LOCATION OF TWO FIRE STATIONS IN FIVE SUB-METROS OF THE KUMASI METROPOLIS

By

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A Thesis Submitted To the Department of Mathematics,

Kwame Nkrumah University of Science and Technology

In partial fulfilment of the requirement for the degree of

MASTER OF SCIENCE

Industrial Mathematics

Institute of Distance Learning

November, 2012

DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.

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ABSTRACT

A lot of studies and research have been done on the location of Emergency Medical Centers, hospitals, fire stations and other such essential public facilities since their services have become an essential part of man's daily life. The services they provide do come in handy to the beneficiaries when the need arises. Several methods have been used to solve these location problems. This thesis uses a new heuristics for placing absolute p-centres on a network which was developed by Damle and Sule (2002) to locate two fire stations in five sub-metros put together as a network of the Kumasi Metropolis of Ghana. The result indicated that the facilities should be sited at Pakoso and Adiebeba with a coverage distance of 14km. The facilities would serve the residents of these sub-metros effectively if they are sited at the places indicated above.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF MATRICES

LIST OF ABBREVIATIONS

ACRONYMS	MEANING
MND	Maximum Nodal Distance
GMND	Greatest of Maximum Nodal Distance
LMND	Least of Maximum Nodal Distance

DEDICATION

This work is dedicated to my family especially my mum, Ms. Esther Pinaman Preko who have always stood by me, inspired and supported me.

ACKNOWLEDGEMENT

I wish to express my profound gratitude to God Almighty for His infinite care, strength and empowerment for the accomplishment of this task.

I am also grateful to Mr Kwaku Darkwah, my supervisor, for his patience, encouragement and guidance without which the work could not have been done.

My heartfelt thanks also go to Mrs Linda Adwoa Abosi of Akropong, Kumasi and Mr Emmanuel Oppong-Gyebi of the Mathematics Department, Kofi Agyei Senior High, Bampenase for their immense contributions towards the successful completion of this piece. I am very much appreciative of their assistance.

Special mention should be made of Mr Moses Oduro of Kumasi, a former Head of Mathematics Department of Kofi Agyei Senior High, Bampenase, for his fatherly love and show of concern to seeing to the completion of this work. Daddy I am grateful.

Finally, I am highly indebted to all my friends for their concern and support in seeing to it that the work was completed. May the good Lord richly bless you all and replenish every energy and resource lost for my sake.

Amo-Asante Kwadwo

CHAPTER ONE

1.0 INTRODUCTION

Fire has always played a vital role in the lives of humans since creation. The ancient Greeks believed that fire-along with earth, water and air was one of the four essential elements that made up the world. Early Chemists believed that fire was caused by the liberation of a substance, phlogiston, found in every combustible material. But observations made in the late 18^{th} century revealed decidedly less mysterious ingredient: oxygen. It's one of our most important tools, and holds a prominent place in many ancient philosophies and religions.

Fire is a rapid oxidation of a combustible material releasing heat, light and various reaction products such as carbon dioxide and water. If hot enough, the gases may become ionized to produce plasma. Depending on the substances alight, and any impurities outside, the colour of the flame and the fire's intensity might vary. Fire in its most common form can result in conflagration, which has the potential to cause physical damage through burning.

Fires start when a flammable and/or a combustible material with adequate supply of oxygen or another oxidizer is subjected to enough heat and is able to sustain a chain reaction. This is commonly called the fire tetrahedron. Fire cannot exist without all of these elements being in place (though as previously stated, another strong oxidizer can replace oxygen).

The ability to control fire was a major change in the habits of early humans. Making fire to generate heat and light made it possible for people to cook food, increasing the variety and availability of nutrients. The heat produced would also help people stay warm in cold weather, enabling them to live in cooler climates. Fire also kept nocturnal predators at bay. Evidence of cooked food is found from 1.9 million years ago, although fire was probably not used in a controlled fashion until 400,000 years ago. Evidence becomes widespread around 50 to 100 thousand years ago, suggesting regular use from this time; interestingly, resistance to air pollution started to evolve in human populations at a similar point in time. The use of fire became progressively more sophisticated, with it being used to create charcoal and to control wildlife from 'tens of thousands' of years ago.

By the Neolithic Revolution, during the introduction of grain-based agriculture, people all over the world used fire as a tool in landscape management. These fires were typically controlled burns or "cool fires", as opposed to uncontrolled "hot fires" which damage the soil. Hot fires destroy plants and animals, and endanger communities. This is especially a problem in the forests of today where traditional burning is prevented in order to encourage the growth of timber crops. Cool fires are generally conducted in the spring and fall. They clear undergrowth, burning up biomass that could trigger a hot fire should it get too dense. They provide a greater variety of environments, which encourages game and plant diversity. For humans, they make dense, impassable forests traversable.

The first technical application of the fire may have been the extracting and treating of metals. There are numerous modern applications of fire. In its broadest sense, fire is used by nearly every human being on earth in a controlled setting every day. Users of internal combustion vehicles employ fire every time they drive. Thermal power stations provide electricity for a large percentage of humanity.

The use of fire in warfare has a long history. Hunter-gatherer groups around the world have been noted as using grass and forest fires to injure their enemies and destroy their ability to find food, so it can be assumed that fire has been used in warfare for as long as humans have had the knowledge to control it. Fire was the basis of all early thermal weapons. Homer detailed the use of fire by Greek commandos who hid in a wooden horse to burn Troy during the Trojan War. Later the Byzantine fleet used Greek fire to attack ships and men. In the First World War, the first modern flamethrowers were used by infantry, and were successfully mounted on armoured vehicles in the Second World War. In the latter war, incendiary bombs were used by Axis and Allies alike, notably on Rotterdam, London, Hamburg and, notoriously, at Dresden, in the latter two cases firestorms were deliberately caused in which a ring of fire surrounding each city was drawn inward by an updraft caused by a central cluster of fires. The United States Army Air Force also extensively used incendiaries against Japanese targets in the latter months of the war, devastating entire cities constructed primarily of wood and paper houses. In the Second World War, the use of napalm and Molotov cocktails was popularized, though the former did not gain public attention until the Vietnam War. More recently many villages were burned during the Rwandan Genocide.

Although fire has been very useful to the daily activities of man, the ability to control it has been a major challenge to mankind and this has resulted in a lot of damage to both life and property with huge cost of damage as well. The damaging effects of fire could be traced from the days of Adam. In Genesis 19:23-25, it is written that the

Lord destroyed the cities of Sodom and Gomorrah and all its inhabitants with fire for the sins they committed. In 1Kings 18:38, it is written that the Prophet Elijah prayed to God and the Lord sent down fire from the Heavens to burn a sacrifice to show the people of Israel that the Lord alone is God.

One major damaging effect of fire to have occurred in the world is the damage that resulted from the Great Fire of London. This fire started on 2nd September, 1666, devastating 13200 houses, 87 parish churches, the Royal Exchange, the Guild Hall, the original St Paul's Cathedral and many other buildings. The death toll was six people; yet a great many others died through indirect causes.

As a result of the fire, London had to be almost totally reconstructed. Initially this meant temporary buildings, which were makeshift, ill-equipped and disease easily spread. Many people died from this. The cost of the fire was £10m, and at a time when London's annual income was £12000. Many people were financially ruined and debtors' prisons became overcrowded.

1.1 THE FIRE SERVICE

The Macmillan English Dictionary for Advanced Learners International Student (2006 Edition) defines the fire service as an organization that deals with fires and helps people to escape from dangerous situations. The organization operates from offices called the fire station. It also defines the fire station as the building where fire-fighters have their office and keep their vehicles and equipment.

A typical fire station has a room called the watch-room from where any information on any fire outbreaks is directed to and recorded in the occurrence book which is kept for the recording of fire occurrences. It also has a Tender and a Timetable Tender for fire fighting and a Tanker for water storage.

The crew on a fire tender comprises the Driver, the Officer in Charge and four other Officers each with a specific assignment.

1.1.1 HISTORICAL BACKGROUND OF FIRE BRIGADES AND STATIONS

One of the first fire fighting organizations was established in ancient Rome. Augustus, who became emperor in 27 B.C, formed a group called the vigiles. The vigiles patrolled the streets to watch for fires. Scholars know little else about the development of fire fighting organizations in Europe until after the Great Fire of London. In 1666, when this fire occurred, there was no organized fire brigade. Fire fighting at that time was very basic and there was little skill involved. They used leather buckets, axes and water squirls which had very little effect on the fire. Following a public outcry during the aftermath of probably the most famous fire ever, a property developer named Nicholas Barbon introduced the first kind of insurance against fire. Soon after the formation of this insurance company, and in a bid to help reduce the cost and number of claims, he formed his own Fire Brigade. Other similar companies soon followed his lead and this was how property was protected until the early 1800s. Many of these insurance companies were to merge, including those of London, which merged in 1833 to form the London Fire Engine Establishment (LFEE).

A major change in the way fires were fought came into being in the mid 1850s when the first reliable steam powered appliances were adopted by brigades. These appliances replaced the manual engines and allowed a far great quantity of water to be directed onto a fire. These steam powered appliances were only to last slightly longer than 50 years due to the introduction of the internal combustion engine in the early 1900s.

1.1.2 THE GHANA NATIONAL FIRE SERVICE

Ghana like any other country in the world has had its fair share of adverse effects of fire outbreaks prior to independence. Although a few institutions and cites had their own fire fighting outfits, they were not very conversant with fire fighting skills and strategies. It took several hours for them to combat fire outbreaks and this increased the cost of damage resulting from these fires. The fire fighting services of these institutions and cities were heavily influenced by the British from being a British colony and its fire technology was primarily British based.

After independence, the government of the day deemed it fit to bring all the fire fighting outfits under one umbrella. As a result a Technical Aid Assistance Pact was signed with the British government to assist Ghana in bringing these organizations together. G. S. Leader, a British expatriate and Fire Adviser was contracted by the government to conduct feasibility studies on how the various fire service organizations could be amalgamated. Leader presented a report, which gave a road map for establishing the Ghana National Fire Service by Act 219 of 11th December, 1963. The service was placed under the Ministry of Interior. The service was made responsible for the following

a. Fighting and extinguishing of fire

b. Rescue of persons, animals and property from fires or from the threat of effect thereof; and

c. Any other functions which was prescribed or specified by the Minister.

The above act was narrow in its perspective and limited the activities of the service. On the 5th of September 1997, an Act of Parliament, The Ghana National Fire Service Act 537 was passed to re-establish the service and made provision for additional responsibilities which enabled the service to meet the challenges and demands of time such as technological advancement, economic and environmental conditions.

With about 140 offices across the country, the service has over the years been called to deal with various fire outbreaks across the length and breadth of the country, with the sole aim of trying to extinguish the fire and rescuing lives and property from these outbreaks.

However, many people have not been very happy with the way the service has conducted their activities especially the time taken by personnel of the service to get to places of fire outbreaks. Many people believe that if the service could get to places of fire outbreaks on time, a lot of the cost of damage to life and property associated with these fires could be reduced considerably.

On Wednesday 21st October, 2009, the entire building of the Ministry of Foreign Affairs was raised down by a fire outbreak with the estimated cost of damage running into millions of Ghana Cedis. This has compelled the government to relocate the offices of the Ministry for them to be able to continue with their work and this has come with another cost to the taxpayer. Now the government is looking for funds to construct a new complex for the Ministry and this will also come with huge cost to the state. Not too long ago the Electoral Commission and National Commission for Civic Education buildings in Accra caught fire the same day bringing their activities to a halt. This has also come with huge costs of damage as renovations will have to be carried out and new set of office equipment will have to be bought.

In the Kumasi Metropolis, the most recent major fire outbreak was recorded at the Kumasi Central Market, to be specific the French Line, on Thursday May 28^{th} 2009. Many traders lost almost everything including monies, sheds and goods. Some technical people have suggested that the extent of damage to the market demands that the market be reconstructed. This will come with very huge cost to the Metropolitan Assembly.

In all these cases, the service was there to help extinguish the fire and save lives and property. However, it is believed that if they were there early enough or perhaps had offices nearer to these places of the fire outbreaks, the problems associated with these outbreaks could be nipped in the bud before they got out of hand.

1.2 BACKGROUND TO THE STUDY

Kumasi, the second largest city in the country has a population of 1,915,179 according to the 2000 National Population Census. The Metropolis is divided into ten Sub-Metros for effective and efficient management of the Metropolis, namely

Asawase, Manhyia, Suame, Bantama, Kwadaso, Subin, Oforikrom, Nhyiaeso, Asokwa and Tafo.

The Kumasi Metropolis station of the Ghana National Fire Service is under the Ashanti Regional Office of the service. It has three offices and with support from the Regional Office is responsible for attending to all fire outbreaks within the Metropolis. Each of the offices has an area it covers.

The four offices are located at Adum in the Subin Sub-Metro, adjacent the offices of the Kumasi Metropolitan Assembly, Bantama, at the premises of the Komfo Anokye Teaching Hospital (KATH), Manhyia, opposite the Asantehene's Palace and Chirapatre in the Asokwa Sub-Metro, a few metres away from the Kumasi South Government Hospital, Agogo, which is the Regional Office.

The office at Adum serves the Subin, Nhyiaeso, and a part of the Asokwa sub-metros. The KATH office serves the Bantama, Kwadaso and the Suame sub-metros. The Manhyia office is responsible for the Manhyia, Asawase and Tafo sub-metros. The Chirapatre office serves the Oforikrom and remaining part of the Asokwa sub-metros.

The study looks at five of the ten sub-metros namely Asawase, Manhyia, Asokwa, Oforikrom and Nhyiaeso. Four of them have no fire stations of their own. They are served by the existing fire offices in the other sub-metros. Due to the larger areas of coverage and the longer distances from the offices to some of these areas, it results in delays in attending to fire outbreaks.

The Oforikrom and Asokwa sub-metros are considered the industrial hub of the city since they house a large chunk of the industries located in the Kumasi metropolis. The other three are also considered to be residential areas housing a lot of people as well as a number of offices for other corporate institutions. Fire outbreaks in these areas come with their own challenges as the extent of damage to property and lives is huge and takes a toll on the economy of the country. According to the Regional Office, 6881 fire outbreaks were attended to in the Ashanti Region between 1997 and 2008 out of which about 80 percent of this number occurred in the Kumasi Metropolis alone with GH₵19,812,899.80 as the estimated cost of damage. According to the office, in 2008 the Metropolis recorded 331 fire outbreaks with GH₵3,459,455.48 as the estimated cost of damage. Out of this number, about 170 occurred as industrial and residential fires. This is more than 50 percent of the total fire outbreaks that occurred. The accompanying cost of damage is huge. This is largely attributed to the late arrivals of fire tenders to salvage the situation and it is as a result of the long distances between the offices and accident scenes.

Again as per the records of the Regional Office, as at September, 2009, 310 fire outbreaks had been attended to with GH₵1,774,832.00 as the estimated cost of damage. Following the same trend, 167 of them were recorded as industrial and residential fires. That again is more than 50 percent of the total. The trend suggests that industrial and domestic fires outbreaks have been the leading fire outbreaks. Unfortunately the major areas which are prone to these types of fires do not have fire tenders closer enough to respond to these calls on time enough hopefully to help reduce the estimated cost of damage to lives and property.

These suggest that if probably there were fire stations nearer to these places, the number of occurrences of the outbreaks could be reduced to the barest minimum and this could also reduce the cost of damage. Thus huge sums of money could be saved by the Assembly for other developmental projects.

1.3 STATEMENT OF THE PROBLEM

Despite the fact that the Kumasi Metropolis office of the Ghana National Fire Service has made efforts to arrive at scenes of fire outbreaks early enough it is very sad to see the alarming rate at which these outbreaks are causing a lot of damage to both lives and property with huge estimated costs of damage associated with them. This could among other factors be largely attributed to the lack of proximity between places of fire outbreaks and locations of fire stations. It is against this background that I have thought of using a mathematical model to optimally locate sites for the establishment of fire stations to offset delays in attending to fire outbreaks.

1.4 OBJECTIVE OF STUDY

Being motivated by the background to the study, the objectives of the study are

a. To model the location of fire station using the Absolute p-centre model.
b. To find the optimal location of two fire stations.

1.5 METHODOLOGY

Location of facilities such as fire stations can be considered as a centre problem. The P- Centre problem is minimax problem which minimizes the maximum distance between a demand and the nearest facility to the demand point. The problem at hand is to locate 2 centres to site fire stations in five sub-metros put together in the Kumasi Metropolis which could be solved by the absolute P – Centre model. This model will help locate the facilities anywhere on the network (i.e. on the nodes and on the links of the network). A map of the Kumasi Metropolis with the individual sub-metros and their respective towns would be obtained from the Planning Department of the Kumasi Metropolitan Assembly. This was prepared in 2008. The towns or settlements of the five sub-metros would be identified and the ArcGIS software would be used to calculate the distances between the towns to obtain the inter-town distances. A new heuristic for placing absolute *p*-centres on a network (Damle and Sule, 2002) would be used to locate two centres in the five sub metros put together. Search on the internet were used to obtain related literature. The main library of KNUST and the Department of Mathematics library were consulted in the course of the project.

1.6 JUSTIFICATON OF THE STUDY

From the data above it can be seen that the fire outbreaks are resulting in major damages with very huge costs associated with them. Victims of these incidents either lose their lives which is a loss of human resource to the country and also loss of bread-winners to some families, or valuable property. In other instances victims lose their capital for their businesses and this increase the problem of unemployment whiles others become liabilities to the government. These costs of damage are normally borne by the Assembly which is resourced from the limited funds of the Central Government. Such monies could have been invested in other sectors where they are most needed. For example, the government could use these funds in providing conducive classrooms in the rural communities where pupils are found studying under sheds and trees.

Besides, portable water could also be provided in the same rural communities with these funds to reduce the incidence of water borne diseases. From what has been said above it could be seen that if measures were put in place to reduce the incidence of fire outbreaks or salvage them as quickly as possible when they do occur huge sums of money could be saved to be injected to the sectors of the economy where they are most needed.

It is against this background that I find the study very necessary, to bring to the fore the need to have as many fire stations as possible and to site them at vantage places to make combating fire outbreaks easier and to be able to save lives and valuable property.

1.7 ORGANIZATION OF THE THESIS

Chapter one looks at the good and bad uses of fires to mankind, the historical background of fire brigades and how they managed to fight fire outbreaks. It also looks at the establishment of the Ghana National Fire Service and their activities. It also briefly discusses the objectives of the study and the methodology used. Chapter two contains the literature review. Chapter three reviews the various methods for solving the P-Centre problem as well as the method that would be used to solve the problem at hand with examples. Chapter four contains the data collection, analysis and discussion. The last chapter covers the conclusion and the recommendations.

CHAPTER TWO

REVIEW OF LITERATURE

2.0 INTRODUCTION

Facility location problems have been extensively investigated by different researchers and practitioners. Various location models have been proposed to formulate different facility location problems for emergency services. These location models can be classified into covering models, P-median models and P-center models.

The P-center problem finds many applications in every-day life. Locating fire stations, police stations or emergency facilities in a community, determining the optimum location for a warehouse or a retail outlet are all examples of the P-center problem. In all the above examples the system can be viewed as a network consisting of the nodes – which are the points at which demand occurs – and the links connecting the nodes – which are the roads connecting the demand points. The P-center model is a minimax problem because it minimizes the coverage distance with a given number of facilities, while maximizing the coverage of all demand nodes. The objective of this model is to find locations of P facilities so that all demands are covered and the maximum distance between a demand node and the nearest facility (coverage distance) is minimized. This is an improvement over the set covering model which seeks to minimize the number of facilities with a given coverage distance since sometimes the number of facilities needed to cover all demand nodes with a predefined coverage distance may be quite large and thereby making it expensive to implement.

There are two kinds of center problems. Problems in which the facilities can be located anywhere on the network (i.e., on the nodes and on the links of the network) are referred to as Absolute P-center problems whereas problems in which facilities can be located only on the nodes of the network are called Vertex P-center problems. In many location problems, the cost of a service from the customers' point of view is related to the distance between their places of abode and the location of the facilities. The service is deemed adequate if the customer is within reach of a given distance of the facility and is deemed inadequate otherwise.

This chapter reviews the literature in the application of the Center Model, Median Model and the Covering Model and also highlights on the works of some of its contributors that led to this study.

2.1 COVERING MODELS

Covering models are the most widespread location models for formulating the emergency facility location problem. The objective of the covering models is to provide "coverage" to the demand points. A demand point is considered as covered only if a facility is available to service the demand point within a distance limit.
Toregas, et al. (1971) first proposed the location set covering problem (LSCP), aiming to locate the least number of facilities to cover all demand points. The objective is to make sure that all the demand points are covered in the LSCP. This makes the resources required for locating the facilities excessive in most cases.

Recognising this problem, Church and ReVelle (1974) and White and Case (1974) developed another model known as the Maximal Covering Location Problem (MCLP) model. This model does not require full coverage to all demand points. Instead, the model seeks the maximal coverage with a given number of facilities. Benedict (1983) and Hogan and Revelle (1986) are among the many researchers who have extensively used the MCLP and different variants of it to solve various emergency services location problems. Eaton et al., (1985) did an extensive work on the MCLP which has been used to locate medical rescue vehicles in Austin, Texas.

Research on emergency service covering models has also been extended to incorporate the stochastic and probabilistic characteristics of emergency situations so as to capture the complexity and uncertainty of these problems. There are situations where the provision of a service requires more than one "covering" facility since it happens at times when the facility may not always be available. For instance, assume that fire stations are located at points in order to serve demand across an urban area, and the nearest fire station to a particular demand point is busy, then the nearest available fire station will need to be assigned to a call from this particular demand point when it is received. Chapman and White (1974) first used the chance constrained models to deal with such situations.

Daskin (1983) also used an estimated parameter (d) to represent the probability that at least one server is free to serve the requests from any demand point. He formulated the Maximum Expected Covering Location Problem (MEXCLP) to place P facilities on a network with the goal to maximize the expected value of population coverage. He proposed the use of the MEXCL model for locating emergency response vehicles based on the idea that not all vehicles allocated to serve a particular zone in the network would actually be available during times of emergency. ReVelle and Hogan (1986) later enhanced the MEXCLP and proposed the Probabilistic Location Set Covering Problem (PLSCP). In the PLSCP, a server busy fraction and a service reliability factor are defined for the demand points. Then the locations of the facilities are determined such that the probability of service being available within a specified distance is maximized. Revelle and Hogan (1989a), Goldberg, et al., (1990) and other researchers later modified the MEXCLP and PLSCP to solve other emergency medical service location problems.

2.2 P-MEDIAN MODELS

Another important way of measuring the effectiveness of facility location is by find the average (total) distance between the demand points and the location of the facilities. The accessibility and effectiveness of the facilities is enhanced or increased when the average (total) distance increases.

The P-median model is formulated to minimize the average (total) distance between the demands and the selected facilities. Since its formulation, the P-median model has been enhanced and applied to a wide range of emergency facility location problems. Carbone (1974) formulated a deterministic P-median model with the objective of minimizing the distance travelled by a number of users to fixed public facilities such as medical or day-care centres. The author recognised that the number of users at each demand is uncertain and so extended the deterministic P-median model to a chance constrained model. The model seeks to maximize a threshold and meanwhile ensure the probability that the total travel distance s below the threshold is smaller than a specified level α.

According to Francis et al., (1992), the P-median problem is to locate P new facilities, called medians, on a network in order to minimize the sum of the weighted distance from each node to its nearest facility. Paluzzi (2004) tested a P-median based on heuristic location model for placing emergency service facilities for the city of Carbondale, Illinois. The aim of this model is to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites of the fire station.

A major application of the P-median models is to dispatch emergency medical service units such as ambulances during emergencies. For example, Carson and Batta (1990) proposed a P-median model to find the dynamic ambulance positioning strategy for campus emergency service. Mirchandani (1980) examined a P-median problem to locate fire-fighting emergency units with consideration of stochastic travel characteristics and demand patterns. Serra and Marianov (1999) implemented a P-median model and introduced the concept of regret and minimax objectives. The authors explicitly addressed in their model the issue of locating facilities when there are uncertainties in demand, travel time or distance.

There have been several variants and extensions of the P-median problem. Pesamosca (1991) considered the interaction weights between the new facilities as well as the connection scheme as a tree network. This case was treated as a problem

EMFLP on a tree and its optimality conditions were then obtained using the optimality conditions of *P* problems of the type ESFL

2.3 P-CENTER MODELS

The P-center model seeks to minimize the worst performance of the system and this is minimax model. The P-center model considers a demand point is served by its nearest facility and therefore full coverage to all demand points is always achieved. The last several decades have seen the P-center model and its extensions being investigated and applied in the context of locating facilities such as emergency medical service centers, fire stations, hospitals and other public facilities.

According to Hakimi (1964) the *P*-center problem addresses the problem of minimizing the maximum distance of a demand point from its closest facility given that we are siting a pre-determined number of facilities. Garfinkel et al. (1977) carefully studied the fundamental properties underlying the P-center problem to locate a given number of emergency facilities along a network. In using integer programming, he modelled the *P*-center problem and it was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

ReVelle and Hogan (1989b) formulated a *P*-center problem to locate facilities so as to minimize the maximum distance within which EMS service is available with α reliability. The authors considered the possibility of the system being congested and so used a server busy probability to constrain the service reliability level that must be satisfied for all demands.

There have been some EMS location problems that have been solved with the use of stochastic P-center models. Hochbaum and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods. The cost and distance between the locations vary in each discrete time period. The authors used k underlying networks to represent different periods and provided a polynomial time 3-approximation algorithm to obtain the solution for each problem. There is one network for each period of time. It is required to locate permanent facilities.

Perez-Brito et al. (1998) formulated a problem consisting of finding the P points that minimize a convex combination of the P-center and P-median objective function. For example, locating a local branch bank may require the minimization of the average distance travelled by all prospective customers without being located too far away from any customer. Tamir (2001) considered a problem that generalizes and unifies the P-center and P-median problems. The objective of unifying model is to minimize the sum of the k largest service distances. Burkard and Dollani (2003) was the first to introduce the P-center problem with positive and negative weights. The model is a P-center problem in which nodes can have positive or negative weights, which indicate friendly and obnoxious facilities.

The Anti P-center problem was introduced for the first time by Klein and Kincaid (1994). Instead of minimizing the maximum weighted distance between a demand node and its nearest facility, the Anti P-center seeks to maximize the minimum weighted distance between demand nodes and the nearest facility. It is used to locate P obnoxious facilities. In such a model all weights are negative. Tansel et al. (1983)

considered the Continuous P-center problem. In this model each point in the network is a demand point, as opposed only to vertices. Talmar (2002) utilized a P-center model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands from accidents of tourist activities such as skiing, hiking and climbing at Alpine mountain ranges. One of the aims of the model is to minimize the maximum (worst) response times and the author used effective heuristics to solve the problem.

There are several variations of the basic P-center model. The "Vertex" P-center problem restricts the set of candidate facility sites to the nodes of the network while the "Absolute" P-center problem permits the facilities to be anywhere along the arcs. Both versions can be either weighted or unweighted. In the unweighted problem, all demand nodes are treated equally. In the weighted model, the distances between demand nodes and facilities are multiplied by a weight associated with the demand node. This weight could represent a nodes importance or, more commonly, the level of its demand.

Garey and Johnson (1979) indicated that for fixed values of P, the vertex P-center problem can be solved in $O\ (N^P)$ time since we can enumerate each possible set of candidate locations in this amount of time. For moderate values of N and P, such enumeration is not realistic and more sophisticated approaches are required.

According to Handler and Mirchandani (1979) and Handler (1990) if integer-valued distances can be assumed, the unweighted Vertex or Absolute P-center problem is most often solved using a binary search over a range of coverage distances. For each coverage distance, a set covering problem is solved. When the solution to the set covering problem equals P, the minimum associated coverage distance s the solution to the P-center problem.

Daskin (2000) has recently shown how the maximal covering model can be used effectively in the place of the set covering model as a sub-problem in solving the unweighted Vertex P-center problem. Hakimi (1964) defined and solved the absolute 1-center problem by examining the piecewise linear objective function on each edge and finding the edge-restricted minimum at one of the breakpoints. The smallest among the edge-restricted minima is the absolute 1-center of the network. Hakimi et al. (1978) further reduced the computational effort in Hakimi's algorithm.Hakimi (1965) defined the absolute p-center problem and developed a solution procedure based on solving a sequence of set covering problems. Christofides and Viola (1971) also employed the idea of using set covering problem in their algorithm.

Minieka (1970), for the unweighted case, and Kariv and Hakimi (1979), for the weighted case, showed that the optimal solution of the problem is restricted to a finte set of points on the network. Hooker et al. (1991) provided later a unified framework for establishing finite dominating sets for rather general classes of network location problems. Hooker et al.'s results nclude as special cases the dominating set properties of Minieka (1970), and of Kariv and Hakimi (1979).

Since the first appearance of this problem, researchers have studied many different versions of the problem, such as the "conditional" 1-center (Minieka. 1980), 2-center

(Handler, 1978), unweighted *p*-center (Handler, 1973; Hedetniemi et al., 1981; Minieka, 1981), vertex-restrcted *p*-center (Toregas et al., 1971; Hooker. 1989), p-center with continuous demand points (Chandrasekaran and Tamir, 1980; Chandrasekaran and Daughety, 1981; Megiddo et al., 1981 Tamir, 1985) and *p*-center problems in which the weighted distances are replaced by non-linear functions of distances (Tansel et al., 1982; Hooker, 1986, 1989)

CHAPTER THREE

METHODOLOGY OF THE STUDY

3.0 INTRODUCTION

This chapter looks at the methodology and components used in solving P-center problems in order to achieve the objectives under study. These components include weighted and unweighted P-center problem on trees and general graphs.

3.1 REVIEW OF METHODS

3.1.1 GRAPHS

A graph is a set of objects known as nodes or vertices, connected by links known as edges which can be undirected or directed. Two edges are said to be neighbours if they have exactly the same vertex. A connected graph is a non-empty graph G with paths from all nodes to all other nodes in the graph. The order of a graph G is determined by the number of nodes. Graphs are finite or infinite according to their order. A graph having a weight or a number associated with each vertex is called a weighted graph denoted by $G = (V; E; W)$. A graph $G(V,E)$ consists of a set V of n elements called vertices, and a set E of edges. Let $v \in V$ be a vertex of G and $e \in E$ be an edge of G. A ~~network is a~~ connected undirected graph $G(V,E)$ with a nonnegative number $w(v)$ (called the weight of v) associated with each of its $|V| = n$ vertices, and a positive number $l(e)$ (called the length of e) associated with each of its

$|E|$ edges. Let $X_p = \{x_1, x_2, \ldots, x_p\}$ be a set p of points on G where by *a point on G,* we mean a point along any edge of G which may not be a vertex of G. We define *the distance $d(v, X_p)$* between a vertex v of G and a set X_p on G by

$$d(v, X_p) = \min_{1 \le i \le p} \{d(v, x_i)\}$$

where $d(v, x_i)$ is the length of a shortest path in G between vertex v and point x_i

3.1.2 NETWORKS

A network is a physical implementation of a graph and it is conveniently modeled as a graph G which consists of a set of vertices and a set of edges which are regarded as pairs of distinct vertices. For example a network of roads has the vertices as the towns and the edges as the road links.

3.1.3 THE NOTION OF COVERAGE

Proximity (distance or travel time) is one of the fundamental aspects of location analysis. The provision of a service by a facility is considered adequate or acceptable if the facility is located within a pre-determined distance or travel time. Thus a customer is considered covered or just *covered,* if he/she has a facility sited within the preset distance or travel time.

A coverage distance is defined as a preset distance in which a demand is covered if a certain distance referred to as the edge distance (d_{ij}) is less than or equal to the preset distance. The edge distance is the shortest direct distance between a demand node and a facility node. A node could also be referred to as a vertex.

3.2 THE LOCATION SET COVERING MODEL

The first location covering problem was the set covering problem (Toregas et al. 1971). The objective is to locate the minimum number of facilities required to "cover" all of the demand nodes. In other words each and every demand node has at least one facility located within some standard distance or travel time serving its need adequately. The model locates the minimum possible number of facilities in such a way that each demand node has at least one of the facilities located within the standard travel time or distance. The first application of this model was in the area of emergency services (ReVelle et al. 1976).

To formulate the location set covering problem, the following inputs are defined as follows:

Let Dc = Coverage distance

d_{ij} = distance between demand node i and candidate site j

$$a_{ij} = \begin{cases} 1, \text{ if candidate } j \text{ can cover demand at node } i \text{ (ie } d_{ij} \leq Dc) \\ 0, \text{ if not (ie } d_{ij} > Dc) \end{cases}$$

The decision variables are

$$X_{ij} = \begin{cases} 1, \text{ if a facility is located at candidate site } j \\ 0, \text{ if not} \end{cases}$$

With this notation, the Location Set Covering Problem can be formulated as follows

$$\text{Minimize} \quad \sum_j X_j \qquad (3.1)$$

$$\text{subject to} \quad \sum_j a_{ij} X_j \geq 1 \quad \forall i \qquad (3.2)$$

$$X_j = 0, 1 \quad \forall j \qquad (3.3)$$

The objective function (3.1) minimizes the number of facilities located or required (assuming that the costs of siting a facility at any node is the same). Constraint (3.2) ensures that each demand node i is covered by at least one of the facilities located. Constraint (3.3) enforces the yes or no nature of the siting of decision or it is referred to as the integrality constraints. The objective function can be generalized by including site-specified costs as coefficients of the decision variables. In this case the objective function would be to minimize the total cost of the facility that selected rather than the number of facilities located.

If we let

f_j = cost of locating a facility at candidate site j

then the objective function would be

$$\text{Minimze} \quad \sum_j f_j X_j \tag{3.4}$$

Example

To illustrate the formulation of the set covering problem, we consider network shown in figure 3.1 below.

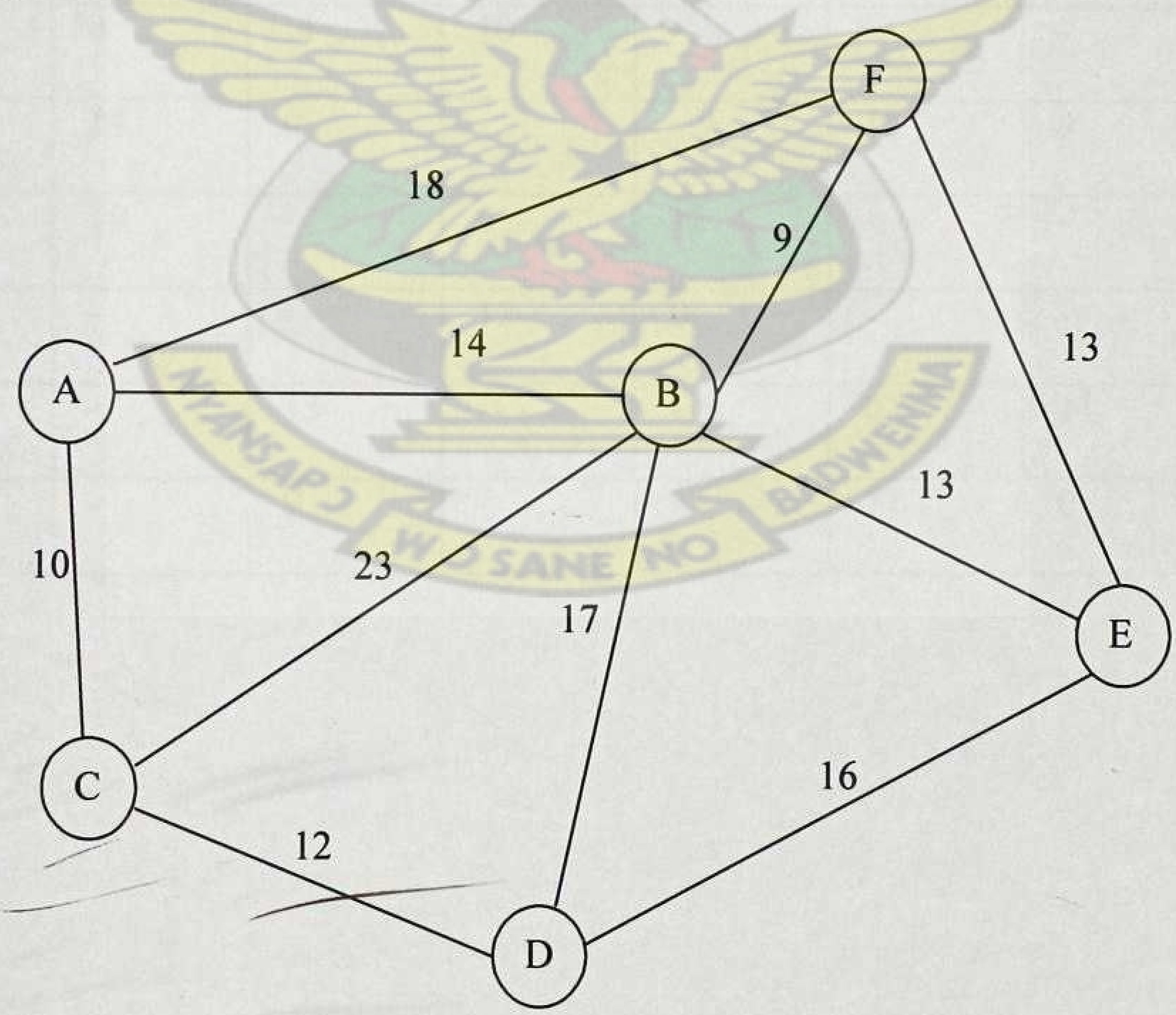

Figure 3.1: Example of a Network

Below is the edge matrix of figure 3.1

$$d_{ij} = \begin{bmatrix} 0 & 14 & 10 & - & - & 18 \\ 14 & 0 & 23 & 17 & 13 & 9 \\ 10 & 23 & 0 & 12 & - & - \\ - & 17 & 12 & 0 & 16 & - \\ - & 13 & - & 16 & 0 & 11 \\ 18 & 9 & - & - & 11 & 0 \end{bmatrix}$$

Matrix 3.1: Matrix of d_{ij} values

A coverage distance (Dc) of 15 units is preset to be used the network above.

The table below shows the elimination of edge distances that are greater than the coverage distance (ie $d_{ij} > Dc$).

Table 3.1 Table of $d_{ij} \leq Dc$ values

	A	B	C	D	E	F
A	0	I4	10	-	-	-
B	14	0	-	-	13	9
C	10	-	0	12	-	-
D	-	-	12	0	-	-
E	-	13	-	-	0	11
F	-	9	-	-	11	0

From Table 3.1 the problem is formulated as

Minimize

$$X_A + X_B + X_C + X_D + X_E + X_F$$

Subject to

$$\begin{array}{llllllll}
\text{(Node } A \text{ covered)} & X_A + & X_B + & X_C & & & & \geq 1 \\
\text{(Node } B \text{ covered)} & X_A + & X_B + & & & X_E + & X_F & \geq 1 \\
\text{(Node } C \text{ covered)} & X_A + & & X_C + & X_D & & & \geq 1 \\
\text{(Node } D \text{ covered)} & & & X_C + & X_D & & & \geq 1 \\
\text{(Node } E \text{ covered)} & & X_B + & & & X_E + & X_F & \geq 1 \\
\text{(Node } F \text{ covered)} & & X_B + & & & X_E + & X_F & \geq 1 \\
\text{(Integrality)} & X_A, & X_B, & X_C, & X_D, & X_E, & X_F & = 0, 1
\end{array}$$

The reduction technique proposed by Toregas and ReVelle (1973) is used in solving this problem. The approach is to reduce the size problem by successive row and column reduction. It states that

- For the column j and k, if $a_{ij} \leq a_{ik}$ for all demand nodes i and $a_{ij} \leq a_{ik}$ for at least one demand node i, then location k covers all demands covered by location j. Location k is said to dominate j and hence j is eliminated.
- For the row reduction, if $\sum a_{ij} = 1$ then, there is only one facility site that can cover node i. In such case, we find location j such that $a_{ij} = 1$ and set $x_j = 1$. Rows containing x_j are then eliminated.

From the problem above, column B dominates columns E and F and so columns E and F are eliminated. Column C dominates column D and therefore column D is also deleted. After the column reduction above, the problem becomes

Minimize

$$X_A + X_B + X_C$$

Subject to

$$\begin{array}{ll}
\text{(Node } A \text{ covered)} & X_A + X_B + X_C \geq 1 \\
\text{(Node } B \text{ covered)} & X_A + X_B \qquad\ \geq 1 \\
\text{(Node } C \text{ covered)} & X_A + \qquad\ X_C \geq 1 \\
\text{(Node } D \text{ covered)} & \qquad\qquad\ \ X_C \geq 1 \\
\text{(Node } E \text{ covered)} & \qquad\ X_B \qquad\ \geq 1 \\
\text{(Node } F \text{ covered)} & \qquad\ X_B \qquad\ \geq 1 \\
\text{(Integrality)} & X_A,\ X_B,\ X_C = 0, 1
\end{array}$$

Applying the row reduction, if $\sum a_{ij} = 1$ we set $x_j = 1$. This holds for the fifth and sixth constraints and so we set $X_B = 1$. We then eliminate all rows containing X_B. We also set $X_C = 1$ and delete rows containing X_C. The solution then becomes $X_B = X_C = 1$ and $X_A = X_D = X_E = X_F = 0$. The objective function is 2. The facilities will be located at nodes X_C and X_B. Facility at node C will cover itself and nodes D and A. Facility at node B will cover itself and nodes F and E.

3.3 THE MAXIMUM COVERING LOCATION MODEL

An underlying assumption of the Location Set Covering problem is that all of the demand nodes must be covered. However it could be recognized that mandatory coverage of all the demand nodes in all cases could require excessive resources. The reality is that due to the scarcity of resources, the solutions of the LSCP are normally not implemented to the fullest. The Maximum Covering Location problem was therefore formulated to address planning situations which have an upper limit on the number of facilities to be sited.

The objective of the MCLP is not to force coverage of all demand nodes but, instead, to locate a fixed number of facilities in such a way that demand covered by the facility is maximized. Thus the MCLP assumes that there may not be enough facilities to cover all of the demand nodes. If not all nodes can be covered, the model seeks the sting scheme that covers the most demand.

To formulate the Maximum Covering Location problem, we augment the definitions used n the LSCP with

Let h_i = demand at node i

P = the number of facilities to locate

$$Z_i = \begin{cases} 1, & \text{if demand node } i \text{ is covered} \\ 0, & \text{if not} \end{cases}$$

The MCLP is now formulated as follows

$$\text{Maximize} \quad \sum_i h_i Z_i \qquad (3.5)$$

$$\text{subject to} \quad Z_i \le \sum_j a_{ij} X_j \qquad \forall i \qquad (3.6)$$

$$\sum_j X_j \le P \qquad (3.7)$$

$$X_j = 0, 1 \qquad \forall j \qquad (3.8)$$

$$Z_i = 0, 1 \qquad \forall i \qquad (3.9)$$

The objective function (3.5) maximizes the total demand covered. Constraint (3.6) ensures that demand at node i is not counted as covered unless at least one of the facility sites that covers node i is selected. Constraint (3.7) limits the number of facilities to be sited. Constraints (3.8) and (3.9) reflect the binary nature of the facility siting decisions and demand node coverage respectively.

Example

The network below is used to illustrate the Maximum Covering Location Model.

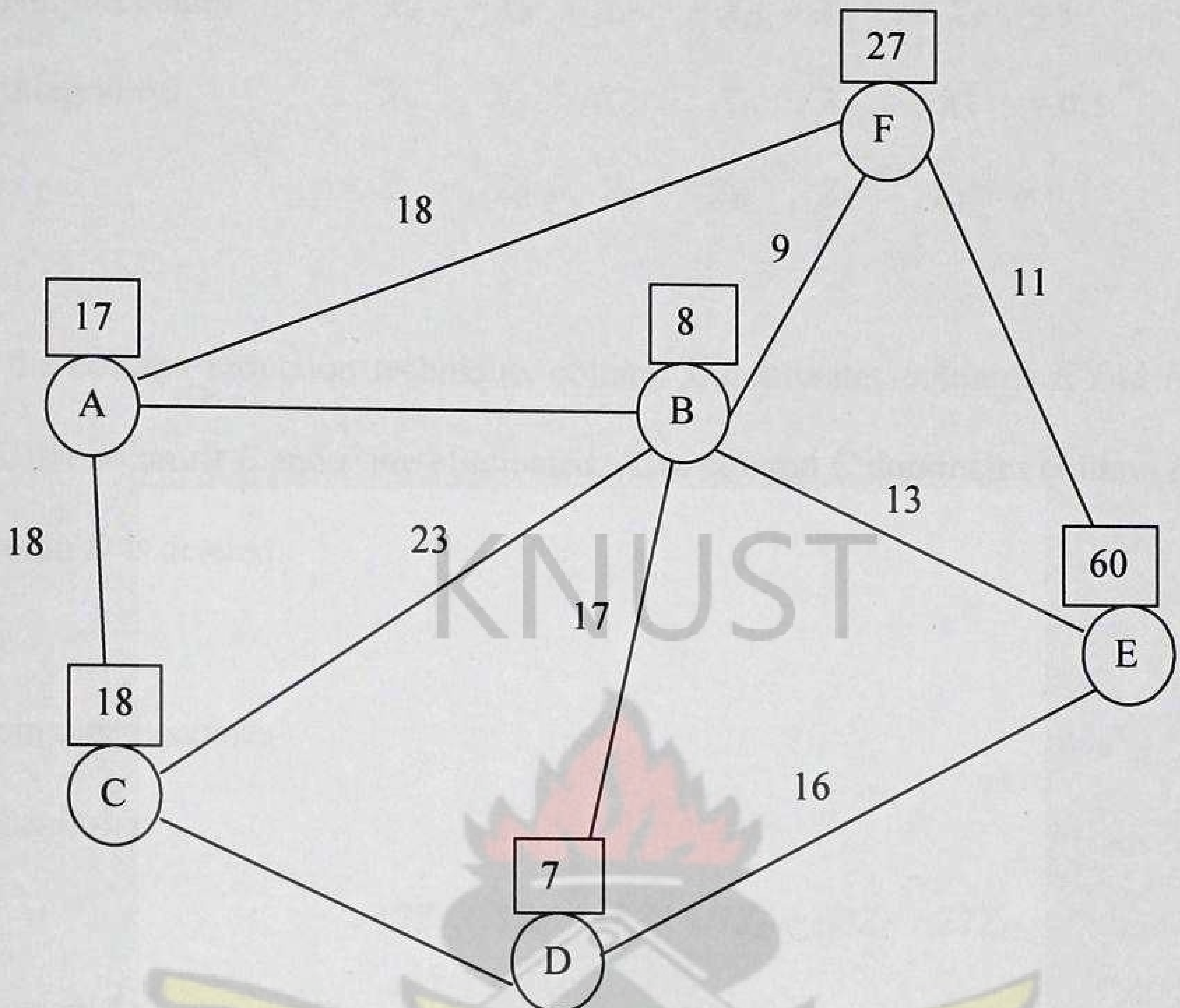

Figure 3.2 Example of a Network with Demands at each node

One facility is to be sited (ie P = 1) and a coverage distance of 15 units is to be used.

The problem becomes

Maximize

$$17Z_A + 8Z_B + 18Z_C + 7Z_D + 60Z_E + 27Z_F$$

subject to

$$\begin{array}{llllllll}
\text{(Node } A \text{ coverage)} & X_A & +X_B & +X_C & & & & \geq Z_A \\
\text{(Node } B \text{ coverage)} & X_A & +X_B & + & & X_E & +X_F & \geq Z_B \\
\text{(Node } C \text{ coverage)} & X_A & + & X_C & +X_D & & & \geq Z_C \\
\text{(Node } D \text{ coverage)} & & & X_C & +X_D & & & \geq Z_D
\end{array}$$

(Node E coverage) $X_B + X_E + X_F \geq Z_E$

(Node F coverage) $X_B + X_E + X_F \geq Z_F$

(No. to Locate) $X_A + X_B + X_C + X_D + X_E + X_F \leq 1$

(Integrality) $X_A, X_B, X_C, X_D, X_E, X_F = 0,1$

$Z_A, Z_B, Z_C, Z_D, Z_E, Z_F = 0,1$

Applying the column reduction technique, column B dominates columns E and F. For that matter columns E and F are eliminated. Also column C dominates column D and so column D is deleted.

The problem then becomes

Maximize

$$17Z_A + 8Z_B + 18Z_C + 7Z_D + 60Z_E + 27Z_F$$

subject to

(Node A coverage) $X_A + X_B + X_C \geq Z_A$

(Node B coverage) $X_A + X_B + \geq Z_B$

(Node C coverage) $X_A + X_C \geq Z_C$

(Node D coverage) $X_C \geq Z_D$

(Node E coverage) $X_B + \geq Z_E$

(Node F coverage) $X_B + \geq Z_F$

(No. to Locate) $X_A + X_B + X_C \leq 1$

(Integrality) $X_A, X_B, X_C = 0,1$

$Z_A, Z_B, Z_C, Z_D, Z_E, Z_F = 0,1$

It can be realised that the row reduction technique cannot be applied to the problem above since the value of Z_i could be 0 or 1. We therefore use an approach called the total enumeration. It can be located at either node A or node B or Node C. In this approach we will set $X_A = 1, X_B = 1, X_C = 1$. That is if

$X_A = 1$ then, $Z_A = Z_B = Z_C = 1$. The objective function = 17(1) + 8(1) + 18(1) + 7(0) + 60(0) + 27(0) = 43. Also if

$X_B = 1$ then, $Z_A = Z_B = Z_E = Z_F = 1$. The objective function = 17(1) + 8(1) + 18(0) + 7(0) + 60(1) + 27(1) = 112. Also if

$X_C = 1$ then, $Z_A = Z_C = Z_D = 1$. The objective function = 17(1) + 8(0) + 18(1) + 7(1) + 60(0) + 27(0) = 42.

We are seeking to maximize the total demand covered and so from the objective function it could be realised that if the facility is sited at X_C, the facility could cover 112 of the demand almost three times what the other two sites can cover. Hence the facility will be located at node C.

We consider locating two facilities (ie P = 2). Then the facilities will be located at either (Nodes A and B) or (Nodes A and C) or (Nodes B and C).

For Nodes A and B we set $X_A = X_B = 1$ then, $Z_A = Z_B = Z_C = Z_E = Z_F = 1$. The objective function = 17(1) + 8(1) + 18(1) + 7(0) + 60(1) + 27(1) = 130.

For Nodes A and C we set $X_A = X_C = 1$ then, $Z_A = Z_B = Z_C = Z_D = 1$. The objective function = 17(1) + 8(1) + 18(1) + 7(1) + 60(0) + 27(0) = 50.

For Nodes B and C we $X_B = X_C = 1$ then, $Z_A = Z_B = Z_C = Z_D = Z_E = Z_F = 1$. The objective function = 17(1) + 8(1) + 18(1) + 7(1) + 60(1) + 27(1) = 137.

If the facilities are located at nodes B and C the objective function is 137, the greatest of the three options. Therefore the facilities will be located at Nodes B and C.

3.4 THE GREEDY ADDING ALGORITHM

Wang et al. (2004) developed the Greedy Adding Algorithm. It One algorithm proposed for solving the Maximum Covering Problem. This is a sequential approach that begins by evaluating each site individually and selecting the one facility site that yields the greatest impact on the objective function. That facility site is then fixed open. The location of the next facility is then identified by enumerating all remaining possible locations and choosing the site that provides the greatest improvement in the objective function. Each subsequent facility is located in an identical manner. The method stops when the required number of facilities, P has been sited.

Assuming we were to locate only one facility (ie $P = 1$) we would solve the problem optimally by simply evaluating how many demands each candidate site covers (candidate site i covers $\sum_i a_{ij}h_j$ demands) and select the site that covers the most demands.

The algorithm is summarized in the flowchart below.

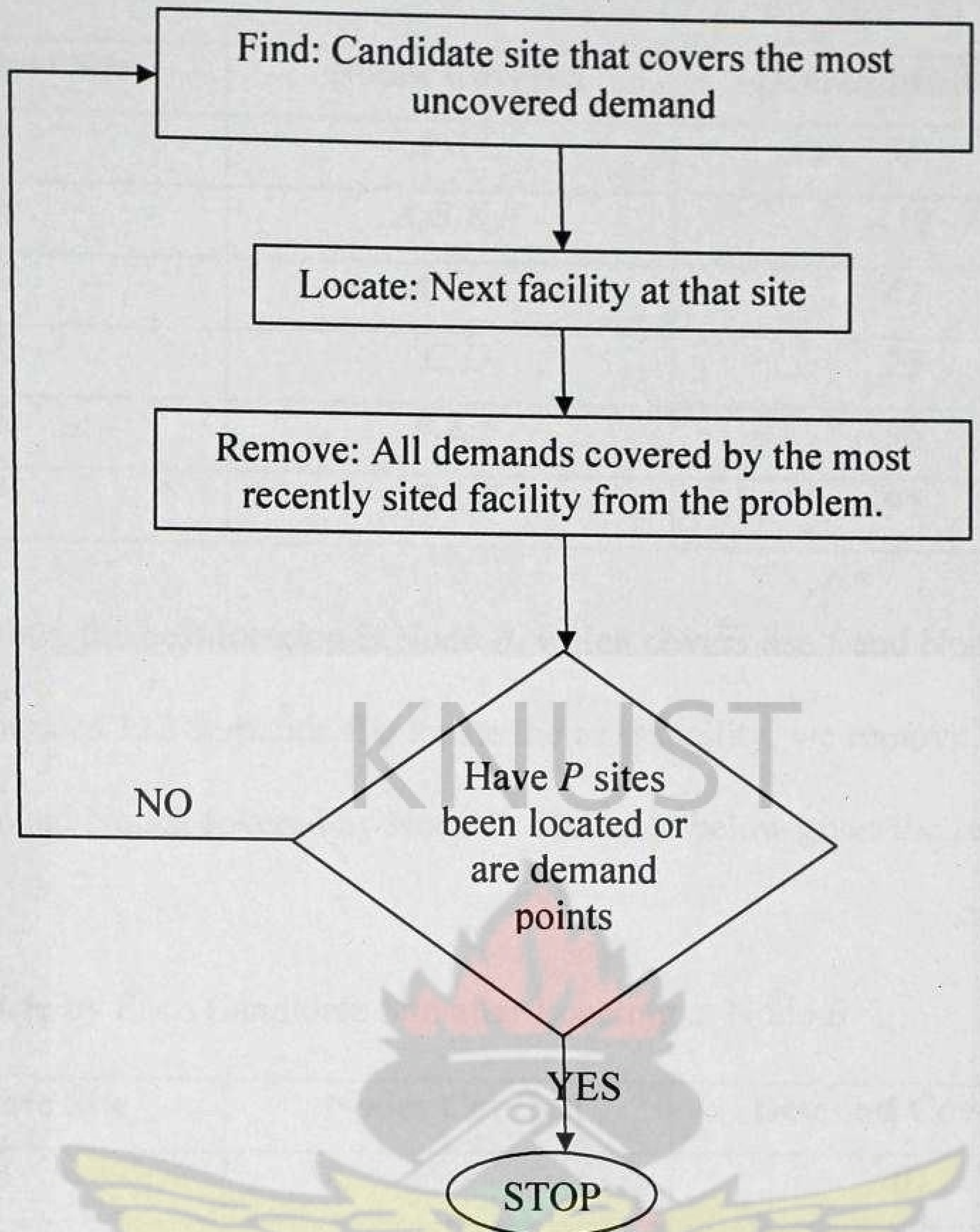

Figure 3.3 Flowchart of the Greedy Adding Algorithm for the Maximum Covering Problem

Example

Figure 3.2 is used to illustrate the algorithm. A coverage distance, D_C of 15 units is used.

The table below lists the coverage by each candidate site.

Table 3.2: Coverage by Each Candidate Site with a Coverage Distance of 15

Candidate Site	Nodes Covered	Demand Covered
A	*A,B,C*	35
B	*A,B,E,F*	112
C	*A,C,D*	42
D	*C,D*	25
E	*B,E,F*	95
F	*B,E,F*	95

As per the algorithm, the best location is Node *B*, which covers itself and Nodes *A*, *E* and *F*. It also generates 112 demands. To locate the next facility, we remove Node *B* and all of the demand Nodes covered by Node *B*. The table below gives the result.

Table 3.3: Coverage by Each Candidate Site after Locating at Node *B*

Candidate Site	Nodes Covered	Demand Covered
A	-	0
B	-	0
C	*A,C,D*	42
D	*C,D*	25
E	-	0
F	-	0

Following this result, the next facility would be located at Node *C* since it has the next greatest demand covered of 42. All demand nodes are now covered since Node *C* will also cover Node *D*. We can then stop the algorithm.

3.5 CENTER PROBLEM

The p-center problem (Hakimi, 1964,1965) addresses the problem of minimizing the maximum distance of demand point from its closet facility given that we are siting a pre-determined number of facilities. There are several possible variations of the basic model. The "vertex" p-center problem restricts the set of candidate facility sites to the nodes of the network while the "absolute" p-center problem permits the facilities to be anywhere along the arcs. Both versions can be either *weighted* or *unweighted.* In the unweighted problem, all demand nodes are treated equally. In the weighted model, the distances between nodes and facilities are multiplied by a weight associated with the demand node. For example, this weight might represent a node's importance or, more commonly, the level of its demand.

3.6 CENTER PROBLEMS ON A TREE NETWORK

3.6.1 FORMULATION OF THE VERTEX P-CENTER PROBLEM

The vertex p-center problem is formulated by defining the following notations.

Let

d_{ij} = distance from demand node i to candidate site j

h_i = demand at node i

p = number of facilities to locate

Decision Variables

$$X_j = \begin{cases} 1, \text{ if we locate at candidate site } j \\ 0, \text{ if not} \end{cases}$$

Y_{ij} = fraction of demand at node i that is served by a facility at node j

W = maximum distance between a demand node and the nearest facility.

The formulation is as follows

$$\text{Minimize} \quad W \tag{3.10}$$

$$\text{Subject to} \quad \sum_{i} Y_{ij} = 1 \qquad \forall i \tag{3.11}$$

$$\sum_{j} X_j = P \tag{3.12}$$

$$Y_{ij} \leq X_j \qquad \forall i, j \tag{3.13}$$

$$W \geq \sum_{j} d_{ij} Y_{ij} \qquad \forall i \tag{3.14}$$

$$X_j = 0,1 \qquad \forall j \tag{3.15}$$

$$Y_{ij} \geq 0 \qquad \forall i, j \tag{3.16}$$

The objective function (3.10) minimizes the maximum distance between a demand node and the closest facility to the node. Constraints (3.11) state that all of the demand at node i must be assigned to a facility at some node j for all nodes i. Constraints (3.12) stipulates that P facilities be located. Constraints (3.13) state that demand at node i cannot be assigned to a facility at node j unless a facility is located at node j. Constraints (3.14) state that the maximum distance between a demand node and the nearest facility to the node (W) must be greater than the distance between any demand node i and the facility j to which it is assigned. Constraints (3.15) and (3.16) are the integrality and nonnegativity constraints, respectively. In some cases, the demand-weighted distance is considered and constraint (3.14) becomes

$$W \geq h_i \sum_{j} d_{ij} Y_{ij} \qquad \forall i \tag{3.14a}$$

3.6.2 THE VERTEX 1-CENTER ON AN UNWEIGHTED TREE NETWORK

A vertex center in a tree network is a node that has the minimum distance to its farthest node. An algorithm to find this node was presented by Hakimi (1964). It is as follows;

Step 1: Compute the square matrix $D = (d_{ij})$ of order n.

$$d_{ij} = \begin{cases} d(v_i, v_j), & \text{for } i, j = 1, 2, \ldots, n \text{ and } i \neq j \\ d(v_i, v_j) = 0 & \text{for } i = j,\ i = 1, 2, \ldots, n \end{cases}$$

Step 2: Let d_i^m be the maximum entry in the i th column of D.

Step 3: Let v_c be the vertex center if

$$d_c^m = \min(d_1^m, d_2^m, \ldots, d_n^m)$$

Example

The figure below is used to illustrate the algorithm

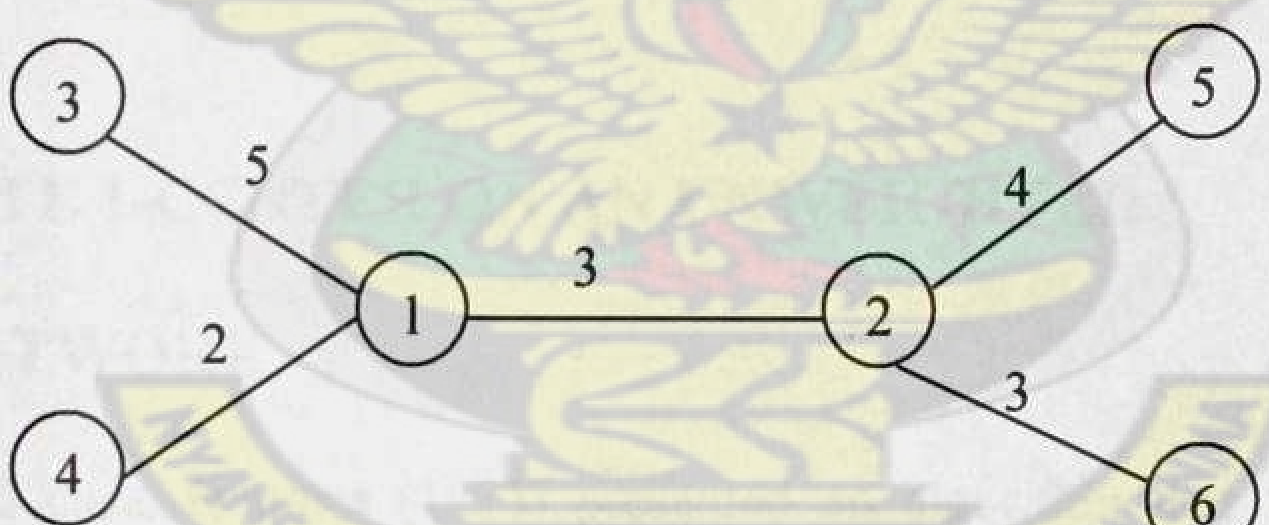

Figure 3.4: Example of a Tree Network.

The (d_{ij}) values are calculated as follows

$$(d_{ij}) = \begin{bmatrix} 0 & 3 & 5 & 2 & 7 & 6 \\ 3 & 0 & 8 & 5 & 4 & 3 \\ 5 & 8 & 0 & 7 & 12 & 11 \\ 2 & 5 & 7 & 0 & 9 & 8 \\ 7 & 4 & 12 & 9 & 0 & 7 \\ 6 & 3 & 11 & 8 & 7 & 0 \end{bmatrix}$$

Matrix 3.2 Matrix of (d_{ij}) values

From the square matrix above the d_i^m values are $d_1^m = 7$, $d_2^m = 8$, $d_3^m = 12$, $d_4^m = 9$, $d_5^m = 12$, $d_6^m = 11$.

Now $d_c^m = \min(7, 8, 12, 9, 12, 11) = 7$. Therefore $v_c = v_1$ and so the v_1 is the vertex center of the tree with $d_1^m = 7$. The location will be at the node 1.

If the tree has weighted demands nodes, the procedure is similar except that each row of (d_{ij}) must be multiplied by its respective node's demand weight h_i.

3.6.3 ABSOLUTE 1-CENTER ON AN UNWEIGHTED TREE NETWORK

For the unweighted case, Handler (1973) presented an algorithm that finds any longest path in the tree and locates the absolute center at the midpoint of the path. The algorithm is as follows:

Step 1: Pick any vertex on the tree and find the farthest vertex from it. Call it vertex v_s.

Step 2: Find a vertex that is farthest from v_s and call it v_t.

Step 3: This path is the longest path and the midpoint is the unique absolute center of the tree.

Example

The algorithm is illustrated using the figure below.

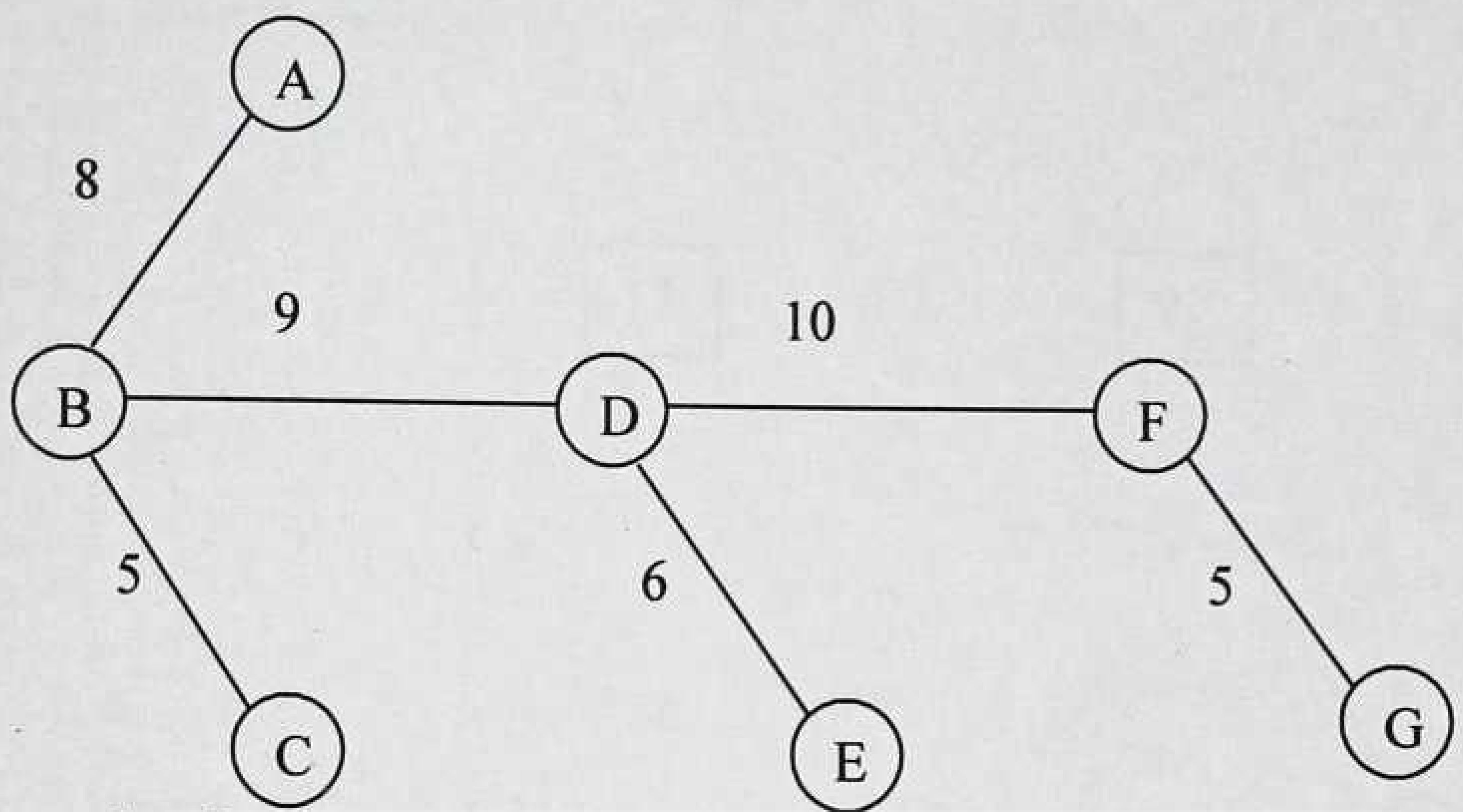

Figure 3.5: Example of an Unweighted Tree Network

We begin by picking vertex D. We then calculate the distance between node D and all vertices on the tree. The figure below shows the result of this calculation.

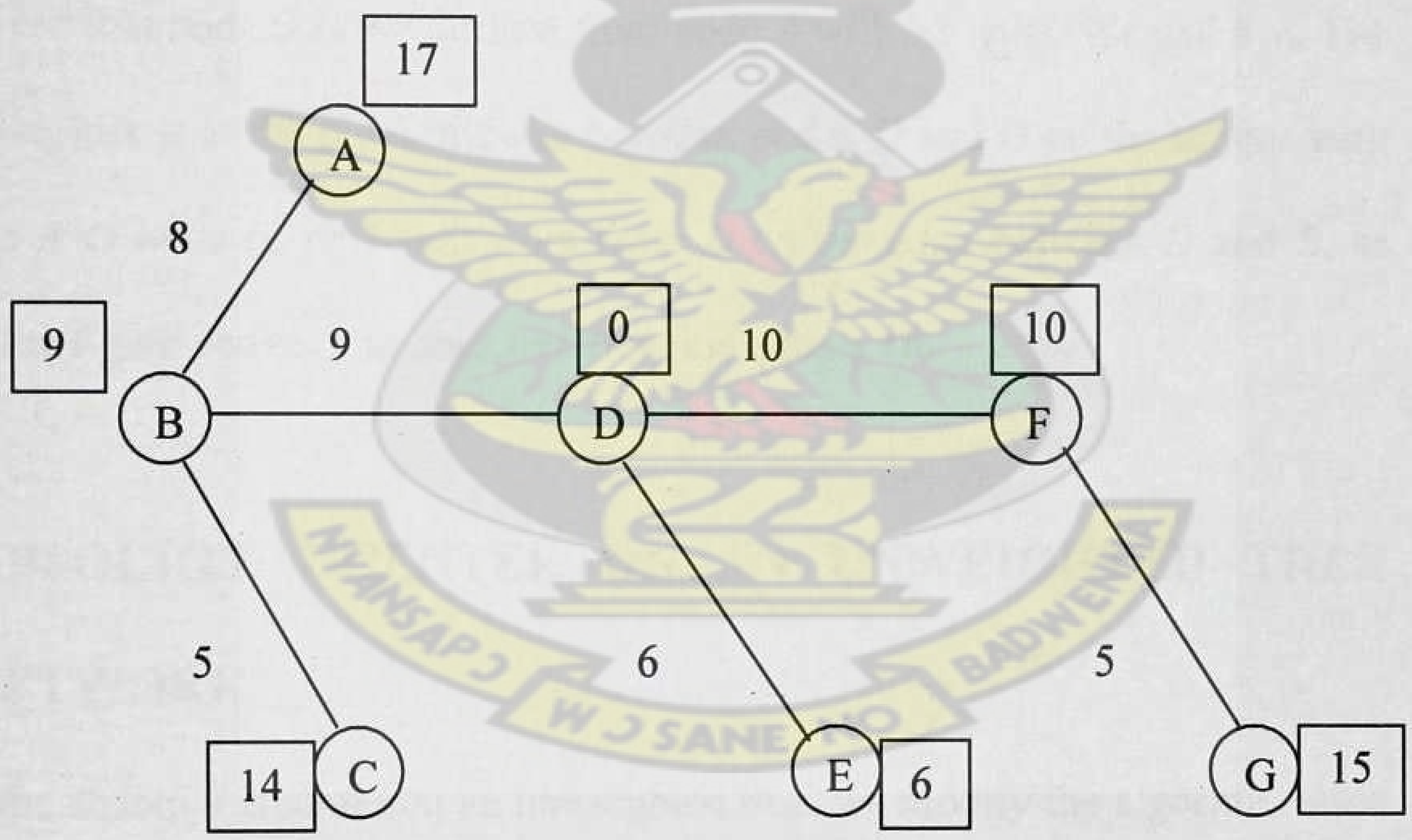

Figure 3.6 Calculated Distances from Node D shown in boxes

From figure 3.6 node A is the farthest from D i.e. 17 units from D, so we let node A be v_s. We then calculate the distances from A as shown in the figure below.

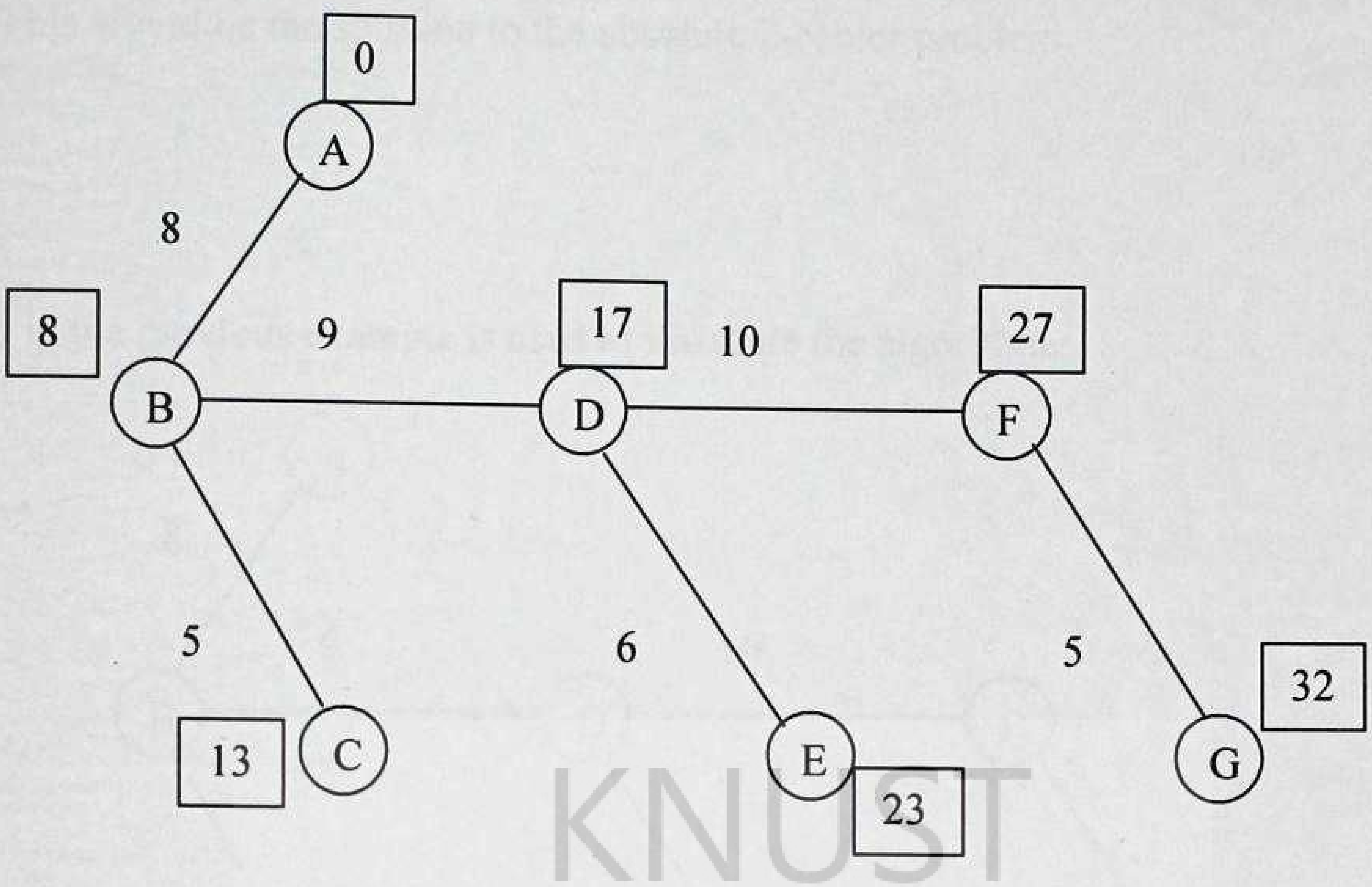

Figure 3.7 Calculated Distances from Node *A* shown in boxes.

It is observed that node *G* is the farthest from node *A* with 32 units. We call it v_t. The absolute 1-center is at the point midway between nodes *D* and *G* on the unique path from node *A* to node *G*, or 1 unit from node *D* on the edge between *D* and *B*, as shown in the figure above. The objective function equals 16.

3.6.4 ABSOLUTE 2-CENTER ON AN UNWEIGHTED TREE NETWORK

To solve the absolute 2-center on an unweighted tree, we modify the algorithm used to find the 1-center on an unweighted tree. The steps are as follows:

Step 1: Use the algorithm for the absolute 1-center and find the absolute 1-center.

Step 2: Delete from the tree the ~~arc containing~~ the absolute center. This divides the tree into two disconnected subtrees.

Step 3: Use the absolute 1-center algorithm to find the absolute 1-center of each subtree. This would be the solution to the absolute 2-center problem.

Example

Figure 3.5 in the previous example is used to illustrate the algorithm.

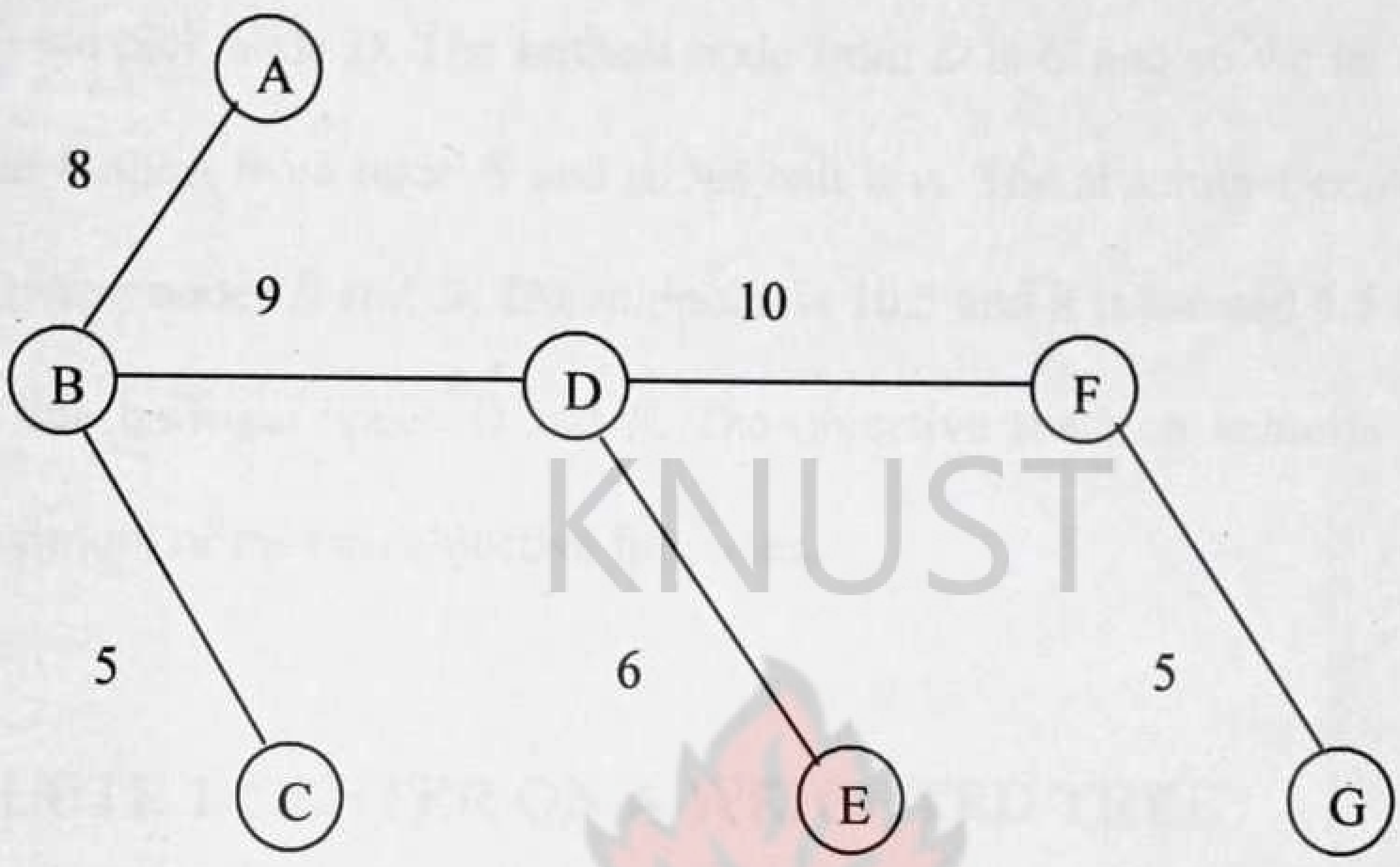

Figure 3.8: Example of an Unweighted Tree Network

As the solution indicated earlier, the absolute 1-center lies on the link *BD*. Removing this link results in the two trees shown in figure 3.8 below.

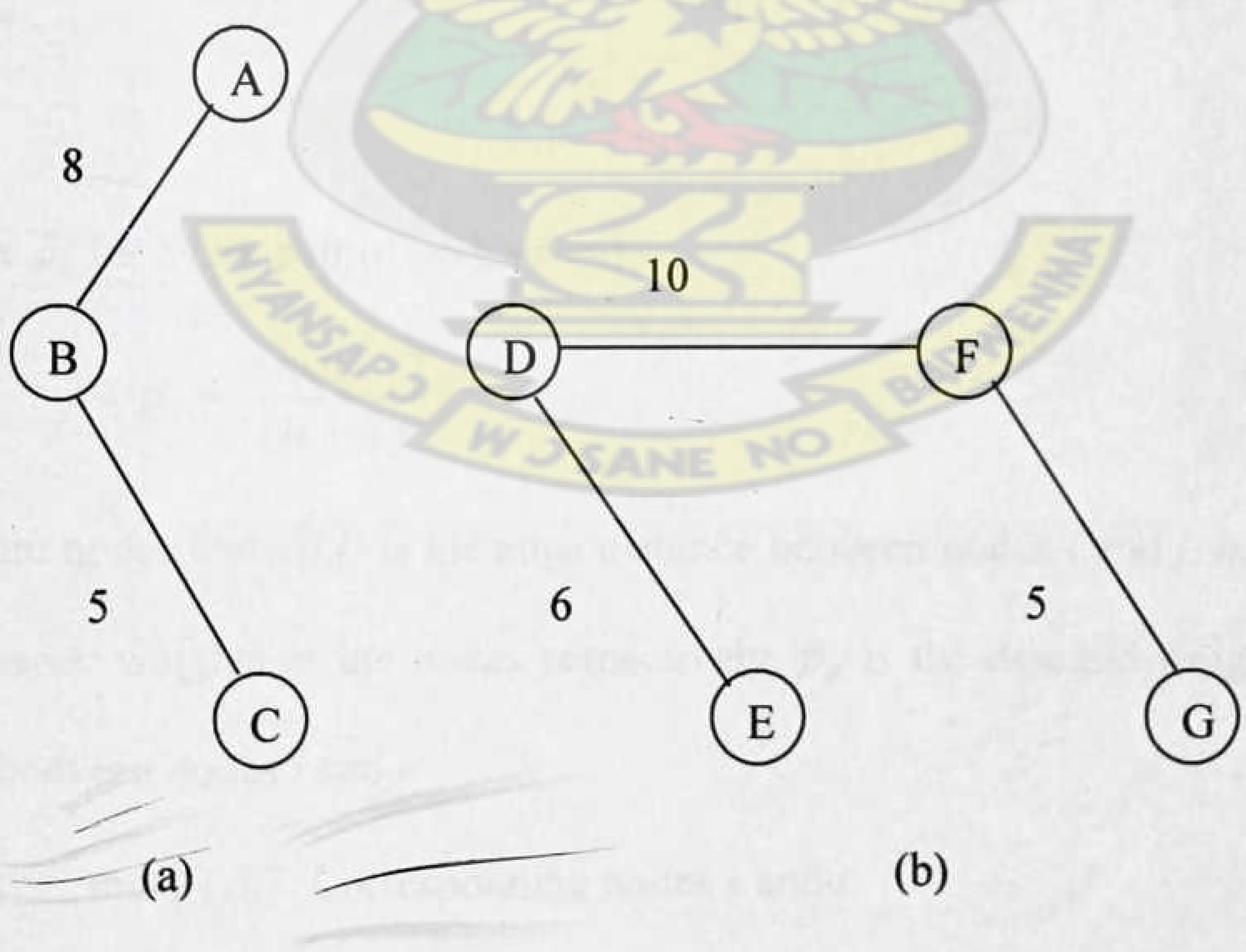

Figure 3.9 Absolute 2-center on an unweighted tree

We apply the absolute 1-center algorithm to each of the subtrees above.

In figure 3.9 (a) we pick node *B*. The farthest node from *B* is node *A* and so we let it be v_s. Since node *C* is the farthest node from node *A*, we call it v_t. Now the absolute 1-center is the point midway between nodes *A* and *C*. The midpoint is 6.5 and this is located 1.5 units from *B* on the link between nodes *A* and *B*.

In figure 3.9 (b) we pick node *D*. The farthest node from *D* is *G* and so we let it be v_s. Node *E* is the farthest from node *G* and so we call it v_t. The absolute 1-center is the midpoint between nodes *E* and *G*. The midpoint is 10.5 and it is located 4.5 units from *D* on the link between nodes *D* and *F*. The objective function value is 10.5 which is the maximum of the two objective functions.

3.6.5 ABSOLUTE 1-CENTER ON A WEIGHTED TREE NETWORK

The objective is find an absolute center on a weighted tree or a tree in which the weights associated with each of the nodes are not equal. The solution is computed as follows:

- Compute β_{ij} for every pair of nodes *i* and *j*

$$\beta_{ij} = \frac{h_i h_j d(i,j)}{(h_i + h_j)}$$

where *i* and *j* are nodes and *d(i,j)* is the edge distance between nodes *i* and *j*. h_i and h_j are the demand weights at the nodes respectively. β_{ij} is the demand-weighted distance units between nodes *i* and *j*.

- Find: $\beta_{st} = \max_{ij}(\beta_{ij})$. Corresponding nodes *s* and *t*.
- We locate at a point $[h_t/(h_s + h_t)]d(s,t)$ from node *s* on the unique path from *s* to *t*.

The algorithm is as follows:

Step 1: Compute one row of the β_{ij} elements in the matrix.

Step 2: Find the maximum element in that row (if the maximum element is in a column that was already computed, stop).

Step 3: Compute the elements β_{ij} in the column in which the maximum β_{ij} element occurred in step 2.

Step 4: Find the maximum element in the column that was just computed (if the maximum element is in the row that was already computed, stop).

Step 5: Compute the element β_{ij} in the row in which the maximum β_{ij} occurred in step 4. Go to step 2.

Example

The figure below is used to illustrate the algorithm above.

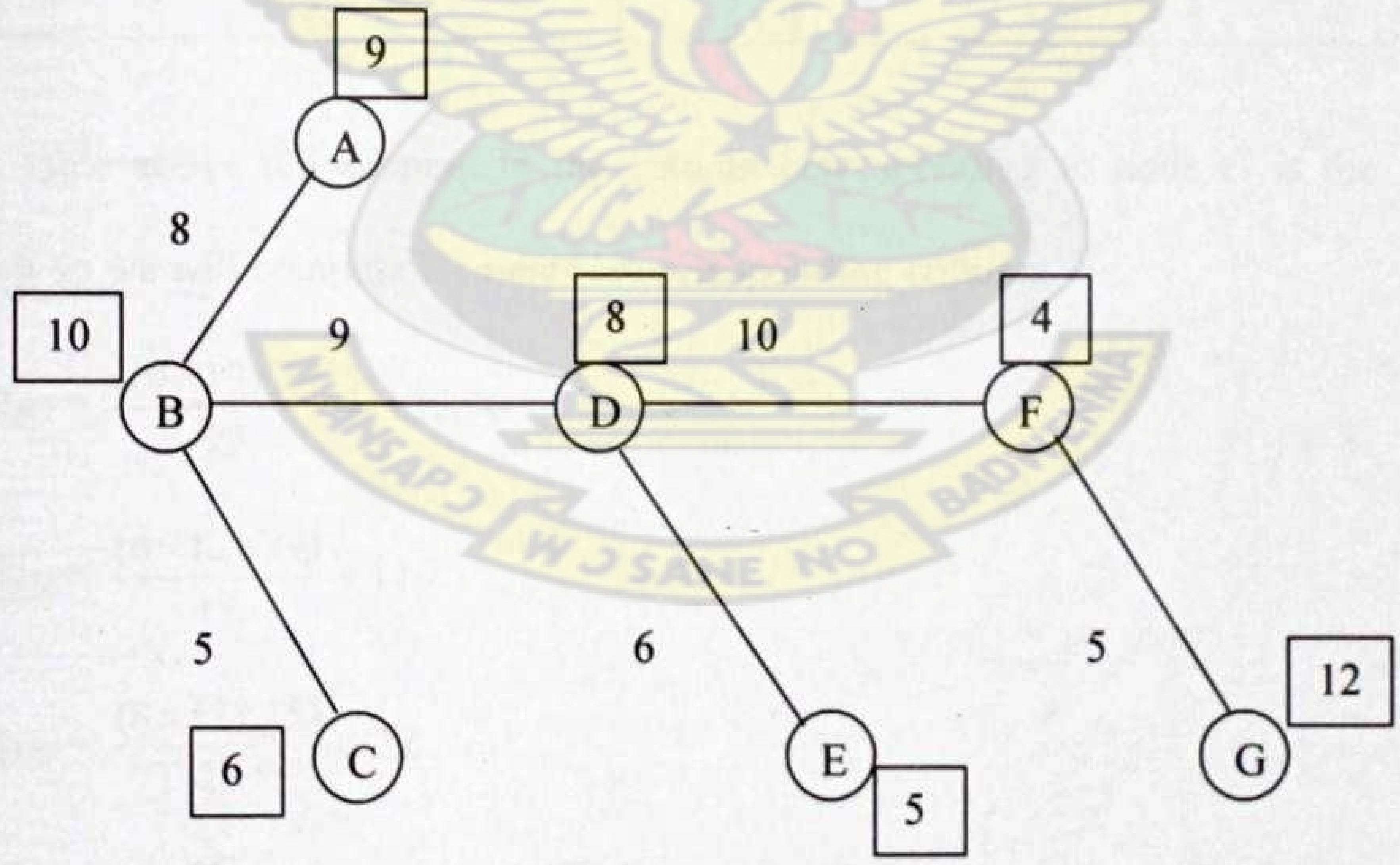

Figure 3.10 A weighted tree with demands shown in boxes.

We first compute elements in the row corresponding to node A as shown below.

$$\beta_{ij} = \frac{h_i h_j d(i, j)}{(h_i + h_j)}$$

$$\beta_{AB} = \frac{(9\times10)(8)}{19} = 37.89$$

$$\beta_{AC} = \frac{(9\times6)(13)}{15} = 46.8$$

$$\beta_{AD} = \frac{(9\times8)(17)}{17} = 72$$

$$\beta_{AE} = \frac{(9\times5)(23)}{14} = 73.92$$

$$\beta_{AF} = \frac{(9\times4)(27)}{13} = 74.76$$

$$\beta_{AG} = \frac{(9\times12)(32)}{21} = 164.57$$

Table 3.4 A partially computed matrix of β_{ij} terms.

β_{ij}	A	B	C	D	E	F	G
A	0	37.89	46.8	72	73.92	74.76	164.57

From the table above the element in the column corresponding to node G is the largest one, so we will compute elements in corresponding column.

$$B_{BG} = \frac{(10\times12)(24)}{22} = 130.90$$

$$B_{CG} = \frac{(6\times12)(29)}{18} = 116$$

$$B_{DG} = \frac{(8\times12)(15)}{20} = 72$$

$$B_{EG} = \frac{(5\times12)(21)}{17} = 74.11$$

$$B_{FG} = \frac{(4\times12)(5)}{16} = 15$$

The values are shown in the table below.

Table 3.5 A partially computed matrix of β_{ij} terms.

β_{ij}	A	B	C	D	E	F	G
A	0	37.89	46.8	72	73.92	74.76	164.57
B							130.90
C							116
D							72
E							74.11
F							15
G							0

The maximum element in the just computed column is n the row associated with node A, so we stop and find $\beta_{st.}$

$$\beta_{st} = \max_{ij}(\beta_{ij})$$

$$\beta_{st} = \beta_{AG} = \beta_{GA} = 164.57$$

$$\text{Point on } \beta_{AG} = \frac{h_A}{(h_G + h_A)} d(A,G)$$

$$\beta_{AG} = \frac{9}{12+9}(32) = 13.71$$

The optimal location is at 13.71 units away from node G on the path from node G or 8.71 units from node F to node D on the link from node F to node D. Alternatively, we could have calculated all the β_{ij} and picked the largest of them as the β_{st} value.

3.7 CENTER PROBLEMS ON A GENERAL GRAPH

3.7.1 VERTEX 1-CENTER ON A GENERAL GRAPH

The algorithm for finding the vertex center is as follows:

Step 1: Compute the matrix $D = (d_{ij})$. The entry in row i and column j in this matrix is $d(v_i,v_j)$ that implicates the shortest path between the node i and node j in the graph (the Floyd's algorithm can be used to compute these distances).

Step 2: Multiply every entry in each column by h_j to obtain a new matrix D'.

Step 3: Determine the maximum entry, $g(v_i)$, of every row of the matrix D'.

Step 4: Locate the row with the smallest value and v_i becomes the vertex center.

Example

The figure below is used to illustrate the algorithm.

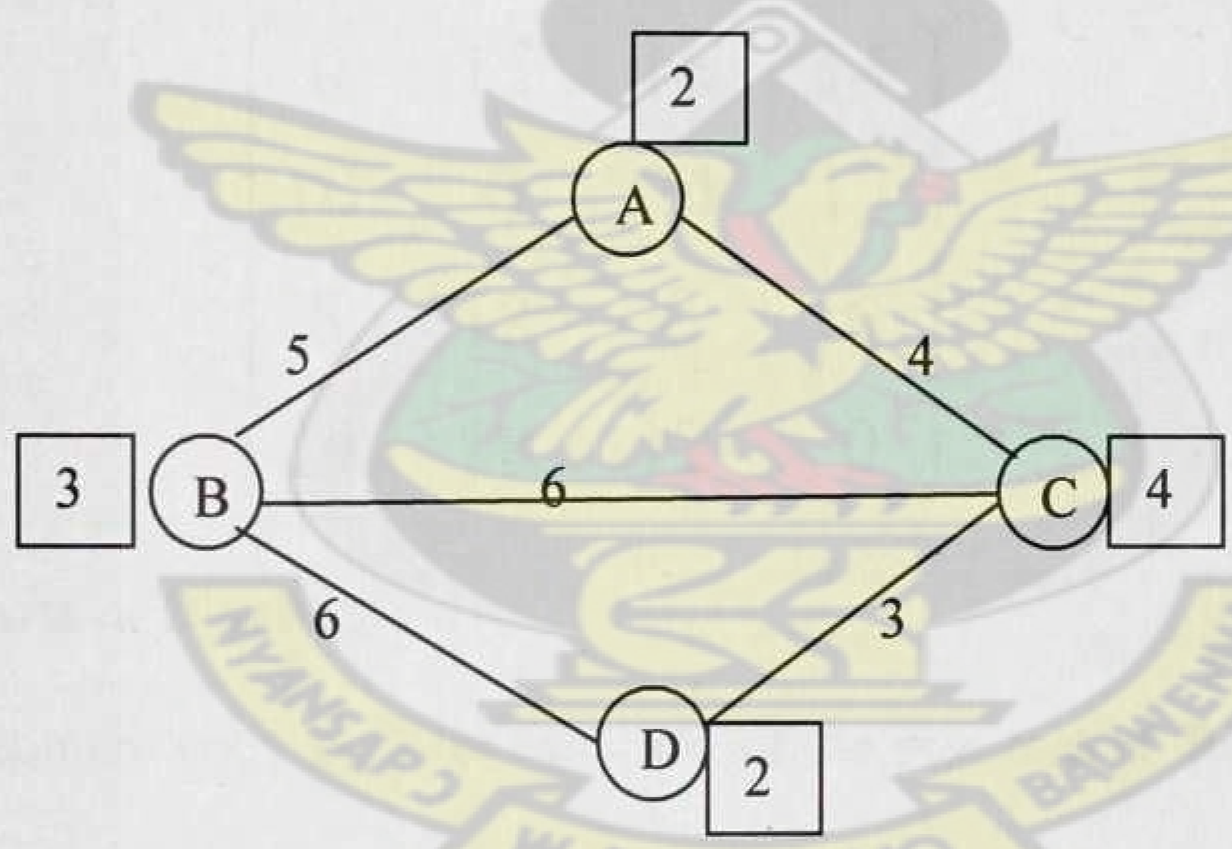

Figure 3.11 Example of a General Graph.

We first compute the shortest path between the nodes using the Floyd's algorithm as shown below.

$$
\begin{array}{cc}
 & v_j \\
v_i & \begin{bmatrix} 0 & 5 & 4 & 7 \\ 5 & 0 & 6 & 6 \\ 4 & 6 & 0 & 3 \\ 7 & 6 & 3 & 0 \end{bmatrix}
\end{array}
$$

Matrix 3.3: Matrix of $d(v_i,v_j)$ values i.e $D = (d_{ij})$

In the next step we multiply every entry in each column by the respective weight, h_j to obtain the new matrix D' as shown below.

Weight of node

$$
\begin{array}{cc}
 & v_j \\
v_i & \begin{bmatrix} 0 & 15 & 16 & 14 \\ 10 & 0 & 24 & 12 \\ 8 & 18 & 0 & 6 \\ 14 & 18 & 12 & 0 \end{bmatrix}
\end{array}
$$

Matrix 3.4: Matrix of D' values.

We find the maximum entry $g(v_i)$ for each row of the new matrix. Row A has 16 as the maximum entry. Row B has 24 as the maximum entry. Row C has 18 as the maximum entry. Row D also has 18 as the maximum entry. Any row with the smallest value identifies a vertex center. With $g(v_A) = 16$, we say that v_A is a vertex 1-center on this graph.

3.7.2 ABSOLUTE P-CENTER ON A GENERAL GRAPH

Tansel et al. (1983) and Mirchandani and Francis (1990) formulated the Absolute P-Center problem as follows:

Let $G = (V, E)$ be a graph where $V = \{v_1, \dots, v_n\}$ is the vertex set consisting of n distinct points and E is the set of edges joining these vertices. A point $x \in G$ is either a vertex or a point in the interior of some edge (v_i, v_j).

Let $X = \{x_1, \dots, x_p\} \subset G$ be any set of p points at which p facilities (servers) will be located. Let $d(x, y)$ be the shortest path distance between any two points $x, y \in G$. The distance of vertex v_i to its closest facility is denoted by

$D(v_i, X) = \min\{d(v_i, x_j)\}$; $x_i \in X$. The Absolute p-center problem of G is

$$\min(G(X)) = \min\{\max_{1 \le i \le n} w_i D(v_i, X)\}$$

where w_i are the non-negative weights that may reflect the relative importance of each vertex.

Damle and Sule (2002) developed an algorithm for placing the absolute p-center on a network as follows:

Step 1: Apply the all-pairs shortest paths algorithm to the network. Create the distance matrix with the distance between each pair of nodes being the length of the shortest path between those nodes.

Step 2: For each node identify the node that is farthest from it and mark the length of this shortest path. This is the Maximum Nodal Distance (**MND**) for this node.

Step 3: Test if $p > 1$ goto step 10, if $p = 1$ continue

Step 4: Identify the Least of the Maximum Nodal Distance (**LMND**) in the matrix. Designate the node from which this path originates as p_1. This is the initial approximation for the 1-centre. The tentative p-radius is the Least of the Maximum Nodal Distance (**LMND**). Designate this as r_1.

Step 5: Designate the node from p_1 at the end of the edge of **LMND** as n_1 and the corresponding distance as d_1. Designate the node that is second farthest from p_1 as n_2 and the corresponding distance as d_2.

REDUCTION OF COVERAGE DISTANCE

Step 6: Displace p_1 toward n_1 by a distance $\dfrac{d_1 - d_2}{2}$

This is the new approximation for the 1-centre. Designate this as the new p_1. The new p-radius is $r_1 - \dfrac{d_1 - d_2}{2}$. Designate this distance as the new r_1.

Step 7: Use Dijkstra's algorithm to check whether all the nodes in the network are served from the new p_1 within the new radius r_1.

Step 8: If all nodes are served withn the new r_1, goto step 6 to attempt further reduction in the coverage distance until no reduction is possible.

Step 9: When no further reduction is possible, stop. The resulting centre(s) and r-value(s) are the p-centre(s) and distances respectively. The p-radius is the highest of these r-value(s).

Step 10: Divide the Greatest of the Maximum Nodal Distance (**GMND**) into $p + 1$ parts.

Step 11: Place the tentative p-centres at the points of the **GMND**. Designate these as p_i (i = 1 to p). The tentative p-radii are the length of **GMND**. Designate these as r_i (i = 1 to p).

Step 12: If the points do not lie on existing nodes, add new nodes into the network at these points. Apply Dijkstra's algorithm to these points to obtain the lengths of the shortest paths from these to all other nodes in the network.

Step 13: For each node in the network, identify the centre that is closest to it and cluster it with that center. Ties may be broken arbitrary.

Step 14: Go to step 1 and apply the algorithm to each of the clusters independently taking p as 1 for each of the clusters.

Example

The figure below is used to illustrate the algorithm.

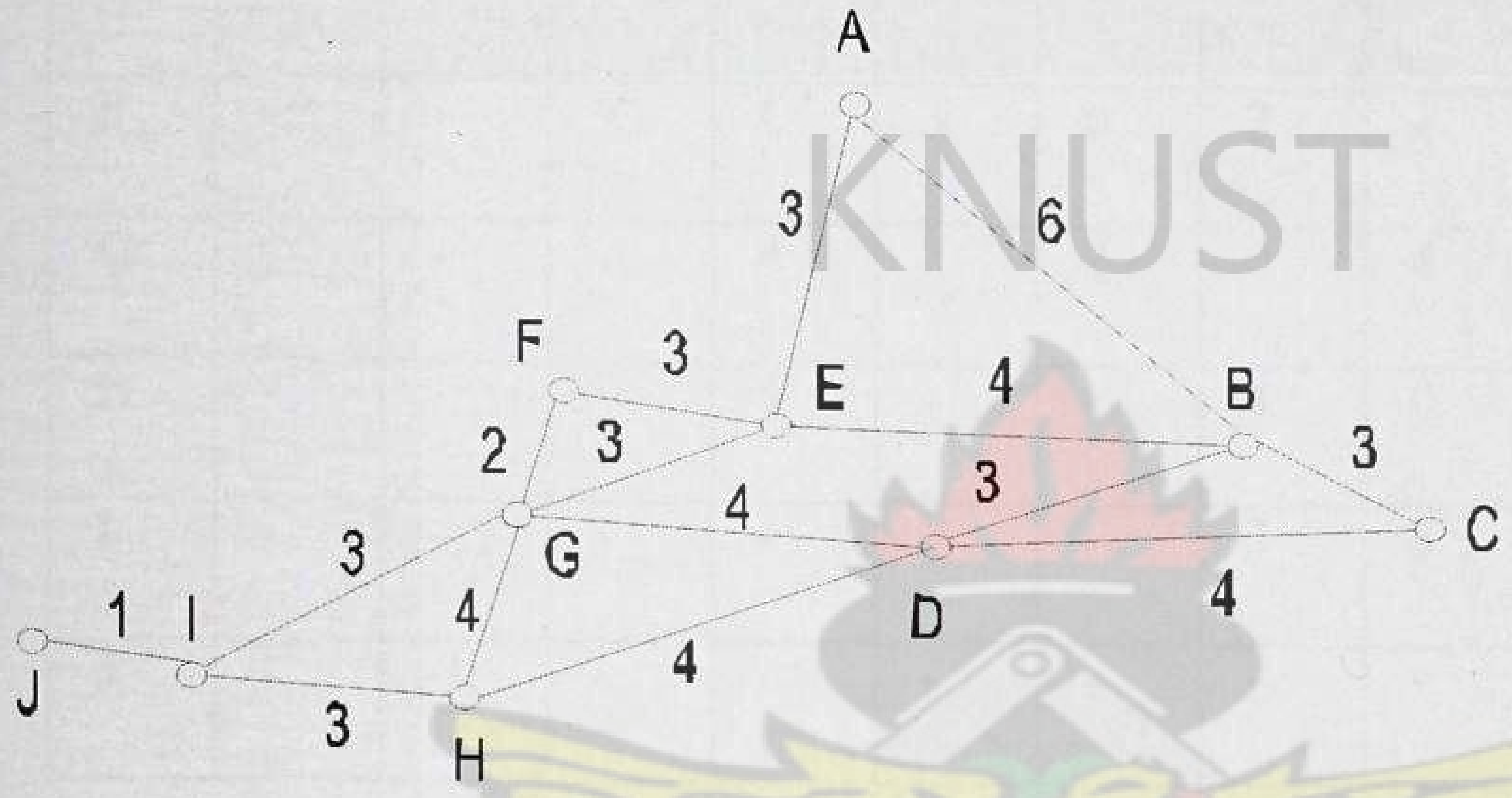

Figure 3.12 Example of a Network

The nodes of the network are put into a 10 by10 square matrix. The weights at each node are all taken to be equal to one. This is shown in Table 3.6 below.

Table 3.6 Matrix of Network Indicating Nodes and their Pair of Distances

	A	B	C	D	E	F	G	H	I	J
A	0	6	-	-	3	-	-	-	-	-
B	6	0	3	3	4	-	-	-	-	-
C	-	3	0	4	-	-	-	-	-	-
D	-	3	4	0	-	-	4	4	-	-
E	3	4	-	-	0	3	3	-	-	-
F	-	-	-	-	3	0	2	-	-	-
G	-	-	-	4	3	2	0	4	3	-
H	-	-	-	4	-	-	4	0	3	-
I	-	-	-	-	-	-	3	3	0	1
J	-	-	-	-	-	-	-	-	1	0

STEPS 1 and 2

The Floyd-Warshall All pairs shortest path algorithm is applied to the matrix in Table 3.6 to obtain the distance matrix between pair of nodes as displayed below.

Table 3.7 Shortest Distance Matrix between Pair of Nodes

	A	B	C	D	E	F	G	H	I	J
A	0	6	9	9	3	6	6	10	9	10
B	6	0	3	3	4	7	7	7	10	11
C	9	3	0	4	7	10	8	8	11	12
D	9	3	4	0	7	6	4	4	7	8
E	3	4	7	7	0	3	3	7	6	7
F	6	7	10	6	3	0	2	6	5	6
G	6	7	8	4	3	2	0	4	3	4
H	10	7	8	4	7	6	4	0	3	4
I	9	10	11	7	6	5	3	3	0	1
J	10	11	12	8	7	6	4	4	1	0
MND	**10**	**11**	**12**	**9**	**7**	**10**	**8**	**10**	**11**	**12**

A row is added to the distance matrix in Table 3.7 and labelled as MND. For each node the MND shows the node that is farthest from it and the corresponding length.

For this example $p = 2$. Since $p = 2$ the algorithm goes to step 10.

From the distance matrix in Table 3.7 above the Greatest of the Maximum Nodal Distance (**GMND**) is 12. This originates from node C. From the network in figure 3.12 it is between nodes C and J. The path is C – D – H – I – J.

The **GMND** is divided into $p + 1 = 2 + 1 = 3$ parts. That is $12/3 = 4$. Each part will be a distance of 4 units.

STEP 11

From the path, one centre is at node D and the other one lies on node H. The two centres are tentatively placed at nodes D and H. Thus p_1 = node D and p_2 = node H. Also $r_1 = 4$ and $r_2 = 8$. This means that the facilities placed nodes D and H will have a maximum coverage distance of 8units.

STEP 13

The nodes in the network are clustered around the 2 centres. Nodes A, B, C, E, and F are clustered around centre node D. Nodes G, I and J are clustered around centre node H.

STEP 14

The algorithm takes the clusters independently and goes to step 1

CENTRE NODE D

The part of the network in figure 3.12 corresponding to the nodes clustered around centre node D is put into a 6 by 6 matrix as shown in Table 3.8 below.

Table 3.8 Matrix of Clustered Nodes of Network and their Pair of Distances

	A	**B**	**C**	**D**	**E**	**F**
A	0	6	-	-	3	-
B	6	0	3	3	4	-
C	-	3	0	4	-	-
D	-	3	4	0	-	-
E	3	4	-	-	0	3
F	-	-	-	-	3	0

STEPS 1 and 2

The Floyd-Warshall All pairs shortest path algorithm is applied to the matrix in Table 3.8 to obtain the distance matrix between pair of nodes as displayed below.

Table 3.9 Shortest Distance Matrix between Pair of Nodes with Centre Node *D*

	A	**B**	**C**	**D**	**E**	**F**
A	0	6	9	9	3	6
B	6	0	3	3	4	7
C	9	3	0	4	7	10
D	9	3	4	0	7	6
E	3	4	7	7	0	3
F	6	7	10	6	3	0
MND	**9**	7	**10**	**9**	7	**10**

A row is added to the distance matrix in Table 3.8 labelled as MND. Again for each node this identifies the node that is farthest from it and the corresponding length.

STEP 3

The value of p is now 1. The algorithm seeks to site one centre in this cluster.

STEP 4

For cluster 1 corresponding to node D, the **LMND** is 7. The node from which this path originates is E. It is between nodes B and F, nodes E and C and nodes E and D. The path between nodes E and C is used. The path is E – B – C.

Therefore p_1 = node E and r_1 = 7. The centre is placed tentatively at node E with a corresponding distance of 7units.

STEP 5

From the path n_1 = node C and $d_1 = 7$. n_2 = node B and $d_2 = 4$.

Reduction of Coverage Distance

Expected distance to be displaced = $\frac{d_1 - d_2}{2} = \frac{7-4}{2} = \frac{3}{2} = 1.5$

STEP 6

Centre node E is displaced towards node C by a distance of 1.5units on the path between nodes E and C. The new 1- centre is therefore placed 1.5units away from node E on the link E – B. This is the new p_1.

The corresponding distance is $r_1 - \frac{d_1 - d_2}{2} = 7 - \frac{7-4}{2} = 5.5$

The new r_1 = 5.5units

STEP 7

The distances between the new 1-centre the nodes and the clustered nodes are shown below.

Table 3.10 Shortest Distance between the Nodes and the New Centre, New P_1

	A	B	C	D	E	F
New p_1	4.5	2.5	5.5	5.5	1.5	4.5

It is seen that all the nodes are served by the new centre within the distance of 5.5units.

The algorithm goes to step 6 to seek a further reduction.

STEP 6

n_1 = node C and $d_1 = 5.5$. n_2 = node B and $d_2 = 2.5$.

Further Reduction

Expected distance to be displaced = $\frac{d_1 - d_2}{2} = \frac{5.5 - 2.5}{2} = \frac{3}{2} = 1.5$

Centre at old p_1 is displaced towards node C by a distance of 1.5units on the path between old p_1 and node C. The new 1-centre is therefore placed 1.5units away from old p_1 on the link old p_1 – B. This is the new p_1.

The corresponding distance is $r_1 - \frac{d_1 - d_2}{2} = 5.5 - \frac{5.5 - 2.5}{2} = 4.0$

The new r_1 = 4.0units.

STEP 7

The distances between the new 1-centre the nodes and the clustered nodes are shown below.

Table 3.11 Shortest Distance between the Nodes and the New Centre, New P_1

	A	B	C	D	E	F
New p_1	6.0	1.0	4.0	4.0	3.0	6.0

It is seen from the Table 3.11 above that not all the nodes are served from the new centre within the distance of 4.0units. If the centre is placed at the new p_1, nodes A and F would not be served.

STEP 9

Since no further reduction is possible the centre is maintained at the previous p_1 which is 1.5units from node E with a corresponding distance of 5.5units. The optimal location of the facility for this cluster is at 1.5units from node E on the link E – ~~B. The~~ coverage distance for this cluster has also been minimised.

CENTRE NODE H

The part of the network in figure 3.12 corresponding to the nodes clustered around centre node H is put into a 4 by 4 matrix as shown in Table 3.12 below.

Table 3.12 Matrix of Clustered Nodes of Network and their Pair of Distances

	G	H	I	J
G	0	4	3	-
H	4	0	3	-
I	3	3	0	1
J	-	-	1	0

STEPS 1 and 2

The Floyd-Warshall All pairs shortest path algorithm was applied to the matrix in Table 3.12 to obtain the distance matrix between pair of nodes as displayed below

Table 3.13 Shortest Distance Matrix between Pair of Nodes with Centre Node *H*

	G	H	I	J
G	0	4	3	4
H	4	0	3	4
I	3	3	0	1
J	4	4	1	0
MND	4	4	3	4

A row is added to the distance matrix in Table 3.13 labelled as the MND. For each node the MND shows the node that is farthest from it and the corresponding length.

STEP 3

The value of p is 1 for this cluster.

STEP 4

For cluster 2 corresponding to node H the **LMND** is 3. The node from which this path originates is I. It is between nodes I and H and nodes I and G. They are all edge distances. The link I – H is used. Therefore p_1 = node I and $r_1 = 3$.

STEP 9

It is seen that no reduction can be made because the path is an edge distance. The 1-centre is placed at node I with a corresponding distance of 3units. The optimal location of the facility for this cluster is at node I.

The 2-centres are thus located at 1.5units away from node E on the link E – B and node I with the 2-radius for the network being 5.5units. The figure below shows the centres.

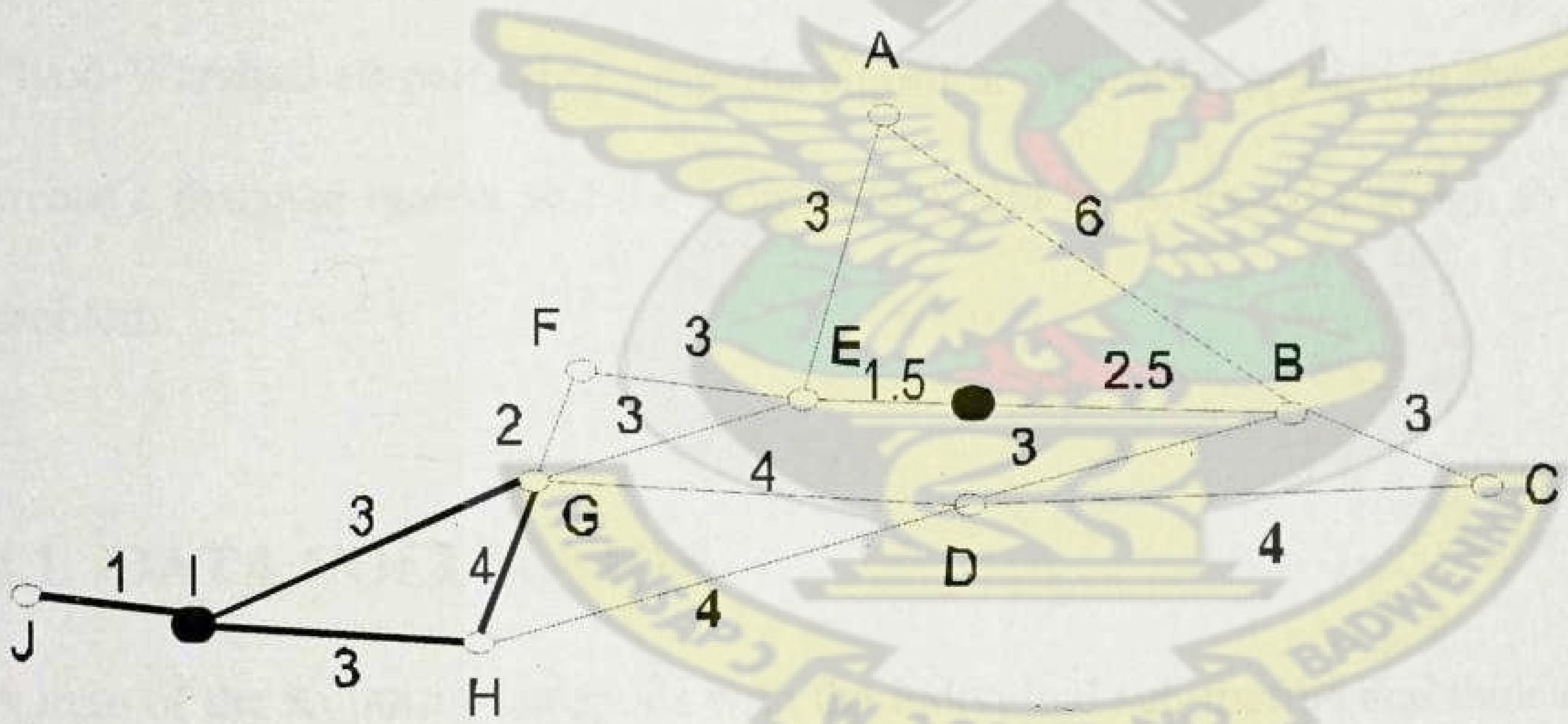

Figure 3.13 Network Indicating Location of 2-centres and their respective Coverage Nodes

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND DISCUSSION OF RESULTS

4.0 INTRODUCTION

In this chapter a new heuristic for placing absolute p-centres on a network (Damle and Sule, 2002) would be used to locate two fire stations in five sub metros put together. They are the Asokwa, Manhyia, Oforikrom, Asawase and Nhyiaeso sub-metros of the Kumasi Metropolitan Assembly. The map of these sub metros will be used to draw a cyclic network with the edges being the inter- town distances. The Flyod-Warshall all-pairs shortest paths algorithm would be applied to the network to create a distance matrix and the heuristics would be followed through to solve the problem.

4.1 DATA COLLECTION

A map of the Kumasi Metropolis with the individual sub-metros and their respective towns was obtained from the Planning Department of the Kumasi Metropolitan Assembly. Figure A1.0 in Appendix 1.0 depict the sub-metro areas of Kumasi.

The map was prepared in 2008. The towns or settlements of the five sub-metros were identified and the ArcGIS software was used to calculate the distances between the towns to obtain the inter-town distances.

A network was formed out of the map. The fifty-six (56) nodes in the network are the towns or settlements of the five sub-metros. The edges are the various roads linking the towns. These are access roads which link the towns. Figure 4.0 below shows the network. The key to the network is shown in Table 4.0 below.

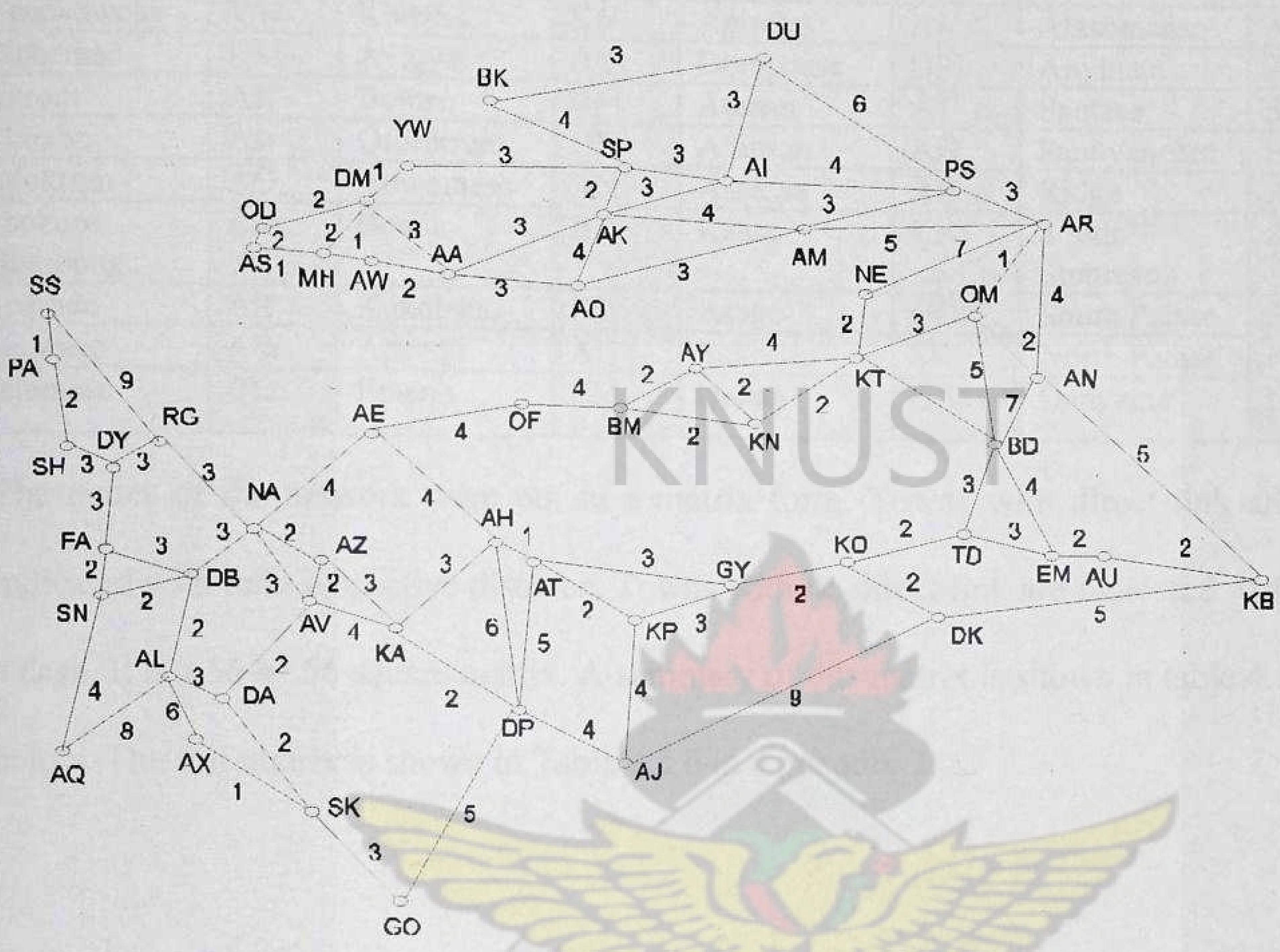

Figure 4.0 Network of towns and settlements in the five sub-metros.

Table 4.0 Key to Network of towns and settlements in Figure 4.0

TOWN	NODE	TOWN	NODE	TOWN	NODE	TOWN	NODE
Buokrom	BK	Manhyia	MH	Twumduase	TD	Ahodwo	AV
Duase	DU	Ashtown	AS	Kotei	KO	Adeibeba	AZ
Pakoso	PS	Nsenie	NE	Deduako	DK	Apramang	AX
Asabi	AI	Oduom	OM	Gyinyase	GY	Nhyiaeso	NA
Sepetimpon	SP	Kentinkrono	KT	Kyirapatre	KP	Adiembra	DB
Yennyawoso	YW	KNUST	KN	Apirabo	AJ	Atasomanso	AL
Dichemso	DM	Ayigya	AY	Dompoase	DP	Anyinam	AQ
Akrom	AK	Bomso	BM	Atonsu	AT	Santase	SN
Aboabo	AA	Oforikrom	OF	Ahinsan	AH	Fankyenebra	FA
Adukrom	AO	Anwomaso	AN	Asokwa	AE	Ridge	RG
Asokore Mampong	AM	Boadi	BD	Kaase	KA	South Suntreso	SS
Aparade	AR	Kokoben	KB	Asago	GO	South Patase	SH
Asawase	AW	Apeadu	AU	Sokoban	SK	North Patase	PA
Odumase	OD	Emena	EM	Daban	DA	Danyame	DY

The nodes of the network were put in a matrix form. Towns with direct link are indicated with their respective distance. Towns with no direct link are indicated with a dash. It is a 56 by 56 square matrix. A summary of this matrix is shown in table 4.1 below. The full matrix is shown in Table A2.0 in Appendix 2.0.

Table 4.1 Summary of Matrix of Network in Fig 4.0 Indicating Towns and their Pair of Distances.

	PA	SS	SH	RG	DY	NA	.	.	.	PS	AR	AK	AO	.	.	.	OF	GY	AE	AH	AT	KP
PA	0	1	2	-	-	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
SS	1	0	-	9	-	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
SH	2	-	0	-	3	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
RG	-	9	-	0	3	3	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
DY	-	-	3	3	0	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
NA	-	-	-	3	-	0	.	.	.	-	-	-	-	.	.	.	-	-	4	-	-	-
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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PS	-	-	-	-	-	-	.	.	.	0	3	-	-	.	.	.	-	-	-	-	-	-
AR	-	-	-	-	-	-	.	.	.	3	0	-	-	.	.	.	-	-	-	-	-	-
AK	-	-	-	-	-	-	.	.	.	-	-	0	4	.	.	.	-	-	-	-	-	-
AO	-	-	-	-	-	-	.	.	.	-	-	4	0	.	.	.	-	-	-	-	-	-
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OF	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	0	-	4	-	-	-
GY	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	0	-	-	3	3
AE	-	-	-	-	-	4	.	.	.	-	-	-	-	.	.	.	4	-	0	4	-	-
AH	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	-	4	0	1	-
AT	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	3	-	1	0	2
KP	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	3	-	-	2	0

4.2 ALGORITHM FOR PLACING THE ABSOLUTE-P CENTER

The algorithm for placing the absolute p-centre on a network is outlined as follows.

Step 1: Apply the all-pairs shortest paths algorithm to the network. Create the distance matrix with the distance between each pair of nodes being the length of the shortest path between those nodes.

Step 2: For each node identify the node that is farthest from it and mark the length of this shortest path. This is the Maximum Nodal Distance (**MND**) for this node.

Step 3: Test if $p > 1$ goto step 10, if $p = 1$ then continue.

Step 4: Identify the Least of the Maximum Nodal Distance (**LMND**) in the matrix. Designate the node from which this path originates as p_1. This is the initial approximation for the 1-centre. The tentative p-radius is the Least of the Maximum Nodal Distance (**LMND**). Designate this as r_1.

Step 5: Designate the node from p_1 at the end of the edge of LMND as n_1 and the corresponding distance as d_1. Designate the node that is second farthest from p_1 as n_2 and the corresponding distance as d_2.

REDUCTION OF COVERAGE DISTANCE

Step 6: Displace p_1 toward n_1 by a distance $\frac{d_1 - d_2}{2}$

This is the new approximation for the 1-centre. Designate this as the new p_1. The new p-radius is $r_1 - \frac{d_1 - d_2}{2}$. Designate this distance as the new r_1.

Step 7: Use Dijkstra's algorithm to check whether all the nodes in the network are served from the new p_1 within the new radius r_1.

Step 8: If all nodes are served within the new r_1, goto step 6 to attempt further reduction in the coverage distance until no reduction is possible.

Step 9: When no further reduction is possible, stop. The resulting centre(s) and r-value(s) are the p-centre(s) and distances respectively. The p-radius is the highest of these r-value(s).

Step 10: Divide the Greatest of the Maximum Nodal Distance (**GMND**) into $p + 1$ parts

Step 11: Place the tentative p-centres at the points of the **GMND**. Designate these as p_i (i = 1 to p). The tentative p-radii are the length of **GMND**. Designate these as r_i (i = 1 to p).

Step 12: If the points do not lie on existing nodes, add new nodes into the network at these points. Apply Dijkstra's algorithm to these points to obtain the lengths of the shortest paths from these to all other nodes in the network.

Step 13: For each node in the network, identify the centre that is closest to it and cluster it with that center. Ties may be broken arbitrary.

Step 14: Go to step 1 and apply the algorithm to each of the clusters independently taking p as 1 for each of the clusters.

4.3 COMPUTATION AND RESULTS

STEPS 1 and 2

The Floyd-Warshall All pairs shortest path algorithm was applied to the matrix in Table A2.0 in Appendix 2.0 to obtain the distance matrix between each pair of node as displayed in Table A3.0 in Appendix 3.0.A summary is shown in Table 4.2 below The matrix shows the length of the shortest path between respective nodes. The Maximum Nodal Distance (**MND**) for each node is identified. A row is created in the distance matrix for the **MND**.

Table 4.2 Summary of Shortest Distance Matrix between Pair of Nodes

	PA	SS	SH	RG	DY	NA	.	.	.	PS	AR	AK	AO	.	.	.	OF	GY	AE	AH	AT	KP
PA	0	1	2	8	5	11	.	.	.	34	31	40	39	.	.	.	19	23	15	19	20	22
SS	1	0	3	9	6	12	.	.	.	35	32	41	40	.	.	.	20	24	16	20	21	23
SH	2	3	0	6	3	9	.	.	.	32	29	38	37	.	.	.	17	21	13	17	18	20
RG	8	9	6	0	3	3	.	.	.	26	23	32	31	.	.	.	11	15	7	11	12	14
DY	5	6	3	3	0	6	.	.	.	29	26	35	34	.	.	.	14	18	10	14	15	17
NA	11	12	9	3	6	0	.	.	.	23	20	29	28	.	.	.	8	12	4	8	9	11
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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PS	34	35	32	26	29	23	.	.	.	0	3	7	6	.	.	.	15	16	19	20	19	19
AR	30	32	29	23	26	20	.	.	.	3	0	9	8	.	.	.	12	13	16	17	16	16
AK	40	41	38	32	35	29	.	.	.	7	9	0	4	.	.	.	21	22	25	26	25	25
AO	49	40	37	31	34	28	.	.	.	6	8	4	0	.	.	.	20	21	24	25	24	24
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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OF	19	20	17	11	14	8	.	.	.	15	12	21	20	.	.	.	0	12	4	8	9	11
GY	23	24	21	15	18	12	.	.	.	16	13	22	21	.	.	.	12	0	8	4	3	3
AE	15	16	13	7	10	4	.	.	.	19	16	25	24	.	.	.	4	8	0	4	5	7
AH	19	20	17	11	14	8	.	.	.	20	17	26	25	.	.	.	8	4	4	0	1	3
AT	20	21	18	12	15	9	.	.	.	19	16	25	24	.	.	.	9	3	5	1	0	2
KP	22	23	20	14	17	11	.	.	.	19	16	25	24	.	.	.	11	3	7	3	2	0
MND	**47**	**48**	**45**	**39**	**42**	**36**	.	.	.	**35**	**32**	**41**	**40**	.	.	.	**28**	**29**	**32**	**33**	**32**	**32**

The objective is to locate two (2) fire stations and so the value of p is 2. From the distance matrix in Table A3.0 in Appendix 3.0, the Greatest of the Maximum Nodal Distances (**GMND**) is 48. This originates from node OD. The path is between nodes OD and SS.

From the network in figure 4.0 the path is

OD – DM – AA – AO – AM – AR – OM – KT – KN – BM – OF – AE – NA – RG – SS.

The **GMND** is divided into $p + 1$ parts. Thus 2 + 1 = 3 parts. This gives 48/3 = 16. Each part will therefore be a distance of 16km.

STEP 11

The first point on this path will be at node AR and the second will be at node AE. Hence the tentative 2-centres will be at nodes AR and AE. Thus p_1 = node AR and p_2 = node AE. Also r_1 = 16km and r_2 = 32km. This indicates that the facilities when placed at nodes AE and AR will have a maximum coverage distance of 32km.

STEP 13

The nodes in the network are clustered around the 2 centres.

Nodes YW, DM, BK, SP, DU, AI, AM, PS, AK, AO, AA, OD, AS, OM, AW, NE, KT, AN, BD, KB, AU, EM, TD, KN, AY, BM and MH are clustered around centre node AR.

Nodes PA, SS, RG, DY, NA, SN, FA, DB, AL, DA, AX, GO, SK, AV, AZ, KA, DP, AJ, KO, OF, GY, AH, AT, KP, AQ, SH and DK are also clustered around centre node AE.

STEP 14

The algorithm takes the clusters independently and goes to step 1

CENTRE NODE AR

The part of the network in figure 4.0 corresponding to the nodes clustered around centre node AR is put into a 28 by 28 square matrix as shown in Table A4.0 in Appendix 4.0. A summary of the matrix is shown in Table 4.3 below.

Table 4.3 Summary of Matrix of Clustered Nodes around Node **AR** in Fig 4.0 Indicating Towns and their Pair of Distances

	YW	DM	BK	SP	.	.	.	OD	AS	OM	AW	.	.	.	KN	AY	BM	MH
YW	0	1	-	3	.	.	.	-	-	-	-	.	.	.	-	-	-	-
DM	1	0	-	-	.	.	.	2	-	-	-	.	.	.	-	-	-	2
BK	-	-	0	4	.	.	.	-	-	-	-	.	.	.	-	-	-	-
SP	3	-	4	0	.	.	.	-	-	-	-	.	.	.	-	-	-	-
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OD	-	2	-	-	.	.	.	0	2	-	-	.	.	.	-	-	-	-
AS	-	-	-	-	.	.	.	2	0	-	-	.	.	.	-	-	-	1
OM	-	-	-	-	.	.	.	-	-	0	-	.	.	.	-	-	-	-
AW	-	-	-	-	.	.	.	-	-	-	0	.	.	.	-	-	-	1
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.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
KN	-	-	-	-	.	.	.	-	-	-	-	.	.	.	0	2	2	-
AY	-	-	-	-	.	.	.	-	-	-	-	.	.	.	2	0	2	-
BM	-	-	-	-	.	.	.	-	-	-	-	.	.	.	2	2	0	-
MH	-	2	-	-	.	.	.	-	1	-	1	.	.	.	-	-	-	0

STEPS 1 and 2

The Floyd-Warshall All pairs shortest path algorithm is applied to the matrix in Table A4.0 to obtain the shortest distance matrix between pair of nodes as displayed in Table A4.1 in Appendix 4.0. A summary of the matrix is in Table 4.4 below. A row is created in the distance matrix for the **MND**.

Table 4.4 Summary of Shortest Distance Matrix between Pair of Nodes of Centre Node **AR**

	YW	DM	BK	SP	.	.	.	OD	AS	OM	AW	.	.	.	KN	AY	BM	MH
YW	0	1	7	3	.	.	.	3	4	14	4	.	.	.	19	21	21	3
DM	1	0	8	4	.	.	.	2	3	15	3	.	.	.	20	22	22	2
BK	7	8	0	4	.	.	.	10	11	13	11	.	.	.	18	20	20	10
SP	3	4	4	0	.	.	.	6	7	11	7	.	.	.	16	18	18	6
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
OD	3	2	10	6	.	.	.	0	2	17	4	.	.	.	22	24	24	3
AS	4	3	11	7	.	.	.	2	0	16	2	.	.	.	21	23	23	1
OM	14	15	13	11	.	.	.	17	16	0	14	.	.	.	5	7	7	15
AW	4	3	11	7	.	.	.	4	2	14	0	.	.	.	19	21	21	1
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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KN	19	20	18	16	.	.	.	22	21	5	19	.	.	.	0	2	2	20
AY	21	22	20	18	.	.	.	24	23	7	21	.	.	.	2	0	2	22
BM	21	22	20	18	.	.	.	24	23	7	21	.	.	.	2	2	0	22
MH	3	2	10	6	.	.	.	3	1	15	1	.	.	.	20	22	22	0
MND	**23**	**24**	**22**	**20**	.	.	.	**26**	**25**	**17**	**23**	.	.	.	**22**	**24**	**24**	**24**

STEP 3

The value of p is now 1. The algorithm seeks to site one centre in this cluster.

STEP 4

From the matrix in Table A4.1 the Least of the Maximum Nodal Distances (**LMND**) is 13.

This originates from node PS. It is between nodes PS and OD, PS and AS, PS and AU and PS and EM.

CONSDERING THE PATH BETWEEN NODES PS AND OD

From the network in figure 4.0, the path between nodes PS and OD is PS – AI – SP – YW – DM – OD.

Node PS is designated as p_1. Therefore the initial approximation for the 1-centre is node PS. The corresponding radius r_1 = 13km.

STEP 5

From the path n_1 = node OD, d_1 = 13, n_2 = node DM, d_2 = 11

REDUCTION OF COVERAGE DISTANCE

Expected distance to be displaced = $\frac{d_1 - d_2}{2} = \frac{13-11}{2} = 1$

STEP 6

Centre node PS is displaced towards node OD by a distance of 1km on the path between nodes PS and OD. The new 1-centre is therefore placed 1km away from node PS on the link between PS and AI. This is the new p_1.

The corresponding distance is $r_1 - \frac{d_1 - d_2}{2} = 13 - \frac{13-11}{2} = 12$

The new r_1 = 12km.

STEP 7

Table 4.5 below gives the distances between the new 1-centre and the clustered nodes.

Table 4.5 Distances between the Clustered Nodes and the New Centre, New P_1

	NEW p_1		NEW p_1		NEW p_1		NEW p_1
YW	9	**PS**	1	**OM**	5	**AU**	14
DM	10	**AR**	4	**AW**	11	**EM**	14
BK	10	**AK**	6	**NE**	10	**TD**	13
SP	6	**AO**	7	**KT**	8	**KN**	10
DU	6	**AA**	9	**AN**	7	**AY**	12
AI	3	**OD**	12	**BD**	10	**BM**	11
AM	4	**AS**	13	**KB**	12	**MH**	12

Since the distances between the new p_1 and nodes AS, AU, EM and TD are beyond the new r_1 this reduction cannot hold and so the 1-ccentre is maintained at node PS with corresponding distance as 13km.

CONSDERING THE PATH BETWEEN NODES PS AND AU

From the network in figure 4.0 the path between nodes PS and AU is PS – AR – OM – AN – KB – AU.

Node PS is designated as p_1. Therefore the initial approximation for the 1-centre is node PS. The corresponding radius $r_1 = 13$.

STEP 5

From the path n_1 = AU, $d_1 = 13$, n_2 = KB, $d_2 = 11$

REDUCTION OF COVERAGE DISTANCE

Expected distance to be displaced = $\dfrac{d_1 - d_2}{2} = \dfrac{13-11}{2} = 1$

STEP 6

Centre node PS is displaced towards node AU by a distance of 1km on the path between PS and AU. The new 1-centre is therefore placed 1km away from node PS on the link between PS and AR. This is the new p_1.

The corresponding distance is $r_1 - \frac{d_1 - d_2}{2} = 13 - \frac{13-11}{2} = 12$

The new r_1 = 12km.

STEP 7

Table 4.6 below gives the distances between the new 1-centre and the clustered nodes.

Table 4.6 Distances between the Clustered Nodes and the New Centre, New P_1

	NEW p_1		NEW p_1		NEW p_1		NEW p_1
YW	11	**PS**	1	**OM**	3	**AU**	12
DM	12	**AR**	2	**AW**	12	**EM**	12
BK	10	**AK**	2	**NE**	8	**TD**	11
SP	8	**AO**	7	**KT**	6	**KN**	8
DU	7	**AA**	10	**AN**	5	**AY**	10
AI	5	**OD**	14	**BD**	8	**BM**	10
AM	4	**AS**	14	**KB**	10	**MH**	13

Since the distances between the new p_1 and nodes OD, AS and MH are beyond the new r_1 this reduction cannot hold and so the 1-ccentre is maintained at node PS with corresponding distance as 13km.

CENTRE NODE AE

The part of the network in figure 4.0 corresponding to the nodes clustered around centre node AE is put into a 28 by 28 square matrix as shown in Table A4.2 in Appendix 4.0. A summary is shown in Table 4.7 below.

Table 4.7 Summary of Matrix of Clustered Nodes around Node **AE** in Fig 4.0 Indicating Towns and their Pair of Distances

	PA	SS	SH	RG	.	.	.	PS	AR	AK	AO	.	.	.	AE	AH	AT	KP
PA	0	1	2	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-
SS	1	0	-	9	.	.	.	-	-	-	-	.	.	.	-	-	-	-
SH	2	-	0	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-
RG	-	9	-	0	.	.	.	-	-	-	-	.	.	.	-	-	-	-
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PS	-	-	-	-	.	.	.	0	3	-	-	.	.	.	-	-	-	-
AR	-	-	-	-	.	.	.	3	0	-	-	.	.	.	-	-	-	-
AK	-	-	-	-	.	.	.	-	-	0	4	.	.	.	-	-	-	-
AO	-	-	-	-	.	.	.	-	-	4	0	.	.	.	-	-	-	-
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AE	-	-	-	-	.	.	.	-	-	-	-	.	.	.	0	4	-	-
AH	-	-	-	-	.	.	.	-	-	-	-	.	.	.	4	0	1	-
AT	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	1	0	2
KP	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	-	2	0

STEPS 1 and 2

The Floyd-Warshall All pairs shortest path algorithm is applied to the matrix in Table A4.2 to obtain the shortest distance matrix between pair of nodes as displayed in Table A4.3 in Appendix 4.0. A summary is shown in Table 4.8 below. A row is created in the distance matrix for the **MND**

Table 4.8 Summary of Shortest Distance Matrix between Pair of Nodes of Centre Node **AE**

	PA	SS	RG	DY	.	.	.	GO	SK	AV	AZ	.	.	.	KP	AQ	SH	DK
PA	0	1	8	5	.	.	.	21	18	14	13	.	.	.	22	14	2	27
SS	1	0	9	6	.	.	.	22	19	15	14	.	.	.	23	15	3	28
RG	8	9	0	3	.	.	.	13	10	6	5	.	.	.	14	12	6	19
DY	5	6	3	0	.	.	.	16	13	9	8	.	.	.	17	9	3	22
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
GO	21	22	13	16	.	.	.	0	3	7	9	.	.	.	12	16	19	17
SK	18	19	10	13	.	.	.	3	0	4	6	.	.	.	14	13	16	19
AV	14	15	6	9	.	.	.	7	4	0	2	.	.	.	10	12	12	15
AZ	13	14	5	8	.	.	.	9	6	2	0	.	.	.	9	11	11	14
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
KP	22	23	14	17	.	.	.	12	14	10	9	.	.	.	0	20	20	7
AQ	14	15	12	9	.	.	.	16	13	12	11	.	.	.	20	0	12	25
SH	2	3	6	3	.	.	.	19	16	12	11	.	.	.	20	11	0	25
DK	27	28	19	22	.	.	.	17	19	15	14	.	.	.	7	25	25	0
MND	**27**	**28**	**19**	**22**	.	.	.	**22**	**19**	**15**	**14**	.	.	.	**23**	**25**	**25**	**28**

STEP 3

The value of p is now 1. The algorithm seeks to site one centre in this cluster.

STEP 4

From the matrix in Table A4.3 the Least of the Maximum Nodal Distances (**LMND**) is 14. This originates from node AZ. It is between nodes AZ and DK and AZ and SS.

CONSDERING THE PATH BETWEEN NODES AZ AND DK

From the network in figure 4.0, the path between nodes AZ and DK is AZ – KA – AH – AT – GY – KO – DK.

Node AZ is designated as p_1. Therefore the initial approximation for the 1-centre is node AZ. The corresponding radius r_1 = 14km.

STEP 5

From the path n_1 = node DK, d_1 = 14, n_2 = node KO, d_2 = 12

REDUCTION OF COVERAGE DISTANCE

Expected distance to be displaced = $\dfrac{d_1 - d_2}{2} = \dfrac{14-12}{2} = 1$

STEP 6

Centre node AZ is displaced towards node DK by a distance of 1km on the path between AZ and DK. The new 1-centre is therefore placed 1km away from node AZ on the link between AZ and KA. This is the new p_1.

The corresponding distance is $r_1 - \dfrac{d_1 - d_2}{2} = 14 - \dfrac{14-12}{2} = 13$

The new r_1 = 13km.

STEP 7

Table 4.9 below gives the distances between the new 1-centre and the clustered nodes.

Table 4.9 Distances between the Clustered Nodes and the New Centre, New P_1

	NEW p_1		NEW p_1		NEW p_1		NEW p_1
PA	14	**DB**	6	**AV**	3	**GY**	9
SS	15	**AL**	8	**AZ**	1	**AH**	5
RG	6	**DA**	5	**KA**	2	**AT**	6
DY	9	**AX**	8	**DP**	4	**KP**	8
NA	3	**AE**	7	**AJ**	8	**AQ**	12
SN	8	**GO**	10	**KO**	11	**SH**	12
FA	9	**SK**	7	**OF**	11	**DK**	13

Since the distances between the new p_1 and nodes PA and SS are beyond the new r_1 this reduction cannot hold and so the 1-ccentre is maintained at node AZ with corresponding distance as 14km.

4.4 DISCUSSION OF FINDINGS

At the beginning of the procedure it is realised that when the **GMND** was 48 the absolute 2-centres would have been located at nodes AR and AE with a coverage distance of 32km for the network. When the network is then clustered into two, it is seen that one centre is located at node PS and the other is located at node AZ. The coverage distance is now reduced to 14km.

When further reductions are carried out it is seen from Tables 4.5, 4.6 and 4.9 that certain nodes are not covered by facilities. The optimal location of the facilities is at nodes PS and AZ with an objective function value of 14km.

The 2-centres are thus located at node **PS** and node **AZ** with the 2-radius for the network in figure 4.0 being 14km.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

The objective of the study was to determine two optimal locations for siting fire stations. The results as discussed in chapter four, section 4.4 show that the two fire stations should be located at Pakoso (PS) in the Asawase sub-metro and Adiebeba (AZ) in the Nhyiaeso sub-metro. The objective function value (coverage distance) is 14km.

The facility at Pakoso will serve the following towns optimally; Yennyawoso, Dichemso, Buokrom, Sepetimpom, Duase, Asabi, Asokore Mampong, Aprade, Akrom, Adukrom, Aboabo, Odumase, Ashtown, Oduom Asawase, Nsenie, Kentinkrono, Anwomaso, Boadi, Kokoben, Apeadu, Emena, Twumduase, KNUST, Ayigya, Bomso, Manhyia and Pakoso itself.

The facilitiy at Adiebeba will also serve the following towns; North Patasi, South Suntreso, Ridge, Nhyiaeso, Santase, Fankyenebra, Adiembra, Atasomanso, Daban, Apramang, Asokwa, Asago, Sokoban, Ahodwo, Kaase, Dompoase, Apirabo, Kotei, Oforikrom, Gyinyase, Ahinsan, Atonsu, Kyirapatre, Anyinam, South Patasi, Deduako and Adiebeba itself.

Figure 5.0 below shows the sites for the facilities on the network of towns and settlements in the five sub-metros. The figure also shows the clustered towns and settlements and their respective facility.

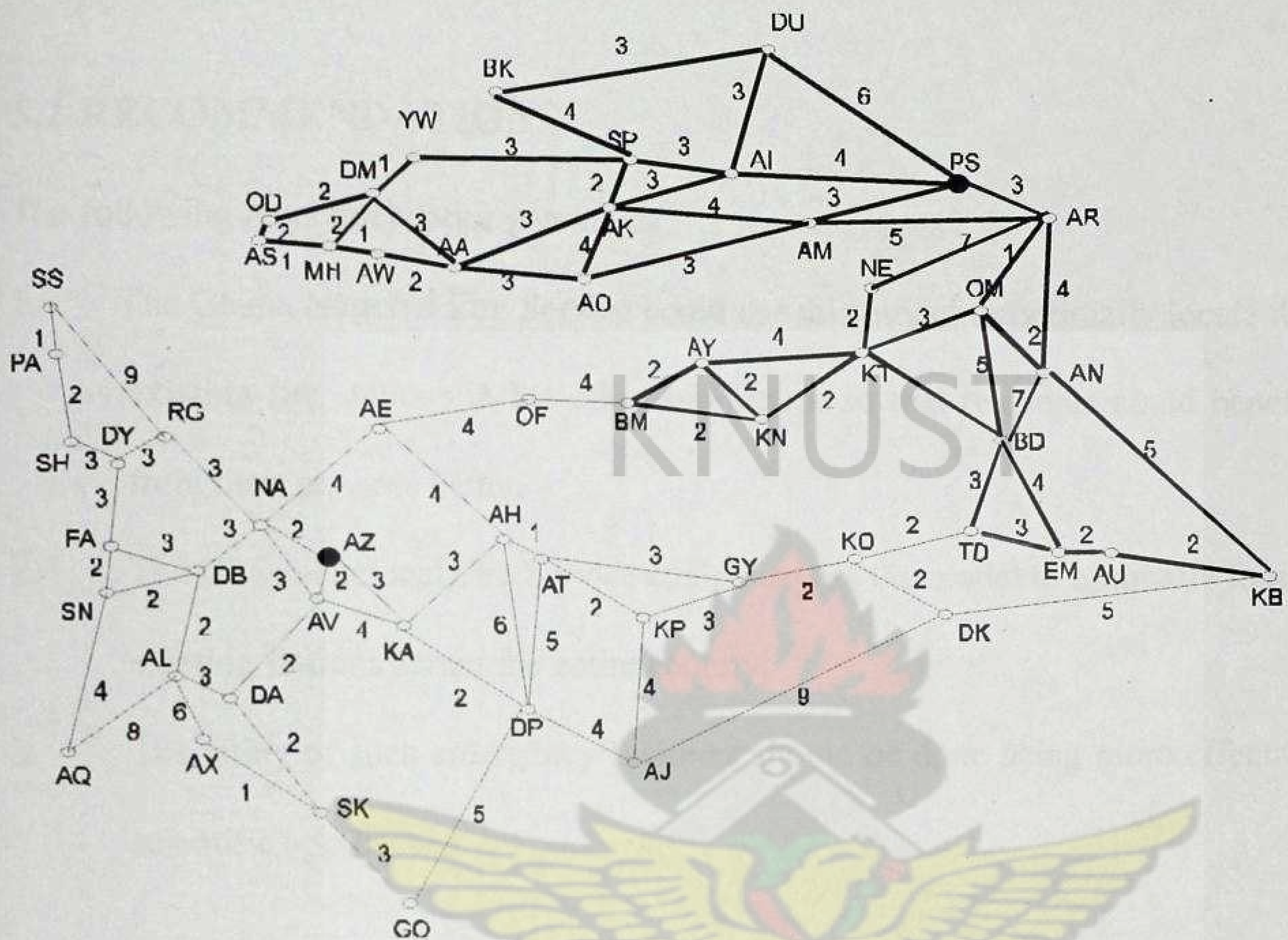

Figure 5.0 Network of towns and settlements in the five sub-metros indicating the location of the 2-centres and their respective nodes they serve.

The facilities should be located at an open space in the two towns with no obstruction to their way to and out of the two towns. Each of the sites should have a watch-room from where information on fire outbreaks would be directed with qualified personnel to man the offices. The offices of these facilities should be stocked with modern fire fighting gadgets and possibly modern fire tenders to make it a bit easier for officers stationed at these offices to fight various fire outbreaks effectively. The offices should have modern communication gadgets so that residents are able to reach them easily when the need arises. Opinion leaders in the towns where these facilities are should assist the officers to locate possible sites to serve as water hydrants.

5.2 RECOMMENDATIONS

The following recommendations are made.

1. The Ghana National Fire Service could use this model to optimally locate the existing fire stations in the other sub-metros so that residents could benefit from their services better.
2. The Ghana National Fire Service could also use the model to optimally locate existing stations across the entire country.
3. The siting of such emergency facilities should be done using more effective scientific approaches.

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APPENDIX 1.0

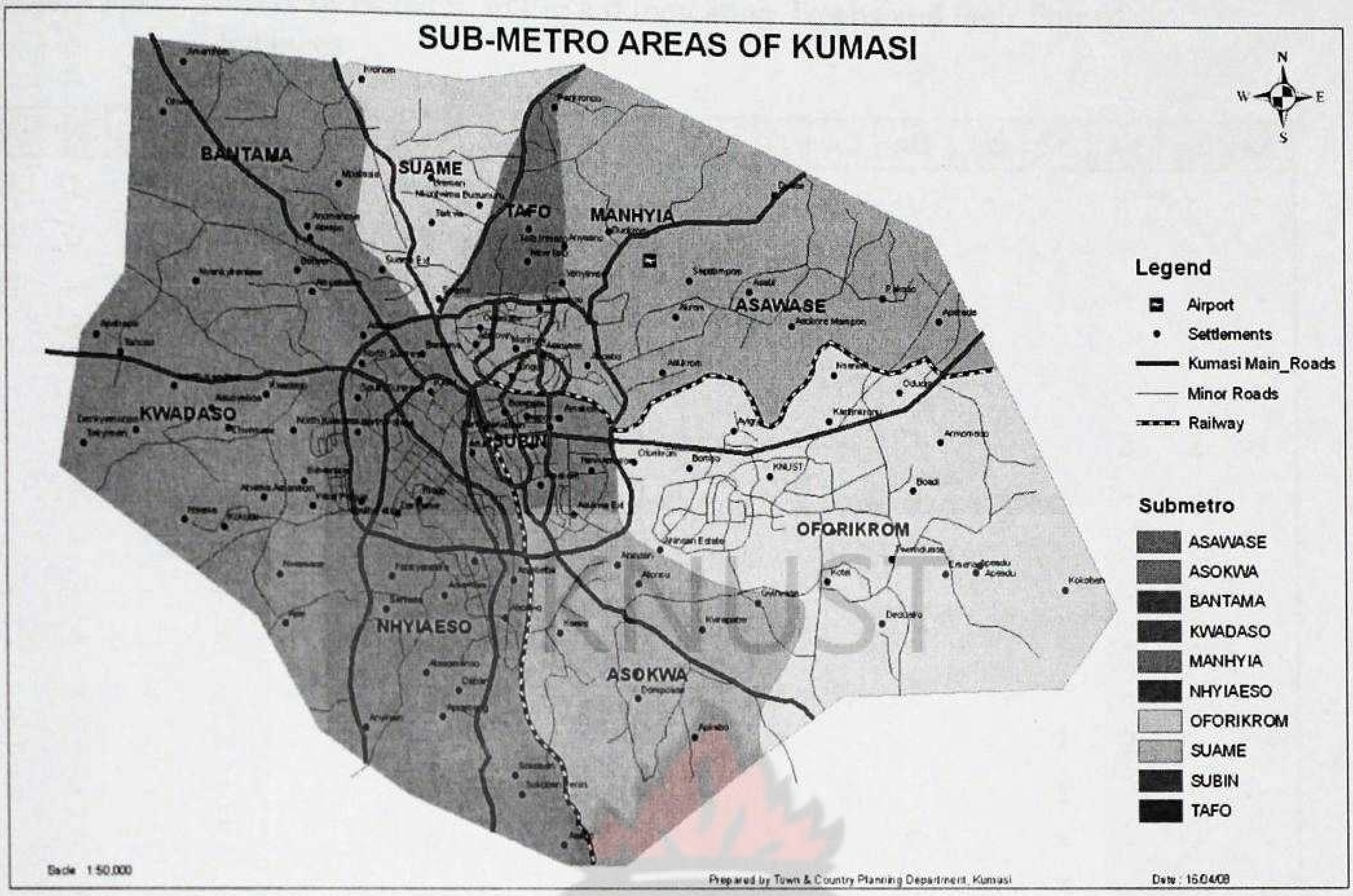

Figure A1.0 Sub-Metro Areas of Kumasi Metropolitan Assembly

APPENDIX 2.0

Table A2.0 Matrix of Network in Fig 4.0 Indicating Towns and their Pair of Distances

	PA	SS	SH	RG	DY	NA	SN	FA	YW	DB	AL	DA	AX	AQ	GO	SK	AV	AZ	KA
PA	0	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SS	1	0	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SH	2	-	0	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
RG	-	9	-	0	3	3	-	-	-	-	-	-	-	-	-	-	-	-	-
DY	-	-	3	3	0	-	-	3	-	-	-	-	-	-	-	-	-	-	-
NA	-	-	-	3	-	0	-	-	-	3	-	-	-	-	-	-	3	2	-
SN	-	-	-	-	-	-	0	2	-	2	-	-	-	4	-	-	-	-	-
FA	-	-	-	-	3	-	2	0	-	3	-	-	-	-	-	-	-	-	-
YW	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-
DB	-	-	-	-	-	3	2	3	-	0	2	-	-	-	-	-	-	-	-
AL	-	-	-	-	-	-	-	-	-	2	0	3	6	8	-	-	-	-	-
DA	-	-	-	-	-	-	-	-	-	-	3	0	-	-	-	2	2	-	-
AX	-	-	-	-	-	-	-	-	-	-	6	-	0	-	-	1	-	-	-
AQ	-	-	-	-	-	-	4	-	-	-	8	-	-	0	-	-	-	-	-
GO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	3	-	-	-
SK	-	-	-	-	-	-	-	-	-	-	-	2	1	-	3	0	-	-	-
AV	-	-	-	-	-	3	-	-	-	-	-	2	-	-	-	-	0	2	4
AZ	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	2	0	3
KA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	3	0
DP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	2
DM	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-
BK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SP	-	-	-	-	-	-	-	-	3	-	-	-	-	-	-	-	-	-	-
DU	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AI	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
PS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
OD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	DP	DM	BK	SP	DU	AI	AM	PS	AR	AK	AO	AA	OD	AS	MH	AW	AJ	OM	NE
PA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
RG	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
NA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
FA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
YW	-	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AL	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AX	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AQ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
GO	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AV	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AZ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KA	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DP	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-
DM	-	0	-	-	-	-	-	-	-	-	-	3	2	-	2	-	-	-	-
BK	-	-	0	4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SP	-	-	4	0	-	3	-	-	-	2	-	-	-	-	-	-	-	-	-
DU	-	-	3	-	0	3	-	6	-	-	-	-	-	-	-	-	-	-	-
AI	-	-	-	3	3	0	-	4	-	3	-	-	-	-	-	-	-	-	-
AM	-	-	-	-	-	-	0	3	5	4	3	-	-	-	-	-	-	-	-
PS	-	-	-	-	6	4	3	0	3	-	-	-	-	-	-	-	-	-	-
AR	-	-	-	-	-	-	5	3	0	-	-	-	-	-	-	-	-	1	7
AK	-	-	-	2	-	3	4	-	-	0	4	3	-	-	-	-	-	-	-
AO	-	-	-	-	-	-	3	-	-	4	0	3	-	-	-	-	-	-	-
AA	-	3	-	-	-	-	-	-	-	3	3	0	-	-	-	2	-	-	-
OD	-	2	-	-	-	-	-	-	-	-	-	-	0	2	-	-	-	-	-
AS	-	-	-	-	-	-	-	-	-	-	-	-	2	0	1	-	-	-	-

	KT	AN	BD	KB	AU	EM	TD	DK	KO	KN	AY	BM	OF	GY	AE	AH	AT	KP
PA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
RG	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
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AL	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AX	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
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SK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AV	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AZ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-
DP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	5	-
DM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
BK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DU	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AI	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
PS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
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AS	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	PA	SS	SH	RG	DY	NA	SN	FA	YW	DB	AL	DA	AX	AQ	GO	SK	AV	AZ	KA
MH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AW	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AJ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
OM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
NE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
BD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AU	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
EM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
TD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
BM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
OF	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
GY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AE	-	-	-	-	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-
AH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
AT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	DP	DM	BK	SP	DU	AI	AM	PS	AR	AK	AO	AA	OD	AS	MH	AW	AJ	OM	NE
MH	-	2	-	-	-	-	-	-	-	-	-	-	-	1	0	1	-	-	-
AW	-	-	-	-	-	-	-	-	-	-	-	2	-	-	1	0	-	-	-
AJ	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-
OM	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	0	-
NE	-	-	-	-	-	-	-	-	7	-	-	-	-	-	-	-	-	-	0
KT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	2
AN	-	-	-	-	-	-	-	-	4	-	-	-	-	-	-	-	-	2	-
BD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-
KB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AU	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
EM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
TD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	-	-
KO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
BM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
OF	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
GY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AH	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AT	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-

	KT	AN	BD	KB	AU	EM	TD	DK	KO	KN	AY	BM	OF	GY	AE	AH	AT	KP
MH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AW	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AJ	-	-	-	-	-	-	-	9	-	-	-	-	-	-	-	-	-	4
OM	3	2	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
NE	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KT	0	-	3	-	-	-	-	-	-	2	4	-	-	-	-	-	-	-
AN	-	0	7	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-
BD	3	7	0	-	-	4	3	-	-	-	-	-	-	-	-	-	-	-
KB	-	5	-	0	2	-	-	5	-	-	-	-	-	-	-	-	-	-
AU	-	-	-	2	0	2	-	-	-	-	-	-	-	-	-	-	-	-
EM	-	-	4	-	2	0	3	-	-	-	-	-	-	-	-	-	-	-
TD	-	-	3	-	-	3	0	-	2	-	-	-	-	-	-	-	-	-
DK	-	-	-	5	-	-	-	0	2	-	-	-	-	-	-	-	-	-
KO	-	-	-	-	-	-	2	2	0	-	-	-	-	2	-	-	-	-
KN	2	-	-	-	-	-	-	-	-	0	2	2	-	-	-	-	-	-
AY	4	-	-	-	-	-	-	-	-	2	0	2	-	-	-	-	-	-
BM	-	-	-	-	-	-	-	-	-	2	2	0	4	-	-	-	-	-
OF	-	-	-	-	-	-	-	-	-	-	-	4	0	-	4	-	-	-
GY	-	-	-	-	-	-	-	-	2	-	-	-	-	0	-	-	3	3
AE	-	-	-	-	-	-	-	-	-	-	-	-	4	-	0	4	-	-
AH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	0	1	-
AT	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	1	0	2
KP	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-	2	0

APPENDIX 3.0

Table A3.0 Shortest Distance Matrix between Pair of Nodes

	PA	SS	SH	RG	DY	NA	SN	FA	YW	DB	AL	DA	AX	AQ	GO	SK	AV	AZ	KA
PA	0	1	2	8	5	11	10	8	44	11	13	16	19	14	21	18	14	13	16
SS	1	0	3	9	6	12	11	9	45	12	14	17	20	15	22	19	15	14	17
SH	2	3	0	6	3	9	8	6	42	9	11	14	17	12	19	16	12	11	14
RG	8	9	6	0	3	3	8	6	36	6	8	8	11	12	13	10	6	5	8
DY	5	6	3	3	0	6	5	3	39	6	8	11	14	9	16	13	9	8	11
NA	11	12	9	3	6	0	5	6	33	3	5	5	8	9	10	7	3	2	5
SN	10	11	8	8	5	5	0	2	38	2	4	7	10	4	12	9	8	7	10
FA	8	9	6	6	3	6	2	0	39	3	5	8	11	6	13	10	9	8	11
YW	44	45	42	36	39	33	38	39	0	36	38	38	41	42	39	40	36	35	33
DB	11	12	9	6	6	3	2	3	36	0	2	5	8	6	10	7	6	5	8
AL	13	14	11	8	8	5	4	5	38	2	0	3	6	8	8	5	5	7	9
DA	16	17	14	8	11	5	7	8	38	5	3	0	3	11	5	2	2	4	6
AX	19	20	17	11	14	8	10	11	41	8	6	3	0	14	4	1	5	7	9
AQ	14	15	12	12	9	9	4	6	42	6	8	11	14	0	16	13	12	11	14
GO	21	22	19	13	16	10	12	13	39	10	8	5	4	16	0	3	7	9	7
SK	18	19	16	10	13	7	9	10	40	7	5	2	1	13	3	0	4	6	8
AV	14	15	12	6	9	3	8	9	36	6	5	2	5	12	7	4	0	2	4
AZ	13	14	11	5	8	2	7	8	35	5	7	4	7	11	9	6	2	0	3
KA	16	17	14	8	11	5	10	11	33	8	9	6	9	14	7	8	4	3	0
DP	18	19	16	10	13	7	12	13	34	10	11	8	9	16	5	8	6	5	2
DM	45	46	43	37	40	34	39	40	1	37	39	39	42	43	40	41	37	36	34
BK	43	44	41	35	38	32	37	38	7	35	37	37	40	41	38	39	35	34	32
SP	41	42	39	33	36	30	35	36	3	33	35	35	38	39	36	37	33	32	30
DU	40	41	38	32	35	29	34	35	9	32	34	34	37	38	35	36	32	31	29
AI	38	39	36	30	33	27	32	33	6	30	32	32	35	36	33	34	30	29	27
AM	36	37	34	28	31	25	30	31	9	28	30	30	33	34	31	32	28	27	25
PS	34	35	32	26	29	23	28	29	10	26	28	28	31	32	29	30	26	25	23
AR	30	32	29	23	26	20	25	26	13	23	25	25	28	29	26	27	23	22	20

	DP	DM	BK	SP	DU	AI	AM	PS	AR	AK	AO	AA	OD	AS	MH	AW	AJ	OM	NE
PA	18	45	43	41	40	38	36	34	31	40	39	42	47	46	45	44	22	30	29
SS	19	46	44	42	41	39	37	35	32	41	40	43	48	47	46	45	23	31	30
SH	16	43	41	39	38	36	34	32	29	38	37	40	45	44	43	42	20	28	27
RG	10	37	35	33	32	30	28	26	23	32	31	34	39	38	37	36	14	22	21
DY	13	40	38	36	35	33	31	29	26	35	34	37	42	41	40	39	17	25	24
NA	7	34	32	30	29	27	25	23	20	29	28	31	36	35	34	33	11	19	18
SN	12	39	37	35	34	32	30	28	25	34	33	36	41	40	39	38	16	24	23
FA	13	40	38	36	35	33	31	29	26	35	34	37	42	41	40	39	17	25	24
YW	34	1	7	3	9	6	9	10	13	5	7	4	3	4	3	4	33	14	19
DB	10	37	35	33	32	30	28	26	23	32	31	34	39	38	37	36	14	22	21
AL	11	39	37	35	34	32	30	28	25	34	33	36	41	40	39	38	15	24	23
DA	8	39	37	35	34	32	30	28	25	34	33	36	41	40	39	38	12	24	23
AX	9	42	40	38	37	35	33	31	28	37	36	39	44	43	42	41	13	27	26
AQ	16	43	41	39	38	36	34	32	29	36	37	40	45	44	43	42	20	28	27
GO	5	40	38	36	35	33	31	29	25	35	34	37	42	41	40	39	9	25	25
SK	8	41	39	37	36	34	32	30	27	36	35	38	43	42	41	40	12	26	25
AV	6	37	35	33	32	30	28	26	23	32	31	34	39	38	37	36	10	22	21
AZ	5	36	34	32	31	29	27	25	22	31	30	33	38	37	36	35	9	21	20
KA	2	34	32	30	29	27	25	23	20	29	28	31	36	35	34	33	6	19	19
DP	0	35	33	31	30	28	26	24	21	30	29	32	37	36	35	34	4	20	20
DM	35	0	8	4	10	7	9	11	14	6	6	3	2	3	2	3	34	15	20
BK	33	8	0	4	3	6	10	9	12	6	10	9	10	11	10	11	32	13	18
SP	31	4	4	0	6	3	6	7	10	2	6	5	6	7	6	7	30	11	16
DU	30	10	3	6	0	3	9	6	9	6	10	9	12	13	12	11	29	10	15
AI	28	7	6	3	3	0	7	4	7	3	7	6	9	10	9	8	27	8	13
AM	26	9	10	6	9	7	0	3	5	4	3	6	11	10	9	8	25	6	11
PS	24	11	9	7	6	4	3	0	3	7	6	9	13	13	12	11	23	4	9
AR	21	14	12	10	9	7	5	3	0	9	8	11	16	15	14	13	20	1	6

	KT	AN	BD	KB	AU	EM	TD	DK	KO	KN	AY	BM	OF	GY	AE	AH	AT	KP
PA	27	32	30	32	32	30	27	27	25	25	25	23	19	23	15	19	20	22
SS	28	33	31	33	33	31	28	28	26	26	26	24	20	24	16	20	21	23
SH	25	30	28	30	30	28	25	25	23	23	23	21	17	21	13	17	18	20
RG	19	24	22	24	24	22	19	19	17	17	17	15	11	15	7	11	12	14
DY	22	27	25	27	27	25	22	22	20	20	20	18	14	18	10	14	15	17
NA	16	21	19	21	21	19	16	16	14	14	14	12	8	12	4	8	9	11
SN	21	26	24	26	26	24	21	21	19	19	19	17	13	17	9	13	14	16
FA	22	27	25	27	27	25	22	22	20	20	20	18	14	18	10	14	15	17
YW	17	16	19	21	23	23	22	26	24	19	21	21	25	26	29	30	29	29
DB	19	24	22	24	24	22	19	19	17	17	17	15	11	15	7	11	12	14
AL	21	26	23	25	25	23	20	20	18	19	19	17	13	16	9	12	13	15
DA	21	26	20	22	22	20	17	17	15	19	19	17	13	13	9	9	10	12
AX	24	29	23	25	25	23	20	20	18	22	22	20	16	16	12	12	13	15
AQ	25	30	28	30	30	28	25	25	23	23	23	21	17	21	13	17	18	20
GO	23	27	20	22	22	20	17	17	15	24	24	22	18	13	14	10	10	12
SK	23	28	22	24	24	22	19	19	17	21	21	19	15	15	11	11	12	14
AV	19	24	18	20	20	18	15	15	13	17	17	15	11	11	7	7	8	10
AZ	18	23	17	19	19	17	14	14	12	16	16	14	10	10	6	6	7	9
KA	17	21	14	16	16	14	11	11	9	17	17	15	11	7	7	3	4	6
DP	18	22	15	17	17	15	12	12	10	19	19	17	13	8	9	5	5	7
DM	18	17	20	22	24	24	23	27	25	20	22	22	26	27	30	31	30	30
BK	16	15	18	20	27	22	22	25	23	18	20	20	24	25	28	29	28	28
SP	14	13	16	18	20	20	19	23	21	16	18	18	22	23	26	27	26	26
DU	13	13	15	17	19	19	18	22	20	15	17	17	21	22	25	26	25	25
AI	11	10	13	15	17	17	16	20	18	13	15	15	19	20	23	24	23	23
AM	9	8	11	13	15	15	14	18	16	11	13	13	17	18	21	22	21	21
PS	7	6	9	11	13	13	12	16	14	9	11	11	15	16	19	20	19	19
AR	4	3	6	8	10	10	9	13	11	6	8	8	12	13	16	17	16	16

	PA	SS	SH	RG	DY	NA	SN	FA	YW	DB	AL	DA	AX	AQ	GO	SK	AV	AZ	KA
AK	40	41	38	32	35	29	34	35	5	32	34	34	37	38	35	36	32	31	29
AO	39	40	37	31	34	28	33	34	7	31	33	33	36	37	34	35	31	30	28
AA	42	43	40	34	37	31	36	37	4	34	36	36	39	40	37	38	34	33	31
OD	47	48	45	39	42	36	41	42	3	39	41	41	44	45	42	43	39	38	36
AS	46	47	44	38	41	35	40	41	4	38	40	40	43	44	41	42	38	37	35
MH	45	46	43	37	40	34	39	40	3	37	39	39	42	43	40	41	37	36	34
AW	44	45	42	36	39	33	38	39	4	36	38	38	41	42	39	40	36	35	33
AJ	22	23	20	14	17	11	16	17	33	14	15	12	13	20	9	12	10	9	6
OM	30	31	28	22	25	19	24	25	14	22	24	24	27	28	25	26	22	21	19
NE	29	30	27	21	24	18	23	24	19	21	23	23	26	27	25	25	21	20	19
KT	27	28	25	19	22	16	21	22	17	19	21	21	24	25	23	23	19	18	17
AN	32	33	30	24	27	21	26	27	16	24	26	26	29	30	27	28	24	23	21
BD	30	31	28	22	25	19	24	25	19	22	23	20	23	28	20	22	18	17	14
KB	32	33	30	24	27	21	26	27	21	24	25	22	25	30	22	24	20	19	16
AU	32	33	40	24	27	21	26	27	23	24	25	22	25	30	22	24	20	19	16
EM	30	31	28	22	25	19	24	25	23	22	23	20	23	28	20	22	18	17	14
TD	27	28	25	19	22	16	21	22	22	19	20	17	20	25	17	19	15	14	11
DK	27	28	25	19	22	16	21	22	26	19	20	17	20	25	17	19	15	14	11
KO	25	26	23	17	20	14	19	20	24	17	18	15	18	23	15	17	13	12	9
KN	25	26	23	17	20	14	19	20	19	17	18	19	22	23	24	21	17	16	17
AY	25	26	23	17	20	14	19	20	21	17	18	19	22	23	24	21	17	16	17
BM	23	24	21	15	18	12	17	18	21	15	17	17	20	21	22	19	15	14	15
OF	19	20	17	11	14	8	13	14	25	11	13	13	16	17	18	15	11	10	11
GY	23	24	21	15	18	12	17	18	26	15	16	13	16	21	13	15	11	10	7
AE	15	16	13	7	10	4	9	10	29	7	9	9	12	13	14	11	7	6	7
AH	19	20	17	11	14	8	13	14	30	11	12	9	12	17	10	11	7	6	3
AT	20	21	18	12	15	9	14	15	29	12	13	10	13	18	10	12	8	7	4
KP	22	23	20	14	17	11	16	17	29	14	15	12	15	20	12	14	10	9	6
MND	**47**	**48**	**45**	**39**	**42**	**36**	**41**	**42**	**45**	**39**	**41**	**41**	**44**	**45**	**42**	**44**	**39**	**38**	**36**

	DP	DM	BK	SP	DU	AI	AM	PS	AR	AK	AO	AA	OD	AS	MH	AW	AJ	OM	NE
AK	30	6	6	2	6	3	4	7	9	0	4	3	8	7	6	5	29	10	15
AO	29	6	10	6	10	7	3	6	8	4	0	3	8	7	6	5	28	9	14
AA	32	3	9	5	9	6	6	9	11	3	3	0	5	4	3	2	31	12	17
OD	37	2	10	6	12	9	11	13	16	8	8	5	0	2	3	4	36	17	22
AS	36	3	11	7	13	10	10	13	15	7	7	4	2	0	1	2	35	16	21
MH	35	2	10	6	12	9	9	12	14	6	6	3	3	1	0	1	34	15	20
AW	34	3	11	7	11	8	8	11	13	5	5	2	4	2	1	0	33	14	19
AJ	4	34	32	30	29	27	25	23	20	29	28	31	36	35	34	33	0	19	19
OM	20	15	13	11	10	8	6	4	1	10	9	12	17	16	15	14	19	0	5
NE	20	20	18	16	15	13	11	9	6	15	14	17	22	21	20	19	19	5	0
KT	18	18	16	14	13	11	9	7	4	13	12	15	20	19	18	17	17	3	2
AN	22	17	15	13	12	10	8	6	3	12	11	14	19	18	17	16	19	2	7
BD	15	20	18	16	15	13	11	9	6	15	14	17	22	21	20	19	14	5	5
KB	17	22	20	18	17	15	13	11	18	17	16	19	24	23	22	21	14	7	12
AU	17	24	22	20	19	17	15	13	10	19	18	21	26	25	24	23	16	9	11
EM	15	24	22	20	19	17	15	13	10	19	18	21	26	25	24	23	14	9	9
TD	12	23	21	19	18	16	14	12	19	18	17	20	25	24	23	22	11	8	8
DK	12	27	25	23	22	20	18	16	13	22	21	24	29	28	27	26	9	12	12
KO	10	25	23	21	20	18	16	14	11	20	19	22	27	26	25	24	9	10	10
KN	19	20	18	16	15	13	11	9	6	15	14	17	22	21	20	19	19	5	4
AY	19	22	20	18	17	15	13	11	8	17	16	19	24	23	22	21	21	7	6
BM	17	22	20	18	17	15	13	11	8	17	16	19	24	23	22	21	19	7	6
OF	13	26	24	22	21	19	17	15	12	21	20	23	28	27	26	25	15	11	10
GY	8	27	25	23	22	20	18	16	13	22	21	24	29	28	27	26	7	12	12
AE	9	30	28	26	25	23	21	19	16	25	24	27	32	31	30	29	11	15	14
AH	5	31	29	27	26	24	22	20	17	26	25	28	33	32	31	30	7	16	16
AT	5	30	28	26	25	23	21	19	16	25	24	27	32	31	30	29	6	15	15
KP	7	30	28	26	25	23	21	19	16	25	24	27	32	31	30	29	4	15	15
MND	**37**	**46**	**44**	**42**	**41**	**39**	**37**	**35**	**32**	**41**	**40**	**43**	**48**	**47**	**46**	**45**	**36**	**31**	**30**

	KT	AN	BD	KB	AU	EM	TD	DK	KO	KN	AY	BM	OF	GY	AE	AH	AT	KP
AK	13	12	15	17	19	19	18	22	20	15	17	17	21	22	25	26	25	25
AO	12	11	14	16	18	18	17	21	19	14	16	16	20	21	24	25	24	24
AA	15	14	17	19	21	21	20	24	22	17	19	19	23	24	27	28	27	27
OD	20	19	22	24	26	26	25	29	27	22	24	24	28	29	32	33	32	32
AS	19	18	21	23	25	25	24	28	26	21	23	23	27	28	31	32	31	31
MH	18	17	20	22	24	24	23	27	25	20	22	22	26	27	30	31	30	30
AW	17	16	19	21	23	23	22	26	24	19	21	21	25	26	29	30	29	29
AJ	17	19	14	14	16	14	11	9	9	19	21	19	15	7	11	7	6	4
OM	3	2	5	7	9	9	8	12	10	5	7	7	11	12	15	16	15	15
NE	2	7	5	12	11	9	8	12	10	4	6	6	10	12	14	16	15	15
KT	0	5	3	10	9	7	6	10	8	2	4	4	8	10	12	14	13	13
AN	5	0	7	5	7	9	10	10	12	1	9	9	13	14	17	18	17	17
BD	3	7	0	8	6	4	3	7	5	5	7	7	11	7	15	11	10	10
KB	10	5	8	0	2	4	7	5	7	12	14	14	18	9	17	13	12	12
AU	9	7	6	2	0	2	5	7	7	11	13	13	17	9	17	13	12	12
EM	7	9	4	4	2	0	3	7	5	9	11	11	15	7	15	11	10	10
TD	6	10	3	7	5	3	0	4	2	8	10	10	14	4	12	8	7	7
DK	10	10	7	5	7	7	4	0	2	12	14	14	16	4	12	8	7	7
KO	8	12	5	7	7	5	2	2	0	10	12	12	14	2	10	6	5	5
KN	2	7	5	12	11	9	8	12	10	0	2	2	6	12	10	14	15	15
AY	4	9	7	14	13	11	10	14	12	2	0	2	6	14	10	14	15	17
BM	4	9	7	14	13	11	10	14	12	2	2	0	4	14	8	12	13	15
OF	8	13	11	18	17	15	14	16	14	6	6	4	0	12	4	8	9	11
GY	10	14	7	9	9	7	4	4	2	12	14	14	12	0	8	4	3	3
AE	12	17	15	17	17	15	12	12	10	10	10	8	4	8	0	4	5	7
AH	14	18	11	13	13	11	8	8	6	14	14	12	8	4	4	0	1	3
AT	13	17	10	12	12	10	7	7	5	15	15	13	9	3	5	1	0	2
KP	13	17	10	12	12	10	7	7	5	15	17	15	11	3	7	3	2	0
MND	**28**	**33**	**31**	**33**	**33**	**31**	**28**	**29**	**27**	**26**	**26**	**24**	**28**	**29**	**32**	**33**	**32**	**32**

APPENDIX 4.0

Table A4.0 Matrix of Clustered Nodes around Node **AR** in Fig 4.0 Indicating Towns and their Pair of Distances

	YW	DM	BK	SP	DU	AI	AM	PS	AR	AK	AO	AA	OD	AS	OM	AW	NE	KT	AN	BD	KB	AU	EM	TD	KN	AY	BM	MH
YW	0	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DM	1	0	-	-	-	-	-	-	-	-	-	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
BK	-	-	0	4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SP	3	-	4	0	-	3	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DU	-	-	3	-	0	3	-	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AI	-	-	-	3	3	0	-	4	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AM	-	-	-	-	-	-	0	3	5	4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
PS	-	-	-	-	6	4	3	0	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR	-	-	-	-	-	-	5	3	0	-	-	-	-	-	1	-	5	-	4	-	-	-	-	-	-	-	-	-
AK	-	-	-	2	-	3	4	-	-	0	4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AO	-	-	-	-	-	-	3	-	-	4	0	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AA	-	3	-	-	-	-	-	-	-	3	3	0	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-
OD	-	2	-	-	-	-	-	-	-	-	-	-	0	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AS	-	-	-	-	-	-	-	-	-	-	-	-	2	0	-	-	-	-	-	-	-	-	-	-	-	-	-	1
OM	-	-	-	-	-	-	-	-	1	-	-	-	-	-	0	-	-	3	2	5	-	-	-	-	-	-	-	-
AW	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	0	-	-	-	-	-	-	-	-	-	-	-	1
NE	-	-	-	-	-	-	-	-	5	-	-	-	-	-	-	-	0	2	-	-	-	-	-	-	-	-	-	-
KT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	2	0	-	3	-	-	-	-	2	4	-	-
AN	-	-	-	-	-	-	-	-	4	-	-	-	-	-	2	-	-	-	0	7	5	-	-	-	-	-	-	-
BD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	3	7	0	-	-	4	3	-	-	-	-
KB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	0	2	-	-	-	-	-	-
AU	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	0	2	-	-	-	-	-
EM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	2	0	3	-	-	-	-
TD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-	3	0	-	-	-	-
KN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	0	2	2	-
AY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-	-	-	-	-	2	0	2	-
BM	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2	0	-
MH	-	2	-	-	-	-	-	-	-	-	-	-	-	1	-	1	-	-	-	-	-	-	-	-	-	-	-	0

Table A4.1 Shortest Distance Matrix between Pair of Nodes of Centre Node **AR**

	YW	DM	BK	SP	DU	AI	AM	PS	AR	AK	AO	AA	OD	AS	OM	AW	NE	KT	AN	BD	KB	AU	EM	TD	KN	AY	BM	MH
YW	0	1	7	3	9	6	9	10	13	5	7	4	3	4	14	4	18	17	16	19	21	23	23	22	19	21	21	3
DM	1	0	8	4	10	7	9	11	14	6	6	3	2	3	15	3	19	18	17	20	22	24	24	23	20	22	22	2
BK	7	8	0	4	3	6	10	9	12	6	10	9	10	11	13	11	17	16	15	18	20	22	22	21	18	20	20	10
SP	3	4	4	0	6	3	6	7	10	2	6	5	6	7	11	7	15	14	13	16	18	20	20	19	16	18	18	6
DU	9	10	3	6	0	3	9	6	9	6	10	9	12	13	10	11	14	13	12	15	17	19	19	18	15	17	17	12
AI	6	7	6	3	3	0	7	4	7	3	7	6	9	10	8	8	12	11	10	13	15	17	17	16	13	15	15	9
AM	9	9	10	6	9	7	0	3	5	4	3	6	11	10	6	8	10	9	8	11	13	15	15	14	11	13	13	9
PS	10	11	9	7	6	4	3	0	3	7	6	9	13	13	4	11	8	7	6	9	11	13	13	12	9	11	11	12
AR	13	14	12	10	9	7	5	3	0	9	8	11	16	15	1	13	5	4	3	6	8	10	10	9	6	8	8	14
AK	5	6	6	2	6	3	4	7	9	0	4	3	8	7	10	5	14	13	12	15	17	19	19	18	15	17	17	6
AO	7	6	10	6	10	7	3	6	8	4	0	3	8	7	9	5	13	12	11	14	16	18	18	17	14	16	16	6
AA	4	3	9	5	9	6	6	9	11	3	3	0	5	4	12	2	16	15	14	17	19	21	21	20	17	19	19	3
OD	3	2	10	6	12	9	11	13	16	8	8	5	0	2	17	4	21	20	19	22	24	26	26	25	22	24	24	3
AS	4	3	11	7	13	10	10	13	15	7	7	4	2	0	16	2	20	19	18	21	23	25	25	24	21	23	23	1
OM	14	15	13	11	10	8	6	4	1	10	9	12	17	16	0	14	5	3	2	5	7	9	9	8	5	7	7	15
AW	4	3	11	7	11	8	8	11	13	5	5	2	4	2	14	0	18	17	16	19	21	23	23	22	19	21	21	1
NE	18	19	17	15	14	12	10	8	5	14	13	16	21	20	5	18	0	2	7	5	12	11	9	8	4	6	6	19
KT	17	18	16	14	13	11	9	7	4	13	12	15	20	19	3	17	2	0	5	3	10	9	7	6	2	4	4	18
AN	16	17	15	13	12	10	8	6	3	12	11	14	19	18	2	16	7	5	0	7	5	7	9	10	7	9	9	17
BD	19	20	18	16	15	13	11	9	6	15	14	17	22	21	5	19	5	3	7	0	8	6	4	3	5	7	7	20
KB	21	22	20	18	17	15	13	11	8	17	16	19	24	23	7	21	12	10	5	8	0	2	4	7	12	14	14	22
AU	23	24	22	20	19	17	15	13	10	19	18	21	26	25	9	23	11	9	7	6	2	0	2	5	11	13	13	24
EM	23	24	22	20	19	17	15	13	10	19	18	21	26	25	9	23	9	7	9	4	4	2	0	3	9	11	11	24
TD	22	23	21	19	18	16	14	12	9	18	17	20	25	24	8	22	8	6	10	3	7	5	3	0	8	10	10	23
KN	19	20	18	16	15	13	11	9	6	15	14	17	22	21	5	19	4	2	7	5	12	11	9	8	0	2	2	20
AY	21	22	20	18	17	15	13	11	8	17	16	19	24	23	7	21	6	4	9	7	14	13	11	10	2	0	2	22
BM	21	22	20	18	17	15	13	11	8	17	16	19	24	23	7	21	6	4	9	7	14	13	11	10	2	2	0	22
MH	3	2	10	6	12	9	9	12	14	6	6	3	3	1	15	1	19	18	17	20	22	24	24	23	20	22	22	0
MND	**23**	**24**	**22**	**20**	**19**	**17**	**15**	**13**	**16**	**19**	**18**	**21**	**26**	**25**	**17**	**23**	**21**	**20**	**19**	**22**	**24**	**26**	**26**	**25**	**22**	**24**	**24**	**24**

Table A4.2 Matrix of Clustered Nodes around Node **AE** in Fig 4.0 Indicating Towns and their Pair of Distances

	PA	SS	RG	DY	NA	SN	FA	DB	AL	DA	AX	AE	GO	SK	AV	AZ	KA	DP	AJ	KO	OF	GY	AH	AT	KP	AQ	SH	DK
PA	0	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-
SS	1	0	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
RG	-	9	0	3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DY	-	-	3	0	-	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-
NA	-	-	3	-	0	-	-	3	-	-	-	4	-	-	3	2	-	-	-	-	-	-	-	-	-	-	-	-
SN	-	-	-	-	-	0	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-
FA	-	-	-	3	-	2	0	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DB	-	-	-	-	3	2	3	0	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AL	-	-	-	-	-	-	-	2	0	3	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	-	-
DA	-	-	-	-	-	-	-	-	3	0	-	-	-	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-
AX	-	-	-	-	-	-	-	-	6	-	0	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AE	-	-	-	-	4	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	4	-	4	-	-	-	-	-
GO	-	-	-	-	-	-	-	-	-	-	-	-	0	3	-	-	-	5	-	-	-	-	-	-	-	-	-	-
SK	-	-	-	-	-	-	-	-	-	2	1	-	3	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AV	-	-	-	-	3	-	-	-	-	2	-	-	-	-	0	2	4	-	-	-	-	-	-	-	-	-	-	-
AZ	-	-	-	-	2	-	-	-	-	-	-	-	-	-	2	0	3	-	-	-	-	-	-	-	-	-	-	-
KA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	3	0	2	-	-	-	-	3	-	-	-	-	-
DP	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	2	0	4	-	-	-	6	5	-	-	-	-
AJ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	0	-	-	-	-	-	4	-	-	9
KO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	2	-	-	-	-	-	2
OF	-	-	-	-	-	-	-	-	-	-	-	4	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-
GY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	0	-	3	3	-	-	-
AH	-	-	-	-	-	-	-	-	-	-	-	4	-	-	-	-	3	6	-	-	-	-	0	1	-	-	-	-
AT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	3	1	0	2	-	-	-
KP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-	3	-	2	0	-	-	-
AQ	-	-	-	-	-	4	-	-	8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-
SH	2	-	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-
DK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	2	-	-	-	-	-	-	-	0

Table A4.3 Shortest Distance Matrix between Pair of Nodes of Centre Node **AE**

	PA	SS	RG	DY	NA	SN	FA	DB	AL	DA	AX	AE	GO	SK	AV	AZ	KA	DP	AJ	KO	OF	GY	AH	AT	KP	AQ	SH	DK
PA	0	1	8	5	11	10	8	11	13	16	19	15	21	18	14	13	16	18	22	25	19	23	19	20	22	14	2	27
SS	1	0	9	6	12	11	9	12	14	17	20	16	22	19	15	14	17	19	23	26	20	24	20	21	23	15	3	28
RG	8	9	0	3	3	8	6	6	8	8	11	7	13	10	6	5	8	10	14	17	11	15	11	12	14	12	6	19
DY	5	6	3	0	6	5	3	6	8	11	14	10	16	13	9	8	11	13	17	20	14	18	14	15	17	9	3	22
NA	11	12	3	6	0	5	6	3	5	5	8	4	10	7	3	2	5	7	11	14	8	12	8	9	11	9	9	16
SN	10	11	8	5	5	0	2	2	4	7	10	9	12	9	8	7	10	12	16	19	13	17	13	14	16	4	8	21
FA	8	9	6	3	6	2	0	3	5	8	11	10	13	10	9	8	11	13	17	20	14	18	14	15	17	6	6	22
DB	11	12	6	6	3	2	3	0	2	5	8	7	10	7	6	5	8	10	14	17	11	15	11	12	14	6	9	19
AL	13	14	8	8	5	4	5	2	0	3	6	9	8	5	5	7	9	11	15	18	13	16	12	13	15	8	11	20
DA	16	17	8	11	5	7	8	5	3	0	3	9	5	2	2	4	6	8	12	15	13	13	9	10	12	11	14	17
AX	19	20	11	14	8	10	11	8	6	3	0	12	4	1	5	7	9	9	13	18	16	16	12	13	15	14	17	20
AE	15	16	7	10	4	9	10	7	9	9	12	0	14	11	7	6	7	9	11	10	4	8	4	5	7	13	13	12
GO	21	22	13	16	10	12	13	10	8	5	4	14	0	3	7	9	7	5	9	15	18	13	10	10	12	16	19	17
SK	18	19	10	13	7	9	10	7	5	2	1	11	3	0	4	6	8	8	12	17	15	15	11	12	14	13	16	19
AV	14	15	6	9	3	8	9	6	5	2	5	7	7	4	0	2	4	6	10	13	11	11	7	8	10	12	12	15
AZ	13	14	5	8	2	7	8	5	7	4	7	6	9	6	2	0	3	5	9	12	10	10	6	7	9	11	11	14
KA	16	17	8	11	5	10	1	8	9	6	9	7	7	8	4	3	0	2	6	9	11	7	3	4	6	14	14	11
DP	18	19	10	13	7	12	13	10	11	8	9	9	5	8	6	5	2	0	4	10	13	8	5	5	7	16	16	12
AJ	22	23	14	17	11	16	17	14	15	12	13	11	9	12	10	9	6	4	0	9	15	7	7	6	4	20	20	9
KO	25	26	17	20	14	19	20	17	18	15	18	10	15	17	13	12	9	10	9	0	14	2	6	5	5	23	23	2
OF	19	20	11	14	8	13	14	11	13	13	16	4	18	15	11	10	11	13	15	14	0	12	8	9	11	17	17	16
GY	23	24	15	18	12	17	18	15	16	13	16	8	13	15	11	10	7	8	7	2	12	0	4	3	3	21	21	4
AH	19	20	11	14	8	13	14	11	12	9	12	4	10	11	7	6	3	5	7	6	8	4	0	1	3	17	17	8
AT	20	21	12	15	9	14	15	12	13	10	13	5	10	12	8	7	4	5	6	5	9	3	1	0	2	18	18	7
KP	22	23	14	17	11	16	17	14	15	12	15	7	12	14	10	9	6	7	4	5	11	3	3	2	0	20	20	7
AQ	14	15	12	9	9	4	6	6	8	11	14	13	16	13	12	11	14	16	20	23	17	21	17	18	20	0	12	25
SH	2	3	6	3	9	8	6	9	11	14	17	13	19	16	12	11	14	16	20	23	17	21	17	18	20	11	0	25
DK	27	28	19	22	16	21	22	19	20	17	20	12	17	19	15	14	11	12	9	2	15	4	8	7	7	25	25	0
MND	**27**	**28**	**19**	**22**	**16**	**21**	**22**	**19**	**20**	**17**	**20**	**16**	**22**	**19**	**15**	**14**	**17**	**19**	**23**	**26**	**20**	**24**	**20**	**21**	**23**	**25**	**25**	**28**