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INSTITUTE OF DISTANCE LEARNING

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MODELLING THE WEST AFRICAN EXAMINATION COUNCIL'S QUESTION PAPER DEPOT INSPECTION AS A TRAVELLING SALESMAN PROBLEM

BY

CHARLES HENYO

BACHELOR OF SCIENCE (HONOURS) PHYSICS

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university except where due acknowledgement has been made in the text.

Charles Henyo, PG3013309	…………………….	…………
Student's Name & ID	Signature	Date

Certified By

Dr. S. K. Amponsah	…………………..	………..
Supervisor's Name	Signature	Date

Certified By

Dr. S. K. Amponsah	…………………..	………..
Head of Department's Name	Signature	Date

ABSTRACT

The travelling salesman problem is considered to be a classic example of what is known as a tour problem. Essentially, any type of tour problem involves making a series of stops along a designated route and making a return journey without ever making a second visit to any previous stop. Generally, a tour problem is present when there is concern on making the most of available resources such as time and mode of travel to accomplish the most in results. Finding a solution to a tour problem is sometimes referred to as discovering the least-cost path, implying that the strategic planning of the route will ensure maximum benefit with minimum expenditure incurred. The concept of the travelling salesman problem can be translated into a number of different disciplines. As a form of optimization that is useful in both mathematical and computer science disciplines, combinatorial optimization seeks to team relevant factors and apply them in a manner that will yield the best results with repeated usage.

This study formulated a real-life problem of WAEC as a TSP, modelled as network problem and applies dynamic programming approach in solving the problem. It was observed that the route that gave minimum achievable inspection plan was 1 – 3 – 6 –9 – 10 at the minimum distance of 183 km, by visiting as many as five centres on the route.

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CHAPTER ONE

1.0 INTRODUCTION

Nowadays, the route management is very important to make sure the user can arrive at the destination the fastest. In the transportation industry, the route that will be generated should consider the cost and time constraints which are dependent on the distance travelled using the route. Although from human logical thinking, the route can be generated easily but the calculation of checking the route whether it is optimal route or not is difficult and will take long time to be implemented.

The travelling salesman or salesperson problem (TSP) is one of the most well-known optimization problems in the literature. It has attracted the attention of many researchers over the last half a century because of its simple problem description but simultaneously its associated difficulty in obtaining an optimal solution efficiently. The travelling-salesman problem involves a salesman who must make a tour of a number of cities using the shortest path available. For each number of cities n, the number of paths which must be explored is n!, causing this problem to grow exponentially rather than as a polynomial.

The travelling salesperson problem (TSP) is a classic model for various production and scheduling problems. Many production and scheduling problems ultimately can be reduced to the simple concept that there is a salesperson who must travel from city to city (visiting each city exactly once) and wishes to minimize the total distance travelled during his tour of all n cities. Obtaining a solution to the problem of a salesperson visiting n cities while minimizing the total distance travelled is one of the most studied combinatorial optimization problems. While there are variations of the TSP, the Euclidean TSP is NP-hard (Schmitt and Amini, 1998; Falkenauer, 1998). The interest in this particular type of problem is due to how common the problem is and how difficult the problem is to solve when n becomes sufficiently large.

In this chapter of the thesis, an overview of the travelling salesman problem would be given; a brief description of the problem statement of the thesis is also presented together with the objectives, the methodology, the justification and the organization of the thesis.

1.1 BACKGROUND OF STUDY

The Travelling Salesman Problem (TSP) is a problem whose solution has eluded many mathematicians for years. Currently there is no solution to the TSP that has satisfied mathematicians. The TSP has a very rich history. Historically, mathematics related to the TSP was developed in the 1800's by Sir William Rowan Hamilton and Thomas Penyngton Kirkman, Irish and British mathematicians, respectively. Specifically, Hamilton was the creator of the Icosian Game in 1857. It was a pegboard with twenty holes that required each vertex to be visited only once, no edge to be visited more than once, and the ending point being the same as the starting point. This kind of path was eventually referred to as a Hamiltonian circuit. However, the general form of the TSP was first studied by Karl Menger in Vienna and Harvard in the late 1920's or early 1930's.

TSPs were first studied in the 1930s by mathematician and economist Karl Menger in Vienna and Harvard. It was later investigated by Hassler Whitney and Merrill Flood at Princeton. Applegate et al., (1994) solved TSP containing 7,397 cities. Later in 1998, they solved it using 13,509 cities in United States. In 2001, the authors found the optimal tour of 15,112 cities in Germany. Later in 2004, TSP of visiting all 24,978 cities in Sweden was solved; a tour of length of approximately 72,500 kilometers was found and it was proven that no shorter tour exists. This is currently the largest solved TSP.

The travelling salesman problem (TSP) is a typical example of a very hard combinatorial optimization problem. The problem is to find the shortest tour that passes through each vertex in a given graph exactly once. The TSP has received considerable attention over the last two decades and various approaches are proposed to solve the problem. As early as in 1954, optimal solution to travelling salesman problem with 49 numbers of cities has been obtained. In 1970's Held and Karp used minimum spanning tree to solve the TSP with 64 cities. In 1971, Bellmore and Malone solved TSP using sub tour elimination .In 1980's, Crowder and Padberg solved the problem with 318 cities using cutting-plane method. In 1991 Grötschel and Holland proposed a solution for large scale TSP. Applegate et al., (1998, 2001 and 2004) proposed solution for TSP using cuts that solved 13509,15112 ,24978 cities respectively. The solutions worked well up to 5000 cities and can be used up to 33,810 cities.

The Travelling Salesman problem (TSP) is one of the benchmark and old problems in Computer Science and Operations Research. It can be stated as: A network with 'n' nodes (or cities), with 'node 1' as 'headquarters' and a travel cost (or distance, or travel time etc.,) matrix C= (C_{ij}) of order n associated with ordered node pairs (i, j) is given. The problem is to find a least cost Hamiltonian cycle.

On the basis of the structure of the cost matrix, the TSPs are classified into two groups – symmetric and asymmetric. The TSP is symmetric if $C_{ij} = C_{ji}$, for all i, j and asymmetric otherwise. For an n-city asymmetric TSP, there are (n -1)! possible solutions, one or more of which gives the minimum cost.

For an n-city symmetric TSP, there is (n-1)! / 2 possible solutions along with their reverse cyclic permutations having the same total cost. In either case the number of solutions becomes extremely large for even moderately large n so that an exhaustive search is impracticable. There are mainly three reasons why TSP has attracted the attention of many researchers and remains an active research area. First, a large number of real-world problems can be modeled by TSP. Second, it was proved to be NP-Complete problem. Third, NP-Complete problems are intractable in the sense that no one has found any really efficient way of solving them for large problem size. Also, NP-complete problems are known to be more or less equivalent to each other; if one knew how to solve one of them one could solve the lot.

Broadly, the TSP is classified as symmetric travelling salesman problem (sTSP), asymmetric travelling salesman problem (aTSP), and multi travelling salesman problem (mTSP). With sTSP: Let $V = \{v_1, .., v_n\}$ be a set of cities, $A = \{(r,s) : r,s \in V\}$ be the edge set, and drs = dsr be a cost measure associated with edge $(r,s) \in A$. The sTSP is the problem of finding a minimal length closed tour that visits each city once. In this case cities $vi \in V$ are given by their coordinates (xi ,yi) and drs is the Euclidean distance between r and s then we have an Euclidean TSP. With aTSP**:** If $drs \neq dsr$ for at least one (r,s) then the TSP becomes an aTSP. With mTSP**:** The mTSP is defined as: In a given set of nodes, let there be m salesmen located at a single depot node. The remaining nodes (cities) that are to be visited are intermediate nodes. Then, the mTSP consists of finding tours for all m salesmen, who all start and end at the depot, such that each intermediate node is visited exactly once and the total cost of visiting all nodes is minimized. The cost metric can be defined in terms of distance, time, etc.

Possible variations of the problem are as follows: Single vs. multiple depots: In the single depot, all salesmen finish their tours at a single point while in multiple depots the salesmen can either return to their initial depot or can return to any depot keeping the initial number of salesmen at each depot the same after the travel. Number of salesmen: The number of salesman in the problem can be fixed or a bounded variable. Cost: When the number of salesmen is not fixed, then each salesman usually has an associated fixed cost incurring whenever this salesman is used. In this case, minimizing the requirements of salesman also becomes an objective. Time frame: Here, some nodes need to be visited in a particular time periods which are called time windows. It is an extension of the mTSP, and referred to as multiple travelling salesman problem with specified time frame (mTSPTW). The application of mTSPTW can be very well seen in the aircraft scheduling problems. Other constraints: Constraints can be on the number of nodes each salesman can visit, maximum or minimum distance a salesman travels or any other constraints. The mTSP is generally treated as a relaxed vehicle routing problems (VRP) where there is no restrictions on capacity. Hence, the formulations and solution methods for the VRP are also equally valid and true for the mTSP if a large capacity is assigned to the salesmen (or vehicles). However, when there is a single salesman, then the mTSP reduces to the TSP (Bektas, 2006).

One example of the usefulness of the TSP is a direct application of the TSP in the drilling problem of printed circuit boards (PCBs) (Grötschel et al., 1991). To connect a conductor on one layer with a conductor on another layer, or to position the pins of integrated circuits, holes have to be drilled through the board. The holes may be of different sizes. To drill two holes of different diameters consecutively, the head of the machine has to move to a tool box and change the drilling equipment. This is quite time consuming. Thus it is clear that one has to choose some diameter, drill all holes of the same diameter, change the drill, drill the holes of the next diameter, etc. Thus, this drilling problem can be viewed as a series of TSPs, one for each hole diameter, where the 'cities' are the initial position and the set of all holes that can be drilled with one and the same drill. The 'distance' between two cities is given by the time it takes to move the drilling head from one position to the other. The aim is to minimize the travel time for the machine head.

In addition to the above application, TSP has been applied to solve a number of real-life problems, including Overhauling gas turbine engines (Plante et al., 1987), X-Ray crystallography(Bland & Shallcross, 1989; Dreissig & Uebach, 1990), Computer wiring(Lenstra & Rinnooy Kan, 1974), The order-picking problem in warehouses(Ratliff & Rosenthal,1983), Vehicle routing (Lenstra & Rinnooy Kan, 1974), and Mask plotting in PCB production(Gottschalk et al., 1991). Thus, TSP has played an important role in supporting managerial decisions in the areas of printing press scheduling, school bus routing, crew scheduling, interview scheduling, hot rolling scheduling, mission planning, and design of global navigation satellite system surveying networks.

1.2 PROBLEM STATEMENT

The specific form of problem that this thesis seeks to solve is to mathematically model a The West African Examinations Council Depot inspection problem as travelling salesman problem (TSP) and solve the problem.

The TSP is one of the most widely studied integer programming problems. The TSP can be easily stated as follows. A salesman wants to visit m distinct cities and then return home. He wants to determine the sequence of the travel so that the overall travelling distance is minimized while visiting each city not more than once. Although the TSP is conceptually simple, it is difficult to obtain an optimal solution.

In an m-city situation, any permutation of m cities yields a possible solution. As a consequence, m! possible tours must be evaluated in the search space. By introducing variables x_{ij} to represent the tour of the salesman travels from city i to city j, one of the common integer programming formulations for the TSP can be written as:

$$\text{Minimize } z = \sum_{\substack{j=1 \\ j \neq i}}^{m} d_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, 2, \ldots, m;\ i \neq j.$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, m;\ i \neq j.$$

$$u_i - u_j + m x_{ij} \leq m - 1 \quad i, j = 2, 3, \ldots, m;\ i^1 j.$$

All $x_{ij} = 0$ or 1, All $u_i \geq 0$ and is a set of integers

The distance between city *i* and city *j* is denoted as d_{ij}. The objective function Z is simply to minimize the total distance travelled in a tour. The first constraint set ensures that the salesman arrives once at each city. The second constraint set ensures that the salesman leaves each city once. The third constraint set is to avoid the presence of sub-tour. Generally, the TSP formulated is known as the Euclidean TSP, in which the distance matrix d is expected to be symmetric, that is $d_{ij} = d_{ji}$ for all i, j, and to satisfy the triangle inequality, that is $d_{ik} \leq d_{ij} + d_{jk}$ for all distinct *i*, *j*, *k*.

1.3 OBJECTIVES

The travelling salesman problem is one of the most commonly studied optimization problem in Operations Research because of its wide applicability. Due to its NP -hard nature, the problem is already complex and difficult to solve.

The goal of this research is to model the Depot inspection problem of The West African Examinations Council as a TSP problem and solve the problem.

1.4 METHODOLOGY

In our methodology, we shall propose the Dynamic Programming approach in solving our problem. First, the algorithm will be presented. A real life computational study is performed and Excel Solver would be employed to analyze our Data.

1.5 JUSTIFICATION

The travelling salesman problems are widely used in modeling most of the real-life industrial applications, and very interestingly from the perspective of computer science because of the time complexities in some of the well-known algorithms used in solving the problems. These have made the studies of travelling salesman problems and their algorithms an important area of research in the contribution to academic knowledge and the benefit of the economy as a whole, hence the reason for solving the travelling salesman problem.

1.6 ORGANIZATION OF THE THESIS

In chapter one, we presented a background study of travelling salesman problem.

In chapter two, related work in the travelling salesman problem will be discussed.

In chapter three, dynamic programming algorithm proposed to solve our problem would be introduced and explained.

Chapter four will provide a computational study of the algorithm applied to our travelling salesman instances.

Chapter five will conclude this thesis with additional comments and recommendations

1.7 SUMMARY

Mathematical programming models are useful tools for modeling and optimizing real-life problems. Unfortunately, the time required to solve mathematical programming models are mostly exponential, so real-life problems often cannot be solved. The travelling salesman problem is a form of mathematical programming problem that are used to model most of the real-life industrial applications. In addition, travelling salesman problems are widely used in industrial decision making, and very interestingly from the perspective of computer science since they are NP-hard. These facts make the studies of travelling salesman problems and their algorithms an extremely important area of research.

This thesis seeks to model a real-life problem in the industry as a travelling salesman problem and proposed a dynamic programming approach in solving the problem.

CHAPTER TWO

LITERATURE REVIEW

Order picking in conventional warehouse environments involves determining a sequence in which to visit the unique locations where each part in the order is stored, and therefore can often be modeled as s TSP. With computer tracking of inventories, parts may now be stored in multiple locations, simplifying the replenishment of inventory and eliminating the need to reserve space for each item. In such an environment, order picking requires choosing a subset of the locations that store an item to collect the required quantity. Thus, both the assignment of inventory to an order and the associated sequence in which the selected locations are visited affect the cost of satisfying an order. Daniels et al. (1998) formulated a model for simultaneously determining the assignment and sequencing decisions, and compared it with the previous models for order picking.

Fagerholt and Christiansen (2000) studied a TSP with allocation, time window, and precedence constraints (TSP-ATWPC). The TSP-ATWPC occurs as a sub problem involving optimally sequencing a given set of port visits in a real bulk ship scheduling problem, which is a combined multi-ship pick up and delivery problem with time windows and multi-allocation problem. Each ship in the fleet is equipped with a flexible cargo hold that can be partitioned into several smaller holds in a given number of ways, thus allowing multiple products to be carried simultaneously by the same ship. The allocation constraints of the TSP-ATWPC ensure that the partition of the ship's flexible cargo hold and the allocation of cargoes to the smaller holds are feasible throughout the visiting sequence.

Calvo and Cordone (2003) introduced the overnight security service problem. The model obtained was a single-objective mixed-integer programming problem. It is NP-hard in the strong sense, and exact approaches are not practicable when solving real-life instances. Thus, the model was solved heuristically, through decomposition into two sub problems. The former was a capacitated clustering problem, the latter a multiple-travelling salesperson problem with time windows.

The time-dependent travelling salesperson problem (TDTSP) is a generalization of the classical TSP, where the cost of any given arc is dependent of its position in the tour. The TDTSP can model several real-life applications (e.g., one-machine sequencing). Gouveia and Vob (1995) presented a classification of formulations for the TDTSP. This framework includes both new and old formulations. All previous literature on the TSP assumed that the sites to be visited are stationary. Motivated by practical applications, Helvig et al. (2003) introduced a time-dependent generalization of TSP, which is called moving target TSP, where a pursuer must intercept in minimum time a set of targets that move with constant velocities.

The TSP has been widely studied in the literature, mainly because of the real-life logistics and transportation problems related to it. Toth and Vigo (2002) reviewed the models and the exact algorithms based on the branch-and bound approach proposed in recent years for the solution of the basic version of the TSP, where the capacity constraints of the vehicle are considered.

Ruiz et al. (2004) proposed a two-stage exact approach for solving a real-life problem of the TSP. In the first stage, all the feasible routes are generated by means of an implicit enumeration algorithm; thereafter, an integer programming model is designed to select in the second stage the optimum routes from the set of feasible routes. The integer model uses a number of 0-1 variables ranging from 2,000 to 15,000 and arrives at optimum solutions in an average time of sixty seconds (for instances up to 60 clients). The developed system was tested with a set of real instances and, in a worst-case scenario (up to 60 clients), the routes obtained ranged from a 7% to 12% reduction in the distance traveled and from a 9% to 11% reduction in operational costs.

Teixeira et al. (2004) conducted a study of route planning for the collection of urban recyclable waste. The aim was to create collection routes for every day of the month, to be repeated every month, as a TSP minimizing the operation cost. Two important features of the problem were the planning of a relatively long period of time and the separate collection of three types of waste. The collection operation was modeled in accordance to the practice of the company that manages the collection system. Preliminary results suggest that significant economies in collection costs are possible.

The Aero medical Airlift Wing of the U. S. Air Force is responsible for the transportation of military personnel in need of specialized medical treatment to and from various military hospitals. Over 8 million active and retired personnel, spouses, and dependents benefit from the system. The system operates under a variety of regulations to ensure timely service and safe operation of the aircraft. Ruland (1999) presented a model of the system to assist the route planners in generating solutions minimizing patient inconvenience. This was achieved by assigning patients to sequence of aircraft while minimizing layovers.

Xiong et al. (1998) used the TSP with time windows (TSPTW) to analyze and model the rolling batch planning problem. Kim and Kim (1999) considered a multi-period scheduling problem in a transportation system where a fleet of homogeneous vehicles delivers products of single type from a central depot to multiple (N) cities. The objective is to minimize transportation costs for product delivery and inventory holding costs at cities over the planning horizon.

Wasner and Zapfel (2004) described why the optimal design of depot and hub transportation networks for parcel service providers makes it necessary to develop a generalized hub location and travelling salesperson model (TSPM). Analogous problems occur for postal, parcel, and piece goods service providers. A generalized hub location and TSPM was developed, which encompassed the determination of the number and locations of hubs and depots and their assigned service areas as well as the routes between the demand points and consolidation points (depots, hubs). The applicability of the model was demonstrated through an Austrian case study. The developed model involved several million binary variables as well as continuous variables and millions of constraints.

Andreea and Camelia-M (2007) presented a model which used a component-based approach to model Ant Colony System (ACS) for the travelling salesman problem (TSP). They used components to solve the TSP with ACS technique and the collaboration rules between components are also described. The internal reasoning about the ant colony component-based system for the TSP gives a better perception of solving this problem using ant-based techniques.

Agarwala et al., (2001) studied a model and used Concorde's TSP solver to construct radiation hybrid maps as part of their ongoing work in genome sequencing. The TSP provides a way to integrate local maps into a single radiation hybrid map for a genome; the cities are the local maps and the cost of travel as a measure of the likelihood tat one local map immediately follows another.

To connect a conductor on one layer with a conductor on another layer, or to position the pins of integrated circuits, holes have to be drilled through the board. The holes may be of different sizes. To drill two holes of different diameters consecutively, the head of the machine has to move to a tool box and change the drilling equipment. This is quite time consuming. Thus it is clear that one has to choose some diameter, drill all holes of the same diameter, change the drill, drill the holes of the next diameter, etc. Thus, this drilling problem can be viewed as a series of TSPs, one for each hole diameter, where the cities are the initial position and the set of all holes that can be drilled with one and the same drill. The distance between two cities is given by the time it takes to move the drilling head from one position to the other. The aim is to minimize the travel time for the machine head. Grotschel et al., (1991) presented a direct application of the TSP in the drilling problem of printed circuit boards (PCBs).

To guarantee a uniform gas flow through the turbines there are nozzle-guide vane assemblies located at each turbine stage. Such an assembly basically consists of a number of nozzle guide vanes affixed about its circumference. All these vanes have individual characteristics and the correct placement of the vanes can result in substantial benefits (reducing vibration, increasing uniformity of flow, reducing fuel consumption). Plante et al., (1987) presented a model on the overhauling gas turbine engines as a TSP problem which occurs when gas turbine engines of aircraft have to be overhauled. The problem of placing the vanes in the best possible way was modeled as a TSP with special objective function.

Analysis of the structure of crystals is an important application of the TSP. Here an X-ray diffractometer is used to obtain information about the structure of crystalline material. To this end a detector measures the intensity of X-ray reflections of the crystal from various positions. Whereas the measurement itself ca be accomplished quite fast, there is a considerable overhead in positioning time since up to hundreds of thousands positions have to be realized for some experiments. In the two examples referred to, the positioning involves moving four motors. The time needed to move from one position to the other can be computed very accurately. The result of the experiment does not depend on the sequence in which the measurements at the various positions are taken. However, the total time needed for the experiment depends on the sequence. Therefore, the problem consists of finding a sequence that minimizes the total positioning time.

This leads to a travelling salesman problem, studied by Bland and Shallcross (1989) and Dreissig and Uebach (1990).

Lenstra and Rinnooy (1974) presented a special case of connecting components on a computer board. Modules are located on a computer board and a given subset of pins has to be connected. In contrast to the usual case where a Steiner tree connection is desired, here the requirement is no more than two wires are attached to each pin. Hence there is the problem of finding a shortest Hamiltonian path with unspecified starting and terminating points. A similar situation occurs for the so-called test bus wiring. To test the manufactured board one has to realize a connection which enters the board at some specified point, runs through all the modules, and terminates at some specified point. For each module there is also a specified entering and leaving point for this test wiring. This problem also amounts to solving a Hamiltonian path problem with the difference that the distances are not symmetric and that starting and terminating point are specified.

Ratliff and Rosenthall (1983) studied a problem of order-picking associated with material handling in a warehouse. Assume that at a warehouse an order arrives for a certain subsets of the items stored in the warehouse. Some vehicle has to collect all items of this order to ship them to the customer. The relation to the TSP is immediately seen. The storage locations of the items correspond to the nodes of the graph. The distance between two nodes is given by the time needed to move the vehicle from one location to the other. The problem of finding a shortest route for the vehicle with minimum pickup time can now be solved as a TSP.

Suppose that in a city n mail boxes have to be emptied every day within a certain period of time, say one hour. The problem is to find the minimum number of trucks to do this and the shortest time to do the collections using this number of trucks. As another example, suppose that n customers require certain amounts of some commodities and a supplier has to satisfy all demands with a fleet of trucks. The problem is to find an assignment of customers to the trucks and a delivery schedule for each truck so that the capacity of each truck is not exceeded and the total travel capacity constraints are combined. These are common in many real-life applications. Lenstra and Rinnooy (1974) studied and solved this problem as a TSP without the time and capacity constraints and a fixed number of trucks (say m). In this case, the authors obtained m-salesmen problem. Nevertheless, one may apply methods for the TSP to find good feasible solutions for this problem.

For the production of each layer of a printed circuit board, as well as for layers of integrated semiconductor devices, a photographic mask has to be produced. In this case for printed circuit boards this is done by a mechanical plotting device. The plotter moves a lens over a photosensitive coated glass plate. The shutter may be opened or closed to expose specific parts of the plate. There are different apertures available to be able to generate different structures on the board. Two types of structures have to be considered. A line is exposed on the plate by moving the closed shutter to one endpoint of the line, then opening the shutter and moving it to the other end of the line. Then the shutter is closed. A point type structure is generated by moving (with the appropriate aperture) to the position of the point then opening the shutter just to make a short flash, and then closing it again. Exact modeling of the plotter control problem leads to a problem more complicated than the TSP and also more complicated than the rural postman problem. Grotschel et al., (1991) presented a real-life application in the actual production environment of the above problem.

One of the major and primary applications of the multiple travelling salesperson problems arises in scheduling a printing press for a periodical with multi-editions. Here, there exist five pairs of cylinders between which the paper rolls and both sides of a page are printed simultaneously. There exist three kinds of forms, namely 4-, 6-, and 8-page forms, which are used to print the editions. The scheduling problem consists of deciding which form will be on which run and the length of each run. In the multiple salesperson problem vocabulary, the plate change costs are the inter-city costs. Gorenstein (1970) and Carter and Ragsdale (2002) presented a real-life application of the above problem.

Angel et al., (1972) investigated the problem of scheduling buses as a variation of the multiple travelling salesperson problems with some side constraints. The objective of the scheduling is to obtain a bus loading pattern such that the number of routes is minimized, the total distance travelled by all buses is kept at minimum, no bus is overloaded and the time required to traverse any route does not exceed a maximum allowed policy.

An application for deposit carrying between different branch banks was presented by Svestka and Huckfeldt (1973) as a multiple travelling salesperson problem. Here, deposits need to be picked up at branch banks and returned to the central office by a crew of messengers. The problem is to determine the routes having a total minimum cost.

Gilbert and Hofstra (1992) studied the application of multiple travelling salesperson problems, having multi period variations, in scheduling interviews between tour brokers and vendors of the tourism industry. Each broker corresponds to a salesman who must visit a specified set of vendor booths, which are represented by a set of T cities.

In the iron and steel industry, orders are scheduled on the hot rolling mill in such a way that the total set-up cost during the production can be minimized. Tang et al., (2000) presented an application of modeling such problem. Here, the orders are treated as cities and the distance between two cities is taken as penalty cost for production changeover between two orders. The solution of the model yielded a complete schedule for the hot strip rolling mill.

Fiechter (1994) proposed a method for the TSP that includes an intensification phase during which each process optimizes a specific slice of the tour. At the end of the intensification phase, processes synchronize to recombine the tour and modify (shift part of the tour to a predetermined neighboring process) the partition. To diversify, each process determines from among its subset of cities a candidate list of most promising moves. The processes then synchronize to exchange these lists, so that all build the same final candidate list and apply the same moves. Fiechter reports near-optimal solutions to large problems (500, 3000 and 10000 vertices) and almost linear speedups (less so for the 10000 vertex problems).

Porto and Ribeiro (1995, 1996) studied the task scheduling problem for heterogeneous systems as a TSP and proposed several synchronous parallel tabu search procedures where a master process determines and modifies partitions, synchronizes slaves, and communicates best solutions. Interesting results were reported, even for strategies involving a high level of communications. Almost linear speedups were observed, better performances being observed for larger problem instances.

Taillard (1993) studied parallel tabu search methods for travelling salesman problems. In Taillard's approach, the domain is decomposed into Polar Regions, to which vehicles are allocated, and each sub problem is solved by an independent tabu search. All processors synchronize after a certain number of iterations (according to the total number of iterations already performed) and the partition is modified: tours, undelivered cities, and empty vehicles are exchanged between adjacent processors. Taillard reports very good results for the epoch.

Rego and Roucairol (1996) proposed a tabu search method for the TSP based on ejection chains and implemented an independent multi-thread parallel version, each thread using a different set of parameter settings but starting from the same solution. The method is implemented in a master-slave setting, where each slave executes a complete sequential tabu search. The master gathers the solutions found by the threads, selects the overall best, and reinitializes the threads for a new search.

Crainic and Gendreau (2001) proposed a co-operative multi-thread parallel tabu search for the fixed cost, capacitated, multi-commodity network design problem as a TSP problem. In their study, the individual tabu search threads differed in their initial solution and parameter settings. Communications were performed asynchronously through a central memory device. The authors compared five strategies of retrieving a solution from the pool when requested by an individual thread. The strategy that always returns the overall best solution displayed the best performance when few (4) processors were used. When the number of processors was increased, a probabilistic procedure, based on the rank of the solution in the pool, appears to offer the best performance. The parallel procedure improves the quality of the solution and also requires less (wall clock) computing time compared to the sequential version, particularly for large problems with many commodities (results for problems with up to 700 design arcs and 400 commodities are reported). The experimental results also emphasize the need for the individual threads to proceed unhindered for some time (e.g., until the first diversification move) before initiating exchanges of solutions. This ensures that local search histories can be established and good solutions can be found to establish the central memory as an elite candidate set. By contrast, early and frequent communications yielded a totally random search that was ineffective. The authors finally report that the co-operative multi-thread procedure also outperformed an independent search strategy that used the same search parameters and started from the same initial points.

The mission planning problem consists of determining an optimal path for each army men (or planner) to accomplish the goals of the mission in the minimum possible time. The mission planner uses a variation of the multiple travelling salesman problems where there are n planners, m goals which must be visited by some planners, and a base city to which all planners must eventually return. Brummit and Stentz (1996) and Yu et al., (2002) studied the application of the mission planning problem as a multiple travelling salesman problem.

Ryan et al., (1998) presented a model of the routing problems arising in the planning of unmanned aerial vehicle applications as a multiple travelling salesman problem, and proposed a tabu search approach for solving the problem.

A very recent and an interesting application of the multiple travelling salesperson problems, presented by Saleh and Chelouah (2004) arise in the design of global navigation satellite system (GNSS) surveying networks. A GNSS is a space-based satellite system which provides coverage for all locations worldwide and is quite crucial in real-life applications such as early warning and management for disasters, environment and agriculture monitoring, etc. The goal of surveying is to determine the geographical positions of unknown points on and above the earth using satellite equipment. These points, on which receivers are placed, are co-ordinated by a series of observation sessions. When there are multiple receivers or multiple working periods, the problem of finding the best order of sessions for the receivers can be formulated as a multiple travelling salesman problem.

The travelling salesman problem is one of a class of difficult problems in combinatorial optimization that is representative of a large number of important scientific and engineering problems. Miller and Pekny (1991) gave a survey of recent applications and methods for solving large problems. In addition, an algorithm for the exact solution of the asymmetric travelling salesman problem was presented along with computational results for several classes of problems. The results show that the algorithm performs remarkably well for some classes of problems, determining an optimal solution even for problems with large numbers of cities, yet for other classes, even small problems thwart determination of a provably optimal solution.

Tobias and Osten (2007) introduced an optical method based on white light interferometer in order to solve the well-known NP–complete travelling salesman problem. According to the authors it was the first time that a method for the reduction of non–polynomial time to quadratic time has been proposed. The authors showed that this achievement is limited by the number of available photons for solving the problem. It turned out that this number of photons is proportional to N^N for a travelling salesman problem with N cities and that for large numbers of cities the method in practice therefore is limited by the signal–to–noise ratio.

Kaur and Murugapan (2008) presented a novel hybrid genetic algorithm for solving Travelling Salesman Problem (TSP) based on the Nearest Neighbour heuristics and pure Genetic Algorithm (GA). The hybrid genetic algorithm exponentially derives higher quality solutions in relatively shorter time for hard combinatorial real world optimization problems such as Travelling

Salesman Problem (TSP) than the pure GA. The hybrid algorithm outperformed the NN algorithm and the pure Genetic Algorithm taken separately. The hybrid genetic algorithm is designed and experimented against the pure GA and the convergence rate improved by more than 200% and the tour distance improved by 17.4% for 90 cities. These results indicate that the hybrid approach is promising and it can be used for various other optimization problems.

Travelling salesman problems with profits (TSPs with profits) are a generalization of the travelling salesman problem (TSP), where it is not necessary to visit all vertices. A profit is associated with each vertex. The overall goal is the simultaneous optimization of the collected profit and the travel costs. These two optimization criteria appear either in the objective function or as a constraint. Dominique et al., (2003) studied a classification of TSPs with profits is proposed, and the existing literature is surveyed. Different classes of applications, modeling approaches, and exact or heuristic solution techniques are identified and compared. Conclusions emphasize the interest of this class of problems, with respect to applications as well as theoretical results.

Cerny (1985) presented a Monte Carlo algorithm to find approximate solutions of the travelling salesman problem. The algorithm generates randomly the permutations of the stations of the travelling salesman trip, with probability depending on the length of the corresponding route. Reasoning by analogy with statistical thermodynamics, we use the probability given by the Boltzmann-Gibbs distribution. Surprisingly enough, using this simple algorithm, one can get very close to the optimal solution of the problem or even find the true optimum. The author demonstrates this on several examples. The author conjectures that the analogy with thermodynamics can offer a new insight into optimization problems and can suggest efficient algorithms for solving them.

Viera et al., (2002) presented an approach to the well-known travelling salesman problem (TSP) via competitive neural networks. The neural network model adopted in this work is the Kohonen network or self-organizing maps (SOM), which has important topological information about its neurons configuration. The author was concerned with the initialization aspects, parameters adaptation, and the complexity analysis of the proposed algorithm. The modified SOM algorithm proposed by the author has shown better results when compared with other neural network based approaches to the TSP.

The travelling salesman problem with precedence constraints (TSPPC) is one of the most difficult combinatorial optimization problems. Chiung (2002) presented an efficient genetic algorithm (GA) to solve the TSPPC. The key concept of the proposed GA is a topological sort (TS), which is defined as an ordering of vertices in a directed graph. Also, a new crossover operation is developed for the proposed GA. The results of numerical experiments showed that the proposed GA produces an optimal solution and shows superior performance compared to the traditional algorithms.

The classical travelling salesman problem involves the establishment of a tour around a set of points in a plane such that each point is intersected only once and the circuit is of minimal total length. When the length of a salesman's tour cannot exceed a specified constant, the problem becomes that of finding the fewest number of salesmen such that every city is visited by a salesman and the length of each salesman's tour does not exceed a specified constant. This is the chromatic travelling salesmen problem. An algorithm for this problem was presented by Milton et al., (2010) which was used to create periodic markets in parts of Sierra Leone. Fifteen rural areas were examined from Sierra Leone, and weekly market places were identified in each area. Salesmen were to be assigned to an area so that each market place was visited and each tour (or periodic ring) did not exceed forty hours. The chromatic travelling salesmen algorithm was used to minimize the number of periodic rings needed for each area and provide the specific tour for each ring.

The Symmetric Circulant Travelling Salesman Problem asks for the minimum cost tour in a symmetric circulant matrix. The computational complexity of this problem is not known – only upper and lower bounds have been determined. Ivan and Federico (2008) presented a characterization of the two-stripe case. Instances where the minimum cost of a tour is equal to either the upper or lower bound are recognized. A new construction providing a tour is proposed for the remaining instances, and this leads to a new upper bound that is closer than the previous one.

The travelling salesman problem (TSP) is known to be a combinatorial optimization problem which belongs to NP-hard (a class of problems which does not allow polynomial time solution). Recently, various types of TSP are studied on the Web and the best solutions up to date are open to the public. The initial solution for a given TSP can be easily obtained by the well-known methods such as greedy, nearest neighbor, and saving method. Murano and Matsumoto (2003) studied on how to improve these initial solutions of TSP with less computation time, and the domain division method for 2-opt and 3-opt methods is proposed. The proposed methods remarkably reduce the number of the candidate edges for trials. By executing appropriate domain division, we can save more than 90 percent computation time for 2-opt and 3-opt methods, and can obtain good solution which is comparative to those obtained by without doing the domain division method and with much more computation time.

Bernd and Peter (1996) presented an approach which incorporates problem specific knowledge into a genetic algorithm which is used to compute near-optimum solutions to travelling salesman problems (TSP). The approach is based on using a tour construction heuristic for generating the initial population, a tour improvement heuristic for finding local optimal in a given TSP search space, and new genetic operators for effectively searching the space of local optima in order to find the global optimum. The quality and efficiency of solutions obtained for a set of TSP instances containing between 318 and 1400 cities are presented.

Gunter (1992) considered the special case of the Euclidean Travelling Salesman Problem where the given points lie on a small number (N) of parallel lines. Such problems arise for example in the fabrication of printed circuit boards, where the distance travelled by a laser which drills holes in certain places of the board should be minimized. By a dynamic programming algorithm, we can solve the N-line travelling salesman problem for *n* points in time n^N, for fixed N, i. e., in polynomial time. This extends a result of Cutler (1980) for 3 lines. The parallelity condition can be relaxed to point sets which lie on N "almost parallel" line segments. The author gave a characterization of the allowed segment configurations by a set of forbidden sub configurations.

The Travelling Salesman Problem (TSP) is a well-studied combinatorial optimization problem with a wide spectrum of applications and theoretical value. Hains (2010) designed a new recombination operator known as Generalized Partition Crossover (GPX) for the TSP. GPX is unique among other recombination operators for the TSP in that recombining two local optima produces new local optima with a high probability. Thus the operator can 'tunnel' between local optima without the need for intermediary solutions. The operator is respectful, meaning that any edges common between the two parent solutions are present in the offspring, and transmits alleles, meaning that offspring are comprised only of edges found in the parent solutions. The author designed a hybrid genetic algorithm, which uses local search in addition to recombination and selection, specifically for GPX. The author showed that this algorithm outperforms Chained Lin-Kernighan, a state-of-the-art approximation algorithm for the TSP. The author next analyzed these algorithms to determine why the algorithms are not capable of consistently finding a globally optimal solution. The results revealed a search space structure which the author called 'funnels' because they are analogous to the funnels found in continuous optimization. Funnels are clusters of tours in the search space that are separated from one another by a non-trivial distance. The author found that funnels can trap Chained Lin-Kernighan, preventing the search from finding an optimal solution. The data used indicated that, under certain conditions, GPX can tunnel between funnels, explaining the higher frequency of optimal solutions produced by the author's hybrid genetic algorithm using GPX.

Travelling salesman problem is a classical complete nondeterministic polynomial problem. It is significant to solve Multiple Travelling Salesman Problems (MTSP). Previous researches on multiple travelling salesman problems are mostly limited to the kind that employed total-path-shortest as the evaluating rule, but little notice is made on the kind that employed longest-path-shortest as the evaluating rule. Hai-Long et al., (2009) studied this problem and employed genetic algorithm to optimize it and decoding method with matrix was proposed. The method could solve symmetric and asymmetric MTSP. Symmetric and asymmetric multiple travelling salesman problems were simulated and different crossover operators were compared.

Logistics Management sometimes involves pickup as well as delivery along the route. Courier service is a typical example. The imposition of precedence constraints among the places to be visited constitutes a variant of the classical Travelling Salesman Problem (TSP). This well-known NP-hard problem is often solved by heuristics. The Precedence-Constrained TSP that incorporates Delivery and Pickup (PCTDP) is a much harder problem to solve. Ganesh and Narendran (2005) studied the PCTDP and presented a three-stage heuristic using clustering and shrink-wrap algorithms for finding an initial path as well as genetic algorithms for the final search to obtain the best solution. The proposed heuristic is tested over a range of trial datasets and is found to give encouraging results. With its ability to provide solutions of good quality at low computing times, the proposed heuristic has ample scope for application as an automated scheduler when implemented at the site of a logistics-planning cell.

Most researches in evolutionary computation focus on optimization of static and no-change problems. Many real world optimization problems however are actually dynamic, and optimization methods capable of continuously adapting the solution to a changing environment are needed. Yan et al., (2004) presented an approach to solving dynamic TSP. A dynamic TSP is harder than a general TSP, which is a NP-hard problem, because the city number and the cost matrix of a dynamic TSP are time varying. The authors proposed an algorithm to solve the dynamic TSP problem, which is the hybrid of EN and Inver-Over algorithm. From the results of the experiment, the authors concluded their algorithm was effective

Vardges (2009) studied an LP relaxation for ATSP. The author introduced concepts of high-value and high-flow cycles in LP basic solutions and show that the existence of this kind of cycles would lead to constant-factor approximation algorithms for ATSP. The existence of high-flow cycles is motivated by computational results and theoretical observations.

The multiple travelling salesmen problem (MTSP) is an extension of the travelling salesman problem with many production and scheduling applications. The TSP has been well studied including methods of solving the problem with genetic algorithms. The MTSP has also been studied and solved with GAs in the form of the vehicle-scheduling problem. Carter (2003) presented a new modeling methodology for setting up the MTSP to be solved using a GA. The advantages of the new model are compared to existing models both mathematically and experimentally. The model is also used to model and solve a multi line production problem in a spreadsheet environment. The new model proves itself to be an effective method to model the

MTSP for solving with GAs. The concept of the MTSP is then used to model and solve with a GA the use of one salesman make many tours to visit all the cities instead of using one continuous trip to visit all the cities. While this problem uses only one salesman, it can be modeled as a MTSP and has many applications for people who must visit many cities on a number of short trips. The method used effectively creates a schedule while considering all required constraints.

The travelling salesperson problem (TSP) is a classic model for various production and scheduling problems. Many production and scheduling problems ultimately can be reduced to the simple concept that there is a salesperson who must travel from city to city (visiting each city exactly once) and wishes to minimize the total distance traveled during his tour of all n cities. Obtaining a solution to the problem of a salesperson visiting n cities while minimizing the total distance traveled is one of the most studied combinatorial optimization problems. While there are variations of the TSP, the Euclidean TSP is NP-hard .Schmitt and Amini (1998) and Falkenauer (1998) studied a model with the interest in this particular type of problem being how common the problem is and how difficult the problem is to solve when n becomes sufficiently large.

The travelling salesman problem (TSP) has been an early proving ground for many approaches to combinatorial optimization, including classical local optimization techniques as well as many of the more recent variants on local optimization, such as simulated annealing, tabu search, neural networks, and genetic algorithms. David and Lyle (1995) studied how these various approaches have been adapted to the TSP and evaluates their relative success in this perhaps a typical domain from both a theoretical and an experimental point of view.

The travelling salesman problem with precedence constraints (TSPPC) is one of the most difficult combinatorial optimization problems. Chiung et al., (2000) presented an efficient genetic algorithm (GA) to solve the TSPPC. The key concept of the proposed GA is a topological sort (TS), which is defined as an ordering of vertices in a directed graph. Also, a new crossover operation is developed for the proposed GA. The results of numerical experiments show that the proposed GA produces an optimal solution and shows superior performance compared to the traditional algorithms.

Many real-life industrial applications involve finding a Hamiltonian path with minimum cost. Some instances that belong to this category are transportation routing problem, scan chain optimization and drilling problem in integrated circuit testing and production. Li-Pei et al., (2001) presented a Bee Colony Optimization (BCO) algorithm for Travelling Salesman Problem (TSP). The BCO model is constructed algorithmically based on the collective intelligence shown in bee foraging behavior. The model is integrated with 2-opt heuristic to further improve prior solutions generated by the BCO model. Experimental results comparing the proposed BCO model with existing approaches on a set of benchmark problems were also presented.

Zakir (2010) presented a new crossover operator, Sequential Constructive crossover (SCX), for a genetic algorithm that generates high quality solutions to the travelling salesman Problem (TSP). The sequential constructive crossover operator constructs an offspring from a pair of parents using better edges on the basis of their values that may be present in the parents' structure maintaining the sequence of nodes in the parent chromosomes. The efficiency of the SCX is compared as against some existing crossover operators; namely, edge recombination crossover (ERX) and generalized N-point crossover (GNX) for some benchmark TSPLIB instances. Experimental results show that the new crossover operator is better than the ERX and GNX.

The aim of the Travelling Salesman Problem (TSP) is to find the cheapest way of visiting all elements in a given set of cities (nodes) exactly once and returning to the starting point. In solutions presented in the literature costs of travel between nodes are based on Euclidean distances, the problem is symmetric and the costs are constant and crisp values. Practical application in road transportation and supply chains are often uncertain or fuzzy. The risk attitude depends on the features of the given operation. Foldesi et al., (2010) presented a model that handles the fuzzy, time dependent nature of the TSP and also gives a solution for the asymmetric loss aversion by embedding the risk attitude into the fitness function of the eugenic bacterial memetic algorithm. Computational results are presented for different cases. The classical TSP is investigated along with a modified instance where some costs between the cities are described with fuzzy numbers. Two different techniques are proposed to evaluate the uncertainties in the fuzzy cost values. The time dependent version of the fuzzy TSP is also investigated and simulation experiences are presented.

Iridia (1996) presented an artificial ant colony model capable of solving the travelling salesman problem (TSP). Ants of the artificial colony are able to generate successively shorter feasible tours by using information accumulated in the form of a pheromone trail deposited on the edges of the TSP graph. Computer simulations demonstrate that the artificial ant colony is capable of generating good solutions to both symmetric and asymmetric instances of the TSP. The method is an example, like simulated annealing, neural networks, and evolutionary computation, of the successful use of a natural metaphor to design an optimization algorithm.

The travelling salesman problem and the quadratic assignment problem are the two of the most commonly studied optimization problems in Operations Research because of their wide applicability. Due to their NP -hard nature, the individual problems are already complex and difficult to solve. Ping and William (2005) studied a model which integrated the two hard problems together, that is called the integrated problem of which the complexity is absolutely much higher than that of the individual ones. Not only a complete mathematical model which integrates both the travelling salesman and the quadratic assignment problems together is built, but also a genetic algorithm hybridized with several improved heuristics is developed to tackle the problem.

The Travelling Salesman Problem (TSP) is one of the most intensively studied problems in computational mathematics. To solve this problem a number of algorithms have been developed using genetic algorithms. But these algorithms are not so suitable for solving large-scale TSP. Kalyan et al., (2010) proposed a new solution for TSP using hierarchical clustering and genetic algorithm.

Time-constrained deliveries are one of the fastest growing segments of the delivery business, and yet there is surprisingly little literature that addresses time constraints in the context of stochastic customer presence. Ann and Barrett (2007) studied the probabilistic travelling salesman problem with deadlines (PTSPD). The PTSPD is an extension of the well-known probabilistic travelling salesman problem (PTSP) in which, in addition to stochastic presence, customers must also be visited before a known deadline. The authors presented two recourse models and a chance constrained model for the PTSPD. Special cases are discussed for each model, and computational experiments are used to illustrate under what conditions stochastic and deterministic models lead to different solutions.

Kenneth and Ruth (2007) studied a new multi-period variation of the M-travelling salesman problem. The problem arises in efficient scheduling of optimal interviews among tour brokers and vendors at conventions of the tourism and travel industry. In classical travelling salesman problem vocabulary, a salesman is a tour broker at the convention and a city is a vendor's booth. In this problem, more than one salesman may be required to visit a city, but at most one salesman per time period can visit each city. The heuristic solution method presented is polynomial and is guaranteed to produce a non-conflicting set of salesmen's tours. The results of an implementation of the method for a recent convention are also reported.

Valentina et al., (2010) studied the equality generalized travelling salesman problem (E-GTSP), which is a variant of the well-known travelling salesman problem. We are given an undirected graph G = (V, E), with set of vertices V and set of edges E, each with an associated cost. The set of vertices is partitioned into clusters. E-GTSP is to find an elementary cycle visiting exactly one vertex for each cluster and minimizing the sum of the costs of the travelled edges. The authors proposed a multi-start heuristic, which iteratively starts with a randomly chosen set of vertices and applies a decomposition approach combined with improvement procedures. The decomposition approach considers a first phase to determine the visiting order of the clusters and a second phase to find the corresponding minimum cost cycle. We show the effectiveness of the proposed approach on benchmark instances from the literature. On small instances, the heuristic always identifies the optimal solution rapidly and outperforms all known heuristics; on larger instances, the heuristic always improves, in comparable computing times, the best known solution values obtained by the genetic algorithm.

June and Sethian (2006) studied a problem in which given a domain, a cost function which depends on position at each point in the domain, and a subset of points ("cities") in the domain. The goal is to determine the cheapest closed path that visits each city in the domain once. This can be thought of as a version of the travelling salesman problem, in which an underlying known metric determines the cost of moving through each point of the domain, but in which the actual shortest path between cities is unknown at the outset. The authors proposed algorithms for both a heuristic and an optimal solution to this problem. The complexity of the heuristic algorithm is at worst case M·N log N, where M is the number of cities, and N the size of the computational mesh used to approximate the solutions to the shortest paths problems. The average runtime of the heuristic algorithm is linear in the number of cities and *O* (N log N) in the size N of the mesh.

Many companies have travelling salesmen that market and sell their products. This results in much travelling by car due to the daily customer visits. This causes costs for the company, in form of travel expenses compensation, and environmental effects, in form of carbon dioxide pollution. As many companies are certified according to environmental management systems, such as ISO 14001, the environmental work becomes more and more important as the environmental consciousness increases every day for companies, authorities and public. Torstensson (2008) presented a model which computes reasonable limits on the mileage of the salesmen; these limits are based on specific conditions for each salesman's district. The objective is to implement a heuristic algorithm that optimizes the customer tours for an arbitrary chosen month, which will represent a "standard" month. The output of the algorithm, the computed distances, will constitute a mileage limit for the salesman. The algorithm consists of a constructive heuristic that builds an initial solution, which is modified if infeasible. This solution is then improved by a local search algorithm preceding a genetic algorithm, which task is to improve the tours separately. This method for computing mileage limits for travelling salesmen generates good solutions in form of realistic tours. The mileage limits could be improved if the input data were more accurate and adjusted to each district, but the suggested method does what it is supposed to do.

Davoian and Gorlatch (2005) presented a new modification of the Genetic Algorithm (GA) for solving the classical Travelling Salesman Problem (TSP), with the objective of achieving its efficient implementation on multiprocessor machines. The authors described the new features of our GA as compared to existing algorithms, and developed a new parallelization scheme, applicable to arbitrary GAs. In addition to parallel processes and iterative data exchanges between the involved populations, our parallel implementation also contains a generation of new possible solutions (strangers), which eliminates typical drawbacks of GA and extends the search area. The proposed algorithm allows for acceleration of the solution process and generates solutions of better quality as compared with previously developed GA versions.

Marco and Luca (1997) presented an artificial ant colony capable of solving the travelling salesman problem (TSP). Ants of the artificial colony are able to generate successively shorter feasible tours by using information accumulated in the form of a pheromone trail deposited on the edges of the TSP graph. Computer simulations demonstrate that the artificial ant colony is capable of generating good solutions to both symmetric and asymmetric instances of the TSP. The method is an example, like simulated annealing, neural networks, and evolutionary computation, of the successful use of a natural metaphor to design an optimization algorithm.

An analogy with the way ant colonies function has suggested the definition of a new computational paradigm, which we call Ant System. Marco et al., (1996) proposed it as a viable new approach to stochastic combinatorial optimization. The main characteristics of this model are positive feedback, distributed computation, and the use of a constructive greedy heuristic. Positive feedback accounts for rapid discovery of good solutions, distributed computation avoids premature convergence, and the greedy heuristic helps find acceptable solutions in the early stages of the search process. The authors applied the proposed methodology to the classical Travelling Salesman Problem (TSP), and report simulation results. The authors also discussed parameter selection and the early setups of the model, and compare it with tabu search and simulated annealing using TSP. To demonstrate the robustness of the approach, the authors showed how the Ant System (AS) can be applied to other optimization problems like the asymmetric travelling salesman.

Durbin and Willshaw (1987) studied the ant colony system (ACS), a distributed algorithm that is applied to the travelling salesman problem (TSP). In the ACS, a set of cooperating agents called ants cooperate to find good solutions to TSP's. Ants cooperate using an indirect form of communication mediated by a pheromone they deposit on the edges of the TSP graph while building solutions. The authors studied the ACS by running experiments to understand its operation. The results showed that the ACS outperforms other nature-inspired algorithms such as simulated annealing and evolutionary computation, and we conclude comparing ACS-3-opt, a version of the ACS augmented with a local search procedure, to some of the best performing algorithms for symmetric and asymmetric TSP's.

Kenneth and Ruth (1992) presented a new multiperiod variation of the M-travelling salesman problem. The problem arises in efficient scheduling of optimal interviews among tour brokers and vendors at conventions of the tourism and travel industry. In classical travelling salesman problem vocabulary, a salesman is a tour broker at the convention and a city is a vendor's booth. In this problem, more than one salesman may be required to visit a city, but at most one salesman per time period can visit each city. The heuristic solution method presented is polynomial and is guaranteed to produce a nonconflicting set of salesmen's tours. The results of an implementation of the method for a recent convention are also reported.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This chapter provides an explanation of the dynamic programming algorithm which we proposed to solve our problem.

In order to have a good understanding of the dynamic programming algorithm, it is necessary to first have a good understanding of some key terms as used in dynamic programming problems

3.1 CHARACTERISTICS OF DYNAMIC PROGRAMMING PROBLEMS

One way to recognize a situation that can be formulated as a dynamic programming problem is to notice its basic features.

These basic features that characterize dynamic programming problems are presented and discussed here.

1. The problem can be divided into stages, with a policy decision required at each stage. Dynamic programming problems require making a sequence of interrelated decisions, where each decision corresponds to one stage of the problem.

2. Each stage has a number of states associated with the beginning of that stage.

In general, the states are the various possible conditions in which the system might be at that stage of the problem. The number of states may be either finite or infinite.

3. The effect of the policy decision at each stage is to transform the current state to *a* state associated with the beginning of the next stage (possibly according to a probability distribution).

This procedure suggests that dynamic programming problems can be interpreted in terms of the networks. Each node would correspond to a state. The network would consist of columns of nodes, with each column corresponding to a stage, so that the flow from a node can go only to a node in the next column to the right. The links from a node to nodes in the next column correspond to the possible policy decisions on which state to go to next. The value assigned to each link usually can be interpreted as the immediate contribution to the objective function from making that policy decision. In most cases, the objective corresponds to finding either the shortest or the longest path through the network.

4. The solution procedure is designed to find an optimal policy for the overall problem, i.e., a prescription of the optimal policy decision at each stage for each of the possible states.

For any problem, dynamic programming provides this kind of policy prescription of what to do under every possible circumstance (which is why the actual decision made upon reaching a particular state at a given stage is referred to as a policy decision). Providing this additional information beyond simply specifying an optimal solution (optimal sequence of decisions) can be helpful in a variety of ways, including sensitivity analysis.

5. Given the current state, an optimal policy for the remaining stages is independent of the policy decisions adopted in previous stages. Therefore, the optimal immediate decision depends on only the current state and not on how you got there. This is the principle of optimality for dynamic programming.

For dynamic programming problems in general, knowledge of the current state of the system conveys all the information about its previous behavior necessary for determining the optimal policy henceforth. Any problem lacking this property cannot be formulated as a dynamic programming problem.

6. The solution procedure begins by finding the optimal policy for the last stage.

The optimal policy for the last stage prescribes the optimal policy decision for each of the possible states at that stage. The solution of this one-stage problem is usually trivial, as it was for the stagecoach problem.

7. A recursive relationship that identifies the optimal policy for stage n, given the optimal policy for stage n + 1, is available.

Therefore, finding the optimal policy decision when you start in state *s* at stage *n* requires finding the minimizing value of x_n.

This property is emphasized in the next (and final) characteristic of dynamic programming.

8. When we use this recursive relationship, the solution procedure starts at the end and moves backward stage by stage - each time finding the optimal policy for that stage - until it finds the optimal policy starting at the initial stage. This optimal policy immediately yields an optimal solution for the entire problem.

3.2 The Algorithm

• Identify the decision variables and specify objective function to be optimized under certain limitations, if any.

• Decompose or divide the given problem into a number of smaller sub-problems or stages. Identify the state variables at each stage and write down the transformation function as a function of the state variable and decision variables at the next stage.

• Write down the general recursive relationship for computing the optimal policy. Decide whether forward or backward method is to follow to solve the problem.

• Construct appropriate stage to show the required values of the return function at each stage.

• Determine the overall optimal policy or decisions and its value at each stage. There may be more than one such optimal policy.

The basic features, which characterize the dynamic programming problem, are as follows:

(*i*) Problem can be sub-divided into stages with a policy decision required at each stage. A stage is a device to sequence the decisions. That is, it decomposes a problem into sub-problems such that an optimal solution to the problem can be obtained from the optimal solution to the sub-problem.

(*ii*) Every stage consists of a number of states associated with it. The states are the different possible conditions in which the system may find itself at that stage of the problem.

(*iii*) Decision at each stage converts the current stage into state associated with the next stage.

(*iv*) The state of the system at a stage is described by a set of variables, called state variables.

(*v*) When the current state is known, an optimal policy for the remaining stages is independent of the policy of the previous ones.

(*vi*) To identify the optimum policy for each state of the system, a recursive equation is formulated with '*n*' stages remaining, given the optimal policy for each stage with (n – 1) stages left.

(*vii*) Using recursive equation approach each time the solution procedure moves backward, stage by stage for obtaining the optimum policy of each stage for that particular stage, still it attains the optimum policy beginning at the initial stage.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter, we shall consider a computational study of the Travelling Salesman Problem. Emphasis will be placed on TSP, which is modelled as a network problem. Data from the Test Administration Department (TAD) of WAEC shall be examined.

4.1 Data Collection and Analysis

WAEC Ghana, during its various Examination seasons, sends officers to inspect the various question paper depots and examination centres where security materials are kept to ascertain whether the regulations regarding the safety of the materials are complied with in the various regions in Ghana.

An officer moves from the various regional capitals and is expected to visit as many examination depots and centres as possible on each route within each journey in a day. Table 4.1 is the distance matrix table, taken from Transport Department of WAEC and it shows the various links of connecting question paper depots and examination centres for officer assigned to some parts of eastern region of Ghana in kilometers (km).

Table 4.1 Distance matrix table connecting question paper depots in km

	1	2	3	4	5	6	7	8	9	10
1	0	66	46	71	0	0	0	0	0	0
2	66	0	0	0	29	0	0	0	0	0
3	46	0	0	0	29	67	0	0	0	0
4	71	0	0	0	0	0	49	0	0	0
5	0	29	29	0	0	43	0	109	0	0
6	0	0	67	0	43	0	0	28	49	0
7	0	0	0	49	0	0	0	87	108	0
8	0	0	0	0	109	28	87	0	41	62
9	0	0	0	0	0	49	108	41	0	21
10	0	0	0	0	0	0	0	62	21	0

The zeroes signify no direct link between the two towns.

Table 4.2 shows the names of the towns where depots and centres are located.

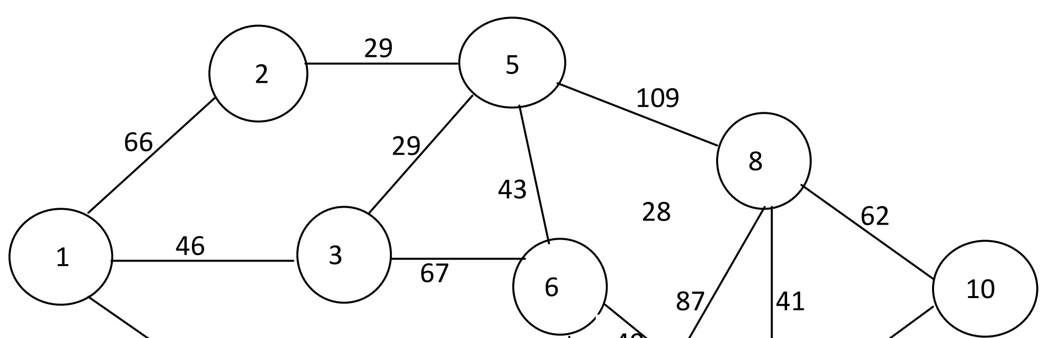
Table 4.2 Names of Towns of Examination centres

1	Koforidua

2	Aburi
3	Suhum
4	Begoro
5	Nsawam
6	Asamankese
7	Kibi
8	Akwatia
9	Akim Oda
10	Akim Swedru

The problem at hand is to find the minimum distance that an officer could cover and visit a maximum number of examination centres and depots as possible.

Modeling the above problem as a Network problem, we obtain Figure 4.1, which shows the route map of the various ways of reaching the examination depots and centres, with each node representing a depot and centre. The numbers on the lines indicate the distances in kilometres (km).



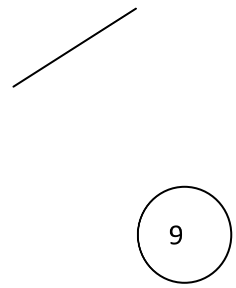


Figure 4.1: Route map of the various ways of reaching question paper depots and examination centres.

By applying dynamic programming, the problem may be considered as 4-stage problem. This is shown in Figure 4.2.

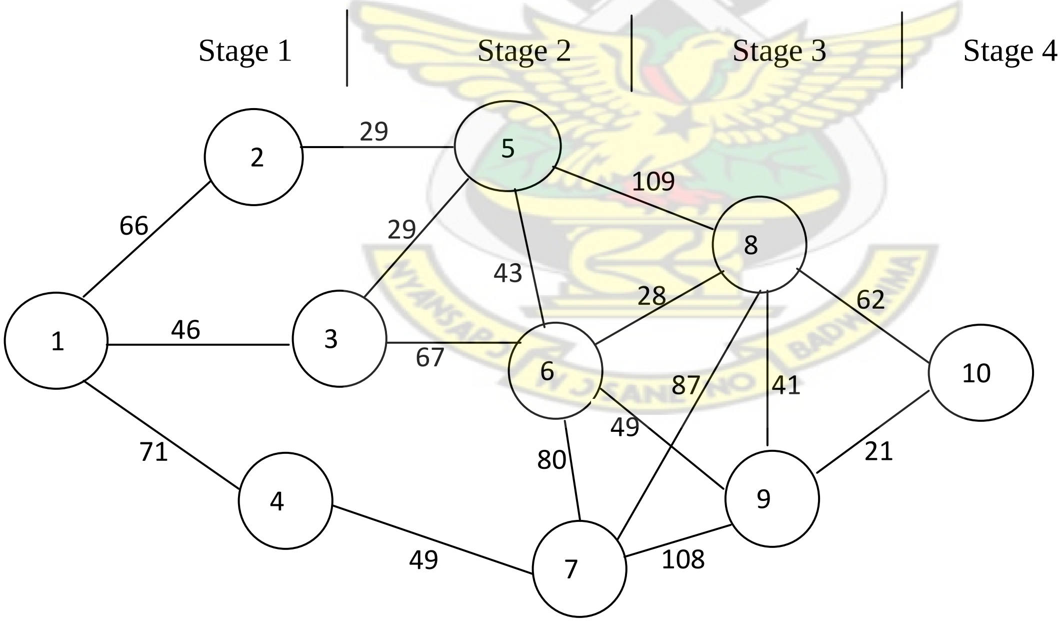


Figure 4.2: Route maps of the various ways of reaching examination depots and centres in Stages

Let x_i be the state variable in the i^{th} stage, and $d_i(x_i)$ be the distance covered in the i^{th} stage. Our model then becomes;

$$d_{(i)} = \min \{\sum_{k=i-1}^{i} d_k(x_k)\}$$

In the first stage, i = 1, and the officer leaves from examination centre 1 (node number 1) and can reach examination centres 2, 3, and 4 directly.

We shall therefore have $d_{(1)} = \min \{\sum_{k=0}^{1} d_k(x_k)\}$.

Considering the distances: 1 to 2, which is 66 km, 1 to 3, which is 46 km, and 1 to 4, which is 71 km, as this is minimization problem and the goal of an officer is to visit more number of examination centres and travel less distance. We can show the routes, which give the minimum distance in bold lines and the rest of the lines we can neglect or we can show in normal lines. In this problem, lines 1 – 3 will be in bold and the rest normal as shown in Figure 4.3.

$d_{(1)} = \min \{66, 46, 71\}$, which is 46 km. The distance covered up to that stage is written just above the node.

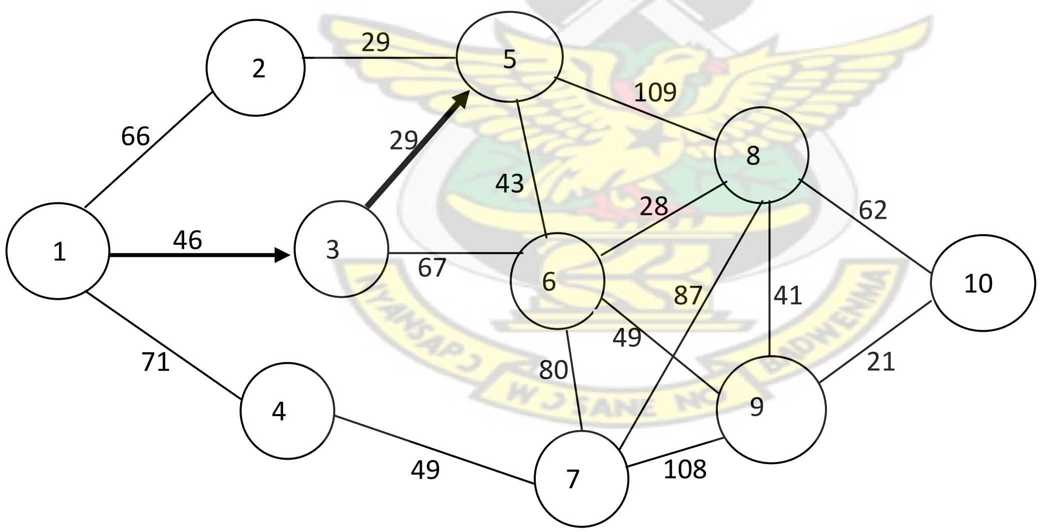


Figure 4.3: Distance travelled between examination depots and centres

In the second stage, i = 2 and the inspection officer can reach examination centre 5 directly from examination centres 2 and 3, examination centre 6 directly from examination centre 3 and examination centre 7 from 4.

We have $d_{(2)} = \min\{\sum_{k=1}^{2} d_k(x_k)\}$.

Considering the distance from examination centre 2 to 5, we have 66 + 29 = 95,

Similarly, from examination centre 3 to 5 is 46 + 29 = 75 km.

The distance from examination centre 3 to 6 = 46 + 67 = 113 km.

The distance from examination centre 4 to 7 = 71 + 49 = 120 km.

The distance from examination 5 to 6 is 75 + 43 = 118 km.

The distance from examination 6 to 7 is 113 + 80 = 193 km.

$d_{(2)} = \min\{95, 75, 113, 118, 120, 193\}$.

The minimum distance is 75 km. Hence, the inspection officer will travel from examination centre 1 to 3 and from examination centre 3 to 5 covering 75 km on the route 1-3-5.

We then move on to the next stage, which is stage 3, with i = 3.

We have $d_{(3)} = \min\{\sum_{k=2}^{3} d_k(x_k)\}$.

In the third stage, the inspection officer may be at examination centre 5 or at examination centre 6 or at examination centre 7. From there the officer can directly go to examination centre 8 or examination centre 9.

Working out the minimum distance from examination centres 5, 6 and 7 to 8 and 9, we have:

The distance from examination centre 5 to 8 = 75 + 109 = 184 km.

The distance from examination centre 6 to 8 = 113 + 28 = 141 km.

The distance from examination centre 7 to 8 = 120 + 87 = 207 km.

The distance from examination centre 6 to 9 = 113 + 49 = 162 km.

The distance from examination centre 7 to 9 = 120 + 108 = 228 km.

The distance from examination centre 8 to 9 = 141 + 41 = 182 km.

$d_{(3)}$ = min {184, 141, 207, 162, 228, 182}.

The minimum of all these is 141 km. Hence, the inspection officer can go from examination centres 6 to 8 at the distance of 141 km only on the routes 1-3-6-8.

Next, we consider our final stage which is stage 4, thus i = 4.

Thus, we have $d_{(4)} = \min \{\sum_{k=3}^{4} d_k(x_k)\}$.

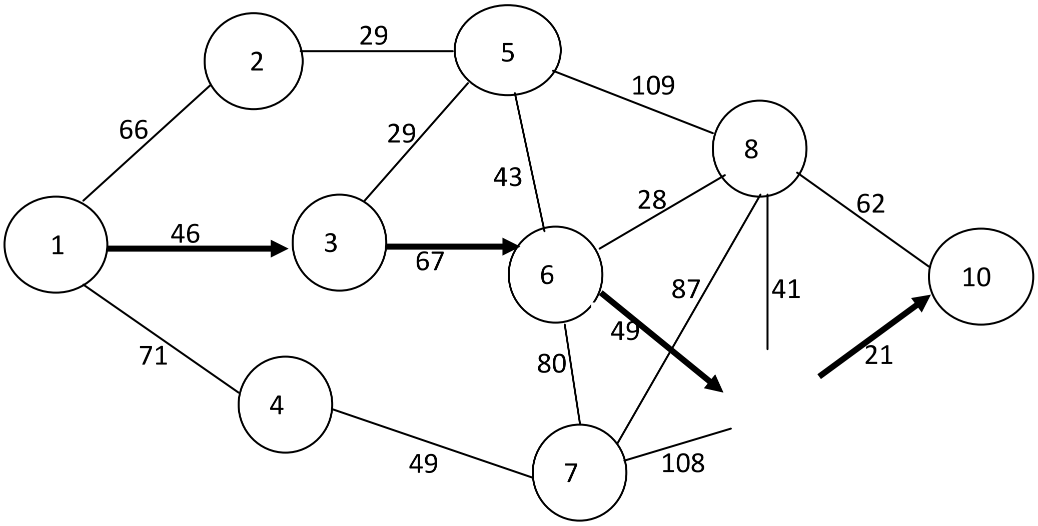
In the 4th stage the inspection officer can reach examination centre 10 from examination centres 8 or 9. Calculating the minimum distances from examination centres 8 and 9 to examination centre 10 we have:

The distance from examination centre 8 to 10 = 141 + 62 = 203 km.

The distance from examination centre 9 to 10 = 162 + 21 = 183 km.

$d_{(4)}$ = min {203, 183}.

The minimum of these is 183 km. This is shown in Figure 4.4 with bold lines and the distances written on top of the nodes.



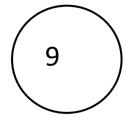


Figure 4.4: Distance covered by officer moving between examination depots and centres

Hence the minimum distance from examination centres 1 to 10 on the path is 183 km, and the routes are 1 – 3 – 6 –9 – 10**.**

This implies that the inspection officer can use any of the above routes and visit as many as five examination centres on the route.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

The travelling salesman problem is a traditional problem that has to do with making the most efficient use of resources while at the same time spending the least amount of energy in that utilization. The designation for this type of problem hails back to the days of the travelling salesman, who often wished to arrange travel distances in a manner that allowed for visiting the most towns without having to double back and cross into any given town more than once.

In a wider sense, the travelling salesman problem is considered to be a classic example of what is known as a tour problem. Essentially, any type of tour problem involves making a series of stops along a designated route and making a return journey without ever making a second visit to any previous stop. Generally, a tour problem is present when there is concern on making the most of available resources such as time and mode of travel to accomplish the most in results. Finding a solution to a tour problem is sometimes referred to as discovering the least-cost path, implying that the strategic planning of the route will ensure maximum benefit with minimum expenditure incurred.

The concept of the travelling salesman problem can be translated into a number of different disciplines. For example, the idea of combinatorial optimization has a direct relationship to the travelling salesman model. As a form of optimization that is useful in both mathematical and computer science disciplines, combinatorial optimization seeks to team relevant factors and apply them in a manner that will yield the best results with repeated usage.

In a similar manner, discrete optimization attempts to accomplish the same goal, although the term is sometimes employed to refer to tasks or operations that occur on a one-time basis rather than recurring. Discrete optimization also is helpful in computer science and mathematical disciplines. In addition, discrete optimization has a direct relationship to computational complexity theory and is understood to be of use in the development of artificial intelligence.

While the imagery associated with a travelling salesman problem may seem an oversimplification of these types of detailed options for optimization, the idea behind the imagery helps to explain a basic fundamental to any type of optimization that strives for efficiency. The travelling salesman problem that is solved will yield huge benefits in the way of maximum return for minimum investment of resources.

TSP is a very attractive problem for the research community because it arises as a natural sub-problem in many applications concerning everyday life. Indeed, each application, in which an optimal ordering of a number of items has to be chosen in a way that the total cost of a solution is determined by adding up the costs arising from two successive items, can be modelled as a TSP instance. Thus, studying TSP can never be considered as an abstract research with no real importance.

5.1 CONCLUSIONS

This thesis seeks to model a real-life problem of WAEC as a network problem and apply dynamic programming approach in solving the problem. It was observed that the route that gave minimum achievable inspection plan was

1 – 3 – 6 –9 – 10 at the minimum distance of 183 km, by visiting as many as five centres on the route.

At the time of this work, records show that there is no laid down procedure for determining which routes to be used by inspection officers. The routes are chosen arbitrarily and sometimes the driver's discretion is the determining factor. The maximum number of centres they normally visit were three on a route.

5.2 RECOMMENDATIONS

The use of mathematical models has proved to be efficient in the computation of optimum results and gives a systematic and transparent solution as compared with an arbitrary method. Operation has become one of the key competitive advantages with optimization-based approaches being expected to play an important role. Using optimization-based approaches to model industrial problem gives a better result. Management will benefit from the proposed approach for officers who would be assigned to inspect various examination centres in order to visit more centres on a route at a minimized travel distance. We therefore recommend that our TSP model should be adopted by WAEC for its depot inspection planning.

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