DECLARATION

I hereby declare that this submission is my own work towards the Mphil and that to the best of my knowledge, it contains neither material previously published by another person nor material which has been accepted for the award of any other degree of the university or elsewhere, except where due acknowledgement has been made in the text.

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DEDICATION

This work is humbly dedicated to the almighty Allah, the most gracious, the most merciful.

It is as well dedicated to the members of my entire family through whose prayers and support we have reached the level we are today.

ACKNOWLEDGEMENT

First and foremost, my sincere thanks go to the almighty Allah for his showers of blessing up on us up to date.

Next to the almighty Allah are my parents, without whom I would not have come this far.

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I am, indeed, grateful to them.

ABSTRACT

Telecommunication and mobile phone usage for that matter have assumed a centre stage in the Ghanaian economy like any other economy the world over to a degree that its possession is increasingly becoming a necessity. The objectives of this thesis are to model the selection of the best telecom network as a decision problem as well as identify the best telecommunication network out of five available telecom networks in the Greater Accra region. These five networks are represented as: A_1, A_2, A_3, A_4, and A_5. The model used to represent this problem is the decision model and the method employed is the PROMETHEE method. The National Communications Authority (NCA) has provided four major criteria for measuring the performance of the various networks. They include: Call Setup Time; Call Completion Rate; Call Congestion Rate and Call Drop Rate. Using the PROMETHEE methodology as a multiple criteria optimization technique, the five mentioned alternative networks are analyzed based on the above mentioned four criteria. However, a simple illustration of the single criterion optimization technique on this multicriteria decision problem is made to expose the inefficiency of the approach in solving such a multicriteria decision problem. The generalized preference function that is chosen for this study is the Gaussian preference function. Through this, partial and complete ranking of the alternatives are made to depict their performance from best to worst. Finally, upon the outcome of the analysis, the ranking is as follows: A_2, A_4, A_5, A_1, and A_3. Hence, the best network is A_2 and the worst is A_3.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the study

Geographically, Greater Accra region is the smallest in terms of land mass, in Ghana (Wikipedia, 2011). It has an area cover of three thousand two hundred and forty-five (3245) km square, representing 1.4 percent of the total land surface of Ghana (Wikipedia, 2011).

Historically, in the sixties, Greater Accra, which was then known as Accra capital district, was formally and geographically part of the eastern region. It was managed separately by the minister in charge of local government until 23rd July, 1982 when the greater Accra was made a separate region legally mandated by the greater Accra region law (PNDCL 26). This region was carved to include the Ada local council area.

In terms of location and size, it is bordered to the west by the central region, to the north by the eastern region, to the east by the Volta region and south by the gulf of guinea. It is the smallest among the ten regions of Ghana and made up of ten (10) districts.

Though, regarded as the smallest, it is the second most populated region next to the Ashanti region of Ghana. It had a population of two million, nine hundred and five thousand, seven hundred and twenty-six (2,905726) (Ghana statistical service, 2000) which account for 15.4 percent of the total population of Ghana. This is again the region that harbors the national capital and the seat of government as well. Due to a high growth rate and in-migration, the region has the highest population density in the country (Ghana statistical service, 2000).

To the educational front, the region can boast of one public university, university of Ghana, and twelve (12) private universities and university colleges. Some of these include: Ghana Telecom University College, Tesano, Accra, Islamic University College, East Legon, Accra, Methodist University College Dansoman, Accra, Ashesi University College Mataheko, Accra, the Regional Maritime University Nungua, Accra etc.

On the part of sports and football in particular, the region has prided itself with four (4) premier clubs namely, Accra Hearts of Oak, Accra Hearts of Lions, Liberty Professionals, and Accra Great Olympics (Wikipedia, 2011)

The Greater Accra region as the main business centre of Ghana; the second most populated region next to Ashanti region; the region with the highest number of educational institutions; numerous recreational areas, monuments, parks, national historic sites for tourists and a host of others make the impact of communication and telecommunication for that matter on this region and its people a matter worthy of discussion Having realized the importance of telecommunication to the government, businesses and people of this region and even beyond by cell phone operators, five major competitors currently compete among themselves keenly in the region for subscribers. They include, Vodafone, Mtn, Tigo, Airtel, and Expresso.

Subscribers on the other hand, are faced with the challenge of identifying the best network in terms of service quality visa vise cost to either join or switch to.

The telecom sector of Ghana has been adjudged one of the most liberalized in Africa. As already mentioned, the market has two national fixed network operators and five operating mobile telephone companies. The fixed-line telephone segment is more or less a monopolistic market as Vodafone Ghana controls about ninety-eight percent (98%) of the market where as Airtel, the subsidiary provider of the fixed-line telephone, controls only two percent (2%) of the market shares. (Frempong, 2010).

Contrary to the fixed-line telephone market, mobile telephone market is highly competitive and this has led to a high penetration rate of mobile services in the country. As of 2008, the telephone penetration was 52.4%, and out of this ninety-nine percent (99%) contribution was made by mobile telephones. Recent studies have shown a decrease in the use of fixed-line telephones (Frempong, 2010).

At the moment, MTN Ghana is still the leading telephone operator in the market; controlling a market share of fifty-three percent (53%), next by Tigo with twenty-three percent (23%), Vodafone mobile with fourteen percent (14%) and Kasapa (Expresso) as the least holder commands two percent (2%) (Frempong, 2010).

Airtel Ghana has made giant strides into the mobile telephone market. A company that commenced business in the last quarter of 2008 now has a market share of eight percent (8%). This success story of the company had been due to the implementation of more effective business strategies. At the end of 2009, the country recorded over fifteen (15) million mobile subscribers (Frempong, 2010)

1.1.1 Advantages of Using Mobile Phone

The benefits of using mobile phone cannot be underestimated.

Carrying mobile phone with you where ever you may be reduces the likelihood of missing important messages and calls (wikipedia, 2011). If an individual happens to lose his way, he can easily gain direction back to the right course by means of a phone call. on emergencies such as arm robbery, accidents, fire outbreak, and such like, one can readily accessed the responsible agency – police, ambulance, fire service, the army and so on- for support. Cell phones can also be used to take pictures of special scenes namely, ceremonies, accidents and other events deemed so memorable. Text messages, games, and music can be played using mobile phones, perhaps for ridding oneself of boredom. As part of the benefits, individuals and groups can use it to communicate with their customers and clients on business, and communicate with the boss when getting late to duty. People use phones to perform basic arithmetic and also for storage of vital information. Through cell phones, people get access to the internet and use it to their advantage as a powerful search engine. on some aspects, mobile phones minimize cost of transportation involving the communication of information to friends, relatives and love ones provided those items of information do not necessarily require face to face contact.

1.1.2 Disadvantages of Using Mobile Phones

Seven main disadvantages of mobile phone usage have been identified (controlyour impact.com, 2011). These are:

(i) **Unsafe safety standards:** safety standards have been set aside to limit the microwave radiation exposure from mobile phones and base stations. However, the current safety standards for micro wave radiation emitted by mobile phones only take thermal heating into account: microwave radiation is considered to be safe if your body temperature does not increase by more than one degree. The non-thermal effects are not taken into account, making the 'safety standards' mockery of public health. Hence, new standards should be developed in order to protect the public from the possible health consequences of exposure to microwave radiation.

(ii) **Negative health effects:** micro wave radiations have been detected to have the effect of altering the electric activity of the brain and bring about disturbance in sleep. The radiations lead to lack of concentration, build up fatigue and the resulting ailments such as headache. The waves result in increased reaction time in a time-dependent manner. They increase the resting blood pressure and reduce the production of melatonin. They equally implicated in DNA strand breaks. It has also been established that mobile phone s damage key brain cells and could cause the early development of a disease called Alzheimer's disease. In this regard, recent researchers have discovered that radiations from mobile phones handsets damage areas of the brain associated with learning, memory and movement. not all, it is found that people who chart on mobile phone for hours a day are fifty percent (50%) more likely to develop mouth cancer than those who do not talk on them at all. The same way, people with mobile phones were two and half times more prone to a temporal brain tumor on the side of the head where they held their phone. For tumors of the auditory nerve, which connects the ear to the brain, the risk increased to more than three times for cell phone users.

(iii) **Male infertility:** At least fifteen percent (15%) of couples of reproductive age suffer from infertility. fifty percent (50%) of this figure is a result of men's infertility. Research results suggested that the use of cell phones by men is associated with a decrease in semen quality. The decrease in sperm count, motility, viability, and normal morphology is a function of the duration of man's exposure to a mobile phone. Again, storage of cell phones close to the testes has a significant negative impact on sperm concentration and the percentage of motile sperm. These trends suggest therefore that recent concerns over long term exposure to the electromagnetic irradiation emitted by mobile phones should be taken more seriously in view of the growing trend for deterioration in the male. Hence, an increase in the percentage of sperm cells with abnormal morphology is associated with the direction exposure to the waves emitted by mobile phone equipment. Research has also established that microwave radiation from mobile phones has the potential of damaging cellular DNA of maturing sperm. So mistakes at this point are likely to create mutations that could disrupt the normality of embryonic development and the health and the well being of the offspring.

(iv) **The effect on children:** preadolescent children have been found to be more vulnerable to microwave radiation because of their thinner skulls, their still developing nervous systems, their increased levels of cell division, and their less robust immune systems (Hyland, 2011). A one year old could absorb around double [The radio frequency (RF) radia], and five year old around sixteen percent (60%), more than an adult (steward, 2007). they added that since children were being exposed to RF radiation from base stations (and from mobile phones) from a younger age than adults, they would have a longer time in which to accumulate exposure over the course of their lives, and a longer time for any delayed effects of exposure to develop. A German study published in 2004 revealed that people who lived within 4oo meters of a cell phone tower for a period of five years have three (3) times higher risk of developing cancer. A similar Spanish research showed that people living in the close vicinity of a cell phone tower have the following health problems:

Depression increased by up to 64-fold; fatigue increased by up to 37-fold; appetite loss increased by up to 25-fold. It has also been found that leukemia is the most common cancer in children and is caused by microwave radiation (Us Natural Resources Defense Council, 2011). According to them, cancers of the central nervous system are the second most common form of cancer in children. Due to the negative consequences of the intense use of mobile phones especially by children and teenagers the usage of mobile must be seriously looked at. In view of this it is believed that wide spread use of mobile phones by children for unproductive calls should be discouraged. It is as well recommended that the mobile phone industry should desist from promoting the use of mobile phones by children therefore it has been suggested that parents take the following precautionary measures to minimize children's exposure to RF radiations.

a. Do not allow your children to use a cell phone, unless there is an emergency.
b. Limit the use of mobile phones around children to a minimum
c. Ensure there is no cell phone mast in the vicinity of your home and your child's school
d. Pass these items of information to adults that spend time with your children especially teachers and other parents.

(v) **Driving safety:** The risk of collision is about four times greater when the driver is using the mobile or soon after a call (Rothman 2011). The author added that the use of the telephone in hands-free mode is no less risky than holding the telephone to ear with one hand while talking. A study in Canada revealed that conversation on mobile phones, both hand – held and hands free, was found to influence driving performance. According to the researchers, epidemiological findings consistently showed an increase in crashes associated with the use of mobile phones.

(vi) **Effects on environment:** Some farmers have observed that cows grazing near cell towers are more likely to experience still births, spontaneous abortions, birth deformities and behavioral problems not to mention general declines in over all health. However moving cattle herds away from such towers has reportedly led to immediate health improvement. Also the environmental effects of radio frequencies have been found to be unclear. According to research migrating birds have been known to fly right into cell phone and other communication towers. While others blame the radiations emitted by those towers for disorienting the birds and undermining their navigational abilities some attribute such cases to poor visibility caused by bad weather.

(vii) **Increased stress level:** According to (Dr. Z, 2011), frequent use of cell phones results in significant disturbances to sleep patterns, increased stress symptoms and an increased incidence of clinical depression.

1.1.3 The Impact of Mobile Phones on the Development of Ghana

The mobile telecom industry has been found in recent times to be one of the formidable contributors to economy of Ghana (Iddrisu, 2011). The sectoral performance of the economy has been shown as the first of its kind that the services sector has taken over from the agriculture as the highest contributor to country's gross domestic product, GDP, (Iddrisu, 2011). According a sector minister, the services sector has consistently grown by 6.1% as compared to a growth of 4.8% by the agriculture sector. This strong performance has been attributed greatly to the development of the telecommunication sector which currently commands a regular growth of 23%. In Ghana mobile phones now connect individuals to one another and help people to access information on concerning their businesses, academic and other social transactions (Aker et al., 2008).

Influx of mobile phones into the Ghanaian market of late has significantly reduced cost of communication and by so doing people are now able than before to send and receive information on various issues of interest ranging from business to academic, politics, social and such like more quickly. Thus the increased in communication brings along an improve production.

Mobile phones create jobs to provide market for their products and thereby providing income generation avenues for the people (Aker, et al., 2008).

Mobile phone – based applications and development projects – sometimes known as "m-development" – have the potential to facilitate the delivery of financial, agricultural-health in educational services (Aker et al., 2008).

1.1.4 National Communications Authority (NCA)

The National Communications Authority (NCA) was founded through the accelerated development programme (ADP, 1994-2000) under the auspices of the World Bank's structural adjustment programme to regulate the communications industry, by passing the National Communications Act 524 in 1996.

The object of the authority is to regulate the provision of communications services in Ghana (Profile of National Communications Authority, 2011)

1.1.5 Mission

The mission of the National Communications Authority is to regulate the communications industry by setting and enforcing high standards of competence and performance to enable it to contribute significantly and fairly to the nation's prosperity through the provision of efficient and competitive services.

1.1.6 Vision

The vision of the body is to become the most forward-looking and innovative Communications Regulatory Authority in the sub-region, by creating and maintaining an efficient, transparent and business friendly environment to enable Ghana become the premier destination of Internet Communication and Technology (ICT) investment in the sub-region.

Decision analysis is the quantitative field that deals with modeling, optimizing, and analyzing decisions made by individuals, groups, and organizations. It is the chronological way of addressing problems of choice in environments besieged with uncertainties. In recent times, the giant development in computer technology coupled with the advance in theory has made decision analysis an indispensable tool in both government and in business as far as the making of multi-criteria decisions is concerned. The most fundamental challenge faced by managers in both public and private sectors is the making of optimal decisions on problems that are multi-criteria in nature. For example, an individual, poised to purchase a car will be faced with multi-criteria challenges. Even though, price is important and must be minimized, people consider other criteria such as reputation, comfort, speed, reliability, consumption etc. and since no car optimizes all the criteria at the same time, a compromise solution has to be sought. Furthermore, it is worthwhile to note that the solution of a multi-criteria problem does not only depend on the fundamental data employed in the evaluation table but also on the decision maker. (Brans, 1986)

Due to individual differences individuals do not have the same taste for a particular car hence they go in for different models all together. Indeed, there exists only a compromise solution, which partly depends on the preferences of each decision maker (i.e. the knowledge, insight, and perception of each decision maker etc.) and as a result additional information representing these preferences is required to provide the decision maker with useful decision aid.

In the last few decades, the field of multi-criteria decision analysis has been further developed to a level that today different mathematical methods are available to facilitate the resolution of problems involving decision making (Brans, 1986). Recent advance in both theory and computer technology have made multi-criteria decision software programs not only indispensable but sophisticated enough to handle more complex decision making problems with ease. One of the developed methods of interest is the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) and its corresponding software – Decision Lab, which concerns itself with the ranking of a finite set of alternative courses of action from the best to the worst ones using a set of criteria which are often conflicting (Brans, 1986).

1.2 Statement of the problem

Mobile telecommunication has now assumed a centre stage in the Ghanaian society to the extent that mobile phone possession is becoming a necessity as a result of people's appreciation of the immense contribution of mobile phone to the betterment of life. Government liberalization of the telecom market has brought about private sector participation to meet the changing needs of Ghanaians. The outcome of this strategy is the keen competition the sector experiences today among the six mobile telephone operators: Mtn, Vodafone, Tigo, Kasapa, Airtel and of course GLO that is yet to commence operation.

The effect of this open market policy is to make subscribers choose among the network alternatives.

Now, individuals and groups are always unable to tell, which amongst these operators best serve them in terms of quality and efficient service delivery, to be able to join that network.

1.3 Objectives of the study

The objectives of this thesis are to:

1) Model the selection of the best telecom operator as a decision problem
2) Identify the best mobile telecom operator in the region in terms of quality and efficient service delivery using the PROMETHEE method.

1.4 Methodology

Generally, methodology consists of the study population, sampling procedure, sample size and how the data is to be analyzed.

The mobile telecommunication operators in Ghana will be used for this case study. Greater Accra region, however, has been selected as the reference region with five mobile networks for study.

The model to be used to calculate for the optimal network will be the PROMETHEE (I and II).

The study will make use of both primary and secondary sources of data for the analysis. Data to be used will be quantitative and the source will be from the National

Communications Authority (NCA). Well designed questionnaires will be the medium for reliable data acquisition from the NCA.

Resources for the study include the college of science library and the internet.

1.5 Justification

The outcome of this thesis will:

(i) Reveal to mobile phone subscribers in the region the network with the best and efficient service which will subsequently inform their decisions on the network to switch or subscribe to.

(ii) Aid the National Communications Authority (NCA) in their duty to enforce high standards of competence and performance in the telecom sector and thereby contributing significantly and fairly to the nation's growth and development.

(iii) Help government identify the best performing telecom operator in the region for perhaps reward and other incentives offer in the future.

1.6 Organization of the Thesis

The thesis is organized in five chapters

Chapter one consists of the introduction to a mathematical model on optimal decision making over five telecommunication networks in Greater Accra region. The background, Problem statement, objective, methodology, and justification are discussed. In Chapter two, we shall put forward pertinent literature in the field of PROMETHEE and its application. Chapter three presents and gives a detailed explanation of the PROMETHEE model.

Chapter four consists of data collection, analysis and results. Chapter five, which is the final chapter focuses on conclusions and recommendations

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Telecommunication is the technology of sending signals and messages over long distances using electronic equipment, for example by radio and telephone (Collins, 2003). Telephone, an electronic device, has two key components, namely, a microphone and a speaker, which normally convert sound and electrical waves into audible relays for communication (Business Dictionary, 2010). These two components facilitate the making and the receiving of calls by both senders and receivers. Due to the impressive progress in technology nowadays, calls connections are automated and sound transmissions propagated mostly by digital than analog. The dominant form of telephone among the various categories available in the system is the cell phone (mobile phone). A cell phone is a telephone that you can carry with you and use to make or receive calls wherever you are (Collins, 2003).

Telephone communication allows users who are by distance away from each other to chat. Scientifically, the sound waves of a person's speech is converted to electric signals via the transmitter of his telephone set; transmitted through the communication channels and then get converted back to sound waves in the receiver of the recipient's phone for him to listen to as speech (Business Dictionary, 2010).

Based on functional divisions of telephone networks, telephone communication has been put into categories namely; local for both rural and urban communication, long distance and international (Business Dictionary, 2010). Apart from inter departmental communication, intra departmental communication is as well feasible. People in motion whether both or one person is involved in the motion can have a successful conversation through the telephone. For instance, people in automobile, aircraft or on board ships can chat with one another through radio communications.

Indeed, this study is restricted to the mobile phone as a telecommunication channel.

Due to the fact that the telecommunication industry in Ghana has not yet been given the needed attention to warrant intensive search and investigation into the sector for rapid development by experts including practitioners and those in the academics, literature on the country's telecommunication industry especially on performance of various networks is scarcely available. Hence, the literature has been selected to cover applications of PROMETHEE method as well as methods for measuring performance of telecommunication operators.

Northern region is the region in Ghana with the largest land mass that covers an area of seventy thousand (70,000) km square, representing 30% of the land surface of Ghana.

Within the last decade, conscious efforts had been made by government to transform the Internet Communication and Technology (ICT) market to a level that could meet the needs of Ghanaians at all times while measuring up to international standard for integration into the global context (Frempong, 1999). In this regard, government adopted the Accelerated Development Plan for Telecommunications (ADP) of 1994 in which Westel was duly registered and mandated to operate as the country's second national network operator (Frempong, 2010).

Sooner than later, the National Telecom Policy (NTP) was enacted to ensure the opening up of the ICT industry for private sector participation, where all private mobile telecom operators are given a level playing field to compete among themselves since this was seen as a way to revamp and sustain the sector through quality service delivery to customers. To this end, National Communications Authority (NCA) was mandated to monitor and regulate the activities of these competitors to bring about quality service and development through healthy competition (Atubra et al., 1999)

As of now, due to the liberalization effect that has hit the telecom industry in Ghana – one of the most liberalized telecom markets in Africa (Frempong et al., 1999), the country can now boast of six mobile telecom operators running alongside the two national fixed lines as stated already (i.e. Ghana telecom and Westel). The six mobile operators are namely Vodafone Ghana limited, MTN Ghana, Tigo Ghana limited, Zain (Airtel), Kasapa (expresso) telecom and Glo mobile. But Glo for now has not started active business yet as it is still engaged in the most important task of infrastructural development. It is worthwhile also to mention that the national duopoly network (Ghana telecom and westel) has been leased out to Vodafone and Zain who now control 98% and 2% respectively of the shares. (Frempong, 2010).

2.2 A Brief History of PROMETHEE

The PROMETHEE methodology is a family of six outranking methods, which are the PROMETHEE I, PROMETHEE II, PROMETHEE III, PROMETHEE IV, PROMETHEE V and PROMETHEE VI (Behzadian et al., 2010).

The first two – PROMETHEE I and PROMETHEE II, which respectively deal with partial and complete ranking of alternatives were propounded by Brans and presented for the first time in 1982 at a conference organized by Nadeau and Landry at the University Laval, Quebec, Canada (Brans, 1982).

Few years afterwards, PROMETHEE III for ranking based on interval, PROMETHEE IV for complete or partial ranking of alternatives when the set of viable solutions is continuous was developed (Brans et al., 2011). The remaining two – PROMETHEE V for multicriteria problems involving segmentation constraints and PROMETHEE VI for the representation of the human brain were proposed between 1992 and 1994 (Brans et al., 2010). Other multicriteria decision aids (MCDA) such as the PROMETHEE group decision support system (GDSS) for group decision-making (Brans et al., 2010), and the visual interactive module GAIA (Geometrical Analysis for Interactive Aid) for pictorial representation to complement the algebraic methodology were developed to facilitate the analysis of more complex decision-making problems (Brans et al, 2010).

Two extensions of PROMETHEE have recently been proposed as PROMETHEE TRI for multicriteria decision-making problems involving sorting and the PROMETHEE CLUSTER for problems dealing with nominal classification (Figueira et al., 2004)

2.3 Applications of PROMETHEE

(Doumpos and Zopounidis, 2010) presented a case study on the implementation of a multi criteria approach to bank rating, especially in Greece. Their proposed methodology was based on the PROMETHEE II method. A rich set of evaluation criteria was used in the analysis and was selected in accordance with widely accepted bank rating principles.

Special emphasis was put on the sensitivity of the results with regard to the relative importance of the evaluation criteria and the parameters of the PROMETHEE method such as the criteria, weights and parameters of the preference functions. Analytic and Monte Carlo simulation techniques were used for this purpose. The data involved detailed information for all Greek banks during the period of 2005-2007. Overall, sixteen (16) banks were considered. The banks were evaluated on a set of thirty one (31) criteria. The criteria had been selected in close co-operation with export analysts of the bank of Greece, who were responsible for monitoring and evaluating the performance of the banks. The criteria were organized into six (6) categories (capital, assets, management, earnings, liquidity, sensitivity to market risks), in accordance with the camels frame work. Overall, seventeen (17) quantitative and fourteen (14) qualitative criteria were used. All qualitative criteria were evaluated on an interval 0.5-5.5 scale, defined by the analysts of the bank of Greece, with lower values indicating higher performance. The weights of each category of criteria and the criteria therein had been defined by the expert analysts of the bank of Greece. The quantitative criteria were assigned a weight of 70%, with the remaining 30% involving qualitative criteria. In all, evaluation results from both the relative assessment procedure and the absolute evaluation process were similar. The results indicated that most banks achieved a rating grade of 2 or 3, each corresponding to performance scores in (1.5, 2.5] and (2.5, 3.5] respectively. There was no bank in the first (best) grade (score≤ 1.5) nor in the highest (5^{th}) risk grade (score >4.5). the dynamics of the performance scores of the banks, indicated that no significant changes were observed between the 5 years of the analysis.

PROMETHEE II method had been used to solve a facility location problem in which there were eight (8) criteria against four (4) alternative solutions (Athawale and Chakraborty, 2010). In the end, the most cost-effective and highest yielding location alternative was identified and selected. They remarked that the PROMETHEE method as a multi criteria decision making approach is a viable tool in solving the location selection decision problems and that it allows the decision maker to rank the candidate alternatives more efficiently and easily.

(Maragoudaki and Tsakiris, 2005) identified PROMETHEE methodology as one of the most efficient multi criteria decision analysis (MCDA) outranking techniques that could be used to arrive at the optimal flood mitigation plan for a river basin. The criteria used to rank alternatives consisted of the cost of flood defense works and their maintenance cost (quantitative assessment) together with environmental and socioeconomic factors representing flood impacts to the environment and the society of the river basin district (qualitative assessment). Alternative scenarios were formulated and evaluated by different stakeholders. The PROMETHEE method was used for aggregating the various criteria and various stakeholder evaluations and proposing the final ranking of the alternative plans.

Four alternative irrigation projects for the east Macedonia-Thrace district – had been evaluated using AHP and PROMETHEE multi criteria methods (Anagnostopouls et al., 2005). The projects goal was the rational water resources management of Nestos River in relation to the operation of two recently constructed dams. They proposed that the management of the water supply system should balance the needs for irrigation, the needs of the public electrical corporation for hydropower generation, as well as environmental requirements given the presence of valuable natural ecosystems in the area.

A preventive maintenance decision model based on integrating PROMETHEE method and the Bayesian approach was developed to help decision makers establish replacement intervals (Ferreira et al., 2008). Finally, a numerical application was given to illustrate the proposed decision model and showed the effectiveness of the model in terms of the decision maker's preferences.

In multi criteria decision making (MCDM) problems dealing with qualitative criteria and uncertain information, the use of linguistic values is suitable for the experts in order to express their judgments (Halouani et al., 2009) . To them, it was common that the group of experts involved in such problems had different degrees of knowledge about the criteria, so they proposed a multi-granular linguistic frame work such that each expert could provide his/her evaluations in different linguistic term sets according to his or her knowledge. The authors were concerned about developing tools and operators for the PROMETHEE method to deal with multigranular linguistic information. They later presented an investment scenario to show the integration between the aggregation operators of PROMETHEE method and the linguistic hierarchies. In this scenario, an investment company wanted to invest a sum of money in the best option. There was a panel with four possible alternatives$A = \{a_1, a_2, \dots, a_4\}$ of investment possibilities. a_1 Was a car industry, a_2 was a food company, a_3 was a computer company and a_4 was an arms industry. They investment company chose four experts $E = \{e_i, \dots, e_4\}$ from four consultancy departments: risk analysis, growth analysis, social-political analysis, and environmental impact analysis departments respectively, to construct a decision group throughout a set of three criteria $c = \{c_1, c_2, c_3\}$ where c_1 was profit, c_2 denoting pollution and c_3 denoting employment. These experts used different linguistic term sets from the linguistic hierarchy

(LH) to provide their preferences over the set of alternatives. In the end, based on the ranking of the alternatives by the experts, the company was advised to choose alternative, a_4 (an arms industry) for its investment.

(Albadvi, 2004) formulated national information technology strategies: a preference ranking model using PROMETHEE method. The sole purpose of his research was to define a national strategy model for information technology (IT) development in developing countries and to apply the model in a real case of Iran. Albadvi research was structured around a three dimensional configuration of strategy development process. These dimensions were key technologies (a set of technology clusters, which have high impact on the development of IT), socio–economic sectors (major economic and social sectors with potential use of IT opportunities); and applications (IT application flagships to provide different strategic choices). The model was a multi-criteria decision making and in order to solve it and select a set of IT application flagships in different budgeting levels, they used the PROMCALC and GAIA decision support system. Finally, it was discovered that by allocating 1% of GDP, four major IT applications for investing were identified. E-education, e-research, e-office and e-information services were ranked as highly important for the realization of long term objectives in the economical, social and cultural development of the country.

The PROMETHEE technique had been applied to determine depression outlet location and flow direction in Digital Elevation Model (DEM) in northern Taiwan (Chou et al., 2003). In their study, the authors proposed depression water shed method coupled with the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEES) theory to determine the optimal outlet and calculate the flow direction in depressions. The method therein developed was used to delineate the Shihmen reservoir water shed located in northern Taiwan. The results they had, however, showed that the depression watershed method could effectively solve the shortcomings such as depression outlet differentiating and looped flow direction between depressions. The suitability of the proposed approach was verified.

A fuzzy based pipe condition assessment model using PROMETHEE II was developed by (Zhou et al., 2010). This method was used to calculate pipe breakage risk to reflect the condition assessment in order to enable them rehabilitate the deteriorated pipes in a planned and proactive way. The numerous influential factors they identified as responsible for pipe breakage included ground load, pipe material, soil corrosion, pipe age, construction quality, pipe length, soil condition, breakage history etc. They argued that the proposed model was different from previous model being used in that it only required usually available data, and that it gave an insight into expert opinion's uncertainty and preference that had a pipe breakage signification in each criterion. The model developed was meant to apply as a new method to some pipes in a water distribution system. This application demonstrated both the stability of the new method and its ability to generate results that will greatly assist decision makers in the development of their rehabilitation strategies.

A PROMETHEE based uncertainty analysis of UK police force performance rank improvement was designed for a periodic comparison of the police forces in the UK with each other in terms of performance by both government and non-government bodies (Barton and Beynon, 2009). The study demonstrated the employment of PROMETHEE in an investigation of the targeted performance rank improvement of individual UK police forces. The graphical representations presented offered an insight into the implications of such a PROMETHEE based series of perceived improvement analysis. The goals of their study were two folds, namely to exposit PROMETHEE based uncertainty analysis in rank improvement and secondly, how the subsequent results could form part of the evidence to aid in their performance strategies.

A new sorting method (Flow Sort) based on the ranking methodology of PROMETHEE for assigning actions to completely ordered categories, defined either by limiting profiles or by central profiles was established by (Nemery and Lamboray, 2007) . The Flow Sort assignment rules were based on the relative position of an action with respect to the reference profiles in terms of the incoming, leaving and/or net flows. The authors added that for a better understanding of the issues involved, a graphical representation was given. An explicit relationship between the assignments obtained when working either with limiting or central profiles was formalized. Finally, an empirical comparison with ELECTRE-TRI was made to compare the resulting assignments.

(Schwartz and GÖthner, 2009) applied for the first time the multi-criteria outranking technique PROMETHEE in incubator evaluations. Based on data from four hundred and ten (410) graduate firms, their evaluation procedure was aimed at comparing the long-term effectiveness of five technology-oriented Business Incubators (BI's) in Germany. In particular, they investigated whether PROMETHEE was a well-suited methodological approach for the evaluation and comparisons in the specific context of business incubation. In the end, they arrived at the conclusion that in using PROMETHEE for incubator comparisons required a set of incubators with sufficient homogeneity regarding major objectives, a set of multiple criteria that cover both incubator and incubator-incubatee dimension of BI performance and, ultimately, a strong participation of the local decision makers to avoid a black-box effect.

(Grau et al., 2010) proposed a mathematical model to select the optimal alternative for an integral plan to desertification and erosion control for the Chaco area in Salta province (Argentine). They used three multi criteria decision methods – ELECTRE, PROMETHEE and AHP for different sub zones which were established based on previous studies. In the development of the model, they took into consideration economical, environmental, cultural and sociological criteria. Their multi-criteria model to select among different alternatives to prepare an integral plan to ameliorate or /and solve this problem in each area has been elaborated taking into account eight criteria and six alternatives. Their results indeed, showed a high level of consistency among the three different multi criteria methods in spite of the complexity of the system studied.

(Manzano et al., 2005) conducted an economic evaluation of the Spanish port system using the PROMETHEE multi criteria decision method. The work established an ordering relationship among twenty-seven Spanish port authorities at different strategically considered time points. They developed various ratios to evaluate the different port authorities. These ratios were referred to economic management, port traffic and labor productivity. Overall, they used six criteria: Economic Yield, Dynamism of Port Activity, Specialization in Containers, Capitalization, Harbor Business and Productivity of the Labor Factor –to order the ports under consideration.

Call quality measurement for telecommunication network and proposition of tariff rates research was conducted by (Aburas et al., 2009). The idea of their research was basically the measurement of call quality from the end users perspective and could be used by both end user and operator to benchmark the network. The call quality was measured based on certain call parameters as average signal strength, the successful call rate, drop rate, handover success rate, handover failure rate, and Location Area Code (LAC). The quality parameters were derived from active calls and the results were analyzed and plotted for detailed analysis and benchmarking as well as used as a base for charging the customer by the operators. They suggested the charging rates in work based on the signal quality and the call statistics recorded.

(Michailidis and Chatzitheodoridis, 2006) proposed a model based on PROMETHEE – a multi-criteria decision aid – to be used to evaluate and rank three tourism destinations, located in the northern and central Greece. Additionally, innovatory elements were the incorporation of differing levels of socioeconomic data (destination image and destination personality) within the decision frame work and the direct determination of the PROMETHEE II preference thresholds. According to them, the developed methodology provides a user- friendly approach, promotes the synergy between different stakeholders, and could pave a way towards consensus. They identified the act of describing the design implementation and use of a Decision Support System (D.S.S), which applied new methodological approaches for the evaluation and ranking of several tourism destinations as the main focus of their study.

PROMETHEE I was introduced to cope with interval criteria introduced for the evaluation of the environmental quality of building products through Life Cycle Assessment (LCA).

Of course, this procedure could be applied to any situation where the decision matrix is an interval matrix.

(Téno and Mareschal, 1997) developed an interval version of PROMETHEE for the comparison of building products' design with ill-defined date on environmental quality. They observed that the Life Cycle Assessment (LCA) is a powerful technique used to calculate total input and output flows of materials and energy from and to the environment during every step of a product life. They added that a measure of a product Environmental Quality (EQ) could then be derived and helped in the selection and in the design of more environmentally friendly design alternatives.

EQ is a multi criteria measurement. In the construction field, LCA flows could not be known with precision without loss of realism. Hence, intervals were introduced to model them. Thus, different designs were characterized by interval multi criteria measures. According to (Mareschal et al., 1997) manipulation of such environmental performances called for a multi criteria decision analysis method which;

(i) did not allow for trade – offs between criteria

(ii) preserved as much information as possible and

(iii) was simple enough to be understood by non – specialist users.

PROMETHEE I was considered as the most suitable method introduced to cope with interval criteria incorporated into the model for the evaluation of the environmental quality of building products through Life Cycle Assessment (LCA).

(Pirdashti and Behzadian, 2009) applied AHP and PROMETHEE to the selection of the best module design for Ultra Filtration (UF) membrane in dairy industry. The authors noted that membrane with a type module had been expressed one of the key areas of interest in diary industry. According to them, although recent publications had given a chance to academic and practitioners to prove successful applications of membrane process to the vast areas; a small number of publications had been devoted to the problem of capital equipment decision making. To facilitate the process of decision – making process in the membrane separation, their study focused on the application of Analytical Hierarchy Process (AHP) and preference ranking organization method for Enrichment Evaluations (PROMETHEE), from a group decision – making view point. They use the Delphi technique to evaluate available alternatives according to the criteria elicited from expert's opinions. A real case study on the ultra filtration membrane area was put forward to determine the best module design based on the five (5) criteria expressed by decision makers: Sanitation design, clean – in – place, packing density, resistance to faulting and shear stress, and relative cost. Finally, expert choice and DECISION LAB soft wares were utilized to facilitate calculations.

The PROMETHEE methodology has been identified as the most sophisticated multi-criteria evaluation methods with deep intrinsic logic and wide flexibility; capable of transforming values of criteria via so – called preference functions (Podvezko and Podviezko, 2010). The authors focus was on the use and choice of preference functions for evaluation of characteristics of socio-economical processes. According to them, all given alternative courses of action were mutually compared pair wise for each criterion. Choice of preference functions and their parameters was important, since it has influence on results of ranking, in which ranks of alternatives may considerably differ. Various preference functions were analyzed, their features described and applications were shown for various socio-economical characteristics.

The institute curie which is a hospital located in Paris, France, with its specialty in oncology seeking enhanced continuity of care inside and outside its walls by using computerized applications relies on two e - health tools as the heart of the institutes ICT systems – Elios and PROMETHEE (E-business watch, 2001).

Elios is a comprehensive electronic patient record system, allowing patient data access during consultations, diagnosis and treatment. PROMETHEE to them is a sophisticated, yet simple to use search engine that enabled the health care professionals to classify medical questions across the hospital`s databases, including Elios. They added that Elios and PROMETHEE together fundamentally transformed health care processes. They improved the continuity and quality of care but offering access to patient data anytime from anywhere in the hospital and from outside. Both tools could be accessed by all members of the health care team involved in their treatment.

In the case of Elios this included external partners such as other hospitals or general practitioners. Both tools also led to considerable economic benefits. Some of the main benefits they identified from PROMETHEE included:

(i) prompt answers to questions on demand

(ii) activity reporting

(iii) faster completion of research and evaluation studies leading to earlier implementation

(iv) rapid evaluation of medical procedures reducing the cost of studies and

(v) audits permitting faster adjustments of the hospital`s organization.

These benefits were achieved through evaluation of medical practices, medical pathways, and medical information quantity.

(Wen-jun et al., 2008) appraised enterprise technology innovation project method based on PROMETHEE. In view of the question on the choice of the iron and steel enterprise technology innovation project, their research established the technology innovation project appraisal index system on the iron and steel enterprise. As mentioned, they used the PROMETHEE method – a class of outranking methods in multi criteria analysis, and it ranked various projects reasonable with the indefinite weight information. When compared with the TOPSIS method, it illuminated that the conclusion of this method was valid and credible.

The collaborative environmental planning in river management in the white river water shed in Vermont adopted the PROMETHEE as a multi criteria decision analysis methodology (Hermans et al, 2006). Their research presented the frame work and results of a structured decision process using the PROMETHEE. The PROMETHEE was used to frame multi- stakeholder discussions of river management alternatives for the upper white river of central Vermont, in the North eastern United States. Stakeholders met over ten (10) months to create a shared vision of an ideal river and its services to communities, develop a list of criteria by which to evaluate river management alternatives, and elicit preferences to rank and compare individual and group preferences. The MCDA procedure helped to frame a group process that made stakeholder preferences explicit and substantive discussions about long – term river management possible.

(Kodikara, 2008) in his thesis on multi–objective optional operation of urban water supply systems made an appropriate use of the PROMETHEE methodology. Kodikara's study attempted to develop and assess the potential of a generic decision support framework to assist in evaluating alternative operating rules for multi–purpose, and multi–reservoir urban water supply systems.

The multi–objective outranking approach (PROMETHEE), which facilitated the incorporation of stakeholder preferences in the decision making process was a main focus area in his study. The main elements of the framework were illustrated on a case study of the Melbourne water supply system, demonstrating its capabilities for evaluating alternative operating rules under single or group decision–making situations. Eight (8) Performance Measures (PMS) were identified under four main objectives to evaluate the system performance related to sixteen pre – selected alternative operating rules. Three (3) major stakeholder groups: resource managers, water users and environmental interest groups were represented in hypothetical decision making situations. An interview- assisted questionnaire survey was used to derive stakeholder preferences on PMS in terms of preference functions and weights as required by the PROMETHEE / GAIA method and its computer software tool – decision lab 2000. A total of ninety-seven (97) personnel selected from Melbourne water and Victoria University participated in the survey expressing their preferences on the eight performance measures. Finally, an overall ranking for alternative operating rules was obtained together with other output results, which focused on the best compromises between the objectives considered.

According to the author, the method yielded reliable and robust results in terms of varying group compositions considered in the study. The authors added that the major innovation of this project was the development of a transparent and intuitive multi – objectives decision support framework that has the potential to be developed for evaluating alternative operating rules for urban water supply systems.

(Mani et al., 2008) adopted the PROMETHEE method in their streamlined life cycle analysis of biomass densification process. They considered mechanical densification to be the process of transforming loose biomass into dense pellets. In their study, a wood pelleting plant was chosen to evaluate the total energy consumption, environmental emissions and cost of pellet production using different alternative fuel for the drying process. The fuels compared were natural gas, coal, dry and wet saw dust, and ground wood pellets. The process models were developed and applied to predict the energy consumption and emissions during combustion process. A streamlined life cycle analysis approach was used to quantify emissions. The authors used average emission factors from published literature to estimate the emissions of trace metals and toxic pollutants. The environmental impacts of the emissions were evaluated based on greenhouse gases, acid rain formation, smog formation and human toxicity impact potentials. A detailed engineering cost analysis was conducted to estimate the pellet production cost using different process options and fuel sources. The PROMETHEE methodology was used to rank fuel alternatives. The best fuel source was selected based on four main criteria –

energy, environmental impacts, economics and fuel quality. Their results showed that wood pellet or dry sawdust might be the best alternative when compared to natural gas, followed by coal and wet sawdust, when all the criteria were weighed equally. If the weighing factor for cost was doubled, coal ranked highest followed by dry sawdust, wet sawdust, wood pellet and natural gas respectively.

(Tzeng et al., 1992) applied two multi – criteria decision–making methods –
AHP and Promethee–to the evaluation of new energy system development in Taiwan. The energy crisis in the 1970s and the recent rise in environmental protectionism had heightened interest in the introduction of new energy systems and the development of techniques to ensure the stability of the energy supply in Taiwan, where more than 90% of the supply was imported. In their study, multi criteria evaluation methods as mentioned above were employed to evaluate comprehensively the alternatives for new energy–system development. Energy technology, environmental impacts, sociology and economic factors were evaluated and development directions and strategy for future energy systems in Taiwan were proposed.

(Martel, 1998) proposed a multi criteria approach for selecting a portfolio manager. The PROMETHEE II method was applied for the selection of a portfolio manager. According to Martel, such application involved four main steps:

(i) Defining the list of potential actions or solutions to the problem
(ii) Defining the list of relevant criteria
(iii) Evaluating the performance of each action based on each criteria
(iv) Aggregating these performances with the multi criteria method PROMETHEE II.

The author underscored the appropriateness of the use of a multi criteria approach to this problem as multiple criteria seemed to be used by decision–makers in the selection of a portfolio manager. The criteria applied to this model were derived from a set of depth interviews with managers of the twelve (12) major pension funds in the province, of Quebec. They ended up with nine criteria that turned out to be heterogeneous and conflicting in their nature. These criteria were then grouped into four: Past performance, Investment philosophy, Staff criteria and Organizational criteria

The richness of data collected through the interview allowed them to specify accurately the decision–maker's preference functions. It was thus possible to choose an outranking technique as a multi criteria aggregating procedure. The choice was limited to one technique of the ELECTRE family and one of the PROMETHEE family of methods. The PROMETHEE II was thus used because the interview revealed that no veto thresholds were applicable to the model. Furthermore, the application was a ranking problem where it was necessary to prioritize a set portfolio managers of from "best" to "worse". Finally, they concluded the analysis by applying their proposed model to the selection of a small capitalization stock portfolio manager.

(Plazibat et al., 2006) adopted a multi criteria approach to credit risk assessment in a significant area of financial management which demands of credit/financial analysts to investigate a large number of financial indicators of firms and make crucial decisions regarding the financing of firms. The focus of their study was on the ranking of firms according to the credit risk assessment using the PROMETHEE method and Analytic

Hierarchy Process (AHP). The PROMETHEE method was used for final ranking of great member of Croatian firms and AHP to determine the importance of the eleven criteria from the three main criteria groups: profitability, liquidity and solvency of the firms.

The lean improvement of the chemical emissions of motor vehicles based on preference ranking PROMETHEE uncertainty analysis has been considered (Beynon and wells, 2006). The authors observed that the motor vehicle had provided mobility and individual freedom for millions of people. Vehicles embodied the dilemma of contemporary industrialization in that the environmental costs of automobiltiy were equally large. Their non – country specific study under took a PROMETHEE–based preference ranking of a small set of motor vehicles based on constituents of their exhaust emissions. As a model of an interested party's preference ranking of the motor vehicles, the subsequent uncertainty (sensitivity) analysis considered here, related to what minimal (lean) changes would be necessary to the emissions of a vehicle so that their preference ranking is improved. For a particular manufacturer, it could identify the necessary engineering performance modification to be made to improve their perceive consumer based ranking. This was compounded by a further consideration of different levels of importance conferred on the criteria (vehicle emissions) and analogue analysis undertaken. The visual elucidation of the results rankings and changes to criteria values, offered a clear presentation of the findings to the interested parties.

(Maragoudaki and Tsakiris, 2005) developed an effective flood mitigation plan using PROMETHEE. The research indeed demonstrated the application of PROMETHEE, one of the most efficient Multi-Criteria Decision Analysis (MCDA) outranking techniques in order to achieve the optimal flood mitigation plan for a river basin. The criteria they used to rank alternatives consisted of the cost of flood defense works and their maintenance cost (quantitative assessment) together with environmental and socio economic factors representing flood impacts to the environment and the society of the river basin district (qualitative assessment). Alternative scenarios were formulated and evaluated by different stakeholders. The PROMETHEE method was used for aggregating the various criteria and various stakeholder evaluations and proposing the final ranking of the alternative plans.

(Kalogeras et al; 2008) used the multi criteria decision aid approach –PROMETHEE method to determine whether or not the ownership structure of cooperative firms drive their financial success. According to these authors, research in finance regarding the impact of ownership structure on the performance of the competing forms of firm organization was scarce. In their study, the ownership structures of co-operatives (co-ops) were analyzed in order to examine whether new models of co-op ownership perform better than the more traditional ones. The assessment procedure introduces a newly developed financial decision – aid approach, which was based on data analysis techniques in combination with a preference ranking organization method for enrichment evaluations (PROMETHEE II). The application of this multi-criteria decision – aid approach allowed the rank ordering of the co-operatives on the basin of the most prominent financial ratios. The authors selected the financial ratios using principal components analysis. This analytical procedure reduced the dimensionality of large member of interrelated financial performance measures. The authors assessed the financial success of fourteen (14) Dutch agribusiness co-ops for the period 1999-2007. The outcome of the research showed that there was no clear–cut evidence that co-op models used to attract outside equity performed better than the more traditional models. This suggested that ownership structure of co-ops was not a decisive factor for their financial success.

(Khiabani, 2006) adopted PROMETHEE to aid him in his studies of business–to–business E-commerce attributes and adoption.

Khiabani observed that understanding intention of businesses to adopt e-commerce was important for researchers and firms. This could be studied with different research strategies and from different perspectives. The authors study was conducted on business–to–business relationship (B2B) e-commerce adoption at firm level from the business–to–business relationship point of view. The respondents were asked to validate and assess the importance of attributes identified for business–to–business relationships. The second part of the study investigated the impacts of adoption of e-commerce on business–to–business relationship. Three different relationships validated the findings of the collected data by using PROMETHEE. The results were showing that business–to–business e-commerce would have certain impacts, with different magnitudes, on the relationship of businesses with each other. It also could guide businesses on how to prioritize their e-commerce projects roll out in the business–to–business context. This according to the author would help businesses to maximize their investment on their relationships deploy an effective business–to–business e-commerce and increase their business– to–business relationship efficiency by enabling electronic aspects in their relationships.

(Ayoko et al., 2004) applied multi criteria decision making methods – PROMETHEE and GAIA to air quality in the micro environment of residential houses in Brisbane, Australia.

Their study centered on the application of the multi criteria decision making methods, PROMETHEE and GAIA, to indoor and outdoor air quality data. Fourteen (14) residential houses in a suburb of Brisbane, Australia were investigated for twenty-one (21) air quality – influencing criteria, which included the characteristics of the houses as well as the concentrations of volatile organic compounds, fungi, bacteria, sub micrometer, and super micrometer particles in their indoor and outdoor air samples. Ranking information necessary to select one house in preference to all others and to assess the parameters influencing the differentiation of the houses was found with the aid of PROMETHEE and GAIA. The outcome of their analysis showed that there was no correlation between the rank order of each house and the health complaints of its occupants. Patterns in GAIA plots showed that indoor air quality in these houses was strongly dependent on the characteristics of the houses (construction materials, distance of the house from a major road, and the presence of an in – built garage). Also, marked similarities were observed in the patterns obtained when GAIA and factor analysis were applied to the data. This to the authors underscored the potential of PROMETHEE and GAIA to provide information that could assist source apportionment and elucidation of effective remedial measures for indoor air pollution.

(Rao and Rajesh, 2009) suggested an effective decision making framework for software selection in manufacturing industries using a multiples criteria decision making method, preference ranking organization method for enrichment evaluations (PROMETHEE). The method was improved in that work by integrating with analytical hierarchy process (AHP) and the fuzzy logic. The fuzzy logic, however, was introduced to handle the imprecision of the human decision making process. The proposed decision making framework was practical for ranking competing software product in terms of their overall performance with respect to multiple criteria.

The methodology to be used for this thesis is the PROMETHEE methodology and it will largely depend on the work of (Podvezko and Podviezko, 2009) on the dependence of multi-criteria evaluation results on choice of preference functions and their parameters. According to them, a considerable usage increase of multi criteria methods was recently observed in the area of quantitative analysis of social or economical phenomena. The PROMETHEE methods were discerned from other multi-criteria methods by depth of their intrinsic logic and by using preference functions, which make up a foundation of the methods. Shapes of functions and their parameters were chosen by decision-makers thus exerting clear advantages and features of the methods. This work revealed the influence of the choice of preference functions and the corresponding parameters on the outcome of evaluation. Along with already recently described by the authors PROMETHEE I method the other PROMETHEE II method was described and examples of its application were provided. New types of preference functions were as well proposed.

Due to its reach acceptance and capability to share information, the World Wide Web has become an important tool for business (Villota, 2009). According to Villota, millions of websites had been developed and so inherently they could come across every kind of website from easy to hard-to-use. The authors added that there were some so-called usability criteria, which should be respected by web designers in order to make websites useful. As a result, using a multicriteria decision making approach, they evaluated the performance, based on seven (7) usability criteria, of five (5) websites from which one could by books online.

They explained that the complexity of multicriteria decision making was based on the fact that those multiple criteria were often contradicting with each other, and so a solution that optimizes every criterion simultaneously, or an ideal solution, was generally unfeasible. In that situation making a decision implied giving an answer which without being optimal was still satisfactory.

Considering usability as a subjective matter, they used two well-known methodologies that deal with this issue: Analytic Hierarchy Process (AHP) and PROMETHEE. Through PROMETHEE they related the preference of a decision maker with specially defined criterion functions

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

In this chapter, we shall put forward the details of the PROMETHEE methodology for the multicriteria decision-making.

3.1 MULTICRITERIA PROBLEMS

Multicriteria problems are the ones bothering on decision-making over a set of decision alternatives that have multicriteria evaluations. They have multicriteria evaluations because a reasonable choice of an alternative must take into consideration a set of criteria (often conflicting) which could be technological, economical, environmental and social in nature (Brans et al., 2011) .

Mathematically, a given multicriteria problem has been expressed as:

$$\text{Max}\{C_1(A_j), C_2(A_j), C_3(A_j), \dots, C_i(A_j), \dots, C_k(A_j) \,/\, A_j \in A\} \text{ -----} \qquad (3.1)$$

Where *A* is a finite set of possible alternatives $\{A_1, A_2, \dots, A_n\}$ and

$\{C_1(.), C_2(.), C_3(.), \dots, C_j(.), \dots, C_k(.)\}$ is a set of evaluation criteria. Although, some criteria may be for maximizing while others for minimizing, the motive of the decision-maker is to get from the set *A* the best compromise alternative. The basic data of multicriteria problems are often expressed in tabular form for evaluation. In every multicriteria decision problem there can be no alternative course of action optimizing all the criteria at the same time and as a result only a compromise alternative is opted for (Brans et al., 2011). It is interesting to note that the evaluation table with the basic data is not the only input for the solution to a multicriteria problem but the decision maker himself especially his/her preference.

3.2 A SINGLE CRITERION OPTIMIZATION

This involves the ranking of a finite set of alternatives on a criterion. In multicriteria problems, this approach ranks existing alternatives based on one criterion at a time giving rise to the generation of more than one optimal solution – each for a criterion. This approach, though less cumbersome does not provide the decision maker with one optimal solution but a set of optimal solutions some of which often conflict themselves by appearing in other ranks at the same time as either less optimal or worst solutions. This feature of the single criterion approach defeats the decision maker's quest to find one optimal solution to a given multicriteria problem. This therefore renders the approach inappropriate for analyzing multicriteria problems.

Let us consider for instance a decision problem with four (4) criteria:

$C = \{C_i\} = \{C_1, C_2, C_3, \text{and } C_4\}$

versus four (4) alternatives

$A = \{A_j\} = \{A_1, A_2, A_3, \text{and } A_4\}$.

The decision matrix is presented in Table 3.1.

Table 3.1: Decision matrix

Criteria	Criteria of Type	Alternatives			
		A_1	A_2	A_3	A_4
C_1	Max	41	50	45	38
C_2	Max	70	65	49	40
C_3	Min	1	2	2.5	3
C_4	Max	85	71	68	65

In Table 3.1, there are three columns. The first column labeled, criteria contains the four criteria $\{C_1, C_2, C_3, \text{and } C_4\}$. The second column identified as type of criteria specifies whether each of the four (4) criteria is a minimizing or maximizing criterion. The third column labeled alternatives has four sub-columns, one sub-column for an alternative for the four alternatives $\{A_1, A_2, A_3, \text{and } A_4\}$. The third column therefore forms a 4x4 matrix with the entries denoted by x_{ij}. The entries x_{ij} are the scores of the various alternatives under each of the four criteria (C_i)

Ranking these alternatives for each of the four criteria, the results are as shown in Table 3.2 in which the column labeled ranking 1 denotes the optimal solution column, ranking 2 as the second optimal solution column and the last column (ranking 4) is the column for the worst solution.

The ranking of the four alternatives for each criterion using the data provided in decision Table

3.1 is presented in Table: 3.2.

The Table 3.2: Ranking of the four alternatives for each of the four criteria

Criterion	Ranking 1	Ranking 2	Ranking 3	Ranking 4
C_1 (Max)	A_2: 50	A_3 : 45	A_1: 41	A_4: 38
C_2 (Max)	A_1: 70	A_3 : 65	A_4: 49	A_2: 40
C_3 (Min)	A_4: 1	A_2: 2	A_3: 2.5	A_1: 3
C_4 (Max)	A_3 : 85	A_4: 71	A_2: 68	A_1: 65

From Table 3.2, it is observed that all the four alternatives have appeared in ranking 1 (i.e. second column) as the best solutions. The same time, alternatives A_1, A_2, and A_4 have appeared in the last column (ranking 4) as worst solutions as well. These outcomes make the single criterion approach not only unreliable but ineffective in helping the decision maker identify the optimal solution.

However, applying the PROMETHEE as one of the known multiple criteria optimization techniques such as Analytic Hierarchy Process (AHP), the decision maker is able to identify one alternative amongst the four as the most compromise solution to the problem.

3.3 THE SCOPE OF PROMETHEE METHODS

The PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) is a family of methods:

PROMETHEE I is a partial ranking of alternatives to aid in the selection of the best alternative.

PROMETHEE II provides complete ranking for the same purpose of selecting one alternative – the most preferred alternative.

PROMETHEE III is for ranking of alternatives based on interval.

PROMETHEE IV is for partial or complete ranking of alternatives when the set of viable solutions is continuous.

PROMETHEE V is an extension of PROMETHEE II and is meant for identification or selection of a subset of alternatives under a certain set of constraints. These could be budget, returns, investment, marketing etc. They are constrains affecting some or all of the alternatives. The main objective here is to maximize the total net outranking flow value while taking into account the existing constraints.

PROMETHEE VI however, represents the "human brain". It provides the decision maker with further information about his multi criteria problem and therein creates the opportunity for the decision-maker to express his or her appreciation of the degree of the problem as to whether or not it is hard. This method becomes applicable when the decision maker is not able to assign specific values of weight to the criteria. The decision maker instead provides intervals within which the true weights of the criteria are expected to fall.

This method is often employed for the analysis of group decision problems since it has a unique feature of incorporating the variation margins of criteria weights arising from the judgment of the group members. PROMETHEE VI is also used as a tool for sensitivity analysis especially when the decision maker is not able to predetermine the weights of criteria to a problem. This is made possible because this method allows the limited area containing all the weight variations to be projected on the GAIA (Geometrical Analysis for Interactive Aid) plane for a clearer visualization and to find out whether the decision axis (decision stick) is in the direction of the selected alternatives or not (Brans and Mareschal , 2011)

3.3.1 PROMETHEE I AND II METHODS

The PROMETHEE method has been recently developed for the analysis of decision problems that have multi criteria evaluations (Brans et al., 1986). The simplicity, clarity, the imbedded mathematical properties and stability of the method make it unique in the midst of other Multi criteria Decision Making Methods (MCDM). This is a ranking method designed to address decision problems involving two or more alternative courses of action against several conflicting criteria.

The PROMETHEE methodology only becomes operational after the determination of the weights or values of the criteria in the given multi criteria decision problem. Weights of evaluated criteria are indispensable ingredients for the application of PROMETHEE. Weights (w_i) show the relative importance of the various criteria (C_i) and must therefore be well defined. Thus, $\{C_1(.), C_2(.), C_3(.), \dots, C_i(.), \dots, C_k(.)\}$, as set of evaluation criteria must be assigned weights $\{w_1, w_2, w_3, \dots, w_j, \dots, w_k\}$ respectively. The weights must be non-negative real values and independent from the unit measure of every criterion. Normalized weights (w_i) are often used such that $\sum_{i=1}^{m} w_i = 1$ for $i = 1,2,\ldots,m$. The higher the weight the more important its criterion. Once the weights are established, alternatives are evaluated based on each criterion, and then ranked. It is the outcome of this ranking that informs the decisions of the decision maker.

On the whole, the PROMETHEE methodology comprises the following four (4) main steps (Brans et al., 1986):

(i) Choice of a generalized preference function.

(ii) Calculation of the value of the generalized preference function.

(iii) Determination of the aggregated preference index for each alternative and finally

(iv) Ranking of the alternatives, of course from best to worst.

3.3.2 CONCEPTS OF PROMETHEE TECHNIQUE

We now consider the following concepts that are fundamental to the PROMETHEE technique.

a) The set of alternatives: this refers to the possible courses of action at the disposal of the decision maker. Mathematically, it may be denoted by the symbol A where A_j for $j = 1,\ldots,$ m becomes a course of action or an alternative in A, ($A_j \in A$). The set of alternative is therefore defined as $A = \{A, \ldots, Am\}$

b) A preference relation: this is a binary relation (Δ) expressed between two alternatives say A_k and A_l. The preference of alternative A_k over A_l is expressed as $A_k \ \Delta \ A_l$ and satisfies the following properties:

(i) The binary relation Δ is reflexive, if $A_j \Delta A_j \quad \forall A_j \in A$

(ii) It is transitive if $A_j \Delta A_k$ and $A_k \Delta A_l \Longrightarrow A_j \Delta A_l \quad \forall A_j, A_k, A_l \in A$

(iii) It is complete if $A_j \Delta A_k$ or $A_k \Delta A_j \quad \forall A_j, A_k \in A$ (Villota, 2009)

c) Standard deviation: this shows how values of a given data are spread around the mean especially when the data is represented by a normal distribution. Standard deviation is denoted by the symbol σ and it is calculated using the relation:

$$\sigma^2 = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n-1} \qquad (3.2)$$

Where x_i for $i = 1, \ldots, n$ is a value of the variable of a distribution and μ is the mean of the distribution calculated as:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i. \qquad (3.3)$$

d) Criterion value: a criterion is a factor to be used in the selection of alternatives. The alternatives are compared pair wise on each criterion to find out the preferences between them. Based on the values of the comparisons, pairs of alternatives are declared indifferent, strict preference, weak preference, etc and thereby assigned the values 0, 1, 0.5 etc respectively. Every criterion must be clearly stated to either be maximizing or minimizing the quantitative measure of an alternative (Podvezko and Podviezko, 2009). This comparison is made using the entries of a decision table.

e) Decision table: this is a table that consists of at least three components namely:

i. The criteria

ii. The type of criteria as to whether maximizing or minimizing

iii. The set of alternatives. The table has an *n x m* matrix consisting of the evaluation of every alternative across all criteria.

The alternatives and various entries of the matrix are denoted respectively by A_j x_{ij} for $i = 1, \ldots, n$ and $j = 1, \ldots, m$ for all criteria. See the decision Table 3.1

In decision Table 3.1, the first column denoted by $C_1, \ldots, C_n$ is the set of criteria while the row denoted by $A_1, \ldots, A_m$ are the alternatives. The second column after the set of criteria is the type of criteria which shows whether a given criterion is a maximizing or minimizing one. Each entry matrix x_{ij} corresponds to the score of an alternative on a criterion

Table 3.3: Decision table of matrix

Criteria	Criteria of Type	Alternatives				
		A_1	•	•	•	A_m
C_1	Max/Min	X_{11}	•	•	•	X_m
•	•	•				•
•	•	•				•
•	•	•				•
C_n	Max/Min	X_{n1}	•	•	•	X_{mn}

So, once the decision table is provided we calculate deviations between pairs of alternatives under each criterion C_i.

f) Deviation: the deviation in the evaluation of alternatives from the decision table is obtained by the relation

$$d_i(A_K, A_l) = x_{ik} - x_{il}, \tag{3.4}$$

for x_{ik} and x_{il} being the entries (scores) of two successive alternatives, A_K and A_l on the same criterion.

After the deviations have been established, we then calculate the criterion values via the preference functions.

g) Preference values: a preference value is denoted by

$P_i(A_K, A_l)$ or $P_i(d_i)$ or $P_i\left(d_i(A_K, A_l)\right), i = 1, \dots n$ and

$$d_i(A_K, A_l) = C_i(A_K) - C_i(A_l), \forall\, A_k, A_l \in A\,. \tag{3.5}$$

Also, $0 \le d_i\,(A_K, A_l) \le 1$.

$d_i(A_K, A_l)$ Shows the intensity of the decision maker's preference of the alternative A_K or A_l on the same criterion C_i . For a maximizing criterion the larger the preference value the better the alternative A_K and the smaller the preference value the less attractive the alternative A_K. A minimizing criterion on the other hand, the smaller the preference value the better the alternative A_K and the bigger the preference value the more unattractive the alternative A_k becomes.

This preference function implies that for a maximizing criterion, preference is given to the alternative A_K over A_l for the observed positive deviation between their evaluations on criterion C_j while for the minimizing criteria preference is assigned to alternative A_K over A_l if the emerging deviations are negative:

$$p_i\,(A_K, A_l) = P_i[-d_i(A_k, A_l)] \quad \forall\, A_k, A_l \in A \tag{3.6}$$

Note however that the preference function:

$p_i(A_K, A_l) = 0,$ (3.7)

for negative deviations holds only for maximizing criteria.

The shape of such a maximized criterion is as shown in Figure 3.1.

From Figure 3.1, the horizontal axis denoted by d (A_k, A_l) is the axis for deviation (d) between alternatives (A_k, A_l) while the vertical axis labeled p(A_k ,A_l) is the preference axis showing the preference between alternatives taking two alternatives at a time.

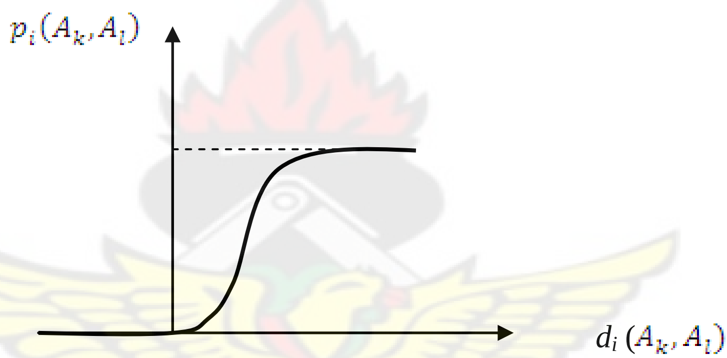


Figure 3.1: Preference Function

So, if d (A_k, A_l)≤ 0 then $p_i(A_k, A_l) = 0$ and if d (A_k, A_l) > 0 then $p_i(A_k, A_l) \leq 1$

The values of preference functions are real values within the interval 0 and 1 and they indicate the degree of preference of one alternative over another (Podvezko and Podviezko, 2009)

3.3.3 AGGREGATED PREFERENCE INDEX

If in a given multicriteria problem such as problem 3.1, the decision maker identifies the preference function as p_i and weight for each criterion C_i to be denoted by w_i where *i*=1, 2, 3,…,k then the preference index π is defined as the weighted average of the preference functions p_i:

$$\pi(A_K, A_l) = \frac{\sum_{i=1}^{k} p_i(A_K, A_l) w_i}{\sum_{i=1}^{k} w_i} \quad \forall A_K, A_l \in A \,. \tag{3.8}$$

And

$$\pi(A_l, A_k) = \frac{\sum_{i=1}^{k} p_i(A_l, A_k) w_i}{\sum_{i=1}^{k} w_i} \quad \forall A_K, A_l \in A \tag{3.9}$$

(Brans et al., 1986)

$\pi(A_K, A_l)$ Denotes the decision maker's degree of preference of alternative A_K to A_l for all the criteria and

$\pi(A_l, A_k)$ Indicates the preference of alternative A_l over A_K for all the criteria. The preference value is always between 0 and 1. The weight w_i is a measure of value or importance of a criterion in relation to the other criteria under consideration. If the decision maker so desires based on judgment that all the criteria have the same value or importance then the weights of these criteria are made equal. But in reality this is not always the case and fixing these weights for the various criteria is often a big challenge for decision makers. The sum of the entire weights equals to one, using normalized values ($\sum_{i=1}^{k} w_i = 1$). In most multicriteria problems, there are criteria that are in favor of alternative A_l over A_K . Thus making $\pi(A_K, A_l)$ and $\pi(A_l, A_K)$ to record positive real values. Hence, the properties below are true for all (A_K, A_l) *in* which $A_k, A_l \in A$

Table 3.4: Properties of aggregated preference indices for all pairs of alternatives

Pair of Alternatives	Properties
(A_k, A_l) $\forall A_k, A_l \in A$	$\pi(A_k, A_k) = 0$ $0 \leq \pi(A_k, A_l) \leq 1$ $0 \leq \pi(A_l, A_k) \leq 1$ $0 \leq \pi(A_k, A_l) + \pi(A_l, A_k) \leq 1$ $\pi(A_l, A_l) = 0$

In Table 3.2, the first column is the column for all possible pairs of alternatives and the second column, labeled properties, gives the range of values for the corresponding preference indices and their sum. And when:

(i) $\pi(A_K, A_l) \approx 0$ (3.10)

Means a weak global preference of A_K over A_l where " $\approx$ " denotes a value of the preference index: $\pi(A_K, A_l)$ closed to zero (0) and

(ii) $\pi(A_K, A_l) \approx 1$ (3.11)

Means a strong global preference of A_K over A_K for which " $\approx$ " stands for a value of $\pi(A_K, A_l)$ that is closed to 1.

3.3.4 RANKING OF ALTERNATIVES

As mentioned before, the preference index $\pi(A_K, A_l)\ \forall A_K\ A_l, \in A$ where A is the finite set of alternatives indicates the degree of preference expressed by the decision maker for the alternative A_K over alternative A_l for all the criteria. Conversely, there are some criteria too on which the alternative A_K may be preferred to the alternative A_K giving rise to the preference index $\pi(A_l, A_K)$. These in other words, manifest how two alternatives have a comparative advantage over each other over a given finite criteria.

These two indices $\pi(A_K, A_l)$ and $\pi(A_l, A_k)$ connect every pair of alternatives say A_K, A_l to each other. Such a connection or relation is known as the outranking relation. Graphically, the relation is often represented by two nodes denoting the two alternatives linked to each other by a corresponding two arcs each for a preference index as presented in Figure 3.2

From Figure 3.2, the alternatives A_K and A_l in rings are the nodes. The preference index $\pi(A_K, A_l)$ which links node A_K to node A_l as indicated by the arrow of the upper arc of Figure 3.2, shows the magnitude of the preference of the alternative A_K over A_l. The preference index $\pi(A_l, A_k)$ on the other hand, connects node A_l to A_k and is indicated by the arrow of the lower arc of Figure 3.2 showing the magnitude of preference of the alternative A_l to A_k

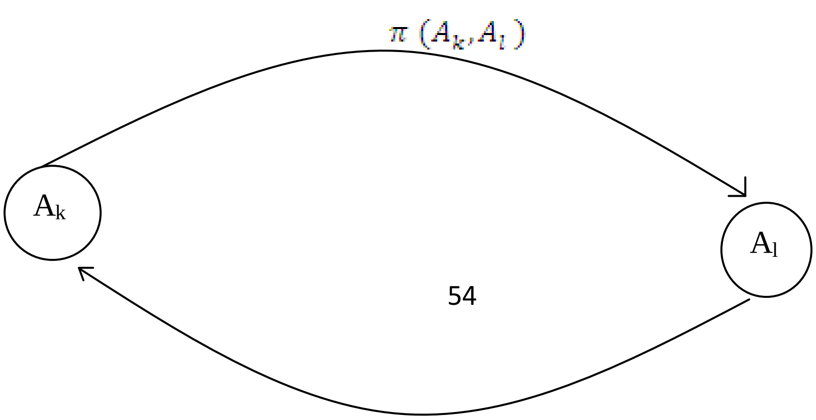


Figure 3.2: Outranking flow relation $\pi\ (A_l, A_k\ )$

Now, given the set of possible alternatives in *A*, each alternative $A_K \in A$ faces (n-1) other alternatives in *A*, where n connotes the number of alternatives in *A*. The PROMETHEE method sums up all preference indices that are in favor of the alternative A_K , $\pi(A_K\ A_l,)$ to get what is referred to as the positive outranking flow expressed as:

$$\phi^+(A_K) = \frac{1}{n-1}\sum_{A_l \epsilon A} \pi(A_K\ , A_l) \qquad (3.12)$$

It sums up all preference indices which are not in favor of A_K to be the negative outranking flow: $\phi^-(A_K) = \frac{1}{n-1}\sum_{A_l \epsilon A} \pi(A_l, A_K).$ (3.13)

So, the positive outranking flow $(\phi^+(A_K))$ shows how an alternative A_K is outranking all else in *A* over all the criteria. It is called the power of A_K or the strength or the outranking characterof A_K. On the other hand, the negative outranking flow indicates how an alternative A_K is being outranked by all other alternatives in *A*. This measure represents the weakness or the outranked character of A_K. The higher the positive outranking flow $\phi^+(A_K)$ and the lower the negative outranking flow $\phi^-(A_K)$ the better the alternative A_K.

In graphical representation, the positive outranking flow $(\phi^+(A_k))$ is represented by Figure 3.3:

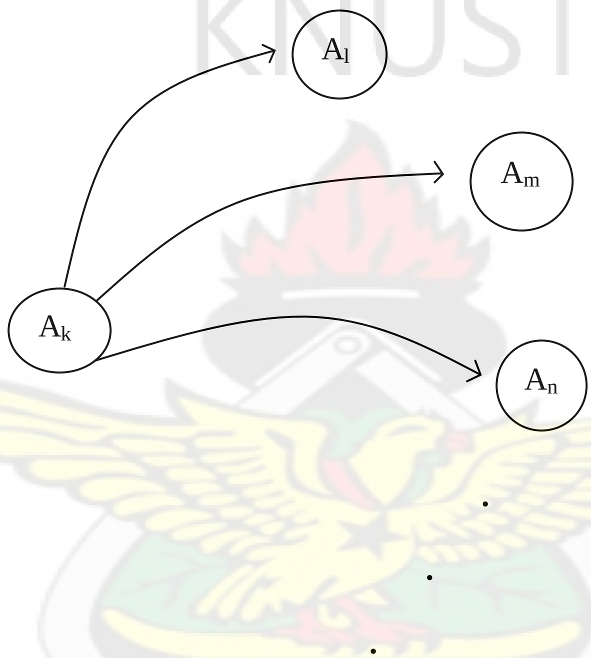


Figure 3.3: Positive outranking flow $(\phi^+(A_k))$

From Figure 3.3, the arrows directed at nodes $A_l, A_m, A_n, \ldots$ from node A_K show how the alternative A_K outranks all other alternatives. These directed arrows from A_K are called the positive outranking flows (leaving flows) denoted by $\phi^+(A_K)$ as shown in Figure 3.3.

The negative outranking flow $(\phi^-(A_K))$ is represented graphically by Figure 3.4:

In Figure 3.4, the arrows from nodes A_l, A_m, A_n etc. directed at node A_K are called the negative outranking (entering) flows and they show how the alternative A_K is outranked by the other alternatives.

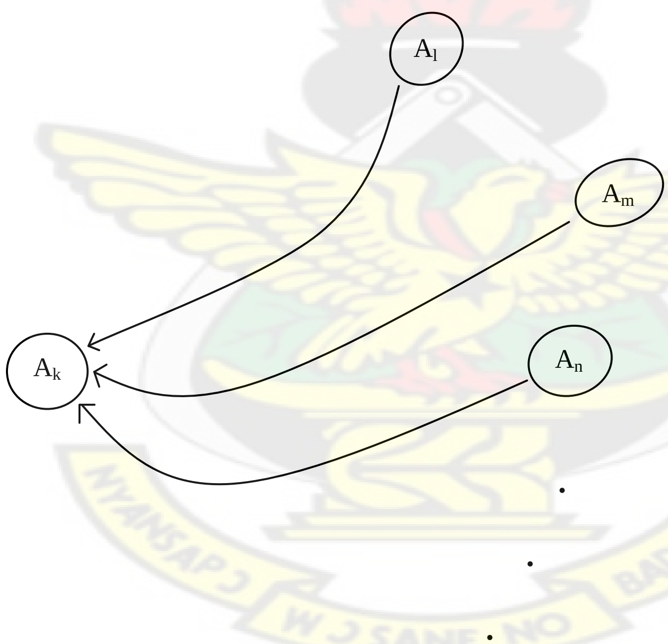


Figure 3.4: the negative outranking flow $(\phi^{-}(A_K))$

The net flow, denoted by $\phi(.)$, is the difference between the positive flow (leaving flow) and the negative flow (entering flow). So, considering the alternatives, $A_K, A_l \in A$, the net flow for the alternative A_K ,

$\phi(A_K) = \phi^{+}(A_K) - \phi^{-}(A_K)$.

(3.14)

Essentially, the net flow is used for PROMETHEE II (complete ranking).

The ranking of a finite set of alternatives under PROMETHEE methodology may involve two ranking processes which are namely:

(i) The partial ranking process and

(ii) The complete ranking process

3.3.5 PROMETHEE I - THE PARTIAL RANKING METHOD

The partial ranking (PROMETHEE I) establishes the outranking relation existing between various alternatives via the leaving $(\phi^{+}(A_K))$ and the entering $(\phi^{-}(A_K))$ flows. The possible outcomes may be denoted by *P*, *I*, and *R*. where *P*, often placed between two alternatives as $A_K P A_l$ signifies the preference of the alternative A_K over A_l ; $A_K\, I A_l$ signifies the indifference between alternatives $\square_K$ and A_l and $A_K\, R\, A_l$ signifies the incomparability of the two alternatives A_K and A_l over all criteria.

These three cases are identified using the following preorders as shown in Table 3.3

Table 3.5: Relations between alternatives in PROMETHEE partial preorder ranking

Preference Relation	**Cases**	**Graphical Representation**
$A_K P\ A_l$	$\phi^{+}(A_K) > \phi^{+}(A_l)$ and $\phi^{-}(A_K) < \phi^{-}(A_l)$	$A_K \longrightarrow A_l$

	$\phi^+(A_K) > \phi^+(A_l)$ and $\phi^-(A_K) = \phi^-(A_l)$ $\phi^+(A_K) = \phi^+(A_l)$ and $\phi^-(A_K) < \phi^-(A_l)$	
$A_K I A_l$	$\phi^+(A_K) = \phi^+(A_l)$ and $\phi^-(A_K) = \phi^-(A_l)$	_
$A_k R A_l$	$\phi^+(A_K) > \phi^+(A_l)$ and $\phi^-(A_K) > \phi^-(A_l)$ $\phi^+(A_l) > \phi^+(A_k)$ and $\phi^-(A_l) > \phi^-(A_k)$	_

The first column in Table 3.3 represents the preference relation which indicates the three possible outcomes when alternatives are compared pair wise. The possible outcomes are

(i) $A_k P A_l$ means A_K is preferred to A_l

(ii) $A_k I A_l$ means A_K is indifferent to A_l

(iii)$A_k R A_l$ means A_K is incomparable to A_l

The second column of Table 3.3 labeled, cases, give the conditions under which a given pair wise comparison of alternatives can be regarded as preference (P), indifference (I) or incomparable (R).

The third column is the graphical representation column which shows how one alternative, A_K

Is preferred to A_l by means of a directed arrow from A_K to A_l (i.e.$A_{k\rightarrow}A_l$). However, indifference or incomparable relations are shown by means of a dash (-)

It can be concluded from the above that:

(i) $A_K P A_l$ implies a higher power of alternative A_K is matched to a lower weakness of A_K, in relation to A_l. In such a consistency the alternative A_K is automatically preferred to A_l.

(ii) $A_K I A_l$ implies the respective leaving flows and entering flows are the same.

(iii) $A_K R A_l$ mplies a higher power of the alternative A_K is associated to a lower weakness of A_l. This type of situation arises when out of a set of criteria, alternative A_K is better than A_l on some, and conversely, the alternative A_l is better than A_K on other criteria. When the flows experience such an inconsistency the alternatives therein are declared incomparable. Over here, PROMETHEE I does not decide which alternative is better than the other. The choice is left to the decision maker to make based on his or her perception, priorities, knowledge, experience etc. This is the reason why PROMETHEE I is regarded a partial preorder ranking method. It only compares alternatives that are comparable (i.e. only those under *p* and *I*) and thus making the whole ranking incomplete.

The partial ranking can be represented graphically using the leaving and the entering flows. Decision to be made according to this ranking is done by considering the alternative with the highest number of leaving flows. This indicates the alternative most preferred in comparison to other alternatives.

3.3.6 PROMETHEE II - COMPLETE RANKING

At this stage it is the PROMETHEE II (preorder complete ranking) method which completes the whole ranking process, establishing a relation that links all alternatives be they comparable or incomparable and placing them in their right perspective in a hierarchy from best to worst.

If after partial ranking some alternatives are found to be incomparable then we apply PROMETHEE II (the complete ranking) method to finish the ranking process for an optimal decision to be made.

It makes use of only the parameter *P* and *I* (preference and indifference respectively). This approach makes use of what is called the net outranking flow symbolically represented by $\phi(A_K)$ for the alternative A_K such that

$$\phi(A_K) = \phi^+(A_K) - \phi^-(A_K).$$

The alternative A_l in terms of the net outranking flow becomes:

$$\phi(A_l) = \phi^+(A_l) - \phi^-(A_l) \quad \forall A_K, A_l \in A. \tag{3.15}$$

So the higher the net flow the better is the alternative.

i. The alternative A_K *is* preferable to A_l if and only if $\phi(A_K) > \phi(A_l)$.

ii. The alternative A_K is indifferent to the alternative A_l if and only if

$$\phi(A_K) = \phi(A_l). \tag{3.16}$$

Table 3.6: Two existing relations between alternatives in complete ranking

Preference Relation	**Cases**	**Graphical representation**

$A_k \; P \; A_l$	$\phi(A_k) > \phi(A_l)$	$A_k \longrightarrow A_l$
$A_k \; I \; A_l$	$\phi(A_k) = \phi(A_l)$	_

The two properties below are also true about PROMETHEE II:

(a) $-1 \leq \phi(A_K) \leq 1$
(b) $\sum_{x \in A} \phi(A_K) = 0$

For $\phi(A_K) > 0$, implies alternative A_K is more outranking the rest of the alternatives over all the criteria and $\phi(A_K) < 0$, implies A_K is more outranked.

The net flow $\phi(A_K)$ can therefore be defined in terms of the leaving and entering flows together with the aggregated indices as:

$$\phi(A_K) = \phi^+(A_K) - \phi^-(A_K) = \frac{1}{n-1}\sum_{i=1}^{k}\sum_{x \in A}[P_i(A_K, x) - P_i(x, A_K)]w_i. \quad (3.17)$$

Finally,

$$\phi(A_K) = \phi^+(A_K) - \phi^-(A_K) = \sum_{i=1}^{k} \phi_i(A_K) w_i \quad (3.18)$$

Where

$$\phi_i(A_K) = \frac{1}{n-1}\sum_{x \in A}[P_i(A_K, x) - P_i(x, A_K)] \text{ for } i = 1, \dots k$$

3.3.7 PREFERENCE FUNCTIONS AND THEIR FEATURES

The preference function *p* (*d*) is the function of deviation or difference (*d*) between values of two evaluated alternatives on the same criterion perhaps over a set of criteria. Mathematically, written as

$$P_i(A_K, A_l) = P_i\left(d_i(A_K, A_l)\right), i = 1, \ldots n.$$

The main features of preference functions are

a) Values of the preference functions: these values are within the interval zero to one such that $0 \leq P_i(A_K, A_l) \leq 1$

b) Preference functions are functions that maximize criteria through normalized values such that the higher the value of the function *p* (*d*), the higher the preference of A_K to A_l.

c) Most preference functions have one or more of the following parameters: *p*, *q*, *σ*. Values of these parameters are always determined by the decision maker and thereby aid in determining the intensity of preference of one alternative over the other on a criterion. The parameter *q*, indicated along the deviation axis, is the greatest point of

d) deviation (*d*) between two evaluations below which the decision maker regards the corresponding alternatives (A_k, A_l) as indifferent. *p* which is fixed to the right of the parameter *q* on the deviation axis measures the lowest point of deviation (*d*) between two alternatives above which the decision maker expresses strict preference for the first alternatives A_k over the second alternative A_l $(P_i(A_k, A_l))$. when the deviation *d* between two evaluations falls in between *q* and *p*, preference for the alternative A_k over alternativeA_l ranges between 0 and 1.

The value of a preference function p (d) equals zero when the deviation or difference (d) is below the lower boundary q. in other words, when the value of deviation is less than the value of q: p (d) = 0 if $d \leq q$ (in case however, the value of q is not specified it is regarded as zero, $q = 0$) So long as the deviation value remains a value in between the thresholds q and p, the following conclusions are worth noting:

(i) $p\,(A_K, A_l) = 0$ (3.19)

implies indifference between A_K and A_l or no preference of A_K over A_l.

(ii) $p\,(A_K, A_l) \approx 0$ (3.20)

implies there is a weak preference of A_K over A_l where the symbol

" $\approx$ " denotes a value of $p\,(A_K, A_l)$ closed to zero (0)

(iii) $p\,(A_K, A_l) \approx 1$ (3.21)

implies a strong preference of A_K over A_l where the symbol " $\approx$ " denotes a value of $p(A_K, A_l)$ closed to 1

(iv) $p\,(A_K, A_l) = 1$ (3.22)

implies a strict preference of A_K over A_l. (Brans et al., 1986)

There is also a parameter σ which is regarded as an intermediate value between q and p. Therefore, the choice of a generalized criterion is preceded by the selection of the appropriate parameters.

e) If the upper boundary of deviation p is defined then p (d) = 1 if and only if $d \geq p$. also, there are times the value of p is not explicitly stated and in such cases

$\lim_{d \to \infty} p\,(d) = 1$ (Podvezko and Podviezko, 2010)

There are known eight preference functions p (d) namely:

(i) Usual criterion (preference function)

(ii) U-shape criterion (preference function) - quasi criterion

(iii) v –shape criterion (preference function) - criterion with linear preference

(iv) Level criterion (preference function)

(v) V –shape with indifference preference function - criterion with linear preference and indifference area

(vi) Gaussian criterion or preference function

(vii) Multistage preference function and

(viii) C-shape preference function. (Podvezko and Podviezko, 2010)

1. Usual criterion or preference function:

This function is applicable to cases when the decision maker is only interested in the difference between criteria values. Here, there is no allocation of importance for the differences between criteria values. The decision maker only has strict preference for an alternative with the greatest criteria value. In short, their preference judgment is based on the principle that the "more the better". This type of function is boundary free (neither q nor p is defined).

The decision maker's focus is only on the evaluation difference and so p (d) = 1 if and only if $d_i\ (A_K, A_l) = C_i\ (A_K) - C_i\ (A_K)$ is positive and p (d) = 0 if $d_i\ (A_K, A_l) = C_i\ (A_K) - C_i\ (A_K)$ is negative and value of the difference does not matter (Podvezko and Podviezko, 2010)

For example, one job offer is preferred over another if offered salary is higher without assigning any importance to the difference; it is important if distance to the office is higher or smaller; if one candidate for a job knows more languages than another etc. the usual preference function is defined as:

$$p(d) = \begin{cases} 0, \text{when } d \leq 0 \\ 1, \text{when } d > 0 \end{cases} \quad (3.23)$$

The graph of the preference function is presented in Figure 3.5

In the graph, the horizontal axis is the deviation axis, *d* which is the difference between values of two evaluated alternatives on a criterion. The vertical axis labeled *p* (*d*) measures the degree of preference. The meeting point of the two axes is labeled 0 as the point of origin. The upper horizontal line that originates from point 1 on the *p* (*d*) axis and runs parallel to the deviation (*d*) axis marks the maximum value the degree of preference can take

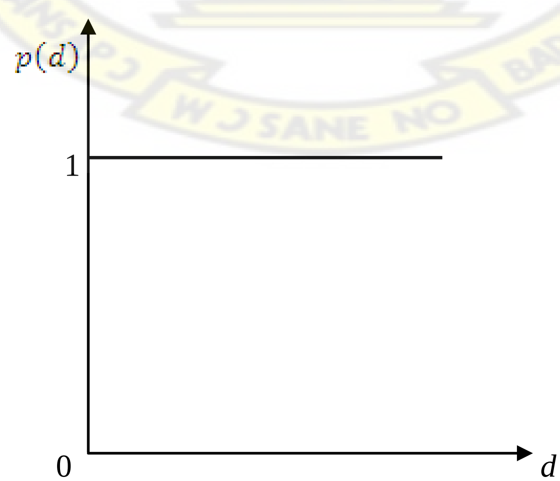


Figure 3.5: Graph of usual preference function

2. U-shape preference function otherwise called the quasi criterion:

This differs from the usual preference function by the establishment of the indifference threshold q. this indifference threshold marks the lower boundary of the evaluation difference such that when the difference (d) between the evaluation of two alternatives is below q the decision maker considers the two alternatives indifferent and the preference function p (d) = 0 since $d \leq q$. On the other hand, if the evaluated difference between the two alternatives is above q then there is a strict preference of one alternative over the other and the preference function p (d) = 1 since $d > q$. Though, the function is u-shape our focus is on the right side of it. Hence, to use the u-shape criterion the decision maker has to determine only the value of q and this has an economic signification – the greatest value of deviation between two alternative actions below which the decision maker declares the affected alternatives indifferent. For example a new job will have strict preference [p (d) =1] over another if only the salary difference exceeds 500 Ghana cedis (q = 500) otherwise the difference will be of no value to the employee and [p (d) = 0]. The same way, a candidate becomes preferable to another if the work experience of that candidate is more than another by 4 years (q = 4) or that candidate correctly answered at least 4 questions more than another and so on. The algebraic definition of the function is:

$$p(d) = \begin{cases} 0, \text{when } d \leq q \\ 1, \text{when } d > q \end{cases} \tag{3.24}$$

The graph of the u-shape preference function is shown in Figure 3.6

Given the graph of this preference function, the horizontal axis is the deviation (d) axis which is the difference between the values of two alternatives evaluated on the same criterion. The vertical axis, p (d), measures the intensity of preference for one alternative over the other. The least value on this axis is zero and the highest is 1. The bold zigzag line that stretches from the origin (0) to point q (the point of indifference) on the deviation axis (d) and upwards parallel to p (d)-axis up to point 1 before changing direction to move horizontal parallel to d describes the preference of the decision maker with respect to different values of deviation.

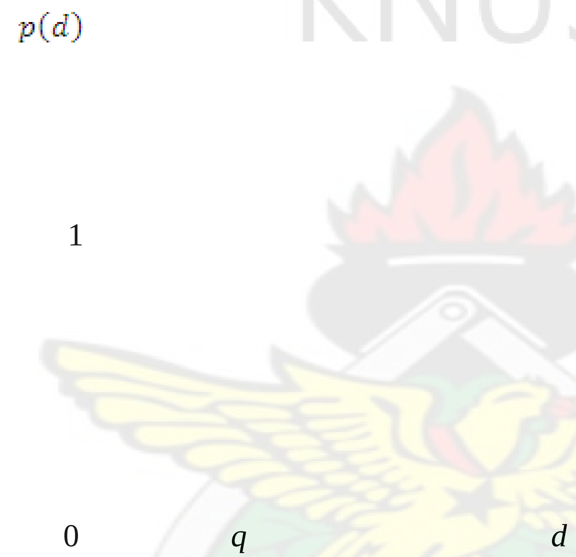


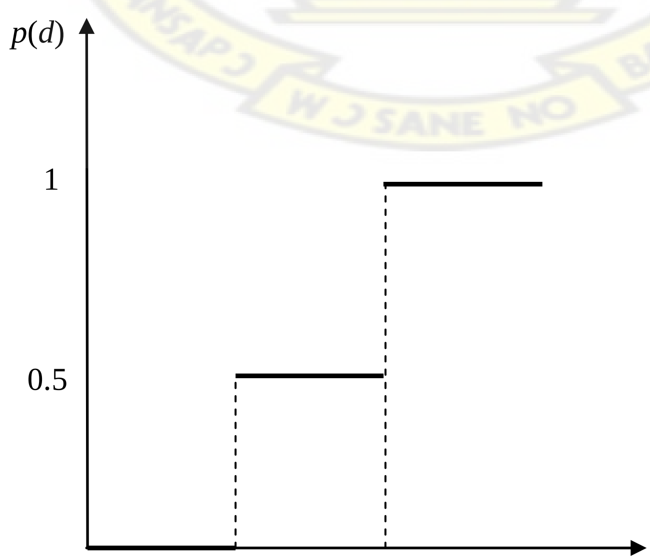
Figure 3.6: Graph of U-shape preference function

3. **Level preference function:**

This function makes use of the indifference and preference thresholds, q and s respectively which must therefore be defined simultaneously by the decision maker. As usual, if the value difference between two evaluated alternatives is below indifference threshold q then the two alternatives concerned are regarded indifferent and [p (d) = 0] by the decision maker. If the difference (d) is above the preference threshold s, the decision maker expresses a strict preference [p (d) =1] for one alternative over another. And if the difference d is in between q and s then there is a weak preference of one alternative over another denoted by [p (d) $=\frac{1}{2}$] as the value of the preference function. The analytical expression is as shown below and graph of the level preference function is presented in Figure 3.7

$$p(d) = \begin{cases} 0, \text{when } d \le q \\ 0.5, \text{when } q < d \le s \\ 1, \text{when } d > s \end{cases} \tag{3.25}$$

The graph of the level preference function has zero (0), q, and s along the deviation (d)-axis. Points q and s are the thresholds. The preference function axis p (d) has points 0, 0.5 and 1. The three bold but short lines that run parallel to the d-axis starts from zero (0) to q, then from q to s at the point p (d) = 0.5 and finally from s to infinity at the point p (d) = 1. The illustration describes the behavior of the decision maker's preferences at different values of deviation.



0 q s d

Figure 3.7: Graph of level preference function

For example an applicant for a job will have no advantage if he has fewer years of working experience than another applicant (p (d) $=0$, d is negative). He has some advantage if he has one year working experience more than another ($p(d) = 0.5$), and will get strict preference over another applicant if he has at least two years working experience more than another applicant [$p(d) = 1$]. When more discrete options are involved additional step gradations are made using a similar preference function. It estimates the linear function as the number of gradations increases.

4. **Multistage preference function:**

According to (Podvezko and Podviezko, 2010) some alternatives do possess discrete criteria values. Discrete because they are natural values (mostly positive natural values). Cases such as the number of children in a given household, number of cattle in a given kraal, etc all give rise to criteria values that are natural numbers. In addition, precise data may be produced in real numbers while evaluation is made and discussed in integers. It is however, important to note that the level preference function with its values, 0, 0.5 and 1 is inadequate to deal with all identified cases. The authors added that for integer criteria values, we must have the largest difference $d = s$, where s is an integer. Where s is not available, we consider s = the maximum criterion value, $\max(r_{ij})$ or any lower value which seems appropriate. The definition of the multistage preference function is:

$$p(d) = \begin{cases} 0, \text{when } d \leq 0 \\ \frac{1}{s}, \text{when } 0 < d \leq 1 \\ \frac{2}{s}, \text{when } 1 < d \leq 2 \\ \ldots \\ \frac{s-1}{s}, \text{when } s-2 < d \leq s-1 \\ 1, \text{when } s-1 < d \leq s \leq \max_j r_{ij} \end{cases} \tag{3.26}$$

The graph of the preference function is displayed in Figure 3.8

The graph of this function has the points, 1, 2, *s*-1 and *s* on the *d*-axis, where the variable *s* denotes the upper boundary on the axis. On the *p* (*d*)-axis, we have the points $\frac{1}{s}$ and 1. The four bold arrow- short lines directed at *p* (*d*) - axis describes the levels of preference at the various points on the *d*-axis.

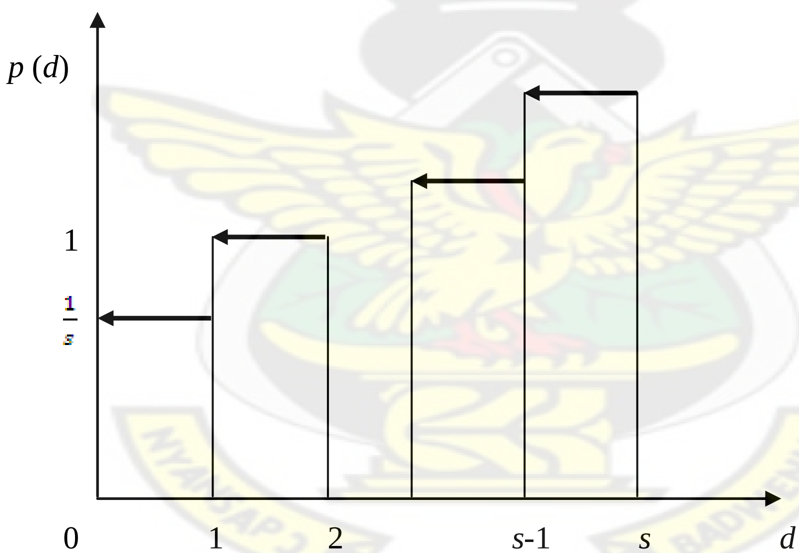


Figure 3.8: Graph of multistage preference function

5. **V-shape (or criterion with linear preference) function:**

This has a boundary parameter *s* such that if the evaluation difference *d* is below *s* then the preference of the decision maker increases linearly with the difference *d*. if *d* is above *s* then the decision maker will have a strict (constant) preference for one option over another. This function is therefore different from the u-shape function in the interval 0 to *s* where the link between the point of indifference $p(d) = 0$ and the point of strict preference of one alternative over another [$p(d) = 1$] is linear but not a shift.

This linear preference function has only an upper boundary s, a preference threshold above which there is a strict preference for one alternative over another. In effect, the preference threshold s is the lowest value of difference (d) above which the decision maker has strict preference for one of the corresponding alternatives.

The analytical expression for the v-shape preference function is as follows:

$$p(d) = \begin{cases} 0, \text{when } d \leq 0 \\ \frac{d}{s}, \text{when } 0 < d \leq s \\ 1, \text{when } d > s \end{cases} \tag{3.27}$$

The corresponding graph is given in Figure 3.9

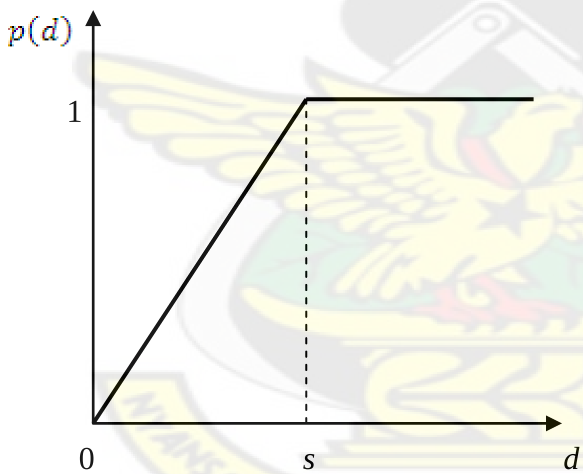


Figure 3.9: Graph of V-shape preference function

An example of practical application of this preference function is when a new job offer will have a strict preference [$p(d) = 1$] over another if only the salary difference is at least 500 cedis.

$s = 500$. However, when the salary difference (d) is negative the employee will show no interest in the offer at all and the preference function $p\ (d) = 0$. As d begins to take positive values so does the employee's interest rise but linearly up to a difference (d) of 500 ($0<d\leq 500$). Value of the preference function is expressed as $p(d) = \frac{d}{500}$.

6. **V-shape with indifference preference function (and level preference function):**

This function too has the parameters q and s as defined before and the decision maker has to determine their values. In this case the preference of the decision maker increases but linearly from the point of indifference threshold (q) to the point of strict preference threshold (s). in other words, the preference function increases steadily and linearly from zero to one based on the formula $\frac{d-q}{s-q}$. the value of this formula suggests the degree of preference of one alternative over another. In view of this, when $q = 0$ the function turns to v –shape preference function. For example, a job seeker already into another job will be indifferent over the job he is engaged in and a new one if the salary difference of these two jobs is less than 500 cedis ($p(d) = 0$). On the other hand, the seeker expresses strict preference for the new job if the salary of the new job offer exceeds 1000 cedis ($p(d) = 1$) and there will be a preference of some sort for the new job over his current job if the salary offer of the new one falls within 500 and 1000 cedis. The preference level is calculated by the formula $p(d) = \frac{d-500}{1000-500} = \frac{d-500}{500}$.

The algebraic definition of this function is as given below and the corresponding graph shown in Figure 3.10

$$p(d) = \begin{cases} 0, when\ d \leq q \\ \frac{d-q}{s-q}, when\ q < d \leq s \\ 1, when\ d > s \end{cases} \qquad (3.28)$$

The graph of this preference function has the indifference and the preference thresholds, *q* and *s*, respectively on the d-axis. On the *p* (*d*)-axis, there is the maximum preference point 1. The origin 0 marks the meeting point of these axes. The bold line that moves from the origin (0) to the point *q* in the first quadrant, and to 1 on the *p* (*d*) axis which corresponds to *s* on the *d*-axis horizontally and parallel to the deviation axis illustrates the linearity of the decision maker's preference.

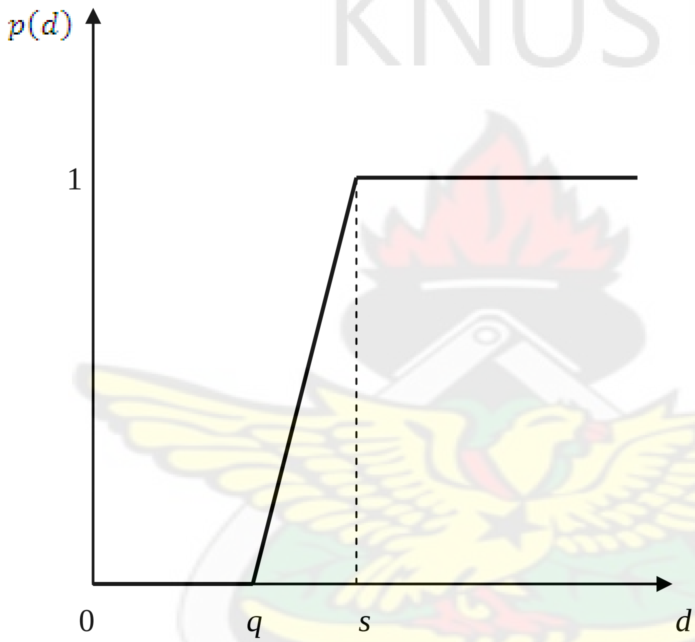


Figure 3.10: Graph of V-shape with indifference preference function

This function has been highly recognized and applied by many users for evaluation of several courses of action using PROMETHEE methods.

7. C-shape preference function:

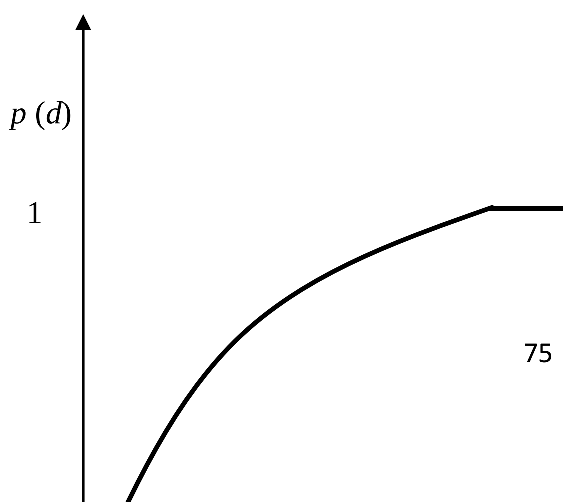
This function increases rapidly at small differences of criteria values (*d*) starting from zero (Podvezko and Podviezko, 2010). As the values of difference (*d*) gradually become larger the increase in preference function becomes smaller. This function exhibits higher sensitivity to low values of difference (d) and relatively lower sensitivity to high values of difference (d)

The definition of the preference function is given by

$$p(d) = \begin{cases} 0, \text{when } d \leq 0 \\ \sqrt{\frac{d}{s}}, \text{when } 0 < d \leq s \\ 1, \text{when } d > s \end{cases} \tag{3.29}$$

The graph of this function is given in Figure 3.11

The graph of c-shape preference function has only the preference threshold s located on the deviation, d-axis. As usual, p (d)-axis, which measures the degree of preference has a maximum point 1. The bold curve that originates from the point 0 and increases towards the point 1 on the p (d) describes the behavior of the decision maker's preference at various levels of deviation.



0 s d

Figure 3.11: Graph of C-shape preference function

It is observed that this function could be used in place of the v-shape preference function when especially small difference *d* between the pair wise evaluation of alternatives induce more relative importance than large differences. (Podvezko and Podviezko, 2010)

Where the deviation values are discrete, the multistage preference function is defined differently as below:

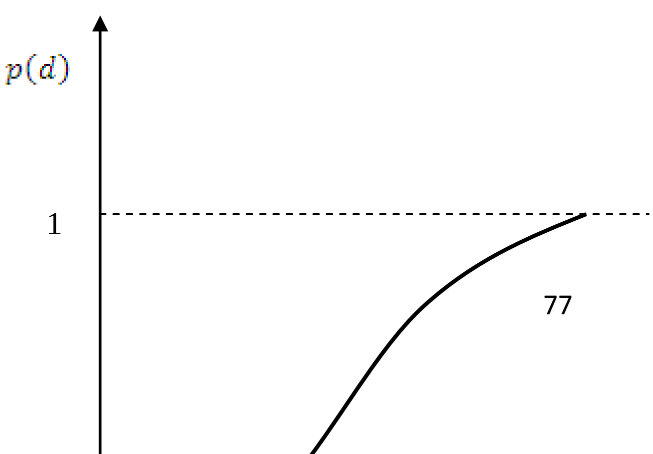
$$p(d) = \begin{cases} 0, \text{when } d \le 0 \\ \frac{1}{s}, \text{when } d = 1 \\ \frac{2}{s}, \text{when } d = 2 \\ \dots \\ \frac{s-1}{s}, \text{when } d = s-1 \\ 1, \text{when } d = s \le \max_j r_{ij} \end{cases} \tag{3.30}$$

8. Gaussian preference function:

This makes use of statistical data involving random values with normal distribution. The decision maker requires only to determine the parameter σ of standard deviation of the given random values. The function increases most considerably at values of difference close to the parameter σ. Preference increases gradually from point zero along with the gradual increase in (d). As the difference (d) in criteria values becomes considerably large so does the preference increase towards the preference threshold 1 but never hit on the exact mark. The algebraic definition is presented below while the graph is provided in Figure 3.12

$$p(d) = \begin{cases} 0, \text{when } d \leq 0 \\ 1 - \exp\left(-\frac{d^2}{2\sigma^2}\right), \text{when } d > 0 \end{cases} \tag{3.31}$$

In this graph are found the two axes, namely the deviation (d) axis which stretches horizontally towards the right from the point zero (0) and the level of preference, p (d) axis, which has point 1 as the maximum point of preference. The shape of the curve is that of a normal distribution and originates from 0 and rises steadily towards the point 1 on the p (d) axis.



0 σ d

Figure 3.12: Graph of Gaussian preference function

Other preference functions proposed by (Podvezko and Podviezko, 2010) are defined as follows

- $p(d) = \sqrt[3]{\frac{d}{p}}$ (its shape looks similar to the graph of c-shape preference function)
- $p(d) = \frac{2}{\pi} arctg\ d$ (Its shape looks similar to the Gaussian preference function, but applicable to non-statistical data.

3.3.8 DEPENDENCE OF EVALUATION RESULT ON CHOICE OF PREFERENCE FUNCTION TYPES AND THEIR PARAMETERS.

The dependence of evaluation results on choice of preference function types and their corresponding parameters had been demonstrated using the growing economies of the Baltic states and Poland for the year of 2003 (Podvezko and Podviezko, 2010). The statistical data they used is shown in the Table 3.7.

Table 3.7: Criteria values of economical growth of different countries

	Criteria	Types of criteria	Estonia	Latvia	Lithuania	Poland
1	Annual growth of the GDP, %	max	5.1	7.5	9.7	3.8
2	Annual growth of production,%	max	9.8	6.5	16.1	8.4
3	Average annual salary in euro, %	max	430	298	306	501
4	Unemployment rate, %	min	9.3	10.3	11.6	19.3
5	Export/ import ratio,%	max	0.70	0.55	0.73	0.79

According to these authors, experts chose the following weights of these criteria values:

$w_1 = 0.28; w_2 = 0.19; w_3 = 0.15; w_4 = 0.18; w_5 = 0.20.$ They then explored the dependence of evaluation results using PROMETHEE I and PROMETHEE II methods – the focus of this thesis – on the choice of the type of the preference function $p\ (d)$ among the five used in practice and described above and its parameters. The sixth Gaussian function was not used as the given data did not contain standard deviation parameter σ, and could not be derived either.

In order to choose parameters q and s as the difference and preference thresholds respectively for preference functions first they found out the smallest module of differences between given criteria values

$\min_{1\leq j,k\leq n}\left|d_i(A_j,A_k)\right|$ and the largest module of differences $\max_{1\leq j,k\leq n}\left|d_i(A_j,A_K)\right|$ using the following algorithm. The largest module of difference could be obtained using the formula:

$\max_{1\leq j,k\leq n}\left|d_i(A_j,A_K)\right| = \max_j r_{ij} - \min_j r_{ij}$. For the first criterion, for example, it yielded:

$\max_{1\leq j,k\leq 4}\left|d_1(A_j,A_K)\right| = 9.7 - 3.8 = 5.9$. To obtain the smallest module of difference, the data was sorted in the descending order, difference of nearby criteria values was calculated and the smallest difference is therefore taken. For example, the sorted list of values of values of the criterion in the first row is the following: (9.7; 7.5; 5.1; 3.8). The smallest module of differences for this criterion is equal

$\min\limits_{1\leq j,k\leq 4(j\neq k)}\left|d_i(A_j,A_k)\right| = min|(9.7-7.5);(7.5-5.1);(5.1-3.8)| =$

$min|(2.2);(2.4);(1.3)| = 1.3$

Values of parameters *q* and *s* for preference functions felt to the interval between the smallest and the largest modules of differences of values of criterion:

$\min_{1\leq j,k\leq n}\left|d_i(A_j,A_k)\right| \leq q \leq s \leq \max_{1\leq j,k\leq n}\left|d_i(A_j,A_K)\right|$.

It was clear that setting parameter *q* lower than just obtained the smallest value $\min_{1\leq j,k\leq n}\left|d_i(A_j,A_k)\right|$ and parameter s larger than the largest obtained value $\max_{1\leq j,k\leq n}\left|d_i(A_j,A_K)\right|$ would not make sense.

The smallest $\min_{1\leq j,k\leq n}\left|d_i(A_j,A_k)\right|$ and the largest $\max_{1\leq j,k\leq n}\left|d_i(A_j,A_K)\right|$

Differences of values of criteria describing development of economies of countries (see Table3.7) were shown in the Table 3.8

Table 3.8: The smallest and the largest modules of differences between given criteria values

	Criteria	$\min\limits_{1\le j,k\le n}\lvert d_i(A_j, A_k)\rvert$	$\max\limits_{1\le j,k\le n}\lvert d_i(A_j, A_K)\rvert$
1	Annual growth of the GDP	1.3	5.9
2	Annual growth of production	1.4	9.6
3	Average annual salary in euros	8	203
4	Unemployment rate	1.0	10.0
5	Export/import ratio	0.03	0.24

To demonstrate the dependence of evaluation results on the choice of preference functions and their parameters, six examples as shown below were proposed.

The first example was already studied (Podvezko and Podviezko, 2010) in which

$p_5(d_1)(q = 2; s = 3.5);\ p_3(d_2)(s = 7);\ p_4(d_3)(s = 150);\ p_2(d_4)(q = 2);\ p_1(d_5).$ this them meant that for the first criterion the fifth preference function was used with parameters q = 2 and s = 3.5; similarly, for other criteria. They aimed to use all the five preference functions here, different for every criterion. In the second example, the first preference function was used for all criteria. It does not have q and s parameters. In the third example, only the second preference function was used with parameters:

$q_1 = 2.5;\ q_2 = 2;\ q_3 = 150;\ q_4 = 2.2; q_5 = 0.1$. In the fourth example the third preference function was used for all criteria with the following parameters:

$s_1 = 5;\ s_2 = 8; s_3 = 100;\ s_4 = 10; s_5 = 0.1$. In the fifth example the fourth preference function was used for all the criteria with the following parameters:

$q_1 = 2.5;\ s_1 = 5;\ q_2 = 2; s_2 = 8; q_3 = 130;\ s_3 = 195;\ q_4 = 2.3;\ s_4 = 10; q_5 = 0.06;\ s_5 = 0.15$.

In the sixth example the fifth preference function was used for all the criteria with the following parameters:

$q_1 = 2.5;\ s_1 = 5;\ q_2 = 2; s_2 = 8;\ q_3 = 130;\ s_3 = 195;\ q_4 = 2.3; s_4 = 10; q_5 = 0.06; s_5 = 0.15$

In different fourth and fifth preference functions used in fifth and sixth examples, they chose the same parameters *q* and *s*.

Now they found out dominance relation $\pi(A_j, A_k)$ between all pairs of alternatives, preference, indifference and incomparability by using the formula: $\pi(A_j, A_k) = \sum_{i=1}^{m} w_i\, p_t(d_i(A_j, A_k))$ where w_i is the weight of the i-th criterion$(\sum_{i=1}^{m} w_i = 1)$; $d_i(A_j, A_k) = r_{ij} - r_{ik}$ is the difference between values r_{ij} and r_{ik} of the criterion R_i for the alternative A_j and A_k; $P_t(d) = p_t(d_i(A_j, A_k))$ is the t-th preference function chosen by the decision maker for the i-th criterion from the set of available preference functions

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 DATA COLLECTION

The data for the study was obtained through a set of questionnaire that was prepared and sent to the National Communications Authority (NCA). The data, quantitative, was on the performance of five (5) telecommunication networks in the Greater Accra region based on the standard criteria set by the (NCA) for measuring performance.

The data sourced from the (NCA) is a measure of the performance of five (5) telecom networks as of June, 2010.

4.2 COMPONENTS OF DATA

4.2.1. ALTERNATIVES

In this study, the five (5) telecommunication networks (alternatives) identified in the region are represented as:

$A_1, A_2, A_3, A_4, \ A_5$

We define these networks as the set of alternatives

$A = \{A_j\} = \{A_1, A_2, A_3, A_4, A_5,\}$

4.2.2. CRITERIA (C_i)

The criteria identified by the National Communications Authority for measuring performance are

(i) Call Setup Time (C_1)

(ii) Call Completion Rate (C_2)

(iii) Call Congestion Rate (C_3)

(iv) Call Drop Rate (C_4)

Call Set Up Time (CST): (C_1)

This is the elapsed time between sending of a complete destination address (target telephone number) and connecting of call by the network. It is calculated as

$$\text{CST [s]} = t_{address-sending} - t_{calling-signal}. \qquad (4.1)$$

Where $t_{calling-signal}$ Stands for moment when the user presses the send button and

$t_{address-sending}$ Is the moment one hears the call signal on the caller terminal.

Condition: CST should be less than ten seconds (< 10 secs) in 95% of cases.

Hence, this is a minimizing criterion.

This criterion (C_1) has the following set of associated data

$\{ A_j \} = \{ x_{11}, x_{12}, x_{13}, x_{14}, x_{15} \}$

= {15.12, 12.09, 11.67, 13.86, and 15.25},

where x_{ij} = score of alternative *j* on criterion *i*

Call Completion Rate (CCR): (C_2)

This is the probability that after being successfully set up, a call can be maintained for a period of time, until normal ending (i.e. ending according to the users will)

$$CCR\ [\%] = \frac{\text{Number of Normally ended calls}}{\text{Total number of call attempts}} \times 100\%. \quad (4.2)$$

Condition: CCR should be equal or better than seventy percent (70%).

This is a maximizing criterion

Criterion C_2 has the following set of associated data:

$\{A_j\} = \{x_{21}, x_{22}, x_{23}, x_{24}, x_{25}\}$

$= \{80, 96, 41, 81, \text{and } 88\}$

Call Congestion Rate (CGR): (C_3)

This is the probability of failure of accessing a traffic channel during Call Set up.

$$CGR\ [\%] = \frac{\text{Number of connect failed calls}}{\text{Total number of call attempts}} \times 100\%. \quad (4.3)$$

Condition: CGR should be equal or less than one percent (1%).

This is a minimizing criterion.

The data set for this criterion, C_3, is

$\{A_j\} = \{x_{31}, x_{32}, x_{33}, x_{34}, x_{35}\}$

$= \{17, 3, 27, 12, \text{and } 10\}$

Voice Call Drop Rate (VCDR): (C_4)

Voice Call Drop Rate is the probability of a call terminating without any of the users will;

$$\text{Drop Rate (\%)} = \frac{\text{Number of calls terminated unwillingly}}{\text{Total number of call attempts}} \times 100\%. \quad (4.4)$$

Condition: VCDR should be equal or less than three percent (3%)

It is a minimizing criterion.

The data set for this criterion C_4 is

$\{A_j\} = \{x_{41}, x_{42}, x_{43}, x_{44}, x_{45}\}$

= {3, 1, 32, 8, and 2}

4.2.3 WEIGHT OF A CRITERION

Weights of the criteria w_i, for $i = 1, \dots 4$ are taken by the (NCA) to be the same. Thus, each criterion was weighed 0.25 according to NCA's measure. Summing all together for the four (4) criteria gives 1 as expected. This signifies that all the criteria were of equal value or importance

4.2.4 THE DECISION TABLE

The decision table showing the performance of each network in the Greater Accra region as of June, 2010 is shown in Table 4.1.

The first column labeled criteria is the column for the four criteria $(C_1, \dots, C_4)$.

The second column, type of criteria, indicates whether a given criterion C_i is a maximizing or minimizing criterion

The third column, alternatives, is a 4x5 matrix in which each of the four rows represent respectively $C_1, C_2, C_3, \text{and} C_4$ while each of the five columns represents one of the five alternatives, $A_1, A_2, A_3, A_4, \text{and } A_5$.

The entries of the matrix x_{ij} where i = 1,2,3,4; j = 1,2,3,4,5 are the scores of the various alternatives under each criterion for all the criteria.

Table 4.1: Decision table showing the performance of each network in the Greater Accra Region as of June, 2010

Criteria	Type of Criteria	Alternatives				
		A_1	A_2	A_3	A_4	A_5
C_1	Min	15.12	12.09	11.67	13.86	15.28
C_2	Max	80	96	41	81	88
C_3	Min	17	3	27	12	10
C_4	Min	3	1	32	8	2

4.3 DATA ANALYSIS AND RESULTS

4.3.1 SINGLE CRITERION OPTIMIZATION

Using the single criterion approach, the five alternatives are ranked on one criterion at a time for all the four criteria as shown in Table 4.2. In other words, the alternatives are ranked on each criterion separately using the decision table.

The ranking of the five (5) alternatives for each criterion based on the data given in the decision Table 4.1 is shown in Table 4.2

Table 4.2: ranking of the five alternatives for each criterion

Criterion	Ranking 1	Ranking 2	Ranking 3	Ranking 4	Ranking 5
C_1 (Min)	A_3: 11.67	A_2 :12.09	A_4: 13.86	A_1:15.12	A_5: 15.28
C_2 (Max)	A_2: 96	A_5 :88	A_4: 81	A_1: 80	A_3: 41
C_3 (Min)	A_2: 3	A_5:10	A_4: 12	A_1: 17	A_3:27
C_4 (Min)	A_2: 1	A_5:2	A_1: 3	A_4: 8	A_3: 32

From Table 4.2, it is observed that alternatives A_3 and A_2 have both appeared in ranking 1 (second column) as the best solutions. At the same time A_2 and A_5 have appeared in ranking 2 (third column) as the second best while A_5 and A_3 have occupied ranking 5 (last column) as worst solutions and so on.

The two or more alternatives often obtained as optimal solutions (in ranking 1) together with the contradictions in the various outcomes makes the single criterion approach not only unreliable but ineffective in aiding the decision maker to come out with the unique but optimal solution.

4.3.2. MULTIPLE CRITERIA OPTIMIZATION

The multiple criteria optimization involves the evaluation and ranking of the five alternatives on the four criteria concurrently. In this case, instead of separate ranking as is done under single criterion optimization, ranking is done by considering all the criteria at the same time. The PROMETHEE method which is one of the multiple criteria approaches is applied here and involves the following steps:

1.**The Preference Function**: Our data was sampled from a continuous set and as a result we use the Gaussian preference function since the generalized preference function used is the choice of the decision maker based on his priorities and the Gaussian preference function is often chosen in PROMETHEE methodology for evaluating criteria on continuous data (Villota, 2009). As noted before, the Gaussian Criterion Function is defined by

$$p(d) = \begin{cases} 0, & d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}}, & d > 0 \end{cases} \tag{4.5}$$

Using this function, the only parameter we have to define is the Standard Deviation $\boldsymbol{\sigma}$.

This is calculated using the decision matrix of Table 4.1 via the formula

$$\sigma_i^2 = \sum_{j=1}^{n} \frac{(x_{ij} - \mu)^2}{n-1}, \; i = 1, \dots 4, \tag{4.6}$$

Where

$$\mu_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \qquad (4.7)$$

is the mean of the data

The Standard Deviation (σ_i) and the mean (μ_i) for each of the four criterion C_i are found in Table 4.3. The values are set to two decimal places

Table 4.3: The mean and standard deviation of the four criteria.

Criterion	Mean (μ)	Standard Deviation (σ)
C_1	13.60	1.67
C_2	77.20	21.23
C_3	13.80	8.93
C_4	10.20	15.22

2. Calculation of Deviations

We calculate the deviations, $d_i(A_k, A_l)$ through pair wise comparison of the values of the alternatives, $A_k, A_l \in A$ on each criterion over all the criteria. It is recalled that the deviation,

$d_i(A_k, A_l)=$

$$\begin{cases} x_{ik} - x_{il} & - \text{maximisation} \qquad (4.8a) \\ -(x_{ik} - x_{il}) & - \text{minimisation} \qquad (4.8b) \end{cases}$$

where x_{ik} and x_{il} correspond to values of two alternatives on a criterion as provided in the decision matrix of Table 4.1

Table 4.4 presents all possible deviation $d_i(A_k, A_l)$ emanating from the pair wise comparison of all the alternatives on each criterion $C_i \in C$.

Table 4.4a: Deviations $d_1(A_k, A_l)$ on the minimizing criterion C_1

Min C_1	A_1	A_2	A_3	A_4	A_5
A_1	0	-3.03	-3.45	-1.26	0.16
A_2	3.03	0	-0.42	1.77	3.19
A_3	3.45	0.42	0	2.19	3.61
A_4	1.26	-1.77	-2.19	0	1.42
A_5	-0.16	-3.19	-3.61	-1.42	0

Table 4.4b: Deviations $d_2(A_k, A_l)$ on the maximizing criterion C_2

Min C_2	A_1	A_2	A_3	A_4	A_5
A_1	0	-16	39	-1	-8
A_2	16	0	55	15	8
A_3	-39	-55	0	-40	-47
A_4	1	-15	40	0	-7
A_5	8	-8	47	7	0

Table 4.4c: Deviations $d_3(A_k, A_l)$ on the minimizing criterion C_3

Min C_3	A_1	A_2	A_3	A_4	A_5
A_1	0	-14	10	-5	-7
A_2	14	0	24	9	7
A_3	-10	-24	0	-15	-17
A_4	5	-9	15	0	-2
A_5	7	-7	17	2	0

Table 4.4d: Deviations $d_4(A_k, A_l)$ on the minimizing criterion C_4

Min C_4	A_1	A_2	A_3	A_4	A_5
A_1	0	-2	29	5	-1
A_2	2	0	31	7	1
A_3	-29	-31	0	-24	-30
A_4	-5	-7	24	0	-6
A_5	1	-1	30	6	0

The entries are expressed to 2 decimal places

3. Preference Evaluation

Now that the deviation $d_i(A_k, A_l)$ has been computed, we evaluate $P_i(A_k, A_l)$ which measures the intensity of the decision maker's preference of A_k over A_l. Thus,

$$P_i(A_k, A_l) = \begin{cases} 0, & d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}}, & d \geq 0 \end{cases} \tag{4.5}$$

Recall that $P_i(A_k, A_l) = P_i(d) \Rightarrow d = d_i(A_k, A_l)$

Table 4.5 Contains the summary of the values of $P_i(A_k, A_l)$, $\forall A_k, A_l \in A$ for each of the criterion $C_i \in C$.

Table4.5a: Values of $P_1(A_k, A_l)$. For criterion C_1

C_1	l=1	l=2	l=3	l=4	l=5
k=1	0.00	0.00	0.00	0.00	0.00
k=2	0.81	0.00	0.00	0.43	0.84
k=3	0.88	0.03	0.00	0.58	0.90
k=4	0.25	0.00	0.00	0.00	0.30
k=5	0.00	0.00	0.00	0.00	0.00

Table 4.5b: Values of $P_2(A_k, A_l)$, for criterion C_2

C_2	l=1	l=2	l=3	l=4	l=5
k=1	0.00	0.00	0.81	0.00	0.00
k=2	0.25	0.00	0.97	0.22	0.07
k=3	0.00	0.00	0.00	0.00	0.00
k=4	0.00	0.00	0.83	0.00	0.00
k=5	0.07	0.00	0.91	0.05	0.00

Table 4.5c: Values of $P_3(A_k, A_l)$, for criterion C_3

C_3	l=1	l=2	l=3	l=4	l=5
k=1	0.00	0.00	0.47	0.00	0.00
k=2	0.71	0.00	0.97	0.40	0.26
k=3	0.00	0.00	0.00	0.00	0.00
k=4	0.15	0.00	0.76	0.00	0.00
k=5	0.26	0.00	0.84	0.02	0.00

Table 4.5d: Values of $P_4(A_k, A_l)$, for criterion C_4

C_4	l=1	l=2	l=3	l=4	l=5
k=1	0.00	0.00	0.84	0.00	0.00
k=2	0.01	0.00	0.87	0.10	0.00
k=3	0.00	0.00	0.00	0.00	0.00
k=4	0.00	0.00	0.71	0.00	0.00
k=5	0.00	0.00	0.86	0.07	0.00

4. Aggregated Preference Index

We already mentioned that the aggregated preference index denoted by $\pi(A_k, A_l)$ is defined as

$$\pi(A_k, A_l) = \sum_{i=1}^{n} w_i \ P_i(A_k, A_l) \quad \forall A_k, A_l \in A \tag{4.10}$$

$with\ k, l = 1, \dots, m,$ w_i= 0.25 is the weight of each criterion

$C_i \ \forall \ C_i \ \in C$. The values of $\pi(A_k, A_l)$ for all the five (5) alternatives in A are shown in Table 4.6

Table 4.6: Aggregated preference indices $\pi(A_k, A_l)$

	A_1	A_2	A_3	A_4	A_5
A_1	0.00	0.00	0.53	0.00	0.00
A_2	0.45	0.00	0.70	0.29	0.29
A_3	0.22	0.01	0.00	0.15	0.23
A_4	0.1	0.00	0.58	0.00	0.08
A_5	0.08	0.00	0.65	0.04	0.00

1. Partial Ranking

We obtain the Partial Ranking of our finite set of alternatives through the equations,

(i) $\phi^{+}(A_i) = \frac{1}{n-1}\sum_{k\in A}\pi(A_i,\ A_k), i = 1, \dots, 5$ (4.11)

(ii) $\phi^{-}(A_i) = \frac{1}{n-1}\sum_{k\in A}\pi(A_k,\ A_i), i = 1, \dots, 5$ (4.12)

Table 4.7 presents values of the positive $\phi^{+}(A_j)$ and negative outranking flows $\phi^{-}(A_j)$ for the five alternatives

Table 4.7 Values of $\phi^{+}(A_j)$ and $\phi^{-}(A_j)$

A_j	$\phi^{+}(A_j)$	$\phi^{-}(A_j)$
A_1	0.13	0.21
A_2	0.43	0.0
A_3	0.15	0.62
A_4	0.19	0.12
A_5	0.19	0.15

1. A_k is preferred to A_l if and only if one of the following three conditions is satisfied.

$i)\ \phi^+(A_k) > \phi^+(A_l)$ and $\phi^-(A_k) < \phi^-(A_l)$

$ii)\ \phi^+(A_k) > \phi^+(A_l)$ and $\phi^-(A_k) = \phi^-(A_l)$

$iii)\ \phi^+(A_k) = \phi^+(A_l)$ and $\phi^-(A_k) < \phi^-(A_l)$

Table 4.8a is a table of 5×5 matrix M, where the five columns and five rows represent the five alternatives. The entries, m_{ij}, denoted by dash (-) indicate no preference between any pair of alternatives while entries with the value one (1) show preference of alternative A_k over A_l

Table 4.8a: The preference matrix for the five alternatives

$$\begin{array}{c c}
 & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \left(\begin{array}{ccccc} - & - & - & - & - \\ 1 & - & 1 & 1 & 1 \\ - & - & - & - & - \\ 1 & - & 1 & - & 1 \end{array}\right.
\end{array}$$

A_5	1	-	1	-	-

2. Indifference: indifference exists between any pair of the five alternatives if and only if the condition below is satisfied.

$\phi^+(A_k) = \phi^+(A_l)$ and

$\phi^-(A_k) = \phi^-(A_l)$

In our case, no indifference exists and so its table of matrix has been left out.

3. Incomparability: two of the alternatives are considered incomparable if and only if

$\phi^+(A_k) > \phi^+(A_l)$ and

$\phi^-(A_k) > \phi^-(A_l)$

Table of matrix 4.8b presents the matrix Q of the incomparability between pairs of the five alternatives. In the matrix, Q_{ij} = 1 implies A_k is incomparable to A_l while Q_{ij} being dash (-) implies A_k is comparable to A_l

Matrix Q of Table 4.8b is the complement of matrix M of Table 4.8a

Table 4.8b: Complement of matrix M

	A_1	A_2	A_3	A_4	A_5
A_1	-	1	1	1	1
A_2	-	-	-	-	-

A_3	1	1	-	1	1
A_4	-	1	-	-	-
A_5	-	1	-	1	-

4. Table 4.8c which is the same as table 4.8a is now considered the incidence matrix for the alternatives.

Table 4.8c: Incidence matrix of alternatives

$$
\begin{array}{c c}
 & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} &
\left(\begin{array}{ccccc}
- & - & - & - & - \\
1 & - & 1 & 1 & 1 \\
- & - & - & - & - \\
1 & - & 1 & - & 1 \\
1 & - & 1 & - & -
\end{array}\right)
\end{array}
$$

From Table 4.8c, the row with the highest number of ones is the row with the highest number of directed arcs and the corresponding alternative in that row is the best alternative. Graph of the partial ranking of the five (5) alternatives according to Table 4.8c is shown in Figure 4.1

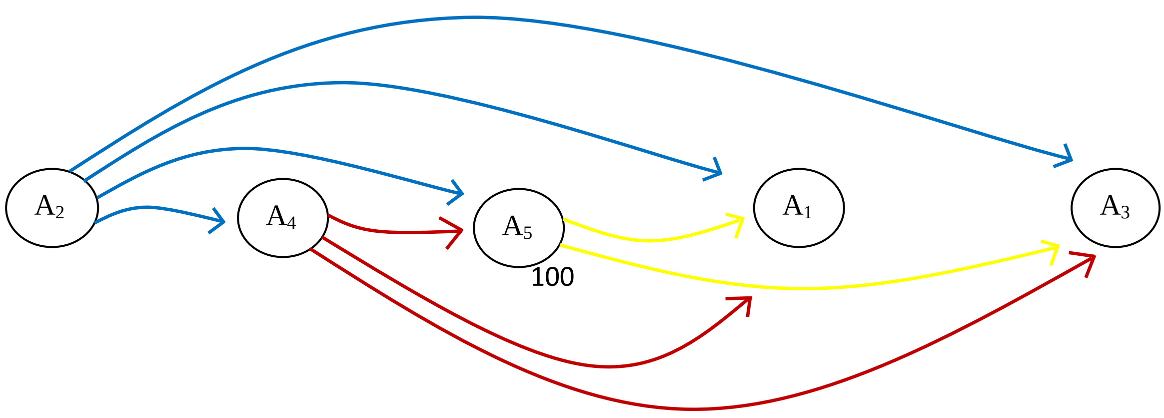


Figure 4.1: Graph of partial ranking

From Figure 4.1, it is realized that there is no connection between

A_1 and A_3 Indicating that the two alternatives are incomparable. Hence, we apply the complete ranking method.

5.Complete Ranking

In Complete Ranking, we analyze pairs of alternatives using their net flows ($\phi(A_j)$). This is achieved using the equation

$$\phi(A_j) = \phi^+(A_j) - \phi^-(A_j) \quad (4.13)$$

The net flows for the five (5) alternatives are presented in Table 4.9

Table 4.9: Net flows for the five (5) alternatives

Table 4.9: Net flows for the five (5) alternatives

A_j	$\phi^+(A_j)$	$\phi^-(A_j)$	$\phi(A_j)$
A_1	0.13	0.21	-0.08
A_2	0.43	0.00	0.43
A_3	0.15	0.62	-0.47
A_4	0.19	0.12	0.07

A_5	0.19 0.15	0.04

Table 4.10a.shows the pair-wise calculation of the net flows $\phi(A_j)$ of the five alternatives

Table 4.10a: pairs of calculated values $(\phi(A_k), \phi(A_l))$ of the net flows $\phi(A_j)$

$$\begin{array}{cc|c} & & \underline{A_l} \\ & & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ A_k & \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \begin{pmatrix} - & (-0.08, 0.43) & (-0.08, 0.47) & (-0.08, 0.07) & (-0.08, 0.04) \\ (0.43, -0.08) & - & (0.43, -0.47) & (0.43, 0.07) & (0.43, 0.04) \\ (0.47, -0.08) & (0.47, 0.43) & - & (0.47, 0.07) & (0.47, 0.04) \\ (0.07, -0.08) & (0.07, 0.43) & (0.07, -0.47) & - & (0.07, 0.04) \\ (0.04, -0.08) & (0.04, 0.43) & (0.04, -0.47) & (0.04, 0.07) & - \end{pmatrix} \end{array}$$

1. Preference exists between a pair of alternatives (A_k, A_l) if

$\phi(A_k) \neq \phi(A_l)$

So, for the pair (A_k, A_l), alternative A_k is preferred to alternative A_l if and only if $\phi(A_k) > \phi(A_l)$. Otherwise A_k is not preferred to alternative A_l

Table 4.10b shows the pair-wise comparison of the calculated net flows $\phi(A_j)$

Table 4.10b: Matrix for the pair wise comparison of the net flows $\phi(A_j)$

$$
\begin{array}{ccc}
 & & \underline{A_l} \\
 & & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\
A_k & \begin{array}{c} A_1 \\ A_2 \\ \backslash A_3 \\ A_4 \\ A_5 \end{array} & \begin{pmatrix} - & - & 1 & - & - \\ 1 & - & 1 & 1 & 1 \\ - & - & - & - & - \\ 1 & - & 1 & - & 1 \\ 1 & - & 1 & - & - \end{pmatrix}
\end{array}
$$

From Table 4.10b, the value 1 is assigned to the former case and dash (-) to the latter case

2. Indifference between pairs of alternatives exists if

$$\phi(A_k) = \phi(A_l). \tag{4.14}$$

In our case, no indifference exists

Table 4.10c shows the matrix of the preference of A_k over A_l used for complete ranking. In this matrix, every value 1 is considered a directed arc. So, the row with the highest number of ones (1s) has the highest number of directed arcs and the corresponding alternative (A_k) in that row is regarded the best alternative

Table 4.10c: Matrix of the preference A_k over A_l for complete ranking

$\underline{A_l}$

$$\begin{array}{cc} & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ A_k \; \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \begin{pmatrix} - & - & 1 & - & - \\ 1 & - & 1 & 1 & 1 \\ - & - & - & - & - \\ 1 & - & 1 & - & 1 \\ 1 & - & 1 & - & - \end{pmatrix} \end{array}$$

The graph of the complete ranking based on the content of Table 4.10c is presented by Figure 4.2.

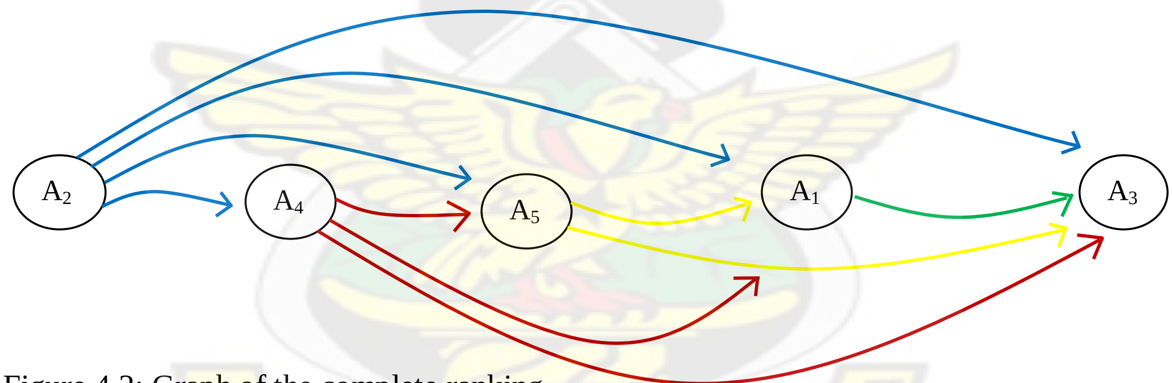


Figure 4.2: Graph of the complete ranking

The ranking is done based on the number of directed arcs that is recorded by each alternative such that the best alternative (A_2) is the one with the highest number of directed arcs and the alternative A_3 with no directed arc becomes the worst one .

By means of PROMETHEE methodology the ranking of the five (5) alternatives are displayed in Table 4.11

Table 4.11: ranking of the five alternatives using PROMETHEE

Alternatives	Number of directed arcs	Ranking position
A_1	1	4
A_2	4	1
A_3	0	5
A_4	3	2
A_5	2	3

From the table, it is observed that $A_2 > A_4 > A_5 > A_1 > A_3$

where ">" means "is better than"

Hence, the alternative A_2 is the best alternative.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

a) By means of PROMETHEE methodology the ranking of the five (5) alternatives in our multicriteria optimization problem has been established. The ranking is presented in Table 5.1

Table 5.1: Ranking position of the five (5) alternatives

Ranking position	PROMETHEE
1	A_2
2	A_4
3	A_5
4	A_1
5	A_3

Hence, the alternative A_2 is the best alternative followed by A_4 A_5, A_1 and A_3

5.2 RECOMMENDATIONS

It is recommended that experts, analysts and academics further investigate the PROMETHEE method and other multicriteria techniques like Analytic Hierarchy Process (AHP).

It is as well recommended to NCA to use PROMETHEE method. They could use the software: Decision Lab that is based on PROMETHEE.

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APPENDIX I

The data used by Villota on the usability of the 5 websites namely:

A_1: Amazon

A_2: Blackwell

A_3: Bookstore

A_4: Borders

A_5: Water stone's

Based on 7 criteria:

C_1: Accessibility

C_2: Customization and Personalization

C_3: Download speed

C_4: Ease of use

C_5: Errors

C_6: Navigation

C_7: Site content,

are shown in the Decision Table 4.10.

Table 4.10: Score of each alternative on each criterion

$$
\begin{array}{c c}
 & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\
\begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} &
\begin{pmatrix}
19 & 23 & 6 & 13 & 39 \\
27 & 28 & 5 & 16 & 24 \\
12 & 24 & 29 & 26 & 10 \\
32 & 10 & 15 & 11 & 32 \\
29 & 25 & 11 & 13 & 21 \\
11 & 19 & 13 & 11 & 46 \\
34 & 11 & 7 & 9 & 39
\end{pmatrix}
\end{array}
$$

Now, ranking the alternatives as given in Table 4.10 using the one criterion optimization approach, the results are indicated in Table 4.11

Table 4.11: Ranking of the alternatives for each criterion $C_{i,}$ for $i = 1, \ldots, 7$

Criterion	Ranking 1	Ranking 2	Ranking 3	Ranking 4	Ranking 5
C_1	A_5 39	A_2 23	A_1 19	A_4 13	A_3 6
C_2	A_2 28	A_1 27	A_5 24	A_4 16	A_3 5
C_3	A_3 29	A_4 26	A_2 24	A_1 12	A_5 10
C_4	A_1 32	A_5 32	A_3 15	A_4 11	A_2 10
C_5	A_1 39	A_2 25	A_5 21	A_4 13	A_3 11
C_6	A_5 46	A_2 19	A_3 13	A_1 11	A_4 11
C_7	A_5 39	A_1 34	A_2 11	A_4 9	A_3 7

Table 4.12: Ranking of the alternatives using PROMETHEE

Ranking Position	PROMETHEE
1	A_5: Amazon
2	A_1: Blackwell
3	A_2: Bookstore
4	A_4: Borders
5	A_3: Water stone's

APPENDIX II

The Following Function may also be used in place of the Gaussian preference function:

$$p(d) = \begin{cases} 0, \text{ when } d \leq 0 \\ \dfrac{1-e^{\left(\frac{-d^2}{\sigma^2}\right)}}{1+e^{\left(\frac{-d^2}{\sigma^2}\right)}}, \quad \text{when } d > 0 \end{cases}$$