OPTIMAL REVENUE GENERATION USING LINEAR PROGRAMMING

A CASE STUDY OF DORMAA EAST DISTRICT ASSEMBLY

By

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DECLARATION

I hereby declare that this dissertation is the true account of my own research work towards the award of MSc. And that, to the best of my knowledge, it contains no material previously published by another person or any material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

In the bid to assist in the improvement of revenue generation efforts at the Dormaa East District Assembly, the Primal-dual method, which is one of the interior-point methods, was used. The data was collected from the Dormaa East District Assembly. The data was modeled into objective function and subject constraints. Matrices generated were run on Matlab 7.5.0(R2007b) Code. The results obtained showed a remarkable and efficient income generation strategy compared to the existing one being used by the Assembly for the past four years.

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LIST OF ABBREVIATIONS

L.P:	Linear Programming
A.R:	Actual Revenue
E.R:	Estimated Revenue
PP:	Primal Problem
DP:	Dual Problem
R.H.S:	Right Hand Side

DEDICATION

I wish to specially dedicate this piece of work to my darling wife, Mrs. Francisca Ofori ; my dear sons Ofori-Amanfo Asiedu and Ofori-Amanfo Adjei , my dear mother Mad. Abena Adjei and my late father Opanin Kwadwo Asiedu; may his soul rest in peace.

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CHAPTER 1

INTRODUCTION

1.1 Background to the Study

The role of government in raising revenue and the capacity of governments to raise taxes for the purpose of financing economic development have preoccupied economists and policy makers for a long time. More than forty years ago, Kaldor (1963) raised the very important question of whether underdeveloped countries will "learn to tax", with the underlying view that for these countries to reach higher levels of living standards, they would need to achieve levels of tax effort that are significantly higher than observed at that time.

Kaldor was in fact echoing an earlier call by Sir Arthur Lewis who posited that "the government of an underdeveloped country needs to be able to raise revenue of about 17-19 percent of Gross National Product(GNP) in order to give a better than average standard of service" (Martin and Lewis 1956).

Indeed, the evidence clearly shows that tax effort rates are much higher for high-income countries than in low-income countries, supporting the notion that performance in tax mobilization is essential for reaching higher levels of income. A low level of government revenue is a constraint on the capacity to finance essential public investment programs and undertake adequate levels of spending on social services, which are essential for improving living standards.

What is less straightforward is what makes a country or a government capable of achieving high levels of revenue performance. As Bird, Vazquez and Torgler (2008) point out, most of the attention in the analyses of tax effort has traditionally been focused on the supply side (or "tax handles" in their words), mainly the availability of readily taxable activities such as trade/commerce and natural resources.

However, as these authors rightly point out, "telling a country that wants to raise its tax levels to find and tax natural resources is not a particularly promising piece of policy advice."

In reality, however, the problem is even much more complicated than presented by Bird and his colleagues. In fact, even finding natural resources does not necessarily guarantee a high level of revenue performance.

Many countries have found natural resources but not all those that were lucky to find a bounty in their underground have been able to take advantage of the resources in raising government revenue.

African countries have generally performed poorly in tax revenue mobilization.

The average tax-to-GDP ratio in sub-Saharan Africa increased only moderately over the past two decades. Two key problems are evident from the evidence. First, African countries have been unable to harness natural resource endowment for the purpose of revenue mobilization. Second, African countries have been unable to develop their capacity to mobilize non-resource sources of tax revenue. In the case of resource-rich countries, this is a result of failure to utilize the natural resource bonanza to promote activities outside the natural resource industry, so as to diversify their production and export base. The problem goes beyond the issue of value addition in the natural resource industry – or moving up the value chain. It also lacks capabilities to innovate within and outside the natural resource value chain.

1.1.0 Relevant Terminologies/Definitions

Revenue: Revenue is a calculation or estimation of periodic income based on a particular standard accounting practice or the rules established by a government or government agency. **Mathematical Model:** A mathematical model is a mathematical representation of an ideal or a real life situation in which all factors contributing to the idea or situation are represented by variables and sometimes, constant figures.

Boundary Line: It is a line or border around outside of a shape. The boundary line defines the space or area.

Corner Points: These are a set of points in the feasible region, which are the intersection of two or more boundary lines. The optimal solution of an objective function if it exists occurs at the corner point of the feasible region.

Feasible Region: The feasible region for an LP is the set of all points satisfying all the LP's constraints and all other sign restrictions.

Bounded Solution: A feasible solution is said to be bounded if it can be contained in a closed figure. If the solution cannot be contained in a closed figure, it is said to be unbounded. Under such condition, the optimal solution to the objective function may not exist.

Optimal Solution: This is the set of points of all feasible regions that produces the optimal value (maximum or minimum) of the objective function.

Linear Programming: Linear programming (LP) is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some lists of requirements represented as linear relationships.

1.1.1 Sources of Revenue

The pool of district financial resources in many developing countries might come from seven main sources: independent revenue sources or own sources (if any) assigned to the district (receipts from these sources accrue directly to the district), central government financial transfers to the district (which can have different forms), voluntary contributions by community or beneficiary groups, profits from public enterprises or rents from public properties etc, financial assistance from donor agencies, short and long term loans and other sources like penalties, selling property. (Kroes, 2008).

1.1.2 Types of Revenue

Following the decentralization process, District Assemblies in Ghana now have the responsibility to plan and implement their own projects or programs. The Dormaa East District largely depends on internal sources for the day-to-day running of the district administration. These include rates and receipts, royalties from lands, fees and tolls, licenses, rent, investments and other miscellaneous activities that accrue as a result of its own effort at revenue mobilization and generation.

Externally, revenue also comes to the District Assembly from the Central Government in the form of Grant-in-aid and the District Assemblies' Common Fund (Dormaa East District medium term 2010-2013 draft plan). However Kessey and Kroes (1992) have noted that financing local development programmes in Ghana has become so problematic that the survival of the decentralized development process, in operation, appears to be threatened. Kazentet (2011), examines the poor revenue generated by most assemblies in Ghana as a result of insufficient revenue base, existence of two institutions working for internally generated funds, poor organizational structure and revenue administrational mechanisms, gap on knowledge and understanding of revenue, weak voluntary compliance as well as revenue leakages /corruption.

1.1.3 Challenges in Revenue Collection

According to the district medium term draft plan (2010-2013), poor data base on revenue/ratable items, inadequate qualified revenue collectors, inadequate and poor marketing facilities, high rate of tax evasion, inadequate logistics to promote education on the need to pay taxes, lack of permanent internal auditors/local government inspectors, inadequate revenue mobilization capacity and weak tax/revenue collection mechanism are the major problems of the district revenue mobilization. Hence this research examines the trends and contribution of internally generated funds for the development expenditure of the district in the broader framework of fiscal decentralization program of the country.

1.1.4 District Profile

The Dormaa East District was carved out of the Dormaa Municipal in November, 2007, through Legislative Instrument (L I) 1881, 2007. It was inaugurated on 29th February, 2008. The Administrated capital of the District is Wamfie. It covers a total land area of 456 Square Kilometres. According to the 2000 Population and Housing Census, the

District has a population of 58,172 and a growth rate of 2.1 percent per annum. The population is however projected to reach 76,070 in 2013.The predominant occupation in the District is agriculture which employs about 66.4 percent of the active labour force. Services employ 8.2 percent, Industry 4.9 percent and Commerce 0.6 percent. The District shares common boundaries with Dormaa Municipal to the West, Berekum to the North, Sunyani to the East and South by Asunafo North Municipal and Asutifi District.

1.2 Problem Statement

The standard of living in the Dormaa East District keeps on deteriorating as the Assembly is not able to provide the citizenry with the basic social amenities such as portable water, better healthcare facilities, quality education, good roads, improved sanitation, infrastructural development and so on .The reason being that the District Assembly is not able to mobilize sufficient revenue to execute its projects and programmes aimed at bettering the lots of its people. The Assembly since its inception in February, 2008 has never met its revenue target, and therefore has to rely heavily on the central government for its basic expenditure financing .This problem has been a major headache to the Assembly as it is hampering the effective growth of the district. This research work is basically targeted at developing a mathematical programme that will help the Assembly to optimize its revenue mobilization strategy.

1.3 Objectives of the Study

The study seeks to:

1. Mode l mobilization strategy by the Assembly as Linear Programming Problem.

2. Determine optimal strategy for revenue mobilization using Primal-Dual Interior-Point.

1.4 Methodology

The problem of revenue maximization will be modeled as a linear programming problem. Primal–Dual, one of the interior point algorithms will be used to solve the Mathematical Model. The interior-point method is preferred over simplex method because interior – point methods approach the boundary of the feasible set in the limit.

Quarterly secondary data, spanning between4th quarter of 2008 and 3rd quarter of 2011 will be collected from the Dormaa East District Assembly for this research work. Software programme on MatLab will be developed using the matlab code. Sources of information for this project would include the Dormaa East District Assembly, the internet, library books, journals, and reports.

1.5 Significance of the Study

The Dormaa East District Assembly, since its inception has been under performing in its revenue mobilization efforts. This state of affair has made it difficult for the Assembly to provide basic social services such as schools, healthcare, access roads, places of convenience, portable water supply etc. This project is geared towards finding a lasting solution to help the Assembly to optimize its revenue collection so that it can support its inhabitant to improve upon their standard of living with the provision of many social amenities such as schools, hospitals, provision of portable water etc. It is also envisaged that some other districts in the country with revenue mobilization challenges can use the findings from this research to improve upon their revenue generation strategy.

1.6 Limitations of the Study Limitations of the Study

The major limitations of this study are:

1. The research did not cover all the districts in Ghana, and therefore might not give a very general outlook of all the assemblies in the country.
2. It was quite cumbersome due to the volume of data involved and insufficient time available.
3. The research work was hampered by information outflow since many assemblies were not willing to give out relevant pieces of information.

In summary, it is my fervent hope that the Dormaa District Assembly, which is the major stakeholder in this research work, will implement the findings of this research work and adopt this method as an optimal revenue mobilization strategy for the Assembly.

1.7 Organization of the Study

The thesis consists of five chapters, including this chapter. Chapter 2 is on Literature Review which takes stock of what has already been written on the topic in terms of theories or concepts, scientific research studies and the overall goal of clarifying how the present study intends to address the gap silence or weakness in the existing literature. Chapter 3 explains the methodology that is being used for the study. The findings and discussion will be presented in Chapter 4. Lastly, Chapter 5 will discuss on the conclusion and recommendations

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

In this chapter we review some important literature in the field of linear programming algorithms. This study is aimed at maximizing the revenue generated by the Dormaa District Assembly focusing on the use of linear programming approach.

2.1 LITERATURE REVIEW ON LINEAR PROGRAMMING

Brief History

The problem of solving a system of linear inequalities dates back at least as far as Fourier, after whom the method of Fourier-Motzkin elimination is named. The three founders of the subject are considered to be Leonid Kantorovich, the Russian mathematician who developed the earliest linear programming problems in 1939, George Dantzig, who published the simplex method in 1947, and John von Neumann, who developed the theory of the duality in the same year. The earliest linear programming was first developed by Leonid Kantorovich, a Russian mathematician, in 1939. It was used during World War II to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy. The method was kept secret until 1947 when George B. Dantzig published the simplex method and John von Neumann developed the theory of duality as a linear optimization solution, and applied it in the field of game theory. Postwar, many industries found its use in their daily planning.

The linear-programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979, but a larger theoretical and practical breakthrough in the field came in 1984 when Narendra Karmarkar introduced a new interior-point method for solving linear-programming problems.

Dantzig's original example of finding the best assignment of 70 people to 70 jobs exemplifies the usefulness of linear programming. The computing power required to test all the permutations to select the best assignment is vast; the number of possible configurations exceeds the number of particles in the universe. However, it takes only a moment to find the optimum solution by posing the problem as a linear program and applying the Simplex algorithm. The theory behind linear programming drastically reduces the number of possible optimal solutions that must be check.

The Simplex Method. The simplex method has been the standard technique for solving a linear program since the 1940's. In brief, the simplex method passes from vertex to vertex on the boundary of the feasible polyhedron, repeatedly increasing the objective function until either an optimal solution is found, or it is established that no solution exists. In principle, the time required might be an exponential function of the number of variables, and this can happen in some contrived cases. In practice, however, the method is highly efficient, typically requiring a number of steps which is just a small multiple of the number of variables. Linear programs in thousands or even millions of variables are routinely solved using the simplex method on modern computers. Efficient, highly sophisticated implementations are available in the form of computer software packages.

Interior-Point Methods: In 1979, Leonid Khaciyan presented the ellipsoid method, guaranteed to solve any linear program in a number of steps which is a polynomial function of the amount of data defining the linear program. Consequently, the ellipsoid method is faster than the simplex method in contrived cases where the simplex method performs poorly. In practice, however, the simplex method is far superior to the ellipsoid method. In 1984, Narendra Karmarkar introduced an interior-point method for linear programming, combining the desirable theoretical properties of the ellipsoid method and practical advantages of the simplex method. Its success initiated an explosion in the development of interior-point methods. These do not pass from vertex to vertex, but pass only through the interior of the feasible region. Though this property is easy to state, the analysis of interior-point methods is a subtle subject which is much less easily understood than the behavior of the simplex method. Interior-point methods are now generally considered competitive with the simplex method in most, though not all, applications, and sophisticated software packages implementing them are now available. Whether they will ultimately replace the simplex method in industrial applications is not clear. An essential component of both the simplex and interior-point methods is the solution of systems of linear equations, which use techniques developed by C.F. Gauss and A. Cholesky in the 18th and 19th centuries. This can be seen in linear equations and matrix. Important generalizations of linear programming include integer programming, quadratic programming, nonlinear and stochastic programming. (Wikipedia, the free encyclopedia).

Many researchers have solved and continue to solve numerous practical problems using linear programming methods. Some of these landmark works are being reviewed in my research work as follows:

According to Branson and Knox (2001) the annual growth rates of real GDP in New Zealand have varied widely, from 18% to –8%, since World War II. During the period the tax burden (the ratio of tax revenue to GDP) has trended upward from 23% to 35%. The tax mix (the ratio of indirect taxes to direct taxes) has varied between 0.31 and 0.75, having increased recently with the introduction of the goods and services tax. In that paper they estimated a combination of the tax burden and the tax mix which would maximize the rate of growth of real GDP. They found out that such a tax structure would have a time-varying tax burden with a mean of 22.5%, and a time-varying tax mix with a mean of 0.54, which implies a mean share of direct taxes in total tax revenue of 65%. They also found that a move to such a tax structure would generate nearly a 17% increase in real a GDP, and while this increase would yield a 6% reduction in tax revenue to the Treasury, it would deliver a 27% increase in purchasing power to the remainder of the economy.

Sharp(2007) concluded that for linear programming models the effects of income taxes on the optimal activity levels and the optimal values of the dual variables were considered. Neither a fixed percentage tax nor a progressive tax, based on the net pretax contribution, changes the optimal activity levels. However, the optimal dual variables were changed in value, proportional to the highest tax rate actually in effect.

Jagannathan (1967) resolved that the portfolio selection problem faced by a mutual fund manager can be formulated following the Markowitz approach by finding those portfolios that were efficient in terms of predicted expected return and standard deviation of return, subject to legal constraints in the form of upper bounds on the proportion of the fund invested in any single security. The paper suggested that such problems be re-formulated as parametric linear-programming problems, utilizing a linear approximation to the true (quadratic) formula for a portfolio's risk. Limited empirical evidence suggested that the approximation was acceptable. Moreover, it allowed the use of an extremely simple and efficient special-purpose solution algorithm. With appropriate modifications, the algorithm may prove useful to the managers of mutual funds with a wide variety of objectives.

Milind et al. (2002) explored a design where, the market signal provided to a supplier was based on the current cost of procurement for the buyer. At the heart of this design lied a fundamental sensitivity analysis of linear programming. Each supplier was required to submit bid proposals that reduced the procurement cost (assuming other suppliers keep their bids unchanged) by some large enough decrement d > a. They showed that, for each supplier, generating a profit maximizing bid that decreased the procurement cost for the buyer by at least d could be done in polynomial time. This implied that in designs where the bids were not common knowledge, each supplier and the buyer could engage in an "algorithmic conversation" to identify such proposals in a polynomial number of steps. In addition, they showed that such a mechanism converged to an "equilibrium solution" where all the suppliers were at their profit maximizing solution given the cost and the required decrement d.

Nyikal and Adhiambo (2008) stated that financing smallholder farming has been one of the major concerns of Kenyans development efforts. They lamented that many credit programs have evolved over the years but with dismal performance. In a study that sought to find the best way to fund smallholder agriculture, it became necessary to analyze and document smallholders' effective demand for credit. Of particular interest was the comparison of the existing production plans and production plans under strictly profit maximization. Linear programming model was used to formalize observed plans and determine those under profit maximization. Both the activities and the values of outputs under different objectives were compared. Farm Investment Analysis was undertaken to determine the suitability of funding farm activities through credit. The study was undertaken in selected zones of Muranga and Kisumu districts, being typical smallholder areas. Sample farmers were visited and structured questionnaires administered to cover farm events and physical resources of short rains 1995 and long rains 1996. This formed a basis of formulating the farm plans. Ten years down the road, objectives of smallholders have not changed as have been observed during outreach programs. The results showed that: (i) farmers activities in the observed plans were different from those under strictly profit maximization; (ii) the observed plans had significantly lower profit than those under profit maximization; and (iii) meeting constraints through credit was only feasible when the objective was profit maximization. Smallholder agriculture, characterized by subsistence production, does not exhibit effective demand for credit, and funding it therefore requires means other than the competitive market.

Matthews (2005) evaluated and optimized the utility of the nurse personnel at the Internal Medicine Outpatient Clinic of Wake Forest University Baptist Medical Center. Linear programming (LP) was employed to determine the effective combination of nurses that would allow for all weekly clinic tasks to be covered while providing the lowest possible cost to the department. A specific sensitivity analysis was performed to assess just how sensitive the outcome was to the stress of adding or deleting a nurse to or from the payroll. The nurse employee cost structure in this study consisted of five certified nurse assistants (CNA), three licensed practicing nurses (LPN), and five registered nurses (RN). The LP revealed that the outpatient clinic should staff four RNs, three LPNs, and four CNAs with 95 percent confidence of covering nurse demand on the floor.

Kuo et al. (2003) stated that from the period of December 1, 2000, to July 31, 2002, the following individualized data were obtained for the Division of General Surgery at Duke University Medical Center: allocated OR time (hours), case mix as determined by CPT codes, total OR time used and normalized professional charges and receipts. Inpatient, outpatient, and emergency cases were included. The Solver linear programming routine in Microsoft Excel (Microsoft Corp.) was used to determine the optimal mix of surgical OR time allocation to maximize professional receipts. Their model of optimized OR allocation maximized weekly professional revenues at 237,523 US dollars, a potential increase of 15% over the historical value of 207,700 US dollars or an annualized increase of approximately 1.5 million US dollars. Their results suggested that mathematical modeling techniques used in operations research, management science, or decision science might rationally optimize OR allocation to maximize revenue or to minimize costs. These techniques may optimize allocation of scarce resources in the context of the goals specific to individual academic departments of surgery.

Mullan (2008) used linear programming to solve the ice cream mix calculations and provided a “proof of calculation” showing that the mass of the mix sums to the correct value and all the components e.g. fat also sum to the correct value. During the development of the calculator the researcher produced 11 Excel spreadsheets that covered many of the ice-cream formulation challenges that commercial manufacturers may encounter.

The metabolic pathway and the properties of many of the enzymes involved in the citric acid biosynthesis in the mould Aspergillus niger are well known. This fact, together with the availability of new theoretical frameworks aimed at quantitative analyses of control and dynamics in metabolic systems, from Torres et al (1996) has allowed them to construct a mathematical model of the carbohydrate metabolism in Aspergillus nigerunder conditions of citric acid accumulation. The model makes use of the S-system representation of biochemical systems, which renders it possible to use linear programming to optimize the process. It was found that maintaining the metabolite pools within narrow physiological limits (20% around the basal steady-state level) and allowing the enzyme concentrations to vary within a range of 0.1 to 50 times their basal values it was possible to triple the glycolytic flux while maintaining 100% yield of substrate transformation. To achieve these improvements it was necessary to modulate seven or more enzymes simultaneously. According to them although that seemed difficult to implement at the time, the results were useful because they indicated what the theoretical limits were and because they suggested several alternative strategies.

To investigate how farmers could sustain an economically viable agricultural production in salt-affected areas of Oman, Naifer et al (2010), divided a sample of 112 farmers into three groups according to the soil salinity levels, low salinity, medium salinity and high salinity. Linear programming was used to maximize each type of farm's gross margin under water, land and labor constraints. The economic losses incurred by farmers due to salinity were estimated by comparing the profitability of the medium and high salinity farms to the low salinity farm's gross margin. Results showed that when salinity increased from low salinity to medium salinity level the damage was US$ 1,604 ha^{-1} and US$ 2,748 ha^{-1} if it increased from medium salinity to high salinity level. Introduction of salt-tolerant crops in the cropping systems showed that the improvement in gross margin was substantial thus attractive enough for medium salinity farmers to adopt the new crops and/or varieties to mitigate the effect of water salinity.

Anderssen and Ive (1982) observed that the utility of linear programming for land use planning was firmly established. To them it allowed realistic models of complex planning situations to be formulated and solved computationally. However, because many objectives were qualitative and conflicting and many of the constraints may not be clearly defined, the actual construction of a linear-programming formulation to model any specific land-use planning problem would not be easy. In addition, even on large computers real problems could not in general be solved in an acceptable time. As a consequence, the availability and utility of the technique to people responsible for planning was restricted. In fact, in many planning contexts, what the practitioner needs most is the ability to experiment with alternative plans cheaply and easily.That lead naturally to a search for 'simplifications' of the linear-programming formulations for land-use planning which yielded effective and implementable systems for the practitioner and which allowed him to experiment with realistic alternatives relevant to his planning responsibilities. In their paper, they examined how the structure of a particular linear-programming formulation for land-use planning could be exploited to yield such simplifications. On the one hand, it was shown that linear-programming formulations which allocated used to the zones that made up a given planning region could be classified as generalized upper bounding because of their special structure. On the other hand, that special structure was exploited to show how such linear-programming formulations could be solved more simply than by the direct use of the simplex method. In addition, it was used to motivate the use of the LUPLAN procedure and established its relationship to linear-programming methods.

Linear Programming (LP) models and technique among various mathematical optimization techniques have evolved through the years to optimize the crude blending and refining operations. Hassan et al. (2011) stated that the operations may include the crude evaluation, selection, and scheduling and product logistics planning. The objective of that study was to develop a mathematical programming model for solving a blending problem in a major refinery in Alexandria, Egypt with the objective of maximizing Naphtha productivity. Refinery planning and optimization was basically addressed through special purpose linear programming software packages that remain a black box for the users and that are very costly for the organizations. The model developed in that work was proved to be highly effective at the level of solving the blending problem. This study has shown that the developed linear programming model for the blending problem has yielded better overall Naphtha productivity for the case of the oil refinery studied, as compared to results obtained by the commercial software.

According to Bruce and Foulger (2009) the amplitudes of radiated seismic waves contain far more information about earthquake source mechanisms than do first-motion polarities, but amplitudes are severely distorted by the effects of heterogeneity in the Earth. This distortion they say can be reduced greatly by using the ratios of amplitudes of appropriately chosen seismic phases, rather than simple amplitudes, but existing methods for inverting amplitude ratios are severely nonlinear and require computationally intensive searching methods to ensure that solutions are globally optimal. Searching methods are particularly costly if general (moment tensor) mechanisms are allowed. Efficient linear-programming methods, which do not suffer from these problems, had previously been applied to inverting polarities and wave amplitudes. They extended these methods to amplitude ratios, in which formulation on inequality constraint for an amplitude ratio takes the same mathematical form as a polarity observation. Three-component digital data for an earthquake at the Hengill-Grensdalur geothermal area in southwestern Iceland illustrated the power of the method. Polarities of *P*, *SH*, and *SV* waves, unusually well distributed on the focal sphere, could not distinguish between diverse mechanisms, including a double couple. Amplitude ratios, on the other hand, clearly ruled out the double-couple solution and required a large explosive isotropic component.

Bassam (2009) researched into how to use the local feedstuffs to formulate least cost rations for broilers using Linear Programming (LP) technique. To investigate, analyze and indicate how best the available local ingredients could be combined effectively and efficiently to formulate least-cost ration for broilers, a linear programming technique was employed to determine the most efficient way of combining these locally available ingredients. Mathematical models were constructed by taking into consideration nutrient requirements of the broilers, nutrient composition of the available ingredient and any other restriction factor of the available ingredients for the formulation. The result of the study showed that the least cost ration for starter broiler produced by linear programming model consists of 68.0% yellow corn, 25.07% soybean, 4% wheat bran, 0.5% fish meal, 0.5% Ca diphosphate, 0.1% lysine, 0.32% methoinine, 0.3% limestone, 0.3% NaCl, 0.5% ready premix, 0.4% soya oil and 0.01% vitamins and mineral mix. For the finisher ration the results showed that the ration consists of 67.5% yellow corn, 20.45% soybean, 5% wheat bran, 0.25% fish meal, 1.5% ca diphosphate, 0.25% lysine, 0.35% methoinine, 0.3% limestone, 0.5% NaCl, 3% ready premix, 0.75% soya oil and 0.15% vitamins and mineral mix

Jansen and Wilton (1984) described linear programming as a tool for selecting breeding stock in a production unit facing constraints of resources, marketing, or preference. The predicted performance of an animal for major input and output traits was incorporated into the objective function reflecting, for example, farm profits, and into a matrix of coefficients specifying the constraints. An example demonstrated the method and contrasts the selection decision indicated by a simple profit equation ignoring constraints to that of the linear programming solution. Direct consideration of constraints and alternative production possibilities was the chief advantage of linear programming over a profit equation.

Bouras and Engle (2007) developed a multi-period linear programming model to identify the optimal size of fingerling to understock to maximize multi-period returns on a catfish grow-out farm. Grow-out production alternatives included understocking three different sizes (7.6 cm, 12.7 cm, and 17.8 cm) of fingerlings in multiple-batch production at 15,000 fingerlings per hectare. Fingerlings were produced either with or without thinning at different stocking densities. Results showed that the optimal size of fingerling to understock was 12.7 cm. On-farm production of fingerlings was optimal across all farm sizes but the fingerling production technique selected varied with farm size. Models of larger farm sizes indicated that it is optimal to thin fingerlings, while for smaller farm sizes, producing fingerlings without thinning was optimal. When farm size was treated as an endogenous variable in the farmer's profit-maximizing decisions, the optimal size of a catfish farm was 404 water-ha. Sensitivity analyses suggested that net returns were sensitive to changes in the key parameters of the model (such as interest rates, feed conversion ratios, survival rates, catfish prices, harvesting costs, and the availability of operating capital), whereas the optimal size of fingerlings to under stock was robust to variations in the model's parameters.

Campbell et al. (1992) focused on the application of linear programming (LP) in combining with a geographic information system (GIS) in planning agricultural land-use strategies. The first step of the proposed methodology was to obtain an assessment of the natural resources available to agriculture. The GIS was used to delineate land-use conflicts and provided reliable data information on the natural-resource database. This was followed by combining the on natural resources with other quantifiable information on available labor, market forecasts, technology, and cost information in order to estimate the economic potential of the agricultural sector. Linear Programming was used in this step. Finally, the GIS were applied again to map the crop and land-allocation patterns generated by the Linear Programming model. The results gave concrete suggestions for resource allocation, farm-size mix, policy application, and implementation projects.

Isa (1990) used of Linear Programming (LP) and other mathematical procedures to evaluate watershed and perpetuity constraints on forest land use for a selected scenario in Terengganu, Peninsular Malaysia. The LP model provided a range of feasible solutions for decision making. Equations were derived for the model to show interaction of sedimentation due to road construction, timber harvesting, and other related forest management activities. Sensitivity analysis was used to test model behavior. Results indicated the constraining effects of sedimentation upon forest revenues when sedimentation was allowed to vary within the feasible region of the model (i.e., from 600,000 m^3/decade up to 1,150,000 m^3/decade.

The energy crisis is one of the deterrents of economic growth in a developing country like India. Rapid industrialization and poor capacity utilization of power plants make the operations of energy consuming industries like integrated steel plants extremely difficult. Dutta et al (1994) assessed the development and implementation of a mixed integer linear programming model for optimal distribution of electrical energy in an integrated steel plant. The model considered the balance equations of capacity, material, thermal and electrical energy, and oxygen. It also considered the constraints of yields, product routes, net realizations, variable costs, market demands and commitments to decide not only the hierarchy of shutdowns in the event of a power crisis but also the optimal product mix in each level of power availability. The round-the-clock implementation of the model increased the net profit per ton of saleable steel by 58% in 1986. Since then, the model, which is generic in nature, has been successfully integrated into the decision-making process. The cumulative benefit from that work will be at least 73 million US dollars.

Mousavi et al. (2004) presented a long-term planning model for optimizing the operation of Iranian Karoon-Dez reservoir system using an interior-point algorithm. The system is the largest multi-purpose reservoir system in Iran with hydropower generation, water supply, and environmental objectives. The focus was on resolving the dimensionality of the problem of optimization of a multi-reservoir system operation while considering hydropower generation and water supply objectives. The weighting and constraints methods of multi-objective programming were used to assess the trade-off between water supply and hydropower objectives so as to find no inferior solutions. The computational efficiency of the proposed approach was demonstrated using historical data taken from Karoon-Dez reservoir system.

Turgeon (1986) developed a parametric mixed-integer linear programming (MILP) method for selecting the sites on the river where reservoirs and hydroelectric power plants were to be built and then determining the type and size of the projected installations. The solution depended on the amount of money the utility was willing to invest, which itself was a function of what the new installations would produce. This method was used based on the fact that the branch-and-bound algorithm for selecting the sites to be developed (and consuming most of the computer time) was solved a minimum number of times. Between the points where the MILP problem was solved, LP parametric analysis was applied.

Preckel et al. (1997) presented economically rational behavior by fertilizer retailers. Tests were developed to measure efficiency of variable cost minimization, revenue maximization, and profit maximization. These tests included standard linear programming-based nonparametric efficiency tests and simpler but less conclusive tests which were performed using only simple arithmetic. Results indicated that fertilizer retailers acted as variable cost minimizers, but not as revenue and profit maximizers. Additional tests isolated whether inefficiency in cost minimization was due to a perception of variable input fixity. Management could then take steps to focus efforts on input control.

Yan and Lam (1997) stated that the urban road networks in Hong Kong are highly congested, particularly during peak periods. Long vehicle queues at bottlenecks, such as the harbor tunnels, have become a daily occurrence. At the time, tunnel tolls were charged in Hong Kong as one means to reduce traffic congestion. In general, flow pattern and queue length on a road network were highly dependent on traffic control and road pricing. An efficient control scheme was, therefore to take into account the effects of traffic control and road pricing on network flow. They presented a bi-level programming approach for determination of road toll pattern. The lower-level problem represented a queuing network equilibrium model that described the users' route choice behavior under conditions of both queuing and congestion. The upper-level problem was to determine road tolls to optimize a given system's performance while considering users' route choice behavior. Sensitivity analysis was also performed for the queuing network equilibrium problem to obtain the derivatives of equilibrium link flows with respect to link tolls. The derivative information was then applied to the evaluation of alternative road pricing policies and to the development of heuristic algorithms for the bi-level road pricing problem.

Network operators are facing hard competition for opportunities in the telecommunications market, forcing network investments to be carefully evaluated before the decision-making process. Velasco et al. (2011) emphasized that a great part of core network operators' revenues comes from the provisioned connectivity services. Taking this premise as their starting point, they first examined the provisioning of differentiated services in current shared-path protection environments. Their analysis revealed that current resource assignment policies were only able to provide a very poor grade of service to the supported best-effort traffic. Aiming to improve this performance, a novel resource partitioning scheme called diff-WS was proposed, which differentiated those wavelengths supporting each class of service in the network. As a major goal of the paper, the benefits of diff-WS over current resource assignment policies were assessed from an economic perspective. For that purpose, the network operator revenues maximization problem (NORMA) was presented to design the optical network such that the operator's revenues were maximized. To solve NORMA, they derived statistical models to obtain, given a certain grade of service, the highest traffic intensity for each class of service and resource partitioning scheme. These models turn NORMA into a nonlinear problem, which was finally addressed as an iterative approach, solving an integer linear programming (ILP) sub problem at each iteration. The obtained numerical results on several network topologies illustrate that diff-WS maximizes resource utilization in the network and, thus, the network operator's profit.

Gassenfert and Soares (2006) presented a practical proposition for the application of the Linear Programming quantitative method in order to assist planning and control of customer circuit delivery activities in telecommunications companies working with the corporative market. Based upon data provided for by a telecom company operating in Brazil, the Linear Programming method was employed for one of the classical problems of determining the optimum mix of production quantities for a set of five products of that company: Private Telephone Network, Internet Network, Intranet Network, Low Speed Data Network, and High Speed Data Network, in face of several limitations of the productive resources, seeking to maximize the company's monthly revenue. By fitting the production data available into a primary model, observation was made as to what number of monthly activations for each product would be mostly optimized in order to achieve maximum revenues in the company. The final delivery of a complete network was not observed but the delivery of the circuits that made it up, and that was a limiting factor for the study herein, which, however, brought an innovative proposition for the planning of private telecommunications network.

According to Hasan et al. (2010) the Saudi Public Transport Company (SAPTCO) intercity bus schedule comprise a list of 382 major trips per day to over 250 cities and villages with 338 buses. SAPTCO operates Mercedes 404 SHD and Mercedes 404 RI-IL fleet types for the intercity trip. The fleet assignment model developed by American Airlines was adapted and applied to a sample of the intercity bus schedule. The results showed a substantial saving of 29% in the total number of needed buses. This encourages the decision makers at SAPTCO to use only Mercedes 404 SHD fleet type. Hence, the fleet assignment model was modified to incorporate only one fleet type and applied to the sample example. Due to the increase in the problem size, the model was decomposed by stations. Finally, the modified decomposed model was applied to the whole schedule. The model results showed a saving of 16.5% in the total number of needed buses of Mercedes 404 SHD. A sensitivity analysis was carried out and showed that the predefined minimum connection time is critical for model efficiency. A modification to the connection time for 11 stations showed a saving of 14 more buses. Considering their recommendation of performing a field study of the trip connection time for every station, the expected saving of the total number of needed buses would be about 27.4% (90 buses). That would yield a net saving of 16.44 million Saudi Riyals (USD 4.4 million) per year for SAPTCO in addition to owing to the growth of air traffic. Better coordination of hiring new employees. The revenue analysis showed that these 90 surplus buses would yield about USD 20,744,000 additional revenue yearly. Dritan et al. (1998) studied the route and level flight assignment problem aiming at global flight plan optimization, which has already become a key issue all existing flights for all airlines, was becoming an increasingly desirable goal. A number of related problems appeared in the operations research literature, notably vehicle routing, scheduling and other transportation problems. Several studies had been especially devoted to the problem of aircraft scheduling and routing. Aircraft routing requires the generation of non-colliding, time-dependent routes through a specified airspace that they called the airspace network. The problem considered there could be modeled as a specific flow problem in a given space-time network. The study aimed at estimating the effects of routing capabilities at a quantitative level (the congestion level, i.e. the number of potential en-route conflicts), and at a qualitative level (Traffic smoothing). They presented a deterministic model based on a Linear Programming approach for optimizing the level route assignment in a trajectory-based Air Traffic Management (ATM) environment. The problem could be seen as a multi-period (dynamic) problem where the time dimension was an essential ingredient to consider when constructing flight plans. The dynamic problem could be transformed into a static one by using standard technique of time-expanding the underlying network.

They proposed there a model to consider the airspace congestion in a finer way: they considered the number of aircraft involved in potential en-route conflicts rather than the number of aircraft in a sector, sometimes understood as en-route capacities in ATM.

Chung et al. (2008) considered a municipal water supply system over a 15-year planning period with initial infrastructure and possibility of construction and expansion during the first and sixth year on the planning horizon. Correlated uncertainties in water demand and supply were applied on the form of the robust optimization approach of Bertsimas and Sim to design a reliable water supply system. Robust optimization aims to find a solution that remains feasible under data uncertainty. It was found that the robust optimization approach addressed parameter uncertainty without excessively affecting the system. While they applied their methodology to hypothetical conditions, extensions to real-world systems with similar structure were straightforward. Therefore, their study showed that this approach was a useful tool in water supply system design that prevented system failure at a certain level of risk.

Hoesein and Limantara (2010) studied the optimization of water supply for irrigation at Jatimlerek irrigation area of 1236 ha. Jatimlerek irrigation scheme was intended to serve more than one district. The methodology consisted of optimization water supply for irrigation with Linear Programming. Results were used as the guidance in cropping pattern and allocating water supply for irrigation at the area.

A ground water management model based on the linear systems theory and the use of linear programming was formulated and solved by Heidari (1982). The model maximized the total amount of pound water that could be pumped from the system subject to the physical capability of the system and institutional constraints. The results were compared with analytical and numerical solutions. This model was then applied to the Pawnee Valley area of south-central Kansas. The results of this application supported the previous studies about the future ground water resources of the Valley. These results provided a guide for the ground water resources management of the area over the next ten years.

Khaled (2004) developed four models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The first model (Dynamic programming model (DP)) allocated a given amount of water optimally over the different growth stages to maximize the yield per hectare for a given crop, accounting for the sensitivity of the crop growth stages to water stress. The second model (Single Crop Model) tried to find the best allocation of the available water both in time and space in order to maximize the total expected yield of a given crop. The third model (Multi crop Model) was an optimization model that determined the optimal allocation of land and water for different crops. It showed the importance of several factors in producing an optimal cropping plan. The output of the models was prepared in a readable form to the normal user by the fourth model (Irrigation Schedule Model).

Frizzone et al. (1997) developed a separable linear programming model, considering a set of technical factors which might influence the profit of an irrigation project. The model presented an objective function that maximized the net income and specified the range of water availability. It was assumed that yield functions in response to water application were available for different crops and described very well the water-yield relationships. The linear programming model was developed genetically, so that, the rational use of the available water resource could be included in an irrigation project.

Specific equations were developed and applied in the irrigation project "Senator Nilo Coelho" (SNCP), located in Petrolina – Brazil. Based on the water-yield functions considered, cultivated land constraints, production costs and products prices, it was concluded that the model was suitable for the management of the SNCP, resulting in optimal cropping patterns.

Becker (1995) explored the implications of the transformation of the system of water resources allocation to the agricultural sector in Israel from a one in which allotments were allocated to the different users without any permission to trade with water rights. A mathematical planning model was used for the entire Israeli agricultural sector, in which an 'optimal' allocation of the water resources was found and compared to the existing one. The results of the model were used in order to gain insight into the shadow price of the different water bodies in Israel (about eight). These prices could be used to grant property rights to the water users themselves in order to guarantee rational behaviour of water use, since no one could sell their rights at the source itself. From the dual prices of the primal problem they could forecast the equilibrium prices and their implications for the different users. The results showed that there was a potential budgetary benefit of 28 million dollars when capital cost was not included and 64 millions dollars when it was included.

Gill et al. (1994) hinted that many interior-point methods for linear programming were based on the properties of the logarithmic barrier function. After a initial discussion of the convergence of the (primal) projected by Newton barrier method, three types of barrier methods were analyzed. These methods may be categorized as primal, dual and primal-dual, and may be derived from the application of Newton's method to different variants of the same system of nonlinear equations. A fourth variant of the same equations leads to a new primal-dual method.

In each of the methods discussed, convergence was demonstrated without the need for a non-degeneracy assumption or a transformation that makes the provision of a feasible point trivial. In particular, convergence was established for a prim al-dual algorithm that allowed a different step in the primal and dual variables and did not require primal and dual feasibility. Finally, a new method for treating free variables was proposed.

CHAPTER 3

METHODOLOGY

3.0 Overview

This chapter presents the methodology used for developing optimal revenue generation model. The first part of this chapter defines some terminologies, the linear programming model, theoretical methods used in solving it (the graphical method, the simplex, and duality algorithms) and software for solving linear programming.

3.1.0 LINEAR PROGRAMMING

Linear programming (LP) is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear equations.

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Given a polytope and a real-valued affine function defined on this polytope, a linear programming method will find a point on the polytope where this function has the smallest (or largest) value if such point exists, by searching through the polytope vertices.

General Linear Programming Problem

Linear programs are problems that can be expressed in the form:

$$\text{Optimize } c^T x \qquad \ldots\ldots 3.1$$

$$\text{Subject to: } Ax \le b, \ldots\ldots 3.2$$

$$Ax = b \text{ or}$$

$$Ax \ge b$$

$$x_j \ge 0, \; j = 1, 2 \ldots. n.$$

where x represents the decision variables , c is the coefficient of the objective function, A is a (known) matrix of coefficients and b is a constant-valued vector. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case). The equations $Ax \le b$, $Ax=b$ and $Ax \ge b$ are the constraints which specify a convex polytope over which the objective function is to be optimized.

Linear programming can be applied to various fields of study. It is used most extensively in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design.

3.1.1 The Standard Form

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following four parts:

A linear function

Problem constraints

Non-negative variables

Non-negative right hand side constants

Given an m - vector, $b = (b_1,...,b_m)^T$, an n - vector, $c = (c_1,...,c_n)^T$, and an $m \times x$

matrix,

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

of real numbers.

3.1.2 The Standard Maximum Problem

In general, a maximum Linear Programme (LP) is formulated as follows;

Maximize

$$c^T x = c_1 x_1 + ... + c_n x_n$$

Subject to the constraints:

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \le b_2$$

$$.$$

$$.$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \le b_m$$

and $\qquad x_1 \ge 0, x_2 \ge 0, ..., x_n \ge 0$

3.1.3 The Standard Minimum Problem

In general, minimum Linear Programme is formulated as follows;

Minimize

$$y^T b = y_1 b_1 + \ldots + y_m b_m$$

Subject to the constraints

$$y_1 a_{11} + y_2 a_{12} + \ldots + y_m a_{1m} \geq c_1$$

$$y_1 a_{12} + y_2 a_{22} + \ldots + y_m a_{m2} \geq c_2$$

$$\vdots$$

$$y_1 a_{1n} + y_2 a_{2n} + \ldots + y_m a_{mn} \geq c_n$$

and

$$y_1 \geq 0, y_2 \geq 0, \ldots, y_m \geq 0$$

3.2.0 THE CORNER- POINT METHODS

This work examines some of the strengths and weaknesses of some corner-point methods. The following corner-point methods will be looked at: 1.The graphical method and 2.The simplex method.

3.2.1 The Graphical Method

A linear programming problem involving one or two variables can be solved by using the graphical approach.

Steps involved in the Graphical Method

Step 1: Write down the inequalities according to the construction of the problem.

Step 2: Represent the inequalities as equations.

Step 3: Draw linear graphs from the equations to obtain the corner points satisfying the region of intersection where the solution set will be determined from.

Step 4: Substitute the coordinates of the corner points into the objective function to obtain the maximum or minimum value of the problem.

3.2.2 ILLUSTRATIVE EXAMPLE

Suppose a situation gives the following objective function and constraints

Maximize: $2x_1 + 3x_2$

Subject to: $2x_1 + x_2 \leq 8$

$x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

Equality form:

$2x_1 + x_2 = 8$ ………………… (1)

$x_1 + 2x_2 = 6$ ……………….. (2)

$x_1, x_2 \geq 0$

Plotting the graphs of the two equations, the following corner points are obtained:

A(0,0) , B(4,0) , C(6,0), D(3.4,1.3), E(8,0) , F(0,3) .

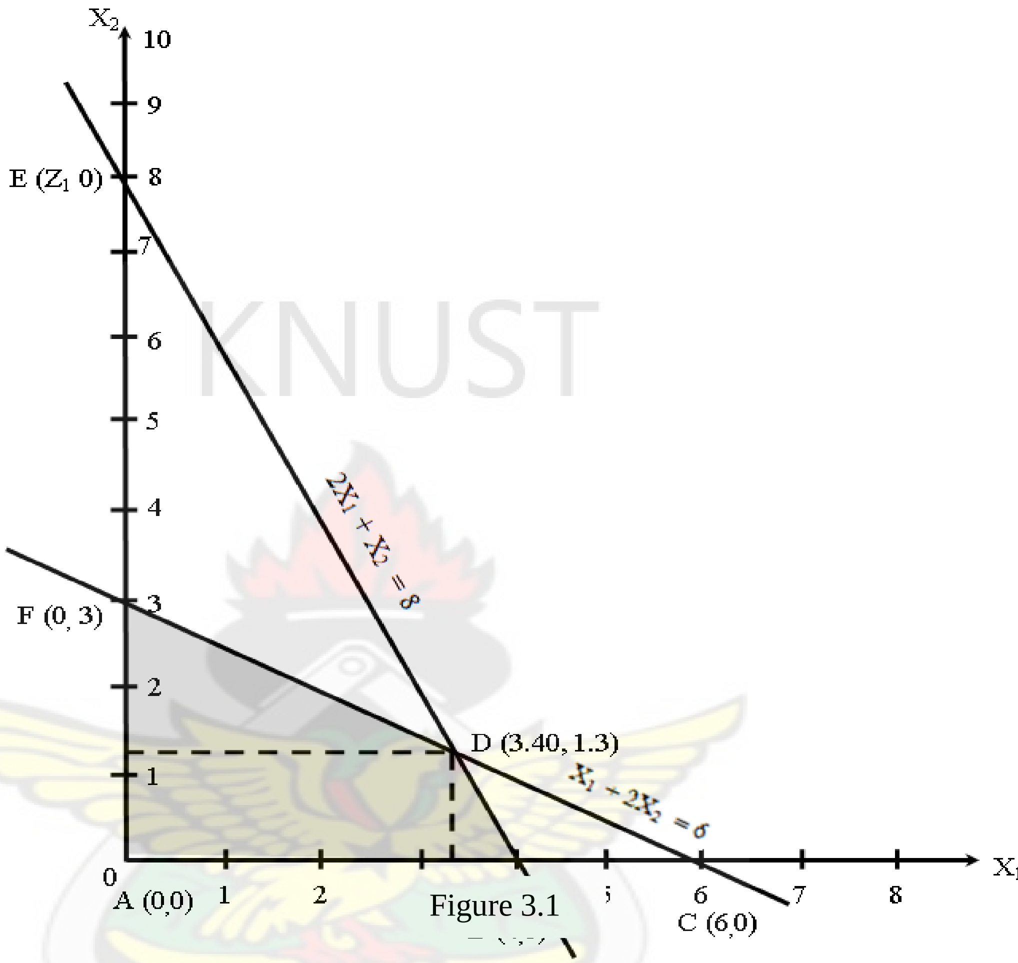

Figure 3.1: A graph of maximum function.

The shaded region represents all set of points which satisfy all the constraints.

Its corresponding corner points are:

A (0, 0)

B (4, 0)

D (3.4, 1.3)

F (0, 3)

Evaluating the objective function at each of the corner points yields the result shown on the table below.

Table 3.1 Objective Function Value at Each Corner Point

Corner Point Coordinates	Net Income Function
(x_1, x_2)	$2x_1 + 3x_2$
A(0, 0)	0
B(4 ,0)	8
D(3.4,1.3)	10.7
F(0,3)	9

Since the objective is to maximize, from the above table we read off the optimal value to be z=10.7 with corner points of x_1=3.4 and x_2=1.3.

3.3.0 THE SIMPLEX METHOD

The simplex algorithm is an iterative procedure that examines the vertices of the feasible region to determine the optimal value of the objective function.

It usually starts at the corner that represents doing nothing. It moves to the neighbouring corner that best improves the solution. It does this over and over again improving the objective function each time until the optimal solution is found at the most attractive corner.

3.3.1 The Standard Maximization Procedure

A standard maximum problem of a linear program in which the objective is to maximize is of the form:

Maximize $\quad z = c^T x$

subject to:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m$$

$$\text{where} \quad x_1, x_2, \ldots, x_n \ge 0$$

$$\text{and} \quad b_j \ge 0 \text{ for } j = 1, 2, \ldots, m$$

3.3.2 The Standard Problem L.P involving the Slacks.

To represent the problem in a standard form a non- negative slack variable s_i is added to the given objective function and its constraints. This is to convert the constraints into equations. The constraints therefore become:

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$$

where $x_i \geq 0$ for $i = 1, 2, ..., n$

3.3.3 Steps involved in the Simplex Method

Step1: Setting up the Initial Simplex Tableau

In developing the initial simplex tableau, we will be consistent with the set of notations as defined below:

C_j=objective function coefficients for variable $_j$.

C_B = objective function coefficients of the basic variables

b_i= right-hand-side coefficients (value) for constraint $_i$

a_{ij} =coefficients of variable $_j$ in constraint $_i$

a= mxn matrix

Z_j is the decrease in the value of the objective function that will result if one unit of the variable corresponding to the j^{th} column of the matrix formed from the coefficients of the variables in the constraints is brought into the basis (thus if the variable is made a basic variable with a value of one).

Table 3.2: Table for formulating simplex tableau

	c_j	c_1	c_2	$\cdots$	c_n	0	0	$\cdots$	0	
C_B	B. V.	x_1	x_2	$\cdots$	x_n	s_1	s_2	$\cdots$	s_n	R.H.S
0	s_1	a_{11}	a_{12}	$\cdots$	a_{1n}	1	0	$\cdots$	0	b_1
0	s_2	a_{21}	a_{22}	$\cdots$	a_{2n}	0	1	$\cdots$	0	b_2
0	s_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}	0	0	$\cdots$	1	b_m
	z_j	0	0	$\cdots$	0	0	0	$\cdots$	0	0
	$c_j - z_j$	c_1	c_2	$\cdots$	c_n	0	0	0	0	

c_i-z_j called the Net Evaluation Row, is the net change in the value of the objective function if one unit of the variable corresponding to $_j{}^{th}$ column of the matrix (formed from the coefficient of the variables in the constraints), is brought into solution.

Step2: Optimality Process

From the $C_j - Z_j$ row we locate the column that contains the largest positive number and this becomes the Pivot Column. In each row we now divide the value in the R.H.S by the positive entry in the pivot column (ignoring all zero or negative entries) and the smallest one of these ratios gives the pivot row. The number at the intersection of the pivot column and the pivot row is called the pivot.

We then divide the entries of that row in the matrix by the pivot and use row operation to reduce all other entries in the pivot column, apart from the pivot, to zero.

Step3: The Stopping Criterion

The simplex method will always terminate in a finite number of steps when the necessary condition for optimality is reached.

The optimal solution to a maximum linear program problem is reached when all the entries in the net evaluation row, that is $c_{j-}z_j$, are all negative or zero.

3.3.4 ILLUSTRATIVE EXAMPLE

Maximize $Z = 6x_1 + 8x_2$

Subject to: $5x_1 + 10x_2 \le 60$

$4x_1 + 4x_2 \le 40$

where $x_1 \ge 0,\ x_2 \ge 0.$

The equations then become:

Maximize: $Z = 6x_1 + 8x_2 + 0s_1 + 0s_2$

Subject to: $5x_1 + 10x_2 + s_1 = 60$

$4x_1 + 4x_2 + s_2 = 40$

where $x_1 \ge 0,\ x_2 \ge 0,\ s_1 \ge 0,\ s_2 \ge 0.$

This is the first table that is generated from the coefficients of the objective function and its constraint variables.

Table 3.3: The initial simplex tableau

	c_j	6	8	0	0	R.H.S	RATIO
c_B	Basic Variables	x_1	x_2	s_1	s_2	Solution	
0	s_1	5	10	1	0	60	6 $\rightarrow$
0	s_2	4	4	0	1	40	10
	z_j	0	0	0	0	0	
	$c_j - z_j$	6	8 $\uparrow$	0	0		

The row in which the smallest positive ratio is obtained is the pivot row.

From the above $\frac{60}{10} = 6$ and $\frac{40}{4} = 10$. Since 6 is the least ratio, row 1 becomes the pivot row.

The intersection of the pivot column and the pivot row gives the pivot element. x_2 now becomes the entry variable and therefore goes into the basis.

The horizontal variable, s_1 leaves the current basis. The second simplex tableau now becomes:

Table 3.4: The second simplex tableau

	c_j	6	8	0	0	R.H.S	RATIO
c_B	Basic variables	x_1	x_2	s_1	s_2	Solution	
8	x_2	½	1	1/10	0	6	
0	s_2	2	0	-2/5	1	16	→
	z_j	4	8	4/3	0	48	
	c_j - z_j	2 ↑	0	- 4/3	0		

There is still a $c_j - z_j = 2$ (positive), so optimality has yet not been attained.

Following the same procedure as above gives the third simplex tableau. Since the result from the table optimizes the objective function, it is called the final simplex tableau.

Table 3.5: The final simplex tableau

	c_j	6	8	0	0	
g_B	Basic variables	x_1	x_2	s_1	s_2	Solution
4	x_2	0	1	1/3	-1/4	2
3	x_1	1	0	- 1/5	½	8
	z_j	6	8	2/5	1	64
	c_j - z_j	0	0	- 2/5	- 1	

Since $c_j - z_j$ contains zeros and negatives, it implies that optimality has been reached.

The objective function will therefore have its maximum value, z= 64 when $x_1 = 2$ and $x_2 = 8$

3.4 THE INTERIOR - POINT METHOD

The simplex method of linear programming finds the optimum solution by starting at the origin and moving along the adjacent corner points of the feasible solution space. Since this is exponential time algorithm, the numbers of iterations become prohibitive for some huge problems. On the contrary, the interior point Linear Programming algorithm is a polynomial time algorithm. This new approach finds the optimum solution by starting at a trial solution and shooting through the interior of the feasible solution space. It is ideal for solving very large LP problems; however it is not quite convenient for solving smaller Linear Programming problems. Interior point methods algorithms can also be used to solve nonlinear convex optimization problems. It is also known as the barrier method. (Wikipedia, the free encyclopedia).

3.5.0 PRIMAL-DUAL INTERIOR- POINT METHOD

In general, the primal-dual linear programming problems algorithm is formulated as follows:

$$\text{PP:} \quad \underset{x}{\text{maximize}} \quad c^T x \qquad \text{subject to} \quad Ax \leq b \quad x \geq 0, \qquad \ldots\ldots\ldots\ldots(1)$$

Where c, x ∈ $\mathbb{R}^m$ and A is an m x n matrix. This problem is called the primal problem. Associated with it, is the dual problem, which can be formulated as

$$\text{DP:} \quad \underset{y}{\text{minimize}} \quad b^T y \qquad \text{subject to} \quad A^T y \geq c, \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

In developing an algorithm for this method, work will be done on primal and dual problems simultaneously as defined in Table 3.6. The primal problem is assumed to consist of m non-redundant equations in n variables, and is given in equality form. This means that the n dual variables are unrestricted in sign. In general, m-dimensional vector of nonnegative slack variables, z, transforms the dual inequalities to equations as follows:

Table 3.6: Primal and dual problems

(P) Maximize $z_P = C^T x$	(D) Minimize $z_D = yb$
Subject to $Ax = b$ $x \geq 0$	Subject to $yA - z = c$ $y \geq 0$, $z \geq 0$

3.5.1 FUNDAMENTAL STEPS IN THE PRIMAL-DUAL METHOD

The use of primal-dual algorithms to solve linear programs is based on three steps:

(i) The application of the Lagrange multiplier approach of classical calculus to transform an equality constrained optimization problem into an unconstrained one.

(ii) The transformation of an inequality constrained optimization problem into a sequence of unconstrained problems by incorporating the constraints in a logarithmic barrier function that imposes a growing penalty as the boundary ($x_j = 0$, $z_j = 0$ for all j) is approached.

(iii) The solution of a set of nonlinear equations using Newton's method, thereby arriving at a solution to the unconstrained optimization problem.

When solving the sequence of unconstrained problems, as the strength of the barrier function is decreased, the optimum follows a well-defined path that ends at the optimal solution to the original problem.

Step1: Finding the Lagrangian of the Function

A well-known procedure for determining the minimum or maximum of a function subject to equality constraints is the Lagrange multiplier approach.

Consider the general problem;

Maximize $f(\mathrm{x})$

Subject to $g_i(\mathrm{x}) = 0$, $i = 1 \ldots$ m,

where f(x) and $g_i(\mathrm{x})$ are scalar functions of the n-dimensional vector x.

The Lagrangian for this problem is

$$L(x, y) = f(x) - \sum_{i=1}^{n} y_i g_i(x)$$

where the variables y= ($y_1, y_2, \ldots y_m$) are the Lagrange multipliers.

Necessary conditions for a stationary point (maximum or minimum) of the constrained optimization of $f(\mathbf{x})$ are that the partial derivatives of the Lagrangian with respect to the components of x and y be zero; i.e.,

$\frac{\partial \mathbf{L}}{\partial \mathbf{x_j}} = \mathbf{0}, \mathbf{j} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n}$ **and** $\frac{\partial \mathbf{L}}{\partial \mathbf{y_i}} = \mathbf{0}, \mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{m}$. For linear constraints ($a_i\mathrm{x} - b_i = 0$), the conditions are sufficient for a maximum if the function $f(\mathrm{x})$ is concave and sufficient for a minimum if $f(\mathrm{x})$ is convex.

Step2: Constructing a Barrier in the Interior Region

The idea of the barrier approach is to start from a point in the strict interior of the inequalities

(x_j >0, z_j >0 for all $_j$) and construct a barrier that prevents any variable from reaching a boundary (e.g., x_j = 0). Adding "log(x_j)" to the objective function of the primal, for example, will cause the objective function to decrease without bound as x_j approaches 0. The difficulty with this idea is that if the constrained optimum is on the boundary (that is, one or more $x^*_j = 0$, which is always the case in linear programming), then the barrier will prevent the optimum from being reached on the boundary. To get around this difficulty, a barrier parameter μ is added to balance the contribution of the true objective function with that of the barrier term. This is shown in the table below:

Table 3.7: Primal and dual barrier problems

(P)	(D)
Maximize $B_P(\mu) = c^T x + \mu \sum_{j=1}^{41} (x_j)$ Subject to $: Ax = b$	Minimize $B_P(\mu) = yb - \mu \sum_{j=1}^{41} (z_j)$ subject to $: yA - z = c$

The parameter μ is required to be positive and controls the magnitude of the barrier term. Because function log(x) takes on very large negative values as x approaches zero from above, as long as x remains positive the optimal solution to the barrier problems will be interior to the nonnegative orthants ($x_j > 0$ and $z_j > 0$ for all $_j$). The barrier term is added to the objective function for a maximization problem and subtracted for a minimization problem. The new formulations have nonlinear objective functions with linear equality constraints, and can be solved with the Lagrangian technique for μ>0 being fixed. The solution to these problems will approach the solution to the original problem as μ approaches zero.

Table 3.8 shows the development of the necessary optimal conditions for the barrier problems. These conditions are also sufficient because the primal Lagrangian is concave and the dual Lagrangian is convex. Note that the dual variables y is the Lagrange multipliers of the primal, and the primal variables x are the Lagrange multipliers of the dual.

Table 3.8 Necessary conditions for the barrier problems

Lagrangian $L_{P=} c^T x + \mu \sum_{j=1}^{41} \log(x_j) - y^T(Ax - b)$	Lagrangian $L_D = yb - \mu \sum_{j=1}^{41} \log(z_j) - (yA - z - c)x$
$\frac{\partial L_P}{\partial y_i} = 0$ $\sum_{j=1}^{n} a_{ij}\ x_j = b_i,\ \ j = 1, 2,..,n.$ (primal feasibility)	$\frac{\partial L_D}{\partial x_j} = 0$ $\sum_{i=1}^{m} a_{ij}\ y_j - z_j = c_j\ ,\ i = 1,2,..,m$ (dual feasibility)
	$\frac{\partial L_D}{\partial z_j} = 0$ $\frac{\mu}{-Z} + x_j = 0$ $z_j x_j = \mu,\ \ j = 1,\ldots,n$ (μ-complementary slackness)

Thus the optimal conditions are nothing more than primal feasibility, dual feasibility, and complementary slackness satisfied to within a tolerance of μ.

Theory shows that when μ goes to zero the solution to the original problem would be attained; however, we cannot just set μ to zero because that would destroy the convergence properties of the algorithm.

To facility the process, a two n x n diagonal matrices containing the components of x and z, respectively are defined. That is;

$$X = \text{diag}\{x_1, x_2, \dots, x_n\}$$

$$Z = \text{diag}\{z_1, z_2, \dots, z_n\}$$

Also, let $e = (1, 1, \dots, 1)^T$ be a column vector of size *n*. Using this notation, the necessary and sufficient conditions derived in Table 3.8 for the simultaneous solution of both the primal and dual barrier problems can be written as:

Primal feasibility: $Ax - b = 0$ (*m* linear equations)

Dual feasibility: $A^T y^T - z - c^T = 0$ (*n* linear equations)

μ-Complementary slackness: $X Z e - \mu e = 0$ (*n* non-linear equations)

There is therefore the need to solve this set of nonlinear equations for the variables (x, y, z).

Step3 (a): Finding the Stationary Solutions using Newton's Method

Newton's method is an iterative procedure for numerically solving a set of nonlinear equations. For instance; consider a single variable problem of finding h to satisfy the nonlinear equation

$f(h) = 0$ where f is once continuously differentiable. Let h^* be the unknown solution. At some point h^k, one can calculate the functional value, $f(h^k)$, and the first derivative, $f'(h^k)$. Using the derivative as a first order approximation for how the function changes with h, one can predict the amount of change $\Delta = h^{k+1} - h^k$ required to bring the function to zero.

Taking the first order Taylor series expansion of f(h) around h^k gives

$$f(h^{k+1}) \approx f(h^k) + \Delta f'(h^k).$$

Setting the approximation of $f(h^{k+1})$ to zero and solving for Δ gives

$$\Delta = -f(h^k)/f'(h^k)$$

The point $h^{k+1} = h^k + \Delta$ is an approximate solution to the equation. It can be shown that if one starts at a point h^0 sufficiently close to h^*, the value of h^k will approach h^* as $k \to \infty$.

The method extends to multidimensional functions. Consider the general problem of finding the r-dimensional vector h that solves the set of r equations $f_i(h) = 0$, $i = 1 \ldots r$ or $f(h) = 0$.

Let the unknown solution to the equations be h^*. The n x n Jacobian matrix describes the first order variations of these functions with the components of h. The Jacobian at h^k is

$$J(h^k) = \begin{pmatrix} \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} \ldots & \frac{\partial f_1}{\partial h_n} \\ \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} \ldots & \frac{\partial f_2}{\partial h_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial h_1} & \frac{\partial f_n}{\partial h_2} \ldots & \frac{\partial f_n}{\partial h_n} \end{pmatrix}$$

All the partial derivatives are evaluated at h^k. Now, taking the first order Taylor series expansion around the point h^k, and setting it to zero gives $f(h^k) + J(h^k)d = 0$ where $d = h^{k+1} - h^k$ is an n-dimensional vector whose components represent the change of position for the k+1st iteration. Solving for d leads to $d = -J(h^{k)-1}f(h^k)$

The point $h^{k+1} = h^k + d$ is an approximation for the solution to the set of equations. Once again, if one starts at an initial point h^0 sufficiently close to h^*, the value of h^k will approach h^* for large values of k.

Step3 (b): Using Newton's Method for solving Barrier Problems

The stage is now set for Newton's method to be used to solve the optimality conditions for the barrier problems given in Table 3.8 for a fixed value of μ. For h = (x, y, z) and r = 2n+m, the corresponding equations and Jacobian are:

$$Ax - b = 0$$
$$A^T y^T - z - c^T = 0$$
$$XZe - \mu e = 0$$

$$J(h) = \begin{pmatrix} A & 0 & 0 \\ 0 & A^T & -I \\ Z & 0 & X \end{pmatrix}$$

Assuming that a starting point (x^0, y^0, z^0) satisfying $x^0 > 0$, $z^0 > 0$, $y^0 > 0$ and denoted by

$$\delta_p = b - Ax^0$$

$$\delta_D = c^T - A^T (y^0)^T + z^0$$

are the primal and dual residual vectors at this starting point. The optimality conditions can be written as

$$J(h)d = -f(h)$$

$$\begin{pmatrix} A & 0 & 0 \\ 0 & A^T & -I \\ Z & 0 & X \end{pmatrix} \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \begin{pmatrix} \delta_P \\ \delta_D \\ \mu e - XZe \end{pmatrix}$$

Where the (2n+m)-dimensional vector $d \equiv (dx, dy, dz)^T$ is called the Newton direction. The d will now be solved.

In explicit form, the above system is

$$Ad_x = 0$$
$$A^T d_y - d_z = 0$$
$$Zd_x + Xd_z = \mu e - XZe$$

The first step is to find dy. In making dy a subject the following equation is obtained;

$$(AZ^{-1}XA^T)dy = -b + \mu AZ^{-1}e + AZ^{-1}X\delta_D \text{ or}$$

$$dy = (AZ^{-1}XAZ)^{-1}(-b + \mu AZ^{-1}e + AZ^{-1}X\delta_D) \ldots\ldots\ldots\ldots\ldots (8)$$

It is worth noting that $Z^{-1} = \mathrm{diag}\{1/z_1, 1/z_2, \ldots, 1/z_n\}$ and is trivial to compute. Further multiplications and substitutions give

$$dz = -\delta_{D+}A^T dy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$\text{and } dx = Z^{-1}(\mu e - XZe - Xdz) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

From these results, it is obvious in part why it is necessary to remain in the interior of the feasible region. In particular, if either Z^{-1} or X^{-1} does not exist the procedure breaks down.

Once the Newton direction has been computed, dx is used as a search direction in the x-space and (dy, dz) as a search direction in the (y, z)-space. That is, the iterant moves from the current point (x^0, y^0, z^0) to a new point (x^1, y^1, z^1) by taking a step in the direction (dx, dy, dz). The step sizes, δ_P and δ_D, are chosen in the two spaces to preserve $x > 0$ and $y > 0$.

This requires a ratio test similar to that performed in the simplex algorithm. The simplest approach is to use

$$\alpha_P = \gamma \min_j \left\{ \frac{-x^k{}_j}{(d_x)^k{}_j} : (d_x)^k{}_j < 0 \right\}$$

$$\alpha_D = \gamma \min_j \left\{ \frac{-z^k{}_j}{(d_z)^k{}_j} : (d_z)^k{}_j < 0 \right\}$$

where γ is the step size factor that keeps the iterant from actually touching the boundary. Typically, $\gamma = 0.995$. The notation $(dx)_j$ refers to the $_j$th component of the vector dx. The new point is

$$x^1 = x^0 + \alpha_P dx$$

$$y^1 = y^0 + \alpha_D dy$$

$$z^1 = z^0 + \alpha_D dz$$

which completes one iteration. Ordinarily, one would now resolve equations (8) - (10) at (x^1, y^1, z^1) to find a new Newton direction and hence a new point. Rather than iterating in this manner until the system converges for the current value of μ, it is much more efficient to reduce μ after every iteration. The primal-dual method itself suggests how to update μ. It is straightforward to show that the Newton step reduces the duality gap (θ), which is the difference between the dual and primal objective values at the current point. Assume that is primal feasible and (y^0,z^0) is dual feasible, then in general, case let "θ(0)" denote the current duality gap, $\theta(0) = y^0 b - cx^0$

$$= y^0 (Ax^0) - (y^0 A - z^0)^T x^0 \quad \text{(primal and dual feasibility)}$$

$$= (z^0)^T x^0$$

If we let $\alpha = \min\{\alpha_P, \alpha_D\}$ then $\theta(\alpha) = (z^0 + \alpha dz)^T (x^0 + \alpha dx)$ and with a little algebra, it can be shown that $\theta(\alpha) < \theta(0)$ as long as

$$\mu < \frac{\theta(0)}{n}$$

The following formula was used in the computations made;

$$\mu^k = \frac{\theta(\alpha^k)}{n^2} = \frac{(z^k)^T x^k}{n^2}$$

which indicates that the value of μ^k is proportional to the duality gap,(θ).

Termination Criteria

Due to the presence of the barrier term that keeps the iterant away from reaching the boundary, they can never produce an exact solution. Feasibility and complementarity can therefore be attained only within a certain level of accuracy.

For this reason, termination criteria for the algorithm to be used have to be decided on. The most common criterion is the use of the duality gap .That is, at optimality the duality gap is zero (0).

Iterative Procedure for Newton's Method

Step 0: In summarizing the basic steps of the algorithm the following inputs are assumed:

(i)The data of the problem (A, b, c), where the m x n matrix A has full row rank

(ii) Initial primal and dual feasible solutions $x^0 > 0$, $z^0 > 0$, $y^0 > 0$.

(iii) The optimality tolerance $\varepsilon > 0$ and the step size parameter $\gamma \in (0, 1)$.

Step 1: (Initialization). Start with some feasible point $x^0 > 0$, $z^0 > 0$, $y^0 > 0$.

Choose(x^0, y^0, z^0) such that$(x^0, z^0, y^0) > 0$ and set the iteration counter k = 0.

Step 2: (Optimality test). If $(z^k)^T x^k < \varepsilon$ stop; otherwise, go to Step 3.

Step 3: (Compute Newton directions). Let

$$X^k = \mathrm{diag}\left\{x^k{}_1, x^k{}_{2,\ldots,}x^k{}_n\right\}$$

$$Z^k = \mathrm{diag}\left\{z^k{}_1, z^k{}_{2,\ldots\ldots,}z^k{}_n\right\}$$

$$\mu^k = \frac{(z^k)^T x^k}{n^2}$$

Solve the following linear system equivalent to (7) to get d^k_x, d^k_y, and d^k_z.

$$A d_x = 0$$
$$A^T d_y - d_z = 0$$
$$Z d_x + X d_z = \mu e - XZe$$

Note that $\delta_P = 0$ and $\delta_D = 0$ due to the feasibility of the initial point.

Step 4: (Find step lengths). Let

$$\alpha_P = \gamma \min_j \left\{ \frac{-x^k{}_j}{(d_x)^k{}_j} : (d_x)^k{}_j < 0 \right\}$$

$$\alpha_D = \gamma \min_j \left\{ \frac{-z^k{}_j}{(d_z)^k{}_j} : (d_z)^k{}_j < 0 \right\}$$

Step 5: (Update solution). Take a step in the Newton direction to get;

$$x^{k+1} = x^k + \alpha_P (dx)^k$$

$$y^{k+1} = y^k + \alpha_D (dy)^k$$

$$z^{k+1} = z^k + \alpha_D (dz)^k$$

Put k =k + 1 and go to Step 2.

3.5.2 NUMERICAL EXAMPLE

PROBLEM

Primal

Maximize $z_P = 2x_1 + 3x_2$

Subject to: $2x_1 + x_2 \leq 8$

$x_1 + 2x_2 \leq 6$

$x_j \geq 0, j = 1, 2$

Dual

Minimize $z_D = 8y_1 + 6y_2$

Subject to:

$$
\begin{aligned}
2y_1 + y_2 &\geq 2 \\
y_1 + 2y_2 &\geq 3 \\
y_1 \quad &\geq 0 \\
y_2 &\geq 0 \\
y_i \geq 0&, i = 1,2
\end{aligned}
$$

Table 3.9: Problem Formulation involving the Slacks

Primal model	Dual model
Maximize $z_P = 2x_1 + 3x_2$ subject to $2x_1 + x_2 + x_3 = 8$ $x_1 + 2x_2 + x_4 = 6$ x_j □□0, j = 1, … , 4	Minimize $z_D = 8y_1 + 6y_2$ subject to $2y_1 + y_2 - z_1 = 2$ $y_1 + 2y_2 - z_2 = 3$ $y_1 - z_3 = 0$ $y_2 - z_4 = 0$ z_j □□0, j = 1, … , 4

Step 0:

From the above problem n = 4 and m = 2.

$$\mathbf{A} = \begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{1} \end{pmatrix}, \mathbf{b} = \begin{pmatrix} \mathbf{8} \\ \mathbf{6} \end{pmatrix} \ \mathbf{and}\ \mathbf{c} = (\mathbf{2} \quad \mathbf{3} \quad \mathbf{0} \quad \mathbf{0})$$

$$\mathrm{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \mathrm{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \text{ and } \mathrm{y} = (y_1 \quad y_2).$$

ITERATIVE PROCEDURE

First Iteration (Initial Solution)

Step 1: Given that the initial assumptions are satisfied, then initial conditions become the update solution as given below;

$$\begin{pmatrix} x^0{}_1 \\ x^0{}_2 \\ x^0{}_3 \\ x^0{}_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 5 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} z^0{}_1 \\ z^0{}_2 \\ z^0{}_3 \\ z^0{}_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} y^0{}_1 \\ y^0{}_2 \end{pmatrix}^T = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

Second Iteration

Step 1: Set k=1, ε < 0 and $\mu^0 = \frac{\theta(0)}{n^2} = \frac{23}{4^2} = 1.4375.$

Step 2: Optimality Test

$$(z^0)^T x^0 = (4\ \ 3\ \ 2\ \ 2)\begin{pmatrix} 1 \\ 1 \\ 5 \\ 3 \end{pmatrix}$$

$$= 23$$

Also;

Since $(z^0)^T x^0 \nless \varepsilon$ the direction vectors are computed.

Step 3: Computing the Direction Vectors

From $$X = \text{diag}\{x_1, x_2, \ldots, x_n\}$$

$$Z = \text{diag}\{z_1, z_2, \ldots, z_n\}$$

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$$Z = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Solving the equations below give the direction vectors as follows;

$$dy = (AZ^{-1}XA^T)^{-1}(-b + \mu AZ^{-1}e + AZ^{-1}X\delta_D)$$

$$dz = -\delta_D + A^T dy \quad dx = Z^{-1}(\mu e - XZe - Xdz)$$

Hence,

$$\begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \\ dx_4 \end{pmatrix} = \begin{pmatrix} 0.2286 \\ 0.4457 \\ -0.903 \\ -1.12 \end{pmatrix}, \begin{pmatrix} dz_1 \\ dz_2 \\ dz \\ dz_4 \end{pmatrix} = \begin{pmatrix} -3.477 \\ -2.49 \\ -1.351 \\ -0.774 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} dy_1 \\ dy_2 \end{pmatrix}^T = \begin{pmatrix} -1.351 \\ -0.77 \end{pmatrix}$$

Step 4: Finding the Step Length using the Ratio Test

Let $\gamma = 0.8$; then from

$$\alpha_p = \gamma \min \left\{ \frac{-x^1_j}{(dx)_j^1} \quad : (dx)_j^1 < 0 \right\}$$

$$\alpha_p = \left\{ \begin{matrix} -4.3744 \\ -2.2437 \\ 5.5378 \\ 2.6785 \end{matrix} \right\} = 2.1428$$

$$\alpha_D = \gamma \min \left\{ \frac{-z_j^1}{(dz)_j^1} : \quad (dz)_j^1 < 0 \right\}$$

$$\alpha_D = 0.8\min \left\{ \begin{matrix} 1.1505 \\ 1.0346 \\ 1.4800 \\ 2.5835 \end{matrix} \right\} = 0.8277$$

Step 5: Update solution

$$x^1 = x^0 + \alpha_P (d_x)^0 = \begin{pmatrix} 1.4898 \\ 1.9551 \\ 3.0653 \\ 0.6000 \end{pmatrix}$$

$$y^1 = y^0 + \alpha_D (y)^0 = \begin{pmatrix} 0.8815 \\ 1.3592 \end{pmatrix}$$

Third Iteration

Step1: Set k=2, ε >0 and μ^1= 0.3977

Step2: Optimality Test.

From the 2nd iteration

$$\theta(1) = (z^1)^T x^1 = \begin{pmatrix} 1.1223 \\ 1.6000 \\ 3.8815 \\ 1.3519 \end{pmatrix}^T \begin{pmatrix} 1.4898 \\ 1.9551 \\ 3.0653 \\ 0.6000 \end{pmatrix}$$

$$\theta(1) = 6.3626$$

Also;

Since$(z^1)^T x^1 \nless \varepsilon$, the direction vectors can be computed for.

Step3: Computing the Newton Directions

From

$$x^2 = \text{diag}\{x^2{}_1, \quad x^2{}_2, \quad x^2{}_3, \quad x^2{}_4\}$$

$$z^2 = \text{diag}\{z^2{}_1, \quad z^2{}_2, \quad z^2{}_3, \quad z^2{}_4\}$$

$$X^2 = \begin{pmatrix} 1.4898 & 0 & 0 & 0 \\ 0 & 1.9551 & 0 & 0 \\ 0 & 0 & 53.0653 & 0 \\ 0 & 0 & 0 & 30.6000 \end{pmatrix}$$

$$z^2 = \begin{pmatrix} 1.1223 & 0 & 0 & 0 \\ 0 & 1.6000 & 0 & 0 \\ 0 & 0 & 3.8815 & 0 \\ 0 & 0 & 0 & 1.3519 \end{pmatrix}$$

Feasibility Vectors
From

$$\delta_p = b - Ax^0$$

$$\delta_D = c^T - A^T(y^0)^T + z^0, \text{ we have}$$

$$\delta_{P=}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\delta_D = \begin{pmatrix} 2.2445 \\ 1.1995 \\ 4.7630 \\ 2.7184 \end{pmatrix}$$

From

$$dy = (AZ^{-1} XA^T)^{-1} (-b + \mu AZ^{-1}e + AZ^{-1} X\delta_D)$$

$$dz = - \delta_D + A^T dy$$

$$dx = Z^{-1} (\mu e - XZe - Xdz)$$

$$d_x = \begin{pmatrix} 0.0121 \\ -0.3925 \\ 0.3680 \\ 0.7722 \end{pmatrix}, d_z = \begin{pmatrix} -0.8645 \\ -1.0757 \\ -4.2177 \\ -2.4289 \end{pmatrix} \text{ and } d_y = \begin{pmatrix} 0.5453 \\ 0.2895 \end{pmatrix}$$

Step 4: Finding the Step Length using the Ratio Test

Let $\gamma = 0.8$; then from

$$\alpha_p = \gamma \min \left\{ \frac{-x^2_j}{(dx)_j^2} \quad : (dx)_j^2 < 0 \right\}$$

$$\alpha_p = 0.8\min \left\{ \begin{matrix} -4.3744 \\ -2.2437 \\ 5.5378 \\ 2.6785 \end{matrix} \right\} = 2.1428$$

$$\alpha_D = \gamma \min \left\{ \frac{-z_j^2}{(dz)_j^2} \; : \; (dz)_j^2 < 0 \right\}$$

$$\alpha_D = 0.8\min \left\{ \begin{matrix} 1.2982 \\ 0.14 \\ 0.209 \\ 0.5595 \end{matrix} \right\}$$

Hence $\alpha_D = 0.14$ and $\alpha_P = 3.4903$

Step 5: Update Solution

We then take a step in the Newton's direction to get

$$x^3 = x^2 + \alpha_P (x)^2 \begin{pmatrix} 1.8872 \\ 1.9964 \\ 2.2292 \\ 0.1200 \end{pmatrix}$$

$$y^3 = y^2 + \alpha_D (y)^2 \begin{pmatrix} 0.3964 \\ 1.4317 \end{pmatrix}$$

$z_P = 9.764$ and $z_D = 11.761$

$\theta = z_D - z_P = 11.761 - 9.764 = 1.997$

Table 3.10: Solution after 3rd iteration:

Primal Solution						Dual Solution		
Iteration	z_P	x_1	x_2	x_3	x_4	z_D	y_1	y_2
1	5	1	1	5	3	28	2	2
2	8.845	1.4898	1.9551	3.0653	0.6	15.208	0.8815	1.3592
3	9.764	1.8872	1.9964	2.2292	0.12	11.761	0.3964	1.4317

Since θ (2) ≠0, it means optimality has not yet been achieved.

The process was continued till the 9th iteration when optimality was attained.

An optimality is achieved if the duality gap (θ) is zero, that is $Z_P - Z_D = 0$. Hence optimality was attained at the 9th iteration with $Z_P = Z_D = 10.667$, and with constraints $x_1 = 3.3332$ and $x_2 = 1.3334$ or $y_1 = 0.3333$ and $y_2 = 1.3334$. This is shown in the table below:

Table 3.11: Solution for all the 9 iterations.

Primal Solution						Dual Solution		
Iteration	z_P	x_1	x_2	x_3	x_4	z_D	y_1	y_2
1	5	1	1	5	3	28	2	2
2	8.845	1.4898	1.9551	3.0653	0.6	15.208	0.8815	1.3592
3	9.764	1.8872	1.9964	2.2292	0.12	11.761	0.3964	1.4317
4	10.419	3.0608	1.4325	0.4458	0.0741	11.025	0.3111	1.4228
5	10.598	3.2836	1.3437	0.0892	0.029	10.744	0.3276	1.3538
6	10.651	3.3201	1.337	0.0227	0.0058	10.68	0.3328	1.3371
7	10.664	3.3306	1.3341	0.0046	0.0012	10.67	0.3332	1.3341
8	10.666	3.3328	1.3335	0.0009	0.0002	10.667	0.3333	1.3335
9	10.667	3.3332	1.3334	0.0002	5E-05	10.667	0.3333	1.3334

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.1.0 Data Collection

The Dormaa East District Assembly has its way of collecting revenue on taxable items using a policy called The Fee Fixing Policy. This Fee Fixing Policy is a document on rates and fixing resolutions and it focuses on taxes such as; Rates, Fees and Fines, Licenses and Lands. Each category has sub taxes which constitute the group. For instance Rates which is one of the main categories is constituted by basic and property rates. Similarly, registration of building plots, stool land revenue, building permit and revenue from concession constitute the category of lands.

4.1.1 Type of Data and Source

The data for this project work is a secondary quarterly data obtained from the offices of the Dormaa East District Assembly in the Brong Ahafo region of Ghana, and it spans between 4th quarter 2008 and 3rd quarter 2011.

4.1.2 The Raw Data:

The tables 4.1 to 4.4 share similar characteristics. The 1st column is made up of the tax item number whiles the 2nd column is made up of the revenue sub-heads. The remaining columns form the estimated revenue (E.R) and the actual revenue (A.R) for the various quarters and their respective averages as shown in the respective tables below:

Table 4.1: Table showing the estimated revenue (E.R) and the actual revenue (A.R) generated by the assembly for the last quarter of 2008.

TAX NO. ITEM	REVENUE SUB-HEAD	ESTIMATED REVENUE	ACTUAL REVENUE	AVERAGE OF ESTIMATED REVENUE	AVERAGE OF ACTUAL REVENUE
1	Basic rate	1,200.00	113.00	1200.00	113.00
2	Property rate	10,00.00	9,979.79	10,000.00	9,979.79
3	Development Levy	12,542.00	00.00	12,542.00	00.00
4	Stool Land Revenue	45,000.00	10,000.00	45,000.00	10,000.00
5	Building Permit	1,780.00	1,168.00	1,780.00	1,168.00
:	:	:	:	:	:
40	Cold Stores	21.60	60.00	124.00	62.00
41	Market Stores/Stalls	2160.00	215.70	21.60	215.70

Table 4.2: Table showing the (E.R) and the (A.R) generated by the assembly for the four quarters of 2009.

TAX ITEM NO.	REVENUE SUB-HEAD	Q1 E.R	Q1 A.R	Q2 E.R	Q2 A.R	Q 3 E.R	Q3 A.R	Q 4 E.R	Q 4 A.R	MEAN E.R	MEAN A.R
1	Basic Rate	1,200.00	35.00	1,200.00	46.00	1,200.00	56.00	1,200.00	56.00	1,200.00	48.25
2	Property Rate	25,140.00	17,266.60	25.140.00	20.285.60	25,140.00	25,197.60	25,140.00	27,870.60	25,140.00	22,655.1
3	Development Levy	0.00	0.00	0.00	00.00						
4	Stool Land Revenue	45,000.00	0.00	45,000.00	00.00	45,000.00	8,000.00	45,000.00	20,000.00	45,000.00	7000.00
5	Building Permit	4,170.00	305.00	4,170.00	643.00	4,170.00	1,271.00	4,170.00	1,790.00	4,170.00	1002.25
:	;	:	:	:	:	:	:	:	:	:	:
41	Market Stores/Stalls	2160.00	0.00	2160.00	80.00	2160.00	339.00	2160.00	445.00	2160.00	216.00

The rest of the raw data up to the 3rd quarter of 2011 can be found at the appendix.

4.2 Data Analysis

Steps involved in Processing the Raw Data

Step1: The averages of the raw data were determined to get the estimated revenue average (E.R) and actual revenue average (A.R) for the data

Step 2: The various revenue sub-heads were assigned variable names.

Step 3: The summation of the actual revenue became the constraint column matrix.

Step 4: The unit charge for each of the items was also determined from the Fee-fixing Policy Document given by the Assembly. This formed the coefficient matrix.

Step 5: The ratio of the actual revenue (A.R) to the corresponding unit charge from the Fee-Fixing Policy were determined. These values formed the coefficients of the objective function.

This information has been illustrated in the tables below:

Table 4.5: This table is made up of 4 columns. Column 1 is made up of the tax item number. Column 2 is made up of the revenue sub-heads. Column 3 is made up of the averages of all the estimated revenues (E.R) and column 4 forms the averages of all the actual revenues (A.R).

Table 4.5: Table showing the average E.R and A.R generated by the assembly for the past 12 quarters of the assembly.

	REVENUE SUB-HEAD	E.R (AVERAGE)	A.R (AVERAGE)
1	Basic rate	1112.70	307.08
2	Property rate	10933.17	13338.90
3	Development Levy	18999.80	1832.13
4	Stool Land Revenue	26042.50	7646.40
5	Building Permit	3723.75	985.21
:	:	:	:
40	Cold Stores	95.20	131.65
41	Market Stores/Stalls	1200.00	299.38

Table 4.6: This table is made up of 2 columns. Column 1 forms the decision variables whiles column 2 is made up of the revenue sub-heads. The remaining part of the table can be found in appendix 3.

Table 4.6: Table showing the number of people paying for each category of tax as a variable, xj, j = 1, 2, 3 … 41.

DECISION VARIABLE (Xi)	REVENUE SUB-HEAD
x_1	Basic Rate
x_2	Property Rate
x_3	Development levy
x_4	Stool Land Revenue
x_5	Building Permit
:	:
x_{40}	Cold Stores
x_{41}	Market Stores/Stalls

Table 4.7**:** This table serves as the pivot for the whole problem formulation .It is made up of 7 columns. The first column deals with the tax item number. The 2nd column talks about the broad Revenue Heads. These include the Rates, Lands, Fees and Fines, Licenses and Rent. This broad category has been sub-grouped into the next basic unit called revenue sub-heads in the 3rd column. The 4th column is made up of the decision variables (x_j) .Column 5 is made up of the unit charge which forms the coefficient matrix, A. The right hand side (R.H.S) matrix is obtained from column 6 of the table. Finally, the coefficient of the objective function c_j is obtained from column 7 of table 4.7.The complete form of the table can be found in the appendix 4.

Table 4.7: Table represents the tax payers (x_j), average (A.R) and the unit charge from the fee-fixing policy document of the assembly.

TAX ITEM NO.	REVENUE HEAD	REVENUE SUB-HEAD	TAX PAYERS' VARIABLE (X_j)	UNIT CHARGE (A)	AVERAGE A.R FOR THE PAST TWELVE QUARTERS	THE RATIO OF A.R TO UNIT CHARGE(C_j)
1	Rates	Basic Rate	x_1	0.10	307.08	3070.80
2	Rates	Property Rate	x_2	10.00	13338.90	1333.89
3	Rates	Development Levy	x_3	10.00	1832.13	183.21
4	Lands	Stool Land Revenue	x_4	10.00	7646.40	764.64
5	Lands	Building Permit	x_5	35.62	985.21	27.66
:	:	:	:	:	:	:
40	Licenses	Cold Store	x_{40}	9.83	131.65	13.39
41	Rent	Market Stores/Stalls	x_{41}	3.00	299.38	99.97

4.3 Data Input Format

In expressing the above information in terms of matrices the following matrix equation will be obtained; **Ax**=b**;** where A is the coefficient matrix of the taxpayers function x, b is the constraint column matrix and c is the coefficient of the objective function. The complete form of the data in A, c and x can be found in table 4.7 of appendix 4.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 8.07\ldots\ldots\ldots\ldots\ldots0 \\ 0 & 0 & 0 & 0 & 0 & 0.2\ldots\ldots\ldots\ldots\ldots0 \\ 0 & 0 & 0 & 0 & 10 & 0\ldots\ldots\ldots\ldots\ldots0 \\ 0.1 & 10 & 10 & 0 & 0 & 0\ldots\ldots\ldots\ldots\ldots0 \\ 0 & 0 & 0 & 0 & 0 & 0\ldots\ldots\ldots\ldots\ldots3 \end{pmatrix}$$

$$b = \begin{pmatrix} 17055.18 \\ 11123.20 \\ 8926.28 \\ 15478.11 \\ 299.38 \end{pmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{41} \end{bmatrix}$$

$$x_1, x_2, x_3, \ldots, x_{41} \geq 0$$

$$c = (3070.8 \quad 1333.89 \quad 183.21 \quad \ldots\ldots \quad 99.97)$$

4.4.0 Model Formulation

The various taxes collected by the district assembly are broadly categorized into five groups. The groups are Rates, Lands, Fees and Fines, Licenses and Rent. These broad categories are shown in column 2 of table 4.7. The unit charge is obtained from the fee-fixing policy of the district. For instance, each member of the Dormaa East district is required by law to pay a basic rate of Ghs0.10 per annum.

4.4.1 Formulation of the Objective Function

At this point we seek to maximize revenue from an objective function generated from the data collected. As stated in chapter three, the function f(x) being maximized is called the objective function and conditions associated with the problem are called the constraints. In using the variable representing number of people paying each tax (x_j) and the ratio of (A.R) to each unit charge (c_i), we model an objective function represented by z from table 4.7 as follows:

$$z = \sum_{j=1}^{41} c_j x_j$$

$$z =$$

$(3070.8x_1+1333.89x_2+183.21x_3+\ldots\ldots\ldots\ldots\ldots\ldots 99.97x_{41})\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$

The c_j represents the coefficients of the objective function .The full data can be found in column 7 of table 4.7 in appendix 4.

From the table 4.7, five constraints are generated for the objective function, z, based on the broad categories of the taxes collected. One constraint is generated for Rates, Licenses, Fees and Fines, Rent and Lands.

4.4.2 Formation of Constraints

The right hand side of each of the constraints represents the respective sum of the actual revenue (A.R) generated by the respective variables. This information can be found in column 6 of table 4.7. Under listed below shows the constraints formed by the broad category of the revenue collected.

Rates: The constraint for the rate is obtained from 3 consecutive decision variables, x_1 to x_3 with corresponding R.H.S value of 15478.11.

$$0.1x_1 + 10x_2 + 10x_3 \leq 15478.11$$

Lands: The constraint for lands is obtained from 4 consecutive decision variables, x_4 to x_7 and with respective R.H.S value of 8926.28.

$$10x_4+35.62x_5+450x_6+21.67x_7 \leq 8926.28$$

Fees and Fines: The constraint for fees and fines is obtained from 9 consecutive decision variables from x_8 to x_{17} and with R.H.S value of 11123.29.

$$0.2x_8+10x_9+0.89x_{10}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + 13.89x_{17} \leq 11123.20$$

Licenses: The constraint for licenses is made up of consecutive decision variables from x_{18} to x_{40} and R.H.S value of 17055.18.

$$8.07x_{18}+0.28x_{19}+9.67x_{20}+\ldots\ldots\ldots\ldots\ldots\ldots+ 9.83x_{40} \leq 17055.18$$

Rent: The constraint for the rent is obtained from a unit decision variable, x_{41} and with corresponding R.H.S value of 299.38.

$$3x_{41} \leq 299.38$$

4.4.3 Formulation of the Problem

Maximize $z=3070.8x_1+ 1333.89x_2+183.21x_3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+99.97x_{41}$

Subject to : $8.07x_{18}+0.28x_{19}+9.67x_{20}+\ldots\ldots\ldots\ldots..+9.85x_{40} \leq 17055.18$

$$0.2x_8+10x_9+0.89\ldots\ldots+13.89x_{17} \leq 11123.20$$

$$10x_4+35.62x_5+450x_6+21.67x_7 \leq 8926.28$$

$$0.1x_1 + 10x_2 + 10x_3 \leq 15478.11$$

$$3x_{41} \leq 299.3$$

Problem Formulation involving the Slacks.

Expressing the above inequalities in the equality form we have the following equations:

$$0.1x_1 + 10x_2 + 10x_3 + s_1 = 15478.11 \quad \ldots\ldots\ldots\ldots\ldots\ldots 4.2a$$

$$10x_4+35.62x_5 + 450x_6+ 21.67x_7 + s_2 = 8926.28\ldots\ldots\ldots\ldots\ldots\ldots\ldots..4.2b$$

$$0.2x_8+10x_9+\ldots+13.89x_{17} + s_3 = 11123.20 \ldots\ldots 4.2c$$

$$8.07x_{18}+0.28x_{19}+\ldots\ldots+9.83x_{40} + s_4 = 17055.18 \ldots\ldots 4.2d$$

$$3x_{41}+s_5 = 299.38 \ldots\ldots 4.2e$$

$$x_j, s_j \geq 0 \qquad j=1, 2, 3\ldots 41$$

4.5 Iterative Primal-Dual Interior-Point Algorithm

In summarizing the basic steps of the algorithm the following inputs are assumed:

Step 0:

(i) The data of the problem (A, b, c), where the m x n matrix A has full row rank,

(ii) Initial primal and dual feasible solutions $x^0 > 0$, $z^0 > 0$, $y^0 > 0$.

(iii) The optimality tolerance $\varepsilon > 0$ and the step size parameter $\gamma \in (0, 1)$.

Step 1: (Initialization) .Start with some feasible point $x^0 > 0$, $z^0 > 0$, $y^0 > 0$.

Choose(x^0,y^0,z^0) such that$(x^0,z^0, y^0)>0$ and set the iteration counter k = 0.

Step 2: (Optimality test). If $(z^k)^T x^k < \varepsilon$ stop; otherwise, go to Step 3.

Step 3: (Compute Newton directions). Let

$$X^k = \text{diag}\left\{x^k_1, \; x^k_2, \ldots\ldots, x^k_n\right\}$$

$$Z^k = \text{diag}\left\{z^k_1, \; z^k_2, \ldots\ldots, z^k_n\right\}$$

$$\mu^k = \frac{(z^k)^T x^k}{n^2}$$

Solve the following linear system equivalent to (7) to get d^k_x, d^k_y, and d^k_z.

$$Adx = 0$$

$$A^T dy - dz = 0$$

$$Zdx + Xdz = \mu e - XZe$$

Note that $\delta_P = 0$ and $\delta_D = 0$ due to the feasibility of the initial point.

Step4: (Find step lengths).Let

$$\alpha_P = \gamma \min_j \left\{ \frac{-x^k{}_j}{(d_x)^k{}_j} : (d_x)^k{}_j < 0 \right\} \text{ and } \alpha_D = \gamma \min_j \left\{ \frac{-z^k{}_j}{(d_z)^k{}_j} : (d_z)^k{}_j < 0 \right\}$$

Step 5: (Update solution). Take a step in the Newton direction to get

$$x^{k+1} = x^k + \alpha_P(dx)^k$$

$$y^{k+1} = y^k + \alpha_D(dy)^k$$

$$z^{k+1} = z^k + \alpha_D(dz)^k$$

Put k = k + 1 and go to Step 2.

4.6 Computational Method

The coefficients of the tax functions, left-hand side constraint inequalities and right-hand side constants were written in matrices form. Matlab program software was used for coding the primal-dual algorithm.

The matrices were inputted in the Matlab. program code and ran on Intel(R) Core (TM) 2 Duo CPU T5750 @ 2.00GHz 2.00GHz, 32-bit operating system, Windows7 HP laptop computer.

The code ran successfully on ten trials with hundred iterations for each trial.

4.7 Results

Table 4.9 gives the primal solution and the dual solution. The x_j; j=1, 2, 3…41 gives the total amount that each revenue item contributes in arriving at the optimal solution. After 20 successful trials with 100 iterations for each of them, an optimal value of f= 2.8474e+010 was achieved. The results of the final test run for the total revenue generated after 100 iterations are shown below:

Table 4.6 Primal Solution and Dual Solution

Primal Solution			Dual Solution	
X	1.0e+006 *		Y	1.0e+007 *
x_1	0.9164		y_1	0.0044
x_2	0.0331		y_2	0.3451
x_3	0.0331		y_3	0.3450
x_4	0.2988		y_4	0.0177
x_5	0.1150		y_5	0.0610
x_6	0.0184		y_6	0.7620
x_7	0.1668		y_7	0.0374
x_8	0.8455		y_8	0.0049
x_9	0.0761		y_9	0.1069
x_{10}	0.4577		y_{10}	0.0104
x_{11}	0.0342		y_{11}	0.3230
x_{12}	0.0507		y_{12}	0.1866
x_{13}	1.0170		y_{13}	0.0039
x_{14}	0.1561		y_{14}	0.0409
x_{15}	0.0139		y_{15}	1.1459
x_{16}	0.0573		y_{16}	0.1577
x_{17}	0.0599		y_{17}	0.1483
x_{18}	0.5467		y_{18}	0.0082
x_{19}	2.9441		y_{19}	0.0011
x_{20}	0.4768		y_{20}	0.0097
x_{21}	0.1385		y_{21}	0.0455
x_{22}	1.9353		y_{22}	0.0018
x_{23}	3.0906		y_{23}	0.0010
x_{24}	0.2424		y_{24}	0.0223
x_{25}	0.1074		y_{25}	0.0633
x_{26}	0.3305		y_{26}	0.0152
x_{27}	0.0317		y_{27}	0.3275
x_{28}	0.9186		y_{28}	0.0044
x_{29}	0.2299		y_{29}	0.0239
x_{30}	0.2821		y_{30}	0.0185
x_{31}	0.0976		y_{31}	0.0718
x_{32}	0.1843		y_{32}	0.0316
x_{33}	0.0077		y_{33}	2.4130
x_{34}	1.2562		y_{34}	0.0030
x_{35}	0.0356		y_{35}	0.2790
x_{36}	1.1392		y_{36}	0.0034
x_{37}	0.2264		y_{37}	0.0244
x_{38}	0.2729		$y38$	0.0193
x_{39}	0.8074		y_{39}	0.0255
x_{40}	0.4709		y_{40}	0.0098
x_{41}	0.0368		y_{41}	0.3034

The f=2.8474e+010 represents the optimal value if the primal solution contributes x=1.0e+006 or if the dual solution contributes y=1.0e+007 in attaining the optimal solution.

4.8 Discussion

The available data by the Dormaa District Assembly which was used for this research work reveals that the average total revenue by the Assembly for the past four years has been GHS52, 882.10. The functional value of f= 2.8474e+010 gives a total of GHS62, 718.16 annually, based upon the primal-dual algorithm code. Hence with this research work, the Assembly can raise its revenue to GHS62, 718.16 annually which represents an appreciable 18.6% increase in the Assembly's revenue collection strategy.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The revenue data collected from the Dormaa East District Assembly was modeled into Linear Programming Problem. An optimal revenue mobilization strategy was then developed out of the Linear Programming Problem. The data was then run on a matlab software code. The analysis done in chapter four using primal-dual interior-point algorithm showed that average annual revenue generated by the Assembly between 2008 and 2011 was GHS52, 882.15. Based upon this research work, the Assembly can raise its revenue to GHS62, 718.16 which represents an 18.6% increase in the Assembly's revenue. The results also revealed that the tax item which performed well was the palm wine/pito sellers and the tax item which performed badly was the telecom companies.

5.2 Recommendations

The Dormaa East District Assembly as aforementioned in the problem statement has not been performing well in revenue mobilization. This state of affairs has contributed immensely in the Assembly's inability in providing basic social amenities such as schools, hospitals, portable water, improved sanitation facilities etc. This research work has come at an opportune time and it is a sigh of relief for most of the indigenes in the Dormaa East District Assembly. This reason stems from the fact that internally generated revenue which has being the assembly's major headache can now be addressed by this research work. I hereby recommend the following results and findings of this thesis to the Dormaa East District Assembly:

1. The work should serve as basis for further research works in improving revenue mobilization strategy by the Assembly and other District Assemblies in Ghana.
2. The research work also reveals that the contribution of basic rate showed a significant impact on the overall revenue generation, but many of the citizens' default in its payment. It is however recommended that this tax will be linked up with the national health insurance registration and renewal. This will take care of citizens who evade this tax, and will also widen the tax bracket. It is also recommended that the basic rate should be increased from its current form of GHS0.10 to GHS0.20 with attractive commission for the tax collectors.
3. The researcher is of the view that the assembly will benefit a lot by way of addressing revenue leakages if they can acquire automated tax collection machines for tax collection by the assembly.
4. Geographically, the scope of the research was limited to Dormaa East district of the Brong Ahafo Region of Ghana. The findings and recommendations may be used for similar districts of the country with low revenue generation capacity.

The study was carried out on secondary data obtained from the Assembly spanning between 2008 and 2011. The focus of the subject area of the study was internally generated revenue of the district.

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APPENDICES

APPENDIX 1

The raw data

Table 4.1: Table showing the estimated revenue (E.R) and the actual revenue (A.R) generated by the assembly for the last quarter of 2008.

TAX NO. ITEM	REVENUE SUB-HEAD	ESTIMATED REVENUE	ACTUAL REVENUE	AVERAGE OF ESTIMATED REVENUE	AVERAGE OF ACTUAL REVENUE
1	Basic rate	1,200.00	113.00	1200.00	113.00
2	Property rate	10,00.00	9,979.79	10,000.00	9,979.79
3	Development Levy	12,542.00	00.00	12,542.00	00.00
4	Stool Land Revenue	45,000.00	10,000.00	45,000.00	10,000.00
5	Building Permit	1,780.00	1,168.00	1,780.00	1,168.00
6	Revenue from Concession	300.00	00.00	300.00	00.00
7	Registration of Building Plot	390.00	00.00	390.00	00.00
8	Market Tolls	1,824.00	5,081.50	1,824.00	5,081.50
9	Court fires		43.00	500.00	43.00
10	Exportation of Farm produce	500.00	1,470.10	278.00	1,479.10
11	Marriage and Divorce	278.00	00.00	100.00	00.00
12	Toilet Management Revenue	100.00	00.00	324.00	00.00
13	Cattle Kraals	324.00	47.00	240.00	47.00
14	Larry Park	32.00	2,946.60	950.00	2,946.60
15	Poultry Farmers	240.00	1,388.60	80.00	1,388.60
16	Burial Fees	950.00	00.00	00.00	00.00
17	Ground Rent	80.00	00.00	20.00	00.00
18	Herbalists	00.00	43.00	288.00	43.00
19	Hawkers	20.00	22.00	216.00	22.00
20	Traditional Caterers	288.00	120.00	100.00	120.00
21	Registration of chainsaws	216.00	123.00	10.80	123.00

22	Corn Mill	100.00	133.28	8.40	133.28
23	Palm Wine / Pito Sellers	10.80	49.50	30.00	49.50
24	Beer / Spirits	8.40	568.00	126.00	568.00
25	Petroleum Products	30.00	40.00	32.00	40.00
26	General Goods Stores	126.00	202.00	400.00	202.00
27	Financial Institutions	32.00	00.00	19.20	00.00
28	Kiosks	400.00	464.00	190.00	464.00
29	Chemical Sellers	19.00	00.00	220.00	00.00
30	Private Schools	220.00	00.00	540.00	00.00
31	Sale of Bid Documents	540.00	120.00	52.00	120.00
32	Adverts / Bill Boards	52.00	00.00	1,200.00	00.00
33	Telecom Companies	1,200.00	00.00	00.00	00.00
34	Sale of Stickers	00.00	00.00	370,00	00.00
35	Award of Contracts	370.00	250.00		250.00
36	Registration of Motor Cycles	145.00	00.00		00.00
37	Lotto Operators	730.00	34.00	145.00	34.00
38	Registration of Business	00.00	00.00	730.00	00.00
39	Self-Employed Artisans	124.00	547.00	00.00	547.00
40	Cold stores	21.60	62.00	124.00	62.00
41	Market Stores / Stalls	2160.00	215.70	21.60	215.70

Table 4.2: Table showing the estimated revenue (E.R) and the actual revenue (A.R) generated by the assembly for the four quarters of 2009**.**

TAX ITEM NO.	REVENUE SUB-HEAD	Q1 E.R	Q1 A.R	Q2 E.R	Q2 A.R	Q 3 E.R	Q3 A.R	Q 4 E.R	Q 4 A.R	MEAN E.R	MEAN A.R
1	Basic Rate	1,200.00	35.00	1,200.00	46.00	1,200.00	56.00	1,200.00	56.00	1,200.00	48.25
2	Property Rate	25,140.00	17,266.60	25.140.00	20.285.60	25,140.00	25,197.60	25,140.00	27,870.60	25,140.00	22,655.1
3	Development Levy	0.00	0.00	0.00	00.00						
4	Stool Land Revenue	45,000.00	0.00	45,000.00	00.00	45,000.00	8,000.00	45,000.00	20,000.00	45,000.00	7000.00
5	Building Permit	4,170.00	305.00	4,170.00	643.00	4,170.00	1,271.00	4,170.00	1,790.00	4,170.00	1002.25
6	Revenue from Concession	300.00	0.00	300.00	00.00	300.00	0.00	300.00	0.00	300.00	0.000
7	Registration of Buildings Plots										
8	Market Tolls	1,824.00	1,540.80	1,824.00	3,142.90	1,824.00	5,322.00	1,824.00	6,442.90	1,824.00	4112.15
9	Court Fines	500.00	240.00	500.00	360.00	500.00	400.00	500.00	400.00	500.00	350.00
10	Exportation Farm Produce	2,278.00	790.20	2,278.00	1,201.70	2,278.00	2,129.70	2,278.00	2,852.70	2,278.00	1743.58
11	Marriage and Divorce	100.00	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00	0.000
12	Toilet Management Revenue	324.80	53.00	324.00	53.00	324.00	53.00	324	53.00	324.00	53.00
13	Cattle Kraals	32.00	0.00	32.00	0.00	32.00	0.00	32.00	0.00	32.00	0.000
14	Lorry Parks	240.00	0.00	240.00	1,970.00	240.00	4,080.00	240.00	4,900.00	240.00	2737.50
15	Burial Fees	950.00	390.00	950.00	690.00	950.00	711.10	950.00	1,251.00	950.00	760.53
16	Ground Rent										
17	Herbalists										
18	Hawkers	20.00	0.00	20.00	0.00	20.00	15.00	20.00	15.00	20.00	7.50
19	Traditional Caterers	288.00	0.00	288.00	0.00	288.00	102.00	288.00	404.00	288.00	126.00
20	Registration of	216.00	0.00	216.00	0.00	216.00	0.00	216.00	0.00	216.00	0.00

	Chainsaws										
21	Corm Mills	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
22	Palm wine / Pito	10.80	6.00	10.80	30.00	10.80	77.00	10.00	103.00	10.80	54.00
23	Beer /Spirits	8.40	0.00	8.40	0.00	8.40	5.00	8.40	29.50	8.40	8.63
24	Petroleum Products	30.00	170.00	30.00	42 0.00	30.00	427.00	30.00	427.00	30.00	361.00
25	General Goods	126.10	0.00	126.00	0.00	126.00	200.00	126.00	200.00	126.00	100.00
26	Financial Institutions.	32.00	18.00	32.00	154.00	32.00	419.00	32.00	474.00	32.00	266.25
27	Kiosks	400.00	0.00	400.00	590.00	400.00	590.00	400.00	590.00	400.00	442.50
28	Chemical Sellers	19.20	41.00	19.20	74.00	19.20	338.00	19.20	615.00	19.20	267.00
29	Chemical Sellers	190.00	0.00	190.00	43.00	190.00	81.00	190.00	81.00	190.00	51.25
30	Private Schools	220.00	0.00	220.00	0.00	220.00	0.00	22.00	60.00	220.00	15.00
31	Sale of Bid Documents	540.00	0.00	540.00	0.00	540.00	315.00	540.00	1,275.00	540.00	397.50
32	Adverts / Bill Boards	52.00	0.00	52.00	0.00	52.00	0.00	52.00	0.00	52.00	0.00
33	Telecom Companies										
34	Sale of Stickers										
35	Award of Contracts										
36	Registration of Motor Cycles										
37	Lotto Operators	730.00	0.00	730.00	0.00	730.00	0.00	730.00	0.00	730.00	0.00
38	Registration. of Business										
39	Self Employed Artisans	2,202.00	106.30	2,202	303.30	2,202.00	447.00	2,202.00	646.84	2,202.00	375.86
40	Cold Stores	21.60	0.00	21.60	54.00	21.60	74.00	21.60	147.00	21.60	68.75
41	Market Stores	2,160.00	0.00	2,160.00	80.00	2,160.00	339.00	2,160.00	445.00	2,160.00	216.00

Table 4.3: Table showing the estimated revenue (E.R) and the actual revenue (A.R) generated by the assembly for the four quarters of 2010.

TAX ITEM NO.	REVENUE SUB-HEAD	Q1 E.R	Q1 A.R	Q2 E.R	Q2 A.R	Q 3 E.R	Q3 A.R	Q 4 E.R	Q 4 A.R	MEAN E.R	MEAN A.R
1	Basic Rate	1125.40	192.20	1125.40	592.20	1125.40	676.20	1125.40	1331.20	1125.40	697.95
2	Property Rate	11254.00	3748.00	11254.00	17534.86	11254.00	18337.16	11254.00	18806.46	11254.00	14,606.62
3	Development Levy	10128.60	317.50	10128.60	317.50	10128.60	339.50	10128.60	33950	10128.60	328.50
4	Stool Land	30000.00	0.00	30000.00	20000.00	30000.00	20000.00	30000.00	25000.00	30000.00	16,250.00
5	Building Permit	3740.00	275.00	3740.00	1061.00	3740.00	1481.00	3740.00	2237.00	3740.00	1,263.50
6	Revenue from Concession	470.00	0.00	470.00	0.00	470.00	184.00	470.00	546.00	470.00	182.50
7	Registration of Buildings	705.00	0.00	705.00	0.00	705.00	27.00	705.00	27.00	705.00	13.50
8	Market Tolls	5064.00	1690.00	5064.00	3158.00	5064.00	4829.20	5064.00	5949.20	5064.00	3,906.60
9	Court Fines	1100.00	0.00	1100.00	30.00	1100.00	140.00	1100.00	230.00	1100.00	100.00
10	Exportation	4975.00	717.00	4975.00	1400.00	4975.00	3100.00	4975.00	5103.50	4975.00	2,580.13
11	Marriage/Divoce	370.00	20.00	370.00	20.00	370.00	60.00	370.00	100.00	370.00	50.00
12	Toilet Management Revenue	960.00	15.00	960.00	215.00	960.00	657.00	960.00	777.00	960.00	416.00
13	Cattle Kraals	65.00	16.00	65.00	180.00	65.00	180.00	65.00	180.00	65.00	139.00
14	Lorry Park	48.00	1277.00	48.00	2537.00	48.00	3863.00	48.00	5084.00	48.00	3190.25
15	Poultry Farmers	2740.00	60.00	2740.00	457.00	2740.00	1206.00	2740.00	2371.00	2740.00	1,023.50
16	Burial fees	685.00	0.00	685.00	25.00	685.00	89.00	685.00	119.00	685.00	58.25
17	Ground Rent	1440.00	70.00	1440.00	290.00	1440.00	570.00	1440.00	825.00	1440.00	438.75
18	Herbalists	180.00	0.00	180.00	25.00	180.00	35.00	180.00	690.00	180.00	46.25
19	Hawkers	240.00	308.00	240.00	547.00	240.00	592.00	240.00	1127.00	240.00	534.25
20	Traditional	1200.00	9.00	1200.00	326.00	1200.00	634.00	1200.00	387.00	1200.00	524.00

21	Registration of Chainsaws	750.00	35.00	750.00	35.00	750.00	137.00	750.00	207.00	750.00	148.5
22	Corn mill	72.00	41.00	72.00	178.00	72.00	198.00	72.00	474.50	72.00	156.00
23	Palm Wine/ Pito	400.00	8.50	400.00	131.50	400.00	275.50	400.00	428.50	400.00	222.5
24	Beer/Spirits	1060.00	0.00	1060.00	70.00	1060.00	338.50	1060.00	428.50	1060.00	209.25
25	Petroleum Products	890.00	100.00	890.00	330.00	890.00	337.00	890.00	337.00	890.00	276.00
26	General Goods	895.00	337.00	895.00	767.00	895.00	985.00	895.00	1076.00	895.00	791.25
27	Financial Institution	2600.00	0.00	2600.00	200.00	2600.00	200.00	2600.00	700.00	2600.00	275.00
28	Kiosks	2121.00	220.00	2121.00	660.00	2121.00	1152.00	2121.00	2110.00	2121.00	1,035.50
29	Chemical Sellers	240.00	66.00	240.00	161.00	240.00	202.00	240.00	310.00	240.00	184.75
30	Private Schools	240.00	30.00	240.00	201.00	240.00	241.00	240.00	271.00	240.00	185.75
31	Sale of Bid Documents	600.00	400.00	600.00	400.00	600.00	2300.00	600.00	4600.00	600.00	1,925.00
32	Adverts/ Bill Boards	810.00	0.00	810.00	20.00	810.00	30.00	810.00	40.00	810.00	22.50
33	Telecom Companies	27000.00	13000.00	27000.00	20500.00	27000.00	20500.00	27000.00	23500.00	27000.00	21,625
34	Sale of Stickers	156.50	110.00	156.50	266.00	156.50	266.00	156.50	326.00	156.50	242.00
35	Award of Contracts	2500.00	0.00	2500.00	0.00	2500.00	0.00	2500.00	0.00	2500.00	0.00
36	Registration of Motor	915.00	0.00	915.00	0.00	915.00	0.00	915.00	41.50	915.00	10.38
37	Lotto Operators	840.00	0.00	840.00	5.00	840.00	105.00	840.00	217.00	840.00	81.75
38	Registration of Business	60.00	0.00	60.00	3090.00	60.00	6730.00	60.00	6800.00		4,155.00
39	Self Employed Artisans	3583.00	543.00	3583.00	1057.00	3583.00	1518.00	3583.00	2410.50	3583.00	1,382.13
40	Cold stores	132.00	67.00	132.00	167.00	132.00	202.00	132.00	254.00	132.00	172.50
41	Market stores	720.00	82.00	720.00	294.00	720.00	520.00	720.00	806.00	720.00	425.50

Table 4.4: Table showing the estimated revenue(E.R) and the actual revenue (A.R) generated by the assembly for the first three quarters of 2011.

TAX ITEM NO.	REVENUE SUB-HEAD	Q1 E.R	Q1 A.R	Q2 E.R	Q2 A.R	Q 3 E.R	Q3 A.R	Q 4 E.R	Q 4 A.R	MEAN E.R	MEAN A.R
1	Basic Rate	925.40	369.10	925.40	369.10	925.40	369.10			935.4	369.10
2	Property Rate	11,545.50	1964.34	11,545.50	7,475.43	11,545.50	8,920.50			11545.50	6114.09
3	Dev't Levy	8,328.60	00.00	8,328.50	0.00	8,328.60	0.00			8328.60	0.00
4	Stool Land	25,000.00	0.00	25,000.00	5,000.00	25,000.00	5,000.00			25000,00	3333.33
5	Building Permit	9075.00	1,118.00	9,075.00	1,568.00	9,075.00	1,842.00			9075.00	1509.33
6	Revenue from Concession	270.00	548.00	270.00	753.00	270.00	753.00			270.00	684.67
7	Registration of Building Plot	1,190.00	0.00	270.00	0.00	1,190.00	10.00			1190.00	3.33
8	Market Tolls	5,556.00	1,850.00	1,190.00	3,696.00	5,556.00	5,636.00			5556.00	3727.33
9	Court Fines	700.00	50.00	5,556.00	100.00	700.00	100.00			700.00	83.33
10	Exportation of Farm Produce	4,230.	1,645.00	700.00	2,695.00	4,230.00	3,676.00			4230.00	2672.00
11	Marriage and Divorce	320.00	60.00	4,230.00	60.00	320.00	150.00			320.00	90.00
12	Toilet Management Revenue	2,520.00	662.60	320.00	712.00	2,520.00	712.60			2520.00	695.53
13	Cattle Kraals	100.00	0.00	2,520.00	59.00	100.00	169.00			100.00	76.00
14	Lorry Park	4,320.00	1,420.00	100.00	2,780.00	4,320.00	4,085.00			4320.00	2671.67
15	Burial Fees	2,350.00	210.00	4,320.00	760.00	2,350.00	1,420.00			2350.00	139.00
16	Ground Rent	347.00	139.00	2350.00	139.00	347.50	139.00			347.17	762.33
17	Herbalists	1980.00	190.00	347.00	480.00	1,980.00	670.00			1980.00	446.67
18	Hawkers	82.00	55.00	1,980.00	65.00	82.00	105.00			82.00	75.00
19	Traditional Caterers	420.00	484.00	82.00	729.00	420	1,074.00			420.00	762.33

20	Registration of Chainsaw	800.00	297.00	420.00	592.00	800.00	766.00			800.00	551.67
21	Corn Mill	470.00	215.00	800.00	335.00	470.00	343.00			470.00	297.67
22	Palm Wine / Pito	144.00	80.00	470.00	177.00	144.00	279.00			144.00	178.67
23	Beer / Spirits	400.00	137.00	144.00	346.00	1,060.00	402.00			400.00	295.00
24	Petroleum Products	1,060.00	68.00	400.00	475.00	410.00	615.00			1060.00	386.00
25	General Goods	410.00	150.00	1,060.00	210.00	1,045.00	250.00			410.00	203.33
26	Financial Institution	1,045.00	955.00	410	1,426.00	1,900.00	1,856.00			1045.00	1412.33
27	Kiosks	1,900.00	430.00	1045.00	630.00		630.00			1900.00	563.33
28	Chemical Sellers	2,533.20	983.00	2,533.20	1628.00	2,533.20	2,040.00			2533.2	1550.33
29	Private Sellers	240.00	139.00	240.00	215.00	240.00	335.00			240.00	229.67
30	Private School	240.00	20.00	240.00	120.00	240.00	277.00			240.00	139.00
31	Sale of Bid Documents	3,000.00	0.00	3,000.00	0.00	3,000.00	32.00			3000.00	10.67
32	Adverts / Bill Boards	760.00	0.00	760.00	117.00	760.00	117.00			760.00	78.00
33	Telecom Company	2,5200.00	0.00	25,200.00	1,500.00	25,200.00	9,500.00			25200.00	3666.67
34	Sale of Stickers	315.50	137.00	315.50	413.00	315.50	479.00			315.50	343.00
35	Award of Contractor	2,500.00	0.00	2,500.00	0.00	2,500.00	0.00			2500.00	0.00
36	Registration of Motor	402.50	21.50	402.50	1,200.00	402.50	1,200.00			402.50	807.17
37	Lotto Operation	420.00	53.00	420.00	139.00	420.00	139.00			420.00	110.33
38	Registration of Business	60.00	273.00	60.00	4,798.00	60.00	5,068.00			60.00	3379.67
39	Self Employed Artisans	1,235.00	1,467.00	1,235.00	1909.00	1,235.00	2,146.00			1235.00	1840.67
40	Cold Stores	132.00	113.00	132.00	223.00	132.00	334.00			132.00	223.33
41	Market Stores	720.00	232.00	720.00	340.00	720.00	449.00			720.00	340.33

APPENDIX 2

Table 4.5: Table showing the average E.R and A.R generated by the assembly for the past 12 quarters of the assembly.

	REVENUE SUB-HEAD	E.R (AVERAGE)	A.R (AVERAGE)
1	Basic rate	1112.70	307.08
2	Property rate	10933.17	13338.90
3	Development Levy	18999.80	1832.13
4	Stool Land Revenue	26042.50	7646.40
5	Building Permit	3723.75	985.21
6	Revenue from concession	716.00	289.06
7	Registration of Building Plot	696.25	5.61
8	Market Tolls	3680.50	4206.90
9	Court Fines	600.00	144.08
10	Exportation of Farm Produce	2451.75	2118.70
11	Marriage and Divorce	205.50	35.00
12	Toilet Management Revenue	1011.00	291.13
13	Cattle Kraals	286.75	65.50
14	Lorry Park	2345.00	2909.00
15	Poultry Farmers	1582.00	992.32
16	Burial Fees	303.04	65.75
17	Ground Rent	857.70	295.14
18	Herbalists	72.60	42.94
19	Hawkers	244.50	361.15
20	Traditional Caterers	585.50	298.92
21	Registration of chainsaws	388.00	167.29
22	Corn Mill	156.50	130.49
23	Palm Wine / Pilot sellers	206.80	143.91
24	Beer / Spirits	585.00	381.06
25	Petroleum Products	411.50	154.83
26	General Goods Stores	628.00	667.96
27	Financial Institutions	1238.00	320.21
28	Kiosks	1557.73	829.21
29	Chemical Sellers	223.33	116.42
30	Private Schools	725.50	84. 94
31	Sale of Bid Documents	1575.00	613.17
32	Adverts / Bill Boards	540.67	25.13
33	Telecom Companies	17800.00	8430.56
34	Sale of Stickers	157.17	195.00
35	Award of Contracts	1790.00	83.33
36	Registration of Motor Cycles	487.50	272.52
37	Lotto Operators	663.33	56.52
38	Registration of Businesses	40.00	2511.56
39	Self-Employed Artisans	1647.33	1036.41
40	Cold Stores	95.20	131.65
41	Market Stores / Stalls	1200.00	299.38

APPENDIX 3

Table 4.6: Table showing the number of people paying for each category of tax as a variable, x_j, j = 1, 2, 3 … 41.

DECISION VARIABLE (XI)	REVENUE SUB-HEAD
x_1	Basic Rate
x_2	Property Rate
x_3	Development levy
x_4	Stool Land Revenue
x_5	Building Permit
x_6	Revenue from Concession
x_7	Registration of Building Plots
x_8	Market Tolls
x_9	Court Fines
x_{10}	Exportation of Farm Produce
x_{11}	Marriage and Divorce
x_{12}	Toilet Management Revenue
x_{13}	Cattle Kraals
x_{14}	Lorry Park
x_{15}	Poultry Farmers
x_{16}	Burial Fees
x_{17}	Ground Rent
x_{18}	Herbalists
x_{19}	Hawkers
x_{20}	Traditional Caterers
x_{21}	Registration of Chainsaws
x_{22}	Corn mill
x_{23}	Palm Wine / Pito Sellers
x_{24}	Beer / Spirits
x_{25}	Petroleum Products
x_{26}	General Goods Stores
x_{27}	Financial Institutions
x_{28}	Kiosks
x_{29}	Chemical sellers
x_{30}	Private Schools
x_{31}	Sale of Bid Documents
x_{32}	Adverts / Bill Boards
x_{33}	Telecom Documents
x_{34}	Sale of Stickers
x_{35}	Award of Contracts
x_{36}	Registration of Motor Cycles
x_{37}	Lotto Operators
x_{38}	Registration of Business
x_{39}	Self-Employed Artisans
x_{40}	Cold Stores
x_{41}	Market Stores / Stalls

APPENDIX 4

Table 4.7: Table representing the tax items and the unit charges used to model the L.P for the problem.

TAX ITEM NO.	REVENUE HEAD	REVENUE SUB-HEAD	TAX PAYERS' VARIABLE (x_i)	UNIT CHARGE (ci)	AVERAGE A.R FOR THE PAST TWELVE QUARTERS	THE RATIO OF A.R TO UNIT CHARGE(Ci)
1	Rates	Basic Rate	x_1	00.10	307.08	3070.80
2	Rates	Property Rate	x_2	10.00	13338.90	1333.89
3	Rates	Development Levy	x_3	10.00	1832.13	183.21
4	Lands	Stool Land Revenue	x_4	10.00	7646.40	764.64
5	Lands	Building Permit	x_5	35.62	985.21	27.66
6	Lands	Revenue from Concession	x_6	450.00	289.06	0.64
7	Lands	Registration of Building Plots	x_7	21.67	5.61	0.26
8	Fees and Fines	Market Tolls	x_8	0.20	4206.90	21034.90
9	Fees and Fines	Court Fines	x_9	10.00	144.08	14.41
10	Fees and Fines	Exportation of Farm Produce	x_{10}	0.89	2118.70	2380.56
11	Fees and Fines	Marriage / Divorce	x_{11}	30.33	35.00	1.15
12	Fees and Fines	Toilet Management Revenue	x_{12}	17.50	291.13	16.64
13	Fees and Fines	Cattle Kraals	x_{13}	0.30	65.50	218.33
14	Fees and Fines	Lorry Park	x_{14}	3.78	2909.00	769.58
15	Fees and Fines	Poultry Farm	x_{15}	107.78	992.32	9.21
16	Fees and Fines	Burial Fees	x_{16}	14.78	65.75	4.45
17	Fees and Fines	Ground Rent	x_{17}	13.89	295.14	21.25

18	Licenses	Herbalists	x_{18}	8.07	42.94	5.32
19	Licenses	Hawkers	x_{19}	0.28	361.15	1289.82
20	Licenses	Traditional Caterers	x_{20}	9.67	298.92	30.91
21	Licenses	Registration of Chainsaw	x_{21}	48.33	167.29	3.46
22	Licenses	Corn Mills	x_{22}	1.17	130.49	111.53
23	Licenses	Palm Wine/ Pito Sellers	x_{23}	0.30	143.91	479.70
24	Licenses	Beer / Spirits	x_{24}	23.33	381.06	16.33
25	Licenses	Petroleum Products	x_{25}	67.50	154.83	2.29
26	Licenses	General Goods Stores	x_{26}	15.61	667.96	42.79
27	Licenses	Financial Institutions	x_{27}	352.22	320.21	0.91
28	Licenses	Kiosks	x_{28}	3.94	829.21	210.46
29	Licenses	Chemical Sellers	x_{29}	25.00	116.42	4.66
30	Licenses	Private Schools	x_{30}	19.17	84.94	4.43
31	Licenses	Sale of Bid Documents	x_{31}	76.67	613.17	8.00
32	Licenses	Adverts / Bill Boards	x_{32}	33.13	25.13	0.76
33	Licenses	Telecom Companies	x_{33}	2600.00	8430.56	3.24
34	Licenses	Sale of Stickers	x_{34}	2.48	195.00	78.63
35	Licenses	Award of Contracts	x_{35}	300.00	83.33	0.28
36	Licenses	Registration of Motor Cycles	x_{36}	2.88	272.52	94.63
37	Licenses	Lotto Operators	x_{37}	25.50	56.52	2.22
38	Licenses	Registration of Business	x_{38}	20.00	2511.56	125.58
39	Licenses	Self-Employed Artisans	x_{39}	4.73	1036.41	219.11
40	Licenses	Cold Stores	x_{40}	9.83	131.65	13.39
41	Rent	Market Stores / Stalls	x_{41}	3.00	299.38	99.97

APPENDIX 5

Number of iteration run for the optimal value.

iter 1: mu = 4.42e+012, resid = 1.05e+009
iter 2: mu = 4.22e+012, resid = 9.97e+008
iter 3: mu = 4.02e+012, resid = 9.50e+008
iter 4: mu = 3.83e+012, resid = 9.07e+008
iter 5: mu = 3.66e+012, resid = 8.65e+008
iter 6: mu = 35
.49e+012, resid = 8.26e+008
iter 7: mu = 3.33e+012, resid = 7.89e+008
iter 8: mu = 3.18e+012, resid = 7.54e+008
iter 9: mu = 3.04e+012, resid = 7.21e+008
iter 10: mu = 2.90e+012, resid = 6.89e+008
iter 11: mu = 2.77e+012, resid = 6.60e+008
iter 12: mu = 2.65e+012, resid = 6.31e+008
iter 13: mu = 2.53e+012, resid = 6.05e+008
iter 14: mu = 2.42e+012, resid = 5.79e+008
iter 15: mu = 2.32e+012, resid = 5.55e+008
iter 16: mu = 2.22e+012, resid = 5.32e+008
iter 17: mu = 2.12e+012, resid = 5.10e+008
iter 18: mu = 2.03e+012, resid = 4.89e+008
iter 19: mu = 1.94e+012, resid = 4.69e+008
iter 20: mu = 1.86e+012, resid = 4.50e+008
iter 21: mu = 1.78e+012, resid = 4.33e+008
iter 22: mu = 1.71e+012, resid = 4.16e+008
iter 23: mu = 1.64e+012, resid = 4.00e+008
iter 24: mu = 1.57e+012, resid = 3.85e+008
iter 25: mu = 1.51e+012, resid = 3.70e+008
iter 26: mu = 1.45e+012, resid = 3.56e+008
iter 27: mu = 1.39e+012, resid = 3.43e+008
iter 28: mu = 1.33e+012, resid = 3.30e+008
iter 29: mu = 1.28e+012, resid = 3.18e+008
iter 30: mu = 1.23e+012, resid = 3.07e+008
iter 31: mu = 1.18e+012, resid = 2.96e+008
iter 32: mu = 1.14e+012, resid = 2.85e+008
iter 33: mu = 1.09e+012, resid = 2.75e+008
iter 34: mu = 1.05e+012, resid = 2.66e+008
iter 35: mu = 1.01e+012, resid = 2.56e+008
iter 36: mu = 9.71e+011, resid = 2.47e+008
iter 37: mu = 9.34e+011, resid = 2.39e+008
iter 38: mu = 8.99e+011, resid = 2.31e+008
iter 39: mu = 8.65e+011, resid = 2.23e+008
iter 40: mu = 8.33e+011, resid = 2.15e+008
iter 41: mu = 8.02e+011, resid = 2.08e+008

iter 42: mu = 7.72e+011, resid = 2.01e+008
iter 43: mu = 7.43e+011, resid = 1.94e+008
iter 44: mu = 7.16e+011, resid = 1.87e+008
iter 45: mu = 6.89e+011, resid = 1.81e+008
iter 46: mu = 6.64e+011, resid = 1.74e+008
iter 47: mu = 6.39e+011, resid = 1.68e+008
iter 48: mu = 6.16e+011, resid = 1.63e+008
iter 49: mu = 5.93e+011, resid = 1.57e+008
iter 50: mu = 5.71e+011, resid = 1.51e+008
iter 51: mu = 5.50e+011, resid = 1.46e+008
iter 52: mu = 5.30e+011, resid = 1.41e+008
iter 53: mu = 5.10e+011, resid = 1.36e+008
iter 54: mu = 4.91e+011, resid = 1.31e+008
iter 55: mu = 4.73e+011, resid = 1.27e+008
iter 56: mu = 4.55e+011, resid = 1.22e+008
iter 57: mu = 4.38e+011, resid = 1.18e+008
iter 58: mu = 4.22e+011, resid = 1.13e+008
iter 59: mu = 4.06e+011, resid = 1.09e+008
iter 60: mu = 3.91e+011, resid = 1.05e+008
iter 61: mu = 3.76e+011, resid = 1.01e+008
iter 62: mu = 3.62e+011, resid = 9.74e+007
iter 63: mu = 3.48e+011, resid = 9.38e+007
iter 64: mu = 3.35e+011, resid = 9.02e+007
iter 65: mu = 3.22e+011, resid = 8.68e+007
iter 66: mu = 3.09e+011, resid = 8.35e+007
iter 67: mu = 2.97e+011, resid = 8.03e+007
iter 68: mu = 2.86e+011, resid = 7.71e+007
iter 69: mu = 2.74e+011, resid = 7.41e+007
iter 70: mu = 2.64e+011, resid = 7.12e+007
iter 71: mu = 2.53e+011, resid = 6.84e+007
iter 72: mu = 2.43e+011, resid = 6.57e+007
iter 73: mu = 2.34e+011, resid = 6.30e+007
iter 74: mu = 2.24e+011, resid = 6.05e+007
iter 75: mu = 2.15e+011, resid = 5.81e+007
iter 76: mu = 2.07e+011, resid = 5.57e+007
iter 77: mu = 1.98e+011, resid = 5.34e+007
iter 78: mu = 1.90e+011, resid = 5.12e+007
iter 79: mu = 1.82e+011, resid = 4.91e+007
iter 80: mu = 1.75e+011, resid = 4.71e+007
iter 81: mu = 1.68e+011, resid = 4.51e+007
iter 82: mu = 1.61e+011, resid = 4.32e+007
iter 83: mu = 1.54e+011, resid = 4.14e+007
iter 84: mu = 1.48e+011, resid = 3.97e+007
iter 85: mu = 1.42e+011, resid = 3.80e+007
iter 86: mu = 1.36e+011, resid = 3.64e+007
iter 87: mu = 1.30e+011, resid = 3.48e+007

iter 88: mu = 1.24e+011, resid = 3.33e+007
iter 89: mu = 1.19e+011, resid = 3.19e+007
iter 90: mu = 1.14e+011, resid = 3.05e+007
iter 91: mu = 1.09e+011, resid = 2.92e+007
iter 92: mu = 1.05e+011, resid = 2.79e+007
iter 93: mu = 1.00e+011, resid = 2.67e+007
iter 94: mu = 9.59e+010, resid = 2.55e+007
iter 95: mu = 9.17e+010, resid = 2.44e+007
iter 96: mu = 8.78e+010, resid = 2.34e+007
iter 97: mu = 8.40e+010, resid = 2.23e+007
iter 98: mu = 8.04e+010, resid = 2.13e+007
iter 99: mu = 7.69e+010, resid = 2.04e+007
iter 100: mu = 7.35e+010, resid = 1.95e+007s

APPENDIX 6

Matlab Code for the Algorithm

```
function [x,y,s,f] = pdip(A,b,c)
% primal-dual interior-point method for problem
%
% min c'x s.t. Ax=b, x>=0,
%
% whose dual is
%
% max b'y  s.t. A'y+s=c,  s>=0.
%
% calling sequence:
%
% [x,y,s,f] = pdip(A,b,c)
%
% input: A is an m x n SPARSE constraint matrix.
%        b is an m x 1 right-hand side vector
%        c is an n x 1 cost vector.
%
% output: x is the  n x 1 solution of the primal problem
%         y is the m x 1 dual solution
%         s is the n x 1 vector of "dual slacks"
%         f is the optimal objective value
if margin ~= 3
  error('must have three input arguments');
end

if ~issparse(A)
  error('first input argument A must be a SPARSE matrix; possibly use sparse() to
convert');
end

t0=cputime;
[m,n] = size(A);
if m <= 0 or n <= 0
  error('input matrix A must be nontrivial');
end

if n ~= length(c)
  error('size of vector p must match number of columns in A');
end
if m ~= length(b)
  error('size of vector b must match number of rows in A');
end
```

```
% set initial point, based on largest element in (A,b,c)
bigM = max(max(abs(A)));
bigM = max([norm(b,inf), norm(p,inf), bigM]);
x = 100*bigM*ones(n,1); s = x; y = zeros(m,1);

% find row/column ordering that gives a sparse Cholesky
% factorization of ADA'
ordering = symmmd(A*A');
bp = 1+max([norm(b), norm(c)]);

for iter=1:100

% compute residuals
  Rd = A'*y+s-c;
  Rc = A*x-b;
  Rp = x.*s;
  mu = mean(Rp);
  relResidual = norm([Rd;Rc;Rp])/bp;
%  fprintf('iter %2i: mu = %9.2e, resid = %9.2e\n', iter, mu, relResidual);
  fprintf('iter %2i: mu = %9.2e, resid = %9.2e\n', iter, full(mu), ...
            full(relResidual));
  if(relResidual <= 1.e-7 & mu <= 1.e-7) break; end;
  Rp = Rp - min(0.1,100*mu)*mu;

  % set up the scaling matrix, and form the coefficient matrix for
  % the linear system
  d = min(5.e+15, x./s);
  B = A*sparse(1:n,1:n,d)*A';
  % use the form of the Cholesky routine "cholinc" that's best
  % suited to interior-point methods
  R = cholinc(B(ordering,ordering),'inf');

  % set up the right-hand side
  t1 = x.*Rd-Rp;
  t2 = -(Rc+A*(t1./s));

  % solve it and recover the other step components
  dy = zeros(m,1);
  dy(ordering) = R\(R'\t2(ordering));
  dx = (x.*(A'*dy)+t1)./s;
  ds = -(s.*dx+Rp)./x;
   tau = max(.9995,1-mu);
  ac= -1/min(min(dx./x),-1);
  ad = -1/min(min(ds./s),-1);
  ac = tau*ac;
```

```
  ad = tau*ap;
  x = x + ac*dx;
  s = s + ad*ds;
  y = y + ad*dy;
end

f = c'*x;

% convert x,y,s to full data structures
x=full(x); s=full(s); y=full(y);

fprintf('Done!\t[m n] = [%g %g]\tCPU = %g\n', m, n, cputime-t0);
return;
```