FEASIBILITY OF ESTIMATING SOIL TEMPERATURE USING KALMAN FILTERING METHOD

By

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Declaration

t ereby declare that this submission is my own vork towards the award of of Philosophy and that, to the best of my knowledge, it contains no materials previously published by person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in tbc text.

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Abstract

The feasibility of improving soil temperature estimation using the Kalman filter data assimilation scheme was investigated. The formulation for this algorithm was based on the discretization of the governing partial differential equation for transfer of temperature through the soil. The data assimilation scheme was designed to incorporate the knowledge of the uncertainties in both the model and the measurement, hereby producing a better estimate (prediction) for the state. Model uncertainty was also estimated by quantifying the model drift from observations when the model is initialized using the observed values. Experimental results using the Kalman filter were found to be more accurate than the other estimates. Also, the Kalman filter was found to be sensitive to the process noise and the measurement noise.

Dedication

I dedicate this research work to the Almighty God for his mercy and blessing over me. To my supervisor E. Owusu Ansah for helping me in this endeavor. Moreover a friend, I will always remember for his support, time and energy, by name Gaston E. Awashie. God richly bless you.

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Table of Contents

Chapter 1

Introduction

This chapter discusses the background of the study and the problem statement relating to the thesis. This section discusses the capabilities of the Kalman filtering scheme to retrieve the soil temperature and the effect of varying observation frequency, observation noise, process noise and initial error covariance as the objective of the study.

Finally, justification and the outline of the thesis are also discussed.

Soil temperature is a very important variable in land processes which greatly influences the water and energy contents of the land-atmosphere system. Soil moisture is a function of soil temperature, so the soil temperature mostly influences the transfer of soil moister in different soil layers. The temperature profile of the soil can be obtained through observation or modeling. Accurate prediction of soil temperature requires a realistic understanding of the soil thermal properties.

Data assimilation is a methodology that combines the observation and the prediction of underlying models to produces best estimates of the state. Different algorithms have been developed to achieve this purpose, one of which is the Kalman filter proposed by (Kalman,

1960). The Kalman filter is a variance minimization algorithm which produces best estimates of the-State when the state-space equations are linear.

Background of Study

Scientific estimations are mostly obtained by observations and by using available models. Estimates from these methods come with a certain degree of error. In the absence of one of these

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methods, the other is taken to be the best estimate. When both estimates are available, then it is mostly not clear which one of the pack is the best estimate.

Data assimilation methodology was introduced to make use of model prediction and observations to obtain an estimate for the state and the error in this estimate smaller than there is in the observed data and the models. The algorithms for data assimilation are developed to combine measured data and dynamical model to obtain best estimates of the state and their uncertainties.

The Kalman filter (Kalman, 1960) is a widely used sequential data assimilation algorithm and is optimal in estimating the state. The method is assumed to work only when the process and measurement equations are linear. The Kalman filter is a variance minimization algorithm, thus it uses the method of least squares to minimize the trace of the covariance matrix. The Kalman filter is used in this thesis to obtain best estimates of the temperature distribution in the soil.

The temperature distribution in the soil is governed by a partial differential equation (PDE) with boundary conditions. The finite volume method which is a numerical scheme for solving boundary value-problems was employed to discretize the PDE to obtain systems of equation and also to solve the PDE.

Problem Statement

Soil temperature plays an important role in land processes. Accurate prediction of soil temperature requires a realistic understanding of the soil thermal properties. The dynamics of the soil

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temperature is assumed to obey the heat diffusion equation. This model is used to estimate the dynamics of temperature distribution as well as the profile in the soil.

The thermal conductivity is based on heat transfer conduction. Change in the soil temperature is mostly impacted by the convection and conduction heat transfer. The thermal diffusivity is a function of the soil texture and the soil moisture content. This parameter is assumed to be constant along the vertical columns of the soil making the model linear.

In this study, we employ the Kalman filter which is a sequential data assimilation scheme introduced by Kalman (1960) for the estimation problem. This algorithm accurately estimates the states and compared to other assimilation methods, it produces an optimal solution when the state-space equations are linear.

1.3 Objectives

The objectives of this study is

- to evaluatethe capabilities of the Kalman filtering scheme to retrieve soil temperature
- to simulate the soil temperature profile using data in calibration with the underlying mathematical model -to ascertain the effect of varying observation frequency, observation noise, process noise and initial error covariance.

Methodology

The data assimilation methodology is employed to aid achieve the set objectives of the study. The Kalman filter algorithm is used as the data assimilation scheme in estimating the soil temperature distribution and profile.

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The heat diffusion model which describes the dynamics of the soil temperature as well as the soil moisture is used. In the discretization and solving the underlying boundary value problem, we employ the finite volume method.

All algorithms employed in this study were implemented using MatLab and used for various experiments.

1.5 Justification

Temperature and the accuracy of its prediction play a major role in determining the capacity of these models for predictions. A failure to properly predict the temperature state in the model can result in an inadequate characterization. A model with a tendency to over-predict or underpredict the soil temperature can result in excessive evaporation and subsequently affect the atmospheric dynamics.

Given the importance of adequate representation of soil temperature, it has become necessary to use methodologes such as data-GGRiÎÅåon which constrain the error growth in predicting the tempyature state resulting from both model estimates and observations.

1.6 Outline of the Thesis

The thesis is organized in five chapters. Chapter 1 is the introductory chapter which discusses the background of the study, the problem statement, objectives, methodology and justification of the study. The current section is also part of the first chapter and it contains the outline of the thesis.

Chapter 2 discusses the literature review and framework of studies relevant to this study. The methodology employed in the study is discussed in chapter 3. Presented in chapter 4 are the results from numerical simulations and the chapter also contains the discussion of results.

Finally summary of findings, conclusion, recommendation and appropriate references are presented in chapter 5.

Chapter 2

Literature Review

2.1 Introduction

This section deals with the review of literature from published and unpublished books, internet and any other relevant information others have said about the subject of study: theories and researches that address the issues. The chapter discusses the movement of heat through the soil which takes three classical mechanisms.

In addition, estimating soil temperature profile and data assimilation in estimating soil temperature is discussed.

The thermal behavior of the soil is important in many fields: whether soil is considered as a growth medium, an insulating blanket, a sink or source of heat or a component of the hydrological cycie, its temperature and thermal properties have immediate relevance. The flux of hcat into and out of the soil is a significant component of the surface energy budget, which should enter into calculations of evaporation using methods such as that of Penman (1948) ,particularly in temperate climates where there is seasonal storage of heat in the soil profile (Edwards & Rodda, 1970; Thorn & Oliver, 1977; Simmers, 1977).

This soil heat flux is usually estimated by measuring temperature gradients in soil heat flux plates, or by intensive temperature measurements, but both approaches involve heavy investment in equipment and field work. Soil temperature is of critical importance to agriculture, particularly in the uplands where the growing season is short. Russell (1977) refers to a number

of experimental studies showing the control exerted by root temperature on growth, and if, as seems likely, the growing season for upland pasture grasses depends in part on soil temperature, then the prediction of soil temperatures in the uplands takes on more than academic interest.

2.2 Movement of Heat in the Soil

Soil is one of the most complexes of inert materials, both in physical structure and in variety of constituents. Movement of heat through the soil can take place by any of the three classical mechanisms, conduction, convection and radiation. There is yet another possibility in the transport of latent heat by water in the vapor phase.

According to Parlange et al (1998), heat transport has enjoyed extensive focus in soil physics and hydrology and yet, there has not been a satisfactory measure in the field with theory until recently. In their work, they reviewed a new theoretical development which explained field observations. The mechanism seems to have important consequences for transport in the soil.

The use of air temperature as a driving mechanism for a soil temperature model was proposed by Hasfurther and Buypan (1974). In their work, they used the Fourier transform model to relate air temperature andÃðil tempera e. The research made use of a 25mm depth soil temperature data which_was available at the time of the study. Hasfurther and Bur-man noted that many other factors enter into the mechanism of soil heat transfer, particularly the soil temperature which depends partly on the warming of the ground surface by solar radiations and air movement bringing advective energy. The instantaneous soil heat flux at the surface was found to be strongly and linearly related to the net radiation, (Fuchs and Hadas, 1972).

Veronez et al. (2010) presented an alternative method for the extrapolation of land surface temperature through the use of Artificial Neural Network. The positional variables (UTM coordinates and altitude), temperature and air relative humidity was considered in their study.

2.3 Estimating Soil Temperature Profile

A methodology for the estimation of daily soil temperature at continental scales using daily air temperature and precipitation data for bare ground was developed by Zheng et al. (1993). In their work, they demonstrated how the soil temperature affected annual soil respiration. Their work revealed that changes in soil temperature under snow cover were smaller than those without snow cover. Soil temperature under vegetation cover was also simulated using an I Iday running average of daily mean air temperature at a depth of 1 Ocm using linear regression.

Soil temperature is also an important parameter in energy balance applications. Holmes et al. (2008) proposed an approach for modeling the surface soil temperature profile from a single observation depth—They used-tWõTIÄÚdata sets in modelling near-surface soil temperature profile in a bare soil and showed that the commonly used solutions to the heat flow equations perform well when applied at deeper soil layers, but results come with large errors when applied to near surface layers, where there are extreme temperature variations.

Land surface temperature is an important factor in global studies, estimating radiation budgets in heat balance studies and as a control for cliamte models, (Mallick et al., 2008). In their study, they found that the use of minimum noise fraction components classifies the uncertainties. The Landsat-7 ETM+ satellite data over Delhi area was used in the study to obtain estimate of the surface temperature. The study revealed a strong correlation between surface temperature with normalized difference vegetation index over different land cover classes.

Methods for the estimation of soil heat flux from soil temperature require more information on the distribution of temperature within the soil profile than can be obtained from measurements at one depth, (Hanks & Jacobs, 1971; Kimball & Jackson, 1975). It was proposed that digital model be used to build up complete temperature profiles with measured soil temperatures and meteorolgical data. Their study revealed that the initial error covariance has a significant influence on the performance of the extended Kalman filter.

Gomez et al. (2007) in their study, presented a method for estimating the annual and monthly mean values of temperature and precipitation, taking in account elements from simple interpolation methods and complementing them with some characteristics of more sophisticated methods. Simple linear regression equations were also generated which associated temperature with altitude ofweather stations-in-thvstwdy region in order to determine the temperature.

The sensitivity of the soil temperature in a "force-restore" model was studied by Mihailovic et al. (1999) to the changes of soil heat flux, soil water evaporation and variations of deep soil temperature. They also discussed the impact of the deep soil temperature variations on partitioning the surface energy and land surface water. Mihailovic et al. proposed a new procedure for the calculation of the deep soil temperature in the deep soil layers. The method was found to be reliable enough especially for long-term integration.

2.4 Data Assimilation in Estimating Soil Temperature

Kumar and Kaleita (2003) used the extended Kalman filter which is a sequential data assimilation scheme to improve the soil temperature profile predictions in land surface models. The formulation was based on the diffusion equation of heat transfer discretized through the soil column.

The performance of the ensemble Kalman filter for land surface assimilation was assessed by Zhou et al. (2006). In their study, the ensemble Kalman filter was generally able to reproduce non-normal soil moisture behaviour, including the skewness. The ensemble filter was compared to the sequential importance re-sampling particle filter and it was found that the mean estimates from the ensemble Kalman filter were very close to those generated by the sequential importance re-sampling particle filter. The ensemble Kalman filter provided a good approximation for nonlinear, non-normal land surface problems, despite its dependence on the normality assumptions.

A one-dimensionálíand data a~~ssimilatio sch~~eme was developed based on ensemble Kalman filter by Huang et al. (2006). The scheme they developed is used to improve the estimation of soil temperature profile. Observations from four automatic weather stations were used to test and validate the scheme. Results from their simulation revealed that data assimilation improves the estimation of soil temperature profile.

Chapter 3

Methodology

3.1 Introduction

This chapter discusses the methodology employed in the study. The chapter discusses the model governing the heat transfer through the soil and a numerical solution to the temperature distribution model, considering the finite volume scheme used in discretizing and finding numerical solution to the heat transfer model. Secondly the Data assimilation types were discussed with focus on the Kalman filter which is a sequential data assimilation scheme.

3.2 The Soil Temperature Distribution Model

Differential equations are used to model, understand and predict systems that change with time.

Thus, many physical laws that have been successfully modeled in nature are expressed in the form of differential equations. The dynamics of the soil temperature considered in this study is modeled using the heat diffusion equation for a one-dimensional vertical column. The heat equation governing the temperature distribution in the absence of internal phase changes is a partial differential given by

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) \quad (3.1)$$

where p is the bulk density, c is the specific heat capacity, k is the thermal conductivity of the soil (W m—1K—l), T is the soil temperature (K) at a point in space within a mass of material, t represents time and z is the soil depth in meters (m). pc is referred to as the volumetric heat capacity mostly denoted by C, measured in (J m—3 K—l).

The temperature is assumed to have a constant value at every depth in the vertical column of the soil layer. This implies that the heat flux occurs only in the direction of the soil depth. The soil properties p, c and k mostly vary through time, chiefly through the transfer of water, but assuming these parameters to be spatially uniform through the soil layers then equation (3.1) becomes

(3.2)

ðt ðzk D)ôz

where D = k/ pc is the thermal diffusivity of the soil. The thermal diffusivity is a function of the soil texture and the soil moisture content. The heat equation in equation (3.2) is classified as a parabolic differential equation. A number of detailed set of mathematical techniques can be used to obtain the solutions for equation (3.2).

Among the several variable that are involved in the hydrological and climatological model studies, surface temperature and the accuracy of its prediction plays a crucial role in determining the predictive capability of such models. It influences the partitioning of incoming radiant energy into ground, sensible and latent heat fluxes. Outgoing long wave flux is a function of the surface temperature.

The heat equation is mostly solved as an initial-boundary value problem. The time and space domains are clearly described in this instance and conditions imposed on the system to be solved. For the equation in (3.2), exact solutions may be obtained. In most cases the exact solutions cannot be obtained and so we resort to numerical solution techniques.

3.3 Solutions of Soil Temperature Distribution Model

The transfer of heat has been of great interest within the engineering and scientific communities for a very long time. It is important to understand the dynamics of temperature distribution through the various mediums considered. The equation governing the dynamics of soil temperature is a partial differential equation assumed to obey the heat diffusion equation for onedimensional vertical column given in equation (3.2).

The analytic solution and numerical solution can be obtained for the governing model. The two solutions can be compared in instances where both solutions can be obtained. In most instances where the model becomes more complex, the analytic solution is virtually impossible to obtain. In such instances we can approximate the solution numerically to obtain the solution to the governing model.

The governing model considered in equation (3.2) is a simple model and its exact solution can be obtained using analytical methods as well as the numerical solution. We can then compare the two solutions obtained from the various methods.

3.4 Analytic Solution to Temperature Model

The behavior of mathematical models can be understood by finding solution to the models, or set of equations. These se solutions ~~can be~~ found using calculus, trigonometry, and other math techniques. The solution can tell absolutely how the model will behave under any circumstance. This type of solution obtained by using analysis is referred to as the analytic solution. But this tends to work only for simple models. For more complex models, it becomes virtually impossible to obtain the analytic solution.

We consider the model governing the temperature distribution through the soil column as in equation (3.2). This is a simple model whose analytic solution can be found. We start finding the solution the temperature distribution equation by the method of separation of variables. This method assumes that the solution (Temperature) can be expressed explicitly as a product of two different functions. Thus

$$T(z,t) = F(z)G(t) \tag{3.3}$$

From equation (3.3) we can obtain expressions for the first derivative with respect to time and the second derivative with respect to soil depth.

$$\frac{\partial T(z,t)}{\partial t} = F(z)\,\frac{\partial G(t)}{\partial t} \tag{3.4}$$

$$\frac{\partial^2 T(z,t)}{\partial z^2} \quad \frac{\partial^2 F(z)}{\partial z^2} \tag{3.5}$$

These expressions are substituted into equation (3.2) and it is assumed that the thermal diffusivity of the soil is constant, which gives the resulting equation

$$\frac{\partial G(t)}{\partial t} = D \cdot G(t)\frac{\partial^2 F(z)}{\partial z^2} \tag{3.6}$$

Rewriting equation (3.6), we have

1

ðG(t) D ð2F(z) (3.7)

G (t) at F(z) ðz2

The right side of the equation (3.7) is just a function of depth of the soil whereas the left hand side of the same equation is explicitly a function of time. The two sides of equation (3.7) are functions of different variables can only be equal if they evaluate to a constant. We set both sides of equation (3.7) to a constant, and rearranging gives the following set of equations

——— - RG(t) = 0 (3.8)

at

$$- \lambda \frac{F(z)}{D} = 0$$

ð2F(z)

(3.9) ðZ2

The solution to the temperature distribution model can be obtained by solving equations (3.8) and (3.9). In general, the linear combination of solutions is also a solution to this model. Equation (3.10) gives the general representation of the solution to the temperature model considered above.

n2Tt2 Dt

t) =an sin (3.10) n=l where

an f (z) sin d

3.3 Numerical Solution to Temperature Model

There are different schemes for finding the numerical solution to partial differential equations, examples of which are the finite difference method, the finite volume method and the finite element method. The sequence of—approximations gcnerated by the numerical algorithm is required to converge to the correct solution.

In this research, the finite volume method among many others is presented for discretizing and finding the numerical solution to the heat equation considered in equation (3.2). The finite volume method is an increasingly popular numerical method for approximating the solution of partial differential equations and it has an excellent numerical capability for capturing changes in conserved quantities such as mass, momentum, energy, among others.

3.3.1 Finite Volume Method (FVM)

In this section, we discuss the finite volume method as a numerical technique for obtaining a numerical solution to partial differential equation. The finite volume method transforms the heat equation from the form of a partial differential equation into a set of algebraic equations that can easily be solved. The finite volume method relies on integration. For a finite volume discretization, the computational domain for the heat equation is partitioned into a number of control volumes, with the value at the centre of the control volume considered to be a representative for the valve over the entire control volume. These cells are number 1 to N, for a one dimensional domain.

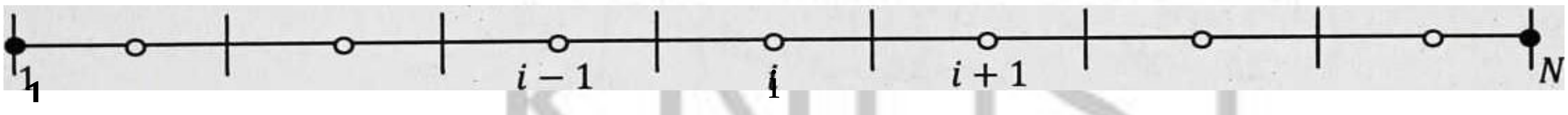


Figure 3.1: The discretization of one dimensional domain into finite volumes

The space is discretized into N equal size grid cells of size h = I/N. By integrating the original PDE over the control volume the equation is cast into a form that ensures conservation and the derivatives at the faces of the volume are approximated by finite difference equations.

Consider the heat transfer equation given in equation (3.1). Integrating both sides with respect to time and space, we have

$$\int_t^{t+\Delta t}\left(\int_{cv}\frac{\partial T}{\partial t}\,dz\right)dt = \int_t^{t+\Delta t}\left(\int_{cv}\frac{\partial}{\partial z}\left(D\frac{\partial T}{\partial z}\right)dz\right)dt \qquad (3.11)$$

Solving for the left hand side, we have

$$\int_t^{t+\Delta t}\left(\int_{cv}\frac{\partial T}{\partial t}\,dz\right)dt = \int_{cv}\int_t^{t+\Delta t}\frac{\partial T}{\partial t}\,dt\,dz$$

$$= \int_{cv}\int_t^{t+\Delta t}(dT)\,dz$$

$$= \int_{cv}\left(T(t+\Delta t) - T(t)\right)dz \qquad (3.12)$$

Thus from equation (3.11), we can then say that

[Tik+l - Tik]Az $\int_t^{t+\Delta t}\left(\left(D\frac{\partial T}{\partial z}\right)\right.$ =dt i− $-\left(D\frac{\partial T}{\partial z}\right)$ i+1/2 1/2 ðz

$\leq \theta \leq 1$

$\int_k^{k+1} T\,\partial t =$

t+At

$\Big($ Di+1/2 $\frac{T_{i+1}-T_i}{\Delta z}$ − Az Di_1/2 Az T)dt (3.13)

Using the theta method, for 0

(6Tk+1 + (1 — 0)Tk$)\Delta t$ (3.14)

When e = 0, the scheme becomes explicit scheme which is conditionally stable.

9 = 1, the scheme is an implicit scheme (unconditionally stable) and

=we solve using the Crank Nicolson scheme.

[Ti^{k+1} — Ti^k]Az

$= \theta$ Dk+l i+1/2 $\Big)\left(\frac{T_{i+1}^{k+1}-T_i^{k+1}}{\Delta z}\right)-\left(D_{i-1/2}^{k+1}\right)\left(\frac{T_i^{k+1}-T_{i-1}^{k+1}}{\Delta z}\right)\Big]$ At

$$+ (1-\theta)\left[\left(D^{k}_{i+1/2}\right)\left(\frac{T^{k}_{i+1}-T^{k}_{i}}{\Delta z}\right)-\left(D^{k}_{i-1/2}\right)\left(\frac{T^{k}_{i}-T^{k}_{i-1}}{\Delta z}\right)\right]\text{At} \quad (3.15)$$

The implicit schemes are considered to be unconditionally stable. Thus, they simply permit the use of larger time steps. Implicit schemes can tolerate a great deal of error, yet it does not allow this error to grow and they mostly involve solving systems of linear equations. Equation (3.15) tends implicit when e = 1 giving

$$[\text{Tk+l} - \text{Tik}]\text{Az} = \left[\left(D^{k+1}_{i+1/2}\right)\left(\frac{T^{k+1}_{i+1}-T^{k+1}_{i}}{\Delta z}\right)-\left(D^{k+1}_{i-1/2}\right)\left(\frac{T^{k+1}_{i}-T^{k+1}_{i-1}}{\Delta z}\right)\right]\text{At} \quad (3.16)$$

Let = (D X At) /(Az)2, since D is assumed to be constant in this research. Equation (3.16) then reduced to

$$\_ = [(T^{k+1}_{i+1} - \text{Tik+l}) - (\text{TLk+1} - \text{Tik-+ll})] \quad (3.17)$$

Rearranging equation (3.17) gives an algebraic system presented in equation (3.15) below.

$$\lambda T^{k+1}_{i-1} + (1 + 2\text{Ä})\text{Ti}^{k+1} - \quad - \text{Ti}^{k} \quad (3.18)$$

Demoñštrating the scheme in equation (3.18) for the discretized space domain with N discrete points, we have

RT{+I + (1 + 2Ä)T1k+1 - RT2k+1=

ATIk+1 + (1 + 2Ä)T2 - AT}+I=

RT2k+1 + (1 + 2Ä)T3k+1 - λT_4^{k+1}

$$\lambda T^{k+1}_{N-3} + (1+2\lambda)T^{k+1}_{N-2} - \lambda T^{k+1}_{N-1}$$

/17'{+_} + (1 + 2R)TÆ+I 1 _RTk+1═

This system can also be written in matrix notation and is presented below.

$$\begin{pmatrix}1+2\lambda \\ \lambda \\ 0 \\ \vdots \\ 0 \\ 0\end{pmatrix}$$

λ 1+2Ä — (3.19)

$$\left.\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\lambda & 0 & 0 \\ \ddots & \ddots & \vdots \\ \lambda & 1+2\lambda & -\lambda \\ 0 & \lambda & 1+2\lambda \end{matrix}\right) \begin{pmatrix} T_1^{k+1} \\ T_2^{k+1} \\ T_3^{k+1} \\ \vdots \\ T_{N-2}^{k+1} \\ T_{N-1}^{k+1} \end{pmatrix} = \begin{pmatrix} T_1^{k} \\ T_2^{k} \\ T_3^{k} \\ \vdots \\ T_{N-2}^{k} \\ T_{N-1}^{k} \end{pmatrix} - \begin{pmatrix} T_0^{k+1} \\ 0 \\ 0 \\ \vdots \\ 0 \\ -T_N^{k+1} \end{pmatrix}$$ R

The system is now represented as a system of linear algebraic equations of the form AT = b ,

where T = T

$$A = \begin{pmatrix} 1+2\lambda & -\lambda & 0 & 0 & 0 & 0 \\ \lambda & 1+2\lambda & -\lambda & 0 & 0 & 0 \\ 0 & \lambda & 1+2\lambda & -\lambda & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda & 1+2\lambda & -\lambda \\ 0 & 0 & 0 & 0 & \lambda & 1+2\lambda \end{pmatrix}$$

and

$$b = \begin{pmatrix} T_1^{k} \\ T_2^{k} \\ T_3^{k} \\ \vdots \\ T_{N-2}^{k} \\ T_{N-1}^{k} \end{pmatrix} - \begin{pmatrix} T_0^{k+1} \\ 0 \\ 0 \\ \vdots \\ 0 \\ -T_N^{k+1} \end{pmatrix}$$

The linear algebraic equations are then solved to obtain the solution to the partial differential equation (in our case the heat equation) at various times. More interestingly, the system in equation (3.19) can be expressed as a state space equation where the state is the temperature. The state equation can be represented using the autoregressive model given as

Tk+l = FTI, + (.01, (3.20)

where F is just the same as the inverse of the coefficient matrix, A in the system above, thus

F = A—l. The temperature, Tk = b and is the process noise assumed to be white Gaussian.

3.4 Data Assimilation

Information used to estimate the state of a system is generally obtained from observations and models. Observations are mostly sparse and in most cases come with some degree of uncertainty. These uncertainties may be due to the mode of collecting these observations, instrumental errors, errors due to implementation of these instruments and many more. Models on the other hand help in interpolating information from observations to unobserved regions or quantities.

In land-atmosphere models, the prediction of surface temperature is subject to several errors. There is usually an uncertainty associated with specifying soil texture at the scales at which these models are for predictions. The soil-moisture dynamics are often inadequately represented resulting in significant errors in the estimates of soil-moisture states in the different soil layers, which adversely impacts the thermal conductivity, estimates. The adequate representation of surface temperature—in land-atmõšñã4Gdies requires using methodologies, such as data assimilation, that constrain the error growth resulting from these uncertainties.

Data assimilation basically combines information available from different sources to obtain at best, estimates of the state of a system. The combination of noisy data is an efficient way of filtering out the noise to obtain more accurate estimates. Data assimilation is therefore a technique used to minimize model forecast errors by periodically incorporating observations and updating the model predicted states in a way that the dynamics in the conservation laws is maintained.

A data assimilation system comprises a set of observations, a dynamical model and a data assimilations scheme. Several algorithms exist for data assimilation which aid in obtaining the best estimates. In this study we use the Kalman filter for the assimilation of near-surface temperature measurements.

3.5 The Kalman Filter

The Kalman filter method is a sequential data assimilation scheme developed by Kalman, 1960. It is a recursive variance minimizing process that optimally predicts the state when the statespace equations are linear. The method explicitly accounts for the dynamical propagation of errors in the model.

The Kalman filter uses the measurements that are observed over time that contain noise (random variables) and other inaccuracies, together with model predictions and produce values that tend to be closer to the true values of the measurements and their associated calculated values. It makes use of the method of least squares to minimize the trace of the error covariance matrix.

The Kalman filter is-applicable on-StãŒGFÉe equations. The state-space equations are made up of the process equation and the measurement equation. These equations are assumed to be linear to allow for the use of the Kalman filter. The state equation can be represented as

$$Xk+l = FkXk + \omega_k \quad (3.21)$$

where Fk is the transition operator, (.ðk is the process noise. The measurement equation is also given as

$$Yk = HkXk + \upsilon_k \quad (3.22)$$

where Hk is the observation operator which relates the state to the observed and b'k is the measurement (observation) noise. The nature of the observation operator is determined by the number of states being observed.

The Kalman filter was developed based on certain assumptions. These assumptions are given below:

1. The process noise 0k and the measurement noise Vk are Gaussian noises, with zero mean.

2. The state xo, Wk and Vk are uncorelated
3. xo N(xo+, Po+)
4. The state and the measurement dynamics are linear.

The Kalman filter is categorized into two steps: Predict and Update. The predict step uses the state estimate from the previous time step to produce an estimate of the state at the current time step. This predicted state estimate is known as the a prior state estimate and is denoted by xi.

The update step, the available observation information is combined with the a priori estimate. This is done to improve the a pnori estimate producing the a posteriori state estimate which is denoted-byxk+.

3.5.1 Least Square Derivation of the Kalman filter

We consider the state and measurement equations in equations (3.21) and (3.22) respectively.

The errors in both the a priori and a posteriori estimates are given as

$$e_k^- = x_k - x_k^- \qquad \text{ek} = x_k$$

These error estimates can be used to estimate corresponding error covariance given as

$$P_k^- = \text{E[eî (eî)}^T] \qquad \text{E[et (e: Y]}$$

The a priori estimates are obtained based on the fact that observations are available until the k — Ith time step. This gives an estimate of the a priori estimate as

$$x_k^- = E[x_k|Y_{k-1}] = \qquad + \text{IYk-1]} = E[F_k x_k|Y_{k-1}] + E[\omega_k|Y_{k-1}] \tag{3.23}$$

The state noise is said to have zero mean, thus = O and hence we obtain the a priori estimate as

$$x_k^- = \text{FkXk+} \tag{3.24}$$

The error in estimating the a priori estimate is computed and subsequently used to obtain the corresponding covariance. The error can be computed as

$$e_k^- = x_k - x_k^- = F_k x_{k-1} + Wk — FkXk^-$$

$$= Fk(Xk\text{-}1 \text{ -}xt) + Ok$$

$$F_k e_k^+ + \omega_k \tag{3.25}$$

or

Having obtained an estimate of the a priori state, then the corresponding covariance can be estimated as

$$P_k^- = E[e\hat{\imath}\,(e\hat{\imath})^T] = E[(F_k e_k^+ + \omega_k)(F_k e_k^+ + \omega_k)^T]$$

$$E[(F_k e_k^+)(F_k e_k^+)^T] + E[\omega_k(\omega_k)^T]$$

$$F_k E[e_k^+(e_k^+)^T]F_k^T + Q_k$$

where Qk is the covariance of the process noise 0k and e: and Ok are uncorrelated. Thus the a priori covariance matrix is obtained as

$$F_k P_k^+ F_k^T + Q_k \tag{3.26}$$

The a priori estimates are updated when observation become available. We consider the measurement dynamics in equation (3.22). Estimate ofthe observation can then be obtained as

$$Yk = E[YkIYk\text{-}1] = E[HkXk + IYk\text{-}11 = E[H_k x_k IYk\text{-}1] + \quad IYk\text{-}1]$$

Again E [Vk] = 0. Therefore

$$y_k^- = \mathrm{HkX\hat{\imath}} \tag{3.27}$$

The covariance associated with this estimate is given as

$$\psi_k = H_k P_k^- H_k^T + R_k \tag{3.28}$$

where Rk is the covariance of the measurement noise Vk.
The update step is carried out when observations become available at the current time step k. At this point the a priori estimates are updated with the available observations to obtain the a posteriori estimate. The update equation is given as

$$x_k^+ = x_k^+ + K_k(y_k - y_k^-) \tag{3.29}$$

where Kk is referred to as the Kalman gain. We now compute the error associated with the a posteriori estimate and this is given as

$$\mathrm{ek} = x_k - x_k^+ = x_k - x_k^+ - K_k(y_k - y_k^-)$$

$$= x_k - x_k^+ - K_k[H_k(x_k - x_k^-) + v_k]$$

$$= (1 - K_kH_k)(x_k - \mathrm{x\hat{\imath}}) - \mathrm{1<kVk}$$

$$= (I - K_kH_k)\, e_k^- - \mathrm{1<kVk} \tag{3.30}$$

By the Kalman filter, the trace of the a posteriori covariance is to be minimized. We then estimate the covariance which gives

$$P_k^+ = E[e_k^+(e_k^+)^T] = (1 - H_k)P_k^-(I - \mathrm{KkHk})\,\mathrm{T} + \tag{3.31}$$

We then differentiate the trace of equation (3.31) with respect to Kk and equate the result to zero.

$$\frac{\partial\, tr(P_k^+)}{\partial K_k} = 2(I - K_k H_k)P_k^-(-H_k^T) + 2K_kR_k - 0 \quad (3.32)$$

Solving equation (3.32) for 1<1, we have

$$K_k = (HkPFHkT + Rk)\ 1 \quad (3.33)$$

Algorithm 1 : Kalman Filter

---

1. Set the initial parameters

2. fork = 1 : T 0/oT is the assimilation period

3. for i = 1 : n % n is number of model predictions in between observations

    <u>Time update</u>

    $x_k^- =$ Fk Xk-l

    $P_k^- =$ Fk pi_1 Ft—l $+ Q_k$

    end loop for i

4. <u>Measurement update</u>

    $K_k =$ PF_IHÈT (H p- 11-11 $+ R_k)^{-1}$ % Kalman gain

    $x_k^+ = x_k^- + K_k\left(y_k - y_k^-\right)$

    $P_k^+ = (I - K_k H_k) P_k^-$

5. end loop for k

Chapter 4

Results and Discussion 4.1 Introduction

An assimilation scheme is used in order to improve the soil temperature profile in the heat model governing the distribution of temperature through the soil layer. The assimilation scheme is implemented on the discretized heat transfer equation through the soil column. Scientific estimations are obtained from observations (measurements) or by using models developed to obtain this estimates. Either of these estimates is the best in the absence of the other. But when both model estimates and measurements are available, we are faced with the issue of deciding among the two, which is the best estimate.

Data assimilation was developed to incorporate the two estimates to obtain the best estimate for the states thereby minimizing the forecast error in a way that the dynamics in the conservation laws is maintained. This improved estimate of the model states provides better prediction. The research is centered on estimating the soil temperature profile through the soil column.

The Kalman filter method is used to improve the soil temperature profile prediction through the soil column in this study due to its-abtfitÿîõcorrect the model states and also provide estimates of the associated error. The Kalman filter is applicable on state space formulation of the problem. The governing differential equation of temperature distribution through the soil column was discretized using the finite volume to obtain the state space formulation of the problem.

The Kalman filter is designed to incorporate knowledge of the uncertainties in both the measurements and the models. In order to ascertain the performance of the Kalman filter algorithm, simulation studies was carried out for the different experimental cases. This chapter discusses the results and findings of the research.

4.2 Experimental Setup

Soil temperature and the accuracy of its prediction play a major role in the determination of the predictive capacity of such models. The governing equation for the prediction of the soil temperature profile is shown in equation (3.1). Under the assumption that the parameter values in the governing equation do not vary with time, equation (3.2) was obtained, which can be simplified to give

$$\frac{T}{\text{at}} \qquad \frac{\text{ð2T}}{\text{ôZ2}} \tag{4.1}$$

The boundary conditions considered for the model in equation (4.1) is the Dirichlet boundary

conditions which state the solution (temperature) of the problem on the boundary. These boundary conditions were stated as follows:

$$T(x_o, t) = T_o \qquad \text{and} \qquad T(x_L, t) = T_L \tag{4.2}$$

whereas the initial condition defined-FÉFoblem is stated as

$$T(x, t_o) = f(x) \tag{4.3}$$

In order to formulate the Kalman filter equations, the vertical column of the soil was discretized into layers of equal depth. Synthetic measurements of the soil temperature were obtained from the layers of the soil column. We use the finite volume scheme to obtain a system of equations represented by equation (3.17).

This problem is solved at various times, and equation (3.21) is the state space formulation of the problem on which the Kalman filter is applicable. The state space can generally be represented as in equation (3.21). All the states were observed in this study, and equation (3.22) represents the measurement equation.

The study investigates the performance of the Kalman filter algorithm by considering various cases that influence the performance. First, we consider the impact of the initial conditions (error covariance) on the performance of the algorithm. We also investigate the effect of the model error (process noise), observation error (measurement noise), and frequency of observation on the performance of the Kalman filter. The assimilation of the soil temperature in this study is based on synthetic data as well as the numerical solution to the governing equations.

4.3 Experimental Results

The results from the prediction of the soil temperature dynamics from the Kalman filter algorithm are presented in this section. The performance is measured by using the root mean squared errors (RMSE). Starting with the propagation of the error covariance which demonstrates how we I the Kalman filte tructs the estimates.

Figure 4.1 show a cross section of a time series propagation of the error covariance by the Kalman filter. As indicated in the algorithm, the prior estimates are updated when observations become available. From the result in Figure 4.1, there is a constant increase-in the error covariance in the absence of observations.

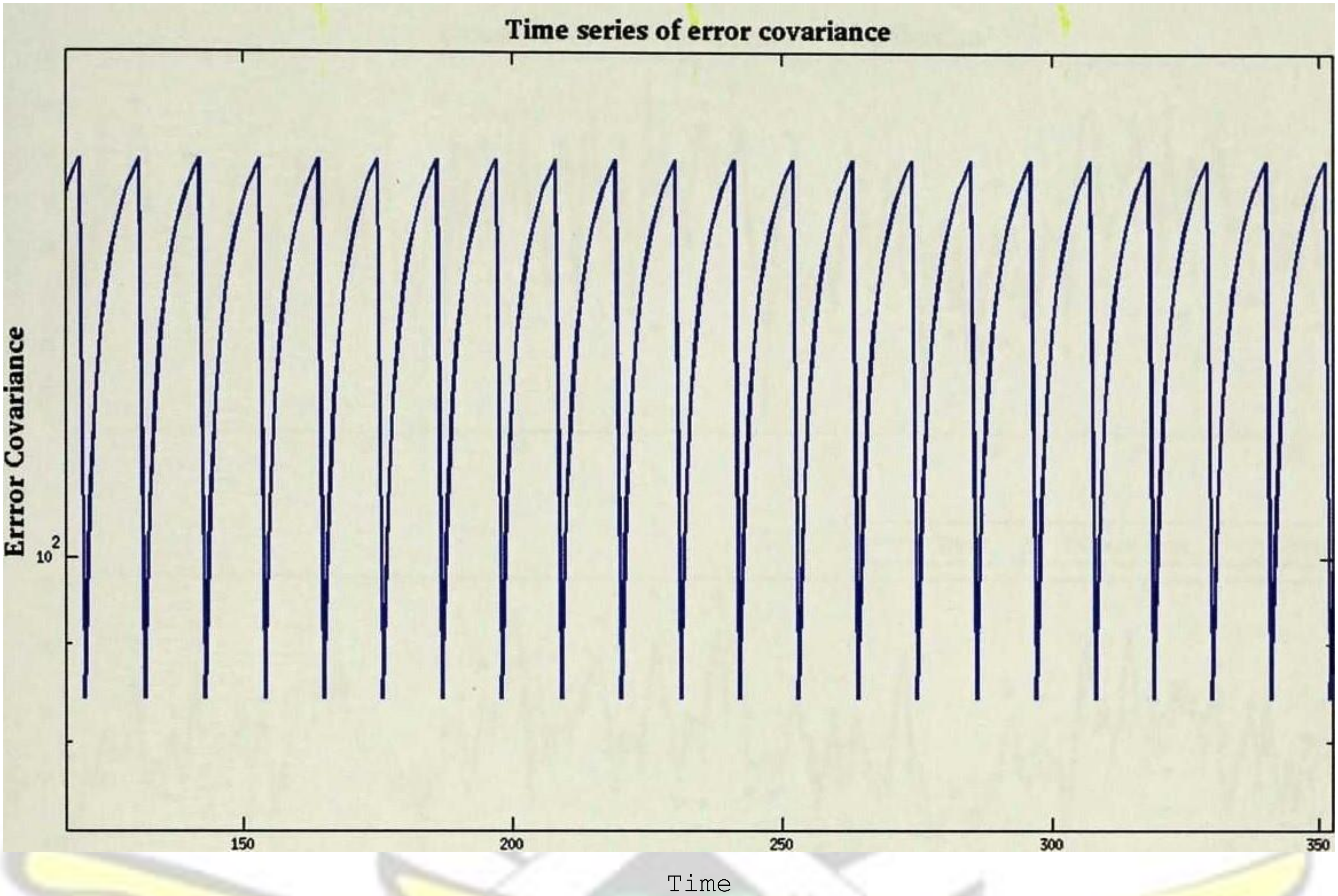


Figure 4.1: Time series propagation of the error covariance from using the Kalman filter algorithm

But when observations are made available to the Kalman filter and the prior estimates are updated, there is a rapid decrease in the error covariance indicating that the estimate as a result of the update has the minimum noise and therefore is the best of the estimates.

The temperature distribution is estimated for different soil layers and the results are shown in Figures42—and 4.3 below. The research considered four (4) inner layers for which the temperature distribution is to be estimated. Figure 4.2 demonstrates the dynamics of the temperature distribution through layers 1 and 2 whereas Figure 4.3 shows the temperature dynamics across layers 4 and 5.

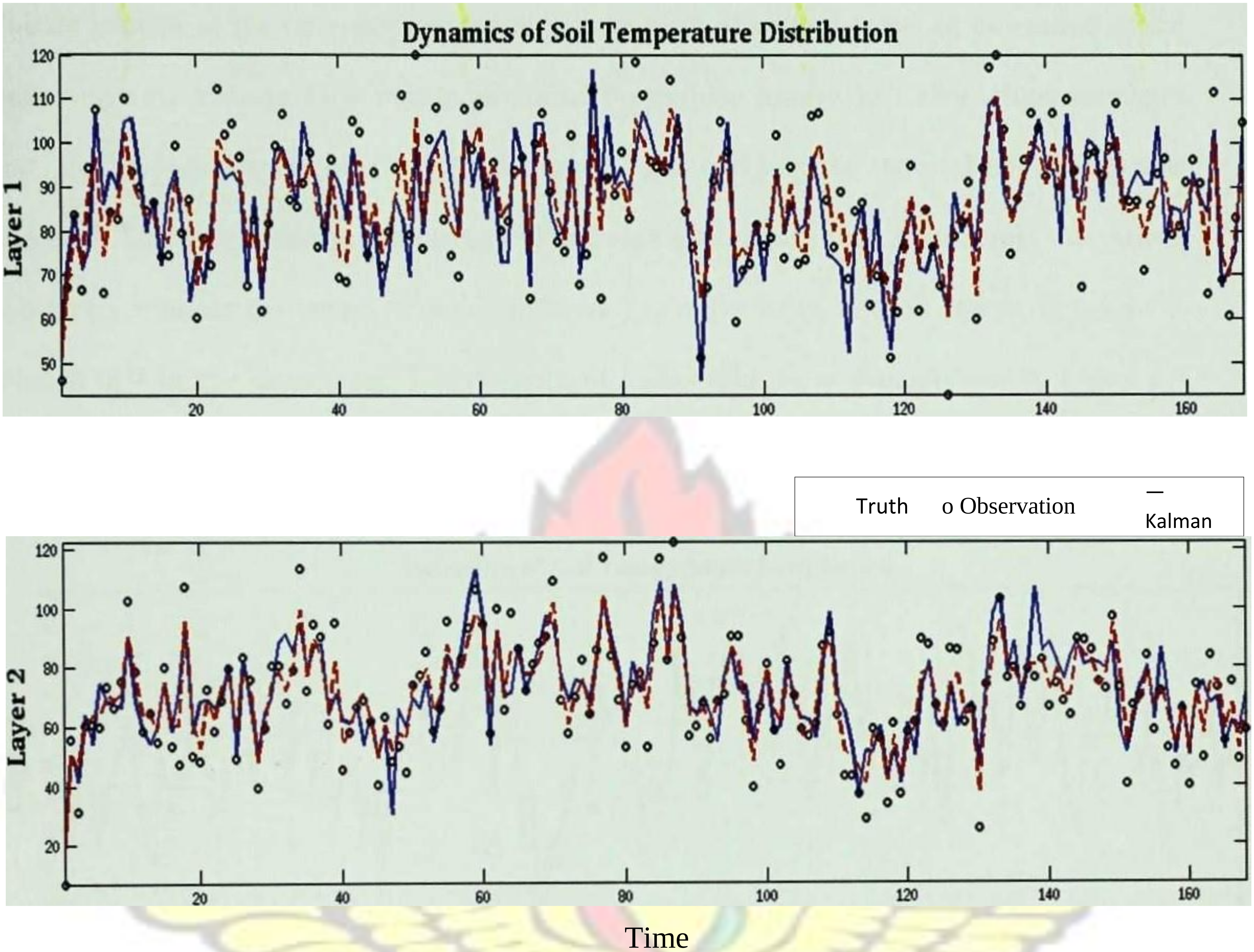


Figure 4.2: Plot of the truth, observations and estimates of the temperature distribution (temperature dynamics) through soil layers 1 and 2 using the Kalman filter.

In both Figures 4.2 and 4.3, the solid blue lines are plots of the true estimates (Truth) which are reconstructed using the Kalman fllEAlgouthm. The available observations are plotted with the black circles whereas the Kalman filter estimates are the plots indicated with the broken red lines. As shown by these results, the Kalman filter estimate coincide with the truth in some instances, but in the other instances, the estimates by the Kalman filter algorithm are different from that from the truth.

The differences in the estimates are due to the presence of various errors in estimation of the truth which the Kalman filter tries to minimize. From these results the Kalman filter estimates are closely predicting the observed. In the case of this study all the states (Temperature) was observed. The temperature is increasing and decreasing at various time steps across the various soil layers whereas the temperature decreases as you move down the soil layers from layer 1 through to 4 in this experiment. The evidence of this conclusion is demonstrated in Figure 4.4 below.

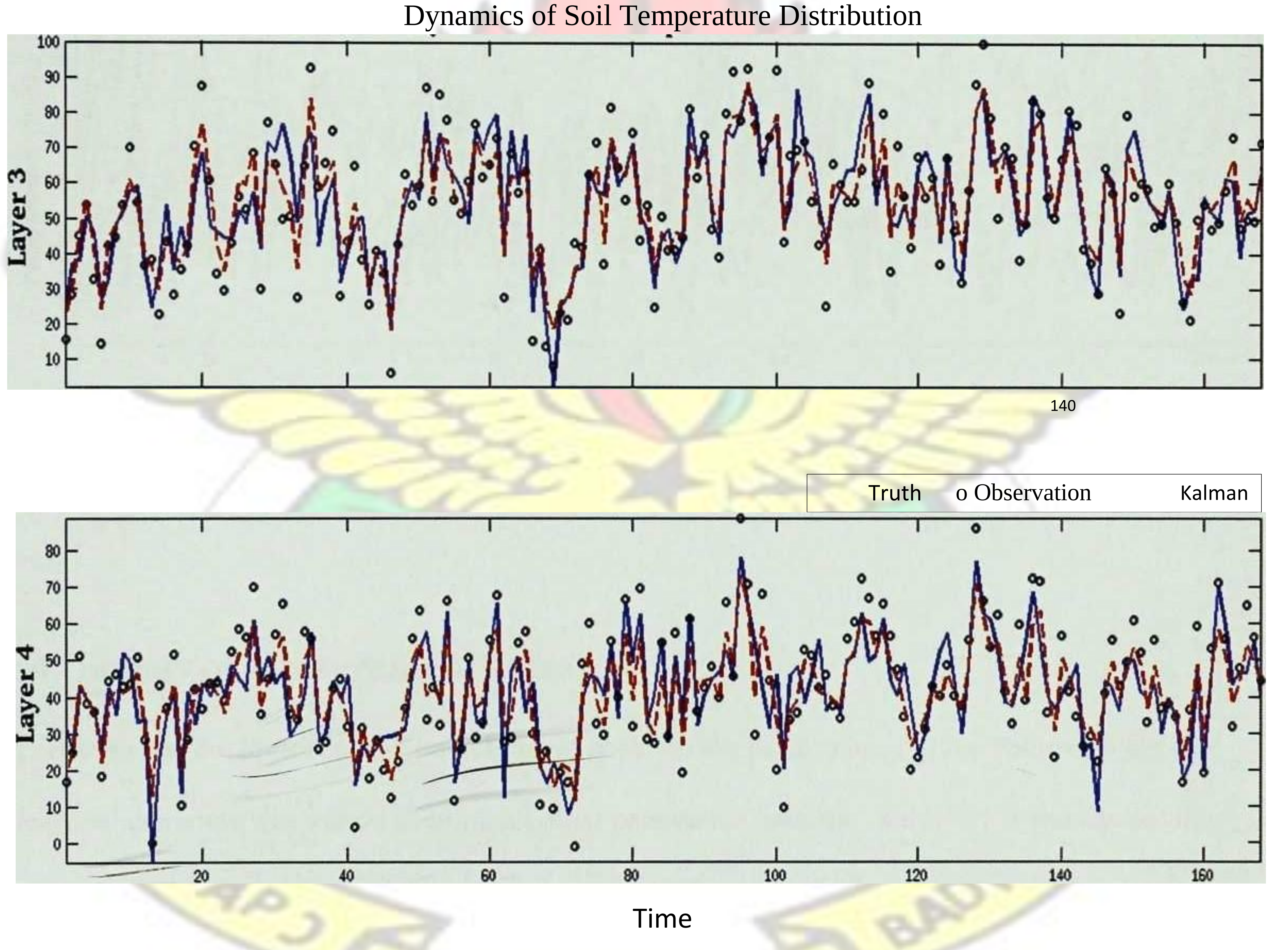
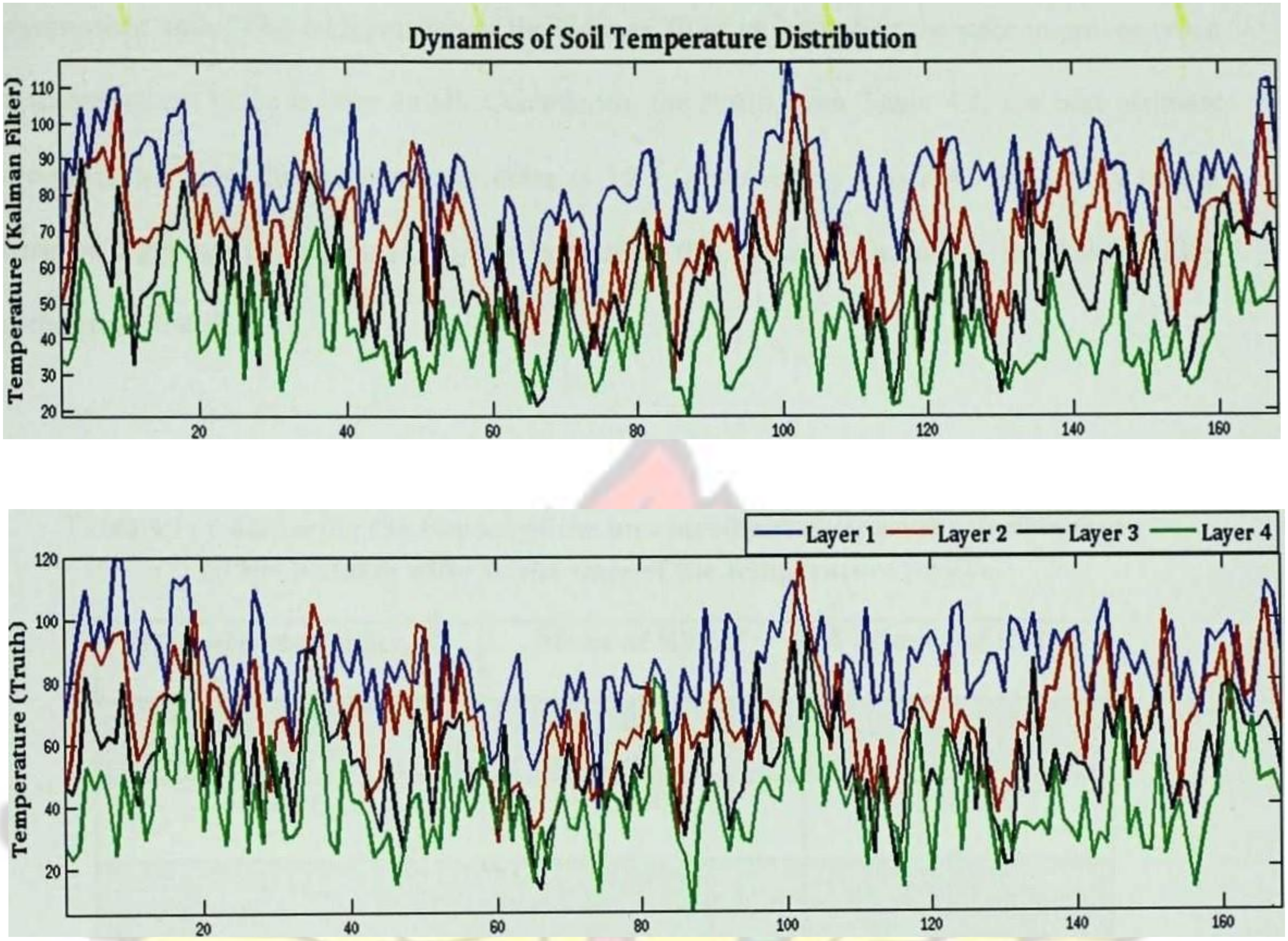


Figure 4.3: Plot of the truth, observations and estimates of the temperature distribution (temperature dynamics) through soil layers 3 and 4 using the Kalman filter.



Time

Figure 4.4: Plot of temperature distribution for the four layers. The first set is from the Kalman filter and the second from the Truth.

4.4 Impact of measurement noise

In order to test the impact of the ~~measuremen~~t noise on the performance of the Kalman filter, the measurement noise was varied keeping all other parameters constant. Table 4. I demonstrates the results from the experiment which varied the measurement noise. The root mean squared errors were measured for the various experimental options with regards the measurement noise and from this results, it can be concluded-That the performance of the

Kalman filter algorithm in reconstructing the solution (temperature distribution) through the soil column is sensitive to the

measurement noise. The performance of the Kalman filter in predicting the state improves when the measurement noise is very small. Considering the result from Table 4.1, the best estimate were obtained when the measurement noise is 10-514 where the 14 is a 4 x 4 identity matrix wherever it appears in this thesis. This is because at this measurement noise, the Kalman filter has the least RMSE.

Table 4.1: Comparing the impact of the measurement noise on the performance of the Kalman filter to the state of the temperature profile

Measurement Noise, R	Mean of RMSE	Variance of RMSE
10-514	0.0621	0.000004
10-214	1.9709	0.002840
o. 114	6.1335	0.026656
o. 514	13.2377	o. 178980
14	17.8733	0.385220
514	33.6353	2.238810
1014	41.6133	3.665593

The performance of the Kalman filter deteriorates as you increase the measurement noise. Measures of the mean and variance of the RMSE increase significantly with increase in the measurement noise. The highest values of the RMSE occur at 10/4 as per the values considered for the measurement noise in this study.

4.5 Impact of process noise

The next experimental setup investigates the impact of the process noise on the Kalman filter algorithm. Presented in Table 4.2, are the results from the experiment based on varying the process noise with all other parameters keep fixed. The measure Of the root mean squared error kept increasing whenever there was an increase in the process noise. That is to say, the performance of the Kalman filter improves with decrease in the process noise resulting in a decrease in the RMSE value.

Table 4.2: Comparing the impact of the process noise on the performance of the Kalman filter to the state of the temperature profile

Process Noise, Q	Mean of RMSE	Variance of RMSE
10-714	0.1609	0.00010513
10-514	0.2163	0.0001716
10-214	0.6116	0.00020426
o. 114	0.6216	0.00039199
o. 514	0.6234	0.00032448

According to the result from Table 4.2, the least RMSE was recorded for a process noise of 10 — 7/4 while the highest value of-the—R-MSE was recorded for a process noise of 0.5/4. The performance of the Kalman filter based on the results from Tables 4.1 and 4.2 is more sensitive to the measurement noise than the process noise due to the fact that increasing the measurement noise recorded higher values of the RMSE compared to the process noise.

Chapter 5

Conclusion and Recommendation

5.1 Conclusion

In order to improve the soil temperature distribution or profile obtained using the temperature model, the Kalman filter which is a sequential data assimilation scheme was used. The formulation is based on the discretized governing equation of heat transfer through the soil column. The Kalman filter is designed to incorporate the knowledge of the uncertainty in both the model and the measurement to obtain a better estimate of the state.

The Kalman filter was used to reconstruct the available information on the state and it was noted that the estimates from the Kalman filter best predicts the state. Model uncertainty was found to play an important role in the performance of the Kalman filter in reconstructing the state. The impact was measured using the root mean square error from which it was deduced that the small the process noise the better the performance of the Kalman filter.

Similar results were obtained when the impact of the measurement noise on the performance of the Kalman filter was investigated, and the results followed that of the process noise. Conclusions were4hat the Kalman—ffter-WáS more sensitive to the measurement noise than the process noise judging from the values recorded for the RMSE in both cases.

Estimates of the error covariance indicates that there is a constant increase in the error covariance of the state until the point measurement is made available and used in updating the aprior estimate. At this point the results show that the error covariance drops rapidly.

5.2 Recommendation

The following recommendations are made as a result of the findings from this study. The Kalman filter algorithm is recommended in reconstructing the solution of the state when both model estimates and observations are available. It is recommended that the process noise and measurement noise be kept minimal especially the measurement noise since the Kalman filter was found to be more sensitivity to the measurement noise. Further work is recommended to investigate other parameters that may be affecting this algorithm.

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