KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

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ANALYSIS OF INVENTORY CONTROL SYSTEMS: A CASE STUDY AT AIR-MATE GAS FACTORY, GHANA.

BY

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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ABSTRACT

Many Organizations face great challenges in managing inventories. Poor inventory management may result in under-stocking, overstocking as well as high inventory total cost.

This thesis examines inventory situation at Air-Mate Gas Factory, Ghana. The objective of this paper is to develop the Economic order Quantity (mathematical) model that will be used to determine number of units of gases to be ordered at a time and the re-order point, that is the level to which stocks are allow to fall before ordering for the various imported gases. The resulting EOQ for each gas is compared to the actual ordered quantities so as to see whether there is any relationship between them in operational cost reduction. The study used secondary data from Air-Mate Gas Factory, Ghana.

The results show that the relationship between the EOQ's and the ordered quantities at Air-Mate Gas Factory, Ghana in terms of cost reduction was significant. Therefore it was concluded that the ordered quantities of gas at Air-Mate Gas Factory, Ghana were not optimal.

Therefore it is recommended that in order to manage inventory effectively, Air-Mate Gas Factory, Ghana needs to employ inventory control model such as the EOQ model to obtain reasonable ordered quantities for its gases.

TABLE OF CONTENTS

CONTENT

PAGE

CHAPTER 3 METHODOLOGY

LIST OF TABLES

LIST OF FIGURES

DEDICATION

This work is dedicated to the Almighty God for his mercy, guidance and protection, for seeing me through this programme.

Finally to my dearest wife, Mrs Agnes Mensah and child, Michael Osei Mensah and my sweet mum Mrs Christiana Anane. I thank you for the support, understanding and your sacrifices. I love you all.

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I say may God bless you all. Amen

CHAPTER ONE

INTRODUCTION

1.0 OVERVIEW

The chapter focuses on the background of the study, problem statement, objectives of study, the methodology, the justification of the study and finally, the organization of study.

1.1 BACKGROUND

Inventory is defined as a stock of items kept on hand by an organization to use in meeting customers demand (Russell and Taylor 1995).

The importance of inventory to a firm stems from two points of view: financial and operational. First, inventory represents a major financial investment for any company. Inventories represent 25 to 50 percent of total assets in manufacturing firms and 75 to 80 percent in wholesalers and retailers (Johnson et al). On the other hand, from the operational perspective, inventories add an operating flexibility. Adequate inventories kept in manufacturing companies will smooth the production process. The wholesalers and retailers can offer good customer services and gain public image by holding sufficient inventories. The basic objective of inventory management is to achieve a balance between the low inventory and high return on investment.

1.1.1 THE FUNCTIONS OF INVENTORY

The functions that inventory performs can be summarized as follows (Evans et al 1990):

First of all, the fundamental function for carrying inventories is to meet customer demand for a product. In fact, it is physically impossible and economically impractical for each stock item to arrive exactly where it is needed and exactly when it is needed. Therefore, a reasonable level of inventory is normally maintained that will meet anticipated or expected customer or user demand.

Secondly, since demand is usually not known with certainty, additional amounts of inventory, called safety or buffer stocks, are often kept on hand to meet unexpected variations in excess of expected demand.

Thirdly, additional stocks of inventories are sometimes built up to meet demand that is seasonal or cyclical in nature. Companies will produce items when demand is low in order to meet high seasonal demand for which their production capacity is insufficient. Correspondingly, retailers might find it necessary to keep large stocks of inventory on their shelves to meet peak seasonal demand or for display purposes to attract buyers.

Finally, inventory can also be carried out to take the advantage of price changes. A company will often purchase large amounts of inventory to take advantage of price discounts, as a hedge against anticipated price increase in the future, or because they can get a lower price by purchasing in volume.

1.1.2 CLASSES OF INVENTORY PROBLEMS

Distinction in inventory management is made according to the nature of demand for items. In 1965 a very useful classification of demand was proposed by (Orlicky, 1975). He used the term "independent demand" to describe any demand of items that is influenced by market conditions and unrelated to demand for other items in a company's inventory. This includes the demand for finished goods and spare parts. He also used the term "dependent demand" to describe any demand for items directly determined by other associated items. Typical of this are raw materials, purchased or manufactured parts or ingredients, and manufactured subassemblies, attachments and accessories.

It is Orlicky's classification that provides the real key to selection and applicability of inventory control techniques. Dependent demand, by definition, can be precisely determined from the demand of related items, the methods of material requirements planning (MRP) and just-in time (JIT) are the appropriate techniques to treat this kind of inventory problem encountered in manufacturing companies (Plossl, 1985 and Schroeder, 1989). The key ingredients of MRP are master production schedule, bills of materials, and inventory recorders. Using information from these sources, the MRP system identifies actions such as releasing new production orders, adjusting order quantities, and expediting last orders. JIT systems are designed to produce or deliver goods or services as needed, using minimal inventories. It is actually a philosophy that focuses on reducing inefficiencies and unproductive time in the production process. Both MRP and JIT are more than inventory control system; they also involve process design and scheduling issues.

1.1.4 INVENTORY DECISIONS

There are three primary decisions that must be made in regard to independent demand inventories. These are:

1. How to monitor the inventory.
2. How much should be ordered.
3. When should orders be placed.

1.1.6 COSTS INVENTORY

Most of the inventory models are built around the assumption that the objective is to minimize inventory costs. In this thesis that inventory costs are:

1. Item cost is the cost of buying and/or producing the individual items. The cost of items is often an important consideration when quantity discounts are offered. The item cost can usually be obtained from vendors.

2. Ordering cost is incurred because of the work involved in placing purchase orders with vendors or to organize for production within a plant. This cost should include the costs of acquiring the data necessary for making decisions, computational cost, stationary, telephone calls, transportation, receiving and inspection. The ordering cost can be estimated from the company's records. However, difficulties are sometimes encountered in separating the fixed and the variable ordering-cost components.

3. Holding cost is associated with keeping items in inventory for a period of time. The holding cost usually consists of electricity and heat, insurance and tax, spoilage and obsolescence, the cost of capital and the expenses of running the warehouse. This cost is more difficult to determine accurately in terms of historical information (Rhodes, 1981). In practice, however, the estimation of holding cost is often based substantially on managerial judgment.

4. Stockout cost reflects the economic consequences of running out of stock. It is the most difficult of all inventory costs to estimate. One approach is simply to specify an acceptable stockout risk level. Another interesting method used by (Reimans et al 1972) is to treat the unit shortage cost as a function of the gross profit of an item. This approach has the advantage of resulting in better service for higher profit items.

It is relatively easy to list the contents of each category of inventory costs as mentioned above. However, their measurement in practice is a very difficult task. In particular, accounting information, primarily collected and recorded for a financial purpose, is usually inappropriate for estimating inventory costs. Furthermore, shortage costs are often not shown in accounting records. There is no satisfactory solution to this problem and more research work is needed to establish methods for estimating inventory costs.

1.2 PROFILE OF STUDY AREA

Air-Mate Gas Factory Ghana was established in 2000 and today employs hundreds of people. With presence in all the major cities and towns in Ghana, supported by a network of over 40 distributors, Air-Mate Gas Factory Ghana has since its establishment played a key role in the socio economic development of Ghana. Air-Mate Gas Factory Ghana engages in the production and sales of air gases for industry, health, and the environmental sectors. The company primarily offers Oxygen, Nitrogen, Hydrogen, Argon and rare gases. They also market and provide services in allied products such as welding, fire fighting, medical equipment and on-site solutions. Airmate continuously reinvents its business, anticipating the needs of the current and future markets.

The group innovates to enable the progress, to achieve dynamic growth and a consistent performance.

Air-Mate Gas Factory Ghana explores the best that air can offer to preserve life, staying true to its sustainable development approach.

1.3 PROBLEM STATEMENT

Inventory management is practiced from the smallest organization such as fruit stands to multimillion dollar industries. Effective inventory management allows an organization to reduce total costs by achieving wide-scale operational efficiencies. It also acts as insurance by improving product availability and buffering against everyday uncertainties the organization faces.

The major decisions in inventory control of any organization concerns the time to replenish an order and the quantity of such an order. The failure to manage these two concerns can significantly increase the total cost of an organization.

Numerous studies have developed inventory- ordering models, but none has applied these models to improve the inventory ordering systems of Airmate Company Ghana Limited.

As an organization with large investments in inventory, Airmate Company Ghana Limited could reduce their inventory cost through maintaining more effective inventory management systems.

The remaining question is "How can Air-Mate Gas Factory Ghana organize their inventory systems in order to reduce total inventory cost while still meeting consumer demands?"

1.4 OBJECTIVES

The main objectives of this study are:

1) To model the inventory cost as an Economic Order Quantity Problem.
2) To optimize the total costs associated with the carrying or ordering costs.
3) To find an optimal re-order level to which stocks can be allowed to fall before placing a new order.

1.5 METHODOLOGY

The data for the study was collected by interacting with the Accounts Manager, Supply Chain Manager. The data regarding the present systems of controlling inventory were collected.

The literature review of the study was obtained from internet and books. EOQ models were used to determine how much gas to order and when to place the order.

1.6 JUSTIFICATION

Many companies including Air-Mate Gas Factory Ghana do not have effective method for managing their inventories. Therefore, the findings of this study are expected to help the management of Air-Mate Gas Factory Ghana and other such companies to formulate good inventory policies. This could help the managers of Air-Mate Gas Factory Ghana to know when to place an order for new items without allowing the inventory level to fall below an expected point.

1.7 ORGANIZATION OF THE STUDY

The study is organised in five chapters as follows. Chapter one provides general background of the study. It also provides the statement of problem and it sets out the objectives of the study, provides methodology of the study, the justification of the study and organization of the study. Chapter two reviews pertinent literature related to the study. Chapter three considers the model and its development. Chapter four discusses the methodological issues of the study and also discusses the results and interprets the results. The final chapter, which is chapter five, summarizes the main findings of the study and provides suggestions and recommendations

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter reviews some of the research work that has been conducted so far in the field of Inventory Management, Just-in Time (JIT), Economic Order Quantity (EOQ), Supply Chain Management.

2.1 Inventory Management

According to Chase et al. (2004), inventory is the stock of any item or resource used in any organization. An inventory system is the set of policies and controls that monitor levels of inventory and determine what levels should be maintained, when stock should be replenished, and how large orders should be.

Inventory management is one of the important key activities of business logistics.

Because of its role in business organizations, Schonsleben (2000) adds that inventory is one of the most important instruments of logistics planning and control. While inventory on work in progress is linked to the production process, physical inventory on stock or buffer storage is necessary from the standpoint of added value and is considered as waste of time and money (tied-up capital).

According to Hill (2002), inventory is a significant asset in most organizations. Its effective management, therefore, is a key task within the auspices of operations. But controlling inventory is far from easy. It involves a complex set of decisions due to the many forms inventory and functions it provides. In addition, inventories are the result of functional policies within an organization as well as the short and long term decisions in purchasing, operations and sales.

Bertolini et al., (2002). The optimal management of inventories is a primary objective for all the firms manufacturing make to stock finished goods. As a matter of fact, inventories have important implications for both the financial and the economic performance of the company, therefore it is widely acknowledged that an optimal inventory management policy allows companies to achieve higher profitability levels. In general terms, inventory management policies should be aimed at lowering the holding costs through higher inventory rotation, but without triggering substantial stockouts and backorders, caused by demand peaks and / or lead time delays.

Bowersox et al., (2002) are of the opinion that inventory typical represent the second largest component of logistics cost next to transportation. The risks associated with holding inventory increase as products move down the supply chain closer to the customer because the potential of having the product in the wrong place or form increases and costs have been incurred to move the product down the channel. In addition to the risk of lost sales due to stockouts because adequate inventory is not available, other risks include obsolescence, pilferage and damage.

Krajewski and Ritzman (1999) discussed inventory management as an important concern for managers in all types of businesses. For companies such as J. C. Penny Limited, which operates on relatively low profit margins, poor inventory management can seriously undermine the business.

The challenge isn't to pare inventories to the bone to reduce costs or to have plenty around to satisfy all demands, but to have the right amount to achieve the competitive priorities for business most efficiently.

As all organizations are concerned with inventory management, a particular accent has to be put to it. A sane inventory management implies the coordination of strategic functions (production, finance, and marketing) of the organization in order to reach objectives.

The achievement of any organization's objectives is linked to the relationships of functional goals. That's the reason why strategic policies related to inventory management to be arrested or conceived in order to achieve the organizational goals. Because failure to do that, an organization will grind to a halt.

2.2 Type of Inventory

According to Stock and Lambert(2001) inventories can be categorized into six distinct forms, that are:

Cycle stock, In-transit inventories, Safety or buffer stock, Speculation stock, Seasonal stock and Dead stock.

1. Cycle Stock is inventory that result from the replenishment of inventory sold or used in production. It is required in order to meet demand under conditions of certainty, that is when a firm can predict demand and replenishment times (lead times) almost perfectly.
2. In-transit inventories are items that are en route from one location to another. They may be considered part of cycle stock even though they are not readily available for sale and / or shipment until after they arrive at the destination.
3. Safety or buffer stock is held in excess of cycle stock because of uncertainty in demand or lead time. The notion is that a portion of average inventory should be devoted to cover short-range variations in demand and lead time. Average inventory at a stock-keeping location that experiences demand or lead time variability is equal to half the order quantity plus the safety stock.
4. Speculation stock is inventory held for reasons other than satisfying current demand. For example, materials may be purchased in volumes larger than necessary in order to receive

quantity discounts, because of a forecasted price increase or materials shortage, or protect against the possibility of a strike.

5. Seasonal stock is a form of speculation stock that involves the accumulation of inventory before a season begins in order to maintain a stable labour force and stable production runs.
6. Dead stock is inventory that no one wants, at least immediately.

2.3 Motivation of Holding Inventory

2.3.1 Rationale for Having Inventory

There are many reasons that motivate companies to have stock. Bloomberg et al., (2002) have identified five reasons for holding stock, namely:

a. Economies of scale. A firm can realize economies of scale in manufacturing, purchasing and transportation by holding inventory. If the business buys large amounts, it gets quantity discounts. In turn, transportation can move larger volumes and get economies of scale through better equipment utilization. Manufacturing can have longer production runs if more material is inventoried, allowing per unit fixed cost reductions.
b. Balancing supply and demand is another important reason for having inventory. If supply is seasonal, inventory can help meet demand when materials or products are not available. If there is an occurrence of seasonal demand, firms must accumulate inventory in advance to meet demand in the future.
c. Specialization. Inventory allows firms with subsidiaries to specialize. Instead manufacturing a variety of products, each plant can manufacture a product and then ship the finished products directly to customers or warehouse for storage. By specializing, each plant can gain economies of scale through long production runs.

d. Production from uncertainties. A primary reason to hold inventory. Having stock on hand can reduce risk of shortage or stockout situation which might lead to lost sales and lack of reliability. Customers can possibly buy products from competitors instead.

e. Buffer interface. Inventory can buffer key interfaces, creating time and place utility. Key interfaces include:

i) Supplier and purchasing

ii) Purchasing and production

iii) Production and marketing

iv) Marketing and distribution

v) Distribution and intermediary and

vi) Intermediary and customers.

Having inventory at these interfaces helps ensure that demand is met and stock outs are minimized.

2.4 Symptoms of Poor Inventory Management

A certain number of symptoms allow discovering poor inventory management. Lambert and Stock (2001) mention the following elements in order to diagnose poor inventory management:

a. Increasing number of back orders.

b. Increasing dollar investment in inventory with back orders remaining constant.

c. High customer turnover.

d. Increasing number of orders cancelled.

e. Periodic lack of sufficient storage-space.

f. Wide variance in inventory turnover among distribution centers and among major inventories items.

g. Deteriorating relationships with intermediaries as typified by dealer cancellations and declaring orders.

h. Large quantities of obsolete items**.**

2.5 Just-in-Time Inventory Management

Harber et al.,(in Biggart and Gargeya 2002) mention that the just-in-time (JIT) production system (as the Toyota production System) was introduced by Shigeo Shing and Taichi Ohno at the Toyota Motor plant in the mid-1970. JIT production is called by many names: zero inventory system (ZIPS), minimum inventory production system (MIPS), kanban production, kaizen production, stockless production, pull-through production and quick response (QR) inventory systems. JIT manufacturing, both as philosophy and a discipline method of production, has received much attention since its introduction. The JIT production philosophy is founded upon three fundamental principles: elimination of waste, continuous quality improvement and encouragement of worker participations planning and execution.

Gourdin (2001) adds that this just-in-time manufacturing philosophy requires manufacturers to work in concert with suppliers and transportation providers to get required items to the assembly line at the precise time they are needed for production.

2.5.1 Basic Tenets of JIT

Harber et al.,(in Biggart and Gargeya 2002) mention that a successful JIT system is based upon the following key concepts:

a. Quality. With JIT, the customer must receive high quality goods. One of the historical roles of inventory has been to protect the customer against defective items; if a bad product is received it can be discarded and a new one drawn from inventory. With a JIT system, however, poor quality means the production line stops or the external customer gets a defective item. There are no "extra "items to replace the poor.

b. Vendors as Partners. Generally, firms using JIT rely on fewer vendors rather than more. Purchases are concentrated with a limited number of suppliers in order to give the buyer leverage with respect to quality and service. Purchasers also include vendors in the planning process, sharing information regarding sales and production forecasts so that vendors then have a clear idea of what their customers need.

c. Vendor co-location with customer. Ideally, suppliers should be located in close proximity to their customers. As the distance between vendors and buyers increases, so does the opportunity for system disruption and stock-outs. In order to minimize this risk, customers often demand that vendor facilities be co-located on the same site or at least in the same geographical era as their own.

2.5.2 Advantages of JIT

Harber et al.,(in Biggart and Gargeya 2002) came out with following as advantages of JIT :

1. More inventory turns. Because there is less on hand, the inventory that is maintained stay for a shorter period of time. The problem with an extremely high number of turns is that it can raise the probability of stocking out to an unacceptable high level while raising ordering costs as well.
2. Better quality. As mentioned earlier, high quality products must be received with a JIT system or else the entire benefits production process collapses. Customers concentrate their purchases with a small number of vendors in exchange for receiving high quality items and requisite service.

3. Less Warehousing space needed. When there is less inventory, fewer and / or smaller warehouses are required.

2.5.3 Disadvantages of JIT

Harber et al.,(in Biggart and Gargeya 2002) cited the following as disadvantages of JIT:

1. Risk of stock-outs. When firms eliminate inventory, the risk of stock-outs can rise. Managers attempt to minimize this occurrence by demanding very high levels of service from their vendors and logistics service providers. However, when co-location of customer and vendor is not feasible, for example, the resultant variability in the pipeline can lead to stock-outs despite management's best effort to prevent them.
2. Increased transportation costs. Since JIT requires frequent shipments of small quantities, transportation costs almost always rise. As long as these costs are more than offset by the inventory savings, it is advantageous for the organization to permit them.
3. Increased purchasing costs. Purchasing discounts are generally associated with buying large quantities at a time. JIT means foregoing those price-breaks in favour of obtaining smaller amounts more frequently. Managers must sure that purchasing costs are not rising more than what inventory costs are falling.
4. Small channel members may suffer. JIT is sometimes criticized as a system that allows strong organizations to unload their inventory on smaller firms in the channel.
5. Environmental issues. In a micro sense, JIT can lead to high levels of traffic congestion and air pollution because additional transportation is often required to maintain customer service levels in the absence of inventory.

2.6 Economic Order Quantity (EOQ)

Economic order quantity is the number of units which a company is supposed to add to the inventory for each order to minimize the total cost of the inventory.

Piasecki (2001) presents an inventory model for calculating optimal order quantity that used the Economic Order Quantity (EOQ) method. He points out that many companies are not using the EOQ method due to poor results received resulted from inaccurate data input. He clarifies that many errors resulted in the calculation of EOQ in the computer software package are due to the failure of the users in understanding how the data inputs and system setup that control the output. He says that the EOQ is an accounting formula that determines the point at which the combination of order costs and inventory cost are the least. He highlights that the EOQ method would not conflict with the Just in Time (JIT) concept. In fact, he explains that JIT is actually a quality initiative to eliminate wasted steps, wasted material, wasted labor and other costs; EOQ method is used to determine which components would fits into the JIT model and what level is economically advantageous for the operation.

Piasecki further elaborates the EOQ formula that includes the parameters such as annual usage in unit, orders cost and carrying cost. Finally, he proposes several steps to follow in implementing the EOQ method. These include the testing of the formula by manually checking the result obtained, run a simulation by using a sampling of items, and maintain the EOQ formula by reviewing the interest rates, storage costs and operational cost periodically.

Liberatore, (1979) discussed an EOQ model, with a few alterations to the assumptions on the basis of which the traditional EOQ model had been developed. Typically, demand always followed a pattern that could be traced by probability distribution for analysis. The basic EOQ model, however, assumed that this demand was deterministic to simplify the calculations involved.

The traditional EOQ model also assumed that if the inventory is zero when the order was received then that particular order was lost. This was not the scenario in real life as orders may be backordered and fulfilled when the inventory was available. Liberatore, (1979) considered a more realistic situation for his model and developed an equation for the order size based on stochastic lead times and backlogged demand. The traditional equations of inventory theory with deterministic lead times and no backlogging were special cases of this model.

Silver, (1976) extends the classical EOQ model to include supply uncertainty. Two problems are analyzed: One in which the standard deviation of quantity received is independent of quantity ordered, and another in which it is proportional to the quantity ordered. For both cases the optimal order quantity is shown to be a simple modification of the EOQ.

Shih, (1980) considered a production system where yield uncertainty is a result of defective items. It is assumed that the percentage defective in a lot is a random variable with a known distribution. A deterministic EOQ model and a stochastic single period model analyzed. The optimal ordering quantities are proved to be greater for the case of uncertain supply. Moreover, the optimal order-up-to levels decrease with the variance of the yield rate.

2.6.1 EOQ with Quantity Discount

Quantity discounts are price reductions that are offered to the retailer when they place an order that is beyond a certain specific level. It is an incentive to the retailer to buy larger quantities. When quantity discounts are offered the retailer is forced to consider the possible benefit of ordering larger number of items with a lower price per item over the increase in the inventory costs that would be incurred by the retailer.

Weng (1995) presented the models for determining optimal all-unit and incremental quantity discount policies. He also investigated the effect of the quantity discounts on increasing demand and ensuring Pareto efficient transactions under general price- sensitive demand functions. Optimal quantity discount policies, their interrelationships and their benefits to the supplier and the buyer were developed in this paper. The gains of the managerial insights for the scenarios of maximizing the supplier’s profit and joint profit are illustrated in this paper. He developed a simple and efficient solution approach for determining the all-unit and the incremental optimal decision policies for general price –sensitive demand functions. The main findings were:

1. With price-sensitive demand there are two incentives in offering quantity discounts: increasing demand and ensuring Pareto-efficient transactions. In most cases increasing demand dominates in justifying the offering of quantity discount,

2. Using a single lot-size associated with all efficient transaction as with the constant demand does not hold with price sensitive demand cases,

3. The optimal all-unit quantity policy is equivalent to the optimal incremental quantity discount policy function benefiting both supplier and the buyer.

Kim (1989) argued how the supplier can formulate the terms of a quantity discount-pricing schedule assuming that the buyer always behaves optimally. Formulas were derived for price and order –size, which maximize:

1. The economic gain of supplier resulting from revising price and order size;
2. The gain of the buyer; and

3. The sum of gains of both the parties.

It was suggested that how the supplier can induce the buyer to pre-determined price and order-size, level of mutual benefit by utilizing all-units and incremental quantity discounts in the system.

Min (1992) studied the profit maximizing EOQ model for monopolistic seller. A monopolistic seller determines both the order quantity and the quantity discount price schedule simultaneously where as buyer have the preference on the purchase quantities.

Followill et al., (1990) studied managerial decision to accept a quantity discount, if total, per period inventory and acquisition costs are reduced. They developed an EOQ model within wealth maximization framework, when volume discounts were unavailable. They established that the traditional method of analyzing volume discount opportunities may invoke wealth decreasing decisions.

Martin (1993) provides an alternative perspective on the quantity discount-pricing problem. He considered the multiple price breaks excluding the buyer's operating parameter from consideration, with the exception of price dependent demand.

Dada et al., (1987) studied quantity discounts from a seller's point of view. The authors characterized the rand of order quantities and prices that would lower costs for both the buyer 24 and the seller. Pricing policies that helped with balancing the savings for both the buyer and the seller were develop according to these characteristics.

This principle of offering quantity discounts is similar to the principle discussed in this research but the benefit of ordering large quantities is implicitly included in the model as opposed to explicitly considering the purchasing cost per unit and providing discounted rates to buyers when they order larger quantities. The discount is obtained by the retailer when large quantities are ordered that larger unit's loads are used.

2.7 Supply Chain Management

The supply chain management literature defines a supply chain as a set of facilities, technologies, suppliers, customers, products, and methods of distribution (Arntzen et., 1995).

Arntzen et al., (1995) said of the supply chain optimization models found in the literature, the most inclusive was a mixed integer programming model that optimized multiple products, facilities, production stages, technologies, time periods, and transportation models for Digital Equipment Corporation's global operation. The model minimizes total cost and activity days subject to service (inventory), local content requirements, and other constraints. However, this model is limited to the internal logistics of Digital Equipment Corporation and is computationally intense.

Cavinto (1991) proposed another method identifying six interfirm total cost factors in supply chain relationships that need to be addressed: labor rate, productivity, capital availability, capital cost, tax rate, and depreciation or other tax elements.

Cavinto (1991) suggested firms have different cost structures, factor inputs, management skills, and buying powers that provide opportunities to evaluate jointly which firm should perform each task. His theory is that firms within a supply chain should determine where each activity should take place in the value-chain based on the lowest total cost across themselves compared against another set of competing firms.

Coyle et al., (2003) discussed that as the definition implied; supply chain management had been developed for customers who played the most important role in businesses. Especially in the globalization era, customers, ever more demanding and powerful than before, were seeking for products and services with higher criteria. In order to meet customers' requirements and satisfactions, companies had to be proactive against globalized markets which could be changed and influenced by several factors. With an increase of use of technology like internet, some claim that there was no more geography in business nowadays. Offshore production, collaboration between international companies, and openness of the global market were the significance of the global environment. Supply chain management could therefore be labeled as global supply chain management in today's environment.

Supply chain management evolved soon after lean manufacturing and Just-in-Time system were implemented in the 1970's. This was after manufactures realized the impact carrying excess inventory and work in progress had on the quality of the products and lead time. Excess inventory along the manufacturing line leads to congestion and consequently affects the quality of the products. Once the quality is affected, the rework rate increases and hence lead time increase. Carrying smaller inventories required fostering a better relationship with the suppliers so that the manufacturers could expect a better response time from the suppliers. This led to development of supplier partnership. The manufacturer also realized that close relationships with the customers helped the manufacture of the products that conformed to customer's needs and helped the manufacturers decide on their next product line based on what the customer wanted. Thus customer partnerships were promoted. These new dimensions in the manufacturing chain led to supply chain management.

According to Quinn (1997), the supply chain includes all of those activities associated with moving goods from the raw-materials stage through to the end user. This includes sourcing and procurement, production scheduling, order processing, inventory management, transportation, warehousing and customer service.

2.8 Inventory Control

Inventory control is the activity which organizes the availability of items to the customers.

It co-ordinates the purchasing, manufacturing and distribution functions to meet the marketing needs. This role includes the supply of current sales items, new products, consumables, spare parts, obsolescent items and all other supplies (Wild 2002).

Wild (2002) adds that the purpose of the inventory control function in supporting the business activities is to optimize the following three targets:

- Customer service
- Inventory cost
- Operating cost

The most profitable policy is not to optimize one of these at the expense of others.

The inventory controller has to make value judgements. If profit is lacking, the company goes out of business in the short term. If the customer service is poor, then the customers disappear and the company goes out of business in longer term. Balancing the financial and marketing aspects is the answer: the stock controller has a fine judgement to make.

The first target, customer service, can be considered in several ways, depending on the type of demand. In a general stores environment the service will normally be taken as “availability ex stock”, whereas in supply to customer specification, the service expected would be delivery on time against customer requested date.

The second target, inventory cost, requires a minimum of cash tied up in stock. This has to be considered carefully, since there is often the feeling that having any stock in stores for a few month is bad practice. In reality, minimizing the stock usually means attending to the major costs: very low-value items are not considered a significant problem.

Low inventory can also be considered in terms of space, or other critical resource. Where the item is voluminous, or the store space restricted, the size of the items will also be a major consideration.

The third target, avoiding operating cost, has become more of an issue as focus has been placed on inventory management. The prime operating costs are those associated with the stores operations, inventory control, purchasing and the associated services. The development of logistics, linking distribution costs with inventory, has added this new set of transportation costs to the analysis.

The research done by Smaros et al (2003) on the impact of increasing demand visibility on production and inventory control efficiency reveals that for products with stable demand a partial improvement of demand visibility can improve production and inventory control efficiency, but that the value of visibility greatly depends on the target products' replenishment frequencies and the production planning cycle employed by the manufacturer.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

The Economic order Quantity model is widely used based on its simple nature. Simplicity and restrictive modeling assumption usually go together, and the EOQ model is not an exception. However, the presence of these modeling assumptions does not mean that the model cannot be used in practice. There are many situations in which this will produce good results. For example, these models have been efficiently employed in automotive, pharmaceutical, retail, wholesale and distribution sectors of the economy of Ghana for many years. Another advantage is that the model gives the optimal solution in closed form. This allows us to gain insights about the behavior of the inventory system. The closed form solution is also easy to compute compared to, for example, an iterative method of computation. In this chapter, we will develop models for a single –stage system in which we manage inventory of single item. The purpose of these models is to determine how much to purchase (order quantity) and when to place the order (the reorder point). The common thread across these models is the assumptions that demand occurs continuously at constant and known rate. We begin with the simple model in which all demand is satisfied on time. Secondly we develop a model in which some of the demand could be backordered. Thirdly, we consider the EOQ model again however, the unit purchasing cost depend on the order size. Finally, we briefly discuss Economic production model with backorder.

3.1 Economic Order Quantity (EOQ) model.

The following notations are used in this chapter.

D: annual demand rate (units/year)

K: purchaser's ordering cost ($/order)

Q: order size (units)

H: inventory carry or holding cost rate (%/year)

C: unit purchase cost ($)

Economic order quantities enable organization to maintain a regular inventory of products which have a uniform and independent demand (Tersine 1994). It is widely used deterministic model which assumes that the demand rate for an item is constant and continuous. The order lead time and the inventory holding cost are also presumed to be known and constant. With the previously mentioned conditions the order quantity as well as the time between orders are always constant and remain unchanged.

Assumptions

The following assumptions are used in the economic order quantity (EOQ) model.

i. Deterministic, constant and continuous demand. The demand rate is assumed to be known with certainty.

ii. Lead time is known and constant and it is independent of demand.

iii. Only a single item is involved in the model and it does not interact with any other products in the inventory(there is no joint orders)

iv. All the model parameters are constant or unchanging over time.

v. There is an infinite time horizon. This is a policy that will be continuously implemented.

vi. All demand is satisfied on time

The ideal situation of the (EOQ) model is illustrated in fuqure.1, where the negative sloping lines represent the constant demand rate D and T is the time between order arrivals or cycle of length. L is the lead time which indicate how long it takes for the products to arrive

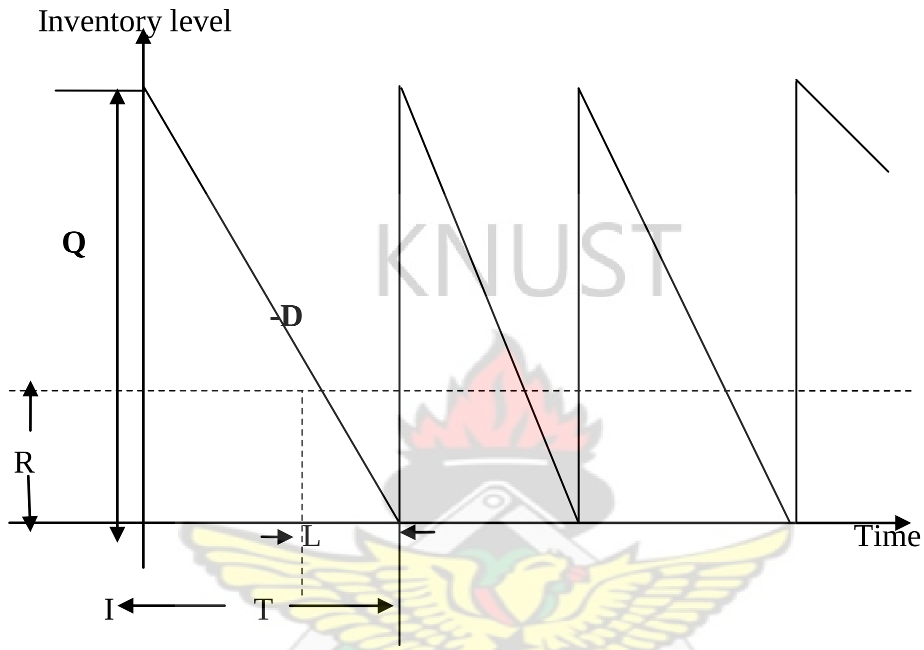

Fig.3.1 Classical Inventory Model (Tersine 1994)

The reorder point R is determined by calculating the demand that will occur during the lead time period, **L**, the inventory system operates as follows; when an order is received, the inventory level is Q units. Thus, the maximum inventory level is Q units at a constant demand rate D; which is indicated by the negative sloping line. Once the inventory level reaches the reorder point R, a new order is placed for Q units. The entire order is received and placed into inventory after a certain time period L. Once the inventory is depleted, a new order of Q units is received.

3.1.1 Model Formulation

Since the inventory level varies between a minimum of zero and a maximum of Q units, the average inventory (AI) in a cycle is given by

$$A1 = \frac{(Q+O)}{2} = \frac{Q}{2} \qquad (3.01)$$

The total annual inventory cost is the sum of the holding cost, the purchase cost and the order cost. The annual purchase cost is the annual cost that the buyer pays for the item. This is the purchase cost per unit (C) times the demand rate (D), times T. the order cost during each cycle is the cost incurred for placing an order say (K). The holding cost during each cycle is the holding cost per unit time say (H) times T, times the average inventory $\frac{Q}{2}$

Therefore, the sum of the three cost components (purchase, order and holding) is the total inventory cost during each cycle.

Total cost = purchase cost + order cost + holding cost

$$TC(Q) = CQ + K + HT\frac{Q}{2} \qquad (3.02)$$

Moreover, the total cost per unit time TUC is equal to $\frac{TC(Q)}{T}$

Or

$$TCU(Q) = \frac{CQ}{T} + \frac{K}{T} + \frac{HT}{T}\frac{Q}{2} \qquad (3.03)$$

But, N= $\frac{1}{T}$ is the number of order per unit time and D = $\frac{Q}{T}$ so we have

$$TCU(Q) = CD + \frac{KD}{Q} + \frac{HQ}{2} \qquad (3.04)$$

The necessary condition for having a minimum TCU (Q) is taking the first derivative of the total inventory cost with respect to q and setting it equal to zero.

$$\frac{dTCU(Q)}{dQ} = 0 \qquad (3.05)$$

$$\frac{dTCU(Q)}{dQ} = 0 = -\frac{KD}{Q^2} + \frac{H}{2} = 0 \tag{3.06}$$

Which after solving for Q, yields the (EOQ)

$$\frac{KD}{Q^2} = \frac{H}{2} \tag{3.07}$$

$$Q^2 = \frac{2KD}{H} \tag{3.08}$$

$$Q^* = \sqrt{\frac{2KD}{H}} \tag{3.09}$$

Noting that $\frac{d^2TCU(Q)}{dQ^2} = \frac{2KD}{Q^3} > 0, \forall \; Q > 0$

We conclude that the value Q* is the unique global minimum of TCU (Q).

To determine the reorder point, we recall that the demand is constant at a rate of D units.

Therefore, the total demand during lead-time of L time units, where L< T* is simply given by

R= DL

Hence, if an order is placed when the inventory level is R= DL, the order will arrive precisely when the inventory is depleted.

Note that if the lead-time L = 0, then this corresponds to an instantaneous delivery.

The minimum total cost per unit time is obtained by substituting Q* for Q in the total inventory cost equation that is

$$TCU(Q^*) = CD + \frac{HQ^*}{2} + \frac{KD}{Q^*} \quad (3.10)$$

Where Q^* is given by $Q^* = \sqrt{\frac{2KD}{H}}$

$$TCU\left(Q^*\right) = CD + \frac{2KD}{Q^*} \quad (3.11)$$

$$TCU\left(Q^*\right) = CD + 2K\frac{D}{\sqrt{\frac{2KD}{H}}} \quad (3.12)$$

$$TCU(Q^*) = CD + \sqrt{2KDH} \quad (3.13$$

Illustrative Example of the Economic order Quantity

Determine optimal number of needles to order if
D = 1,000 *units*
K = $10 *per order*
H = $.50 *per unit per year*

$Q^* = \sqrt{\frac{2KD}{H}}$

$Q^* = \sqrt{\frac{2*10*1000}{0.50}}$

$Q^* = \sqrt{40{,}000}$

Q*=200 units

Expected number of orders(N) = $\frac{Demand}{ordered\ quantity}$

$= \frac{D}{Q*}$

$= \frac{1{,}000}{200}$

N = 5 orders per year

$$T = \frac{number\ of\ working\ days\ per\ year}{N}$$

$$T = \frac{250}{5} = 50\ days\ between\ orders$$

The total annual cost

$$TCU(Q) = CD + \frac{KD}{Q} + \frac{HQ}{2}$$
$$= 1 * 1000 + \frac{10*1000}{200} + \frac{0.50*200}{2}$$
$$= 1000 + 50 + 50$$
$$= \$1100$$
$$\approx Gh₵3080$$

3.3 Economic Order Quantity (EOQ) with Backordering (Shortages)

We will relax one of the assumptions we have made about satisfying all demand on time. We will now allow some of the demand to be backordered but there will be a cost penalty incurred. The rest of the modeling assumptions remain unchanged. As a result, the cost function now consists of four components, that is, the purchasing cost, the fixed order cost, the inventory holding cost and the backlog penalty cost or shortage cost. If an item required by a customer is not currently available on the shelves of the organization, then the customer either goes to another place (a lost customer) or alternatively, places a backorder for the item. Some organizations are either sole supplier, providing a competitive price or offering a discount for delaying the delivery of certain item. If this is the case an organization does not lose the sale when its inventory is depleted. Instead the customer has to wait for his order to be filled whenever a new order arrives. Therefore, backordering or shortages are the demand that will be filled some time later than desired.

Figure 3.3 depicts a typical inventory model in which shortages are allowed to occur at constant demand rate D during time t_2

A maximum shortage level of X unit is assumed. Once the inventory level reaches the reorder point B, an order is place for Q units. When the lot size or the order is received, the maximum shortage X units are filled immediately, and the rest of the lost size is place into inventory. Thus inventory level is $(Q-X)$ unit

Inventory level

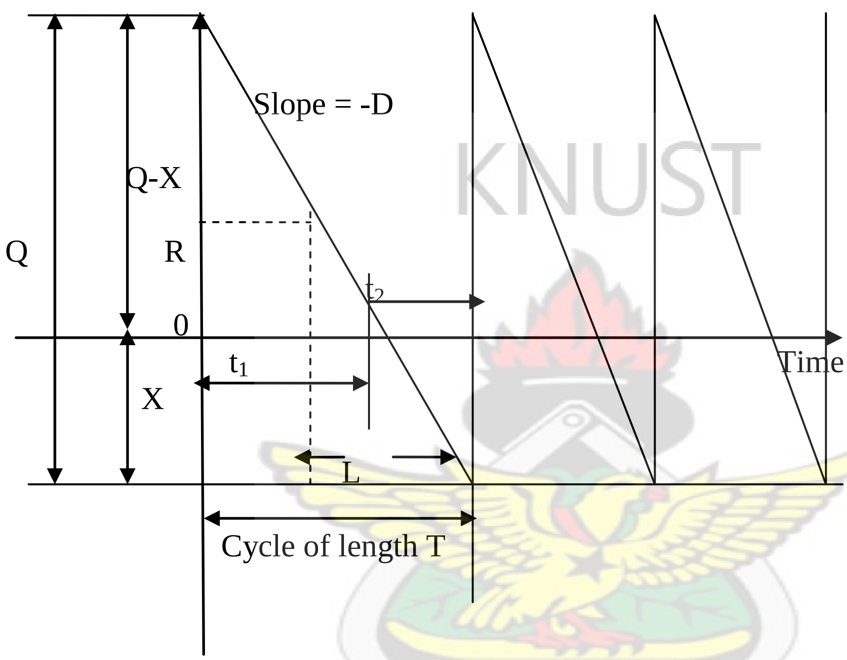

Figure 3.3 A typical inventory variation of an (EOQ) model with shortage.

There are two decisions to be made

(i) How much to order whenever an order is placed

(ii) How large the maximum backlog level should be in each case**.**

3.3.1 Model formulation

There is a positive inventory of $(Q-x)$ units occurs during time $t_{1,}$ so the average inventory (AI) is given by

$$AI = \frac{(Q-x)}{2} \tag{3.14}$$

By keeping the notations of the previous model then the holding cost during time t_1 is given by

$$\text{Holding cost } = \frac{H(Q-x)t_1}{2} \tag{3.15}$$

Noting that $t_1 = \dfrac{(Q-x)}{D}$, then, the holding cost during time t_1 is given

$$\text{Holding cost } = \frac{H(Q-x)^2}{2} \tag{3.16}$$

Shortage or negative inventory occurs during time t_2, so the average shortage inventory (AS) during time t_2 is given by

$$AS = \frac{x}{2} \tag{3.17}$$

Noting that $t_2 = \dfrac{x}{D}$ thus, the shortage cost during time t_2 is given by

$$\text{shortage cost } = \frac{Sxt_2}{2} = \frac{Sx^2}{2D} \tag{3.18}$$

Where S is the shortage cost per unit per unit time.

The total inventory cost for each cycle of length T, ($T = t_1+t_2$) is given by

Total cost =purchase cost + order cost +holding cost +shortage (backorder) cost.

$$TC(Q,X) = CQ + K + \frac{H(Q-X)^2}{2D} + \frac{SX^2}{2D} \tag{3.19}$$

Recalling that N= D/Q =1/T, Then, the per unit time total cost is

$$TC(Q,X) = CD + \frac{KD}{Q} + \frac{H(Q-X)^2}{2Q} + \frac{SX^2}{2Q} \tag{3.20}$$

Before obtaining the optimal solution, it is anticipated what properties to expect the optimal solution to possess. As before, if the fixed order cost K increase, fever order will be placed, which will increase the order quantity. An increase in the holding cost rate should drive the order quantity to lower values. The effect of the backorder cost on the maximum possible number of units on the backordered should be as follows the higher the backorder cost the lower the maximum desirable number of backorders.

To obtain the optimal solution, we take the first partial derivatives of TCU (Q, X) above with respect to q and x set them equal to zero

that is

$$\frac{\partial TCU(Q,X)}{\partial Q}=0 \text{ and } \frac{\partial TCU\left(Q,X\right)}{\partial X}=0$$

This yields two simultaneous equations in Q and X

$$\frac{\partial TCU(Q,X)}{\partial Q}=\frac{KD}{Q^2}+\frac{2H(Q-X)2Q-2H(Q-X)^2}{4Q^2}-\frac{2SX^2}{4Q^2}=0 \quad (3.21)$$

$$\frac{\partial TCU\left(Q,X\right)}{\partial Q}=\frac{KD}{Q^2}+\frac{H\left(Q-X\right)}{Q}-\frac{H\left(Q-X\right)^2}{2Q^2}-\frac{SX^2}{2Q^2}=0 \quad (3.22)$$

$$\frac{\partial TCU\left(Q,X\right)}{\partial Q}=\frac{KD}{Q^2}+H-\frac{H}{Q}+\frac{HX^2}{2Q^2}-\frac{SX^2}{2Q^2}=0 \quad (3.23)$$

Which after solving for Q, gives

$$\frac{-2Q2KD}{Q^2}+2Q^2H-\frac{2Q^2H}{2}-\frac{HX^2}{2Q^2}.2Q^2-\frac{SX^2}{2Q^2}.2Q^2=0 \quad (3.24)$$

$$-2KD + 2Q^2H - Q^2H - HX^2 - SX^2 = 0 \quad (3.25)$$

$$Q^2 = \frac{2KD}{H} + (H+S)X^2 \quad (3.26)$$

$$Q = X\sqrt{\frac{H+S}{H}} + \sqrt{\frac{2KD}{H}} \quad (3.27)$$

Now,

$$\frac{\partial TCU(Q,X)}{\partial X} = -\frac{4QH(Q-X)}{4Q^2} + \frac{4QSX}{4Q^2} = 0 \quad (3.28)$$

$$= -\frac{4Q^2H + 4QHX}{4Q^2} + \frac{SX}{Q} = 0 \quad (3.29)$$

$$= -H + \frac{HX}{Q} + \frac{SX}{Q} \quad (3.30)$$

$$= -\frac{HQ + HX}{Q} + \frac{SX}{Q} \quad (3.31)$$

$$= -\frac{H(Q-X)}{Q} + \frac{SX}{Q} = 0 \quad (3.32)$$

Which, after solving for X, gives

$$X = \frac{QH}{H+S} \quad (3.33)$$

Substituting (3.33) into (3.27) yields

$$Q^* = \frac{QH}{H+S}\sqrt{\frac{H+S}{H}} + \sqrt{\frac{2KD}{H}} \quad (3.34)$$

$$Q^* = \sqrt{\frac{H+S}{S}}\sqrt{\frac{2KD}{H}} \tag{3.35}$$

Also substituting (3.35) into (3.33) yields,

$$X^* = Q^* \frac{H}{H+S} = \sqrt{\frac{2KDS}{(H+S)H}} \tag{3.36}$$

$$T^* = \frac{Q^*}{D} = \sqrt{\frac{2K\left(H+S\right)}{DHS}} \tag{3.37}$$

Noting that

$$A = \frac{\partial^2 TCU\left(Q,X\right)}{\partial Q^2} = \frac{2KD}{Q^3} + \frac{2HX^2}{Q^3} + \frac{2SX^2}{Q^3} > 0, \forall Q > 0, \forall X > 0, \tag{3.38}$$

$$L = \frac{\partial TCU\left(Q,X\right)}{\partial X^2} = \frac{H}{Q} + \frac{S}{Q^2} > 0, \forall Q > 0 \tag{3.39}$$

$$E = \frac{\partial TCU\left(Q,X\right)}{\partial Q \partial X} = -\frac{X\left(H+S\right)}{Q^2} < 0, \forall Q > 0, X > 0, \text{ and} \tag{3.40}$$

$$E^2 = \frac{X^2\left(H+S\right)^2}{Q^4} > 0, \forall Q > 0, \forall > 0 \tag{3.41}$$

Then the Hessian matrix is given by

$$H(Q,X)=\begin{pmatrix} A & E \\ E & L \end{pmatrix} \tag{3.42}$$

The principal minors of order 1 and 2 are $h_{11}=A>0,$ and it is easy to show that the determinant of H (Q, X) $=h_{22}=A*L-E^2>0$. Hence, $H(Q,X)$ is positive definite $\Rightarrow TCU(Q,X)$ is strictly convex, we conclude that the pair (Q^*,X^*) forms the unique global minimum of $TCU(Q,X)$.

Now, the minimum per unit time total cost is

$$TCU(Q^*,X^*)=CD+\frac{KD}{Q^*}+\frac{H(Q^*-X^*)^2}{2Q^*}+\frac{S(X^*)^2}{2Q^*} \tag{3.43}$$

But $Q^*-X^*=\dfrac{Q^*S}{H+S}$. Substituting into (3.43) we obtain

$$TCU(Q^*,X^*)=CD+\frac{KD}{Q^*}+\frac{H\left(\dfrac{Q^*S}{H+S}\right)^2}{2Q^*}+\frac{S(X^*)^2}{2Q^*} \tag{3.44}$$

This can be reduced to

$$TCU(Q,X)=CD+\frac{Q^*SH}{H+S} \tag{3.45}$$

To determine the reorder point, we note that, the demand is of constant rate of D units. Also, note that when the lot size or the order is received, the maximum shortage of X^* units is filled immediately. Therefore, the total demand during lead-time of L time units, where $L<T^*$ is given by $R=DL-X^*$

Hence, if an order is placed when the inventory level is $R = DL - X^*$, the order will arrive precisely when the inventory reaches the maximum shortage level of X^* units.

Note that if the lead-time L = 0, then this corresponds to an instantaneous delivery. In this case, the order is placed when a maximum shortage level of X^* units is reached; where the then placed order is received instantaneously.

Illustrative Example of the Economic order Quantity

From the table below, calculate the quantity of pencils and the total cost using the EOQ with Backordering(Shortages);

Annual demand (D)	12000 units	Amount in GhȻ
Production cost (C)	\$12	33.6
Holding cost (H)	\$0.50	1.4
Set up Cost (K)	\$10	28
Shortage Cost (S)	\$2.2	6.16

$$Q^* = \sqrt{\frac{H+S}{S}}\sqrt{\frac{2KD}{H}}$$

$$Q^* = \sqrt{\frac{0.5+2.2}{2.2}} * \sqrt{\frac{2*10*12000}{0.5}}$$

$$Q^* = \sqrt{1.23} * \sqrt{480000}$$

Q*= 768.4 *or* 768 pencils

$$TCU(Q,X) = CD + \frac{Q^* SH}{H+S}$$

$$TCU(Q,X) = 12 * 12000 + \frac{768.37 * 2.2 * 0.5}{0.50+2.2}$$

= 144000+313.04

= \$144313.04

≈GhȻ404076.511

3.4 Quantity Discount Model.

In the models previously discussed, it was assumed that the unit purchasing cost is constant and independent of the quantity purchased.

However, it is a common practice that suppliers provide incentive to their customers for large ordering quantity by offering discounts. When offering discount to their customer, suppliers usually quote their prices in the form of discount schedules, where the list of reduced unit prices depend on the quantity purchased. This form of pricing is very common in business-to-business transactions. Quantity discounts are generally provided for.

(i) Increasing the sales of the product.

(ii) Reducing the in-hand inventory, by increasing sales

(iii) Better production planning

(iv) Lower order processing cost

(v) Reducing the transportation cost, by making use of the discount offered by the trucking industry.

A discount is lot size-based if the pricing schedule offers discounts based on the quantity ordered in a single lot. A discount is volume-based if the discount is based on the total quantity purchased over a given period (e.g. a year) regardless of the number of lots purchased over that period.

Although there are a wide variety of discount schedules, the two most commonly used discount schedules are:

(i) All- unit quantity discounts

(ii) Incremental quantity discounts.

For the purpose of this thesis, we look at the All- unit quantity discount.

3.4.1 All Unit Quantity Discounts.

The all-unit discount applies the reduced price to all units purchased; that is the purchase of larger lot size results in a lower price for all units. This types of discount is characterized by the breakpoints $1 = Q_0 < Q_1 < Q_2 < \ldots < Q_m = \infty$ such that if the lot size Q is within the i^{th} discount (price-break) interval, $Q_{i-1} \le Q < Q_i$, then the purchasing cost is $C_i\left(Q\right) = C_i Q$,where

$C_1 > C_2 \ldots > C_m$ and m is the number of discounts levels.

Therefore the cost of purchasing a lot size Q is given by $C\left(Q\right) = C_i Q$ if $Q_{i-1} \le Q < Q_{i,i=1,2\ldots m}$

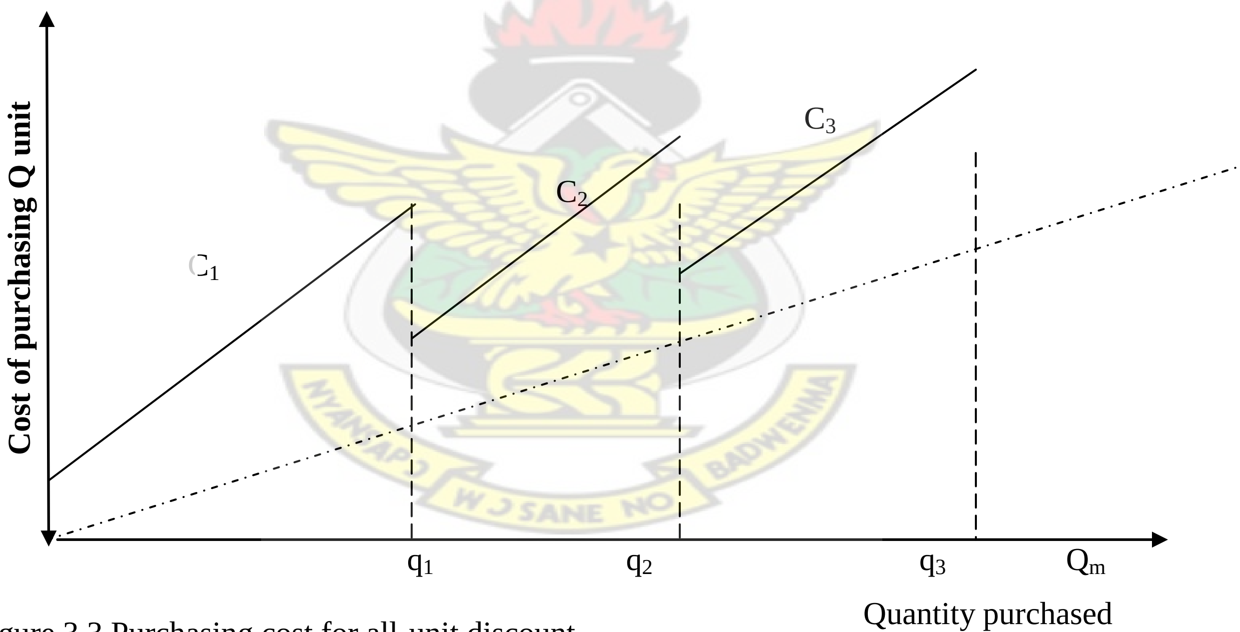

Figure 3.3 Purchasing cost for all-unit discount

3.4.2 All Units Discount Model with no shortages.

For the i^{th} discount level, the total system cost per units of time

$$TCU_i\left(Q\right) = C_i D + \frac{KD}{Q} + \frac{H_i}{2} Q$$

Where

$$= Q_{i-1} \leq Q \leq Q_i$$

Hence

$$Q^* = \sqrt{\frac{2KD}{H_i}}$$

3.5 Lot sizing when constraints Exist.

In the earlier portions of chapter 3, we focused on determining the optimal ordering policy for a single item. In many if not most real –world situations, decisions are not made for each item independently. There may be limitations on space to store items in warehouses; there may be constraints on the number of orders that can be received per year; there may be monetary limitation on the value of inventories that are stocked.

Each of these situations requires stocking decisions to be made jointly among the many items managed at a location. Holding costs are often set to limit the amount of space or investment consumed as a result of the lot sizing decisions. Rather than assuming a holding cost rate is used to calculate the lot sizes, suppose a constraint is placed on the average amount of money invested in inventory. Thus a budget constraint is imposed that limits investment across items.

Let, Q_i be the procurement lot size for item i

C_i the per-unit purchasing cost for item i, $i = 1...n$ where n is the number of items being managed.

The sum measured the average amount invested in inventory over times

$$\sum_{i=1}^{n} c_i \frac{Q_i}{2}$$

Let b be the maximum amount that can be invested in inventory on average. Furthermore, suppose our goal is to minimize the average annual total fixed procurement cost over all item types while adhering to the budget constraint. Let D_i and K_i represent the average annual demand rate and fixed order cost for item i respectively.

$D_i K_i / Q_i$ measures the average annual fixed order cost incurred for item i given Q_i is the lot size for item i.

Define $F_i = D_i K_i / Q_i$

This procurement problem can be stated as follows:

Minimize

$$\sum_{i=1}^{n} \frac{D_i K_i}{Q_i} = \sum_{i=1}^{n} F_i\left(Q_i\right)$$

$$\sum_{i=1}^{n} C_i \frac{Q_i}{2} \le b$$

$$Q_{i\ge 0}$$

3.6 Economic Production Quantity (EPQ).

If the items are to be manufactured internally, then the problem of inventory in the manufacturing system increases in magnitude and complexity. The problem of inventory exists because production and consumption are difficult to manage, since in most cases production and consumption differ in the rates. So, either they provide or require stock.

Thus, in manufacturing system especially in batch-type production systems, where units are often produced and added to inventory in lot size (batches), it is required to determine the optimum number of units to be produced in each production run (each inventory cycle) so as to minimize total inventory cost. The economic production quantity formulation assumes gradual additions to stock over the production time. With this assumption, the inventory level is always less than the lot size, since production and consumptions occur simultaneously during the production time. However, the production cannot continue forever.

Rather there are in general two stages, production and consumption stage, and pure consumption stage. The goal of inventory management is to minimize the total inventory cost and to satisfy the decision making objectives. Having too much inventory, though it reduces set up costs, it may tie up capital, which many lead to unnecessary holding cost and possibility of deteriorating items. On the other hand too little inventory even if it reduces the holding cost can result in lost customers or interrupted production lot size for any organization which minimizes its total inventory cost.

In the EPQ model C is redefined as the production cost and K as the setup cost and all other notations, which have been used in the above EOQ models are kept with the addition that $P(P > D)$ is the production rate (which is equal to the number of units produced per unit time).

The system operates as follow. It starts at time t=0 at a demand rate D up to time $t=t_1$ to allow shortages of X units to occur. Then production starts where the inventory level increases at a rate(P-D) in order to satisfy the demand and to eliminate the entire shortage of X units up to the inventory level at time t = t_{1+} t_2 . At this time, the inventory level starts to go up with a rate (P-D) until time $t_1+ t_2+ t_3$ where production ceases and the inventory level reaches its maximum. Then the inventory level declines continuously at a demand rate D and becomes zero at time $t_1+ t_2+ t_{3+} t_4$ = (the end of the cycles).

Now the cost's components consist of shortage, holding, setup and production costs.

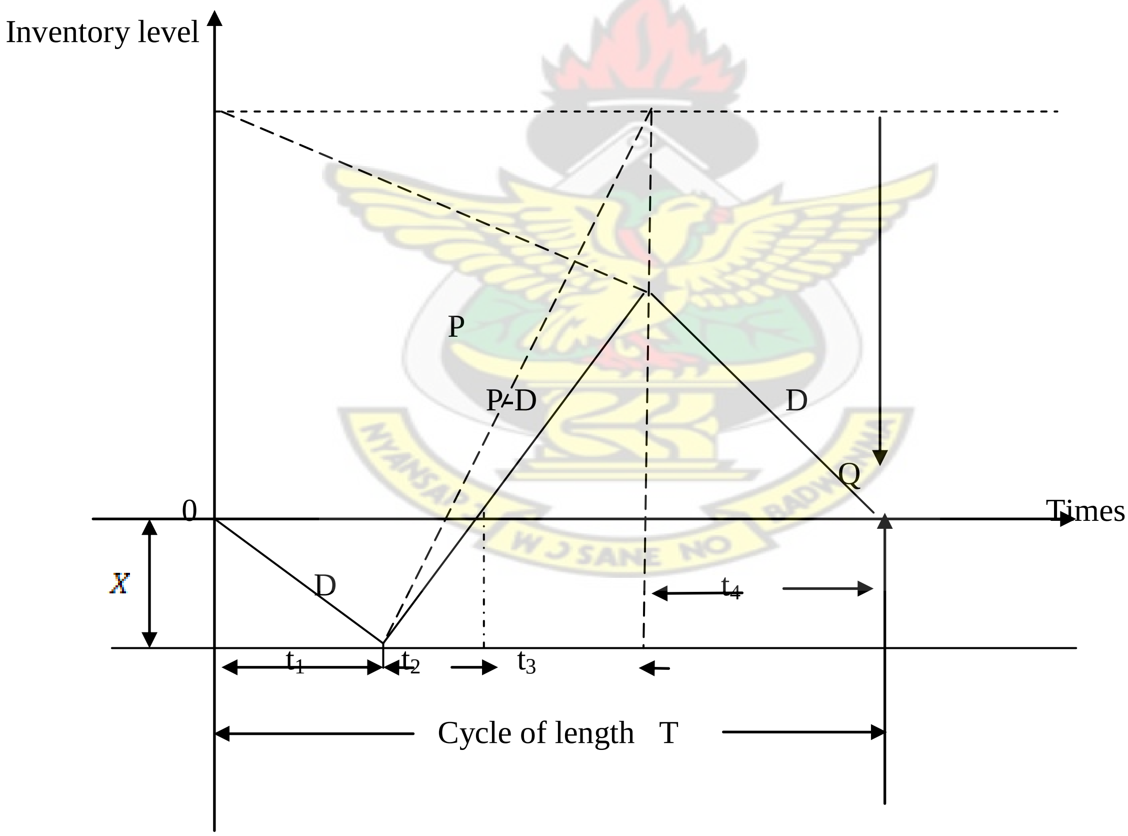

Figure 3.4; A typical inventory variation of an EPQ model with shortages.

3.6.1 Model

From figure 3.4 shortages or negative inventory occurs during time interval $t_1 + t_2$. The average shortage inventory (AS) during t_1 is given by

$$AS = \frac{(0+X)}{2} = \frac{X}{2} \tag{3.48}$$

But $t_1 = \frac{X}{D}$, thus the shortage cost during time interval t_1 is given by

$$\text{Shortage cost } = \frac{SXt_1}{2} = \frac{SX^2}{2D} \tag{3.49}$$

Note that $Pt_2 = Dt_1 + Dt_2$

$$t_2 = \frac{X}{P-D} \tag{3.50}$$

Thus, the shortage cost during time interval t_2 is given by

$$\text{Shortage cost } = \frac{SXt_2}{2} = \frac{SX^2}{2(P-D)} \tag{3.51}$$

Where S is the shortage cost per unit quantity per unit time.

Hence, the shortage cost during time interval $t_1 + t_2$ is given by

$$\text{Shortage cost } = \frac{SX^2}{2}\left[\frac{1}{D} + \frac{1}{P-D}\right] = \frac{SX^2}{2}\left[\frac{P-D+D}{D(P-D)}\right] = \frac{SX^2 P}{2D(P-D)} \tag{3.52}$$

We also note that positive inventory occurs during time interval $t_3 + t_4$ or during time interval T-($t_1 + t_2$), where $T = t_1 + t_2 + t_3 + t_4$. Since the average inventory (AI) during time interval $t_3 + t_4$ varies between a minimum and zero and a maximum of (P-D) t_3 units, the average inventory (AI) during time $t_3 + t_4$ is given by $AI = \frac{(P-D)t_2}{2}$ (3.53)

But the production phase occurs during time interval $t_2 + t_3 = \frac{Q}{P} \Rightarrow t_3 = \frac{Q}{P} - t_2$

Where Q is the (EPQ) recalling, that $t_2 = \dfrac{X}{P-D}$, then $t_3 = \dfrac{Q}{P} - \dfrac{X}{P-D} \Rightarrow t3 = \dfrac{Q(P-D)-XP}{P(P-D)}$

but $T = \dfrac{Q}{D}$. In addition, the time interval t_1+t_2 is equal to $\dfrac{XP}{D(P-D)}$ therefore $= \dfrac{Q(P-D)-XP}{D(P-D)}$

Thus, the holding cost during T - $(t_1 + t_2)$ is given by

$$\text{Holding cost} = \frac{H(P-D)t_3\left[T-(t_1+t_2)\ \right]}{2} \tag{3.54}$$

$$\frac{H(P-D)}{}\left\{\frac{Q(P-D)-XP}{2P(P-D)}\ \right\}\left\{\frac{Q(P-D)-XP}{D(P-D)}\right. = \frac{H\left[Q(P-D)-XP\ \right]^2}{2PD(P-D)} \tag{3.55}$$

The total inventory cost for a cycle of length T, is given by

Total cost = production cost + setup cost + Holding cost + shortage cost

$$TC(Q,X) = CQ + K\,\frac{H\left[Q(P-D)-XP\ \right]^2}{2PD(P-D)} + \frac{SX^2P}{2D(P-D)} \tag{3.56}$$

Recalling that $N = \dfrac{1}{T} = \dfrac{D}{Q}$, the per unit time total relevant cost is given by

$$TCU(Q,X) = CQ + \frac{KD}{Q} + \frac{H\left[Q(P-D)-XP\ \right]^2}{2QP(P-D)} + \frac{SX^2P}{2Q(P-D)} \tag{3.57}$$

To obtain the optimal solution, we take the first partial derivation of $TCU\left(Q,X\right)$ above with respect to Q and X and set them to zero.

$$\text{That is}\quad \frac{\partial TCU(Q,X)}{\partial Q} = 0, \frac{\partial TCU(Q,X)}{\partial X} = 0 \tag{3.58}$$

Now, $\dfrac{\partial TCU(Q,X)}{\partial Q} = \dfrac{-KD}{Q^2} + \dfrac{2H\left[Q(P-D)-XP\ \right]^2 QP(P-D)^2}{4Q^2P^2(P-D)^2}\ \dfrac{-H\left[Q(P-D)-XP\ \right]^2 2P(P-D)}{4Q^2P^2(P-D)^2}\ \dfrac{-SX^2P}{2Q^2(P-D)} = 0$

$$\frac{\partial TCU(Q,X)}{\partial Q} = \frac{-KD}{Q^2} + \frac{H(P-D)}{2p} - \frac{XP^2}{2Q^2(P-D)}(H+S) = 0 \qquad (3.59)$$

Which after solving for Q, gives

$$Q^2 = \frac{X^2P^2(H+S)}{H(P-D)^2} + \frac{2KD}{H(P-D)} \qquad (3.60)$$

$$Q = \sqrt{\frac{(H+S)}{H}}\frac{XP}{P-D} + \sqrt{\frac{2KDP}{H(P-D)}} \qquad (3.61)$$

Now,

$$\frac{\partial TCU(Q,X)}{\partial X} = -H + \frac{HXP}{Q(P-D)} + \frac{SXP}{Q(P-D)} = 0 \qquad (3.62)$$

Which after solving for X , gives $X = \dfrac{QH(P-D)}{P(H+S)}$ (3.63)

Substituting X into

$$Q = \sqrt{\frac{(H+S)}{H}}\frac{XP}{P-D} + \sqrt{\frac{2KDP}{H(P-D)}} \qquad (3.64)$$

we obtain $Q^* = \dfrac{\sqrt{HP}}{\sqrt{H+S}} + \sqrt{\dfrac{2KDP}{H(P-D)}}$ (3.65)

or $Q^* = \dfrac{\sqrt{H+S}}{\sqrt{S}} + \sqrt{\dfrac{2KDP}{H(P-D)}}$ (3.66)

Noting that $L = \dfrac{\partial TCU(Q,X)}{\partial X^2} = \dfrac{P(H+S)}{Q(P-D)} > 0, \forall Q > 0$ (3.67)

$$E = \frac{\partial TCU(Q,X)}{\partial Q \partial X} = -\frac{XP(H+S)}{Q^2(P-D)} < 0, \forall Q > 0, \forall X > 0 \qquad (3.68)$$

and $E^2 = \frac{2XP^2\left(H+S\right)^2}{Q^4\left(P-D\right)^2} > 0, \forall > 0, \forall X > 0$ (3.69)

Then the hessian matrix is given by $H\left(Q,X\right)=\begin{pmatrix} A & E \\ E & L \end{pmatrix}$

$h_{11} = A > 0$ and that $h_{22} = AL - E^2 > 0$

Hence, $H\left(Q,X\right)$ is positive definite.

$\Rightarrow TCU\left(Q,X\right)$ is strictly convex and we conclude that the pair $\left(Q^*,X^*\right)$ forms the unique global minimum of $TCU\left(Q,X\right)$

The minimum total unit cost per unit time is

$$TCU\left(Q^*,X^*\right)=CD+\frac{KD}{Q^*}+\frac{H\left[Q^*\left(P-D\right)-XP\right]^2}{2Q^*P\left(P-D\right)}+\frac{SX^{*2}P}{2Q\left(P-D\right)} \quad (3.70)$$

But $Q^*\left(P-D\right)-XP=\frac{Q^*S\left(P-D\right)}{\left(H+S\right)}$ (3.71)

Then $TCU\left(Q^*,X^*\right)=CD+\frac{Q^*SH\left(P-D\right)}{P\left(H+S\right)}$ (3.72)

Illustrative Example of the Economic Production Quantity

From the table below, calculate the quantity of hubcaps and the total cost of production

Annual demand (D)	1000 units	Amount in GhȻ
Production rate (P)	8000 units	
Production cost (C)	\$10	28
Holding cost (H)	\$0.50	1.4
Set up Cost (K)	\$10	672
Shortage Cost (S)	\$2	5.6

$$Q^* = \sqrt{\frac{H+S}{S}} + \sqrt{\frac{2KDP}{H(P-D)}}$$

$$TCU = 10 * 1000 + \frac{214.93 * 0.50 * 2(8000 - 1000)}{8000(0.50+2)}$$

Q*= $\sqrt{1.25} + \sqrt{45714.2857}$

Q*= 214.93 *or* 215 hubcaps

$$TCU\left(Q^*, X^*\right) = CD + \frac{Q^* HS(P-D)}{P(H+S)}$$

$$TCU = 10 * 1000 + \frac{214.93 * 0.50 * 2(8000 - 1000)}{8000(0.50+2)}$$

= 10000+752255

= \$10075.2255

≈GhȻ28210.63

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 Introduction

In this Chapter, we analyze and discuss sample data of Air-Mate Gas Factory Ghana.

We use Economic Order Quantity models to determine the optimal quantity of gases that should be ordered and when to place the order. For the purpose of the study, we will consider four gases imported by Air-Mate Gas Factory Ghana, namely, Ammonia gas, Argon gas, Nitrous Oxide and Atal gas.

4.1 Types of Data used and their collection.

In this study, secondary data for 2011 from Air-Mate Gas Factory Ghana was collected.

The data was collected from the Accounts Department and the supply chain Department. The data collected from the company includes; the holding cost, ordering cost, unit purchase price and the annual demand of the four imported gases.

4.1.1 Display of Data for Order Quantities.

Table 4.1 is the Quarterly demand of four different gases imported by Air-Mate Gas Factory Ghana for one year.

It shows the unit and the capacity of each gas. Argon and Atal gas are measured in cubic meters whilst Nitrous oxide and Ammonia gases are measured in kilograms.

Table 4.1 History of the Four imported Gases in Quarters in 2011

	Gas Products			
	Ammonia gas	Argon gas	Nitrous oxide gas	Atal gas
2011 Year	kg	m^3	kg	m^3
First Quarter	12000	1000	3000	550
Second Quarter	6000	13000	2500	500
Third Quarter	10000	4000	1500	350
Fourth Quarter	4400	9000	2000	600

Table 4.2 summarizes the annual demand of four different imported gases. It also shows the parameters used in inventory planning. The unit of the inventory parameter is in dollar. This is the unit at which all the gases are imported.

Table 4.2 Annual Demand and parameters on the imported gases for 2011

Product Line	Annual Demand (D)	Fix order Cost(K)	Unit cost (C)	Holding rate(I) %	Unit Holding Cost H =(C *I)	Lead Time(L) days	Number Of Times ordered (N)
Ammonia	32000kg	$320	$1.00	0.25	$ 0.25	40	5
Argon	42000m3	$407	$1.08	0.25	$ 0.27	40	5
Nitrous oxide	9000kg	$250	$2.19	0.25	$ 0.55	40	5
Atal	2000m3	$20	$1.60	0.25	$ 0.4	40	5

Table 4.3 shows the fix order cost, unit cost and holding cost been converted from dollar to the Ghana cedi equivalent. ($1≈Gh₵2.8)

Table 4.3 Currency($) in Ghana cedi equivalent

Product Line	Fix Order Cost (K) in Gh₵	Unit Cost (C)) in Gh₵	Unit holding cost (H) in Gh₵
Ammonia	896	2.8	0.7
Argon	1139.6	3.024	0.756
Nitrous Oxide	700	6.132	1.54
Atal	56	4.48	1.12

4.2 Model of EOQ problem yields the following as captured in chapter 3 of the thesis

$$TCU(Q) = CD + \frac{KD}{Q} + \frac{HQ}{2} \qquad (4.21)$$

$$\frac{dTCU(Q)}{dQ} = 0 \qquad (4.22)$$

$$Q^* = \sqrt{\frac{2KD}{H}} \qquad (4.23)$$

Noting that $\frac{d^2TCU(Q)}{dQ^2} = \frac{2KD}{Q^3} > 0, \forall \; Q > 0$

$$TCU(Q^*) = CD + \sqrt{2KDH} \qquad (4.24)$$

$$T^* = \frac{Q^*}{D} \qquad (4.25)$$

$$R = D * L \qquad (4.26)$$

Computation of Ammonia as an illustrative example

Economic order quantity of Ammonia

$Q^* = \sqrt{\frac{2KD}{H}}$

Q*= $\sqrt{\frac{2*320*32000}{0.25}}$

Q*= $\sqrt{\frac{20480000}{0.25}}$

Q*= $\sqrt{81920000}$

Q*=9051kg

Total cost after EOQ of Ammonia

$TCU(Q) = CD + \frac{KD}{Q} + \frac{HQ}{2}$

TCU(Q)= 1*32000+ $\frac{320*32000}{9051} + \frac{0.25*9051}{2}$

= 32000+1131.3667+1131.375

TCU(Q)= $34262.74
TCU(Q)≈Gh₵95935.68

Time cycle

$$T^{*} = \frac{Q^{*}}{D}$$

T*=$\frac{9051}{32000}$

T*=0.28284

T*≈0.3

Reorder point of Ammonia

R = D * L

R = 32000*40

R = 1280000

There is 250 working days in the year

R = $\frac{128000}{250}$

R = 5120kg

Number of order after EOQ of Ammonia

N= $\frac{D}{Q*}$

N = $\frac{32000}{9051}$

N = 3.5355

N ≈ 4

4.3 Results

The data was provided by the company to calculate for the results obtained in table 4.4. In the data gathered, an EOQ is used to minimize stock outs and find the optimal order quantity while minimize total cost associated with each gas. The results in table 4.4 also indicate the total cost, the time to place order, the reorder point and the number of orders to be made.

Table 4.4: Results after applying the EOQ model

Name of Product	Q*	TCU	T*=(Q*/D) in years	R=(D*L/250)	Number of order after EOQ(D/Q*)
Ammonia	9051kg	34262.74	0.28	5120kg	4
Argon	11252m^3	48398.22	0.27	6720 m^3	4
Nitrous oxide	2860kg	21283.20	0.34	1440kg	3
Atal	447 m^3	3378.89	0.22	72 m^3	4

The EOQ in table 4.4 indicates that in order for the holding cost and the ordering cost to be equal, the amount in table 4.4 should be ordered every time an order is placed to minimize the inventory cost.

Cost Comparison

Table 4.5 is the variance of the operational cost of the various imported gas before and after applying the Economic Order Quantity model. The values in Table 4.5 is from Appendix B

Table 4.5: Comparison of cost before and after applying the EOQ model

Name of Product	Total cost before EOQ model	Total cost after the EOQ model	Difference between TC(Q) and TC(EOQ)$
Ammonia	34400	34262.74	137.26
Argon	48529	48398.22	130.78
Nitrous oxide	21455	21283.20	171.78
Atal	3380	3378.89	1.12
Total	107764	107323.06	440.94

Total savings	$440.94	Gh₵1234.63
Percentage (%)	0.41	

From Table 4.5, it is observed that the total cost of an inventory before applying the EOQ model was higher than after applying the model. This means that if the company employed the EOQ model, it would reduce its annual total cost as shown in the table 4.5. The difference in operation costs could be attributed to ordering cost.

Orders

Table 4.6 is a comparison between the number of orders placed by the company before applying the EOQ model and after the application of the EOQ model.

Table 4.6 number of orders before and after applying the EOQ model

Name of Product	Number of orders before applying the EOQ model provided by the company	Number of order after applying the EOQ model (D/Q*)
Ammonia	5	4
Argon	5	4
Nitrous oxide	5	3
Atal	5	4

Table 4.6 above shows that the number of orders was much higher before applying the EOQ model than it was after applying it. This applies to all types of gases dealt with in this study. By having a large number of orders, the company increases ordering costs hence increasing the annual total cost of inventory.

The re-order point

The second aspect regarding the time to place new orders gives an answer to the minimum stock level at which additional quantities are ordered. After making calculations, the value of the re-order point was obtained and Table 4.7 summarizes the results.

Table 4.7: Reorder point

Name of Product	Re-order point (R)
Ammonia	5120kg
Argon	6720 m^3
Nitrous oxide	1440kg
Atal	72 m^3

The reorder point took into consideration the annual demand and the lead time. The reorder point states that an order needs to be placed once the product falls below a certain amount of unit as indicated in the table 4.7. Furthermore, the reorder point maintains enough stock to satisfy the demand between orders.

4.4 Economic Production Quantity of Nitrogen

Table 4.8 is annual demand, the production rate and the cost of production of Nitrogen gas locally by Air-Mate Gas Factory Ghana.

It shows the unit and the capacity of the gas. The demand and production rates are measured in cubic meters whilst cost of production is in dollars.

Table 4.8 Parameters of the Economic Production Quantity per year

Annual demand (D)	62500m^3	Amount in GhC̸
Production rate (P)	188400m^3	
Production cost (C)	\$10	28
Holding cost (H)	\$0.5	1.4
Set up Cost (K)	\$240	672
Shortage Cost (S)	\$2	5.6

The following equations were used to produce the results of the Economic Production Quantity as captured chapter 3 of this thesis

$$TC(Q,X)=CQ+K\,\frac{H\left[Q(P-D)-XP\ \right]^2}{2PD(P-D)}+\frac{SX^2P}{2D(P-D)} \tag{4.41}$$

Nothing, $\frac{\partial TCU(Q,X)}{\partial Q}=0,\frac{\partial TCU(Q,X)}{\partial X}=0$ (4.42)

$$Q^*=\frac{\sqrt{H+S}}{\sqrt{S}}+\sqrt{\frac{2KDP}{H\left(P-D\right)}} \tag{4.43}$$

$$TCU\left(Q^*,X^*\right)=CD+\frac{KD}{Q^*}+\frac{H\left[Q^*\left(P-D\right)-XP\right]^2}{2Q^*P\left(P-D\right)}+\frac{SX^{*2}P}{2Q\left(P-D\right)} \tag{4.44}$$

Then $TCU\left(Q^*, X^*\right) = CD + \dfrac{Q^* SH\left(P-D\right)}{P\left(H+S\right)}$ (4.45)

Substitute the date from table 4.8 into the equation (4.46)

$$Q^* = \sqrt{\frac{H+S}{S}} + \sqrt{\frac{2KDP}{H\left(P-D\right)}} \quad (4.46)$$

$$Q^* = \sqrt{\frac{0.5+2}{2}} + \sqrt{\frac{2*240*62500*188400}{0.5\left(188400-62500\right)}}$$

$$Q^* = \sqrt{\frac{2.5}{2}} + \sqrt{\frac{5.652*10^{12}}{0.5\left(125900\right)}}$$

$$Q^* = \sqrt{1.25} + \sqrt{89785544.08}$$

$$Q^* = 9477m^3$$

$$X^* = \frac{Q^* H\left(P-D\right)}{P\left(H+S\right)} = \frac{9477*0.5\left(188400-62500\right)}{188400\left(0.5+2\right)} \quad (4.83)$$

$$X^* = 1267m^3$$

$$T^* = \frac{Q^*}{D} \quad (4.84)$$

$$T^* = \frac{9477}{62500}$$

$T^* = 0.152$ =0.152 year

There is 250 working days in 2012

Therefore

T^* = 0.152 year x 250

T^* =37.9 days

Production time $= \frac{Q^*}{P} = \frac{9477m^3}{188400m^3 / year}$ (4.85)

$= 0.050$

$= 12.57$

Total cost of production

$$TCU\left(Q^*, X^*\right) = CD + \frac{Q^* HS\left(P - D\right)}{P\left(H + S\right)} \tag{4.86}$$

$$TCU\left(Q^*, X^*\right) = 10 * 62500 + \frac{9477 * 0.5 * 2\left(188400 - 62500\right)}{188400\left(0.5 + 2\right)}$$

$$TCU\left(Q^*, X^*\right) = 625000 + \frac{1193154300}{471000}$$

$$= 625000 + 2533.24$$

$$= \$627,533.24$$

4.4.1 Results of the Economic Production Quantity

Table 4.9 indicates the Economic Production Quantity models, which shows that an average of 9477m^3 of Nitrogen gas should be produced in every 38 days. Each production time will require 13days to complete.

Table 4.9: Results of the Economic Production Quantity of Nitrogen

Product	Q*	X*	T*	Production time(Q*/P)	TCU(Q*,X*)	TCU(Q*,X*) in Ghana cedis
Nitrogen	$9477m^3$	$1267m^3$	37.9 days	12.57 days	$627,533.24	GhȻ1757093.072

Thus, according to table 4.9, we should plan a production run of 9477m^3 of the Nitrogen about every 38 working days at cost of $627533.24 ≈ GhȻ1757093.072

4.4.2 Testing for Minimal Global Cost.

For a stationary point to be an extreme point, the matrix of second partial derivatives (Hessian matrix) of TCU (Q*, X*) evaluate at (Q*, X*) is

(i) Relative minimum if the Hessian is positive definite and

(ii) Relative maximum if the Hessian is negative definite.

Then the Hessian matrix is given by

$$H\left(Q^*, X^*\right) = \begin{pmatrix} A & E \\ E & L \end{pmatrix}$$

$$A = \frac{dTCU\left(Q, X\right)}{dQ^2} = \frac{2KD}{Q^3} + \frac{2PX^2\left(H+S\right)}{Q^3\left(P-D\right)}$$

$$L = \frac{dTCU(Q,X)}{dX^2} = \frac{P\,(H+S)}{Q\,(P-D)}$$

$$E = \frac{-XP(H+S)}{Q^2(P-D)}$$

The Hessian Matrix H will be positive definite if all its eigenvalves are positive. That is all values of λ that satisfy the equation

$$\left| H - \lambda I \right| = 0$$

$$H\left(Q^*, X^*\right) = \begin{vmatrix} 4.94*10^{-5} - \lambda & -5.28*10^{-5} \\ -5.28*10^{-5} & 3.95*10^{-5} - \lambda \end{vmatrix} = 0$$

$$\left(4.94*10^{-5} - \lambda\right)\left(3.95*10^{-5} - \lambda\right) - \left(-5.28*10^{-5}\right) = 0$$

Should be positive. I is the identity matrix.

$A = 4.94*10^{-5}$, $L = 3.95*10^{-5}$, $E = -5.28*10^{-5}$

$$\lambda_1 = 7.82*10^{-5}$$

$$\lambda_2 = 1.07*10^{-5}$$

The Eigen values are all positive, hence TCU (Q*, X*) is minimum and we conclude that the pair (Q*, X*) forms the unique global minimum of TCU (Q*, X*).

4.5 Discussion

Table 4.5 presents the Economic Order Quantity for each gas. The optimal order quantity was higher compared to their current quantity ordered with a difference of $440.94 (0.41%).

Table 4.6 shows the variance in operational cost. A comparison with their current inventory cost shows a decrease of 0.41%.

Table 4.7 presents the number of orders placed by Air-Mate Gas Factory Ghana before and after the application of the EOQ model. The result shows that the number of orders placed by Air-Mate Gas Factory Ghana before applying the EOQ model was much higher. The frequency of the orders contributed to high rate of increase in ordering cost and consequently increasing the annual total cost of inventory.

According to the Economic Production Quantity models, an average of $9477m^3$ of Nitrogen gas should be produced in every 38 days. Each production time will require 13days to complete.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

This thesis provides an Economic Order Quantity Model in which we reviewed the importance of inventory cost minimization with a view to increasing Air-Mate Gas Factory Ghana profitability.

If the Economic Order Quantity model is objectively used, with the aid of some judgment by the management of Air-Mate Gas Factory Ghana, the holding cost and the ordering cost will become low. This would enable the company to reduce their total cost by approximately 0.41% for the four selected imported gases.

The use of this model will help the company to know the optimal number of gases to order (thus, Ammonia-4, Argon-4, Nitrous xoide-3 and Atal-4) within a year and when to place new orders for each gas.

According to the Economic Production Quantity (EPQ), all the Eigen values were found to be positive, we then conclude that, the total cost of production of the Nitrogen gas forms the unique global minimum cost that will be incurred by Air-Mate Gas Factory Ghana.

5.2 Recommendations

1. We recommend that in order to manage inventory effectively, the management of Air-Mate Gas Factory Ghana needs to employ inventory control model such as the Economic Order Quantity (EOQ) model to obtain optimal ordered quantities for its imported gases.
2. We also recommend that Air-Mate Gas Factory Ghana adopt the Economic Order Quantity (EOQ) model which will help in trying to reduce the number of orders made within a year.
3. Alternate procedure may be used to create feasible solutions for the re-order point. In this case study, this was calculated by lead time and demand. (D * L). Future research may study the optimal re-order point problem which may be the function of inventory level and demand rates.

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APPENDICES

APPENDIX A (i)

Table 4.1 Historical Data for 4 Products in Quarters

Product Line	Ammonia gas	Argon gas	Nitrous oxide gas	Atal gas
Quarters	kg	m 3	kg	m 3
Qrt1 (2011)	12000	1000	3000	550
Qrt2 (2011)	6000	13000	2500	500
Qrt3 (2011)	10000	4000	1500	350
Qrt4 (2011)	4400	9000	2000	600

(ii)

Table 4.2 Summary of data on gas for 2011

Product Line	Annual Demand (D)	Fix order Cost(k) $	Unit cost Price(c) $	Holding rate(I) %	Holding Cost(H) $ (c.I)	Lead Time(L) days	Number of Times ordered (N)
Ammonia	32000kg	$320	$1	0.25	0.25	40	5
Argon	42000m3	$407	$1.08	0.25	0.27	40	5
Nitrous oxide	9000kg	$250	$2.19	0.25	0.55	40	5
Atal	2000m3	$20	$1.6	0.25	0.4	40	5

Appendix A (iii)

Annual production Data for Nitrogen gas

Annual demand	62500m3
Production rate	188400m3
Production cost	$10
Holding cost	$0.5
Ordering cost	$240
Shortage cost	$2

Appendix B

RESULTS BEFORE EMPLOYING EOQ MODEL

	Ammonia gas	Argon gas	Nitrous oxide	Atal
Time ordered (Annual)	5	5	5	5
Annual Demand (D) $	32000kg	42000m3	9000kg	2000m3
Unit cost Price (C) $	1.0	1.08	2.19	1.6
Holding rate (I) $	25%	25%	25%	25%
Holding cost H (C.I)	0.25	0.27	0.55	0.4
Fix order cost (k) $	320	407	250	20
Lead Time(days)	40	40	40	40
Q=Demand/Number Times ordered				
Q	6400	8400	1800	400
Annual Holding cost $	800	1134	495	80
Annual order cost $	1600	2035	1250	100
Total Purchase cost T (CD)	32000	45360	19710	3200
Total annual cost				
TCU=CD + $\frac{HQ}{2}$ +$\frac{KD}{Q}$	34400.00	48529	21455	3380

RESULTS AFTER APPLYING EOQ MODEL

	Ammonia	Argon gas	Nitrous oxide	Atal
Annual Demand	32000	42000	9000	2000
Unit cost price (C) $	1	1.08	2.19	1.6
Holding cost rate I %	25%	25%	25%	25%
Holding cost H (C.I) $	0.25	0.27	0.55	0.4
Fix order cost (k) $	320	407	250	20
Lead Time	40	40	40	40
EOQ=Q*=$\frac{\sqrt{2KD}}{H}$	9051	11253	2860	447
ROP=DL(Days) /working Days in Year	5120	6720	1440	72
Optimal orders n* = $\frac{D}{Q*}$	4	4	3	4
Annual Holding cost	1131.37	1519.11	786.61	89.44
Annual order cost	1131.37	1519.11	786.61	89.44
Total purchase cost	32000	45360	19710	3200
Total Annual cost $=\mathrm{CD}+\frac{HQ*}{2}+\frac{KD}{Q*}$	34262.74	48398.22	21283.2	3378.89

Appendix C

Total Cost

BEFORE EMPLOYING EOQ				AFTER EMPLOYING EOQ MODEL		
Products line	Annual holding cost $	Annual Order cost $	Annual Total Cost $	Holding cost $	Order cost $	Annual Total cost $
Ammonia Gas	800	1600	2400	1131.37	1131.37	2262.74
Argon Gas	1134	2035	3169	1519.11	1519.11	3038.22
Nitrous Gas	495	1250	1745	786.61	786.61	1573.22
Atal Gas	80	100	180	89.44	89.44	178.88
Total Cost	2576.50	5125.00	7701.50	3623.74	3623.74	7247.48
					Total savings	$454.02
					Percentage (%)	5.90

Appendix D

$$A=\frac{dTCU\left(Q,X\right)}{dQ^{2}}=\frac{2KD}{Q^{3}}+\frac{2PX^{2}\left(H+S\right)}{Q^{3}\left(P-D\right)}$$

$$A=\frac{2*240*62500}{\left(9477\right)^{3}}+\frac{2*188400*\left(1267\right)^{2}\left(0.5+2\right)}{\left(9477\right)^{3}\left(188400-62500\right)}$$

$$A=4.94*10^{-5}$$

$$L=\frac{dTCU\left(Q,X\right)}{dX^{2}}=\frac{P\left(H+S\right)}{Q\left(P-D\right)}=\frac{188400\left(0.5+2\right)}{\left(9477\right)\left(188400-62500\right)}$$

$$L=3.95*10^{-5}$$

$$E=\frac{-XP\left(H+S\right)}{Q^{2}\left(P-D\right)}=\frac{-1267*188400\left(0.5+2\right)}{\left(9477\right)^{2}\left(188400-62500\right)}$$

$$E=-5.28*10^{-5}$$

$$H\left(Q^{*},X^{*}\right)=\begin{vmatrix}4.94*10^{-5}-\lambda & -5.28*10^{-5}\\ -5.28*10^{-5} & 3.95*10^{-5}-\lambda\end{vmatrix}=0$$

$$\left(4.94*10^{-5}-\lambda\right)\left(3.95*10^{-5}-\lambda\right)-\left(-5.28*10^{-5}\right)=0$$

$$\lambda^{2}-8.89*10^{-5}\lambda-8.365*10^{-10}$$

$$\lambda_{1}=7.82*10^{-5}$$

$$\lambda_{2}=1.07*10^{-5}$$