KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY –KUMASI

COLLEGE OF SCIENCE

FACULTY OF PHYSICAL SCIENCES

DEPARTMENT OF MATHEMATICS



# INFERENTIAL ANALYSIS OF HOURLY SOLAR IRRADIATION IN KUMASI, GHANA

BY

YARHANDS DISSOU ARTHUR

JULY 2009

**INFERENTIAL ANALYSIS OF HOURLY SOLAR IRRADIATION IN KUMASI, GHANA**

**A THESIS PRESENTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, IN PARTIAL FULLFILLMENT OF THE REQUIREMENT FOR THE AWARD**

OF

MASTERS OF SCIENCE (Msc) MATHEMATICS



BY

YARHANDS DISSOU ARTHUR

**JULY 2009**

# ACKNOWLEDGEMENT

*But thanks be to God who gives us victory through our Lord Jesus Christ. Therefore, my brethren, be steadfast, immovable, always abounding in the work of the Lord ,knowing that your labor is not in vain in the Lord.(1st Corinthians 15:57-58)*

I praise my God true His son Jesus Christ for his infinite blessings and mercies towards my life. May His name be glorified.

I want to thank my parents Mr. Charles Kwaku-vi and Miss Esther Botchwey for their parental support during my school period. I also want to thank Mr. and Mrs. Odei-Asiedu for their financial, spiritual and their moral support given to me during my period of study. Indeed words cannot express my gratitude but God who knows the heart of every man knows how grateful I am to Mr. and Mrs. Odei-Asiedu.

God does not just bring people in our ways. I always thank God and my Lord Jesus Christ for bringing my supervisor, Dr. F.T. Oduro and Lecturers Dr S.K Amponsah and Mr. S.K Appiah in my way. Dr. F.T. Oduro selflessly gave me enough attention for discussions and corrections. He was always willing to offer a helping hand when it is needed. He really took me as a son "Doc" I say God richly bless you. Mr. S.K Appiah was very helpful. He was enormously helpful in the data analysis and discussion of results. He was always willing to read and correct whatever I gave him. Sir, may God richly bless you and grant you your heart desires. Dr. S.K. Amponsah always encouraged me to move ahead. Indeed he was a mentor.

Other thanks go to the Head of Department, Dr. Edward Prempeh and the Dean of Physical Science, Professor I.K. Dontwi for their fatherly encouragement during my period of study.

I thank all Senior Members of the Department of Mathematics for all their input during my studies, may God richly bless them and their family.

I also appreciate the support of Professor F.O. Akuffo for his support and help offered during my thesis. Other thanks goes to the office staff in the mathematics department and the solar laboratory worker mechanical engineering KNUST.

Thank you SAM, for all support which words cannot express. May God give you in-depth knowledge in the field you want to work..

And to you my special friends: Jemima Owusua Awenseba, Alexander Kwame Kwarting, Ebenerzer Kwaku Wilmot, Thywil Dzoveho and all friends of Yarhands, I say a big thank you all and God bless you where ever you are.

To God be the glory.

DECLARATION

I hereby declare that this submission is my own work toward the award of the M.Sc. Mathematics, and that to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.



YARHANDS DISSOU ARTHUR ............................ 24/02/10 ............................

Signature Date

Dr. F.T ODURO ............................ 25/02/10 ............................

(SUPERVISOR) Signature Date

Dr S.K AMPONSAH ............................ 7/04/10 ............................

(HEAD) Signature Date



Dedication

This thesis is dedicated to Mr. and Mrs. Odei-Asiedu.and my mother.

## CHAPTER ONE

## INTRODUCTION



## CHAPTER TWO

## LITERATURE REVIEW

CHAPTER THREE

## CHAPTER FOUR



## LIST OF FIGURES

LIST OF TABLES

# ABSTRACT

The inferential analysis of hourly solar irradiation for Kumasi –Ghana was conducted using 14 years of data of measured values by the KNUST SOLAR LABORATORY. The analysis was carried out to find out the probability distributions that best fit the data of a given month of the year. Further analysis was carried out to find out the cluster for the month of the year. From the analysis conducted, it was found that solar irradiation for January, March and May can be fitted with the lognormal probability distribution. The month of April can be fitted with the Exponential, Weibull, lognormal Geometric and Gamma distribution while the months of June to December can be fitted with Exponential, Weibull, Geometric and Gamma. From the hypothesis testing carried out it was discovered that the months of the year can be put into two clusters. The first cluster is from the month of January to October and the second cluster is the November and December. Further analysis was conducted to find out if there are significant difference between the two clusters of the year and it was found that the result of the test showed significant different between the clusters.

Bayesian analysis on high or low (i.e. respectively above or below a threshold of $120kWhm^{-2}$ hourly solar irradiation) for each month with given prior beta distribution converged to posterior beta distribution after 20 iteration with average mean of 0.86. This shows that the on average the solar irradiation patterns in Kumasi tends to be high frequently. Also the prior variance of the various months of the year converged to the posterior tolerance level of 0.000001

Comparing the sunshine output of the two clusters indicated that cluster one have the highest hourly solar irradiation output than that of cluster two. The Bayesian analysis of the clusters also confirms that the first cluster have a higher possibility (0.8600) of solar irradiation output as compared with the second cluster (0.76000).

# CHAPTER ONE

# INTRODUCTION

## 1.1 BACKGROUND

The mobilization of adequate national financial resources for the planning and development of the local solar energy resource depends on the availability of solar radiation data which could be used to evaluate available resources and to assess the probable long term performance of systems and hence their economic viability.

The solar radiation received at the earth's surface is subject to daily, seasonal and annual variations and hence many years of observation (perhaps at least 20 years) must be acquired in order to obtain a fairly accurate estimate of long term availability and distribution. However many locations in the developing countries do not have the facilities for continuous and accurate measurements of solar radiation and it is then necessary to use empirical methods which are based on easily measured meteorological parameters such as temperature, relative humidity, rainfall, cloudiness and duration of bright sunshine.

Many such formulae have been documented in the literature (Knight et al, 1991) although the most widely used correlation and perhaps the simplest, is the Angstrom (B) linear regression equation as modified by Page (1964) and others. This correlation relates the monthly average, daily global irradiation on the horizontal to the relative duration of sunshine, and it has been applied to a variety of climates including tropical locations. Except for the recent work of Neba-Fabs et al., (1988) and of Exel(1978) nearly all the work done for locations in the West Africa sub-region and other tropical locations have sought to determine a single regression equation which could be used for all months and hence all seasons of the year. The results of Eze and Ododo (1988) and of other researchers however tend to indicate that the Angstom-Page correlation coefficients depend on both the local climate and the season. Furthermore, it is anticipated that more accurate estimates of monthly average global irradiation would be obtained from correlations for particular months.



Liu and Jordan (1963) as well as Bendt et al (1981) conducted extensive statistical analysis of daily global irradiation on the horizontal particularly investigating possible variations of the frequency distribution with both location and season. Their results showed that frequency distribution of daily global irradiation on the horizontal for the monthly period corresponding to a specified value of a monthly mean clearness index, is almost independent of the location and the time of the year. Bendt *et al* (1981), moreover, went ahead and showed that the generalized cumulative distribution function may be obtained from a probability density function which assumed among others random daily insolation sequences.

In this study, we shall not dwell on the regression methods used to estimate the monthly global averages which, in any case, has already comprehensively been dealt with by Jackson et al (1990) but rather, taking advantage of the currently abundant data on solar irradiation data for Kumasi, we undertake the determination of the pertinent probability density curves based on randomly selected samples in respect of monthly or seasonal variations.

## 1.2 STATEMENT OF THE PROBLEM

There is the need to investigate the variability or otherwise of solar irradiation in terms of daily monthly seasonal or annual distributions. For example, one may need to determine or test whether there is a significant change from month to month or whether climatological conditions will affect the probability density functions. For purposes of applications knowledge of how universal are the probability distribution characterizations for a particular location is very important.



## 1.3 OBJECTIVES

The major objectives of the study are as follows:

1. To determine the pertinent standard probability distributions functions in respect of the daily solar irradiation in Kumasi
2. To investigate the existence of significant differences in the pattern of solar irradiation within the year
3. To undertake a Bayesian analysis of the frequency of high solar irradiation.

## 1.4 JUSTIFICATION OF STUDY

The knowledge of the probability distribution function of solar irradiation will assist engineers to design systems based on months or seasons that have similar solar irradiation patterns.

## 1.5 METHODOLOGY

### 1.5.1 Nature and Source of data

Data on hourly solar irradiation in Kumasi was collected from the Solar Energy Laboratory of the Mechanical Engineering department of KNUST, Kumasi. The irradiation data which was measured in kilowatt hours per meter squared was collected by means of a pyranometer. The data consists of fourteen years of hourly solar irradiation data from 1995-2008.

### 1.5.2 Methods of data analysis

Analysis was conducted on the daily solar radiation data obtained in the past years to gain much insight of the data to constructively solve the problems as stated. The statistical methods adopted included

1. Descriptive statistics.
2. Curve fitting in respect of the various months of the year.
3. Inferential statistical analysis.
4. Bayesian analysis.

### 1.5.1 Computational tools

The various computational tools used included

1. Visual Basic
2. Microsoft Excel
3. INSTAT climatic software
4. SPSS

## 1.6 ORGANISATION OF THE STUDY

The thesis consists of four chapters .The first chapter comprises the introduction, background and methodology.

Chapter two seeks to review the relevant literature on the theory of statistics and probability as well as some relevant research work done on the statistical analysis of solar irradiation

In chapter three we analyze the data and present the results of the analysis.

Finally in chapter four conclusions and recommendation are drawn based on the findings.

# CHAPTER TWO

# LITERATURE REVIEW

Knowledge of the solar irradiation climate of an area is of paramount importance in assessing the potential use of a solar energy system, converted into either thermal or electrical energy, as a power source in that area. Such information is a prerequisite for the design of such solar energy conversion system.



## 2.1 DEFINITION OF TERMS

Sunshine arrives on the earth as a type of energy called radiation. Radiation is composed of millions of high energy particles called photons. This energy is easily converted into heat energy (objects placed in direct sunshine gain energy and become hot). It can also be converted into stored chemical energy (as plants do through photosynthesis) or it can be converted into electricity using solar cells.



### 2.1.1 Solar Irradiance

Solar irradiance refers to the solar radiation actually striking a surface, or the power received per unit area from the sun. This is measured in watts or kilowatts per square meter ($W/m^2$ or $kW/m^2$). If the solar module is facing the sun directly, irradiance will be much higher than if the module is at a large angle to the sun.

### 2.1.2 Insolation

Insolation (incident solar radiation) is a measure of the solar energy received on a specified area over a specified period of time. It is normally measured in kilowatt hours per square meter per day (kWh/ $m^2$ per day) or peak sun hours per day.It measures the amount of radiant energy collected at a site.

### 2.1.3 Pyranometer



It is the device used to record the sunlight data. It is made up of solar cell modules which harvest energy from the sun. The output of the solar cell modules depends on the amount of sunlight (or solar radiation) falling on them and it is affected by seasonal and daily solar radiation changes. It also changes depending on how cloudy or dusty the site is. It records two types of radiations: global average and diffused average.

The global average radiation is the hourly average irradiance of the direct solar energy reaching the earth's surface and the diffused average radiation is as a result of the direct solar energy being blocked by a black ring, clouds or dust before reaching the solar cell modules. However, this research work makes use of the global average radiation which is useful in the production of solar energy.

Clouds and dust absorb and scatter radiation, reducing the amount that reaches the ground. On a sunny day most of the radiation is direct, but on a cloudy day, up to100% of the radiation is diffuse.

## 2.2 REVIEW OF RELEVANT SOLAR IRRADIATION RESEARCH

Jackson *et al* studied Angstrom - Page type correlation between monthly average, daily global irradiation on the horizontal and the monthly average relative duration of sunshine has been derived for Kumasi, Ghana, using 20 years data of measured values by the Ghana Meteorological Services Department. The correlations were carried out at three levels: for each month of the year; for the dry, wet and harmattan seasons and for the full year. At the three levels, the maximum absolute errors in the estimated global irradiation are 0.59%, 4.24% and 8.34% respectively. The analyses were repeated after incorporating multiple reflections between the ground and sky and non-burning of the sunshine recorder chart when the solar elevation was below 4°. These latter considerations did not significantly affect their results.

The frequency distribution of insolation values for any location is essential for predicting long term performance of solar energy systems. Liu and Jordan (1963) first conducted statistical analysis of daily global irradiation on the horizontal for 27 locations. In approximately 5 years of data collection they produced cumulative distribution curves which were considered to have universal validity. Bendt et al (1981) conducted a study on the frequency distribution of daily insolation. In their study they observed that the data used by Liu and Jordan (1963) were limited both in location and duration of observation and they repeated the analysis using extensive data from 90 stations each for approximately 20 years of observation. They particularly investigated possible variations of the frequency distribution with both location and season. Their results truly confirmed Liu and Jordan's observation, showing that frequency distribution of daily global irradiation on the horizontal for the monthly period corresponding to a specified value of a monthly mean clearness index, is almost independent of the location and the time of the year. Bendt *et al* (1981), moreover, went ahead and showed that the generalized cumulative distribution function may be obtained from a probability density function which assumed among others random daily insolation sequences.

An obvious consequence of the generalized cumulative distribution curves is that the maximum and the minimum values of the clearness index are also independent of location and season Bendt *et al* (1981) suggested a constant value for the Kmin = 0.05 which corresponds to overcast sky conditions. Hollans and Huget published a report in 1983 titled 'a probability density function for the clearness index with application. In their paper they proposed that an expansion for the maximum value (kmax) depends on only the monthly mean clearness index.

$$K_{max} = .6313k' - 11.9(k' - .75)^8$$

Saunier *et al (1987)* noted that the earlier analysis of the Liu and Jordan (1963) and also Bendt *et al* (1981) had used irradiation data from North America and they conducted similar analysis using 5years of data for Bangkok, Thailand. Their results disagreed with the generalized CDC as proposed by Bendt *et al* (1981), and also proposed by Hollands and Hugets (1983) for the generalized CDC. Saunier *et al* (1987) observed that for other locations in Thailand as well as India, and they concluded that the generalized CDC may be invalid for tropical locations. However, satisfactory agreement was obtained when suitable values of Kmax for Bangkok were used by the Bendt et al (1981). Consequently they proposed an expression for Kmax, based on observations in Thailand, to be used with the Bendt et al. CDC:

$K_{max}$= 0.362 +0.59K'............................................. (2)

They also suggested that the Kmax defined by eqn(2) might be suitable for other tropical locations. Following the method used by Bendt *et al* (1981), Saunier *et al* (1987) also derived a higher order probability density of function for the daily global irradiation which provided better agreement with observation, as compared with the model by Bendt *et al* (1981). Again they suggested that their model may be valid for both tropical and temperate climates.



Saunier *et al* (1987) conclusion concerning the validity of the generalized CDC for tropical locations has been corroborated by the analysis of from other tropical locations. Feuillard *et al* (1989) noted that the CDC for tropical locations exhibits "a more pronounced S-shape" than that for temperate climates as a consequence of more pronounced peaks in the corresponding probability density functions. They also proposed a new probability density function and suggested that Kmax be calculated from the annual correlation equation of daily clearness index with relative sunshine. This proposal is not in agreement with general observation, which clearly indicates that Kmax varies and depends on the season.

The work of Olseth and Skartveit (1984) has also shown that the disagreement with the generalized CDC is not only confined to tropical climates. They observed that universal values of the maximum daily clearness index, as suggested by the generalized CDC are invalid; rather, the maximum daily clearness index is climatologically dependent.

Gordon and Reddy (1988) proposed an expression for the probability density function of daily global irradiation which they claim had universal validity. As noted by Knight *et al* (1991) this claim needs investigated further.

Akuffo and Hammond (1993) presented the results on statistical analysis of daily global irradiation for Kumasi, Ghana, also located in the tropics. Their study objective was to obtain the CDC and compare their findings with the generalized curves. Their work was to ascertain the validity of generalized curves for Ghana's conditions and consequently the applicability of available design procedures which are based on these curves. In addition, they compared their results with the model of Saunier et al and with the observations of IDeriah and Suleman (1989) for Ibadan, which is also located in the West Africa sub region and has similar agro-climatic conditions as pertains in Kumasi. The work by Akuffo and Hammond in 1993 demonstrated that Ibadan and Kumasi are all in the tropics though Kumasi had cloudier conditions. Again cumulative frequency curves for Kumasi differed from that of Ibadan in cases where there were discrepancies in Kmax and Kmin.

F. Youcef Ettoumi and *et al* (2002) published a paper on Statistical analysis of solar measurements in Algeria using beta distributions. In their paper a method of smoothing solar data by beta probability distributions was implemented. The method was used to process daily sunshine duration data recorded at thirty-three meteorological stations in Algeria for eleven year periods or more. Secondly the method has been applied to hourly global solar irradiation flux measured in Algiers during the 1987/89 period. For each location and each month of the year, beta probability density functions fitting the monthly frequency distributions of the daily sunshine duration measurements are obtained. Both the parameters characterising the resulting beta distributions are then mapped, enabling them to build the frequency distributions of sunshine duration for every site in Algeria. In the case of solar radiation for Algiers, the recorded data have been processed following two different ways. The first one consists in sorting the hourly global solar irradiation data into eight typical classes of the daily clearness index. The second one is based on the repartition of these data per month. The results of the first classification show that for each class of daily clearness index, the hourly data under consideration are modeled by only one beta distribution. When using the second classification, linear combinations of two beta distributions are found to fit the monthly frequency distributions of the hourly solar radiation data.

## 2.3 REVIEW OF PROBABILITY THEORY

A Probability Space is a triple $(\Omega, F, P)$ consisting of a set $\Omega$ called the sample space, a $\sigma$-algebra $F$ consisting of subsets of $\Omega$ (these subsets are called events) and a measure $P$ (called probability measure) on $(\Omega, F)$ such that $P(\Omega) = 1$ called the probability measure.

### 2.3.1 Random Variables

Let $(\Omega, F, P)$ be a probability space and $(Y, G)$ be a measurable space then the random variable $X$ is defined as a measurable function $X : \Omega \to Y$. Where $Y \subseteq \Re$ and G is a $\sigma$-algebra of $Y$ consisting of subsets of $Y$

A random variable $X$ is said to be continuous if there exists a function $f$, called the probability density function or (probability distribution function (pdf)) of X and a function F called the cumulative distribution function (cdf), such that $F(x_2)-F(x_1)=\int_{x_1}^{x_2} f(x)dx = P\{x_1 \le X \le x_2\}$

A random variable $X$ will be called discrete if there exists a finite or countable set U of real numbers with $U=\{x_1, x_2, \ldots x_n\}$ such that $P\{U\}=\sum_{x_i \in U} P(X=x_i)=1$. The probability distribution function (pdf) of a discrete random variable X and the cumulative distribution function (cdf), F are given by $f(x_i)=P(X=x_i)$ and $F(x_n)=\sum_{k=0}^{n} f(x_k)$



If $X$ is a random variable such that $f(x)$ is the probability distribution function then the expectation $E$ and the variance $Var$ of $X$ are given as

$$E(X)=\begin{cases}\sum_x xf(x) \text{ if X is discrete}\\ \int_{\infty}^{\infty} xf(x)dx \text{ if X is continious}\end{cases}$$

$$Var(X)=\begin{cases}\sum_x (x-E(X))^2 f(x) \text{ if X is discrete}\\ \int_{\infty}^{\infty} (x-E(X))^2 f(x)dx \text{ if X is continious}\end{cases}$$

### 2.3.3 Properties of cdf of a Continuous Random Variable

1. $F(X)$ is a monotonically non decreasing function.

2. $\lim_{x\to-\infty} F(x)=0$

3. $\lim_{x\to\infty} F(x)=1$

If $F(X)$ is absolutely continuous if its derivative exists. Integrating the derivative gives back the $(cdf)$ again. The random variable $X$ is then said to have a probability density function (pdf).

$$f(x) = \frac{\partial F(x)}{\partial x}.$$

For a set $E \subseteq \Re$, the probability of the random variable $X$ being in $E$ is

$$P(X \in E) = \int_{x\in E} dF(x) = \int_{x\in E} f(x)dx$$



### 2.3.3 Partition and total probability rule

#### 2.3.3.1 Definition

A class of events $H = \{H_1, H_2, ....\}$ forms a partition of the sample space $\Omega$ if the event excludes one another and one of them must occur. That means that

$$H_i \cap H_j = \phi \ \forall i, j \text{ and}$$
$$H_1 \cup H_2 \cup .... \cup = \Omega$$



A partition is called finite or countable if it contains a finite or countable many infinite number of events.

#### 2.3.3.2 Definition

Given two partitions $H = \{H_1, H_2, ....\}$ and $K = \{K_1, K_2, ....\}$ we say that $H$ is finer than K or $H$ is refinement of K if for every $H_i$ in $H$ there exist an event $K_j$ such that $H_i \subset K_j$.

#### 2.3.3.3 Theorem

Let $H=\{H_j, j=1,2,....\}$ be a positive finite or infinite partition and let A be an arbitrary event. Then

$$P(A)=P(A|H_1)P(H_1)+P(A|H_2)P(H_2)+.....+P(A|H_n)P(H_n)$$

in the case of partition $H$ into a finite number $n$ of sets.

And

$$P(A)=P(A|H_1)P(H_1)+P(A|H_2)P(H_2)+.....$$

in the case of partition $H$ into a countable number of set.

#### 2.3.3.4 Conditional Probability

Let $H=\{H_1,H_2,....\}$ be a positive partition of the sample space $\Omega$ and let $A,B$ be two event with $P(B)>0$, then $P(A|B)=\sum_{j=1}P(A|B\cap H_j)P(H_j|B)$

**Proof**

Using the expression on the right hand side of the above equation it can be deduced that

$$\sum_{j=1}P(A|B\cap H_j)P(H_j|B)=\sum_{j=1}\frac{P(A\cap B\cap H_j)}{P(B\cap H_j)}\times\frac{P(B\cap H_j)}{P(B)}$$

$$\frac{1}{P(B)}\sum_{j=1}P(A\cap B\cap H_j)=\frac{1}{P(B)}P(A\cap B)\cap\bigcup_{j=1}H_j=\frac{P(A\cap B)}{P(B)}=P(A|B)$$

#### 2.3.3.5 Bayes' Formula

We shall now consider a question opposite to the total probability formula. Given that the event occurred what is the probability of the event. of the partition ? The answer is contained is contained in the theorem below.

#### 2.3.3.6 Theorem (Bayes' formula)

*Let* $H=\{H_1,H_2,....\}$ be a positive partition of $\Omega$ and $A$ be any event with $P(A)>0$ Then for any event $H_k$ of the partition $H$ we have



$$P(H_k|A)=\frac{P(A|H_k)P(H_k)}{P(A|H_1)P(H_1)+P(A|H_2)P(H_2)+...+P(A|H_n)P(H_n)}$$

## 2.4 EXAMPLES OF CONTINUOUS PROBABLILITY DISTRIBUTIONS

### 2.4.1 The Uniform Distribution

The random variable X is uniformly distributed over the interval, [a, b] if it has probability density function defined as:

$$f(X)=\begin{cases}\frac{1}{a+b}, & a<X<b\\ 0 & \text{otherwise}\end{cases}$$

So that the mean and variance are:

$$E(X)=\frac{a+b}{2} \text{ and } Var(X)=\frac{(b-a)^2}{12}$$

The uniform distribution provides a simple probability model to describe a continuous random variable that can randomly assume any value between two points a and b (a < b ) on a line. The uniform probability density function has a rectangular shape over the interval [a, b] with height $\frac{1}{b-a}$ so that the area under the density function equal to 1. Thus, the uniform distribution provides a good model for a continuous random variable whose values are uniformly distributed over an interval. For example:



1. If buses arrive at a given bus stop over 15 minutes and you arrive at the bus stop at a random time, the time you must wait for the next bus to arrive could be describe by the uniform distribution over the interval from 0 to 15.
2. In a milling operation, pieces of lumber less than 1 meter length are considered scrap. The distribution of the lengths of scrap lumber would have a uniform distribution on the interval from 0 to 1.



**2.4.2 The Exponential Distribution**

The exponential probability distribution for the continuous random variable, X which represents an interval of time or space is defined as:

$$f(X)=\begin{cases}\theta e^{-\theta x} & x>0\\ 0 & \text{otherwise}\end{cases}$$

Where, the parameter $\theta > 0$ is the mean number of events that occur in the given unit of time or space. The mean value of X (that is, the mean length of time or space between successive occurrences of the event and the variance are:

$$E(X) = \frac{1}{\theta} \qquad Var(X) = \frac{1}{\theta^2}$$

Typical situations resulting in this random variable are:

1. The length of time until a machine or a component of it fails.
2. The length of time between arrivals at a car wash.
3. The length of time in a service line or queue.
4. The length of time between successive filing of claims in an insurance office.

From the above examples, the exponential distribution models situations in which the random variable represents waiting time or measurements of length of time between successive occurrences of an event. It plays an important role in Reliability Theory where we try to find the reliability of a system at time t.



### 2.4.3 The Normal Distribution.

The probability density function for the normal random variable, $X$ which is simply called normal distribution is defined by:

$$f(X) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(X-\mu)}{\sigma^2}\right)}, & -\infty \le X \le \infty \\ 0 & \text{otherwise} \end{cases}$$

$\sigma > 0$, $-\infty < \mu < \infty$ and the mean and variance of the measurements, x are;

$E(X) = \mu$ and $Var(X) = \sigma^2$

If the random variable is modeled by the Normal distribution with mean μ and variance, $\sigma^2$ then it is simply denoted as $x \sqcup N(\mu, \sigma^2)$. The graph of a normal distribution is a bell-shaped smooth curve.

The Normal distribution is one of the most widely used probability distributions for modeling random experiments. It provides a good model for continuous random variables involving measurements such as time, heights/ weight of persons, marks scored in an examination, amount of rainfall, growth rate and many other scientific measurements.



The normal curve has the following desirable properties, accounting for the wide-spread applications of the Normal distribution.

1. The normal curve is symmetrical about its mean, μ.
2. The mean, median and mode of $x$ are the same.
3. The total area under the normal curve is equal to 1.
4. The probability distribution of the normal random variable, $x$ is completely determined by its two parameters μ and $\sigma$.
5. The curve is asymptotic to its horizontal axis, $x$.
6. The probabilities of the normal random variable, $x$ are given by the areas under the curve.

### 2.4.4 The Weibull Distribution

The continuous random variable $X$ has a Weibull distribution, with parameters $\alpha$ and $\beta$ if its density function is given by

$$f(X;\alpha,\beta)=\begin{cases}\alpha\beta X^{\beta-1}e^{-\alpha X^{\beta}} & ,X>0\\ 0, elsewhere\end{cases}$$

where $\alpha>0$ and $\beta>0$.

The mean and variance of the Weibull distribution are given by



$$\mu = \alpha^{-1/\beta}\,\Gamma\left(1+\frac{1}{\beta}\right) \text{ and } \sigma^2=\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\}$$

Modern technology has enabled us to design many complicated systems whose operation, or perhaps safety, depends on the reliability of various components. The Weibull distribution has been used extensively in recent times to deal with such problems. This is applied to reliability and life-testing problems such as time to failure or life length of a component, measured from specific time until it fails.

### 2.4.5 The Gamma Distribution

The continuous random variable $X$ has a gamma distribution, with parameter $\alpha$ and $\beta$, if its density function is given by

$$f(X;\alpha,\beta)=\begin{cases}\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}X^{\alpha-1}e^{-x/\beta} & ,X>0\\ 0, elsewhere\end{cases}$$

Where, $\alpha > 0$ and $\beta > 0$.

The gamma distribution for which $\alpha = 1$ is the exponential distribution. The mean and variance of the gamma distribution are

$$\mu = \alpha\beta \quad \text{and} \quad \sigma^2 = \alpha\beta^2$$

The exponential thus turns out to be a special case of the gamma distribution. The exponential gamma distribution plays an important role in both queuing theory and reliability problems. Time between arrivals at service facilities, and time to failure of component parts and electrical systems, often is nicely modeled by the exponential distribution. The relationship between the gamma and exponential allows the gamma to be involved in similar types of problems



### 2.4.6 Lognormal Distribution

The continuous random variable X has a lognormal distribution if the random variable Y=ln(X) has normal distribution with mean $\mu$ and standard deviation $\sigma$. The resulting density function of X is

$$f(X;\mu,\sigma) = \begin{cases} \dfrac{1}{\sqrt{2\Pi\sigma X}} e^{-\frac{1}{2\sigma^2}[\ln(X)-\mu]^2}, x \geq 0 \\ 0, elsewhere, X < 0 \end{cases}$$

The mean and variance of the lognormal distribution are

$$\mu = e^{\mu + \frac{\sigma^2}{2}} \text{ and } \sigma^2 = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$$

The lognormal distribution is used for a wide variety of applications. The distribution applies is cases where a natural log transformation results in a normal distribution. The cumulative distribution function is quite simple due to its relationship to the normal distribution

### 2.4.7 Beta Distribution

The family of distribution most commonly used to model researchers' uncertainty about the unknown probability ***p*** of some event is the family of beta distributions, defined as follows: A random variable $X$ is said to have a beta distribution with parameters $\alpha > 0$ $\beta > 0$ if the density of $X$ is $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ for $0 \le x \le 1$ and $f(x) = 0$ outside the interval $[0,1]$

The mean and the variance of the beta distribution are given below

*Mean* denoted $E(X) = \mu = \frac{\alpha}{\alpha+\beta}$ and variance denoted $\mathrm{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

## 2.5.0 METHODS OF PARAMETER ESTIMATION

### 2.5.1 The Maximum Likelihood Estimator

The Maximum likelihood estimator represents one of the most important approaches to estimation in all statistical inference. The method of maximum likelihood is that for which the likelihood function is maximized. The likelihood function is best illustrated through the use of a discrete distribution and a single parameter.

Let $x_1, x_2, \ldots, x_n$ denote independent random variables taken from a discrete probability distribution represented by $f(x,\theta)$, where $\theta$ is a single parameter of the distribution. $L(x_1,x_2,\ldots,x_n;\theta)=f(x_1,x_2,\ldots,x_n;\theta)= f(x_1,\theta)f(x_2,\theta)\ldots,f(x_n,\theta)$ is the joint distribution of the random variables. The quantity $L(x_1,x_2,\ldots,x_n;\theta)$, the likelihood of the sample, is the joint probability: $P(X_1=x_1,X_2=x_2,\ldots,X_n=x_n;\theta)$, which is the probability of obtaining the sample values $x_1,x_2,\ldots,x_n$. For the discrete case the maximum likelihood estimator is one that results in a maximum value for this joint probability or maximizes the likelihood of the sample.

Although the interpretation of the likelihood function as a joint probability is confined to the discrete case, the notion of the maximum likelihood extends to the estimation of parameters of a continuous distribution. The maximum likelihood estimation is as follows:

Given independent observation $x_1,x_2,\ldots x_n$ from a probability density function (continuous) or probability mass function (discrete case) $f(x,\theta)$, the maximum likelihood estimator $\hat{\theta}$, is that which maximizes the likelihood function $L(x_1, x_2,....x_n;\theta) = f(x_1,\theta) f(x_2,\theta)...f(x_n,\theta)$.

$$L(\theta) = f(x_1,\theta).f(x_2,\theta).....f(x_n,\theta) = \prod_{i=1}^{n} f(x_i,\theta) \ ,\theta \in \Omega$$

$$\frac{\partial \ln L(\theta)}{\partial \theta_1} = 0, \frac{\partial \ln L(\theta)}{\partial \theta_2} = 0,..., \frac{\partial \ln L(\theta)}{\partial \theta_k} = 0.$$

Solving the resulting system of equation will give

$$\hat{\theta} = h(x_1, x_2,..., x_n)$$



### 2.5.1.1 Properties of Maximum Likelihood Estimation

### 2.5.1.2 Unbiasedness

Let $X_1, X_2,....X_n$ be a random sample from a distribution with probability distribution $f(X,\theta)$.An estimator $\hat{\theta} = h(x_1, x_2,...., x_n)$ is said to be unbiased for $\theta$ if $E(\hat{\theta}) = \theta$

### 2.5.1.3 Efficiency

Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be two unbiased estimators for $\theta$ with variance $Var(\hat{\theta}_1)$ and $Var(\hat{\theta}_2)$ respectively, $\hat{\theta}_1$ is said to be relatively more efficient than $\hat{\theta}_2$ if $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$

#### 2.5.1.4 Definition

An unbiased estimator, $\hat{\theta} = h(x_1, x_2, ..., x_n)$ is said to be efficient if and only if

$Var(\hat{\theta}) = \frac{1}{nE\left[\left(\frac{\partial \ln f(X,\theta)}{\partial\theta}\right)^2\right]}$, Thus the efficient estimator for $\theta$ is the minimum variance unbiased estimator whose variance attains the Crammer-Rao Lower bound.

#### 2.5.1.5 Consistency:

Let $\hat{\theta} = h(x_1, x_2, ..., x_n)$ be an estimate of the parameter $\theta$. The estimator $\hat{\theta}$ is said to be consistent for $\theta$ if $\lim_{n\to\infty} P\{|\theta - \theta| > \varepsilon\} = 0$ OR $\lim_{x\to\infty} P\{|\theta - \theta| \le \varepsilon\} = 1$ $\forall\theta$ and $\varepsilon > 0$

#### 2.5.1.6 Theorem:

Let $\hat{\theta} = h(x_1, x_2, ..., x_n)$ be an estimate of the parameter $\theta$ based on sample of size $n$ .if $\lim_{n\to\infty} E(\hat{\theta}) = 0$ and $\lim_{n\to\infty} Var(\hat{\theta}) = 0$ then $\hat{\theta}$ is a consistent estimator for $\theta$

## 2.6 LEAST SQUARE ESTIMATES

For any given set of data, our problem or main objective is to draw the particular straight line that best reflects the linear trend indicated by the points. In other words, the problem is simply that of determining appropriate constants $a$ and $b$ associated with the straight line. The principle involved in obtaining "the line of best fit" is called the method of least squares. Suppose $\hat{y} = a + bx$ is the line that best fits the data consisting of $n$ pairs of values $(x_1, y_1), (x_2, y_2), \ldots\ldots, (x_n, y_n)$, this is called the prediction equation.

For instance $\hat{y}_i = a + bx_i$ represents the predicted value of dependent variable when the value of the independent variable is $x_i$. $y_i$ is the corresponding observed $y$-value in the data. The quantities $y_1 - \hat{y}_1, y_2 - \hat{y}_2 \ldots y_n - \hat{y}_n$, that is $y_1 - (a + bx_1), y_2 - (a + bx_2) \ldots y_n - (a + bx_n)$, give deviations of the observed y- values and each is called the residual or error denoted $e$. Thus

$$e_i = y - \hat{y}_i .$$



The principle of the method of least squares is stipulated as follows. Of all possible straight lines that can be drawn on a scatter diagram, choose as the line of best fit the one for which the sum of the squares of the deviations of the observed $y$-*values* from the predicted $y$-*values* is a minimum. That is determining the constants $a$ and $b$ in such a way that $\sum e_i^2 = \sum [y_i - (a + bx_i)]^2$ is the minimum.

The straight line obtained using this criterion is called the least squares regression line. The line of $a$ and $b$ that determine the line of best fit can be obtained from the following formulas. The least squares formulas for the slope of best fit is given by

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$



And its y-intercept by

$$a = \overline{y} - b\overline{x}$$

Where n is the number of pairs of observations in the data, $\overline{y}$ is the mean of the observed $y$-values and $\overline{x}$ is the mean of the $x$-*values*.

## 2.7 PARAMETRIC HYPOTHESIS TESTING

In some practical problem of statistical inference we may be required to take decision concerning the parameters of the population instead of finding estimates for them. The following are some situations requiring such decisions:

1. A health personnel may claim may a drug is effective in 90% cases it is administered.
2. The mean life span of a type of an electric bulb is at least 8,000 hours.
3. An accused, in a criminal trial, is always assumed to be innocent until proven otherwise.
4. An educationist may claim that two methods of teaching are equally effective.
5. An educational programme will result in improved communication between parents and children.
6. A medical researcher may hypothesis that a new drug is more effective that another one in curing a disease.

The above statements can be subjected to statistical verification using the sample observations.

Thus by means of hypothesis testing we are able to determine whether or not the statements are consistent with available data or evidence.

### 2.7.1 Definitions

1. A hypothesis is a statement, assertion or conjecture about the nature of one or more situations (or populations) to be studied.
2. Hypothesis Testing is a statistical procedure that uses a random sample data to determine whether a statement about a population should be rejected or not. Hypothesis testing involving population parameters are called Parametric or Classical tests.



### 2.7.2 Types of Hypothesis

In testing for the validity of a hypothesis we usually propose two types of hypotheses, namely;

1. Null hypothesis, denoted $H_o$, which is the tentative statement assumed to be true.
2. Alternative hypothesis denoted $H_1$, which contradicts the null hypothesis. It is accepted only when sufficient evidence exists to establish its truth.



### 2.7.3 Formulation of $H_0$ and $H_1$.

When we wish to establish the validity of a statement about a population using the evidence obtained from a random sample data, the negative of the statement is what we take as the null hypothesis. The statement itself constitutes the alternative hypothesis. In some applications, it is not obvious how $H_0$ and $H_1$ should be formulated. The following guidelines for developing hypotheses of three types of situations are suggested:

1. Testing Research Hypothesis: This is formulated as alternative hypothesis.
2. Testing the validity of a claim: This generally corresponds to the "*innocent until proven guilty*" analogy. The claim made is chosen as the null hypothesis while the challenge to the claim is taken as the alternative hypothesis.
3. Testing in decision-making situations: This occurs when the decision-maker must choose between two courses of action, one associated with $H_0$ and the other with $H_1$. If example, the decision involves the population parameter, $\theta$ we should have the two hypotheses formulated as: $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$,

where $\theta_0$ is a particular value of $\theta$ and an instant action is taken if $H_0$ is rejected.

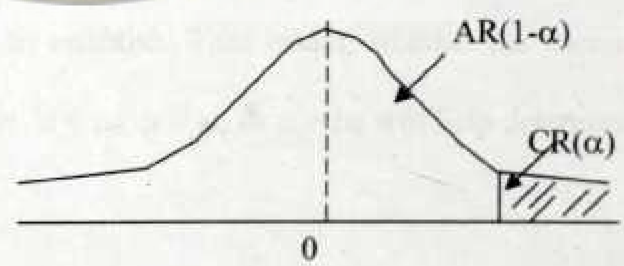
### 2.7.4 Forms of Tests

In general a hypothesis testing involving a parameter, say $\mu$ takes one of the following forms:

(i) *One-Tailed test* to the right which is formulated as:

$H_0: \mu \leq \mu_0$

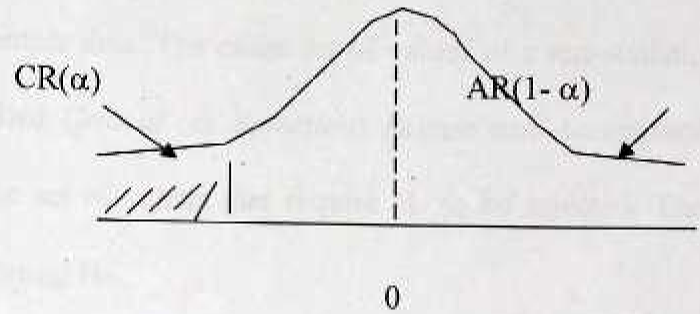
$H_1: \mu > \mu_0$



(ii) *One-tailed test* to the left, formulated as:

$H_o$: $\mu \geq \mu_o$

$H_0$: $\mu < \mu_0$

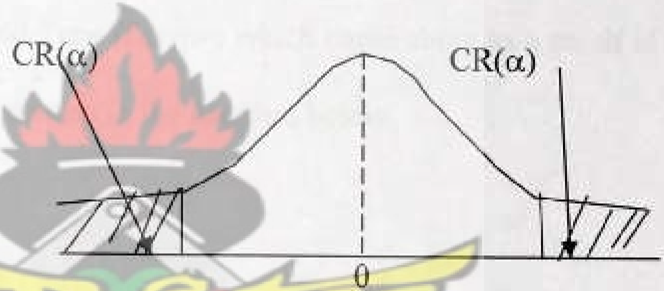



(iii) *Two-tailed Test*, which takes the form of choice between two courses of action.



$H_0$: $\mu = \mu_0$

$H_1$: $\mu \neq \mu_0$




(iv) *Remark*: In many situations, the choice of $H_0$ and $H_1$ is not obvious. In such cases, judgment on the part of the user is needed to select the proper form of $H_0$ and $H_1$. However, as the three forms show, the equality part of expression (either≥, ≤ or =) always appear in the null hypothesis. In selecting the proper form of $H_0$ and $H_1$ keep in mind that the alternative hypothesis is what the sampling study is attempting to establish. Thus asking whether the user is looking for evidence to support $\mu < \mu_0$, $\mu > \mu_0$ or $\mu \neq \mu_0$ will help determine $H_1$.

### 2.7.5 Test-Statistic

The test-statistic is a decision rule which leads to a rejection or acceptance of $H_0$. Its value is obtained from the randomly selected sample data. The entire set of values of a test-statistic is divided into two sets or regions called *Critical* (or *Rejection*) *Region* and *Acceptance Region*. The critical region contains the set of values that require $H_0$ to be rejected. The acceptance region contains values supporting $H_0$.

### 2.7.6 Errors in Hypothesis Testing



When $H_0$ is tested against $H_1$ using a randomly selected sample data, two possible errors may be committed. These are the *Types I* and *Type II errors* which come about as a result of the decisions which are taken. These are illustrated in the diagram below.

| Decision | Actual Situation | |
|---|---|---|
| | $H_0$ is true | $H_0$ is false ($H_1$ is true) |
| Accept $H_0$ | Correct decision<br>$(1-\alpha)$ | Type II error<br>$(\beta)$ |
| Reject $H_0$ | Type I error<br>$(\alpha)$ | Correct decision<br>$(1-\beta)$ |

(i) Type I error is committed when the null hypothesis, $H_0$ is rejected when in actual situation it is true. The probability of committing this error is $\alpha$, which is also referred to *level of significance* and indicates the size of the critical region.

(ii) Type II error is committed when $H_0$ is accepted when in actual situation it is false. The probability of committing this error is $\beta$.

### 2.7.6 P -Value



The *p-value* is the smallest level of significance for which the observed data would call for rejection of $H_0$ in favour of $H_1$. The *p-value* gives additional insight into the strength of the decision taken. A very small *p-value*, such as 0.0001, indicates that there virtually no likelihood that $H_0$ is true. On the other hand, a high *p-value* such as 0.20 means that $H_0$ is not rejected and there is little likelihood that it is false. The *p-value* is often referred to as the *observed level of significance*. For a given level of significance, $\alpha$, the null hypothesis, $H_0$

1. is rejected if *p-value* $\leq \alpha$.
2. fails to be rejected if *p-value* $> \alpha$.

### 2.7.7 Elements of Statistical Tests

We note from the above discussion that a typical statistical test of hypothesis involves the following elements:

1. The Null and Alternative Hypotheses
2. The Test-Statistic
3. Critical or Rejected Region

### 2.7.8 Tests for the Population Means and Proportions



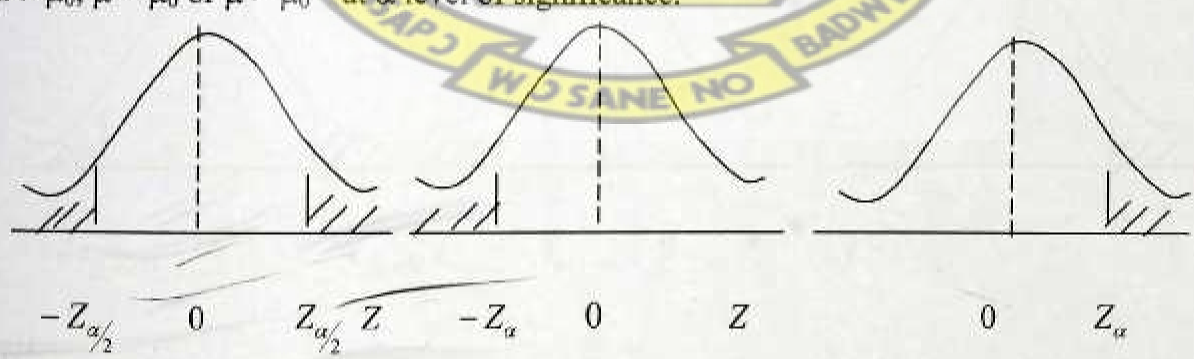
We shall discus the tests for the population mean, $\mu$ and the difference between two population means, $(\mu_1 - \mu_2)$ taking into consideration for each situation the size of the sample(s). In addition, tests for population $P$ and difference between two proportions, $(P_1 - P_2)$ shall be discussed.

### 2.7.9 Testing for the Population Mean, $\mu$

(a) <u>*Case I:*</u> When the sample size, $n$ is large.

(i) The hypotheses are $H_0: \mu = \mu_0, \mu \geq \mu_0$ or $\mu \leq \mu_0$

$H_1: \mu \neq \mu_0, \mu < \mu_0$ or $\mu > \mu_0$ at $\alpha$ level of significance.




$H_1: \mu \neq \mu_0$ $H_1: \mu < \mu_0$ $H_1: \mu > \mu$

(ii) The test-statistic is $Z = \dfrac{\bar{x} - \mu_0}{\sigma\sqrt{n}}$,

where σ is replaced by its estimate s if it is unknown.

(iii) The decision rule is to reject $H_0$ if the test-statistic $(Z)$ falls in the critical region. That is, $H_0$ is rejected if $|Z| \geq Z_{\alpha/2}$, for two-tailed test; or $Z \leq -Z_\alpha$ or $Z \geq Z_\alpha$, for one-tailed test.

(iv) Assumptions: The sample observations are randomly and independently selected from a normally distributed population. The sample size, $n \geq 30$.



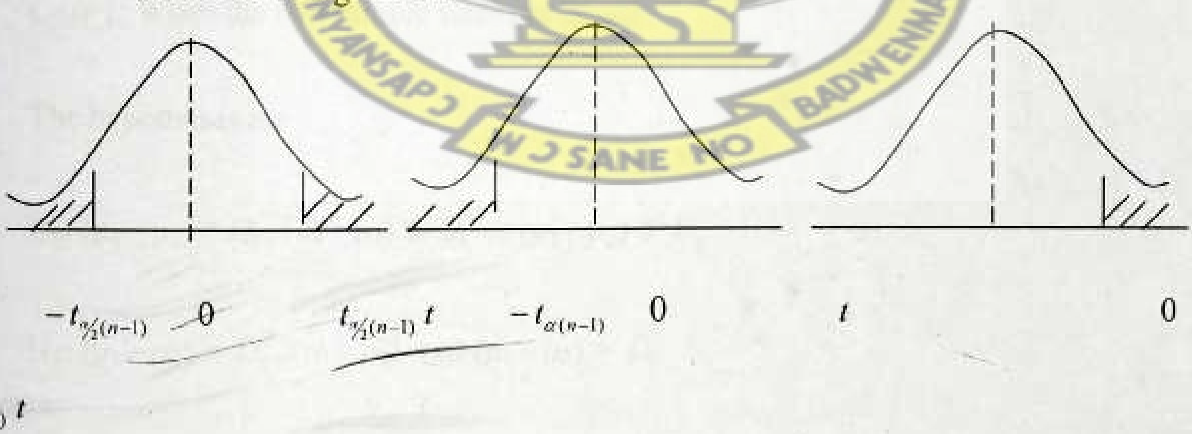
(b) *Case II:* When the sample size, $n$ is small

(i) The hypotheses are

$H_0$: $\mu = \mu_0$, $\mu \geq \mu_0$ or $\mu \leq \mu_0$

$H_1$: $\mu \neq \mu_0$, $\mu < \mu_0$ or $\mu > \mu_0$

at α-level of significance



$H_1$: $\mu \neq \mu_0$ $H_1$: $\mu < \mu_0$ $H_1$: $\mu > \mu_0$

(ii) The test-statistic is: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

which has $t$-distribution with degrees of freedom, $(n-1)$.

(iii) The decision rule is to reject $H_0$ if the test-statistic $(t)$ falls in the critical region. That is, $H_0$ is rejected if $|t| \geq t_{\alpha/2(n-1)}$, for two-tailed test; or $t \leq -t_{\alpha(n-1)}$ or $t \geq t_{\alpha(n-1)}$, for one-tailed test.

(iv) Assumptions: Same as large sample size except that $n < 30$ and $\sigma^2$ is unknown.

### 2.7.10 Testing For The Difference Between Two Population Means, $(\mu_1 - \mu_2)$

(a) *Case I:* When the two sample sizes are large.

(i) The hypotheses are

$H_0$: $(\mu_1 - \mu_2) = D_0$, $(\mu_1 - \mu_2) \geq D_0$ or $(\mu_1 - \mu_2) \leq D_0$

$H_1$: $(\mu_1 - \mu_2) \neq D_0$, $(\mu_1 - \mu_2) <$ or $(\mu_1 - \mu_2) > D_0$

at α - level of significance, where $D_0$ is the difference between the two means.

(ii) The test-statistic is: $Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

where $\sigma_1^2$ *and* $\sigma_2^2$ are replaced by $s_1^2$ *and* $s_2^2$ respectively if they are unknown.

(iii) The decision rule is to reject $H_0$ if the test-statistic $(Z)$ falls in the critical region. That is, $H_0$ is rejected if $|z| \geq Z_{\alpha/2}$, for two-tailed test; or if $Z \leq -Z_\alpha$ or $Z \geq Z_\alpha$, for one-tailed test.

(b) *Case II:* When the same sizes are small.

(i) The hypotheses are

$H_0$: $(\mu_1 - \mu_2) = D_0$, $(\mu_1 - \mu_2) \geq D_0$ or $(\mu_1 - \mu_2) \leq D_0$

$H_1$: $(\mu_1 - \mu_2) \neq D_0$, $(\mu_1 - \mu_2) < D_0$ or $(\mu_1 - \mu_2) > D_0$

at α-level of significance.

(ii) The test-statistic is: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$,

which has t-distribution with degrees of freedom, $(n_1 + n_2 - 2)$ and

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)}{(n_1 + n_2 - 2)},$$

which is the estimate of a common variance assumed for the two populations

(iii) The decision rule is to reject $H_0$ if the test-statistic $(t)$ falls in the critical region. That is, $H_0$ is rejected if $|t| \geq t_{\alpha/2(n_1+n_2-2)}$, for two-tailed test; or $t \leq -t_{\alpha(n_1+n_2-2)}$ or , $t \geq t_{\alpha(n_1+n_2-2)}$ for one-tailed test.



(iii) The assumptions are that the samples are randomly and independently selected from two normally distributed populations whose variances are the same. The sample sizes, $n_1 < 30$ and $n_2 < 30$.

### 2.7.11 Testing for Population Proportions

(a) Test for the population proportion, $P$:

(i) The hypotheses are

$H_0: P = p_0,\ P \geq p_0$ or $P \leq p_0$

$H_1: P \neq p_0,\ P < p_0$ or $P > p_0$

at α-level of significance

(ii) Test-statistic is: $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$,

where $\hat{p} = \frac{x}{n}$ and $n > 30$.

(iii) The decision rule is to reject $H_0$ if the test-statistic $(Z)$ falls in the critical region. That is, $H_0$ is rejected if $|Z| \geq Z_{\alpha/2}$, for two-tailed test; or $Z \leq -Z_\alpha$ or $Z \geq Z_\alpha$, for one-tailed test.



(b) Test for the difference between two Populations Proportions, $(P_1 - P_2)$:

(i) The hypotheses are

$H_0$: $(P_1 - P_2) = D_0$, $(P_1 - P_2) \geq D_0$ or $(P_1 - P_2) \leq D_0$

$H_1$: $(P_1 - P_2) \neq D_0$, $(P_1 - P_2) < D_0$ or $(P_1 - P_2) > D_0$

at α-level of significance



(ii) The test-statistics is: $Z = \frac{(\hat{p}_1 - \hat{p}_2) - D_0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$

If however, $P_1 = P_2 = P$, then $D_0 = 0$ and the test-statistic becomes,

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$, the estimate for the common proportion $P$, $n_1 > 30$ and $n_2 > 30$.

(iii) The decision rule is to reject $H_0$ if the test-statistic $(Z)$ falls in the critical region. That is, $H_0$ is rejected if $|Z| \geq Z_{\alpha/2}$, for two-tailed test; or $Z \leq -Z_{\alpha}$ or $Z \geq Z_{\alpha}$, for one-tailed test.



## 2.8 HISTORY OF BAYESIAN ESTIMATION

Though several key concepts were developed earlier-such as mathematical expectation (Huygens 1657), significance testing (Arbuthnot 1711), and the approximation of the binomial by the normal distribution (de Moivre 1718), many of the earliest statistical methods were developed in the latter part of the nineteenth century. For example, the concept of linear regression first appeared in the work of Galton (1889), building on the concept of least squares introduced by Legendre (1805) and arguably also by Gauss (who claimed that he had been using the method since 1795 (Gauss1809)). Galton (1888) also introduced the concept of statistical correlation, which was later developed by Edgeworth (1893), Yule (1897) and Pearson (1896), who also began the development of goodness-of-fit measures (e.g. Pearson 1900) at around the turn of the century. The field really took off in the 1920s and 1930s when Fisher (1922, 1925) developed the notion of likelihood for general estimation; Neyman & Pearson (1933) developed the basis for frequentist (often termed classical) hypothesis testing; and Yates & Cochran (1938) established the principles for the analysis of variance. Since then, the field has continued to grow and statistical inference now plays a key role in nearly every area of scientific research. Bayesian methods date to the original paper by the Reverend Thomas Bayes, read to the Royal Statistical Society in 1763 (the paper was in fact read by Richard Price several years after Bayes' death). The area generated interest from Laplace (1774), Gauss (1809) and Pearson (Pearson & Filon 1898) amongst others and dominated statistical thinking throughout the nineteenth century. The Bayesian approach fell out of favour at the beginning of the twentieth century due, in part, to the domination of the field by staunch opponents such as Neyman and Fisher, who held philosophical objections to the subjectivity of the Bayesian approach. Nonetheless, Bayesian stalwarts such as Jeffreys (1939), Savage (1954), Lindley (1965) and de Finetti (1970) kept the Bayesian flame alive by continuing to advocate Bayesian methods as remedies for certain deficiencies in the frequentist approach. It was not until the late 1980s that the Bayesian approach began to re-emerge, motivated both by rapid recent developments in computing and by the growing desire to describe increasingly complex scientific phenomena that older sampling theories were ill equipped to address. Since then the Bayesian approach has begun to dominate statistical research once more with new computational tools providing a far more flexible framework for statistical inference matching exactly the increasing complexity of scientific research as we move into the twenty first century.

### 2.8.1 Two Approaches to Statistical Inference

The two competing approaches to statistical inference are perhaps best explained in the context of a simple example. Suppose that we wish to toss a single coin $X$ . Our statistical model places a probability, say $\theta$ on it coming down heads $(X=1)$ and a probability, $1-\theta$ of it coming down tails. $(X=0)$ This is a statistical model parameterized by the unknown probability $\theta$ which simply says that $\Pr ob(X=1)=0$ and $\Pr ob(X=0)=1-\theta$ In fact, we can write down a general formula which covers both cases, i.e. $P(X=x)=\theta^x(1-\theta)^{1-x}=f(x|\theta)$ for $x=0$ and $x=1$



Given a whole series of $n$ coin tosses $X_1$...............$X_n$ we can write down a joint probability for all $n$ outcomes: $f(x_1,x_2,...,x_n|\theta)$, which is read aloud as `the joint probability distribution, $f$, of $x_1$ up to $x_n$ given the value of $\theta$ and explicitly states the dependence of the series of observed coin tosses on the value of the parameter $\theta$. This joint probability distribution then forms the basis for statistical inference, whichever inferential approach is used.

### 2.8.2 The Objective Approach (Frequentist)

Statistical inference is essentially an inversion problem. The probability of an event is the proportion of times the event would occur if the experiment were repeated many times under identical conditions.

Given a parametric model and associated parameter values, it is possible to predict potential outcomes. For example, given the probability distribution above and the value of $\theta$ it would be possible to simulate or predict the outcomes of the coin-tossing experiment. However, in practice, it is the outcome of the experiment that is observed and the values of the parameters that are unknown. For example, we might observe three coin tosses each of which were heads and wish to infer from these observations what the value of $\theta$ (i.e. the probability of a head) might be. In practical terms, this means that the frequentist approach to inference involves the maximization of a function of the model parameters. In some simple cases, this maximization can be performed analytically: by taking derivatives of the likelihood function, for example. However, for more complex examples numerical optimization procedures are required, which treat the likelihood function as a multi-dimensional surface where the height of the surface at any vector point $\theta$ is given by the likelihood at that point. The frequentist approach to this problem relies on the strong repeated sampling principle and assesses the performance of our statistical estimation procedure for a single sample of data on the basis of the expected long-run performance given a hypothetical series of datasets collected under identical conditions.



### 2.8.3 The Bayesian Approach

The Bayesian approach to statistical inference is based upon Bayes' theorem (Bayes1763), Consider the problem of finding a point estimate of the parameter $\theta$ for the population with distribution $f(x|\theta)$ given $\theta$. Denote $\pi(\theta)$ the prior distribution of $\theta$.

Suppose that a random sample of size n, denoted by $X=(x_1,x_2,\ldots,x_n)$ is observed. For continuous distribution the bayes' theorem is defined as follows $\pi(\theta|x) \propto f(x|\theta)\pi(\theta)$

Definition: The distribution of $\theta$, given data $x$, which is called the posterior distribution is given by

$$\pi(\theta|x)=\frac{f(x|\theta)\pi(\theta)}{g(x)}$$ Where $g(x)$ is the marginal distribution of $x$.

The marginal distribution in the above definition can be calculated using the following formula

$$g(x)=\begin{cases}\sum_{\theta} f(x|\theta)\pi(\theta), & \theta \text{ is discrete}\\ \int_{-\infty}^{\infty} f(x|\theta)\pi(\theta), & \theta \text{ is continous}\end{cases}$$



### 2.8.4 Differences in Approach

As in the frequentist case above, the Bayesian approach ends up with a function of the model parameters given the observed data, but this time our inference is in the form of a probability distribution for $\theta$ rather than a simple point estimate. This illustrates one of the fundamental differences between the beliefs of the Bayesian and frequentist statisticians. The frequentist statistician believes that there exists a fixed true value for the model parameters and tries to estimate this value by maximizing the likelihood. The Bayesian believes that the unknown parameters have a fixed but unknown distribution which represents their beliefs about those parameters having observed the data. Note that the Bayesian may also believe that a true value exists, but since this can never be known with absolute certainty they prefer to think instead in terms of a distribution which reflects this uncertainty. As their level of information increases, the width of this distribution decreases and, in the limit

### 2.8.5 Principles of Bayesian Statistical Analysis

There are four basic principles of Bayesian statistical analysis

1. Specify an objective probability model of the trials in terms of some unknown parameters.

2. Form subjective beliefs about the unknown parameters. These original beliefs are called the prior probabilities.



3. Update beliefs in light of the information contained in the sample by applying Bayes' Rule. The updated beliefs are called the posterior probabilities.

4. Base decisions on the updated beliefs (i.e., the posterior probabilities).

Remember that the prior distribution shows your beliefs about different parameter values before seeing any data. The likelihood essentially shows what the observed data tells about how probable different parameter values are. The posterior probability combines the information in these two distributions, and shows your updated beliefs about parameter values after having seen the data.



In frequentist statistics, the goal is to obtain good point estimates of parameter values.

In Bayesian statistics, the goal is instead to obtain a probability distribution over all possible parameter values. This is the posterior probability distribution. It shows how uncertain we are about the parameter value, and can be used as the basis for asking many different questions.

### 2.8.6 Bayesian Prior, Posteriors and Estimators

If $Y_1, Y_2, ..., Y_N$ denote the random variables associated with a sample of size $n$. Let the notation $L(y_1, y_2, ..., y_n | \theta)$ denote the likelihood of the sample. In the discrete case, this function is defined to be the joint probability $P(Y_1 = y_1, Y_2 = y_2, ... Y_n = y_n)$ and in the continuous case, it is the joint density of $Y_1, Y_2, ..., Y_N$ evaluated at $y_1, y_2, ..., y_n$. The parameter $\theta$ is included among the argument of $L(y_1, y_2, ..., y_n | \theta)$ to denote this function depends explicitly on the value of some parameter $\theta$.

In Bayesian approach, the unknown parameter $\theta$ is viewed to be a random variable with a probability distribution called the prior distribution of $\theta$. This prior distribution of $\theta$ is specified before any data are collected and provides a theoretical description of information about $\theta$ that was available before any data were obtained. In our initial discussion we will assume that the parameter $\theta$ has a continuous distribution with density $g(\theta)$ that has no unknown parameters. Using the likelihood of the data and the prior on $\theta$, it follows that the joint likelihood $Y_1, Y_2, ... Y_n, \theta$ is

$f(y_1, y_2 ... y_n, \theta) = L(y_1, y_2 ... y_n | \theta) \times g(\theta)$ And that the marginal density or mass function of $Y_1, Y_2, ..., Y_N$ is $m(y_1, y_2 ... y_n, \theta) = \int_{-\infty}^{\infty} L(y_1, y_2 ... y_n | \theta) \times g(\theta) d\theta$ also the posterior density function of $\theta | y_1, y_2 ... y_n$ is $g^*(\theta | y_1, y_2 ... y_n) = \dfrac{L(y_1, y_2 ... y_n | \theta) \times g(\theta)}{\int_{-\infty}^{\infty} L(y_1, y_2 ... y_n | \theta) \times g(\theta) d\theta}$

Let $Y_1, Y_2, ..., Y_N$ denote a random sample from a Bernoulli distribution of $n$ observed morning where $P(Y_i = 1) = p$ and $P(Y_i = 0) = 1 - p$ and assume that the prior distribution for $p$ is $beta(\alpha, \beta)$



## 2.9 BETA AS CONJUGATE PRIOR

What is the posterior distribution for $p$ where $p$ is the probability of sunshine?

Since the Bernoulli probability function can be written as $P(y_i | p) = p^{y_i}(1-p)^{1-y_i}$, $y_i = 0, 1$, the likelihood $L(y_1, y_2, ..., y_n | p)$ is

$$\begin{aligned} L(y_1, y_2, ..., y_n | p) &= p(y_1, y_2, ..., y_n | p) \\ &= p^{y_1}(1-p)^{1-y_1} \times p^{y_2}(1-p)^{1-y_2} \times .... \times p^{y_n}(1-p)^{1-y_n} \\ &= p^{\sum y_i}(1-p)^{1-\sum y_i}, \qquad y_i = 0, 1 \text{ and } 0 < p < 1 \end{aligned}$$

The joint probability distribution is

$$
\begin{aligned}
f(y_1, y_2, \ldots, y_n, p) &= L(y_i, y_2, \ldots, y_n | p) \times g(p) \\
&= p^{\sum y_i}(1-p)^{n-\sum y_i} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1} \\
&= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1}
\end{aligned}
$$

And the marginal density function is

$$
\begin{aligned}
m(y_1, y_2, \ldots, y_n) &= \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1} dp \\
&= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{\Gamma\left(\sum y_i+\alpha\right)\Gamma\left(n-\sum y_i+\beta\right)}{\Gamma(n+\alpha+\beta)}
\end{aligned}
$$

The posterior density of $p$ is given by

$$
g^*(\theta | y_1, y_2 \ldots y_n) = \frac{L(y_1, y_2 \ldots y_n | \theta) \times g(\theta)}{\int_{-\infty}^{\infty} L(y_1, y_2 \ldots y_n | \theta) \times g(\theta) d\theta}
$$

Hence

$$
\begin{aligned}
g^*(p | y_1, y_2, \ldots, y_n) &= \frac{\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1}}{\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \dfrac{\Gamma\left(\sum y_i+\alpha\right)\Gamma\left(n-\sum y_i+\beta\right)}{\Gamma(n+\alpha+\beta)}} \\
&= \frac{\Gamma(n+\alpha+\beta)}{\Gamma\left(\sum y_i+\alpha\right)\Gamma\left(n-\sum y_i+\beta\right)} \times p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1}, \quad 0<p<1
\end{aligned}
$$

The posterior beta density function is with the parameters

$\alpha^* = \sum y_i + \alpha$ and $\beta^* = n - \sum y_i + \beta$

## CHAPTER THREE

## 3.1 ANALYSIS OF DATA

### 3.1.1 Nature and Source of Data

The solar radiation data was obtained from the Department of Mechanical Engineering KNUST. It comprised 14 years (1995-2008) of daily global irradiation, compiled from hourly measurements at the KNUST Solar laboratory. The observations were made with Bellan distillation pyranometers. These radiometers are robust and simple and are therefore appropriate for the prevailing conditions in many developing countries. In spite of these observations, the data for Kumasi is consistent with measurements from locations in the sub region with similar climatic and vegetation characteristics

### 3.1.2 Data Quality Information

A lot of data is usually lost due to malfunctioning of the Pyranometer sensor at certain periods. The data from 2005 – 2007 for instance were lost, because during that period the calibrator of the Pyranometer was undergoing repairs. Moreover, there are few missing data points in the 1995 and 1996 data. In 1997, some of the sunlight measurements were negatives so these data may not be reliable since their true accuracy is unknown. It is therefore highly probable that useful information and details might have been lost in compiling data, which could affect results of the analysis.

### 3.1.2 Scope of Analysis

Sunlight has a lot of uses, especially in Africa and for that matter Ghana where it is in abundance. It is useful in hybrid lighting, drying clothes and bricks, etc. This research work however, focuses mainly on fitting a probability distribution function for the month of the year and also cluster the various months of the year.



Moreover analysis will also include Bayesian analysis of the daily solar irradiation in Kumasi-Ghana.

### 3.1.3 Compilation of Data

Due to the large size of the secondary data, and also the fact that total amount of energy available changes considerably from month to month, the data was grouped into months ranging from 1995 – 2008. With the help of the data analysis tool in excel, one thousand five hundred random samples were selected from the population for each month for the from 1995-2008 available data. (Consisting of over 7,000 data points) and analyzed as follows. A detailed presentation of the primary data is provided in the appendix of this document.

### 3.1.4 Analysis of Data

The analysis adopted both descriptive and inferential methods. The descriptive methods used are frequency and relative frequency distribution tables, probability density curves, and histograms. Hypothesis testing, least squares estimation, and maximum likelihood estimation are the only inferential statistics used.

### 3.1.5 Frequency Distribution

Since the sampled data were too many to gain an insight into, and impossible for one to convey much information about its characteristics, it became necessary to organize and reduce it into meaningful forms as follows:

The data for each month was organized into groups, called classes or categories using the Sturges (approximation) rule, which states that:

The number of classes, $K = 1 + 3.322 \log_{10} n$. and class width, $C = \frac{Range}{K}$

Where n is the total number of observations or frequencies and

Range = (maximum – minimum) of the observations.

### 3.1.7 Probability Distributions

To make inferences with respect to the population distribution of each month, some standard continuous probability distributions were selected based on their shape parameters to fit the data for each month. The distributions are the exponential, Weibull, Gamma, Lognormal and Beta, Least square and maximum likelihood estimation procedures were then applied to estimate the parameter for each distribution from the data. The probability density function and their respective histogram for the various months are shown below.

### 3.1.9 Mean Squared Errors

The mean squared error of the monthly data and each distribution selected were computed to come up with the best candidate fitting the data as follows: $MSE = \frac{\sum_{i=1}^{n}(rf_i - f(x_i))^2}{n}$

Where $rf_i$ = each relative frequency for the month

$f(x_i)$ = values computed using the probability distribution under consideration.

$x_i$ = the class mark(x), i.e. the hourly solar irradiance for the month under consideration.



## 3.2 PRESENTATION OF RESULTS

### 3.2.1 Histogram and Probability Density Function

A random sample of size one thousand five hundred was selected from each month of the year and probability density function and histogram in respect of hourly solar irradiance for the various months of the year was produced.

#### 3.2.1.1 Probability Density Function and Histogram for January

The histogram and the probability density function for the month of January are presented in the Fig 3.1 and Fig 3.1a below respectively.

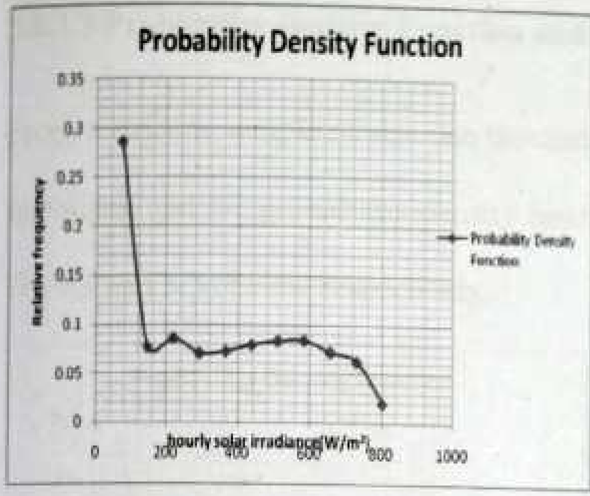



Fig 3.1:Probabilility density function for Jan

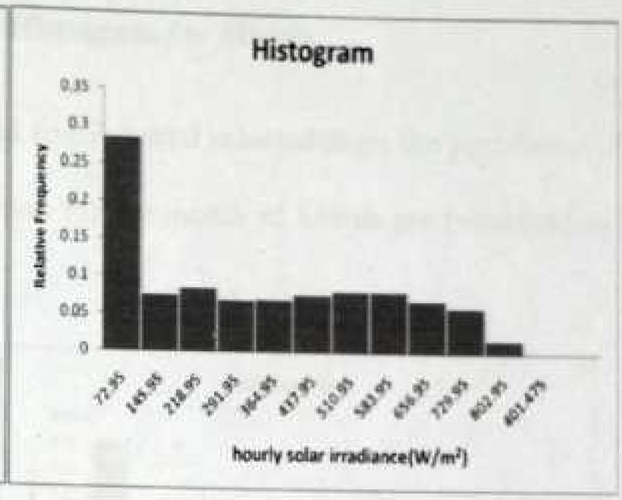



Fig 3.1a:Histogram for the month of Jan



### 3.2.1.2 Probability Density Function and Histogram for February

From a random sample of size one thousand five hundred was selected from the population .The histogram and the probability density function for the month of February are presented in the Fig 3.2 and Fig 3.2a respectively.

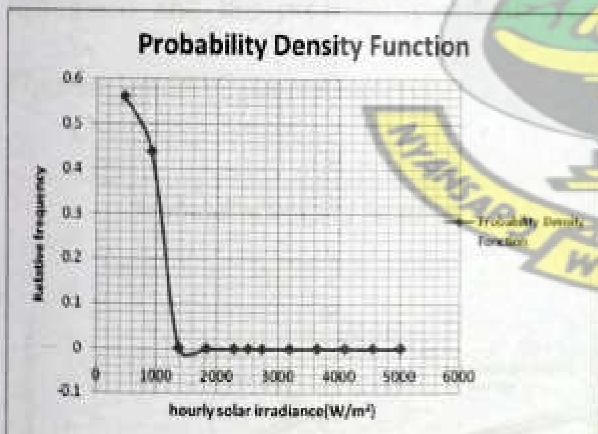



Fig 3.2:Probabilility density function for Feb

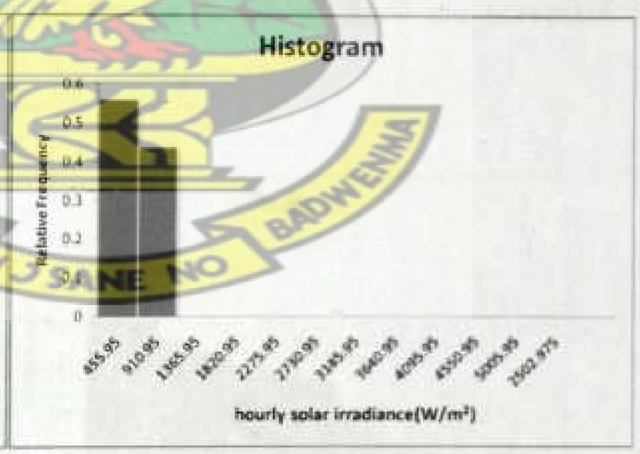
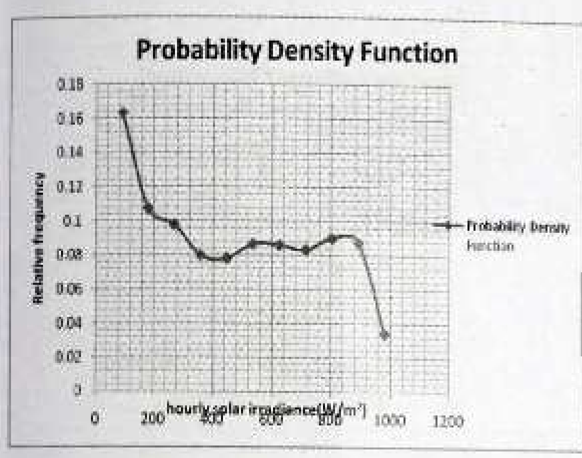



Fig 3.1a:Histogram for the month of Feb

### 3.2.1.3 Probability Density Function and Histogram for March

From a random sample of size one thousand five hundred selected from the population .The histogram and the probability density function for the month of March are presented in the Fig 3.3 and 3.3a below respectively



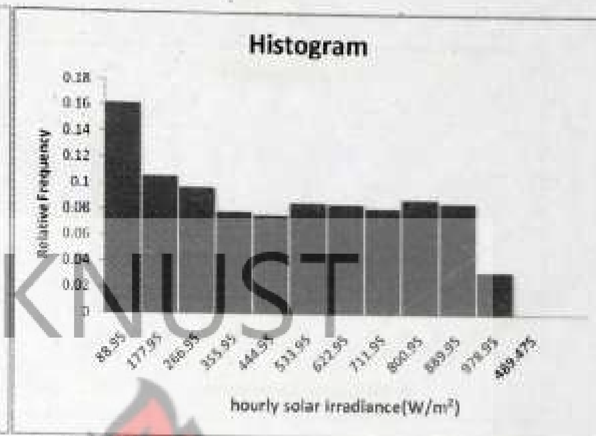
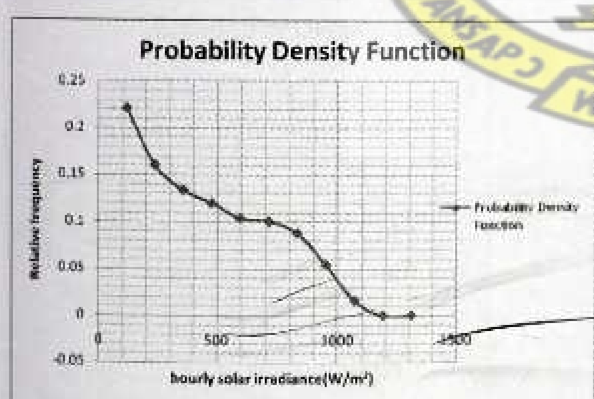




Fig 3.3:Probabilility density function for march

Fig 3.3a:Histogram for the month of march

### 3.2.1.4 Probability Density Function and Histogram for April

From a random sample of size one thousand five hundred selected from the population .The histogram and the probability density function for the month of April are presented in Figures 3.4 and 3.4a respectively.



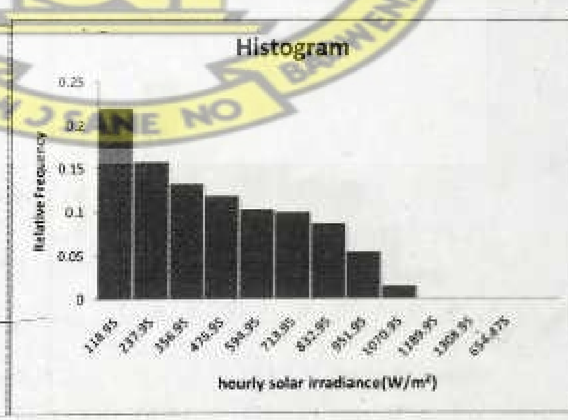




Fig 3.1: Probability density function for April

Fig 3.11: Histogram for the month of April

### 3.2.1.5 Probability Density Function and Histogram for May

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function for the month of May were obtained and are presented in Figures 3.5 and 3.5a respectively.

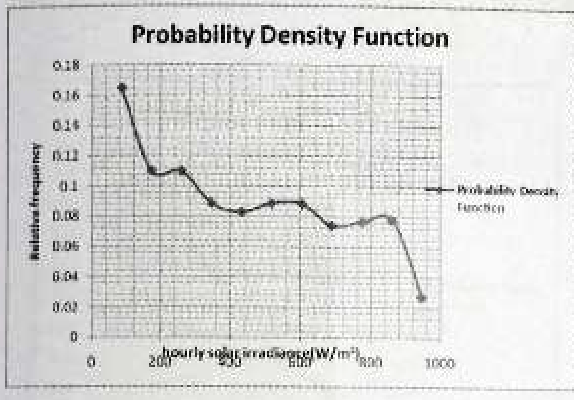



Fig 3.5: Probability density function for May

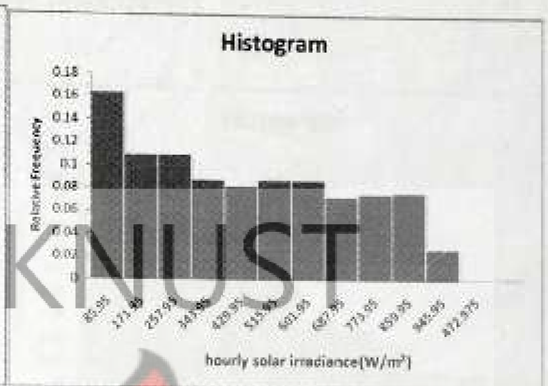



Fig 3.5a: Histogram for the month of May

### 3.2.1.6 Probability Density Function and Histogram for June

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function for the month of June were obtained and are here presented in Figures 3.6 and 3.6a below respectively.

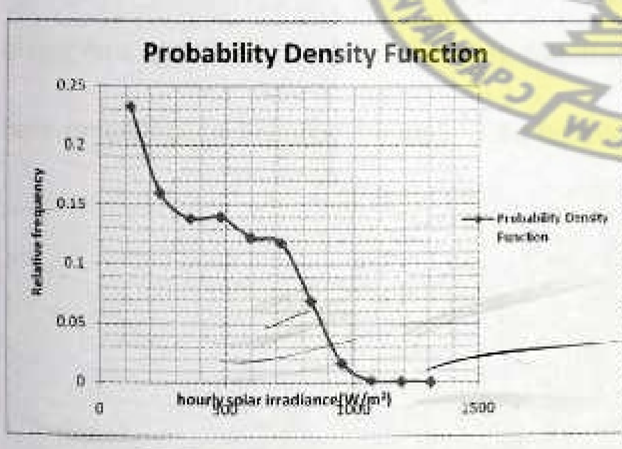



Fig 3.6: Probability density function for June

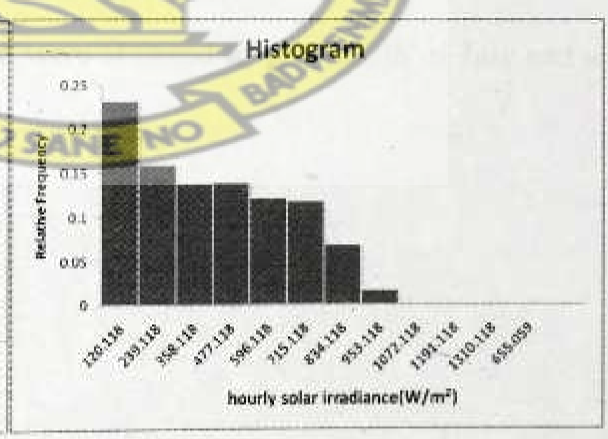
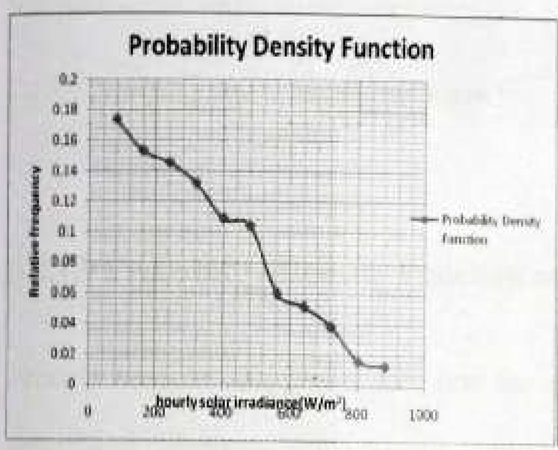



Fig 3.6a: Histogram for the month of June

### 3.2.1.7 Probability Density Function and Histogram for July

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function were obtained for the month of July and are here presented in Figures 3.7 and 3.7a below respectively



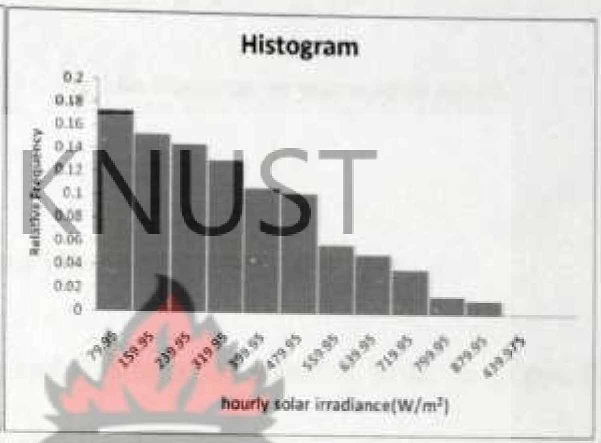




Fig 3.7: Probability density function for July

Fig 3.7a: Histogram for the month of July



### 3.2.1.8 Probability Density Function and Histogram for August

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function were obtained for the month of July and are here presented in Figures 3.8 and 3.8a below respectively.

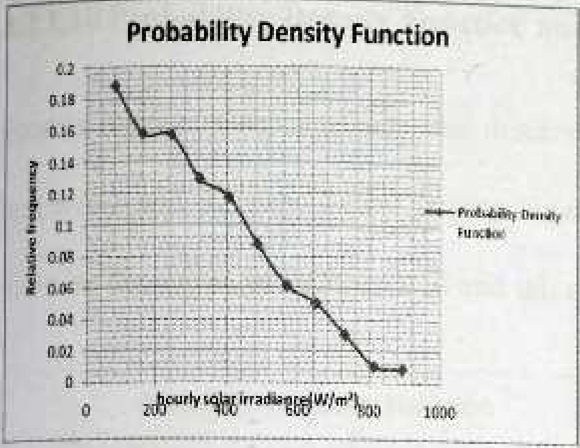



Fig 3.8: Probability density function for August

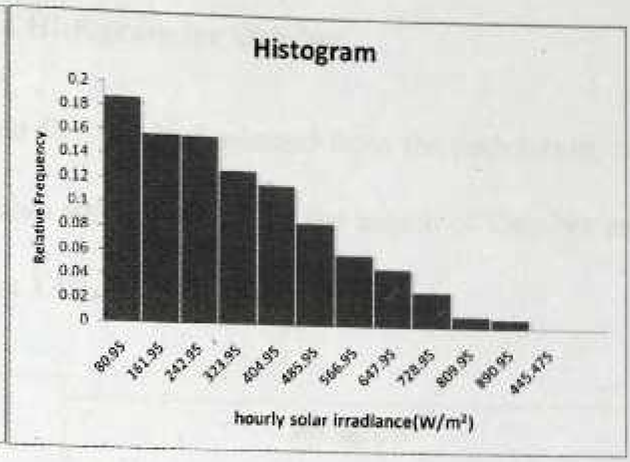



Fig 3.8a: Histogram for the month of August



### 3.2.1.9 Probability Density Function and Histogram for September

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function were obtained for the month of September and are here presented in Figures 3.9 and 3.9a below respectively.

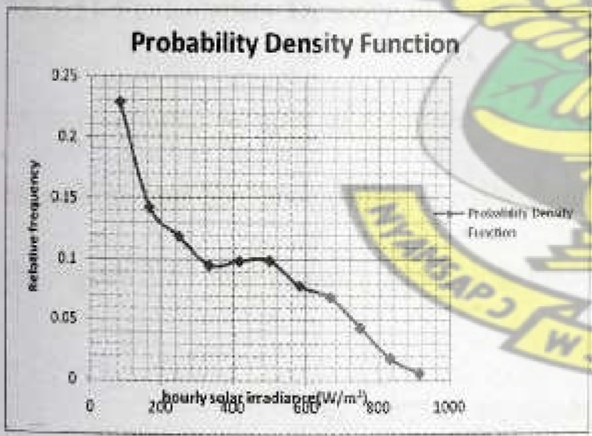



Fig 3.9: Probability density function for September

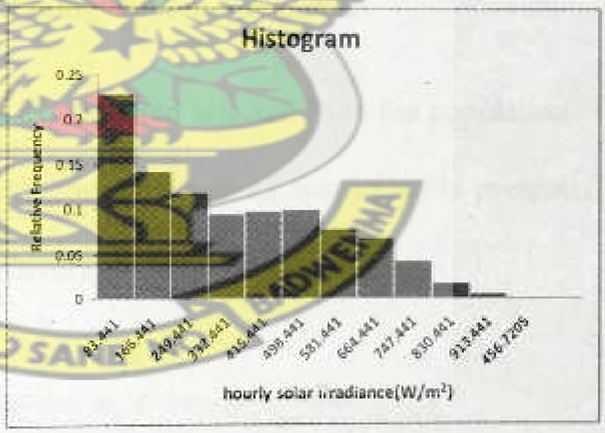



Fig 3.9a: Histogram for the month of September

### 3.2.1.10 Probability Density Function and Histogram for October

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function were obtained for the month of October and are here presented in Figures 3.10 and tables 3.10a below respectively

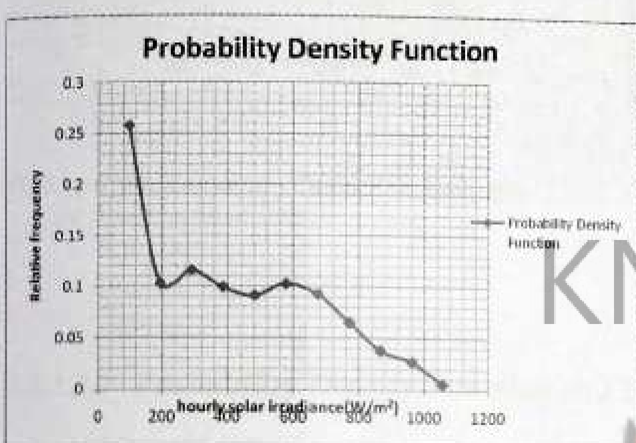



Fig 3.10:Probabilility density function for October

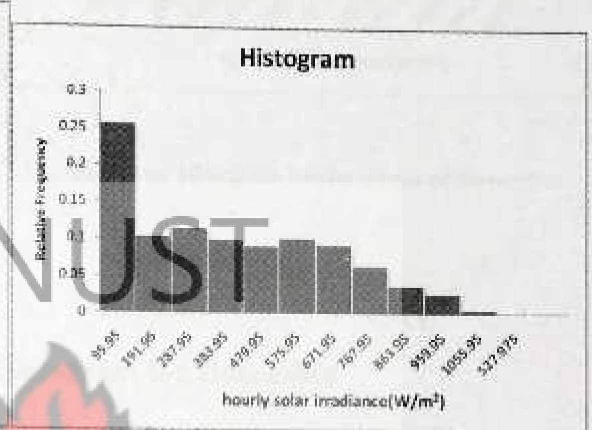



Fig 3.10a:Histogram for the month of October

### 3.2.1.11 Probability Density Function and Histogram for November

From a random sample of size one thousand five hundred selected from the population .The histogram and the probability density function for the month of November is presented in the Figure 3.11 and tables 3.11a below respectively.

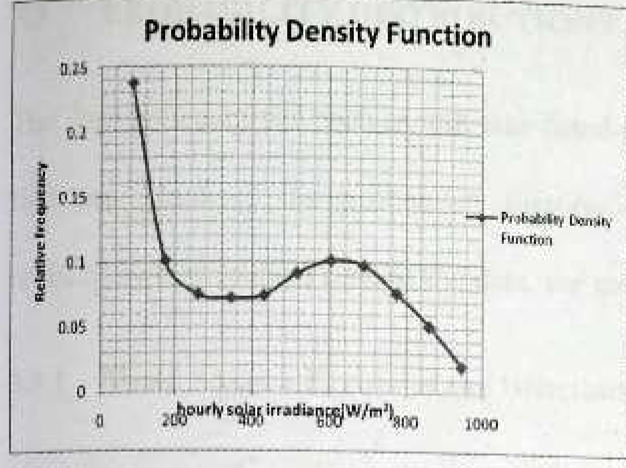



Fig 3.11: Probability density function for November

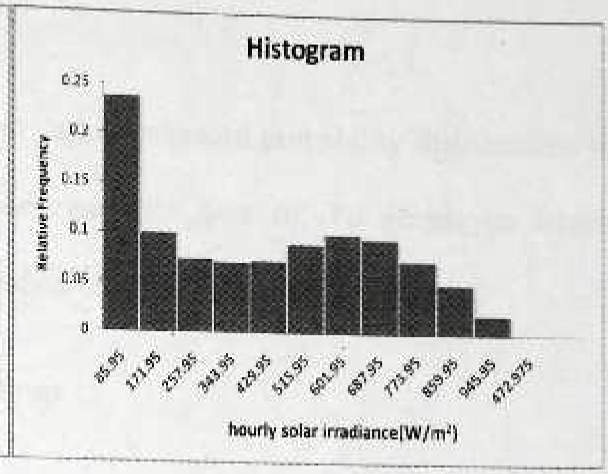



Fig 3.11a: Histogram for the month of November



### 3.2.1.12 Probability Density Function and Histogram for December

From a random sample of size one thousand five hundred selected from the population, the histogram and the probability density function were obtained for the month of December and are here presented in Figures 3.12 and 3.12a respectively.

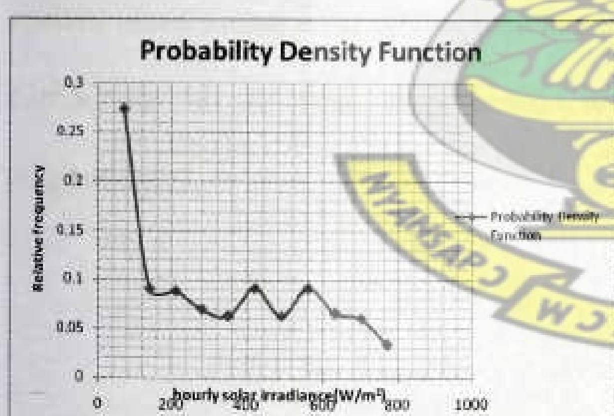



Fig 3.12: Probability density function for Dec

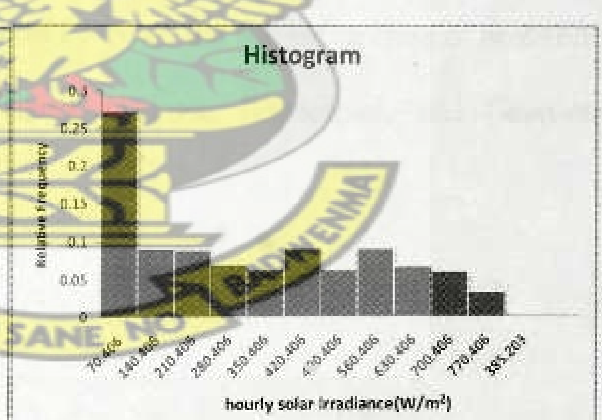



Fig 3.12a: Histogram for the month of December

## 3.3 PROBABILITY DISTRIBUTIONS

The data collected for each month was fitted to various standard probability distributions to find out which of the probability distributions gave the best fit. To decide on which probability distribution best fit the data, the method of mean square error was used.

### 3.3.1 Mean Square Errors of the Distributions

The standard distribution with the smallest mean square error with respect to the data was considered the best candidate distribution for the particular month under consideration.

Due to the large number of distributions dealt with, the remaining tables and charts have been omitted here and rather presented in the appendix.

Tables 3.1 and 3.2 below show the mean square errors for various standard probability distributions for the months of January and February respectively. We find that the lognormal distribution is the best fitted distribution for the month of January since it has the smallest mean square error while in the month of February the following class of probability distributions all fit best: Exponential, Weibull, Gamma, Lognormal, and Geometric distributions.

### 3.3.1.1 Mean Square Errors of the Distributions for Jan and Feb

Table 3.1: Distribution Mean Square Error for Jan

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 0.000003 |
| Weibull Distribution | 0.000003 |
| Gamma Distribution | 0.000003 |
| Lognormal Distribution | 2.54456E-06 |
| Beta Distribution | 2.005E+191 |
| Geometric Distribution | 0.000003 |

Table 3.2: Distribution Mean Square Error for Feb

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 3.7037E-08 |
| Weibull Distribution | 3.7037E-08 |
| Gamma Distribution | 3.7037E-08 |
| Lognormal Distribution | 3.59166E-08 |
| Beta Distribution | 2.88371E+34 |
| Geometric Distribution | 3.7037E-08 |



### 3.3.1.2 Mean Square Errors of the Distributions for March and May

Tables 3.3 and 3.4 below show the mean square errors for various standard probability distributions for March and May respectively. The log normal distribution can be seen to be the best fitted distribution for the month of March and May since it has the smallest mean square error in both cases.

Table 3.3: Distribution Mean Square Error for March

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 0.000003 |
| Weibull Distribution | 0.000003 |
| Gamma Distribution | 0.000003 |
| Lognormal Distribution | 5.61688E-06 |
| Beta Distribution | 1.79165E+60 |
| Geometric Distribution | 0.000003 |

Table3.4: Distribution Mean Square Error form May

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 2.5037E-05 |
| Weibull Distribution | 2.5037E-05 |
| Gamma Distribution | 2.5037E-05 |
| Lognormal Distribution | 4.98008E-07 |
| Beta Distribution | 1.29291E+81 |
| Geometric Distribution | 2.5037E-05 |

### 3.3.1.3 Mean Square Errors of the Distributions for April

The table 3.5 below shows the mean square errors for the various standard probability distributions under consideration for the month of April. The Weibull, exponential, gamma and the geometric distribution as can be seen below is the best fitted distribution for the month of April since it has the smallest mean square error.

Table 3.5: Distribution Mean Square Error for April

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 3.7037E-08 |
| Weibull Distribution | 3.7037E-08 |
| Gamma Distribution | 3.7037E-08 |
| Lognormal Distribution | 7.10438E-06 |
| Beta Distribution | 4.0449E+248 |
| Geometric Distribution | 3.7037E-08 |



### 3.3.1.4 Mean Square Errors of the Distributions for June and Sept

Tables 3.6 and 3.7 below show the mean square errors for the various standard probability distributions under consideration for June and September respectively. The Exponential, Gamma, Weibull and Geometric distributions as can be seen below were the best fitted distributions for both the months of June and September since they all have the smallest mean square error

Table 3.6: Distribution Mean Square Error for June

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 3.7037E-08 |
| Weibull Distribution | 3.7037E-08 |
| Gamma Distribution | 3.7037E-08 |
| Lognormal Distribution | 6.79067E-06 |
| Beta Distribution | 1.0862E+237 |
| Geometric Distribution | 3.7037E-08 |

Table3.7: Distribution Mean Square Error for Sept

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 9.25926E-07 |
| Weibull Distribution | 9.25926E-07 |
| Gamma Distribution | 9.25926E-07 |
| Lognormal Distribution | 7.11702E-06 |
| Beta Distribution | 8.0732E+235 |
| Geometric Distribution | 9.25926E-07 |

### 3.3.1.5 Mean Square Errors of the distributions for July



Table 3.8 below shows the mean square errors for the various standard probability distributions under consideration for July. The Exponential, Gamma, Weibull and Geometric distributions were the best fitted distributions for both the months of June and September since they each have the smallest mean square error.

Table 3.8: Distribution Mean Square Error for July

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 1.33333E-06 |
| Weibull Distribution | 1.33333E-06 |
| Gamma Distribution | 1.33333E-06 |
| Lognormal Distribution | 7.5403E-06 |
| Beta Distribution | 1.2338E+233 |
| Geometric Distribution | 1.33333E-06 |

### 3.3.1.6 Mean Square Errors of the Distributions for August

Table 3.9 below shows the mean square errors for the various probability distributions under consideration for August best fitted distribution for the month of August since it has the smallest mean square error.

Table 3.9: Distribution Mean Square Error for August

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 9.25926E-07 |
| Weibull Distribution | 9.25926E-07 |
| Gamma Distribution | 9.25926E-07 |
| Lognormal Distribution | 7.49618E-06 |
| Beta Distribution | 5.7038E+234 |
| Geometric Distribution | 9.25926E-07 |



### 3.3.1.7 Mean Square Errors of the Distributions for Oct and Nov

Tables 3.10 and 3.11 below show the mean square errors for various standard probability distributions for October and November respectively. The Exponential, Gamma, Weibull and Geometric distributions were the best fitted distributions for both the months of October and November since they all have the smallest mean square error.

Table 3.10: Distribution Mean Square Error for October

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 3.7037E-08 |
| Weibull Distribution | 5.7201E-128 |
| Gamma Distribution | 3.7037E-08 |
| Lognormal Distribution | 6.37252E-07 |
| Beta Distribution | 1.13913E+81 |
| Geometric Distribution | 3.7037E-08 |

Table3.11:Distribution Mean Square Error for

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 3.33333E-07 |
| Weibull Distribution | 3.33333E-07 |
| Gamma Distribution | 3.33333E-07 |
| Lognormal Distribution | 7.33666E-06 |
| Beta Distribution | 1.8199E+191 |
| Geometric Distribution | 3.33333E-07 |



### 3.3.1.8 Mean Square Errors of the Distributions for December

Table 3.12 below shows the mean square errors for the various standard probability distributions under consideration for December. The Exponential, Gamma, Weibull Lognormal and Geometric distributions were the best fitted distributions for the month of December since they all have the smallest mean square error.

Table 3.12: Distribution Mean Square Error for December

| DISTRIBUTION | MEAN SQUARE ERROR |
|---|---|
| Exponential Distribution | 2.37037E-06 |
| Weibull Distribution | 2.37037E-06 |
| Gamma Distribution | 2.37037E-06 |
| Lognormal Distribution | 4.14557E-06 |
| Beta Distribution | 1.0932E+178 |
| Geometric Distribution | 2.37037E-06 |

### 3.3.2 Monthly Probability Distributions

The randomly selected solar irradiation data was fitted into the various probability distributions and the resulting graphs of their probability distributions are presented for the various months of January march and May.

As discussed in section in section 3.3 the lognormal probability distribution can best fit the month of January march and May. Fig 3.16 below gives the graph of the lognormal distribution for the month of January march and May solar irradiation.



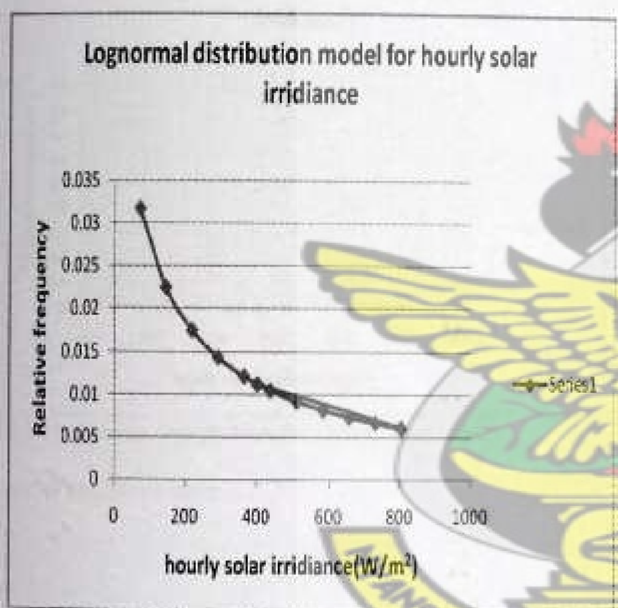



Fig 3.16 Lognormal distribution for January, March and May

As discussed in section in section 3.3 the Exponential, Gamma, Weibull Lognormal and Geometric distributions can best fit the month of April June September October and November . Fig 3.16 to fig 3.21 below gives the graph of the above mentioned probability distribution for the month of April, June, September, October and November

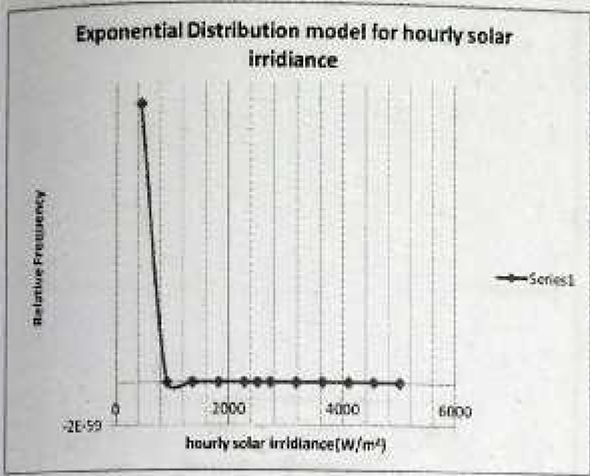



Fig 3.17Exponenti al distribution for April, June, September, September,October and November

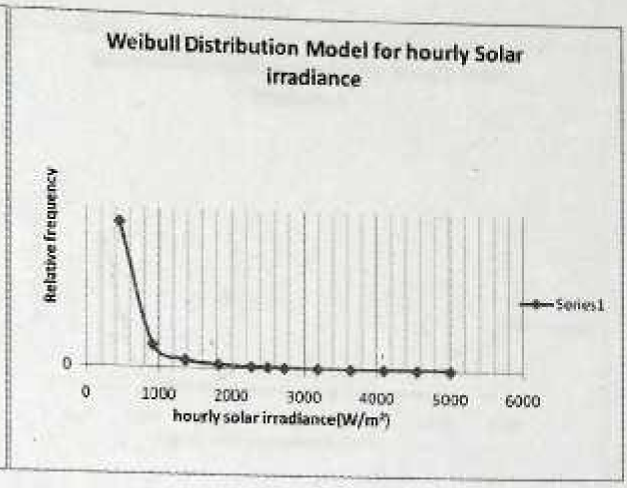



Fig 3.18 Weibull distribution for April, June, October and November Fig 1



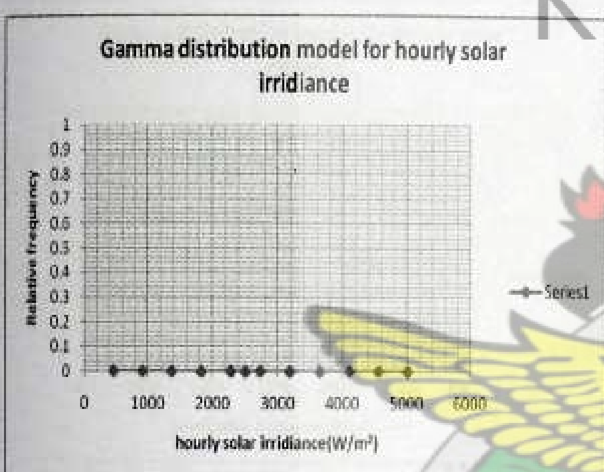



Fig 3.19 Gamma distribution for, June, September, October and November

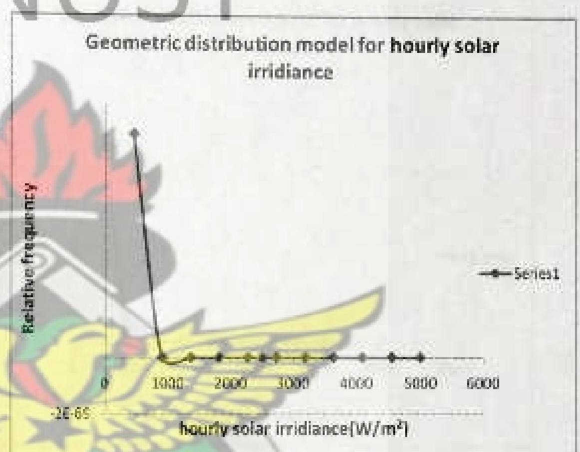



Fig 3.20 Geometric distribution for, June, September, October and November

As discussed in section in section 3.3 the Exponential, Gamma, Weibull Lognormal and Geometric distributions can best fit the month of February, July and December. Fig 3.21 to fig 3.25 below gives the graph of the above mentioned probability distribution for the month of February, July and December

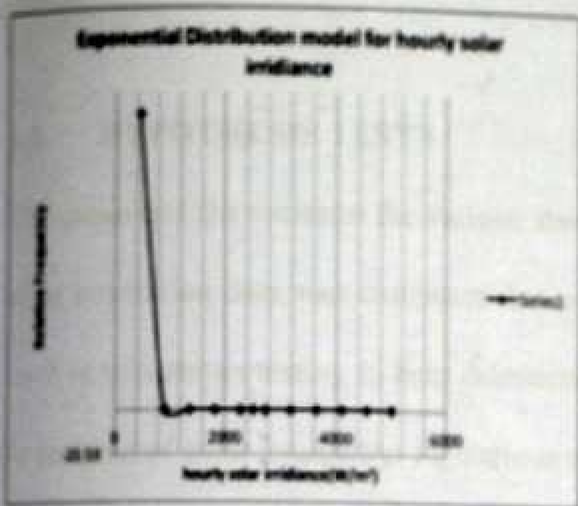



Fig 3.21Exponenti al distribution for February, July and December.

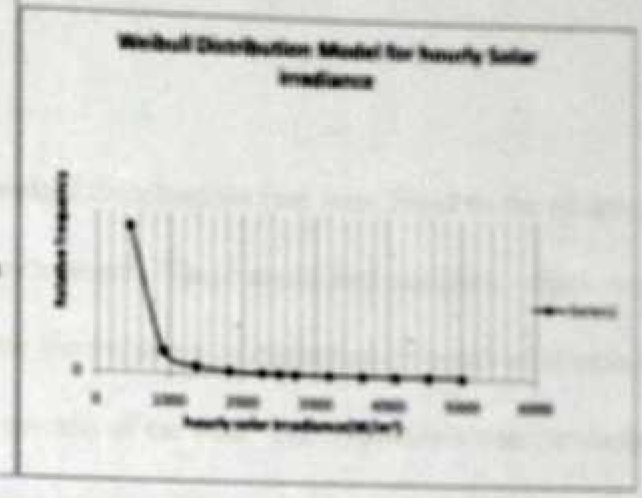



Fig 3.22 Weibull distribution for February, July and December



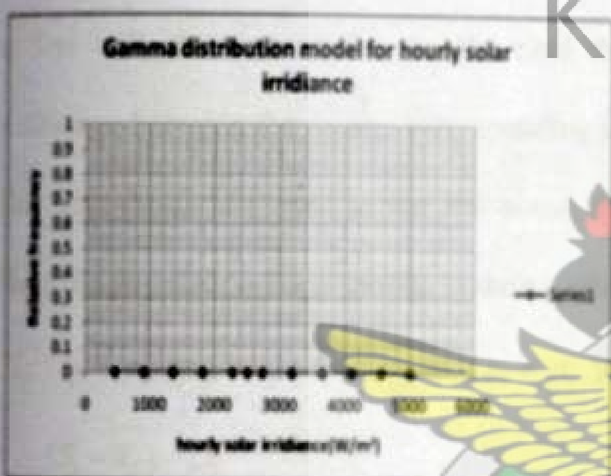



Fig 3.23 Gamma distribution for, February, July and December.


Fig 3.24 Geometric distribution for, February, July and December .

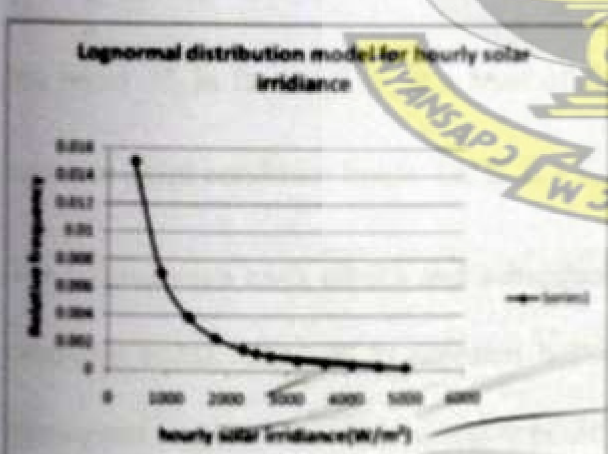



Fig 3.25 Geometric distribution for, February, July and December .

## 3.5 HYPOTHESIS TESTS

The mean and the variance the various standard distributions that were fitted to the observed solar irradiation data was computed for each month. These mean and variance values were used in hypothesis testing to help determine the existence of clustering in terms of similarity in patterns of solar irradiation for various months of the year. The hypothesis was conducted to test the difference between the population means of the various months of the year solar irradiation based on the sample selected. January being the first month of the year was used to test with the rest of the months until there was a rejection (existence of significant difference between the means corresponding to the months).

The hypothesis test used in the test about the difference between two populations means. Here the population corresponds to the month of the year. Testing:

$H_O : \mu_1 = \mu_2 \text{ or } \mu_1 - \mu_2 = 0$ $\quad H_a : \mu_1 \neq \mu_2$ With the test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_o}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

We reject $H_O$ in favor of $H_a$ at a level of significance α=0.05 if and only if the appropriate rejection point condition holds. I.e. if $Z > Z_{\alpha/2} \text{ or } Z < -Z_{\alpha/2}$

The test between each month and subsequent months is performed until the rejection point condition holds. The test is repeated between the last month used in the latter test and subsequent months until the rejection point condition holds again. This procedure grouped the months into clusters, shown in the tables below.

### 3.5.1 Testing of Hypothesis between Mean Solar Irradiation of January and February

The month of January was used to test against the month of February, it was noticed that there is no significant difference between mean solar output for January and February. Thus January and February belong to the same cluster. The Table 3.20 below shows the results of the test.



Table3.20: Test of Hypothesis between Mean Solar Irradiation of January and February

| | Month | sample size (n) | Mean($\bar{x}$) | Variance($s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | FEB | 1500 | 675 | 50625 |

| RESULTS | | | |
|---|---|---|---|
| Z = | -0.448966081 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -592.2117801 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |



### 3.5.2 Testing of Hypothesis between Mean Solar Irradiation of January and March

The month of January was tested with March and it was found that there is no significant difference between the two months which indicate that January and March can also be in the same cluster. The table 3.21 below shows the result of the test.

Table3.21: Test of Hypothesis between Mean Solar Irradiation of January and March

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | MARCH | 1500 | 37.14205451 | 200752.5614 |

**RESULTS**

| | | | |
|---|---|---|---|
| Z = | 0.008801078 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | 45.64616544 |
| Decision: | Accept Ho ( since Z < 1.96) | | |
| Conclusion: | There is no difference between the population means | | |



### 3.5.3 Testing of Hypothesis between Mean Solar Irradiation of January and April

The month of January was tested with April and it was found that there is no significant difference between the two months which indicate that January and April can also be in the same cluster. The table 3.22 below shows the result of the test.

Table3.22: Test of Hypothesis between Mean Solar Irradiation of January and April

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | APRIL | 1500 | 36.42145334 | 148625.8932 |

**RESULTS**

| | | | |
|---|---|---|---|
| Z = | 0.012233982 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | 46.36676661 |
| Decision: | Accept Ho ( since Z < 1.96) | | |
| Conclusion: | There is no difference between the population means | | |

### 3.5.4 Testing of Hypothesis between Mean Solar Irradiation of January and May

The month of January was tested with May and it was found that there is no significant difference between the two months which indicate that January and May can also be in the same cluster. The table 3.23 below shows the result of the test.

Table3.23: Test of Hypothesis between Mean Solar Irradiation of January and May

| | Month | sample size (n) | Mean($\bar{x}$) | Variance($s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | MAY | 1500 | 34.30471912 | 169874.2951 |

| RESULTS | | | |
|---|---|---|---|
| Z = | 0.011044822 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | 48.48350083 |
| Decision: | Accept Ho ( since Z < 1.96) | | |
| Conclusion: | There is no difference between the population means | | |

### 3.5.5 Testing of Hypothesis between Mean Solar Irradiation of January and June

The month of January was tested with June and it was found that there is no significant difference between the two months which indicate that January and June can also be in the same cluster. The table 3.24below shows the result of the test.

Table 3.24: Test of Hypothesis between Mean Solar Irradiation of January and June

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| **Sample 1** | JAN | 1500 | 82.78821995 | 6853.889362 |
| **Sample 2** | JUNE | 1500 | 404 | 1616 |

| RESULTS | | | |
|---|---|---|---|
| $Z$ = | -1.766656266 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -321.2117801 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |



### 3.5.6 Testing of Hypothesis between Mean Solar Irradiation of January and July

The month of January was tested with July and it was found that there is no significant difference between the two months which indicate that January and July can also be in the same cluster. The table 3.25 below shows the result of the test.

Table3.25: Test of Hypothesis between Mean Solar Irradiation of January and February

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| **Sample 1** | JAN | 1500 | 82.78821995 | 6853.889362 |
| **Sample 2** | JULY | 1500 | 308 | 616 |

| RESULTS | | | |
|---|---|---|---|
| $Z$ = | -1.267513639 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -225.2117801 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |

### 3.5.7 Testing of Hypothesis between Mean Solar Irradiation of January and August

The month of January was tested with August and it was found that there is no significant difference between the two months which indicate that January and August can also be in the same cluster. The table 3.26 below shows the result of the test.

Table3.26: Test of Hypothesis between Mean Solar Irradiation of January and August

| | Month | sample size (n) | Mean($\bar{x}$) | Variance($s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | AUG | 1500 | 312 | 936 |



| RESULTS | | | |
|---|---|---|---|
| Z = | -1.283314174 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$ = | 1.96 | Mean difference | -229.2117801 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |

### 3.5.8 Testing of Hypothesis between Mean Solar Irradiation of January and September

The month of January was tested with September and it was found that there is no significant difference between the two months which indicate that January and September can also be in the same cluster. The table 3.27 below shows the result of the test.

Table3.27: Test of Hypothesis between Mean Solar Irradiation of January and September

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | SEPT | 1500 | 306 | 612 |

| RESULTS | | | |
|---|---|---|---|
| Z = | -1.256322605 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -223.2117801 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |



### 3.5.9 Testing of Hypothesis between Mean Solar Irradiation of January and October

The month of January was tested with October and it was found that there is no significant difference between the two months which indicate that January and October can also be in the same cluster. The table 3.28 below shows the result of the test.

Table3.28: Test of Hypothesis between Mean Solar Irradiation of January and October

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | OCT | 1500 | 241 | 868 |

**RESULTS**

| | | | |
|---|---|---|---|
| Z = | -0.886935958 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -158.2117801 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |



### 3.5.10 Testing of Hypothesis between Mean Solar Irradiation of January and November

The month of January was tested with November and it was found that there is a significant difference between the two months which indicate that January and November cannot be in the same the same cluster. The table 3.29 below shows the result of the test.

Table 3.29: Test of Hypothesis between Mean Solar Irradiation of January and November

| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | JAN | 1500 | 82.78821995 | 6853.889362 |
| Sample 2 | NOV | 1500 | 494 | 988 |

**RESULTS**

| | | | |
|---|---|---|---|
| Z = | -2.299895326 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -411.2117801 |
| Decision: | Reject Ho in favour of Ha ( since Z < -1.96) | | |
| Conclusion: | There is a difference between the population means | | |

### 3.5.11 Testing of Hypothesis between Mean Solar Irradiation of November and December

Since the test for January and November shows a significant difference, we will now test November with December to check the result. It was found that the there is no significant difference between the solar irradiation for November and December and these two month can also form another cluster. Table 3.31 below shows the test results.

Table3.30: Test of Hypothesis between Mean Solar Irradiation of November and December



| | Month | sample size (n) | Mean($\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | NOV | 1500 | 494 | 988 |
| Sample 2 | DEC | 1500 | 512 | 2048 |

RESULTS

| | | | |
|---|---|---|---|
| Z = | -0.30658722 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | -18 |
| Decision: | Accept Ho ( since Z > -1.96) | | |
| Conclusion: | There is no difference between the population means | | |

### 3.5.12 Testing of Hypothesis between Mean Solar Irradiation of November and January

The month of November was tested with January and it was found that there is a significant difference between the two months which confirms the fact that January and November cannot be in the same cluster. Table 3.31 below shows the result of t

Table 3.31: Test of Hypothesis between Mean Solar Irradiation of November and January

| | Month | sample size (n) | Mean( $\bar{x}$) | Variance( $s^2$) |
|---|---|---|---|---|
| Sample 1 | NOV | 1500 | 494 | 988 |
| Sample 2 | JAN | 1500 | 82.78821995 | 6853.889362 |

**RESULTS**

| | | | |
|---|---|---|---|
| $Z$ = | 2.299895326 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$= | 1.96 | Mean difference | 411.2117801 |
| Decision: | Reject Ho in favour of Ha ( since Z > 1.96) | | |
| Conclusion: | There is a difference between the population means | | |

### 3.5.4 Testing of Hypothesis between Mean Solar Irradiation of the Clusters.

The result of the series of tests indicate that the there are two clusters in the year in respect of similar patterns of solar irradiation, namely, one from January to October and the other from November to December.

Further hypothesis testing analysis was conducted on these two clusters of the year and it was also found that there is a significant difference between the mean outputs of solar irradiation for the two clusters of the year. The result of the cluster analysis is shown inTable 3.33 below.

Table 3.32: Test of Hypothesis between Mean Solar Irradiation of Jan-Oct and Nov-Dec

| INPUTS | | | | |
|---|---|---|---|---|
| | | Level of Significance | 0.05 | |
| | Month | sample size (n) | Mean($\bar{x}$) | Variance($s^2$) |
| Sample 1 | jan-oct | 1500 | 428 | 856 |
| Sample 2 | nov-dec | 1500 | 326 | 652 |

| RESULTS | | | |
|---|---|---|---|
| Z = | 3.671313969 | Hypothesized difference (Do) | 0 |
| $\lvert Z_{\alpha/2} \rvert$ = | 1.96 | Mean difference | 102 |
| Decision: | Reject Ho in favour of Ha ( since Z > 1.96) | | |
| Conclusion: | There is a difference between the population means | | |

## 3.6 BAYESIAN ANALYSIS

In the Bayesian analysis we use the random variable $X$ which represents the event of having a high or low sunshine.

Thus $X = \begin{cases} 0 \text{ low sunshine} \\ 1 \text{ high sunshin} \end{cases}$

The values of $X$ are generated based on a given threshold below which we describe the sunshine level as being low and as high otherwise. The threshold value used in this analysis is $120kWhm^{-2}$. A random sample of 1500 sunshine hours were selected and based on the above mentioned threshold the monthly average number of high and low sunshine levels was computed.

The beta distribution was used as prior distribution and the Bernoulli probability distribution as a likelihood function. The posterior probability distribution is derived in section 2.46.

Bayesian analysis for the various months of the year was conducted to estimate the posterior expectation and variance of the beta distribution for number of iterations until these iterations are converging to a certain value given a tolerance level. The analysis started with initial parameter values of alpha and beta to compute the prior means and prior variance .The posterior parameters are then computed based on the sample size and the number of high sunshine hours. The posterior parameters were then used as the prior parameters to compute the mean and variance. The procedure is repeated for twenty iterations for all the months of the year.



### 3.6.1 Beta Distribution

As discussed in the section 2.4.7 a random variable $X$ is said to have beta distribution with parameters $\alpha > 0$ $\beta > 0$ if the density of $X$ is

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \text{ for } 0 \le x \le 1 \text{ and } f(x) = 0 \text{ outside the interval } [0,1]$$

The mean and the variance of the beta distribution is given below

*Mean* denoted $E(X) = \mu = \frac{\alpha}{\alpha+\beta}$ and variance denoted $\mathrm{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$



### 3.6.2 Bayesian Methodology

The prior beta distribution is used to together with the likelihood function to obtain the posterior distribution. The beta distribution being a conjugate prior always has a posterior beta distribution with parameters

$$\alpha^{*} = \sum x + \alpha$$
$$\beta^{*} = n - \sum x + \beta$$

Where $n$ the sample is size and $\sum x$ is the total number of sunshine hours in the sampled data under the threshold of 120kwh/m^2.The alpha and beta values were computed and used as the initial guess. The posterior parameters were computed using the formula above. The posterior alpha and beta values were used as the prior alpha and beta values to compute the posterior parameters for that iteration.

The posterior mean and variance for the posterior probability distribution was computed as below for the number of iterations.

$$E(X) = \frac{\alpha^{*}}{\alpha^{*} + \beta^{*}}$$

$$Var(X) = \frac{\alpha^{*}\beta^{*}}{(\alpha^{*} + \beta^{*})^{2}(\alpha^{*} + \beta^{*} + 1)}$$

### 3.6.3 Presentation of Results

A sample of size 1500 was selected from each month of the year. Based on the threshold value the total number of sunshine hours is calculated to help in the computation of posterior beta distribution parameters.

### 3.6.4 Bayesian Results for January

The sample selected from the month of January was analyzed using Bayesian approach.

Twenty iterations were performed and the results are shown on the table 3.34 below

The results in table 3.34 indicate that the prior and posterior means converges to 0.87000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 4.0E-06 at 0.000001 tolerance level. The analysis shows that the month of January has a high possibility (.87000) of sunshine.

Table 3.34 Bayesian analysis for January

| ITERATION S | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 27 | 7 | 0.794117647 | 0.00467128 | 1332 | 202 | 0.868318123 | 7.44897E-05 |
| 2 | 1332 | 202 | 0.868318123 | 7.44897E-05 | 2637 | 397 | 0.869149637 | 3.74723E-05 |
| 3 | 2637 | 397 | 0.869149637 | 3.74723E-05 | 3942 | 592 | 0.869430966 | 2.50321E-05 |
| 4 | 3942 | 592 | 0.869430966 | 2.50321E-05 | 5247 | 787 | 0.869572423 | 1.87931E-05 |
| 5 | 5247 | 787 | 0.869572423 | 1.87931E-05 | 6552 | 982 | 0.869657552 | 1.50436E-05 |
| 6 | 6552 | 982 | 0.869657552 | 1.50436E-05 | 7857 | 1177 | 0.869714412 | 1.25414E-05 |
| 7 | 7857 | 1177 | 0.869714412 | 1.25414E-05 | 9162 | 1372 | 0.869755079 | 1.07528E-05 |
| 8 | 9162 | 1372 | 0.869755079 | 1.07528E-05 | 10467 | 1567 | 0.869785607 | 9.41077E-06 |
| 9 | 10467 | 1567 | 0.869785607 | 9.41077E-06 | 11772 | 1762 | 0.869809369 | 8.36653E-06 |
| 10 | 11772 | 1762 | 0.869809369 | 8.36653E-06 | 13077 | 1957 | 0.869828389 | 7.53089E-06 |
| 11 | 13077 | 1957 | 0.869828389 | 7.53089E-06 | 14382 | 2152 | 0.869843958 | 6.84702E-06 |
| 12 | 14382 | 2152 | 0.869843958 | 6.84702E-06 | 15687 | 2347 | 0.869856937 | 6.27701E-06 |
| 13 | 15687 | 2347 | 0.869856937 | 6.27701E-06 | 16992 | 2542 | 0.869867923 | 5.79461E-06 |
| 14 | 16992 | 2542 | 0.869867923 | 5.79461E-06 | 18297 | 2737 | 0.869877341 | 5.38107E-06 |
| 15 | 18297 | 2737 | 0.869877341 | 5.38107E-06 | 19602 | 2932 | 0.869885506 | 5.02262E-06 |
| 16 | 19602 | 2932 | 0.869885506 | 5.02262E-06 | 20907 | 3127 | 0.869892652 | 4.70894E-06 |
| 17 | 20907 | 3127 | 0.869892652 | 4.70894E-06 | 22212 | 3322 | 0.869898958 | 4.43214E-06 |
| 18 | 22212 | 3322 | 0.869898958 | 4.43214E-06 | 23517 | 3517 | 0.869904565 | 4.18608E-06 |
| 19 | 23517 | 3517 | 0.869904565 | 4.18608E-06 | 24822 | 3712 | 0.869909582 | 3.9659E-06 |
| 20 | 24822 | 3712 | 0.869909582 | 3.9659E-06 | 26127 | 3907 | 0.869914097 | 3.76772E-06 |

### 3.6.5 Bayesian Analysis for February

The sample selected from the month of February was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.35. The results in the table 3.35 below indicate that the prior and posterior means converge to 0.83000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of February has a high possibility (0.82000) of sunshine.

Table 3.35: Bayesian analysis for February

| Iterations | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 6 | 0.142857143 | 0.015306122 | 1046 | 461 | 0.694094227 | 0.000140801 |
| 2 | 1046 | 461 | 0.694094227 | 0.000140801 | 2091 | 671 | 0.757060101 | 6.65654E-05 |
| 3 | 2091 | 671 | 0.757060101 | 6.65654E-05 | 3136 | 881 | 0.780682101 | 4.26126E-05 |
| 4 | 3136 | 881 | 0.780682101 | 4.26126E-05 | 4181 | 1091 | 0.793057663 | 3.11241E-05 |
| 5 | 4181 | 1091 | 0.793057663 | 3.11241E-05 | 5226 | 1301 | 0.800674123 | 2.44478E-05 |
| 6 | 5226 | 1301 | 0.800674123 | 2.44478E-05 | 6271 | 1511 | 0.805833976 | 2.01035E-05 |
| 7 | 6271 | 1511 | 0.805833976 | 2.01035E-05 | 7316 | 1721 | 0.809560695 | 1.70582E-05 |
| 8 | 7316 | 1721 | 0.809560695 | 1.70582E-05 | 8361 | 1931 | 0.812378546 | 1.48081E-05 |
| 9 | 8361 | 1931 | 0.812378546 | 1.48081E-05 | 9406 | 2141 | 0.814583875 | 1.30791E-05 |
| 10 | 9406 | 2141 | 0.814583875 | 1.30791E-05 | 10451 | 2351 | 0.816356819 | 1.17096E-05 |
| 11 | 10451 | 2351 | 0.816356819 | 1.17096E-05 | 11496 | 2561 | 0.817813189 | 1.05986E-05 |
| 12 | 11496 | 2561 | 0.817813189 | 1.05986E-05 | 12541 | 2771 | 0.819030825 | 9.67931E-06 |
| 13 | 12541 | 2771 | 0.819030825 | 9.67931E-06 | 13586 | 2981 | 0.820063983 | 8.90627E-06 |
| 14 | 13586 | 2981 | 0.820063983 | 8.90627E-06 | 14631 | 3191 | 0.820951633 | 8.24721E-06 |
| 15 | 14631 | 3191 | 0.820951633 | 8.24721E-06 | 15676 | 3401 | 0.821722493 | 7.67872E-06 |
| 16 | 15676 | 3401 | 0.821722493 | 7.67872E-06 | 16721 | 3611 | 0.82239819 | 7.18337E-06 |
| 17 | 16721 | 3611 | 0.82239819 | 7.18337E-06 | 17766 | 3821 | 0.822995321 | 6.74792E-06 |
| 18 | 17766 | 3821 | 0.822995321 | 6.74792E-06 | 18811 | 4031 | 0.823526837 | 6.36214E-06 |
| 19 | 18811 | 4031 | .0.823526837 | 6.36214E-06 | 19856 | 4241 | 0.824002988 | 6.01801E-06 |
| 20 | 19856 | 4241 | 0.824002988 | 6.01801E-06 | 20901 | 4451 | 0.824431997 | 5.70914E-06 |



### 3.6.6 Bayesian Analysis for March

Table 3.36: Bayesian analysis for March

| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 13 | 1 | 0.928571429 | 0.004421769 | 1363 | 151 | 0.900264201 | 5.92664E-05 |
| 2 | 1363 | 151 | 0.900264201 | 5.92664E-05 | 2713 | 301 | 0.900132714 | 2.98155E-05 |
| 3 | 2713 | 301 | 0.900132714 | 2.98155E-05 | 4063 | 451 | 0.900088613 | 1.99179E-05 |
| 4 | 4063 | 451 | 0.900088613 | 1.99179E-05 | 5413 | 601 | 0.900066511 | 1.49537E-05 |
| 5 | 5413 | 601 | 0.900066511 | 1.49537E-05 | 6763 | 751 | 0.900053234 | 1.19704E-05 |
| 6 | 6763 | 751 | 0.900053234 | 1.19704E-05 | 8113 | 901 | 0.900044375 | 9.97942E-06 |
| 7 | 8113 | 901 | 0.900044375 | 9.97942E-06 | 9463 | 1051 | 0.900038045 | 8.55631E-06 |
| 8 | 9463 | 1051 | 0.900038045 | 8.55631E-06 | 10813 | 1201 | 0.900033294 | 7.48842E-06 |
| 9 | 10813 | 1201 | 0.900033294 | 7.48842E-06 | 12163 | 1351 | 0.900029599 | 6.65752E-06 |
| 10 | 12163 | 1351 | 0.900029599 | 6.65752E-06 | 13513 | 1501 | 0.900026642 | 5.99259E-06 |
| 11 | 13513 | 1501 | 0.900026642 | 5.99259E-06 | 14863 | 1651 | 0.900024222 | 5.44842E-06 |
| 12 | 14863 | 1651 | 0.900024222 | 5.44842E-06 | 16213 | 1801 | 0.900022205 | 4.99485E-06 |
| 13 | 16213 | 1801 | 0.900022205 | 4.99485E-06 | 17563 | 1951 | 0.900020498 | 4.611E-06 |
| 14 | 17563 | 1951 | 0.900020498 | 4.611E-06 | 18913 | 2101 | 0.900019035 | 4.28193E-06 |
| 15 | 18913 | 2101 | 0.900019035 | 4.28193E-06 | 20263 | 2251 | 0.900017767 | 3.9967E-06 |
| 16 | 20263 | 2251 | 0.900017767 | 3.9967E-06 | 21613 | 2401 | 0.900016657 | 3.7471E-06 |
| 17 | 21613 | 2401 | 0.900016657 | 3.7471E-06 | 22963 | 2551 | 0.900015678 | 3.52685E-06 |
| 18 | 22963 | 2551 | 0.900015678 | 3.52685E-06 | 24313 | 2701 | 0.900014807 | 3.33104E-06 |
| 19 | 24313 | 2701 | 0.900014807 | 3.33104E-06 | 25663 | 2851 | 0.900014028 | 3.15584E-06 |
| 20 | 25663 | 2851 | 0.900014028 | 3.15584E-06 | 27013 | 3001 | 0.900013327 | 2.99815E-06 |

The sample selected from the month of March was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.36. The results in the table 3.35 indicate that the prior and posterior means converges to 0.9000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 3.0E-06 at 0.000001 tolerance level. The analysis shows that the month of February has a high possibility (0.9000) of sunshine.

### 3.6.7 Bayesian Analysis for April

The sample selected from the month of April was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3..37 The results in the table 3.37 indicate that the prior and posterior means converge to 0.86000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of April has a high possibility (0.86000) of sunshine

Table 3.37: Bayesian analysis for April

| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 26 | 16 | 0.619047619 | 0.005484364 | 1316 | 226 | 0.853437095 | 8.10643E-05 |
| 2 | 1316 | 226 | 0.853437095 | 8.10643E-05 | 2606 | 436 | 0.856673241 | 4.03497E-05 |
| 3 | 2606 | 436 | 0.856673241 | 4.03497E-05 | 3896 | 646 | 0.857771907 | 2.68543E-05 |
| 4 | 3896 | 646 | 0.857771907 | 2.68543E-05 | 5186 | 856 | 0.858325058 | 2.0123E-05 |
| 5 | 5186 | 856 | 0.858325058 | 2.0123E-05 | 6476 | 1066 | 0.858658181 | 1.60897E-05 |
| 6 | 6476 | 1066 | 0.858658181 | 1.60897E-05 | 7766 | 1276 | 0.858880779 | 1.34031E-05 |
| 7 | 7766 | 1276 | 0.858880779 | 1.34031E-05 | 9056 | 1486 | 0.85904003 | 1.14854E-05 |
| 8 | 9056 | 1486 | 0.85904003 | 1.14854E-05 | 10346 | 1696 | 0.859159608 | 1.00477E-05 |
| 9 | 10346 | 1696 | 0.859159608 | 1.00477E-05 | 11636 | 1906 | 0.859252695 | 8.92989E-06 |
| 10 | 11636 | 1906 | 0.859252695 | 8.92989E-06 | 12926 | 2116 | 0.859327217 | 8.03589E-06 |
| 11 | 12926 | 2116 | 0.859327217 | 8.03589E-06 | 14216 | 2326 | 0.859388224 | 7.30461E-06 |
| 12 | 14216 | 2326 | 0.859388224 | 7.30461E-06 | 15506 | 2536 | 0.859439087 | 6.69531E-06 |
| 13 | 15506 | 2536 | 0.859439087 | 6.69531E-06 | 16796 | 2746 | 0.859482141 | 6.17984E-06 |
| 14 | 16796 | 2746 | 0.859482141 | 6.17984E-06 | 18086 | 2956 | 0.859519057 | 5.73806E-06 |
| 15 | 18086 | 2956 | 0.859519057 | 5.73806E-06 | 19376 | 3166 | 0.85955106 | 5.35523E-06 |
| 16 | 19376 | 3166 | 0.85955106 | 5.35523E-06 | 20666 | 3376 | 0.85957907 | 5.02029E-06 |
| 17 | 20666 | 3376 | 0.85957907 | 5.02029E-06 | 21956 | 3586 | 0.85960379 | 4.72478E-06 |
| 18 | 21956 | 3586 | 0.85960379 | 4.72478E-06 | 23246 | 3796 | 0.859625767 | 4.46213E-06 |
| 19 | 23246 | 3796 | 0.859625767 | 4.46213E-06 | 24536 | 4006 | 0.859645435 | 4.22714E-06 |
| 20 | 24536 | 4006 | 0.859645435 | 4.22714E-06 | 25826 | 4216 | 0.859663138 | 4.01566E-06 |

### 3.6.8 Bayesian Analysis for May

The sample selected from the month of February was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.38. The results in the table 3.38 indicate that the prior and posterior means converge to 0.92000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 2.0E-06 at 0.000001 tolerance level. The analysis shows that the month of February has a very high possibility (0.92000) of sunshine.

Table 3.38: Bayesian analysis for May



| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 15 | 1 | 0.9375 | 0.003446691 | 1400 | 116 | 0.92348285 | 4.65803E-05 |
| 2 | 1400 | 116 | 0.92348285 | 4.65803E-05 | 2785 | 231 | 0.923408488 | 2.34422E-05 |
| 3 | 2785 | 231 | 0.923408488 | 2.34422E-05 | 4170 | 346 | 0.923383525 | 1.56623E-05 |
| 4 | 4170 | 346 | 0.923383525 | 1.56623E-05 | 5555 | 461 | 0.923371011 | 1.17595E-05 |
| 5 | 5555 | 461 | 0.923371011 | 1.17595E-05 | 6940 | 576 | 0.923363491 | 9.41378E-06 |
| 6 | 6940 | 576 | 0.923363491 | 9.41378E-06 | 8325 | 691 | 0.923358474 | 7.84824E-06 |
| 7 | 8325 | 691 | 0.923358474 | 7.84824E-06 | 9710 | 806 | 0.923354888 | 6.72917E-06 |
| 8 | 9710 | 806 | 0.923354888 | 6.72917E-06 | 11095 | 921 | 0.923352197 | 5.8894E-06 |
| 9 | 11095 | 921 | 0.923352197 | 5.8894E-06 | 12480 | 1036 | 0.923350104 | 5.23598E-06 |
| 10 | 12480 | 1036 | 0.923350104 | 5.23598E-06 | 13865 | 1151 | 0.923348428 | 4.71307E-06 |
| 11 | 13865 | 1151 | 0.923348428 | 4.71307E-06 | 15250 | 1266 | 0.923347057 | 4.28512E-06 |
| 12 | 15250 | 1266 | 0.923347057 | 4.28512E-06 | 16635 | 1381 | 0.923345915 | 3.92841E-06 |
| 13 | 16635 | 1381 | 0.923345915 | 3.92841E-06 | 18020 | 1496 | 0.923344948 | 3.62653E-06 |
| 14 | 18020 | 1496 | 0.923344948 | 3.62653E-06 | 19405 | 1611 | 0.923344119 | 3.36774E-06 |
| 15 | 19405 | 1611 | 0.923344119 | 3.36774E-06 | 20790 | 1726 | 0.9233434 | 3.14342E-06 |
| 16 | 20790 | 1726 | 0.9233434 | 3.14342E-06 | 22175 | 1841 | 0.923342771 | 2.94712E-06 |
| 17 | 22175 | 1841 | 0.923342771 | 2.94712E-06 | 23560 | 1956 | 0.923342217 | 2.77389E-06 |
| 18 | 23560 | 1956 | 0.923342217 | 2.77389E-06 | 24945 | 2071 | 0.923341723 | 2.6199E-06 |
| 19 | 24945 | 2071 | 0.923341723 | 2.6199E-06 | 26330 | 2186 | 0.923341282 | 2.4821E-06 |
| 20 | 26330 | 2186 | 0.923341282 | 2.4821E-06 | 27715 | 2301 | 0.923340885 | 2.35808E-06 |

### 3.6.9 Bayesian Analysis for June

The sample selected from the month of June was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.39. The results in table 3.39 indicates that the prior and posterior means converge to 0.77000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of February has a high possibility (0.77000) of sunshine

Table 3.39: Bayesian analysis for June

| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 21 | 18 | 0.538461538 | 0.006213018 | 1075 | 464 | 0.698505523 | 0.00013675 |
| 2 | 1075 | 464 | 0.698505523 | 0.00013675 | 2129 | 910 | 0.700559395 | 6.90052E-05 |
| 3 | 2129 | 910 | 0.700559395 | 6.90052E-05 | 3183 | 1356 | 0.701255783 | 4.61445E-05 |
| 4 | 3183 | 1356 | 0.701255783 | 4.61445E-05 | 4237 | 1802 | 0.701606226 | 3.46614E-05 |
| 5 | 4237 | 1802 | 0.701606226 | 3.46614E-05 | 5291 | 2248 | 0.701817217 | 2.77546E-05 |
| 6 | 5291 | 2248 | 0.701817217 | 2.77546E-05 | 6345 | 2694 | 0.701958181 | 2.3143E-05 |
| 7 | 6345 | 2694 | 0.701958181 | 2.3143E-05 | 7399 | 3140 | 0.702059019 | 1.98456E-05 |
| 8 | 7399 | 3140 | 0.702059019 | 1.98456E-05 | 8453 | 3586 | 0.702134729 | 1.73706E-05 |
| 9 | 10346 | 1696 | 0.859159608 | 1.00477E-05 | 11400 | 2142 | 0.841825432 | 9.83204E-06 |
| 10 | 11400 | 2142 | 0.841825432 | 9.83204E-06 | 12454 | 2588 | 0.827948411 | 9.46951E-06 |
| 11 | 12454 | 2588 | 0.827948411 | 9.46951E-06 | 13508 | 3034 | 0.816588079 | 9.0535E-06 |
| 12 | 13508 | 3034 | 0.816588079 | 9.0535E-06 | 14562 | 3480 | 0.807116728 | 8.62824E-06 |
| 13 | 14562 | 3480 | 0.807116728 | 8.62824E-06 | 15616 | 3926 | 0.799099376 | 8.21468E-06 |
| 14 | 15616 | 3926 | 0.799099376 | 8.21468E-06 | 16670 | 4372 | 0.792225074 | 7.82229E-06 |
| 15 | 16670 | 4372 | 0.792225074 | 7.82229E-06 | 17724 | 4818 | 0.786265637 | 7.45473E-06 |
| 16 | 17724 | 4818 | 0.786265637 | 7.45473E-06 | 18778 | 5264 | 0.781049829 | 7.11271E-06 |
| 17 | 18778 | 5264 | 0.781049829 | 7.11271E-06 | 19832 | 5710 | 0.776446637 | 6.79549E-06 |
| 18 | 19832 | 5710 | 0.776446637 | 6.79549E-06 | 20886 | 6156 | 0.772354116 | 6.50162E-06 |
| 19 | 20886 | 6156 | 0.772354116 | 6.50162E-06 | 21940 | 6602 | 0.768691753 | 6.22936E-06 |
| 20 | 21940 | 6602 | 0.768691753 | 6.22936E-06 | 22994 | 7048 | 0.765395114 | 5.97695E-06 |



## 3.6.10 Bayesian Analysis for July

Table 3.40: Bayesian analysis for July

| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 28 | 14 | 0.666666667 | 0.005167959 | 1082 | 427 | 0.717031146 | 0.000134369 |
| 2 | 1082 | 427 | 0.717031146 | 0.000134369 | 2136 | 840 | 0.717741935 | 6.80512E-05 |
| 3 | 2136 | 840 | 0.717741935 | 6.80512E-05 | 3190 | 1253 | 0.717983345 | 4.55633E-05 |
| 4 | 3190 | 1253 | 0.717983345 | 4.55633E-05 | 4244 | 1666 | 0.718104907 | 3.42464E-05 |
| 5 | 4244 | 1666 | 0.718104907 | 3.42464E-05 | 5298 | 2079 | 0.718178121 | 2.74327E-05 |
| 6 | 5298 | 2079 | 0.718178121 | 2.74327E-05 | 6352 | 2492 | 0.718227047 | 2.28804E-05 |
| 7 | 6352 | 2492 | 0.718227047 | 2.28804E-05 | 7406 | 2905 | 0.71826205 | 1.96239E-05 |
| 8 | 7406 | 2905 | 0.71826205 | 1.96239E-05 | 8460 | 3318 | 0.718288334 | 1.71789E-05 |
| 9 | 8460 | 3318 | 0.718288334 | 1.71789E-05 | 9514 | 3731 | 0.718308796 | 1.52757E-05 |
| 10 | 9514 | 3731 | 0.718308796 | 1.52757E-05 | 10568 | 4144 | 0.718325177 | 1.37521E-05 |
| 11 | 10568 | 4144 | 0.718325177 | 1.37521E-05 | 11622 | 4557 | 0.718338587 | 1.25048E-05 |
| 12 | 11622 | 4557 | 0.718338587 | 1.25048E-05 | 12676 | 4970 | 0.718349768 | 1.1465E-05 |
| 13 | 12676 | 4970 | 0.718349768 | 1.1465E-05 | 13730 | 5383 | 0.718359232 | 1.05849E-05 |
| 14 | 13730 | 5383 | 0.718359232 | 1.05849E-05 | 14784 | 5796 | 0.718367347 | 9.83022E-06 |
| 15 | 14784 | 5796 | 0.718367347 | 9.83022E-06 | 15838 | 6209 | 0.718374382 | 9.17601E-06 |
| 16 | 15838 | 6209 | 0.718374382 | 9.17601E-06 | 16892 | 6622 | 0.718380539 | 8.60344E-06 |
| 17 | 16892 | 6622 | 0.718380539 | 8.60344E-06 | 17946 | 7035 | 0.718385973 | 8.09813E-06 |
| 18 | 17946 | 7035 | 0.718385973 | 8.09813E-06 | 19000 | 7448 | 0.718390805 | 7.64889E-06 |
| 19 | 19000 | 7448 | 0.718390805 | 7.64889E-06 | 20054 | 7861 | 0.718395128 | 7.24687E-06 |
| 20 | 20054 | 7861 | 0.718395128 | 7.24687E-06 | 21108 | 8274 | 0.71839902 | 6.885E-06 |

The sample selected from the month of July was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.40. The results in the table 3.40 below indicates that the prior means and posterior mean converges to 0.72000 tolerance level of 0.0001 while the prior and posterior variances are converging to 7.0E-06 at 0.000001 tolerance level. The analysis shows that the month of July has a greater possibility (0.72000) of sunshine

### 3.6.11 Bayesian Analysis for August

Table 3.41: Bayesian analysis for August



| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 26 | 16 | 0.619047619 | 0.005484364 | 1277 | 265 | 0.828145266 | 9.22363E-05 |
| 2 | 1277 | 265 | 0.828145266 | 9.22363E-05 | 2528 | 514 | 0.831032216 | 4.61445E-05 |
| 3 | 2528 | 514 | 0.831032216 | 4.61445E-05 | 3779 | 763 | 0.832012329 | 3.07655E-05 |
| 4 | 3779 | 763 | 0.832012329 | 3.07655E-05 | 5030 | 1012 | 0.832505793 | 2.30746E-05 |
| 5 | 5030 | 1012 | 0.832505793 | 2.30746E-05 | 6281 | 1261 | 0.83280297 | 1.84598E-05 |
| 6 | 6281 | 1261 | 0.83280297 | 1.84598E-05 | 7532 | 1510 | 0.833001548 | 1.53832E-05 |
| 7 | 7532 | 1510 | 0.833001548 | 1.53832E-05 | 8783 | 1759 | 0.833143616 | 1.31856E-05 |
| 8 | 8783 | 1759 | 0.833143616 | 1.31856E-05 | 10034 | 2008 | 0.833250291 | 1.15373E-05 |
| 9 | 10034 | 2008 | 0.833250291 | 1.15373E-05 | 11285 | 2257 | 0.833333333 | 1.02554E-05 |
| 10 | 11636 | 1906 | 0.859252695 | 8.92989E-06 | 12887 | 2155 | 0.856734477 | 8.15931E-06 |
| 11 | 12887 | 2155 | 0.856734477 | 8.15931E-06 | 14138 | 2404 | 0.854672954 | 7.50814E-06 |
| 12 | 14138 | 2404 | 0.854672954 | 7.50814E-06 | 15389 | 2653 | 0.852954218 | 6.95136E-06 |
| 13 | 15389 | 2653 | 0.852954218 | 6.95136E-06 | 16640 | 2902 | 0.851499335 | 6.47026E-06 |
| 14 | 16640 | 2902 | 0.851499335 | 6.47026E-06 | 17891 | 3151 | 0.850251877 | 6.05064E-06 |
| 15 | 17891 | 3151 | 0.850251877 | 6.05064E-06 | 19142 | 3400 | 0.849170437 | 5.68159E-06 |
| 16 | 19142 | 3400 | 0.849170437 | 5.68159E-06 | 20393 | 3649 | 0.848223941 | 5.35458E-06 |
| 17 | 20666 | 20393 | 3649 | 6.08838E-06 | 21917 | 20642 | 0.514979205 | 5.86879E-06 |
| 18 | 21917 | 20642 | 0.514979205 | 5.86879E-06 | 23168 | 20891 | 0.52584035 | 5.65893E-06 |
| 19 | 23168 | 20891 | 0.52584035 | 5.65893E-06 | 24419 | 21140 | 0.535986303 | 5.45885E-06 |
| 20 | 24419 | 21140 | 0.535986303 | 5.45885E-06 | 25670 | 21389 | 0.545485454 | 5.2684E-06 |

The sample selected from the month of August was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown in table 3.41. The results in table 3.41 indicates that the prior and posterior means converge to 0.55000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of August has a low possibility (0.55000) of sunshine

## 3.6.12 Bayesian Analysis for September

The sample selected from the month of September was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.42. The results in the table 3.42 below indicate that the prior and posterior means converge to 0.79000 with tolerance level of 0.0001 while the prior and posterior variances are converge to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of September has a high possibility (0.79000) of sunshine



Table 3.42: Bayesian analysis for September

| iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 29 | 13 | 0.690476 | 0.004970205 | 1207 | 335 | 0.782749676 | 0.000110209 |
| 2 | 1207 | 335 | 0.78775 | 0.000110209 | 2385 | 657 | 0.784023669 | 5.56459E-05 |
| 3 | 2385 | 657 | 0.784024 | 5.56459E-05 | 3563 | 979 | 0.784456187 | 3.72187E-05 |
| 4 | 3563 | 979 | 0.784456 | 3.72187E-05 | 4741 | 1301 | 0.784673949 | 2.79597E-05 |
| 5 | 4741 | 1301 | 0.784674 | 2.79597E-05 | 5919 | 1623 | 0.784805091 | 2.23898E-05 |
| 6 | 5919 | 1623 | 0.784805 | 2.23898E-05 | 7097 | 1945 | 0.784892723 | 1.86704E-05 |
| 7 | 7097 | 1945 | 0.784893 | 1.86704E-05 | 8275 | 2267 | 0.784955416 | 1.60107E-05 |
| 8 | 8275 | 2267 | 0.784955 | 1.60107E-05 | 9453 | 2589 | 0.785002491 | 1.40142E-05 |
| 9 | 9453 | 2589 | 0.785002 | 1.40142E-05 | 10631 | 2911 | 0.785039137 | 1.24605E-05 |
| 10 | 10631 | 2911 | 0.785039 | 1.24605E-05 | 11809 | 3233 | 0.785068475 | 1.12169E-05 |
| 11 | 11809 | 3233 | 0.785068 | 1.12169E-05 | 12987 | 3555 | 0.785092492 | 1.0199E-05 |
| 12 | 12987 | 3555 | 0.785092 | 1.0199E-05 | 14165 | 3877 | 0.785112515 | 9.35049E-06 |
| 13 | 14165 | 3877 | 0.785113 | 9.35049E-06 | 15343 | 4199 | 0.785129465 | 8.63231E-06 |
| 14 | 15343 | 4199 | 0.785129 | 8.63231E-06 | 16521 | 4521 | 0.785143998 | 8.01658E-06 |
| 15 | 16521 | 4521 | 0.785144 | 8.01658E-06 | 17699 | 4843 | 0.785156597 | 7.48284E-06 |
| 16 | 17699 | 4843 | 0.785157 | 7.48284E-06 | 18877 | 5165 | 0.785167623 | 7.01574E-06 |
| 17 | 18877 | 5165 | 0.785168 | 7.01574E-06 | 20055 | 5487 | 0.785177355 | 6.60353E-06 |
| 18 | 20055 | 5487 | 0.785177 | 6.60353E-06 | 21233 | 5809 | 0.785186007 | 6.23706E-06 |
| 19 | 21233 | 5809 | 0.785186 | 6.23706E-06 | 22411 | 6131 | 0.78519375 | 5.90914E-06 |
| 20 | 22411 | 6131 | 0.785194 | 5.90914E-06 | 23589 | 6453 | 0.785200719 | 5.61397E-06 |

## 3.6.13 Bayesian Analysis for October

A sample selected from the data for October was analyzed using the Bayesian approach. Iterations performed and the results (see table 3.43) indicate the prior and posterior means converge to 0.7200 at tolerance level of 0.0001 while the prior and posterior variances converge to 7.0E-06 at 0.000001 tolerance level. Thus the month of October has a high possibility of sunshine.

Table 3.43: Bayesian analysis for October

| iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 29 | 9 | 0.763157895 | 0.004634562 | 1104 | 434 | 0.717815345 | 0.000131616 |
| 2 | 1104 | 434 | 0.717815345 | 0.000131616 | 2179 | 859 | 0.71724819 | 6.67335E-05 |
| 3 | 2179 | 859 | 0.71724819 | 6.67335E-05 | 3254 | 1284 | 0.717055972 | 4.46985E-05 |
| 4 | 3254 | 1284 | 0.717055972 | 4.46985E-05 | 4329 | 1709 | 0.716959258 | 3.3603E-05 |
| 5 | 4329 | 1709 | 0.716959258 | 3.3603E-05 | 5404 | 2134 | 0.716901035 | 2.69205E-05 |
| 6 | 5404 | 2134 | 0.716901035 | 2.69205E-05 | 6479 | 2559 | 0.716862138 | 2.2455E-05 |
| 7 | 6479 | 2559 | 0.716862138 | 2.2455E-05 | 7554 | 2984 | 0.716834314 | 1.92602E-05 |
| 8 | 7554 | 2984 | 0.716834314 | 1.92602E-05 | 8629 | 3409 | 0.716813424 | 1.68612E-05 |
| 9 | 8629 | 3409 | 0.716813424 | 1.68612E-05 | 9704 | 3834 | 0.716797164 | 1.49936E-05 |
| 10 | 9704 | 3834 | 0.716797164 | 1.49936E 05 | 10779 | 4259 | 0.716784147 | 1.34985E-05 |
| 11 | 10779 | 4259 | 0.716784147 | 1.34985E-05 | 11854 | 4684 | 0.716773491 | 1.22746E-05 |
| 12 | 11854 | 4684 | 0.716773491 | 1.22746E-05 | 12929 | 5109 | 0.716764608 | 1.12541E-05 |
| 13 | 12929 | 5109 | 0.716764608 | 1.12541E-05 | 14004 | 5534 | 0.716757089 | 1.03903E-05 |
| 14 | 14004 | 5534 | 0.716757089 | 1.03903E-05 | 15079 | 5959 | 0.716750642 | 9.64966E-06 |
| 15 | 15079 | 5959 | 0.716750642 | 9.64966E-06 | 16154 | 6384 | 0.716745053 | 9.00757E-06 |
| 16 | 16154 | 6384 | 0.716745053 | 9.00757E-06 | 17229 | 6809 | 0.716740161 | 8.4456E-06 |
| 17 | 17229 | 6809 | 0.716740161 | 8.4456E-06 | 18304 | 7234 | 0.716735845 | 7.94963E-06 |
| 18 | 18304 | 7234 | 0.716735845 | 7.94963E-06 | 19379 | 7659 | 0.716732007 | 7.50868E-06 |
| 19 | 19379 | 7659 | 0.716732007 | 7.50868E 06 | 20454 | 8084 | 0.716728572 | 7.11408E-06 |
| 20 | 20454 | 8084 | 0.716728572 | 7.11408E-06 | 21529 | 8509 | 0.716725481 | 6.75888E-06 |

### 3.6.14 Bayesian Analysis for November

A sample selected from data for November was analyzed using the Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.44. The results in the table 3.44 below indicate that the prior and posterior means converge to 0.9000 with tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of November has a high possibility (0.9000) of sunshine.

Table 3.44: Bayesian analysis for November

| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 28 | 6 | 0.823529412 | 0.004152249 | 1406 | 128 | 0.916558018 | 4.98237E-05 |
| 2 | 1406 | 128 | 0.916558018 | 4.98237E-05 | 2784 | 250 | 0.917600527 | 2.49126E-05 |
| 3 | 2784 | 250 | 0.917600527 | 2.49126E-05 | 4162 | 372 | 0.917953242 | 1.66075E-05 |
| 4 | 4162 | 372 | 0.917953242 | 1.66075E-05 | 5540 | 494 | 0.918130593 | 1.24551E-05 |
| 5 | 5540 | 494 | 0.918130593 | 1.24551E-05 | 6918 | 616 | 0.918237324 | 9.96384E-06 |
| 6 | 6918 | 616 | 0.918237324 | 9.96384E-06 | 8296 | 738 | 0.918308612 | 8.30303E-06 |
| 7 | 7766 | 1276 | 0.858880779 | 1.34031E-05 | 9144 | 1398 | 0.867387592 | 1.09102E-05 |
| 8 | 9144 | 1398 | 0.867387592 | 1.09102E-05 | 10522 | 1520 | 0.87377512 | 9.1582E-06 |
| 9 | 10522 | 1520 | 0.87377512 | 9.1582E-06 | 11900 | 1642 | 0.8787476 | 7.86755E-06 |
| 10 | 11900 | 1642 | 0.8787476 | 7.86755E-06 | 13278 | 1764 | 0.882728361 | 6.88154E-06 |
| 11 | 13278 | 1764 | 0.882728361 | 6.88154E-06 | 14656 | 1886 | 0.885987184 | 6.10614E-06 |
| 12 | 14656 | 1886 | 0.885987184 | 6.10614E-06 | 16034 | 2008 | 0.888704135 | 5.48185E-06 |
| 13 | 16034 | 2008 | 0.888704135 | 5.48185E-06 | 17412 | 2130 | 0.891003991 | 4.96934E-06 |
| 14 | 17412 | 2130 | 0.891003991 | 4.96934E-06 | 18790 | 2252 | 0.892975953 | 4.54165E-06 |
| 15 | 18790 | 2252 | 0.892975953 | 4.54165E-06 | 20168 | 2374 | 0.894685476 | 4.17972E-06 |
| 16 | 20168 | 2374 | 0.894685476 | 4.17972E-06 | 21546 | 2496 | 0.896181682 | 3.86974E-06 |
| 17 | 21546 | 2496 | 0.896181682 | 3.86974E-06 | 22924 | 2618 | 0.897502153 | 3.60146E-06 |
| 18 | 22924 | 2618 | 0.897502153 | 3.60146E-06 | 24302 | 2740 | 0.898676133 | 3.36713E-06 |
| 19 | 24302 | 2740 | 0.898676133 | 3.36713E-06 | 25680 | 2862 | 0.899726719 | 3.16079E-06 |
| 20 | 25680 | 2862 | 0.899726719 | 3.16079E-06 | 27058 | 2984 | 0.900672392 | 2.97779E-06 |



### 3.6.15 Bayesian Analysis for December

The sample selected from data pertaining to the month of December was analyzed using the Bayesian approach. Twenty iterations were performed and the results are shown on the table 3.45. The results in the table 3.45 below indicate that the prior and posterior means converge to 0.87000 at a tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level. The analysis shows that the month of December has a high possibility (.87000) of sunshine.

Table 3.45: Bayesian analysis for December

| Iteration | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 27 | 7 | 0.794117647 | 0.00467128 | 1332 | 202 | 0.868318123 | 7.44897E-05 |
| 2 | 1332 | 202 | 0.868318123 | 7.44897E-05 | 2637 | 397 | 0.869149637 | 3.74723E-05 |
| 3 | 2637 | 397 | 0.869149637 | 3.74723E-05 | 3942 | 592 | 0.869430966 | 2.50321E-05 |
| 4 | 3942 | 592 | 0.869430966 | 2.50321E-05 | 5247 | 787 | 0.869572423 | 1.87931E-05 |
| 5 | 5247 | 787 | 0.869572423 | 1.87931E-05 | 6552 | 982 | 0.869657552 | 1.50436E-05 |
| 6 | 6552 | 982 | 0.869657552 | 1.50436E-05 | 7857 | 1177 | 0.869714412 | 1.25414E-05 |
| 7 | 7857 | 1177 | 0.869714412 | 1.25414E-05 | 9162 | 1372 | 0.869755079 | 1.07528E-05 |
| 8 | 9162 | 1372 | 0.869755079 | 1.07528E-05 | 10467 | 1567 | 0.869785607 | 9.41077E-06 |
| 9 | 10467 | 1567 | 0.869785607 | 9.41077E-06 | 11772 | 1762 | 0.869809369 | 8.36653E-06 |
| 10 | 11772 | 1762 | 0.869809369 | 8.36653E-06 | 13077 | 1957 | 0.869828389 | 7.53089E-06 |
| 11 | 13077 | 1957 | 0.869828389 | 7.53089E-06 | 14382 | 2152 | 0.869843958 | 6.84702E-06 |
| 12 | 14382 | 2152 | 0.869843958 | 6.84702E-06 | 15687 | 2347 | 0.869856937 | 6.27701E-06 |
| 13 | 15687 | 2347 | 0.869856937 | 6.27701E-06 | 16992 | 2542 | 0.869867923 | 5.79461E-06 |
| 14 | 16992 | 2542 | 0.869867923 | 5.79461E-06 | 18297 | 2737 | 0.869877341 | 5.38107E-06 |
| 15 | 18297 | 2737 | 0.869877341 | 5.38107E-06 | 19602 | 2932 | 0.869885506 | 5.02262E-06 |
| 16 | 19602 | 2932 | 0.869885506 | 5.02262E-06 | 20907 | 3127 | 0.869892652 | 4.70894E-06 |
| 17 | 20907 | 3127 | 0.869892652 | 4.70894E-06 | 22212 | 3322 | 0.869898958 | 4.43214E-06 |
| 18 | 22212 | 3322 | 0.869898958 | 4.43214E-06 | 23517 | 3517 | 0.869904565 | 4.18608E-06 |
| 19 | 23517 | 3517 | 0.869904565 | 4.18608E-06 | 24822 | 3712 | 0.869909582 | 3.9659E-06 |
| 20 | 24822 | 3712 | 0.869909582 | 3.9659E-06 | 26127 | 3907 | 0.869914097 | 3.76772E-06 |

### 3.6.16 Bayesian Analysis for Clusters

Samples were selected in respect of months pertaining to the two clusters found and analyzed using the Bayesian approach. Twenty iterations were performed and the results are shown in tables 3.46 and 3.47 for cluster one and two respectively. The results in the table 3.46 and 3.47 indicates that the prior and posterior means converge respectively to 0.86000 and .76000 at the tolerance level of 0.0001 while the prior and posterior variances for cluster one and two converged to 6.0E-06 and 4.0E-06 respectively at 0.000001 tolerance level. The analysis shows that the clusters indicate a high potential (.86000 and 0.76000) of sunshine. This is also an indication that Kumasi generally has a high possibility of sunny days

Table 3.46: Bayesian analysis for cluster one

| ITERATION S | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 23 | 9 | 0.71875 | 0.00612571 | 1103 | 429 | 0.71997389 | 0.000131514 |
| 2 | 1103 | 429 | 0.71997389 | 0.000131514 | 2183 | 624 | 0.777698611 | 6.15682E-05 |
| 3 | 2408 | 624 | 0.794195251 | 5.38903E-05 | 3488 | 819 | 0.809844439 | 3.57466E-05 |
| 4 | 3713 | 819 | 0.819285084 | 3.2662E-05 | 4793 | 1014 | 0.825383158 | 2.4815E-05 |
| 5 | 5018 | 1014 | 0.831896552 | 2.318E-05 | 6098 | 1209 | 0.83454222 | 1.88946E-05 |
| 6 | 6323 | 1209 | 0.839484865 | 1.7888E-05 | 7403 | 1404 | 0.840581356 | 1.52139E-05 |
| 7 | 7628 | 1404 | 0.844552702 | 1.45338E-05 | 8708 | 1599 | 0.844862715 | 1.27153E-05 |
| 8 | 8933 | 1599 | 0.848176984 | 1.22257E-05 | 10013 | 1794 | 0.848056238 | 1.09127E-05 |
| 9 | 10238 | 1794 | 0.850897606 | 1.05436E-05 | 11318 | 1989 | 0.850529796 | 9.55281E-06 |
| 10 | 11543 | 1989 | 0.853015075 | 9.26479E-06 | 12623 | 2184 | 0.852502195 | 8.4915E-06 |
| 11 | 12848 | 2184 | 0.854709952 | 8.26055E-06 | 13928 | 2379 | 0.854111731 | 7.64072E-06 |
| 12 | 14153 | 2379 | 0.856097266 | 7.45144E-06 | 15233 | 2574 | 0.855450104 | 6.9438E-06 |
| 13 | 15458 | 2574 | 0.857253771 | 6.78588E-06 | 16538 | 2769 | 0.856580515 | 6.36267E-06 |
| 14 | 16763 | 2769 | 0.858232644 | 6.22891E-06 | 17843 | 2964 | 0.857547941 | 5.87079E-06 |
| 15 | 18068 | 2964 | 0.85907189 | 5.75607E-06 | 19148 | 3159 | 0.85838526 | 5.44917E-06 |
| 16 | 19373 | 3159 | 0.859799396 | 5.34968E-06 | 20453 | 3354 | 0.859117066 | 5.08379E-06 |
| 17 | 20678 | 3354 | 0.860436085 | 4.99671E-06 | 21758 | 3549 | 0.859762121 | 4.76415E-06 |
| 18 | 21983 | 3549 | 0.860997963 | 4.68729E-06 | 23063 | 3744 | 0.860334987 | 4.4822E-06 |
| 19 | 23288 | 3744 | 0.861497484 | 4.41385E-06 | 24368 | 3939 | 0.86084714 | 4.23164E-06 |
| 20 | 24593 | 3939 | 0.861944483 | 4.17048E-06 | 25673 | 4134 | 0.861307747 | 4.00754E-06 |

Table 3.47: Bayesian analysis for cluster two

| ITERATION S | alpha | beta | prior mean | prior variance | posterior alpha | posterior beta | posterior mean | posterior variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 24 | 9 | 0.727272727 | 0.005833738 | 1171 | 362 | 0.763861709 | 0.000117586 |
| 2 | 1171 | 362 | 0.763861709 | 0.000117586 | 2318 | 715 | 0.764259809 | 5.93826E-05 |
| 3 | 2318 | 715 | 0.764259809 | 5.93826E-05 | 3465 | 1068 | 0.764394441 | 3.97211E-05 |
| 4 | 3465 | 1068 | 0.764394441 | 3.97211E-05 | 4612 | 1421 | 0.764462125 | 2.98409E-05 |
| 5 | 4612 | 1421 | 0.764462125 | 2.98409E-05 | 5759 | 1774 | 0.764502854 | 2.38968E-05 |
| 6 | 5759 | 1774 | 0.764502854 | 2.38968E-05 | 6906 | 2127 | 0.764530056 | 1.99274E-05 |
| 7 | 6906 | 2127 | 0.764530056 | 1.99274E-05 | 8053 | 2480 | 0.764549511 | 1.70888E-05 |
| 8 | 8053 | 2480 | 0.764549511 | 1.70888E-05 | 9200 | 2833 | 0.764564115 | 1.49581E-05 |
| 9 | 9200 | 2833 | 0.764564115 | 1.49581E-05 | 10347 | 3186 | 0.764575482 | 1.32998E-05 |
| 10 | 10347 | 3186 | 0.764575482 | 1.32998E-05 | 11494 | 3539 | 0.764584581 | 1.19725E-05 |
| 11 | 11494 | 3539 | 0.764584581 | 1.19725E-05 | 12641 | 3892 | 0.764592028 | 1.08861E-05 |
| 12 | 12641 | 3892 | 0.764592028 | 1.08861E-05 | 13788 | 4245 | 0.764598237 | 9.98047E-06 |
| 13 | 13788 | 4245 | 0.764598237 | 9.98047E-06 | 14935 | 4598 | 0.764603492 | 9.21393E-06 |
| 14 | 14935 | 4598 | 0.764603492 | 9.21393E-06 | 16082 | 4951 | 0.764607997 | 8.55675E-06 |
| 15 | 16082 | 4951 | 0.764607997 | 8.55675E-06 | 17229 | 5304 | 0.764611903 | 7.98707E-06 |
| 16 | 17229 | 5304 | 0.764611903 | 7.98707E-06 | 18376 | 5657 | 0.764615321 | 7.48851E-06 |
| 17 | 18376 | 5657 | 0.764615321 | 7.48851E-06 | 19523 | 6010 | 0.764618337 | 7.04853E-06 |
| 18 | 19523 | 6010 | 0.764618337 | 7.04853E-06 | 20670 | 6363 | 0.764621019 | 6.65738E-06 |
| 19 | 20670 | 6363 | 0.764621019 | 6.65738E-06 | 21817 | 6716 | 0.764623418 | 6.30737E-06 |
| 20 | 21817 | 6716 | 0.764623418 | 6.30737E-06 | 22964 | 7069 | 0.764625579 | 5.99232E-06 |

# CHAPTER FOUR

# CONCLUSION AND RECOMMENDATIONS

## 4.1 SUMMARY OF RESULTS

### 4.1.1 Probability Distribution

It is well known that the solar irradiation reaching the surface of the earth is not normally distributed. There is the need to find the probability density function that best describes the sunshine hours in the various month of the year. In the present study, the method of mean square error was used to obtain the best distribution among a selection of standard distributions for each month of the year. In each case the standard probability distribution with the smallest mean square error relative to observed data was considered the best probability density function.

It was thus found that the solar irradiation for the month of April was Weibull distribution January, March and May is Lognormal distributed, and those for the months of June, August, September October and November follows Exponential, Weibull, Geometric and gamma distribution while February July and December were Lognormal, Exponential, Weibull, Geometric and gamma distributed.

Table 4.1 below summarizes the results for the various months indicating the relevant probability density function

### 4.1.1 Classification of Distribution by Months

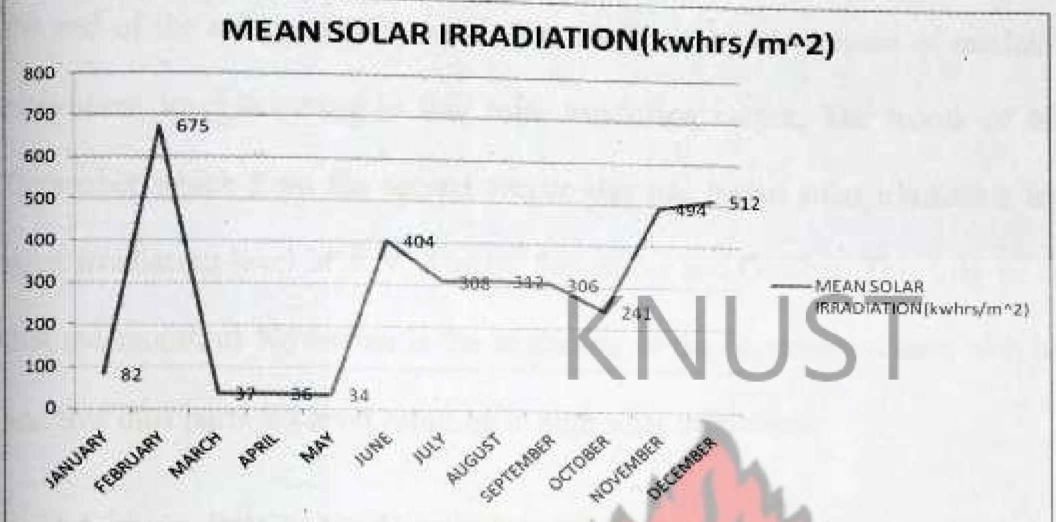
**Table 4.1 summary of monthly probability distributions**

| Month | Probability distributions |
|---|---|
| January | Lognormal |
| February | Lognormal, Exponential, Weibull, Geometric and gamma |
| March | Lognormal |
| April | Weibull, exponential, gamma and geometric |
| May | Lognormal |
| June | Exponential, Weibull, Geometric and gamma |
| July | Exponential, Weibull, Geometric, Lognormal and gamma |
| August | Exponential, Weibull, Geometric and gamma |
| September | Exponential, Weibull, Geometric and gamma |
| October | Exponential, Weibull, Geometric and gamma |
| November | Exponential, Weibull, Geometric and gamma |
| December | Exponential, Weibull, Geometric, Lognormal and gamma |

Detailed charts and tables for the various probability density functions are displayed in the Appendix (II)

### 4.1.2 Mean hourly solar irradiation of the Months

The figure 4.1 Mean hourly solar irradiation of the Months




There are certain climatic factors that are accounting for the rise and fall of the hourly solar irradiation pattern in the year. The factors include cloudiness and the presence of dust particles in the atmosphere. Fig 4.1 above indicates that the maximum mean solar irradiation received occurred in February. This perhaps may be due to the fact that February marks the end of the harmattan and the beginning of the rainy season where both cloudiness and dust particles level in the atmosphere are low and hence the solar irradiation level reaching the earth is high. The months of March, April and May were found to be the months with the least mean solar irradiation. Perhaps this may be due to the fact by March, the rainy season would have begun for some time and the cloudiness level would be quite high resulting in a low level of solar irradiation. In the month of June there was a rise in solar irradiation even though the solar irradiation level falls in July, August, September and October which may perhaps be due to the fact that in June there is intermittent cloudiness and sunshine level. In some cases we have a cloudy day throughout and also sunshine through certain day's amount of rainfall is not as in march April and May. The cloudiness level reduces and so the solar irradiation output is higher. The month of July, August, September and October marks the end of the raining season which also results in higher amount of rainfall hence higher cloudiness level resulting in low solar irradiation output. The month of November and December which form the second cluster also had higher solar irradiation level above the solar irradiation level of July, August, September and October. This may be due to the fact that the month of November is the beginning of the harmattan season with low cloudiness and low dust particles level resulting in high solar irradiation.



### 4.1.3 Putting the Months into Clusters

Based on the theory of hypothesis testing the various months of the year with similar solar irradiation patterns were put into clusters. From the hypothesis testing carried out it was discovered that the months of the year can be put into two such clusters. The first cluster extends from the month of January to October and the second cluster is from November to December.



### 4.1.4 Bayesian Estimation for Months

As indicated in table 4.2, the Bayesian analysis was conducted for the various months of the year. The summary of the results are given in table 4.2. The results presented are the means and the variance for twenty iterations for the various months of the year. The results in table 4.2 indicate that the beta distribution which was used as the prior distribution converged to the mean and variance of the posterior distributions shown. The posterior means of various months indicates that despite the rain fall patterns in Kumasi there is a higher potential sunshine region

Table 4.2 Final iterative values of the Bayesian analysis

| MONTH | PRIOR MEAN | PRIOR VARIANCE | POSTERIOR MEAN | POSTERIOR VARIANCE |
|---|---|---|---|---|
| JANUARY | 0.869 | 3.97E-06 | 0.899 | 3.77E-06 |
| FEBRUARY | 0.824 | 6.02E-06 | 0.824 | 5.71E-06 |
| MARCH | 0.9 | 3.16E-06 | 0.9 | 2.99E-06 |
| APRIL | 0.859 | 4.22E-06 | 0.859 | 4.02E-06 |
| MAY | 0.923 | 2.48E-06 | 0.923 | 2.35E-06 |
| JUNE | 0.769 | 6.22E-06 | 0.769 | 5.96E-06 |
| JULY | 0.718 | 7.25E-06 | 0.718 | 6.88E-06 |
| AUGUST | 0.536 | 5.45E-06 | 0.536 | 5.27E-06 |
| SEPTEMBER | 0.785 | 5.91E-06 | 0.785 | 5.61E-06 |
| OCTOBER | 0.717 | 7.11E-06 | 0.717 | 6.76E-06 |
| NOVEMBER | 0.899 | 3.16E-06 | 0.899 | 2.98E-06 |
| DECEMBER | 0.869 | 3.97E-06 | 0.869 | 3.77E-06 |
| | BAYESIAN SUMMARY OF THE CLUSTERS | | | |
| CLUSTER ONE | 0.86 | 4.17048E-06 | 0.86 | 4.00754E-06 |
| CUSTER TWO | 0.765 | 6.30737E-06 | 0.764 | 5.99232E-06 |



## 4.2 CONCLUSION

The hourly solar irradiation data for Kumasi, Ghana, was analyzed in order to determine standard probability distribution models pertaining to the various months of the year, as well as for clusters of months of the year with similar patterns. From the analysis conducted it was found out that the month of February has the highest mean hourly solar irradiation of $675Whm^{-2}$ and the month of May has least mean hourly solar irradiation of $34Whm^{-2}$.From the analysis conducted it was also found that hourly solar irradiation for January, March and May could be fitted to the lognormal probability distribution while the month of April could be equally well fitted to the Weibull, exponential, gamma and the geometric distributions. The months of June, July, August, September, October, November and December could be equally well be fitted to the Exponential, Weibull, Geometric and Gamma distributions.

From hypothesis testing carried out, it was discovered that the months of the year could be put into two clusters. The first cluster is from the month of January to October and the second cluster comprises November and December. It was observed that there is a significant difference between the mean hourly solar irradiation pertaining to the two clusters. The first cluster had a higher mean hourly solar irradiation of $428Whm^{-2}$ while the second cluster had a mean hourly solar irradiation of $326Whm^{-2}$.

Bayesian analysis on high or low (i.e. respectively above or below a threshold of $120kWhm^{-2}$ hourly solar irradiation) for each month with given prior beta distribution converged to posterior beta distribution after 20 iteration with average mean of 0.86. This shows that the on average the solar irradiation patterns in Kumasi tends to be high frequently. Also the prior variance of the various months of the year converged to the posterior tolerance level of 0.000001

Comparing the sunshine output of the two clusters indicated that the first cluster has a higher hourly solar irradiation output than the second cluster. The Bayesian analysis of the clusters confirms that this and we find that the first cluster has a higher possibility (0.8600) of high solar irradiation output as compared with the second cluster (0.76000).

## 4.3 RECOMMENDATIONS

The following are the recommendations made after the study.

1. That a similar and more comprehensive study should be conducted in respect of other geographical locations to ascertain and compare hourly solar irradiation distributions.
2. That Scientists and Engineers take cognizance of the results obtained in this work when analyzing or designing photovoltaic solar energy systems in areas with weather patterns similar to that of Kumasi.
3. That other natural energy resource such as wind should similarly investigated to determine the viability of energy production.

**REFERENCES**

Akuffo F.O. and A. Brew-Hammond, (1993) The frequency distribution of daily global irradiation at Kumasi, Solar Energy 50 (Pages 145-154)

Anderson et al (1993); Introduction to statistics, concepts and applications third edition. West publishing company. New York

Bowerman B., et al., (1997) Applied Statistics, Von Hoffmann Press Incorporated. New Jersey

Bendt P, M.Collares-Pereira and A.Rabl, (1981).The frequency distribution of daily insulation values Solar Energy 27, (Page 1)

Crawshaw J and J Chambrs (2004) Concise course in advanced level statistics, fourth edition.Stanley Thorns(publishers) Ltd. Cheltenham

Devore J. L.; (2007) Probability and statistics for Engineering and the Sciences, Duxbury Press, third Edition. New Jersey

Feuillard T., J.M. Abillon, and C. Martias, (1989) The probability density function of the clearness index Solar Energy 43, (Pages 363-372)

Gorden J.M. and T.A Reddy, (1988) Time series analysis of daily horizontal solar radiation Solar Energy 41, (Pages 215-226)

Hankins Mark, (1995) Small Solar electric System for Africa. Second Edition Commonwealth Secretariat

Hollands K.G. and R.G. Huget (1983) Probability density function for the clearness index with application. Solar Energy 30, (Pages 195-209)

Ideriah F.J.K. and S.O. Suleman, (1989) Sky conditions at Ibadan during 1975-1980 Solar Energy 43, (Pages 325-330)

Jackson E. and F.O. Akuffo, ( 1990 ) Angstrom - Page type correlation between monthly average daily global irradiation on the horizontal and the monthly average relative duration of sunshine. Kumasi,Energy conservation and management.

Knight, K.M. S.A. Klein and J .A. Duffie, (1991) A methodology for the synthesis of hourly weather data, Solar Energy 46, (Pages 109-120)

Khazanie (2001); Statistics in a world of Applications, Harper Collins college Publishers, Fourth Edition New Jersey

Laurence. D. Hoffman (1995); Finite mathematics with calculus, Prudential Securities, Inc., second Edition.New Jersey

Levine et al (2000) Business statistics, A first course second edition,Prenttice Hall,

Liu B.Y.U., and R.C. Jordan, (1963) A rational procedure for predicting the long term average performance of flat plate energy collector. Solar Energy 7, (Page 53)

Oturanc and *et al (*2003) Statistical analysis on daily solar radiation Energy source volume 25. Pages 89 – 97

Olseth J.A. and A. Skartvieit, (1984) A probability density function for daily insolation within the temperate storm belts, Solar Energy 33, (Pages 533-542)

Pearson Education, Inc., 8th Edition. New Jersey

Rosen; Kenneth H. , Discrete Mathematics and its Application, Fourth edition MacGraw-Hill.

Rosner, B., (2000), Fundamentals of biostatistics Fifth edition, Duxbury Thomson learning

Saunier G.Y., T.A. Reddy and S. Kumar, (1987) Monthly probability distributions for both tropical and temperate locations Solar Energy 38, (Pages 325-330)

Ussher A.K.E., (1986) Distribution of solar and wind energy in Ghana, Bulletin of Energy Research Group 1, 25, Ghana

Walpole R. E., et al., (2007) Probability and statistics for Engineering & Sciences, Ramakant

Yilmaz.E, and *et al* (2006). Statistical analysis of solar radiation data for the city of Valparaiso in the coast region of Chile. Energy Sources, Volume 29. Pages 71 - 83

APENDIXSAMPLED DATA OF THE VARIOUS MONTHS

| SAMPLED DATA FOR THE MONTH OF APRIL | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 19.9 | 18.72 | 12.88 | 19.9 | 81.9 | 1.171 | 12.88 | 16.36 | 598.2 | 873 |
| 83.1 | 150.9 | 180.2 | 83.1 | 289 | 125.3 | 147.5 | 268.7 | 414.8 | 981 |
| 165 | 293.5 | 426.8 | 165 | 449 | 243.4 | 369.5 | 58.43 | 21.09 | 47.99 |
| 267.8 | 500.1 | 552.6 | 267.8 | 898 | 464.4 | 614.4 | 133.3 | 112.5 | 121.7 |
| 383.5 | 729 | 780 | 383.5 | 768 | 661.6 | 769 | 133.3 | 120.6 | 558 |
| 653.3 | 763 | 819 | 653.3 | 875 | 519.9 | 890 | 17.56 | 237.7 | 374.8 |
| 697.3 | 842 | 750 | 697.3 | 745 | 586.4 | 751 | 226.9 | 657.4 | 120.3 |
| 462.3 | 792 | 673.2 | 462.3 | 777 | 731 | 822 | 491.9 | 782 | 1.168 |
| 454.3 | 664.7 | 821 | 454.3 | 577.8 | 727 | 663.7 | 721 | 994 | 271.3 |
| 312 | 354.6 | 525.1 | 312 | 330.4 | 421.5 | 459.7 | 877 | 1006 | 268.9 |
| 355.2 | 206.5 | 301.1 | 355.2 | 105.1 | 191.5 | 290.6 | 600.4 | 986 | 332 |
| 210.3 | 50.18 | 126 | 8.17 | 590.1 | 33.88 | 117.9 | 657.6 | 578.9 | 260.2 |
| 4.673 | 210.3 | 9.37 | 22.25 | 861 | 76.2 | 24.57 | 47.99 | 481.6 | 428.4 |
| 169.5 | 4.673 | 165.1 | 133.5 | 722 | 364.7 | 157.9 | 121.7 | 608.8 | 19.85 |
| 26.94 | 169.5 | 345.2 | 498.1 | 737 | 4.685 | 16.39 | 177.9 | 356.9 | 57.38 |
| 148.7 | 406 | 53.84 | 531.8 | 451.9 | 113.6 | 17.56 | 197.7 | 241.5 | 172.1 |
| 265.6 | 589.1 | 147.5 | 262.1 | 303.6 | 300.9 | 30.42 | 271.3 | 60.68 | 88.6 |
| 53.84 | 791 | 839 | 181.3 | 68.91 | 361.6 | 132.2 | 268.9 | 168.3 | 862 |
| 147.5 | 440.9 | 974 | 366 | 308.1 | 590.5 | 288.8 | 332 | 260.7 | 848 |
| 262.1 | 543.5 | 934 | 358.9 | 821 | 687.2 | 509.4 | 168.3 | 187.1 | 861 |
| 181.3 | 555 | 665.2 | 448.8 | 842 | 599.3 | 630.4 | 260.7 | 79.5 | 606.9 |
| 366 | 858 | 535.7 | 337.7 | 848 | 634.2 | 789 | 187.1 | 14.03 | 552.1 |
| 358.9 | 867 | 403.9 | 308.4 | 621.1 | 345.7 | 878 | 79.5 | 9.37 | 148.4 |
| 448.8 | 741 | 142.4 | 199.8 | 364.3 | 181.4 | 650.8 | 14.03 | 105.4 | 5.844 |
| 337.7 | 542.8 | 4.671 | 0 | 133.2 | 438.3 | 439.8 | 404.8 | 432.6 | 325.1 |
| 308.4 | 294.2 | 137.9 | 565.2 | 15.19 | 644.5 | 289.4 | 680 | 698.7 | 319.8 |
| 199.8 | 53.71 | 452.9 | 93.7 | 11.72 | 412.1 | 4.679 | 93.6 | 764 | 199.7 |
| 137.9 | 1.168 | 731 | 188.5 | 105.4 | 157.6 | 127.5 | 159.1 | 773 | 135.4 |
| 1.168 | 602.2 | 773 | 280.8 | 746 | 11.68 | 230.3 | 255 | 16.39 | 687.2 |
| 532.8 | 608.9 | 497 | 248.1 | 171.5 | 5.859 | 476.8 | 88.6 | 121.7 | 941 |
| 446.3 | 680.9 | 285.9 | 271.4 | 771 | 118.3 | 713 | 333.5 | 318.7 | 345 |
| 508.3 | 608.7 | 91 | 662.9 | 785 | 338.5 | 705 | 701 | 143.6 | 341.4 |
| 116.8 | 584.4 | | 70.2 | 884 | 624.7 | 751 | 810 | 17.51 | 475.8 |
| 0 | 495.5 | 0 | 76.1 | 566.3 | 821 | 797 | 719 | 76.1 | 455.7 |
| 116.8 | 280.5 | 4.682 | 154.4 | 379.5 | 1022 | 607.7 | 431.9 | 209.5 | 114.5 |
| 1.169 | 119.2 | 70.2 | 386.1 | 133.1 | 756 | 497 | 622.3 | 270.2 | 225.6 |
| 0 | 2.335 | 395.2 | 582.9 | 149.5 | 676.1 | 381.6 | 541.6 | 414.4 | 99.3 |
| 97.2 | 140.5 | 529.5 | 621.4 | 9.35 | 512.8 | 108.6 | 37.46 | 223 | 47.93 |
| 35.12 | 291.4 | 557.4 | 508 | 525.4 | 378.5 | 11.7 | 141.6 | 120.3 | 664.9 |
| 142.8 | 382.5 | 592.4 | 456.1 | 524 | 265.3 | 114.6 | 225.8 | 649 | 781 |
| 228.1 | 466.5 | 209.1 | 436.1 | 979 | 118.1 | 278.3 | 523.9 | 638.5 | 980 |
| 4.682 | 829 | 2.338 | 496.7 | 802 | 8.19 | 438.1 | 507.2 | 2.343 | 259.2 |
| 70.2 | 748 | 404.6 | 93.7 | 598.7 | 12.89 | 549.9 | 397.3 | 105.4 | 102.9 |
| 70.2 | 1051 | 514.3 | 325.3 | 266.1 | 141.7 | 734 | 310.8 | 15.22 | 28.09 |
| 76.1 | 853 | 523.6 | 378.9 | 112.2 | 394.4 | 520.3 | 273.4 | 139.3 | 270.2 |
| 154.4 | 655.6 | 231.2 | 620.8 | 100.7 | 608.2 | 714 | 415.9 | 368.5 | 318.9 |
| 386.1 | 445.7 | 72.4 | 980 | 184.9 | 698.9 | 467.7 | 262.9 | 520 | 467.1 |
| 404.6 | 197.2 | 5.844 | 928 | 453.8 | 752 | 260.1 | 148.4 | 438 | 444.8 |



SAMPLED DATA FOR THE MONTH OF AUGUST

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 23 | 967 | 245 | 96 | 268 | 269 | 20 | 725 |
| 117 | 763 | 428 | 33 | 355 | 439 | 138 | 734 |
| 279 | 612 | 475 | 27 | 131 | 491 | 246 | 675 |
| 414 | 458 | 330 | 172 | 47 | 615 | 182 | 64 |
| 390 | 269 | 373 | 213 | 22 | 733 | 268 | 24 |
| 450 | 40 | 315 | 378 | 86 | 505 | 551 | 7 |
| 551 | 46 | 183 | 684 | 181 | 491 | 395 | 28 |
| 341 | 98 | 61 | 902 | 292 | 266 | 489 | 142 |
| 305 | 178 | 16 | 837 | 546 | 211 | 498 | 375 |
| 360 | 307 | 87 | 640 | 719 | 20 | 333 | 621 |
| 153 | 504 | 236 | 536 | 705 | 22 | 202 | 700 |
| 55 | 666 | 277 | 432 | 697 | 102 | 61 | 653 |
| 15 | 686 | 532 | 238 | 518 | 267 | 23 | 775 |
| 61 | 680 | 670 | 45 | 346 | 422 | 82 | 643 |
| 181 | 579 | 398 | 44 | 180 | 690 | 134 | 516 |
| 370 | 445 | 611 | 189 | 73 | 563 | 161 | 165 |
| 361 | 222 | 430 | 315 | 30 | 650 | 261 | 215 |
| 134 | 31 | 437 | 472 | 137 | 511 | 306 | 17 |
| 177 | 46 | 109 | 660 | 310 | 294 | 368 | 25 |
| 517 | 150 | 30 | 594 | 267 | 215 | 384 | 113 |
| 634 | 161 | 13 | 891 | 195 | 124 | 280 | 250 |
| 277 | 344 | 57 | 769 | 187 | 32 | 244 | 313 |
| 187 | 568 | 99 | 598 | 280 | 43 | 182 | 658 |
| 46 | 572 | 153 | 327 | 219 | 243 | 44 | 682 |
| 15 | 680 | 206 | 61 | 147 | 441 | 6 | 828 |
| 93 | 490 | 169 | 8 | 146 | 608 | 47 | 763 |
| 149 | 451 | 415 | 48 | 109 | 758 | 98 | 603 |
| 370 | 301 | 335 | 99 | 52 | 839 | 312 | 387 |
| 505 | 179 | 356 | 431 | 21 | 934 | 489 | 200 |
| 432 | 32 | 233 | 679 | 98 | 729 | 412 | 23 |
| 216 | 19 | 261 | 797 | 108 | 565 | 359 | 5 |
| 515 | 135 | 51 | 895 | 324 | 67 | 127 | 32 |
| 444 | 350 | 7 | 877 | 584 | 19 | 188 | 61 |
| 545 | 627 | 51 | 842 | 440 | 2 | 61 | 208 |
| 225 | 758 | 87 | 655 | 666 | 47 | 193 | 163 |
| 94 | 870 | 199 | 96 | 296 | 139 | 93 | 167 |
| 11 | 887 | 172 | 88 | 332 | 235 | 7 | 238 |
| 70 | 719 | 196 | 40 | 257 | 406 | 42 | 248 |
| 196 | 545 | 204 | 60 | 86 | 676 | 133 | 251 |
| 216 | 426 | 282 | 199 | 40 | 693 | 155 | 300 |

SAMPLED DATA FOR THE MONTH OF DECEMBER

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 120.4 | 427 | 58.42 | 100.7 | 577.2 | 161.2 | 63.2 | 411.4 | 173.1 |
| 14.05 | 557.4 | 133.4 | 207.1 | 392.5 | 25.7 | 165 | 584.2 | 235.1 |
| 194.3 | 421.8 | 3.512 | 349.7 | 473.1 | 201.6 | 314.7 | 301.4 | 293.5 |
| 403.3 | 504.7 | 76.1 | 384.6 | 364.5 | 4.685 | 443.1 | 340 | 265.4 |
| 406.6 | 478.9 | 188.4 | 594.8 | 334.2 | 119.4 | 386.8 | 130.9 | 68.98 |
| 477.8 | 443.8 | 446.8 | 827 | 163.6 | 116.5 | 657.7 | 254.9 | 308.8 |
| 453.2 | 348.1 | 495.6 | 665.4 | 11.69 | 8.19 | 418.1 | 90.1 | 186 |
| 678.6 | 244.1 | 642.6 | 309.3 | 153.7 | 78.4 | 127.3 | 110 | 292.4 |
| 843 | 206.8 | 598 | 206.7 | 47.99 | 201.3 | 164.7 | 320.6 | 128.6 |
| 735 | 158.9 | 425.1 | 150.6 | 148.6 | 263.2 | 2.337 | 188.4 | 2.339 |
| 504.4 | 9.35 | 399.4 | 95.8 | 260.9 | 168.4 | 134.1 | 65.53 | 90.9 |
| 429.7 | 163.5 | 596.8 | 5.841 | 286.6 | 244.4 | 32.77 | 28.09 | 3.513 |
| 204.4 | 79.6 | 362.1 | 162.6 | 302.9 | 329.7 | 108.8 | 74.7 | 69.09 |
| 24.53 | 200.1 | 144.9 | 14.04 | 277.1 | 210.4 | 438.6 | 62.06 | 254.1 |
| 223.7 | 216.4 | 16.36 | 101.8 | 203.4 | 251.3 | 412.6 | 118.2 | 577.9 |
| 15.22 | 229.2 | 183.2 | 142.7 | 194.1 | 168.3 | 342.4 | 133.4 | 466.4 |
| 106.5 | 406.9 | 2.342 | 248 | 143.8 | 162.5 | 483.7 | 272.6 | 358.8 |
| 205.9 | 694.3 | 76.1 | 339.1 | 116.9 | 74.8 | 295.5 | 419.9 | 348.3 |
| 327.5 | 724 | 158 | 372.9 | 54.95 | 1.169 | 275.7 | 675.6 | 382.3 |
| 593.8 | 667 | 238.7 | 351.8 | 84.9 | 90.1 | 267.5 | 576 | 422 |
| 728 | 526.8 | 658.4 | 168.3 | 15.23 | 21.07 | 213.8 | 686.7 | 167.2 |
| 480 | 397.2 | 559.8 | 327.3 | 125.3 | 118.2 | 68.95 | 572.2 | 15.2 |
| 607.2 | 142.6 | 583 | 239.6 | 344 | 93.6 | 8.18 | 370.2 | 127.7 |
| 567.5 | 11.69 | 410 | 78.3 | 431.4 | 472.6 | 122.9 | 120.3 | 40.98 |
| 502.1 | 179 | 568.8 | 18.7 | 527 | 707 | 78.4 | 3.505 | 66.72 |
| 360.9 | 5.854 | 463.7 | 100.1 | 575.8 | 735 | 136.9 | 167.1 | 107.7 |
| 164.7 | 77.3 | 357.5 | 14.05 | 491.6 | 671.6 | 156.8 | 14.05 | 374.5 |
| 25.7 | 246.9 | 181.1 | 104.2 | 556.9 | 578.1 | 361.4 | 100.7 | 508.8 |
| 195.2 | 349.7 | 24.54 | 195.4 | 545.2 | 506.9 | 217.5 | 235.2 | 653.4 |
| 1.171 | 533.1 | 178.4 | 342.7 | 300 | 311.8 | 514.4 | 324 | 313.1 |
| 151 | 590 | 58.53 | 556.4 | 217.2 | 123.8 | 463.9 | 192.9 | 307.5 |
| 246.8 | 573.5 | 243.4 | 691.6 | 103.9 | 1.169 | 564.3 | 325.1 | 91.2 |
| 325.1 | 450.8 | 260.9 | 575.7 | 7.01 | 180.9 | 650.8 | 814 | 47.97 |
| 433.7 | 316.4 | 523.9 | 597.8 | 176.7 | 67.88 | 320.2 | 574.8 | 16.38 |
| 305 | 233.5 | 538.8 | 731 | 52.68 | 117 | 59.61 | 502.3 | 105.3 |
| 550.4 | 293.1 | 587.6 | 575.6 | 84.3 | 175.5 | 146.8 | 370.4 | 18.74 |
| 341.1 | 137.8 | 854 | 332.8 | 112.3 | 214.1 | 39.8 | 158.9 | 106.5 |
| 319 | 22.19 | 718 | 174 | 156.8 | 376.5 | 122.9 | 150.5 | 179.1 |
| 294.4 | 159.6 | 565.1 | 8.18 | 297.1 | 329.6 | 232.8 | 83.1 | 253.9 |
| 175.3 | 4.682 | 301.3 | 204.1 | 323.9 | 566.8 | 340.3 | 143.9 | 345.1 |

SAMPLED DATA FOR THE MONTH OF FEBRUARY

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 216.6 | 421 | 556.8 | 512 | 496 | 156 | 348 | 781 | 764 |
| 298.4 | 222 | 344.4 | 717 | 808 | 39 | 155 | 779 | 820 |
| 455.1 | 48 | 67.73 | 790 | 772 | 32 | 20 | 654 | 702 |
| 641.9 | 96 | 48 | 840 | 876 | 140 | 113 | 414 | 597 |
| 769 | 265 | 230.6 | 788 | 849 | 378 | 304 | 324 | 369 |
| 680 | 427 | 479.5 | 634 | 787 | 669 | 510 | 72 | 194 |
| 428.8 | 653 | 460.4 | 472 | 548.8 | 776 | 666 | 88 | 54 |
| 331.8 | 764 | 555.9 | 183 | 350.3 | 803 | 720 | 181 | 118 |
| 130.9 | 809 | 879 | 76 | 234.7 | 720 | 702 | 475 | 307 |
| 1.169 | 735 | 662.9 | 68 | 30.37 | 632 | 660 | 589 | 501 |
| 70.3 | 615 | 743 | 167 | 113.6 | 453 | 586 | 646 | 674 |
| 268 | 413 | 575.5 | 412 | 313.7 | 236 | 417 | 769 | 753 |
| 381.2 | 202 | 369 | 708 | 535.6 | 61 | 207 | 738 | 760 |
| 417.3 | 46 | 162.3 | 798 | 697.5 | 111 | 34 | 469 | 656 |
| 735 | 107 | 10.51 | 840 | 691.4 | 276 | 102 | 473 | 538 |
| 613.1 | 293 | 63.22 | 816 | 672.7 | 485 | 285 | 273 | 365 |
| 698.2 | 492 | 177.9 | 699 | 845 | 668 | 482 | 76 | 168 |
| 527.8 | 651 | 354.5 | 490 | 757 | 737 | 643 | 55 | 25 |
| 578 | 716 | 584.5 | 263 | 493.9 | 800 | 701 | 187 | 109 |
| 346.8 | 724 | 644.9 | 54 | 314.2 | 729 | 701 | 296 | 295 |
| 121.5 | 691 | 780 | 109 | 196.3 | 635 | 626 | 462 | 496 |
| 60.89 | 562 | 694.6 | 322 | 17.53 | 446 | 511 | 772 | 663 |
| 244.6 | 379 | 634 | 524 | 229 | 232 | 320 | 839 | 702 |
| 288.9 | 193 | 458.9 | 693 | 411 | 60 | 152 | 759 | 702 |
| 464.2 | 35 | 224.2 | 794 | 562 | 66 | 23 | 638 | 674 |
| 649.7 | 45 | 225.4 | 839 | 665 | 165 | 95 | 418 | 538 |
| 631.8 | 104 | 97.3 | 756 | 701 | 278 | 276 | 212 | 360 |
| 785 | 304 | 292.8 | 648 | 621 | 499 | 477 | 63 | 166 |
| 572 | 514 | 474.9 | 466 | 506 | 648 | 629 | 94 | 21 |
| 396.9 | 677 | 698.9 | 249 | 347 | 816 | 719 | 171 | 103 |
| 272 | 809 | 820 | 64 | 162 | 751 | 721 | 182 | 282 |
| 98.1 | 815 | 799 | 93 | 26 | 642 | 679 | 256 | 494 |
| 83.1 | 735 | 733 | 276 | 86 | 468 | 559 | 717 | 643 |
| 259.7 | 609 | 677.2 | 407 | 242 | 242 | 404 | 525 | 736 |
| 493.3 | 425 | 559.4 | 668 | 442 | 67 | 209 | 764 | 771 |
| 703 | 217 | 405.3 | 752 | 596 | 55 | 41 | 649 | 703 |
| 720 | 46 | 228.9 | 785 | 698 | 180 | 83 | 512 | 588 |
| 455.1 | 51 | 15.19 | 720 | 701 | 308 | 266 | 323 | 402 |
| 659.5 | 135 | 111.3 | 594 | 672 | 709 | 439 | 43 | 192 |
| 696.9 | 336 | 292.7 | 407 | 530 | 707 | 639 | 114 | 32 |

SAMPLED DATA FOR THE MONTH OF JANUARY

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 726 | 121 | 388 | 452 | 35 | 395 | 118 | 277.4 | 737 |
| 752 | 48 | 182 | 647 | 101 | 353 | 230 | 458.8 | 760 |
| 681 | 132 | 41 | 713 | 191 | 432 | 370 | 485 | 781 |
| 546 | 213 | 9 | 601 | 433 | 288 | 463 | 540.6 | 305 |
| 360 | 497 | 78 | 619 | 672 | 111 | 698 | 323.3 | 360 |
| 166 | 304 | 222 | 546 | 696 | 65 | 655 | 249.6 | 244 |
| 35 | 492 | 494 | 370 | 624 | 192 | 689 | 157.3 | 41 |
| 10 | 745 | 652 | 237 | 499 | 462 | 554 | 47.25 | 9 |
| 101 | 732 | 717 | 86 | 339 | 365 | 415 | 0.814 | 74 |
| 263 | 725 | 738 | 36 | 163 | 582 | 135 | 18 | 283 |
| 451 | 499 | 680 | 152 | 38 | 605 | 57 | 14 | 529 |
| 614 | 277 | 566 | 216 | 8 | 814 | 58 | 3.642 | 547 |
| 699 | 63 | 421 | 369 | 119 | 865 | 173 | 56.35 | 776 |
| 707 | 56 | 223 | 505 | 145 | 476 | 396 | 141.1 | 720 |
| 640 | 224 | 52 | 579 | 285 | 129 | 281 | 287.1 | 783 |
| 500 | 463 | 15 | 727 | 485 | 101 | 437 | 432.8 | 639 |
| 327 | 575 | 133 | 597 | 635 | 40 | 660 | 410.6 | 442 |
| 155 | 847 | 308 | 603 | 655 | 39 | 820 | 411.3 | 249 |
| 31 | 869 | 509 | 213 | 647 | 116 | 655 | 615.1 | 68 |
| 12 | 834 | 671 | 106 | 521 | 211 | 627 | 402.8 | 5 |
| 99 | 728 | 613 | 44 | 356 | 359 | 399 | 311 | 46 |
| 246 | 676 | 744 | 17 | 169 | 553 | 222 | 99.3 | 89 |
| 449 | 482 | 679 | 60 | 35 | 747 | 68 | 28.03 | 219 |
| 615 | 273 | 548 | 184 | 5 | 824 | 44 | 1.05 | 317 |
| 705 | 83 | 372 | 382 | 40 | 766 | 185 | 3.194 | 440 |
| 710 | 56 | 187 | 466 | 132 | 549 | 490 | 3.24 | 716 |
| 650 | 222 | 44 | 622 | 322 | 450 | 464 | 3.383 | 671 |
| 526 | 297 | 10 | 503 | 357 | 272 | 520 | 3.303 | 543 |
| 356 | 430 | 106 | 354 | 486 | 94 | 721 | 3.043 | 466 |
| 170 | 732 | 288 | 369 | 606 | 59 | 888 | 0.209 | 253 |
| 38 | 752 | 487 | 163 | 765 | 514 | 47 | 2.664 | 31.42 |
| 13 | 605 | 640 | 54 | 661 | 650 | 174 | 2.961 | 161 |
| 106 | 597 | 679 | 29 | 537 | 755 | 320 | 2.47 | 298.4 |
| 274 | 466 | 703 | 45 | 263 | 650 | 421 | 1.857 | 474 |
| 449 | 357 | 677 | 174 | 84 | 443 | 496 | 0.568 | 683 |
| 599 | 87 | 539 | 242 | 33 | 213 | 460 | 0.401 | 2.819 |
| 679 | 23 | 368 | 574 | 126 | 101 | 542 | 1.655 | 2.394 |
| 699 | 36 | 199 | 670 | 130 | 11 | 36.92 | 21.51 | 0.502 |
| 640 | 115 | 43 | 541 | 530 | 133 | 133.9 | 81.2 | 0.31 |
| 526 | 297 | 4 | 609 | 424 | 291 | 250.6 | 116.1 | 2.087 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 62 | 371 | 449 | 659 | 141 | 188 | 13 | 241 | 489 |
| 121 | 503 | 584 | 742 | 200 | 74 | 106 | 373 | 709 |
| 295 | 489 | 590 | 567 | 100 | 37 | 212 | 489 | 508 |
| 504 | 587 | 465 | 406 | 45 | 136 | 358 | 639 | 400 |
| 474 | 570 | 367 | 245 | 58 | 211 | 466 | 612 | 170 |
| 522 | 610 | 336 | 86 | 150 | 257 | 728 | 676 | 171 |
| 519 | 499 | 218 | 54 | 325 | 492 | 547 | 518 | 90 |
| 428 | 399 | 42 | 201 | 465 | 623 | 455 | 420 | 25 |
| 666 | 217 | 45 | 218 | 447 | 540 | 394 | 260 | 17 |
| 446 | 82 | 205 | 410 | 649 | 478 | 265 | 89 | 112 |
| 251 | 24 | 180 | 654 | 599 | 347 | 137 | 18 | 208 |
| 71 | 95 | 200 | 688 | 656 | 333 | 61 | 39 | 247 |
| 50 | 194 | 456 | 599 | 558 | 165 | 55 | 117 | 325 |
| 93 | 330 | 549 | 548 | 537 | 59 | 145 | 205 | 270 |
| 342 | 333 | 592 | 442 | 295 | 16 | 247 | 368 | 387 |
| 407 | 517 | 651 | 427 | 52 | 95 | 359 | 529 | 255 |
| 236 | 521 | 606 | 289 | 37 | 128 | 503 | 697 | 250 |
| 608 | 354 | 259 | 88 | 139 | 342 | 548 | 382 | 327 |
| 589 | 454 | 183 | 12 | 236 | 372 | 516 | 361 | 193 |
| 228 | 250 | 88 | 26 | 173 | 325 | 565 | 317 | 53 |
| 109 | 253 | 18 | 81 | 219 | 386 | 411 | 156 | 18 |
| 24 | 61 | 74 | 216 | 448 | 573 | 370 | 76 | 89 |
| 7 | 34 | 136 | 138 | 446 | 525 | 236 | 14 | 219 |
| 5 | 118 | 186 | 220 | 307 | 266 | 77 | 78 | 227 |
| 38 | 179 | 257 | 243 | 154 | 286 | 7 | 167 | 390 |
| 175 | 338 | 281 | 267 | 64 | 72 | 62 | 266 | 541 |
| 294 | 460 | 539 | 273 | 73 | 31 | 308 | 364 | 602 |
| 435 | 746 | 553 | 217 | 31 | 97 | 384 | 289 | 552 |
| 331 | 723 | 601 | 142 | 28 | 199 | 443 | 353 | 550 |
| 348 | 721 | 480 | 65 | 83 | 463 | 338 | 653 | 468 |
| 307 | 582 | 248 | 34 | 305 | 656 | 455 | 258 | 255 |
| 534 | 376 | 105 | 169 | 417 | 743 | 408 | 108 | 95 |
| 389 | 116 | 28 | 321 | 513 | 688 | 362 | 52 | 37 |
| 515 | 54 | 136 | 416 | 636 | 719 | 301 | 24 | 87 |
| 100 | 26 | 213 | 497 | 687 | 591 | 159 | 13 | 121 |
| 41 | 105 | 294 | 469 | 632 | 463 | 68 | 65 | 152 |
| 37 | 255 | 440 | 461 | 687 | 266 | 30 | 147 | 282 |
| 101 | 286 | 443 | 649 | 447 | 89 | 59 | 174 | 258 |
| 328 | 463 | 710 | 389 | 288 | 34 | 166 | 247 | 400 |
| 454 | 496 | 617 | 280 | 93 | 124 | 327 | 376 | 492 |

| SAMPLED DATA FOR THE MONTH OF JUNE | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 37 | 31 | 436 | 618 | 382 | 124 | 42 | 221 | 512 |
| 88 | 104 | 733 | 851 | 368 | 193 | 74 | 424 | 680 |
| 146 | 242 | 815 | 663 | 169 | 87 | 239 | 581 | 735 |
| 325 | 278 | 732 | 570 | 58 | 39 | 386 | 627 | 404 |
| 338 | 246 | 612 | 444 | 63 | 120 | 539 | 674 | 446 |
| 405 | 417 | 388 | 360 | 165 | 195 | 627 | 509 | 279 |
| 537 | 582 | 252 | 266 | 459 | 347 | 832 | 231 | 229 |
| 472 | 557 | 77 | 89 | 629 | 485 | 726 | 156 | 50 |
| 408 | 324 | 19 | 29 | 748 | 736 | 634 | 91 | 39 |
| 298 | 275 | 77 | 161 | 797 | 701 | 363 | 35 | 100 |
| 149 | 177 | 170 | 229 | 740 | 782 | 179 | 38 | 276 |
| 62 | 41 | 232 | 385 | 676 | 675 | 62 | 131 | 418 |
| 75 | 49 | 454 | 396 | 677 | 414 | 41 | 202 | 433 |
| 234 | 181 | 577 | 733 | 433 | 249 | 162 | 201 | 513 |
| 318 | 208 | 620 | 752 | 248 | 75 | 233 | 318 | 533 |
| 371 | 283 | 609 | 748 | 80 | 36 | 414 | 604 | 729 |
| 518 | 691 | 616 | 633 | 30 | 165 | 471 | 692 | 504 |
| 697 | 737 | 444 | 270 | 100 | 205 | 453 | 330 | 241 |
| 905 | 657 | 209 | 128 | 314 | 457 | 909 | 444 | 117 |
| 700 | 743 | 74 | 32 | 499 | 572 | 609 | 259 | 75 |
| 420 | 708 | 71 | 107 | 557 | 720 | 315 | 71 | 41 |
| 455 | 498 | 115 | 262 | 623 | 657 | 251 | 60 | 123 |
| 203 | 253 | 217 | 359 | 846 | 726 | 164 | 49 | 298 |
| 86 | 47 | 372 | 409 | 676 | 674 | 57 | 175 | 265 |
| 39 | 82 | 738 | 643 | 622 | 492 | 35 | 243 | 400 |
| 169 | 224 | 678 | 825 | 470 | 261 | 95 | 406 | 428 |
| 322 | 208 | 862 | 775 | 249 | 33 | 156 | 690 | 540 |
| 418 | 285 | 608 | 450 | 96 | 93 | 277 | 636 | 439 |
| 66 | 407 | 663 | 512 | 55 | 135 | 523 | 456 | 438 |
| 45 | 576 | 398 | 261 | 133 | 331 | 459 | 818 | 398 |
| 106 | 869 | 228 | 87 | 258 | 585 | 754 | 515 | 178 |
| 143 | 824 | 81 | 25 | 459 | 440 | 841 | 296 | 46 |
| 151 | 621 | 66 | 87 | 674 | 596 | 463 | 193 | 61 |
| 217 | 448 | 182 | 260 | 758 | 529 | 588 | 35 | 209 |
| 159 | 186 | 207 | 354 | 811 | 640 | 93 | 43 | 406 |
| 51 | 53 | 1110 | 562 | 606 | 313 | 12 | 122 | 378 |
| 26 | 33 | 76 | 777 | 707 | 193 | 36 | 268 | 625 |
| 88 | 143 | 290 | 747 | 374 | 36 | 143 | 453 | 504 |
| 169 | 284 | 986 | 621 | 218 | 39 | 340 | 491 | 343 |
| 246 | 508 | 492 | 309 | 70 | 106 | 388 | 579 | 754 |

SAMPLED DATA FOR THE MONTH OF MARCH

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 162 | 881 | 508 | 432 | 522 | 102 | 158 | 557 | 783 |
| 405 | 883 | 713 | 283 | 258 | 283 | 276 | 562 | 865 |
| 628 | 880 | 851 | 32 | 52 | 619 | 463 | 651 | 777 |
| 715 | 680 | 682 | 20 | 9 | 707 | 697 | 360 | 588 |
| 828 | 440 | 456 | 110 | 138 | 729 | 460 | 314 | 464 |
| 840 | 248 | 191 | 231 | 357 | 829 | 600 | 204 | 282 |
| 681 | 75 | 40 | 535 | 553 | 785 | 722 | 72 | 88 |
| 242 | 24 | 6 | 747 | 687 | 703 | 643 | 21 | 22 |
| 342 | 117 | 83 | 862 | 636 | 473 | 486 | 198 | 158 |
| 176 | 299 | 188 | 876 | 441 | 246 | 279 | 418 | 387 |
| 59 | 359 | 408 | 829 | 678 | 79 | 88 | 589 | 605 |
| 10 | 531 | 539 | 672 | 724 | 44 | 14 | 756 | 727 |
| 102 | 660 | 705 | 386 | 538 | 202 | 89 | 877 | 895 |
| 373 | 791 | 679 | 270 | 232 | 417 | 218 | 884 | 835 |
| 685 | 706 | 778 | 77 | 55 | 575 | 301 | 794 | 766 |
| 740 | 647 | 507 | 22 | 18 | 780 | 507 | 673 | 631 |
| 611 | 363 | 414 | 143 | 117 | 837 | 557 | 493 | 478 |
| 575 | 256 | 304 | 344 | 303 | 646 | 500 | 261 | 274 |
| 824 | 61 | 64 | 538 | 413 | 756 | 651 | 89 | 94 |
| 666 | 46 | 9 | 745 | 523 | 651 | 662 | 17 | 12 |
| 473 | 190 | 99 | 905 | 741 | 377 | 321 | 85 | 93 |
| 194 | 177 | 290 | 885 | 687 | 128 | 121 | 265 | 265 |
| 65 | 291 | 562 | 860 | 785 | 54 | 40 | 510 | 587 |
| 20 | 477 | 501 | 729 | 478 | 33 | 26 | 771 | 745 |
| 162 | 742 | 656 | 488 | 429 | 202 | 184 | 675 | 730 |
| 383 | 578 | 573 | 297 | 247 | 325 | 408 | 744 | 767 |
| 583 | 627 | 471 | 94 | 87 | 619 | 552 | 542 | 946 |
| 713 | 689 | 565 | 20 | 6 | 522 | 565 | 612 | 752 |
| 878 | 486 | 447 | 168 | 44 | 700 | 870 | 437 | 362 |
| 944 | 126 | 236 | 407 | 123 | 635 | 774 | 285 | 227 |
| 897 | 55 | 67 | 570 | 241 | 828 | 872 | 72 | 40 |
| 542 | 25 | 14 | 841 | 276 | 448 | 761 | 169.3 | 17 |
| 452 | 116 | 84 | 805 | 399 | 447 | 556 | 66.7 | 56 |
| 276 | 202 | 204 | 767 | 672 | 244 | 306 | 180.2 | 167 |
| 82 | 587 | 361 | 633 | 757 | 77 | 94 | 191.8 | 442 |
| 7 | 845 | 704 | 531 | 535 | 34 | 24 | 236.2 | 569 |
| 34 | 812 | 769 | 429 | 439 | 129 | 171 | 377.6 | 793 |
| 55 | 885 | 895 | 181 | 230 | 197 | 367 | 527 | 688 |
| 87 | 894 | 736 | 89 | 93 | 456 | 600 | 546.7 | 565 |
| 190 | 704 | 667 | 15 | 19 | 524 | 784 | 437.9 | 617 |

| SAMPLED DATA FOR THE MONTH OF MAY | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 20 | 270 | 740 | 126 | 211 | 645 | 351 | 16 | 111 |
| 74 | 57 | 721 | 301 | 503 | 123 | 457 | 8 | 46 |
| 220 | 79 | 618 | 544 | 485 | 244 | 207 | 36 | 39 |
| 542 | 211 | 663 | 429 | 547 | 256 | 146 | 168 | 31 |
| 665 | 347 | 691 | 561 | 784 | 113 | 104 | 532 | 133 |
| 578 | 443 | 526 | 582 | 503 | 9 | 72 | 616 | 310 |
| 329 | 466 | 329 | 659 | 724 | 32 | 26 | 805 | 446 |
| 435 | 358 | 26 | 534 | 603 | 99 | 56 | 916 | 743 |
| 398 | 396 | 36 | 275 | 432 | 194 | 194 | 868 | 879 |
| 395 | 391 | 154 | 62 | 289 | 355 | 459 | 756 | 682 |
| 219 | 401 | 327 | 38 | 145 | 437 | 559 | 510 | 639 |
| 82 | 214 | 328 | 87 | 51 | 431 | 705 | 470 | 443 |
| 24 | 105 | 490 | 232 | 173 | 462 | 700 | 271 | 252 |
| 131 | 15 | 582 | 344 | 351 | 791 | 575 | 92 | 111 |
| 148 | 59 | 661 | 701 | 511 | 477 | 756 | 88 | 65 |
| 467 | 236 | 600 | 761 | 619 | 393 | 552 | 298 | 53 |
| 646 | 236 | 403 | 507 | 696 | 257 | 525 | 516 | 170 |
| 784 | 312 | 329 | 404 | 758 | 91 | 301 | 607 | 443 |
| 837 | 468 | 343 | 383 | 686 | 27 | 99 | 597 | 506 |
| 801 | 570 | 52 | 72 | 592 | 182 | 37 | 633 | 655 |
| 611 | 466 | 252.9 | 32 | 434 | 321 | 218 | 595 | 646 |
| 442 | 774 | 0 | 28 | 234 | 234 | 362 | 440 | 733 |
| 251 | 698 | 39.78 | 48 | 75 | 644 | 530 | 373 | 828 |
| 92 | 556 | 203.5 | 91 | 39 | 253 | 898 | 288 | 686 |
| 39 | 286 | 407.8 | 196 | 189 | 755 | 743 | 224 | 445 |
| 265 | 104 | 589.5 | 394 | 459 | 786 | 599 | 79 | 80 |
| 312 | 50 | 794 | 530 | 613 | 535 | 627 | 21 | 16 |
| 445 | 140 | 19 | 780 | 796 | 107 | 721 | 109 | 28 |
| 554 | 285 | 148 | 746 | 821 | 16 | 485 | 159 | 99 |
| 626 | 420 | 362 | 581 | 752 | 3 | 243 | 326 | 319 |
| 826 | 674 | 443 | 459 | 571 | 23 | 55 | 557 | 588 |
| 546 | 629 | 721 | 458 | 396 | 39 | 56 | 388 | 649 |
| 710 | 708 | 568 | 132 | 203 | 147 | 193 | 336 | 851 |
| 430 | 318 | 648 | 18 | 110 | 205 | 324 | 351 | 651 |
| 220 | 518 | 647 | 33 | 46 | 475 | 523 | 480 | 715 |
| 43 | 342 | 654 | 202 | 38 | 478 | 697 | 311 | 492 |
| 44 | 98 | 419 | 422 | 165 | 560 | 477 | 205 | 411 |
| 280 | 24 | 293 | 605 | 213 | 445 | 472 | 111 | 167 |
| 513 | 31 | 59 | 496 | 252 | 493 | 595 | 24 | 28 |
| 635 | 68 | 29 | 601 | 438 | 204 | 254 | 83 | 323 |

SAMPLED DATA FOR THE MONTH OF NOVEMBER

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 29 | 134 | 651 | 655 | 631 | 253 | 56 | 262 | 529 |
| 149 | 226 | 717 | 636 | 401 | 47 | 29 | 359 | 640 |
| 304 | 239 | 853 | 503 | 150 | 82 | 96 | 409 | 313 |
| 712 | 544 | 728 | 326 | 27 | 224 | 178 | 643 | 193 |
| 747 | 698 | 596 | 140 | 23 | 208 | 296 | 825 | 36 |
| 815 | 667 | 376 | 20 | 80 | 285 | 570 | 775 | 39 |
| 819 | 492 | 172 | 34 | 152 | 407 | 532 | 450 | 106 |
| 735 | 289 | 28 | 150 | 172 | 576 | 546 | 512 | 221 |
| 437 | 129 | 21 | 284 | 150 | 869 | 530 | 261 | 424 |
| 390 | 22 | 108 | 475 | 197 | 824 | 358 | 87 | 581 |
| 175 | 25 | 190 | 538 | 489 | 621 | 295 | 25 | 627 |
| 29 | 198 | 388 | 546 | 681 | 448 | 105 | 87 | 674 |
| 53 | 393 | 517 | 570 | 513 | 186 | 9 | 260 | 509 |
| 208 | 549 | 770 | 580 | 365 | 53 | 28 | 354 | 231 |
| 333 | 661 | 502 | 479 | 160 | 33 | 140 | 562 | 156 |
| 363 | 787 | 710 | 248 | 25 | 143 | 238 | 777 | 91 |
| 399 | 800 | 439 | 120 | 17 | 284 | 307 | 747 | 35 |
| 597 | 699 | 287 | 15 | 80 | 508 | 484 | 621 | 38 |
| 775 | 511 | 179 | 27 | 170 | 618 | 660 | 309 | 131 |
| 601 | 329 | 38 | 113 | 436 | 851 | 652 | 124 | 202 |
| 447 | 141 | 50 | 221 | 526 | 663 | 690 | 193 | 201 |
| 360 | 22 | 157 | 347 | 558 | 570 | 526 | 87 | 318 |
| 174 | 36 | 285 | 465 | 749 | 444 | 320 | 39 | 604 |
| 28 | 147 | 293 | 584 | 672 | 360 | 158 | 120 | 692 |
| 40 | 352 | 582 | 581 | 482 | 266 | 22 | 195 | 330 |
| 92 | 490 | 582 | 545 | 239 | 89 | 30 | 347 | 444 |
| 200 | 725 | 573 | 325 | 102 | 29 | 72 | 485 | 259 |
| 235 | 789 | 715 | 186 | 22 | 161 | 657 | 736 | 71 |
| 352 | 730 | 546 | 56 | 17 | 229 | 726 | 701 | 60 |
| 423 | 690 | 324 | 7 | 78 | 385 | 674 | 782 | 49 |
| 654 | 533 | 153 | 49 | 236 | 396 | 492 | 675 | 175 |
| 672 | 332 | 24 | 181 | 442 | 733 | 261 | 414 | 243 |
| 452 | 141 | 25 | 208 | 418 | 752 | 9 | 249 | 406 |
| 372 | 23 | 87 | 283 | 613 | 748 | 33 | 75 | 690 |
| 170 | 32 | 168 | 691 | 566 | 633 | 93 | 36 | 636 |
| 269 | 132 | 249 | 737 | 536 | 270 | 135 | 165 | 456 |
| 14 | 274 | 366 | 657 | 541 | 128 | 331 | 205 | 818 |
| 115 | 398 | 656 | 743 | 273 | 9 | 585 | 457 | 515 |
| 268 | 574 | 666 | 708 | 99 | 32 | 440 | 572 | 296 |
| 521 | 607 | 560 | 498 | 16 | 107 | 596 | 720 | 193 |

| SAMPLED DATA FOR THE MONTH OF OCTOBER | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 50.09 | 3.571 | 341.7 | 255 | 280.9 | 517.9 | 17.88 | 319.8 | 220.5 |
| 110.9 | 28.62 | 645.2 | 250.1 | 434.5 | 580.7 | 82.3 | 335.3 | 288.3 |
| 101.3 | 110.8 | 621.2 | 296.6 | 115.5 | 552 | 193.1 | 221.2 | 508.5 |
| 137.1 | 226.3 | 735 | 403.6 | 5.964 | 423.5 | 452.7 | 91.6 | 491.6 |
| 473.1 | 345.3 | 401 | 412.9 | 73.9 | 265.3 | 567.8 | 67.82 | 456.9 |
| 318 | 565.3 | 273.7 | 317.6 | 163.4 | 147.5 | 395 | 103.7 | 722 |
| 346.5 | 767 | 107.1 | 342.6 | 236 | 69.03 | 604.3 | 201.4 | 584.1 |
| 78.7 | 596.9 | 4.762 | 207 | 159.7 | 29.81 | 399.6 | 290.5 | 480.6 |
| 144.3 | 632.7 | 48.9 | 99.9 | 537.3 | 140.7 | 614.9 | 314 | 235.6 |
| 143.1 | 396.1 | 140.7 | 70.2 | 524 | 234.8 | 403.2 | 324.7 | 46.42 |
| 91.8 | 325.9 | 153.8 | 4.76 | 627.4 | 233.5 | 192.7 | 344.8 | 54.86 |
| 59.68 | 172.5 | 272.9 | 1.193 | 571.5 | 459.7 | 67.82 | 305.6 | 195.5 |
| 163.5 | 70.2 | 402.6 | 76.3 | 585.8 | 509.3 | 2.381 | 214.1 | 349.2 |
| 189.7 | 2.38 | 673.7 | 199.1 | 360.8 | 499.5 | 15.5 | 214.1 | 414.5 |
| 430.4 | 15.5 | 633.1 | 284.9 | 98.8 | 409.1 | 66.78 | 108.3 | 514.2 |
| 601.7 | 89.4 | 690.1 | 506.3 | 4.771 | 398.5 | 158.6 | 64.25 | 672.2 |
| 831 | 181.2 | 512.9 | 503.5 | 121.6 | 236.7 | 378.9 | 1.19 | 705 |
| 632.1 | 214.5 | 336.8 | 434.3 | 201.4 | 163 | 520.3 | 2.381 | 622 |
| 423.7 | 331.2 | 105.9 | 352.1 | 259.8 | 69.03 | 405.9 | 28.62 | 346.1 |
| 502.4 | 397.7 | 5.952 | 296.2 | 380 | 1.19 | 806 | 125.2 | 71.4 |
| 170.2 | 491.5 | 28.62 | 198.7 | 529.8 | 9.54 | 554.3 | 319.4 | 63.09 |
| 66.68 | 416.4 | 79.9 | 75 | 499.9 | 89.4 | 584 | 367.9 | 3.577 |
| 2.382 | 387.9 | 207.4 | 65.46 | 815 | 166.9 | 532.9 | 335.6 | 88.2 |
| 31.01 | 276.1 | 218.1 | 1.193 | 556.9 | 244.3 | 343.8 | 356.9 | 131.1 |
| 110.9 | 132.1 | 305 | 109.7 | 512.9 | 413.4 | 63.06 | 344.9 | 417 |
| 226.5 | 67.84 | 178.7 | 161 | 333.3 | 439.2 | 5.951 | 242.6 | 488.1 |
| 313.4 | 93 | 263.3 | 214.6 | 104.8 | 492.5 | 3.578 | 214.1 | 572.3 |
| 470.6 | 210.9 | 354.9 | 195.4 | 4.762 | 468.6 | 106.1 | 190.4 | 705 |
| 690.6 | 366.8 | 341.7 | 170.4 | 81.1 | 340.2 | 141.9 | 67.83 | 574.5 |
| 556.9 | 454.5 | 229.8 | 556.3 | 107.3 | 218.9 | 188.3 | 2.38 | 534.1 |
| 541.4 | 530.5 | 92.9 | 639.3 | 290.8 | 165.4 | 554 | 108.5 | 467.5 |
| 466.4 | 545.8 | 1.191 | 463 | 427.6 | 71.4 | 828 | 236 | 258.2 |
| 171.4 | 486.4 | 82.3 | 317.8 | 603.6 | 1.19 | 567.4 | 298.9 | 70.2 |
| 114.3 | 350.9 | 271.9 | 160.7 | 953 | 9.54 | 422.2 | 354.6 | 3.571 |
| 5.953 | 226 | 257.5 | 58.34 | 924 | 88.2 | 735 | 369.9 | 21.47 |
| 1.193 | 147.5 | 389.6 | 22.67 | 697.1 | 214.4 | 538.9 | 355.6 | 122.8 |
| 112.1 | 70.2 | 605 | 115.7 | 484.2 | 278.5 | 310.6 | 347.3 | 232.4 |
| 263.4 | 5.95 | 561.8 | 224.1 | 352.2 | 347.4 | 89.3 | 236.7 | 472.8 |
| 331.2 | 10.73 | 480.8 | 358.7 | 124.9 | 390 | 1.19 | 190.4 | 609.4 |
| 409.7 | 116.8 | 357.1 | 358.5 | 22.61 | 403 | 63.2 | 135.7 | 72.6 |

SAMPLED DATA FOR THE MONTH OF SEPTEMBER

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 193 | 140.1 | 16.36 | 163.5 | 85.5 | 35.12 | 56.2 | 64.37 | 177.9 |
| 19.9 | 185.3 | 133.3 | 66.71 | 184.9 | 145.1 | 144 | 70.2 | 197.7 |
| 83.1 | 36.28 | 47.99 | 73.7 | 273.8 | 153.3 | 172 | 108.8 | 196.5 |
| 165 | 84.3 | 121.7 | 193.1 | 236.2 | 133.4 | 118.2 | 256.2 | 417.5 |
| 267.8 | 117 | 177.9 | 326.3 | 304 | 168.4 | 324.1 | 251.5 | 424.3 |
| 383.5 | 157.9 | 197.7 | 290 | 392.8 | 257.3 | 315.8 | 276 | 492 |
| 653.3 | 237.4 | 271.3 | 233.8 | 169.5 | 437.2 | 281.8 | 598.6 | 400.7 |
| 697.3 | 353.1 | 268.9 | 275.8 | 90.1 | 531.7 | 76 | 696.5 | 384.3 |
| 462.3 | 624.1 | 332 | 439.4 | 169.6 | 523.4 | 174.3 | 463.8 | 234.8 |
| 454.3 | 565.4 | 168.3 | 407.8 | 60.84 | 241.8 | 253.8 | 364.5 | 73.6 |
| 312 | 327.1 | 260.7 | 319 | 1.17 | 122.7 | 65.48 | 98.1 | 8.18 |
| 355.2 | 244.2 | 187.1 | 147.2 | 83.9 | 1.169 | 82.6 | 135.4 | 127.8 |
| 210.3 | 130.9 | 79.5 | 22.2 | 1.171 | 114.6 | 37.46 | 166.2 | 49.16 |
| 4.673 | 16.36 | 14.03 | 116.5 | 69.08 | 2.341 | 132.3 | 236.3 | 88.9 |
| 169.5 | 120.6 | 88.6 | 9.36 | 199 | 67.88 | 380.3 | 283 | 329.9 |
| 53.84 | 32.79 | 37.46 | 79.6 | 280.8 | 218.8 | 373.1 | 289.9 | 460.6 |
| 147.5 | 70.3 | 141.6 | 424.6 | 410.5 | 499.2 | 566.9 | 497.8 | 528.1 |
| 262.1 | 302.1 | 225.8 | 327.2 | 588 | 580.6 | 470.9 | 542 | 412.3 |
| 181.3 | 885 | 523.9 | 586.4 | 512.9 | 550 | 467.3 | 886 | 262.8 |
| 366 | 868 | 507.2 | 676 | 620.3 | 740 | 495.3 | 537 | 670.5 |
| 358.9 | 586.5 | 397.3 | 626.7 | 567.6 | 487.9 | 371.5 | 319.9 | 520.9 |
| 448.8 | 808 | 310.8 | 863 | 211.4 | 692.1 | 219.6 | 175.1 | 280.3 |
| 337.7 | 658.7 | 273.4 | 729 | 145 | 298.8 | 92.3 | 129.6 | 100.5 |
| 308.4 | 419.3 | 415.9 | 512.3 | 2.338 | 434.3 | 8.18 | 5.854 | 154.3 |
| 199.8 | 185.7 | 262.9 | 234.6 | 150.3 | 190.3 | 150.6 | 81.9 | 8.19 |
| 137.9 | 52.58 | 148.4 | 8.17 | 7.03 | 3.503 | 10.54 | 263.3 | 80.8 |
| 1.168 | 202.9 | 5.844 | 211.6 | 72.6 | 198.6 | 72.6 | 478.3 | 166.2 |
| 116.8 | 5.852 | 135.4 | 30.44 | 89 | 8.19 | 100.7 | 534.1 | 224.6 |
| 4.682 | 69.05 | 56.19 | 166.2 | 167.4 | 83.1 | 100.6 | 358.7 | 507.4 |
| 70.2 | 180.2 | 307.8 | 271.4 | 382.6 | 345 | 232.8 | 504.7 | 363.4 |
| 70.2 | 336.8 | 345 | 374.2 | 391.7 | 317.9 | 651.2 | 608.5 | 460.3 |
| 76.1 | 365.9 | 341.4 | 340.2 | 427.9 | 446.3 | 337.7 | 594.5 | 488.2 |
| 154.4 | 544.5 | 475.8 | 785 | 499 | 200.9 | 357.5 | 552.4 | 464.7 |
| 386.1 | 467.2 | 455.7 | 869 | 260.6 | 431.1 | 442.7 | 283.8 | 286.1 |
| 404.6 | 380.7 | 114.5 | 932 | 294.5 | 606.1 | 281.5 | 107.5 | 106.3 |
| 514.3 | 745 | 225.6 | 650.1 | 278.1 | 637.5 | 278 | 21.07 | 127.4 |
| 523.6 | 566.3 | 99.3 | 300 | 146.1 | 134.3 | 86.5 | 217.7 | 8.18 |
| 527.1 | 175.1 | 47.93 | 213.7 | 7.01 | 42.04 | 1.169 | 259.8 | 137.1 |
| 255.9 | 77.1 | 102.9 | 122.6 | 126 | 1.169 | 123.1 | 205.9 | 21.07 |
| 195.1 | 10.51 | 44.5 | 210.6 | 1.171 | 135.6 | 59.7 | 166.1 | 70.2 |