OPTIMIZING PROFIT IN THE BANK

A CASE STUDY OF STANBIC BANK, TAMALE

BY

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(B.ED MATHEMATICS)

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this University or elsewhere.

Candidate's Name: MUSAH SULEMANA **Signature**………………… **Date:**…11-12-1…

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis was supervised in accordance with the guidelines on thesis laid down by the Kwame Nkrumah University of Science and Technology, Kumasi.

Supervisor's Name: K. F. ADARKWA **Signature**……………….. **Date:**………

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ABSTRACT

The purpose of this study was to determine the Optimal Profit of Stanbic Bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and Asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012.

The **objectives** of the study were to:

i. model the problem of profit optimization of Stanbic Bank, Tamale as Linear Programming Problem;

ii. determine the optimal profit using Revised Simplex Algorithm;

iii. provide Sensitivity Analysis of the problem of profit optimization.

The problem of profit optimization of Stanbic Bank, Tamale was modeled as a Linear Programming Problem (LPP). The resulting LPP was then solved using the Revised Simplex Method (RSM).

The study revealed that the Optimal Profit of Stanbic Bank, Tamale in the areas of interest from ~~loans~~ such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and Asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012 was **GH₵967,405.50**

Therefore, for the bank to achieve the Optimal Profit of **GH₵967,405.5,** Revolving Term Loans x_1 was allocated an amount of GH₵1,123,560; Fixed Term Loans x_2 an amount of

GH₵749,040; Home Loans x_3 an amount of GH₵374,520; Personal VAF x_4 an amount of GH₵0.00; Vehicle and Asset Finance x_5 an amount of GH₵1,498,080.

It was also observed that if Stanbic Bank, Tamale does not allocate any amount to Personal VAF, the bank can still achieve the Optimal Profit of **GH₵967,405.5**

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My next greater debt of gratitude goes to Mr. Wahab Abdul, Lecturer, Statistics Department, Accra Polytechnic for his useful suggestions with regard to data for the thesis.

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I say a big thank you to my lovely wife Mrs. Zenabu Musah and son Musah Mujaab Maltiti for their patience and encouragement given me to accomplish this task.

Above all, my greatest thanks go to Almighty Allah for his provision of life throughout the duration of my course of study in the Kwame Nkrumah University of Science and Technology, Kumasi.

DEDICATION

This dissertation is dedicated to my dear wife Mrs. Zenabu Musah, my son Musah Mujaab Maltiti, my parents Mr. Sulemana Alhassan (late), Mariama Issahaku, my siblings and to all well wishers in society.

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CHAPTER ONE

1.0 INTRODUCTION

According to Britannica Online Encyclopedia (2011): "A bank is an institution that deals in money and its substitutes and provides other financial services". Banks accept deposits, make loans and derive a profit from the difference in the interest rates charged. Banks are critical to our economy. The primary function of banks is to put their account holders' money to use in other to optimize profit, by lending it out to others who can then use it to buy homes, do businesses, send kids to school, etc.

When you deposit money in the bank, your money goes into a big pool of money along with everyone else's and your account is credited with the amount of your deposit. When you write checks or make withdrawals, that amount is deducted from your account balance. Interest you earn on your balance is also added to your account.

Number of studies have examined bank performance in an effort to isolate the factors that account for interbank differences in profitability. These studies fall generally into several categories. One group has focused broadly on the tie between bank earnings and various aspects of bank's operating performance. A second set of studies has focused on the relationship between bank earnings performance and balance sheet structure. Another body has examined the impact of some regulatory, macroeconomic or structural factors on overall bank performance .The term bank structure is frequently used when referring to the characteristics of individual institutions. Individual bank characteristics such as the portfolio composition, and the scale and scope of operations, can affect the costs at which banks produce financial services. Market structure, measured by the relative size and number of firms, can influence the degree of local competition, and by extension, the quality, quantity, and price of financial services ultimately available to bank customers.

Bourke (1986) indicated that the determinants of commercial bank Profitability can be divided into two main categories namely the internal determinants which are management controllable and the external determinants which are beyond the control of the management of these institutions.

The internal determinants can be further subdivided as follows:

- Financial Statements variables
- Non- financial statement variable

The Financial statement variables relate to the decisions which directly affect the items in a balance sheet and profit & loss accounts. On the other hand, the nonfinancial statement variables involve those factors which do not have a direct impact on the financial statements. The external determinants on the other hand can be listed as follows

- Financial De-regulations
- Impact on competitive conditions
- Concentration
- Market Share
- Interest Rate on profitability
- Ownership
- Scarcity of Capital and Inflation

Banks create money in the economy by giving loans to their customers. These loans given to customers are usually associated with risk of non-payment. The amount of money that banks can lend is directly affected by the reserve requirement set by Federal Reserve or the Central Bank (ie. Bank of Ghana). Monetary and Exchange Rate Management in Ghana-Bank of Ghana (2011): "The reserve requirement is a ratio of cash to total deposits that a bank must keep. This is used for both prudential and monetary management purposes. During the period of direct controls, they were used as a supplement to credit controls. The reserve requirement ratio has evolved from its highest of 27% in 1990 to 10% in 1996 and its current level of 8% since 1997". Since banks optimize profit through deposits and loans given to their customers. These loans are restricted by the bank's reserve requirement and the bank also stands the risk of non-payment on the part of the customer. Therefore, there is a need for prudent management of these financial risks though the banks opt to make profit to be operational.

Therefore, the study seek to determine the Optimal Profit of Stanbic bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012. This problem of profit optimization of Stanbic Bank, Tamale is modeled as a Linear Programming Problem (LPP) and the resulting LPP is then solved using the Revised Simplex Method (RSM).

LINEAR PROGRAMMING: Linear Programming is a subset of Mathematical Programming that is concerned with efficient allocation of limited resources to known activities with the objective of meeting a desired goal of maximization of profit or minimization of cost. In Statistics and Mathematics, Linear Programming (LP) is a technique for optimization of linear objective function, subject to linear equality and linear inequality constraint. Informally, linear programming determines the way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model and given some list of requirement as linear equation. Although there is a tendency to think that linear programming which is a subset of operations research has a recent development, but there is really nothing new about the idea of maximization of profit in any organization setting i.e. in a production company or manufacturing company. For centuries, highly skilled artisans have striven to formulate models that can assist manufacturing and production companies in maximizing their profit, that is why linear programming among other models in operations research has determine the way to achieve the best outcome (i.e. maximization of profit) in a given mathematical model and given some list of requirements. Linear programming can be applied to various fields of study. Most extensively, it is used in business and economic situation, but can also be utilized in some engineering problems. Other industries such as transportation, energy, telecommunications and production or manufacturing companies use linear programming for maximization of profit or minimization of cost or materials. To this extent, linear programming has proved useful in modeling diverse types of problems in planning, routing and scheduling assignment.

Tucker (1993) noted linear programming is a mathematical method developed to solve problems related to tactical and strategic operations. Its origins show its application in the decision process of business analysis, funds. Although the practical application of a mathematical model is wide and complex, it will provide a set of results that enable the elimination of a part of the subjectivism that exists in the decision-making process as to the choice of action alternatives Bierman and Bonini (1973).

1.1 BACKGROUND TO THE STUDY

As banks device means to optimize profit, there are always associated financial risk. Ampong (2005) stated in Reserve Requirements in Bank - "Bank of Ghana stated that the Banking Act, 2004 which has passed through parliament indicates that:

- A bank at times while in operation maintains a minimum capital adequacy ratio of 10%.
- The capital adequacy ratio shall be measured as capital base of the bank to its adjusted assets base in accordance with regulations made by the bank of Ghana."

The central bank is concerned about the possibility of a banking crisis resulting from the lack of adequate foresight in the actions of an individual bank. A bank capital reserve can fall dangerously too low hence fail to meet the operational requirements of its customers. When this unfortunate situation happened in the banking sector, the panicked customers will rush to the bank to cash their accounts. Some banks might need a higher capital adequacy requirement while others might need something much lower. In nominal terms 10% capital requirement for, say, Stanbic Bank could be the market capitalization of about five (5) Rural Banks combined.

In the above, it is quite obvious that while a bank opt to optimize profit through deposits, loans, interest rates charged etc., the bank should be mindful of how much is kept in the capital reserve and this must be determined by a specific risk conditions.

1.2 STATEMENT OF THE PROBLEM

Optimizing Profit in Stanbic bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012 became necessary as a result of keen competition among banks and each bank is much interested in determining its profit margin quarterly, half yearly or per year. Also, it is quite obvious that while a bank opt to optimize profit through deposits, loans, interest rates, etc, the bank should be mindful of how much is kept in the capital reserve. These problems have therefore provoked a study on optimizing profit in the bank. A case study of Stanbic Bank, Tamale.

1.3 OBJECTIVES OF THE STUDY

This research aims at:

i. modeling the problem of profit optimization of Stanbic Bank, Tamale as Linear Programming Problem;
ii. determination of the optimal profit using revised simplex algorithm;
iii. providing sensitivity analysis of the problem of profit optimization.

1.4 ORGANIZATION OF THE STUDY

In order to provide a systematic flow of ideas, the study is presented in five (5) chapters. The first chapter focuses on the introduction to the study and deals with the background to the study, the statement of the problem, objective of the study, definition of terms and organization of the study. The second chapter deals with the literature review. This provides a theoretical frame work within which the study is located and some related research findings. Chapter three highlights on the methodology, this includes modeling the problem of profit optimization of Stanbic Bank, Tamale as Linear Programming Problem; determination of the optimal profit using Revised Simplex Algorithm and providing sensitivity analysis of the problem of profit optimization. Chapter four provides for the results of the data analysis and discussion of the result whilst the final chapter provides the summary, conclusion and recommendations of the study.

1.5 METHODOLOGY

The methodology employed in the study includes modeling the problem of profit optimization of Stanbic Bank, Tamale as Linear Programming Problem; determination of the optimal profit using Revised Simplex Algorithm and providing sensitivity analysis of the problem of profit optimization. The data type is secondary data of Stanbic Bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012.

1.6 JUSTIFICATION OF THE STUDY

- This study will serve as a reference guide to the banking industry, how the problem of profit optimization in the bank can be model as a Linear programming problem.
- The study is also very essential for equipping bankers how to determine the optimal profit using Revised Simplex Algorithm.
- The material is also very suitable to be used for in-service training for newly recruited bank officers.

CHAPTER TWO

2.0 RELATED LITERATURE REVIEW

This chapter opens with a theoretical framework within which the study is located and related research findings. The chapter centered on various reviews on Profit Maximization in the Bank, Linear Programming (LP) as an effective tool for Profit Optimization; how the Revised Simplex Method (RSM) is used to solve a Linear Programming problem (LPP) and related research findings on Sensitivity analysis.

2.1 REVIEWS ON PROFIT MAXIMIZATION IN THE BANK

Allen and Mester (1999) investigate the sources of recent changes in the performance of U.S. banks using concepts and techniques borrowed from the cross-section efficiency literature. Their most striking result is that during 1991- 1997, cost productivity worsened while profit productivity improved substantially, particularly for banks engaging in mergers. The data are consistent with the hypothesis that banks tried to maximize profits by raising revenues from the products the bank render (such as Loans, Current Accounts charges, ATM charges, etc) as well as reducing costs, and that banks provide additional services or higher service quality that raised costs ~~but also~~ raised revenues by more than the cost increases.

The U.S. Bureau of Labor Statistics (BLS) (1997) developed a labor productivity measure for maximizing profit in the commercial banking industry . They measure physical banking output using a "number-of-transactions" approach based on demand deposits (number of checks written and cleared, and number of electronic funds transfers), time deposits (weighted index of number of deposits and withdrawals on regular savings accounts, club accounts, CDs, money market accounts, and IRAs), ATM transactions, loans (indexes of new and existing real estate, consumer installment, and commercial loans, and number of bank credit card transactions), and trust accounts (number of these accounts), each weighted by the proportion of employee hours used in each activity. Employee labor hours are used as the denominator of the productivity index, although the BLS also computes an output per employee measure. For later comparison to their results and those of other research studies of labour productivity measure for profit maximization in the bank **[Griliches (1992), Dean and Kunze (1992), Mohr (1992), and Kunze, Jablonski, and Sieling (1998)],** the BLS index for banking productivity per employee hour grew at an annualized rate of 3.25% over 1984-1996, which reflects annualized rates of change of 2.99% and 3.62% over the subintervals 1984-1991 and 1991-1996, respectively. Similarly, **Fixler and Zieschang (1997)** propose a promising financial-firm approach method for measuring bank output, and **Fixler and Hancock (1997) and Fixler Zieschang (1998)** propose interesting new methods of measuring the credit services of banks.

Rwanyaga (1996) carried out research work on Bank of Africa on things that affect profit levels in the bank. Regarding the study findings, it was revealed that cash management in bank of Africa affects profitability levels. The findings also showed that the bank employs several cash management techniques in order to reduce fraud of cash in the bank. The study also showed that profitability levels were high in the bank. The researcher recommended that there is a need for deploying a cash management system which involves support and coordination among multiple departments. Banks need to pick the right cash management provider who can coordinate the physical and technological installation of the system can significantly expedite and smooth the process of ensuring profit maximization. Each department should carefully consider features and functionality that will be required for a successful deployment and utilization of cash management which also increases profitability.

Lassila (1996) using a marginalistic approach that borrowing at high interest rates at the Central Bank in order to grant lower-interest loans can be consistent with straightforward short-term profit maximization for the bank. These phenomena simply result from the fact that the deposits received depend on the loans made, and, consequently, by borrowing at the Central Bank for credit expansion the bank generates sufficient additional deposits (which it invests in proper options) to make the borrowing profitable in spite of the high interest rate. Similarly, **Cohen and Hammer (1996)** Practitioners often argue that profit maximization is not an operational criterion in bank planning, and contend that maximization of a bank's deposits at the end of the planning horizon should be used instead. Their main argument is that short-term profit maximization is not consistent with the banks' long-term objectives. For example, in their opinion, short-term profit maximization gives no motivation for credit expansion.

During the 1990s Data Envelopment Analysis (DEA) has been used extensively to evaluate banking institutions. The relation between efficiency and profits was first addressed by **Oral (1990, 1992)** through two DEA models for analyzing both efficiency and profitability. The profitability model consisted of a desegregation of expenses and income, which were considered as inputs and outputs, respectively.

Laeven and Majnoni (2003) argue that risk can be incorporated into efficiency studies via the inclusion of loan loss provisions. That is, "following the general consensus among risk agent analysts and practitioners, economic capital should be tailored to cope with unexpected losses and loan loss reserves should instead buffer the expected component of the loss distribution. Consistent with this interpretation, loan loss provisions required to build up loan loss reserves should be considered and treated as a cost; a cost that will be faced with certainty over time but that is uncertain as to when it will materialize" (pp. 181). Among the earlier research that incorporated and studied the impact of nonperforming loans on bank efficiency are those of by **Hughes and Mester (1993, 1998), Hughes *et al.* (1996, 1999) and Mester (1997),** who included the volume of non-performing loans as a control for loan quality in studies of U.S. banks. **Berg *et al.* (1993)** on the other hand included loan losses as an indicator of the quality of loan evaluations in DEA study of Norwegian bank productivity.

Shelagh Hefternan (1996) insisted that the banking world is changing rapidly; the strategic priority has shifted away from growth and size alone towards a greater emphasis on profitability, performance and value creation within the banking firm. The performance of the banks decides the economy of nation. If the banks perform successfully, the economy of nation must be sound, growing and sustainable one.

2.2 LINEAR PROGRAMMING (LP)

Linear Programming was developed as a discipline in the 1940's, motivated initially by the need to solve complex planning problems in war time operations. Its development accelerated rapidly in the post war periods as many industries found its valuable uses for linear programming. The founders of the subject are generally regarded as **George B. Dantzig**, who devised the simplex method in **1947**, and **John Von Neumann**, who establish the theory of duality that same year. The noble price in economics was awarded in **1975** to mathematician **Leonid Kantorovich (USSR)** and the economist **Tjalling Koopmas (USA)** for their contribution to the theory of optimal allocation of resources, in which linear programming played a key role.

Many industries use linear programming as a standard tool, e.g. to allocate a finite set of resources in an optimal way. Example of important application areas include Airline crew scheduling, shipping or telecommunication networks, oil refining and blending, stock and bond portfolio selection. The problem of solving a system of linear inequality also dates back as far as **Fourier Joseph (1768 – 1830)** who was a Mathematician, Physicist and Historian, after which the method of Fourier – **Motzkin** elimination is named. Linear programming arose a mathe~~matical~~ model developed during the Second World War to plan expenditure and returns in other to reduce cost to the army and increase losses to the enemy. It was kept secret for years until 1947 when many industries found its use in their daily planning. The linear programming problem was first shown to be solvable in polynomial time **by Leonid Khachiyan in 1979** but a large theoretical and practical breakthrough in the field came in 1984 when **Narendra Karmarkar (1957 – 2006)** introduced a new interior point method for solving linear programming problems.

Many applications were developed in Linear programming these includes: **Lagrange in 1762** solves tractable optimization problems with simple equality constraint. In 1820, **Gauss** solved linear system of equations by what is now called Gaussian elimination method and in 1866, **Whelhelm Jordan** refined the method to finding least squared error as a measure of goodness-of-fit. Now it is referred to as **Gausss-Jordan** method. Linear programming has proven to be an extremely powerful tool, both in modelling real-world problems and as a widely applicable mathematical theory. However, many interesting optimization problems are non linear. The studies of such problems involve a diverse blend of linear Algebra, multivariate calculus, numerical analysis and computing techniques. The simplex method which is used to solve linear programming was developed by **George B. Dantzig in 1947** as a product of his research work during World War II when he was working in the Pentagon with the Mil. Most linear programming problems are solved with this method. He extended his research work to solving problems of planning or scheduling dynamically overtime, particularly planning dynamically under uncertainty. Concentrating on the development and application of specific operations research techniques to determine ~~the optimal~~ choice among several courses of action, including the evaluation of specific numerical values (if required), we need to construct (or formulate) mathematical model **Dantzig(1963), Hiller et al (1995), Adams(1969).**

The development of linear programming has been ranked among the most important scientific advances of the mid-20th century, and its assessment is generally accepted.

2.2.1 APPLICATIONS OF LP FOR PROFIT MAXIMIZATION IN THE BANK

Wheelock and Wilson (1996) used the linear programming approach to investigate bank productivity growth, decomposing the change in productivity into its change in efficiency and shift in the frontiers (profit). They found that larger banks (assets over $300 million) experienced productivity growth between 1984-1993, while smaller banks experienced a decline. Average inefficiency remained high in the industry, since banks were not able to adapt quickly to changes in technology, regulations, and competitive conditions. Similarly, **Alam (1998)** used linear programming techniques to investigate productivity change in banking using a balanced panel of 166 banks with greater than $500 million in assets and uninterrupted data from 1980 to 1989. As in Wheelock and Wilson, productivity change was decomposed into its two components: changes in efficiency and shifts in the frontier (profit). Bootstrapping methods were used to determine confidence intervals for the productivity measure and its components. The findings were that productivity surged between 1983 and 1984, regressed over the next year, and grew again between 1985 and 1989. The main source of the productivity growth was a shift in the frontier rather than a change in efficiency.

Veikko, Timo and Jaako (1996) an intertemporal linear programming model for exploring optimal credit expansion strategies of a commercial bank in the framework of dynamic balance sheet management assuming that it is both technically feasible and economically relevant to establish a linear relationship between the bank's credit expansion and the deposits received by the bank induced by the credit expansion process. The inclusion of the relationship between the credit expansion and the deposits induced thereby in the intertemporal model leads to optimal solutions which run counter to intuitive reasoning. The optimal solution may, e.g., exclude the purchase of investment securities in favor of loans to be granted even in the case where the nominal yield on securities is higher than the yield on loans. The optimal solution may also contain a variable representing the utilization of a source of funds, e.g., funds obtained from the central bank, which implies the payment of a rate of interest on these funds higher than any yield obtained on the bank's portfolio of loans and securities. Since the objective function of the model is the maximization of the difference between the total yield on the securities and loans portfolio and the total interest on the various deposits and other liabilities that the bank obtains, it would be hard to arrive at these results by intuitive reasoning. The explanation for the results obtained is the dynamic relationship between the loans granted and deposits received.

Chambers and Charnes (1961) developed intertemporal linear programming models for bank dynamic balance sheet management determine (given as inputs e.g. forecasts of loan demand, deposit levels, yields and costs of various alternatives over a several-year planning horizon) the sequence of period-by-period balance sheets which will maximize the bank's net return subject to constraints on the bank's maximum exposure to risk, minimum supply of liquidity, and a host of other relevant considerations. Later by **Cohen and Hammer (1967)** who extended the model to cover the relevant set of factors encountered in a real-life application at the Bankers Trust Company. Cohen describes this paper as follows: "It presents an intertemporal linear programming model whose decision variables relate to assets, liabilities, and capital accounts. The model incorporates constraints pertaining to risk, funds availability, management policy, and market restrictions. Intertemporal effects and the dynamics of loan-related feedback mechanisms were considered, and the relative merits of alternative criterion functions were also discussed."

Cohen and Hammer (1967) made three alternative suggestions for the incorporation of the discussed feature into intertemporal linear programming models for bank dynamic balance sheet management: (1) loan making is made a predetermined constant, (2) imputed yield rates on the loans are applied, or (3) loan related feedback mechanisms are incorporated in the intertemporal constraints in order to reflect changes in the market share as the result of the bank's relative performance in meeting its loan demand. Thore [33, pp. 126-127] presented a technique for linking uncertain future changes in deposits to drawing rights created on loans granted by using a linear relationship in his model, which is basically an asset allocation model only. As the discussion this far indicates, a modification of Cohen's and Hammer's third alternative will be adopted along the lines indicated by Thore.

2.3 REVISED SIMPLEX METHOD

The Revised Simplex Method is commonly used for solving linear programs. This method operates on a data structure that is roughly of size *m* by *m* instead of the whole tableau. This is a computational gain over the full tableau method, especially in sparse systems (where the matrix has many zero entries) and/or in problems with many more columns than rows. On the other hand, the revised method requires extra computation to generate necessary elements of the tableau.

In the revised method, the standard form is represented implicitly in terms of the original system together with a functional equivalent of the inverse of the basis *B*. "Functional equivalent" means we have a data structure which makes solving $\pi B = cB$ for π and $BA'_j = A_j$ for $A'j$, easy. Aj represents the jth column of the A matrix and *cb* represents the basic objective coefficients. The data structure need not be B^{-1} or even necessarily a representation of it. For example, an LU decomposition of the basis is often used, **Nash and Sofer, (1996)**; another is to represent the inverse as a product of simple pivot matrices **Nash and Sofer (1996)** ;and **Chvátal (1983).** Given the implicit representation, we recreate the data needed to implement the three parts of the revised simplex iteration. Thus, to "pivot" we update *b*' and our functional equivalent of B^{-1}.

We must have the coefficients c'_j available. We use multiples of the original constraints to eliminate the basic variables in the expression for *z*. Symbolically, we let the component row vector π represent the multiples; that is, we multiply constraint *i* by *πi* and subtract the result from the expression for *z*. To make this work π must have *πB* = *cB* where *cB*, as above, represents the m elements of *c* corresponding to the basis columns. Then *c'j* in the dictionary is given by $c'_j = c - \pi A_j$ where A_j represents the jth column of A.

This computation is called *pricing*. So to select the column using the classical Dantzig rule, the vector π must be calculated and then the inner product of π with each column of A must be subtracted from the original coefficient. The revised method takes more effort than the standard simplex method in this step. However, for sparse matrices, pricing out is speeded up because many ~~of the~~ products have zero factors. Moreover, the revised simplex method can be speeded up considerably by using *partial pricing* **Nash and Sofer (1996)**. Partial pricing is a heuristic of not considering all the columns during the column choice step. On the other hand, alternate column choice rules such as steepest edge **Forrest and Goldfarb (1992)** and greatest change are much more difficult to implement using the revised approach. To implement this we need b'. The b' vector will be updated from iteration to iteration; it does not need to be recreated. A'_j is given by solving $BA'_j = A_j$. In the revised method since we always go back to the original matrix, we still have the original sparsity. Specifically, we have the original sparsity in *Aj*. If we need to pivot, instead of explicitly pivoting as before, we update our functional equivalent of B^{-1}.

2.4 SENSITIVITY ANALYSIS

Porter, et al.(1980) "once an equation, model, or simulation is chosen to represent a given system, there is the question regarding which parts of that equation are the best predictors. Sensitivity analysis is a general means to ascertain the sensitivity of system (model) parameters by making changes in important variables and observing their effects".

Sensitivity analysis involves testing a model with different data sets to determine how different data and different assumptions affect a model. **Porter** further explained that term sensitivity is defined as "the ratio between the fractional change in a parameter that serves as a basis for decision to the fractional change in the simple parameter being tested. He gave an example that " you've developed a model to predict a bank's profit level for some period of time. You have included a number of factors such as interest accrued from given loans, ATM charges, Current Accounts charges, etc that may contribute to the bank profit level. Your model is pretty good with predictions based on the historical data you have supplied it with. You believe the factors noted above are the most important factors, but by eliminating one or two of them from the model, the model gives the same results. It turns out that the sensitivity here is not as high as you had thought".

CHAPTER THREE

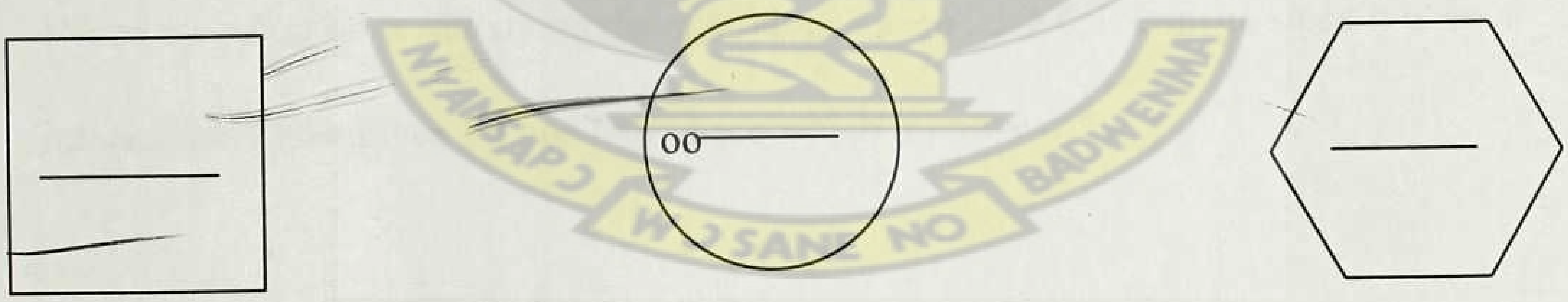
3.0 METHODOLOGY

This chapter opens with a discussion of the mathematical theory, formulations, models or algorithms. ie the methodology employed in the study includes example of modeling a problem as Linear Programming Problem; determination of the optimal solution using Revised Simplex Algorithm and providing sensitivity analysis of the problem.

3.1 CONVEX SETS

Definition: A subset Q of R^n is said to be **convex** if for any two elements x_1, x_2 in Q the line segment $[x_1, x_2]$ is contained in Q. Thus x_1 and x_2 in Q imply $\lambda x_1 + (1-\lambda)x_2 \in Q$ for all $0 \le \lambda \le 1$ if Q is convex. i.e Sets in R^n are convex if they contain no hole, protrusion or indentation and are not convex otherwise.

Examples of Convex Sets are shown below:



Examples below are not Convex Sets:

From, the above diagrams, it can be observed that:

1. The intersection of any family of convex sets in R^n is convex.
2. A closed half-space or open half-space in R^n is convex. Therefore, hyperplane being the intersection of two closed half-space is convex.
3. If A is an $m \times n$ matrix and b is an m-vector, then the set of solution of the Linear System

$$Ax = b,$$

being the intersection of a finite number of hyperplanes in R^n, is convex.

Hence the set of all x satisfying the condition $Ax = b$, $x \geq 0$, is convex since it is the intersection of a convex set and half-space which is convex.

3.2 LINEAR PROGRAMMING

Andrade (1990), Linear Programming deals with special mathematical problems by developing rules and relationships that aims at distribution of limited funds under the restrictions imposed by either technological or practical aspects when an attribution decision has to be made. Linear Programming (LP) is also used as a technique for optimization of linear objective function, subject to linear inequality or equality constraint. Generally, Linear Programming can be written in a canonical form for profit maximization:

Max $\quad Z = cx$

S.t $\quad Ax \leq b$

$\quad x > 0$

From the model above, x represents the vector of variables (to be determined) while c and b are vectors of known matrix of coefficient. The expression to be maximized is called the objective function. Linear Programming is a considerable field of optimization for several reasons. Many practical problems in operations problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming such as network flow problems and multi commodity flow problems are considered important specialised algorithm for their solution. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory such as Duality and Decomposition.

3.2.1 LP PROBLEM FORMULATION PROCESS AND ITS APPLICATIONS

To formulate an LP problem, the following guidelines are recommended after reading the problem statement carefully.

Any linear program consists of four parts: a set of decision variables, the parameters, the objective function, and a set of constraints. In formulating a given decision problem in mathematical form, you should practice understanding the problem (i.e., formulating a mental model) by carefully reading and re-reading the problem statement. While trying to understand the problem, ask yourself the following general questions:

1. **What are the decision variables?** That is, what are controllable inputs? Define the decision variables precisely, using descriptive names. Remember that the controllable inputs are also known as controllable activities, decision variables, and decision activities.
2. **What are the parameters?** That is, what are the uncontrollable inputs? These are usually the given constant numerical values. Define the parameters precisely, using descriptive names.
3. **What is the objective?** What is the objective function? Also, what does the owner of the problem want? How the objective is related to the decision variables? Is it a maximization or minimization problem? The objective represents the goal of the decision-maker.
4. **What are the constraints?** That is, what requirements must be met? Should I use inequality or equality type of constraint? What are the connections among variables? Write them ~~out~~ in words before putting them in mathematical form.

3.2.2 LINEAR PROGRAMMING PROBLEM WITH A CONSTANT TERM IN THE OBJECTIVE FUNCTION

Given any Linear Programming Problem:

Max $\quad Z = cx + d$

S.t $\quad Ax \leq b$

$\quad x > 0$

Where d is a constant term in the objective function.

To solve such problem, apply the Revised Simplex Method to solve the LPP whilst keeping the constant term aside. When the Objective function is determined, we then add the constant term to objective function to obtain the required Optimal objective Function.

3.2.3 METHODS OF SOLVING LINEAR PROGRAMMING PROBLEMS

Linear Programming Problems can be solve using **Simplex Method, Revised Simplex Method, Duality and Integer Programming.**

Linear Programming is also widely applied in the areas of **Transportation Problem, Assignment Problem, Production Problem and Transshipment Problem.**

DUALITY

To every linear program there is a dual linear program with which it is intimately connected. We first state this duality for the standard programs.

Definition. The dual of the standard maximum problem

$$\begin{aligned} &\text{maximize} && c^T x \\ &\text{subject to} && Ax \leq b \\ & && x \geq 0 \end{aligned}$$

is defined to be the standard minimum problem

$$\begin{aligned} &\text{minimize} && y^T b \\ &\text{subject to} && y^T A \geq c^T \quad and \quad y \geq 0 \end{aligned}$$

Where $\boldsymbol{c}$ and $\boldsymbol{x}$ are n-vectors, $\boldsymbol{b}$ and $\boldsymbol{y}$ are m-vectors, and $\boldsymbol{A}$ is an $m \times n$ matrix. We assume $m \geq 1$ and $n \geq 1$.

The Duality Theorem: If a standard linear programming problem is bounded feasible, then so is its dual, their values are equal, and there exists optimal vectors for both problems.

Interpretation of the dual : In addition to the help it provides in finding a solution, the dual problem offers advantages in the interpretation of the original, primal problem. In practical cases, the dual problem may be analyzed in terms of the primal problem.

INTEGER PROGRAMMING

An **integer programming** problem is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers. In many settings the term refers to integer linear programming, which is also known as ***mixed integer programming*** when some but not all the variables are restricted to be integers and is called a ***pure integer programming*** when *all* decision variables must be integers.

Integer Linear Programming Problem is of the form:

Maximize $\sum_{j=1}^{n} c_j x_j$

Subject to $\sum_{j=1}^{n} a_{ij} x_j = bi \qquad (i = 1, 2, \ldots, m)$

$x_j \geq 0 \qquad (j = 1, 2, \ldots, n)$

x_j integer $\qquad (\textit{for some or all } j = 1, 2, \ldots, n)$

Methods of Solving Integer Programming Problems

1. Branch and Bound

The most widely used method for solving integer programs is branch and bound. Sub- problems are created by restricting the range of the integer variables. For binary variables, there are only two possible restrictions: setting the variable to 0, or setting the variable to 1. More generally, a variable with lower bound l and upper bound u will be divided into two problems with ranges l to q and q+1 to u respectively. Lower bounds are provided by the linear-programming relaxation to the problem: keep the objective function and all constraints, but relax the integrality restrictions to derive a linear program. If the optimal solution to a relaxed problem is (coincidentally) integral, it is an optimal solution to the sub- problem, and the value can be used to terminate searches of sub-problems whose lower bound is higher.

2. Branch and Cut

For branch and cut, the lower bound is again provided by the linear-programming (LP) relaxation of the integer program. The optimal solution to this linear program is at a corner of the polytope which represents the feasible region (the set of all variable settings which satisfy the constraints). If the optimal solution to the LP is not integral, this algorithm searches for a constraint which is violated by this solution, but is not violated by any optimal integer solutions. This constraint is called a cutting plane. When this constraint is added to the LP, the old optimal solution is no longer valid, and so the new optimal will be different, potentially providing a better lower bound. Cutting planes are iteratively until either an integral solution is found or it becomes impossible or too expensive to find another cutting plane. In the latter case, a tradional branch operation is performed and the search for cutting planes continues on the sub-problems.

3. Branch and Price

This is essentially branch and bound combined with column generation. This method is used to solve integer programs where there are too many variables to represent the problem explicitly. Thus only the active set of variables is maintained and columns are generated as needed during the solution of the linear program. Column generation techniques are problem specific and can interact with branching decisions.

3.3 REVISED SIMPLEX METHOD

Original simplex method calculates and stores all numbers in the tableau. Revised Simplex Method which is more efficient for computing Linear programming problems operates on a data structure that is roughly of size *m* by *m* instead of the whole tableau.

Max $\quad Z = cx$

S.t $\quad Ax \le b$

$\quad x > 0$

Initially constraints become the standard form:

$$\begin{bmatrix} A & I \end{bmatrix} \begin{bmatrix} x \\ x_s \end{bmatrix} = [b]$$

where x_s = slack variables

Basis matrix: columns relating to basic variables.

$$B = \begin{bmatrix} B_{11} \ldots\ldots\ldots\ldots\ldots\ldots B_{1M} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ B_{M1} \ldots\ldots\ldots\ldots\ldots\ldots B_{MM} \end{bmatrix}$$

(Initially $B = I$)

Basic variable values: $X_B = \begin{pmatrix} X_{B1} \\ . \ . \\ . \ . \\ . \ . \\ X_{BM} \\ . \end{pmatrix}$

At any iteration non-basic variables =0

$$Bx_B = b$$

$$\Rightarrow X_B = B^{-1}b$$

Where B^{-1} is the inverse matrix

At any iteration, given the original b vector and the inverse matrix, X_B (current R.H.S.) can be calculated.

$Z = c_B x_B$

where c_B = objective coefficients of basic variables.

3.3.1 STEPS IN THE REVISED SIMPLEX METHOD

1. Determine entering variable, X_j, with associated vector P_j.

Compute $Y = c_B B^{-1}$

Compute $Z_j - C_j = YP_j - C_j$ for all non-basic variables.

Choose largest negative value (maximization).

If none, stop.

2. Determine leaving variable, x_r, with associated vector P_r.

Compute $x_B = B^{-1}b$ (current R.H.S.)

Compute current constraint coefficients of entering variable:

$\alpha^j = B^{-1}P_j$

X_r is associated with

$\theta = \underset{K}{Min}\{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\}$ i.e. minimum ratio rule

3. Determine next basis i.e. calculate $\overline{B}^{-1}$.

Go to step 1.

Example:

Max $Z = 3x_1 + 5x_2$

$$S.t \quad \begin{aligned} x_1 &\le 4 \\ 2x_2 &\le 12 \\ 3x_1 + 2x_2 &\le 18 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Standard form of constraints:

$$x_1 + s_1 = 4$$
$$2x_2 + s_2 = 12$$
$$3x_1 + 2x_2 + s_3 = 18$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0$$

$$x_B = B^{-1}b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 12 \\ 18 \end{pmatrix}$$

$$c_B = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 12 \\ 18 \end{pmatrix} = 0$$

First Iteration

Step 1

Determine entering variable, x_j , with associated vector p_j .

Compute $Y = c_B B^{-1}$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables (x_1 *and* x_2)

$$Y = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$Z_1 - C_1 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} - 3 = -3$$

and similarly for $Z_2 - c_2 = -5$

Therefore X_2 is entering variable.

Step 2

Determine leaving variable, x_r, with associated vector P_r.

Compute $x_B = B^{-1} b$ (current R.H.S.)

Compute current constraint coefficients of entering variable:

$$\alpha^j = B^{-1}P_j$$

x_r is associated with

$$\theta = \min_K \{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\}$$

$$x_B = \begin{pmatrix} 4 \\ 12 \\ 18 \end{pmatrix} \qquad \alpha^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

$$\theta = Min\{-, 12/2, 18/2\}$$
$$= 12/2$$

Therefore S_2 leaves the basis.

Step 3

Determine new B^{-1}

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

Solution after one iteration:

$$x_B = B^{-1} b$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 12 \\ 18 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 6 \end{pmatrix}$$

Go to step 1

Second Iteration

Step 1

Compute $Y = c_B B^{-1}$

$$Y = \begin{pmatrix} 0 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 5/2 & 0 \end{pmatrix}$$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables (x_1 and S_2):-

$$x_1 : z_1 - c_1 = Yp_1 - c_1 = \begin{pmatrix} 0 & 5/2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} - 3 = -3$$

$$S_2 : z_2 - c_2 = Yp_2 - c_2 = \begin{pmatrix} 0 & 5/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - 0 = 5/2$$

Therefore x_1 enters the basis.

Step 2

Determine leaving variable.

$$x_B = \begin{pmatrix} 4 \\ 6 \\ 6 \end{pmatrix} \qquad \alpha^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

θ = Min { 4/1 , - , 6/3 }

= 6/3

Therefore S_3 leaves the basis.

Step 3

Determine new B^{-1}

$$B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 2 & 3 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{pmatrix}$$

Solution after two iterations:

$$x_B = B^{-1}b$$

$$= \begin{pmatrix} 0 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{pmatrix} \begin{pmatrix} 4 \\ 12 \\ 18 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix}$$

Go to step 1

Third Iteration

Step1

Compute $Y = c_B B^{-1}$

$$Y = (0 \quad 5 \quad 3)\begin{pmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{pmatrix} = (0 \quad 3/2 \quad 1)$$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables (S_2 and S_3):-

$$S_2 : z_4 - c_4 = (0 \quad 3/2 \quad 1)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - 0 = 3/2$$

$$S_3 : Z_5 - C_5 = (0 \quad 3/2 \quad 1)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - 0 = 1$$

No negatives. Therefore stop.

Optimal solution:

$S_1^* = 2$

$X_2^* = 6$

$X_1^* = 2$

$$Z^* = c_B x_B = \begin{pmatrix} 0 & 5 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix} = 36$$

Therefore, the Optimal objective function is 36 where $x_1 = 2$ and $x_2 = 6$.

3.3.2 ALTERNATIVE

The function revised in MATLAB can also be used to solve LPP using revised simplex method. It uses big M method to solve an LPP when there are <=, >= or = constraints present.

Input:

c : The cost vector or the (row) vector containing co-efficient of decision variables in the objective function. It is required parameter.

b : The (row) vector containing right hand side constant of the constraints. It is a required parameter.

a : The coefficient matrix of the left hand side of the constraints. it is a required parameter.

inq ~~: A~~ (row) vector indicating the type of constraints as 1 for >=, 0 for = and -1 for <= constraints. If inq is not supplied then it is by default taken that all constraints are of <= type. It is an optional parameter.

minimize : This parameter indicates whether the objective function is to be minimized. minimized = 1 indicates a minimization problem and minimization = 0 stands for a maximization problem. By default it is taken as 0. It is an optional parameter.

Now, consider LPP

Max $\quad Z = 3x_1 + 5x_2$

$$\begin{aligned} S.t \quad & x_1 \le 4 \\ & 2x_2 \le 12 \\ & 3x_1 + 2x_2 \le 18 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Therefore, in the LPP above, the problem can be written as

$c = [3 \quad 5]$
$b = [4 \quad 12 \quad 18]$
$a = [1 \ 0; 0 \ 2; 3 \ \ 2]$
$inq = [-1 \ \ -1 \ \ -1]$

After supplying these inputs in MATLAB call revised(c,b,a,inq,0).

The following tablaue and optimal solution were obtained:

............The initial tablaue................

	z	3	4	5
1	0	0	0	0
0	1	0	0	4
0	0	1	0	12
0	0	0	1	18

.............The 1th tablaue................

z	3	4	5		
1	0	0	0	0	-5
0	1	0	0	4	0
0	0	1	0	12	2
0	0	0	1	18	2

.............The 2th tablaue................

z	3	2	5		
1.0000	0	2.5000	0	30.0000	-3.0000
0	1.0000	0	0	4.0000	1.0000
0	0	0.5000	0	6.0000	0
0	0	-1.0000	1.0000	6.0000	3.0000

............The 3th tablaue................

z 3 2 1

1.0000	0	1.5000	1.0000	36.0000	1.0000
0	1.0000	0.3333	-0.3333	2.0000	-0.3333
0	0	0.5000	0	6.0000	0
0	0	-0.3333	0.3333	2.0000	0.3333

Reqiured optimization has been achieved!

The optimum objective function value=36.

The optimum solution is:

$x_1 = 2$

$x_2 = 6$

3.4 SENSITIVITY ANALYSIS

Sensitivity analysis helps to study the effect of changes in the parameters of the Linear Programming model in other to determine the optimal solution. i.e. it allows the analyst to study the behaviour of the optimal solution as a result of making changes in the model's parameters.

3.4.1 Changing Objective Function

Suppose in the solution of the LPP above, we wish to solve another problem with the same constraints but a slightly different objective function. So if the objective function is changed, not only will I hold the constraints fixed, but I will change only one coefficient in the objective function. When you change the objective function it turns out that there are two cases to consider. The first case is the change in a non-basic variable (a variable that takes on the value zero in the solution). In the example, the relevant non-basic variables are x_1 and s_2. What happens to your solution if the coefficient of a non-basic variable decreases? For example, suppose that the coefficient of x_1 in the objective function above was reduced from 3 to 2 (so that the objective function is: $Max \quad 2x_1 + 5x_2$). What has happened is this: You have taken a variable that you didn't want to use in the first place (i.e you set x_1 = 0) and then made it less profitable (lowered its coefficient in the objective function). You are still not going to use it. The solution does not change.

Observation: If you lower the objective function coefficient of a non-basic variable, then the solution does not change. What if you raise the coefficient? Intuitively, raising it just a little bit should not matter, but raising the coefficient a lot might induce you to change the value of x_1 in a way that makes $x_1 > 0$. So, for a non-basic variable, you should expect a solution to continue to be valid for a range of values for coefficients of non-basic variables. The range should include all lower values for the coefficient and some higher values. If the coefficient increases enough and putting the variable into the basis is feasible, then the solution changes.

What happens to your solution if the coefficient of a basic variable (like x_2 in the example) decreases? The change makes the variable contribute less to profit. You should expect that a sufficiently large reduction makes you want to change your solution. For example, if the coefficient of x_2 in the objective function in the example was 2 instead of 5 (so that the objective was $Max\ \ 3x_1 + 2x_2$), will change the solution since the reduction in the coefficient of x_2 is large. On the other hand, a small reduction in x_2 objective function coefficient would typically not cause you to change your solution.

So, intuitively, there should be a range of values of the coefficient of the objective function (a range that includes the original value) in which the solution of the problem does not change. Outside of this range, the solution will change. The value of the problem always changes when you change the coefficient of a basic variable.

3.4.2 Changing a Right-Hand Side Constant of constraint

When you changed the amount of resource in a non-binding constraint, i.e small increases will never change your solution and small decreases will also not change anything. However, if you decreased the amount of resource enough to make the constraint binding, your solution could change. But, changes in the right-hand side (RHS) of binding constraints always change the solution.

3.4.3 Adding a Constraint

If you add a constraint to a problem, two things can happen. Your original solution satisfies the constraint or it doesn't. If it does, then you are finished. If you had a solution before and the solution is still feasible for the new problem, then you must still have a solution. If the original solution does not satisfy the new constraint, then possibly the new problem is infeasible. If not, then there is another solution.

CHAPTER FOUR

4.0 DATA ANALYSIS

This chapter opens with the study area, sampling procedure, instruments, data collection procedure, data analysis and the results.

4.1 SAMPLING PROCEDURE

The research was a case study of Stanbic bank, Tamale. The data type is secondary data of Stanbic Bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and Asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012.

Table 4.1 Data of Stanbic bank for the period of six (6) months (Nov. 2011 to April, 2012) of variable products (i.e Loans)

NUMBER OF CUSTOMERS	TYPE OF PRODUCT	INTEREST RATE	PROBABILTY OF BAD DEBT	PROBABILTY OF NO BAD DEBT
150	Revolving term Loans	22%+3%=25%	1%	=1-0.01 =0.99
140	Fixed term Loans	22%+6%=28%	2%	=1-0.02 =0.98
150	Home Loans	22%+1.5%=23.5%	1%	=1-0.01 =0.99
95	Personal VAF	22%+2%=24%	1%	=1-0.02 =0.99
90	Vehicle and Asset Finance	22%+6%=28%	1%	=1-0.01 =0.99
180	Current Account	GHC5 per month	2%	=1-0.02 =0.98
54000	ATM withdrawal	GHC0.44 per withdrawal	2%	=1-0.02 =0.98
180	Cheque books	GHC6 per cheque book	3%	=1-0.03 =0.97
372	Counter cheque	GHC3 per counter cheque	0%	1

The bank is to allocate a total fund of GH₵3,745,200 to the various loan products. The bank is faced with following constraints:

1. Amounts allocated to the various loan products should not more than the total funds.
2. Allocate not more than 60% of the total funds to Fixed term loans, Personal VAF; and Vehicle and Asset loans.
3. Revolving term loans, Fixed term loans and Personal VAF should not be more than 50% of the total funds.
4. Allocate not more than 40% of the total funds to Personal VAF and Vehicle and Asset loans.
5. The overall bad debts on the Revolving term loans, Fixed term loans, Home loans, Personal VAF; and Vehicle and Asset Loans should not exceed 0.03 of the total funds.

Table 4.2 ***Data of Stanbic bank for the period of six (6) months (Nov. 2011 to April, 2012) of constant products.***

Number of Customers	**Type of Products**	**Interest Rate**	**Amount at the End of the Six (6) Months**	**Probability of Bad debt**	**Probability of No Bad Debt**	**Amount at the end of the Six (6) Months**
180	Current Account	GH₵5	GH₵5 $\times$ 180 =GH₵900	2%	0.98	GH₵900 $\times$ 0.98 = GH₵882
54000	ATM Withdrawals	GH₵0.44	GH₵0.44 $\times$ 54000 =GH₵23,760	2%	0.98	GH₵23,760 $\times$ 0.98 =GH₵23,284.8
180	Cheque Books	GH₵6	GH₵6 $\times$ 180 = GH₵1,080	3%	0.97	GH₵1,080 $\times$ 0.97 =GH₵1,047.6
372	Counter Cheques	GH₵3	GH₵3 $\times$ 372 =GH₵1,116	0%	1	GH₵1,116
						Total =GH₵26,330.4

4.2 MATHEMATICAL MODEL

The variables of the model are defined as

x_1 = Revolving term loans (in millions in GH₵)

x_2 = Fixed term loans

x_3 = Home loans

x_4 = Personal VAF

x_5 = Vehicle and Asset finance

The objective of Stanbic Bank, Tamale was to maximize its net returns consisting of the difference between the revenue from interest and lost funds due to bad debts.

The objective function is written as

$$Z = [0.25(0.99x_1) - 0.01x_1] + [0.28(0.98x_2) - 0.02x_2] + [0.235(0.99x_3) - 0.01x_3] + [0.24(0.99x_4) - 0.01x$$

$$+ [0.28(0.99x_5) - 0.01x_5] + \text{[Revenue from Current Accounts + ATM withdrawals +}$$

Cheque Books + Counter Cheque]

$$Z = 0.2375x_5 + 0.2544x_2 + 0.22265x_3 + 0.2276x_4 + 0.2672x_5 + 0.0263304$$

The problem has **six (6) constraints**:

1. Total funds:

   $x_1 + x_2 + x_3 + x_4 + x_5 \leq 3.7452$

2. Allocate not more than 60% of the total funds to Fixed term loans, Personal VAF; and Vehicle and Asset loans:

$$x_2 + x_4 + x_5 \le 0.6 \times 3.7452$$

$$x_2 + x_4 + x_5 \le 2.24712$$

3. Revolving term loans, Fixed term loans and Personal VAF should not be more than 50% of the total funds:

$$x_1 + x_2 + x_4 \le 0.5 \times 3.7452$$

$$x_1 + x_2 + x_4 \le 1.8726$$

4. Allocate not more than 40% of the total funds to Personal VAF and Vehicle and Asset loans:

$$x_4 + x_5 \le 0.4 \times 3.7452$$

$$x_4 + x_5 \le 1.49808$$

5. The overall bad debts on the Revolving term loans, Fixed term loans, Home loans, Personal VAF; and Vehicle and Asset Loans should not exceed 0.03 of the total funds:

$$0.01x_1 + 0.02x_2 + 0.01x_3 + 0.01x_4 + 0.01x_5 \le 0.03 \times 3.7452$$

$$0.01x_1 + 0.02x_2 + 0.01x_3 + 0.01x_4 + 0.01x_5 \le 0.112356$$

6. Non-negativity constraint:

$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0,\ x_5 \ge 0$$

Now the LP is

Max $\quad Z = 0.2375x_5 + 0.2544x_2 + 0.22265x_3 + 0.2276x_4 + 0.2672x_5 + 0.0263304$

s.t $\quad x_1 + x_2 + x_3 + x_4 + x_5 \leq 3.7452$

$x_2 + x_4 + x_5 \leq 2.24712$

$x_1 + x_2 + x_4 \leq 1.8726$

$x_4 + x_5 \leq 1.49808$

$0.01x_1 + 0.02x_2 + 0.01x_3 + 0.01x_4 + 0.01x_5 \leq 0.112356$

4.3 USING THE REVISED SIMPLEX METHOD TO SOLVE THE LPP

Standard form of constraints is:

$x_1 + x_2 + x_3 + x_4 + x_5 + s_1 = 3.7452$

$x_2 + x_4 + x_5 + s_2 = 24712$

$x_1 + x_2 + x_4 + s_3 = 1.8726$

$x_4 + x_5 + s_4 = 1.49808$

$0.01x_1 + 0.02x_2 + 0.01x_3 + 0.01x_4 + 0.01x_5 + s_5 = 0.112356$

$x_1, x_2, x_3, x_4, x_5, s_1, s_2, s_3, s_4, s_5 \geq 0$

$$x_B = B^{-1}b = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix}$$

$$= \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix}$$

$$Z = c_B x_B$$

$$Z = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix} = 0$$

FIRST ITERATION

Step 1

Determine entering variable, x_j, with associated vector p_j.

Compute $Y = c_B B^{-1}$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables.

$$Y = \begin{pmatrix}0 & 0 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & 0 & 0\end{pmatrix}$$

$$Z_1 - C_1 = Yp_1 - c_1 = \begin{pmatrix}0 & 0 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\\ 1\\ 0\\ 0.01\end{pmatrix} - 0.2375 = -0.2375$$

$$Z_2 - C_2 = Yp_2 - c_2 = \begin{pmatrix}0 & 0 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\\ 1\\ 0\\ 0.02\end{pmatrix} - 0.2544 = -0.2544$$

Similarly for $Z_3 - c_3 = -0.22265$

$Z_4 - c_4 = -0.2276$

$Z_5 - c_5 = -0.2672$

Therefore X_5 is entering variable.

Step 2

Determine leaving variable, x_r, with associated vector p_r

Compute $x_B = B^{-1}b$ (current RHS)

Compute current constraint coefficients of entering variable

$$\alpha^j = B^{-1}P_j$$

x_r is associated with

$$\theta = \underset{K}{Min}\{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\}$$

$$x_B = \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix} \qquad \alpha^5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0.01 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0.01 \end{pmatrix}$$

$$\theta = Min\{3.7452, 2.24712, -, 1.49808, \frac{0.112356}{0.01}\}$$
$$= 1.49808$$

Therefore S_4 leaves the basis.

Step 3

Determine new B^{-1}

$$B = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.01 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -0.01 & 1 \end{pmatrix}$$

Solution after one iteration:

$$x_B = \mathrm{B}^{-1}\mathrm{b}$$

$$= \begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -0.01 & 1 \end{pmatrix} \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix}$$

$$= \begin{pmatrix} 2.2471 \\ 0.7490 \\ 1.8726 \\ 1.4981 \\ 0.0974 \end{pmatrix}$$

Go to step 1

SECOND ITERATION

Step 1

Determine entering variable, x_j , with associated vector p_j .

Compute $Y = c_B B^{-1}$

$$Y = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & -1 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -0.01 & 1\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}$$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables $(x_1, x_2, x_3, x_4, s_4)$

$$x_1 : Z_1 - C_1 = Yp_1 - c_1 = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\\ 1\\ 0\\ 0.01\end{pmatrix} - 0.2375 = -0.2375$$

$$x_2 : Z_2 - C_2 = Yp_2 - c_2 = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\\ 1\\ 0\\ 0.02\end{pmatrix} - 0.2544 = -0.2544$$

$$x_3 : Z_3 - C_3 = Yp_3 - c_{31} = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\\0.01\end{pmatrix} - 0.22265 = -0.22265$$

$$x_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\\0.01\end{pmatrix} - 0.2276$$

$$= 0.2672 - 0.2276$$
$$= 0.0396$$

$$s_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix}0 & 0 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} - 0 = 0.2672$$

Therefore X_2 is entering variable.

Step 2

Determine leaving variable, x_r, with associated vector p_r

Compute $x_B = B^{-1}b$ (current RHS)

Compute current constraint coefficients of entering variable

$$\alpha^j = B^{-1} P_j$$

x_r is associated with

$$\theta = \underset{K}{Min}\{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\}$$

$$x_B = \begin{pmatrix} 2.2471 \\ 0.7490 \\ 1.8726 \\ 1.4981 \\ 0.0974 \end{pmatrix} \qquad \alpha^2 = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -0.01 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0.02 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0.02 \end{pmatrix}$$

$$\theta = Min\{2.2471, 0.7490, 1.8726, -, 4.87\}$$
$$= 0.7490$$

Therefore S_2 leaves the basis.

Step 3

Determine new B^{-1}

$$B^{-1} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -0.02 & 0 & 0.01 & 1 \end{pmatrix}$$

Solution after two iterations:

$$x_B = \mathrm{B}^{-1}\,\mathrm{b}$$

$$= \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -0.02 & 0 & 0.01 & 1 \end{pmatrix} \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix}$$

$$= \begin{pmatrix} 7.4904 \\ 3.7452 \\ 4.1197 \\ 1.4981 \\ 0.1723 \end{pmatrix}$$

Go to step 1

THIIRD ITERATION

Step 1

Determine entering variable, x_j , with associated vector p_j .

Compute $Y = c_B B^{-1}$

$$Y = \begin{pmatrix}0 & 0.2544 & 0 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1 & -1 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & -1 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & -0.02 & 0 & 0.01 & 1\end{pmatrix} = \begin{pmatrix}0 & 0.2544 & 0 & 0.0128 & 0\end{pmatrix}$$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables (x_1, x_3, x_4, s_4, s_2)

$$x_1 : Z_1 - C_1 = Yp_1 - c_1 = \begin{pmatrix}0 & 0.2544 & 0 & 0.0128 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\\ 1\\ 0\\ 0.01\end{pmatrix} - 0.2375 = -0.2375$$

$$x_3 : Z_3 - C_3 = Yp_3 - c_{31} = \begin{pmatrix}0 & 0.2544 & 0 & 0.0128 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\\ 0\\ 0\\ 0.01\end{pmatrix} - 0.22265 = -0.22265$$

$$x_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix} 0 & 0.2544 & 0 & 0.0128 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0.01 \end{pmatrix} - 0.2276$$

$$= 0.2672 - 0.2276$$
$$= 0.0396$$

$$s_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix} 0 & 0.2544 & 0 & 0.0128 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} - 0 = 0.0128$$

$$s_2 : Z_2 - C_2 = Yp_2 - c_2 = \begin{pmatrix} 0 & 0.2544 & 0 & 0.0128 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - 0 = 0.2544$$

Therefore X_1 is entering variable.

Step 2

Determine leaving variable, x_r, with associated vector p_r

Compute $x_B = B^{-1}b$ (current RHS)

Compute current constraint coefficients of entering variable

$$\alpha^j = B^{-1}P_j$$

x_r is associated with

$$\theta = \underset{K}{Min}\{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\}$$

$$x_B = \begin{pmatrix} 7.4904 \\ 3.7452 \\ 4.1197 \\ 1.4981 \\ 0.1723 \end{pmatrix} \qquad \alpha^1 = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -0.02 & 0 & 0.01 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0.01 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0.01 \end{pmatrix}$$

$$\theta = Min\{7.4904, -, 4.117, -, 17.23\}$$
$$= 4.117$$

Therefore S_3 leaves the basis.

Step 3

Determine new B^{-1}

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.02 & 0.01 & 0.01 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -0.01 & -0.01 & 0 & 1 \end{pmatrix}$$

Solution after three iterations:

$$x_B = B^{-1} b$$

$$= \begin{pmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -0.01 & -0.01 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3745 \\ 0.7490 \\ 1.1236 \\ 1.4981 \\ 0.0712 \end{pmatrix}$$

Go to step 1

FOUTH ITERATION

Step 1

Determine entering variable, x_j , with associated vector p_j .

Compute $Y = c_B B^{-1}$

$$Y = \begin{pmatrix}0 & 0.2544 & 0.2375 & 0.2672 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & -1 & -1 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & -1 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & -0.01 & -0.01 & 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0.0169 & 0.2375 & 0.2503 & 0\end{pmatrix}$$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables (x_3, x_4, s_4, s_2, s_3)

$$x_3 : Z_3 - C_3 = Yp_3 - c_{31} = \begin{pmatrix}0 & 0.0169 & 0.2375 & 0.2503 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\\ 0\\ 0\\ 0.01\end{pmatrix} - 0.22265 = -0.22265$$

$$x_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix} 0 & 0.0169 & 0.2375 & 0.2503 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0.01 \end{pmatrix} - 0.2276$$

$$= 0.2771$$

$$s_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix} 0 & 0.0169 & 0.2375 & 0.2503 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} - 0 = 0.02503$$

$$s_2 : Z_2 - C_2 = Yp_2 - c_2 = \begin{pmatrix} 0 & 0.0169 & 0.2375 & 0.2503 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - 0 = 0.0169$$

$$s_2 : Z_2 - C_2 = Yp_2 - c_2 = \begin{pmatrix} 0 & 0.0169 & 0.2375 & 0.2503 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - 0 = 0.0169$$

Therefore X_3 is enters the basis.

Step 2

Determine leaving variable, x_r, with associated vector p_r

Compute $x_B = B^{-1}b$ (current RHS)

Compute current constraint coefficients of entering variable

$$\alpha^j = B^{-1}P_j$$

x_r is associated with

$$\theta = \underset{K}{Min}\{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\}$$

$$x_B = \begin{pmatrix} 0.3745 \\ 0.7490 \\ 1.1236 \\ 1.4981 \\ 0.0712 \end{pmatrix} \qquad \alpha^3 = \begin{pmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -0.01 & -0.01 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0.01 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0.01 \end{pmatrix}$$

$$\theta = Min\{0.3745, -, -, -, 7.12\}$$
$$= 0.3745$$

Therefore S_1 leaves the basis.

Step 3

Determine new B^{-1}

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.01 & 0.02 & 0.01 & 0.01 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.01 & -0.01 & 0 & 0 & 1 \end{pmatrix}$$

Solution after four iterations:

$$x_B = B^{-1}b$$

$$= \begin{pmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.01 & -0.01 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3.7452 \\ 2.24712 \\ 1.8726 \\ 1.49808 \\ 0.112356 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3745 \\ 0.7490 \\ 1.1236 \\ 1.4981 \\ 0.0674 \end{pmatrix}$$

Go to step 1

FIFTH ITERATION

Step 1

Determine entering variable, x_j, with associated vector p_j.

Compute $Y = c_B B^{-1}$

$$Y = \begin{pmatrix}0.22265 & 0.2544 & 0.2375 & 0.2672 & 0..\end{pmatrix}\begin{pmatrix}1 & 0 & -1 & -1 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & -1 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ -0.01 & -0.01 & 0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}0.2226 & 0.0169 & 0.0149 & 0.0277 & 0\end{pmatrix}$$

Compute $Z_j - c_j = Yp_j - c_j$ for all non-basic variables (x_4, s_4, s_2, s_3, s_1)

$$x_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix}0.2226 & 0.0169 & 0.0149 & 0.0277 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\\ 1\\ 1\\ 0.01\end{pmatrix} - 0.2276$$

$$= 0.2821 - 0.2276$$
$$= 0.0545$$

$$s_4 : Z_4 - C_4 = Yp_4 - c_4 = \begin{pmatrix} 0.2226 & 0.0169 & 0.0149 & 0.0277 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} - 0 = 0.0277$$

$$s_2 : Z_2 - C_2 = Yp_2 - c_2 = \begin{pmatrix} 0.2226 & 0.0169 & 0.0149 & 0.0277 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - 0 = 0.0169$$

$$s_3 : Z_3 - C_3 = Yp_3 - c_3 = \begin{pmatrix} 0.2226 & 0.0169 & 0.0149 & 0.0277 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - 0 = 0.0149$$

$$s_1 : Z_1 - C_1 = Yp_1 - c_1 = \begin{pmatrix} 0.2226 & 0.0169 & 0.0149 & 0.0277 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} - 0 = 0.2226$$

No negatives. Therefore, stop.

Optimal Solution:

$x_3^* = 0.3745$
$x_2^* = 0.7490$
$x_1^* = 1.1236$
$x_5^* = 1.4981$
$s_5^* = 0.0674$

$$Z^* = c_B x_B = \begin{pmatrix} 0.22265 & 0.2544 & 0.2375 & 0.2672 & 0 \end{pmatrix} \begin{pmatrix} 0.3745 \\ 0.7490 \\ 1.1236 \\ 1.4981 \\ 0.0674 \end{pmatrix} = 0.9410751$$

4.3.1 ALTERNATIVE

The function revised in MATLAB was then used to solve LPP using revised simplex method. It uses big M method to solve an LPP when there are <=, >= or = constraints present.

Input:

c : The cost vector or the (row) vector containing co-efficient of decision variables in the objective function. It is required parameter.

b : The (row) vector containing right hand side constant of the constraints. It is a required parameter.

a : The coefficient matrix of the left hand side of the constraints. it is a required parameter.

inq : A (row) vector indicating the type of constraints as 1 for >=, 0 for = and -1 for <= constraints. If **inq** is not supplied then it is by default taken that all constraints are of <= type. It is an optional parameter.

minimize : This parameter indicates whether the objective function is to be minimized. minimized = 1 indicates a minimization problem and minimization = 0 stands for a maximization problem. By default it is taken as 0. It is an optional parameter.

Therefore, in the LPP above, the problem can be written as

```
c=[0.2375   0.2544   0.22265   0.2276   0.2672];
b=[3.7452   2.24712   1.8726   1.49808   0.112356];
a=[1 1 1 1 1; 0 1 0 1 1; 1 1 0 1 0; 0 0 0 1 1; 0.01 0.02 0.01 0.01 0.01];
inq=[-1 -1 -1 -1 -1];
```

After supplying these inputs in MATLAB call revised(c, b, a, inq, 0).

The following tablaue and optimal solution were obtained:

.......The initial tablaue................

z	6	7	8	9	10	
1.0000	0	0	0	0	0	0
0	1.0000	0	0	0	0	3.7452
0	0	1.0000	0	0	0	2.2471
0	0	0	1.0000	0	0	1.8726
0	0	0	0	1.0000	0	1.4981
0	0	0	0	0	1.0000	0.1124

.............The 1th tablaue................

z 6 7 8 9 10

1.0000	0	0	0	0	0	0	-0.2672
0	1.0000	0	0	0	0	3.7452	1.0000
0	0	1.0000	0	0	0	2.2471	1.0000
0	0	0	1.0000	0	0	1.8726	0
0	0	0	0	1.0000	0	1.4981	1.0000
0	0	0	0	0	1.0000	0.1124	0.0100

.............The 2th tablaue................

z 6 7 8 5 10

1.0000	0	0	0	0.2672	0	0.4003	-0.2544
0	1.0000	0	0	-1.0000	0	2.2471	1.0000
0	0	1.0000	0	-1.0000	0	0.7490	1.0000
0	0	0	1.0000	0	0	1.8726	1.0000
0	0	0	0	1.0000	0	1.4981	0
0	0	0	0	-0.0100	1.0000	0.0974	0.0200

.............The 3th tablaue................

z 6 2 8 5 10

1.0000	0	0.2544	0	0.0128	0	0.5908	-0.2375
0	1.0000	-1.0000	0	0	0	1.4981	1.0000
0	0	1.0000	0	-1.0000	0	0.7490	0
0	0	-1.0000	1.0000	1.0000	0	1.1236	1.0000
0	0	0	0	1.0000	0	1.4981	0
0	0	-0.0200	0	0.0100	1.0000	0.0824	0.0100

.............The 4th tablaue................

z 6 2 1 5 10

1.0000	0	0.0169	0.2375	0.2503	0	0.8577	-0.2226
0	1.0000	0	-1.0000	-1.0000	0	0.3745	1.0000
0	0	1.0000	0	-1.0000	0	0.7490	0
0	0	-1.0000	1.0000	1.0000	0	1.1236	0
0	0	0	0	1.0000	0	1.4981	0
0	0	-0.0100	-0.0100	0	1.0000	0.0712	0.0100

............The 5th tablaue................

z 3 2 1 5 10

1.0000	0.2226	0.0169	0.0149	0.0276	0	0.9411	0.0149
0	1.0000	0	-1.0000	-1.0000	0	0.3745	-1.0000
0	0	1.0000	0	-1.0000	0	0.7490	0
0	0	-1.0000	1.0000	1.0000	0	1.1236	1.0000
0	0	0	0	1.0000	0	1.4981	0
0	-0.0100	-0.0100	0	0.0100	1.0000	0.0674	0

Reqiured optimization has been achieved!

The optimum objective function value= 0.9410751 (in millions GH₵).

The optimum solution is:

x_1 = 1.123560 (in millions)

x_2 =0. 7490400

x_3=0. 3745200

x_4 = 0

x_5 = 1.498080

However, the objective function in the LPP was having a constant term of 0.0263304 which was then added to the optimum objective function value of 0.9410751 to obtain an Optimal Profit of GH₵0.9674055 (in millions). ie. the Optimal Profit of Stanbic Bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and Asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012 was **GH₵967,405.5**

Therefore, for the bank to achieve the Optimal Profit of **GH₵967,405.5,** Revolving Term Loans x_1 must be allocated an amount of GH₵1,123,560; Fixed Term Loans x_2 an amount of GH₵749,040; Home Loans x_3 an amount of GH₵374,520; Personal VAF x_4 an amount of GH₵0.00; Vehicle and Asset Finance x_5 an amount of GH₵1,498,080.

4.4 SENSIVITY ANALYSIS

In the linear programming model of the data of Stanbic Bank, Tamale we may want to see how the answer changes if the problem is changed. In every case, the results assume that only one thing about the problem changes. That is, in sensitivity analysis you evaluate what happens when only one parameter of the problem changes.

Now the LP was

$$\text{Max} \quad Z = 0.2375x_5 + 0.2544x_2 + 0.22265x_3 + 0.2276x_4 + 0.2672x_5 + 0.0263304$$

$$\text{s.t} \quad x_1 + x_2 + x_3 + x_4 + x_5 \le 3.7452$$

$$x_2 + x_4 + x_5 \le 2.24712$$

$$x_1 + x_2 + x_4 \le 1.8726$$

$$x_4 + x_5 \le 1.49808$$

$$0.01x_1 + 0.02x_2 + 0.01x_3 + 0.01x_4 + 0.01x_5 \le 0.112356$$

The solution to this problem was:

x_1 = 1.123560 (in millions)

x_2 =0. 7490400

x_3=0. 3745200

x_4 = 0

x_5 = 1.498080

4.4.1 Changing Objective Function

Suppose in the solution of the LPP above, we wish to solve another problem with the same constraints but a slightly different objective function. When you change the objective function it turns out that there are two cases to consider. The first case is the change in a non-basic variable (a variable that takes on the value zero in the solution). In the LPP above, the relevant non-basic variables are(x_1, x_2, x_3, x_4, s_4). What happens to your solution if the coefficient of a non-basic variable decreases? For example, suppose that the coefficient of x_1 in the objective function above was reduced from 0.2375 to 0.11 so that the objective function is :

Max $\quad Z = 0.11x_5 + 0.2544x_2 + 0.22265x_3 + 0.2276x_4 + 0.2672x_5 + 0.0263304$

What has happened is this: You have taken a variable that you didn't want to use in the first place (i.e you set $x_1 = 0$) and then made it less profitable (lowered its coefficient in the objective function). You are still not going to use it. The solution does not change.

Observation: If you lower the objective function coefficient of a non-basic variable, then the solution does not change. What if you raise the coefficient? Intuitively, raising it just a little bit should not matter, but raising the coefficient a lot might induce you to change the value of x_1 in a way that makes $x_1 > 0$. So, for a non-basic variable, you should expect a solution to continue to be valid for a range of values for coefficients of non-basic variables. The range should include all lower values for the coefficient and some higher values. If the coefficient increases enough and putting the variable into the basis is feasible, then the solution changes.

What happens to your solution if the coefficient of a basic variable (like x_5 in the LPP) decreases? The change makes the variable contribute less to profit. You should expect that a sufficiently large reduction makes you want to change your solution. For example, if the coefficient of x_5 in the objective function in the example was 0.1411 instead of 0.2672 (so that the objective was Max $Z = 0.2375x_5 + 0.2544x_2 + 0.22265x_3 + 0.2276x_4 + 0.1411x_5 + 0.0263304$), will change the solution since the reduction in the coefficient of x_5 is large. On the other hand, a small reduction in x_5 objective function coefficient would typically not cause you to change your solution.

So, intuitively, there should be a range of values of the coefficient of the objective function (a range that includes the original value) in which the solution of the problem does not change. Outside of this range, the solution will change. The value of the problem always changes when you change the coefficient of a basic variable.

4.4.2 Changing a Right-Hand Side (RHS) Constant of a constraint

When you changed the amount of resource in a non-binding constraint, i.e small increases will never change your solution and small decreases will also not change anything. However, if you decreased the amount of resource enough to make the constraint binding, your solution could change. But, changes in the right-hand side of binding constraints always change the solution.

4.4.3 Adding a Constraint

If you add a constraint to a problem, two things can happen. Your original solution satisfies the constraint or it doesn't. If it does, then you are finished. If you had a solution before and the solution is still feasible for the new problem, then you must still have a solution. If the original solution does not satisfy the new constraint, then possibly the new problem is infeasible. If not, then there is another solution.

CHAPTER FIVE

5.0 SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 SUMMARY

The problem of profit optimization of Stanbic Bank (SB), Tamale was examined in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and Asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012.

In order to determine the optimal profit of SB, the data were modeled using LP, the Revised Simplex Method codes in MATLAB was then used to solve the LP.

5.2 CONCLUSION

Based on the results of the data analysis, it can be concluded that Profit Optimization in SB, Tamale can be achieved through LP modeling of the problem and using the Revised Simplex Method to solve the LP. It was realized that the Optimal Profit of SB, Tamale in areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012 was **GH₵967,405.5**

Therefore, for the bank to achieve the Optimal Profit of **GH₵967,405.5,** Revolving Term Loans x_1 must be allocated an amount of GH₵1,123,560; Fixed Term Loans x_2 an amount of

GH₵749,040; Home Loans x_3 an amount of GH₵374,520; Personal VAF x_4 an amount of GH₵0.00; Vehicle and Asset Finance x_5 an amount of GH₵1,498,080.

It was also observed that if Stanbic Bank, Tamale does not allocate any amount to Personal VAF, the bank can still achieve the Optimal Profit of **GH₵967,405.50**

Finally, by comparing the amounts allocated to each product, the bank should allocate more amounts to Vehicle and Asset Finance, Revolving Term Loans and Fixed Term Loans if the bank desire to achieve the Optimal profit of GH**₵967,405.50**

5.3 RECOMMENDATION

It is established in the course of this research work that among other products, Vehicle and Asset Finance, Revolving Term Loans and Fixed Term Loans assure more profit in Stanbic Bank, Tamale. I thereby recommend that the management of SB, Tamale should allocate more amounts to Vehicle and Asset Finance, Revolving Term Loans and Fixed Term Loans than other Loan products in the bank. In the same vein, critical examination should be carried out on other Loan products on their contribution to the ~~growth or~~ success of the bank, if their profit margin is very low, then the bank can stop given such Loan products to customers. Finally, any product having an adverse effect or contributing losses to the profit margin of the bank can be stopped and the bank invest more on the Vehicle and Asset Finance, Revolving Term Loans and Fixed Term Loans to generate more profit.

Also the following recommendations are made for other banks and future researchers in the same field.

5.3.1 Recommendation for Banks

- This study will serve as a reference guide to the banking industry, how the problem of profit optimization in the bank can be model as a Linear programming problem.
- The study is also very essential for equipping bankers how to determine the optimal profit using Revised Simplex Algorithm.
- The material is also very suitable to be used for in-service training for newly recruited bank officers.

Finally, future researchers into the same field should carter for more products or services that the bank renders so that larger sample can be obtained for modeling the profit optimization of the bank.

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APPENDIX

LP: Linear Programming

LPP: Linear Programming Problem

SB: Stanbic bank

OP: Optimal profit

RSM: Revised Simplex Method

RSA: Revised Simplex Algorithm

RS: Revised Simplex

OR: Operations Research

RHS: Right Hand Side

The table below shows the data of Stanbic Bank, Tamale in the areas of interest from loans such as Revolving Term Loans, Fixed Term Loans, Home Loans, Personal VAF, Vehicle and Asset Finance as well as interest derived from Current Accounts, ATM withdrawals, Cheque Books and Counter Cheques of at least 90 customers for the period of six (6) months from November, 2011 to April, 2012:

Table 4.1 ***Data of Stanbic bank for the period of six (6) months (Nov. 2011 to April, 2012) of variable products (i.e Loans)***

NUMBER OF CUSTOMERS	TYPE OF PRODUCT	INTEREST RATE	PROBABILTY OF BAD DEBT	PROBABILTY OF NO BAD DEBT
150	Revolving term Loans	22%+3%=25%	1%	=1-0.01 =0.99
140	Fixed term Loans	22%+6%=28%	2%	=1-0.02 =0.98
150	Home Loans	22%+1.5%=23.5%	1%	=1-0.01 =0.99
95	Personal VAF	22%+2%=24%	1%	=1-0.02 =0.99
90	Vehicle and Asset Finance	22%+6%=28%	1%	=1-0.01 =0.99
180	Current Account	GH₵5 per month	2%	=1-0.02 =0.98
54000	ATM withdrawal	GH₵0.44 per withdrawal	2%	=1-0.02 =0.98
180	Cheque books	GH₵6 per cheque book	3%	=1-0.03 =0.97
372	Counter cheque	GH₵3 per counter cheque	0%	1

The bank is to allocate a total fund of GH₵3,745,200 to the various loan products. The bank is faced with following constraints:

1. Amounts allocated to the various loan products should not more than the total funds.
2. Allocate not more than 60% of the total funds to Fixed term loans, Personal VAF; and Vehicle and Asset loans.
3. Revolving term loans, Fixed term loans and Personal VAF should not be more than 50% of the total funds.
4. Allocate not more than 40% of the total funds to Personal VAF and Vehicle and Asset loans.
5. The overall bad debts on the Revolving term loans, Fixed term loans, Home loans, Personal VAF; and Vehicle and Asset Loans should not exceed 0.03 of the total funds.

Table 4.2 Data of Stanbic bank for the period of six (6) months (Nov. 2011 to April, 2012) of constant products.

Number of Customers	Type of Products	Interest Rate	Amount at the End of the Six (6) Months	Probability of Bad debt	Probability of No Bad Debt	Amount at the end of the Six (6) Months
180	Current Account	GH₵5	GH₵5 $\times$ 180 =GH₵900	2%	0.98	GH₵900 $\times$ 0.98 = GH₵882
54000	ATM Withdrawals	GH₵0.44	GH₵0.44 $\times$ 54000 =GH₵23,760	2%	0.98	GH₵23,760 $\times$ 0.98 =GH₵23,284.8
180	Cheque Books	GH₵6	GH₵6 $\times$ 180 = GH₵1,080	3%	0.97	GH₵1,080 $\times$ 0.97 =GH₵1,047.6
372	Counter Cheques	GH₵3	GH₵3 $\times$ 372 =GH₵1,116	0%	1	GH₵1,116
						Total =GH₵26,330.4

REVISED SIMPLEX METHOD CODE IN MATLAB

```
function revised(c,b,a,inq,minimize)
if nargin<3||nargin>5
    fprintf('\nError:Number of input arguments are
inappropriate!\n');
else
    n=length(c);m=length(b);j=max(abs(c));
    if nargin<4
        minimize=0;
        inq=-ones(m,1);
    elseif nargin<5
        minimize=0;
    end
    if ~isequal(size(a),[m,n])||m~=length(inq)
        fprintf('\nError: Dimension mismatch!\n');
    else
        if minimize==1
            c=-c;
        end
        count=n;nbv=1:n;bv=zeros(1,m);av=zeros(1,m);
        for i=1:m
            if b(i)<0
                a(i,:)=-a(i,:);
                b(i)=-b(i);
            end
            if inq(i)<0
                count=count+1;
                c(count)=0;
                a(i,count)=1;
                bv(i)=count;
            elseif inq(i)==0
                count=count+1;
                c(count)=-10*j;
                a(i,count)=1;
                bv(i)=count;
                av(i)=count;
            else
                count=count+1;
                c(count)=0;
                a(i,count)=-1;
                nbv=[nbv count];
                count=count+1;
                c(count)=-10*j;
                a(i,count)=1;
                av(i)=count;
```

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```
                bv(i)=count;
            end
        end
        A=[-c;a];
        B_inv=eye(m+1,m+1);
        B_inv(1,2:m+1)=c(bv);
        x_b=B_inv*[0; b'];
fprintf('\n.............The initial tablaue................\n')
fprintf('\t z');disp(bv);
fprintf('-----------------------------------------------------\n')
        disp([B_inv x_b])
        flag=0;count=0;of_curr=0;
        while(flag~=1)
            [s,t]=min(B_inv(1,:)*A(:,nbv));
            y=B_inv*A(:,nbv(t));count=count+1;
            if(any(y(2:m+1)>0))
 fprintf('\n........The %dth tablaue................\n',count)
 fprintf('\t z');disp(bv);
 fprintf('-----------------------------------------------------\n')
                disp([B_inv x_b y])
                if count>1 && of_curr==x_b(1)
                    flag=1;
                    if minimize==1
                        x_b(1)=-x_b(1);
                    end
fprintf('\nThe given problem has degeneracy!\n');
fprintf('\nThe current objective function value=%d.\n',x_b(1));
fprintf('\nThe current solution is:\n');
                    for i=1:n
                        found=0;
                        for j=1:m
                            if bv(j)==i
fprintf('x%u = %d\n',i,x_b(1+j));found=1;
                            end
                        end
                        if found==0
fprintf('x%u = %d\n',i,0);
                        end
                    end
                else
                    of_curr=x_b(1);
                    if(s>=0)
                        flag=1;
                        for i=1:length(av)
                            for j=1:m
                                if av(i)==bv(j)
```

```
    fprintf('\nThe given LPP is infeasible!\n');
                                return
                            end
                        end
                    end
                    if minimize==1
                        x_b(1)=-x_b(1);
                    end
  fprintf('\nReqiured optimization has been achieved!\n');
fprintf('\nThe optimum objective function value=%d.\n',x_b(1));
                    fprintf('\nThe optimum solution is:\n');
                    for i=1:n
                        found=0;
                        for j=1:m
                            if bv(j)==i
   fprintf('x%u = %d\n',i,x_b(1+j));found=1;
                            end
                        end
                        if found==0
                            fprintf('x%u = %d\n',i,0);
                        end
                    end
                    if (s==0 && any(y(2:m+1)>0))
    fprintf('\nThe given problem has alternate optima!\n');
                    end
                else
                    u=10*j;
                    for i=2:m+1
                        if y(i)>0
                            if (x_b(i)/y(i))<u
                                u=(x_b(i)/y(i));
                                v=i-1;
                            end
                        end
                    end
                    temp=bv(v);bv(v)=nbv(t);
                    nbv(t)=temp;
                    E=eye(m+1,m+1);
                    E(:,1+v)=-y/y(1+v);
                    E(1+v,1+v)=1/y(1+v);
                    B_inv=E*B_inv;
                    x_b=B_inv*[0; b'];
                end
            end
        else
```

```
   fprintf('\nThe given problem has unbounded solution\n')
                  return
              end
          end
      end
end
```