KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

A Multi-State Reserving Model of Motor Insurance Claims of an Insurance Company in Ghana

By

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A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF MASTER OF SCIENCE, ACTUARIAL SCIENCE

April 15, 2016

Declaration

I hereby declare that this submission is my own work towards the award of the M.Sc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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Dedication

I dedicate this work to my parents, Samuel and Betty who have been a great pillar in my life. To my lovely sisters Lyanne, Irene, Cathy and Keren who have always stood by me.

Abbreviations

IBNR Incurred But Not Reported

RBNS Reported But Not Settled

NIC National Insurance Commission

CIMG Chattered Institute of Marketing Ghana

Abstract

General insurance under which motor insurance falls is perhaps the fastest growing area thus claim numbers and amount are huge (Achieng, 2010). Due to the high rate of growth, number of claims are also overwhelming thus creating a huge backlog of unsettled claims. The objective of the study was to explore how the multistate model underlie the development of motor insurance contracts using a three state model, the states being Incurred But Not Reported (IBNR), Reported But Not Settled (RBNS) and PAID. The three state model was used to derive the expressions for transition probabilities using the generator matrix and the Chapman-Kolmogorov forward differential equations. This was done after the transition rates had been estimated using the method of maximum likelihood. These were then used to compute the number of claims in the development years. It was found that losses arrived at the IBNR state at a rate of 105 losses per month and then waits for 1.88 months before it transitions to the RBNS state with a force of 0.53 claims per month. Claims reported were then seen to be settled (PAID) with the force of settlement of 0.17 claims per month. All the claims in the IBNR states were moved to the RBNS in the accident year, 80 percent of claim in the RBNS state are settled in the accident year with the remaining spread over three more years. 44.6 percent and 47.2 percent of the claims are paid in the accident and first development year respectively. It was concluded that an average of 6 claims are reported per year with 2 claims being settled in the same year. The claims in the IBNR state are run off after a year, claims in the RBNS and PAID states being completely run off after four and five years respectively. Expressions for the expected number of claims for the accident year and development years were derived.

Acknowledgements

I would like to first of all thank God Almighty for seeing me through this work. I also appreciate the the support of my Supervisor, Mr. Yao Elikem Ayekple, for the guide he gave me throughout the work.

To my father (Mr. Samuel Sackey) who has been a source of encouragement to me, I would like to say thank you for being there. You have always been a good father, I appreciate you.

I also appreciate all the authors from whose work I made reference. I also give special thanks to Mr. Nana Kena Frempong (Lecturer, Department of Mathematics, KNUST)for his invaluable contributions to my work. And to Michael and Joshua (Department of Mathematics, KNUST), for their contributions to my work.

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Chapter 1

Introduction

1.1 Introduction

This chapter gives a background to the study which covers motor insurance contracts, claims and claim counts and reserving models. The chapter also presents the problem, purpose of the study and also limitations of the study.

1.2 Background of the study

Motor Insurance which is sometimes also referred to as automobile insurance is a form of security which is purchased to protect the buyer of the product from future loss (Awunyo-Vitor, 2012). The National Insurance Commission (NIC) described the aim of motor insurance as providing compensation for loss or damage to one's motor vehicle. These compensations, usually referred to as claims, however aims at providing compensation for injury or death to third parties arising from vehicular accidents or the car owner in the event of loss arising out of theft of the vehicle or loss from damage caused by fire (NIC, 2015).

General insurance under which motor insurance falls is perhaps the fastest growing area for insurance industries thus resulting in huge amount and number of claims (Achieng, 2010). This calls for claim actuaries to be more concerned with both the claim amount and number since they are likely to have a great impact on the finances of the firm. Due to the fact that data is very important in the insurance industry, a lot of insurance companies employ large numbers of actuarial analysts to understand the large volumes of claims data. Actuaries managing claims need to also concern themselves with the number of claims, the claim amounts the insurance industry will have to pay and the exact situations which gives rise to the claim numbers (Boland, 2006).

A much critical attention to both claim numbers and amounts will enable claim managers to develop suitable models which best forecast future claim amounts and numbers. This is termed reserve techniques (Orr, 2007). This is sometimes referred to as loss reserves, since they are used to forecast and and make provision for future liability. Conventional reserving methods addresses the problem associated with the prediction of future claims as they develop using information readily available (Orr, 2007). The reserving methods consider the information available from the previous years and uses it to forecast the overall development of the claims. This is done with the assumption that the claim development pattern does not change. In practice, factors surrounding the historical factors will be taken into consideration for the development years (Frees, 2014).

The techniques include the basic chain ladder, which is one of the simplest as it was developed in the pre-computer age and does not involve specialized software (Orr, 2007). There are stochastically developed techniques which use proportions in estimating claim developments. In this study, a multistate model will be used to explore the idea that the development of claims of a motor insurance contract is frequently presented as a compressed s-curve of incurred (reported) or paid claims over time. The application of multistate models to actuarial problems was first introduced by Hoem (1969). Significant contributions to the analysis of present values in a multistate model framework were made by Waters (1984). Few studies have been conducted over the years on the Markov loss reserve model which involves states such as the Incurred but not reported (IBNR), Reported but Not Settled (RBNS) and Settled (S). Others include states such as Reported but No Case Reserve and Reported and Case reserve. These could either be a time inhomogeneous markov jump chain or time inhomogeneous Poisson process (Hesselager, 1995).

1.3 Data Source

Claim data of the various motor insurance product was a secondary source data obtained from Vanguard Assurance Company Ltd. The data covered losses which were reported in 2014 regardless of the cause of loss. The various dates (loss incurred, date relevant documents were requested and submitted, date claim was settled and date cheque was issued) and also the policy type and cause of losses were the main parameters.

1.4 Problem Statement

Claim severity forms the largest insurer liability on the insurer's balance sheet. Thus the need for the development of a suitable model, which will consider the number of claims managed by the insurance company.

At Vanguard Assurance Company limited, motor policy records an over-whelming number of claims in any given period. This implies that there is a tendency for a delay in settlement thus claims keep piling up. It is for this reason that there is a need to fit a model that does not just focus on the claims being paid to the clients but also the number of claims. Fitting this model enables the valuation actuaries to best forecast future expected claims and also set reserves which will be adequate to curb these losses.

In most companies, it takes a longer time to settle a claim once it is reported. This creates a huge backlog of unpaid (outstanding) claims . This is a problem as all these claims are not settled in the same accident year but are mostly transferred and added to the subsequent accident years.

The Markov model provides a relatively simpler approach to modeling loss reserves as compared to the traditional chain ladder approach (Hesselager, 1995). Thus knowing the distribution of the claim amounts and claim numbers will aid the process of summarizing and modeling large amounts of claims data. Thus these distributions however are very important to actuaries (Raz, 2000).

1.5 Objectives

To explore how the multistate model underlie claim development of motor insurance contracts.

The main objectives of the study are;

1. To estimate the transition intensities between claim development model.
2. To derive the expressions for the expected claim numbers in each state ateach point in time, *t*.
3. To use the multistate model to compute the expected claim counts in theaccident and development years.
4. To identify the run-off year for claims in each state.

1.6 Research Hypothesis

Given that X is the random variable of the number of losses that arrive in a day, the number losses arrive as a Poisson process with parameter λ and is tested by:

H_0:$X\sim$ Poi(λ)

H_0:$X \nsim$Poi(λ)

1.7 Significance of the study

The motor insurance policy is very essential because the Third Party Insurance Act, 1958 demands that whoever uses a vehicle on any public road must have insurance which covers both his liabilities and others which arise from the use of the vehicle. This makes general insurance, which covers motor insurance, section of the insurance companies is their largest wealth pulling sectors (NIC, 2015).

This results in large number of claims and claim amounts in each year. Claim reserving actuaries who have the task of ensuring claims are taken care of also have the task of advising management on the amount needed to be reserved for future claims. This study intends to provide claim managers an approach to modeling claim reserves which uses volumes of claims in modeling and not just claim amounts. This method is an appropriate method for large number of claims as in this case. Subsequently, it will help claim actuaries to know how much and when claims will be made in the future and also help estimate future losses.

1.8 Scope of Study

This research will cover the various motor insurance policies including the various vehicle type (commercial or private) and each category of insurance policy types (comprehensive, third party ...) of Vanguard Assurance. The number

of claims which are not reported (IBNR), those yet to be settled (RBNS) and those settled
(S)

1.9 Limitations of the study

The study had the following limitations;

1. The data organization structure created a limitation as to the number ofstates the process (claim development) goes through before finally being settled. For that reason, only the three states (IBNR, RBNS and PAID) were considered in the study.

2. Time allocated for the research limited the research to just motor insuranceand in just one insurance company.

3. Data was also a limitation to the study since claims data for just one yearwas received. This focused the research on only one accident year.

1.10 Data Analysis and Presentation

The data received was sorted into the relevant dates to the three state model such as date of loss (accident), date accident was reported and the date the claim was paid. Data sorting and analysis was done mainly by the use of Microsoft excel (2013), and MATLAB.

1.11 Organization

The thesis will be presented in five major chapters. Chapter one is the introduction which presents the background to the study, statement of the problem, objective of the study, significance of the study, scope of the study and the limitations of the study. Chapter two will present a review of relevant literature and comprises the theoretical framework and the different perspectives of researchers of the problem related to reserve models. The methodology to achieve the objectives is outlined in chapter three and here the relationships between the three fundamental traffic variables are developed. Chapter four presents the data analysis, modeling results and the accompanying discussions. The summary of findings conclusions and recommendations are presented in chapter five, then the

References and appendix

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1.12 Definition of Terms

1. **Claim** - A demand or request for something considered one's due (IFE, 2013).

2. **Loss** - A loss is the injury or damage sustained by the insured in consequence of the happening of one or more of the accidents or misfortunes against which the insurer, in consideration of the premium, has undertaken to indemnify the insured (Marshall, 1810)

3. **Random Variable**-A function that associates a unique numerical value with every outcome of an experiment (Bendat, 2011).

4. **Reserve**-Provisions for future liabilities which indicate how much money should be set aside now to reasonably provide for future pay-outs (Achieng 2010).

5. **Severity**-This can either be the amount paid due to a loss or the size of a loss event (Frees, 2014).

6. **Model**-An imitation of a real world system or process (Hartmann 1996).

7. **Actuary**-An actuary is a business professional who deals with the measurement and management of risk and uncertainty (SOA, 2016) .

8. **Maximum Likelihood Estimate**-is a method of estimating the parameters of a statistical model given data (Myung, 2003).

9. **Insurance**-is the equitable transfer of the risk of a loss, from one entity to another in exchange for money. It is a form of risk management primarily used to hedge against the risk of a contingent, uncertain loss. An insurer, or insurance carrier, is selling the insurance; the insured, or policyholder, is the person or entity buying the insurance policy (Chi-Chi, 2013).

Chapter 2

Literature Review

2.1 Introduction

Non-life modeling of claims is one of the most researched areas in the field of actuarial science. This chapter reviews literature on reserve models, the Markov process, and other models for modeling claim reserves.

2.2 Theoretical Review

The conceptual framework on which this work was done was the Markov process. The Poisson process was used in modeling the claim development process from time loss was made through time it was reported until final settlement.

2.3 Markov Process

Many actuarial calculations involve a multistate setup (Frees, 2014). Many authors have therefore used multistate models to analyse actuarial problems (Jones, 1994). IFE (2013) defines a Markov process as a discrete state stochastic process which has the Markov property and states that the probability of a being in a state depends only on the probability of being in the current state. The process can either be a chain or a jump chain depending on the nature of the time component (i.e discrete or continuous). Jones (1994) in a paper on Actuarial Calculations using a Markov model represented the states of an individual at time (age) $t \geq 0$ by $X(t)$ and the stochastic process by $X(t), t \geq 0$. For a finite state space $1,2,...,k$, defined the stochastic process. $X(t), t \geq 0$ as a Markov process if $\forall s,t > 0$ and $i,j,x(u) \in 1,2,...,k$,

$$PrX(s+t) =| X(s) = i, X(u) = x(u), 0 \leq u \leq s \quad (2.1)$$

$$Pr(X \mid s+t) = j \mid X(s) = i \quad (2.2)$$

The above equation represents the Markov property. He further explains that the future of the process (after time s) depends only on the state at time s and not the history of the process up to time s.

Bovier (2012) described Markov processes as the most important stochastic processes that are used to model real life phenomena. He classified the Markov processes into the following categories:

(a) Discrete time, finite state

(b) Discrete time, countable state space

(c) Discrete time, general state space

(d) Continuous time, countable state space

(e) Continuous time, general state space

2.4 Claim Development Process

Wang (2008) studied Modeling Claim Development Processes. The purpose of the report was to suggest a theoretical account that could be applied to analyse active claims side by side with finalised claims for pricing purposes. The processes claims were to break through were; a delay process that models the time between each successive update to the incurred claims cost variable; a binary process that models, whether the claims cost is revised upwards or down at each successive update; a positive process that models the size of the revision. These operations were then modelled and a more robust inference based on all available claims information, rather than only a subset was created.

Antonio (2012 and 2014) studied Micro-level stochastic loss reserving for general insurance. This study looked at the use of Position Dependent Marked Poisson Processes and statistical tools for recurrent events, a dataset was analysed with liability claims from a European insurance company. Detailed data on the time of the occurrence of the claim, delay between the occurrence and reporting to the insurance company and also the occurrences of payments and their sizes and final resolution. The model was calibrated to historical data and was used to visualise the future development of open claims. The resolutions from the case study analysis showed that the microlevel model outperforms the results obtained with traditional loss reserving methods for aggregate information.

Van (2015) studied on Micro-level stochastic loss reserving models for timediscrete data. The paper focused on the development of future cash flows on a claim-by-claim basis. A multiple state framework was adopted by the study such that, the claim development process could be reconstructed as a series of transitions between a given set of states. The claim sizes were modeled by means of a multinomial distribution the probability is determined that the claim size of a certain claim pertains to the slice of interest.

2.5 Multistate Reserving Model

Norberg (1986) in his paper titled A contribution to modeling of IBNR, claims presented some relatively simple structural ideas about how probabilistic modeling, and in particular, the modern theory of point processes and martingales could be used in estimation of claim reserves. He assumed accidents occurred as a Poisson process and reporting delays were iid. His study supported the use of Bayesian paradigm and the Poisson-gamma conjugate distributions for the estimation of number of claims in each state.

Hesselager (1995), studied a markov model for loss reserving. In his research studied the claims generating process for non-life insurance portfolio, which was modelled as a Poisson process where the marker associated with an incurred claim described the development of that claim until final resolution. The Markov principle governed an unsettled claim at any period in time assigned to a state in some state space. He, however developed separate expression for the IBNR and RBNS reserves and the corresponding
prediction errors.

Orr (2007) aimed at exploring how a simple, common process may underlie the growth of claims rising from a portfolio of insurance contracts. This study used a simplified claim number multi state model of three states.
The states being IBNR (State 0), State 1 for Reported But Not Settled (RBNS) claims and paid claims being absorbed into State 2. Losses arrived at the first state as la Poisson process. Using the R statistical package, the estimated number of claims were obtained from the simulated data. Outcomes from the survey demonstrated that simulation of the simplified claims number model was established to generate plausible data, to which existing reserving techniques may be used. Alternative estimation approaches using least squares and Bayesian approaches were established to create similar effects. Though it was not tried out in a material world, the

model was considered to have supplied an alternative base for the estimation of claim development.

Hurlimann (2015) conducted a study on the topic Old and New on some IBNR Methods. This study considered three main categories of the IBNR methods being the: the standard IBNR methods, that is, chain-ladder, Cape Cod, Bornhuetter-Ferguson, the IBNR loss ratio method and the stochastic IBNR methods. This study focused on the stochastic chain ladder models which included the multivariate setting, distribution based reserving models and multistate reserving models. This study assumed that the IBNR followed a log-Laplace distribution and used numerical examples to show that it is comparable in accuracy to the standard and loss ratio IBNR methods.

2.6 Other Reserving Models

Renshaw (1998) studied on the topic a stochastic model underlying the chain-ladder technique. This study presented a statistical model underlying the chain-ladder technique. The statistical models used here were the generalized linear model and a quasi-likelihood approach to the chain ladder technique. Results from this study showed that the method was able to process negative incremental claims. Conclusion from the study revealed that the chain-ladder technique represented a very narrow view of the possible range of models.

The Bootstrap methodology in claim reserving was considered by Pinheiro (2003). In the article, the bootstrap technique was used to obtain prediction errors for different claim-reserving methods, namely, the chain ladder technique and methods based on generalized linear models. de Andres-

Sanchez (2007) studied claim reserving with fuzzy regression and Taylor's geometric separation method. The paper developed a claim reserving method that combined Ishibuchi and Nii's extension. Data analysis was by Fuzzy probabilistic linear models. Fuzzy Regression Analysis with the scheme for claim reserving proposed was conducted and a statistical testing of a non-life insurance run-off model was performed.

Studies on Bayesian estimation of outstanding claim reserves was conducted by De Alba (2002). The paper presented a Bayesian approach to forecasting outstanding claims, that is, either the total number of claims or the total claim amount. The assumption made here was that there was complete information for one or two past years of origin and partial information for some development years of other years of origin. He presented two different models. One for the claim numbers (intensity) and the other on claim amounts (severity). He then derived the complete predictive distribution of the reserve requirements, from which point estimates as well as probability intervals and other summary measures, such as mean, variance, and quantiles were obtained

Pettere (2006) modelled incurred but not reported claim reserving using copulas. The purpose of the study was to calculate IBNR reserve directly without total outstanding reserve. Copula was used as a tool for modelling different dependence structures has been intensively. This was done in six steps and results showed that there was one very large and not traditional paid sum 5000 and the real paid out amount of claims was 21481.28 LVL. If this large amount had not occurred, the amount paid out would have been 16481.28 LVL.

Dahms (2008) conducted a study on a loss reserving method for incomplete claim data. A stochastic model of an additive loss reserving method based on the assumption that the claim reserves are good measures for the remaining exposure was presented. This method was was seen to even work for incomplete triangles. The estimators for the total necessary reserves and estimators for the corresponding standard error were found and further discussions to distinguish between property damage and bodily injury claims.

Jing (2009) conducted a study on the topic claim reserving: performance testing and the control cycle. In his study, estimates of claim liabilities were forecast subjects to estimation errors. Performance testing of alternative methods was used to provide a formal assurance of the usage of the best methods for the given circumstance, and also to provide insight into the appropriate weight to give to the indications produced by each method. This was however an integral part of the actuarial control cycle associated with the loss reserving process.

Bjorkwall (2009) studied on non-parametric and parametric bootstrap techniques for age−to−age development factor methods in stochastic claims reserving'. The aim of the study was to investigate existing bootstrap techniques and suggest two alternative bootstrap procedures, one non−parametric and one parametric, by which the predictive distribution of the claims reserve can be found for other age−to−age development factor methods than the chain−ladder, using some rather mild model assumptions.

Chapter 3

Methodology

3.1 Introduction

This chapter presents the methods used in arriving at the results. It presents any underlying assumptions made in the work and also the sources of data and data analysis and presentation.

3.2 Research Design

This research was based on a Poisson process and a Markov jump process. The study applied Markov application of multistate models to actuarial problems which was introduced by Hoem (1969). From IFE (2013), $[N(S,t)]_{t>0}$ (an increasing integer valued process with starting from 0 with $\lambda > 0$) was said to be a Poisson process since the following conditions could be applied to the process;

(a) The holding times, $T_0, T_1, T_2, \ldots T_{n-1}$ of $[N(S,t)]_t \geq 0$ were independent exponentially distributed random variables with parameter λ and

$N_{T_0+T_1+\ldots+T_n-1} = n$

(b) It was a Markov transition rates were given by:

$$\mu_{ij} = \begin{cases} -\lambda & if\, j = i \\ \lambda & if\, j = i + 1 \\ 0 & otherwise \end{cases}$$

This study was primarily based on Markov process applications to actuarial problems (Frees, 2014).

3.2.1 The Markov Jump Processes

Let Γ be the set of all states of a system and $x, y \in \Gamma$ the states of the system.

A jump process is a random variable $X(t)$ with parameters time $t \in (0, \infty)$. This random variable starts in an initial state x_0 at time $t = 0$ and stays in this state until time t_1 when it makes transition to a different state x_i.

It stays until a later time $t_2 > t_1$ at which it jumps to a different state. Then if $t_1, t_2, \ldots$ are the set of jump times, then $X(t) = x_0$ for $t \in (t_1, t_2)$, and so on. This defines a Markov Jump process $X(t)$. We assume that $\lim_{n\to\infty} t_n = \infty$, so the jump process $X(t)$ is defined for all non negative value of t.

3.2.2 The Poisson Process

The Poisson process is the simplest example of the Markov jump process in continuous time. The standard time – homogeneous Poisson process is a counting process in continuous time, N_t, $t \geq 0$, where N_t records the number of occurrences of some type of event within the time interval from '0' to 't'. The events of interest occur singly and may occur at any time. IFE (2013)

The probability that an event occurs during the short time interval from time t to time $t+h$ is approximately equal to λh for small h; the parameter λ is called the rate of the Poisson process. It is very commonly used to model the occurrence of unpredictable incidents, such as car accidents or arrival of claims at an office.

3.3 The Model

The model was discrete state space, continuous time Markov jump process with arrival following a Poisson process. The model was a three state model namely IBNR as State 0, RBNS as State 1 and PAID as state 2. The state space was IBNR, RBNS, PAID or 0, 1, 2. Hesselager (1994) and Orr (2007) have discussed similar models and applied to modeling actuarial problems.

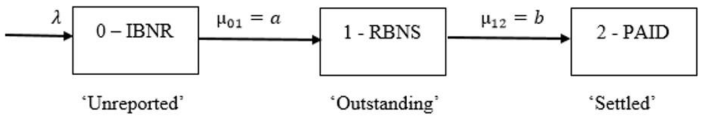


Figure 3.1: The Expected Model

Figure 3.1 The Multistate Model

The above model shows that losses (accidents) arrive as a Poisson process with parameter(λ). These losses arrive at a state IBNR which is often referred to as unreported claims. These losses are now claims which are to be reported to the claims department for processing. They are reported in state RBNS at a rate of $\mu_{01} = a$. These claims are therefore the outstanding claims the claims department has to pay. These claims are then absorbed into the PAID at a rate of $\mu_{12} = b$. The rate at which claims move from state 0 to state 1 are referred to as force of reporting. The ratio at which reported claims are paid was also referred to as force of settlement.

Assumptions made

(a) Only losses and claim numbers in each state were considered. Thisimplies that claims which were considered at any point in time were found in either states.

(b) Losses were assumed to arrive following a Poisson process with a constant rate of arrival λ with exponential waiting times .This is because claim arrivals are rare events.(Orr, 2007).

(c) Claims are assumed to be absorbed only in the PAID state; all claimsreported will eventually be paid. PAID state is therefore an absorbing state which implies that all claims which are incurred will finally be absorbed into the state and will not transit to any other state.

(d) The Markov property holds. The states that the probability of a beingin a state depends only on the probability of being in the current state.

3.4 Estimation transition intensities

Orr (2007) defined the transition intensities as;

$$\mu^{01} = \lim_{h \to 0} \frac{P(\text{Claim in State 1 at t+h}|\text{Loss in state 0 at t})}{h}$$

= force of reporting = a

$$\mu^{12} = \lim_{h \to 0} \frac{P(\text{Claim in State 2 at t+h}|\text{Loss in state 1 at t})}{h}$$

= force of reporting = b

3.4.1 Maximum likelihood estimators (transition intensities)

Let

N_1 = Number of losses in the i^{th} month . Thus

$$N = \sum_{i=1}^{12} n_i$$

V_1 = Waiting time of i^{th} loss in the IBNR state . Thus

$$V = \sum_{i=1}^{N} V_i$$

W_1 = Waiting time of i^{th} loss in the RBNS state . Thus

$$N$$

$$W = \sum_{i=1} W_i$$

R_1 = The number of transitions from IBNR to RBNS by i^{th} claim. Thus

$$R = \sum_{i=1}^{N} R_i$$

S_1 = The number of transitions from RBNS to PAID by i^{th} claim. Thus

$$S = \sum_{i=1}^{N} S_i$$

$$L(\mu_{01},\mu_{12}) = L(a,b) = e^{-av}e^{-bw}a^{r}b^{s}$$

$$(e^{-av}a^{r})(e^{-bw}b^{s})$$

(a) Log likelihood is

$$\ln L = -av - bw + r\ln a + s\ln b$$

(b) Differentiating this with respect to a and b gives

$$\frac{\partial \ln L}{\partial a} = -v + \frac{r}{a}$$

$$\frac{\partial \ln L}{\partial b} = -w + \frac{s}{b}$$

(c) Setting each of these derivatives to 0 gives;

$$a = \frac{r}{v}$$

$$b = \frac{s}{w}$$

(d) Using the second derivative to check for maxima gives

$$\frac{\partial^2 l}{\partial b^2} = -\frac{s}{b^2} < 0$$

Hence the negatives denotes the maximum likelihood estimates of a and b

$$\hat{\mu}^{01} = \hat{a} = \frac{r}{v}$$

$$\hat{\mu}^{12} = \hat{b} = \frac{s}{w}$$

3.4.2 Maximum likelihood estimators (Arrival rate)

The maximum likelihood method was applied to estimate the expected rates of diagnoses of various items as stated above.

The likelihood function of Poisson model in estimating the parameter λ

(a) Log likelihood is

$$L(\lambda) = \prod_{i=1}^{12} \frac{e^{-\lambda}\lambda^{n_i}}{n_i!}$$

$$L(\lambda) = \frac{e^{-12\lambda}\lambda^{\sum_{i=1}^{12} N_i}}{\prod_{i=1}^{12} n_i!}$$

(b) The log likelihood of the likelihood function was then determined forthe parameter estimates

$$ln(L(\lambda)) = -12\lambda + \sum_{i=1}^{12} n_i ln\lambda - ln\prod_{i=1}^{12} n_i!$$

(c) Differentiating this with respect λ

$$\frac{d}{d\lambda} log(L(\lambda)) = -n + \frac{1}{\lambda}\sum_{i=1}^{N} n_i$$

(d) Setting the derivative to 0 gives;

$$-12 + \frac{1}{\lambda}\sum_{i=1}^{12} n_i = 0$$

$$\lambda = \frac{1}{12}\sum_{i=1}^{12} n_i$$

(e) Using the second derivative to check for maxima gives

$$\frac{d^2 l}{d\lambda^2} = -\frac{1}{\lambda^2}\sum_{i=1}^{12} n_i$$

Hence the negatives denotes the maximum likelihood estimates of λ

$$\hat{\lambda} = \frac{1}{12}\sum_{i=1}^{12} n_i = \frac{1}{12}N$$

3.4.3 Average waiting time in each state

The average waiting times in each state were found by the reciprocal of the transition rates since the waiting times follow an exponential distribution. i.e. Average waiting time in IBNR was

$$\frac{1}{a}$$

and that for RBNS was

$$\frac{1}{b}$$

and

$$\frac{1}{\lambda}$$

for average loss arrival time.

3.4.4 Deriving expressions for the expected claim numbers

The expressions for expected number of losses in each state (S), at each point in time (t), for the accident year and the subsequent development run-off years.

During the accident year (t 6 1)

Orr (2007) in his method gave the following;

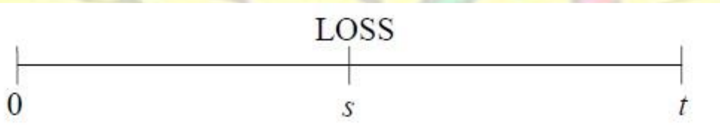
$$E[N(S,t)] = \lambda t * \pi(1)$$

$$\pi(1) = [\alpha(0,t)\alpha(1,t)\alpha(2,t)]$$

Where $\pi(1)$ is the probability vector of remaining in each state within the accident year.

State 0

The expected number of losses in state 0 at an arrival rate of λ at time ${}^{0}s^{0}$ and the probability of staying in state 0 under the force of reporting a was given by;



$$E[N(0,t)] = \lambda t * \alpha(0,t)$$

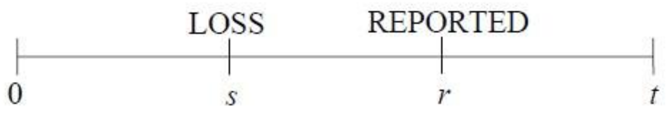
$$\lambda t * \alpha(0,t) = \int_0^t \lambda exp[-a(t-s)ds]$$

$$\frac{\lambda}{a}[1 - exp(-at)]$$

State 1

The expected number of claims in state 1 by time [1]t^0 with losses arriving at time

[1] s^0 and being reported at time ${}^{0}r^{0}$ is given by;


LOSS
REPORTED
0
s
r
t

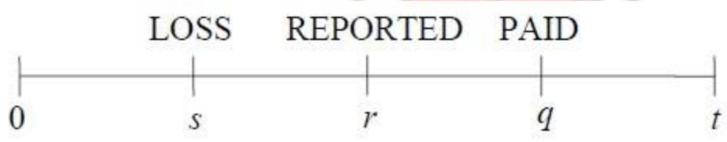
$$E[N(1,t)] = \lambda t * \alpha(1,t)$$

$$\lambda t * \alpha(1,t) = \int_0^t \lambda \int_s^t aexp[-a(r-s)exp[-b(t-r)]drds]$$

$$\frac{a\lambda}{(a-b)b}[1-exp(-bt)] - \frac{\lambda}{(a-b)}[1-exp(-at)]$$

State 2

Lastly, the expected number of claims in state 2 by time ${}^0t^0$ with losses arriving at time ${}^0s^0$ which are then being reported at time ${}^0r^0$ and settled in time ${}^0q^0$ is given by;



$$E[N(2,t)] = \lambda t * \alpha(2,t)$$

$$\lambda t * \alpha(2,t) = \int_0^t \lambda \int_s^t aexp[-a(r-s) \int_r^t exp[-b(q-r)]dqdrds]$$

$$\lambda a\{\frac{t}{a} - \frac{[1-exp(-at)]}{a^2} - \frac{[1-exp(-bt)]}{(a-b)b} + \frac{[1-exp(-at)]}{a(a-b)}\}$$

After the accident year ($t > 1$)

There were no further losses that arrived, thus the multi-state Markov jump chain was used to estimate. The Generator matrix, A, was derived from the transition intensities and then used to derive the transition probabilities using the Chapman-Kolmogorov forward differential equation. This method was also discussed by Orr (2007).

a

$-a$ 0

$-b$

0

$$= \begin{pmatrix} A & b \\ 0 & 0 \\ 0 & \end{pmatrix}$$

The expected number of claims in each state is given as;

$$E[N(S,t)] = \lambda\pi(1)p(t-1)$$

where

$$P(t-1) = \begin{pmatrix} P^{00}(t-1) & P^{01}(t-1) & P^{02}(t-1) \\ 0 & P^{11}(t-1) & P_{12}(t-1) \\ 0 & 0 & 1 \end{pmatrix}$$

Applying the Chapman-Kolmogorov foward differential equation

$$\frac{\partial}{\partial t}P^{ij}(S,t) = \sum_{k} P^{ik}(S,t)\delta^{kj}(t)$$

where $\delta^{ij}(t)$ is the transition rate from state *i* to *j*.

3.4.5 Computations from the model

The expressions derived above were then used to compute the expected number of claims in each state from time t = 1,2,3,4.

Chapter 4

Data Ananlysis and Results

4.1 Introduction

This chapter presents results from data analysis performed based on the methodology stated in the previous chapter.

4.2 Preliminary analysis of losses

Table 4.1: Cause of Loss

Cause of Losses	Frequency	Percentage
Own Damage	965	76.5
TP Bodily Injury and Death	134	10.6
Total Loss	54	4.3
TP Property Damage	31	2.5
Theft/Fire-own vehicle	28	2.2
Partial Theft	23	1.8
T P Liability	11	0.9
Accident to Passengers	5	0.4
Fire	5	0.4
Damage to property	3	0.2
Others	2	0.2
Total	**1261**	**100**

Table 4.1 above shows the cause of claims for the accident year. The table shows that own damage was the most frequent (965) Third party bodily injury and death, Total loss and Third party property damage were also frequently causes of claims with frequencies 134, 54 and 31 respectively.

Table 4.2: Descriptive Statistics

Statistic	Number
Sample size	267
Range	15
Mean	4.723
Variance	7.547
Std. Deviation	2.747
Coef. of Variation	0.17272
Std. Error	0.168
Skewness	0.4558
Excess Kurtosis	-0.173

Table 4.2 above is the descriptive analysis of the daily claim arrivals of the insurance company. It is revealed that the average claims reported per day was 4.7 claims with a minimum of zero claims per day and a maximum of 15 claims in a particular day. The data has a skewness of 0.4558 and variance of 7.547.

Hypothesis Test

H_0:λ=7.54

H_0:λ6= 7.54

The normal approximation to the Poisson gave a Z-score of 1.299 which is less than the $Z_{0.05}$ which is 1.96 thus we fail to reject the null hypothesis. Thus the mean and variance are equal.

Figure 4.1: Distribution of losses in months

Figure 4.1 shows how the losses are incurred per month. The months with the highest number of losses are March (132) and February (120). June and October recorded 111 losses with December recording the least number of losses of 89. These losses follow a Poisson process with a mean of 105 losses per month.

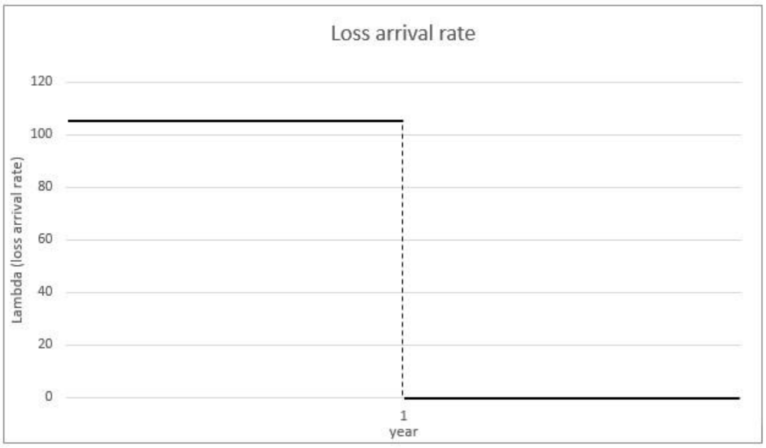


Figure 4.2: Arrival rate per month of losses

Claims arrive as losses (105 per month) as a Poisson process in the accident year 1.

4.3 Development of multi-state model

Table 4.3: Transition of claims within states

Transition(intensity	Number of transitions	Total Waiting time
IBNR-RBNS(a)	1459	2742.53
RBNS-PAID(b)	109	626.2

Table 4.3 above shows the transitions and the waiting times within each state. A total of 1459 transitions are made from the IBNR state to the RBNS state and 109 from RBNS to paid. The total waiting times in the IBNR and RBNS states are 2742.53 months and 626.2 months respectively.

Table 4.4: Force of transition of claims

Transition	Transition Name	Rate per month	Rate per year
IBNR-RBNS	Force of Reporting	0.53	6.36
RBNS-PAID	Force of Settlement	0.17	2.04

The transition from IBNR to RBNS (force of reporting) claims was 0.53 and that from the RBNS to PAID state being 0.17.

Table 4.5: Average waiting times for losses or claims

Event	Average waiting times for losses or claims
Loss(s)	0.01
Reported claims(r)	1.88
Paid claims(q)	5.74

Losses are expected to take 0.01 months before they arrive at IBNR state. They take an average of 1.88 months before they are reported and also an average of 5.74 months before they are settled (paid).

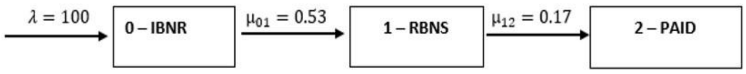


Figure 4.3: The multi-state model

Figure 4.3 above gives a clear description of the claim development process. Claims are seen to enter the IBNR state as losses at a rate of 105 claims per month. These claims are then transitioned to the RBNS state at a rate of 0.53 claims per month. Transition rate from state RBNS to PAID state was 0.17.

4.4 Computation from Multistate model

4.4.1 Expected Number of claims within accident year

Accident Year 1, Time t = 1

IBNR State

$$\lambda t * \alpha(0,t) = \frac{1260}{6.36}[1 - exp(-6.36)]$$

RBNS State

$$\lambda t * \alpha(1,t) = \frac{1260}{(6.36-2.04)2.04}[1 - exp(-2.04)] - \frac{1260}{(6.36-2.04)}[1 - exp(-6.36)]$$

PAID State

$$\lambda t{*}\alpha(2,t) = (1260)(6.36)\{\frac{1}{6.36} - \frac{[1 - exp(-6.36)]}{6.36^2} - \frac{[1 - exp(-2.04)]}{(6.36-2.04)2.04} + \frac{[1 - exp(-6.36)]}{6.36(6.36-2.04)}\}$$

Estimated number of claims at end of accident year

State	Expected number of claims
IBNR	198
RBNS	500
PAID	562

The expected number of claims in the IBNR, RBNS and PAID states at the end of the year respectively are 198, 500 and 562 claims. Thus

$$\pi(1) = \begin{bmatrix} 0.1569 & 0.3967 & 0.44628 \end{bmatrix}$$

The probability vector (π) above shows that the probability of being in the IBNR state after the accident year were 0.16 for IBNR, 0.4 for RBNS and 0.44 for PAID. Thus there is a higher probability of being paid after the accident year. At subsequent accident years (time $t > 1$)

Generator matrix

$$A = \begin{bmatrix} -6.36 & 6.36 & 0 \\ 0 & -2.04 & 2.04 \\ 0 & 0 & 0 \end{bmatrix}$$

4.4.2 Expressions for transition intensities

$$P(t-1) = \begin{bmatrix} P^{00}(t-1) & P^{01}(t-1) & P^{02}(t-1) \\ 0 & P^{11}(t-1) & P^{12}(t-1) \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{00}(t-1) = e^{-6.36(t-1)}$$

$$P_{01}(t-1) = 1.472\left(e^{-2.04(t-1)} - e^{-6.36(t-1)}\right)$$

$$P_{02}(t-1) = 1 - e^{-6.36(t-1)} - 1.472\left(e^{-2.04(t-1)} - e^{-6.36(t-1)}\right)$$

$$P_{11}(t-1) = e^{-2.04(t-1)}$$

$$P_{12}(t-1) = 1 - e^{-2.04(t-1)}$$

4.4.3 Expected Number of claims after accident year, E[N(S,t)],

$(t > 1)$ **Development year**

Year 1, Time $t = 2$

$$P(2) = \begin{bmatrix} 0.02 & 0.189 & 0.809 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{bmatrix}$$

Development year 2, time $t = 3$

$$P(3) = \begin{bmatrix} 0 & 0.025 & 0.975 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Development year 3, time $t = 4$

$$P(4) = \begin{bmatrix} 0 & 0.003 & 0.997 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Development year 4, time $t = 5$

$$P(5) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Table 4.6: Expected number of claims in the IBNR state for the accident year and development years

Year	Number	Cumulative Number	Percentage
1	198	198	100
Total	**198**	**100.0**	

Table 4.6 above estimates the number of claims in the IBNR state in and after the accident year. The expected number of claims at the end of the first was 198 representing 100%. This implies that claims were fully run off in the IBNR state after the first year.

Table 4.7: Expected number of claims in the RBNS state for the accident year and development years

Year	Number	Cumulative Number	Percentage
1	500	500	84.3
2	87	587	14.7
3	5	592	0.8
4	1	593	0.2
Total	**593**		**100.0**

500 (84.3%) of the claims in the RBNS state were in the first year, however, the number in the second to fourth years were 87, 5 and 1 respectively, thus the claim numbers in the RBNS state were fully run off after the fourth year.

Table 4.8: Expected number of claims in the PAID state for the accident year and development years

Year	Number paid	Cumulative Number paid	Percentage paid
1	562	562	44.6
2	610	1172	48.4
3	83	1255	6.6
4	4	1259	0.3
5	1	1260	0.1
Total	**1260**		**100.0**

Table 4.8 above indicates that claims are fully run off after the fifth year. 44.6% of the claims were settled in the accident year, 48.4% settled in the first development year with the remaining paid from the second and fourth development years. Thus claims were fully paid at the end of the fifth year.

Chapter 5

Discussion, Conclusions and Recommendations

5.1 Discussion

General insurance under which motor insurance falls is perhaps the fastest growing area for Actuaries thus resulting in huge amount and number of claims (Achieng, 2010). This however calls for claim actuaries to be more concerned with both the claim amount and number since they are likely to have large control on the finances of the firm (Boland, 2006). Companies controlling large amount of claims need to be concerned about a model for claim numbers also. The purpose of this study was however to explore how the multistate model underlie claim development of motor insurance contracts of Vanguard Assurance Company Limited. The study considers claims in three states; IBNR, RBNS and PAID.

The time homogeneous Markov jump process is the main framework underlying this study. The Poisson process also underlies the study since claim counts are under study. There has however been a lot of studies on reserve models but very few on multistate claim reserving model of claim development. Studies by Orr (2007) constructed a model which was a strapped down version of the claim development by Hachemeister (1980). Norberg (1986) made a contribution to the IBNR model in estimation of claim reserves based on a Poisson process. Bootstrap methodology in claim reserving was considered by Pinheiro (2003) and Bjorkwall (2009). Other models used were on Bayesian estimation of outstanding claim reserves was conducted by De Alba (2002).

Claims data obtained from Vanguard Assurance was used in this study were the claim numbers and dates of occurrences were used in the modeling process. Claims were assumed to arrive as losses following a Poisson distribution. The losses that arrived at IBNR state at a rate λ waited in the state and transitioned to the RBNS state with a force ${}^{0}a^{0}$. These claims are then absorbed into the PAID state at a rate ${}^{0}b^{0}$. The three state model is used to construct closed form expression of transition probabilities in future years using the generator matrix and the Chapman Kolmogorov forward differential equations. These were then used in simulation of expected future development of claims.

It was found that losses which arrive at the rate of 105 losses per month arrive at the IBNR state where its waits for 1.88 months then transitions to the RBNS state with a force of 0.53 claims per month before they arrive at the PAID state at a force of 0.17 claims per month (Table 4.4). This however implies that there were more reported claims in a month than that of settled. This means that it takes a longer time for reported claims to be paid than incurred to be reported. Thus creating room for more outstanding claims. However the losses take the least time to arrive.The ratio of reported to settled claims was however computed to be 3 to 1. This means that to every paid claim, there were 3 of them that were reported. The estimates of the future claim developments from the model shows that the claims in the IBNS state are fully run off after 2 years whilst that in the RBNS and PAID states are fully run off after 4 years.

5.2 Conclusions

An average of 6 claims are reported per year with 2 claims being settled in the same year. The numbers in the IBNR state are run off after a year, claims in the RBNS and PAID states being completely run off after four and five years respectively. Expressions for the expected number of claims for the accident year and development years were derived. The percentages of claims in each state presents a relatively easier way of predicting when claims are paid more and how many expected.The process which provided a means by which claim development from the current year provided a means of estimating future claim development.

5.3 Recommendations

From the studies the accumulative nature of the claims outstanding gave rise to huge numbers in the subsequent year, it is however recommended that the claims department reduce the outstanding claims which creates the need for huge reserves thus making the claim process faster and commendable by the clients.

The amounts associated with the claim numbers in each of the states creates a good opportunity coupled with this model to efficiently manage reserves which will go a long way to safeguard the company against unexpected losses. Thus the need for a similar model of claim amounts.

The researcher recommends that the data associated with claim development process be properly organized and monitored to enable the application of the model to future changes in loss arrivals thus easing the process of developing the model. The process of claims development should also be made known to the clients thus reducing false anticipation. This is to say that the clients should be made away of the expected times for claim development hence to develop packages which will give room for clients to pay for the process to be hastened. For instance, a product where by claim could be processed and settled the same day which will attract a fee. This will help generate more revenue for the claims department.

Recommendations for future studies was to develop the multistate model which includes states which reflects further how most of the claims are developed. For instance including states of complicated claims and/or modified claims. Future studies could also consider modeling claim amounts which will give a clearer picture of the actual reserve.

Chapter 6

Appendix A

6.1 References

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Chapter 7

Appendix B

7.1 Excerpts of Data

INCURRED	REPORTED	SETTLED	INCURRED	REPORTED
5-Mar-13	25-Mar-14	25-Mar-14	30-May-14	30-May-14
13-Mar-13	21-Jan-14	28-Feb-14	30-May-14	30-May-14
3-Apr-13	24-Nov-14	24-Nov-14	30-May-14	30-May-14
10-Apr-13	7-Feb-14	7-Feb-14	30-May-14	30-May-14
16-Apr-13	7-Mar-14	19-Mar-14	30-May-14	2-Jun-14
18-Apr-13	21-Mar-14	21-Apr-15	30-May-14	5-Jun-14
26-Apr-13	2-Sep-14	8-Oct-14	2-Jun-14	2-Jun-14
30-Apr-13	30-Jan-14	30-Jan-14	2-Jun-14	2-Jun-14
30-Apr-13	15-Mar-14	15-Mar-14	2-Jun-14	2-Jun-14
30-Apr-13	5-Aug-14	5-Aug-14	2-Jun-14	9-Jun-14
10-May-13	12-Aug-14	4-Nov-14	2-Jun-14	10-Jun-14
22-May-13	25-Feb-14	25-Feb-14	3-Jun-14	3-Jun-14
28-May-13	20-Jun-14	20-Jun-14	3-Jun-14	3-Jun-14
17-Jun-13	13-Jan-14	13-Jan-14	3-Jun-14	3-Jun-14
24-Jun-13	3-Feb-14	12-May-15	3-Jun-14	3-Jun-14
26-Jun-13	23-Jan-14	23-Jan-14	3-Jun-14	3-Jun-14
26-Jun-13	21-Mar-14	23-Jun-14	3-Jun-14	3-Jun-14
3-Jul-13	31-Dec-14	31-Dec-14	4-Jun-14	4-Jun-14
5-Jul-13	2-Sep-14	9-Dec-14	4-Jun-14	4-Jun-14
12-Jul-13	21-Jul-14	22-Jul-14	4-Jun-14	6-Jun-14
14-Jul-13	7-Nov-14	7-Nov-14	4-Jun-14	24-Jun-14
16-Jul-13	6-Jan-14	6-Jan-14	4-Jun-14	7-Oct-14
18-Jul-13	24-Jan-14	24-Jan-14	5-Jun-14	5-Jun-14

Figure 7.1: Claim data

18-Jul-13	25-Mar-14	25-Mar-14	5-Jun-14	5-Jun-14
23-Jul-13	16-Jan-14	16-Jan-14	5-Jun-14	10-Jun-14
31-Jul-13	25-Sep-14	25-Sep-14	5-Jun-14	10-Jun-14
1-Aug-13	25-Feb-14	25-Feb-14	5-Jun-14	16-Jun-14
1-Aug-13	18-Mar-14	18-Mar-15	5-Jun-14	24-Jun-14
12-Aug-13	31-Jan-14	31-Jan-14	5-Jun-14	10-Jul-14
12-Aug-13	29-Oct-14	29-Oct-14	6-Jun-14	6-Jun-14
13-Aug-13	14-Apr-14	14-Apr-14	6-Jun-14	6-Jun-14
15-Aug-13	7-Apr-14	24-Jul-14	6-Jun-14	6-Jun-14
16-Aug-13	12-Mar-14	26-Feb-14	6-Jun-14	17-Jun-14
20-Aug-13	31-Jul-14	31-Jul-14	7-Jun-14	13-Nov-14
25-Aug-13	10-Feb-14	10-Feb-14	9-Jun-14	9-Jun-14
28-Aug-13	23-Jan-14	24-Jan-14	9-Jun-14	10-Jun-14
29-Aug-13	4-Jun-14	30-Jun-14	9-Jun-14	17-Jun-14
4-Sep-13	3-Apr-14	3-Apr-14	9-Jun-14	19-Jun-14
11-Sep-13	23-Apr-14	23-Apr-14	9-Jun-14	2-Jul-14
11-Sep-13	31-Jul-14	31-Jul-14	9-Jun-14	7-Jul-14
14-Sep-13	15-Jan-14	15-Jan-14	9-Jun-14	8-Sep-14
17-Sep-13	14-Oct-14	5-Jun-15	9-Jun-14	14-Nov-14

Figure 7.2: Claim data

7.2 Waiting times

Waiting Time (IBNR)	Waiting Time (RBN:
1452	10
1427	203
1552	76
1410	56
1265	53
1215	41
1335	14
1414	326
1174	61
1064	114
1032	96
1137	58
1115	14
1094	23
861	141

Figure 7.3: waiting times in days

963	42
958	55
764	19
728	70
798	64
558	21
591	84
700	55
816	31
514	28
556	165
539	29
570	62
454	51
520	48
435	44
506	301
459	44
452	362

Figure 7.4: waiting times in days