KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

COLLEGE OF SCIENCE

BOUNDARY – VALUE PROBLEMS WITH PARTICULAR REFERENCE TO THE AFRICAN RECTANGULAR DRUM

A Thesis submitted to the Department of Mathematics, in partial fulfillment of the requirements for the degree of

Master of Philosophy

(Mathematics)

By

NGONIBI MATTHEW KPABU NGALA

BSc (HONS) MATHS

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DECLARATION

I hereby declare that this submission is my own work towards the MPHILL MATHEMATICS and that, to the best of my knowledge, it contains no material previously published by another person nor materials which have been accepted for the award of any other degree of the University except where due acknowledgement has been made in the text.

Ngonibi Matthew Kpabu Ngala	…………………………….	……………………………..
(Student) PG3942509	Signature	Date

Certified by:

Rev. Dr. G. O. Lartey	…………………………….	……………………………..
(Supervisor)	Signature	Date

Certified by:

Dr. S. K. Amponsah	…………………………….	……………………………..
(Head of Department)	Signature	Date

ABSTRACT

Drums are part of most Africa cultures, and produce different sounds according to their shapes and sizes. The purpose of this study is to explain the mathematics involved in the vibration of a rectangular drum. Hence a rectangular drum was modeled for that purpose. The wave equation in the rectangular Cartesian coordinates was solved with boundary conditions dictated by the fact that the edges of the drum were fixed. The initial condition was that, the velocity was zero for the entire membrane. The double Fourier coefficients were computed and truncated and presented as 3 x 3 matrices. The computed Fourier coefficients enabled the identification of the normal modes. This was done for four cases of solutions for certain polynomials in x and y. The regular symmetry of the drum appears to influence the symmetry of the solutions. The study found that, the sound waves produced from all the four cases, moved crest and trough with cases one, two and four recording nodal points and case three having none.

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DEDICATION

This work is dedicated to my late father Mr. Ngonibi Ngala and my mother Madam Ngonibi Akua.

ACKNOWLEDGEMENT

I thank God for the strength and creative mind to make this work a success.

My sincere thanks go to Rev. Dr. G. O. Lartey for his constructive criticism that helped me to complete this project on time.

My thanks also go to my colleagues teachers for assisting me with their comments. Furthermore, I thank Mr. Daniel Amponsah (Agya Koo Niimo) for teaching me the background of the rectangular drum and making the drums available for me to take measurements. My sincere thanks also go to Mr. Abubakari Yahaya for writing the visual basic programme for me. My thanks again go to Mr. Laweh Augustine for writing sage and mat lab codes for me. One more time, I say thanks to all and sundry.

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Most mathematicians and science academics in Africa in particular have ever seen rectangular drums used in their traditional dances to make music lovely and appeals to their ears. Tribes such as the Akans, the Gas, the Kotokolis often use rectangular drums in their traditional set ups during funerals, marriage ceremonies and high life music's for enjoyments.

However, nobody bothered to consider the mathematics of the sound waves they hear from the rectangular drum, if there is any. The source of any wave is a vibration and it is the vibrations that propagate outward and thus constitute the wave. If the source vibrates sinusoid ally, then the wave itself will have a sinusoidal shape both in space and in time.

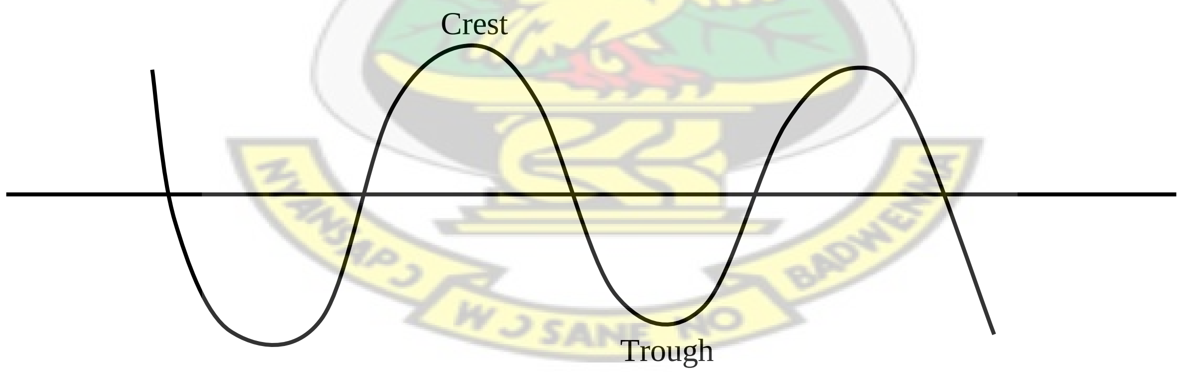


Fig 1.1: A sinusoidal wave

This thesis seeks to explain the mathematics involve in the vibration of a bounded rectangular drum, by transforming a two dimensional wave equation to differential equations, and determine the Fourier sine waves and the vibrations associated with them.

In Africa, people express their culture and feelings through drumming. In fact drumming is said to be the life blood of most cultures in Africa. In the past people modeled different types of drums from trees to use for music and also to communicate to each other. The beating of the drums can tell the circumstance in which a group of people find themselves. The sound of the drums can tell whether a group of people or a tribe is in merriment, at war, or at a funeral. In fact it communicates the feelings or state of a group of people to others as to whether they are in happiness or in sadness. Chiefs in Africa in particular use drums to talk to their subjects at durbars and festivals to show their love and protection for them. As a result of the importance of drums, to culture and for that matter music in general, Africans kept on modeling different types of drums to suit them at different occasions.

This innovation eventually led to the construction of rectangular drums in Africa to boost music. The most widely use of this drum are the Akans, the Gas, the Kotokolis in Ghana. Therefore it is imperative for mathematicians to know the mathematics of this drum and appreciate it better. In appreciating this drum, the culture of Ghanaians will also be appreciated, loved and practiced in order to ebb the western culture that is having a negative tone on our youth today. The study will transform a two dimensional wave equation into differential equations, obtain its general solution, and finally calculate the Fourier coefficients and their corresponding Fourier sine waves.

1.2 Statement of the Problem

Mathematicians and Science academics in Africa have ever seen the rectangular drum used in drumming during musical occasions, with a good sound hearing. However, no body imagines how the waves from this drums that give good sound to the ear are represented mathematically. For this reason, the thesis seeks to explain the mathematics involve in the vibration of a bounded rectangular membrane, to all stakeholders for better appreciation. In appreciating the mathematics of this drum, it is expected that the African cultures will be better appreciated and to nib in the bud the negative Western culture influence on African youths today.

1.3 Objectives of the Study

The general objective is to explain the mathematics involved in the vibration of a rectangular drum.

The specific objectives are:

1. To solve the two dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ with initial boundary conditions.
2. To identify this boundary value problem with the African rectangular drum.
3. To calculate the Fourier coefficients and hence obtain the double Fourier sine waves (normal modes).
4. To calculate the vibrations at a particular fixed point x with varying valves of y and vice versa.

1.4 Methodology

The rectangular drum has its measurements taken from a resource musician by name Mr. Daniel Amponsah. The drums length and breadth were measured in meters as 0.5m and 0.35m as the boundaries.

The two dimensional wave equation represented as $\frac{\partial^2 u}{\partial t^2}$ = C^2 $(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$ was transformed into ordinary differential equations and the general solution obtained.

Based on the Fourier coefficients obtained, the corresponding double Fourier sine waves were obtained to depict the waves being propagated by the drums. A fixed value for x is indicated with some varying values of y to calculate the vibrations at a particular time t and vice versa.

The programming languages used are;

1. Visual Basic 6 2. Mat lab 3.Sage

Visual Basic 6 program was used to calculate the Fourier coefficients (B_{mn}) for values m = 1, 2, 3:, n = 1, 2, 3: whiles Mat lab and Sage languages used to draw the shapes of the rectangular drum as the waves propagate with a chosen f(x, y) in a three dimensional plane.

1.5 Justification

There are many cultural instruments through the years, but not much mathematical academic study has been made into these musical instruments. This thesis find it important to use the rectangular bounded drum as a case study to explain the mathematics of the waves it propagate to fellow mathematicians, scientist and other stakeholders. It is hoped that all stakeholders will appreciate the mathematics of the rectangular drum and in turn appreciate the culture of Africans which will help ebb the foreign culture that had a negative tone on our modern youths.

1.6 Scope and Limitations

The study will use the rectangular drum with boundaries to explain the mathematics of the waves it propagates to all stakeholders. Importance of the drum will be outlined in the literature review and a detailed explanation of the transformation of the wave equation to differential equations which yielded the Fourier coefficients and their Fourier sine waves.

This thesis has limited itself to a boundary value problem with particular reference to the rectangular drum, due to time and economic constraints to cover other musical instruments.

1.7 Organisation of the Thesis

The thesis is organized as follows: Chapter one, covers the general introduction that indicates the need to use the rectangular drum in Africa and for that matter Ghana, and the people who used it. The chapter also indicates the problem statement which emphasized on the need to learn the mathematics of the rectangular drum to ebb the foreign culture tone on our modern youths. It also outlined the methods used to solve the wave equation to obtain Fourier coefficients and the Fourier sine waves. It further stated the justification of chosen the boundary problem, the limitations of the study, organization of the study and the general summary of the chapter.

Chapter two talked about the review of the literature with regards to importance of the rectangular drum to Africans and for that matter Ghanaians. It also outlined the method used to transform a two dimensional wave equation into a differential equation.

Chapter three looked at the mathematical integration of Fourier coefficients with a specified function f(x, y), which eventually led to the Fourier sine waves and their vibrations. Transformation of a two dimensional wave equation was done to obtain ordinary differential equations and hence the general solution.

Chapter four presents the analysis of the thesis and presentation of the pictures of the drum with each specified function f(x y). And finally chapter five dealt with the general summary of the thesis, conclusion and some recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

This chapter of the study looks at the importance of the rectangular drum and other drums to Africans and for that matter Ghanaians. It further looks at the definition, origin and pictures of a rectangular drum used in the transformation of a two dimensional wave equation $\frac{\partial^2 U}{\partial t^2} = C^2 \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right)$ into ordinary differential equations, hence obtains a general solution. The general solution enables the researcher to calculate Fourier expansions of vibrations at particular fixed points. Some types, shapes and sizes of instruments used as drums in Africa are discussed as well as Fourier series and their coefficients. Some review is also made on the importance of differential equations.

2.1 Definition and Origin

The rectangular drum is naturally called a frame drum, with some pegs fixed inside to raise the tension the drummer wish to have for drumming. It is rectangular in shape with a membrane covering. The membrane is made up of an animal skin, either, a goat, a sheep, a cow or that of a bush animal. When well constructed the drum is beating with the hand for the specific music the drummer wants to use for. It originated from the second world war(II) in 1939-1945. African soldiers constructed the drum and used it to drum for marching. From this point some tribes in Africa adopted it for their local music.

For example, “Kokomma” is a form of music among the Ashantis, the Bono Ahafo's, the Kwawu's, the Akyem's and the Sewhis that employ the rectangular drum.

Asaadua is another form of music that made use of the rectangular drum. Asaadua stands for sweet berry with a botanical name of Discoreophyllum cuminsii. It literally means that the music played from the rectangular drum is as sweet as berry fruits. The brain child of Nketia (1971).

2.2 Importance of Drums

"A village without music is a dead place", says an African proverb. And so, in each African town, the drums and bells, flutes and xylophones sing out the rhythms of African life. There are songs for the stages of life, songs for communication and teaching, for work and for worship. At the heart of each song is the drum. What is the first thing to do before you can make a drum? Different tribes make different drums because music is not the same everywhere in Africa. If you live in Nigeria and are a member of the Yoruba tribe, you will carefully choose the tree to cut down to make your drum frame. It must be a tree that has grown near the village and has been "listening" to human voices. This way, when it is made into a drum it will "speak well" (Friedberg, 1993).

In history, drumming and the use of percussive instruments have a significant role in people's lives. Not only do the people who play these instruments enjoy them, but it is said that "there is as much pleasure participating in, as listening to and admiring an expert drummer improvisations". The use of drums has people recognized as being able to put them into spiritual trances throughout history. The drum is a musical instrument with great power and presence that gives the "pulse" or backbone to the music it is incorporated with.

There are three rudimental rhythmic procedures that have been known in drumming for the use of communication, entertainment and both communication and entertainment together. These are:

(1) The use of drum as speech surrogate or a “talking drum”. These methods of playing were used for communicative purposes and often codes were used to be played over long distances for the sending and receiving of messages.

(2) The use of both iconic and symbolic dimensions of communication within music and dance. Throughout many festivals in Africa, depending on the event being celebrated, drum beats are used to dictate the type of dance to be done by the listeners. For example, at the time of a birth of twins there is a different dance than at a birth of single child and the beat of the drum instructs the listeners to do the appropriate dance.

Bobb (1998) in his book indicated that, the drum is a divine tool of the supreme being, a womb or beginning of created life.

Nketia (1968) has this to say: Drums are used during the following occasions.

(1) During merry occasions: when people are happy, then they drum and celebrate. For example during marriage ceremonies, naming ceremonies and general high life for entertainment.

(2) People also drum to mourn their dead. The pain of a departed one expressed in drumming and dancing.

(3) People drum during durbars and festivals to entrench their cultures

(4) During wars, people drum to communicate to each other as to which strategy to adopt for the fight.

(5) People drum during victories. This is seen in the areas of wars or victory on a greater achievement.

Friedberg (1993) indicated in his journal that, Africa drums hold a special place in the history of Africa. In western culture the idea of drumming is nearly always associated with entertainment or just to add to the musical quality of a song. In Africa, drums hold a deeper symbolic and historical meaning. Drums are almost always an accompaniment for any manner of ceremony – birth, deaths and marriages, together with a ritual dance. The vicious sound of many drums pounding together is also a necessary installment to stir up emotions in a battle or war inspires excitement and passion.

The Djembe drum is possibly the most influential and basic of all the Africa drums. Originally, it dates back to 500 A. D. The Djembe was originally created as a sacred drum to be used in healing ceremonies, rites of passage, ancestral worship, warrior rituals, as well as social dances. Friedberg (1993) said, the drum rhythm of the Djembe is performed in the evening for most celebrations, especially during full moon, spring, summer and winter harvesting time, weddings, baptisms, honoring of mothers, immediately after Ramadan (the month of fast of all Muslims) or other celebrations.

Dearling (1996) has this to say: In much of Africa, certain drums symbolize and projects royalty and are often housed in sacred dwellings. In fact you could telephone. Tribes, with use of drum would communicate with other tribes often miles away. Drums were often used to signal meetings, dangers, etc.

The talking drums of Africa imitate the pitch patterns of language and transmit messages over many miles.

The drums in figure 2.1a and 2.1b below were measured and the pictures taken from a resource person, Dr. Daniel Amponsah

Fig 2.1 a: the outer view of a rectangular drum
(*The dimensions are 0.5m by 0.35m*)

Fig 2.1 b: the inner view of a rectangular drum
(*The dimensions are 0.5m by 0.35m*)

2.3 Types, shapes and sizes of instruments used as drums in Africa

There are countless types of drums within the countries of Africa. These ranges from tall drums that have high pitch, to wider drums that add to bass. The most widely used drum throughout Africa is known as the membranophone. This is a hollow body drum with one or two parchment heads at either end. These drum are the standard drum throughout much of the world today. This is the brainchild of Agawu (1995).

Special occasions call for special drums in the African heritage. These drums have more decorations than the average drum and are treated as sacred pieces of art. Most drums in Africa are carved from solid logs of wood or made with several strips of wood bound together by Iron hoops. In some southern regions, drums are made from clay or types of metal for their ritual music, but these are usually the exception and have become more popular over the years. Agawu (1995) further said, another material used to make the drum is from a large gourd or a calabash. These are most often seen in the Savannah Belt of West Africa. Now as technology is improving worldwide, many hollow vessels have substituted the original vessel used. These new materials include tins, light oil drums and discarded trash that can be used as a means to make a simple drum. Drums that are made for children are made from hard fruit shells or discarded tins.

Many children in African villages view drumming as a way to complete the inner self. By the age of 10, children usually realize whether they are capable of the skill of drumming. Becoming a respected drummer is a sign of maturity in many African cultures, and the few who do become selected to represent their villages are treated as royals.

Bobb (1998) discussed that; drums can appear in a wide variety of shapes and sizes. Some of these shapes include conical, cylindrical, or semi-cylindrical. Drums may have a bulge in the middle or a bowl shape top. Other variations are cup-shaped, bottle shaped or shaped in the form of a goblet, vase or even an hourglass. These variations are unlimited and even considered uncountable. Tribes in Western Africa have also been known to make drums in the figure of humans, giving them the appearance of a small totem pole. All drums have frame as well, which also can vary in their shape and size. Frames can either be round or square, fitting over the drum to keep the head in proper place. Often designs are made in the frame that coincides with the design of the rest of the drum to enhance its appearance for different purposes and occasions. This all depends on the artist who constructs it.

Agawu (1995) added that, some of the taller drums can be up to 5 or 6 feet and usually no more than 24 inches across. These drums are known for their high pitch and with the widening of the head come about a deeper sound. The largest drums, known for their explosive bass usually are no more than 4 feet tall and 30 inches in diameter. Again, many variations occur while making the drum, which gives it the unique sound it plays. This is why no 2 drums can be or sound the same.

The drumhead can be placed on by a couple of different techniques. Glue can be used, but the most favored and common today is nailing it directly to the drum itself. This is done by the use of thorns or large nails that are suspended by pegs in order to adjust the tension, which in turn adjusts the tone.

The most important aspect of drum making is the creation of the perfect tone quality and pitch. All of the size and shape determination are based on the desired sound. It is said that anyone can carve wood and design it to be aesthetically pleasing, but only a true artist can create a beautiful drum, with both sight and sound.

Although most drums are played by percussive means, there are a few types played by the use of friction. An example of this occurs among the Akan in Ghana is the Etwie, or friction drum is played by rubbing the drumhead with a stick over a fine layer of powder to create more resonance.

Nketia (1971) said that, drums can be played by one person, in pairs, or in large groups known as ensembles. Drums that are played together usually differ in their tone and pitch, so each can be heard distinctly throughout the performance. There are some instances where one person plays multiple drums, but the most intriguing ensemble to me is the use of15 drums. These are called the Entenga drums and are heard in Uganda. These drums are all tuned to different and distinct pitches and are used for playing tunes that sound incredibly similar to those played by the instrument, xylophone. Twelve of these drums form the melody section and are played by four drummers. Each of these drummers is in reaching distance of playing five drums while the other occupies the remaining two, a big and small drum. I have never heard this form of ensemble which is usually played for Kings, but the sounds and energy from these drums is heard for miles in times of playing.

Each society within Africa has its own unique drumming ceremonies which all have different styles of rhythms and variations of drums themselves. Likewise, African societies differ in kinds of activities for which they provide music. Some societies, for example, celebrate marriage with a great deal of music, while others do not. Similarly, some use music in the rites performed for newborns, while others do not make this a time for music making.

Throughout Africa, one common ground is the time in which drumming is played. Drumming is considered a nighttime activity and may be heard in times of daylight only on days of rest, or periods of mourning which may last up to three months. In one village named Ga, drumming is banned for three weeks prior to their harvest festival.

The role that drumming plays within Africa is obvious, it is not easy to overlook the sounds and energies created by the drum itself. Wilson Harris has a famous line that encloses a womb of space in which silence and identity will emerge out of the darkness and the void.

Burnett (1982) explained that, the most important instrument used in Rasta music is the drum. Ironically, there are over 25 types of drums in Jamaica but Jamaicans usually only use three types within their style of music. The largest of these is the bass drum. This drum is similar to the large drums found in Africa. The bass drum is made from wooden sticks, held together with metal bands and pegs. The heads of the Jamaica drums are made from either Goat or Cow skins and held on by similar methods as African drums by either nails or glue. The bass drum is held on the player's lap and played by the use of a heavy padded stick. This drum is usually no wider than 60 centimeters across and no more than four feet tall. Variations do occur with all drums as seen in the African history section, so generalize all types of drums and their size is not truly possible.

The next largest drum in Jamaica music is about 30 centimeters across and ranges in height from less than a foot to between two and three feet. This drum is called the Fundeh. Similar to the bass drum, the Fundeh is made from wood in the same way standard membranophone are designed. The difference in design lies in the one-sided head feature. One end is left open for a higher resonance than the bass. The head is made from the same skins as the bass, but the tension is greater for less revibration (repercussions). This drum is played by the use of finger or hand tapping and is held between the player's legs.

The smallest Rasta drum is called the Repeater. This drum is usually much skinnier and smaller in stature than the others because of the sound it is designed to create. The high pitch that emanates from this drum creates the unique sound of Reggae Music, covering the high notes at a quicker, more complex rate.

Burnett (1982) further explained that, each drum has its own job in Reggae music. The Fundeh sets the speed of the music and keeps things together. Its role is to play the "Lifeline" of the music. The Bass Drum plays more catchy rhythms than the Fundeh, usually repeated over and over to keep the groove of the tune. The repeater plays the most difficult beats and rhythms. Often notes are played at double the speed of the Bass and the Fundeh to give music a bounce.

Although, there are many other drum variations used in Jamaica, the three spoken of are most common and usually the other drums are variations of the original three. The drum set has also become popular in the latest Jamaica musical forms, but even these drums are members of the membranophone family, just made with a little help of technology. Metal stands connect the drums to keep them in close proximity of the player, which allows for multiple Drum and Cymbal sounds.

Andrian (1995) also puts that, similar to that of Mother Africa, Jamaica is home of frequent drum ensembles called grounations. Within these grounations are drum circles known today as

Niyabinghi circles. These drum circles commonly put people into spiritual trances that Jamaicans refer to as soul cleansers and purifiers. These festivals are the heartbeat of their culture and are treated as sacred as a day in church. Niyabinghi circle have come a long way in Jamaican history. Before the term Niyabinghi, the most common drum circles and methods were referred to as Burra Drumming. The Burra drums are religious survivals from Africa and Burra is probably the oldest Jamaica musical form known. Burra Drumming was originally played for released prisoners coming back to their original communities as a welcoming ceremony.

2.4 Importance of Fourier series and coefficients

Boyce (2005) indicated in his book, that, in mathematics, a Fourier series decomposes periodic functions or periodic signals into the sum of a (possible infinite) set of simple oscillating functions, namely sine and cosines (or complex exponentials) the study of Fourier series is a branch of Fourier analysis. He said Fourier series were introduced by Joseph Fourier (1768 – 1830) for the purpose of solving the heat equation in a metal plate.

The heat equation is a partial differential equation. Prior to Fourier's work, no solution to the heat equation was known in the general case, although particular solutions were known if the heat source behaved in a simple way, in particular, if the heat source was a sine or cosine wave. These simple solutions are now sometimes called eigen solutions. Fourier's idea was to model a complicated heat source as a superposition (or linear combination) of simple sine and cosine waves, and to write the solutions as superposition of the corresponding eigen solution.

This superposition or linear combination is called the Fourier series. Boyce (2005) further stated that, although the original motivation was to solve the heat equation, it later became obvious that the same techniques could be applied to a wide array of mathematical and physical problems, and especially those involving linear differential equations with constant coefficients, for which the eigen solutions are sinusoids.

The Fourier series has many such applications in electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics and thin- walled shell theory.

From a modern point of view, Fourier results are somewhat informal, due to the lack of a precise notion of function and integral in the early nineteenth century. Later, Dirichlet and Riemann expressed Fourier's results with greater precision and formality .Zygmund (2002) also added that, since Fourier's time, many different approaches to defining and understanding the concept of Fourier series have been discovered, all of which are consistent with one another, but each of which emphasizes different aspects of the topic. Some of the more powerful and elegant approaches are based on mathematical ideas and tools that were not available at the time Fourier completed his original work. Fourier originally defined the Fourier series for real-valued functions of real arguments, and using the sine and cosine functions as the basis set for the decomposition many other Fourier-related transforms have since been defined, extending the initial idea to other applications. This general area of inquiry is now sometimes called harmonic analysis. A Fourier series, however, can be used only for periodic functions or for functions on a bounded (compact) interval.

Wilson (1995) also noted that, the Fourier transform is a mathematical operation that decomposes a signal into its constituent frequencies. That is the Fourier transform of a musical chord is a mathematical representation of amplitudes of the individual notes that make it up. The original signal depends on time, and therefore is called the time domain representation of signal, whereas called the frequency domain representation of the signal. The term Fourier transform refers both to the frequency domain representation of the signal and the process that transforms the signal to its frequency domain representation.

In mathematical terms, Bochner (1949) said, the Fourier transform, transforms one complex – valued function of a real variable into another. In effect, the Fourier transform decomposes a function into oscillatory functions. The Fourier transform and its generalizations are the subject of Fourier analysis. In this specific case, both the time and frequency domains are unbounded linear continua. It is possible to define the Fourier transform of a function of several variables, which is important for instance in the physical study of wave motion and optics. It is also possible to generalize the Fourier transform on discrete structures such as finite groups. The efficient computation of such structures, by fast Fourier transform is a cornerstone for high-speed numerical computing.

2.5 Importance of differential equations

A differential equation is a mathematical equation for an unknown function of one or several variables that relates the values of the functions itself and its derivatives of various orders. (Polyanin 2003).

Polyanin (2003) explains that differential equations arise in many areas of science and technology, especially whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics where the motion of a body is described by its position and velocity as the time varies. Newton's laws allow one to relate the position, velocity, acceleration and various forces acting on the body and state this relation as a differential equation for the unknown position of the body as a function of time. In some cases, this differential equation (called an equation of motion) may be solved explicitly.

Polyanin (2003) further explains that, an example of modeling a real world problem using differential equation is the determination of the velocity of a ball falling through the air, considering only gravity and air resistance. The ball's acceleration towards the ground is the acceleration due to gravity minus the deceleration due to air resistance. Gravity is considered constant, and air resistance may be modeled as proportional to the ball's velocity. This means that the ball's acceleration, which is the derivative of its velocity, depends on the velocity. Finding the velocity as a function of time involves solving a differential equation.

He further argues that differential equations are mathematically studied from several differential perspectives, mostly concerned with their solutions – the set of functions that satisfy the equation. Only the simplest differential equation admits solutions given by explicit formulas. However, some properties of solutions of a given differential equation may be determined without finding their exact form. If a self-contained formula for the solution is not available, the solution may be numerically approximated using computers. The theory of dynamical systems puts emphasis on qualitative analysis of systems described by differential equations, while many numerical methods have been developed to determine solutions with a given degree of accuracy.

Devaney (2006) said differential equations are mathematically studied from several different perspectives, mostly concerned with their solutions; the set of functions that satisfy the equation. Only the simplest differential equation admits solutions given by explicit formulas, however, some properties of solutions of a given differential equation may be determined without finding their exact form. If a self-contained formula for the solution is not available, the solution may be numerically approximated using computers. The theory of dynamical systems puts emphasis on qualitative analysis of systems described by differential equations, while many numerical methods have been developed to determine solutions with a given degree of accuracy. The study of differential equations as put by Coddington (1955) is a wide field in pure and applied mathematics, physics, meteorology and engineering. All these disciplines are concerned with the properties of differential equations of various types. Pure mathematics focuses on the existence and uniqueness of solution, while applied mathematics emphasizes the rigorous justification of the methods for approximating solutions. Differential equations play an important role in modeling virtually every physical, technical, or biological process, from celestial motion, to bridge design, to interactions between neurons. Differential equations such as those used to solve real-life problems may not necessarily be directly solvable, that is they do not have closed form solutions. Instead, solutions can be approximated using numerical methods.

Mathematicians also study weak solutions (relying on weak derivatives) which are types solutions that do not have to be differentiable everywhere. This extension is often necessary for solutions to exist, and it also results in more physical reasonable properties of solutions, such as possible presence of shock for equations of hyperbolic type. Hyperboloids are used in challenging designs situations, such as the design of cooling towers for nuclear plants as stated by Robert et al (2002).

CHAPTER 3

METHODOLOGY

THE WAVE EQUATION AND THE RECTANGULAR MEMBRANE

3.0 Introduction

This chapter of the thesis gave some assumptions underlining the vibration of a stretched membrane with regards to a rectangular drum, and partial differentiation of the wave equation to obtain ordinary differential equations and their solutions. The method of separation of variables was used to separate the spatial part made up of (x y) variables from the time part made up of (t) variable. The spatial part again was separated into ordinary differential equations of x components only and that of y components only. Ordinary differential equations of time (t) only, x only and y only were solved to obtain the general solution of the wave equation. A double series was considered and calculations made to obtain Fourier Coefficients B_{mn} and B^*_{mn}

3.1 Assumptions

1. The mass of the membrane per unit area is constant (homogeneous membrane). The membrane is perfectly flexible and is so thin that it does not offer any resistance to bending.
2. The membrane is stretched and fixed along its entire boundary in the xy plane. The tension per unit length T caused by stretching the membrane is the same at all points and in all directions and does not change during the motion.
3. The deflection u, (x, y, t) of the membrane during the motion is small compared with the size of the membrane, and all angles of inclination are small.

3.2 Differentiation of the wave equation to obtain the time part

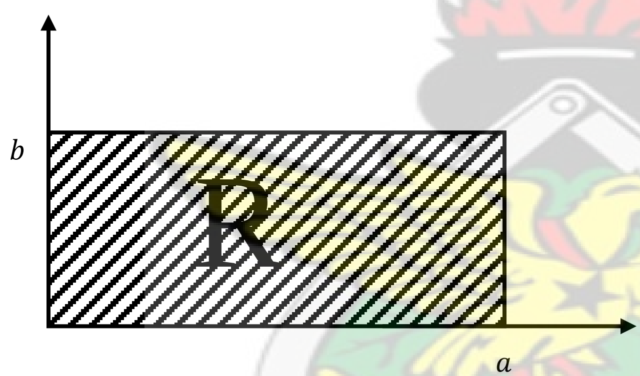
Partial differentiation of the wave equation was done to separate the spatial part from the time part which led to ordinary differential equation in time ‘t’ only

The two dimensional wave equation is given by

$$\frac{\partial^2 U}{\partial t^2} = C^2 \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \qquad (1)$$

We require the following boundary conditions $U(x, y) = o, \forall t \geq 0$ on the rectangle with boundary lines

$x = 0, x = a, y = 0, y = b$.



$$U(x, y) = o, \qquad (2)$$

Figure 3

Fig 3.1: Rectangular Membrane

x

We also require the following initial conditions:

$U(x, y, 0) = f(x, y)\ - (3)$ initial deflection $f(x, y)$

$\left.\frac{\partial U}{\partial t}\right|_{t\,=\,0} = g(x, y)\ - (4)$ initial velocity $g(x, y)$

Solutions of (1) to satisfy condition (2) use the method of separation of variables.

Step 1

$$\begin{cases} U(x,y,t) = F(x,y)G(t) \\ \dfrac{\partial^2 U}{\partial t^2} = F\,\ddot{G} \\ \dfrac{\partial^2 U}{\partial x^2} = F_{xx}G \\ \dfrac{\partial^2 U}{\partial y^2} = F_{yy}G \end{cases} \tag{5}$$

Substitute (5) into (1)

$$F\,\ddot{G} = C^2\left[F_{xx}\,G + F_{yy}\,G\right]$$

Divide through by C^2FG

$$\frac{F\,\ddot{G}}{C^2FG} = \frac{C^2F_{xx}\,G + C^2F_{yy}\,G}{C^2FG} = \frac{C^2F_{xx}\,G}{C^2FG} + \frac{C^2F_{yy}\,G}{C^2FG}$$

$$\frac{\ddot{G}}{C^2G} = \frac{F_{xx}}{F} + \frac{F_{yy}}{F}$$

$$\frac{\ddot{G}}{C^2G} = \frac{1}{F}\left[F_{xx} + F_{yy}\right]$$

Left hand side depends on t and right hand side does not depend on t, the expression must be equal to a constant. A negative constant will lead to a solution which will satisfy condition (2) without identically being zero.

Set the negative constant to $-v^2$. Thus we have

$$\frac{\ddot{G}}{C^2G} = \frac{1}{F}\left[F_{xx} + F_{yy}\right] = -v^2$$

From this $\frac{\ddot{G}}{C^2 G} = -v^2$

$$\ddot{G} = -v^2 C^2 G$$

$$\ddot{G} + v^2 C^2 G = 0$$

But let $\lambda = Cv$

$$\ddot{G} + \lambda^2 G = 0 \qquad (6)$$

3.3 Differentiation of the spatial part (*x y*) of the wave equation.

The spatial part of (*x y*) was differentiated by the use of separation of variable method to obtain ordinary differentia equations in *x* only and that of *y* only.

$$\frac{1}{F}\left[F_{xx} + F_{yy}\right] = -v^2$$

$$F_{xx} + F_{yy} = -v^2 F$$

$$F_{xx} + F_{yy} + v^2 F = 0 \qquad (7)$$

Apply separation of variables for (7). Let the solution be of the form

$$\left.\begin{array}{l} F(x,y) = H(x)Q(y) \\ F_{xx} = \ddot{H}Q = \dfrac{d^2H}{dx^2}Q \\ F_{yy} = \ddot{Q}H = \dfrac{d^2Q}{dy^2}H \end{array}\right) \qquad (8)$$

Put (8) into (7)

$$\frac{d^2H}{dx^2}\,Q + \frac{d^2Q}{dy^2}H + v^2 HQ = 0$$

$$\frac{d^2H}{dx^2}\,Q = -\left[\frac{d^2Q}{dy^2}\,H + v^2 HQ\right]$$

Divide both sides by HQ

$$\frac{1}{H}\frac{d^2H}{dx^2} = -\left[\frac{1}{Q}\frac{d^2Q}{dy^2} + \frac{v^2Q}{Q}\right]$$

$$\frac{1}{H}\frac{d^2H}{dx^2} = -\frac{1}{Q}\left[\frac{d^2Q}{dy^2} + v^2Q\right]$$

Left hand side depends on x and the right hand side depends on y. The solution must be equal to a constant. The constant must be equal to a negative value which satisfies the condition (2) without identically being zero.

Let the constant $= -k^2$

Hence

$$\frac{1}{H}\frac{d^2H}{dx^2} = -\frac{1}{Q}\left[\frac{d^2Q}{dy^2} + v^2Q\right] = -k^2$$

This yields the ordinary differential equations

$$\frac{1}{H}\frac{d^2H}{dx^2} = -k^2$$

$$\frac{d^2H}{dx^2} = -Hk^2$$

$$\frac{d^2H}{dx^2} + Hk^2 = 0 \tag{9}$$

$$-\frac{1}{Q}\left[\frac{d^2Q}{dy^2} + v^2Q\right] = -k^2$$

$$\frac{d^2Q}{dy^2} + v^2Q = Qk^2$$

$$\frac{d^2Q}{dy^2} + v^2Q - Qk^2 = 0$$

$$\frac{d^2Q}{dy^2} + (v^2 - k^2)Q = 0$$

But let $p^2 = v^2 - k^2$

$$\frac{d^2Q}{dy^2} + p^2Q = 0 \qquad (10)$$

Therefore

$$\left.\begin{array}{ll} \frac{d^2H}{dx^2} + k^2H = 0 & (9) \\ \frac{d^2Q}{dy^2} + p^2Q = 0 & (10) \end{array}\right\}$$

3.4 The specific solutions of ordinary differential equations in *x* only and that of *y* only.

Specific solutions were obtained for ordinary differential equations in *x* only and that of *y* only. This was made possible by substituting the boundary conditions into equation ….(9) and equation …………(10). The boundary conditions are:

x = 0, x = a, y = 0, y = b

<u>Step 2</u>

The general solutions of (9) and (10) are

$$H(x) = Acos(kx) + Bsin(kx)$$

and

$$Q(y) = C\cos(py) + D\sin(py)$$

where A, B, C and D are constants.

From (5) and (2) it follows that $F = HQ$ must be zero on the boundary, which corresponds to $x = 0,\ x = a, y = 0$ and $y = b$ from figure 3.1.

This yields the conditions $H(0) = 0,\ H(a) = 0,\ Q(0) = 0,\ Q(b) = 0$

Therefore

$$H(0) = Acos(0) + Bsin\,(0)$$

$$H(0) = A = 0$$

$$H(a) = Acos(ka) + Bsin\,(ka)$$

$$H(a) = 0\ cos(ka) + Bsin\,(ka)$$

$$H(a) = Bsin\,(ka)$$

But $H(a) = 0$ which implies $Bsin\,(ka) = 0$

If $B \neq 0$, then $\sin(ka) = 0$ or $ka = m\,\pi$

That is

$k = \frac{m\,\pi}{a}$ (m integrals)

Similarly $c = 0$ and p must be restricted to the value $p = \frac{n\,\pi}{b}$ with n integrals.

Therefore the solutions are

$Hm(x) = \sin\frac{m\pi x}{a}$ and $Qn(y) = \sin\frac{n\pi y}{b}$

$$m = 1, 2, \dots \qquad n = 1, 2, \dots$$

The function

$$F_{mn}(x, y) = H_m(x)Q_n(y) = \sin\frac{m\pi x}{a}\,\sin\frac{n\pi y}{b}$$

$m = 1, 2, \ldots$ $\qquad$ $n = 1, 2, \ldots$ $\qquad$ are solutions of (7)

Which are zero on the boundary of the rectangular membrane.

Since $p^2 = v^2 - k^2$ and $\lambda = cv$

$$\Rightarrow v^2 = p^2 + k^2$$

$$v = \sqrt{p^2 + k^2}$$

$$\Rightarrow \lambda = c\sqrt{p^2 + k^2}$$

But $k = \frac{m\,\pi}{a}$ and $p = \frac{n\,\pi}{b}$

$$\lambda = \lambda mn = c\sqrt{\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}}$$

$$\lambda = \lambda mn = c\pi \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \tag{12}$$

$$m = 1, 2, \ldots \qquad m = 1, 2, \ldots$$

The corresponding general solution of (6) is $G_{mn}(t) = B_{mn} \cos \lambda mnt + B^*_{mn} \sin \lambda mnt$

It follows that the functions

$$U_{mn}(x, y, t) = F_{mn}(x, y) G_{mn}(t)$$

when written out

$$U_{mn}(x, y, t) = (B_{mn} \cos \lambda mnt + B^*_{mn} \sin \lambda mnt)\left(\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right)$$

λmn in (12) are solutions of the wave equation (1) which are zero on the boundary of the rectangular membrane in figure 3.

These functions are called the eigenfunctions or characteristic functions, and the numbers λmn are called the eigenvalues or characteristic values of the vibrating membrane. The frequency of U_{mn} is $\frac{\lambda mn}{2\pi}$.

It is interesting to note that, depending on a and b, several functions F_{mn} may correspond to the same eigenvalues. Physically this means that there may exist vibrations having the same frequency but entirely different nodal lines (curves of points on the membrane which do not move)

3.5 Calculation of the Fourier Coefficients B_{mn} and B^*_{mn}

Step 3

To obtain the solution that also satisfies the initial conditions in (3) and (4). We consider the double series.

$$U(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}(x,y,t)$$

$$= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} (B_{mn} \cos \lambda mnt$$

$$+ B^*_{mn} \sin \lambda mnt) \left(\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right) \qquad (a)$$

From (3) we obtain

$$U(x,y,0) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \left(\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right) = f(x,y) \qquad (b)$$

This series is called a double Fourier series.

Suppose that $f(x,y)$ can be developed in such a series. Then the Fourier coefficient B_{mn} of $f(x,y)$ in (b) can be determined as follows.

Setting

$$K_m(y) = \sum_{n=1}^{\infty} B_{mn} \sin\frac{n\pi y}{b} \qquad (c)$$

We may write (b) in the form

$$f(x,y) = \sum_{m=1}^{\infty} K_m(y) \sin\frac{m\pi x}{a}$$

For fixed y this is the Fourier *sine* series of $f(x,y)$, considered as a function of x, and from (4) it follows that the coefficients of this expansion are

$$K_m(y) = \frac{2}{a}\int_0^a f(x,y) \sin\frac{m\pi x}{a}\, dx \qquad (d)$$

Furthermore, (c) is the Fourier sin series of $K_m(y)$ and from (4) it follows that the coefficients are

$$B_{mn} = \frac{2}{b}\int_0^b K_m(y) \sin\frac{n\pi y}{b}\, dy$$

From this and (d), we obtain the generalized Euler formular

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\, dxdy \;\;, \;\; m = 1,2,\ldots \;\; n = 1,2,\ldots \qquad (e)$$

For the Fourier coefficient of $f(x,y)$ in the double Fourier series (b)

The B_{mn} in (a) are now determined in terms of $f(x,y)$. To determine the B_{mn}^{*} we differentiate (a) term wise with respect to t. Using (4), we obtain

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \cos\lambda mn \left(\sin\frac{m\pi x}{a}\, \sin\frac{m\pi y}{b}\right) = g(x,y)$$

Suppose that $g(x,y)$ can be developed in this double Fourier series. Then, proceeding as before we find

$$B_{mn}^{*} = \frac{4}{ab\lambda mn}\int_0^b \int_0^a g(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, dxdy\,,\ \ m = 1,2,\ldots\ \ n = 1,2,\ldots \qquad (f)$$

The result is that for (a) to satisfy the initial conditions, the coefficients B_{mn} and B_{mn}^{*} must be chosen according to (e) and (f).

Sufficient conditions: f, $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x \partial y}$ continuous in the rectangle under consideration.

CHAPTER 4

CALCULATIONS OF FOURIER COEFFICIENTS FOR GIVEN INITIAL DEFLECTIONS WITH ZERO VELOCITIES

4.0 Introduction

This chapter considered four selected functions (deflections) for $f\,(x\ y)$ to calculate the Fourier coefficients B_{mm} and set the function for velocity g (x y) to be zero. These four cases were chosen on the basis that they satisfy the boundary conditions of the stretched membrane on the drum. That is x = 0, x = a, y = 0, y = b and u (x, y) = 0.

On these boundaries, the membrane is fixed and the vibration of the waves is zero. This means that the movements of the waves is confined within the area of the membrane, beyond which vibrations does not exist. *f*(x,y) is defined for subsections 4.1, 4.2, 4.3, and 4.4

This chapter seeks to find the Fourier coefficient

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy$$

with given $f(x,y)$ functions. This solution is of the form

$$U_{mn}(x,y,t) = (B_{mn}\cos\lambda mnt + B_{mn}^{*}\sin\lambda mnt)\left(\sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b}\right)$$

which will lead to the determination of Fourier *sine* waves and some vibrations at some specific points.

4.1 A fourth order polynomial as initial deflection

The function $\boldsymbol{f(x\,y) = x\,y(a-x)(\,b-y)}$ was chosen because it does not violate the boundary conditions unlike a function of the form y = (x + 1) (y + 1). In other words there is no discontinuity (breaks) on the graph but continuous uniformly as can be seen from the graph. The values of a, and b are the length and breadth of the rectangular drum. Detailed calculations were done and obtained Umn (x, y, t) function, from which the Fourier Coefficients, sine waves (normal modes) and vibrations at specific points were obtained. See appendix 5 for detailed calculations.

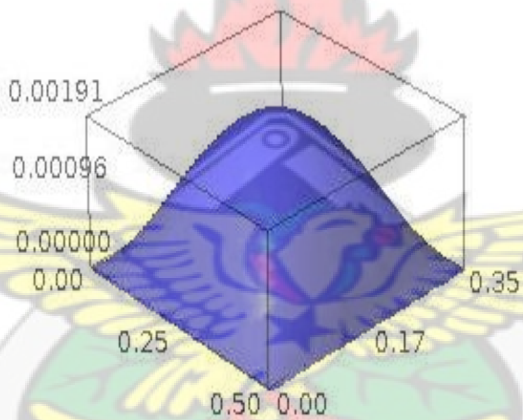


Fig 4.1: The surface $\boldsymbol{f(x\,y) = x\,y(a-x)(\,b-y)}$

$$
\begin{aligned}
B_{mn} = {} & \frac{0.14}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5} \\
& + \frac{0.3}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& - \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.3}{\pi^5 m^3 n^2}\sin\frac{0.35n\pi}{0.35} - \frac{0.6}{\pi^6 m^3 n^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.6}{\pi^6 m^3 n^3}
\end{aligned}
$$

$$U_{mn}(x,y,t) = \Big(\frac{0.14}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$- \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5} + \frac{0.3}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$+ \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.3}{\pi^5 m^3 n^2}\sin\frac{0.35n\pi}{0.35}$$

$$- \frac{0.6}{\pi^6 m^3 n^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.6}{\pi^6 m^3 n^3}\Big)\cos\lambda mnt\left(\sin\frac{m\pi x}{0.5}\ \sin\frac{n\pi y}{0.35}\right)$$

$$\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$\lambda_{11} = 3.50c\pi$ $\lambda_{12} = 6.05c\pi$ $\lambda_{13} = 8.80c\pi$ $\lambda_{21} = 4.92c\pi$ $\lambda_{22} = 6.98c\pi$ $\lambda_{23} = 9.46c\pi$
$\lambda_{31} = 6.65c\pi$ $\lambda_{32} = 8.29c\pi$ $\lambda_{33} = 10.46c\pi$

Hence B_{11} is obtained by substituting m = 1 and n = 1 into the B_{mn} formular. The same is done for B_{12} B_{13}……to obtain the 3 x 3 matrix.

From the programme of Visual Basic 6

B_{mn} values for $m = 1, 2, 3$ and $n = 1,\ 2,\ 3$ a 3×3 matrix values are

$$\begin{pmatrix} 0.00235 & 0 & 0.00009 \\ 0.00002 & 0 & 0 \\ 0.00009 & 0 & 0 \end{pmatrix}$$

Sine Waves (Normal Modes)

$$U_{11} = 0.00235\cos 3.5c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{\pi y}{0.35}$$

$$U_{12} = 0$$

$$U_{13} = 0.00009 \cos 8.80c\pi t \; sin\frac{\pi x}{0.5} \; sin\frac{3\pi y}{0.35}$$

$$U_{21} = 0.00002 \; cos4.92c\pi t \; sin \; \frac{2\pi x}{0.5} \sin\frac{\pi y}{0.35}$$

$$U_{22} = 0$$

$$U_{23} = 0$$

$$U_{31} = 0.00009 \cos 6.65c\pi t \sin\frac{3\pi x}{0.5} \sin\frac{\pi y}{0.35}$$

$$U_{32} = 0$$

$$U_{33} = 0$$

$$U_{11} = 0.00235 \cos 3.5c\pi t \; \sin\frac{\pi x}{0.5} \; \sin\frac{\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1, \; 0.2, \; 0.3$ vibrations are;

$$0.000000807999 \cos 3.5c\pi t$$

$$0.0000016158 \cos 3.5c\pi t$$

$$0.0000024232 \cos 3.5c\pi t$$

For $y = 0.2$ and $x = 0.1, \; 0.2, \; 0.3$ vibrations are;

$$0.00000080794909 \cos 3.5c\pi t$$

$$0.0000016158 \cos 3.5c\pi t$$

$$0.000002423458 \cos 3.5c\pi t$$

$$U_{12} = 0$$

$$U_{13} = 0.00009 \cos 8.80c\pi t \; sin\frac{\pi x}{0.5} \; sin\frac{3\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1, \; 0.2, \; 0.3$ vibrations are;

0.0000000928 cos8.80cπt

0.000000185 cos8.80cπt

0.00001265 cos8.80cπt

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.0000000927 cos8.80cπt

0.000000185 cos8.80cπt

0.000000278 cos8.80cπt

$$U_{21} = 0.00002\ cos4.92c\pi t\ sin\ \frac{2\pi x}{0.5} \sin \frac{\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.0000000137 cos4.92$c\pi t$

0.00000002749 cos4.92$c\pi t$

0.00000004123 cos4.92$c\pi t$

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.00000001375 cos4.92$c\pi t$

0.00000002749 cos4.92$c\pi t$

0.00000004122 cos4.92$c\pi t$

$$U_{22} = 0$$

$$U_{23} = 0$$

$$U_{31} = 00.00009 \cos 6.65c\pi t \sin \frac{3\pi x}{0.5} \sin \frac{\pi y}{0.35}$$

For x= 0.2 and y= 0.1, 0.2, 0.3 vibrations are;

0.00000009277 cos6.65$c\pi t$

0.0000001855 cos6.65$c\pi t$

0.0000002782 cos6.65$c\pi t$

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$0.0000000928 \cos 6.65 c\pi t$$

$$0.0000001855 \cos 6.65 c\pi t$$

$$0.000000278 \cos 6.65 c\pi t$$

$$U_{32} = 0$$

$$U_{33} = 0$$

Discussions

The waves moved to a crest, to a nodal point, to a crest, to a trough, to a nodal point, to a nodal point, to a crest, to a nodal point and finally to a nodal point. This indicates that the drum had a surface pivoted at the origin and uniformly distributed as a normal distribution shape. This shape produced the waves above. For a normal mode of U_{11}, and for a fixed value of x = 0.2 with varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{21}, and U_{31} also witnessed increased in vibrations along the nodal lines, whiles U_{12}, U_{22}, U_{23}, U_{32}, U_{33} showed nodal points.

The membrane with this deflection produced a sound that could hardly be heard and quickly died down. Deleting the principal (leading) diagonal of the matrix and running down the value ≤ 0.00002 saw the matrix to be symmetric. That is, the upper triangular matrix equals the lower triangular matrix.

4.2 A sixth order polynomial of the fist type as initial deflection

The function $\boldsymbol{f(x\,y) = x\,y(a^2 - x^2)(\,b^2 - y^2)}$ was chosen because it does not violate the boundary conditions unlike a function of the form y = xy. In other words there is no discontinuity (breaks) on the graph but continuous uniformly as can be seen from the graph. The values of a, and b are the length and breadth of the rectangular drum. Detailed calculations were done and obtained Umn (x, y, t) function, from which the Fourier coefficients, sine waves (normal modes) and vibrations at specific points were obtained.

See appendix 6 for detailed calculations

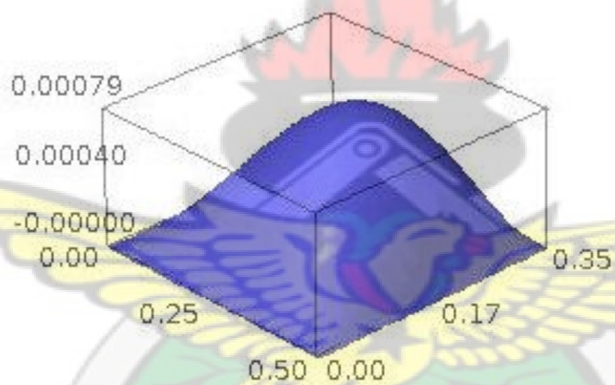


Fig 4.2 The surface $\boldsymbol{f(x\,y) = x\,y(a^2 - x^2)(\,b^2 - y^2)}$

$$
\begin{aligned}
B_{mn} = {} & \frac{0.07}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.3}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& - \frac{0.3}{\pi^6 m^2 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.19}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} \\
& + \frac{0.77}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.77}{\pi^7 m^3 n^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} \\
& - \frac{0.19}{\pi^6 m^4 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.77}{\pi^7 m^4 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& + \frac{0.77}{\pi^8 m^4 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}
\end{aligned}
$$

$B^*_{mn} = 0$ since $g(x,y) = 0$

$$U_{mn}(x,y,t) = (B_{mn}\cos\lambda mnt + B^*_{mn}\sin\lambda mnt)\left(\sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b}\right)$$

$$U_{mn}(x,y,t) = \Big(\frac{0.07}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.3}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$-\frac{0.3}{\pi^6 m^2 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.19}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$+\frac{0.77}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.77}{\pi^7 m^3 n^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$-\frac{0.19}{\pi^6 m^4 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.77}{\pi^7 m^4 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.77}{\pi^8 m^4 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}\Big)\cos\lambda mnt\left(\sin\frac{m\pi x}{0.5}\ \sin\frac{n\pi y}{0.35}\right)$$

$$\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$\lambda_{11} = 3.50c\pi \quad \lambda_{12} = 6.05c\pi \quad \lambda_{13} = 8.80c\pi \quad \lambda_{21} = 4.92c\pi \quad \lambda_{22} = 6.98c\pi \quad \lambda_{23} = 9.46c\pi$$
$$\lambda_{31} = 6.65c\pi \quad \lambda_{32} = 8.29c\pi \quad \lambda_{33} = 10.46c\pi$$

From the programme of Visual Basic 6

B_{mn} values for $m = 1, 2, 3$ and $n = 1,\ 2,\ 3$ a 3×3 matrix values are

$$\begin{pmatrix} 0.0008 & -0.0001 & 0.00003 \\ -0.0001 & 0.00001 & 0 \\ 0.00003 & 0 & 0 \end{pmatrix}$$

Sine Waves (Normal Modes)

$$U_{11} = 0.0008\cos 3.5c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{\pi y}{0.35}$$

$$U_{12} = -0.0001\cos 6.05c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{2\pi y}{0.35}$$

$$U_{13} = 0.00003\cos 8.80c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{3\pi y}{0.35}$$

$$U_{21} = -0.0001\cos 4.92c\pi t\ \sin\frac{2\pi x}{0.5}\ \sin\frac{\pi y}{0.35}$$

$$U_{22} = 0.00001\cos 6.98c\pi t\ \sin\frac{2\pi x}{0.5}\ \sin\frac{2\pi y}{0.35}$$

$$U_{23} = 0$$

$$U_{31} = 0.00003\cos 6.65c\pi t\ \sin\frac{3\pi x}{0.5}\ \sin\frac{\pi y}{0.35}$$

$$U_{32} = 0$$

$$U_{33} = 0$$

$$U_{11} = 0.0008\cos 3.5c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$0.00000027506\cos 3.5c\pi t$$

$$0.00000055005\cos 3.5c\pi t$$

$$0.00000082492\cos 3.5c\pi t$$

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$0.000000275046\cos 3.5c\pi t$$

$$0.000000550059\cos 3.5c\pi t$$

$$0.000000825007\cos 3.5c\pi t$$

$$U_{12} = -0.0001\cos 6.05c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{2\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$-0.00000006875 \cos 6.05c\pi t$$

$$-0.00000013744 \cos 6.05c\pi t$$

$$-0.000000206 \cos 6.05c\pi t$$

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$-0.0000000687278 \cos 6.05c\pi t$$

$$-0.000000137447 \cos 6.05c\pi t$$

$$-0.00000020615 \cos 6.05c\pi t$$

$$U_{13} = 0.00003 \cos 8.80c\pi t\ \sin\frac{\pi x}{0.5}\ \sin\frac{3\pi y}{0.35}$$

For $x = 0.2$ and y= 0.1, 0.2, 0.3 vibrations are;

0.0000000309 cos8.80cπt

0.0000000618 cos8.80cπt

0.0000000925 cos8.80cπt

For $y = 0.2$ and x= 0.1, 0.2, 0.3 vibrations are;

0.0000000309 cos8.80cπt

0.0000000618 cos8.80cπt

0.0000000927 cos8.80cπt

$$U_{21} = -0.0001 \cos 4.92c\pi t\ \sin\frac{2\pi x}{0.5}\ \sin\frac{\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$-0.00000006874938 \cos 4.92c\pi t$$

$$-0.0000001374818 \cos 4.92c\pi t$$

$$-0.0000002061806 \cos 4.92c\pi t$$

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

$$-0.00000006875748 \cos 4.92c\pi t$$

$$-0.0000001374818 \cos 4.92c\pi t$$

$$-0.00000020614007 \cos 4.92c\pi t$$

$$U_{22} = 0.00001 \cos 6.98c\pi t \ \sin\frac{2\pi x}{0.5} \ \sin\frac{2\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.0000000137cos6.98cπt

0.0000000275cos6.98cπt

0.00000004119cos6.98cπt

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.0000000137cos6.98cπt

0.0000000275cos6.98cπt

0.0000000412cos6.98cπt

$$U_{23} = 0$$

$$U_{31} = 0.00003 \cos 6.65c\pi t \ \sin\frac{3\pi x}{0.5} \ \sin\frac{\pi y}{0.35}$$

For $x = 0.2$ and $y = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.0000000309cos6.65cπt

0.0000000618cos6.65cπt

0.0000000927cos6.65cπt

For $y = 0.2$ and $x = 0.1,\ 0.2,\ 0.3$ vibrations are;

0.0000000309cos6.65cπt

0.0000000618cos6.65cπt

0.0000000927cos6.65cπt

$$U_{32} = 0$$

$$U_{33} = 0$$

Discussions

The waves moved to a crest, to a trough, to a crest, to a trough, to a crest, to a nodal point, to a crest, to a nodal point, and finally to a nodal point.

The graph indicates that the drum had a surface pivoted at the origin with a distribution semi uniformly skewed to the right. This shape produced the waves above.

For a normal mode of $U_{11,}$ and for a fixed value of x = 0.2 and varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{22} and U_{31} also witnessed increased vibrations along the nodal lines, whiles U_{12} and U_{21} witnessed decreased vibrations along the nodal lines with U_{23}, U_{32} and U_{33} having nodal points.

The membrane with this deflection produced a sound that could hardly be heard and quickly died down. Deleting the principal (leading) diagonal of the matrix, saw the matrix to be symmetric. That is, the upper triangular matrix equals to the lower triangular matrix.

4.3 A sixth order polynomial of the second type as initial deflection

The function $f(xy) = x^2y^2(a-x)(b-y)$ was chosen because it does not violate the boundary conditions unlike a function of the form y = x+y. In other words there is no discontinuity (breaks) on the graph but continuous as can be seen from the graph. The values of a, and b are the length and breadth of the rectangular drum. Detailed calculations were done and obtained Umn (x, y, t) function, from which the Fourier coefficients, sine waves (normal modes) and vibrations at specific points were obtained.

See appendix 7 for detailed calculations

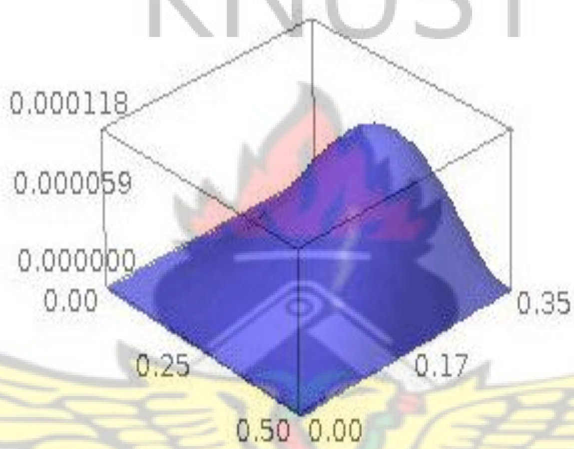


Fig 4.3: The surface $f(xy) = x^2 y^2 (a-x)(b-y)$

$$
\begin{aligned}
B_{mn} = {} & \frac{0.086}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.120}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\,cos\frac{0.35n\pi}{0.35} \\
& - \frac{0.253}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5} - \frac{0.253}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.253}{m^2n^4\pi^6} \\
& \sin\frac{0.5m\pi}{0.5} + \frac{0.129}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.181}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.4071}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5} - \frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& + \frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5} + \frac{0.086}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.120}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\,cos\frac{0.35n\pi}{0.35} + \frac{0.086}{m^3n^3\pi^6} \\
& \cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.086}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} + \frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.3611}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& - \frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.805}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5} + \frac{0.805}{m^3n^4\pi^7} \\
& \sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.805}{m^3n^4\pi^7}\sin\frac{0.5m\pi}{0.5} - \frac{0.086}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35} - \frac{0.120}{m^3n^2\pi^5}\cos\frac{0.25n\pi}{0.35} \\
& - \frac{0.086}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.253}{m^3n^3\pi^6}\sin\frac{0.35n\pi}{0.35} + \frac{0.86}{m^3n^3\pi^6} + \frac{0.253}{m^3n^4\pi^7}\cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^3n^4\pi^7} - \frac{0.253}{m^4n^2\pi^6} \\
& \cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.345}{m^4n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& + \frac{0.805}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5} + \frac{0.805}{m^4n^4\pi^8}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.805}{m^4n^4\pi^8} \\
& \cos\frac{0.5m\pi}{0.5} + \frac{0.253}{m^4n^2\pi^6}\sin\frac{0.35n\pi}{0.35} + \frac{0.345}{m^4n^2\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.253}{m^4n^3\pi^7}\cos\frac{0.35n\pi}{0.35} - \frac{0.805}{m^4n^3\pi^7} \\
& \sin\frac{0.35n\pi}{0.35} - \frac{0.253}{m^4n^3\pi^7} - \frac{0.805}{m^4n^4\pi^8}\cos\frac{0.35n\pi}{0.35} + \frac{0.805}{m^4n^4\pi^8}
\end{aligned}
$$

$B^{*}_{mn} = 0$ since $g(x, y) = 0$

$$U_{mn}(x, y, t) = (B_{mn}\cos\lambda_{mn}t + B^{*}_{mn}\sin\lambda_{mn}t)\left(\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right)$$

$$U_{mn}(x,y,t) = \Big(\frac{0.086}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.120}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}$$

$$cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5} - \frac{0.253}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.253}{m^2n^4\pi^6}$$

$$\sin\frac{0.5m\pi}{0.5} + \frac{0.129}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.181}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}$$

$$\cos\frac{0.35n\pi}{0.35} - \frac{0.4071}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5} - \frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5} + \frac{0.086}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.120}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}cos\frac{0.35n\pi}{0.35} + \frac{0.086}{m^3n^3\pi^6}$$

$$\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.086}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}$$

$$\cos\frac{0.35n\pi}{0.35} + \frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.3611}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$-\frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.805}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5} + \frac{0.805}{m^3n^4\pi^7}$$

$$\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.805}{m^3n^4\pi^7}\sin\frac{0.5m\pi}{0.5} - \frac{0.086}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35} - \frac{0.120}{m^3n^2\pi^5}\cos\frac{0.25n\pi}{0.35}$$

$$-\frac{0.086}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.253}{m^3n^3\pi^6}\sin\frac{0.35n\pi}{0.35} + \frac{0.86}{m^3n^3\pi^6} + \frac{0.253}{m^3n^4\pi^7}\cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^3n^4\pi^7} - \frac{0.253}{m^4n^2\pi^6}$$

$$\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.345}{m^4n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.805}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5} + \frac{0.805}{m^4n^4\pi^8}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.805}{m^4n^4\pi^8}$$

$$\cos\frac{0.5m\pi}{0.5} + \frac{0.253}{m^4n^2\pi^6}\sin\frac{0.35n\pi}{0.35} + \frac{0.345}{m^4n^2\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.253}{m^4n^3\pi^7}\cos\frac{0.35n\pi}{0.35} - \frac{0.805}{m^4n^3\pi^7}$$

$$\sin\frac{0.35n\pi}{0.35} - \frac{0.253}{m^4n^3\pi^7} - \frac{0.805}{m^4n^4\pi^8}\cos\frac{0.35n\pi}{0.35} + \frac{0.805}{m^4n^4\pi^8}\Big)\cos\lambda_{mn}t\left(\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\right)$$

Hence B_{11} is obtained by substituting m = 1 and n = 1 into the B_{mn} formular. The same is done for B_{12} B_{13} ………………to obtain the 3 x 3 matrix

From the programme of visual Basic 6

B_{mn} *values for* $m = 1, 2, 3$ *and* $n = 1, 2, 3$ *a* $3x3$ *matrix values are*:

$$\begin{pmatrix} 0.00195 & -0.00047 & 0.00024 \\ -0.00047 & 0.00011 & -0.00006 \\ 0.00023 & -0.00005 & 0.00003 \end{pmatrix}$$

$$\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$\lambda_{11} = 3.50c\pi \quad \lambda_{12} = 6.05c\pi \quad \lambda_{13} = 8.80c\pi \quad \lambda_{21} = 4.92c\pi \quad \lambda_{22} = 6.98c\pi \quad \lambda_{23} = 9.46c\pi$

$\lambda_{31} = 6.65c\pi \quad \lambda_{32} = 8.29c\pi \quad \lambda_{33} = 10.46c\pi$

The sine waves (normal modes) are:

$$U_{11} = 0.00195 \cos 3.50c\pi t \sin\frac{\pi x}{0.5} \sin\frac{\pi y}{0.35}$$

$$U_{12} = -0.00047 \cos 6.05c\pi t \sin\frac{\pi x}{0.5} \sin\frac{2\pi y}{0.35}$$

$$U_{13} = 0.00024 \cos 8.80c\pi t \sin\frac{\pi x}{0.5} \sin\frac{3\pi y}{0.35}$$

$$U_{21} = -0.00047 \cos 4.92c\pi t \sin\frac{2\pi x}{0.5} \sin\frac{\pi y}{0.35}$$

$$U_{22} = 0.00011 \cos 6.98c\pi t \sin\frac{2\pi x}{0.5} \sin\frac{2\pi y}{0.35}$$

$$U_{23} = -0.00006 \cos 9.46c\pi t \sin\frac{2\pi x}{0.5} \sin\frac{3\pi y}{0.35}$$

$$U_{31} = 0.00023 \cos 6.65c\pi t \sin\frac{3\pi x}{0.5} \sin\frac{\pi y}{0.35}$$

$$U_{32} = -0.00005 \cos 8.29c\pi t \sin\frac{3\pi x}{0.5} \sin\frac{2\pi y}{0.35}$$

$$U_{33} = 0.00003 \cos 10.46c\pi t \sin\frac{3\pi x}{0.5} \sin\frac{3\pi y}{0.35}$$

For x: $0 \le x \le 0.5$ *and for* y: $0 \le y \le 0.35$,
and for a fixed value of $x = 0.2$ *and* y *taking values of* $0.1, 0.2, 0.3$ *and also for a fixed value of* $y = 0.2$ *and* x *taking values of* $0.1, 0.2, 0.3$ *saw the vibrations* increasing *along the fixed* points *with positive* B_{mn} *values and* decreasing *along the fixed* points *with negative* B_{mn} *values as similar to the calculations done in* section 4.1 *and* section 4.2.

Discussions

The waves moved to a crest, to a trough, to a crest, to a trough, to a crest, to a trough, to a crest, to a trough and finally to a crest. The graph indicates that the drum had a surface pivoted at the origin with a distribution semi uniformly skewed to the right.

This shape produced the waves above. For a normal mode of U_{11}, and for a fixed value of x = 0.2 and varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{22}, U_{31} and U_{33} also witnessed increased vibrations along the nodal lines, whiles U_{12}, U_{21}, U_{23} and U_{32} witnessed decreased vibrations along the nodal lines. No nodal point was recorded. The membrane with this deflection produced a sound that could be heard and quickly die down. Deleting the principal (leading) diagonal of the matrix and running down all the values to four decimal places saw the matrix to be symmetric. That is the upper triangular matrix equals the lower triangular matrix.

4.4. A sixth order polynomial of the third type as initial deflection

The function $f(x, y) = xy(a - x)^2(b - y)^2$ was chosen because it does not violate the boundary conditions unlike a function of the form $y = x^2+y^2$. In other words there is no discontinuity (breaks) on the graph but continuous as can be seen from the graph. The values of a, and b are the length and breadth of the rectangular drum. Detailed calculations were done and obtained Umn (x, y, t) function, from which the Fourier coefficients, sine waves (normal modes) and vibrations at specific points were obtained.

See appendix 8 for detailed calculations

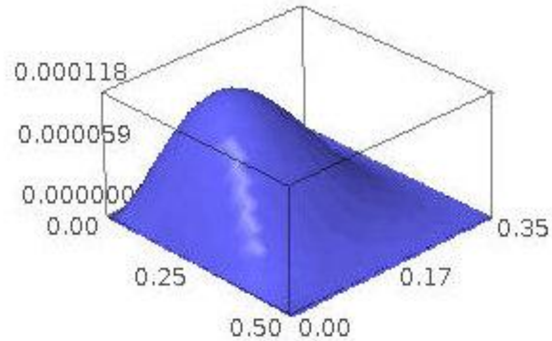


Fig4.4: The surface $f(x, y) = xy(a - x)^2(b - y)^2$

$$
\begin{aligned}
B_{mn} = {} & \frac{-0.0004}{mn\pi^2}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.00023}{mn^2\pi^3}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.00207}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} + \frac{0.0036}{mn^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.0056}{mn^4\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.000403}{m^2n\pi^3}\sin\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.00023}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.0021}{m^3n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.0037}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5} \\
& - \frac{0.0056}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.0145}{m^3n\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.0083}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} \\
& + \frac{0.0745}{m^3n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.13}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.2047}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.0414}{m^3n\pi^4} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.023}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35} + \frac{0.207}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.37}{m^3n^3\pi^6} - \frac{0.575}{m^3n^4\pi^7}\sin\frac{0.35n\pi}{0.35} - \frac{0.0552}{m^4n\pi^5} \\
& \sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.0313}{m^4n^2\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.28}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.4922}{m^4n^3\pi^7} \\
& \sin\frac{0.5m\pi}{0.5} - \frac{0.782}{m^4n^4\pi^8}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}
\end{aligned}
$$

$B^*_{mn} = 0 \sin ce\ g(x, y) = 0$

$$U_{mn}(x, y, t) = (B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t)\left(\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\right)$$

$$
\begin{aligned}
U_{mn}(x,y,t) = \Bigg[& \frac{-0.0004}{mn\pi^2}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.00023}{mn^2\pi^3}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.00207}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} + \frac{0.0036}{mn^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.0056}{mn^4\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.000403}{m^2n\pi^3}\sin\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.00023}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.0021}{m^3n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.0037}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5} \\
& -\frac{0.0056}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.0145}{m^3n\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.0083}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} \\
& +\frac{0.0745}{m^3n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.13}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.2047}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.0414}{m^3n\pi^4} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.023}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35} + \frac{0.207}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.37}{m^3n^3\pi^6} - \frac{0.575}{m^3n^4\pi^7}\sin\frac{0.35n\pi}{0.35} - \frac{0.0552}{m^4n\pi^5} \\
& \sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.0313}{m^4n^2\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.28}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.4922}{m^4n^3\pi^7} \\
& \sin\frac{0.5m\pi}{0.5} - \frac{0.782}{m^4n^4\pi^8}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}\Bigg] \cos\lambda_{mn}t\left(\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\right)
\end{aligned}
$$

Hence B_{11} is obtained by substituting m = 1 and n = 1 into the B_{mn} formular. The same is done for B_{12} B_{13} ………………to obtain the 3 x 3 matrix

From the programme of visual Basic 6

B_{mn} *values for* $m = 1,2,3$ *and* $n = 1,2,3$ *a* 3*x*3 *matrix values are*:

$$
\begin{pmatrix} 0.00033 & -\ 0.00009 & 0.00009 \\ 0.00013 & -0.00003 & 0.00003 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$
\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}
$$

$$\lambda_{11} = 3.50c\pi \quad \lambda_{12} = 6.05c\pi \quad \lambda_{13} = 8.80c\pi \quad \lambda_{21} = 4.92c\pi \quad \lambda_{22} = 6.98c\pi \quad \lambda_{23} = 9.46c\pi$$
$$\lambda_{31} = 6.65c\pi \quad \lambda_{32} = 8.29c\pi \quad \lambda_{33} = 10.46c\pi$$

The sine waves (normal modes) are:

$$U_{11} = 0.00033\cos 3.50c\pi t \sin\frac{\pi x}{0.5}\sin\frac{\pi y}{0.35}$$
$$U_{12} = -0.00009\cos 6.05c\pi t \sin\frac{\pi x}{0.5}\sin\frac{2\pi y}{0.35}$$
$$U_{13} = 0.00009\cos 8.80c\pi t \sin\frac{\pi x}{0.5}\sin\frac{3\pi y}{0.35}$$
$$U_{21} = 0.00013\cos 4.92c\pi t \sin\frac{2\pi x}{0.5}\sin\frac{\pi y}{0.35}$$
$$U_{22} = -0.00003\cos 6.98c\pi t \sin\frac{2\pi x}{0.5}\sin\frac{2\pi y}{0.35}$$
$$U_{23} = 0.00003\cos 9.46c\pi t \sin\frac{2\pi x}{0.5}\sin\frac{3\pi y}{0.35}$$
$$U_{31} = 0$$
$$U_{32} = 0$$
$$U_{33} = 0$$

For x: $0 \le x \le 0.5$ *and for* y: $0 \le y \le 0.35$, *and for a fixed value of* $x = 0.2$ *and* y *taking values of* 0.1, 0.2, 0.3 *and also for a fixed value of* $y = 0.2$ *and* x *taking values of* 0.1, 0.2, 0.3 *saw the vibrations* increasing *along the fixed* points *with positive* B_{mn} *values and* decreasing *along the fixed* points *with negative* B_{mn} *values as similar to the calculations done in* section 4.1 *and* section 4.2.

Discussions

The waves moved to a crest, to a trough, to a crest, to a crest, to a trough, to a crest, to a nodal point, to a nodal point and finally to a nodal point. The graph indicated that the drum had a surface pivoted at the origin with a distribution semi uniformly skewed to the left.

This shape produced the waves above. For a normal mode of U_{11}, and for a fixed value of x = 0.2 and varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{21}, and U_{23} also witnessed increased vibrations along the nodal lines, whiles U_{12} and U_{22} witnessed decreased vibrations along the nodal lines. However, U_{31}, U_{32} and U_{33} recorded nodal points. The membrane with this deflection produced a sound that could hardly be heard and quickly die down. Deleting the principal (leading) diagonal of the matrix and running down all the values to three decimal places, saw the matrix to be symmetric. That is, the upper triangular matrix equals the lower triangular matrix.

CHAPTER 5

CONCLUSION

5.0 Introduction

This chapter explains why the rectangular drum is said to be bounded, considered the possible cases of initial deflections and initial velocities as well as the description of the sine waves (normal modes) of the vibrating membrane. It also described the vibrations at fixed points of x and y and varying values of y and x respectively and finally showed the symmetry of the calculated Fourier coefficients.

5.1 Example of a fourth order polynomial as initial deflection

Considering an example of a fourth order polynomial $f(x, y) = z = x(0.5 - x)\ y(0.35 - y)$ as initial deflection, the following discussions are made. The rectangular drum is said to be bounded in that, the membrane is fixed at the ends of all the four edges of the drum case, beyond these edges the vibrations are zero. The double sine waves (normal modes), or vibrations are propagated within the area of the membrane.

In this study, initial deflections were given to the drum, with the initial velocities set to zero. However, other cases could also be looked at, that is, where both initial deflections and velocities are present, or initial velocities are present and the initial deflections set to zero.

When the membrane was given an initial deflection of Z = x (0.5 – x) y (0.35 – y) and released, the Fourier coefficients that dictated the pattern of the movements of the sine waves (normal modes) are:

$$\begin{pmatrix} 0.00235 & 0 & 0.000\ 09 \\ 0.00002 & 0 & 0 \\ 0.00009 & 0 & 0 \end{pmatrix}$$

This means that the waves moved to a crest, to a nodal point, to a crest, to a trough, to a nodal point, to a nodal point, to a nodal point, to a crest, to a nodal point and finally to a nodal point. The corresponding graph in chapter four indicated that the drum had a surface pivoted at the origin and uniformly distributed as a normal distribution shape. This shape produced the waves above. For a normal mode of $U_{11,}$ and for a fixed value of x = 0.2 with varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{21}, and U_{31} also witnessed increased in vibrations along the nodal lines, whiles U_{12}, U_{22}, U_{23}, U_{32}, U_{33} showed nodal points.

The membrane with this deflection produced a sound that could hardly be heard and quickly died down. Deleting the principal (leading) diagonal of the matrix and running down the value ≤ 0.00002 saw the matrix to be symmetric. That is, the upper triangular matrix equals the lower triangular matrix.

5.2 Example of a sixth order polynomial of the first type as initial deflection.

Considering a sixth order polynomial of the first type $f(x,y) = z = x(0.5^2 - x^2)\ y(0.35^2 - y^2)$ discussions are as follows.

When the membrane was given an initial deflection of $Z = X\ (0.5^2 - X^2)\ y(0.35^2 - y^2)$ and released, the Fourier coefficients that dictated the pattern of the movements of the sine waves (normal modes) are:

$$\begin{pmatrix} 0.0008 & -0.0001 & 0.00003 \\ -0.0001 & 0.00001 & 0 \\ 0.00003 & 0 & 0 \end{pmatrix}$$

This means that the waves moved to a crest, to a trough, to a crest, to a trough, to a crest, to a nodal point, to a crest, to a nodal point, and finally to a nodal point.

The corresponding graph in chapter four indicated that the drum had a surface pivoted at the origin with a distribution semi uniformly skewed to the right. This shape produced the waves above.

For a normal mode of $U_{11,}$ and for a fixed value of x = 0.2 and varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{22} and U_{31} also witnessed increased vibrations along the nodal lines, whiles U_{12} and U_{21} witnessed decreased vibrations along the nodal lines with U_{23}, U_{32} and U_{33} having nodal points.

The membrane with this deflection produced a sound that could hardly be heard and quickly died down. Deleting the principal (leading) diagonal of the matrix, saw the matrix to be symmetric. That is, the upper triangular matrix equals to the lower triangular matrix.

All the waves had the same eigenvalues and the same frequencies, because of the same dimensions used for the calculations.

5.3 Example of a sixth order polynomial of the second type as initial deflection.

Considering a sixth order polynomial of the second type $f(x,y) = x^2y^2(0.5 - x)(0.35 - y)$discussions are as follows.

When the membrane was given an initial deflection of $z = x^2 y^2 (0.5 - x)(0.35 - y)$ and released, the Fourier coefficients that dictated the parttern of the movements of sine waves (normal modes) are:

$$\begin{pmatrix} 0.00195 & - & 0.00047 & 0.00024 \\ -0.00047 & & 0.00011 & -0.00006 \\ 0.00023 & & -0.00005 & 0.00003 \end{pmatrix}$$

This means that the waves moved to a crest, to a trough, to a crest, to a trough, to a crest, to a trough, to a crest, to a trough and finally to a crest. The corresponding graph in chapter four indicated that the drum had a surface pivoted at the origin with a distribution semi uniformly skewed to the right.

This shape produced the waves above. For a normal mode of U_{11}, and for a fixed value of x = 0.2 and varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{22}, U_{31} and U_{33} also witnessed increased vibrations along the nodal lines, whiles U_{12}, U_{21}, U_{23} and U_{32} witnessed decreased vibrations along the nodal lines. No nodal point was recorded. The membrane with this deflection produced a sound that could be heard and quickly die down. Deleting the principal (leading) diagonal of the matrix and running down all the values to four decimal places saw the matrix to be symmetric. That is the upper triangular matrix equals the lower triangular matrix.

5.4 Example of a sixth order polynomial of the third type as initial deflection.

Considering a sixth order polynomial of the third type $f(x, y) = xy(0.5 - x)^2(0.35 - y)^2$discussions are as follows.

When the membrane was given an initial deflection of $z = xy(0.5 - x)^2 (0.35 - y)^2$ *and released, the Fourier coefficients that dictated the parttern of the movements of the* sin*e waves (normal* mod*es) are:*

$$\begin{pmatrix} 0.00033 & - & 0.00009 & 0.00009 \\ 0.00013 & & -0.00003 & 0.00003 \\ 0 & & 0 & 0 \end{pmatrix}$$

This means that the waves moved to a crest, to a trough, to a crest, to a crest, to a trough, to a crest, to a nodal point, to a nodal point and finally to a nodal point. The corresponding graph in chapter four indicated that the drum had a surface pivoted at the origin with a distribution semi uniformly skewed to the left.

This shape produced the waves above. For a normal mode of U_{11}, and for a fixed value of x = 0.2 and varying values of y, and for a fixed value of y = 0.2 with varying values of x respectively, saw the vibrations increased along the nodal lines. With the same parameters above, U_{13}, U_{21}, and U_{23} also witnessed increased vibrations along the nodal lines, whiles U_{12} and U_{22} witnessed decreased vibrations along the nodal lines. However, U_{31}, U_{32} and U_{33} recorded nodal points. The membrane with this deflection produced a sound that could hardly be heard and quickly die down. Deleting the principal (leading) diagonal of the matrix and running down all the values to three decimal places, saw the matrix to be symmetric. That is, the upper triangular matrix equals the lower triangular matrix.

All the waves had the same eigenvalues and the same frequencies, because of the same dimensions used for the calculations.

5.5 RECOMMENDATIONS

This study looked at a situation where the deflections are not zero but the velocities all set to zero. Cases involving both deflections and velocities known could be investigated. Also cases involving the deflections being zero and the velocities known could also be investigated. These suggested cases could also find interesting description of the waves and vibrations.

5.6CORE CONCLUSION

The study looked at a bounded rectangular membrane and four different deflections given to the membrane. The first deflection $Z = x(0.5 – x) y(0.35-y)$ saw the waves vibrating crest and trough with four nodal points. These vibrations are hardly heard and quickly die off. The second deflection $Z = x(0.5^2- x^2) y(0.35^2- y^2)$ recorded vibrations that moved crest and trough with three nodal points. These vibrations are hardly heard and quickly die off. The third deflection $Z = x^2 y^2 (0.5 –x) (0.35 - y)$ saw the waves vibrating crest and trough with no nodal point. These vibrations can be heard and quickly dies off. The fourth deflection $Z = xy (0.5 – x)^2 (0.35 – y)^2$ saw the waves vibrating crest trough with three nodal point. These vibrations are hardly heard and quickly die off.

The first, second and fourth deflections gives sounds that are hardly heard whiles the third deflection gives sounds that could be heard. The rectangular drum has a mathematical background and the best deflection that could be given to membrane for a good sound and symmetry is $Z = x^2y^2 (0.5-x)(0.35-y)$.

REFERENCES

Agawu K. (1995) *Africa Rhythm, Cambridge University Press* Published by Wanye, 24th February, 2009.

Andrian B. (1995) Bob Marley, Songs of Freedom (1995) Published by Viking studio books (1995)

Bracewell R. N (2000) *The Fourier Transforms, and its Applications* (3rd edition) Boston. Published by McGraw-Hill

Bobb D. (1998) *Beating a restless drum, the poetics of Kamau Brath Waife and Derek Walcott*. Published by African World Press Inc.

Boyce W. E (2005) *Elementary Differential Equations and Boundary Valve problem* (8th edition) New Jersey. Published by John Wiley and sons Inc.

Coddington E. A and Levinson N (1995)*Theory of ordinary Differential Equations* Published by McGraw – Hill, 1995

Dearling R. (1996). *The Ultimate Encyclopedia of Musical Instruments.* Published by Wanye, 24th February, 2009.

Devaney G. R and Blanchard P. (2006) *Differential Equation.* Published by Thompson (2006)

Friedberg M. and Lilian K. (1993) *The Djembe drum, a drum with thousand face*. Published by Wanye, 24th February, 2009.

Grafakos L. (2004), *Classical and modern Fourier Analysis*. Published by Prentice-Hall.

Joseph Fourier (2003) translated by Alexander Freeman (Published 1822, translated 1878, re-released 2003) *The Analytical theory of Heat*. Dover publication (2003)

Kreyszig E. (2006) . *Advanced Engineering Mathematics*, *9th Edition*, Chapter 11 and 12 pages 478 – 562 Published by Willey John and Sons Inc.

Michael B. (1982) Oxford University Press Music department. Published by Oxford University Press.

Microsoft Visual Basic 6.0, *Crystal report 8.5*

Nketia J. H. K. (1968) *Our Drums and Drummers.* Published by Ghana Publishing House

Nketia J. H. K. (1971) "*History and the Organisation of Music in West Africa".* In Essays on Music and History in Africa.

Pinsky M. (2002) *Introduction to Fourier Analysis and Wavelets, Brooks and Cole*, Published by Brooks and Cole Inc. (2002)

Pinsky M. (2002), *Introduction to Fourier Analysis and Wavelets*. Published by Brooks and Cole Inc.

Robert T. M. and Roland B. M (2002). Calculus. (2nd edition) Published by McGraw-Hill, a business unit of the McGraw-Hill companies Inc. New York.

Polyanin A. D and Zaitsev V. F. (2003) *Handbook of Exact Solutions for ordinary Differential Equations* (2nd edition) (2003) Published by Chapman and Hall/CRC Press.

Stein E et al (2003) *Fourier Analysis. An introduction. Princeton*. Published by Princeton University Press.

William A. Stein et al. *Sage Mathematics Software(version4.6.2).*The Sage Development Team,(2009) http://www.sagemath.org.

William E. and Richard c. (2005). *Elementary Differential Equations and Boundary Valve Problems,* New Jersey, America. Published by John Wiley and Sons Inc.

Wilson R. G (1995). Fourier series and optical Transform Techniques in contemporary optical, New York. Published by Wiley Inc.

Zygmund A. (2002) *Trigonometric series* (third edition) Cambridge: Cambridge University Press.

APPENDIX 1

Derivation of the Wave Equation

Consider the deflection of a string in xy plane in figure A1. To derive the differential equation which governs the motion of the membrane, we consider the forces acting on a small portion of the membrane are shown in figure A1. Since the deflections of the membrane and the angles of inclination are small, the sides of the portion are approximately equal to Δx and Δy. The tension T is the force per unit length. Hence the forces acting on the edges of the portion are approximately TΔx and TΔy. Since the membrane is perfectly flexible, these forces are tangent to the membrane.

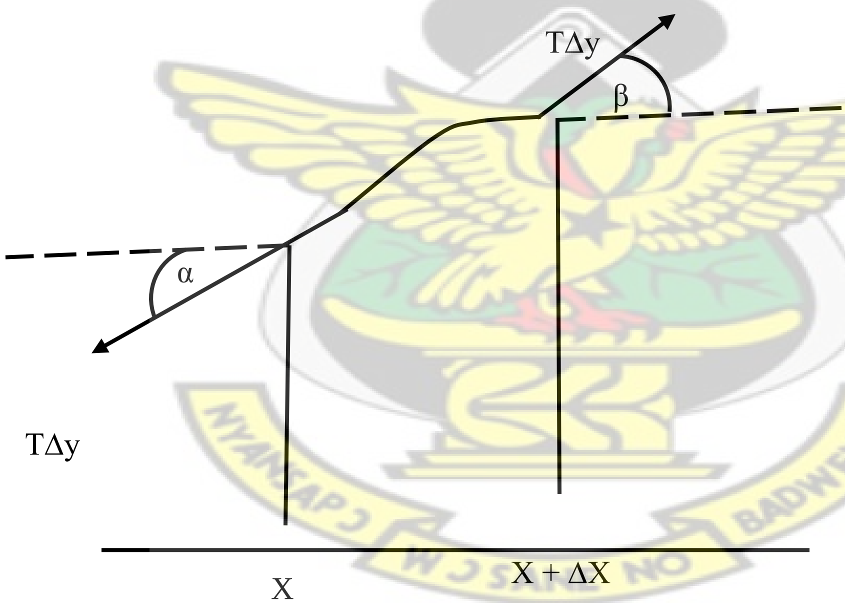


Fig. A1: Deflection of string

Vertical components of the forces along the edges parallel to the xy-plane. $T\Delta y\sin(\beta)$ and $-T\Delta y\sin(\alpha)$.

$$T\Delta y\sin(\beta) - T\Delta y\sin(\alpha) = T\Delta y\,(\sin(\beta) - \sin(\alpha))$$

$$= T\Delta y \left(\frac{\sin(\beta)}{\cos(\beta)} - \frac{\sin(\alpha)}{\cos(\alpha)}\right)$$

$$= T\Delta y\,(\tan(\beta) - \tan(\alpha))$$

$$= T\Delta y\,[U_x(x+\Delta x, y_1) - U_x(x, y_2)] \qquad (1)$$

Subscript $x =$ partial derivatives. y_1 and y_2 are between y and $y + \Delta \mathrm{y}$.

Similarly

$$T\Delta x\left[U_y(x_1, y+\Delta y) - U_y(x_2, y)\right] \qquad (2)$$

where x_1 and x_2 are values between x and $x + \Delta x$.

Newton's second law of motion is applied as $F = ma$ where $m = P\Delta A$ and $A = \frac{\partial^2 U}{\partial t^2}$, $P =$ the mass of the undeflected membrane per unit area and $\Delta A = \Delta x \Delta y$ is the area of the portion when it is undeflected.

Thus

$$P\Delta x\Delta y\frac{\partial^2 U}{\partial t^2} = T\Delta y\,[U_x(x+\Delta x, y_1) - U_x(x, y_2)] + T\Delta x\left[U_y(x_1, y+\Delta y) - U_y(x_2, y)\right]$$

Divided through by $P\Delta x\Delta y$ yields

$$\frac{\partial^2 U}{\partial t^2} = \frac{T}{P}\left[\frac{U_x(x+\Delta x, y_1) - U_x(x, y_2)}{\Delta x} + \frac{U_y(x_1, y+\Delta y) - U_y(x_2, y)}{\Delta y}\right]$$

If we let Δx and $\Delta y \to 0$, we obtain partial differential equation

$$\frac{\partial^2 U}{\partial t^2} = C^2\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right)$$

Where $C^2 = \frac{T}{P}$, $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \nabla^2 U$ (laplacian) and $\frac{\partial^2 U}{\partial t^2} = C^2 \nabla^2 U$.

APPENDIX 2

<u>Programming Language – Visual Basic 6</u>

SOURCE CODES FOR THE PROGRAM

```
Dim M As Integer, N As Integer

'''

Private Sub CmdCLEAR_Click()

    Cls

End Sub


Private Sub CmdExecuteEQUATIONS_Click()

    Dim vPI As Double, vFinaLResult As String

    vPI = 22 / 7

    ''

    ExecuteEQUATION_X vPI

    ''

End Sub


Private Sub CmdMINUS_Click()

    If CInt(Me.TxtNumber.Text) > 1 Then

        Me.TxtNumber.Text = CStr(Val(Me.TxtNumber.Text) - 1)
```

```
        "
        CmdExecuteEQUATIONS_Click
    End If
End Sub

Private Sub CmdPLUS_Click()
    If CInt(Me.TxtNumber.Text) <= 3 Then
        Me.TxtNumber.Text = CStr(Val(Me.TxtNumber.Text) + 1)
        "
        CmdExecuteEQUATIONS_Click
    End If
End Sub

Private Sub Command1_Click()
    Unload Me
End Sub
```

```
Private Sub ExecuteEQUATION_X(PI As Double)

    Dim vResult As String, vRR As Double
'''    Dim M As Integer, N As Integer

    '''AA = (4 / ((PI ^ 2) * M * N)) * [  {(Cos(M * PI)) *  (COS (PI)) }   -  COS ( M * PI )  -   COS
( N * PI )  +  1 ]

    ''

    vResult = vbTab

    For M = 1 To 3
        ''

        For N = 1 To 3
            ''

            Select Case CInt(Me.TxtNumber.Text)
                Case 1
                    Dim X1 As Double, X2 As Double, X3 As Double, X4 As Double, X5 As Double,
X6 As Double, X7 As Double, X8 As Double, X9 As Double
                    ''

                    X1 = (0.14 / ((PI ^ 4) * (M ^ 2) * (N ^ 2))) * ((Sin(M * PI)) * (Sin(N * PI)))

                    X2 = (0.23 / ((PI ^ 5) * (M ^ 2) * (N ^ 3))) * ((Sin(M * PI)) * (Cos(N * PI)))

                    X3 = (0.23 / ((PI ^ 5) * (M ^ 2) * (N ^ 3))) * (Sin(M * PI))

                    X4 = (0.3 / ((PI ^ 5) * (M ^ 3) * (N ^ 2))) * ((Cos(M * PI)) * (Sin(N * PI)))

                    X5 = (0.53 / ((PI ^ 6) * (M ^ 3) * (N ^ 3))) * ((Cos(M * PI)) * (Cos(N * PI)))
```

```
X6 = (0.53 / ((PI ^ 6) * (M ^ 3) * (N ^ 3))) * (Cos(M * PI))

X7 = (0.3 / ((PI ^ 5) * (M ^ 3) * (N ^ 2))) * (Sin(N * PI))

X8 = (0.6 / ((PI ^ 6) * (M ^ 3) * (N ^ 3))) * (Cos(N * PI))

X9 = (0.6 / ((PI ^ 6) * (M ^ 3) * (N ^ 3)))

"

vRR = X1 + X2 - X3 + X4 + X5 - X6 - X7 - X8 + X9

"
```

Case 2

```
Dim FF1 As Double, FF2 As Double, FF3 As Double, FF4 As Double, FF5 As Double, FF6 As Double, FF7 As Double, FF8 As Double, FF9 As Double

Dim SinSIN As Double, SinCOS As Double, CosSIN As Double

"

SinSIN = ((Sin(M * PI)) * (Sin(N * PI)))

SinCOS = ((Sin(M * PI)) * (Cos(N * PI)))

CosSIN = ((Cos(M * PI)) * (Sin(N * PI)))

"

FF1 = (0.07 / ((PI ^ 4) * (M ^ 2) * (N ^ 2))) * SinSIN

FF2 = (0.3 / ((PI ^ 5) * (M ^ 2) * (N ^ 3))) * SinCOS

FF3 = (0.3 / ((PI ^ 6) * (M ^ 2) * (N ^ 4))) * SinSIN

FF4 = (0.19 / ((PI ^ 5) * (M ^ 3) * (N ^ 2))) * CosSIN

FF5 = (0.77 / ((PI ^ 6) * (M ^ 3) * (N ^ 3))) * ((Cos(M * PI)) * (Cos(N * PI)))

FF6 = (0.77 / ((PI ^ 7) * (M ^ 3) * (N ^ 4))) * CosSIN
```

```
    FF7 = (0.19 / ((PI ^ 6) * (M ^ 4) * (N ^ 2))) * SinSIN

    FF8 = (0.77 / ((PI ^ 7) * (M ^ 4) * (N ^ 3))) * SinCOS

    FF9 = (0.77 / ((PI ^ 8) * (M ^ 4) * (N ^ 4))) * SinSIN

    ''

    vRR = FF1 + FF2 - FF3 + FF4 + FF5 - FF6 - FF7 - FF8 + FF9

    ''

Case 3  '''                    ''

    Dim x(1 To 49) As Double, CosCOS As Double, Kn As Integer

    ''

    SinSIN = ((Sin(M * PI)) * (Sin(N * PI)))

    SinCOS = ((Sin(M * PI)) * (Cos(N * PI)))

    CosSIN = ((Cos(M * PI)) * (Sin(N * PI)))

    CosCOS = ((Cos(M * PI)) * (Cos(N * PI)))

    ''

    x(1) = RCOE(0.086, 2, 2, 4) * SinSIN

    x(2) = RCOE(0.12, 2, 2, 4) * SinCOS

    x(3) = RCOE(0.086, 2, 3, 5) * SinCOS

    x(4) = (-1) * RCOE(0.253, 2, 2, 5) * SinSIN

    x(5) = (-1) * RCOE(0.086, 2, 3, 5) * (Sin(M * PI))

    x(6) = (-1) * RCOE(0.253, 2, 4, 6) * SinCOS

    x(7) = RCOE(0.253, 2, 4, 6) * (Sin(M * PI))

    x(8) = RCOE(0.129, 2, 2, 4) * CosSIN
```

```
x(9) = RCOE(0.181, 2, 2, 4) * CosCOS

x(10) = RCOE(0.129, 2, 3, 5) * CosCOS

x(11) = (-1) * RCOE(0.4071, 2, 3, 5) * CosSIN

x(12) = (-1) * RCOE(0.129, 2, 3, 5) * (Cos(M * PI))

x(13) = (-1) * RCOE(0.407, 2, 4, 6) * CosCOS

x(14) = RCOE(0.407, 2, 4, 6) * (Cos(M * PI))

x(15) = RCOE(0.086, 3, 2, 5) * CosSIN

x(16) = RCOE(0.12, 3, 2, 5) * CosCOS

x(17) = RCOE(0.086, 3, 3, 6) * CosCOS

x(18) = (-1) * RCOE(0.253, 3, 3, 6) * CosSIN

x(19) = (-1) * RCOE(0.086, 2, 2, 6) * (Cos(M * PI))

x(20) = (-1) * RCOE(0.25, 3, 4, 7) * CosCOS

x(21) = RCOE(0.25, 3, 4, 7) * (Cos(M * PI))

x(22) = (-1) * RCOE(0.25, 3, 2, 5) * SinSIN

x(23) = (-1) * RCOE(0.3611, 3, 2, 5) * SinCOS

x(24) = (-1) * RCOE(0.25, 3, 3, 6) * SinCOS

x(25) = RCOE(0.805, 3, 3, 6) * SinSIN

x(26) = RCOE(0.25, 3, 3, 6) * (Sin(M * PI))

x(27) = RCOE(0.805, 3, 4, 7) * SinCOS

x(28) = (-1) * RCOE(0.805, 3, 4, 7) * (Sin(M * PI))

x(29) = (-1) * RCOE(0.086, 3, 2, 5) * (Sin(N * PI))

x(30) = (-1) * RCOE(0.12, 3, 2, 5) * (Cos(N * PI))

x(31) = (-1) * RCOE(0.086, 3, 3, 6) * (Cos(N * PI))
```

```
x(32) = RCOE(0.253, 3, 3, 6) * (Sin(N * PI))

x(33) = RCOE(0.086, 3, 3, 6)

x(34) = RCOE(0.253, 3, 4, 7) * (Cos(N * PI))

x(35) = (-1) * RCOE(0.253, 3, 4, 7)

x(36) = (-1) * RCOE(0.253, 4, 2, 6) * CosSIN

x(37) = (-1) * RCOE(0.345, 4, 2, 6) * CosCOS

x(38) = (-1) * RCOE(0.253, 4, 3, 7) * CosCOS

x(39) = RCOE(0.805, 4, 3, 7) * CosSIN

x(40) = RCOE(0.253, 4, 3, 7) * (Cos(M * PI))

x(41) = RCOE(0.805, 4, 4, 8) * CosCOS

x(42) = (-1) * RCOE(0.805, 4, 4, 8) * (Cos(M * PI))

x(43) = RCOE(0.253, 4, 2, 6) * (Sin(N * PI))

x(44) = RCOE(0.345, 4, 2, 6) * (Cos(N * PI))

x(45) = RCOE(0.253, 4, 3, 7) * (Cos(N * PI))

x(46) = (-1) * RCOE(0.805, 4, 3, 7) * (Sin(N * PI))

x(47) = (-1) * RCOE(0.253, 4, 3, 7)

x(48) = (-1) * RCOE(0.805, 4, 4, 8) * (Cos(N * PI))

x(49) = RCOE(0.805, 4, 4, 8)

"

vRR = 0

For Kn = 1 To 49

   vRR = vRR + x(Kn)

Next Kn
```

```
  "
Case 4
  "
  Dim y(1 To 50) As Double, vCounter As Integer
  "
  SinSIN = ((Sin(M * PI)) * (Sin(N * PI)))
  SinCOS = ((Sin(M * PI)) * (Cos(N * PI)))
  CosSIN = ((Cos(M * PI)) * (Sin(N * PI)))
  CosCOS = ((Cos(M * PI)) * (Cos(N * PI)))
  "x(38) = (-1) * RCOE(0.253, 4, 3, 7) * CosCOS
  "
  y(1) = (-1) * RCOE(0.0004, 1, 1, 2) * CosCOS
  y(2) = (-1) * RCOE(0.00023, 1, 2, 3) * CosSIN
  y(3) = RCOE(0.00207, 1, 3, 4) * CosCOS
  y(4) = RCOE(0.0036, 1, 3, 4) * (Cos(M * PI))
  y(5) = (-1) * RCOE(0.0056, 1, 4, 5) * CosSIN
  y(6) = (-1) * RCOE(0.000403, 2, 1, 3) * SinCOS
  y(7) = (-1) * RCOE(0.00023, 2, 2, 4) * SinSIN
  y(8) = RCOE(0.0021, 3, 3, 5) * SinCOS
  y(9) = RCOE(0.0037, 2, 3, 5) * (Sin(M * PI))
  y(10) = (-1) * RCOE(0.0056, 2, 4, 6) * SinSIN
  y(11) = (-1) * RCOE(0.0145, 3, 1, 4) * CosCOS
  y(12) = (-1) * RCOE(0.0083, 3, 2, 5) * CosSIN
```

```
        y(13) = RCOE(0.0745, 3, 2, 6) * CosCOS

        y(14) = RCOE(0.13, 3, 3, 6) * (Cos(M * PI))

        y(15) = (-1) * RCOE(0.2047, 3, 4, 7) * CosSIN

        y(16) = (-1) * RCOE(0.0414, 3, 1, 4) * (Cos(N * PI))

        y(17) = (-1) * RCOE(0.023, 3, 2, 5) * (Sin(N * PI))

        y(18) = RCOE(0.207, 3, 3, 6) * (Cos(N * PI))

        y(19) = RCOE(0.37, 3, 3, 6)

        y(20) = (-1) * RCOE(0.575, 3, 4, 7) * (Sin(N * PI))

        y(21) = (-1) * RCOE(0.0552, 4, 1, 4) * SinCOS

        y(22) = (-1) * RCOE(0.0313, 4, 2, 6) * SinSIN

        y(23) = RCOE(0.28, 4, 3, 7) * SinCOS

        y(24) = RCOE(0.4922, 4, 3, 7) * (Sin(M * PI))

        y(25) = (-1) * RCOE(0.782, 4, 4, 8) * SinSIN

        ''

        vRR = 0

        For vCounter = 1 To 25

            vRR = vRR + y(vCounter)

        Next vCounter

        ''


        ''

    Case Else

        'DO NOTHIGN..

End Select
```

```
        "
        vRR = Round(vRR, CInt(Me.TxtDecimalPLaces.Text))
        vResult = vResult & CStr(vRR) & vbTab & vbTab & vbTab
      Next N
      "
      vResult = vResult & vbCrLf & vbCrLf & vbCrLf & vbTab
      "
    Next M
    "
    Cls
    Print
    Print
    Print
    Print vResult
    "
    Print "NB: VALUES  FROM  EQUATION -    No.   (" & CInt(Me.TxtNumber.Text) & ")"
End Sub
Private Function RCOE(NumeratorXX As Double, pmm As Integer, pnn As Integer, ppi As Integer) As Double
    Dim RR As Double
    "
    RR = NumeratorXX / ((M ^ pmm) * (N ^ pnn) * ((22 / 7) ^ ppi))
```

```
    "
    RCOE = RR
End Function

Private Sub Command2_Click()
    If CInt(Me.TxtDecimalPLaces.Text) <= 4 Then

        Me.TxtDecimalPLaces.Text = CStr(Val(Me.TxtDecimalPLaces.Text) + 1)

        "
        CmdExecuteEQUATIONS_Click
    End If
End Sub

Private Sub Command3_Click()
    If CInt(Me.TxtDecimalPLaces.Text) > 1 Then
        Me.TxtDecimalPLaces.Text = CStr(Val(Me.TxtDecimalPLaces.Text) - 1)

        "
        CmdExecuteEQUATIONS_Click
    End If
End Sub
```

APPENDIX 3

SAGE CODES

```
sage: x, y = var('x,y')

sage: plot3d(x^2*(0.5-x)*y^2*(0.35-y),(x,0,.5),(y, 0,.35))
```



```
sage: x, y = var('x,y')

sage: plot3d(x*(0.5-x)^2*y*(0.35-y)^2,(x,0,.5),(y, 0,.35))
```

APPENDIX 4

```
[x,y] = meshgrid(0:.2:8);

z = x.^2.*(0.5-x).*y.^2.*(0.35-y);

mesh(x,y,z)
```



```
[x,y] = meshgrid(0:.2:8);

z = x.*(0.5-x).^2.*y.*(0.35-y).^2;

mesh(x,y,z)
```

APPENDIX 5

$$a = \text{length} = 0.5\text{metres}; \qquad b = \text{breadth} = 0.35\text{metres}$$

$$f(x, y) = xy(0.5 - x)(0.35 - y)$$

$$f(x, y) = x(0.5 - x)\, y(0.35 - y)$$

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x, y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, dxdy$$

$$B_{mn} = \frac{4}{(0.5)(0.35)}\int_0^{0.35}\int_0^{0.5} x(0.5 - x)\, y(0.35 - y)\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\, dxdy$$

$$B_{mn} = 23\int_0^{0.35}\int_0^{0.5} x(0.5 - x)\ \sin\frac{m\pi x}{0.5}dx\ \ y(0.35 - y)\sin\frac{n\pi y}{0.35}\, dy$$

$$\int_0^{0.5} x(0.5 - x)\ \sin\frac{m\pi x}{0.5}\ dx = \int_0^{0.5} 0.5x\ \sin\frac{m\pi x}{0.5}\ dx - \int_0^{0.5} x^2\sin\frac{m\pi x}{0.5}\ dx$$

$$\int_0^{0.5} x\sin\frac{m\pi x}{0.5}dx, \quad \text{let}\ \ u = x\,, \qquad du = dx \quad \text{and} \quad \int dv = \sin\frac{m\pi x}{0.5}\,, \quad v = \frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}$$

$$0.5\int_0^{0.5} x\sin\frac{m\pi x}{0.5}dx = 0.5\left[x\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)\Bigg|_0^{0.5} - \int_0^{0.5}\frac{-0.5}{m\pi}\sin\frac{m\pi x}{0.5}dx\right]$$

$$= 0.5\left[\frac{-0.25}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{(0.5)^2}{m^2\pi^2}\sin\frac{m\pi x}{0.5}\Bigg|_0^{0.5}\right]$$

$$= 0.5\left[\frac{-0.25}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}\right]$$

$$= \frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.125}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} \tag{1}$$

Next

$$\int_0^{0.5} x^2 \sin\frac{m\pi x}{0.5}\ dx\,,\ \text{ let }\ u = x^2\,,\ \ du = 2xdx\ \ \text{ and }\ \int dv = \sin\frac{m\pi x}{0.5}\,,\quad v = \frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}$$

$$\int_0^{0.5} x^2 \sin\frac{m\pi x}{0.5}\ dx = x^2\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)\Bigg|_0^{0.5} - \int_0^{0.5} 2x\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)dx$$

$$= \frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{1}{m\pi}\int_0^{0.5} x\cos\frac{m\pi x}{0.5}\,dx$$

$$\int_0^{0.5} x\cos\frac{m\pi x}{0.5}\,dx\ \ ,\text{let }\ u = x\,,\ \ du = dx\ \ \text{ and }\ \int dv = \cos\frac{m\pi x}{0.5}\,,\quad v = \frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}$$

$$\int_0^{0.5} x\cos\frac{m\pi x}{0.5}\,dx = x\left(\frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}\right)\Bigg|_0^{0.5} - \int_0^{0.5}\frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}\,dx$$

$$= \frac{0.25}{m\pi}\sin\frac{0.5m\pi}{0.5} + \left(\frac{(0.5)^2}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5}\right)\Bigg|_0^{0.5}$$

$$= \frac{0.25}{m\pi}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^2\pi^2}$$

$$\int_0^{0.5} x^2 \sin\frac{m\pi x}{0.5}\ dx = \frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{1}{m\pi}\left[\frac{0.25}{m\pi}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^2\pi^2}\right]$$

$$= \frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3\pi^3} \qquad (2)$$

$(1) - (2)$

$$= \frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.125}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}$$

$$- \frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^3\pi^3}$$

$$= \frac{-0.125}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^3\pi^3} \qquad (3)$$

Next

$$\int_0^{0.35} y(0.35 - y)\sin\frac{n\pi y}{0.35}\,dy = 0.35\int_0^{0.35} y\,\sin\frac{n\pi y}{0.35}\,dy - \int_0^{0.35} y^2\sin\frac{n\pi y}{0.35}\,dy$$

$0.35\int_0^{0.35} y\,\sin\frac{n\pi y}{0.35}\,dy$, let $u = y$, $du = dy$ and $\int dv = \sin\frac{n\pi y}{0.35}$, $v = \frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}$

$$0.35\int_0^{0.35} y\sin\frac{n\pi y}{0.35}dy = 0.35\left[y\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)\Big|_0^{0.35} - \int_0^{0.35}\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}dy\right]$$

$$= 0.35\left[\frac{-0.1225}{n\pi}\cos\frac{0.35m\pi}{0.35} + \frac{(0.35)^2}{n^2\pi^2}\sin\frac{n\pi y}{0.35}\Big|_0^{0.35}\right]$$

$$= 0.5\left[\frac{-0.1225}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}\right]$$

$$= \frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.04}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} \qquad (4)$$

Next

$\int_0^{0.35} y^2\sin\frac{n\pi y}{0.35}\,dy$, let $u = y^2$, $du = 2ydy$ and $\int dv = \sin\frac{n\pi y}{0.35}$, $v = \frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}$

$$\int_0^{0.35} y^2 \sin\frac{n\pi y}{0.35}\ dy = y^2\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35} - \int_0^{0.35} 2y\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)dy$$

$$= \frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.7}{n\pi}\int_0^{0.35} y\cos\frac{n\pi y}{0.35}dy$$

$$\int_0^{0.35} y\cos\frac{n\pi y}{0.35}dy \ \text{ ,let } \ u = y\,,\ \ du = dy \ \text{ and } \int dv = \cos\frac{n\pi y}{0.35}\,,\quad v = \frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}$$

$$\int_0^{0.35} y\cos\frac{n\pi y}{0.35}dy = y\left(\frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35} - \int_0^{0.35}\frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}dy$$

$$= \frac{0.1225}{n\pi}\sin\frac{0.35n\pi}{0.35} + \left(\frac{(0.35)^2}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35}\right)\Bigg|_0^{0.35}$$

$$= \frac{0.1225}{n\pi}\sin\frac{0.35n\pi}{0.35} + \frac{1225}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} - \frac{0.1225}{n^2\pi^2}$$

$$\int_0^{0.35} y^2\sin\frac{n\pi y}{0.35}dy$$

$$= \frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.7}{n\pi}\left[\frac{0.1225}{n\pi}\sin\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} - \frac{0.1225}{n^2\pi^2}\right]$$

$$= \frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.09}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.09}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} - \frac{0.09}{n^3\pi^3} \qquad (5)$$

$(4) - (5)$

$$= \frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.04}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} - \frac{0.09}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}$$

$$- \frac{0.09}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.09}{n^3\pi^3}$$

$$= \frac{-0.05}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} - \frac{0.09}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.09}{n^3\pi^3} \tag{6}$$

$(3) \times (6)$

$$= 23\Big[\frac{0.006}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.01}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.01}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}$$

$$+ \frac{0.013}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.023}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$- \frac{0.023}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.013}{\pi^5 m^3 n^2}\sin\frac{0.35n\pi}{0.35} - \frac{0.023}{\pi^6 m^3 n^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.023}{\pi^6 m^3 n^3}\Big]$$

$$B_{mn} = \frac{0.14}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}$$

$$+ \frac{0.3}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$- \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.3}{\pi^5 m^3 n^2}\sin\frac{0.35n\pi}{0.35} - \frac{0.6}{\pi^6 m^3 n^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.6}{\pi^6 m^3 n^3}$$

$B^*_{mn} = 0$ since $g(x, y) = 0$

$$U_{mn}(x, y, t) = (B_{mn}\cos\lambda mnt + B^*_{mn}\sin\lambda mnt)\left(\sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b}\right)$$

$$U_{mn}(x, y, t) = \Big(\frac{0.14}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$- \frac{0.23}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5} + \frac{0.3}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$+ \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.53}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.3}{\pi^5 m^3 n^2}\sin\frac{0.35n\pi}{0.35}$$

$$- \frac{0.6}{\pi^6 m^3 n^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.6}{\pi^6 m^3 n^3}\Big)\cos\lambda mnt\left(\sin\frac{m\pi x}{0.5}\ \sin\frac{n\pi y}{0.35}\right)$$

$\lambda = \lambda_{11} = 3.5c\pi$

$$\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$\text{Frequency,} \quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{11} = \frac{\lambda_{11}}{2\pi} = \frac{3.5c\pi}{2\pi} = 1.75c$$

$$U_{11} = 1.75c$$

$$\text{Frequency} = \frac{\lambda_{mn}}{2\pi} = \frac{\lambda_{12}}{2\pi} = \frac{6.05c\pi}{2\pi} = 3.03c$$

$$U_{12} = 3.03c$$

$$\text{Frequency,} \quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{13} = \frac{\lambda_{13}}{2\pi} = \frac{8.80c\pi}{2\pi} = 4.4c$$

$$\text{Frequency,} \quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{21} = \frac{\lambda_{21}}{2\pi} = \frac{4.92c\pi}{2\pi} = 2.46c$$

$$U_{21} = 2.46c$$

$$\text{Frequency,} \quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{22} = \frac{\lambda_{22}}{2\pi} = \frac{6.98c\pi}{2\pi} = 3.49c$$

$$U_{22} = 3.49c$$

$$\text{Frequency,} \qquad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{23} = \frac{\lambda_{23}}{2\pi} = \frac{9.46c\pi}{2\pi} = 4.73c$$

$$\text{Frequency,} \qquad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{31} = \frac{\lambda_{31}}{2\pi} = \frac{6.65c\pi}{2\pi} = 3.325c$$

$$\text{Frequency,} \qquad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{32} = \frac{\lambda_{32}}{2\pi} = \frac{8.29c\pi}{2\pi} = 4.145c$$

$$\text{Frequency,} \qquad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{33} = \frac{\lambda_{33}}{2\pi} = \frac{10.46c\pi}{2\pi} = 5.23c$$

APPENDIX 6

$$a = \text{length} = 0.5\text{metres}; \qquad b = \text{breadth} = 0.35\text{metres}$$

$$f(x,y) = x\,y(a^2 - x^2)(\,b^2 - y^2)$$

$$f(x,y) = x\left(0.5^2 - x^2\right)\,y\left(0.35^2 - y^2\right)$$

$$f(x,y) = x(0.25 - x^2)\,y(0.1225 - y^2)$$

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy$$

$$B_{mn} = \frac{4}{(0.5)(0.35)}\int_0^{0.35}\int_0^{0.5} x(0.25 - x^2)\,y(0.1225 - y^2)\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\,dxdy$$

$$B_{mn} = 23\int_0^{0.35}\int_0^{0.5} x(0.25 - x^2)\,\sin\frac{m\pi x}{0.5}dx\;\;y(0.1225 - y^2)\sin\frac{n\pi y}{0.35}\,dy$$

$$\int_0^{0.5} x(0.25 - x^2)\,\sin\frac{m\pi x}{0.5}\,dx = \int_0^{0.5} 0.25x\,\sin\frac{m\pi x}{0.5}\,dx - \int_0^{0.5} x^3\sin\frac{m\pi x}{0.5}\,dx$$

$$\int_0^{0.5} x\sin\frac{m\pi x}{0.5}dx,\quad \text{let}\;\; u = x\,,\qquad du = dx \quad\text{and}\quad \int dv = \sin\frac{m\pi x}{0.5}\,,\quad v = \frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}$$

$$0.25\int_0^{0.5} x\sin\frac{m\pi x}{0.5}dx = 0.25\left[x\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)\Big|_0^{0.5} - \int_0^{0.5}\frac{-0.5}{m\pi}\sin\frac{m\pi x}{0.5}dx\right]$$

$$= 0.25\left[\frac{-0.25}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{(0.5)^2}{m^2\pi^2}\sin\frac{m\pi x}{0.5}\Big|_0^{0.5}\right]$$

$$= 0.25\left[\frac{-0.25}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}\right]$$

$$= \frac{-0.06}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.06}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} \tag{1}$$

Next

$$\int_0^{0.5} x^3\sin\frac{m\pi x}{0.5}\,dx\,,\;\;\text{let}\;\; u = x^3\,,\;\; du = 3x^2dx \;\;\text{and}\;\; \int dv = \sin\frac{m\pi x}{0.5}\,,\quad v = \frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}$$

$$\int_0^{0.5} x^3 \sin\frac{m\pi x}{0.5}\ dx = x^3\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)\Big|_0^{0.5} - \int_0^{0.5} 3x^2\left(\frac{0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)dx$$

$$= \frac{-0.06}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{1.5}{m\pi}\int_0^{0.5} x^2\cos\frac{m\pi x}{0.5}\,dx$$

Next

$\int_0^{0.5} x^2\cos\frac{m\pi x}{0.5}\ dx$, let $u = x^2$, $du = 2xdx$ and $\int dv = \cos\frac{m\pi x}{0.5}$, $v = \frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}$

$$\int_0^{0.5} x^2\cos\frac{m\pi x}{0.5}\ dx = x^2\left(\frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}\right)\Big|_0^{0.5} - 2\int_0^{0.5} x\left(\frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}\right)dx$$

$$= \frac{0.125}{m\pi}\sin\frac{0.5m\pi}{0.5} - \frac{1}{m\pi}\int_0^{0.5} x\sin\frac{m\pi x}{0.5}\,dx$$

$$= \frac{0.125}{m\pi}\sin\frac{0.5m\pi}{0.5} - \frac{1}{m\pi}\left[\frac{-0.25}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}\right]$$

$$= \frac{0.125}{m\pi}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3\pi^3}\sin\frac{0.5m\pi}{0.5}$$

$$= \frac{-0.06}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{1.5}{m\pi}\left[\frac{0.125}{m\pi}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3\pi^3}\sin\frac{0.5m\pi}{0.5}\right]$$

$$= \frac{-0.06}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.19}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.34}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.34}{m^4\pi^4}\sin\frac{0.5m\pi}{0.5} \qquad (2)$$

$(1) - (2)$

$$= \frac{-0.06}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.06}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.06}{m\pi}\cos\frac{0.5m\pi}{0.5} - \frac{0.19}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}$$

$$- \frac{0.34}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} + \frac{0.34}{m^4\pi^4}\sin\frac{0.5m\pi}{0.5}$$

$$= \frac{-0.13}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} - \frac{0.34}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} + \frac{0.34}{m^4\pi^4}\sin\frac{0.5m\pi}{0.5} \qquad (3)$$

Next

$$\int_0^{0.35} y(0.1225 - y^2)\sin\frac{n\pi y}{0.35}\,dy = 0.1225\int_0^{0.35} y\,\sin\frac{n\pi y}{0.35}\,dy - \int_0^{0.35} y^3\sin\frac{n\pi y}{0.35}\,dy$$

$$0.1225\int_0^{0.35} y\,\sin\frac{n\pi y}{0.35}\,dy\text{, let } u = y\,, du = dy \text{ and } \int dv = \sin\frac{n\pi y}{0.35}\,,$$

$$v = \frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}$$

$$0.1225\int_0^{0.35} y\sin\frac{n\pi y}{0.35}dy = 0.1225\left[y\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)\Big|_0^{0.35} - \int_0^{0.35}\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}dy\right]$$

$$= 0.1225\left[\frac{-0.1225}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{(0.35)^2}{n^2\pi^2}\sin\frac{n\pi y}{0.35}\Big|_0^{0.35}\right]$$

$$= 0.1225\left[\frac{-0.1225}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}\right]$$

$$= \frac{-0.02}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.02}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} \tag{4}$$

Next

$$\int_0^{0.35} y^3\sin\frac{n\pi y}{0.35}\,dy\text{, let } u = y^3\,,\ du = 3y^2dy \text{ and } \int dv = \sin\frac{n\pi y}{0.35}\,,$$

$$v = \frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}$$

$$\int_0^{0.35} y^3\sin\frac{n\pi y}{0.35}\,dy = y^3\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)\Big|_0^{0.35} - 3\int_0^{0.35} y^2\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)dy$$

$$= \frac{-0.02}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{1.1}{n\pi}\int_0^{0.35} y^2\cos\frac{n\pi y}{0.35}dy$$

Next

$$\int_0^{0.35} y^2 \cos\frac{n\pi y}{0.35}\,dy \quad \text{, let} \quad u = y^2\,, \quad du = 2y\,dy \quad \text{and} \quad \int dv = \cos\frac{n\pi y}{0.35}\,, \qquad v = \frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}$$

$$\int_0^{0.35} y^2 \cos\frac{n\pi y}{0.35}\,dy = y^2\left(\frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35} - 2\int_0^{0.35} y\left(\frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}\right)dy$$

$$= \frac{0.04}{n\pi}\sin\frac{0.35n\pi}{0.35} - \frac{0.7}{n\pi}\int_0^{0.35} y\sin\frac{n\pi y}{0.35}\,dy$$

$$= \frac{0.04}{n\pi}\sin\frac{0.35n\pi}{0.35} - \frac{0.7}{n\pi}\left[\frac{-0.1225}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}\right]$$

$$= \frac{0.04}{n\pi}\sin\frac{0.35n\pi}{0.35} + \frac{0.09}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} - \frac{0.09}{n^3\pi^3}\sin\frac{0.35n\pi}{0.35}$$

$$= \frac{-0.02}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{1.1}{n\pi}\left[\frac{0.04}{n\pi}\sin\frac{0.35n\pi}{0.35} + \frac{0.09}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} - \frac{0.09}{n^3\pi^3}\sin\frac{0.35n\pi}{0.35}\right]$$

$$= \frac{-0.02}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.044}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.099}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} - \frac{0.099}{n^4\pi^4}\sin\frac{0.35n\pi}{0.35} \qquad (5)$$

$(4) - (5)$

$$= \frac{-0.02}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.02}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.02}{n\pi}\cos\frac{0.35n\pi}{0.35} - \frac{0.044}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} - \frac{0.099}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.099}{n^4\pi^4}\sin\frac{0.35n\pi}{0.35}$$

$$= -\frac{0.024}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} - \frac{0.099}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} + \frac{0.099}{n^4\pi^4}\sin\frac{0.35n\pi}{0.35} \qquad (6)$$

$(3) \times (6)$

$$= 23\Big[\frac{0.00312}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.01287}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$-\frac{0.01287}{\pi^6 m^2 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00816}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$+\frac{0.03366}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.03366}{\pi^7 m^3 n^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$-\frac{0.00816}{\pi^6 m^4 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.03366}{\pi^7 m^4 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.03366}{\pi^8 m^4 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}\Big]$$

$$B_{mn} = \frac{0.07}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.3}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$-\frac{0.3}{\pi^6 m^2 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.19}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$+\frac{0.77}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.77}{\pi^7 m^3 n^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$-\frac{0.19}{\pi^6 m^4 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.77}{\pi^7 m^4 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.77}{\pi^8 m^4 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$B^*_{mn} = 0$ since $g(x, y) = 0$

$$U_{mn}(x, y, t) = (B_{mn}\cos\lambda mnt + B^*_{mn}\sin\lambda mnt)\left(\sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b}\right)$$

$$U_{mn}(x,y,t) = \Big(\frac{0.07}{\pi^4 m^2 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.3}{\pi^5 m^2 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$- \frac{0.3}{\pi^6 m^2 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.19}{\pi^5 m^3 n^2}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$+ \frac{0.77}{\pi^6 m^3 n^3}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.77}{\pi^7 m^3 n^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$- \frac{0.19}{\pi^6 m^4 n^2}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.77}{\pi^7 m^4 n^3}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+ \frac{0.77}{\pi^8 m^4 n^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}\Big)\cos\lambda mnt\left(\sin\frac{m\pi x}{0.5}\ \sin\frac{n\pi y}{0.35}\right)$$

$$\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

$$\text{Frequency,}\quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{11} = \frac{\lambda_{11}}{2\pi} = \frac{3.5c\pi}{2\pi} = 1.75c$$

$$U_{11} = 1.75c$$

$$\text{Frequency} = \frac{\lambda_{mn}}{2\pi} = \frac{\lambda_{12}}{2\pi} = \frac{6.05c\pi}{2\pi} = 3.03c$$

$$U_{12} = 3.03c$$

$$\text{Frequency,}\qquad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{13} = \frac{\lambda_{13}}{2\pi} = \frac{8.80c\pi}{2\pi} = 4.4c$$

$$\text{Frequency,}\qquad U_{mn} = \frac{\lambda_{mn}}{2\pi}$$

$$U_{21} = \frac{\lambda_{21}}{2\pi} = \frac{4.92c\pi}{2\pi} = 2.46c$$

$$U_{21} = 2.46c$$

Frequency, $\quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$

$$U_{22} = \frac{\lambda_{22}}{2\pi} = \frac{6.98c\pi}{2\pi} = 3.49c$$

$$U_{22} = 3.49c$$

Frequency, $\quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$

$$U_{23} = \frac{\lambda_{23}}{2\pi} = \frac{9.46c\pi}{2\pi} = 4.73c$$

Frequency, $\quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$

$$U_{31} = \frac{\lambda_{31}}{2\pi} = \frac{6.65c\pi}{2\pi} = 3.325c$$

Frequency, $\quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$

$$U_{32} = \frac{\lambda_{32}}{2\pi} = \frac{8.29c\pi}{2\pi} = 4.145c$$

Frequency, $\quad U_{mn} = \frac{\lambda_{mn}}{2\pi}$

$$U_{33} = \frac{\lambda_{33}}{2\pi} = \frac{10.46c\pi}{2\pi} = 5.23c$$

APPENDIX 7

a = length = 0.5 metres b = breadth = 0.35metres

$$f(xy) = x^2 y^2 (0.5 - x)(0.35 - y)$$

$$f(xy)\ x^2 (0.5 - x)\ y^2 (0.35 - y)$$

$$B_{mn} = \frac{4}{ab} \int_0^b \int_0^a f(xy) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$B_{mn} = \frac{4}{(0.5)(0.35)} \int_0^{0.35} \int_0^{0.5} x^2 (0.5 - x)\ \ y^2 (0.35 - y) \sin\frac{m\pi x}{0.5}\ sin\frac{n\pi y}{0.35} dxdy$$

$$B_{mn} = 23 \int_0^{0.35} \int_0^{0.5} x^2 (0.5 - x) \sin\frac{m\pi x}{0.5} dx\ \ y^2 (0.35 - y)\ sin\frac{n\pi y}{0.35} dy$$

$$\int_0^{0.5} x^2 (0.5 - x) \sin\frac{m\pi x}{0.5} dx = \int_0^{0.5} 0.5x^2 \sin\frac{m\pi x}{0.5} dx - \int_0^{0.5} x^3 \sin\frac{m\pi x}{0.5} dx$$

$$0.5\int_0^{0.5} x^2 \sin\frac{m\pi x}{0.5} dx;\ let\ u = x^2,\ du = 2xdx\ and\ \int dv = \sin\frac{m\pi x}{0.5},\ v = \frac{0.5}{m\pi}\cos\frac{m\pi x}{0.5}$$

$$0.5\int_0^{0.5} x^2 \sin\frac{m\pi x}{0.5} dx = (0.5)x^2 \left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right|_0^{0.5} - \int_0^{0.5} 2x\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)dx$$

$$= \frac{0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.5}{m\pi}\int_0^{0.5} x\cos\frac{m\pi x}{0.5} dx$$

$$\int_0^{0.5} x\cos\frac{m\pi x}{0.5} dx,\ let\ u = x,\ du = dx\ and\ \int dv = \cos\frac{m\pi x}{0.5},\ \ V = \frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}$$

$$\int_0^{0.5} x\cos\frac{m\pi x}{0.5} dx = x\left(\frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5}\right)\Big|_0^{0.5} - \int_0^{0.5} \frac{0.5}{m\pi}\sin\frac{m\pi x}{0.5} dx = \frac{0.25}{m\pi}\sin\frac{0.5m\pi}{0.5} + \left(\frac{(0.5)^2}{m^2\pi^2}\cos\frac{m\pi x}{0.5}\right)\Bigg|_0^{0.5}$$

$$= \frac{0.25}{m\pi}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^2\pi^2}$$

$$\int_0^{0.5} x^2 \sin\frac{m\pi x}{0.5}dx = \frac{-0.625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.5}{m\pi}\left[\frac{0.25}{m\pi}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^2\pi^2}\right]$$

$$= \frac{-0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.125}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.125}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.125}{m^3\pi^3} \quad \text{.................(1)}$$

Next

$$\int_0^{0.5} x^3 \sin\frac{m\pi x}{0.5}dx,\ let\ u = x^3,\ du = 3x^2dx\ and\ \int dv = \sin\frac{m\pi x}{0.5},\ V = \frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}$$

$$\int_0^{0.5} x^3 \sin\frac{m\pi x}{0.5}dx = x^3\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)\Bigg|_0^{0.5} - \int_0^{0.5} 3x^2\left(\frac{-0.5}{m\pi}\cos\frac{m\pi x}{0.5}\right)dx = \frac{-0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{1.5}{m\pi}$$

$$\int_0^{0.5} x^2 \cos\frac{m\pi x}{0.5}dx.\ But\ \int_0^{0.5} x^2 \cos\frac{m\pi x}{0.5} = \frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3\pi^3}$$

$\Rightarrow$

$$\frac{-0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{1.5}{m\pi}\left[\frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.25}{m^3\pi^3}\right]$$

$$= \frac{-0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} - \frac{0.1875}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} + \frac{0.375}{m^3\pi^3}\sin\frac{0.5m\pi}{0.5} + \frac{0.375}{m^4\pi^4}\cos\frac{0.5m\pi}{0.5} - \frac{0.375}{m^4\pi^4} \quad \text{...........(2)}$$

.............(1) − ....(2)

$$\frac{-0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.125}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.125}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.125}{m^3\pi^3} + \frac{0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5} + \frac{0.1875}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5}$$

$$-\frac{0.375}{m^3\pi^3}\sin\frac{0.5m\pi}{0.5} - \frac{0.375}{m^4\pi^4}\cos\frac{0.5m\pi}{0.5} + \frac{0.375}{m^4\pi^4}$$

$$= \frac{0.125}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5} + \frac{0.1875}{m^2\pi^2}\cos\frac{0.5m\pi}{0.5} + \frac{0.125}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5} - \frac{0.375}{m^3\pi^3}\sin\frac{0.5m\pi}{0.5} - \frac{0.125}{m^3\pi^3} - \frac{0.375}{m^4\pi^4}$$

$$\cos\frac{0.5m\pi}{0.5} + \frac{0.375}{m^4\pi^4} \quad \text{............. (3)}$$

Next

$$\int_0^{0.35} y^2(0.35 - y)\sin\frac{n\pi y}{0.35}dy = \int_0^{0.35} 0.35y^2\sin\frac{n\pi y}{0.35}dy - \int_0^{0.35} y^3\sin\frac{n\pi y}{0.35}dy$$

$$0.35\int_0^{0.35} y^2\sin\frac{n\pi y}{0.35}dy;\ let\ u = y^2,\ du = 2ydy\ and \int dv = \sin\frac{n\pi y}{0.35},\ v = \frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}$$

$$0.35\int_0^{0.35} y^2 \sin\frac{n\pi y}{0.35}dy = (o.35)y^2\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35} - \int_0^{0.35} 2y\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)$$

$$= \frac{-0.15}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.245}{n\pi}\int_0^{0.35} y\cos\frac{n\pi y}{0.35}dy.$$

$$\int_0^{0.35} y\cos\frac{n\pi y}{0.35}dy,\ let\ u = y,\ du = dy\ and\ \int dv = \cos\frac{n\pi y}{0.35},\ V = \frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35\pi}$$

$$\int_0^{0.35} y\cos\frac{n\pi y}{0.35}dy = y\left(\frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35} - \int_0^{0.35}\frac{0.35}{n\pi}\sin\frac{n\pi y}{0.35}dy$$

$$= \frac{0.1225}{n\pi}\sin\frac{0.35n\pi}{0.35} + \left(\frac{(0.35)^2}{n^2\pi^2}\cos\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35}$$

$$= \frac{0.1225}{n\pi}\sin\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} - \frac{0.1225}{n^2\pi^2}$$

$$\int_0^{0.35} y^2\sin\frac{n\pi y}{0.35}dy = \frac{-0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.245}{n\pi}\left[\frac{0.1225}{n\pi}\sin\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} - \frac{0.1225}{n^2\pi^2}\right]$$

$$= \frac{-0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.03}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.03}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} - \frac{0.03}{n^3\pi^3}\text{...........(4)}$$

Next

$$\int_0^{0.35} y^3\sin\frac{n\pi y}{0.35}dy,\ let\ u = y^3,\ du = 3y^2dy\ and\ \int dv = \sin\frac{n\pi y}{0.35}dy,\ v = \frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}$$

$$\int_0^{0.35} y^3\sin\frac{n\pi y}{0.35}dy = y^3\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)\Bigg|_0^{0.35} - \int_0^{0.35} 3y^2\left(\frac{-0.35}{n\pi}\cos\frac{n\pi y}{0.35}\right)$$

$$= \frac{-0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{1.05}{n\pi}\int_0^{0.35} y^2\cos\frac{n\pi y}{0.35}dy$$

$$But\ \int_0^{0.35} y^2\cos\frac{n\pi y}{0.35}dy = \frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.09}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.09}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} - \frac{0.09}{n^3\pi^3}$$

$\Rightarrow$

$$\frac{-0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{1.05}{n\pi}\left[\frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.09}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.09}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} - \frac{0.09}{n^3\pi^3}\right]$$

$$= \frac{-0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} - \frac{0.042}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35} + \frac{0.0945}{n^3\pi^3}\sin\frac{0.35n\pi}{0.35} + \frac{0.0945}{n^4\pi^4}\cos\frac{0.35n\pi}{0.35} - \frac{0.0945}{n^4\pi^4} \text{.........(5)}$$

..............(4) $-$ ....(5)

$$\frac{-0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.03}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \frac{0.03}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35} - \frac{0.03}{n^3\pi^3} + \frac{0.015}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.042}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35}$$
$$- \frac{0.0945}{n^3\pi^3}\sin\frac{0.35n\pi}{0.35} - \frac{0.0945}{n^4\pi^4}\cos\frac{0.35n\pi}{0.35} + \frac{0.0945}{n^4\pi^4}$$

$$=\frac{0.03}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}+\frac{0.042}{n^2\pi^2}\cos\frac{0.35n\pi}{0.35}+\frac{0.35}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35}-\frac{0.0945}{n^3\pi^3}\sin\frac{0.35n\pi}{0.35}-\frac{0.03}{n^3\pi^3}$$

$$-\frac{0.0945}{n^4\pi^4}\cos\frac{0.35n\pi}{0.35}+\frac{0.0945}{n^4\pi^4}\text{.................}(6)$$

........(3) x ....(6)

$$23\left[\frac{0.00375}{m^2n^2\pi^4}\sin\frac{0.05m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00525}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.00375}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}\right.$$

$$-\frac{-0.011}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.00375}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}-\frac{0.011}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.011}{m^2n^4\pi^6}$$

$$\sin\frac{0.5m\pi}{0.5}+\frac{0.0056}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00787}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.0056}{m^2n^3\pi^5}\cos\frac{5m\pi}{0.5}$$

$$\cos\frac{0.35n\pi}{0.35}-\frac{0.0177}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.00562}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}-\frac{o.0177}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.0177}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}+\frac{0.0375}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00525}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.00375}{m^3n^3\pi^6}$$

$$\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.011}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.00375}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.011}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}$$

$$\cos\frac{0.35n\pi}{0.35}+\frac{0.011}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}-\frac{0.011}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.0157}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$-\frac{0.011}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.035}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.011}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}+\frac{0.035}{m^3n^4\pi^7}$$

$$\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.035}{m^3n^4\pi^7}\sin\frac{0.5m\pi}{0.5}-\frac{0.00375}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35}-\frac{0.00525}{m^3n^2\pi^5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00375}{m^3n^3\pi^6}$$

$$\cos\frac{0.35n\pi}{0.35}+\frac{0.011}{m^3n^3\pi^6}\sin\frac{0.35n\pi}{0.35}+\frac{0.00375}{m^3n^3\pi^6}+\frac{0.011}{m^3n^4\pi^7}\cos\frac{0.35n\pi}{0.35}-\frac{0.011}{m^3n^4\pi^7}-\frac{0.011}{m^4n^2\pi^6}\cos\frac{0.5m\pi}{0.5}$$

$$\sin\frac{0.35n\pi}{0.35}-\frac{0.015}{m^4n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.011}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.035}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}$$

$$\sin\frac{0.35n\pi}{0.35}+\frac{0.011}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}+\frac{0.035}{m^4n^4\pi^8}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.035}{m^4n^4\pi^8}\cos\frac{0.5m\pi}{0.5}+\frac{0.011}{m^4n^2\pi^6}$$

$$\sin\frac{0.35n\pi}{0.35}+\frac{0.015}{m^4n^2\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.011}{m^4n^3\pi^7}\cos\frac{0.35n\pi}{0.35}-\frac{0.035}{m^4n^3\pi^7}\sin\frac{0.35n\pi}{0.35}-\frac{0.011}{m^4n^3\pi^7}-\frac{0.035}{m^4n^4\pi^8}$$

$$\left.\cos\frac{0.35n\pi}{0.35}+\frac{0.035}{m^4n^4\pi^8}\right]$$

$$
\begin{aligned}
B_{mn} = {} & \frac{0.086}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.120}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\,cos\frac{0.35n\pi}{0.35} \\
& -\frac{0.253}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}-\frac{0.253}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.253}{m^2n^4\pi^6} \\
& \sin\frac{0.5m\pi}{0.5}+\frac{0.129}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.181}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35}-\frac{0.4071}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}-\frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& +\frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}+\frac{0.086}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.120}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\,cos\frac{0.35n\pi}{0.35}+\frac{0.086}{m^3n^3\pi^6} \\
& \cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.086}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35}+\frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}-\frac{0.25}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.3611}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& -\frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.805}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}+\frac{0.805}{m^3n^4\pi^7} \\
& \sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.805}{m^3n^4\pi^7}\sin\frac{0.5m\pi}{0.5}-\frac{0.086}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35}-\frac{0.120}{m^3n^2\pi^5}\cos\frac{0.25n\pi}{0.35} \\
& -\frac{0.086}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.253}{m^3n^3\pi^6}\sin\frac{0.35n\pi}{0.35}+\frac{0.86}{m^3n^3\pi^6}+\frac{0.253}{m^3n^4\pi^7}\cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^3n^4\pi^7}-\frac{0.253}{m^4n^2\pi^6} \\
& \cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.345}{m^4n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& +\frac{0.805}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}+\frac{0.805}{m^4n^4\pi^8}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.805}{m^4n^4\pi^8} \\
& \cos\frac{0.5m\pi}{0.5}+\frac{0.253}{m^4n^2\pi^6}\sin\frac{0.35n\pi}{0.35}+\frac{0.345}{m^4n^2\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.253}{m^4n^3\pi^7}\cos\frac{0.35n\pi}{0.35}-\frac{0.805}{m^4n^3\pi^7} \\
& \sin\frac{0.35n\pi}{0.35}-\frac{0.253}{m^4n^3\pi^7}-\frac{0.805}{m^4n^4\pi^8}\cos\frac{0.35n\pi}{0.35}+\frac{0.805}{m^4n^4\pi^8}
\end{aligned}
$$

$$B^*_{mn} = 0 \sin ce\ g(x,y) = 0$$

$$U_{mn}(x,y,t) = (B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t)\left(\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right)$$

$$
\begin{aligned}
U_{mn}(x,y,t) = \Big(& \frac{0.086}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.120}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.086}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}-\frac{0.253}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.253}{m^2n^4\pi^6} \\
& \sin\frac{0.5m\pi}{0.5}+\frac{0.129}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.181}{m^2n^2\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35}-\frac{0.4071}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.129}{m^2n^3\pi^5}\cos\frac{0.5m\pi}{0.5}-\frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& +\frac{0.407}{m^2n^4\pi^6}\cos\frac{0.5m\pi}{0.5}+\frac{0.086}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.120}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.086}{m^3n^3\pi^6} \\
& \cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.086}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35}+\frac{0.25}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}-\frac{0.25}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.3611}{m^3n^2\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& -\frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.805}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.25}{m^3n^3\pi^6}\sin\frac{0.5m\pi}{0.5}+\frac{0.805}{m^3n^4\pi^7} \\
& \sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.805}{m^3n^4\pi^7}\sin\frac{0.5m\pi}{0.5}-\frac{0.086}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35}-\frac{0.120}{m^3n^2\pi^5}\cos\frac{0.25n\pi}{0.35} \\
& -\frac{0.086}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.253}{m^3n^3\pi^6}\sin\frac{0.35n\pi}{0.35}+\frac{0.86}{m^3n^3\pi^6}+\frac{0.253}{m^3n^4\pi^7}\cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^3n^4\pi^7}-\frac{0.253}{m^4n^2\pi^6} \\
& \cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.345}{m^4n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} \\
& +\frac{0.805}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.253}{m^4n^3\pi^7}\cos\frac{0.5m\pi}{0.5}+\frac{0.805}{m^4n^4\pi^8}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.805}{m^4n^4\pi^8} \\
& \cos\frac{0.5m\pi}{0.5}+\frac{0.253}{m^4n^2\pi^6}\sin\frac{0.35n\pi}{0.35}+\frac{0.345}{m^4n^2\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.253}{m^4n^3\pi^7}\cos\frac{0.35n\pi}{0.35}-\frac{0.805}{m^4n^3\pi^7} \\
& \sin\frac{0.35n\pi}{0.35}-\frac{0.253}{m^4n^3\pi^7}-\frac{0.805}{m^4n^4\pi^8}\cos\frac{0.35n\pi}{0.35}+\frac{0.805}{m^4n^4\pi^8}\Big)\cos\lambda_{mn}t\left(\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\right)
\end{aligned}
$$

APPENDIX 8

$$a = length = 0.5metres\ b = breadth = 0.35metres$$

$$f(x,y) = xy(0.5-x)^2(0.35-y)^2$$

$$f(xy) = x(0.5-x)^2 y(0.35-y)^2$$

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy$$

$$B_{mn} = \frac{4}{ab}\int_0^{0.35}\int_0^{0.5} x(0.5-x)^2 y(0.35-y)^2 \sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\,dxdy$$

$$B_{mn} = 23\int_0^{0.35}\int_0^{0.5} x(0.5-x)^2\sin\frac{m\pi x}{0.5}\,dx\ y(0.35-y)^2\sin\frac{n\pi y}{0.35}\,dx$$

$$\int_0^{0.5} x(0.5-x)^2\sin\frac{m\pi x}{0.5}\,dx = \int_0^{0.5} x(0.25-x+x^2)\sin\frac{m\pi x}{0.5}\,dx = \int_0^{0.5}(0.25x-x^2+x^3)sin\frac{m\pi x}{0.5}\,dx$$

$$= 0.25\int_0^{0.5} x\sin\frac{m\pi x}{0.5}\,dx - \int_0^{0.5} x^2\sin\frac{m\pi x}{0.5}\,dx + \int_0^{0.5} x^3\sin\frac{m\pi x}{0.5}\,dx$$

$$= 0.25\left[\frac{-0.25}{m\pi}\cos\frac{0.5m\pi}{0.5}+\frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}\right]-\left[\frac{-0.125}{m\pi}\cos\frac{0.5m\pi}{0.5}+\frac{0.25}{m^2\pi^2}\sin\frac{0.5mx}{0.5}+\frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5}\right.$$

$$\left.-\frac{0.25}{m^3\pi^3}\right]+\left[\frac{-0.06}{m\pi}\cos\frac{0.5m\pi}{0.5}+\frac{0.19}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}+\frac{0.34}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5}-\frac{0.34}{m^4\pi^4}\sin\frac{0.5m\pi}{0.5}\right]$$

$$= \frac{-0.0625}{m\pi}\cos\frac{0.5m\pi}{0.5}+\frac{0.0625}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}+\frac{0.125}{m\pi}\cos\frac{0.5m\pi}{0.5}-\frac{0.25}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}-\frac{0.25}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5}$$

$$+\frac{0.25}{m^3\pi^3}-\frac{0.06}{m\pi}\cos\frac{0.5m\pi}{0.5}+\frac{0.19}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}+\frac{0.34}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5}-\frac{0.34}{m^4\pi^4}\sin\frac{0.5m\pi}{0.5}$$

$$= \frac{0.0025}{m\pi}\cos\frac{0.5m\pi}{0.5}+\frac{0.0025}{m^2\pi^2}\sin\frac{0.5m\pi}{0.5}+\frac{0.09}{m^3\pi^3}\cos\frac{0.5m\pi}{0.5}+\frac{0.25}{m^3\pi^3}-\frac{0.34}{m^4\pi^4}\sin\frac{0.5m\pi}{0.5}\ ....(1)$$

Next

$$\int_0^{0.35} y(0.35-y)^2 \sin\frac{n\pi y}{0.35}dy = \int_0^{0.35} y(0.1225-0.7y+y^2)\sin\frac{n\pi y}{0.35}dy = \int_0^{0.35} y(0.122y-0.7y^2+y^3 \sin\frac{n\pi y}{0.35}dy$$

$$= 0.1225\int_0^{0.35} y\sin\frac{n\pi y}{0.35}dy - 0.7\int_0^{0.35} y^2 \sin\frac{n\pi y}{0.35}dy + \int_0^{0.35} y^3 \sin\frac{n\pi y}{0.35}dy$$

$$= 0.1225\left[\frac{-0.1225}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.1225}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}\right] - 0.7\left[\frac{-0.04}{n\pi}\cos\frac{0.35n\pi}{0.35} + \frac{0.09}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35} + \right.$$

$$\frac{0.09}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35}-\frac{0.09}{n^3\pi^3}\Bigg]+\Bigg[\frac{-0.02}{n\pi}\cos\frac{0.35n\pi}{0.35}+\frac{0.044}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}+\frac{0.099}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35}-\frac{0.099}{n^4\pi^4}$$

$$\sin\frac{0.35n\pi}{0.35}\Bigg]$$

$$=\frac{0.015}{n\pi}\cos\frac{0.35n\pi}{0.35}+\frac{0.015}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}+\frac{0.028}{n\pi}\cos\frac{0.35n\pi}{0.35}-\frac{0.063}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}-\frac{0.063}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.063}{n^3\pi^3}-\frac{0.02}{n\pi}\cos\frac{0.35n\pi}{0.35}+\frac{0.044}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}+\frac{0.099}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35}-\frac{0.099}{n^4\pi^4}\sin\frac{0.35n\pi}{0.35}$$

$$=\frac{0.007}{n\pi}cos\frac{0.35n\pi}{0.35}-\frac{0.004}{n^2\pi^2}\sin\frac{0.35n\pi}{0.35}+\frac{0.036}{n^3\pi^3}\cos\frac{0.35n\pi}{0.35}+\frac{0.063}{n^3\pi^3}-\frac{0.099}{n^4\pi^4}\sin\frac{0.35n\pi}{035}\ldots\ldots\ldots\ldots\ldots(2)$$

$\ldots\ldots\ldots\ldots(1)\ x \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

$$=\frac{-0.0000175}{mn\pi^2}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00001}{mn^2\pi^3}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00009}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5}$$

$$\cos\frac{0.35n\pi}{0.35}+\frac{0.000157}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5}-\frac{0.00025}{mn^4\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.0000175}{m^2n\pi^3}\sin\frac{0.5m\pi}{0.5}$$

$$\cos\frac{0.35n\pi}{0.35}-\frac{0.00001}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00009}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.00016}{m^2n^3\pi^5}$$

$$\sin\frac{0.5m\pi}{0.5}-\frac{0.00025}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.00063}{m^3n\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00036}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}$$

$$\sin\frac{0.35n\pi}{0.35}+\frac{0.00324}{m^3n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.0057}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.0089}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$-\frac{0.0018}{m^3n\pi^4}\cos\frac{0.35n\pi}{0.35}-\frac{0.001}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35}+\frac{0.009}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.016}{m^3n^3\pi^6}-\frac{0.025}{m^3n^4\pi^7}\sin\frac{0.35n\pi}{0.35}$$

$$-\frac{0.0024}{m^4n\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00136}{m^4n^2\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.01224}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.0214}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}-\frac{0.034}{m^4n^4\pi^8}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$B_{mn}=23\Bigg[\frac{-0.0000175}{mn\pi^2}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00001}{mn^2\pi^3}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00009}{mn^3\pi^4}$$

$$\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.000157}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5}-\frac{0.00025}{mn^4\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.0000175}{m^2n\pi^3}$$

$$\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00001}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00009}{m^3n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$+\frac{0.00016}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}-\frac{0.00025}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.00063}{m^3n\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}$$

$$-\frac{0.00036}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00324}{m^3n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.0057}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.0089}{m^3n^4\pi^7}$$
$$\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.0018}{m^3n\pi^4}\cos\frac{0.35n\pi}{0.35}-\frac{0.001}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35}+\frac{0.009}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.016}{m^3n^3\pi^6}$$
$$-\frac{0.025}{m^3n^4\pi^7}\sin\frac{0.35n\pi}{0.35}-\frac{0.0024}{m^4n\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00136}{m^4n^2\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.1224}{m^4n^3\pi^7}$$
$$\left.\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.0214}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}-\frac{0.034}{m^4n^4\pi^8}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}\right]$$

$$B_{mn}=\frac{-0.0004}{mn\pi^2}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.00023}{mn^2\pi^3}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.00207}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5}$$
$$\cos\frac{0.35n\pi}{0.35}+\frac{0.0036}{mn^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.0056}{mn^4\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.000403}{m^2n\pi^3}\sin\frac{0.5m\pi}{0.5}$$
$$\cos\frac{0.35n\pi}{0.35}-\frac{0.00023}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.0021}{m^3n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.0037}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5}$$
$$-\frac{0.0056}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.0145}{m^3n\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.0083}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$
$$+\frac{0.0745}{m^3n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.13}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5}-\frac{0.2047}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}-\frac{0.0414}{m^3n\pi^4}$$
$$\cos\frac{0.35n\pi}{0.35}-\frac{0.023}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35}+\frac{0.207}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35}+\frac{0.37}{m^3n^3\pi^6}-\frac{0.575}{m^3n^4\pi^7}\sin\frac{0.35n\pi}{0.35}-\frac{0.0552}{m^4n\pi^5}$$
$$\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}-\frac{0.0313}{m^4n^2\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}+\frac{0.28}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35}+\frac{0.4922}{m^4n^3\pi^7}$$
$$\sin\frac{0.5m\pi}{0.5}-\frac{0.782}{m^4n^4\pi^8}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}$$

$$B^*_{mn}=0\ \text{since}\ g(x,y)=0$$

$$U_{mn}(x,y,t)=(B_{mn}\cos\lambda_{mn}t+B^*_{mn}\sin\lambda_{mn}t)\left(\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\right)$$

$$
\begin{aligned}
U_{mn}(x,y,t) = \Bigg[& \frac{-0.0004}{mn\pi^2}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.00023}{mn^2\pi^3}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.00207}{mn^3\pi^4}\cos\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} + \frac{0.0036}{mn^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.0056}{mn^4\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.000403}{m^2n\pi^3}\sin\frac{0.5m\pi}{0.5} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.00023}{m^2n^2\pi^4}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.0021}{m^3n^3\pi^5}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.0037}{m^2n^3\pi^5}\sin\frac{0.5m\pi}{0.5} \\
& - \frac{0.0056}{m^2n^4\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.0145}{m^3n\pi^4}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.0083}{m^3n^2\pi^5}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} \\
& + \frac{0.0745}{m^3n^2\pi^6}\cos\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.13}{m^3n^3\pi^6}\cos\frac{0.5m\pi}{0.5} - \frac{0.2047}{m^3n^4\pi^7}\cos\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} - \frac{0.0414}{m^3n\pi^4} \\
& \cos\frac{0.35n\pi}{0.35} - \frac{0.023}{m^3n^2\pi^5}\sin\frac{0.35n\pi}{0.35} + \frac{0.207}{m^3n^3\pi^6}\cos\frac{0.35n\pi}{0.35} + \frac{0.37}{m^3n^3\pi^6} - \frac{0.575}{m^3n^4\pi^7}\sin\frac{0.35n\pi}{0.35} - \frac{0.0552}{m^4n\pi^5} \\
& \sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} - \frac{0.0313}{m^4n^2\pi^6}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35} + \frac{0.28}{m^4n^3\pi^7}\sin\frac{0.5m\pi}{0.5}\cos\frac{0.35n\pi}{0.35} + \frac{0.4922}{m^4n^3\pi^7} \\
& \sin\frac{0.5m\pi}{0.5} - \frac{0.782}{m^4n^4\pi^8}\sin\frac{0.5m\pi}{0.5}\sin\frac{0.35n\pi}{0.35}\Bigg] \cos\lambda_{mn}t\left(\sin\frac{m\pi x}{0.5}\sin\frac{n\pi y}{0.35}\right)
\end{aligned}
$$