KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI.

# MODELLING THE TRANSPORTATION PROBLEM OF COCA-COLA BOTTLING COMPANY, GHANA.


By

Appiagyei Ebenezer

(BSc. Mathematics)


A Thesis submitted to the Department of Mathematics, Kwame Nkrumah University of Science and Technology in partial fulfillment of the requirements

for the degree of

MPhil. Applied Mathematics

College of Science

MAY, 2013.

# Declaration

I hereby declare that this submission is my own work towards the award of the M.Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the University, except where due acknowledgement had been made in the text.

Appiagyei Ebenezer ................... 9-10-2013 ....................

..................... 21/10/13 ....................

Student Signature Date

Certified by: Prof. S. K. Amponsah

..................... Signature

21/10/13 ....................

Supervisor Signature Date

Certified by:

Mr. K. F. Dark-wa

Head_of Departmen Date

# Acknowledgements

I thank The Almighty God for His bountiful blessings and grace that has brought me this far, may His name be praised forever and ever. I would like to acknowledge the support and guidance received from my supervisor, Prof. S. K. Amponsah. I am also grateful to my parents, Mr. and Mrs. Appiah for their moral and financial support during my study and research. I also thank my cousin, Roseevelyn Nsiah Ababio (U.S.A) for her constant support and encouragement. I am also grateful to all the Mphil. students of the 2011/2012 year group of the Mathematics department for their support and sense of brotherliness. I love you all. I would also want to thank the human resource manager and the operations manager of The Coca cola Bottling Company Ghana Limited, Kumasi branch. My special thanks go to my beloved, Elizabeth Serwaa Asare for her assistance, patience and encouragement throughout my work.

# Dedication

This study is dedicated to The Almighty God, my parents and all Ghanaian students.

# Abstract

Manufacturing companies operate with the prime aim of reaching their target of production and transporting their goods or services to consumers on time and at a minimum transportation cost in order to maximize profit. The objective of the transportation problem is to transport various quantities of a single homogeneous commodity, which are initially stored at various origins to various destinations in such a way that the total transportation cost is minimized whilst satisfying the demands at the various destinations. In this study, the transportation problem of Coca-Cola bottling company, Ghana, was considered. The objectives of the study were (i) to identify the transportation problems of Coca-cola bottling company, (ii) to identify the various determinants of transportation costs of the company and, (iii) to use mathematical model to estimate the optimal number of goods to be transported to various destinations at an optimal transportation cost. In pursuit of this objective, secondary data on the transportation cost and number of crates for the second and third quarters in the year 2012 i.e April-September 2012 was collected and analyzed for this study. Based on few assumptions made, a mathematical model was formulated. The data gathered were modelled as a

Linear Programming model of the transportation type. A computer software, POM-ËQ' for wind—yas-used to solve the problem. The study revealed that the maximum total cost of transporting Coca-cola from Kumasi and Accra to the various destination in Ghana from April-September 2012 was two hundred and two thousand eight hundred and ninety (Gh 202,890) Ghana Cedis.

# List of Tables

# List of Figures

# Contents

# Chapter 1

# INTRODUCTION

## 1.1 Introduction

A key problem managers face today is how to allocate scares resources among various activities or projects. Linear Programming is a method of allocating resources in an optimal way. It is one of the most widely used tools in Operation Management. Linear Programming has been a decision making aid in almost all manufacturing industries and in financial service organisation.

## 1.2 Background of the Study

### 1.2.1 The Transportation System In Ghana

Transport in Ghana is accomplished by road, rail, air and water. Ghana's transportation and communications networks are centered in the southern regions, especially the areas in which gold, cocoa, and timber are produced. The Northern and Central areas are connected through a major road system; some areas,

—Thowever, remain relatively isolated. The deterioration of the country's transportation and communications networks has been blamed for impeding the distribution of eco¶omic inputs and food as well as the transport of crucial exports.

Consequently, the first priority of the Economic Recovery Program (ERP) was to repair physical infrastructure. Under the program's first phase (1983-1986), the government allocated US$ 1.5 billion, or thirty-six (36) percent of total investment for that purpose and an additional US$ 222 million in 1987 for road and rail rehabilitation. In 1991, the Ghana government allocated twenty- seven (27) percent of it's budget for various road schemes. Foreign donor support helped to increase the number of new vehicle registrations from eight thousand (8,000) in 1984 to almost twenty thousand (20,000) in 1989. The distribution of vehicles was skewed, however, because, by 1988, more than half of all vehicles were in Accra, which contained approximately seven (7) percent of the country's population. Furthermore, most new vehicles are intended for private use rather than for hauling goods and people, a reflection of income disparities. Transportation is especially difficult in Eastern region, near the coast, and in the vast, underdeveloped Northern regions, where vehicles are scarce. At any one time, moreover, a large percentage of intercity buses and Accra city buses are out of service.

In Ghana, the modern public transportation system started in the late 1800 when the first rail line was constructed for the commercial exploitation of gold, and the first road created from Accra to the Eastern region. Omnibus Services Authority (OSA), State Transport Company (STC), City Express Services (CES), and lately Metro Mass Transit (MMT) Ltd are public transport companies owned by the government to facilitate the mobility of citizens to various destinations. The public transportation companies are also established for the reasons including one of the social services on government to provide for the people, environmental factors, energy consideration and the promotion of efficient public transportation to increase-productivity (Lamptey-Mills, 2009).

### 1.2.2 Means Of Transportation In Ghana

#### Road Transport In Ghana

Road transport is by far the dominant carrier of freight and passengers in Ghana's land transport system. It carries over ninety-five (95) percent of all passenger and freight traffic and reaches most communities, including the rural poor and is classified under three categories of trunk roads, urban roads, and feeder roads. The Ghana Highway Authority is tasked with developing and maintaining the country's trunk road network totalling thirteen thousand three hundred and sixtyseven (13,367) km, which makes up thirty-three (33) percent of Ghana's total road network of forty thousand one hundred and eighty-six (40,186) km .( "Ghana

Highway Authority" , 2011)

## Rail Transport In Ghana

Rail transport facilitates long distance travel and the transport of bulky goods that cannot easily be transported by motor vehicles. Additionally, it is believed to be one of the safest forms of transport. The chances of accidents and breakdown are minimal as compared to other modes of transport. Moreover, it helps in the management of road traffic.

The railway system in Ghana has historically been confined to the plains south of the barrier range of mountains north of the city of Kumasi. However, a thousand and sixty-seven (1,067) mm (3 ft 6 in) narrow gauge railway, totalling ninehundred and thirty-five (935) km, is presently undergoing major rehabilitation and inroads to the interior are now being made. In Ghana, most of the lines are single tracked, and in 1997 it was estimated that thirty-two (32) km were double tracked. There are no rail links with adjoining countries .( "Ghana Railway

Corporation" , 2011)

## Air Transport In Ghana

Domestic air transport in Ghana has in recent times received a little attention.

This is normally patronized by the rich because it is relatively expensive. It is safe and very fast. Ghana has twelve landing fields, six with hard surfaced runaways. The most important are Kotoka International Airport in Accra and airports at Sekondi-Takoradi, Kumasi and Tamale that serve domestic air traffic.

Water Transport In Ghana

Domestic water transport in Ghana is as a result of non availability of road network connecting the source and destination in question, and its cost effectiveness as compared to other modes of transportation. This mode of transport is essential for passenger, liquid and dry cargo. Relatively short distance travel in the farming communities is normally done by canoes often powered by man. Long distance travel often involves passenger and cargo transport. The types of boats used under this mode are normally powered by internal combustion engines assisted by gear box and propellers. Passenger boat is normally used to transport passengers, charcoal and other food items like salt, fish and yams from the southern to the northern part of Ghana and vice versa.

Also cargo boats aided by tug boats normally transport heavy industrial products like cement, fuel and other minerals in the same direction. The Volta, Ankobra and Tano rivers provide one-hundred and sixty- eight (168) km of perennial navigation for launches and lighters. Lake Volta provides one thousand one hundred and twenty- five (1,125) km of arterial and feeder waterway. There are ferries on Lake Volta at Yeji and KwadjoKrom. There are ports on the Atlantic Ocean at Takoradi and Tema for international transactions. ("Ghana Marine Transport" ,
2011)

A major problem facing many businesses in the world is transportation. Especially, in countries where access to transport is a major problem, many industries and companies lose much profit because of transportation. The transportation problem is oneof the most important and successful applications of quantitative analysis to solving business problems. The transportation problem deals with transporting the commodity from the various sources to the various warehouses at minimum cost while satisfying all constraints of productive capacity and demands. A transportation-problem basically deals with the problem, which aims to find the best way to fulfil the demand of n demand points using the capacities



of m supply points.

## 1.2.3 The Transportation Problem In Ghana

The transportation industry facilitates the movement of goods for the purposes of trade, production and consumption. Good transportation systems are often de scribed as satisfying several quality factors such as cost, time and length (Angus, 2005). Transportation problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply or origins) can be transported to a number of warehouses (called demand destinations). Transportation problems arise whenever there is a physical movement of goods through a variety of channels of distribution (wholesalers, retailers, distributors etc.); there is therefore a need to minimize the cost of transportation so as to increase the profit on sales.

It aims at providing assistance to the top management in ascertaining how many units of a particular product should be transported from each supply origin to each demand destination so that the total prevailing demand for the company's product is satisfied, while at the same time the total transportation cost is minimized.

# 1.3 Statement Of The Problem

Manufacturing firms produce and deliver products to their customers using a logistics distribution network. Such networks typically consist of product flows from the producers to the customers through distribution centres (warehouses or depots). The firms enerally face problems in making decisions on production planning, inventory levels, and transportation in each level of the logistics distribution network in such a way that customer's demand is satisfied at a minimum cost. This calls for the introduction of mathematical techniques into the production and transportation planning model.

The transportation problem is to transport the commodity from various sources to the various destinations at a minimum cost while satisfying constraints of productive capacity and demands. This thesis seeks to determine the optimal transportation plan of The Coca-Cola Bottling Company of Ghana Ltd to meet its pre-determined demands of goods to customers at a minimum cost.

## 1.4 Objectives Of The Study

The high cost of transportation is a major hackle to many manufacturing firms. This has called for a study to explore ways to effectively manage transportation related problems. This technique aims at profit maximization and cost minimization. We seek to delve into the transportation of The Coca-Cola Bottling Company Ghana Ltd. The objectives of this thesis among other things are to determine:

(i) Transporting various quantities of a single homogeneous commodity, which are initially stored at various origins to various destinations in such a way that the total transportation cost is minimum.

(ii) Minimizing the cost of transporting goods from one location to another so that the needs of each arrival area are met.

(iii) A supply schedule with minimum cost of transportation.

## 1.5 Justification

This research when completed will help managers, supervisors, directors and all who have responsibility of managing the transportation of goods from supply points to demands points in their operation. The research when completed would enable these decision makers to

(i) Achieve an optimum transportation cost for Coca-Cola Ghana Ltd.
(ii) To help establish a better distribution of product depending on the demands at the various centres so as to maximize profit.
(iii) To help improve efficiency in the transport system.

## 1.6 Research Methodology

The data was obtained from The Coca-Cola Bottling Company of Ghana. Literature on the topic was review which gave insight into what has already been done and the knowledge gab to be filled. Several mathematical concepts and theories were examined and finally the selection of the most appropriate approach.

## 1.7 Limitation Of The Study

The scope of this study is limited to the transportation problem of The CocaCola Bottling Company produced for domestic purposes only. Different cost elements such as: production, inventory, transportation, royalties, advertisement, taxes such as excise duty and VAT and other practical constraints are normally considered in production planning models, the study takes into consideration only the transportation problem. The proposed model under this study is limited to the available data used for the study

## 1.8 Organisation Of The Study

The study consists of five chapters. Chapter 1 discusses the transportation in a. means of transpertatioïin Ghana, problem statement, objectives of the study, justification, methodology, limitation and organization of the thesis. Chapter 2 reviews the detailed literature on the transportation problem/model. Chapter 3 contains the method used to carry out this research. Chapter 4 presents data collection and analysis. Chapter 5, the final chapter, focuses on the conclusions and recommendations of the study.

## 1.9 Summary

In this chapter, we discussed the background, Problem Statement and Objectives of the Study. The methodology, justification and limitations of the study were also put forward. The next chapter presents relevant literature on transportation problem.

# Chapter 2

# LITERATURE REVIEW

## 2.1 Introduction

In this chapter, we shall put forward relevant literature on Transportation problem and its variants.

## 2.2 Review of Existing Literature

The Transportation Problem (TP) is an important Linear Programming (LP) model that arises in several contexts and has deservedly received much attention in literature. The transportation problem is probably the most important special linear programming problem in terms of relative frequency with which it appears in the applications and also in the simplicity of the procedure developed for its solution. ʳfúe following features of the transportation problem are considered to be most important. The TP were the earliest class of linear programs discovered to have totally uni-modular matrices and integral extreme points resulting in considerable simplification of the simplex method. The study of the TP laid the foundation for further theoretical and algorithmic development of the minimal cost network flow problems.

The transportation problem was formalized by the French mathematician Gaspard Monge in 1781. Major advances were made in the field during World War Il by the Russian mathematician and economist Leonid Kantorovich. Consequently, the problem as it is now stated is sometimes Known as the Monge-Kantorovich transportation problem.

Kantorovich (1942) published a paper on continuous version of the problem and later with Gavurian, and applied study of the capacitated transportation problem (Kantorovich & Gavurin, 1949). Many scientific disciplines have contributed towards analysing problems associated with the transportation problem, including operation research, Economics, Engineering, Geographic Information Science and Geography. It is explored extensively in the mathematical programming and engineering literatures. Sometimes referred to as the facility location and allocation problem, the Transportation optimization problem can be modelled as a large-scale mixed integer linear programming problem.

The origin of transportation was first presented by Hitchcock, (1941). The author also presented a study entitled " The Distribution of a Product from Several sources to numerous localities " . This presentation is considered to be the first important contribution to the solution of transportation problems. Hitchcock developed the basic traditional transportation problem, in which the objective is to minimize the cost of transportation of various amounts of a single homogeneous commodity from different sources to different destination.

Koopman (1947) presented an independent study, not related to Hitchcock's and was called " Optimum Utilization of the Transportation System ". These two contributions helped in the development of transportation methods which involve a number of-shipping souFees-and-a number of destinations. The transportation problem, received this name because many of its applications involve determining how to optimally transport goods.

Koopman (1947) based on the work done earlier by Hitchcock, led an independent research on the tendencies of linear programs for the study of problems in Economics. Hence, referring to the classical case of the transportation problem as Hitchcock-Koopman's transportation problem which aims at total transport cost minimization associated with moving a commodity to its final destination. The transportation problem however, could be solved optimally as an answer to complex business problem only in 1951, when Dantzig applied the concept of Linear Programming in solving the transportation model.

Dantzig, then uses the simplex method on transportation problem as the primal simplex transportation method.

The first algorithm to find an optimal solution for the uncapacitated transportation problem was that of Efroymson and Ray . The authors assumed that each of the unit production cost functions has a fixed charge form. But they remarked that their branch - and - bound method can be extended to the case in which each of these functions is concave and consists of several linear segments and each unit transportation cost function is inear.

A decomposition principle was later made by Dantzig and Wolf to the solution of the Hitchcock transportation problem and to several generalizations of it. In this generalizations, the case in which the costs are piecewise linear convex functions is included. They decomposed the problem and reduced to a strictly linear program. In addition they argued that the two problems are the same by a theorem that they called the reduction theorem. The algorithm given by them, to solve the problem, is a variation of the simplex method with " generalized pricing operation ". It ignores the integer solution property of the transportation problem so that some problems of not strictly transportation type, and for which the integer solution property may not hold be solved. They also formulated an algorithm-to solve transpoTtãtïðWproblems taking nonlinear costs. They considered the case when a convex production cost is included at each supply center besides the linear transportation cost. Some of the approaches used to solve the concave transportation problem are presented as follows. The branch and bound algorithm approach is based on using a convex approximation to the concave cost functions. It is equivalent to the solution of a finite sequence of transportation problems. The algorithm was developed as a particular case of the simplified algorithm for minimizing separable concave functions over linear polyhedral. (Falk & Soland, 1969).

Caputo and Pelagagge, (2000) presented a methodology for optimally planning long-haul road transport activities through proper aggregation of customer orders in separate full-truckload or less-than-truckload shipments in order to minimize total transportation costs. The authors demonstrated that evolutionary computation techniques may be effective in tactical planning of transportation activities. The model shows that substantial savings on overall transportation cost may be achieved adopting the methodology in a real life scenario.

Roy and Gelders (1981) solved a real life distribution problem of a liquid bottled product through a 3-stage logistic system; the stages of the system are plantdepot, depot-distributor and distributor-dealer. The authors modelled the customer allocation, depot location and transportation problem as a 0-1 integer programming model with the objective function of minimization of the fleet operating costs, the depot setup costs, and delivery costs subject to supply constraints, demand constraints, truck load capacity constraints, and driver hours constraints. The problem was solved optimally by branch and bound, and Langrangian relaxation.

Hammer (1969) introduced a concept of time-minimizing algorithm for solving the transportation problem. In a related development, a school of taught is of the view that the objective of the transportation problem is to minimize total transportation cost plus expected penalty costs arising from stochastic transportation problems=- This is seelb.as-arfitêfåtive method for the solution of time-minimizing transportation problems.

Fisher and Jaikumar (1981) developed a generalized assignment for vehicle routing. They considered a problem where a multi-capacity vehicle fleet delivers products stored at a central depot to satisfy customer orders. The routing decision involves determining which of the demands will be satisfied by each vehicle and what route each vehicle will follow in servicing its assigned demand in order

to minimize total delivery cost. They claim their heuristics will always find a feasible solution if one exists, something no other existing heuristics (until that time) can guarantee. Further, the heuristics can be easily adapted to accommodate many additional problem complexities.

Jacques et al., (1988) examined a class of asymmetrical multi-depot vehicle routing problems and location-routing problems, under capacity or maximum cost restrictions. The problem was formulated as a Travelling Salesman Problem

(TSP) in which it is required to visit all specific nodes exactly once and all non-specified nodes at most once. There exist capacity and maximum cost constraints on the vehicle routes; plus, all vehicles start and end their journey at a depot, visit a number of customers and return to the same depot.

Bhatia et al., (1974) discussed such a problem with stochastic demand and penalties for over supply and under supply demand.

Wilson (1975) used a linear approximation method to solve the stochastic transportation problem as a capacitated transportation problem.

Sharma (1977) identified classical transportation problem as one of the many well-structured problems in operation research that has been extensively studied along-side other problem such as travelling sales and shortest-route problems. The author stated that different types of transportation problems have been developed and the simplest of them was first presented by Hitchcock (1941) along with a constructive solution in his work "The distribution of products from several sources to numerous localities Sharma further argues that the transportation problemiyprobably the-rnošfiiiiþortant linear programming problem in terms of relative frequency with which it appears in the application and the simplicity of the procedures developed for its solutions.

Transportation problems were the earliest class of linear programming application amongst the many other features to have totally uni-modular matrices and extreme point that resulted in the simplification of the simplex,which was later develop by (Danzig,1947).

An early study of the transportation problem was made by (Tolsto, 1930). The author published in a book on transportation planning issued by the National Commissariat of Transportation of the Soviet Union, an article called Methods of finding the minimal total kilometres in cargo-transportation planning in space, in which he formulated and studied the transportation problem, and described a number of solution approaches, including the now well-known idea that an optimum solution does not have any negative-cost cycle in its residual graph. Tolsto illuminated his approach by application to the transportation of salt, cement and other cargo between sources and destination along the railway network of the Soviet Union. He first considered the transportation problem for the cases where there are only two sources. He observed that in that case, one can order the destinations by the differences between the distances to two sources. Then one source can provide the destination starting from the beginning of the list, until the supply of that source has been used up. The other source supplies the remaining demands. It was observed that the list is independent of the supplies and demands,and hence it is applicable for the whole life-time of factories or sources of production. Next Tolsto studied the transportation problem in the case when all sources and destinations are along one circular railway line in which case the optimum solution is readily obtained by considering the difference of two sums costs. The author called this phenomenon circle dependency. Finally, he combined the two ideas into a heuristic to solve a concrete transportation problem coming from cargo transportation along the Soviet railway network.

~~nes and~~Cooper (1-95Ù—dêfiQed an intuitive presentation of Dantzig's procedure called the stepping-stone method which follows the basic logic of the simplex method but avoids the use of the tableau and the pivot operations required to get the inverse of the basis.

An algorithm was formulated to solve transportation problems taking nonlinear costs. It considered the case when a convex production cost is included at each supply center besides the linear transportation cost. It also assessed the concavity of the cost curve brought about by economics of scale leads to multiple-optima, and thus problems like these are not susceptible to conventional mathematical techniques. The power of the Simplex method in solving linear programs is based on the general theorem which states that the number of variables including slack variables, whose values are positive in an optimal solution, is at most equal to the number of constraints in the problem. Algorithms for the capacitated case have been presented by (Efroymson & Ray, 1966). In all of these the cost functions are assumed to be linear and the production cost is linear where ever the production is and zero where not. This technique allows the easy incorporation of configuration constraints that restrict the allowable combinations of open plants and generalization of the production.

Klingman and Russell (1975) developed an efficient procedure for solving transportation problems with additional linear constraints. Their method exploits the topological properties of basis trees within a generalized upper bound framework. Sharma (1977) developed the same concepts for multi-dimensional transportation problem.

Gass (1990) detailed the practical issues for solving transportation problems and offered comments on various aspects of transportation problem methodologies along with discussions on the computational results, by the respective researchers. Denardo et al ., (1988) researched into the transportation problem in which cost depends on the order of arrival. The authors developed a problem which uses supplied item travel time averages to determine the cost of satisfying the demand at a partictilar location. The authors observed that items that arrived first re-

ceived the greatest weight, and decreasing weight were given to each succeeding item. An equivalent transportation problem was used for problems with a known demand. They concluded that if the demand is stochastic transportation problem whose aim is to minimizeJThe sum of a linear function is used and the function is linearised by substituting the product and a linear term for the convex function.

Equi et al., (1996) modelled a combined transportation and scheduling in one problem where a product such as sugar cane, timber or mineral ore is transported from multi origin supply points to multi destination demand points or transhipment points using carriers that can be ships, trains or trucks. They defined a trip as a full-loaded vehicle travel from one origin to one destination. The authors solved the model optimally using Langrangean Decomposition.

Nagurney (2004) disclosed that the transportation problem is one of the subclasses of linear programming problems for which simple and practical computational procedures have been developed to take advantage of the special structure of the problem with the objective to transport various quantities of a single homogeneous product that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. Transportation problems are complex, large-scale systems, and come in a variety of forms, such as road, rail, air, and waterway networks. Transportation problems provide the foundation for the functioning of our economies and societies through the movement of people, goods, and services.

Mills-Lamptey (2009) considered the transportation problem of Accra Brewery Limited , Ghana. The company produces different beverages with the aim of distributing their products to the ten regions of Ghana at a minimum total cost. The archival data for the past ten years was used. The huge transportation cost in the past showed that there was a need to investigate the cause. The study was aimed at formulating a transportation model to minimize the cost of transporting the beverage within the entire country. The author collected secondary data on the transportation cost-a-rõñüiiiber of beer transported for the past six months in the year 2007 i.e May - October and analysed them. The objective was to develop a mathematical model to optimize the total transportation cost for the Distribution Department of Accra Brewery Limited.

Osman (2010) reviewed the different methods that have been proposed in solving transportation problems and in particular focused on transportation in a supply chain system involving a brewery in which he considered a number of truckloads of the finished product hauled from the brewery to its distribution centres as the unit of transportation. In particular he dealt with the minimum cost principle and hence formulated the model to minimize the total cost of distribution in the par ticular case study where data for the model was obtained. He solved the problem using mathematical algorithms and compared the results with commercial tools to find the optimal schedule that minimized the total cost of transportation.

## 2.3 Summary

In this chapter, we have reviewed some literature on transportation problem .The research methodology is detailed in the next chapter.



# Chapter 3

# METHODOLOGY

## 3.1 Introduction

In this Chapter, the transportation problem and it's variants are discussed. The transportation problem seeks to minimize the total shipping costs of shipping goods from m origins, each with a supply Si, to n destinations, each with a demand dj, when the unit shipping cost from an origin, i, to a destination, j, is Cij. We shall first put forward the profile of the Coca Cola bottling Company.

## 3.2 Profile Of The Coca Cola Bottling Company

The Coca-Cola Bottling Company of Ghana (TCCBCG) was formed from the Divestiture of the Bottling Section of Ghana National Trading Company (GNTC) in March 1995 and con—ngecLoperations on March 7, that same year at the old plant at Adjabeng. Currently the company has acquired an ultra modern premise — bf the Spintex Road, which was commissioned in November 1996.

The new company is wholly owned by the Equatorial Coca-Cola Bottling Company, which is in turn owned by the Cobega Group in Spain and the Coca-Cola Company - Atlanta who own seventy (70) and thirty(30) percent respectively. Since its divestiture the company has invested over USS 70 million in production and marketing equipment, vehicles, glass bottles and plastic crates.

In 1999, the company added a second bottling line at a cost of US$ 10 million, while a PET line was added in 2003 at an additional US$ 1 million for bottling products in plastic bottles. Another pouch line was also commissioned at the Accra plant last year. An US$ 8 million expansion program has also be commenced which include a new bottling line with a capacity of 48,000 bottles per hour and was commissioned in November, 2004. The company directly employs 745 staff currently as compared to the pre-divestiture level of 372. Additionally, TCCBCG has also registered about 60 independent Mini-Depot Operators (Independent Distributors) each of which employs at least four people. By the introduction of the HAWAI product, such operators have employed additional vendors who will sell the product on bicycles provided for that purpose. To properly operationalize its environmental friendly policy the company commissioned a US$ 1 million Waste Water Treatment facility at its Accra Plant in 2001 and followed up with another US$600,000 facility at the Kumasi Plant in January 2004.

These facilities ensure that the effluent from the production process is treated before being discharged into natural water sources. Thus water from the production facility can support marine life while the sludge from the Waste Water Treatment processes serves as manure for local farmers. The company has over the years fulfilled its tax obligations to the state. Taxes, comprising of excise duty and VAT paid to the government in 2003 grossed over US$105 billion as against over US$ 78 billion paid 2002.

Community relations here-afiiäýOeen directed towards support for Health Institutions, Schools and Non-governmental Organizations. Prominent amongst the support are donations to the Otumfuo Education Trust Fund and the First Lady's Mother and Child Foundation.

The company provided a fully furnished six classroom block and an office for the residents of Samsam Odumasi in the Amasaman District of the Greater Accra Region at the cost of about US$30,000 in 2003. That same year it donated computers and printers to four institutions while it sponsored the Coca-Cola Top 4 Soccer tournament to the tune of US$ 1.5 billion in 2004.

## 3.3 The Transportation Problem

The transportation problem is a special case of the Simplex Method. It gets it's name from it's application to problems involving transporting products from several sources to several destinations. A basic transportation problem has fixed supply equal to fixed demand and the goal is to find the least cost way to deliver all the goods from supply nodes to demand nodes. The objective of the transportation problem is to transport various quantities of a single homogeneous commodity, which are initially stored at various warehouses to various destinations in such a way that the total transportation cost is minimum. Basically, the transportation problem is to transport the commodity from the various sources to the various warehouses at a minimum cost while satisfying all constraints of demands. The two common objectives of such problems are either

(i) minimize the cost of shipping n units to m destinations

OR

(ii) maximize the profit of shipping n units to m destinations.

A special form of linear programming, that is Transportation Model, can be used to com are the total transportation cost associated with each alternative site. The-transportatien<öaêÃechnique can be used to determine how many units should be shipped from each plant to each warehouse to minimize total Transportation Cost.

## 3.4 Tabular Representation of the Transportation Problem

A commodity is being produced at the sources Sl , S2 and S3 and are shipped to the various destinations dl , (12 and (13. The unit cost associated with transporting goods from source i to destination j is Cij. For example the unit cost of transporting from source 2 to destination 1 is C21 and so on. The transportation problem is to transport the commodity from the various sources to the various warehouses at a minimum cost while satisfying all constraints of productive capacities and demands. Tabularly, the problem is represented below as

Table 3.1: A Tabular Representation of the Transportation Problem

| | Destination 1 | Destination 2 | Destination 3 | Supply |
|---|---|---|---|---|
| Sources 1 | 11 | 12 | 13 | |
| Sources 2 | 21 | 22 | 23 | $2 |
| Sources 3 | 31 | 32 | 33 | |
| Demand | | | (13 | TOTAL |

# 3.5 Mathematical Model of the Transportation problem

Suppose a Ÿognmodity is being produced at the sources Sl , S2 , S3 , and shipped to the various destinatiofiT,-G, (13 and so on. The transportation problem is to transport the commodity from the various sources to the various destinations at a minimum cost while satisfying all constraints of productive capacities and demands. Mathematically, a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints.

Let

Si quantity of the commodity available at the source i.

dj = demand of the commodity at destination, j.

Cij = transportation cost of a unit of commodity from source i to destination j.

$x_{ij}$ = quantity transported from source i to destination j.

## 3.5.1 Definitions

### The Decision Variable

Decision variables are the variables within a model that one can control. They are not random variables. In other words, a decision variable is a quantity that the decision-maker controls. The variables in the linear programming model of the Transportation Problem will hold the values for the number of units shipped from one source to a destination.

The decision variables are where $i = 1, 2, \ldots$ m and $j = 1, 2, 3, \ldots$ n . This is a set of m x n variables.

### The Objective Function

It is an equation to be optimized given certain constraints and with variables that need to be minimized or maximized using nonlinear programming techniques. An objective function can be the result of an attempt to express a business goal in mathematical terms for use in decision analysis, operations research or optimization studies.(The Objec>ýnctcon.• avaialable at www.businessdictionary.com, 2012). In other words, it is the function to be minimized or maximised. The úbjective function contains costs associated with each of the decision variables.

Under Transportation problem, the objective function is a minimization problem. Consider the shipment from warehouse i to outlet j. For any i and j, the transportation cost per unit is Cij and the size of the shipment is Tij. Then we assume that the total cost of this shipment is given by CijCij. Summing over all i and j now yields the overall transportation cost for all ware house-outlet combinations,

thus our objective function is

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \tag{3.1}$$

## The Constraints

The constraints are the conditions that force supply and demand needs to be satisfied. In a transportation problem, there is one constraint for each node.

Let Si denote a source capacity and dj denote destination needs.

(i) The supply at each source that must be used

$$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad i = 1, 2, 3, ...m \tag{3.2}$$

(ii) The demand at each destination that must be met

$$\sum_{i=1}^{m} x_{ij} \geq d_j \qquad j = 1, 2, 3, ...n \tag{3.3}$$

(iii) Non negativity:

$\forall \quad i$ and j (3.4)

The transportation model will then become

Minimize z=EEcij ij (3.5)

subject to

$$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad i = 1, 2, 3, ...m \tag{3.6}$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j \qquad j = 1, 2, 3, ...n \tag{3.7}$$

Cij > 0 and $j$ (3.8)

This is a linear program with m x n decision variables, rn+n functional constraints and m x n nonnegative constraints.

Each source has a certain supply of units to distribute to the destination and each destination has a certain demand for units to be received from the sources. The model for a transportation problem makes the following assumptions about these supplies and demands.

## The required assumptions are:

(i) Each source has a fixed supply of units where this entire supply of units must be distributed to the destination.

(ii) Each destination has a fixed demand for units where this entire demand must be received from the sources.

## Remarks

The assumption that there is no allowance in the amounts to be sent or received means that there need to be a balance between the total supply from all sources and the total demand at all destinations.

## The cost assumption

The cost of distributing units from any particular source to any particular destination is directly proportional to the number of units distributed. Therefore, this cost is just-the unit cost of distribution times the number of units distributed.

We let Cij denote this unit cost from source i and destination j. The only data — needed for a transportation problem model are the supplies, demands, and unit costs. These are the parameters of the model.

The feasible solutions property

A transportation problem will have feasible solutions if and only if the total supply equals the total demand, that is

$$\sum s_i = \text{Edo} \tag{3.9}$$

## 3.6 Formulating The Transportation Problem as a linear programming model

We consider an example of the formulation of mathematical model of transportation problem of transporting single commodity from three sources of supply to three demand destinations.

Let s denotes the source of the goods and d denotes the demands at various destinations.

Further, let

Tij= quantity shipped from plant Si to the destination dj and

Cij= transportation cost per unit of shipping from plant Si to the destination dj.

The problem can be represented as

Minimize

Z ClIŒ11 + C12X12 + C13Œ13

+C21X21 + C22Œ22 + C23X23 Cost of shipping from factory to warehouses

+C31Œ31 + C32X32 + C33œ33

(3.10)

subject to

Cll •T13 < Sl 1

X21 + C22 + 2 $23 S2supply constraint (3.11)

X31 + Œ32 + X33 < S3 3

C11 + Œ12 + X13 < d1

£21 + C22 + X23 < (12Demand constraint (3.12) x31 + £32 + x33 d3

Tij > 0 V i and j (3.13)

It is further assumed that S1+S2+S3 = d1+d2+d3 (i.e the total supply available at the plants exactly matches the total demand at the destination warehouse). Hence there is neither excess supply nor excess demand. Since number of variables are very high, simplex method is not appropriate. Thus the effective number of constraints on the balanced transportation problem is (m + n — 1). Hence we expect a basic feasible solution of the balanced transportation to have (m -kn — 1) non-negative entries.

## 3.7 Balanced Transportation Problem

A transportation problem is $= \sum_{j=1}^{n}$ balanced if dj . Again if a problem is balanced, $\sum_{i=1}^{m} x_{ij}$ then st — for each i and dj = for each j.

Table 3.2: A Balanced Transportation Problem

| From \ To | A | B | C | Supply |
|---|---|---|---|---|
| 1 | 6 | 8 | 10 | 150 |
| 2 | 7 | 11 | 11 | 175 |
| 3 | 4 | 5 | 12 | 275 |
| Demand | 200 | 100 | 300 | 600 |

In summary, a balanced transportation problem has the total supply equals the total demand at any instant. Hence the balanced transportation problem may be written as

Minimize

z CijXij (3.14)

subject to

Xij = Si $i = 1, 2, 3, ...m$ (3.15)

:tij = d, $j = 1, 2, 3, ...n$ (3.16)

$\geq 0$ V i and j (3.17)

## 3.8 Unbalanced Transportation Problem

For an unbalanced transportation problem Si E dj. This is no restriction since the unbalanced problem can always be converted to an equivalent balanced problem to which the special method may be applied. We introduce a dummy origin in the transportation tableau, the cost associated with this origin is set equal to zero. The availability at this origin is

$$\sum_{j=1}^{n} d_j \quad \text{\_ESi=o} \tag{3.18}$$

—There are two cases:

Case 1:

$$\sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j \tag{3.19}$$

Case 2:

$$\sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j \tag{3.20}$$

If the transportation model is unbalanced, it is balanced by either creating a fictitious (or dummy) source or destination to absorb the difference. When the total supply exceeds the total demand as in equation (3.19), the problem is solved by creating a dummy destination whose demand is exactly the excess. Since shipment to the dummy demand points are not real, they are assigned a cost of zero. In the case of where the total demand exceeds the supply as in equation (3.20) a dummy source is created to transport the difference. The unit cost of transporting from a dummy source or to a dummy destination is taken to be zero.

## 3.9 Redundancy in the Constraints

The constraints are

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad i = 1, 2, 3, ...m \tag{3.21}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = ...n \tag{3.22}$$

### Theorem

There is exactly one redundant equality constraint in equations (3.21) and (3.22). When any one of the c>lntsin equations (3.21) and (3.22) is dropped, the remaining is a linearly independent system of constraint. If a transportation problem has a total supply that is strictly less than total demand the problem has no feasible solution. There is no doubt that in such a case one or more of the demand will be left unmet. Generally, in such situations a penalty cost is often associated with unmet demand and as one guess this time the total penalty is desired to be minimum.

## 3.10 Network Representation of the Transportation Problem

Graphically, transportation problem is often visualised as a network with m source nodes, n sink nodes and a set of m x n " directed arcs " as shown in the Figure 3.1



Units of supply source Units of Destination Deman

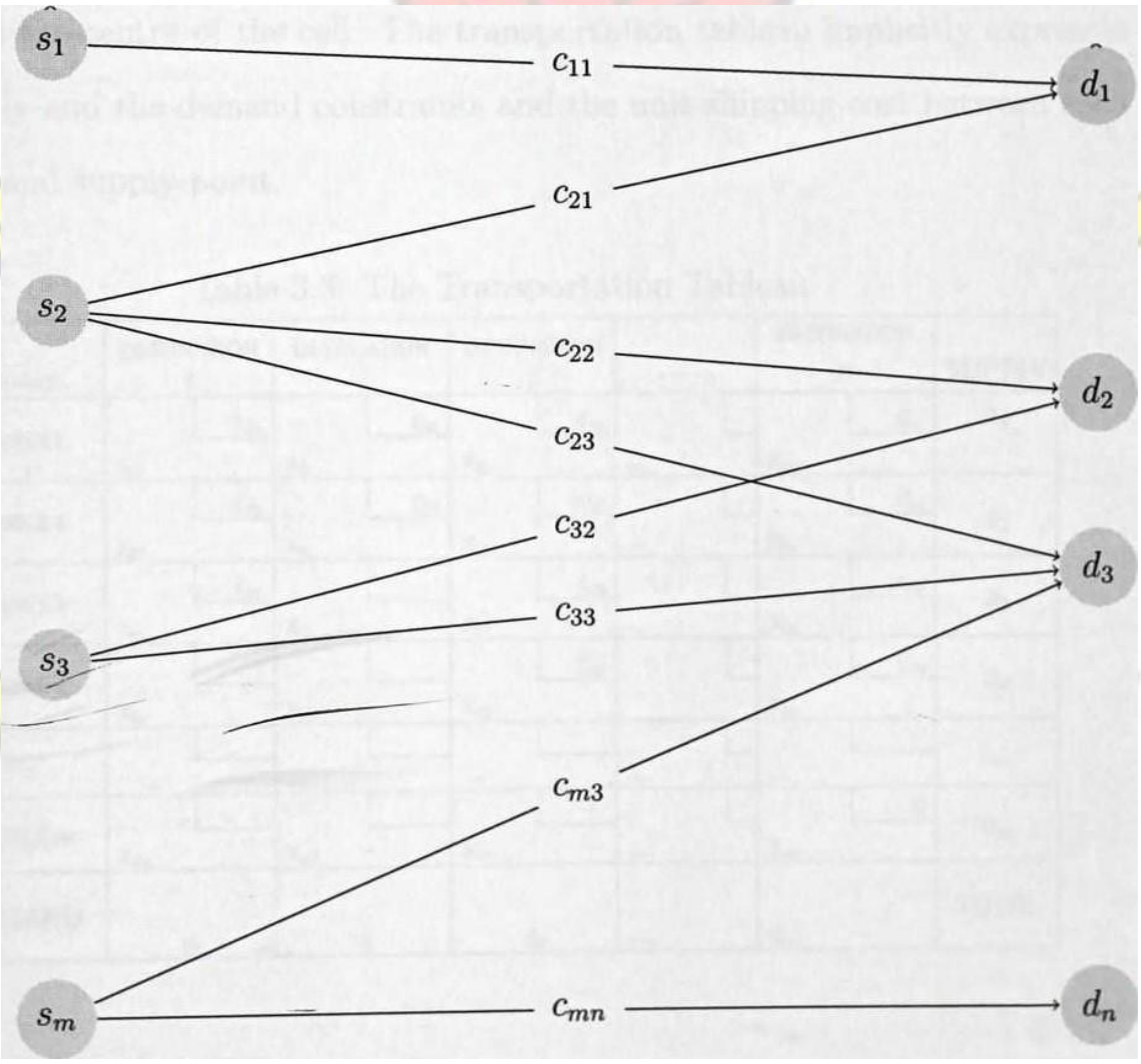



Figure 3.1: Network Representation of the Transportation Problem

In the diagram there are Sl, s2, sources and dl, (12, dn destinations. The arrows show flows of output from source to destination. Each destination is linked to each source by an arrow. The numbers Cl, c2, ...,cn on each arrow represent the unit cost of transporting on that route.

## 3.11 Transportation Tableau

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau. The transportation tableau is represented in standard matrix form, where supply availability (ai) at each source is shown in the far right column and the destination requirement(di) are shown in the bottom row. The unit shipping costs Cij is shown in the upper right corner of the cell, the amount of shipped material is shown in the centre of the cell. The transportation tableau implicitly expresses the supply and the demand constraints and the unit shipping cost between each demand and supply point.

Table 3.3: The Transportation Tableau

| SOURCE | DESTINATION 1 | DESTINATION 2 | DESTINATION 3 | ..... | DESTINATION $n$ | SUPPLY |
|---|---|---|---|---|---|---|
| SOURCE 1 | $C_{11}$ $X_{11}$ | $C_{12}$ $X_{12}$ | $C_{13}$ $X_{13}$ | .... | $C_{1n}$ $X_{1n}$ | $a_1$ |
| SOURCE 2 | $C_{21}$ $X_{21}$ | $C_{22}$ $X_{22}$ | $C_{23}$ $X_{23}$ | .... | $C_{2n}$ $X_{2n}$ | $a_2$ |
| SOURCE 3 | $C_{31}$ $X_{31}$ | $X_{23}$ | $C_{33}$ $X_{33}$ | ... | $C_{3n}$ $X_{3n}$ | $a_3$ |
| SOURCE 4 | $X_{41}$ | $X_{24}$ | $C_{34}$ $X_{43}$ | | $C_{4n}$ $X_{4n}$ | $a_4$ |
| ...... | .... | .... | .... | .... | .... | .... |
| SOURCE m | $X_{m1}$ | $X_{m2}$ | $X_{m3}$ | ..... | C $X_{mn}$ | $a_m$ |
| DEMAND | $d_1$ | $d_2$ | $d_3$ | ..... | $d_{mn}$ | TOTAL |

m —¥ number of sources.

n —Y number of destination.

ai —Y capacity of i — th source.

dj —Y demand of the j — th destination.

Cij —¥ unit material shipping cost between ith source and the jth destination. $x_{ij}$ —Y amount of material shipped between ith source and jth destination.

# 3.12 Solution of Transportation Model

The solution of a transportation model is characterized by three stages:

(i) Obtaining an initial basic feasible solution.

(ii) Checking an optimality criteria that indicates whether or not a termination condition has been met.

(iii) Developing a procedure to improve the current solution if a termination condition has not been met.

The requirement for application of a transportation technique is that the model must be balanced.

## 3.12.1 Definitions

### Cell

It is a small compartment in the transportation tableau.

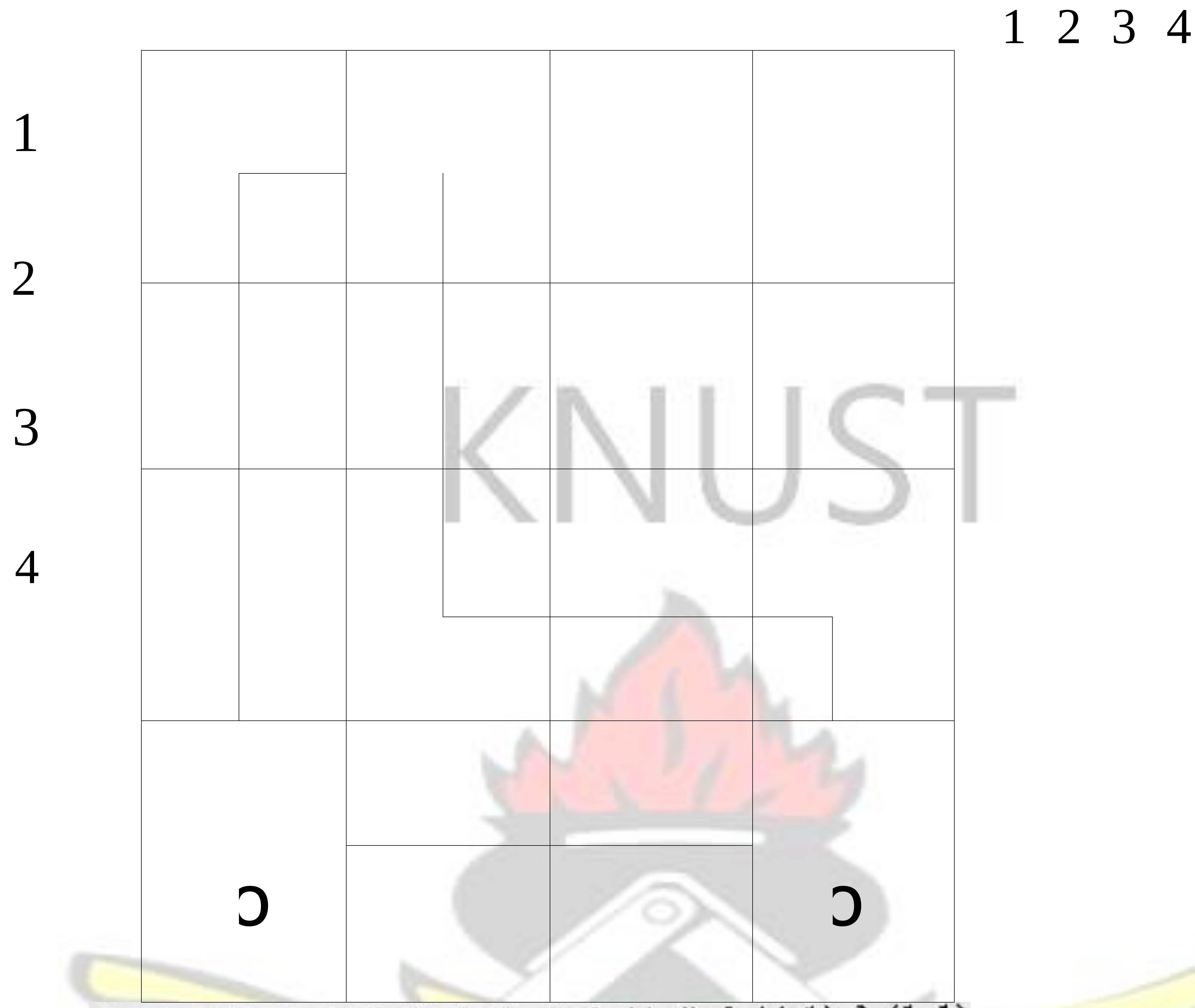
### Allocation

The number of units ofÃ5<ðGßÍ1ipped from a source to a destination which is recorded in a cell in the transportation tableau.

### Circuit

A circuit is a sequence of cells (in the balanced transportation tableau) such that:

(i) It starts and ends with the same cells.
(ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tableau.




(1,1)→(1,2)→(3,2)→(3,4)→(4,4)→(4,1)→(1,1)

Figure 3.2: An example of a circuit

## Basic variables

They are the variables in a basic solution whose values are obtained as the simultaneous solution to the system of equations that conform to the functional constraints.

## Feasible solution

_Aset of non-negative allocation > 0 which satisfies the row and column restrictions is known as feasible solution.

## Basic Feasible Solution(BFS)

A solution is called a basic feasible solution if

(i) It involves (m + n — 1) cells with non-negative allocation:-
(ii) There are no circuits among cells in the solution.

If the number of allocations in a basic feasible solutions are less than (m + n — 1), it is called degenerate basic feasible solution otherwise non-degenerate.



## Optimal Solution

A feasible solution ( not necessary basic) is said to be optimal if it minimizes the total transportation cost.

## Penalty cost

A penalty cost is the difference between the largest and the next largest cell cost in a row or column.

## Degenerate LP

An LP is degenerate if in the basic feasible solution, one of the basic variables takes on a zero value.

## Loop

An ordered sequence of at least four different cells is called a loop if:

(i) any two consecutive cells lie in either the same row or same column.

(ii) no three consecutive cells lie in the same row or column.

(iii) the last cell in the—uence-has a row or column in common with first cell in the sequence.

In a balance transportation problem with m supply points and n demand points, the cells corresponding to a set of (m + n — 1) variables contains no loop iff the (m + n — 1) variables yield a basic solution. This follows from the fact that a set of (m + n — 1) cells contains no loop iff the (m + n — 1) columns corresponding to these cells are linearly independent.

## Degeneracy

Degeneracy in a transportation problem arises when the number of occupied cells is less than (m + n — 1). The degeneracy can develop in two ways

(i) The degeneracy develops while determining an initial assignment via any of the initial basic feasible solution method.

(ii) The degeneracy develops at the iteration stage. This happens when the selection of the entering variable results in the simultaneous drive to zero of two or more current (pre-iteration) basic variables.

Then it is necessary to make one or more zero allocation to (m + n — 1). This means that for cost calculation purposes, one or more cells with zero allocation are treated as occupied. The zero allocation is chosen in such a way that:

(i) the total numbers of cells with allocation is (m + n — 1).

(ii) there is not a circuit among the cells of the solution.

## Prohibited routes

Sometimes one or more of the routes in the transportation model are prohibited. That is, units cannot be transported from a particular source to a particular destination. When this situation occurs, we must make sure that no units in the optimal solution are allocated to the cell representing this route. A value of M is assigned as twe transportation cost for a cell that represents a prohibited route.

Thus, when the prohibited cell is evaluated, it will always contain a large positive —cowchange of M, which will keep it from being selected as an entering variable.

## 3.13 Flow Chart Solution for the Transportation Problem

A flowchart is a type of diagram that represents an algorithm or process, showing the steps as boxes of various kinds, and their order by connecting them with arrows. This diagrammatic representation can give a step-by-step solution to a given problem. Process operations are represented in these boxes, and arrows connecting them represent flow of control. Data flow are not typically represented in a flowchart, in contrast with data flow diagrams; rather, they are implied by the sequencing of operations. Flowcharts are used in analysing, designing, documenting or managing a process or program in various fields.

Figure 3.3 depicts the flow chart solution of the Transportation problem.

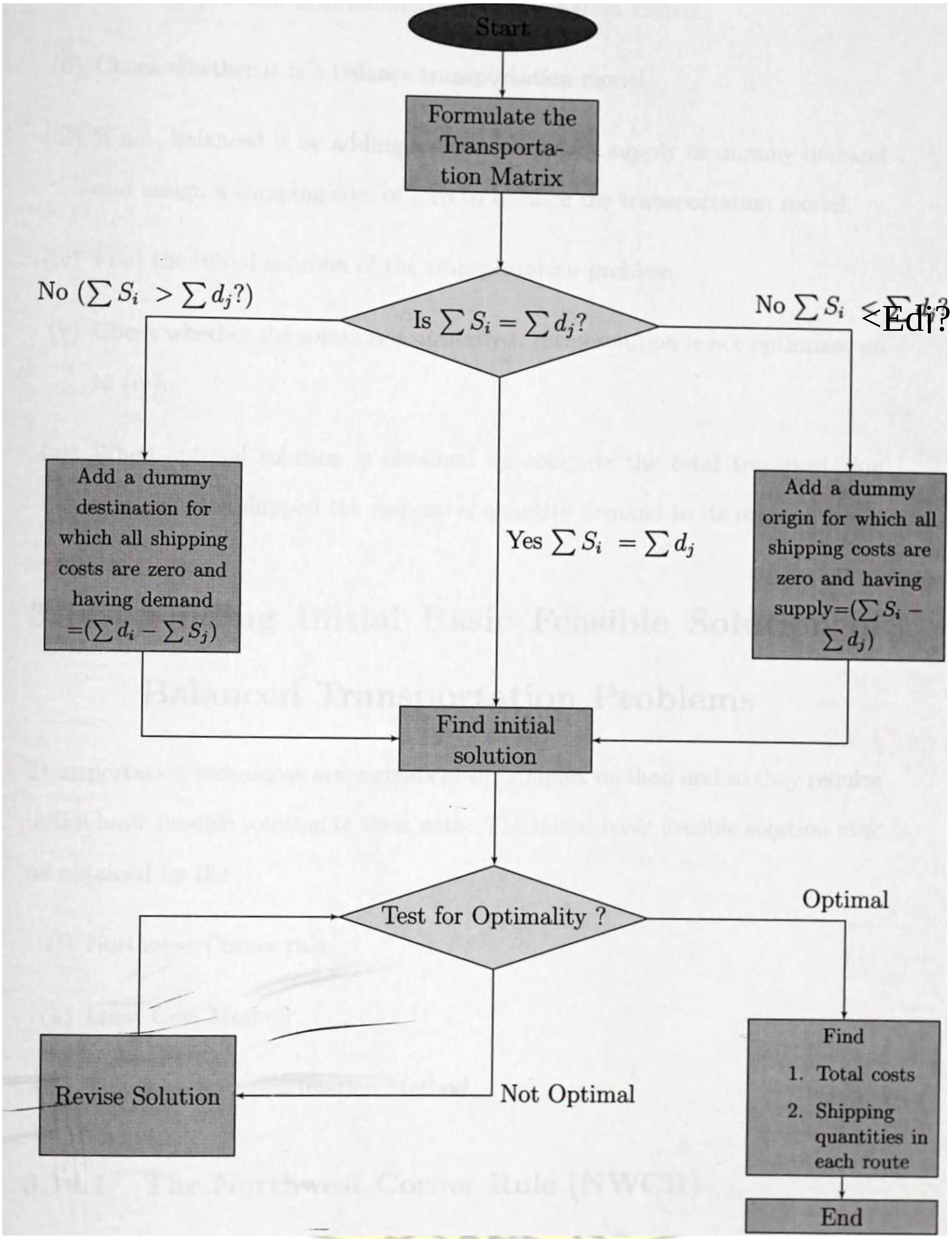



Figure 3.3: Flow Chart Solution for the Transportation Problem

Summary description of the Flow chart

(i) First the problem is formulated as transportation matrix.

(ii) Check whether it is a balance transportation model.

(iii) If not, balanced it by adding a either a dummy supply or dummy demand and assign a shipping cost of zero to balance the transportation model.

(iv) Find the initial solution of the transportation problem.

(v) Check whether the solution is optimized, If the solution is not optimized go to (iv).

(vi) When optimal solution is obtained we compute the total transportation cost and also shipped the respective quantity demand to its route.

## 3.14 Finding Initial Basic Feasible Solution of Balanced Transportation Problems

TYansportation techniques are variants of the simplex method and so they require initial basic feasible solution to start with. The initial basic feasible solution may be obtained by the ,

(i) Northwest Corner rule

(ii) Least ast Method

(iii) The Vogel's Approximation Method

### 3.14.1 The Northwest Corner Rule (NWCR)

The North West Corner Rule is a method for computing a basic feasible solution of a transportation problem, where the basic variables are selected from the NorthWest corner. In the NWCR, the largest possible allocation is made to the cell in

the upper left hand corner of the tableau followed by the allocations to adjacent feasible cells.

The major advantage of the Northwest Corner Rule method is that it is very simple and easy to apply. It's major disadvantage, however, is that it is not sensitive to costs and consequently yields poor initial solution. The steps involved in determining an initial solution using Northwest Corner rule are as follows:

Step 1: Select the north west (upper left hand) corner of the transportation tableau and allocates as many units as possible equal to the minimum between available supply and demand. i.e min (1,1)

Step 2: Adjust the supply and demand numbers in the respective rows and columns.

Step 3: If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.

Step 4: If the supply for the first row is exhausted, then move down to the first cell in the second row.

Step 5: If for any cell supply equals demand, then the next allocation can be made in cell either in the next row or column.

Step 6: Continue the process until all supply and demand values are exhausted.

## Remarks

In case of Degéneracy, the solution obtained by this method is not a Basic Feasible Solution because it has fewer than (m + n — 1) cells in the solution. This happens —because at some point during the allocation when a supply is used up there is no cell with unfulfilled demand in the column. In the non-degenerate case, until the end, whenever a supply is used up there is always an unfulfilled demand in the column. Even in the case of degeneracy the Northwest corner rule still yields a basic feasible solution if it is modified as follows. Having obtained a solution which is not a basic feasible solution choose some empty cells and add there the solution with circled zeros in them, so that it will produce a basic feasible solution, that is

(i) The total number of cells with allocations should be (m -l- n — 1).

(ii) There should be no circuit in any of the cells of the solution.

### 3.14.2 The Least Cost Method

The least cost method identifies the least unit cost in the transportation tableau and allocate as much as possible to the associated cell without violating any of the supply or demand constraints. In the Least cost method, as much as possible is allocated to the cell with the minimum cost. The steps involved in using the Least Cost method are as follows:

Step 1: Select the cell with the least unit transportation cost among all the rows and columns and allocate as many units as possible to that cell.

Step 2: If the minimum cost exists in several cells, select a cell arbitrarily and assign the possible number of goods. Then consider the remaining cells of the same transportation cost.

Step 3: Select a cell with the next higher unit transportation cost and continue the process till all requirement are met.

#### Remarks

The least cosómethod ~~will provide a~~ solution with a lower cost than the North West Corner Rule solution because it considers costs in the allocation process. In —the-minimum cell cost method as much as possible is allocated to the cell with the minimum cost followed by allocation to the feasible cell with minimum cost.

### 3.14.3 The Vogel's Approximation Method (VAM)

It provides a Basic feasible solution which is optimal or close to it and moreover, performs better that the least cost method and the North west Corner Rule. The basic idea of VAM is to avoid shipment that have high cost. This is achieved by computing column penalties by identifying the least unit cost and the next unit cost in that column and taking their positive difference. In a similar way row penalties are computed by taking the positive difference between the least unit cost and the next least unit cost in a row. This method is a variant of the Least Cost Method and is based on the idea that if for some reason, the allocation cannot be made to the least unit cost in a row or column and it is made to the next least unit cost cell in that row or column then the appropriate penalty is paid for not being able to make the best allocation. The steps involve in using the VAM are:

Step 1: Write the given transportation problem in tabular form (if not given).

Step 2: Compute rows and columns penalties (i.e. subtract the least cost element on each row or column from the next least cost element).

Step 3: Identify the row or column with maximum penalty.

Step 4: Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.

Step 5: Repeat steps 2,3 and 4 until all rim requirements have been met.

Step 6: When left with only one row or column use the Least Cost Method.

Remarks:

(i) The Vogel's approximation method usually produces an optimal or near optimal starting solution. It provides a basic feasible solution which is close to optimality and this performs better than the North West Corner Rule or the Least Cost Method.

(ii) Unlike the North West Corner Rule the Vogel's approximation method may lead to an allocation with fewer than (m + n — 1) non-empty cells even in the non-degenerate case.



(iii) Vogel's approximation method and Least Cost Method provide a better initial solutions that the North West Corner Rule.

### 3.14.4 Methods for Improving an Initial Basic Feasible

## Solution of a TYansportation Problem to Optimality

Optimality is achieved when resources are used in a most effective and efficient manner and yields the highest possible return under the circumstances. An op timal solution for a transportation problem is a feasible solution that minimizes the total transportation cost. The solutions obtained from the three methods discussed earlier are feasible but not the optimal. To obtain an optimal solution we make successive improvement to the initial basic feasible solution until no further decrease in the transportation cost is possible. An optimal Transportation solution is one where there is no other set of transportation routes that will further reduce the total transportation cost.

The widely used methods for finding an optimal solution are .

(i) Stepping Stone Method

(ii) Modified Distribution (MODI) method

They differ in their mechanics, but will give exactly the same results and use the same testing strategy.

### Remarks:

Although the transportation problem can be solved using the regular simplex method, it's special properties provide a more convenient method for solving these types of problems. This method is based on the same theory of simplex method. It makes use, however, of some short cuts which provides a less burdensome computational scheme. There is one difference between the two methods.

The simplex method performs the operations on a simplex tableau. The transportation method performs the same operations on a transportation tableau.

## 3.14.5 The Stepping-Stone Solution Method

The Stepping-Stone Method, being a variant of the simplex method, requires an initial basic feasible solution which is then improved to optimality. The SteppingStone Method determines if there is a cell with no allocation that would reduced cost if used. The steps used in the stepping stone solution method are

1. Determine an initial basic feasible solution using any one of the following

   (i) North West Corner Rule (ii)

   Least Cost Method

   (iii) Vogel's Approximation Method

2. Make sure that the number of occupied cells are exactly equal to m + n — 1, where m is the number of rows and n is the number of columns.

3. Select an unoccupied cell. Beginning at this cell, trace a closed path, starting from the selected unoccupied cell until finally returning to that same unoccupied cell. The cells at the turning point are called "Stepping Stones" on the path.

4. Assign plus (+) and (—) signs alternatively on each corner cell of the closed path ju"traced, beginning with the plus sign at unoccupied cell to be evaluated.

5. Add the unit transportation costs associated with each of the cell in the closed path. This will give net change in terms of cost.

6. Repeat steps 3 to 5 until all unoccupied cells are evaluated.

7. Check the sign of each of the net change in the unit transportation costs. If all the net changes computed are greater than or equal to zero, an optimal

solution has been reached. If not, it is possible to improve the current solution and decrease the total transportation cost, so more to step 8

8. Select the unoccupied cell having the most negative net cost change and determine the maximum number of units that can be assigned to this cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the number of units that can be shipped to the entering cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on closed path marked with a minus sign.

If the improvement index of each unoccupied cell in the basic feasible solution is non-negative then the current basic feasible solution is optimal since every re-allocation increases the costs. If there is at least one unoccupied cell with a negative improvement index then a re-allocation to produce a new basic feasible solution with lower cost is possible and so the current basic feasible solution is not optimal.

## Remarks

The current basic feasible solution is optimal if and only if each unoccupied cell has a non-negative improvement index. Once an initial basic feasible solution has been obtained, we use the stepping stone method to achieve optimality. The stepping-stone method determines if there is a cell with no allocation that would reduce cost_if used. The basic solution principle in Stepping Stone Method is to determine whether a transportation route not at present being used (i.e., an empty cell) would result in a lower total cost if it were used.

## 3.14.6 The Modified Distribution Method(MODI)

The MODI is a modified version of the stepping-stone method in which maths equations replace the stepping stone paths.

## Steps Involve in MODI Method

The MODI method can be described as follows:

1. Find a basic solution by any standard method.

2. Test for optimality : The number of occupied cells should equal to m+n—1. If the initial basic feasible solution does not satisfy this rule, then optimal solution cannot be obtained. Such solution is a degenerate solution.

3. Determine a number Ui for each row and a number Vj for each column.

4. Compute the value of Ui and Vj with the formula Ui + Vj = Cij to all basic (occupied) cells.

5. Calculate the value of unoccupied cells using the relation Ui + Vj = Cir

6. Compute the opportunity cost for each unoccupied cell by the Pij = Cij — Ui-Vj.

7. If all Pij > 0, then the solution is optimal. If one or more Pij < 0, then the solution is not optimal. If at least one Pij > 0, then the solution is optimal and unique.

8. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.

9. Draw path or loop for the unoccupied cell selected in the previous step. Please note--tÃiÇóght angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

10. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.

11. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed

path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

## 3.15 Summary

This Chapter presented the research methodology of the study. The next Chapter is devoted for the data collection and analysis of the study.

# Chapter 4

# DATA COLLECTION, ANALYSIS AND RESULTS

## 4.1 Introduction

In chapter three, a number of methods for analysing the Transportation problem to get the initial basic solution were discussed. This included the North West Corner Rule, Least Cost Method and Vogel's Approximation Method. Also methods such as the Stepping Stone Method and Modified Distribution method to help improve the initial basic solution to get optimality were also discussed.

## 4.2 Data Collection

In this chapter, we shall look at how the data for the work was obtained, how it was used for the intended analysis based on the methods discussed in chapter three. The Coca cola Bottling Company has two production plants, one in Kumasi and the other in Accra. After production, the company keeps the products in the depots before transporting them to the various destination. The company faces challenges on how to optimally distribute it's products among the key distributors with a minimum transportation cost. As a results some private individuals are also contracted to help in transporting the products to the various destinations. The data used for the analysis was based on the quantities at the various depots owned by the company. This project is intended to minimize the total transportation cost of distributing the products from the 8 depots to the 10 Regions of Ghana. The study covered data gathered from June 2011- July 2012. The transportation cost for full truckload of cases was known as well as the demand at the various destination. The cost of transportation per unit crate of Coca cola was express in Ghana cedis.

## 4.3 Data Source

The data used for the analysis was collected from the logistics manager of Coca Cola Bottling Company Ltd. The data included the transport cost per crate of full truck Coca cola from sources to the various destinations and quantity demanded of Coca cola by the various destinations.The total cost of transportation during June 2011- July 2012 was around 273,901.5 Ghana cedis. It also includes the quantities at the various depots and the charge per unit of transporting a crate of Coca cola from the depots to the various destinations.

## 4.4 June 2011-July 2012 Transportation Matrix for The Coca cola Bottling Company of Ghana Limited.

—TWeollected data for June 2011-July 2012 on transportation cost is shown in the table below. This data indicate the transportation matrix showing the supply, demand and the unit cost of crate per full truck.

Table 4.1 The Transportation Matrix

| SOURCES | ASHANTI | EASTERN | WESTERN | UPPER WEST | UPPER EAST | VOLTA | BRONG AHAFO | G. ACCRA | NORTHERN | CENTRAL | SUPPLY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| KOFORIDUA | 3 | 0.5 | 0.95 | 2.3 | 2.5 | 1.8 | 1.7 | 0.75 | 2 | 1.4 | 45676 |
| CAPE COAST | 1.9 | 1 | 0.7 | 3 | 3 | 2.5 | 2.2 | 0.5 | 2.8 | 0.3 | 58565 |
| KUMASI | 0.2 | 1.6 | 1.8 | 2.8 | 2.8 | 1.9 | 0.9 | 1 | 2.9 | 1.9 | 78562 |
| TAMALE | 2.9 | 2.4 | 2.4 | 1 | 1 | 1.4 | 1.6 | 2.4 | 0.3 | 2.8 | 69521 |
| TAKORADI | 1 | 1.7 | 0.3 | 3 | 3 | 1.8 | 1.6 | 1.3 | 2.4 | 0.7 | 56215 |
| HO | 1.3 | 2.4 | 1.9 | 2.1 | 2.2 | 0.2 | 1.8 | 2.4 | 1.4 | 2.5 | 46580 |
| SUNYANI | 0.9 | 1.5 | 1.6 | 0.9 | 0.9 | 1.8 | 0.2 | 1.8 | 0.8 | 1.2 | 64606 |
| ACCRA | 1 | 0.7 | 1.3 | 3 | 3 | 2.4 | 2.7 | 0.2 | 3 | 0.5 | 73451 |
| | 74567 | 46734 | 37656 | 27498 | 19568 | 45789 | 57963 | 64974 | 59567 | 58860 | |

# 4.5 Optimal Solution for The Coca cola Bottling Company June 2011-July 2012

Using the POM- QM for Windows, the Optimal solution is obtained below.

Table 4.2: The Optimal Solution

| | ASHANTI REGION | | | UPPER | UPPER EAST | VOLTA | ÅHÅFO | | NORTHERN | REGION | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| KOFORIOUÅ | | 45676 | | | | | | | | | 45676 |
| | | | | | | | | | | 58565 | 58565 |
| KUMASI | 67443 | | | | | | | | | | 78562 |
| TUE | | | | | | | | | 59567 | | 69521 |
| | | | 37656 | | | | 18559 | | | | 56215 |
| HO | | | | | | 45789 | | | | | 46580 |
| SUN | | | | 16753 | 19568 | | 28285 | | | | 64606 |
| | | | | | | | | 64974 | | 295 | 73451 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| TOTAL | 74567 | 46734 | 37656 | 27498 | 19568 | 45789 | 57963 | 64974 | 59567 | 58860 | |

The Optimal Solution results shows that, the total transportation cost is approximately 20,2890 Ghana Cedis. The Koforidua depot has capacity of 45,676 crates

and supplied 45,676 to the Eastern Region. The Cape Coast depot also supplied 58,565 crates to the Central Region. The Kumasi depot with capacity 78,562 supplied 67,443 and 11,119 crates to Ashanti Region and Brong Ahafo Regions respectively. Moreover, the Tamale depot with capacity of 69521 supplied 9,954 and 59,567 crates to the Upper West Region and the Northern Region respectively. With 56,215 as capacity the Takoradi depot supplied Western region with 37,656 and Brong Ahafo region with 18,559. The Ho depot has capacity of capacity of 46,580. It supplied 791 and 45,789 to Upper West Region and Volta Region region respectively. The Sunyani depot has capacity of 64,606 and supplied Upper West with 16,753 Upper West with 19,568 and Brong Ahafo region with 28,285. The Accra depot has capacity of 73,451. It supplied 7,124 to Ashanti Region, 1,058 to Eastern Region, Greater Accra with 64,974 and Central Region with 295.

The results of transactions from depots to destinations and the amount involved are summarized in Table 4.3

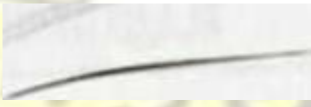


Table 4.3: Shipping List

| From | | Shipment | Costperunit | Shipmentcost(GhanaCedi5) |
|---|---|---|---|---|
| KOFORIDUA | EASTERN | 45676 | 05 | 22838 |
| CAPECOÅST | | 58565 | 0.3 | 175695 |
| KUMASI | ASHANTI REGION | 67443 | | 13488.6 |
| KUMASI | BRONGAHAFO | | | 10007.1 |
| TAMALE | UPPEREAST | 9954 | | 9954 |
| TAMALE | NORTHERN | 59567 | 03 | 17870.1 |
| TAKOUDI | WESTERN | 37656 | 03 | 112961 |
| TAKOUDI | BRONGAHAFO | 18559 | 1.6 | 29594.4 |
| HO | UPPERWEST | | | 1661.1 |
| | VOLTA | 45789 | 0.2 | |
| | UPPERWEST | 26707 | | 24036.3 |
| SUNNI | UPPEREAST | 9614 | | 8652.6 |
| | BRONGAHAFO | 28285 | | 5657 |
| Accu | ASHANTI REGION | 7124 | | 7124 |
| Accu | EASTERN | 1058 | 0.7 | 740.6 |
| ACCU | | 64974 | 01 | |
| Accu | REGION | | 05 | |

# Chapter 5

# CONCLUSIONS AND

# RECOMMENDATIONS

## 5.1 Introduction

In chapter four, the data was collected and analysed to determine how many crates of the Coca cola should be sent to which place and at which cost. Moreover, the data included the quantities at the various depots, the demands at the various destinations and the per unit costs of crate.

## 5.2 Conclusions

The transpoxtation cost is—important element of the total cost structure for any business. The Transportation problem was solved with the standard LP solver that is the PQ 3 for windows to obtain the optimal solution. The computational results provided the minimal total transportation cost. Through the use of this Transportation Model The Coca cola Bottling Company of Ghana can efficiently plan out it's transportation, so that it can not only minimize the cost of transporting goods and services but also create time utility by reaching the goods and services at the right place and at the right time. This intend will enable them to meet the corporative objective such as education fund, entertainment and other support they offered to people of Ghana. The study recorded total minimization of transportation cost during the periods of June 2011- July 2012. The study shows that during June 2011- July 2012, the minimum transportation cost was around 202,890 Ghana Cedis. This is a reduction of the company's transportation cost by approximately 35 percent. Because most supply point are regional based and demand location were on regional bases, the results made it clear that it is better to transport more crates of Coca cola within the same region since regional transportation cost is less costly.

## 5.3 Recommendation

Based on the results and findings of this study, I recommend to the management of The Coca cola Bottling Company, Ghana to seek to the application of mathematical theories into their operations as a necessary tool when it comes to decision making, not only in the area logistics (the transportation Problem), but in production, networking, assignment as well as administration. This study employed mathematical technique to solve management problems and make timely optimal decisions. If the Coca cola Bottling Company managers are to employed the proposed transportation model it will assist them to efficiently plan out the company's transportation schedules at a minimum cost. There are number of programs that can assist in construction of Transportation Problems problems. Probably the best known is GAMS-General Algebraic Modelling System. This provides a high level language for easy representation of complex problems. In Future, I recommend the solution of large-scale transportation problems through aggregation. This proposed method is applicable to any transportation problem.

# References


Ahaja, K. R. (2010). Network optimization trunsportation scheduling.

Ahuja, R. K., Magnati, T. L., & Orlin, J. (1989). Network flows in optimization. Handbooks in Operation Reseamh and Management Science, Vol. 1.

Alexander, S. (2003). Combinatorial optimization.

Amponsah, S. K. (2008). Optimization technique. UPK, KNUST, Kumasi.

Andersion, R. D, Sweeney, J. D, Williams, A. T., & Martin, K. R. (2010). An introduction to management science: Quantitative approach to decision.

Angus, D. (2005). Problems in transport and logistics.

Badra, M. N. (2007). Sensitivity analysis of trunsportation problems. Journal of Applied Sciences Research,.

Bhatia, H. L, Swarup, K., & Puri, M. C. (1975). A procedure for time minimizing trunsportation problem.

Caputo, A. C., & Pelagagge, P. M. (2000). Baghouse system design based on economic optimization. ENVIRON PR, 19(4), 238-245.

Charnes, A., , & Cooper, W. W. (1954). & A constrained game formulation of advertising strategies. Econometrica, 22, 511.

Dantzig, G. B., & Thapa, M. N. (1997). Linear programming 1: Introduction. Springer, New York,.

Dantzig, G. B, & Wolfe, P. (1960). Decomposition principle for linear programs. Operation Research, Vol. 8, 101-111.

Desrosiers, J., Laporte, G., Sauve, M., Soumis, F., & Taillefer, S. (1988). Vehicle routing with full loads. Computers and OR, 15 (3), 219-226.

Doustdargholi, S., & Abasgholipour, D. A. (2009). A sensitivity analysis of right-hand-side parameter in transportation problem, applied mathematical science,. , Vol. 3, 1501 - 1511.

Efroymson, M. A., & Ray, T. L. (1966). A branch and bound algorithm for plant location. Operation Research, 361-368.

Erlander, S. B. (2010). Cost-minimizing choice behavior in transportation planning.

Falk, J. E., & Soland, R. M. (1969). An algorithm for separable nonconvex programming problems. Management Science, 15 (9), 550-569.

Fisher, M. L., & Jaikumar, R. (1981). A generalized assignment heuristic for vehicle routing. Networks, 11 (2), 109-124.

Gaspard, M. (1781). Linear programming problem.

Gass, S. I. (1990). An illustrated guide to linear programming. Dover Publications, Incorporated.

George, D. (1963). Linear progmmming and extensions. Princeton University Press.

Ghana highway authority. (2011). Transportation in Ghana.

Ghana marine transport. (2011). Transportation in Ghana.

Ghana railway corporation. (2011). Transportation in Ghana.

Hammer, P. L. (1969). Time minimizing transportation problems. Naval Research Logistics Quarterly, 16(3), 345-357.

Hay, A. (1977). Linear—programming: elementary geographical applications of the transportation problem.

Hillier, F. , Hillier, M. S., 8t Lieberman, G. J. (2000,). Introduction to management science: A modeling and case studies approach with spreadsheets. Irwin/McGraw-Hill, Burr Ridge.

Hitchcock, F. (1941). The distribution of a product from several sources to numeruos localities. Journal of Mathematics and Physics, 224-230.

Kantorovich, & Gavurin. (1949). Application of mathematical methods in questions of analysis of freight traffic.

Kantorovich, L. (1942). On the translocation of masses.

Kantorovich, L. V. (1939). Mathematical methods of organizing and planing production. Mathematical Science, 366- 422.

Kantorovich, L. V. (1987). My journey in science.

Klingman, D. , & Russel, R. (1975). Solving constrained transportation problems. Opns. Res, 23(1).

Koopman, B. (1936). On distribution admitting a sufficient statistic.

Koopmans, & C., T. (1975). Concept of optimality and their uses.

Kornkoglu, S., & Balli, S. (2010). An improvement vogel's approcimation method for the transportation problem.

Kumar, T. , & Schilling, S. M. (2010). Comparison of optimization technique in large scale transportation problem.

Lampa, M. (2005). Comparison of individual methods for solution of transportation problems on the example of stoneware distribution.

Lamptey-Mills, G. (2009). The optimal transportation problem of accra brewey limited. Unpublished master's thesis, KNUST.

Mikami, T. , Thieullen, M. (2005). Optimal transportation problem by stochastic optimal control.

The objective function: avaialable at www.businessdictionary.com. (2012).

Osman, M. (2010). The transportation problems. Available from

httptf/orms.pefæzŒCïTExt/transProb1em.html

Reeb, J. , & Leavengood, S. (2010). Transportation problem : A special case for linear programming.

Roy, T. V. , & Gelders, L. (1981). Solving a distribution problem with side constraints. Europeandournal of Operational Research, 6(1), 61-66. Sharma, J. K. (1977). Extensions and special cases of transportation problems.

Storoy, S. (2007). The transportation paradox revisited . Borgen.

Sweeney, J., Anderson, R. , Williams, T. A., & Camm, D. J. (2010). Quantitative methods for business.

The Transportation problem. (2013). available at www.bussiness managementcourses.org.

Tjalling, C. , & Koopmans, B. (1949). Optimum utilization of the transportation system.

Wilson, D. (1975). A mean cost approximation for transportation problems with stochastic demand. Naval Research Quarterly, 22.

Winston, L. W. (2010). Transportation, assignment U transhipment problem to accompany operation research: Applied U algorithm 4th edition.

Winston, W. (2003). Operation research applications and algorithms (No. 1).