	25.6293
6	3.8707	25.4068

URM Results for a load of 45.78 for m=1, α=0.3

n	N_R^{URM}	C_E
1	0.1389	40.1957
2	0.1515	39.7569
3	0.1531	39.7000
4	0.1534	39.6924
5	0.1534	39.6914
6	0.1534	39.6913

URM Results for a load of 45.78 for m=1, α=0.6

n	N_R^{URM}	C_E
1	0.1447	39.9935
2	0.1523	39.7291
3	0.1533	39.6962
4	0.1534	39.6919
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 45.78 for m=1, α=0.9

n	N_R^{URM}	C_E
1	0.1503	39.7991
2	0.1531	39.7016
3	0.1534	39.6925
4	0.1534	39.6914
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 45.78 for m=2, α=0.3

n	N_R^{URM}	C_E
1	0.1390	40.1916
2	0.1515	39.7568
3	0.1531	39.7000
4	0.1534	39.6924
5	0.1534	39.6914
6	0.1534	39.6913

URM Results for a load of 45.78 for m=2, α=0.6

n	N_R^{URM}	C_E
1	0.1451	39.9774
2	0.1523	39.7287
3	0.1533	39.6962
4	0.1534	39.6919
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 45.78 for m=2, α=0.9

n	N_R^{URM}	C_E
1	0.1513	39.7636
2	0.1531	39.7006
3	0.1534	39.6925
4	0.1534	39.6914
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 45.78 for m=3, α=0.3

n	N_R^{URM}	C_E
1	0.1390	40.1916
2	0.1515	39.7568
3	0.1531	39.7000
4	0.1534	39.6924
5	0.1534	39.6914
6	0.1534	39.6913

URM Results for a load of 45.78 for m=3, α=0.9

n	N_R^{URM}	C_E
1	0.1513	39.7628
2	0.1531	39.7006
3	0.1534	39.6925
4	0.1534	39.6914
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 45.78 for m=4, α=0.3

n	N_R^{URM}	C_E
1	0.1390	40.1916
2	0.1515	39.7568
3	0.1531	39.7000
4	0.1534	39.6924
5	0.1534	39.6914
6	0.1534	39.6913

URM Results for a load of 45.78 for m=4, α=0.6

n	N_R^{URM}	C_E
1	0.1452	39.9772
2	0.1523	39.7287
3	0.1533	39.6962
4	0.1534	39.6919
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 45.78 for m=4, α=0.9

n	N_R^{URM}	C_E
1	0.1513	39.7627
2	0.1531	39.7006
3	0.1534	39.6925
4	0.1534	39.6914
5	0.1534	39.6913
6	0.1534	39.6913

URM Results for a load of 75.38 for m=1, α=0.3

n	N_R^{URM}	C_E
1	0.5542	48.4995
2	0.7387	43.3538
3	0.8195	41.4291
4	0.8558	40.6191
5	0.8724	40.2590
6	0.8801	40.0945

URM Results for a load of 75.38 for m=1, α=0.6

n	N_R^{URM}	C_E
1	0.6121	46.7519
2	0.7768	42.4239
3	0.8413	40.9392
4	0.8672	40.3710
5	0.8781	40.1370
6	0.8828	40.0351

URM Results for a load of 75.38 for m=1, α=0.9

n	N_R^{URM}	C_E
1	0.6437	45.8607
2	0.8052	41.7562
3	0.8600	40.5260
4	0.8777	40.1441
5	0.8835	40.0207
6	0.8855	39.9783

URM Results for a load of 75.38 for m=2, α=0.3

n	N_R^{URM}	C_E
1	0.5724	47.9389
2	0.7450	43.1986
3	0.8213	41.3870
4	0.8563	40.6080
5	0.8725	40.2561
6	0.8801	40.0938

URM Results for a load of 75.38 for m=2, α=0.6

n	N_R^{URM}	C_E
1	0.6790	44.8654
2	0.8009	41.8578
3	0.8485	40.7783
4	0.8692	40.3274
5	0.8786	40.1256
6	0.8830	40.0328

URM Results for a load of 75.38 for m=2, α=0.9

n	N_R^{URM}	C_E
1	0.7811	42.3230
2	0.8571	40.5894
3	0.8761	40.1797
4	0.8822	40.0481
5	0.8847	39.9953
6	0.8858	39.9718

URM Results for a load of 75.38 for m=3, α=0.3

n	N_R^{URM}	C_E
1	0.5724	47.8857
2	0.7452	43.1916
3	0.8214	41.3861
4	08563	40.6078
5	0.8725	40.2561
6	0.8801	40.0938

URM Results for a load of 75.38 for m=3, α=0.6

n	N_R^{URM}	C_E
1	0.6919	44.5529
2	0.8030	41.8072
3	0.8488	40.7715
4	0.8692	40.3265
5	0.8786	40.1255
6	0.8830	40.0328

URM Results for a load of 75.38 for m=3, α=0.9

n	N_R^{URM}	C_E
1	0.8210	41.3956
2	0.8642	40.4355
3	0.8771	40.1577
4	0.8824	40.0453
5	0.8847	39.9949
6	0.8858	39.9717

URM Results for a load of 75.38 for m=4, α=0.3

n	N_R^{URM}	C_E
1	0.5743	47.8811
2	0.7453	43.1913
3	0.8214	41.3861
4	0.8563	40.6078
5	0.8725	40.2561
6	0.8801	40.0938

URM Results for a load of 75.38 for m=4, α=0.6

n	N_R^{URM}	C_E
1	0.6942	44.4939
2	0.8032	41.8031
3	0.8489	40.7712
4	0.8692	40.3265
5	0.8786	40.1255
6	0.8830	40.0328

URM Results for a load of 75.38 for m=4, α=0.9

n	N_R^{URM}	C_E
1	0.8314	41.1596
2	0.8651	40.4167
3	0.8772	40.1564
4	08824	40.0452
5	0.8847	39.9949
6	0.8858	39.9717

URM Results for a load of 123.75 for m=1, α=0.3

n	N_R^{URM}	C_E
1	1.0350	60.8106
2	1.6968	45.8874
3	2.2090	38.5629
4	2.6051	34.3265
5	2.9114	31.6384
6	3.1486	29.8296

URM Results for a load of 123.75 for m=1, α=0.6

n	N_R^{URM}	C_E
1	1.0488	60.4003
2	1.7177	45.5347
3	2.2547	38.0220
4	2.6146	33.6769
5	2.9974	30.9574
6	3.2428	29.1671

URM Results for a load of 123.75 for m=1, α=0.9

n	N_R^{URM}	C_E
1	0.8415	67.2017
2	1.5027	49.4472
3	2.0890	40.0621
4	2.5702	34.6617
5	2.9474	31.3498
6	3.2341	29.2269

URM Results for a load of 123.75 for m=2, α=0.3

n	N_R^{URM}	C_E
1	1.1766	56.8556
2	1.8434	43.5225
3	2.3389	37.0630
4	2.7112	33.3446
5	2.9940	30.9842
6	3.2107	29.3895

URM Results for a load of 123.75 for m=2, α=0.6

n	N_R^{URM}	C_E
1	1.4735	50.0302
2	2.1879	38.8186
3	2.6925	33.5139
4	3.0461	30.5853
5	3.2947	28.8148
6	3.4713	27.6762

URM Results for a load of 123.75 for m=2, α=0.9

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
4	3.2863	28.8714
5	3.5418	27.2469
6	3.7035	26.3101

URM Results for a load of 123.75 for m=3, α=0.3

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
4	3.2863	28.8714
5	3.5418	27.2469
6	3.7035	26.3101

URM Results for a load of 123.75 for m=3, α=0.6

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
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URM Results for a load of 123.75 for m=3, α=0.9

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
4	3.2863	28.8714
5	3.5418	27.2469
6	3.7035	26.3101

URM Results for a load of 123.75 for m=4, α=0.3

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
4	3.2863	28.8714
5	3.5418	27.2469
6	3.7035	26.3101

URM Results for a load of 123.75 for m=4, α=0.6

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
4	3.2863	28.8714
5	3.5418	27.2469
6	3.7035	26.3101

URM Results for a load of 123.75 for m=4, α=0.9

n	N_R^{URM}	C_E
1	1.4785	47.9298
2	2.2997	37.5035
3	2.8903	31.8097
4	3.2863	28.8714
5	3.5418	27.2469
6	3.7035	26.3101

SHM Results for a load of 45.78 m=1

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.1349	0.1330	22.1861	18.1827	0.0643	40.3045
2	0.0720	0.1507	23.4879	17.9032	0.0084	41.3826
3	0.0212	0.1530	24.6563	17.8667	0.0011	42.5218
4	0.0050	0.1534	25.0537	17.8618	1.4867×10^{-4}	42.9154
5	0.0010	0.1534	25.1538	17.8612	1.9772×10^{-5}	43.0150
6	1.9749×10^{-4}	0.1534	25.1740	17.8611	2.6297×10^{-6}	43.0351

SHM Results for a load of 45.78 m=2

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.1351	0.1330	22.1705	18.1827	0.0648	40.2888
2	0.0720	0.1507	23.4879	17.9032	0.0084	41.3826
3	0.0212	0.1530	24.6563	17.8667	0.0011	42.5218
4	0.0050	0.1534	25.0537	17.8618	1.4867×10^{-4}	42.9154
5	0.0010	0.1534	25.1538	17.8612	1.9772×10^{-5}	43.0150
6	1.9749×10^{-4}	0.1534	25.1740	17.8611	2.6297×10^{-6}	43.0351

SHM Results for a load of 45.78 m=3

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.1358	0.1330	22.1685	18.1827	0.1608	40.1904
2	0.0720	0.1507	23.4879	17.9032	0.0211	41.3100
3	0.0212	0.1530	24.6563	17.8667	0.0028	42.5201
4	0.0050	0.1534	25.0537	17.8618	3.7167×10^{-4}	42.9152
5	0.0010	0.1534	25.1538	17.8612	4.943×10^{-4}	43.0150
6	1.9749×10^{-4}	0.1534	25.1740	17.8611	6.5741×10^{-4}	43.0351

SHM Results for a load of 45.78 m=4

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.1358	0.1330	22.1685	18.1827	0.1608	40.1904
2	0.0720	0.1507	23.4879	17.9032	0.0211	41.3100
3	0.0212	0.1530	24.6563	17.8667	0.0028	42.5201
4	0.0050	0.1534	25.0537	17.8618	3.7167×10^{-4}	42.9152
5	0.0010	0.1534	25.1538	17.8612	4.943×10^{-4}	43.0150
6	1.9749×10^{-4}	0.1534	25.1740	17.8611	6.5741×10^{-4}	43.0351

SHM Results for a load of 75.38 m=1

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.4893	0.4700	23.0755	27.8379	1.0195	49.8939
2	0.9230	0.6909	20.0609	21.5595	0.4166	41.2039
3	0.9512	0.7947	18.9004	21.2479	0.1845	39.9639
4	0.7793	0.8435	18.4001	23.3007	0.0844	41.6165

5	0.5648	0.8665	18.1740	26.4948	0.0392	44.6296
6	0.3789	0.8772	18.0097	30.0667	0.0183	48.1181

SHM Results for a load of 75.38 m=2

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.5754	0.4700	26.3165	23.0755	1.0195	48.3725
2	0.9298	0.6909	21.4836	20.0609	0.4166	41.1279
3	0.9524	0.7947	21.2349	18.9004	0.1845	39.9508
4	0.7795	0.8435	23.2981	18.4001	0.0844	41.6138
5	0.5648	0.8665	26.4948	18.1740	0.0392	44.6296
6	0.3789	0.8772	30.0667	18.0697	0.0183	48.1181

SHM Results for a load of 75.38 m=3

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.5288	0.4700	27.1187	23.0755	2.5487	47.9455
2	0.9314	0.6909	21.4658	20.0609	1.0414	40.4853
3	0.9525	0.7947	21.2338	18.9004	0.4611	39.6731
4	0.7795	0.8435	23.2981	18.4001	0.2110	41.4872
5	0.5648	0.8665	26.4948	18.1740	0.0980	44.5708
6	0.3789	0.8772	30.0667	18.0697	0.0458	48.0906

SHM Results for a load of 75.38 m=4

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.5334	0.4700	27.0373	23.0755	2.5487	47.5641
2	0.9316	0.6909	21.4636	20.0609	1.0414	40.4831
3	0.9525	0.7947	21.2338	18.9004	0.4611	39.6731
4	0.7795	0.8435	23.2981	18.4001	0.2110	41.4872
5	0.5648	0.8665	26.4948	18.1740	0.0980	44.5708
6	0.3789	0.8772	30.0667	18.0697	0.0458	48.0906

SHM Results for a load of 123.75 m=1

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.8046	0.8000	37.7161	30.9375	3.9600	58.7536
2	2.6001	1.4400	18.9057	22.8227	2.3370	35.8858
3	4.6377	1.9520	12.0727	18.8643	1.5454	27.0727
4	6.5225	2.3616	9.0479	16.5658	1.0852	22.8995
5	8.0679	2.6893	7.5059	15.0944	0.7914	20.6218
6	9.2086	2.9574	6.6672	14.0930	0.5911	19.2824

SHM Results for a load of 123.75 m=2

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.8919	0.8000	35.9757	30.9375	3.9600	57.0132

2	2.7376	1.4400	18.2102	22.8227	2.3370	35.1903
3	4.7772	1.9520	11.7812	18.8643	1.5454	26.7821
4	6.6393	2.3616	8.9095	16.5658	1.0857	22.7612
5	8.1557	2.6893	7.4339	15.0944	0.7919	20.5498
6	9.2702	2.9514	6.6272	14.0930	0.5911	19.2424

SHM Results for a load of 123.75 m=3

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	0.9853	0.8000	34.2832	30.9375	9.9000	55.3207
2	2.8531	1.4400	17.6643	22.8227	5.8426	34.6445
3	4.8706	1.9520	11.5938	18.8643	3.8634	26.5947
4	6.7018	2.3616	8.8372	16.5658	2.7141	22.6889
5	8.1933	2.6893	7.4035	15.0944	1.9785	20.5194
6	9.2913	2.9514	6.6136	14.0930	1.4778	19.2289

SHM Results for a load of 123.75 m=4

n	N_F^{SHM}	N_H^{SHM}	C_F	C_H	C_{HF}	C_E
1	1.0402	0.8000	33.3607	30.9375	9.9000	54.3982
2	2.9068	1.4400	17.4215	22.8227	5.8426	34.4017
3	4.9050	1.9520	11.5262	18.8643	3.8634	26.5272
4	6.7201	2.3616	8.8163	16.5658	2.7141	22.6679
5	8.2020	2.6893	7.3965	15.0944	1.9785	20.5124
6	9.2952	2.9514	6.6111	14.0930	1.4778	19.2264