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GARCH Volatility Modeling and Forecasting of Continuously Compounded Returns
Case Study: Ghana Stock Exchange

by

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Declaration

I hereby declare that this submission is my own work towards the MPHIL degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.

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Abstract

This research presents a rudimentary description of the procedures and applications of ARMA specification in financial time series modeling of the Guinness Ghana Limited and further replicates those concepts on 21 listed companies in the Ghana Stock Exchange Databank Stock Index. The data is from the first financial week in January of 2004 to the last financial week in December of 2008 excluding non-trading days and public holidays. Several tests of the statistical significance and accuracy of the most appropriate $SARIMA(p, d, q)(P, D, Q)^S$ and $ARIMA(p, d, q)$ specifications are performed through checks on the asymptotic standard errors, adoption and implementation of the principle of parsimony, examining correlogram plots among other several tests before final selection is made. Analysis is performed to test for ARCH effect presence and a final confirmation is thus achieved by using Engle's Lagrange multiplier test with a null hypothesis of 'no ARCH effects'. All shares except ABL show no presence of ARCH effects. Conditional volatility is thus modeled from the GARCH specification model fitted to ABL which is an ARCH(1) model. The volatility is forecasted for 3 years ahead. These tests are replicated at the sector level (*i.e.*one level higher than the share level), it is discovered that the ARMA specification for all sectors captures all the ARCH effects. Which implies that the volatility of ABL vanishes at the sector level. A further replication of concepts is performed at the industry level(*i.e* one level higher than the sector level), it is also discovered that, the ARMA specification for all industries captures all the ARCH effects. However at the over all DSI returns level, it is discovered that after a $SARIMA(2, 1, 3)(2, 0, 1)^1$ model specification a $GARCH(1, 1)$ model is fit to capture the uncaptured non-linear ARCH effects present. The ARCH coefficient of the $GARCH(1, 1)$, α was found to be positive and statistically significant which indicates significant short run volatility persistence (i.e. there is significant ARCH effects in the series). The estimate of β, the GARCH coefficient, which represents the contribution of shocks to long run volatility persistence, has a positive and statistically significant value. This means that there is significant long run persistence in volatility. It is concluded that since the sum of the ARCH and GARCH coefficients, $\alpha + \beta > 1$, volatility shock is strongly persistent and under the GARCH model, there is no

covariance stationarity.

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List of Abbreviations

AR	AutoRegressive
MA	Moving Average
ARIMA	AutoRegressive Integrated Moving Average
SARIMA	Seasonal AutoRegressive Integrated Moving Average
ARCH	AutoRegressive Conditional Heteroskedasticity
GARCH	Generalised AutoRegressive Conditional Heteroskedasticity

Dedication

This entire thesis is dedicated to my lovely parents, Rev. Peter Kofi Nyarkoh and Mrs. Grace Nyarkoh, who have supported me immensely throughout my entire life; spiritually, emotionally, financially and morally. Thank you for giving me the opportunity of an education. I am all I am because of you.

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Chapter 1

Introduction

1.1 Background of the Ghana Stock Exchange

1.1.1 Brief History

The Ghana Stock Exchange (GSE) is the principal stock exchange of Ghana and was incorporated in July 1989 as a private company limited under Ghana's Companies Code of 1963 (Act 179). One of the main functions of the exchange is to facilitate the fair trading of bonds, stocks and other types of securities. It was limited to governments bills and bonds which were sold in the primary market. Stock markets affect economic activity via creation of liquidity. Liquid equity markets make investments less risky (and more attractive).

The Pearl Report published the first feasibility study on the need for establishment of a stock exchange. The Stock Exchange Act (Act 384) was then passed in 1971 and was incorporated under the Companies Code in 1989. This time lapse was due to frequent changes in government and hence political instability. Until the inauguration of the Stock Exchange and commencement of active trading on the floor of the stock market in November 12, 1990, financial sector development in Ghana was retarded. In April 1994, it was converted into a public company limited by guarantee.From November 2005, capital gains on the listed securities are exempt from taxes and will remain until 2015. According to the GSE official website, as at Thursday, July 14th, 2011, there were 35 listed companies on the GSE.

1.1.2 Structure and Functions

The principal stock index of the GSE is currently the GSE All-Share Index (currently published by Ghana Stock Exchange). This index is calculated from the values of each of the market's listings. Other indexes on the Exchange include the CBL All-Share Index (This is published by the CAL Brokers Limited). According to the 1998 review of the GSE, and as referenced by wikipedia, as of February 2007 there were 32 listings. The GSE is governed by a Council (Board of Directors) with supervisory roles which sets its policies. It is made up of a representation from Licensed Dealing Members, Listed Companies, the banks, Insurance Companies, Money Market and the general public. An ex-officio member serves as the Managing Director of the Exchange and heads the management staff whose responsibility is the day-to-day management of the GSE assisted by the Deputy Managing Director. Granting listings, preventing fraud and malpractices, maintaining good order among members and regulating stock market business is among the functions of the Exchange's Council and any member who contravenes can be expelled or suspended by the Council. The GSE has no owners or shareholders as such, but members are either corporate bodies or individuals grouped into categories. They are the Licensed Dealing Members, Associate Members and Government Securities Dealers (PDs). An LDM is a corporate body licensed by the Exchange to deal in all securities. An Associate member is an individual or corporate body which has satisfied the Exchange's membership requirements but is not licensed to deal in securities. A PD is a corporate body, which is approved by the Bank of Ghana and registered by the Exchange to deal only in government securities.

The GSE was the sixth best index performing emerging stock market in 1993, with a capital appreciation of 116%. It became the best index performing stock market among all emerging markets, gaining 124.3% in its index level. In 1995 index growth was a disappointing 6.3%, partly because of high inflation and interest rates. Growth of the index for 1997 was 42%, and at the end of 1998 it was 868.35 . According to the "Publications" section on the official website of the GSE, as of October 2006 the market capitalization of the Ghana Stock Exchange was about 111,500billion cedis($11.5 billion). As of December 31 2007, the GSE's market capitalization was 131,633.22bil cedis. In 2007, the index appreciated by 31.84% . According to a statement released by the GSE, "The Africa investor Index Series Awards are the only international pan-African awards that recognize and reward Africa's institutional investors, stock exchanges, best-performing listed companies, stockbrokers and capital market regulators." It awarded the GSE as the "Most Innovative African Stock Exchange for 2010" among a group of seven nominated African Stock Exchanges on Friday,17th September 2010. . The GSE was assesed for her performance between April 2009 and April 2010 and this award was held at the New York Stock Exchange (NYSE).

According to a statement released by the GSE and published on their official website on the 4th of January 2004,'GSE will also publish two new indices, namely the GSE Composite Index (GSE-CI) and the GSE Financial Stocks Index (GSE-FSI). The GSE-CI is a market capitalization weighted index and will include all ordinary shares listed on the GSE at total market capitalization with exemptions to those which have shares listed on other markets. It will be calculated using the volume weighted average closing price. With the GSE-CI, each constituent is given weight according to its market capitalization.Its base date is December 31, 2010 and base index value is 1000. However, listed stocks from the financial sector (banks,insurance sector stocks,etc) and all ordinary shares of the financial stocks listed on the GSE are included in the GSE-FI at total market capitalization, with exemptions to those stocks which are listed on other stock markets. The base date is also December 31, 2010 and the base index value is also 1000. These according to the statement 'will be published on each trading day beginning January 4, 2011'. According to the GSE page on wikipedia, as of 10^{th} June, 2011, there were 30 listed companies and 2 corporate bonds.

The types of securities that can be listed are shares (preference or equities); Debt in the form of corporate bonds (and notes), municipal bonds (and notes), & government bonds (and notes); and Close-end unit trusts and mutual funds.

List of licensed brokers of the GSE include the IC Securities, Databank Group, Gold Coast Securities, NTHC, and Securities Discount Brokers.

1.1.3 Official Trading System (Method, Days and Trading Hours)

Trading takes place every working day. Official continuous trading takes place between the hours of 10:00am and 15:00pm each working day of the week except Saturdays, Sundays and holidays declared by the exchange in advance. The exchange however has pre-market sessions from 9:30am to 10:00am.

GSE uses an electronic trading platform called the GSE Automated Trading System (GATS). Trading is carried out on the Floor of the Exchange. Some companies, like the Ashanti Goldfields Company allow over the counter trading. With effect from January 4, 2011, closing prices of listed equities are calculated using the volume weighted average price of each equity for every given trading day.

Settlement of trades is done electronically using a web based application. Settlement occurs three business days (T+3) after the trade date. The system allows for mutual settlement of trade on T+0 or T+1 basis. On settlement dates shares are moved automatically to client's accounts in the depository system and the brokers settlement account debited.

1.1.4 Regulations

The Securities Industry Law(S.I.L.), P.N.D.C.L. 333, of 1993 is by far the most important legislation in this sector of the economy partly due to its intended scope of coverage and its regulations in the securities. As an umbrella legislation it covers all facets of the securities industry. Companies who deal need to operate under the Companies Code, 1963(Act 179) and those operating on the GSE under the enacted trading regulations. Other than these groups of people and their regulations, trading was unregulated until the introduction of the S.I.L. The Securities and Exchange Commission otherwise known as the Securities Regulatory Commission (S.R.C.) serves as the pivotal regulatory body in the securities market. This is according to the S.I.L. In order to ensure orderly, fair and equitable dealing in securities, they are to maintain surveillance over the securities business. They are also to protect the integrity of the securities markets against any abuses arising from the practice of insider trading. They also register, license, authorize or regulate the Stock Exchange, investment advisors, unit trusts, mutual funds and securities dealers. Another yet important thing they do is form principles for the guidance of the industry and creating of a conducive atmosphere for orderly growth and development of the securities market. The S.R.C. operates in powers that allows her to gather information. It can order for the production of books by stock exchanges and particular persons. Upon failure to obey, criminal sanctions can be imposed.

Other laws governing the securities market in Ghana are the Companies Code of 1963 (Act 179), the Bank of Ghana Act of 1963 (Act 182), the Banking Law of 1989 (P.N.D.C.L 225), the Financial Institutions (Non-Banking) Law of 1993 (P.N.D.C.L 328). The Stock Exchange (Ghana Stock Exchange) Listing Regulations of 1990 (L.I. 1509) and the Stock Exchange (Ghana Stock Exchange) Membership Regulations of 1991 (L.I. 1510) are two regulations that regulate transactions on the GSE and members of it. (See Mensah (1997) for details).

The GSE operates within a set of Rules: Membership Rules deals with the criteria for membership of the GSE, code of conduct or ethics for members, among others. ; Listing Rules prescribe among others, criteria for listing securities (local and external), continued obligations of the listed companies as well as Take-over and merger procedures. ; GSE Automated Trading (GATS) Rules govern electronic trading done by the brokers whether on the Floor, from Dealers offices or through the secured Internet. and Clearing and Settlement House Rules ensure time clearing and settlement of trades electronically. The GSE charges regulatory levies of 0.55% of the value of the trade on each trade where as Brokerage Commisions range between 1-1.75% of the value of the trade.

There is a withholding tax which is also the final tax on dividend income for both the resident and non-resident investors , it is 8%. Even though a non-resident foreign portfolio investor, whether individual or institutional can invest in the Exchange without any prior approval because of the Exchange Control permission that has been granted non-resident Ghanaians and non-resident foreigners since 1993, they can only hold up to 10% of any security approved for listing on the Exchange and for total holdings in one listed security, it shall not exceed 74% except by approval from the Bank of Ghana. The Foerign Exchange Act of 2006 (Act 723) phased out these limits. Non-resident investors can now invest without any limit what so ever or prior exchange control approval.

Foreign exchange remittability is full and free and its for original capital plus all capital gains (which were exempt from tax until November of 2010) and related earnings.

Membership Regulations and Listing Regulations are designed to protect investors. Among functions of the Membership regulations are the code of ethics or conduct for members, regulations to be abided by the Licensed Dealing Members in their operations and giving out criteria for members of the Exchange. For the Listing regulations, these however prescribe criteria for listing securities, application procedure, contents of application and prospectus and continued obligations of the listed companies.

There are several difficult situations among the GSE.

1.1.5 Other Stock Markets

Africa has two regional stock exchanges and there are 29 exchanges representing 38 nations' capital markets. The two regional stock exchanges are the Bourse Régionale des Valeurs Mobilières, or BRVM, located in Abidjan, Cote d'Ivoire; and the Bourse Régionale des Valeurs Mobilières d'Afrique Centrale, or BRVM, located in Libreville, Gabon. BVRM serves the following countries, Benin, Burkina Faso, Guinea Bissau, Côte d'Ivoire, Mali, Niger, Senegal and Togo; the BVMAC serves the Central African Republic, Tchad, Congo, Equatorial Guinea and Gabon.

21 of the 29 stock exchanges in Africa are members of the African Securities Exchanges Association (ASEA). This includes the Bourse Régionale des Valeurs Mobiliès, Botswana Stock Exchange, Douala Stock Exchange, Egyptian Exchange, Ghana Stock Exchange, Nairobi Stock Exchange, Libyan Stock Market, Malawi Stock Exchange, Stock Exchange of Mauritius, Casablanca Stock Exchange, Bolsa de Valores de Mocambique, Namibia Stock Exchange, Nigerian Stock Exchange, Johannesburg Stock Exchange, Khartoum Stock Exchange,

Swaziland Stock Exchange, Dar es Salaam Stock Exchange, Bourse des Valeurs Mobiliè de Tunis, Uganda Securities Exchange, Lusaka Stock Exchange and finally the Zimababwe Stock Exchange. The largest stock exchange in Africa is the Johannesburg Stock Exchange Limited. It was previously called the JSE Securities Exchange and the Johannesburg Stock Exchange. It is situated at the corner of the Maude Street and Gwen Lane in Sandton, Johannesburg, South Africa. It had an estimated 472 listed companies and a market capitalisation of US$182.6 billion as well as an average monthly traded value of US$6.399. As of 30 September, 2006, the market capitalisation of the JSE was at US$579.1 billion. The JSE is presently the 16th largest stock exchange worldwide. One of the oldest bourses (exchanges) on the continent is the Casablanca Stock Exchange of Morocco, founded in 1929. The Egyptian Exchange(EGX) was founded in 1883 and the JSE, Ltd, in 1887. The Casablanca Stock Exchange is one of Africa's ten largest exchanges along with JSE, Ltd, EGX , the Nigerian Stock Exchange, the Namibian Stock Exchange and the Zimbabwe Stock Exchange.

As at 31 December 2010, the US and Europe Economy with the NYSE Euronext was toping the ranking with a market capitalization of US$15,970billion and a trade value of US$19,813billion.

1.2 Problem Statement

Volatility modeling and forecasting on the Ghana Stock Exchange(GSE) using GARCH models has not been carried out extensively. There has not been any such work on the GSE considering it at various industry classification levels. This thesis is an attempt to model and forecast volatility on the GSE, considering the GSE at various industry classification levels in aid of understanding and quantifying the instabilities on the GSE.

1.3 Research Objectives

The main objectives of this thesis are as listed below.

1. To identify the underlying structural pattern embedded on the Ghana Stock Exchange by generating ARMA specifications for the various industry classifications (shares, sectors and industries) on it.

2. To test for the presence of ARCH effect and to generate GARCH specification models for those classifications where the ARMA specification does not capture all the ARCH effect presence.

3. To model(quantify) and forecast the volatility of returns of the various shares, sectors and industries on the Ghana Stock Exchange with appropriate GARCH models.

1.4 Methodology

Focus of the analysis is on the weekly closing stock prices/continuously compounded logarithmic returns of the Ghana Stock Exchange Databank Stock Index. Sample data from the first week in January of 2004 to the last week in December of 2008 (02-01-2004 to 31-12-2008), giving a total of 260 data points and comprising a total of 21 different stocks is used. R Development Core Team's language and environment for statistical computing was the software used for the main statistical analysis. The main version adopted was the R version 2.13.0 (2011-04-13), ISBN 3-900051-07-0,Platform: x86_64-pc-mingw32/x64 (64-bit). Microsoft Office Excel 2007 was used in processing the sample data at the initial stages. The final reports were written using the ©2007-2011, Jonathan Kew, Stefan Loffer version 0.4.0 r.759 (MikTeX 2.9)of TeXworks and TeXnicCenter version 1 Beta 7.50. It is a simple environment for editing, typesetting and previewing TeX documents.

ARMA model specifications are fit to the sample data and ARCH effect presence tests performed. The uncaptured ARCH effects present in the ARMA specifications are captured using non-linear symmetric $GARCH(p, q)$ models which is used to model volatility in the stock returns residuals series.

Descriptive statistics of the data are generated to study the mean, skewness, kurtosis (to determine leptokurticity) and other important central tendencies. The Jarque-Bera test is used to test for normality where as the Augmented Dicky-Fuller test (ADF) and the Kwiatkowski Phillips Schmidt Shin (KPSS) test are used to test for stationarity in the data. Quantile-Quantile (QQ) plots and density graphs are all generated to aid in the analysis.

Checks on the asymptotic standard errors, adoption and implementation of the principle of parsimony, examining correlogram plots among other statistical significance and accuracy tests are performed in order to select the most appropriate $SARIMA(p, d, q)(P, D, Q)^s$ and $ARIMA(p, d, q)$ specifications. Several analysis are performed to test for ARCH effect presence and a final confirmation is thus achieved by using Engle's Lagrange multiplier test with a null hypothesis of 'no ARCH effects'. In estimation of the parameters for the GARCH family of models, the robust method of Bolleslev-Wooldridge's Quasi Maximum Likelihood Estimator (QMLE) assuming the Gaussian standard normal distribution is adopted since GARCH models are non-linear.

A combination of information criteria such as the Akaike Information Criteria (AIC), Schwarz Bayesian Information Criteria (BIC) and a set of diagnostic tests (Q-statistics and χ^2- statistic) aid to select the most appropriate $GARCH(p, q)$ models.The resulting squared residuals from the specified equations are tested for optimality.

1.5 Justification

According to financial literature, very little work has been done on the GSE using GARCH models. The majority of the work done on the GSE in relation to modeling and forecasting is limited to ARMA specifications, which only captures the underlying structure of the GSE and fails to explain or capture volatility and the main stylised facts embedded on it.It is essential therefore to use GARCH models to find the presence of nonlinear autoregressive conditional heteroscedasticity (ARCH) effects in ARMA model specifications. It is also of much essence to policy makers, financial analysts, applied economists, researchers and management teams of companies in pursuit of corporate goals to model volatility and accurately predict it, in aid of concrete financial decisions to prepare for unforeseen future events.

This is a necessary step in understanding and quantifying the instabilities in the GSE. It would however, contribute to the economic development and growth of the GSE and Ghana at large. This research would however also contribute to the development of financial time series modeling and analysis.

1.6 Concise Organizational Structure of Thesis

This thesis is structured into five chapters. Chapter 1 gives, the history, geography and background of the GSE.In chapter 2, relevant research are reviewed. In chapter 3, there is a thorough discussion of the related concepts and methodologies needed in this thesis. Analysis and results are presented in chapter 4 and in chapter 5, the thesis is summarized, recommendations are made and a conclusion is drawn.

Chapter 2

Review of Relevant Research

2.1 Introduction

Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) models which capture volatility persistence in inflation. His idea was to relate the conditional variance of the disturbance term to a linear combination of the squared disturbances in the recent past. Bollerslev (1986) however generalized the ARCH model by modeling the conditional variance to depend on its lagged values as well as squared lagged values of the disturbance. Advancements in the ARCH and GARCH literature has been rather dynamic and interesting. Several extensions to these models by several academicians and their applicability to several financial markets earned Robert Engle a Nobel prize in 2003.

Subbotin *et al.* (2009) reviews the empirical properties of stock price dynamics. Bera and Higgins (1993) as can be seen below has extensively studied the GARCH models and their extensions. Christos (2008), Irfan *et al.* (2010),Tran (2011) and Emenike (2010) among other researchers studied the application of these models on the Egyptian, Indian, Israeli, Pakistani and Nigerian stock markets among others.

Among the few published studies on the GSE, Frimpong *et al.* (2006), Alagidede *et al.* (2006), Mensah (1997) and Frimpong (2008)'s papers have been reviewed.

2.2 Review

Subbotin *et al.* (2009), reviews empirical properties of the variability of stock price dynamics and various models proposed to represent it. The focus is on the most recent developments, concerning mainly multi-horizon and multifractal stochastic volatility processes. He introduces his work with the evolution of the interpretation of volatility (historical volatility and implied volatility). They use a continuous-time diffusion (geometric Brownian motion) to model stock prices:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \tag{2.1}$$

with S_t as the stock price, μ the drift parameter and W_t a Brownian motion. The parameter σ is called volatility because it characterizes the degree of variability and defines return at time t to be

$$r_t = ln(S_t) - ln(S_{t-r}) \tag{2.2}$$

The log-returns, computed from the stock prices that follow the equation 2.1 are normally distributed, hence this model is also called a log-normal diffusion. Further developments led to the understanding of volatility as a stochastic process and not merely as a parameter, even time-varying. With reference for details in Cont (2001), they present summaries on the empirical properties of volatility such as excessive volatility (Cutler *et al.*, 1989), absence of linear correlations in returns, Fama (1970), clustering of volatility and long memory in absolute values of returns, Bollerslev *et al.* (1992); Ding *et al.* (1993); Ding *et al.* (1996), the link between the trading volume and volatility, Labato *et al.* 2000, asymmetry and leverage in the dynamic structure of volatility, Black (1976), heavy tails in the distribution of returns, Mandelbrot (1963); Fama (1965) and the form of the probability distribution of returns varies across time intervals, over which returns are computed, Ghashghaie *et al.* (1996), Arneodo *et al.* (1998). They illustrate these empirical properties of returns using two types of stock index data and of particular interest to their work is the properties related to the ACF of returns and the form of the probability distribution of returns and their magnitudes. They consider ARCH/GARCH family of volatility models and extensions, stochastic volatility models, aggregation of returns in time, the hypothesis of multiple horizons in volatility and finally present modeling multiple horizons in volatility and econophysics approach.

Frimpong *et al.* (2006) suggests that, financial and investment decisions are generally based upon the trade-off between risk and return; the econometric analysis of risk is therefore an integral part of asset pricing, portfolio optimization, option pricing and risk management.When the variances of the error terms of a data are not equal and the expectation of the error terms are not the same at some points or ranges of the data, it is said to suffer heteroskedasticity. In such a situation, the coefficients of the ordinary least squares regression models are still unbiased, but the warning lies in the fact that, the standard error and the confidence intervals are narrow. This creates a false impression of the precision of the model. This concern often arises in financial applications where the dependent variable is the return on an asset or portfolio and the variance of the return represents the risk level of those returns. As part of the financial sector reforms in Ghana, there have been renewed efforts aimed at promoting investments and listings on the Ghana stock market to open access to capital for corporate bodies and greater returns for investors. The stock market provides an added dimension of investment opportunity for both individuals and institutional investors with the fall in the returns on government treasury bills and bonds. Thus recent listings on the bourse saw equities being oversubscribed. They model and forecast volatility (conditional variance) on the Ghana Stock Exchange (GSE) using different types of GARCH models and a 'three days a week' data from the GSE Databank Stock Index (DSI) over a period of 10 years and conclude that the symmetric GARCH(1,1) model outperformed the other models (a basic random walk (RW), two asymmetric EGARCH(1,1), and TGARCH(1,1) models) under the assumption that the innovations follow a normal distribution.The random walk hypothesis is rejected.

Alagidede *et al.* (2006) investigates two prominent calendar anomalies- day of the week and month of the year effects on an emerging African market, the GSE which serves as an attempt of modeling seasonality and claims the GSE's three trading day per week (Mondays, Wednesdays, and Fridays) and the increased attention the exchange receives from both academics and practitioners is interesting. They adopt models from the non-linear GARCH family models in a rolling framework to investigate the role of asymmetries using daily closing prices from 15 June 1994 to 28 April 2004, a total of 1508 observations (excluding holidays). They attribute these anomalies to settlement procedures, negative information releases, and bid-ask-spread biases among others. The January effect postulates that stock returns in January are higher than other months of the year and the day of the week effect holds that stock exhibit significantly lower returns over the period between Friday's close and Monday's close. They discovered and concluded an April effect rather than a January effect and claimed its related to the submission of company reports in the late March which causes a build up of momentum which translates into high positive April returns. Again, that TGARCH yields better results but fails to provide support for the existence of seasonalities. This rather supports Efficient Market Hypothesis and the sceptics approach for the existence of seasonalities.

Bera *et al.* (1993), provides an informal account of some of the important developments (theoretical advances) and their impact on applied work in the ARCH model since its inception by Engle in a seminal paper in 1982. They compliment its usefulness in its ability to capture some stylised facts as well as its applicability to numerous and diverse areas such as in, asset pricing, interest rates, optimal dynamic hedging strategies, option pricing and risk modeling still among others. They begin with a short study on the weekly rate of return on the U.S. dollar/British pound exchange rate, changes in the three month treasury bill rate and the growth rate of the NYSE monthly composite index. The basic ARCH models are described. It begins with the original ARCH model of Engle (1982) by defining the ARCH process and heuristically describing and emphasizing the its properties. Then the GARCH model of Bollerslev (1986) is then introduced before formally describing the unconditional moments of the properties of the ARCH process. They discuss extensions to the ARCH model such as the log ARCH model suggested by Geweke (1986) and Milhoj (1987a), the nonlinear ARCH (NARCH) model proposed by Bera *et al.*(1992), the exponential GARCH (EGARCH) proposed by Nelson (1991) with a look at its properties, the threshold ARCH (TARCH) suggested by Glosten, Jagannathan *et al.* (1991) and Zakoian (1990), the qualitative TARCH (QTARCH) model proposed by Gourieroux *et al.* (1992) among several others. A further look at multivariate ARCH models

According to Engle (2001), traditionally, applied econometricians and financial analysts were required to study change in one variable in response to change in some other variables mainly by the use of the ordinary least squares model. In contemporary times, the nature of their work however demands that, they forecast and analyse the size of errors of the models . The accuracy of the predictions of the model is another natural concern for the applied econometrician and financial analyst. The variance of the error term and what makes them large is the key issue in this case. Even a simple look at financial data suggests that some time periods are riskier than others. Moreover, these risky times are not scattered randomly across quarterly or annual data. Instead, there is a degree of autocorrelation in the riskiness of financial returns. The goal of such models is to provide a volatility measure like a standard deviation or a variance that can be used in financial decisions concerning risk analysis and portfolio selection . The rolling standard deviation was the primary descriptive tool to model and attempt to forecast variance before the introduction of the ARCH models. The GARCH model parameterization is however complemented because it gives parsimonious models and are easy to estimate. Even in its simplest form, it has surprisingly successful abilities in predicting conditional variances. Engle (2001) suggests using maximum likelihood in the normal likelihood to maximize the parameters in order to estimate the parameters. He even suggests softwares to aid in this. He then uses a value-at-risk example to make illustrations and ends with extensions and modifications of the GARCH model.

According to Engle *et al.* (2001) predictability of volatility is a very essential financial ability and it is what is required in each of the following situations. The knowledge of the future behaviour of a risk manager's portfolio is very essential to him as the future volatility of a life contract is to an option trader. To hedge this contract he will also want to know how volatile this forecast volatility is. A portfolio manager may want to sell a stock or a portfolio before it becomes too volatile. Their focus is on the properties that volatility models should satisfy in as much as they discuss the properties that they do not appear to satisfy, all in a univariate context. Focus on the volatility of asset returns is more than for expected returns. They talk about GARCH type of models being formulated in terms of the conditional moments, stochastic volatility models in terms of latent variables and multi-fractals or stochastic structural break models in terms of the unconditional distributions. These models often require information to give forecasting relations. They further discuss the stylised facts that a good volatility model should capture and reflect. It includes persistence, mean reversion, asymmetry as an impact of innovations, exogenous variables, tail probabilities and forecast evaluation. They illustrate the above with daily closing price data on the Dow Jones Industrial index from 23 August, 1988 to 22 August, 2000, representing 3,131 observations. Conditional volatility was found to be quite persistent, with a volatility half-life of about 73 days yet non-stationarity tests showed that its mean reverting. It was again found that a negative lagged return innovation impacted conditional variance about four times as large as a positive return innovation. Finally and among other conclusions, it was found that the empirical results obtained were dependent on the sampling frequency.

Christos (2008) suggests that one of the most prominent tools for capturing changing variance is the Autorgressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH) models developed by Engle (1982), and extended by Bollerslev (1986) and Nelson (1991). Two important characteristics within financial time series, the fat tails and volatility clustering (or volatility pooling), can be captured by the GARCH family models. A series with some periods of low volatility and some periods of high volatility is said to exhibit volatility clustering. Volatility clustering can be thought of as clustering of the variance of the error term over time: if the regression error has a small variance in one period, its variance tends to be small in the next period too. In other words, volatility clustering implies that the error exhibits time-varying heteroskedasticity (unconditional standard deviations are not constant). The use of GARCH-type models on daily data from Egypt (CMA General index from 1997-2007 comprising 1987 daily observations) and Israel (TASE-100 index from 1997-2007 comprising 2063 daily observations) to model volatility and explain financial market risks illustrated strong evidence that, the simple GARCH model

GARCH(p,q), exponential GARCH (EGARCH), threshold GARCH (TGARCH), asymmetric component GARCH (AGARCH), component GARCH (CGARCH) and the power GARCH (PGARCH) model could characterise daily returns. There was a conclusion to indicate that increased risk does not necessarily lead to a rise in the returns. The uncertainties in the prices in the CMA index of Egypt for the period under study makes it more volatile than the TASE-100 index of Israel.

Irfan *et al.* (2010), estimates volatility of short term interest rates with GARCH, EGARCH, TGARCH and PGARCH models using 1639 daily data obsrvations from the Karachi Inter Bank Offering Rate (KIBOR, three month bid rates covering the period 2001 to 2008) and 2318 daily data observations Mumbai Inter Bank Offering Rate (MIBOR, three month bid rate covering the period 2001 to 2008) in Pakistan and India respectively with the aim to search out the best inter bank offering rate. Weekends and holidays have been exempted. A simple GARCH(1,1) was able to capture the persistence in volatility in both returns, with KIBOR returns indicating high volatility shocks and non stationarity in variance where as MIBOR returns indicated moderate presence of volatility shocks. The asymmetric EGARCH(1,1) model was used to capture leverage effects and showed positive and significant parameters for both returns. This indicates the continuationf of leverage effect and that bad news increases the volatility term. The TGARCH(1,1) indicated negative and insignificant parameter in the MIBOR returns case. A PARCH(1,1) confirms the presence of asymmetry in both returns. Comparison tests are performed for within sample forecasting performance using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE), based on these criteria, MIBOR gave smaller error values than the KIBOR in terms of the comparison. Hence, the MIBOR from India was concluded to be the best inter bank offering rate. KIBOR's unpredictability was assumed to be because of the doubt in prices.

Tran (2011) analyses the asymmetric effect on the Vietnamese stock market by emphasizing that a negative shock to returns will create more volatility than a positive shock of equal magnitude. The Vietnamese stock index is rather young with Hochiminh Stock Exchange (HOSE)(listed stock on HOSE is called Vn-index) commencing trade on 28/07/2000 with just two securities and Hanoi Stock Exchange (HNX)(listed stock on HNX is called HNX-index) on 14/07/2005. As at January 2011, it has652 listed companies with a market capitalization around US$50 billion approximately 45% of GDP of vietnam. Tran (2011) employed GARCH, EGARCH, TGARCH and GARCH-in-Mean models to examine whether stock return volatility changes over time is predictable. He studied again the relationship between market risk and expected return. He again takes a look at the Bull, Friday and low transaction effect in the market. He concludes that although the evidence on the examination of the time series features of stock returns and volatility suggests that volatility is prevalent on this market, the effects of shocks on volatility are symmetric. He again discovered the Bull effect on the market, however, the Friday and Low-transaction effect could not be discovered on the market. The best model to describe the return dynamics was the GARCH(1,0) model. The GARCH-M model however do not show any relation between expected returns and expected risk. Forecasts for the daily closing index for the next trading session was generated and it seemed suitable.

Mensah (1997) uses a frame work that reviews various stages of financial development against three basic attributes of an effective financial system to review how Ghana has developed in terms of financial markets and institutions. He states that the attributes are a monetary system, the savings-investment process and a claims-to-wealth structure. He observes that, the financial system which was in use prior to the independence era only satisfied the attribute of the monetary system. Mensah (1997) remarked that since liquid equity markets allow investors to acquire an asset which they can sell quickly if they need access to their savings, investments are less risky and hence more attractive. The permanent capital raised through the equity markets is however of great use to the companies. It is however preferred, for the sake of faster growth, for countries to have both liquid stock markets and well-developed banks than both illiquid markets and undeveloped banks. Notwithstanding,faster future growth regardless of the level of banking development is associated with greater stock market liquidity.The financial system was brought under a planned economy when there was the recognition that, there was a need for a more effective financial system. This was in the immediate post-independence era. However, due to the macroeconomic instability in the early 1980's, the financial system ceased to function. In an attempt to save the situation by restructuring the banking sector, encourage the growth of nonbank financial institutions and to liberalize markets, the Financial Sector Adjustment Program (FINSAP) in 1988 commenced. After a discourse of detailed information he concludes with the following points, that Ghana's financial system has undergone dramatic restructuring in the early days, that other areas within the financial sector requires further attention and support, that the policy and regulatory agencies are slow to respond to new developments in the market. Chronology of the financial market developments in detailed in the Appendix for appreciation of the pace of change.

Frimpong (2008) remarked that the financial crisis prior to this, since 1983 to 1988 led to several drastic restructuring programs aimed at liberalizing and opening up access to long-term capital for investments. The Financial Sector Adjustment Program (FINSAP) which was launched in 1988, established the

Ghana Stock Exchange in 1989. In November 1990, it began full operations with 12 listed companies and one Government bond With an increase in market capital from GH3 billion to GH4.3 billion between 1991 and 1992, the listed companies increased to 15,to 19 in 1995 and to 34 in June 2007. The main indices are the GSE All Share Index and the Databank Stock Index (DSI) but the later was the first major share index on the GSE (which means its computation began in November 1990). The efficiency of capital markets is another issue of great importance. Financial literature mostly captures allocational, operational and informational efficiencies. However,as referenced by Frimpong (2008), Muslumov *et al.* (2004) remarks that capital markets with higher informational efficiency are more likely to retain higher operational and allocational efficiencies.

Emenike (2010), investigates the behaviour of stock return volatility of the Nigerian Stock Exchange returns using GARCH(1,1) and the GJR-GARCH(1,1) models assuming the Generalized Error Distribution (GED) using data from the monthly all share indices of the NSE from January 1999 to December 2008. He sought to do this by examining the NSE return series for evidence of volatility clustering, fat-tails distribution and leverage effects, because they provide essential information about the riskiness of assets on the market. He discovered that there exists volatility clustering on the NSE and used GARCH(1,1) to model that. He captured the existence of leverage effects in the series with the GJR-GARCH(1,1) model. The GED shape test also revealed a leptokurtic returns distribution. By the overall results of the study, there is evidence of volatility persistence, fat-tail distribution, and leverage effects present in the NSE. He concluded that the volatility of the stock returns is persistent in Nigeria and that the shape parameter estimated from GED reveals evidence of leptokurtosis in the NSE returns distribution.

2.3 Chapter Summary

Extensive use of the ARCH and GARCH family of models in the area of volatility modeling can clearly be seen from above review of related papers. It can be discovered however from these publications that, whilst low order ARCH(p) models and the much simple GARCH(1,1) models are good enough to capture volatility clustering, it fails to capture the leptokurticity and asymmetry effects of the volatility.

Chapter 3

Methodology and Relevant Concepts

3.1 Time Series Analysis

3.1.1 Introduction

A basic assumption of classical time series analysis is that successive values in the data represents consecutive measurements taken at equally spaced time intervals. The techniques of time series analysis aid in the identification and understanding of an already existing pattern or structure embedded in a time series data. It again aids to fit a model to this identified underlying pattern or structure and proceed to forecast, monitor or even feed-back and feed-forward control. Its range of applications covers virtually all areas of Statistics. Some of the most important includes economic and financial time series analysis such as economic forecasting, sales forecasting, budgetary analysis, stock market analysis among many others. The main purposes of time series analysis are general exploration, description and prediction and forecasting.

Notation 3.1 *The different notations in use for a time series X, includes;*
$X = X_1, X_2, ...$ *Or* $X = \{X_t : t \in T\}$, *where 1,2,...,T denotes the time steps and is assumed to be equally spaced.*

A difficulty in identifying this pattern is the assumption that the data consists of a set of identifiable components and random noise (which must mostly be filtered to make the pattern more salient). The trend and seasonality components are the two most basic components used to describe most patterns embedded in time series data. The trend component is a nonrecurring function of time that represents a systematic linear or (most often) nonlinear component whereas the seasonality component represents the regularly repeating patterns (periodic patterns).

Also for the sake of making future predictions with the time series data, the general class of the autoregressive moving average (ARMA) models can be used to represent the data. Basic forecasting techniques are able to make short term predictions whereas rather advanced forecasting techniques are used for long term predictions (mostly serving as decision making basics) see Alicia.Statistics (2011).

3.1.2 Trend Analysis

The first step in trend identification in the presence of considerable error is smoothing. Smoothing mostly involves some form of local averaging; this cancels out the non-systematic components of the individual observations. Fairly simple smoothing or averaging methods include the Ordinary Moving Average and the Exponentially Weighted Moving Average see Alicia.Statistics (2011), the former, which was adopted in this research. It is expressed as;

$$y_t = \frac{y_t + y_{t_1} + y_{t_2} + \cdots + y_{t_k}}{k}$$

$i.e.$the moving average through time t.
According to Alicia.Statistics (2011) , the moving average smoothing replaces each element of the series by either a simple or weighted average of n surrounding observations (n is the width of the smoothing window). Instead of using the means, medians which does not allow for weighting could also be used. As a major disadvantage, it may however produce even more unsmooth (jagged) curves than the moving average in the absence of clear outliers. This implies on the contrary that, medians produce a much smoother curve than the moving averages in the presence of outliers. The distance weighted least squares smoothing or negative exponentially weighted smoothing techniques filter out noise when the measurement error is very large. The smooth curve that is outputted is relatively unbiased by outliers. Bi cubic splines smoothing, can also be used when the series is systematically distributed and has relatively few points, see StatSoft (2011).

3.1.3 Seasonal Dependency Analysis(Seasonality Analysis)

The concept of seasonal dependency is formally defined as the correlational dependency of order k between each j'th element of the series and the (j-k)'th element see Kendal *et al* (1990) and measured by autocorrelation (i.e. a correlation between the two terms); k is usually called the lag or time lag in some cases.

It is possible, in the presence of minimal measurement error to visually identify seasonal dependency as a pattern that repeats every k elements in the data.

3.1.4 Concepts Used in Seasonal Dependency Examination and for Trend Analysis

- **Autocovariance ($\gamma(h)$) and Autocorrelation ($\hat{\rho}(h)$):** This is a preliminary step to the construction of an appropriate model. Whereas the mean and auto covariances completely characterize Gaussian processes, it only gives a fair idea of the temporal independence structure in non-Gaussian processes.$\gamma(h)$, is estimated using sample observations. To estimate $\gamma(h)$, we generally use the sample autocovariance denoted as $\gamma(\hat{h})$ and defined for $0 \leq h < n$, by

$$\gamma(\hat{h}) = \frac{1}{n}\sum_{j=1}^{n-h}(X_j - \overline{X})(X_{j+h} - \overline{X}) := \gamma(\hat{-h}) \tag{3.1}$$

  where $\overline{X}$=$(\frac{1}{n})\sum_{j=1}^{n} X_j$ denotes the sample mean. For the definition of the sample autocovariance, $\hat{\rho}(h) = \dfrac{\gamma(\hat{h})}{\gamma(\hat{0})}$ for $|h| < n$ see Francq *et al* (2010)

- **Partial Autocorrelation:**
  An extension of the autocorrelation is the partial autocorrelation which is an essential tool in seasonal dependency examination. With the partial autocorrelation, the effect of the correlation of intermediate observations (*i.e.* those within the lags) is set to zero see Box *et al*,(1976), McDowall *et all*, (1980) and Velleman *et al*, (1981). Which means, in the event that there are no intermediate elements between the lags, the autocorrelation is identical to the partial autocorrelation (*i.e.* at a lag of 1).

- **Correlogram:**
  A plot of correlations (autocorrelations or partial autocorrelations) against its corresponding lags is called the correlogram (autocorrelogram). Which means it displays numerically as well as graphically the autocorrelation function or the partial autocorrelation function. In examination of the correlogram, serial dependencies in the time series data can be identified.

3.1.5 Stationarity

Two standard notions of stationarity are strict stationarity (strictly stationary or strong stationarity) and second-order stationarity. A strictly stationary process is a stochastic process whose joint probability distribution does not change when shifted through time or space. Consequently, parameters such as the mean and variance, if they exist, also do not change over time or position.

Definition 3.1 (Strict stationarity) *Let X_t be a stochastic process and let $F_X(x_t1+\tau,\cdots,x_tk+\tau)$ represent the cumulative distribution function of the joint distribution of X_t at times $t_{1+\tau},\cdots,t_{k+\tau}$. Then, X_t is said to be strictly stationary if, for all k, for all τ, and for all $t_1,\cdots,t_k$, $F_X(x_t1+\tau,\cdots,x_tk+\tau)=F_X(x_t1,\cdots,x_tk)$. Since τ does not affect $F_X(.)$,F_X is not a function of time. See Francq et al (2010).*

Second order stationarity may seem less demanding but it requires the existence of second order moments and classical time series analysis is centered on the second-order structure of the process.

Definition 3.2 (Second-order stationarity) *The process (X_t) is said to be second-order stationary if:*

1. $E(X_t^2)<\infty$, $\forall t\in Z$;
2. $E(X_t)=m$, $\forall t\in Z$;
3. $Cov(X_t,X_{t+h})=\gamma_x(h)$, $\forall t,h\in Z$

The function $\gamma_x(.)(\rho_x(.):=\dfrac{\gamma_x(.)}{\gamma_x(0)})$ is called the autocovariance function (autocorrelation function) of (X_t). See Francq et al (2010).

The ability of stationary processes to replace independent and identically distributed (iid) observations is of crucial importance in time series analysis see Kendall *et al*, (1990). White noise is an important example of second-order stationary process as it permits the construction of more complicated stationary processes. Again, two standard notions of white noise are the weak white noise and the strong white noise.

Definition 3.3 (Weak white noise) *The process (η) is called weak white noise if ,for some positive constant σ^2:*

1. $E(\eta_t)=0$, $\forall t\in Z$;
2. $E(\eta_t^2)=\sigma^2$, $\forall t\in Z$;

3. $Cov(\eta_t, \eta_{t+h}) = 0$, $\forall t, h \in Z, h \neq 0$.

see Francq et al (2010).

Remark 3.1 (Strong white noise) *No independence assumption is made in the definition of weak white noise. The variables at different dates are only uncorrelated and the distinction is particularly crucial for financial time series.The variables η_t and η_{t+h} are independent and identically distributed.*
The process (η_t) is then said to be strong white noise, see Francq et al (2010).

Crucial to financial time series is the distinction between uncorrelation and independence of variables at different times of which no such independence assumption is made in the weak white noise definition (Francq, *et al* (2010)). To achieve stationarity, differencing the series is mostly required. The order of differencing to achieve stationarity is represented by the d in the order of the ARIMA models as treated below. Examination of the plot of the data and the correlograms helps to identify the order of the differencing. If for moderate and large k, the autocorrelogram shows that the estimated autocorrelation function dies off slowly and very nearly linearly, it implies that there is at least a root which approaches unity. Consequently, failure of the estimated autocorrelation function to die out rapidly suffices to mean that the underlying stochastic process is non-stationary. In other words, if none of the roots lie close to the boundary of the unit circle, the autocorrelation function will quickly die out or decay for moderate and large k, meaning it is stationary (Agyemang *et al*, (2010)). A first order differencing is most likely to achieve stationarity. A major assumption in autocorrelogram examination is that, stationarity is achieved if the estimated autocorrelation function dies out fairly quickly. When there is a linear trend about the mean (mostly strong fluctuations evident in the plot of the data), first order non-seasonal (lag=1) differencing is necessary to achieve stationarity. Quadratic trend about the mean (usually strong changes of slope, mostly evident in the plot of the data), requires second order non-seasonal (lag=2) differencing in order to achieve stationarity. If there is trend in the variance, log transformation is necessary to achieve stability.
Tests for stationarity also exist. These include the Dickey-Fuller Test, the Augmented Dickey-Fuller Test, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test and the Phillips-Perron Unit Root Test. Improvements made by recent software developers have captured these advanced tests for stationarity. Some of these tests are used in this research to test for stationarity of the series.

3.1.6 Differencing

As an important technique, differencing aids in the elimination of serial dependency from the series. For a particular lag of k, converting each j^{th} element into its difference results in a series, k elements less the number of original elements initially in the series. Mainly, differencing as a technique helps to identify the embedded nature of serial dependencies in the data. It is very essential in achieving stationarity in the time series which is necessary for the ARIMA modelling and related techniques.

Many economic series display trends, making the stationarity assumption unrealistic. Such trends often vanish when the series is differentiated, once or several times. Let $\nabla X_t = X_t - X_{t-1}$ denote the first-difference series, and let $\nabla^d X_t = \nabla(\nabla^{d-1})X_t$ $(with \nabla^0 X_t = X_t)$ denote the differences of order d.

3.1.7 ARMA and ARIMA Models

An important aim of time series analysis is to construct a model for the underlying stochastic process.This model is then used to analyse the causal structure of the process or to obtain optimal predictions. The ARMA methodology has amassed vast popularity in academia, research and industry, mainly due to its confirmed power and flexibility see Hoff, (1983), Pankratz, (1983) and Vandaele, (1983) for more details. Nonetheless, it is still a rather complex technique and to use effectively and efficiently to provide satisfactory results requires enormous exposure and experience. Interpretation of results is mostly subject to the researcher's experience Bails *et al*, (1982). The class of ARMA models is the most widely used for the prediction of second-order stationary processes. As referenced by Francq *et al* (2010), according to Wold (1938), this can be viewed as a natural consequence of a fundamental result.

3.1.7.1 Autoregressive/AR(p) Processes

The concept of the autoregressive process, is to generate consecutive elements of the series from a linear combination of time-lagged (previous) elements of the series see Statsoft (2011). The number of previous elements used is usually the order of the process. An autoregressive model of order p denoted $AR(p)$ is given by;

$$X_t = \sum_{k=1}^{p} \alpha_k X_{t-k} + \mu + \epsilon_t$$

X_t	represents the consecutive terms of the AR(p) model.
α_k	represents the parameters of the AR(p) model.
μ	represents the intercept of the AR(p) model.
ϵ_t	represents the linear innovation process of the AR(p) model.
X_{t-k}	represents the time-lagged observations of the AR(p) model.
t	represents the period.
k	represents the time lag.

Correlogram examination for an AR(p) process shows that, the autocorrelation function will trail off to zero and the partial autocorrelation function will cut off after lag p.

Stationarity Requirement

For an autoregressive process to be stationary, certain conditions must be satisfied. If an AR(1) process is under consideration, then, $-1 < \alpha_1 < 1$, for an AR(2) process, α_1+α_2 <1, α_1-α_2 <1, α_2-α_1 <1, -1< α_2 <1. For an AR(p) process with $p > 2$ general restrictions can be defined see Francq *et al* (2010).

3.1.7.2 Moving Average/MA(q) Processes

The concept of the moving average process is to generate consecutive elements of the series based on a linear combination of past errors(random shocks) that cannot be accounted for by the autoregressive component.

Definition 3.4 *Any centered, second-order stationary, and 'purely nondeterministic' process admits an infinite moving-average representation of the form*

$$X_t = \eta_t + \sum_{i=1}^{\infty} \theta_i \eta_{t-i}, \tag{3.2}$$

where (η_t) is the linear innovation process of (X_t), that is

$$\eta_t = X_t - E(X_t|H_X(t-1)) \tag{3.3}$$

where $H_X(t-1)$ denotes the Hilbert space generated by the random variables $X_{t-1}, X_{t-2}, \cdots$ and $E(X_t|H_X(t-1))$ denotes the orthogonal projection of X_t onto $H_X(t-1)$. The sequence of coefficients (c_i) is such that $\sum_i \theta_i^2 < \infty$. Note that (η_t) is a weak white noise.

A moving average model of order q, denoted $MA(q)$ is given by truncating the infinite sum in 3.2;

$$X_t = \sum_{k=1}^{q} \theta_k e_{t-k} + \mu + \epsilon_t$$

X_t	represents the consecutive terms of the $MA(q)$ model.
θ_k	represents the parameters of the $MA(q)$ model.
μ	represents the mean of the series.
ϵ_t	represents the linear innovation process of the $MA(q)$ model.
e_{t-k}	represents the time-lagged forecast errors of the $MA(q)$ model.
t	represents the period.
k	represents the time lag.

We have $||X_t(q) - X_t||_2^2 = E\eta_t^2 \sum_{i>q} \theta_i^2 \to 0$, as $q \to \infty$. It follows that the set of all finite-order moving averages is dense in the set of second-order stationary and purely nondeterministic processes. The class of ARMA models is often preferred to the MA models for parsimony reasons, because they generally require fewer parameters. Correlogram examination shows that, an MA(q) process will have an autocorrelation function cutting of at laq q and partial autocorrelation function traiIling of to zero.

Invertibility Requirement

Certain conditions must be satisfied for the invertibility condition to hold. For an MA(1) model, $-1 < \theta_1 < 1$. For an MA(2) model $\theta_1 + \theta_2 < 1, \theta_2 - \theta_1 < 1, \theta_2 < 1$. For an $MA(q)$ process with $q > 2$, general restrictions can be defined see Francq *et al*, (2010) and Montnegro *et al*, (1990).

3.1.7.3 Mixed/ARMA(p,q) Processes

A general mixture of the AR(p) models and the MA(q) models are called the AutoRegressive Moving Average (ARMA) models of order (p,q).

Definition 3.5 (ARMAp,q process) *A second-order stationary process (X_t) is called ARMA(p,q), where p and q are integers, if there exist real coefficients $\mu, \alpha_1, \cdots, \alpha_p, \theta_1, \cdots, \theta_q$ such that,*

$$X_t = \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + \mu + \epsilon_t$$

X_t	*represents the consecutive terms of the ARMA(p,q) model.*
α_i	*represents the parameters of the AR(p) part.*
θ_j	*represents the parameters of the MA(q) part.*
μ	*represents the intercept of the series.*
ϵ_t	*represents the linear innovation process of the ARMA(p,q) model.*
e_{t-j}	*represents the time-lagged forecast errors of the MA(q) part.*
X_{t-i}	*represents the time-lagged observations of the AR(p) part.*
t	*represents the period.*
i & j	*represents the time lags.*

An important characteristic of the ARMA models is that both the autocorrelation function (ACF) and the partial autocorrelation function (PACF) do not cut off as in AR and MA models.

For estimation to be possible on a data with non-stationarity, the time series needs to be transformed (differenced).Before the forecasts are generated, the series needs to be integrated (integration is the inverse of differencing) so that the forecasts are expressed in values compatible with the input data. The integration part is represented in the naming of the model as I and its order represented as d (thus the order of differencing). We thus have Auto-Regressive Integrated Moving Average(ARIMA) models, denoted as ARIMA(p,d,q).

Writing $W_t = \nabla^d Y_t = (1-B)^d Y_t$ the general ARIMA process is of the form

$$W_t = \sum_{i=1}^{p} W_i Y_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + \mu + \epsilon_t$$

Where p ,d and q are as defined above. μ represents the mean or intercept of the differenced series. For example, if the series is differenced once, and there are no autoregressive parameters in the model, then the constant represents the mean of the differenced series, and therefore the linear trend slope of the un-differenced series see Statsoft (2011).

3.1.8 Box-Jenkins Methodology

Box-Jenkins methodology aims to chose the most appropriate ARIMA(p,d,q) model and to use it for forecasting. It adopts a six-stage iterative scheme. It is as described below;

1. a priori identification of the differentiation order d (or choice of another transformation); this is essentially based on examining the graph of the series despite the upsurge of many unit root tests.;

2. a priori identification of the orders p and q;the primary tool here is the autocorrelation function and it often results in a selection of more than one ARMA model.

3. estimation of the parameters $(\alpha_1, \cdots, \alpha_p, \theta_1, \cdots, \theta_q$ and $\theta^2 = Var(\epsilon_t))$;using for instance the least-squares method.

4. validation;examining the residuals to gauge the compatiblity of the estimated model to the data is the aim of this step. The model is satisfactory

when the residuals are or at least close to white noise. So correlograms and portmanteau tests on the residuals are performed.

5. choice of a model; If after the tests on the residuals we fail to reject a model the significance of the estimated coefficients are studied. This step could lead to the rejection of all models or to consideration of other models, at that stage, repeat step 1 or 2. Information criteria are used if several models pass step 4. Akaike(AIC) and Schwartz-Bayesian(BIC) information criteria are rather popular. Among other extra considerations, the predictive properties of the models and the principle of parsimony can be studied.

6. prediction.Upon selection of an appropriate model, linear predictions $X_t\hat{(}h)$ at horizon $h = 1, 2, \cdots$ can be computed quite easily. It is based on Gaussian assumptions that the interval predictions are obtained

See Francq *et al* (2010),pages 5-7 for a detailed explanation.

3.2 ARCH/GARCH: Analysis and Model Specification

3.2.1 Introduction

The complexity of modeling financial time series mostly lies in the artificial reproduction of statistical regularities (stylised facts), existent in a large number of financial series. The nature of the series as well as its frequency heavily affects how evident these properties appear in the data. Weekly stock prices/returns are considered. Let p_t denote the price of an asset at time t and let $\epsilon_t = \log(\frac{p_t}{p_{t-1}})$ be the continuosly compounded return or log return (also simply called the return). The series (ϵ_t) is often close to the series of relative price variations $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$, since $\epsilon_t = \log(1 + r_t)$. In contrast to the prices, the returns or relative prices do not depend on monetary units which facilitates comparisons between assets (Francq *et al*, (2010)).

3.2.2 The ARCH and GARCH Models

The Autoregressive Conditional Heteroskedasticity (ARCH) processes are univariate conditionally heteroskedastic white noises, mostly useful in finance and econometrics for modeling conditional heteroskedasticity and volatility clustering. It was proposed by Engle in 1982. ARCH processes assume that conditional variances in time-dependent data are subject to influences from previous unexpected factors which are functions of error terms,allowing them to change over time. It is mostly variance of the current innovation related to squares of the previous innovations. It is defined by the interrelated formulas given below. We first write returns, r_t, as:

$$
\begin{gathered}
r_t = \mu + \epsilon_t \\
\epsilon_t = \sigma_t \eta_t \\
var(\epsilon_t) = \sigma_t^2 = \omega + \textstyle\sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \\
\omega > 0 \\
\alpha_i \geq 0 \; ; i = 1, \cdots, q \text{ and } \textstyle\sum_{i-1}^{q} \alpha_i < 1
\end{gathered}
$$

Where

- r_t is the mean equation representing certain stock market returns series (in this case the weekly returns series).
- ϵ_t is the innovation(which is split into a stochastic part and a time dependent standard deviation) of the r_t process(return residuals).
- η_t is a sequence of iid centered variables with zero mean and unit variance(a standard normal Gaussian white noise process). This means that the time t distributions of ϵ_t, conditional on information available at time $t-1$, is normal, with constant mean 0 and a conditional variance σ_t^2, conditional on information obtained at time $t-1$.
- σ_t^2 is the conditional variance of η_t, conditional on information obtained at time $t-1$.
- q specifies the depth of memory in the variance of the process.

The condition $\omega > 0$ and $\alpha_i \geq 0$ are set to ensure strictly positive variance. Typically, due to the volatility persistence in financial markets, q is of high order. From the formulation of volatility above, σ_t^2 is known at time t-1. The one-step ahead forecast is readily available. The multi-step ahead forecast can also be formulated by assuming $E[\epsilon_{t+r}^2] = \sigma_{t+r}^2$

Unconditional variance of r_t is

$$\sigma^2 = \frac{\omega}{1 - \sum_{j=1}^{q} \alpha_j}$$

The process is covariance stationary if and only if the sum of the autoregressive parameters is less than one ($i.e.$ $\sum_{j=1}^{q} \alpha_j < 1$.)

The Ordinary Least Squares(OLS) method is used in the estimation of the parameters of the ARCH(q) process. After a statistically significant and appropriate ARMA specification, it is useful to implement the Lagrange multiplier(LM) test proposed by Engle(1982) to test for the presence of the ARCH effect that was not captured by the ARMA specification. The test assumes a null hypothesis of 'no ARCH effect presence' in the ARMA specification. The test statistic ($n * R^2$, n is the sample size of the residuals and R^2 is the coefficient of determination) in the LM test follows a χ^2- distribution with q degrees of freedom. If the value of the test statistic is greater than the Chi-square table value, then reject the null hypothesis, otherwise do not reject it.

According to Hung, (2009), Engle's ARCH model(1982) is subject to influences from previous unexpected factors, particularly it states that the conditional variances are functions of the error terms (they are allowed to change over time). In financial time series, the word 'Conditional' stands for a technique with explicit dependence on previous/past observations where as 'Heteroskedastic' talks about time-varying or time-dependent variance see [http//www.riskglossary.com/link /ARCH_GARCH.htm],(accessed 13 May.2011).

Generally, stock market performance is time-varying and nonlinear,and exhibits properties of clustering among other statistical regularities (stylised facts.) Clustering simply means that other large changes tend to follow large changes and small changes tend to follow other small changes.This notion introduced the idea that, variance might not be homogenous (constant) through time see Hung, (2009). Traditional econometric models are associated with the assumption of homogeneity which is not consistent with the conditional variances which are often present in the analysis of time-dependent data (Hung, (2009)).

According to Glyn (2010) if r_t takes on large positive or negative values at some point in time,its conditional variance will be elevated for subsequent points in time, thereby making it likely that r_t will also take on large positive or negative values at those times too. In this manner, an ARCH process models volatility clustering 'periods of high or low volatility. Hung, (2009) agrees by saying that conditional variance is a very important phenomenon in modeling volatility clustering.

Borrowing from the idea of the Autoregressive Moving Average (ARMA) models, Bollerslev in 1986, generalised Engle's ARCH(q) model to introduce the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models. According to Hung, (2009), Bollerslev assumed 'that the conditional variances are influenced not only by the squared error terms, but also by previous conditional variances.' Again for high orders of ARCH(q) process, it is more parsimonious to model volatility as GARCH(p,q). The resulting GARCH(p,q) model is defined by the interelated formulae as given below.

$$r_t = \mu + \epsilon_t$$

$$\epsilon_t = \sigma_t \eta_t$$
$$var(\epsilon_t) = \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$
$$\omega > 0$$
$$\alpha_i \geq 0 \; ; \; i = 1, \cdots, q$$
$$\beta_j \geq 0 \; ; \; j = 1, \cdots, p$$
$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j \leq 1$$

Where the parameter definitions as well as restrictions are as defined above.

For GARCH(1,1), the constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ are needed to ensure σ_t^2 is strictly positive. See Nelson and Cao (1992) for the details of the constraints on β_j and α_i higher orders of GARCH

The unconditional variance equals

$$\sigma^2 = \frac{\omega}{1 - \sum_{j=1}^{p} \beta_j - \sum_{i=1}^{q} \alpha_i}$$

The GARCH(p,q) model is covariance stationary if and only if

$$\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i < 1$$

As discussed in details by Poon (2008), volatility forecasts from GARCH(1,1) can be made by repeated substitutions. For an estimate for the expected squared residuals,

$$E[\epsilon_t^2] = \sigma_t^2 E[\eta_t^2] = \sigma_t^2 \tag{3.4}$$

The conditional variance σ_{t+1} and 1-step ahead forecast is known at time t,

$$\hat{\sigma}_{t+1} = \omega + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_t$$

The forecast of σ_{t+2} makes use of the fact that $E[\epsilon_{t+1}^2] = \sigma_{t+1}$ and we get

$$\hat{\sigma}_{t+2} = \omega + \alpha_1 \epsilon_{t+1}^2 + \beta_1 \sigma_{t+1} = \omega + (\alpha_1 + \beta_1)\sigma_{t+1}^2$$

Similarly,

$$\hat{\sigma}_{t+3} = \omega + (\alpha_1 + \beta_1)\sigma_{t+1}^1 \; \omega + \omega(\alpha_1 + \beta_1) + (\alpha_1 + \beta_1)^2 \sigma_{t+1}^2$$
$$\omega + \omega(\alpha_1 + \beta_1) + \omega(\alpha_1 + \beta_1)^2 + (\alpha_1 + \beta_1)^2[\alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2]$$

As the forecast horizon τ lengthens,

$$\hat{\sigma_{t+r}^2} = \frac{\omega}{1 - (\alpha_1 + \beta_1)} + (\alpha_1 + \beta_1)^\tau [\alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2]$$

If $\alpha_1 + \beta_1 < 1$, the second term to the RHS of $\hat{\sigma^2_{t+r}}$ dies out eventually and $\hat{\sigma^2_{t+r}}$ converges to $\frac{\omega}{1-(\alpha_1+\beta_1)}$, the unconditional variance.

If we write $\upsilon_t = \epsilon^2 - \sigma_t^2$ and substitute $\sigma_t^2 = \epsilon_t^2 - \upsilon_t$ into 3.4, we get

$$\begin{aligned}\epsilon_t^2 - \upsilon_t &= \omega + \alpha_1\epsilon_{t-1}^2 + \beta_1\epsilon_{t-1}^2 - \beta_1\upsilon_{t-1}\\ \epsilon_t^2 &= \omega + (\alpha_1+\beta_1)\epsilon_{t-1}^2 + \upsilon_t - \beta_1\upsilon_{t-1}\end{aligned}$$

Hence, ϵ_t^2, the squared residual returns follow an ARMA process with autoregressive parameter $(\alpha_1 + \beta_1)$. If $\alpha_1 + \beta_1$ is close to 1, the autoregressive process in ϵ_t^2 above dies out slowly,this implies that volatility shock is moderately present. If $\alpha_1 + \beta_1 > 1$, it indicates that the volatility shock is very high and the variances are not stationary under GARCH model.

The GARCH(p,q) model's ability to model fat tail and excess kurtosis (leptokuritsity) are among the reasons why it is frequently used to explore returns and transmissions of volatility in time-dependent financial data sets featuring time-varying conditional variance. White test is the best test in testing for heteroskedasticity. It assumes a null hypothesis of homoskedasticity(constant / unconditional variance) and rejects the null hypothesis to imply that GARCH models can be used to model the heteroskedasticity that is in the series. The Ljung-Box's Q-statistic which follows a χ^2 distribution with n degrees of freedom can also be used to indicate the presence of GARCH errors for large uncorrelated squared residuals with asymptotic standard deviation of the autocorrelation given by $\frac{1}{\sqrt{n}}$. At least $\frac{n}{4}$ values of n should be considered. It assumes a null hypothesis of 'no GARCH error' presence in the conditional variance.

To test for optimality in the specified GARCH model, the resulting squared residuals should look like a white noise process (i.e. the ACF/PACF of the resulting residuals squared must lie within the significance band).

Leverage effect is a phenomenon that relates to high volatility being induced by negative return and its often modeled with a sign based return variable in the conditional volatility equation.

3.3 Ghana Stock Exchange

The trading benchmark currently in use by the Ghana Stock Exchange is the Volume Weighted Average Price (V.W.A.P.). Frequently used in pension plans, it is calculated by adding up the dollars traded for every transaction (price multiplied by number of shares traded) and then divided through by the total shares traded for the day. The theory states that, if the price of a buy trade is less than the V.W.A.P., then it is a good trade. If the price of a buy trade is greater than the V.W.A.P., then it is a bad trade. Calculation starts when trading opens and ends when trading closes. Because it is good for the current trading day only, intraday periods and data are used in the calculation.

$$V.W.A.P. = \frac{\sum NumberofSharesBought * SharePrice}{TotalSharesBought}$$

VWAP is primarily used by institutional traders and represents the total value of shares traded in a particular stock on a given day, divided by the total volume of shares traded in that stock on that day. Calculation techniques vary: some will use data from all markets or just the primary market, and may or may not adjust for resubmits and other error corrections. VWAP is a method of pricing transactions and also a benchmark to measure the efficiency of institutional trading or the performance of traders themselves.

V.W.A.P. is used to identify liquid and illiquid price points for a specific security over a very short time period and reflects price levels weighted by volume. It can help institutions with large orders. The idea is not to disrupt the market when entering large buy or sell orders. V.W.A.P. can also be used to measure trading efficiency. After buying or selling a security , institutions or individuals can compare their price to a V.W.A.P. values. A buy order executed below the V.W.A.P. value would be considered a good fill because the security was bought at a below average price. Conversely, a sell order executed above the V.W.A.P. would be deemed a good fill because it was sold at an above average price.

It serves as a reference point for prices for one day. As such, it is best suited for intraday analysis. Chartists can compare current prices with the V.W.A.P. values to determine the intraday trend. VWAP can also be used to determine relative value. Prices below VWAP values are relatively low for that day or specific time. Prices above VWAP values are relatively high for that day or specific time. Keep in mind that VWAP is a cumulative indicator, which means the number of data points progressively increases throughout the day. On a 1 minute chart, IBM will have 90 data points (minutes) by 11AM, 210 data points by 1PM and 390 data points by the close. The number dramatically increases as the day extends. This is why VWAP lags price and this lag increases as the day extends.

3.4 Modern Portfolio Theory (MPT)

3.4.1 Introduction

This theory was pioneered by Harry Markowitz in his seminal paper 'Portfolio Selection', published in 1952 by the Journal of Finance for which he was awarded a Nobel Prize in 1990. 'Portfolio theory', 'portfolio management theory' and 'mean-variance optimization' are among other aliases MPT is known by.The fundamental goal of portfolio theory is to optimally allocate investments between different assets (diversification) by optimizing or maximizing expected return of the assets based on a given level of market risk of the individual assets and the correlation between the assets. Each stock has its own standard deviation from the mean, which MPT calls 'risk'. Provided the risks of various stocks are not directly related, the risk in a portfolio of diverse individual stocks will be less than the risk inherent in holding anyone of the individual stocks. The risk for individual stock returns has two components;

1. Systematic Risk- These are the market risks that cannot be avoided by way of diversification. Examples are effects of war on the market and recession.

2. Unsystematic Risk- This risk is specific to individual stocks and can be diversified away as you increase the number of stocks in your portfolio.

Investors benefit more from holding diversified portfolios instead of individual stocks. The four 'basic' steps involved in portfolio construction are:

1. Security valuation

2. Asset allocation

3. Portfolio optimization

4. Performance measurement

Mean variance optimization (MVO) is a quantitative tool which allows allocation by considering the trade-off between risk and return. Markowitz's conventional MVO considers a single period within which portfolio allocation is made for a single upcoming period. The objective is to maximize expected return subject to a select level of risk. Multi-period MVO, considers strategies in which the portfolio is rebalanced to a specified allocation at the end of each period.

3.4.2 Expected Return and Risk

According to Graeme (2006), suppose a portfolio with n assets has the i^{th} asset giving a return of $R_{t,i}$ at time t. The mean on the return is $\mu_{t,i}$ and it has a variance of $\sigma^2_{t,i}$. Now let us assume that, the proportion of the value of the portfolio that the asset i makes is ω_i. Hence, $\sum_{i=1}^{n} \omega_i = 1$. At an implicit time of t, bearing in mind that estimates must be updated on a daily bases;

$$\mu := E[R] = E[\sum_{i=1}^{n} \omega_i R_i] = \sum_{i-1} \omega_i E[R_i] = \sum_{i=1}^{n} \omega_i \mu_i \tag{3.5}$$

and

$$\begin{aligned}
\sigma^2(R) &= E[(R-\mu)^2] \\
&= E[(\sum_{i=1}^{n} \omega_i (R_i - \mu_i))^2] \\
&= E[\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j (R_j - \mu_i)(R_j - \mu_j)] \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} E[\omega_i \omega_j (R_i - \omega_i)(R_j - \mu_j)] \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j covar(R_i, R_j) \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{i,j} \\
&= \omega' \sum \omega
\end{aligned}$$

where $\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}$ and $\Sigma = [\sigma_{i,j}] = \begin{pmatrix} \sigma_{11} & \dots & \dots & \dots & \sigma_{1n} \\ \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ \sigma_{n1} & \dots & \dots & \dots & \sigma_{nn} \end{pmatrix}$

This is called the covariance matrix. So, the return on the portfolio has

$$E[R] = \omega' \mu$$
$$\sigma(R) = \sqrt{\omega' \Sigma \omega}$$

Note that

- σ_{ij} is the covariance between R_i the return on asset i and R_j the return on asset j.
- $\sigma^2 = \sigma_{ii}$ is the variance of R_i
- $\rho_{ij} = \dfrac{\sigma_{ij}}{\sigma_i \sigma_j}$ is the correlation of R_i and R_j

Denote the

- the covariance matrix by Σ;

- the correlation matrix $[\rho_{ij}]=\begin{bmatrix} \rho_{11} & \cdots & \cdots & \cdots & \rho_{1n} \\ \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ \rho_{n1} & \cdots & \cdots & \cdots & \rho_{nn} \end{bmatrix}$ by P ;

- the diagonal matrix of standard deviations $\begin{bmatrix} \sigma_i & 0 & \cdots & \cdots & 0 \\ 0 & \sigma_2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \sigma_n \end{bmatrix}$ by D

Then

$$\Sigma = DPD$$

and so

$$\sigma(R) = \sqrt{\omega' DPD\omega}$$

3.4.3 Diversification and its Benefits

Graeme (2006) considers a case in which assets are all independent and uncorrelated so $\rho_{ij} = \delta_{ij}$. (δ_{ij} is the indicator function). Then

$$\sigma^2(R) = \sigma_{i=1}^{n} \omega_i^2 \sigma_i^2.$$

Let us assume here that the portfolio is equally weighted, such that $\omega_i = \frac{1}{n}$ for every i. Then

$$\omega^2(R) = \Sigma_{i=1}^{n} \frac{1}{n^2} \sigma_i^2 = \frac{1}{n} \Sigma_{i=1}^{n} \frac{\sigma_i^2}{n} \rightarrow 0$$
$$\text{as n} \rightarrow \infty$$

Once variance is accepted as a risk measure, then risk goes to 0 as more and more assets are obtained.

$$\omega^2(R) = \Sigma_{i=1}^n \Sigma_{j=1}^n \frac{1}{n^2}\sigma_{ij}$$
$$= \frac{1}{n}\Sigma_{i=1}^n \frac{\sigma_i^2}{n} + \frac{n-1}{n}\Sigma_{i=1}^n \Sigma_{j=1,j\neq i}^n \frac{\sigma_{ij}}{n(n-1)}$$
$$= \frac{1}{n}\,\overline{\sigma_i^2} + \frac{n-1}{n}\overline{\sigma_{ij}, i \neq j}$$
$$\rightarrow \overline{\sigma_{ij,i\neq j}} asn \rightarrow \infty$$

As a measure of undiversifiable market risk, the average covariance serves as the limit.

3.4.4 Efficient Frontier Construction

In Graeme (2006), there is one portfolio that offers the lowest possible risk, for every level of return and for every level of risk, there is a portfolio that offers the highest return. When these combinations are plotted on a graph of the standard deviation (risk) against the return , the resulting line is the efficient frontier. A portfolio above the curve is impossible and portfolios below the curve is not efficient, because there is a greater return for the same level of risk. Optimal portfolios should lie on the curve (low risk/low return are at the bottom, medium risk/mediun return are in the middle, and high risk/high return are at the top of the line). A rational investor will only ever hold a portfolio that lies on the efficient frontier. MPT suggests that, borrowing to acquire a risk-free stock could make your portfolio a riskier profile and hence give a higher return than might otherwise have been chosen. The efficient frontier can be estimated in manipulating the concept of Optimum Portfolio of Risky Assets. Considering the scenario where short sales are possible, hypothetically varying the risk free rate can be done. You obtain an $OPRA_r$ for each risk free rate r(simply, this is an asset with $\sigma = 0$ and known return say r). The reduced problem is now in finding the OPRA for any risk free rate r.

$$\frac{\delta}{\delta\omega_i}\omega'\Sigma\omega{=}2\Sigma_{j=1}^n \omega_j \sigma_{ij}$$

Let $\theta = \dfrac{\mu_p - r}{\sigma_p}$. Maximizing θ is necessary, in fact it is known as the market price of risk of the portfolio. The constraint is $\Sigma_{j=1}^n \omega_j = 1$.

$$\theta = \frac{\Sigma_{j=1}^{n}\omega_j(\mu_j - r)}{\sqrt{\omega'\Sigma\omega}}$$

$$\Rightarrow \frac{\delta\theta}{\delta\omega_i} = \frac{\sqrt{\omega'\Sigma\omega}(\mu_i - r) - \Sigma_{j=1}^{n}\omega_j(\mu_j - r)\frac{2\Sigma_{j=1}^{n}\omega_j\sigma_{ij}}{2\sqrt{\omega'\Sigma\omega}}}{\omega'\Sigma\omega}$$

$$\Rightarrow 0 = \mu_i - r - \frac{\Sigma_{j=1}^{n}\omega_j(\mu_j - r)}{\omega'\Sigma\omega}\Sigma_{j=1}^{n}\omega_j\sigma_{ij}$$

$$\underline{0} = \begin{pmatrix}\mu_1\\ \mu_2\\ \vdots\\ \mu_n\end{pmatrix} - r\begin{bmatrix}1\\ 1\\ \vdots\\ 1\end{bmatrix} - \lambda\Sigma\begin{pmatrix}\omega_1\\ \omega_2\\ \vdots\\ \omega_n\end{pmatrix}$$

where it happens that

$$\lambda = \frac{\Sigma_{j=1}n\omega_j(\mu_j - r)}{\omega'\Sigma\omega}$$

λ is known as the Merton proportion. Thus

$$\lambda\begin{pmatrix}\omega_1\\ \omega_2\\ \vdots\\ \omega_n\end{pmatrix} = \Sigma^{-1}\begin{bmatrix}\mu_1 - r\\ \mu_2 - r\\ \vdots\\ \mu_n - r\end{bmatrix}$$

Solution to $\omega_1, \omega_2, \ldots, \omega_n$ can be achieved using $\Sigma_{j=1}^{n}\omega_j = 1$. (σ, μ),is a point on the efficient frontier.

Chapter 4

Analysis and Results

4.1 ARMA Model Specification

4.1.1 Introduction

The analysis focused on the weekly closing stock prices/continuously compounded returns of the Ghana Stock Exchange Databank Stock Index. The sample data begins from the first week in January that is 02 January 2004 and ends in the last week of December that is 31 December 2008, giving a total of 260 data points and comprising a total of 21 different stocks. The volatility analysis herein includes fitting of the GARCH specification models to the return residuals series of the returns in events where the ARMA specifications are unable to capture all the ARCH effects present in the series. Volatility measures are thus modeled from such GARCH specifications and used for forecasting where applicable.

4.1.2 Descriptive Statistics and Graphical Presentation

Table 4.1 on the following page gives the descriptive statistics with a mean of 1.8181 and a standard deviation of 0.4507, this indicates a slight presence of instability .

Time plot of prices of Guinness Ghana Limited (GGL) depicts that, from the beginning of 2004, there is an appreciably sharp rise in the prices from approximately 0.56 to 1.26 up until the middle of 2004 and fairly level prices from then until the middle of 2005. This is followed by a sharp decline within a matter of a few weeks and followed by an approximate gradual rise throughout the rest of the series until almost three quarters into 2007. Another very sharp rise occurs in the last quater of 2007 and a fairly stable rise between the end of 2007 and a few weeks into 2008. There is a very sharp rise soon after, up until the series attains its maximum value at 2.5, three quarters into 2008. The series ends with a steep decline.

The time plot of the continuously compounded returns of the GGL shown in Figure 4.2 on page 39 indicates slight presence of instability. These dynamics are as described in the plot of the price series on page 38 above. Tables 4.2 and 4.3 on page 39 gives a summary of descriptive statistics of the GGL returns. Sample mean, standard deviation, skewness and kurtosis, and the Jarque-Bera normality statistic and p-value have been reported. A mean value of 0.00208 and a standard deviation of 0.0152 shows that there is indeed slight presence of instability.

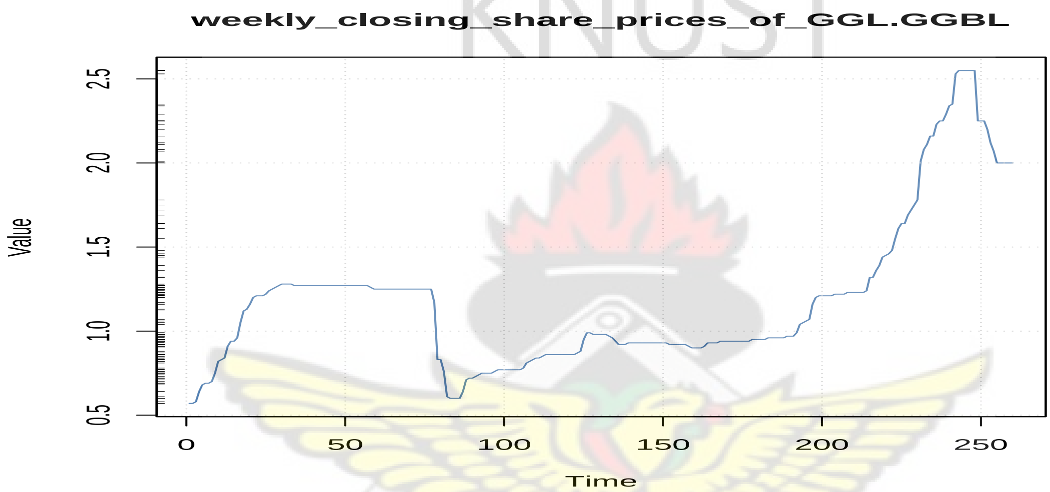

Figure 4.1: Time plot of GGL

	Mean	Maximum	Minimum	Std. Dev.
estimates	1.181154	2.55	0.57	0.4507002

Table 4.1: Descriptive Statistics of GGL prices

	Mean	Maximum	Minimum	Std. Dev.
estimates	0.002086659	0.05205523	-0.1504928	0.01519946

Table 4.2: Descriptive Statistics of GGL log returns

4.1.3 Stationarity Tests:

The unit root stationarity test on the returns series using the Augmented Dickey Fuller (ADF) test which assumes a null hypothesis that the series has a unit root gives the results given in Table 4.4 on the facing page. It implies that we reject the null hypothesis that there exists a unit root in the series.

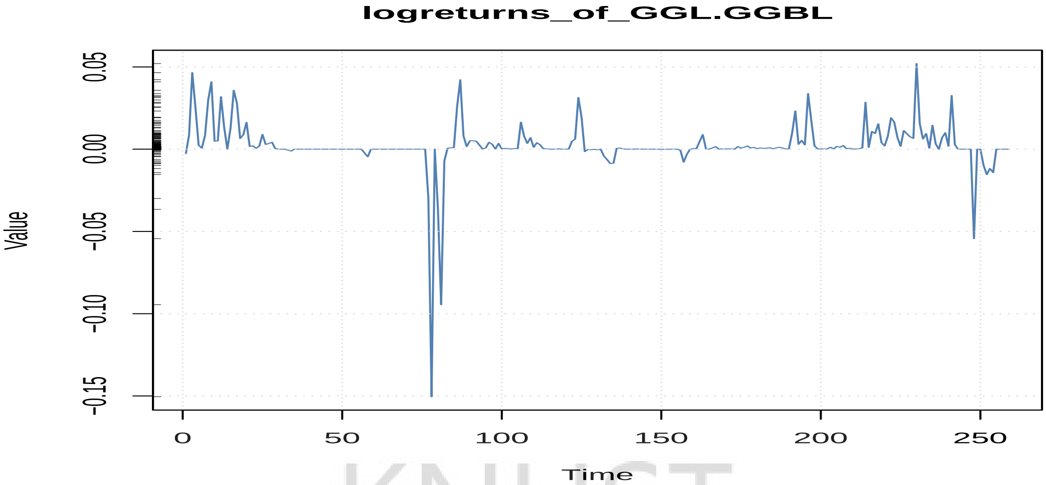

Figure 4.2: Time plot of logreturns of GGL

	Skewness	**Kurtosis**	**Jarque-Bera**	**Probability**
estimates	-4.521365	44.95264	23150.3449	<2.2e-16

Table 4.3: Descriptive Statistics of GGL log returns *continued.*

As a complimentary test, the Kwiatkowski Phillips Schmidt Shin (KPSS) test which assumes a null hypothesis of stationarity is also performed. This gives the results given in Table 4.4. Due to these results, we fail to reject the null hypothesis.

All the above tests prove that the returns series of GGL is stationary.

Stationarity Test	**Test Statistic**	**Lag Order**	**p-value**	**α-value**
ADF	-5.2792	6	0.01	0.05
KPSS.	0.1969	3	0.1	0.05

Table 4.4: Stationarity Test

4.1.4 Trend Analysis:

Judging from the steep downward straight line from 2004 to 2005 and the steep straight line from 2005 to 2006 and other such similar lines to 2007 and to 2008 as shown in Figure 4.3 above, there is absence of any form of trend embedded in the underlying stochastic process of the returns series of the GGL data series.

4.1.5 Seasonality Analysis:

The box-and-whisker plot shown in Figure 4.4 below depicts a generally flat plot, with a few minimal rises in weeks 8, 9 and 20 up to 22. This shows no significant

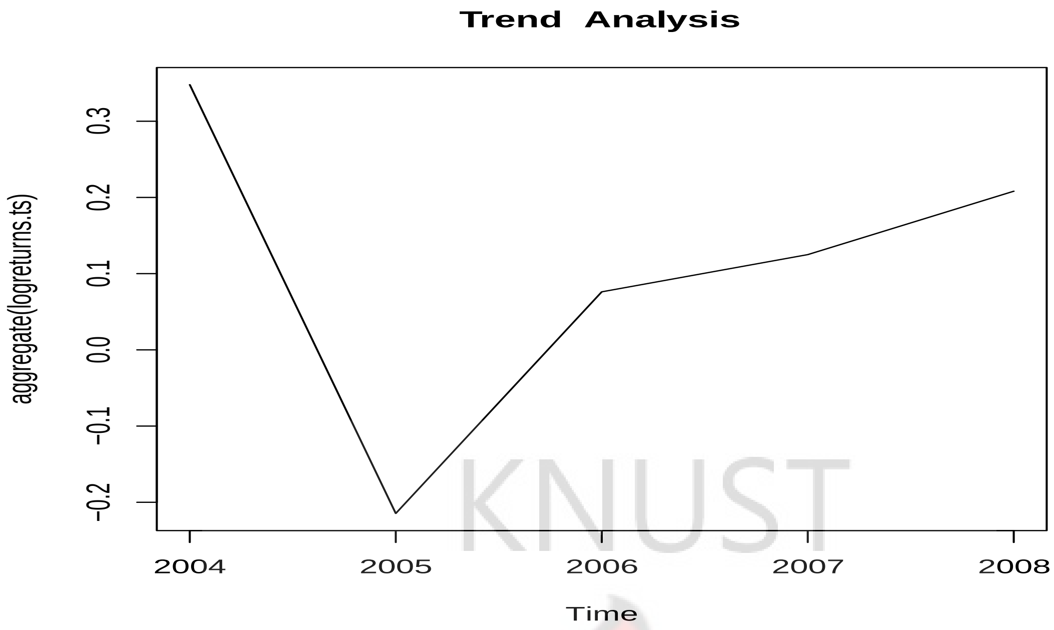

Figure 4.3: Aggregate plot of GGL

serial dependence and hence no seasonal component.

4.1.6 Model Specification: ARIMA(3,0,3)

"auto.arima"is a command in R, version 2.13.0, from the forecast package which returns the best ARIMA model that is to be fit to a univariate time series. It was adopted in this model specification. It is used in accordance with particular specifications of information criteria, specifying which types of stationarity tests to adopt and initiating whether to search among seasonal and non-seasonal models or not.(See Hyndman, (2010))

The stationarity tests used are the KPSS test and the Augmented Dickey Fuller's unit root test. Information criteria used are the Akaike Information Criteria, the Corrected Akaike Information Criteria and the Schwartz-Bayesian Information Criteria.

	ar1	**ar2**	**ar3**	**ma1**	**ma2**	**ma3**
estimates	0.6387	-0.1181	0.0421	-0.3618	0.0134	0.3505
s.e.	0.2030	0.1933	0.1893	0.1955	0.1712	0.1450

Table 4.5: Parameter Estimates

$E(X_t) = \mu_{X_t} = 0$
$Var(X_t) = E(X_t)^2 - (E(X_t))^2 = E(X_t)^2 = \sigma^2_{X_t} = 0.0001759;$
AIC $= -1495.29$; AICc $= -1494.84$; BIC $= -1470.36$

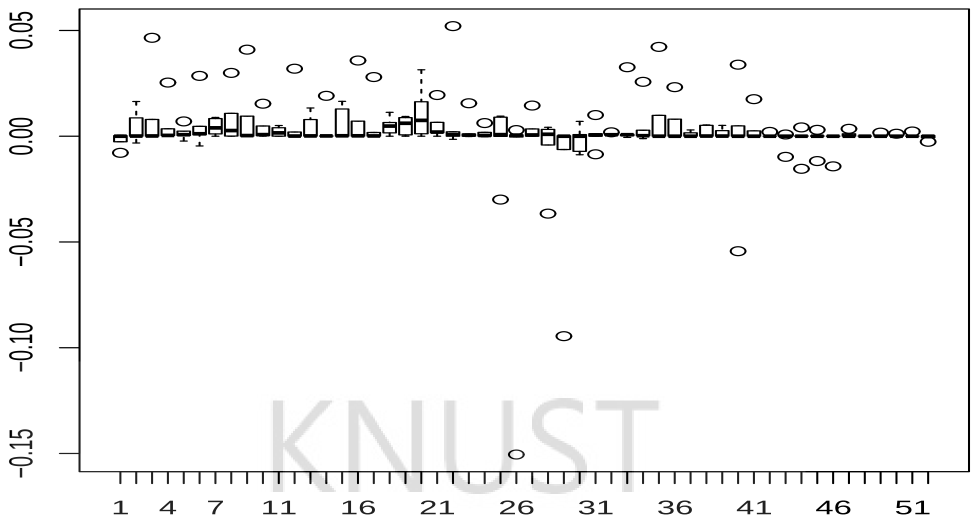

Figure 4.4: Box and Whisker plot of the GGL sample data

The select, hence best model is ARIMA(3,0,3).

$X_t = \mu + \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$

$\Rightarrow X_t = 0.6387X_{t-1} - 0.1181X_{t-2} + 0.0421X_{t-3} - 0.3618\epsilon_{t-1} + 0.0134\epsilon_{t-2} + 0.3505\epsilon_{t-3}$

4.1.7 Test of Model Adequacy and Forecasts:

4.1.7.1 Error Analysis

In-sample error measures:

ME= 0.0008993084 RMSE= 0.0132639011 MAE= 0.0055838565

1. As shown in Table 4.5, the asymptotic standard errors of the parameter estimates are all statistically not far from zero as expected for adequacy.

2. The model has 3 AR parameters and 3 MA parameters. Based on the principle of parsimony, it is concluded that the model is parsimonious.

3. Comparing the forecasted values to the original values from the observed data , it can be seen that the forecasts are accurate (values not shown here).

4. The plot of the model residuals is shown in Figure 4.6 on page 43.

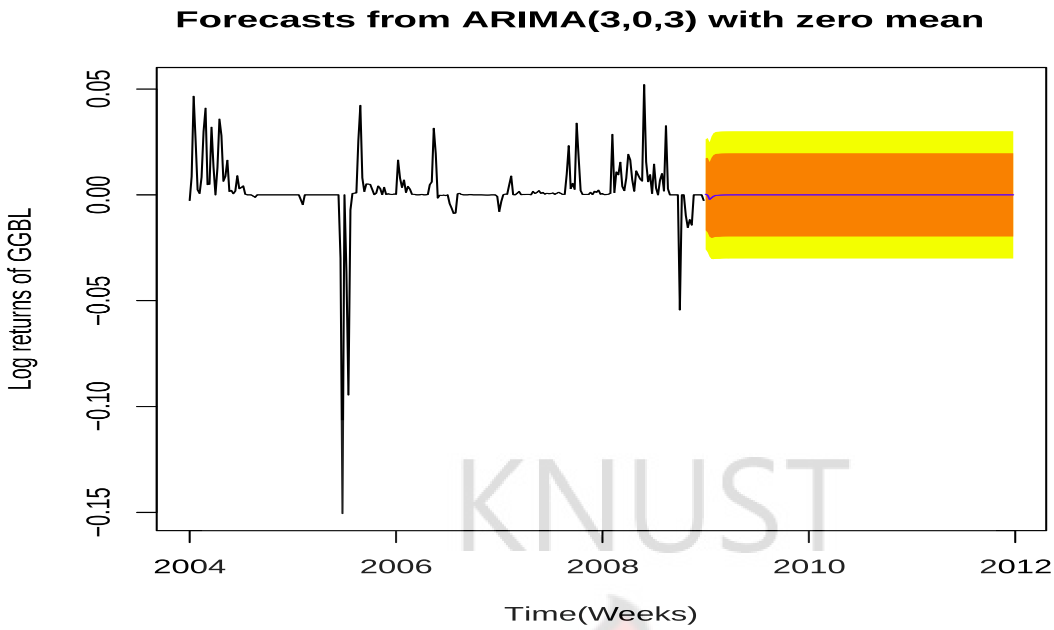

Figure 4.5: Three year forecast using ARIMA(3,0,3)

5. The autocorrelogram of the residuals shown in Figure 4.7 on page 44 shows no sign of significant autocorrelation or serial dependence, this however means that the residuals are purely random as expected for adequacy.

6. The plot of the autocorrelation of the squared residuals as shown in Figure 4.8 on page 45 shows no sign of significant autocorrelation or serial dependence. This means that there is no ARCH effect left uncaptured as will be discussed in a later section.

A Box-Ljung test for autocorrelation among the residuals with a χ^2 statistic of 0.168, degrees of freedom of 6 and a p-value of 0.0099, leads to a rejection of the null hypothesis of correlation.

All above strictly imply that, indeed the model is adequate and statistically significant.

4.1.8 Test for ARCH effect presence

The following portmanteau (Box-Pierce and Ljung-Box) tests, test for independence in the residuals of the model. From Table 4.6 below, there is a failure to reject the null hypothesis of independence in the residuals of the model.

Jarque-Bera's omnibus test for normality which is derived as a Lagrange multiplier test for normal distribution is used to test for normality in the residuals.

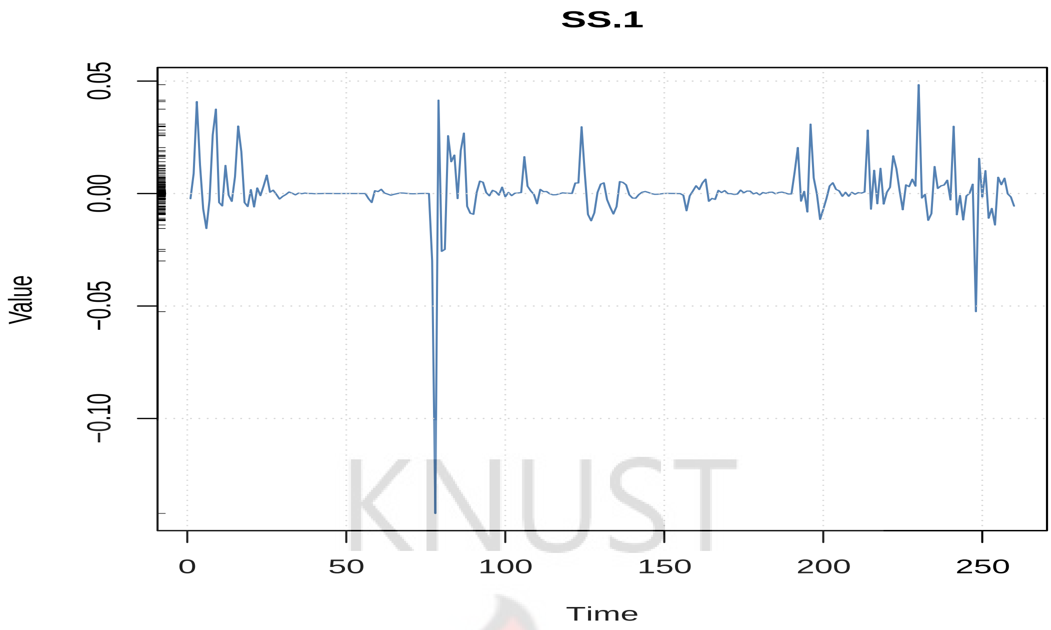

Figure 4.6: Time plot of residuals of ARIMA(3,0,3) fit to the logreturns of GGL

	χ^2-**test statistic**	**degrees of freedom**	**p-value**
Box-Pierce test	0.0003	1	0.9853
Box-Ljung test	0.0003	1	0.9853

Table 4.6: Portmanteau Tests

The Lagrange multiplier test statistic of 31414.54 and a p-value of 0 together with excess kurtosis of 52.68174, provides clear evidence to reject the null hypothesis of normality, which implies the residuals are not normally distributed but leptokurtic.

Again, Ljung-Box's test for randomness among the residuals is implemented to a maximum lag of 30 which rejects the null hypothesis of randomness. Implementing the Ljung-Box test for an ARCH effect at a maximum lag of 30, a test statistic of 27.43 and a p-value of 0.60056970 is discovered. The normal Q-Q plot shown in Figure 4.9 on page 46 agrees with these tests.

A formal and final confirmation is thus achieved by using Engle's Lagrange multiplier test with a null hypothesis of 'no ARCH effects'. With a χ^2 statistic of 3.8705 , a degrees of freedom of 12 and a p-value of 0.9857 , we thus fail to reject the null hypothesis. This ARCH test shows that there is no ARCH effect that the ARMA specification was unable to capture.

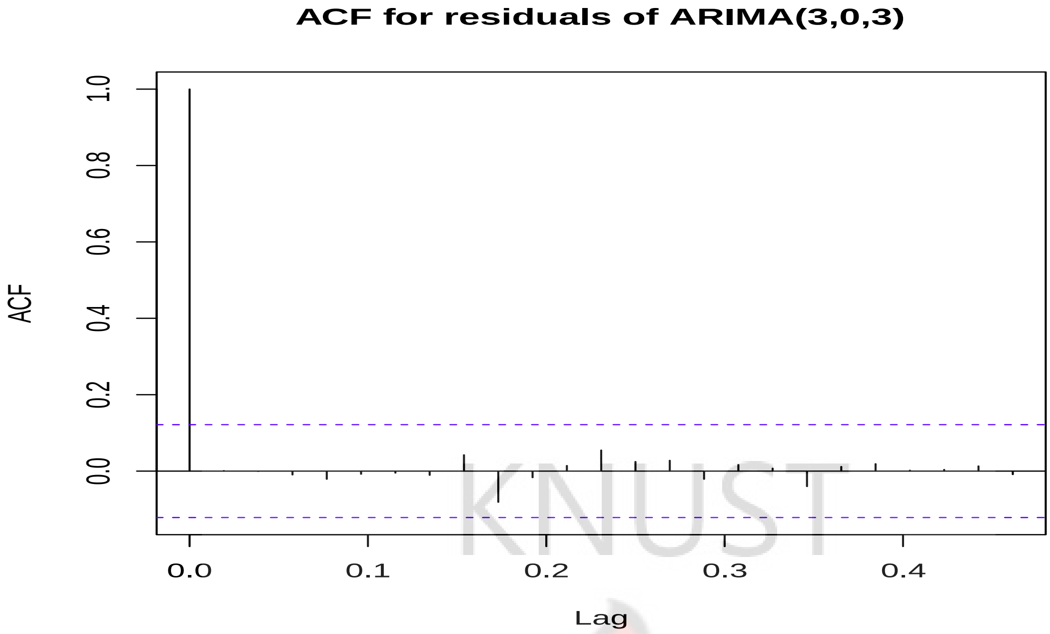

Figure 4.7: ACF of residuals of ARIMA(3,0,3) model fit to the logreturns of GGL

4.2 Application of Concepts Over Entire Share Index

As can be clearly observed from Table 4.8 on page 48, only ABL shows presence of ARCH effects and upon GARCH specification, an ARCH(1) model is most appropriate and hence specified, based on information criteria and asymptotic standard error checks for significance. The parameters of the various ARMA specifications are estimated and given in Table 5.6 in the Appendix on page 67 and the parameter estimates of the ARCH specification is also given in Table 4.9 on page 48.

The ARCH(1) model has an ω estimate of $2.603e-05$ which is small, we expect it to be small any way. All parameters of the variance equation must be positive and they all are in this case. We also expect μ to be small which has a value of $3.422e-05$ which is small. News about the volatility from the previous period is measured with the coefficient of the laq of the squared innovations in the mean equation, which is the ARCH term and in this case the α. Its values are all positive and statistically significant,we conclude that there is significant ARCH effect. Which means that there is significant news about the volatility from the previous period. Volatility (conditional variance) is modeled from the ARCH(1) specification and a plot of it is given in Figure 4.10 on page 49. Forecasts from this volatility are generated for 3 financial years ahead and a plot of it displayed in Figure 4.11 on page 50.

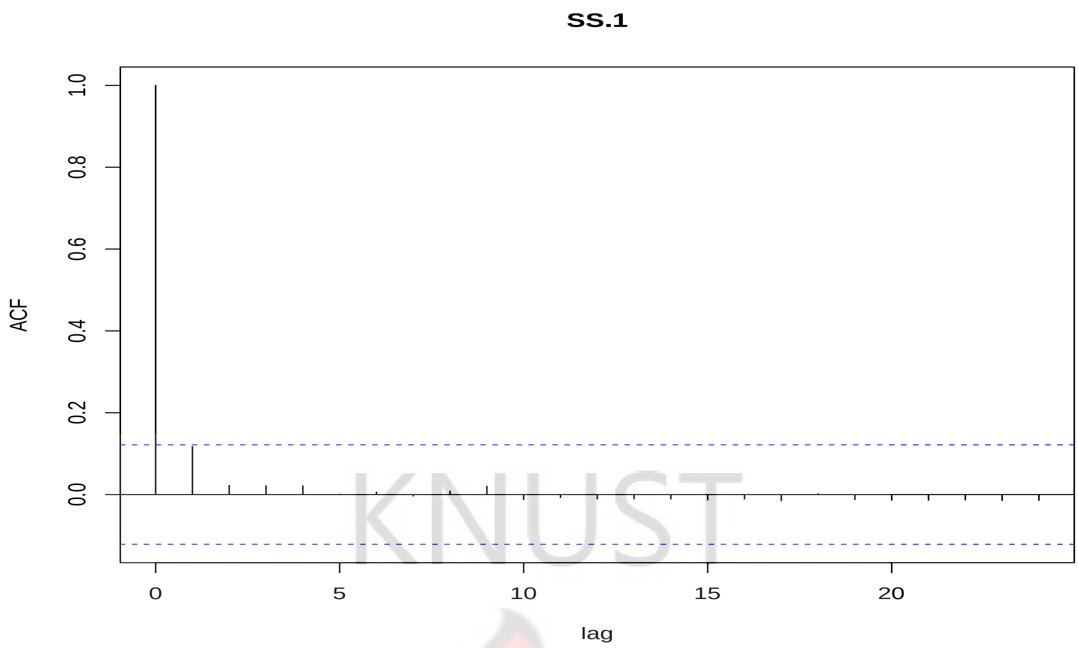

Figure 4.8: ACF of squared residuals ARIMA(3,0,3) model fit to the logreturns of GGL

4.3 Application of Concepts at the Sector level

Applying the concepts at the sector level (which is one level higher than the share level) results in Tables 4.10 and 4.11 on pages 49 and 50. The sector level only considers sectors with more than one share index. As categories in this case, we use Automobiles(comprising CFAO and MLC), Media and Publishing(comprising SWL and CMLT), Breweries(comprising ABL and GGL), Food Products(comprising CPC, UNIL and FML) and Banks(comprising GCB, HFC, SCB, SSB and TBL).

4.4 Application of Concepts at the Industry level

Application of the concepts at the industry level (which is one level higher than the sector level) results in Tables 4.12 and 4.13 on pages 51 and 51. The industry level only considers industries with more than one sector. As categories in this case, we use Consumer Discretionary(comprising Automobiles, Media and Publishing and Household Durables), Consumer Staples (comprising Breweries, Food Products and Personal Products), Financials(comprising Banks and Insurance) and Materials(comprising Metals & Mining and Paper & Forest).

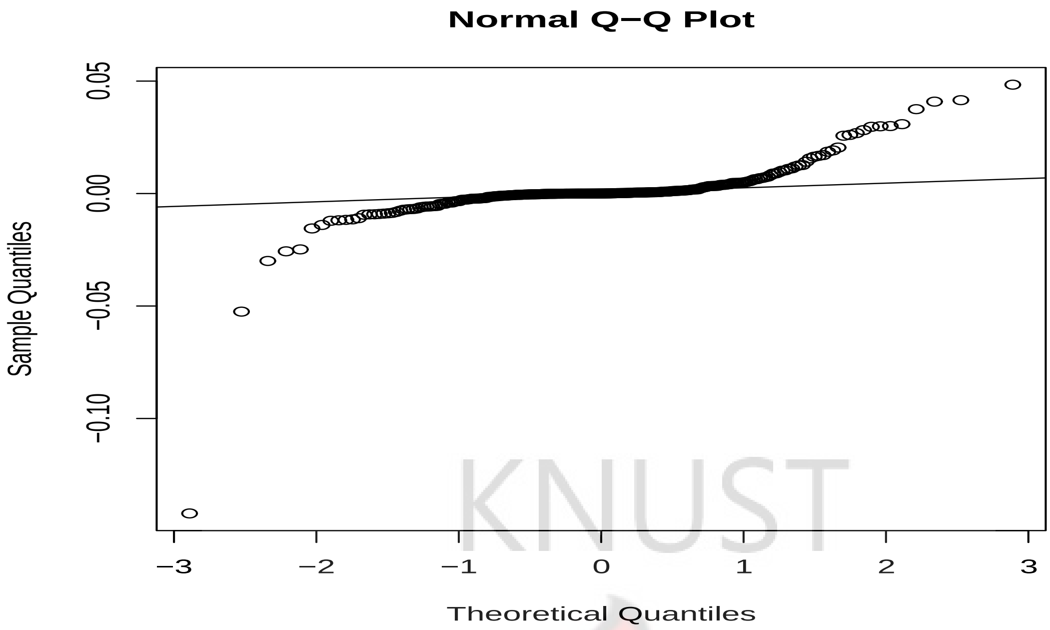

Figure 4.9: Normal Q-Q plot of residuals of ARIMA(3,0,3) model fit to the logreturns of GGL

4.5 Application of Concepts on DSI returns

The mean continuously compounded weekly returns for the DSI ranges from -0.0007647477 to 0.004128573. It is expected for a time series of returns to have a mean close to zero, which is so in this case. It has standard deviation 0.001281724,with respect to the mean, and its an indication of some volatility in the market returns.

Plot of the DSI prices and logreturns are given in Figures 4.12 and 4.13 respectively on pages 51 and 52 respectively. This indicates some amount of volatility. ARMA specifications modeled for this DSI returns generated a $SARIMA(2,1,3)(2,0,1)^1$. Parameter estimates are given in Table 4.14 on page 53. Engle's Lagrange Multiplier tests for ARCH effects presence are performed on the selected ARMA specifications. With a χ^2 value of 31.3592 at a degrees of freedom of 12 and a p-value of 0.001736, the null hypothesis of 'no ARCH effects' is rejected. This means that there indeed is some non-linear ARCH effects presence that the ARMA specification was unable to capture. A GARCH(1,1) model is fit to capture this remaining non-linear ARCH effect presence. The estimates to this GARCH(1,1) specifications are given in Table 4.15 on page 53. All parameters in the variance equation must be positive. We expect to get a positive

Company	Stationarity Test	ARMA Specification
MLC	Non-stationary	$SARIMA(3,1,5)(2,0,1)^1$
CFAO	Non-stationary	$SARIMA(4,1,1)(1,0,0)^1$
PAF	Stationary	$ARIMA(1,0,1)$ with 0 mean
CMLT	Stationary	$ARIMA(2,0,0)$with non-zero mean
SWL	Stationary	$ARIMA(1,0,1)$ with 0 mean
GGL	Stationary	$ARIMA(3,0,3)$ with 0 mean
ABL	Non-stationary	$SARIMA(1,1,1)(2,0,1)^1$
UNIL	Non-stationary	$SARIMA(1,1,3)(2,0,1)^1$
CPC	Stationary	$SARIMA(0,0,2)(2,0,0)^1$ with 0 mean
FML	Stationary	$SARIMA(2,1,2)(2,0,2)^1$ with drift
PZC	Non-stationary	$SARIMA(3,1,1)(2,0,1)^1$ with drift
MOGL	Non-stationary	$ARIMA(0,0,1)$ with non-zero mean
GCB	Stationary	$SARIMA(2,0,2)(2,0,0)^1$ with non-zero mean
TBL	Non-stationary	$SARIMA(0,1,1)(1,0,1)^1$ with drift
SG-SSB	Non-stationary	$ARIMA(1,0,0)$ with zero mean
HFC	Non-stationary	$SARIMA(0,1,3)(2,0,1)^1$with drift
SCB	Stationary	$SARIMA(1,0,1)(2,0,1)^1$ with non-zero mean
EIC	Stationary	$SARIMA(3,0,0)(2,0,0)^1$ with non-zero mean
PBC	Non-stationary	$SARIMA(0,1,1)(2,0,1)^1$
ALW	Non-stationary	$SARIMA(5,0,2)(2,0,0)^1$ with zero mean
SPPC	Non-stationary	$ARIMA(1,1,2)$

Table 4.7: Replication of concepts over entire share index

and small mean value (μ). We find out that it is $1.784e-5$, which is positive and small. We also expect the value of ω to be small. It is $6.691e-8$, which is as well positive and small. It can be seen from Table 4.15 on page 53 that they all are. News about volatility from the previous t periods can be measured as the lag of the squared residuals from the mean equation (ARCH term), which indicates the short run persistence of shocks, i.e. α. It has a positive value and is statistically significant, implying that news about volatility from the previous t periods has an explanatory power on current volatility. Also the estimate of β shows the persistence of volatility to a shock which represents the contribution of shocks long run persistence (alternatively, the impact of old news on volatility). It has a positive and statistically significant value, indicating that, the impact of old news on volatility is significant and that there is significant ARCH effects in the series . It can be observed that , the sum of the ARCH and GARCH coefficients, $\alpha + \beta > 1$, this indicates that volatility shock is strongly persistent (very high) and the variances are not stationary under GARCH model.

Company	AIC	BIC	TEST OF ARCH EFFECT PRESENCE
MLC	-2487.55	-2441.31	No Presence of Arch Effects
CFAO	-1474.87	1449.97	No Presence of Arch Effects
PAF	-2221.57	-2210.89	No Presence of Arch Effects
CMLT	-1190.66	1176.41	No Presence of Arch Effects
SWL	-2422.7	-2412.02	No Presence of Arch Effects
GGL	-1495.29	-1470.36	No Presence of Arch Effects
ABL	-1850.19	-1825.29	Presence of Arch Effects
UNIL	-1735.17	-1703.16	No Presence of Arch Effects
CPC	-1456.18	-1438.38	No Presence of Arch Effects
FML	-1863.71	-1828.14	No Presence of Arch Effects
PZC	-2105.38	-2073.37	No Presence of Arch Effects
MOGL	-1752.85	1742.17	No Presence of Arch Effects
GCB	-1669.99	-1641.5	No Presence of Arch Effects
TBL	-1447.79	-1426.45	No Presence of Arch Effects
SG-SSB	-1406.95	-1399.89	No Presence of Arch Effects
HFC	-1768.52	-1740.07	No Presence of Arch Effects
SCB	-1625.67	-1600.75	No Presence of Arch Effects
EIC	-1802.85	-1777.93	No Presence of Arch Effects
PBC	-1636.44	-1615.1	No Presence of Arch Effects
ALW	-2185.72	-2150.11	No Presence of Arch Effects
SPPC	-1435.36	-1421.13	No Presence of Arch Effects

Table 4.8: Replication of concepts over entire share index continued

Model	Parameters	Estimate	Std Error	t-statistic	p-value
$ARCH(1)$	μ	3.422e-05	3.344e-04	0.102	0.9185
	ω	2.603e-05	2.875e-06	9.053	2e-16
	α	8.275e-01	3.964e-01	2.087	0.0369
Log likelihood = 970.2559			normalized log likelihood = 3.731754		

Table 4.9: Estimates of ARCH(1) specification model

4.6 Chapter Summary, Findings and Implications

This section summarizes and compiles the analysis , the findings and attempts to give interpretations.

It was found at the share level that, only the ABL share showed presence of non-linear ARCH effects after the ARMA model specification. An ARCH(1) model was the most appropriate model to capture the remaining ARCH effects. Conditional variance (volatility) was modeled from the ARCH(1) specification.

Upon extension to the sector level, this volatility vanished. The ARMA specifications in the sector levels unanimously captured all the ARCH effects, implying

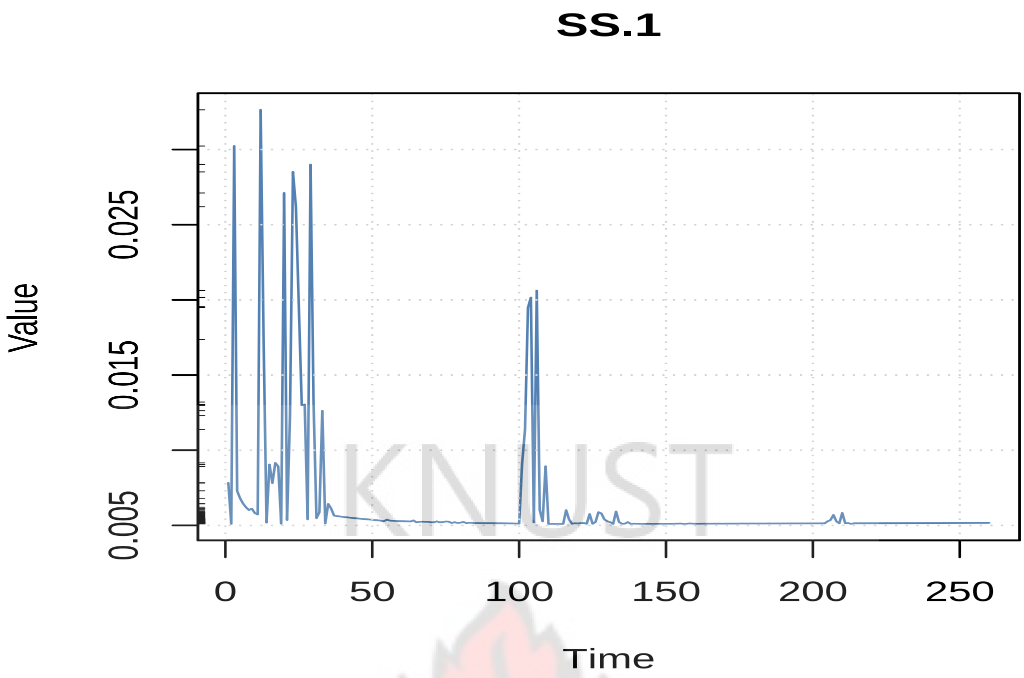

Figure 4.10: Plot of conditional variance of ABL

Sectors	Stationarity Test	ARMA Specification	AIC	BIC	Test of ARCH Effect Presence
AUTOMOBILES	Non-stationary	$SARIMA(1,1,2)(2,0,1)^1$ with drift	-1964.96	-1936.5	No Presence of ARCH Effects
BANKS	Non-stationary	$SARIMA(2,1,3)(2,0,0)^1$	-1843.87	-1815.42	No Presence of ARCH Effects
BREWERIES	Non-stationary	$SARIMA(5,1,3)(0,0,1)^1$	-1787.74	-1752.17	No Presence of ARCH Effects
FOOD PRODUCTS	Stationary	$SARIMA(2,0,3)(2,0,0)^1$ with non-zero mean	-1820.4	-1788.36	No Presence of ARCH Effects
MEDIA AND PUBLISHING	Non-stationary	$ARIMA(2,1,1)$	-1578.53	-15622.3	No Presence of ARCH Effects

Table 4.10: ARMA model specifications for the sector level

there was no need for GARCH model specifications. At the industry level, the ARMA specifications fit to the various industries captured all the ARCH effects present and there was no need for GARCH modeling. However, the ARCH effect presence test on the DSI returns series showed that, there is some non-linear ARCH effect still to be captured. Fitting a GARCH(1,1) was the most appropriate specification. The ARCH and GARCH terms in the GARCH(1,1) model specification were all statistically significant. This implies on the market that, news about volatility from the previous t periods has an explanatory power on current volatility and that there exists strong volatility persistence which is rather predictable in the case of the GSE.

In effect, stock prices on the GSE will be either undervalued or overvalued.

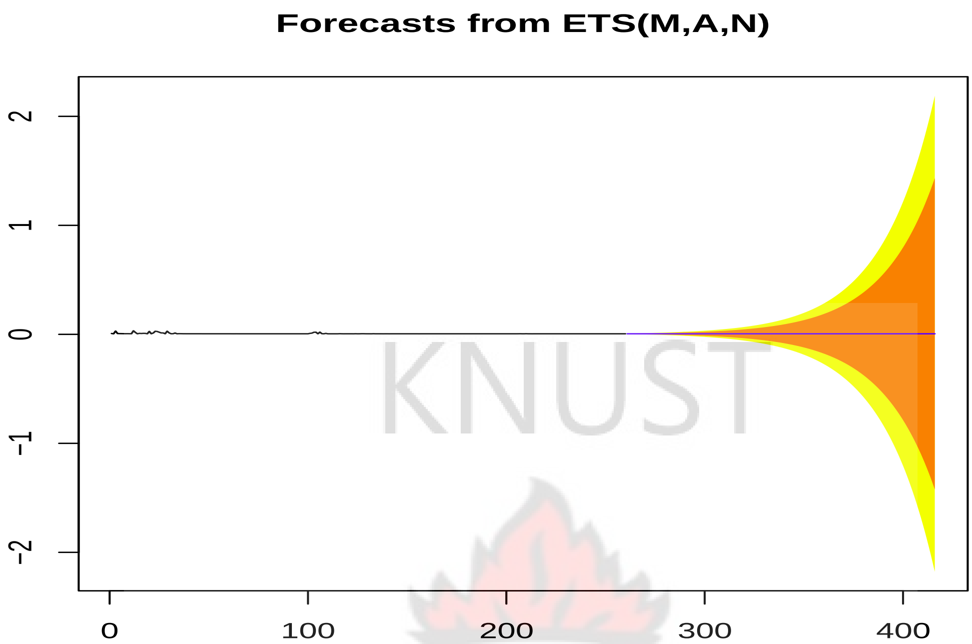

Figure 4.11: Three years forecast of conditional variance of ABL

Sector	ARMA Specification Model									
AUTOMOBILES		ar1	ma1	ma2	sar1	sar2	sma1			
		0.0221	-0.3140	-0.7875	-0.0012	0.0110	-0.623			
	s.e	0.0040	0.0854	0.0857	0.0009	0.0001	0.1722			
BANKS		ar1	ar2	ma1	ma2	ma3	sar1	sar2		
		-0.8438	-0.2907	0.2619	-0.3068	-0.3381	0.0552	0.1028		
	s.e.	0.0010	0.0021	0.0019	0.0064	0.0012	0.0017	0.0032		
BREWERIES		ar1	ar2	ar3	ar4	ar5	ma1	ma2	ma3	sma1
		0.4777	-0.9896	0.6657	-0.0021	0.3260	-0.1823	0.1823	-0.9871	0.0226
	s.e.	0.0629	0.0686	0.0835	0.0692	0.0635	0.0254	0.0312	0.0345	0.0706
FOOD PRODUCTS		ar1	ar2	ma1	ma2	ma3	sar1	sar2	intercept	
		0.3641	-0.4405	-0.8186	-0.0374	0.0858	4e-04	0.0868	0.0016	
	s.e.	0.0010	0.0011	0.0007	0.0013	0.0009	0.0021	0.0323	0.0015	
MEDIA AND PUBLISHING		ar1	ar2	ma1						
		0.0672	-0.2213	-0.8060						
	s.e.	0.1901	0.1840	0.1052						

Table 4.11: Parameter estimates for the sector level

Industry	ARMA Specification Model								
CONSUMER DISCRETIONARY		ar1	ar2	ar3	ma1	ma2	sar1		
		-0.2374	0.0428	0.0225	-0.2785	-0.5872	-0.0046		
	s.e	0.1562	0.1174	0.1083	0.1447	0.1218	0.0561		
CONSUMER STAPLES		ar1	ar2	ar3	sar1	sar2	sma1	sma2	
		-0.5303	-0.4491	-0.3276	0.1471	0.0168	-0.6222	-0.4953	
	s.e.	0.0613	0.0628	0.0613	0.0356	0.0281	0.1084	0.1986	
FINANCIALS		ar1	ma1	ma2	ma3	ma4	sar1	sar2	intercept
		0.8948	-0.1095	-0.6245	-0.0594	-0.0576	0.1596	-0.0622	0.0014
	s.e.	0.1116	0.1249	0.1087	0.0579	0.0618	0.1128	0.0969	0.0014
MATERIALS		ma1	sar1						
		0.4703	0.0253						
	s.e.	0.0514	0.0765						

Table 4.12: Parameter estimates for the industry level

Industry	Stationary Test	ARMA Specification	AIC	BIC	Test of ARCH Effect Presence
CONSUMER DISCRETIONARY	Non-stationary	$SARIMA(3,1,2)(1,0,0)^1$	-2326.43	-2301.53	No Presence of ARCH Effects
CONSUMER STAPLES	Non-stationary	$SARIMA(3,2,0)(2,0,2)^1$	-2350.15	-2321.69	No Presence of ARCH Effects
FINANCIALS	Stationary	$SARIMA(1,0,4)(2,0,0)^1$with non-zero mean	-1467.45	-1435.4	No Presence of ARCH Effects
MATERIALS	Stationary	$SARIMA(0,0,1)(1,0,0)^1$with zero mean	-1625.65	-1614.97	No Presence of ARCH Effects

Table 4.13: ARMA Specification for the industry level

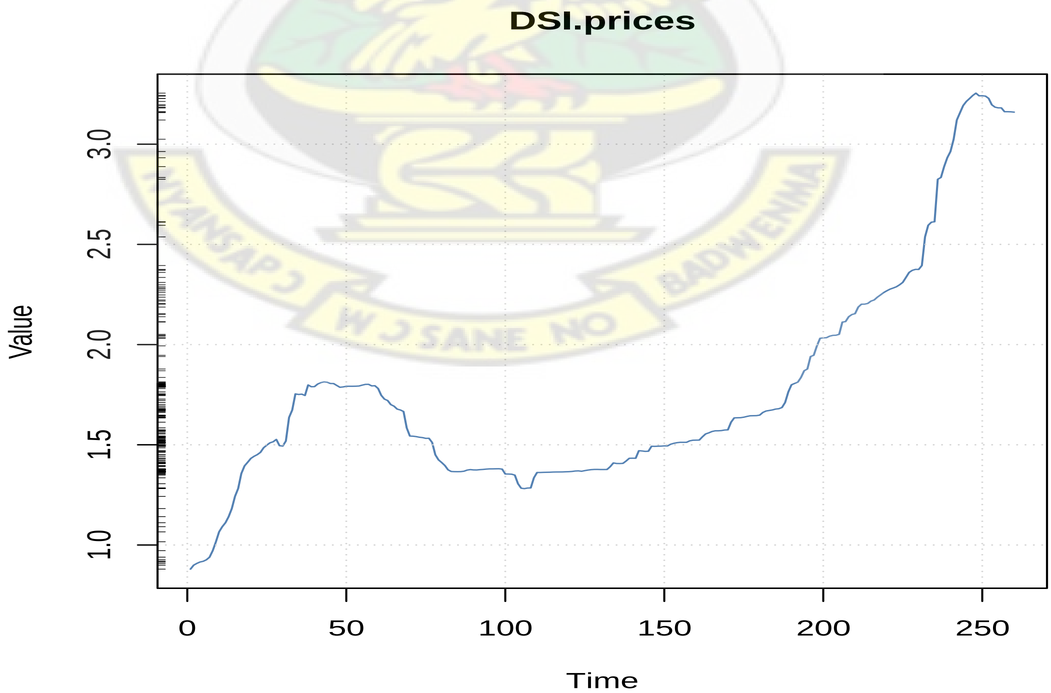

Figure 4.12: Plot of DSI prices

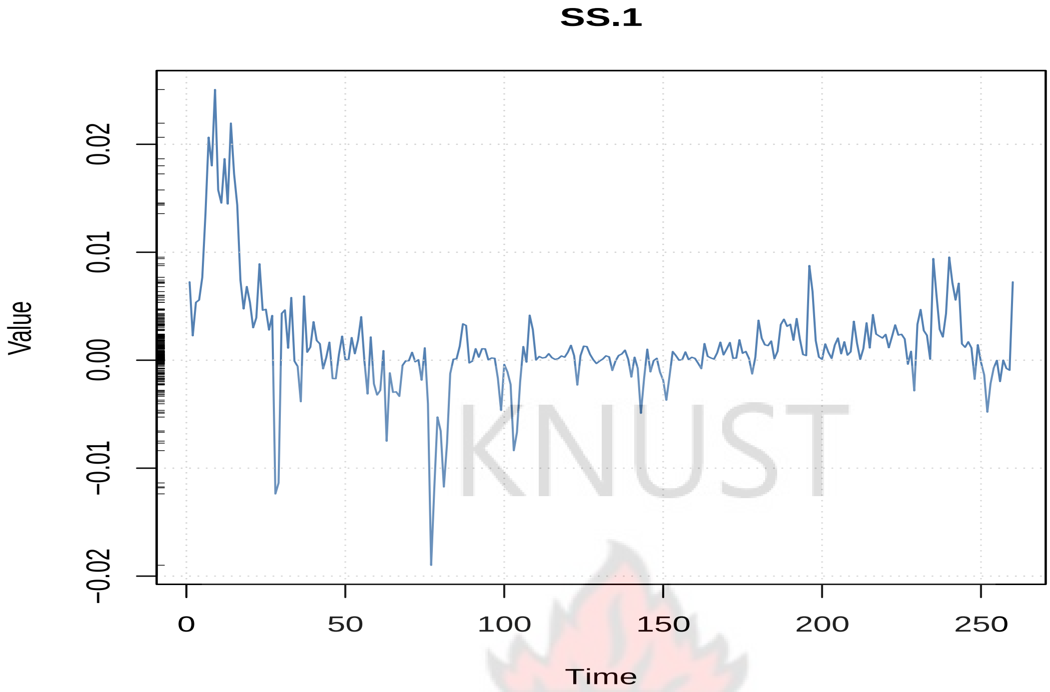

Figure 4.13: Plot of DSI logreturns

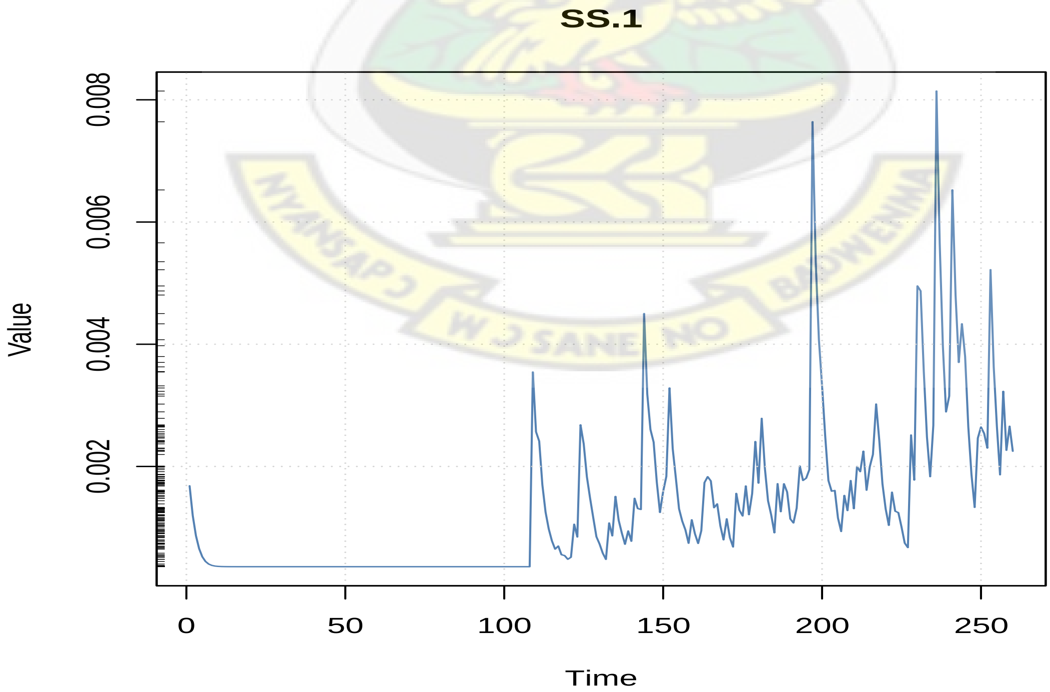

Figure 4.14: Conditional variance plot of DSI returns

Model	Parameters	Estimate	Std Error
$SARIMA(2,1,3)(2,0,1)^1$	ar1	0.0667	0.0023
	ar2	0.2702	0.0325
	ma1	-0.5561	0.0236
	ma2	-0.6188	0.0634
	ma3	0.1015	0.0330
	sar1	0.0105	0.0002
	sar2	0.0395	0.0316
	sma1	-0.0837	0.0988
AIC = -2514.8	AICc = -2514.07	BIC = -2482.79	$\sigma^2 = 3.171e-06$
log likelihood=1272.16			

Table 4.14: Estimates of ARMA specification model for DSI returns

Model	Parameters	Estimate	Std Error	t-value	p-value
$GARCH(1,1)$	μ	1.784e-05	3.218e-05	0.554	0.579356
	ω	6.691e-08	1.506e-08	4.442	8.92e-06
	α_1	1	2.702e-01	3.702	0.000214
	β_1	4.804e-01	8.292e-02	5.793	6.91e-09
log likelihood=1455.348			normalised log likelihood=5.597492		

Table 4.15: Estimates of GARCH specification model for DSI returns

Chapter 5

Summary, Recommendations and Conclusion

5.1 Introduction

A very crucial aspect of the financial and investment decision making process is the concern for the risk and instability of expected returns. The focus for this thesis included the modeling of the GARCH volatility from the GARCH specification models fit to the ARMA specifications that tested positive for the ARCH effect presence test on the Ghana Stock Exchange Databank Stock Index. Where applicable, forecasts were generated for volatility from GARCH model specifications.

5.2 Summary

ARMA specification models were fit to data (at the share, sector, industry and overall DSI levels) and their accuracy tested. Tests like non-correlation of model residuals, statistical insignificance of asymptotic standard errors, adoption and application of the principle of parsimony and comparison of observed values with forecast values were used, thus objective 1 has been achieved in by using these methods . Box-Pierce and Ljung-Box portmanteau tests among several other tests were used to test for the independence among the residuals of the selected ARMA specification models and in investigation to test for the presence of non-linear ARCH effects. Jarque-Bera's omnibus normality test rejected the null hypothesis of normality among the residuals. A confirmatory test using Engle's Lagrange Multiplier test with a null hypothesis of 'no ARCH effect presence' was used to finalize these tests of ARCH effects presence, thus objective 2 has been achieved in so doing. Further analysis was done at the sector level (one level higher than the share level). At the industry level (one level higher than the sector level), ARMA specifications were also fit to the various industries. Final analysis is performed on the entire DSI returns and a final analysis and inference drawn. GARCH model specifications were fit in appropriate instances where there was non-linear ARCH effects still to be captured after ARMA specifications, thus objective 3 has been achieved.

5.3 Recommendations

1. Experts and researchers should intensify their research to find the undervalued and overvalued stocks. Hardworking analysts could take advantage of the situation and do better than the market averages. On the other hand, people such as stock brokers, bankers and investors (especially foreign ones) with inside information can also consistently outperform the market . To increase their returns, they are therefore very likely to be attracted to invest more or diversify in the GSE bourse.

2. As studied in Frimpong (2008), further works of study on the efficiency of Ghana's financial markets is recommended.

3. Asymmetric models like the EGARCH, TGARCH ,GJR-GARCH, PGARCH, AGARCH and CGARCH models as have been studied by Lee(1991), Cao *et al.*(1992) and Bera *et al.* (1993) are recommended for further studies on the GSE with the aim of capturing the volatility asymmetry (leverage effects), the fat-taildness (leptokurtiscity) among other standardized regularities on the GSE which is a major component of volatility dynamics.

4. Extensions of this work using a combination of concepts from the modern optimal portfolio selection, efficient frontier construction and the unconditional variance extracted from the fitted variance models would be worthwhile. It would improve upon investors knowledge on their investing ability and chances.

5.4 Conclusion

This thesis contributes and extends the existing literature on modeling stock returns volatility in Ghana using past data by concluding that, there is strong persistence of volatility evident in the GSE at the overall DSI returns level.

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Appendix

The GSE is closed on the following Public Holidays. It is for the general public to note that where a holiday falls on a weekend, the following working day will be observed as a holiday (if declared by Government).Dates with * are subject to visibility of the New Moon.

Number	Holiday	Date
1	New Year's Day	January 1
2	Independence Day	March 6
3	Good Friday	April 22
4	Easter Monday	April 25
5	May Day	May 1
6	Africa Unity Day	May 25
7	Republic Day	July 1
8	Eid-Al-Fitr	August 30*
9	Founder's Day	September 21
10	Eid-Al-Adha	November 6*
11	Farmers' Day	December 2
12	Christmas Day	December 25
13	Boxing Day	December 26

Table 5.1: Public Holidays for the year 2011

The landmarks and chronology of financial evens that earmark Ghana's financial development from 1953 to 1994 has been tabulated below.

DATE	EVENT
1969	Pearl report by Commonwealth Development Finance Co. Ltd recommended the establishment of a Stock Exchange in Ghana within two years and suggested ways of achieving it.
1971	The Stock Exchange Act was enacted
1971	The Accra Stock Exchange company incorporated but never operated
February, 1989	PNDC government set up a 10-member National Committee on the establishment of the Stock Exchange under the chairmanship of Dr. G.K. Agama, the then Governor of the Bank of Ghana.
July 1989	Ghana Stock Exchange was incorporated as a private company limited by guarantee under the Companies Code 1963
October, 1990	Executive Instrument No. 20 giving recognition to Ghana Stock Exchange as authorized Stock Exchange signed.
November.1990	Council of the Exchange adopted operational regulations namely, GSE Membership Regulations L.I. 1510, Listing Regulations L.I 1509 and Trading and Settlement Regulations.
November, 12, 1990	First Council of the Exchange with Mrs. Gloria Nikoi as Chairperson inaugurated.
November, 12, 1990	Trading commenced on the floor of the Exchange
January, 11, 1991	Ghana Stock Exchange was officially launched
September. 1993	The Exchange moved to its present offices, 5th Floor, Cedi House, Liberia Road, Accra
April 1994	A resolution passed at the AGM changed the Exchange from a private company limited by guarantee to that of a public company limited by guarantee under the Company Code 1963 (Act 179)

Table 5.2: Landmarks of the Ghana Stock Exchange

Date	Event
1953	Bank of the Gold Coast (now Ghana Commercial Bank) established
1961	Promulgation of Exchange Control Act which puts the inflow and outflow of foreign exchange under Bank of Ghana regulation
1963	National Investment Bank set up to provide medium and long-term finance for the promotion of industrial, commercial, agricultural and other enterprises.
1965	Agricultural Development Bank set up.
1972	Bank for Housing and Construction set up to promote housing and civil engineering projects.
1972	Merchant Bank Ghana Limited Established. This was the first merchant bank in Ghana
1983	Ghana launches World Bank/IMF backed structural Adjustment Program
1985	Financial Institutions Sector Adjustment Program (FINSAP) launched. The main objectives were: to deregulate interest rates and remove ceilings on deposit and lending rates to privatize government-owned financial institutions and commercial Banks to enhance the soundness of the banking institutions by improving prudential regulation and supervision by the bank of Ghana to improve deposit mobilization and increase efficiency in credit allocation to develop money and securities markets
September 1986	A weekly foreign exchange auction system was introduced
1986(September)	A TWO-TIER EXCHANGE RATE SYSTEM WAS ADOPTED. THE WINDOW 1 RATE WAS FIXED AT 90/US$ WHILE THE WINDOW 2 RATE WAS DETERMINED AT THE WEEKLY FOREIGN EXCHANGE AUCTION.
1987	FINSAC I launched
September	Maximum lending rates aminimum deposit rates decontrolled
October	Weekly auction of Treasury Bills introduced
November	Consolidated Discount House starts operations
1988	The two foreign exchange windows were unified. Window 1 was abolished and the marginal rate established at the weekly auction became the rate for transactions for the week. Bank of Ghana could now authorize dealers other than banks to set up Bureaux de Change to buy and sell currency
February	Minimum bank savings rate decontrolled; Sectoral credit controls, except for agriculture removed
April	Foreign exchange bureaus established
September	90-day Bank of Ghana Bills for banks introduced
1989 July	Comprehensive restructuring plan for banks adopted

Table 5.3: A Chronology of Financial Market Developments

August	Banking Law PNDCL 225 passed. The law covered capital adequacy, reserve requirements, loan limits and reporting requirements. The new banking law strengthened the Bank of Ghana"s supervisory role, including a) periodic on-site examination of banks, b) regular standard reporting, c) issuance of new accounting standards, d) audit guidelines and e) authority to impose fines and punitive actions in case of violations.
September	Insurance Law enacted
December	Non-rediscountable, medium-term Bank of Ghana instruments for banks with 180 day, 1 year and 2 year maturities introduced
1990	Ecobank (Ghana) Limited established. Ecobank is a subsidiary of Ecobank International Limited. The parent holds 58% of the shares while a number of institutions and individuals resident in Ghana hold the remaining 42%.
January	New managers for public sector banks appointed; two new merchant banks licensed
March	Bank cash reserve requirements on demand, savings and time deposits unified.
April	Foreign exchange market unified
May	Restructuring of three state-owned banks begun; SOE non-performing loans swapped with Bank of Ghana FINSAP bonds
September	NPART and the Non-Performing Assets Tribunal created
November	Ghana Stock Exchange Commences operations; 30-day BOG Bills and 180-Day, 1 and 2 year treasury bills introduced; 5 Year Government Stock introduced; BOG instruments made available to the non-bank sector; lending targets for the agricultural sector and prescribed bank charges and fees abolished; cash reserve ratio reduced to 22%; secondary reserves ratio increased to 20%; bank restructuring extended to three additional banks.
December	Payment of interest on cash reserves at 3% introduced; private sector non-performing loans of state-owned banks swapped with BOG-issued FINSAP bonds; capital adequacy standards enforced
1991 March	Private sector nonperforming loans in sound banks swapped for non-issued FINSAP bonds
June	Securities Discount House Commences operations
July	Cash reserve ratio reduced to 18%; secondary reserves increased to 24% and remuneration on cash reserves increased to 5%
1992 January	Leasing company licensed
October	Bank of Ghana Law (PNDC Law 291) providing for tougher supervisory and regulatory powers enacted
1993 March	Cash reserve ratio reduced to 10%; secondary reserve ratio increased to 32%

Table 5.4: A Chronology of Financial Market Developments continued

May	Financial Institution(Non-Banking) Law PNDCL 328 enacted to provide the supervisory and regulatory framework for nonbank financial institutions and to encourage competition among commercial banks; Home Finance Mortgage Law enacted to support development of housing finance.
June	Finance Lease Law enacted to further the development of the leasing industry September Cash reserve ratio reduced to 5%; temporary additional 15% secondary reserve ratio imposed bringing secondary reserve ratio up to 15%
1993	Securities Industry Law PNDCL333 promulgated
1994	May NSCB merged with Social Security bank
August	Approval granted for 3 new banks; Approval granted for new leasing company
1995 January	Changes in payment arrangements for inter-bank dealings introduced by BOG
February	Appointment of advisor for the partial divestiture of SSB
March	Appointment of advisor for the partial divestiture of GCB
April	Appointment of advisors for the partial divestiture of NIB
October	SSB listed on the Ghana Stock Exchange
December	Bank of Ghana notice BG/GOV/SEC/95/29 11/12/95 Remedial measures at GSE. Foreigners participation in listed stocks permitted after the stock has been offered to the Ghanaian public for three consecutive days.
1996 May	Ghana Commercial Bank Listed on the Ghana Stock Exchange

Table 5.5: A Chronology of Financial Market Developments continued

Company	ARMA SPECIFICATION MODEL											
MLC		ar1	ar2	ar3	ma1	ma2	ma3	ma4	ma5	sar1	sar2	sma1
		-0.0003	0.0811	0.0004	-0.4515	-0.6509	1.1084	-0.4833	-0.6950	0.0032	-7e-04	0.0300
	s.e.	0.0908	0.0876	0.0435	0.0625	0.0712	0.0020	0.0700	0.0784	0.0002	3e-04	0.0626
CFAO		ar1	ar2	ar3	ar4	ma1	sar1					
		0.1237	-0.1895	0.0979	-0.021	-0.8862	-0.0837					
	s.e.	0.0011	0.0006	0.0006	0.0005	0.0014	0.0006					
PAF		ar1	ma1									
		0.0356	0.4212									
	s.e.	0.1775	0.1614									
CMLT		ar1	ar2	intercept								
		0.2606	-0.0072	0.0045								
	s.e.	0.1918	0.1886	0.0022								
SWL		ar1	ma1									
		0.0011	0.4735									
	s.e.	0.1762	0.1548									
GGL		ar1	ar2	ar3	ma1	ma2	ma3					
		0.6387	-0.1181	0.0421	-0.3618	0.0134	0.3505					
	s.e.	0.2030	0.1933	0.1893	0.1955	0.1712	0.1450					
ABL		ar1	ma1	sar1	sar2	sma1						
		0.6549	-1.0000	-0.002	0.1016	0.0108						
	s.e.	0.1284	0.0134	0.0321	0.1947	0.0558						
UNIL		ar1	ma1	ma2	ma3	sar1	sar2	sma1				
		-0.8732	0.2426	-0.9499	-0.4596	0.0221	-0.0247	0.2818				
	s.e.	0.0214	0.1212	0.0156	0.0317	0.0010	0.0008	0.0949				
CPC		ma1	ma2	sar1	sar2							
		0.5388	0.1261	-0.1118	0.0723							
	s.e.	0.0609	0.0592	0.0814	0.0953							
FML		ar1	ar2	ma1	ma2	sar1	sar2	sma1	sma2			
		-0.0326	0.0874	-0.5356	-0.5631	0.0493	-0.0054	0.3780	0.0104			
	s.e.	0.2782	0.2211	0.1998	0.0027	0.0004	0.0003	0.1227	0.2528			
PZC		ar1	ar2	ar3	ma1	sar1	sar2	sma1	drift			
		0.2725	-0.1753	0.3255	-1.0438	-0.0296	0.0230	0.0091				
	s.e.	0.0588	0.0633	0.0565	0.0045	0.0036	0.0191	0.0860				
MOGL		ma1	intercept									
		0.4117	0.0013									
	s.e.	0.0569	0.0007									
ar1	ar2	ma1	ma2	sar1	sar2	intercept						
		-0.6927	-0.2815	1.0896	0.6982	-0.0196	0.0183	0.0014				
	s.e.	0.0935	0.0898	0.0745	0.0887	0.0358	0.0334	0.0008				
TBL		ma1	sar1	sma1	drift							
		-0.3604	0.0051	.0025	0e+00							
	s.e.	0.1605	0.2032	0.0021	6e-04							
SG-SSB		ar1										
		0.5331										
	s.e.	0.0522										
HFC		ma1	ma2	ma3	sar1	sar2	sma1	drift				
		-0.5454	-0.0576	-0.1794	0.8504	0.0129	-0.9696	-1e-04				
	s.e.	0.0638	0.0714	0.0613	0.4714	0.1083	1.0185	1e-04				
SCB		ar1	ma1	sar1	sar2	sma1	intercept					
		0.6948	-0.4280	0.6766	0.2320	-0.8755	0.0031					
	s.e.	0.1299	0.1656	0.4468	0.1234	0.4891	0.0011					
EIC		ar1	ar2	ar3	sar1	sar2	intercept					
		0.5020	-0.2217	0.2871	-0.0206	0.0048	0.0043					
	s.e.	0.0594	0.0657	0.0596	0.0545	0.0134	0.0010					
PBC		ma1	sar1	sar2	sma1							
		-0.4427	-0.0794	0.0054	-0.0297							
	s.e.	0.0574	36.4828	3.7191	36.5066							
ALW		ar1	ar2	ar3	ar4	ar5	ma1	ma2	sar1	sar2		
		0.2996	0.6990	0.1744	-0.0984	-0.1328	-0.0913	-0.8103	0.0366	-0.0048		
	s.e.	0.1158	0.1242	0.0733	0.0787	0.0686	0.0956	0.0940	0.0235	0.0116		
SPPC		ar1	ma1	ma2								
		-0.1255	-0.3520	-0.5764								
	s.e.	0.1250	0.1049	0.0890								

Table 5.6: Estimates of ARMA specification model at the share level

Weeks	DSI logreturns	DSI prices
1	0.007249233	0.87952381
2	0.0022677	0.899273016
3	0.005356368	0.908087302
4	0.005578916	0.915120635
5	0.007677858	0.918693651
6	0.013579297	0.926203175
7	0.02064921	0.939285714
8	0.018013232	0.971760317
9	0.025067434	1.016228571
10	0.015769962	1.065634921
11	0.014563027	1.091265079
12	0.018659178	1.111104762
13	0.014472467	1.141385714
14	0.02195641	1.18161746
15	0.017260471	1.24258254
16	0.014364088	1.28187619
17	0.007421111	1.356815873
18	0.004756074	1.394722222
19	0.006818117	1.413022222
20	0.005355603	1.432188889
21	0.003013077	1.442609524
22	0.003932432	1.450798413
23	0.008921968	1.462269841
24	0.004629845	1.485225397
25	0.004711503	1.498334921
26	0.002800105	1.509861905
27	0.004137622	1.515109524
28	-0.012382691	1.526807937
29	-0.011369543	1.495477778
30	0.004331302	1.493596825
31	0.004648468	1.51865873
32	0.00112283	1.636344444
33	0.005799658	1.673446032
34	-7.97055E-05	1.754093651
35	-0.000571845	1.751106349
36	-0.003840354	1.752785714
37	0.005939883	1.745971429
38	0.000745496	1.798598413
39	0.001208633	1.789779365
40	0.00355922	1.790252381
41	0.001822919	1.803065079
42	0.001516827	1.810225397
43	-0.000794171	1.813820635
44	0.000246658	1.812131746
45	0.001662497	1.805925397
46	-0.001699605	1.805687302
47	-0.001707598	1.796190476
48	0.000542225	1.78697619
49	0.002222155	1.788826984
50	6.69964E-05	1.791892063
51	2.59199E-05	1.792047619
52	0.002089575	1.792107937

Table 5.7: 2004 Weekly Sample Data at the DSI returns level

MATERIALS	CONSUMER DISCRETIONARY	CONSUMER STAPLES	FINANCIALS	MATERIALS
-0.005497692	0.028174675	-0.003905864	0.003400419	-0.005497692
0	0.001026233	0.002841522	0.004202391	0
0	0.001778197	0.011727554	0.00293003	0
0	0.002187143	0.007239829	0.000890965	0
0	0.000801157	0.004607007	0.002600088	0
0	0.019718242	0.003578556	0.010181348	0
-0.01889428	0.007008367	0.016722888	0.03977645	-0.01889428
0.001965147	0.000449977	0.017244886	0.051571945	0.001965147
0.050402528	0.002346197	0.016633049	0.042234868	0.050402528
0.054572235	0.00776496	0.00492528	0.007876483	0.054572235
0.012504931	0.008960543	0.012375929	0.01182532	0.012504931
0.05532637	0.020967743	0.008558417	0.011154411	0.05532637
0.024593248	0.013052254	0.012798172	0.008813819	0.024593248
0.042461602	0.014028686	0.01877045	0.019204302	0.042461602
0.02449323	0.023619162	0.009946271	0.013025016	0.02449323
0	0.009481314	0.016727199	0.016129004	0
0	0.004061211	0.006467485	0.006012196	0
0	0.001312984	0.004182238	0.006034051	0
0	0.002951595	0.009676531	0.005856658	0
0	0.00342549	0.010719658	0.002119465	0
0	0	0.009159724	0.001135882	0
0	0.000538162	0.012151625	0.001250476	0
-0.001744164	0.01514297	0.012898233	0.002640369	-0.001744164
-0.001170608	0.001493291	0.01029957	0.000710852	-0.001170608
-0.000587679	0.002910629	0.010809867	0.002765762	-0.000587679
-0.001180151	0.005131643	0.004262827	0.005662327	-0.001180151
-0.000592489	0	0.018599545	0.000379825	-0.000592489
-0.00059411	0.004960126	0.013100443	-0.166473219	-0.00059411
-0.00119312	0.002844779	0.019215886	-0.182378273	-0.00119312
0	0.004722266	0.005768609	0.004536786	0
0	0.001880168	0.004660718	0.009626688	0
0	0	0.000157147	0.01377892	0
0	0	0.003266664	0.039929005	0
-0.026700796	0.001832564	0.011589166	0.010744458	-0.026700796
-0.014691889	0.003619875	-0.009659063	0.046306946	-0.014691889
0	0.001744239	-0.015276573	0.034218458	0
0	0.006898781	0.00394115	0.016488212	0
0	0.002766544	0.004582635	0.003891548	0
0	0.000300967	0.004942685	-0.002093391	0
0	0.000598205	0.007759823	-0.000135675	0
0	0.00118178	0.005357245	-0.002009444	0
0	0.002307205	0.002434318	-0.000186393	0
0	0	0.001381816	-0.002286288	0
0	0.005465419	-0.001969863	-0.001189922	0
0	0.006298093	0	-0.000287613	0
0	0.005514754	-0.001361797	-0.005259776	0
0	0.002201916	-0.002682077	-0.002802872	0
0	0	0.001977408	0.000239647	0
0	0.006230288	0.000978666	5.17977E-05	0
0	0	0	0.000140693	0
0	0	0	5.44318E-05	0
0	0	2.05326E-05	0.000242679	0

Table 5.8: 2004 Weekly Sample Data at the Industry level

AUTOMOBILES	Media and Publishing	BREWERIES	FOOD PRODUCTS	BANKS
-0.062365989	0.122756334	-0.019418004	0.007700413	0.006800839
0.003078699	0	0.004338409	0.001513566	0.004288216
0.005334592	0	0.023265359	0.006620965	0.005860061
0.00656143	0	0.012684135	0.009035352	0.00178193
0.00240347	0	0.001206435	0.012614585	0.005200175
0.024556214	0.034598513	0.00032691	0.005176231	0.014933945
0.01389206	0.00713304	0.004174632	0.013437997	0.037180419
0.000349252	0.00100068	0.015229236	0.036505421	0.025494549
0.00703859	0	0.020724682	0.029174464	0.032465489
0.023294879	0	0.002680205	0.012095635	0.015752966
0.02688163	0	0.019850206	0.01727758	0.020936292
0.062903229	0	0.017765622	0.007909628	0.018268844
0.039156762	0	0.006674731	0.022080066	0.014946166
0.039590623	0.000543548	0.003559871	0.048010688	0.024426366
0.067831301	0.001083031	0.011670849	0.013570738	0.019656141
0.026509444	0	0.025350808	0.015779804	0.026784519
0.012183634	0	0.014653512	0.004748945	0.012024393
0.001053263	0	0.0032426	0.009304114	0.012059734
0.007897134	0	0.019159879	0.009869712	0.01169658
0.008367479	0	0.017261722	0.012641171	0.004138532
0	0	0.013421679	0.005148577	0.002271763
0	0.001614485	0.025697446	0.004150231	0.002442397
0.020628495	0.024800415	0.0320713	0.004371143	0.005213827
0.000677527	0.002850989	0.012841583	0.018057128	0.001421705
0.00402136	0.002814043	0.020362837	0.012066763	-0.000200997
0.011635671	0.000929968	0.006661849	0.005608536	-0.000320201
0	0	0.005469907	0.013728388	-4.31133E-05
0.003744011	0	0.021893151	0.003167739	-0.002021958
0.003071441	0	0.008087371	0.02240804	-0.000464256
0.014166799	0	0.007897134	0.009408693	0.002132285
0.005640505	0	0.00841722	0.005564933	0.019253377
0	0	0.00069519	-0.00022375	0.004694957
0	0	0.001935544	-0.001233858	0.008164784
0	0.005497692	0.000165654	-0.005826812	0
0.010859625	0	0	-0.029170165	-0.003696681
0.005232717	0	0	-0.045829719	-0.006141133
0.020696343	0	0	0.010282713	0.003505212
0.008299631	0	0	0.010682734	-0.010971658
0.0009029	0	0	0.014669001	-0.004186781
0.001794616	0	0	0.022961536	0
0.00354534	0	0	0.016071736	-0.004018889
0.006921614	0	0	0.005716779	-0.000372786
0	0	0	0.001462173	-0.004572577
0.016396257	0	0	-0.005909589	-0.002379845
0.01889428	0	0	0	-0.000575227
0	0.016544261	0	-0.00408539	-0.010519553
0	0.006605749	0	-0.008518035	-0.005605745
0	0	0	0	0.000479294
0	0.018690864	0	0	0.000103595
0	0	0	0	0.000281385
0	0	0	0	0.000108864
0.020696343	0	0.000485358	0	0.000203

Table 5.9: 2004 Weekly Sample Data at the Sector Level