

**KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY,  
KUMASI, GHANA**

**KNUST**

**Modelling and Prediction of Maximum Vibration Amplitude for the Evaluation  
of Gas Turbine Performance.**

**by**

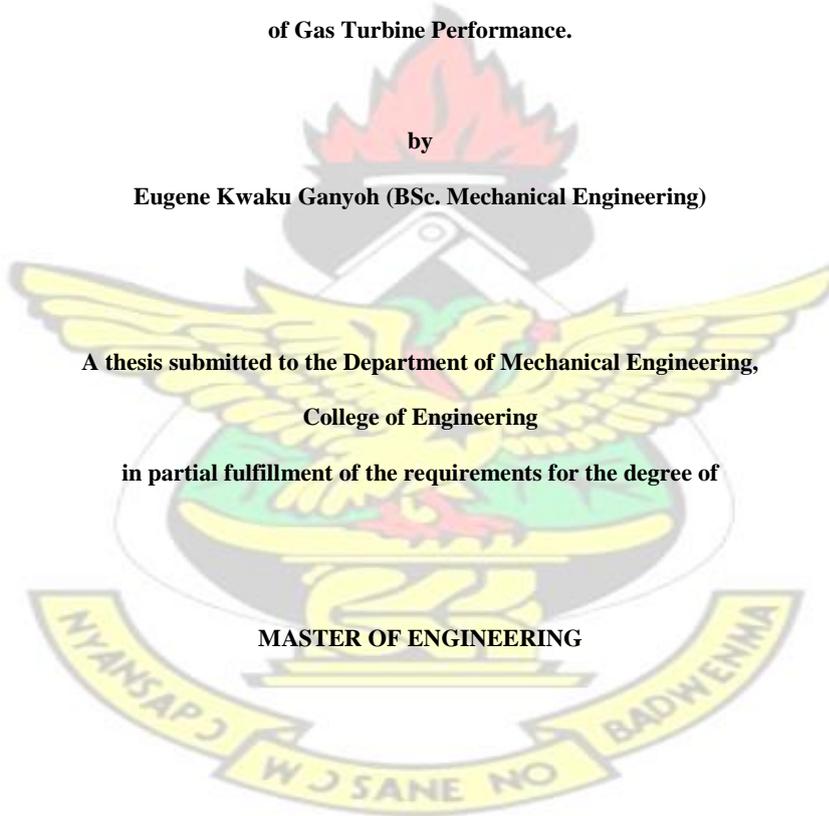
**Eugene Kwaku Ganyoh (BSc. Mechanical Engineering)**

**A thesis submitted to the Department of Mechanical Engineering,  
College of Engineering**

**in partial fulfillment of the requirements for the degree of**

**MASTER OF ENGINEERING**

**NOVEMBER, 2019**



## DECLARATION

I hereby declare that this submission is my own work towards the award of Professional Master of Engineering (Industrial Operation) degree. and that, to the best of my knowledge, it contains no material previously published by another person, nor material which has been accepted for the award of any other degree of this University or any other, except where due acknowledgement has been made in the text.

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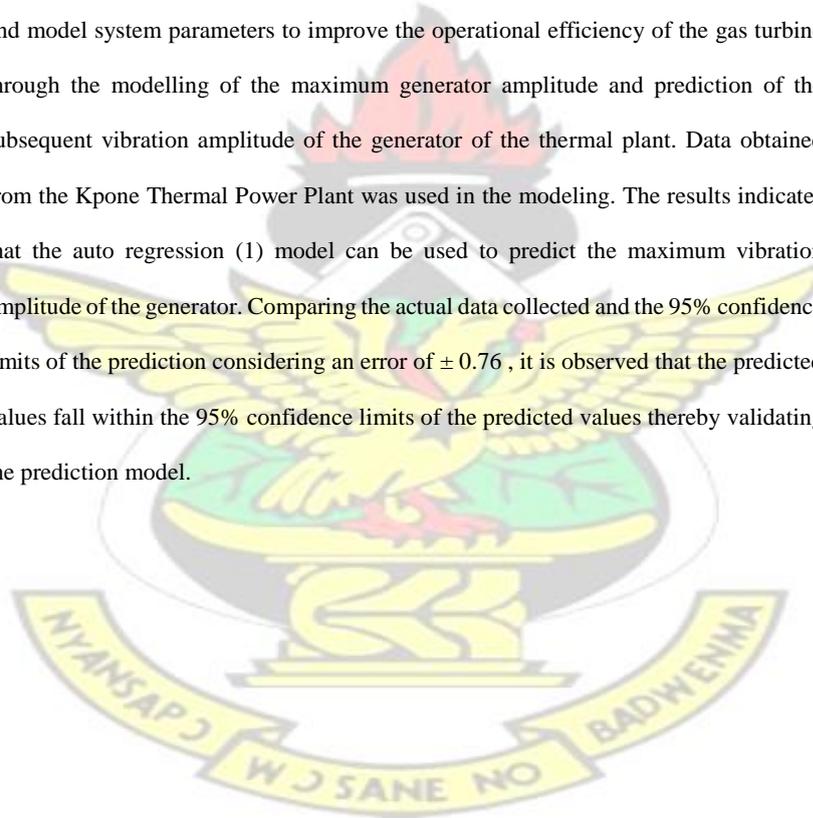
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## ABSTRACT

Electric power generation is largely dependent on the generator. The generator is, however, found to have lateral vibration, torsional vibration or stator vibration problems which can affect the blade, bearings, shaft and speed of the generator among others. This ultimately affects the performance of the generator and thus production of power would be affected. It is therefore very essential to monitor the maximum vibration amplitude to ensure that it stays within range. The purpose of this research is to study and model system parameters to improve the operational efficiency of the gas turbine through the modelling of the maximum generator amplitude and prediction of the subsequent vibration amplitude of the generator of the thermal plant. Data obtained from the Kpone Thermal Power Plant was used in the modeling. The results indicates that the auto regression (1) model can be used to predict the maximum vibration amplitude of the generator. Comparing the actual data collected and the 95% confidence limits of the prediction considering an error of  $\pm 0.76$  , it is observed that the predicted values fall within the 95% confidence limits of the predicted values thereby validating the prediction model.



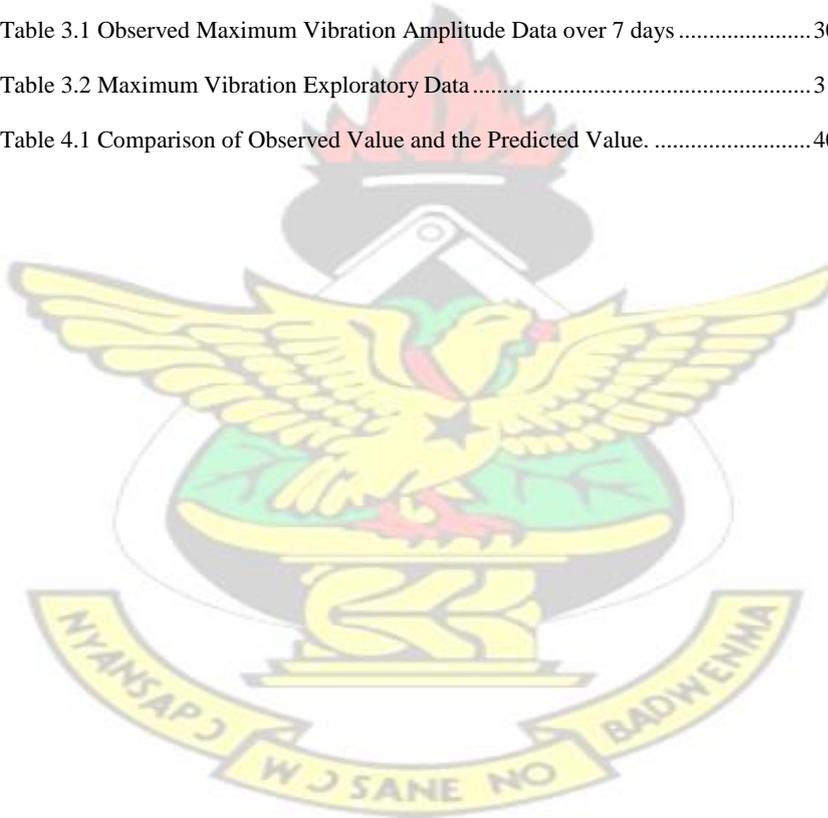
## TABLE OF CONTENTS

<b>DECLARATION</b> .....	<b>ii</b>
<b>ACKNOWLEDGEMENT</b> .....	<b>iii</b>
<b>ABSTRACT</b> .....	<b>iv</b>
<b>TABLE OF CONTENTS</b> .....	<b>v</b>
<b>LIST OF TABLES</b> .....	<b>vii</b>
<b>LIST OF FIGURES</b> .....	<b>viii</b>
<b>CHAPTER ONE</b> .....	<b>1</b>
<b>INTRODUCTION</b> .....	<b>1</b>
1.1 Background to the study.....	1
1.2 Problem Statement.....	4
1.3 Objectives of the study.....	5
1.4 Justification for the study.....	5
1.5 Scope of the Study.....	5
1.6 Organization of the Chapters.....	6
<b>CHAPTER TWO</b> .....	<b>7</b>
<b>LITERATURE REVIEW</b> .....	<b>7</b>
2.1 Thermal power plant.....	7
2.1.1 Brayton-type cycle.....	11
2.2 Theory of Thermal Power Station.....	13
2.3 Thermal Power Plant Types.....	15
2.3.1 Steam Turbine Power Plant.....	15
2.3.2 Gas Turbine Power Plant.....	16
2.3.3 Combined Cycle.....	17
2.4 Generator and Monitored Parameters.....	18

2.5 Gas Turbine Generators and Operation Principles.....	19
2.6 Gas Turbines and Generator Vibrations .....	21
2.7 Analysis of the Vibration Problems.....	25
2.8 Mitigation of the vibration problems.....	26
2.9 Mathematical Modeling .....	27
<b>CHAPTER THREE.....</b>	<b>29</b>
<b>METHODOLOGY AND DATA COLLECTION .....</b>	<b>29</b>
3.1 Data Collection .....	29
3.2 Time Series Modeling and Analysis.....	32
3.3 Estimation of Model Order and Parameters .....	33
<b>CHAPTER FOUR.....</b>	<b>35</b>
<b>DATA ANALYSIS AND DISCUSSIONS.....</b>	<b>35</b>
4.1 Data Modeling .....	35
4.2 Check of Model Adequacy .....	37
4.3 Prediction of Maximum Vibration Amplitude .....	39
<b>CHAPTER FIVE .....</b>	<b>41</b>
<b>CONCLUSIONS AND RECOMMENDATIONS.....</b>	<b>41</b>
5.1 Conclusions .....	41
5.2 Recommendations .....	41
<b>REFERENCE.....</b>	<b>42</b>
<b>APPENDIX A- Monitored Data for Various Parameters of the Kpone Thermal Power Plant.....</b>	<b>48</b>
<b>APPENDIX B – Development of the Maximum Vibration Amplitude Model .....</b>	<b>52</b>

## LIST OF TABLES

Table 2.1: Grouping of vibrations into six different lateral vibration phenomena .....	22
Table 2.2: Grouping of vibrations into three different torsional vibration phenomena.....	25
Table 2.3: Grouping of vibrations into three different Stator vibration phenomena ...	25
Table 2.4: Applied Analysis Techniques and Mitigation Methods in the three power plants to solve Lateral Vibrations of the shaft trains.....	27
Table 3.1 Observed Maximum Vibration Amplitude Data over 7 days .....	30
Table 3.2 Maximum Vibration Exploratory Data.....	31
Table 4.1 Comparison of Observed Value and the Predicted Value. ....	40



## LIST OF FIGURES

Figure 2.1 Schematic representation of ideal Rankine cycle.....	8
Figure 2.2 T-S diagram of a Rankine cycle .....	9
Figure 2.3. Brayton cycle (left) and its associated TS diagram (right).....	12
Figure 2.4 Schematic diagramme of thermal power plant.....	14
Figure 2.5 Steam Turbine Power Plant.....	16
Figure 2.6 Gas Turbine Power Plant.....	17
Figure 2.7 A schematic diagram of a Combined Cycle.....	18
Figure 2.8 Cross section of Generator .....	19
Figure 2.9 Locations of the Vibration transducers in a shaft train.....	26
Figure 2.10: Typical presentations of vibrations for shaft trains in power plants .....	26
Figure 3.1: Time Series Plot of Representative maximum vibration data.....	31
Figure 3.2: Scatter Plot of $X_t$ versus $X_{t-1}$ for exploratory maximum vibration Data .....	33
Figure 4.1 Maximum Vibration Amplitude Observed Data .....	35
Figure 4.2 A plot of Compressed Data of Maximum Vibration Amplitude.....	36
Figure 4.3. Scatter Plot of $X_t$ versus $X_{t-1}$ .....	36
Figure 4.4 Scatter Plot of $X_t$ versus $X_{t-2}$ .....	38
Figure 4.5. Comparison of the Data Series and the Model Series .....	38

## CHAPTER ONE

### INTRODUCTION

#### 1.1 Background to the study

Electric power is considered as one of the key elements that drove the Industrial Revolution of the last quarter of the 19<sup>th</sup> century and the 20<sup>th</sup> century. It is still very essential in the health sector, the food sector, and the general development of any nation (Rosenberg, 1998, Shahbaz *et al.*, 2014). It was estimated that more than 80% of electricity generated worldwide in the year 2009 was from thermal power plants (IEA., 2011, Linnerud *et al.*, 2011).

For development or any manufacturing entity to invest in a country, investors will first look at access to power because there will be large dependency on power. Luckily, Ghana as a country has power, which is dependent on water (the chunk of it is dependent on water). With population increase over the period, overdependence on water for power supply is unsustainable so the nation has had to look to other sources of power such as Thermal. Thermal power generation is reliable and can be controlled and managed. Though hydro is the main source it is unreliable and cannot be continually depended on due to the depletion of the ozone layer and changes in rainfall patterns. The efficiency of a thermal power plant is generally defined as the useful electricity produced per unit of energy input (Kotas, 2013). Thermal power is a complete system comprising the gas turbine and the steam turbine with their output driving the electric generator. The generator is the last stage of the plant where power is generated by converting mechanical energy into electrical energy. In the production of electricity the generator converts motive power (mechanical energy) into electrical power for use in a circuit (Golding, 1976, Hall and Bain, 2008). Steam, gas and water

turbines are some of the sources of mechanical energy. Almost all the power for electric power grids are provided by generators. The mechanical structure of the generator is made up of a rotating part and a stationary part. These generate a magnetic field and electric current form a changing field. The armature can be on either the rotor or the stator, depending on the design, with the field coil or magnet on the other part (Brunekreeft *et al.*, 2005).

The generator is subject to vibration as a result of its bulky size and high rotational speed. Vibration is defined as the continuous, repetitive or periodic oscillations relative to a certain fixed reference. The physical motion of rotating machines generates vibration, which gives a physical indication of the health of equipment and the generated vibration frequencies and magnitudes represent the machine vibration signature. This vibration must be within a safety range of operation. The generator rotor can experience thermal sensitivity which may take place on the generator rotor and lead to the rotor vibration changing as the field current is increases. This thermal sensitivity can be caused by irregular temperature distribution circumferentially around the rotor, or by axial forces, not uniformly distributed in the circumferential direction. The primary driver of this second cause is the large difference in coefficient of thermal expansion between the copper coils and the steel alloy rotor forging and components. If the rotor winding is not balanced both electrically and mechanically in the circumferential direction, the generator rotor will be unevenly loaded which can cause the rotor to bow and cause the vibration to change in amplitude (Brunekreeft *et al.*, 2005, Ameri and Ahmadi, 2007, Farrahi *et al.*, 2011, Osintsev *et al.*, 2017). A thermally sensitive rotor usually will not inhibit a generator from running, but may reduce the operation at high field currents or volt – ampere reactive (VAR) loads from its excessive

rotor vibration. As the vibration of the generator cannot be prevented, it is imperative that it remains within acceptable limits and runs smooth at all operating speeds and under all operating conditions. There can be several causes of high vibration on a generator field such as mechanical imbalance, thermal sensitivity, misalignment and bearing degradation. Other causes may include rubbing, bent overhangs, rotor stiffness dissymmetry, out-of-round journals and other design deviations caused by abnormal in-service operation. Each of these causes has a pre dominate frequency and a characteristic response.

The cause of the vibration can be diagnosed by a thorough analysis of the vibration data. For example, the most often observed cause of vibration is mechanical unbalance. This type of vibration is synchronous; that is, the vibration frequency equals the rotor rotational speed frequency. The rotor winding inter-turn short circuit is one of the common faults, and the causes of this fault have several effects, which in turn will lead to rotor vibration, develop rotor earth, rotor winding burn-out, generator loss of excitation and generator components magnetization.

Generators are often subjected to high currents and voltages caused by electrical disturbances in the power system and these considerably contribute to the reduction of the machine's operating life. Faults in the generator cause immense stress usually above the design limits leading to high temperature which lowers the mechanical strength, alter the air gap torques, build imbalance flux densities in the air gap, loss of load, fault clearance and reclosing. All these culminate in material fatigue, insulation and structural failure and ultimately equipment breakdown. Mechanically, the unusual forces built up excite and amplify the rotor oscillatory motion and result in severe

machine vibration. Generator vibration is normally monitored by the plant's condition monitoring system that serves as back up to its electrical protection system. Vibration measurement provides a very efficient way of monitoring the dynamic conditions of a machine such as imbalance, misalignment, mechanical looseness, structural resonance, soft foundation and shaft bow (Al-Badour, 2011). The vibrations which are either lateral, torsional or stator can affect the blade, bearings, shaft and speed of the generator. This ultimately affects the performance of the generator and thus production of power would be affected. It is therefore very essential to monitor the Maximum Vibration Amplitude to ensure that it stays within range. This study therefore seeks to model and predict the maximum vibrational amplitude of the generator for evaluation of gas turbine performance.

## **1.2 Problem Statement**

The generation of electricity is essential in the development of a nation. Key in the production of electricity is the generator. The generator is, however, found to have lateral vibration, torsional vibration or stator vibration problems which can affect the blade, bearings, shaft and speed of the generator among others. The detection of torsional vibrations is important in practical rotors where unnoticed large torsional vibrations can lead to fatigue induced cracks and failure (Mokhtar *et al.*, 2017). Also, Michel *et al.*, 2017 suggested that the problems caused by excessive vibration are fatigue cracks and fretting-wear damage. The maximum vibration amplitude of the generator is important because as vibration exceeds the maximum value of the designed vibration, the generator overheats, wears out and can break down. This ultimately affects the performance of the generator and thus production of power would be impaired.

### **1.3 Objectives of the study**

The goal of this work is to improve the operational efficiency of thermal power generation by monitoring, modeling and prediction of the behavior of the maximum vibration amplitude of the generator. The specific objectives are:

1. To assess the maximum vibration amplitude data from the generating system
2. To model the maximum vibration amplitude of the generator.
3. To use the model to predict future maximum vibration amplitude values

### **1.4 Justification for the study**

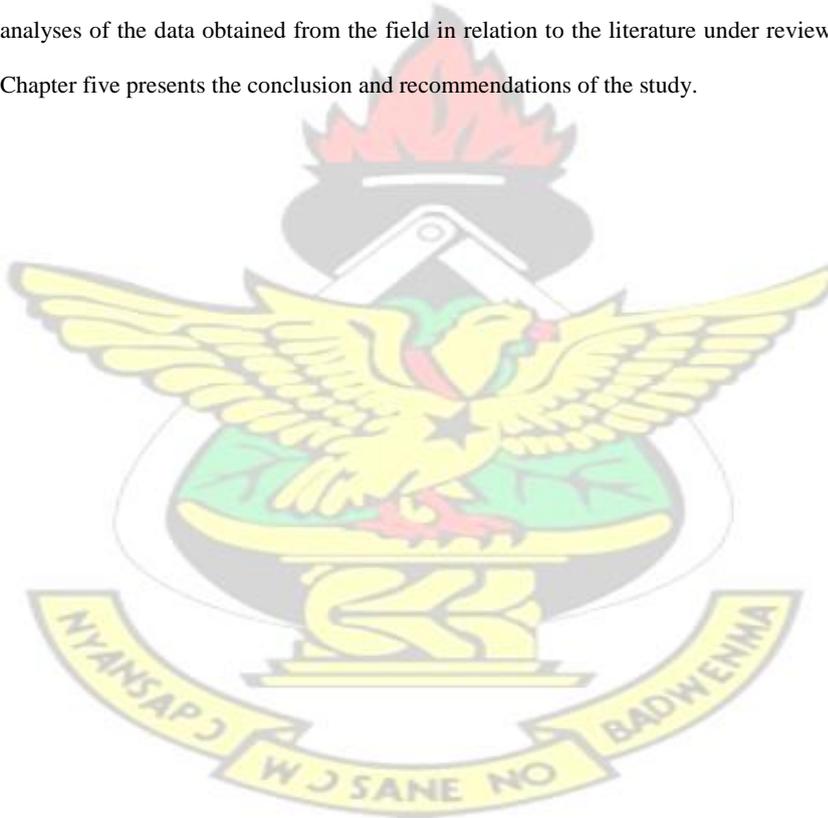
A thermal power plant ideally operates at steady state design load, but it is also required to operate on so-called “off-design load” conditions due to the fluctuations of supply and demand as well as the increased penetration of renewable energy sources. Operating flexibility of thermal power plants is therefore an essential factor for reliable grid stability as well as for economic operation. Dynamic simulation offers an effective tool for optimising the power plant performance and control structures as well as for assessing capabilities and limitations of the system with regard to process, materials, emissions or economics. Findings from this study will be useful in improving some of the existing thermal power plants and in developing more efficient operational strategies and related policy-making for a better future thermal power plant management in Ghana.

### **1.5 Scope of the Study**

The work was built on daily data obtained from the Kpone Thermal Power Plant for the month of June 2016. The result though based on this data can however be applicable to other systems.

### **1.6 Organization of the Chapters**

The thesis is organised into five (5) chapters. Chapter one covers the general introduction that includes the background to the study, objectives of the study as well as scope and limitations and the study. Chapter two looks at the relevant literature on Thermal Power Plant and its generator. The third chapter covers the methodology of the study in detail, highlighting the various approaches and means of obtaining the data. Chapter four looks at the results and discussions of findings. This chapter presents the analyses of the data obtained from the field in relation to the literature under review. Chapter five presents the conclusion and recommendations of the study.



**CHAPTER TWO**  
**LITERATURE REVIEW**

The chapter provides information on thermal power plant, theory of thermal power station as well as generator and monitored parameters. The chapter is grouped under various key sub headings.

**2.1 Thermal power plant**

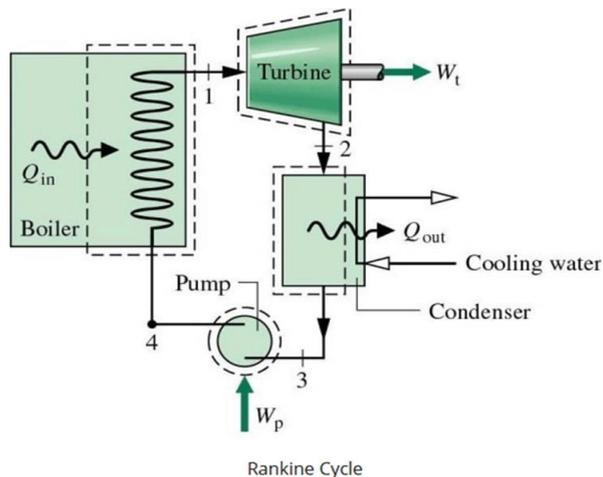
A thermal power plant is a plant which produces electricity by means of a heat source. It employs the use of a working fluid, most often water (liquid and vapour) or natural gas although pressurized gases and liquid salts are sometimes used due to their high thermal conductivity. This is heated and pressurized causing it to expand in a turbine and thereby creating work which can be used to power an electricity generator. Thermal power plants usually refer to only nuclear, oil, gas, coal and biomass-fueled power plants though other technologies such as solar and geothermal can be referred to as thermal but are hardly as their respective share in the global electricity mix is currently under 1% (IEA., 2011). Thermal plants work on the principles of theoretical heat engines and their thermodynamic cycles, Rankine (Hirn)-type cycle as shown in figure 2.1. In most thermal power plants, the turbine is actioned by steam. The steam can be generated in a boiler either directly (fossil/organic fuels or concentrated solar type) or indirectly (nuclear type) or extracted from the ground (geothermal) (Kotas, 2013, Goswami, D. Y., 1998). The thermal efficiency of a theoretical Rankine cycle,  $\eta_{Rankine}$  is given by

$$\eta_{Rankine} = 1 - \frac{H_4 - H_1}{H_3 - H_2} \dots \dots \dots \text{Eqn 2.1}$$

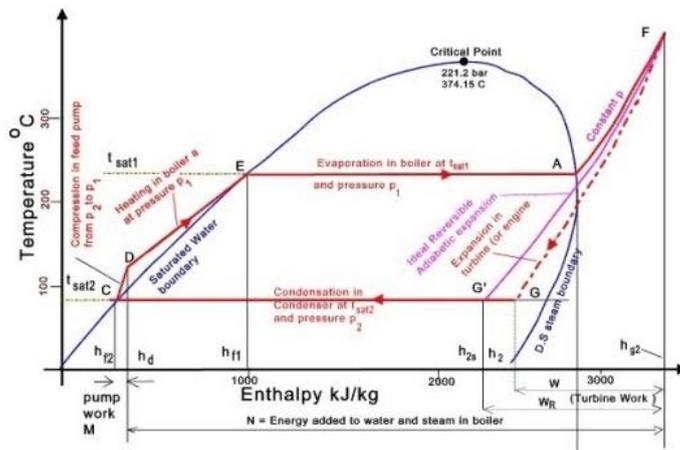
Where  $H_1, H_2, H_3, H_4$  are the specific enthalpy of the working fluid at the different steps of its transformation as shown in figure 2.1. The specific enthalpies are by definition

dependent on the temperature and pressure at the different steps of the cycle. The efficiency of a Rankine cycle is generally low, 15-20% (Hung, 2001, Liu *et al.*, 2004, Quoilin *et al.*, 2011).

In real conditions, condensation and compression are non-isentropic. It also would be very inefficient to use only partially vaporized water as working fluid, as it would increase erosion of the turbine and decrease efficiency. After the boiler, the water is therefore superheated past the critical temperature and fully vaporized. Such modified operating principles are referred as ‘Hirn cycle.’ Due to the superheating, the overall efficiency of a Hirn cycle will be higher than those of a Rankine cycle, typically 30-40%. At full load (optimum burning of combustible) enthalpy can be shown to be constant and fixed by technology, except in the condenser, where it depends upon various parameters, mostly temperature, heat capacity (itself temperature dependent) and flow of the coolant (Curzon and Ahlborn, 1975, Liu *et al.*, 2004, Moran *et al.*, 2010).



**Figure 2.1 Schematic representation of ideal Rankine cycle.**



**Figure 2.2 T-S diagram of a Rankine cycle**

The roles of the Rankine cycle outlined in figure 2.1, are 1-2: work done by turbine, 2-3: heat rejection in cooling tower, 3-4: pump, 4-1: heat addition in boiler. Water is the working fluid in Rankine Cycle. Water goes through the following processes in the Cycle. C-D, D-F, F-G, G-C are the defining process in Rankine Cycle.

**C-D Process:** This is an isentropic process, where water is pumped from low pressure to high pressure with a Centrifugal Pump. This process has no change in entropy and water remains in liquid phase only.

**D-F Process:** It happens inside a boiler. Water changes from liquid to steam.

**F-G Process:** The process occurs in Steam Turbine. The steam from the boiler enters the turbine and undergoes an isentropic-expansion process. The energy stored in water vapor converts to kinetic energy in the turbine.

**G-C Process:** It is an Isobaric Compression process and happens in a Condenser. The phase change of the working fluid happens here from steam to water.

The thermal efficiency of the Rankine cycle is the ratio between the work produced by the steam turbine that has been reduced by the pump work, with the incoming heat energy from the boiler. The heat energy from the fuel is transferred to the working fluid i.e. water. The calorific value absorbed by water vapor can be calculated using the following formula:

$$Q_{in} = m(h_F - h_D) \dots \dots \dots \text{Eqn 2.2}$$

The superheated steam produced by the boiler then goes to the steam turbine. Heat energy from water vapor is then converted into kinetic energy, shown by the F-G line in figure 2.2. The reduction of the enthalpy can be used to calculate the magnitude of the motion energy produced by the steam turbine using the following formula:

$$W_{out} = m(h_F - h_G) \dots \dots \dots \text{Eqn 2.3}$$

The steam coming out from the steam turbine enters the condenser to be condensed back into liquid phase. Here the heat energy not converted into kinetic energy, because the energy is used to convert the water into steam (latent heat). The decreases of the enthalpy (G-C line) can be used to calculate the thermal energy of condensed water using the following formula:

$$Q_{out} = m(h_G - h_C) \dots \dots \dots \text{Eqn 2.4}$$

In the next process, the condensate water is pumped to the boiler to increase its pressure. Shown by the C-D line, water does not experience much increase in enthalpy. This means that the energy given to the air is not too significant. Incoming energy values can be calculated using the following formula:

$$W_{in} = m(h_D - h_C) \dots \dots \dots \text{Eqn 2.5}$$

So now the thermal efficiency can be calculated by using the formula below:

$$\text{thermal efficiency} = \frac{\text{Work output} - \text{Work input}}{\text{Heat recovered into the system}} \dots \dots \dots \text{Eqn 2.6}$$

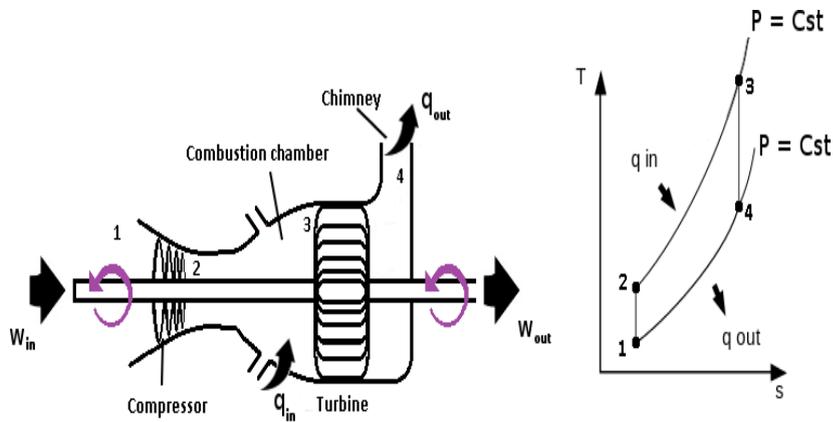
$$\text{and the rankine cycle efficiency} = \frac{m(h_F - h_G) - m(h_D - h_C)}{m(h_F - h_D)} \dots \dots \dots \text{Eqn 2.7}$$

### 2.1.1 Brayton-type cycle

Gas fuelled boiler-type power plant are rare, as their thermal efficiency is low. Most are used only to satisfy peak demand of electricity, except in countries whose gas resources are important. In order to use the combustible more efficiency, gas fuelled reactors operate a turbine under the Brayton cycle (figure 2.3). The thermal efficiency of an ideal Brayton cycle is given by:

$$\eta_{\text{Brayton}} = 1 - \frac{T_0}{T_{\text{compressor exit}}} = 1 - \frac{T_0}{T_3} = 1 - \frac{T_0}{T_{t0}} \dots \dots \dots \text{Eqn 2.8}$$

Where  $T_0$  is the atmospheric temperature and  $T_{\text{compressor exit}}$  is the temperature at the exit of the compressor (Chen *et al.*, 2002, Viteri and Anderson, 2003, Wu and Kiang, 1991) In reality, the processes are neither perfectly adiabatic nor isobaric, leading to an actual efficiency, typically 30%.



**Figure 2.3. Brayton cycle (left) and its associated TS diagram (right).**

As depicted in Figure 2.3, an Ideal Brayton cycle can be decomposed in 4 transformations: 1-2: Adiabatic compression. Air is injected and compressed into the cycle in the combustion chamber. 2-3: Isobaric heat transfer. The air-gas mix is burned, increasing its temperature and pressure. 3-4: Adiabatic expansion. The burned gases expand in the turbine, creating work ( $W_{out}$ ), which will be used for electricity production. Some of the work is used to drive the compressor ( $W_{in}$ ). 4-1: transfers The pressure heat transfers to the gas from an external source, since the chamber is open to flow in and out. In an open ideal Brayton cycle, the compressed air then runs through a combustion chamber which is known Isobaric heat transfer. Waste heat from the hot fumes is transferred to the atmosphere.

In many cases the gas turbine of a Brayton cycle is combined with a heat recovery scheme or a secondary (boiler-type) cycle to increase efficiency (Viteri and Anderson, 2003). The hot fumes can be collected and used for electricity production through a Rankine cycle. Such reactors are called combined cycle gas turbines, or CCGT

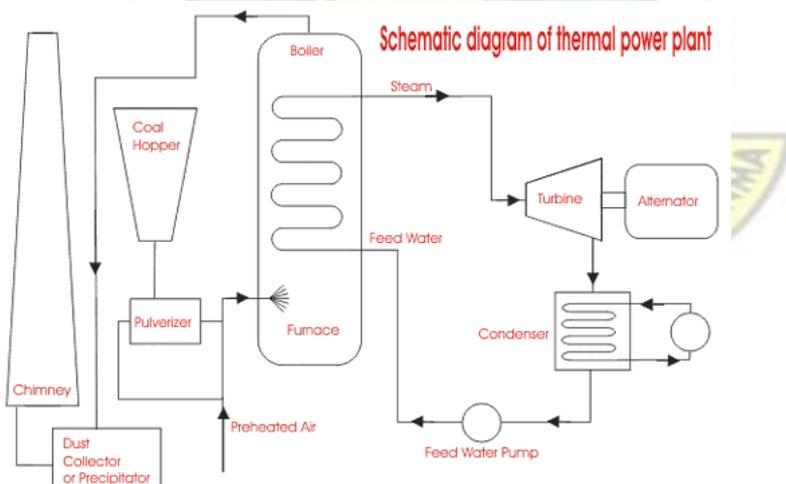
(Kehlhofer *et al.*, 2009, Valdés *et al.*, 2003). Their overall efficiency can reach 60% and more. They also present the advantage to allow for the recycling of the evacuated fumes to avoid dispersion of harmful gases in the atmosphere and increase overall efficiency (Kehlhofer *et al.*, 2009, Massardo and Lubelli, 1998, Valdés *et al.*, 2003).

## 2.2 Theory of Thermal Power Station

The theory of thermal power station or working of thermal power station is very simple. A power generation plant mainly consists of alternator runs with help of steam turbine. The steam is obtained from high pressure boilers. Generally in India, bituminous coal, brown coal and peat are used as fuel of boiler. The bituminous coal is used as boiler fuel has volatile matter from 8 to 33% and ash content 5 to 16%. To increase the thermal efficiency, the coal is used in the boiler in powder form. In coal thermal power plant, the steam is produced in high pressure in the steam boiler due to burning of fuel (pulverized coal) in boiler furnaces. This steam is further super heated in a super heater. This super heated steam then enters into the turbine and rotates the turbine blades. The turbine is mechanically so coupled with alternator that its rotor will rotate with the rotation of turbine blades (Avila-Marin, 2011, Kotas, 2013). After entering in turbine the steam pressure suddenly falls and corresponding volume of the steam increases. After imparting energy to the turbine rotor the steam passes out of the turbine blades into the condenser.

In the condenser the water is circulated with the help of pump which condenses the low pressure wet steam. This condensed water is further supplied to low pressure water heater where the low pressure steam increases the temperature of this feed water, it is again heated in high pressure (Kaushik *et al.*, 2011, Kuravi *et al.*, 2013). For better

understanding every step of function of a thermal power station as shown in Figure 2.4 as follows; (1) First, the pulverized coal is burnt into the furnace of steam boiler. (2) High pressure steam is produced in the boiler. (3) This steam is then passed through the super heater, where it further heated up. (4) This super heated steam is then entered into a turbine at high speed. (5) In turbine this steam force rotates the turbine blades that means here in the turbine the stored potential energy of the high pressured steam is converted into mechanical energy. After rotating the turbine blades, the steam has lost its high pressure, passes out of turbine blades and enters into a condenser. (6) In the condenser the water is circulated with help of pump which condenses the low pressure wet steam. (7). This condensed water is then further supplied to low pressure water heater where the low pressure steam increases the temperature of this feed water, it is then again heated in a high pressure heater where the high pressure of steam is used for heating. (8) The turbine in thermal power station acts as a prime mover of the alternator (Avila-Marin, 2011, Kaushik *et al.*, 2011, Kotas, 2013, Kuravi *et al.*, 2013, Goswami, 1998)



**Figure 2.4 Schematic diagramme of thermal power plant.**

### 2.3 Thermal Power Plant Types

Thermal energy of fuel is converted to rotational energy in Thermal Power Plants. There are several types of Thermal Power Plant. A short brief of these power plants are described below.

#### 2.3.1 Steam Turbine Power Plant

About 90% of the total electric powers of the world are produced from steam turbines. Efficiency of steam turbine depends on the maximum temperature of the steam. It uses the dynamic pressure of expanding steam to rotate the blades of the turbine. Schematic diagram of such power plants principle and working cycles is represented in Figure 2.5 Heat is added in the boiler which converts water to steam. The steam flows over the turbine and rotates the shaft of the turbine, which in turn rotates the shaft of the generator and produce power. After the turbine steam flows to condenser and condense to water, the water then flows to the boiler to be pumped and the cycle continues and the plant produces power (Itou *et al.*, 2004, Kehlhofer *et al.*, 2009, Takahashi *et al.*, 2007). A portion of the produced in the plant is used to operate the machines and equipment in the system. Most of the power plants have the efficiency ranging in between 20 to 40% (Cengel and Boles, 2002, Moran *et al.*, 2010).

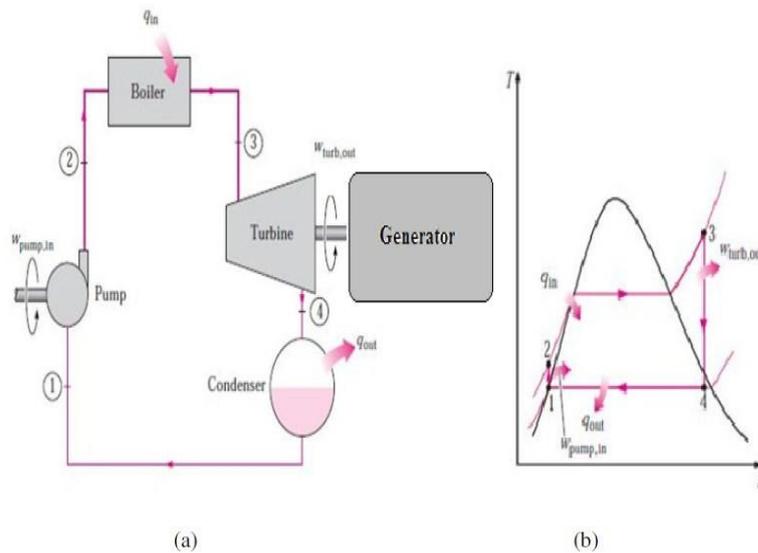
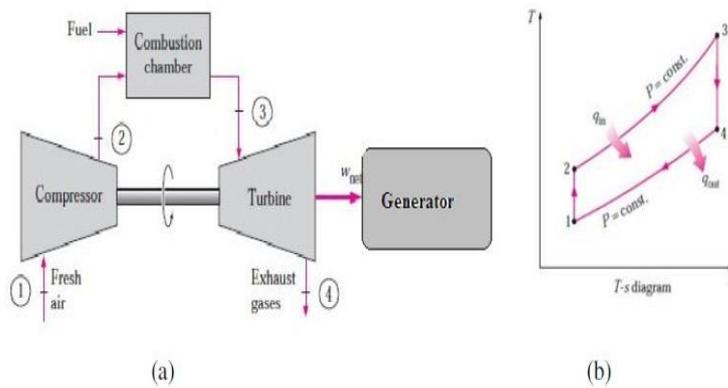


Figure 2.5 Steam Turbine Power Plant

### 2.3.2 Gas Turbine Power Plant

In gas turbine power plant dynamic pressure of flowing gases (air and combustion products) rotates turbine and generate electricity. The principle of generation of power in this type of power plant is Brayton cycle. In Figure 2.6, a schematic diagram is represented to describe the basic working principle in gas turbine power plant. Compressed air and fuel (Oil/Gas) makes chemical reaction and burns in combustion chamber, which in turn produce high temperature (900 to 1500°C) exhaust gas. This high speed exhausts gas flows over turbine and rotates it to produce power through a generator. Natural-gas fueled power station has the capability to supply power quickly. For this reason it is mostly used in peak load power plants. However, the production cost is high in this type of power plants (Ahmadi and Dincer, 2011, Chan *et al.*, 2002, Rehman *et al.*, 2012).

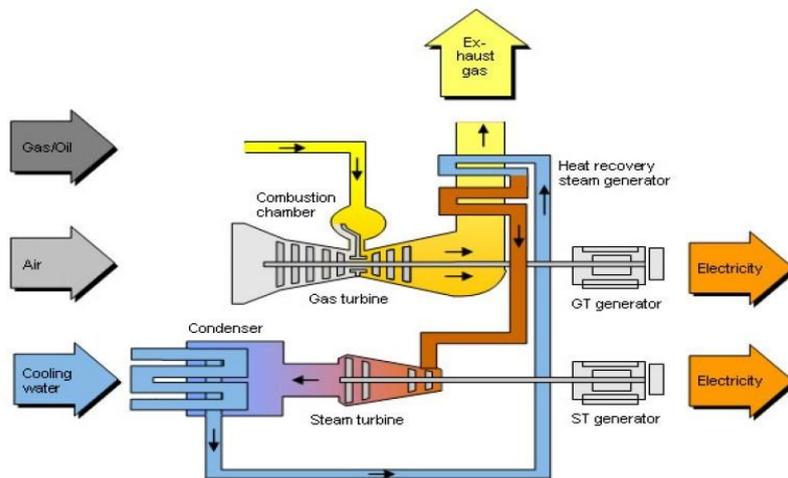


**Figure 2.6 Gas Turbine Power Plant**

### 2.3.3 Combined Cycle

Combined cycle power plants are the combination of both steam turbine power plants and gas turbine power plants. The main parts of a combined cycle power plant are a gas turbine, a steam generator and a steam turbine. The working principle of combined cycle power plant is represented in Figure 2.7. Exhaust gas coming from the gas turbine is reused to boil water and produce steam, which rotates turbine and generate electricity (Kehlhofer et al., 2009, Massardo and Lubelli, 1998).

The exhaust gas from gas turbines comes at a temperature of 450 to 650°C. This passes through a heat recovery steam generator unit which generates steam at a typical temperature of about 550°C and high pressure (30-120 bar). The steam turbine utilizes 30 to 40% energy of the exhaust gas and increase the overall efficiency. As the overall efficiency of this type of power plant is high, they are used in most base load power stations currently and mostly operated with natural gas (Fragaki et al., 2008, Pilavachi, 2000)



**Figure 2.7 A schematic diagram of a Combined Cycle**

#### 2.4 Generator and Monitored Parameters

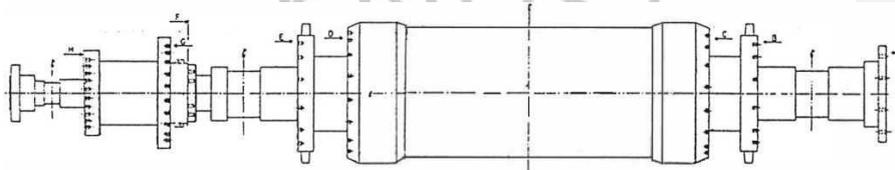
On the gas turbine, the generator is considered as a key component because it produces the power by the fundamental principle that if the line of force of magnet is made to cross through a coil, a voltage is induced (Cengel and Boles, 2002, Moran *et al.*, 2010).

The synchronous three phase generator, driven directly by a turbine, is an electromechanical equipment used to convert mechanical power to electrical power.

The generator is built in package-style and consists of these major components that are supplied in assembled condition:

- (i.) Stator: Stationary component that houses the coils
- (ii.) Rotor: Rotating part that produces the magnetic field
- (iii.) Bearings: Support to the rotor.
- (iv.) Slip-ring unit and brush gear: The brush gear with the terminal clamp is for the rotor earth fault protection and the capacitive shaft grounding.
- (v.) Cooling air-system: Removes the loss heat by the air coolant, which flows through various channels to the air-water cooler (Rolan *et al.*, 2009, Senjyu *et al.*, 2006).

For precise control of voltage, frequency, VAR and WATT, synchronous generators are more applicable. This control is achieved through the use of voltage regulators and governors. Every generator requires a good and reliable monitoring system, for optimum performance (Moran *et al.*, 2010). Figure 2.8 shows a cross-section of a generator.



**Figure 2.8 Cross section of Generator**

## 2.5 Gas Turbine Generators and Operation Principles

There are different forms of operation of Gas Turbine Generators. The under listed are some of the working principles:

One of the common gas turbine generator modes of operations is based on compression of air and fuel flows introduced into the combustion chamber. The air-fuel mixture is allowed to burn to obtain high temperature combustion gases, decompression of the last in gas turbine output and output end products of combustion in the boiler (Shnee, 1969)

However, this method of operation has some disadvantages such as low reliability of combustor elements and gas turbine, operating in conditions of high temperature and thermal stress. The temperature in the combustion chamber can be reduced by introducing excess air, which leads to high overrun compression power and reduces efficiency of gas turbine cycle. There is also the release of nitrogen oxides in high concentrations from the boiler (Ferreira *et al.*, 1995, Peirs *et al.*, 2003, Osintsev *et al.*, 2017)

Gas turbine generator also works by compressing air and fuel streams followed by their introduction into combustion chamber. The air-fuel mixture burns to obtain high temperature combustion gases, decompression of the last in the gas turbine output and output end products of combustion in the boiler. A unique feature of this method is the injection of water into the combustion chamber to protect the combustion chamber and gas turbine components from high heat flow and temperature of the combustion products (Osintsev *et al.*, 2017). Remaining gaps relating to large overruns power air compression and a high concentration of nitrogen oxides from the boiler.

Another method of operating gas turbine generator is by compressing air and fuel streams followed by their introduction into the combustion chamber. High temperature combustion gases, decompression of latter in the gas turbine output and output end products of combustion in the boiler are produced as a result of the burning of the air-fuel mixture, with gas recirculation flue between the chimney and furnace. The concentration of nitrogen oxides from exhaust flue gases into atmosphere from the boiler is reduced by recycling the products of as flue gases (Amano *et al.*, 2001, Maurya and Agarwal, 2011). However, this method has some disadvantages namely high cost of electricity on air compression and flue gas recirculation in the boiler, as well as low reliability of elements of the combustion chamber and the gas turbine, operating under high thermal stress and temperature (Maurya and Agarwal, 2011).

Gas turbine generator also works by sequential compression of air and fuel flow, entering the combustion chamber and burning. This results in the decompression of received high temperature combustion gases in the gas turbine. The output from the gas turbine is separated from the end products of combustion into two streams. One of the

streams is fed into boiler flue gas recirculation system between its chimney and furnace and the other is used for cooling and returning to the combustion chamber (Osintsev *et al.*, 2017)

Another known method by which gas turbine generator works, are through the combustion of natural gas, carbon dioxide and oxygen under pressure to form high temperature combustion gases. The product of combustion is divided into two streams. One stream is fed to the gas flue of the boiler, and the other in the recirculate flue combustion gases to cool and maintain the regime of combustion chamber (Osintsev *et al.*, 2017). The disadvantage of the method is that there is larger power loss and combustion gases change into refrigerant under cooling conditions when it returns (Kehlhofer *et al.*, 2009, Noor *et al.*, 2014, Shnee, 1969).

## **2.6 Gas Turbines and Generator Vibrations**

Gas turbines and generators vibrate in the course of their operation for the generation of electrical power in thermal plants. Vibration is the continuous, repetitive or periodic oscillations relative to a certain fixed reference. The physical motion of rotating machines generates vibration, which gives a physical indication of the health of equipment and the generated vibration frequencies and magnitudes represent the machine vibration signature. These vibrations if not detected and prevented will affect the performance of the gas turbines and the generators thereby impacting negatively on the output of power plants. Over the years there have been different problems related to turbine and generator vibrations. It is therefore important to improve the performance and minimize the risk of vibration problems (Nordmann, 2016).

In a study by Nordmann (2016), three power plants in Sweden were used as case study to map up vibration problems in three different power plants in the cities of Forsmark, Oskarshamn and Olkiluoto. The vibration problems and mitigating action described was the focus of this research, to gather knowledge and experience in the area of turbine and generator vibration problems. Nordmann (2016), found out that vibrations can be can be grouped into three main areas independent from the power plants as;

(a) Lateral Vibration Problems of the turbine-generator rotor trains which consist of 1X Lateral Vibrations in shaft trains due to Unbalance, Cyclic or Spiral Lateral Vibrations, Friction Induced Lateral Vibrations in Generators, Lateral Vibrations due to changes of Seawater temperature, Lateral Vibrations due to Unequal Moments of Inertia, Unstable Lateral Vibrations,

(b) Torsional Vibration Problems of the turbine-generator rotor trains also made up of 1X Lateral Vibrations in shaft trains due to Unbalance, Cyclic or Spiral Lateral Vibrations, Friction Induced Lateral Vibrations in Generators, Lateral Vibrations due to changes of Seawater temperature, Lateral Vibrations due to Unequal Moments of Inertia, Unstable Lateral Vibrations, and Torsional Vibration Problems are divided as follows:

(c) Stator Vibration Problems comprising of, 2X End Winding Vibrations in the Generator Stator, 2X Stator Core Axial Vibration, 2X Stator Cooling Pipe Vibrations  
 Table 2.1 to 2.3 show a summary of the different types of vibrations from the three power plants

**Table 2.1: Grouping of vibrations into six different lateral vibration phenomena**

Vibrations	Oskarshamn	Olkiluoto	Forsmark
------------	------------	-----------	----------

1X Lateral Vibrations in Shaft Trains due to Unbalance	1X Vibrations are normal. Observe Run up curves. Critical speeds. Damping Long term behavior	1X Vibrations are normal. Observe Run up curves. Critical speeds. Damping Long term behavior	Problem with I-3 Gen.-rotor during warming in the balance machine before dismantling and assembly
Cyclic or Spiral Vibrations	03: Cyclic vibrations since 2011 due to Oil seal system in Generator. 10 to 50 um. 12 hours.	OL1: Strong unstable cyclic vibrations after R108 at 1800 rpm. Labyrinth Seals?	F21: Cyclic vibrations during Runout tests (RA10) at 500 rpm 100um at B03. 1 hour.Steam Seal.
Friction Induced Lateral Vibrations in Generators	Not reported during visit	OL2/R7: problem appeared with R7 Rotor Friction between. Windings and Generator Rotor	Not reported
Lateral Vibrations due to change in sea water temperature/ condenser vacuum	1 X vibration increase due to water and rotor with increasing sea water temperature thermal bow	Change of 1 X vibration during the year due to changing condition i.e pressure and temperature, stat deformation	1X vibration showing cyclic behavior. Rubbing period 1 day.
Unstable lateral vibrations	Not reported	Not reported	Bearing instability at first bending mode (8Hz) of the LP turbine
Lateral Vibrations due to Unequal Moments of Inertia	Not at 03, because of 4 pole Generator 1500 rpm . 01,02?	2X vibrations of 150um at weight resonance of 2 pole Generator 350 rpm.	Not reported

Vibrations	Oskarshamn	Olkiluoto	Forsmark
1X Lateral Vibrations in Shaft Trains due to Unbalance	1X Vibrations are normal. Observe Run up curves. Critical speeds. Damping Long term behavior	1X Vibrations are normal. Observe Run up curves. Critical speeds. Damping Long term behavior	Problem with I-3 Gen.-rotor during warming in the balance machine before dismantling and assembly
Cyclic or Spiral Vibrations	03: Cyclic vibrations since 2011 due to Oil seal system in Generator. 10 to 50 um. 12 hours.	OL1: Strong unstable cyclic vibrations after R108 at 1800 rpm. Labyrinth Seals?	F21: Cyclic vibrations during Runout tests (RA10) at 500 rpm 100um at B03. 1 hour.Steam Seal.
Friction Induced Lateral Vibrations in Generators	Not reported during visit	OL2/R7: problem appeared with R7 Rotor Friction between. Windings and Generator Rotor	Not reported
Lateral Vibrations due to change in sea water temperature/ condenser vacuum	1 X vibration increase due to water and rotor with increasing sea water temperature thermal bow	Change of 1 X vibration during the year due to changing condition i.e pressure and temperature, stat deformation	1X vibration showing cyclic behavior. Rubbing period 1 day.
Unstable lateral vibrations	Not reported	Not reported	Bearing instability at first bending mode (8Hz) of the LP turbine
Lateral Vibrations due Unequal Moments of Inertia	Not at 03, because of 4 pole Generator 1500 rpm . 01,02?	2X vibrations of 150um at weight resonance of 2 pole Generator 350 rpm.	Not reported

Source: Energiforsk Report, 2016

**Table 2.2: Grouping of vibrations into three different torsional vibration phenomena**

Vibrations	Oskarshamn	Olkiluoto	Forsmark
Torsional vibration due to electrical faults	Not Reported	Not Report	Disturbance in electrical grid
Torsional vibrations due to SSR	Not investigated	There is a SSR protection	Due to SSR a blade resonance of F3 was excited
Blade vibrations in last stage LPT	Not Reported	Not Reported	Strong LS blade at run down after vacuum breaking

Source: Energiforsk Report, 2016

**Table 2.3: Grouping of vibrations into three different Stator vibration phenomena**

Vibrations	Oskarshamn	Olkiluoto	Forsmark
2X end winding vibrations in generator	Not Reported	Strong end winding vibrations 100 Hz up to 60 mm/sec	2X 100 Hz end winding vibrations in F2. cracks found
2X stator core axial vibrations	Not Reported	hHigh axial stator core vibrations up to 50 mm/sec	F1 and F2 had problems after many years running in old Gen. Design
2 X stator cooling Pipe vibrations	Not Reported	High axial and horizontal cooling pipe vibrations up to 48 mm/sec	Problems on old stator design of F2 with high pipe vibrations, resonance and leakage

Source: Energiforsk Report, 2016

## 2.7 Analysis of the Vibration Problems

Absolute vibration velocities measured in mm/sec form the basis for the analysis of vibration problems in power plants. The vibration velocities are measured on the bearings and relative shaft vibrations in displacements ( $\mu\text{m}$ ) close to the bearings or at shaft ends. Measurements are taken in all directions, for example, horizontal, vertical,

axial and in other directions, (e.g. 45°) on the bearings (Nordmann, 2016) Figure 2.9 shows the location of vibrations in a shaft train while Figure 2.10 shows the typical vibrations for shaft trains in power.

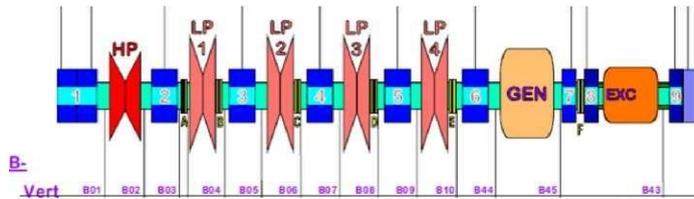
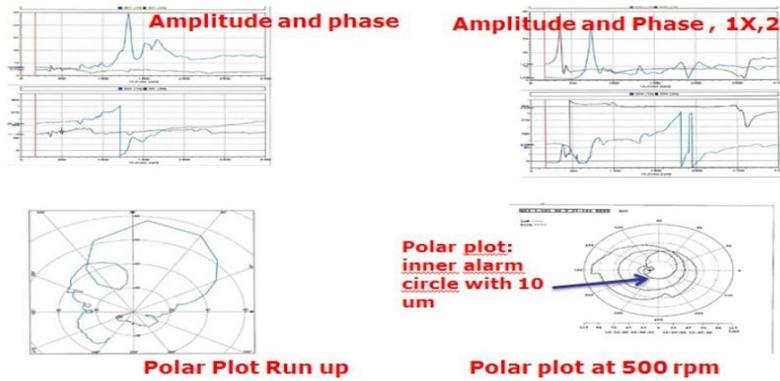


Figure 2.9 Locations of the Vibration transducers in a shaft train

Figure 2.10: Typical presentations of vibrations for shaft trains in power plants



## 2.8 Mitigation of the vibration problems

Table 2.4 is a summary of mitigation methods (Mitigation) applied to overcome vibration problems with corresponding appropriate analysis techniques (Investigation).

**Table 2.4: Applied Analysis Techniques and Mitigation Methods in the three power plants to solve Lateral Vibrations of the shaft trains**

Grouping	Investigation	Mitigation
1X Lateral Vibrations in Shaft Trains due to Unbalance	Run up and Run down curves. Critical Speeds (peaks). Damping? Short term and long term behavior	Balancing with influence coefficients. Improve damping. Shift resonances by tuning.
Cyclic or Spiral Vibrations	Amplitude and phase versus time Determine change of amplitudes and period. Polar plot. Rotational vector	Avoid rubbing by Pressure control in generator. Change load or Condense r vacuum
Friction Induced Lateral Vibrations in Generators	Heat runs. Influence of pressure and speed on vibrations (Generator).	circumference for friction, pressure winding temperature,
Lateral Vibrations due to change in Sea water temperature	Investigate vibration versus time depending' on condenser temp., pressure and power	Protect rotor by sheet metal against water Reduce active Power. Check process parameter
Unstable Lateral Vibrations	Investigate Frequency spectra and look for half frequency components, e.g. from bearings or HP seals	Change bearing parameters, improve damping. Decrease power in case of HP seal instability
Lateral Vibrations due Unequal Moments of Inertia	Investigate lateral 2X and 1X vibration	Slots in 2 pole Generator rotor. Improve damping

Source: Energiforsk Report, 2016

## 2.9 Mathematical Modeling

Mathematical models are tools used in engineering and science to predict functional relationships between certain input and output variables. Mathematical modeling thus means to produce operators in quantitative agreement with physical experience. It is a representation in mathematical terms of the behavior of real devices and objects (Magnussen, B.F. and Hjertager, B.H., 1977). Typical mathematical models involve: (a) deterministic ODE models (b) stochastic models (c) optimality principles (e.g.

principles of utility, properties of materials, minimization of energy consumption, trading strategies) (d)discrete or continuous flow models (e.g. queuing problems, traffic, logistics, load balancing in parallel computers) (e) statistical models (e.g. distribution of votes, change in the precipitation rate, wage justice, criminal statistics)(FitzHugh, R., 1955).

# KNUST



## CHAPTER 3

### METHODOLOGY AND DATA COLLECTION

This chapter outlines the research design methods that were applied in addressing the research objectives and looks at the methodology of the study, describes the source of the data and the mode of data collection, and the modeling tool used in the study.

#### 3.1 Data Collection

The operation of the thermal plant is continuously monitored via a number of parameters such as the fuel/NO<sub>x</sub> water supply system, generator, compressor, turbine, lube system, bearings and the excitation system. These systems were monitored over a period of 13 days. Appendix A contains the monitored data for each of these systems. However, the generator is of interest in this study. The vibrational amplitude of the generator ultimately affects the performance of the generator and eventually the power output. For this study, the Kpone Thermal Power Plant (KTPP) was monitored and all data obtained from the power plant. The hourly maximum vibration data collected over a 7 day period by means of a data logger is used for the modeling and analysis. This set consists of 168 data points. This maximum vibration data is provided in Table 3.1

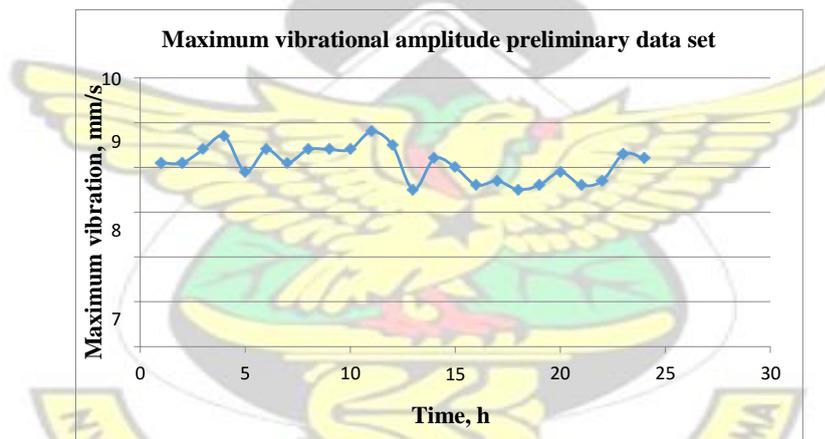
**Table 3.1 Observed Maximum Vibration Amplitude Data over 7 days**

Day 1		Day 2		Day 3		Day 4		Day 5		Day 6		Day 7	
Time, h	Max vib, mm/s												
1	8.1	25	7.7	49	8.4	73	7.7	97	7.9	122	8.6	145	8.1
2	8.1	26	6.7	50	8.2	74	6.5	98	6.6	123	8.2	146	6.7
3	8.4	27	7.1	51	7.7	75	6.6	99	6.6	124	8.5	147	7.1
4	8.7	28	6.9	52	8.0	76	6.7	100	6.6	125	8.4	148	7.1
5	7.9	29	6.9	53	6.8	77	6.6	101	6.6	126	8.3	149	6.5
6	8.4	30	6.8	54	7.4	78	6.7	102	6.6	127	8.6	150	6.6
7	8.1	31	6.9	55	8.1	79	6.8	103	6.6	128	8.4	151	6.6
8	8.4	32	6.9	56	7.7	80	6.9	104	6.5	129	8.1	152	6.7
9	8.4	33	6.8	57	7.6	81	6.7	105	7.5	130	7.8	153	6.5
10	8.4	34	6.9	58	6.6	82	6.6	106	7.6	131	8.2	154	6.5
11	8.8	35	6.9	59	6.8	83	6.3	107	7.5	132	8.6	155	6.5
12	8.5	36	6.8	60	6.7	84	6.7	108	8.2	133	8.4	156	6.6
13	7.5	37	6.9	61	6.7	85	6.3	109	7.9	134	8.5	157	6.5
14	8.2	38	6.9	62	6.8	86	6.8	110	7.8	135	8.0	158	6.5
15	8.0	39	7.1	63	6.8	87	6.9	111	7.5	136	8.3	159	6.6
16	7.6	40	7.3	64	6.9	88	6.9	112	7.3	137	8.2	160	6.7
17	7.7	41	7.4	65	6.8	89	6.7	113	8.2	138	7.9	161	6.8
18	7.5	42	7.9	66	7.0	90	6.6	114	7.9	139	7.9	162	6.6
19	7.6	43	8.0	67	7.1	91	6.6	115	7.5	140	8.0	163	6.5
20	7.9	44	7.9	68	7.8	92	6.9	116	7.9	141	8.0	164	7.0
21	7.6	45	6.8	69	8.0	93	7.6	117	7.8	142	8.4	165	7.5
22	7.7	46	7.3	70	8.1	94	8.2	118	8.2	143	8.3	166	7.7
23	8.3	47	8.6	71	8.1	95	8.0	119	8.1	144	8.3	167	8.3
24	8.2	48	7.9	72	8.3	96	7.9	120	8.2	121	8.2	168	7.8

Displays. Table 3.2 shows the data for 2<sup>nd</sup> June measured at one hour interval which is also displayed in figure 3.1.

**Table 3.2 Maximum Vibration Exploratory Data**

time, h	Max vib, mm/s	time, h	Max vib, mm/s
1	8.1	13	7.5
2	8.1	14	8.2
3	8.4	15	8.0
4	8.7	16	7.6
5	7.9	17	7.7
6	8.4	18	7.5
7	8.1	19	7.6
8	8.4	20	7.9
9	8.4	21	7.6
10	8.4	22	7.7
11	8.8	23	8.3
12	8.5	24	8.2



**Figure 3.1: Time Series Plot of Representative maximum vibration data**

Figure 3.1, which represents the data for maximum vibration over a 24 hour period seems to exhibit random behavior. Therefore, the work explores the application of stochastic models to analyze the data.

### 3.2 Time Series Modeling and Analysis

Stochastic models lend themselves to a sequence of ordered observed data. This type of data is typically referred to as time series data. The analysis of such data with the aim of obtaining the representative stochastic model is referred to as time series analysis. The following requirements as outlined by Das, S., 1994 are necessary for the application of time series analysis.

- a. The observations are obtained at uniform intervals in a discrete form.
- b. There must be a statistical dependence of the present data on the previous data.

This is expressed by autocorrelation between successive observations. The first requirement means that observations made at times  $t, t-1, t-2, t-3, t-4$  etc. are made at uniform times whereby the intervals ( $\Delta t$ ) is constant. The second requirement means that an observation made at time  $t$  ( $X_t$ ) has some degree of dependence on past observations ( $X_{t-1}, X_{t-2}, X_{t-3}, \dots, X_{t-j}$ ). Mathematically this is expressed as:

$$X_t = \sum \phi_j X_{t-j} + a_t \quad 3.1$$

Where  $\phi_j$  are autoregressive parameters,  $a_t$  is the error associated with the observation,  $X_t$  and  $j = 1, 2, 3, 4, \dots, n$ . These Autoregressive Models are shorthanded as **AR(n)** where  $n$  is the order of the model and  $n$  is how far back  $X_t$  depends on its past values. Specifically;

AR (1) model represents the model:  $X_t = \phi_1 X_{t-1} + a_t$

AR (2) model represents the model:  $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + a_t$

AR (3) model represents the model:  $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + a_t$

Since these equations express the dependence of  $X_t$  on its past values, the models are referred to as autoregressive models. Once the autoregressive parameters are determined, the model can be used to predict future values of  $X_t$  (that is  $X_{t+1}, X_{t+2},$

$X_{t+3}, \dots, X_{t+j}$ )

### 3.3 Estimation of Model Order and Parameters

The first order model ( $n=1$ ) or AR(1) is the simplest model that recognizes the dependence of the current data on the immediate past data. This expresses dependence of  $X_t$  on  $X_{t-1}$ ,  $X_{t-1}$  on  $X_{t-2}$ ,  $X_{t-2}$  on  $X_{t-3}$  creating a pairwise comparison  $(X_t, X_{t-1})$ ,  $(X_{t-1}, X_{t-2})$ ,  $(X_{t-2}, X_{t-3})$ , and so on. Each of the following represent an AR(1) model:

$$X_t = \phi_1 X_{t-1} + a_t$$

$$X_{t-1} = \phi_1 X_{t-2} + a_{t-1} \quad X_{t-2} = \phi_1 X_{t-3} + a_{t-2}$$

To verify this dependence, a scatter plot of  $X_t$  versus  $X_{t-1}$  must intuitively show a straight line trend. Figure 3.2 is a plot of  $X_t$  versus  $X_{t-1}$  for the maximum vibration of the generator from KTPP

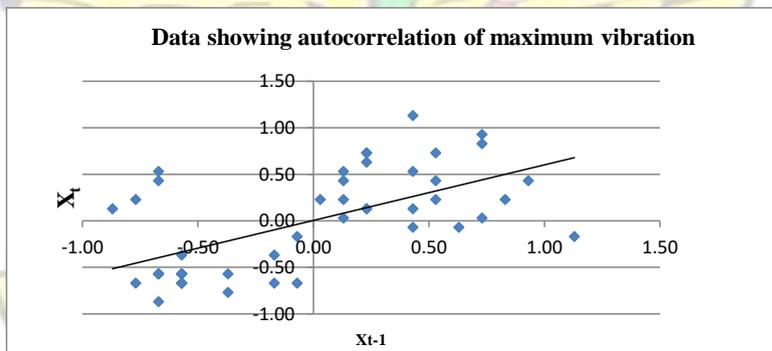


Figure 3.2: Scatter Plot of  $X_t$  versus  $X_{t-1}$  for exploratory maximum vibration

#### Data

Observation of Figure 3.2 indicates that there is a linear correlation in the data points. This project investigated the use of first order autoregressive model, AR (1) for analysis and prediction. The next step was to estimate the autoregressive parameter  $\phi_1$  and the probabilistic parameters that define the system. As with any modeling technique, the

best set of parameters are those that minimize the sum of squares of the errors (residuals).

Using this approach, the value of the first order autoregressive parameter  $\phi_1$  was found as:

$$\phi_1 = \frac{\sum_{t=2}^N X_t \cdot X_{t-1}}{\sum_{t=2}^N X_{t-1}^2} \text{ from } t = 2 \text{ to } t = N \text{ (number of observations).}$$

For the best fit, the errors must have a mean of Zero and a variance of  $\sigma_a^2 = \frac{1}{N} \sum_{t=2}^N a_t^2$ .



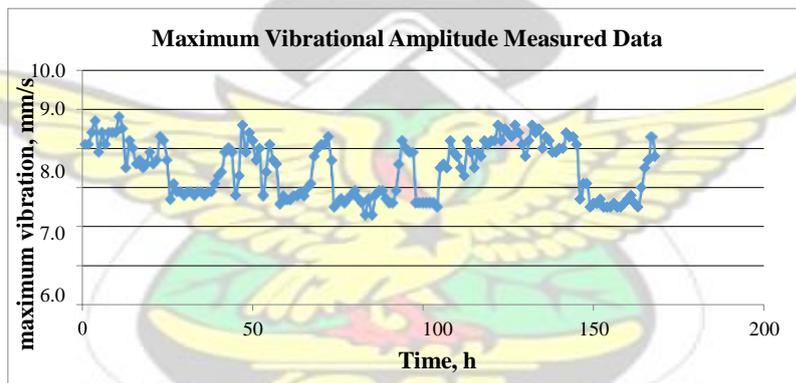
## CHAPTER FOUR

### DATA ANALYSIS AND DISCUSSIONS

This chapter tests the validity of the proposed autoregressive model for the data showed in Table 3.1. The data is then modeled to obtain its parameters. The model is then verified using residual analysis. Using a different set of data collected at another day, the predicted values are compared with the observed value to demonstrate the utilization of the model.

#### 4.1 Data Modeling

The data obtained in Chapter 3 and presented in Table 3.2 is now modeled according to the discussion in section 3.2. This set of data is plotted in Figure 4.1.



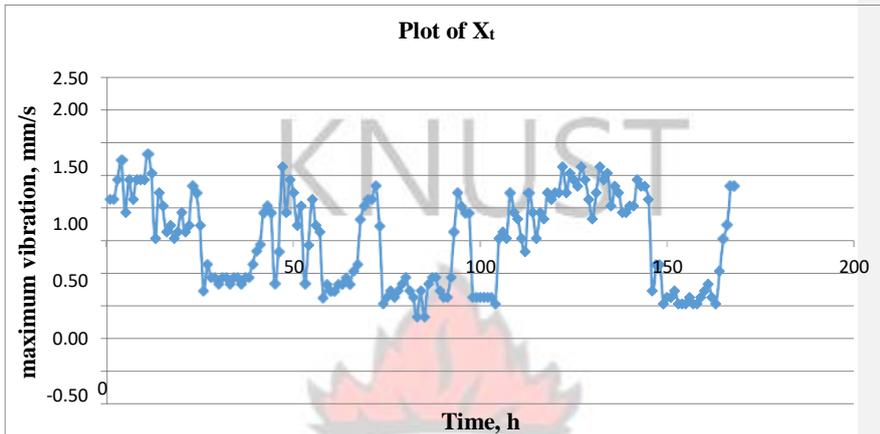
**Figure 4.1 Maximum Vibration Amplitude Observed Data**

For modeling purposes, it is convenient to use *average subtracted data*. The result allows one to avoid dealing with rather large numbers and rather complex analysis.

Thus definition of model  $X_t = \text{Observed Data} - \text{Data Average}$

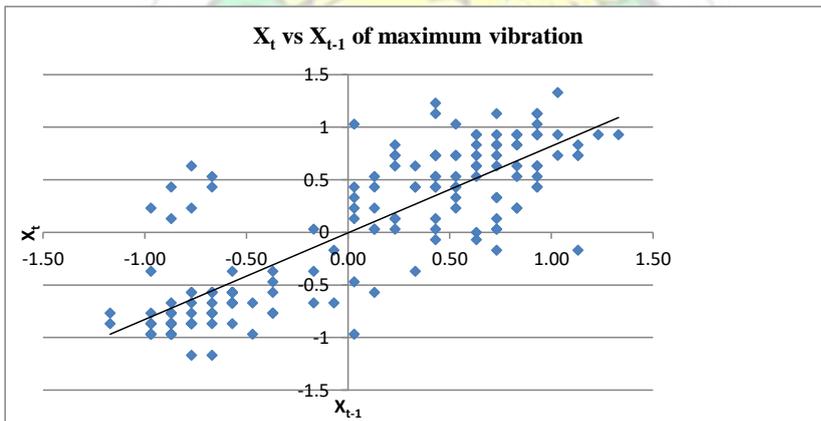
Figure 4.2 is a plot of the model data. It can be observed that both Figure 4.1 which uses the observed data and Figure 4.2, which uses the model data are identical.

The system behavior remains the same. It may also be observed that Figure 4.1 is centered on the mean value of the data while Figure 4.2 is centered on Zero.



**Figure 4.2 A plot of Compressed Data of Maximum Vibration Amplitude**

In order to determine the validity of using an autoregressive model, a scatter plot of  $X_t$  versus  $X_{t-1}$  results is obtained. This is presented in Figure 4.3.



**Figure 4.3. Scatter Plot of  $X_t$  versus  $X_{t-1}$**

In Figure 4.3, it is observed that the points are scattered around a straight line through the origin. The straight line trend shows that  $X_t$  is dependent on  $X_{t-1}$ . Specifically,  $X_2$  is dependent on  $X_1$ ,  $X_3$  is dependent on  $X_2$ ,  $X_4$  is dependent on  $X_3$ ,  $X_5$  is dependent on

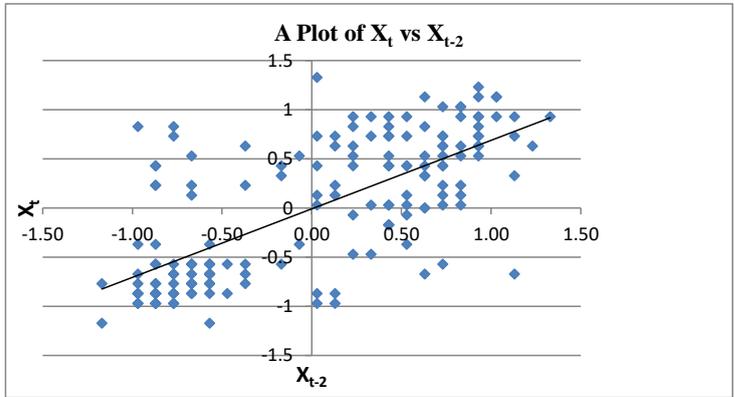
$X_4$ ,  $X_6$  is dependent on  $X_5$  and so on. Using a simple regression method that minimizes the sum of squares of the errors, the autoregressive parameter  $\phi$  is obtained to be 0.83128. Appendix B presents an Excel Spreadsheet developed for the analysis. Thus the maximum vibration may now be described by:

$$X_t = 0.83128X_{t-1} + \text{at} \dots\dots\dots (4.1)$$

The value of  $\phi$  indicates a strong dependence on relation between  $X_t$  and  $X_{t-1}$ .

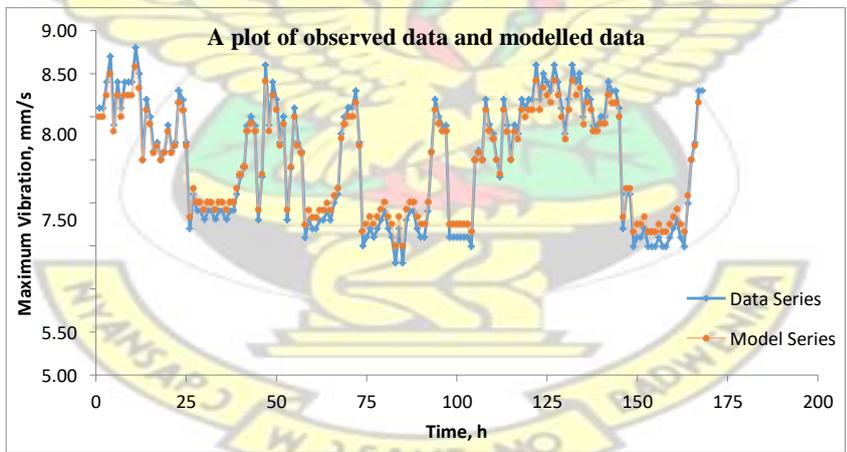
#### 4.2 Check of Model Adequacy

It is established in section 4.1 that there is a strong dependence of  $X_t$  on  $X_{t-1}$ . However, there could be dependence of  $X_t$  on  $X_{t-2}$ ,  $X_{t-3}$ , and so on leading to higher order AR (n) models as discussed in section 3.2. Thus there is the need to check such dependence. Figure 4.4 shows a plot of  $X_t$  versus  $X_{t-2}$ . The data seems to be scattered and still shows a strong linear relationship of  $X_t$  on  $X_{t-2}$  similar to what was shown in Figure 4.3 which shows the dependence of  $X_t$  on  $X_{t-1}$ . Therefore, this is an indication that a higher order model might be applicable as well. For the purpose of this work though it has been established that a higher order model is applicable only the AR (1) model was used.



**Figure 4.4 Scatter Plot of  $X_t$  versus  $X_{t-2}$**

Figure 4.5 shows the  $X_t$  data obtained from the observation with  $X_t$  generated from the model superimposed on it. The similarity of the two plots is one of the proofs of the adequacy of the AR(1) model to describe the system. Section 4.3 quantifies the degree of adequacy of the model.



**Figure 4.5. Comparison of the Data Series and the Model Series**

**Commented [EA1]:** Not in list of figures

### 4.3 Prediction of Maximum Vibration Amplitude

Inspection of Figure 4.5 shows deviations of the data series from the model series. These deviations are the errors or residuals. The statistical nature of these errors allow for the estimation of the confidence intervals of each measurement. For the model to be complete, the variance of the errors ( $a_t$ ) is needed. From equation 3.1, the errors are assumed to be random with a mean value of Zero and a variance,  $\sigma_a^2 = (1/N) \sum a_t^2$ . The variance of the errors was obtained to be 0.15. Thus the errors are Normally Independently Distributed with Mean of Zero and Variance of 0.15 (that is  $a_t \sim \text{NID}(0, \sigma_a^2)$ ).

Thus the complete statistical model for the prediction of the vibration is:

Predicted model value  $X_t = 0.83128X_{t-1} + a_t$  and  $a_t = \text{NID}(0, 0.15)$

Since the number of observations are statistically large, Z-Distribution is used to estimate the error at each observation. The error at the 95% Confidence is then,

$$e_t = \pm 1.96 \cdot \sqrt{0.15} = \pm 0.76$$

Finally, the prediction of the observed value is:  $(X_t + \text{Average}) \pm 0.76$

Table 4.1 shows the observed values and the predicted values for a the 5<sup>th</sup> of June and whether the observed values fall within the predicted range at 95% confidence.

**Table 4.1 Comparison of Observed Value and the Predicted Value.**

Hour	observed data (June 5th)	predicted value (xt+average)	95% Confidence Limits of Predicted Values; -0.765	95% Confidence Limits of Predicted Values;+ 0.765	Accuracy Y/N
1	7.7	-	-	-	-
2	6.5	7.4	6.6	8.2	Y
3	6.6	7.9	7.2	8.7	Y
4	6.7	7.9	7.2	8.7	Y
5	6.6	8.2	7.4	9.0	Y
6	6.7	8.4	7.7	9.2	Y
7	6.8	7.8	7.0	8.5	Y
8	6.9	8.2	7.4	9.0	Y
9	6.7	7.9	7.2	8.7	Y
10	6.6	8.2	7.4	9.0	Y
11	6.3	8.2	7.4	9.0	Y
12	6.7	8.2	7.4	9.0	Y
13	6.3	8.5	7.8	9.3	Y
14	6.8	8.3	7.5	9.0	Y
15	6.9	7.4	6.7	8.2	Y
16	6.9	8.0	7.3	8.8	Y
17	6.7	7.9	7.1	8.6	Y
18	6.6	7.5	6.8	8.3	Y
19	6.6	7.6	6.8	8.4	Y
20	6.9	7.4	6.7	8.2	Y
21	7.6	7.5	6.8	8.3	Y
22	8.2	7.8	7.0	8.5	Y
23	8	7.5	6.8	8.3	Y
24	7.9	7.6	6.8	8.4	Y
<b>Average</b>	6.65				

In order to check the accuracy of the model, data for June 5th was used. The results are shown in Table 4.1. Column two in the table shows the actual data collected on June 5<sup>th</sup>. Column three shows the predicted values based on the AR (1) model. Column four and five show the 95% confidence limits of the prediction considering an error of – 0.767 and +0.765 respectively. Comparing column two with columns four and five, it is observed that the predicted values fall within the 95% confidence limits of the predicted values. This validates the prediction model.

## CHAPTER FIVE

### CONCLUSIONS AND RECOMMENDATIONS

The main aim of this work is to improve the operational efficiency of the gas turbine through the monitoring, modeling and prediction of the maximum vibration amplitude at the Kpone Thermal Power Plant.

#### 5.1 Conclusions

The following conclusions may be drawn from the results and analysis of maximum vibration amplitude data:

- i. There is a linear dependence of the maximum vibration amplitude on its past values as  $X_t$  depends on  $X_{t-1}$ .
- ii. An autoregressive model of order one, AR (1) was used to describe the dynamics of the maximum vibration amplitude at any time.
- iii. When the current maximum vibration amplitude is known, the vibration amplitude at the next hour may be predicted within a margin of error of  $\pm 0.76$  with 95% confidence using the equation  $X_t = 0.83128X_{t-1} \pm 0.76$ .
- iv. By using the model developed, any vibration amplitude that would exceed the desired value can be seen before hand and corrective measures put in place before any damage is caused.

#### 5.2 Recommendations

Monitoring of the maximum vibration amplitude with the designed model can be done to ensure the efficiency of power generation. Further study in relation to modelling of other parameters of the thermal plant can be undertaken as the maximum vibration amplitude is not the only essential aspect of the thermal plant. The model can be implemented as a valid detection system on other thermal plants.

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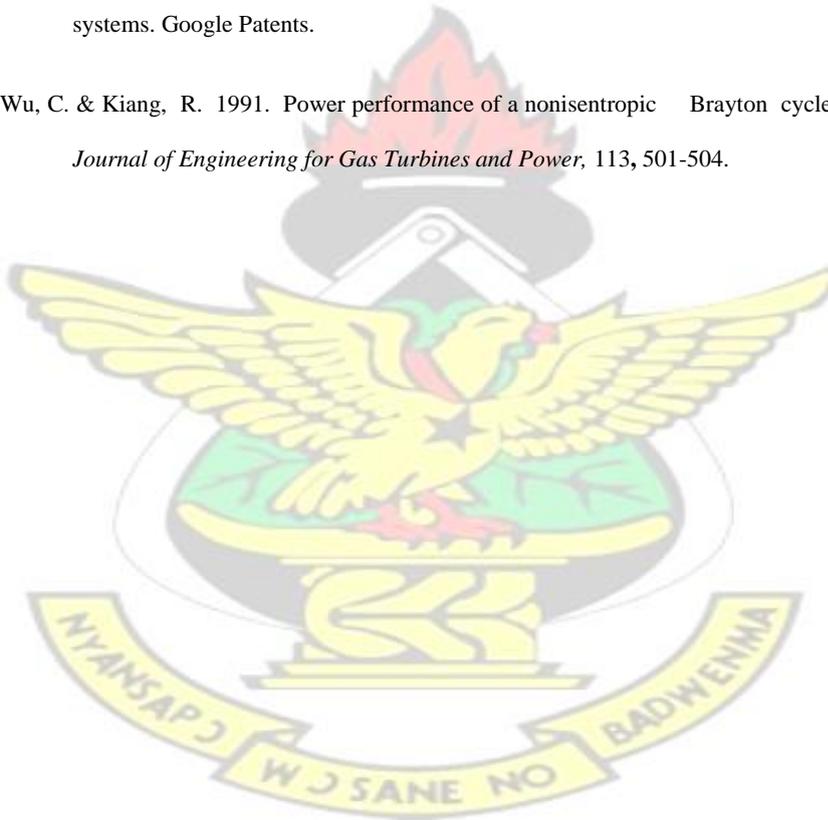
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**APPENDIX A- Monitored Data for Various Parameters of the Kpone**

**Thermal Power Plant**

MW	MVAR	CPD	BB_Max	LIQ FUEL FLOW (Kg/s)	AMB TEMP OC	EXHT TEMP OC
107	18	11	8.1	8.2	26	553.0
107	22	11	8.1	8.3	27	555.0
108	21	11	8.4	8.3	26	554.0
107	24	11	8.7	8.4	27	555.0
108	25	11	7.9	8.4	26	555.0
109	30	11	8.4	8.4	26	554.0
108	15	11	8.1	8.1	27	555.0
106	19	11	8.4	8.1	29	558.0
105	19	11	8.4	8.2	31	558.0
104	19	11	8.4	8.1	33	559.0
103	23	10	8.8	8.1	34	559.0
103	25	10	8.5	7.9	34	559.0
61	19	9	7.5	5.3	34	412.0
59	18	9	8.2	5.3	34	409.0
60	19	9	8.0	5.3	32	412.0
111	12	10.7	7.6	8.2	28	556
110	11	10.7	7.7	8.2	28	556
110	11	10.7	7.5	8.2	28	557
111	10	10.7	7.6	8.2	27	556
111	13	10.7	7.9	8.3	27	555
111	11	10.7	7.6	8.3	27	556
112	11	10.7	7.7	8.3	27	556
111	12	10.7	8.3	8.2	27	556
112	13	10.8	8.2	8.3	26	555
60	13	9.3	7.7	4.9	27	369
60	12	10.0	6.7	5.2	27	386
61	12	9.5	7.1	5.1	26	385
60	13	9.5	6.9	5.2	27	386
60	10	10.0	6.9	5.2	26	340
60	12	10.0	6.8	5.2	27	387
60	13	9.6	6.9	5.2	27	414
59	13	9.5	6.9	5.1	27	382
60	11	9.5	6.8	5.1	28	387
58	12	9.3	6.9	4.9	28	375
60	12	9.4	6.9	5.2	28	377
59	11	9.4	6.8	5.2	28	385
60	12	9.4	6.9	5.2	28	386
62	16	9.4	6.9	5.2	28	388
61	16	9.3	7.1	5.1	28	390

110	21	10.7	7.3	8.3	28	556
110	20	10.6	7.4	8.3	28	557
111	21	10.7	7.9	8.3	27	556
111	29	10.7	8.0	8.3	27	556
112	30	10.7	7.9	8.3	27	555
60	20	9.5	6.8	5.1	27	382
112	21	10.7	7.3	8.3	27	556
111	20	10.8	8.6	8.4	27	556
111	19	10.8	7.9	8.3	27	555
112	20	10.8	8.4	8.3	27	555
112	21	10.8	8.2	8.3	27	555
112	21	10.9	7.7	8.2	27	554
113	20	10.8	8.0	8.3	25	554
86	19	10.2	6.8	6.5	25	460
113	21	10.9	7.4	8.4	25	554
113	15	10.9	8.1	8.4	26	553
111	16	10.7	7.7	8.3	28	556
110	16	10.7	7.6	8.3	28	556
86	14	10.2	6.6	6.7	28	477
60	16	9.5	6.8	5.2	28	391
60	14	9.5	6.7	5.0	28	389
60	15	9.5	6.7	5.2	29	388
60	15	9.5	6.8	5.1	29	390
60	15	9.5	6.8	5.2	28	390
53	15	8.9	6.9	4.6	28	372
60	16	9.5	6.8	5.1	28	387
62	15	9.4	7.0	5.0	27	381
112	16	10.8	7.1	8.3	26	555
111	13	10.7	7.8	8.3	27	556
112	12	10.8	8.0	8.4	27	554
111	13	10.8	8.1	7.9	27	555
111	14	10.8	8.1	8.4	27	554
111	14	10.8	8.3	8.4	27	554
62	16	9.6	7.7	5.2	25	385
60	16	9.6	6.5	5.2	26	382
60	14	9.6	6.6	5.2	25	380
59	14	9.6	6.7	5.2	25	379
60	14	9.5	6.6	4.9	25	375
60	16	9.6	6.7	5.1	25	377
60	15	9.6	6.8	5.2	25	383
59	15	9.4	6.9	5.2	27	386
59	14	9.5	6.7	5.2	28	389
60	15	9.5	6.6	5.1	28	389
58	14	9.5	6.3	5.1	29	388

60	14	9.4	6.7	5.1	30	394
61	15	9.5	6.3	5.2	29	394
59	16	9.4	6.8	5.2	29	393
61	20	9.4	6.9	5.1	29	392
60	20	9.4	6.9	5.2	29	395
58	17	9.5	6.7	5.2	29	388
60	19	9.5	6.6	5.2	28	389
109	25	10.6	6.6	8.1	29	558
110	24	10.6	6.9	8.2	28	557
109	25	10.6	7.6	8.2	28	557
110	25	10.7	8.2	8.2	28	556
111	26	10.8	8.0	8.3	27	555
111	21	10.8	7.9	8.3	27	555
61	11	10.8	7.9	8.3	27	555
61	11	9.6	6.6	5.3	26	381
60	10	9.6	6.6	5.1	25	381
61	11	9.6	6.6	5.1	25	381
61	12	9.6	6.6	5.1	25	381
59	10	9.6	6.6	5.1	25	381
60	9	9.6	6.6	5.2	24	376
60	10	9.5	6.5	5.2	25	386
110	10	10.7	7.5	8.3	28	556
110	9	10.8	7.6	8.3	27	555
109	21	10.7	7.5	8.3	28	556
109	20	10.7	8.2	8.2	29	556
109	19	10.7	7.9	8.2	29	556
108	18	10.6	7.8	8.2	29	559
109	17	10.7	7.5	8.2	28	558
108	19	10.5	7.3	8.1	28	558
110	20	10.7	8.2	8.3	28	556
110	21	10.8	7.9	8.2	28	555
109	20	10.5	7.5	8.1	27	559
110	20	10.8	7.9	8.2	27	555
110	19	10.8	7.8	8.3	27	555
110	20	10.8	8.2	8.3	27	555
110	20	10.8	8.1	8.3	27	555
110	15	10.8	8.2	8.3	27	555
112	15	10.8	8.2	8.3	25	554
111	10	10.8	8.6	8.3	25	554
110	9	10.9	8.2	8.3	25	554
113	13	10.9	8.5	8.3	25	553
112	11	10.9	8.4	8.3	25	554
113	12	10.9	8.3	8.3	26	553
112	10	10.9	8.6	8.3	27	554

112	11	10.9	8.4	8.4	25	553
111	10	10.9	8.1	8.3	26	554
110	10	10.8	7.8	8.2	27	555
109	7	10.7	8.2	8.1	29	556
109	9	10.7	8.6	8.3	28	556
107	9	10.7	8.4	8.3	29	557
108	10	10.7	8.5	8.2	29	556
108	10	10.7	8.0	8.1	29	557
107	9	10.5	8.3	8.0	29	558
107	12	10.5	8.2	8.0	28	559
109	14	10.7	7.9	8.3	28	556
109	19	10.7	7.9	8.2	28	556
109	17	10.5	8.0	8.1	28	559
110	20	10.7	8.0	8.3	27	555
111	21	10.8	8.4	8.2	27	554
111	16	10.8	8.3	8.2	26	554
111	14	10.8	8.3	8.3	26	554
110	8	10.8	8.1	8.3	27	555
60	5	9.6	6.7	5.2	27	387
60	6	9.6	7.1	5.2	27	387
60	6	9.6	7.1	5.2	27	387
59	4	9.5	6.5	5.1	26	383
60	5	9.5	6.6	5.1	26	383
59	4	9.6	6.6	5.1	26	385
60	5	9.6	6.7	5.2	27	385
60	6	9.5	6.5	5.3	28	392
60	5	9.5	6.5	5.1	28	387
61	6	9.5	6.5	5.2	28	391
60	5	9.5	6.6	5.3	28	392
61	5	9.4	6.5	5.3	28	387
58	6	9.5	6.5	5.2	28	392
60	6	9.5	6.6	5.2	28	391
60	15	9.5	6.7	5.3	28	390
60	14	9.5	6.8	5.2	28	390
60	21	9.5	6.6	5.2	28	389
109	21	10.6	6.5	8.1	28	558
110	19	10.7	7.0	8.3	27	556
109	21	10.6	7.5	8.1	27	557
111	21	10.8	7.7	8.4	27	555
110	19	10.8	8.3	8.4	27	555
110	21	10.8	8.3	8.4	27	555

**APPENDIX B – Development of the Maximum Vibration Amplitude Model**

SN	X Dot	Xtee	Xtee-1	(Xtee-1)SQ	Xtee*Xtee- 1	PHI*Xtee-1	Xtee-PHI*Xtee-1
1	8.10	0.63	0	0	0	0	0.63
2	8.10	0.63	0.63	0.3969	0.39	0.5236938	0.1063062
3	8.40	0.93	0.63	0.3969	0.58	0.5236938	0.4063062
4	8.70	1.23	0.93	0.8649	1.14	0.7730718	0.4569282
5	7.90	0.43	1.23	1.5129	0.52	1.0224498	-0.5924498
6	8.40	0.93	0.43	0.1849	0.39	0.3574418	0.5725582
7	8.10	0.63	0.93	0.8649	0.58	0.7730718	-0.1430718
8	8.40	0.93	0.63	0.3969	0.58	0.5236938	0.4063062
9	8.40	0.93	0.93	0.8649	0.86	0.7730718	0.1569282
10	8.40	0.93	0.93	0.8649	0.86	0.7730718	0.1569282
11	8.80	1.33	0.93	0.8649	1.23	0.7730718	0.5569282
12	8.50	1.03	1.33	1.7689	1.36	1.1055758	-0.0755758
13	7.50	0.03	1.03	1.0609	0.03	0.8561978	-0.8261978
14	8.20	0.73	0.03	0.0009	0.02	0.0249378	0.7050622
15	8.00	0.53	0.73	0.5329	0.38	0.6068198	-0.0768198
16	7.60	0.13	0.53	0.2809	0.06	0.4405678	-0.3105678
17	7.70	0.23	0.13	0.0169	0.02	0.1080638	0.1219362
18	7.50	0.03	0.23	0.0529	0.00	0.1911898	-0.1611898
19	7.60	0.13	0.03	0.0009	0.00	0.0249378	0.1050622
20	7.90	0.43	0.13	0.0169	0.05	0.1080638	0.3219362
21	7.60	0.13	0.43	0.1849	0.05	0.3574418	-0.2274418
22	7.70	0.23	0.13	0.0169	0.02	0.1080638	0.1219362
23	8.30	0.83	0.23	0.0529	0.19	0.1911898	0.6388102
24	8.20	0.73	0.83	0.6889	0.60	0.6899458	0.0400542
25	7.70	0.23	0.73	0.5329	0.16	0.6068198	-0.3768198
26	6.70	-0.77	0.23	0.0529	-	0.1911898	-0.9611898
27	7.10	-0.37	-0.77	0.5929	0.28	-0.6400702	0.2700702
28	6.90	-0.57	-0.37	0.1369	0.21	-0.3075662	-0.2624338
29	6.90	-0.57	-0.57	0.3249	0.32	-0.4738182	-0.0961818
30	6.80	-0.67	-0.57	0.3249	0.38	-0.4738182	-0.1961818
31	6.90	-0.57	-0.67	0.4489	0.38	-0.5569442	-0.0130558
32	6.90	-0.57	-0.57	0.3249	0.32	-0.4738182	-0.0961818
33	6.80	-0.67	-0.57	0.3249	0.38	-0.4738182	-0.1961818
34	6.90	-0.57	-0.67	0.4489	0.38	-0.5569442	-0.0130558
35	6.90	-0.57	-0.57	0.3249	0.32	-0.4738182	-0.0961818
36	6.80	-0.67	-0.57	0.3249	0.38	-0.4738182	-0.1961818
37	6.90	-0.57	-0.67	0.4489	0.38	-0.5569442	-0.0130558
38	6.90	-0.57	-0.57	0.3249	0.32	-0.4738182	-0.0961818
39	7.10	-0.37	-0.57	0.3249	0.21	-0.4738182	0.1038182

SN	X Dot	Xtee	Xtee-1	(Xtee-1)SQ	Xtee*Xtee-1	PHI*Xtee-1	Xtee-PHI*Xtee-1
40	7.30	-0.17	-0.37	0.1369	0.06	-0.3075662	0.1375662
41	7.40	-0.07	-0.17	0.0289	0.01	-0.1413142	0.0713142
42	7.90	0.43	-0.07	0.0049	-	-0.0581882	0.4881882
43	8.00	0.53	0.43	0.1849	0.22	0.3574418	0.1725582
44	7.90	0.43	0.53	0.2809	0.22	0.4405678	-0.0105678
45	6.80	-0.67	0.43	0.1849	-	0.3574418	-1.0274418
46	7.30	-0.17	-0.67	0.4489	0.11	-0.5569442	0.3869442
47	8.60	1.13	-0.17	0.0289	-	-0.1413142	1.2713142
48	7.90	0.43	1.13	1.2769	0.48	0.9393238	-0.5093238
49	8.40	0.93	0.43	0.1849	0.39	0.3574418	0.5725582
50	8.20	0.73	0.93	0.8649	0.67	0.7730718	-0.0430718
51	7.70	0.23	0.73	0.5329	0.16	0.6068198	-0.3768198
52	8.00	0.53	0.23	0.0529	0.12	0.1911898	0.3388102
53	6.80	-0.67	0.53	0.2809	-	0.4405678	-1.1105678
54	7.40	-0.07	-0.67	0.4489	0.04	-0.5569442	0.4869442
55	8.10	0.63	-0.07	0.0049	-	-0.0581882	0.6881882
56	7.70	0.23	0.63	0.3969	0.14	0.5236938	-0.2936938
57	7.60	0.13	0.23	0.0529	0.02	0.1911898	-0.0611898
58	6.60	-0.87	0.13	0.0169	-	0.1080638	-0.9780638
59	6.80	-0.67	-0.87	0.7569	0.58	-0.7231962	0.0531962
60	6.70	-0.77	-0.67	0.4489	0.51	-0.5569442	-0.2130558
61	6.70	-0.77	-0.77	0.5929	0.59	-0.6400702	-0.1299298
63	6.80	-0.67	-0.67	0.4489	0.44	-0.5569442	-0.1130558
64	6.90	-0.57	-0.67	0.4489	0.38	-0.5569442	-0.0130558
65	6.80	-0.67	-0.57	0.3249	0.38	-0.4738182	-0.1961818
66	7.00	-0.47	-0.67	0.4489	0.31	-0.5569442	0.0869442
67	7.10	-0.37	-0.47	0.2209	0.17	-0.3906922	0.0206922
68	7.80	0.33	-0.37	0.1369	-	-0.3075662	0.6375662
69	8.00	0.53	0.33	0.1089	0.17	0.2743158	0.2556842
70	8.10	0.63	0.53	0.2809	0.33	0.4405678	0.1894322
71	8.10	0.63	0.63	0.3969	0.39	0.5236938	0.1063062
72	8.30	0.83	0.63	0.3969	0.52	0.5236938	0.3063062
73	7.70	0.23	0.83	0.6889	0.19	0.6899458	-0.4599458
74	6.50	-0.97	0.23	0.0529	-	0.1911898	-1.1611898
75	6.60	-0.87	-0.97	0.9409	0.84	-0.8063222	-0.0636778
76	6.70	-0.77	-0.87	0.7569	0.66	-0.7231962	-0.0468038
77	6.60	-0.87	-0.77	0.5929	0.66	-0.6400702	-0.2299298
78	6.70	-0.77	-0.87	0.7569	0.66	-0.7231962	-0.0468038
79	6.80	-0.67	-0.77	0.5929	0.51	-0.6400702	-0.0299298
80	6.90	-0.57	-0.67	0.4489	0.38	-0.5569442	-0.0130558
81	6.70	-0.77	-0.57	0.3249	0.43	-0.4738182	-0.2961818

SN	X Dot	Xtee	Xtee-1	(Xtee-1)SQ	Xtee*Xtee-1	PHI*Xtee-1	Xtee-PHI*Xtee-1
82	6.60	-0.87	-0.77	0.5929	0.66	-0.6400702	-0.2299298
83	6.30	-1.17	-0.87	0.7569	1.01	-0.7231962	-0.4468038
84	6.70	-0.77	-1.17	1.3689	0.90	-0.9725742	0.2025742
85	6.30	-1.17	-0.77	0.5929	0.90	-0.6400702	-0.5299298
86	6.80	-0.67	-1.17	1.3689	0.78	-0.9725742	0.3025742
87	6.90	-0.57	-0.67	0.4489	0.38	-0.5569442	-0.0130558
88	6.90	-0.57	-0.57	0.3249	0.32	-0.4738182	-0.0961818
89	6.70	-0.77	-0.57	0.3249	0.43	-0.4738182	-0.2961818
90	6.60	-0.87	-0.77	0.5929	0.66	-0.6400702	-0.2299298
91	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
92	6.90	-0.57	-0.87	0.7569	0.49	-0.7231962	0.1531962
93	7.60	0.13	-0.57	0.3249	-	-0.4738182	0.6038182
94	8.20	0.73	0.13	0.0169	0.09	0.1080638	0.6219362
95	8.00	0.53	0.73	0.5329	0.38	0.6068198	-0.0768198
96	7.90	0.43	-0.53	0.2809	0.22	0.4405678	-0.0105678
97	7.90	0.43	0.43	0.1849	0.18	0.3574418	0.0725582
98	6.60	-0.87	0.43	0.1849	-	0.3574418	-1.2274418
99	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
100	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
101	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
102	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
103	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
104	6.50	-0.97	-0.87	0.7569	0.84	-0.7231962	-0.2468038
105	7.50	0.03	-0.97	0.9409	-	-0.8063222	0.8363222
106	7.60	0.13	0.03	0.0009	0.00	0.0249378	0.1050622
107	7.50	0.03	0.13	0.0169	0.00	0.1080638	-0.0780638
108	8.20	0.73	0.03	0.0009	0.02	0.0249378	0.7050622
109	7.90	0.43	0.73	0.5329	0.31	0.6068198	-0.1768198
110	7.80	0.33	0.43	0.1849	0.14	0.3574418	-0.0274418
111	7.50	0.03	0.33	0.1089	0.00	0.2743158	-0.2443158
112	7.30	-0.17	0.03	0.0009	-	0.0249378	-0.1949378
113	8.20	0.73	-0.17	0.0289	-	-0.1413142	0.8713142
114	7.90	0.43	0.73	0.5329	0.31	0.6068198	-0.1768198
115	7.50	0.03	0.43	0.1849	0.01	0.3574418	-0.3274418
116	7.90	0.43	0.03	0.0009	0.01	0.0249378	0.4050622
117	7.80	0.33	0.43	0.1849	0.14	0.3574418	-0.0274418
118	8.20	0.73	0.33	0.1089	0.24	0.2743158	0.4556842
119	8.10	0.63	0.73	0.5329	0.45	0.6068198	0.0231802
120	8.20	0.73	0.63	0.3969	0.45	0.5236938	0.2063062
121	8.20	0.73	0.73	0.5329	0.53	0.6068198	0.1231802
122	8.60	1.13	0.73	0.5329	0.82	0.6068198	0.5231802

SN	X Dot	Xtee	Xtee-1	(Xtee-1)SQ	Xtee*Xtee-1	PHI*Xtee-1	Xtee-PHI*Xtee-1
123	8.20	0.73	1.13	1.2769	0.82	0.9393238	-0.2093238
124	8.50	1.03	0.73	0.5329	0.75	0.6068198	0.4231802
125	8.40	0.93	1.03	1.0609	0.95	0.8561978	0.0738022
126	8.30	0.83	0.93	0.8649	0.77	0.7730718	0.0569282
127	8.60	1.13	0.83	0.6889	0.93	0.6899458	0.4400542
128	8.40	0.93	1.13	1.2769	1.05	0.9393238	-0.0093238
129	8.10	0.63	0.93	0.8649	0.58	0.7730718	-0.1430718
130	7.80	0.33	0.63	0.3969	0.20	0.5236938	-0.1936938
131	8.20	0.73	0.33	0.1089	0.24	0.2743158	0.4556842
132	8.60	1.13	0.73	0.5329	0.82	0.6068198	0.5231802
133	8.40	0.93	1.13	1.2769	1.05	0.9393238	-0.0093238
134	8.50	1.03	0.93	0.8649	0.95	0.7730718	0.2569282
135	8.00	0.53	1.03	1.0609	0.54	0.8561978	-0.3261978
136	8.30	0.83	0.53	0.2809	0.43	0.4405678	0.3894322
137	8.20	0.73	0.83	0.6889	0.60	0.6899458	0.0400542
138	7.90	0.43	0.73	0.5329	0.31	0.6068198	-0.1768198
139	7.90	0.43	0.43	0.1849	0.18	0.3574418	0.0725582
140	8.00	0.53	0.43	0.1849	0.22	0.3574418	0.1725582
141	8.00	0.53	0.53	0.2809	0.28	0.4405678	0.0894322
142	8.40	0.93	0.53	0.2809	0.49	0.4405678	0.4894322
143	8.30	0.83	0.93	0.8649	0.77	0.7730718	0.0569282
144	8.30	0.83	0.83	0.6889	0.68	0.6899458	0.1400542
145	8.10	0.63	0.83	0.6889	0.52	0.6899458	-0.0599458
146	6.70	-0.77	0.63	0.3969	-	0.5236938	-1.2936938
147	7.10	-0.37	-0.77	0.5929	0.28	-0.6400702	0.2700702
148	7.10	-0.37	-0.37	0.1369	0.13	-0.3075662	-0.0624338
149	6.50	-0.97	-0.37	0.1369	0.35	-0.3075662	-0.6624338
150	6.60	-0.87	-0.97	0.9409	0.84	-0.8063222	-0.0636778
151	6.60	-0.87	-0.87	0.7569	0.75	-0.7231962	-0.1468038
152	6.70	-0.77	-0.87	0.7569	0.66	-0.7231962	-0.0468038
154	6.50	-0.97	-0.97	0.9409	0.94	-0.8063222	-0.1636778
155	6.50	-0.97	-0.97	0.9409	0.94	-0.8063222	-0.1636778
156	6.60	-0.87	-0.97	0.9409	0.84	-0.8063222	-0.0636778
157	6.50	-0.97	-0.87	0.7569	0.84	-0.7231962	-0.2468038
158	6.50	-0.97	-0.97	0.9409	0.94	-0.8063222	-0.1636778
159	6.60	-0.87	-0.97	0.9409	0.84	-0.8063222	-0.0636778
160	6.70	-0.77	-0.87	0.7569	0.66	-0.7231962	-0.0468038
161	6.80	-0.67	-0.77	0.5929	0.51	-0.6400702	-0.0299298
162	6.60	-0.87	-0.67	0.4489	0.58	-0.5569442	-0.3130558
163	6.50	-0.97	-0.87	0.7569	0.84	-0.7231962	-0.2468038
164	7.00	-0.47	-0.97	0.9409	0.45	-0.8063222	0.3363222
165	7.50	0.03	-0.47	0.2209	-	-0.3906922	0.4206922

SN	X Dot	Xtee	Xtee-1	(Xtee-1)SQ	Xtee*Xtee-1	PHI*Xtee-1	Xtee-PHI*Xtee-1
166	7.70	0.23	0.03	0.0009	0.00	0.0249378	0.2050622
167	8.30	0.83	0.23	0.0529	0.19	0.1911898	0.6388102
168	8.30	0.83	0.83	0.6889	0.68	0.6899458	0.1400542
<b>Average</b>	7.47			81.8183	68.0		
		<b>PHI</b>	0.83128				
		<b>SIGMA A SQUARE</b>					
	plus/minus	<b>ERROR</b>	0.76467				

