ON THE SUSTAINABILITY OF GHANA NATIONAL HEALTH INSURANCE SCHEME: AN ACTUARIAL APPROACH. A CASE STUDY OF THE KPANDO DISTRICT

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DECLARATION

I, the undersigned, declare that this submission is the result of my own research work carried out in the Department of Mathematics, KNUST, towards the MSc. Mathematics and that to my best knowledge, under the supervision of Dr. Francis T. Oduro.

All references have been duly acknowledged.

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ABSTRACT

There is considerable interest at present in exploring the potential of health insurance to increase access to and affordability of health care in Ghana. Ghana's National Health Insurance Scheme (NHIS) was passed into law in 2003 but fully implemented from late 2005. It has already reached impressive coverage levels.

This study seeks to check the sustainability of Ghana's National Health Insurance Scheme (NHIS) vis-à-vis the treatment of the top nine diseases from the designated health institutions within the Kpandu District of the Volta Region.

It was on the above background that the research was conducted with the main objective of critically analysing the prevailing situation taking into account the available statistics.

To achieve this result, probability theory was used to estimate the total cost of health care. A sample of both the insured and staff of the health insurance scheme were interviewed in addition to the secondary data collected and other literature on the subject area.

The research finding indicated that the NHIS is heavily reliant on tax funding for its revenue. This has permitted quick expansion of coverage, partly through the inclusion of large exempted population groups. Policyholders (Membership) increased from 7% of the population in 2005 to 53.3% in March 2008 within the Kpando District of the Volta Region. The premium was estimated to be GHC272.40 with loadings and GHC82.40 with without loadings.

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LIST OF ABBREVIATIONS

ABBREVIATIONS	MEANING
APV	Actuarial Present Value
DMHIS	District Mutual Health Insurance Scheme
GDP	Gross Domestic Product
DRG	Diagnosis-Related Group
GHS	Ghana Health Service
GPRS	Ghana Poverty Reduction Strategy
HFAC	Health Facilities Attendance Cards
KDMHIS	Kpando District Mutual Health Insurance Scheme
MHIS	Mutual Health Insurance Scheme
MHO	Mutual Health Organisation
MOH	Ministry of Health
NHIA	National Health Insurance Authority
NHIF	National Health Insurance Fund
NHIL	National Health Insurance Levy
NHIS	National Health Insurance Scheme
NIC	National Insurance Commission
OPD	Out Patient Department
PUD	Pelvic Ulcer Disease
SHI	Social Health Insurance
SIC	State Insurance Company
SSNIT	Social Security & National Insurance Trust
URI	Upper Respiratory Infection

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To my lovely mother, Irene Abra Ninnette Kpikpi.

You have laid the foundation for many generations yet to come.

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CHAPTER ONE

1.0 INTRODUCTION

Insurance may simply be defined as "pooling of risk". It is a means of transferring risk from the insured or policyholder (the person who is protected against risk) to the insurer (the person or company who assumes or takes the risk). Insurance is a practice which allows interested persons to contribute periodic funds (premium) towards a central "pool" which may be used to compensate people who suffer the actual loss for which the contributions were made. Healthcare is insurable since it has a high probability of occurrence and can be predetermined or calculated according to rules of probability.

The Government through the Ghana Poverty Reduction Strategy (GPRS) has outlined its policy strategies for dealing with poverty in the country. One of the major components of this GPRS is the strategy to deliver quality, accessible and affordable healthcare to all residents in Ghana especially the poor and vulnerable. The method of financing healthcare determines its quality, accessibility and affordability. As part of the GPRS, the previously operated system of "Cash and Carry" was phased out to give way to the newly established system of Health Insurance. This policy framework allows for the establishment of multiple health insurance schemes across the nation with a focus on the social-type which is called District Mutual Health Insurance and this is to address the needs of the poor with the districts. Government is aimed at achieving its set health goal within the Poverty Reduction Strategy by instituting the National Health Insurance Scheme. This insurance does not prevent ill health (sickness). The basic principle is to compensate the policyholder(s) by spreading out the risk of health cost on the shoulders of the entire community (district). The scheme thus, acts as a middleman of such a social co-operation. The more subscribers the scheme has, the more likelihood of available funds to support members when they require healthcare. The point to note here is that individuals still make payment for services consumed but in a more humane manner as they do not have to carry the burden of healthcare all alone. This underscores the policy of making it compulsory among others for every resident in Ghana to belong to a health insurance scheme of his or her choice.

Access to healthcare is made easier for those who readily need it. Nonetheless, access is a function of location of providers of services, cost of care and the ability to pay, quality of care and socio-cultural aspects of service provision. Financial barrier to health care is dependent on the payment mechanism that is put in place at the time of use of service. Out of pocket payment (cash and carry) at the time of use reinforces non-access to healthcare. Pre-payment schemes minimize or remove entirely the financial barrier to accessing healthcare. Thus, access to healthcare becomes independent of the individuals ability to pay out of pocket at the time of ill health. Direct out of pocket payment is regressive in that a higher proportion of income of the poor and lower income groups goes into healthcare.

Moreover, people are expected to pay for services consumed at the time of ill health when in fact they are non-productive during the period.

The National Health Insurance Scheme (NHIS) as the name suggests is a government operated system of insurance that seeks to eliminate the financial constraints in acquiring basic healthcare across the country. The management of the scheme entails keeping information of beneficiaries and their contributions.

The stakeholders of the scheme thus (the government, the board of trustees, the management team, medical service providers and the beneficiaries of the scheme), will therefore be concerned with its efficient management hence the need to establish defined procedures or structures to oversee the program.

The National Health Insurance ACT 2003 is an ACT to secure the provision of basic healthcare services to persons resident in the country through mutual and private health schemes, to put in place a body to register, license and regulate health insurance schemes and accredit and monitor healthcare providers operating under health insurance schemes; to establish a National Health Insurance Fund that will provide subsidy to licensed district mutual health insurance schemes, to impose a health insurance levy and to provide for purposes connected with these.

There is established by this ACT a body corporate known as the National Health Insurance Council referring to in this ACT as the "Council". This Council is headed by a Chairperson who together with the other members of the council is appointed by the President of the Republic of Ghana in consultation with the Council of State.

At the district level where the scheme is mainly operated, the following are the departments that play the role of seeing to the successful management of the scheme.

1.1 THE SCHEME MANAGERS DEPARTMENT

This is made up of the Managers of the local office of the scheme. Its functions are to supervise the daily activities of the scheme. It also serves as a mediatory body between the local office and other secretariats, health service providers and the Council. Decisions and actions concerning the local office are addressed by this department. The Scheme Manager who also acts as the Member-Secretary to the Board of Director is the head of the management at the local office.

1.2 THE ACCOUNTING DEPARTMENT

This is where the accountant of the scheme is. All financial related issues (In-and-Out financial flows) of the local office are handled by this department financial decisions of the local office are taken with the mail consent of this unit.

1. 3 THE CLAIMS DEPARTMENT

This is an investigative body that seeks to verify various claims by beneficiaries of scheme within the scheme's catchment area. Since the scheme entails different levels of contributions, subscribers with various claims would have to go through some form of vetting or scrutiny and this is done by the claims department.

1.4 PUBLICITY / MARKETING DEPARTMENT

It takes care of information and the human relation component of the secretariat (interactions with visitors are done here). Information on history and progress of the scheme can be obtained from this department. The NHIA gave a band of GHC7.10 to GHC48.00 and the Kpando District subscribers are made to pay a premium of GHC12.00 which is within the given range.

1.5 DATA ENTRY OPERATIONS

This department takes and enters subscribers' records. Scanning of pictures and issuing of ID cards to subscribers are done here.

1.6 RECEPTION / SECRETARY SECTION

This section is situated close to the main entrance of the secretariat and receives people before introduction to various departments within the local office.

1.7 PROBLEM STATEMENT

Several problems have been identified by the clients and administrators of National Health Insurance Scheme. The effective management of this scheme involves the timely and judicious disbursement of funds to provide quality, accessible and affordable healthcare which will go a long way to help achieve the set goal on health in the GPRS. To do this, the policyholders (the insured) must be ready and pay the appropriate premium then also the government comes in with the needed subsidy. Presently in the Kpando District of the Volta Region, out of a district population of 73,778 (from the National Census of 2000) the scheme has registered 39,343 as at 1st March, 2008 which is 53.3% of the district population. The breakdown of the total number registered is as follows;

- Formal Sector (SSNIT Contributors) 3,535
- Informal Sector (18 – 69 years) 8,958
- Dependants (under 18 years of age) 19,502
- Aged (70 years and above) 7,295
- Pensioner (SSNIT Retired) 389
- Indigents 629
- Partially Paid-up Members 283
- Number of expired membership 217

From the above figures, it is clear that non-contributors are more than contributors to the scheme. This suggests that there will be financial burden on the scheme which might eventually collapse the scheme.

The following is the breakdown of the claims management.

- Total premium collected from the formal sector as at April, 2008 GHC100,716.20
- Total subsidy from National Health Insurance Authority (NHIA) for exempt groups GHC489,291.45
- Distress Fund from NHIA GHC198,767.46
- Total claims paid from Jan. 2006 to April 2008 GHC881,240.50
- Total amount in arrears as at April, 2008 GHC377,503.40
- The actual amount for the Kpando District Scheme as at April, 2008 GHC267,102.15
- Total amount owned by Sister-Schemes as at April, 2008 GHC 110,401.25
- Monthly average of claims paid to service providers GHC 28,860.76
- Average per capita of claims paid GHC15.71
- Total attendance at facilities as at 28th February, 2008 49,616 persons
- Monthly average attendance 1,837 persons.

1.7.1 CHALLENGES TO FINANCIAL SUSTAINABILITY IN NHIS IN GHANA

Underlying the cash flow difficulties now emerging are some fundamental design issues which threaten the long-term sustainability of the NHIS. These are summarised as follows;

Dimension	Current challenges
Funding sources	Majority of income grows with growth in consumption, not with membership. Very low premia for informal sector in relation to cost of care.
Benefits package	Benefits package comprises an estimated 95% of all treatments in Ghana, with no limit to consumption.
Coverage	Large proportion of population is exempt and these categories continue to grow. Membership is growing and with a growing rate of utilization by members.
Payment systems	Prices have risen with new Diagnosis-Related Group (DRG) payment system Drug costs additional-incentive to over-prescribe Anecdotal evidence of "tariff creep" and gaming by providers Reported increase in fraudulent claims Increasing role of private sector (increase access but also raises costs)
Cost-control	No co-payments Gate-keeping not effective-patients self-refer to secondary hospitals and tertiary ones use their polyclinics as an entry point into specialist care
Monitoring	Poor monitoring and control systems within the NHIS, although a new IT system is being introduced which may improve the situation.

Table 1.1 Challenges to Financial Sustainability if NHIS in Ghana

While in a 'normal' insurance system, increased membership would bring in increased income from premia, in the NHIS, the income is largely de-coupled - 90-95%, according to the CEO in 2008, coming from SSNIT and the VAT levy. The bulk of its income will therefore grow with national income rather than membership numbers. GDP growth (6.2% in 2008) is below the rate of growth of membership (from 36% in 2007 to 45% of valid card holders in 2008). This means that the more successful the NHIS (in terms of coverage), the greater the risk of financial difficulties.

In 2008 a new tariff structure was introduced by the NHIA, based on a Diagnosis-Related Group (DRG) system (paying per episode of care, according to disease groups, but also differentiated by level of care and sector). This replaced the previous payment system which was based on fees for service.

Although financial data is not yet available from the NHIA to analyse the overall impact of the new tariffs, scrutiny of selected health facilities' claims shows an immediate jump in NHIS claims, sometimes a doubling within the month of the new tariff being introduced.

Drug costs are currently billed separately on top of the fixed DRG payment per episode, and it is reported by the NHIA that the number of drugs per prescription have increased, from 4.5 in 2004 to 6 now, with some more expensive drugs being particularly favoured by some doctors.

Another common result of the introduction of DRG systems is 'tariff creep' – shifting to diagnoses which attract a higher tariff - which is being reported by NHIA informants. ('We don't get simple malaria cases any more – all malaria is complicated'.)

In addition to the increase in tariffs and increase in members, there has been an increase in utilisation of services by members, which is the expected result of any reduction in financial barriers to care. While this is a positive development (OPD per capita visits remain under the expected norm), it is also something to monitor carefully, in terms of the implications for cash flows and, ultimately, sustainability. Increased utilisation of curative care is not self-evidently positive and care patterns can be distorted by provider interests and also unequal access by different groups. In addition, improving the quality of care is critical to realising health gains from increased utilisation.

The poor gate-keeping in the health system, which is a general issue in Ghana, not limited to care provided under the NHIS, also raises prices, as it means people frequent higher level facilities more, which results in higher reimbursement per episode.

An increase in accredited providers widens access, which is positive, but also has an impact on the cash flow of the NHIS. In 2008, there were 1,551 accredited private providers, providing one-third of all services reimbursed by the NHIS. Given that the tariff for private providers is higher (and consumers in urban areas have no price disincentive to visiting them), this is likely to be another driver of cost escalation.

There is anecdotal evidence of various types of fraud (against schemes but also, some allege, by and with schemes). It is not easy to assess their scale, not least because some mis-billing reflects lack of understanding of the new tariff. The shift from fee for service, which health facilities are accustomed to, to a DRG-based payment system is not simple. The payment is per illness episode, but the definition of episodes, and the rules about return visits (designed to control costs) are quite complex for providers to follow. As a result of this in one region, an estimated 20-25% of claims presented were rejected, for a variety of reasons.

1.7.2 CHALLENGES TO SUSTAINABILITY IN NHIS IN THE KPANDO DISTRICT

Other problems facing the Kpando Mutual Health Insurance Scheme are divided into for (4) main categories.

1. **Clients and Facilities Management.**

Some accredited Healthcare Providers and Community Premium Collectors do not render accurate accounts to the scheme's secretariat after selling out copies of the

Health Facilities Attendance Cards (HFAC) that were given them to sell out within the communities where they operate.

Some Sister-Schemes delay in making payment on behalf of their members who by no fault of theirs had to enjoy the facility from the Mutual Scheme under consideration. In the second half of April, 2008, Management embarked on an aggressive debts recovery exercise and an amount of GHC103,180.69 was recovered from some Sister-Schemes

2. **Provider Side**

Non-adherence to the Health Insurance Tariff Manual which include

i. Over prescription of drugs

ii. Taking money from clients for drugs even in the National Health Insurance Drug List.

iii. Refusal to fill the Health Insurance Prescription Form for the National Health Insurance Client who could not get all the drugs at the Service Providers' drug section (dispensary or pharmacy).

For the scheme's inability to pay its debts owed to Margret Marquart Catholic Hospital in Kpando, Health Insurance Clients from outside the region are not been treated unless they make payment for the services provided them.

3. **Clients Side**

Over-utilisation of the facilities by the clients and this include multiple reviews, moving from one hospital to another daily, taking drugs from non-health insurance clients by their relatives and many others. Several cases have been reported where a whole family or household intentionally visits the hospital when they are not sick but because they have been registered under the NHIS.

High rate of adverse selection. Thus, only people with particular medical problem rush to enrol with the scheme.

4. **In the Scheme**

Re-imbursement from Sister-Schemes for the Cross-Border Arrangement had not been timely coupled with delay in disbursement of subsidy from National Health Insurance Authority, inadequacy of subsidy and human resource constraints are stressing the Kpando District Mutual Scheme.

1.8 OBJECTIVES OF STUDY

This research work is to establish a practical National Health Insurance Scheme Funding Policy after carefully analysing the existing funding approach coupled with the other numerous existing challenges the scheme faces. The careful analysis will include estimating the financial and economic bearing of the diseases. The Margret Marquart Catholic Hospital which serves as the Kpando District Hospital was sourced for this information together with information from the Scheme's Secretariat. This funding policy will take into account how much should be paid as premium and how much government subsidy is needed in order to sustain the scheme.

1. 9 ORGANISATION OF THE STUDY

The study is organised into five (5) chapters. Chapter one has to do with a brief background information and introduction of the National Health Insurance Scheme where Chapter two looks at the concept and literature review, Chapter three has the methodology employed in the research work. Chapter four focuses on the analysis of the data obtained and modelling of the data. Finally, the conclusion and recommendations are contained in the chapter five.

1.10 LIMITATIONS

There is no information on the morbidity rates in Ghana therefore no information on mortality as well and very scanty research has been conducted in Ghana regarding the National Health Insurance Scheme (NHIS) in the area of premium determination or fixing. Another challenge was the change in government just at the beginning of the study which obstructed the sourcing of information from the various government departments and agencies. Improper record keeping culture also made it difficult to source for information on time and accurately.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 INTRODUCTION

With just a little percentage of the Ghanaian population living in cities - metropolitan areas, a greater number of the population live in towns and villages. The economic and social fabric of these high-density clusters is elaborately interwoven, with the well-being of each citizen intricately enmeshed with the activities of others. Strong interdependencies arise in all areas of human need: health, food, shelter, safety, clothing, recreation, maintenance, energy provision, and so on. Servicing these needs requires highly structured transportation and communication networks throughout the city for effective provision of a variety of services: emergency medical, police, mail collection and delivery, fire protection, street and highway maintenance, utility repair, street cleaning, refuse collection, bus and train transportation, taxi transportation, and so on.

Increasingly, citizens are demanding more services, by type, quantity, and quality. Yet the ability to pay for additional services has been severely strained by the economic capability of these same citizens who are demanding for the services. The resulting pressure, between the demands for more and better services, on the one hand, and decreased costs, on the other, has created a strong need for improved management decision making in services. It is a primary purpose of this research to provide methods for assisting these decisions specifically that of health – Health Insurance.

For our purposes, a decision is an irrevocable allocation of resources. A decision is irrevocable, in the sense that it is impossible or extremely costly to change back to the situation that existed before making the decision. Thus for our purposes a decision is not a mental commitment to follow a course of action but rather the actual pursuit of the course of action. R. A. Howard Thus, we deal with the allocation or deployment of the resources of the health service systems, including personnel, equipment, and various service-providers. From this viewpoint, operations research can be thought of as a decision-aiding technology, one to assist managers and stakeholders in improving the deployment of their resources. Most deployments occur spatially throughout the city, so much of our work will have a strong spatial component.

2.2 HISTORY OF INSURANCE

Insurance has been in existence for thousands of years as far back as the Babylonians, the early Greeks and the Romans.

To protect themselves against the dangers to travel, the traders of Babylon often agreed with the owners of the goods traded that the traders would not be held responsible if the goods were stolen.

Making cargo loans that would be rapid contingency upon safe arrival was a practice among the Phoenicians and Greeks, and was recognised under Roman law.

Life insurance was very common in the ancient religious societies of the Egyptians and the Greeks. The members of these societies made periodic contributions to provide burials for themselves in accordance with existing religious rites.

The earlier known insurance policy contract originated from Pisa, Italy in 1343. Maritime commerce flourished in the Italian republics and Merchants dealing internationally sought the maritime coverage provided by this Italian policies. The body of rules known as the "law merchants" was developed for the settlement of disputes by tribunals of Merchants. Maritime insurance was the first great step in the history of the insurance industry. The second was insurance against fire. Insurers of the periods worked without statistics or calculation of probability relying solely on their personal assignment of these risks. In 1706, the founding of the amicable or perpetual assurance in London finally signalled the breakthrough of actuarial science as the means of assessing risks and setting rates. The increased need for security which this new age brought was the origin of the insurance business today.

Insurance can be defined from the view points of several disciplines, including risk theory, law, history, sociology, economics and actuarial science.

After careful study, the commission on insurance terminology of the Americans risks and insurance association has defined insurance as: insurance is the pooling of fortuitous losses by transfer of such risks to insurers, who agrees to indemnify insured for such losses, to provide other pecuniary benefits on their occurrences or to render services connected with the risks. Although this definition may not be acceptable to all insurance scholars, it is useful for analysing the common elements of a time insurance plan.

According to Microsoft Encarta 2009, insurance is defined as a financial protection against loss or harm, an arrangement by which a company gives customers financial protection against loss or harm such as theft or illness in return for payment premium

2. 3 TYPES OF INSURANCE

Insurance can be classified as either private or government insurance. Private insurance includes life and health insurance and property and liability insurance.

Government insurance includes social insurance programs and all other government insurance plans.

2. 4 PRIVATE INSURANCE

Health insurance pays benefits to the insured in time of ill health in the form of, the insured receives free healthcare when he/she goes to an accredited health facility and the insurance company reimburses the health facility later. Life insurance pays death benefits to designated beneficiaries when the insured dies. The benefits include bills, estate taxes, and other expenses. The death process can also provide periodic income payment to the deceased beneficiary or beneficiaries. In addition, life insurers sell group and individual retirement plans that pay retirement benefits. Life and health insurers also sell individual and group health insurance plans that cover medical expenses because of sickness or injury. Both life and health insurers sell liability income plans that pay benefits during a period of disability.

Property insurance indemnifies property owners against the loss or damage of real or personal property caused by various perils, such as fire, lightening, wind storm or tornado. Liability insurance covers the insured legal liability arising out of property damage or bodily injury to others.

Property and liability insurance is also called property and casualty insurance. In practice, non life insurers typically use the term property and casualty in insurance (rather than property and liability insurance) to describe the various coverages and operating results. Casualty lines include auto liability, burglary and theft, workers compensations and health insurance. The various coverages can be grouped into two major categories – personal lines and commercial lines. This division however is not completely precise and accurate because of the overlap of certain coverage. For example, although inland marine insurance is commercial lines, it also covers certain personal property such as experience jewellery and furs.

1. Personal lines: this refers to coverage that insure the real estate and personal property of individual and families or provide protection against legal liability. Major personal lines include:
    - Personal umbrella liability insurance
    - Private passenger auto insurance
    - Boat owners insurance
    - Homeowners insurance
2. Commercial lines: refers to property and casualty coverage for business firms, non-profit organisations or government agencies. Major commercial lines include but the following:
    - Health and Accident insurance
    - Fire insurance
    - General liability insurance
    - Workers compensation insurance
    - Inland marine insurance
    - Commercial auto insurance
    - Professional liability insurance
    - Ocean marine insurance, etc.

2.5 GOVERNMENT INSURANCE

Government insurance can be divided into social insurance programs and other government insurance. A number of these government insurance programs are in operation at the moment.

2.6 SOCIAL INSURANCE

Social insurance programs are government insurance programs with certain characteristics that distinguish them from other government insurance plans. These programs are financed entirely in large part by mandatory contributions from employers, employees or both and not primarily by the general revenues of government. The contributions are usually earmarked for special trust funds, the benefits in turn, are paid from these funds. Moreover, most social insurance programs are compulsory. Covered workers and employers are required by law to pay contributions and participate in the programs. Finally eligibility requirements and benefits rights are usually prescribed exactly by status, leaving little room for administrative discretion in the award of benefits.

In the United States, major social insurance programs are Old Age, Survivors and Disability insurance, Health insurance (Medicare), Unemployment insurance, Workers compensation, Railroad Retirement Act, and Compulsory temporal disability insurance.

In Ghana, the major social insurance program is the National Health Insurance Scheme (NHIS).

2.7 OTHER GOVERNMENT INSURANCE PROGRAMS

Some of the other social insurance include retirement system, various life insurance programs for veterans, pension termination insurance, insurance on checking and savings accounts in commercial banks and saving and loan associations. Government insurance programs include the Automobile Insurance Fund. In addition competition and monopoly workers compensation funds are also in operation. Finally, the majority of states have special health insurance pools that make health insurance available to persons who are uninsurable or substandard in health.

In Ghana, there are many groups (cooperatives, unions and associations) that run insurance packages on behalf of the government at the grass-root levels.

2.8 BENEFITS OF INSURANCE TO SOCIETY

The social and economic benefits of insurance include;

- **Indemnification For Loss**

Indemnification permits individuals and families to be restored to their former financial positions after a loss occurs. As a result, they can maintain their financial security. Because the insured are restored either in part or in whole after a loss occurs, they are likely to apply for public assistance or welfare benefits or to seek financial assistance from relatives and friends.

To firms it also permits firms to remain in business and employees to keep their jobs. Suppliers continue to receive orders, and customers can still receive goods and services they desire. The community also benefits because its tax base is not eroded. In short, the indemnification function contributes greatly to family and business stability and therefore is one of the most importance of insurance.

- **Reduction of Worry and Fear**

Worry and fear are reduced both before and after a loss. For instance, if family heads have adequate amounts of life insurance, they are less likely to worry about their dependents in the case of premature death. Persons insured for long-term disability do not have to worry about loss of earnings if a serious illness or accident occurs; and property owners who are insured enjoyed greater peace of mind because they know they are covered if a loss occurs. Worry and fear are also reduced after a loss occurs, because the insured know that they have insurance that will pay for the loss.

- **Source of Investment Fund**

The insurance industry is an important source of fund for capital investment and accumulation. Premiums are collected in advance of the loss, and funds not need to pay immediate losses and expenses can be loaned to business firms. These funds typically are invested in shopping centres, hospitals, factories, housing developments, and new machinery and equipments. The investment increases society's stock of capital goods, and promote economic growth and full employment. Insurers also invest in social investments, such as housing, nursing homes, and economic development projects. In addition, because the total supply of loan able funds is increased by the advance payment of insurance premiums, the cost of capita to business firms that borrow lower than it would be in the absence of insurance.

- **Enhancement of Credit**

A final benefit is that insurance enhances a person's credit. Insurance makes a borrower a better credit risk because it guarantees the value of the borrower's collateral or gives greater assurance that the loan will be repaid. For instance, when a house is purchased, the lending institution normally requires property insurance on the house before the mortgage loan is granted. The property insurance protects the lender’s financial interest if the property is damaged or destroyed.

Similarly, a business firm seeking a temporary loan for Christmas or seasonal business may be required to insure its inventories before the loan is made. If a new car is purchased and financed by a bank or other lending institution, physical damage insurance on the car may be required before the loan is made. Thus, insurance can enhance a person’s credit.

- **Loss Prevention**

Insurance companies are actively involved in numerous loss prevention programs and also employ a wide variety of loss-prevention personnel including safety engineers and specialist in fire prevention, occupational safety and health product liability. Some important loss prevention activities that property and liability insurers strongly support are;

- ✓ Fire prevention
- ✓ High way safety and reduction of automobile deaths
- ✓ Prevention of defective products that could injure the user
- ✓ Educational program on loss prevention
- ✓ Reduction of work related disabilities

The loss prevention activities reduce both direct and indirect, or consequential losses. Society benefits, since both types of losses are reduced.

- **Cost of Insurance to Society**

Although the insurance industry provides enormous social and economic benefits to society, the social cost of insurance are Fraudulent claims, Cost of doing business and Inflated claims, to mention but a few.

2.9 COST OF DOING BUSINESS

One important cost is the cost of doing business. Insurance consume scarce resources – land, labour, capital and business enterprises in providing insurance to society. In financial term, an expense loading must be added to the pure premium to cover the expenses incurred by insurance companies in their daily operations. An expense loading is the amount needed to pay all expenses including commission, general administrative expenses, state premium taxes, acquisition expenses, and all allowance for contingencies and profit. Scales and administrative expenses of property liability insurers consume 22 cents of each premium dollar, where as operating expenses of life insurer's account for 13 percent (13%) of total expenditures. As a result, total costs to society are increased. For instance, assume that a small country with no property insurance has an average of $100 million of fire losses each year. Also assume that property insurance later becomes available, and the expense loading is 35 percent of losses. The total cost to the country is increased to $135 million.

However, this additional cost can be justified for several reasons. First, from the insured's viewpoint, uncertainty concerning the payment of a covered loss is reduced because of insurance. Second, the cost of doing business is not necessarily wasteful, because insurers engage in a wide variety of loss prevention activities. Finally, because economic resources are used up in providing insurance to society, a real economic cost is insured.

Fraudulent Claims

A second cost if insurance comes from the submission of fraudulent claims. Examples of fraudulent claims are;

- False health insurance claims are submitted to collect benefits

- Auto accidents are faked or staged to collect benefits
- Dishonest claimant fake slip and fake accidents
- Dishonest policy owners take out life insurance policies on insured who are reported as having died

The payment of such fraudulent claims results in higher premiums to all insured. The existence of insurance also prompts some insured to deliberately cause a loss so as to profit from insurance. These social costs fall directly on society.

Inflated Claims

Another cost of insurance relates to the submission of inflated or “padded” claims. Although the loss is not intentionally caused by insured, the amount of the claims -value may exceed the actual financial loss. Examples of inflated claims are

- Insured exaggerate the amount and value of property stolen from a home or business
- Insured presents extra symptoms so as to collect extra medication to sell out to persons uninsured with these symptoms.
- Insured (or Health Facilities) inflate the cost of healthcare by asking for prescriptions to diseases they are not presenting just to sell out or give these medications to uninsured persons.
- Insured inflate the amount of damage in automobile collision claims so that the insurance payments will cover the collision deductible.
- Attorneys for plaintiffs sue for high-liability judgements that exceed the true economic loss of the victim
- Disabled persons often malinger to collect disability income benefits for longer duration

Inflated claims must be recognised as an important social cost of insurance. Premium must be increased to pay the additional losses; as a result, disposable income and the consumption of goods and services are reduced.

Cost of Fraudulent and Inflated Claims

According to the Insurance Information Institute, property and casualty fraud totalled \$31 billion in 2002. When insurance fraud of all types is considered including life and health insurance, the total cost of insurance fraud is estimated to be between \$85 billion annually. To put these figures in prospective, the National Insurance Crime Bureau estimates that an average household pays an additional \$200 to \$300 in insurance premiums each year because of fraud.

2.10 THE AFRICAN PERSPECTIVE

To develop an insurance culture in Africa, particularly for those in rural areas living from agriculture, will undoubtedly require time and patience. If poverty is not an inescapable trap, sustainable development can be appreciated over the long-run. This horizon is several decades away; far beyond one term of government.

Micro-insurance is emerging as a critical pillar in poverty reduction in Africa. “Poverty is a worldwide phenomenon complex and multidimensional that requires that strategies designed to fight it be transversal and supported by long-term commitments at the international level.

Traditional approaches to fighting poverty based on direct subsidies have shown their limitations and inefficiencies in the African context.

Fighting poverty is a key element to maintaining peace world wide, both at the national and international level" (Pascal Koupaki June 2007).

However, if those in poverty are hit by one or more of the generally insurable perils – such as ill health, cattle death, crop loss, flood and fire – the likelihood of recovery is small. In reality, life perils and hazards are the most frequent cause of default in such schemes hence it is essential to find insurance solidarity solutions to mutualise those risks.

One major cause of poverty in Sub-Saharan Africa is the fact that low income households and markets do not have the same opportunities to finance investments, accumulate capital, and protect their health and properties. In order to facilitate good health, economic development and alleviate poverty, the financial systems must be made more inclusive by improving access to savings, credit and insurance services to the poor. The distribution systems of most insurers are not designed to serve the low income market. The contribution of insurance industry to the Gross Domestic Product (GDP) of countries in Sub-Saharan Africa is low and this is because most of the people operate informal sector as petty traders, artisans, subsistence farmers, fishermen, etc. Such people, who ironically happen to be the majority, usually have the low education, low and irregular income and therefore do not have access to regular financial services. The only way to reach out to these people is to use appropriate micro-insurance mechanisms or government (or state) driven insurance schemes.

These appropriate mechanisms include insurance products that are tailored to the needs of these low-income households such as mutual health insurance schemes or national health insurance. These mechanisms must be designed with appropriate and affordable insurance premiums and should factor in the irregular cash flows of those who work in the informal sector. The policy wording should be in very simple language, which can easily be understood by low-income clients who are often illiterate. Regulations should be formulated to protect low income clients from the two main forms of abuse and these are from agents who may mislead them in order to increase sales and to prevent MFIs from forcing clients to purchase insurance when borrowing. Despite these challenges, there is a surge of micro-insurance initiatives in Africa as of late.

2.11 THE GHANAIAN PERSPECTIVE

Protection against the risk associated with everyday life is essential to everyone's wellbeing and peace of mind. Insurable risks surround us in all walks of life, in some ways that we may be aware of and others that may not be, such as:

- Insuring your home and property against risk of burglary
- Insuring on the unexpected in the case of an accident
- Preparing for retirement by saving for a pension
- Protecting employers and providing assurance for employees

Insurance is a useful intervention capable of securing people against unforeseen risks. As a social device, it allows the individual to contractually transfer the potential financial consequences of a loss exposure to an insurer. An important benefit of insurance is the reduction of uncertainty and worry. It helps businesses to protect themselves from risk and provides a wide range of services to ordinary people from car and home insurance to pensions, health, among others.

The insured, in buying insurance, transfers some of the losses he/she cannot avoid and which are too expensive to prevent and too large to be retained to an insurer.

The insurance industry contributes immensely to economic growth by converting savings made by individuals into portfolios of assets and smoothing investment return, as well as allowing individuals to share in the prosperity of the economy. The funds raised by the industry are long-term in nature, especially that of the life insurance business, making it the most critical fund for economic development.

Insurance companies also assist economic growth by using the income they receive to provide long-term capital for investment and by providing a large pool of investment funds, cuts the risk and cost of investing, allowing businesses to invest in a wider range of activities. This indicates that with stable economic conditions in Ghana – low inflation, low interest rates and high income – the industry could fulfil its important role as the favourite savings vehicle in the country's economic development by providing financial security for businesses and individuals, since people would have more disposable income to start thinking positively about insurance. In view of this, the industry needs to be innovative in engineering products to attract the public's interest in areas such as Health and Life to increase gross premium income.

There is need for a change in perception and attitude of Ghanaians towards insurance. Although insurance is a useful tool for health and business risk management and social protection, the insurance culture of Ghanaians is still low. The industry needs to create public awareness, not only through advertisement of products but also by providing education on the importance of insurance as well as trying to dispel people's mistrust of the industry.

For the public to imbibe the culture of insurance and have confidence in the industry, it is imperative that the public know that insurance companies are not there to take advantage of them, but rather that they are established to fill an important gap in the socio-economic development of every nation.

Discrepancies in the payment of insurance claims arise due to the lack of understanding of insurance products by the insured. Bureaucracy is sometimes a problem in the industry because administration approvals for instance, take longer than they should. This can be curbed through the collaborative effort of the companies, service providers and the insuring public to enhance transparency and efficiency. Often times, many of the insuring public failed to painstakingly read and understand the policies bought, hence when their expectations are not met, they lose confidence in the entire business of insurance.

To overcome the challenges facing the sector, there is the need to educate the public on the importance of insurance. The NIC must also ensure Insurance Companies endeavour to make their products meaningful and easily understandable to the public by the use of simple language. They must also improve their marketing skills to encourage more people to take up insurance policies especially in the area of Health Insurance. The public should try to understand the products of the insurance companies and the policies they buy so that they do not get disappointed when it comes to payment of claims. This should be the case in the Social Insurance Schemes especially, because not everything is covered under such policies, thus it is intended to cover some specific issues.

Public confidence in the industry should be enhanced with the presence of a good regulatory body, the NIC, ensuring that insurance companies and service providers transact business to the satisfaction of the public.

CHAPTER THREE

3.0 METHODOLOGY

3.1 SURVIVAL MODELS

A probabilistic model of a random variable that represents the time until the occurrence of an event is called a **survival model**. Examples of this include the health expectancy of an individual, life expectancy of a newborn baby, the future working lifetime of a machine until it fails etc.

The concept of "survival" may mean, for example the time until a patient in a coma recovers from the coma, given that he recovers, the time until a claim is made on an automobile insurance policy, the time until a worker leaves employment to mention but a few.

The time until the specified event is known as a waiting time or a random time-to-event variable. Probabilities associated with these models play a central role in actuarial calculation such as pricing insurance contracts.

3.2 THE HEALTH TABLE FOR A DISCRETE SURVIVAL MODEL

The number of persons (or lives) expected to survive without falling sick to a number of days x from a group of l_0 insured persons is l_x. A health table displays in a table format the values l_x at days x for $x = 0,1,2,\ldots,\omega$, where ω is the first whole number day at which there are no remaining insured persons in the group.

The number of persons among l_0 insureds that fall sick in the day range $[x, x+1)$ is h_x. It is given as $h_x = l_x - l_{x+1}$

x	0	1	2	3	4	5	6	7	8	9
L_x	1,000	991	985	982	979	976	972	968	964	959
h_x	9	6	3	3	3	3	4	4	4	6

Table 3.1 Hypothetical Health Table

3. 3 PROBABILITIES COMPUTED FROM THE HEALTH TABLE ($_nP_x$), ($_nq_x$) and ($_{m\,|\,n}q_x$)

The probability that an individual currently on day *x* will survive without disease for *n* quarters is denoted by $_nP_x$ and is given by $_nP_x = \frac{l_{x+n}}{l_x}$ It is standard to omit the n subscript when n=1. The probability that an insured person currently on day *x* will fall sick within *n* quarters is denoted by $_nq_x$ and is given by $_nq_x = 1 - {}_np_x = 1 - \frac{l_{x+n}}{l_x} = \frac{l_x - l_{x+n}}{l_x}$ It is standard to omit the n subscript when n=1. The probability that an insured person currently on day *x* will survive without disease for m quarters and then fall sick within *n* quarters after is denoted $_{m|n}q_x$ and is given by $_{m|n}q_x = \frac{l_{x+m} - l_{x+m+n}}{l_x}$. It is standard to omit the n subscript when n=1.

For instance, the probability that an insured person currently on day *x* will survive without disease 1 quarter is $P_x = \frac{l_{x+1}}{l_x}$

For example, the probability that an insured person on day 5 survives for 2 quarters to day 7 is

$$ {}_{2}P_5 = \frac{l_{5+2}}{l_5} = \frac{l_7}{l_5} = \frac{968}{976} $$

The probability that an insured person currently on the first day will fall sick within 3 quarters is ${}_{3}q_1 = \frac{l_1 - l_4}{l_1} = \frac{991-979}{991} = \frac{12}{991}$

- Note that the probability that an insured person currently on day *x* will fall sick within 1 quarter is $q_x = \frac{l_x - l_{x+1}}{l_x} = \frac{h_x}{l_x}$
- The probability that an insured person on day 4 survives without disease for 3 quarters and then falls sick within the following 2 quarters is ${}_{3|2}q_4 = \frac{l_7 - l_9}{l_4} = \frac{968-959}{979} = \frac{9}{979}$
- The probability that an insured person currently on day *x* will survive without sickness for *m* quarters and then fall sick within 1 quarter is

$$ {}_{m|}\,q_x = \frac{l_{x+m} - l_{x+m+1}}{l_x} = \frac{h_{x+m}}{l_x} $$

3.4 RANDOM VARIABLE L(x)

Let the number of survivors on/at day *x* be a random variable L(x). The insured person function l_x represents the expected number of survivors without disease (persons who did not fall sick). Thus $l_x = E[L(x)]$

Assume L(x) follows a binomial distribution. Each insured person is viewed as an independent Bernoulli trial with number of trials n= l_0. Survival without disease to day *x*, with $p = {}_xP_0$ is "success".

Example

Using the health table, the mean and variance of L(5) are

$$E[L(5)] = np = l_0\,{}_5P_0 = (1000)(0.976) = 976 = l_5$$
$$Var[L(5)] = npq = l_0\,{}_5P_0\,{}_5q_0 = (1,000)(0.976)(0.024) = 23.424$$

The distribution of X, the random health-time of an insured person may be specified using any of the following

- the cumulative distribution function of X
- the probability density function of X
- the survival function
- the health table function
- the force of morbidity

3.5 CUMULATIVE DISTRIBUTION FUNCTION OF X

The cdf, $F_X(x)$ represents the probability that an insured person will fall sick at or before day *x* and is represented by $F_X(x) = \Pr(X \le x)$. $F_X(x)$ is continuous and non-decreasing with $F_X(0) = 0$ and $F_X(\omega) = 1$ where ω is the first day at which an insured person is certain to have fallen sick.

3.6 PROBABILITY DENSITY FUNCTION OF X

The probability density function is $f_X(x) = F_X\,'(x)$ and $\int_0^{\omega} f_X(x)dx = 1$. The pdf $f_X(x)$ is non-negative and continuous on the interval $[0, \omega)$. The probability that an insured person falls sick between days *a* and *b* is

$$\Pr(a \le X \le b) = \int_a^b f_x(u)du = F(b) - F(a)$$

3.7 PROPERTIES OF pdf AND cdf

The probability that an insured person will fall sick in the interval $[x, x + \Delta x]$ can be estimated as $\Pr(x \le X \le x + \Delta x) = f_X(x).\Delta x$

There is also the need to note that $F_X(x) = \Pr(X \le x) = \int_0^x f_x(u)du$

3.8 THE SURVIVAL FUNCTION

The survival function is $s_X(x) = \Pr(X > x)$. The survival function gives the probability that an insured person falls sick after day *x*. This is the same as saying that the insured person survives without falling sick to day *x*, or is in good health at day *x*.

3.9.1 PROPERTIES OF THE SURVIVAL FUNCTION

From the distributions of the health-time variable X, the following properties of the survival function can be deduced.

1. $s_X(x) = 1 - F_X(x)$

2. $\Pr(a \le X \le b) = \int_a^b f_X(x)dx = s_X(a) - s_X(b)$

3. $f_X(x) = -s_X{}'(x)$

$s_X(x)$ is continuous and non-increasing with $s_X(0) = 1$ and $s_X(\omega) = 0$

Example

Suppose that the health-time X of an insured person is exponentially distributed with mean 75 days. Identify the survival function $s_X(x)$, calculate the probability that an insured person is still not fallen sick at the 100 and calculate the probability that a person was insured between the 60^{th} and 75th days.

Solution

The survival function $s_X(x)$ is

$F_X(x) = 1 - e^{-x/75}$ implies $s_X(100) = e^{-100/75}$

The probability that an insured person has not fallen sick at the 100 is

$s_X(100) = e^{-100/75} = 0.26360$ this is very unlikely

The probability that a person was insured between the 60^{th} and 75th days is

$s_X(60) - s_X(75) = e^{-75/75} = 0.08145$

3.9 THE HEALTH TABLE FUNCTION

The health table function l_x is defined as the expected number of survivors who are healthy at day x and is given as $l_x = E[L(x)] = np = l_0 s_X(x)$

Here, the random variable L(x) follows a binomial distribution with n = l_0 trials. Survival in good health to day *x* is defined as "success", with probability $p = \Pr(X > x) = s_X(x)$.

Example

Suppose that the health-time X of an insured person is uniformly distributed on [0,100]. Identify the survival function $s_X(x)$ and identify the table function l_x if $l_0 = 100$

Solution

The survival function is given by

$$s_X(x) = \Pr(X > x) = \int_x^{100} f_X(u)du = \int_x^{100} 0.01.du$$
$$= 0.01(100 - x) = 1 - 0.01x$$

The health-table function is $l_x = l_0 s_X(x) = 100(1 - 0.01)x = 100 - x$ for $0 \le x \le 100$

3.9.1 PROPERTIES OF THE HEALTH TABLE FUNCTION

1. l_x is the expected number of survivors in good health at day *x* from a group of l_0 insured persons

2. $l_x = l_0 s_X(x)$ is continuous and non-increasing with $l_\varpi = 0$

3. $\quad s_X(x) = \dfrac{l_x}{l_0}$

Example

Let $l_x = 10,000(100-x)^2$ for $0 \le x \le 100$

Find the cdf and the pdf for the associated health-time variable X.

Solution

For the cdf;

$$s_X(x) = \frac{l_x}{l_0} = \frac{10,000(100-x)^2}{10,000(100)^2} = \frac{(100-x)^2}{100^2}$$

$$F_X(x) = 1 - s_X(x) \qquad = 1 - \frac{(100-x)^2}{100^2}$$

The pdf is worked for as;

$$f_X(x) = F_X\,'(x)$$

$$= \frac{d}{dx}\left(1 - \frac{(100-x)^2}{100^2}\right) \qquad = \frac{2(100-x)}{100^2} \qquad = \frac{100-x}{5,000}$$

3.10 FORCE OF MORBIDITY

The force of the morbidity is denoted by $\mu(x)$ and is an instantaneous measure of morbidity at day *x*. It is defined as

$$\mu(x) = \frac{f_X(x)}{s_X(x)} = -\frac{s'_X(x)}{s_X(x)} = -\left[\ln(s_X(x))\right]' = -\frac{l'_x}{l_x}$$

Example

Using the data in the previous example, it gives

$$\mu(x) = -\frac{s_X\,'(x)}{s_X(x)} = -\frac{-2(100-x)/100^2}{(100-x)^2/100^2} = \frac{2}{100-x}$$

A previous example with $F_X(x) = 1 - e^{-x/75}$ gives

$$\mu(x) = -\ \ In(s_X(x)\ \ ' = -\left(-\frac{x}{75}\right)' = \frac{1}{75}$$

3.10.1 SURVIVAL FUNCTION FROM FORCE OF MORBIDITY

Survival function is written in terms of the force of morbidity as

$$\int_0^x \mu(y)dy = -In(s_X(x))$$

$$\Rightarrow s_X(x) = \exp\ \ -\int_0^x \mu(y)dy$$

Example

Let the force of morbidity be given by $\mu(x) = \dfrac{0.9}{90-x}$ for $0 \le x < 90$. Calculate the survival function.

Solution

$$s_X(x) = \exp\ \ -\int_0^x \mu(y)dy \qquad = \exp\left(-\int_0^x \frac{0.9}{90-y}dy\right) \qquad = \exp\ \ 0.9\ln(90-y)\Big|_0^x$$

$$= \exp\left(\left(0.9\ln\left(\frac{90-x}{90}\right)\right)\right) \qquad = \left(\frac{90-x}{90}\right)^{0.9}$$

3.10.2 THE MEANING OF FORCE OF MORBIDITY

The defining formula is in the form

$$f_X(x)\Delta x = \Pr(x < X \le x + \Delta x)$$

$$= \Pr(X \le x + \Delta x \mid X > x)\Pr(X > x) \quad = \Pr(X \le x + \Delta x \mid X > x)s_X(x)$$

$$\Rightarrow s_X(x)\mu(x)\Delta x = \Pr(X \le x + \Delta x \mid X \ge x)s_X(x)$$

$$\Rightarrow \mu(x)\Delta x = \Pr(X \le x + \Delta x \mid X > x)$$

So, $\mu(x)\Delta x$ is approximately equal to the conditional probability that an insured person that has survived in good health to day x subsequently falls sick during the next Δx quarters.

Example

Let the force of morbidity be given by the formula: $\mu(x) = \dfrac{0.9}{90 - x}$ for $0 \le x < 90$.

Calculate the approximate probability that a person insured on day 40 falls sick within the next quarter.

Solution

Setting $\Delta x = 1/100$ the required probability is: $\mu(40)\Delta x = \left(\dfrac{0.9}{90 - x}\right)\left(\dfrac{1}{100}\right) = 0.00018$

3.11 STANDARD PROBABILITIES

The probability that an individual currently on day x will survive t quarters.

$${}_tP_x = \frac{l_{x+t}}{l_x} = \frac{s_X(x+t)}{s_X(x)} = \frac{\Pr(X > x+t)}{\Pr(X > x)} = \Pr(X > x+t \mid X > x)$$

The probability that an individual currently on day *x* will fall sick within the next *t* quarters.

$${}_tq_x = \frac{l_x - l_{x+t}}{l_x} = \frac{\Pr(X > x) - \Pr(X > x+t)}{\Pr(X > x)} = \Pr(X \le x+t \mid X > x)$$

The probability that an individual currently on day *x* will survive *s* quarters but will fall sick within the following *t* quarters.

$${}_{s|t}q_x = \frac{l_{x+s} - l_{x+s+1}}{l_x} = \Pr(x+s < X \le x+t \mid X > x)$$

Key relations concerning standard probabilities are deduced as stated below and these are

$${}_tP_x +_t q_x = 1$$

$${}_{s+t}P_x =_s P_x \; {}_sP_x \; {}_tP_{x+s}$$

$${}_{s|t}q_x =_s P_x \; {}_tq_{x+s} =_s P_x -_{s+t} P_x =_{s+t} P_x =_{s+t} P_x =_{s+t} q_x -_s q_x$$

$${}_nP_x = P_x P_{x+1} ......P_{x+n-1}$$

$${}_nq_x = 0 \mid q_x +_1 \mid q_x + ... +_{n-1} \mid q_x$$

3.12 CONTINUOUS FUTURE HEALTH-TIME AFTER DAY *x*

The conditional distribution of the time lived after day *x*, given survival without falling sick to day *x*, is $T(x) = X\text{-}x|X>x$. The continuous random variable T(x) is a survival model defined on the interval $[0, \omega - x]$. The corresponding survival function is

$$s_{T(x)}(t) = P(T(x) > t) = {}_t P_x = \frac{l_{x+t}}{l_x} = \frac{s_X(x+t)}{s_X(x)} \qquad \text{since} \quad l_x = l_0 s_X(x)$$

Example

Compute the table survival function for newly insured persons and the survival function for persons currently on day 20, where $l_x = 10{,}000\,(100 - x)^2$.

Solution

The survival function for newly insured persons $s_X(x)$ is

$$s_X(x) = {}_x P_0 = \frac{l_x}{l_0} = \frac{10{,}000(100-x)^2}{10{,}000(100-0)^2} = \left(\frac{100-x}{100}\right)^2 \qquad \text{since} \quad 0 \le x \le 100$$

The survival function for persons currently on day 20, $s_{T(20)}(t)$ is

$$s_{T(20)}(t) = {}_t P_{20} = \frac{l_{20+t}}{l_{20}} = \frac{10{,}000(100-(20+t))^2}{10{,}000(100-20)^2} = \left(\frac{80-t}{80}\right)^2 \qquad \text{since} \quad 0 \le x \le 80$$

3.13 cdf AND pdf FOR THE DISTRIBUTIONS OF X AND T(x)

$$F_T(t) = \Pr(T(x) \le t) = \Pr(X - x \le t \mid X > x)$$

$$= \frac{\Pr(x < X \le x+t)}{\Pr(X > x)} = \frac{F_X(x+t) - F_X(x)}{1 - F_X(x)}$$

Thus

$$f_T(t) = (F_T(t))' = \frac{d}{dt}\left(\frac{F_X(x+t) - F_X(x)}{1 - F_X(x)}\right) = \frac{f_X(x+t)}{1 - F_X(x)} = \frac{f_X(x+t)}{s_X(x)}$$

$$= -\frac{s_X{}'(x+t)}{s_X(x)} = -\frac{l'_{x+t}}{l_x}$$

3.14 $f_{T(x)}(t)$ AND FORCE OF MORBIDITY

It is given as

$$f_X(x) = s_X(x)\mu(x) = {}_x P_0 \mu(x)$$

$$\Rightarrow f_T(t) = \frac{f_X(x+t)}{s_X(x)}$$

$$= \frac{{}_{x+t}P_0\mu(x+t)}{{}_xP_0} = \frac{{}_xP_0\,{}_tP_x\mu(x+t)}{{}_xP_0}$$

$$= {}_t P_x \mu(x+t)$$

Example

Compute the cdf and pdf for the future health-time of an individual on day 20 for l_x = 10,000(100 - x)^2 , $0 \le x \le 100$.

Solution

From the previous problem, the survival function $s_{T(20)}(t)$ on day 20 for individuals, is

$$s_{T(20)}(t) = {}_t P_{20} = \frac{l_{20+t}}{l_{20}} = \frac{10,000(100-(20+t))^2}{10,000(100-20)^2} = \left(\frac{80-t}{80}\right)^2 \quad \text{since} \quad 0 \le x \le 80$$

Thus

$$F_{T(20)}(t) = 1 - s_{T(20)} = 1 - \left(\frac{80-t}{80}\right)^2$$

$$f_{T(20)}(t) = F_{T(20)}(t)\ ' = \left(1 - \left(\frac{80-t}{80}\right)^2\right)' = 2\frac{(80-t)}{80^2} = \frac{(80-t)}{3,200}$$

Alternative solution to $f_{T(20)}(t)$

$$\mu(x) = \frac{l_x'}{l_x} = -\frac{2}{100-x}$$

$$f_{T(20)}(t) = {}_tP_{20}\mu(20+t) = \left(\frac{80-t}{80}\right)^2 \frac{2}{80-t} = \frac{80-t}{3,200}$$

3.19 KEY RELATIONS BETWEEN T(x) AND X

These are;

$$s_T(t) = \Pr(T(x) > t) = {}_tP_x = \frac{l_{x+t}}{l_x} = \frac{s_X(x+t)}{s_X(x)}$$

$$F_T(t) = {}_tq_x = 1 - s_X(t) = \frac{F_X(x+t) - F_X(x)}{s_X(x)}$$

$$f_T(t) = \frac{f_X(x+t)}{s_X(x)} = {}_tP_x\mu(x+t)$$

3.16 CURTATE FUTURE HEALTH-TIME K(x) AFTER DAY *x*

The curtate health-time is a discrete random variable defined by K(x) = [T(x)] thus the integer part of T(x). Since it is a function of T(x), it is simple to calculate the probability function of K(x) from T(x). The values of K(x) are the numbers 0,1, 2,…,*w*-1. If the health (70) eventually falls sick on day 85.8, then the continuous future health-time is T(70) = 15.8 and the curtate health-time is K(70) = [15.8]=15.

3.17 PROBABILITY FUNCTION FOR K(x)

Note that if K(x) = k, then

$$k \le T(x) < k+1$$

$$\Rightarrow \Pr(K(x)=k) = \Pr(k \le T(x) < k+1)$$

$$f_{K(x)}(k) = \Pr(K(x)=k) = {}_{k|}q_x = \frac{h_{x+k}}{l_x} \qquad \text{for} \qquad k = 0,1,2,\ldots,\omega - x - 1$$

Example

Given $l_x = 100 - x$, we compute the probability function for K (75) as follows

$$\Pr(K(75)=k) = \frac{h_{75+k}}{l_{75}} = \frac{l_{75+k} - l_{75+k+1}}{l_{75}} = \frac{(100-75-k)-(100-75-k-1)}{(100-75)}$$

$$= \frac{1}{25}$$

Thus K(75) has 25 possible values (0,1,2,…,24) that are equally likely to occur.

Example

Given $l_x = 1000e^{-0.015x}$, we compute the probability function for K (75) as follows

$$\Pr(K(75)=k) = \frac{h_{75+k}}{l_{75}} = \frac{l_{75+k} - l_{75+k+1}}{l_{75}} = \frac{e^{-0.015(75+k)} - e^{-0.015(75+k+1)}}{e^{-015x75}}$$

$$= e^{-0.015k} \quad 1 - e^{-0.0l5} \qquad \text{this is a geometric distribution}$$

3.18 SURVIVAL FUNCTION FOR THE CURTATE FUTURE HEALTH-TIME

For any random variable $F_X(x) = \Pr(X \le x)$. Thus

$$F_{K(x)}(k) = \Pr(K(x) \le k)$$

$$= \Pr(K(x)=0) + \Pr(K(x)=1) + \ldots + \Pr(K(x)=k)$$

$$= \frac{h_x}{l_x} + \frac{h_{x+1}}{l_x} + \frac{h_{x+2}}{l_x} + \ldots + \frac{h_{x+k}}{l_x} \quad = \frac{l_x - l_{x+k+1}}{l_x}$$

$$= {}_{k+1}q_x \quad = 1 - {}_{k+1}P_x \qquad \text{for } k = 0,1\ldots,\omega - x - 1$$

$$\Rightarrow s_{K(x)}(k) = \Pr(K(x) > k)$$

$$= 1 - F_{K(x)}(k) \quad = 1 - {}_{k+1}q_x$$

$$= {}_{k+1}P_x = \frac{l_{x+k+1}}{l_x} \qquad \text{for } k = 0,1\ldots,\omega - x - 1$$

Example

Given $l_x = 100 - x$, the survival function for K (75) is calculated as follows

$$s_{K(75)}(k) = {}_{k+1}P_{75} \quad = \frac{l_{75+k+1}}{l_{75}} \quad = \frac{100-(75+k+1)}{100-75}$$

$$= \frac{24-k}{25} \qquad \text{for} \quad \text{k} = 0,1,\&,24$$

Example

Given $l_x = 1000e^{-0.015x}$, the survival function for K (75) is calculated as follows

$$s_{X(75)}(k) = {}_{k+1}P_{75} \quad = \frac{l_{75+k+1}}{l_{75}} \quad = \frac{e^{-0.015(75+k+1)}}{e^{-0.015(75)}}$$

$$= e^{-0.015(k+1)} \qquad \text{for} \qquad \text{k} = 0,1,2,\ldots$$

3.19 HEALTH ANNUITY MODELS

A health annuity is a series of payments made at regular intervals over the future of an individual. For example, an individual may purchase a health annuity from an insurance company to provide a regular treatment anytime he/she visits the health facility with ill health for the duration agreed upon in the contract. Since the future health status of the policyholder is unknown, the present value of the annuity benefit payments is also uncertain. The random present value of benefits payable is a function of either the complete future benefit variable, T(x), or the curtate K(x).

The objectives are to compute

1. The expected present value of benefits under a health annuity policy
2. The probability that the actual cost of benefits exceeds the expected amount
3. The probability that a certain fund of money will be sufficient to pay all the annuity benefits for a group of insured persons.

3.19.1 DISCRETE HEALTH ANNUITY MODEL

Payments are made at regular time intervals during the future of (x). There are two main types of annuities, according to the timing of the payments. These are

1. for an annuity due, the payments are made at the start of each period
2. for an annuity immediate, the payments are at the end of each period

Example

For a discrete model of an annuity due of 1 per day for health (x), payments of 1 will be made at the start of each, for as long as (x) is healthy. The series of payments associated with this annuity are illustrated in the diagram below

Payment:	1	1	1	..…	1	1
Day:	x	x+1	x+2	..…	x+K(x)	x+K(x)+1

3.19.2 DISCRETE MODEL OF ANNUITIES DUE

The random present value Y, and its expectation of this series of payments contingent on the survival of (x) is

$$Y \quad = a_{K(x)+1} \qquad \text{for } K(x) = 0,1,2,\ldots,\omega - x - 1$$

$$a_x = E[Y] \; = E\left[a_{K(x)+1}\right] \quad = \sum_{k=0}^{\omega-x-1} a_{k+1} \Pr(K(x)=k) = \sum_{k=0}^{\omega-x-1} a_{k+1} \; {}_{k|}q_x$$

3.20 CURRENT PAYMENT FORMULA

The current payment formula is derived as follows

$$a_x = E[Y] = \sum_{k=0}^{\omega-x-1} a_{k+1}\, {}_{k|}q_x = \sum_{k=0}^{\omega-x-1} \left(1+v+\ldots+v^k\right)\left({}_kP_x - {}_{k+1}P_x\right)$$

$$= (1)\left(1-P_x\right) + \left(1+v\right)\left(P_x - {}_2P_x\right) + \left(1+v+v^2\right)\left({}_2P_x - {}_3P_x\right) + \ldots + \left(1+v+\ldots+v^{\omega-x-1}\right)\left({}_{\omega-x-1}P_x - {}_{\omega-x}P_x\right)$$

$$= 1 + P_x\left(1+v-1\right) + {}_2P_x\left(1+v+v^2-1-v\right) + \ldots + {}_{\omega-x-1}P_x\left(1+v+\ldots+v^{\omega-x-1} - 1 - v \ldots - v^{\omega-x-2}\right)$$

$$= 1 + vP_x + v^2\,{}_2P_x + \ldots + v^{\omega-x-1}\,{}_{\omega-x-1}P_x = \sum_{k=0}^{\omega-x-1} v^k\,{}_kP_x$$

Example

A whole health annuity due of 1 is paid on day 90. The effective daily rate of interest is 0.1%. Calculate E[Y] and the probability that Y exceeds E[Y]. Morbidity is assumed to follow the model

Day x	l_x	h_x
90	100	25
91	75	35
92	40	40
93	0	0

Solution

On day 90, the present value variable for the annuity payments is

$$Y = a_{K(90)+1}$$

$$\Rightarrow E[Y] = \sum_{k=0}^{2} a_{k+1} \Pr(K(90) = k) \quad = a_1 \; {}_{0|}q_{90} + a_2 \; {}_{|}q_{90} + a_3 \; {}_{2|}q_{90}$$

$$= 1\left(\frac{25}{100}\right) + 1.999\left(\frac{35}{100}\right) + 2.998\left(\frac{40}{100}\right) \qquad = 2.14885$$

Where $\qquad a_{k+1} = 1 + v + ... + v^k \qquad = \frac{1 - v^{k+1}}{h} \qquad = \frac{1 - v^{k+1}}{iv}$

And the three possible values of Y are

if K = 0, 1

if K = 1, 1.999

and if K = 2, 2.998

From the calculations, it seen that only the last of these exceeds E[Y], therefore

$$\Pr(Y > E[Y]) = \Pr(K = 2) = \frac{h_{92}}{l_{90}} = \frac{40}{100} = 0.4.$$

Example

A health annuity due of 1 per day is paid on the 20th day with an effective daily rate of interest of 0.1%. Morbidity is assumed to follow the model $l_x = 100 - x$ at whole number days $x \le 100$.

i. Calculate E[Y]

ii. Calculate the probability that Y exceeds E[Y]

Solution

i. Using the current method to compute the APV

$$a_{20} = \sum_{k=0}^{100-20-1} v^k \, {}_kP_{20} = \sum_{k=0}^{79} v^k \, \frac{80-k}{80}$$

Now $\quad (Ha)_{n:i} = n + v(n-1) + v^2(n-2) + \ldots + 2v^{n-2} + v^{n-1} = \dfrac{n - a_n}{h}$

$$\Rightarrow a_{20} = \sum_{k=0}^{79} v^k \, \frac{80-k}{80} = \frac{(Ha)_{80:0.1\%}}{80}$$

$$= \frac{1}{80}\left(\frac{80 - a_{80:0.1\%}}{h}\right)$$

$$= \frac{1}{80}\left(\frac{80 - 19.5968}{0.001/1.001}\right)$$

$$= 755.7958$$

The probability that Y exceeds the actuarial present value is

$$\Pr\ Y > E[Y] \ = \Pr\ a_{K+1} > 755.7958 \qquad = \Pr\left(\frac{1 - 1.001^{-(K+1)}}{0.001/1.001} > 755.7958\right)$$

$$= \Pr\ 1.001^{-(K+1)} < 0.24496 \qquad = \Pr\left(K > \frac{\ln(0.24496)}{\ln(1.001)} - 1\right)$$

$$= \Pr(K > 27.83) \quad = {}_{28}P_{20} \quad = \frac{l_{48}}{l_{20}} \quad = \frac{100-48}{100-20} \quad = \frac{52}{80} \quad = 0.65$$

3.21 ANNUITY INSURANCE RELATION

The relations here are

$$Y = a_{K(x)+1} = \frac{1-v^{K(x)+1}}{h}$$

$$\Rightarrow a_x = E[Y] = \frac{1-E[Z]}{h} \quad = \frac{1-A_x}{h}$$

$$\Leftrightarrow 1 = A_x + ha_x$$

From the previous example it is observed that

$$A_{20} = 0.24496$$

$$\Rightarrow a_x = \frac{1-A_x}{h} = \frac{1-0.24496}{0.001/1.001} = 755.7958$$

This result agrees with the result of the last example.

3.22 PROBABILITY THAT Y EXCEEDS SOME VALUE y_0

The relationship between annuities and insurances is employed to calculate a result for the probability that Y exceeds some value y_0

$$\Pr(Y > y_0) = \Pr\left(\frac{1-v^{K(x)+1}}{h} > y_0\right) = \Pr\ 1-hy_0 > v^{K(x)+1}$$

$$= \Pr\left(K(x) > \frac{\ln(1-hy_0)}{\ln(v)} - 1\right)$$

3.23 FUND AVAILABLE

The corresponding fund (F) is given by

$$F = l_x a_x = \sum_{k=0}^{\omega-x-1} v^k l_{x+k}$$

Since there are K(x) payments, random present value of a standard health annuity immediate is

$Y_1 = a_{K(x)}$ for $K(x) = 0,1,2,...,\omega - x - 1$

Now $Y_1 = Y - 1$

$\Rightarrow E[Y_1] = E[Y-1] = E[Y] - 1$

3.24 CONTINUOUS HEALTH ANNUITY MODEL

Consider a health annuity that provides a continuous stream at the rate of 1 per day while the individual person (x) is surviving with good health

$Y = a_{T(x)}$ for $0 < T(x) < \omega - x$

$$\bar{a}_x = E[\bar{Y}] = E\left[\bar{a}_{T(x)}\right] = \int_0^{\omega-x} a_t f_{T(x)}(t)dt$$

$\bar{a}_x = \int_0^{\omega-x} v^t {}_tP_x dt$ which the current payment form obtained by integration by parts.

3. 31 CONTINUOUS ANNUITY INSURANCE RELATION

Continuous equivalent of the annuity-insurance relation can be derived as;

$$1 = \overline{A}_x + \delta\overline{a}$$

$$\Pr(Y > y_0) = \Pr\left(\frac{1-e^{-\delta T(x)}}{\delta} > y_0\right)$$

$$= \Pr\left(1-\delta y_0 > e^{-\delta T(x)}\right)$$

$$= \Pr\left(T(x) > -\frac{\ln(1-\delta y_0)}{\delta}\right)$$

Example

A continuous health annuity due of 1 per day is paid on the 20th day with an effective daily interest rate of 0.1%. Morbidity is assumed to follow the model $l_x = 100 - x$ at whole number days $x \leq 100$. Calculate E[Y] and the probability that Y exceeds E[Y]

Solution

i. By de Moivre's law, the distribution of future health-time is uniform on the interval [0, $\omega - x$]

$$\Rightarrow f_{T(20)}(t) = \frac{1}{80} \quad \text{for } 0 < t < 80$$

$$\overline{a}_{20} = E[Y] = \int_0^{\omega-x} a_t\, f_{T(20)}(t)dt = \int_0^{80} \frac{1-1.001^{-t}}{\ln(1.001)}\left(\frac{1}{80}\right)dt$$

$$= \frac{1}{80\ln(1.001)}\left(80 + \left(\frac{1.001^t}{\ln(1.001)}\Big|_0^{80}\right)\right) = \frac{1}{80\ln(1.001)}\left(80 - \frac{1-1.001^{-80}}{\ln(1.001)}\right)$$

$$= \frac{1}{0.07996}\left(80 - \frac{0.07684}{9.995\times10^{-4}}\right)$$

$$E[Y] = 39.0390$$

ii. The probability that this is insufficient to make all of the annuity payments is given by

$$\Pr(Y > 39.0390)$$

$$= \Pr\left(T(20) > -\frac{\ln(1-39.0390\delta)}{\delta}\right) \quad = \Pr(T(20) > 28.329)$$

$$= {}_{28.329}P_{20} = \frac{l_{48.329}}{l_{20}} \quad = \frac{100-48.329}{100-20} \quad = 0.6459$$

Alternatively, using the annuity insurance relation it becomes

$$\bar{a}_{20} = \frac{1-\bar{A}_{20}}{\delta} = \frac{1-0.9609}{\ln(1.001)} = 39.0390$$

Moreover, it can be shown that

$$\Pr(Z > E[Z]) + \Pr(Y > E[Y]) = 1$$

Now a previous problem gave

Pr (Z>E[Z])= 0.35411

Thus $\Pr(Y > E[Y]) = 1\text{-}0.35411 = 0.6459$

3.26 n-QUARTER TEMPORARY HEALTH ANNUITY FOR DISCRETE MODEL

Payments cease on the earlier of the expiration of the policy of the policyholder, the expiration of n days after the date of issue.

$$
Y = \begin{cases} a_{K(x)+1} = \dfrac{1 - v^{K(x)+1}}{h} & \text{if} \quad K(x) \le n-1 \\ \\ a_{n} = \dfrac{1 - v^{n}}{h} & \text{if} \quad K(x) \ge n \end{cases}
$$

In relation to n-day endowment, the random present value can be written as

$$
Y = \frac{1 - Z}{h} \quad \text{where} \quad Z = \begin{cases} v^{K(x)} & \text{if} \quad K(x) \le n-1 \\ v^{n} & \text{if} \quad K(x) \ge n \end{cases}
$$

Where Z is the random present value of an *n*-day endowment insurance.

3.26.1 APV OF n-QUARTER TEMPORARY ANNUITY FOR DISCRETE MODEL

The actuarial present value is given as

$$
\begin{aligned}
a_{x:\,n} &= E[Y] \\
&= \sum_{k=0}^{n-1} a_{k+1}\; {}_{k|}q_x + \sum_{k=n}^{\omega-x-1} a_n\; {}_{k|}q_x && \text{(aggregate payment)} \\
&= \sum_{k=0}^{n-1} v^k \; {}_kP_x && \text{(current payment)} \\
&= \frac{1 - A_{x:\,n}}{h} && \text{(annuity-insurance)}
\end{aligned}
$$

3.27 n-QUARTER TEMPORARY ANNUITY FOR CONTINUOUS MODEL

Payments cease on the earlier of the expiration of the policy of the policyholder, the expiration of *n* days after the date of issue.

$$Y = \begin{cases} a_{T(x)} = \dfrac{1-v^{T(x)}}{\delta} & \text{if} \quad T(x) \le n \\ \\ a_{n} = \dfrac{1-v^{n}}{\delta} & \text{if} \quad T(x) > n \end{cases}$$

In relation to n-day endowment, the random present value can be written as

$$Y = \frac{1-Z}{\delta} \qquad \text{where } Z = \begin{cases} v^{T(x)} & \text{if} \quad T(x) \le n-1 \\ v^{n} & \text{if} \quad T(x) > n \end{cases}$$

Where Z is the random present value of an *n*-day endowment insurance.

3.27.1 APV OF n-QUARTER TEMPORARY ANNUITY FOR CONTINUOUS MODEL

The actuarial present value is given by

$$\bar{a}_{x:n} = E[Y]$$

$$= \int_0^n a_t f_{T(x)}(t)dt + \int_n^{\omega-x} a_n f_{T(x)}(t)dt \qquad \text{(aggregate payment)}$$

$$= \int_0^n v^t {}_tP_x dt \qquad \text{(current payment)}$$

$$= \frac{1-\bar{A}_{x:n}}{\delta} \qquad \text{(annuity-insurance)}$$

Example

The effective daily rate of interest is 0.1% and morbidity is assumed to follow the model $l_x = 100 - x$ at all days $x \le 100$. Calculate the actuarial present value of a continuous 10-quarter temporary annuity of 1,000 per quarter payable by (20).

Solution

For the actuarial present value of a continuous 10-quarter temporary annuity of 1,000 per quarter payable by (20), we have

$$1,000\bar{a}_{20:10} = 1,000 \int_0^{10} a_t f_{T(2)}(t)dt + \int_{10}^{80} a_{10} f_{T(2)}(t)dt$$

$$= 1,000\left(\int_0^{10} \frac{1-100.1^{-t}}{80\ln(1.001)}dt + \int_0^{10} \frac{1-100.1^{-10}}{80\ln(1.001)}dt \right)$$

$$= 1,000\left(\left(\frac{10}{80\ln(1.001)} + \frac{1.001^{-1}}{80\ln(1.001)^2} \right)\Big|_0^{10} + \frac{70(1-1.001^{-10}}{80\ln(1.001)} \right)$$

$$= 1,000\left(\left(\frac{10}{0.07996} + \frac{1.001^{-1}}{7.992\times 10^{-5}} \right)\Big|_0^{10} + 870.60301 \right)$$

$$= 13495.6656$$

3.28 DEFERRED ANNUITY FOR DISCRETE MODEL

The random present value of an n-day deferred annuity due is a function given by

$$Y = \begin{cases} 0 & if \quad K(x) \le n-1 \\ a_{K(x)+1} - a_n & if \quad K(x) \ge n \end{cases}$$

The actuarial present value is thus given by

$${}_n|a_x = E[Y]$$

$$= \sum_{k=n}^{\omega-x-1} a_{k+1} - a_n \quad k|q_x \qquad \text{(aggregate payment)}$$

$$= \sum_{k=n}^{\omega-x-1} v^k {}_kP_x \qquad \text{(current payment)}$$

Let j=k-n, then the last result becomes $\quad {}_{n|}a_x \;=\; v^n\,{}_nP_x \sum_{j=0}^{\omega-x-n-1} v^j\,{}_jP_{x+n} \;=\; v^n\,{}_nP_x a_{x+n}$

3.29 DEFERRED ANNUITY FOR CONTINUOUS MODEL

The random present value of an n-day deferred annuity due is a function is given by

$$Y = \begin{cases} 0 & if \quad T(x) \le n \\ a_{T(x)} - a_n & if \quad T(x) > n \end{cases}$$

The actuarial present value is given by

$$\begin{aligned} {}_{n|}\bar{a}_x &= E[Y] \\ &= \int_n^{\omega-x} a_t - a_n \quad f_{T(x)}(t)dt && \text{(aggregate payment)} \\ &= \int_n^{\omega-x} v^t\,{}_tP_x dt && \text{(current payment)} \end{aligned}$$

Example

Morbidity is governed by a constant force =0.02 and the effective daily rate of interest is assumed to be 0.1% . Calculate the actuarial present value of a 10-quarter temporary annuity due of 1 per quarter payable by (20) and the actuarial present value of a 10-quarter deferred annuity due of 1 per quarter payable by (20).

Solution

From the annuity-insurance relation the actuarial present value of the deferred and the temporary annuities can be computed.

$$A_x = \frac{q_x}{q_x + 1} = \frac{1 - e^{-\mu}}{1 - e^{-\mu} + i} = \frac{1 - e^{-0.02}}{1 - e^{-0.02} + 0.001} = \frac{0.019802}{0.020802} = 0.9519$$

$$\Rightarrow a_x = \frac{1 - A_x}{h} = \frac{1 - 0.9519}{0.001/1.001} = 48.1481$$

$$\Rightarrow {}_{10}|\, a_{20} = v^{10}\,{}_{10}P_{20}a_{30} = \frac{e^{-0.02\times 10}(48.1481)}{1.001^{10}} = \frac{39.4203}{1.01004} = 39.0283$$

$$\Rightarrow a_{20:10} = a_{20} - {}_{10}|\, a_{20} = 48.1481 - 39.0283 = 9.1198$$

3.30 VARIANCE OF HEALTH ANNUITY

As a result of the linear relation, the variance relation comes out as

$$\operatorname{var}(Y) = \operatorname{var}\left(\frac{1-Z}{h}\right) = \left(\frac{1}{h}\right)^2 \qquad \operatorname{var}(z) = \frac{1}{h^2}\; E[Z^2] - (E[Z])^2$$

$$= \frac{{}^2A_x - (A_x)^2}{h^2} \qquad \text{(discrete)}$$

$$\operatorname{var}(Y) = \operatorname{var}\left(\frac{1-Z}{\delta}\right) = \left(\frac{1}{\delta}\right)^2 \qquad \operatorname{var}(z) = \frac{1}{\delta^2}\; E[Z^2] - (E[Z])^2$$

$$= \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2} \qquad \text{(continuous)}$$

The variance of a temporary annuity from the linear relation is also given as

$$\operatorname{var}(Y) = \operatorname{var}\left(\frac{1-Z}{d}\right) = \left(\frac{1}{d}\right)^2 \qquad \operatorname{var}(z) = \frac{1}{d^2}\; E[Z^2] - (E[Z])^2$$

$$= \frac{{}^2A_{x:n} - A_{x:n}^{\;\;2}}{d^2} \qquad \text{(discrete)}$$

$$\operatorname{var}(Y) = \operatorname{var}\left(\frac{1-Z}{\delta}\right) = \left(\frac{1}{\delta}\right)^2 \qquad \operatorname{var}(z) = \frac{1}{\delta^2}\; E[Z^2] - (E[Z])^2$$

$$= \frac{{}^2\bar{A}_{x:n} - \ \bar{A}_{x:n} \ {}^2}{\delta^2} \qquad \text{(continuous)}$$

The variance of a deferred annuity is given as

From $\mathrm{var}(Y_1 + Y_2) = \mathrm{var}(Y_1) + \mathrm{var}(Y_2) + 2\,\mathrm{cov}(Y_1, Y_2)$ and $\mathrm{cov}(Y_1, Y_2) = E[Y_1Y_2] - E[Y_1]E[Y_2]$ the relation becomes

$\frac{{}^2A_x - (A_x)^2}{d^2} - \frac{{}^2A_{x:n} - (A_{x:n})^2}{d^2} - 2\left[{}_{n|}a_x \;\; a_n - a_{x:n} \right]$ is the variance of an n-quarter deferred health annuity due.

3.31 AGGREGATE HEALTH ANNUITY

What fund F is needed so that there is a 100(1-*a*)% chance of providing the annuity payments for all *n* individual persons? Such a fund is the 100(1-*a*)% percentile of the distribution of S:

$$F = E[S] + z_a = nE[Y] + z_a\sqrt{\mathrm{var}(S)} \quad \text{where } \alpha = \Pr(N(0,1) > z_\alpha)$$

and $\frac{F}{n} = E[Y] + z_\alpha\sqrt{\frac{\mathrm{var}(Y)}{n}}$

Example

Suppose that 100 independent persons on day x purchase a continuous whole health annuity of 1 per quarter. Suppose that 0.05 and 0.02 are values of the forces of interest and morbidity respectively. Determine the probability that a fund of $1,500 will be able to cover the benefits for all 100 persons.

Solution

The mean and variance of Y are

$$E[Y] = a_x = \frac{1}{\mu+\delta} = \frac{1}{0.07} = 14.28571$$

$$\text{var}(Y) = \frac{{}^2A_x - (A_x)^2}{\delta^2} = \frac{\left(\frac{\mu}{\mu+2\delta}\right) - \left(\frac{\mu}{\mu+\delta}\right)^2}{\delta^2}$$

$$= \frac{\left(\frac{0.02}{0.12}\right) - \left(\frac{0.02}{0.07}\right)}{0.05^2} = 34.01361$$

$$\Rightarrow \Pr(S \le F) = \phi\left(\frac{F - E[S]}{\sqrt{\text{var}(S)}}\right) = \phi\left(\frac{1.500 - 100(14.28571)}{\sqrt{100(34.01631)}}\right)$$

$$= \phi(1.225) = 0.89$$

Example

Using the information above, calculate the fund per an individual necessary to have a 90% chance of providing the annuity payments for a group of 100 independent persons on day (x).

$$\frac{F}{n} = E[Y] + z_\alpha \sqrt{\frac{\text{var}(Y)}{n}} \qquad (z_{0.1} = 1.282)$$

$$= 14.28571 + 1.282\sqrt{\frac{34.01361}{100}}$$

$$= 15.03339$$

3.32 HEALTH INSURANCE MODELS

Health insurance policies provide benefits that are contingent on the health of the policyholder for a certain period. Since the timing of the benefit payment is unknown at issue, the present value of the benefit payment is also unknown and are thus random variables.

The random present value of benefits payable is a function of either the complete future lifetime variable, T(x), or the curtate lifetime K(x).

The objectives include computing:

- The expected present value of benefits under a health insurance policy
- The probability that the actual cost of benefits exceeds the expected amount
- The probability that a certain fund of money will be sufficient to pay all the benefits for a group of insured persons.

3.32.1 TERMINOLOGY

The day x on which the policy is issued to the policyholder is called the **day at issue**. It is assumed this refers to an exact day (within a 100 day period)**.**

Policy anniversaries occur at one-day intervals after the contract is issued. E.g. the second policy anniversary occurs at day x+2.

A **policy day** runs from one policy anniversary to the next. E.g. the first policy day coincides with the day interval [x, x+1].

3.33 RANDOM PRESENT VALUE

The random variable denoted Z, is the present value at the day of issue of the health insurance benefit payment. In continuous models, Z is a function of the complete health-time T(x) and it is assumed that a health benefit is paid immediately an insured visits a health facility for treatment. In discrete models, Z is a function of the curtate future health-time K(x), and it is assumed a health benefit is paid at the end of the quarter of an insured's visit to a health facility for treatment.

Present values are calculated at a constant effective daily rate of interest denoted i. The discount factor associated with this interest rate is denoted $v = \frac{1}{1+i}$. Where compound interest annuity is given by $a_{n:i} = v + v^2 + \ldots + v^n = \frac{1-v^n}{i}$

3.34 DISCRETE MODEL OF HEALTH INSURANCE

An assumed payment of GHC1.00 is made at K(x) + 1 quarters after the contract is issued at day x. Z is the present value of the benefit payment at the time the contract is issued and is given by $Z = v^{K(x)+1} \qquad K(x) = 0,1,\ldots,\omega - x - 1$

The expected value, which is the value on that day (or actuarial present value, A_x) is given by $E[Z] = E\left[v^{K(x)+1}\right] = \sum_{k=0}^{\omega-x-1} v^{k+1}{}_{k|}q_x = \frac{1}{l_x}\sum_{k=0}^{\omega-x-1} v^{k+1} d_{x+k}$

Example

A health insurance policy with a health benefit of GHC1.00 is issued on an insured on the 20th day. The benefit is to be paid on the policy anniversary immediately the insured falls sick and accesses healthcare from an accredited health facility. The effective daily rate of interest is assumed to be 0.1%. Morbidity is assumed to follow the model $l_x = 100 - x$

i. Calculate E[Z], the expected value.

ii. Calculate the probability that the present value Z exceeds the expected value E[Z].

Solution

At issue, the present value of the benefit is given as $Z = v^{K(20)+1}$

$$\Pr(K(20) = k) = {}_{k|}q_{20} = \frac{l_{20+k} - l_{20+k+1}}{l_{20}} = \frac{1}{80}$$

$$\Rightarrow E[Z] = \sum_{k=0}^{79} v^{k+1} \Pr(K(20) = k)$$

$$= \frac{1}{80}\sum_{k=0}^{79} v^{k+1} = \frac{1}{80}\left(\frac{1-v^{80}}{i}\right) = \frac{1}{80}\left(\frac{1-1.001^{-80}}{0.001}\right)$$

$$= \frac{1}{80}(76.8467)$$

$$= 0.9606$$

To determine $\Pr(Z > E[Z]) = \Pr(Z > 0.9606)$

Now Z > 0.9606

$$\Rightarrow \frac{1}{1.001^{K+1}} > 0.9606$$

$$1.001^{K+1} < 1.04101$$

$$K < \frac{\ln(1.04101)}{\ln(1.001)} - 1$$

$$= 39.2115$$

So Pr (Z>0.9606) = Pr(20) becomes ill before 48 days within the 100 days for heath insurance.

$$= {}_{39}q_{20} = \frac{l_{20} - l_{59}}{l_{20}} = \frac{80-41}{80}$$

$$= 0.49$$

Example

A whole health insurance of GHC1.00 is issued on a health on the 90th day. The health benefit is paid on the policy anniversary after the insured falls sick and accesses healthcare from an accredited health facility. The effective daily rate of interest is 0.1%. Determine the expected value E[Z], and the probability that the present value Z exceeds the expected value E[Z] given the model

Day x	l_x	h_x
90	100	25
91	75	35
92	40	40
93	0	0

Solution

At issue, the present value of the benefit is: $Z = v^{K(90)+1}$

$$E[Z] = \sum_{k=0}^{2} v^{k+1}{}_{k}|q_{90} = v\frac{25}{100} + v^2\frac{35}{100} + v^3\frac{40}{100} = 0.90109$$

$$\Pr(Z > E[Z] = \Pr\ 1.001^{-(K+1)} > 0.90109$$

$$= \Pr\left(K < \frac{\text{-}(\ln(0.90109))}{\ln(1.001)} - 1 = 103.2022 \right)$$

$$= \Pr(K = 0) + \Pr(K = 1) = \frac{25}{100} + \frac{35}{100}$$

$= 0.60$

3.35 CONTINUOUS MODEL OF HEALTH INSURANCE

A payment of GHC1.00 is made at time of accessing healthcare is assumed, i.e. T(x) days after the contract is issued at day *x*. Z is the present value of the benefit payment at the time the contract is issued $Z = v^{T(x)}$ where $0 < T(x) < \omega - x$. The expected value (or actuarial present value, A_x) is given by $E[Z] = E[v^{T(x)}] = \int v^t f_T(t)dt = \int v^t {}_tP_x \mu(x+t)dt$

Example

A health insurance of \$1 issued on the 20th day with the benefit to be paid at the time of accessing healthcare at an accredited health facility. The effective daily rate of interest is 0.1%. Morbidity is assumed to follow the model $l_x = 100 - x$, $x \leq 100$

i. Calculate the expected value E[Z]

ii. Calculate the probability that the present value Z exceeds the expected value E[Z].

Solution

At issue, the present value of the benefit is given as $Z = v^{T(20)}$

$$f_{T(20)}(t) = - \ S_{T(20)}(t)\ ' = \left(\frac{l_{20+i}}{l_{20}}\right)' = \left(\frac{100 - 2 - t}{100 - 20}\right)'$$

$$\bar{A}_{20} = \int_0^{80} v^t f_T(t)dt = \int_0^{80} 1.05^{-t} \frac{1}{80} dt = \frac{1-1.001^{-80}}{80\ln(1.001)} = 0.9611$$

The probability that the present value Z exceeds the expected value E[Z] then becomes

$$\Pr(Z > E[Z]) = \Pr\ 1.001^{-T(2)} > 0.9611\ = \Pr\left(T(20) < \frac{\ln(0.9611)}{\ln(1.001^{-1})}\right)$$

$$= \Pr(T(20) < 39.6511) = {}_{39.651}q_{20} = 1 - \frac{l_{59.651}}{l_{20}}$$

$$= 1 - \frac{100-59.651}{100-20} = 0.4956$$

Example

A health insurance of 1 issued on an insured on day 20 with the benefit to be paid at the time of accessing healthcare from an accredited health facility. The effective daily interest rate is 0.1%. Morbidity is assumed to follow the model $l_x = 100 - x$, $x \le 100$. Find the 90th percentile of Z.

Solution

In general, a decreasing relation between random variable will "flip" percentiles. Thus the 90th percentile of Z is $z_{0.9} = 1.001^{-t_{0.1}}$

$$0.1 = \Pr\ T(20) \le t_{0.1})\ = {}_{t_{0.1}}q_{20} = 1 - \frac{l_{20+t_{0.1}}}{l_{20}}$$

$$\Rightarrow 0.9 = \frac{100-20-t_{0.1}}{100-20} = \frac{80-t_{0.1}}{80}$$

$$\Rightarrow t_{0.1} = 8$$

$\Rightarrow z_{0.9} = 1.001^{-t_{0.1}} = 0.8000$

3.36 SIGNIFICANCE OF E[Z]

For an insurance company that charges each policyholder a premium equal to E[Z];

- If Z > E[Z] then the insurance company makes a loss on the policy
- If Z < E[Z] then the insurance company makes a profit on the policy
- If Z = E[Z] then the insurance company breaks even on the policy, with zero profit.

3.37 TERM INSURANCE FOR DISCRETE MODEL

Under an n-quarter term insurance policy, benefit is only paid if the healthcare is accessed within n quarters of issue

$$b_{K+1} = \begin{cases} 1 & K = 0,1,...,n-1 \\ 0 & K \geq n \end{cases}$$

$$\Rightarrow Z = \begin{cases} v^{K+1} & K = 0,1,,...,n-1 \\ 0 & K \geq n \end{cases}$$

$$E[Z] = \sum_{k=0}^{n-1} v^{K+1} \Pr(K = k) = \sum_{k=0}^{n-1} v^{K+1}$$

Example

Morbidity is assumed to follow the model $l_x = 100 - x$, $x \leq 100$. Using an effective daily rate of interest of 0.1%, calculate $A^{1}_{20:10}$

Solution

$$A^{1}_{20:10} = \sum_{k=0}^{9} 1.001^{-(k+1)} \Pr(K(x) = k) = \frac{a_{10:0.1}}{80} = 0.1243$$

3.38 TERM INSURANCE FOR CONTINUOUS MODEL

Under an-n-quarter term insurance policy, benefit is only paid if the healthcare is accessed within n quarters of issue.

$$b_T = \begin{Bmatrix} 1 & T < n \\ 0 & T \geq n \end{Bmatrix}$$

$$\Rightarrow Z = \begin{Bmatrix} v^T & T < n \\ 0 & T \geq n \end{Bmatrix}$$

$$A^{1}_{x:n} = E[Z] = \int v^t f_T(t)dt$$

Example

Morbidity is assumed to follow the model $l_x = 100 - x$, $x \leq 100$. Using an effective daily rate of interest of 0.1%, calculate $A^{1}_{20:10}$

Solution

$$A^{1}_{20:10} = \int v^t f_T(t)dt = \int \frac{1.001^{-t}}{80} dt = \frac{1 - 1.001^{-10}}{80 \ln(1.001)} = 0.1243$$

3.39 DEFERRED INSURANCE FOR DISCRETE MODEL

Under an n-quarter deferred insurance policy, benefit is only paid if the healthcare is accessed more than n quarters after issue

$$b_{K+1} = \begin{cases} 0 & K = 0,1,...,n-1 \\ 1 & K \geq n \end{cases}$$

$$\Rightarrow Z = \begin{cases} 0 & K = 0,1,,...,n-1 \\ v^{K+1} & K \geq n \end{cases}$$

$$n\,|\,A_x = E[Z] = \sum_{k=n}^{\omega-x-1} v^{k+1}\Pr(K = k) = \sum_{k=n}^{\omega-x-1} v^{k+1}{}_k\,|\,q_x$$

Example

Morbidity is assumed to follow the model $l_x = 100 - x$, $x \leq 100$. Using an effective daily rate of interest of 0.1%, calculate e ${}_{10}|\,A_{20}$

Solution

$${}_{10}|\,A_{20} = \sum_{k=10}^{79} 1.001^{-(k+1)}\Pr(K = k) = \frac{a_{80:0.1} - a_{10:0.1}}{80} = 0.8367$$

3.40 DEFERRED INSURANCE FOR CONTINUOUS MODEL

Under an n-quarter deferred insurance policy, benefit is only paid if the healthcare is accessed more than n quarters after issue

$$b_T = \begin{cases} 0 & T \leq n \\ 1 & T > n \end{cases}$$

$$\Rightarrow Z = \begin{cases} 0 & T \le n \\ v^T & T > n \end{cases}$$

$$n|A_x = E[Z] = \int v^t f_X(t)dt$$

Example

Morbidity is assumed to follow the model $l_x = 100 - x$, $x \le 100$. Using an effective daily rate of interest of 0.1%, calculate ${}_{10|}A_{20}$

Solution

$${}_{10|}A_{20} = \int v^t f_T(t)dt = \int \frac{1.001^{-t}}{80} dt = \frac{1.001^{-10} - 1.001^{-80}}{80\ln(1.001)} = 0.8367$$

3.41 PURE ENDOWMENT INSURANCE

Under an n-quarter pure endowment insurance policy, benefit is paid n quarters after issue if and only if the policyholder did not access healthcare at that time (For a benefit of 1, the expression is) $A^{1}_{x:n} = v^n {}_nP_x$. For an n-quarter endowment insurance policy, benefit is paid at the earlier of

- the time of accessing healthcare, if the policyholder accesses healthcare within n quarters of issue, and
- n quarters after issue, if the policyholder accesses healthcare n quarters after issue.

i.e an n-quarter time insurance policy or an n-quarter pure endowment insurance policy

3.41.1 ENDOWMENT INSURANCE FOR DISCRETE MODEL

Any healthcare benefit due will be paid at the policy anniversary immediately following accessing healthcare. Hence the following deductions

$b_{K+1} = 1 , \quad K \geq n$

$\Rightarrow Z = \begin{cases} v^{K+1} & K = 0,1,,...,n-1 \\ v^{n} & K \geq n \end{cases}$

$E[Z] = \sum_{k=0}^{n-1} v^{k+1}\Pr(K = k) + \sum_{k=n}^{\omega-x-1} v^{n}\Pr(K = k) = \sum_{k=0}^{n-1} v^{k+1}\ {}_{n|}q_x + v^{n}\ {}_{n}P_x$

Example

Morbidity is assumed to follow the model $l_x = 100 - x , \quad x \leq 100$. Using an effective daily rate of interest of 0.1%, calculate $A_{20:10}$

Solution

$A_{20:10} = A^{1}_{20:10} \quad + \quad v^{10}\ {}_{10}P_{20}$

$= 0.1243 \ + \ 1.001^{-10}\left(\frac{l_{30}}{l_{20}}\right) \qquad = 0.1243 \ + \ 1.001^{-10}\left(\frac{70}{80}\right)$

$= 0.9905$

3.41.2 ENDOWMENT INSURANCE FOR CONTINUOUS MODEL

Any healthcare benefit will be paid immediately on accessing healthcare. Therefore $b_T = 1$ for $T > 0$

$$\Rightarrow Z = \begin{cases} v^T & T \le n \\ v^n & T > n \end{cases}$$

$$A_{x:n} = E[Z] = \int_0^n v^t f_T(t)dt + \int_n^{\omega-x} v^n f_T(t)dt + v^n\,{}_nP_x$$

Example

Morbidity is assumed to follow the model $l_x = 100 - x$, $x \le 100$. Using an effective daily rate of interest of 0.1%, calculate $A_{20:10}$

Solution

$$A_{20:10} = A^{1}_{20:10} \quad + \quad v^{10}\,{}_{10}P_{20}$$

$$= 0.1243 \quad + \quad 1.001^{-10}\left(\frac{l_{30}}{l_{20}}\right) \qquad = 0.1243 \quad + \quad 1.001^{-10}\left(\frac{70}{80}\right)$$

$$= 0.9905$$

Some other formulae can be deduced from the previous calculations. The actuarial present value (APV) can be written for a health insurance as $A_x = A^{1}_{x:n} \ + \ {}_n|A_x$

This is equal to the sum of an n-quarter term insurance policy which provides a benefit on accessing healthcare if and only if the access to healthcare is less than n quarters after issue and an n-quarter deferred insurance policy, which provides a benefit on accessing healthcare if and only if access to healthcare occurs at least n quarters after issue.

But intuitively, an n-quarter deferred insurance for (x) is equivalent to a whole health policy that starts at day x+n, discounted for interest and survival from day x to day x+n by $v^n\,{}_nP_x$.

Thus ${}_n|A_x = v^n\,{}_nP_x A_{x+n}$

Example

Morbidity is assumed to follow the model $l_x = 100e^{-0.0015x}$ with force of interest $\delta = 0.05$.

i. Compute $1,000\bar{A}_{50:\,20}$ (i.e. value a 20-quarter endowment insurance policy for a health on day 50)

ii. Compute the actuarial present value (APV) of a 10-quarter term insurance policy with a benefit of 500 for 65 days.

Solution

i. Note the existence a constant force of mortality

$$l_x = 100e^{-0.015x} \Rightarrow \mu(x) = -\frac{l_x'}{l_x} = -\frac{-0.015x100e^{-0.015x}}{100e^{-0.015x}} = 0.015$$

$$\Rightarrow f_{T(x)}(x) = {}_tP_x\mu(x+t) = 0.015\frac{100e^{-0.015(x+t)}}{100e^{-0.015x}} = 0.015e^{-0.015t}$$

$$1,000A_{50:20} = 1,000 \;\; A^1_{50:20} + A_{50:\overset{1}{20}} \;\; = 1,000 \;\; \int v^t f_{X(50)}(t)dt + v^{20}\;{}_{20}P_{50}$$

$$= \;\; 1,000 \;\; \int e^{-0.05t}(0.015)e^{-0.015t}dt + e^{-0.05x20}e^{-0.015x20}$$

$$= \;\; 15\left(\frac{1-e^{-1.30}}{0.065}\right) + 1,000e^{-1.30} = 167.88 + 272.53 = 440.41$$

ii. The actuarial present value (curtate) is calculated as

$$500A^{1}_{65:10} = 500\sum_{k=0}^{9} v^{k+1}\Pr(K(65)=k) = 500\sum_{k=0}^{9} e^{-0.05(k+1)}{}_{k}\,|\,q_{65}$$

$$= 500\sum_{k=0}^{9} e^{-0.05(k+1)}{}_{k}P_{65}q_{65+k} = 500\sum_{k=0}^{10} e^{-0.05(k+1)}e^{-0.015k}(1-e^{-0.015})$$

$$= 500e^{-0.05}\quad 1-e^{-0.015}\quad \sum_{k=0}^{9} e^{-0.650k}$$

$$= 500e^{-0.05}(1-e^{-0.015})\left(\frac{1-e^{-0.065}}{1-e^{-0.065}}\right)$$

$$= 53.78$$

3.42 VARIANCE OF THE RANDOM PRESENT VALUE

Suppose that Z is the random present value of the benefits, E[Z], to calculate the variance of Z, the standard formula is adopted (variance is a measure of risk) $\mathrm{var}[Z] = E[Z^2] - (E[Z])^2$

Example

A health insurance policy with a healthcare benefit of 1 is issued on day 20 with benefit to be paid at the time of accessing healthcare. The effective daily interest rate is assumed to be 0.1%. Morbidity is assumed to follow the model $l_x = 100 - x$

i. Calculate $E[Z^2]$

ii. Calculate the variance of Z

Solution

i. To calculate $E[Z^2]$,

$$Z^2 = 1.1025^{-T(20)}$$

$$\Rightarrow E[Z^2] = E\ 1.1025^{-T(20)}$$

$$\int_0^{80} 1.1025^{-t} f_{T(20)}(t) = \int_0^{80} 1.1025^{-t} \frac{1}{80} dt = \frac{1}{80}\left(-\frac{1.1025^{-t}}{\ln(1.1025)} \Big|_0^{80} \right)$$

$$= \frac{1-1.1025^{-80}}{80\ln(1.1025)} = 0.12805$$

ii. The variance of Z then becomes

$$\text{var}(Z) = [Z^2] - (E[Z])^2 \quad = 0.12805 - (0.25103)^2 \quad = 0.06503$$

3.43 THEOREM OF THE BENEFIT FUNCTION b_T

In the continuous model, if the benefit function b_T is always 0 or 1, and the force of interest is δ then $E[Z^2] = E\left[b_{T^2} v^T\right]$. Where ${}^2v^T$ is the discount function calculated at a force of interest of 2δ

PROOF

For the continuous case (quite similar to the discrete case) we have

$$v_T^2 = {}^2 v_T \quad \text{since} \quad \left(e^{-\delta t}\right)^2 = e^{-(2\delta)t}$$

$$\Rightarrow E[Z^2] = E\left[b_T^2\ v_T^2\right] = E\left[b_{T^2}\ v_T\right]$$

To determine formulae for both E[Z] and E[Z^2] given a force of interest δ and the mortality model $f_{T(x)}(t) = \mu e^{-\mu t}$. The approaches are;

For this morbidity model

$$f_{T(x)}(t) = {}_t P_x \mu(x+t) = \mu e^{-\mu t}$$

$$\Longrightarrow E[Z] = E\left[e^{-\delta T}\right]$$

$$\int_0^\infty e^{-\delta t} f_{T(x)}(t)dt = \int_0^\infty e^{-\delta t}\mu e^{-\mu t}dt$$

$$= \mu\left(-\frac{e^{-((\mu+\delta)t}}{\mu+\delta}\Big|_0^\infty\right) = \frac{\mu}{\mu+\delta}(1-0) = \frac{\mu}{\mu+\delta}$$

$$\Rightarrow E[Z^2] = \frac{\mu}{\mu+2\delta}$$

Notation for actuarial present values calculated using double the original force of interest.

Health insurance: ${}^2A_x$

n-quarter term: ${}^2A^1_{x:n}$

n-quarter deferred: ${}_{n|}{}^2A_x$

n-quarter endowment: ${}^2A_{x:n}$

3.44 THEOREM OF THE RANDOM PRESENT VALUE (RPV)

Let Z be the random present value variable for a discrete health insurance pays healthcare benefit, having no pure endowment. Let $\bar{Z}$ be the random present value variable for a continuous health insurance that pays identical health benefits at the time of accessing healthcare. Let UDD be assumed for calculating the number accessing healthcare at fractional days. Then the expression becomes $E[\bar{Z}] = \frac{i}{\delta}E[Z]$

Example

Consider a one-day term insurance for GHC1.00 issued on (x), with any healthcare benefit payable on the policy anniversary immediately following accessing healthcare. Hence,

$$Z = \begin{cases} v & K = 0 \\ 0 & K \geq 1 \end{cases}$$

Solution

The pdf given by (UDD assumption)

$${}_tP_x = 1 - tq_x \Rightarrow f(t) = -({}_tP_x)' = q_x \quad \text{for } 0 \leq t \leq 1$$

$$\Rightarrow E[\bar{Z}] = A^1_{x:1} = \int e^{-\delta t} f_T(t)dt = q_x \int e^{-\delta t}\, dt$$

$$= q_x \frac{1 - e^{-\delta}}{\delta}$$

$$q_x e^{-\delta} \frac{e^{\delta} - 1}{\delta} = vq_x \frac{1 + i - 1}{\delta} = \frac{i}{\delta} vq_x$$

$$= \frac{i}{\delta} E[Z]$$

Example

A health insurance of GHC1.00 is issued on day 90, with benefits to be paid at the time of accessing healthcare. The effective daily rate of interest is 0.1%. Morbidity is assumed to follow the model

Day x	**l_x**	**h_x**
90	100	25
91	75	35
92	40	40
93	0	0

Calculate var(Z) for the continuous health insurance model

Solution

There is no need to integrate the APV for the continuous model

$$E[\bar{Z}] = \bar{A}_{90} = \frac{i}{\delta} A_{90}$$

$$= \frac{0.001}{\ln(1.001)} 0.90109$$

$$= 0.9015$$

The var(Z) is given as $\text{var}(Z) = E[Z^2] - \left(E[Z]\right)^2$, therefore

$$E[Z^2] = {}^2A_{90} = \sum_{k=0}^{2} v^{2(k+1)}\, {}_{k|}q_{90} = v^2 \frac{25}{100} + v^4 . \frac{35}{100} + v^6 . \frac{40}{100} = 0.81319$$

Now $e^{2\delta} = (1+i)^2 = 1 + (2i + i^2)$

$$\Rightarrow {}^2\bar{A}_{90} = \frac{2i + i^2}{2\delta}\, {}^2A_{90}$$

$$\Rightarrow E[\bar{Z}^2] = {}^2\bar{A}_{90} = \frac{2i + i^2}{2\delta}\, {}^2A_{90} = \frac{2(0.001) + (0.001)^2}{2\delta}\, {}^2A_{90} = \frac{0.002001}{2\ln(1.001)}$$

$$E[\bar{Z}^2] = 1.001$$

$$\Rightarrow \text{var}(Z) = E[Z^2] - E[Z]^2$$

$$= 1.001 - 0.9015^2 = 0.1883$$

3.45 RATE MAKING

Rate making, or insurance pricing, has several basic objectives. Because insurance rates are regulated by the government, certain statutory or regulatory requirements must be satisfied. Also, due to the overall goal of probability, certain objectives must be stressed in rate making.

3.46 REGULATORY OBJECTIVES

The goal of insurance regulation is to protect the public. The government has rating laws that require insurance rates to meet certain standards. In general, rates charged must be adequate, nor excessive and not unfairly discriminatory.

3.46.1 ADEQUATE RATES

The rates charged by insurers should be high enough to pay all benefits. If rates are inadequate, an insurer may become insolvent and fail. As a result, policyholders, beneficiaries, and third-party claimants (service providers) may be financially harmed if their claims are not paid. However, rate adequacy is complicated by the fact that the insurer does not know its actual costs when the policy is sold. The premium is paid in advance, but it may not be sufficient to pay all claims and expenses during the policy period. It is only after the period of protection has expired that an insurer can determine its actual costs.

3.46.2 NOT EXCESSIVE

This means that the rates should not be so high that policy owners are paying more than the actual value of their protection. Exorbitant prices are not in the public interest.

3.46.3 NOT UNFAIRLY DISCRIMINATORY

This means that exposures that are similar with respect to losses and expenses should not be charged substantially different rates. For example, if two healthy males, age 30, buy the same type and amount of health insurance from the same insurer, they should not be charged two different rates. However if the loss exposures are substantially different, it is not unfair rate discrimination to charge different rates. Thus if two males, age 30 and age 65, apply for the same type and amount of health insurance, it is not unfair to charge the older male a higher rate because of the higher probability of complicated age related diseases. But this is not the case in social health insurance. For instance, in social health insurance both males aged 30 and age 65 will be charged the same rate.

3.47 PROBABILITY

Probability is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur. “The chance of rain today is 30%’’ is a statement that quantifies our feeling about the possibility of rain. The likelihood of an outcome is quantified by assigning a number from the interval [0, 1] to the outcome (or a percentage from 0 to 100%). Higher numbers indicate that the outcome is more likely than lower numbers. A probability of 0 indicates an outcome will not occur. A probability of 1 indicates an outcome will occur with certainty.

3.48 CLASSICAL DEFINITION OF PROBABILITY

If an experiment can result in any one of N different equally likely outcomes, and exactly n of these outcomes corresponds to event A, then the probability of occurrence of event A denoted by P(A) is defined as;

$$P(A) = \frac{n}{N}$$

Example: If a card is drawn from an ordinary deck, find the probability that it is a heart.

Solution: Let H be the event a heart is drawn. There are 13 of them. Total number of cards = 52

$$P(H) = \frac{n}{N} = \frac{13}{52} = \frac{1}{4}$$

If E denotes the occurrence of event E then $\overline{\mathrm{E}}$ denotes the non-occurrence of event E.

Let $\quad P(E) = \frac{n}{N} \quad$ then, $\quad P(\bar{E}) = \frac{N-n}{N} \quad = 1 - \frac{n}{N}$

Hence $P(\bar{E}) = 1 - P(E)$

This means that $P(E) + P(\bar{E}) = 1$

The probability of an event is a number between 0 and 1 inclusive. If the event cannot occur, thus to say will never happen (i.e. impossible event), its probability is zero (0). If it must occur (i.e. certain event), its probability is 1.

3.49 FREQUENCY INTERPRETATION OF PROBABILITY

If a sample space of an experiment consist of n(s) possible equally likely outcomes, and if n(E) of these outcomes belong to the event E, then the probability of the occurrence of event E is the proportion of outcomes in the event relative to the sample space.

$$i.e. \qquad P(E) = \frac{n(E)}{N(S)}$$

3.50 AXIOMS OF PROBABILITY

1. For any event E of sample space S, $0 \leq P(E) \leq 1$
2. $P(\emptyset) = 0$, impossible event
3. $P(S) = 1$, certain event
4. $P(E) + P(\bar{E}) = 1$

3.51 FINITE PROBABILITY SPACE

Let $S = \{a_1, a_2, \dots\dots, a_n\}$ such that probabilities of each a_i is $P(a_i)$ with the following conditions;

1. $P(a_i) > 0$
2. $\sum_{i=1}^{n} P(a_i) = 1$

Then S is called a finite probability space.

Example: Suppose a fair coin is tossed three times and the number of heads observed and noted. Then the possible number of heads is given by $S = \{0, 1, 2, 3\}$. Find $P(0)$, $P(1)$ $P(2)$, $P(3)$ and hence show that S is a finite probability space.

Solution: Sample space s, for tossing a fair coin three times is;

$\mathrm{S} = \{\mathrm{HHH}, \mathrm{HHT}, \mathrm{HTH}, \mathrm{HTT}, \mathrm{THH}, \mathrm{THT}, \mathrm{TTH}, \mathrm{TTT}\}$

Now, $S = \{0, 1, 2, 3\}$, where $P(0) = \frac{1}{8}$, $P(1) = \frac{3}{8}$, $P(2) = \frac{3}{8}$, $P(3) = \frac{1}{8}$ and $P(a_i) > 0$

$\sum_{i=1}^{4} P(a_i) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

Hence S is a finite probability space.

3.52 LAWS OF PROBABILITY

Sometimes a probability problem may be a bit more complicated. For instance, we may wish to know not only the probability of occurrence of an event E but perhaps the probability of the occurrence of two events E, and F or the occurrence of three events E, F, and G. There are several simple rules for the calculation of such probabilities.

3.53 ADDITION RULE

3.53.1 MUTUALLY EXCLUSIVE EVENTS

If two events E and F are mutually exclusive, then the probability of either E or F occurring denoted by $P(E\ or\ F)$ or $P(E \cup F)$ is the sum of their separate probabilities.

i.e. $P(E \cup F) = P(E) + P(F)$

Proof

Let there be **n** equally likely cases such that event E occurs in **r** ways of them and event F occurs in **s** of them. Since events E and F are mutually exclusive, there is no overlapping between $(r + s)$ cases; that is, the events occur in only the $(r + s)$ cases.

Hence, $P(E \cup F) = P(E + F) = \frac{r+s}{N} = \frac{r}{n} + \frac{s}{n}$

$$\Rightarrow P(E \cup F) = P(E) + P(F)$$

The same rule applies to the occurrence of three or more mutually exclusive events;

i.e. $P(E \cup F \cup G) = P(E) + P(F) + \mathrm{P(G)}$

3.53.2 EVENTS NOT MUTUALLY EXCLUSIVE

If E and F are not mutually exclusive, then $P(E \cup F) = P(E) + P(F) - P(E\ and\ F)$ where $P(E\ and\ F) = P(E \cap F) = P(E) \times P(F)$

For three events E, F, G that are not mutually exclusive, we have;

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G)$$
$$+ P(E \cap F + \cap G)$$

3.54 MULTIPLICATION RULE

3.54.1 INDEPENDENT EVENTS

If events E and F are independent, the probability of both E and F occurring is the product of their separate probabilities.

i.e. $P(E \cap F) = P(E) \,.\, P(F)$

For three independent events E, F, G;

$$P(E \cap F \cap G) = P(E) \,.\, P(F) \,.\, P(G)$$

3.54.2 DEPENDENT EVENT – CONDITIONAL PROBABILITY

If E and F are dependent events then the joint occurrence of E and F is given by;

$$P(E \cap F) = P(E) \,.\, P(F/E)$$

where $P(F/E)$ is the probability of F, given the occurrence of E. $P(F/E)$ is called the conditional probability of F.

Note:

- If E and F are not independent, then $P(E \cap F) = P(E) \,. P(F/E) = P(E/F)$
- If $P(E/F) = P(E)$ and $P(F/E) = P(F)$

  Then E and F are independent

$$P(E/F) = \frac{P(E \cap F)}{P(F)} \ \ if\ P(E)\ > 0$$

Example. The probability that a student passes Mathematics is $\frac{2}{3}$, and the probability that he passes English is $\frac{4}{9}$. If the probability of passing at least one course is $\frac{4}{5}$ what is the probability that he will pass both subjects?

Solution: Let $M = event\ that\ he\ passes\ Math$

$$E = \ event\ that\ he\ passes\ English$$

But M, E are independent.

$P(M) = \frac{2}{3}, P(E) = \frac{4}{9}, P(M \cup E) = \frac{4}{5}$

Now, $P(M \cap E) = P(M) + \ P(E) - \ P(M \cup E)$

$$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5} \ = \frac{14}{45}$$

Example: A basket contains 10 oranges of which 4 are rotten. Find the probability that if the oranges are taken from the basket one at a time without replacement, the third orange will be rotten.

Solution: Let $R\ be\ the\ event\ that\ a\ rotten\ orange\ is\ picked$.

The required probability is then given as;

$$P(R_1\, R_2 R_3) \cup P(R_1 \bar{R}_2 R_3) \cup P(\bar{R}_1 R_2 R_3) \cup P(\bar{R}_1 \bar{R}_2 R_3)$$

$$= \frac{4}{10}.\frac{3}{9}.\frac{2}{8} + \frac{4}{10}.\frac{6}{9}.\frac{3}{8} + \frac{6}{10}.\frac{4}{9}.\frac{3}{8} + \frac{6}{10}.\frac{5}{9}.\frac{4}{8}$$

$$= \frac{1}{30} + \frac{1}{10} + \frac{1}{10} + \frac{1}{6} \quad = \frac{2}{5}$$

3.55 TOTAL PROBABILITY THEOREM

Suppose $B_1, B_2, \ldots.. B_n$ are non-null mutually exclusive events and A is an event subset of B_i $(i = 1, 2, \ldots.., n)$, then the probability of the occurrence of A is given as:

$P(A) = P(B_1)P(A/B_1) + P(B_2)P(A/B_2) + \cdots + P(B_n)P(A/B_n)$; which can be written as;

$$P(A) = \sum_{i=1}^{n} P(B_i)P(A/B_i)$$

Example: Box A has three red and four blue balls, box B has two red and five blue balls and box C has four red and three blue balls. All the balls are identical except colour. If a box is chosen at random and ball picked from it, what is the probability that;

i. A red ball is chosen
ii. A blue ball is chosen

Solution: $Let\ R\ be\ event\ a\ red\ ball\ is\ chosen$

$Let\ K\ be\ event\ a\ blue\ ball\ is\ chosen$

(i) $P(R) = P[(A \cap R) \cup (B \cap R) \cup (C \cap R)]$

$$P(R) = P(A)\ P(R/A) + P(B)\ P(R/B) + P(C)\ P(R/C)$$

$$= \frac{1}{3} \cdot \frac{3}{7} + \frac{1}{3} \cdot \frac{2}{7} + \frac{1}{3} \cdot \frac{3}{7} \quad = \frac{3}{7}$$

(ii) $P(K) = P(A)\ P(K/A) + P(B)\ P(K/B) + P(C)\ P(K/C)$

$$= \frac{1}{3} \cdot \frac{4}{7} + \frac{1}{3} \cdot \frac{5}{7} + \frac{1}{3} \cdot \frac{3}{7}$$

$$= \frac{4}{7}$$

CHAPTER FOUR

4.0 DATA ANALYSIS AND DISCUSSION

4.1 INTRODUCTION AND SOURCE OF INFORMATION

Information on the thesis is sourced through the library and the internet through search engines such as google.com and wikipedia.com for Journals and relevant materials on queuing analytic theory. The literature obtained was carefully reviewed and appropriately acknowledged.

Some staffs of the National Insurance Commission (NIC), National Health Insurance Authority (NHIA), Vanguard Assurance Company Limited and The Kpando District Mutual Health Insurance Scheme were interviewed together with some Insured Persons (Registered Members of the Scheme) to clarify certain issues and observations that came up during the cause of taking data.

4.2 SOURCE OF DATA

Secondary data was collected from the institutions of concern such as the National Health Insurance Authority (NHIA), Margret Marquart Catholic Hospital – Kpando, Kpando District Mutual Health Insurance Scheme and the internet.

4.3 TOOLS

There was Inter-Personal Interview to clarify issues that came up after a careful study of the secondary data collected from the various institutions.

The data collected was then subjected to statistical analysis to give the pictorial view of the data in the form of graphical plots using Microsoft Excel to ascertain empirically the nature of the data as a guide.

4.4 MATHEMATICAL OR STATISTICAL METHODS

Survival function, health annuities, health insurance and probability were employed to obtain the various estimates.

Some assumptions were made in order to fine tune the estimates and also arrive at more scientific deductions. It is assumed within a period of 100 days no individual person from the target population, Kpandu district accessed healthcare more than once to avoid multiple entries. Everybody within the target population would have accessed healthcare by the 100^{th} day. The effective rate of interest per annum was pegged at 20% which then translated into a quarterly rate of 5%. It is assumed premium was paid on quarterly (taken to be 100 days) bases.

4.5 PRELIMINARY ANALYSIS OF DATA

The Health Insurance Scheme will concentrate on the various quarters into which the data collected has been classified. Healthcare in the study includes the consultation cost, laboratory tests, treatment cost and medication.

Table 4.1 *Number of insured persons who accessed healthcare in 2008 in Kpando District*

Month	Malaria	Hypert-ension	URI	Rheumatisn & Joint Pains	Skin Disease & Ulcers	PUD	Intestinal Worms	Diarrhoea	Anaemia	TOTAL
January	2,984	894	493	176	163	193	179	125	122	**5,329**
February	2,868	963	465	252	377	247	164	111	141	**5,588**
March	2,922	774	368	187	235	185	198	141	109	**5,119**
Sub-Total(1st Qtr)	**8,774**	**2,631**	**1,326**	**615**	**775**	**625**	**541**	**377**	**372**	**16,036**
April	2,654	861	438	243	193	127	193	163	123	**4,995**
May	4,457	854	558	269	155	184	154	219	125	**6,975**
June	3,678	745	614	274	189	147	198	194	116	**6,155**
Sub-Total(2nd Qtr)	**10,789**	**2,460**	**1,610**	**786**	**537**	**458**	**545**	**576**	**364**	**18,125**
July	3,865	717	321	326	376	376	184	157	134	**6,456**
August	2,589	614	296	217	217	217	204	125	145	**4,624**
September	2,721	557	447	304	203	213	197	126	136	**4,904**
Sub-Total(3rd Qtr)	**9,175**	**1,888**	**1,064**	**847**	**796**	**806**	**585**	**408**	**415**	**15,984**
October	1,990	704	626	288	288	288	169	109	129	**4,591**
November	2,620	755	595	214	194	254	176	105	128	**5,041**
December	3,806	648	587	285	185	245	184	114	141	**6,195**
Sub-Total(4th Qtr)	**8,416**	**2,107**	**1,808**	**787**	**667**	**787**	**529**	**328**	**398**	**15,827**
TOTAL	37,154	9,086	5,808	3,035	2,775	2,676	2,200	1,689	1,549	65,972

Figure 4.1 Reported Attendance of the top 9 Diseases in a Bar-chat.

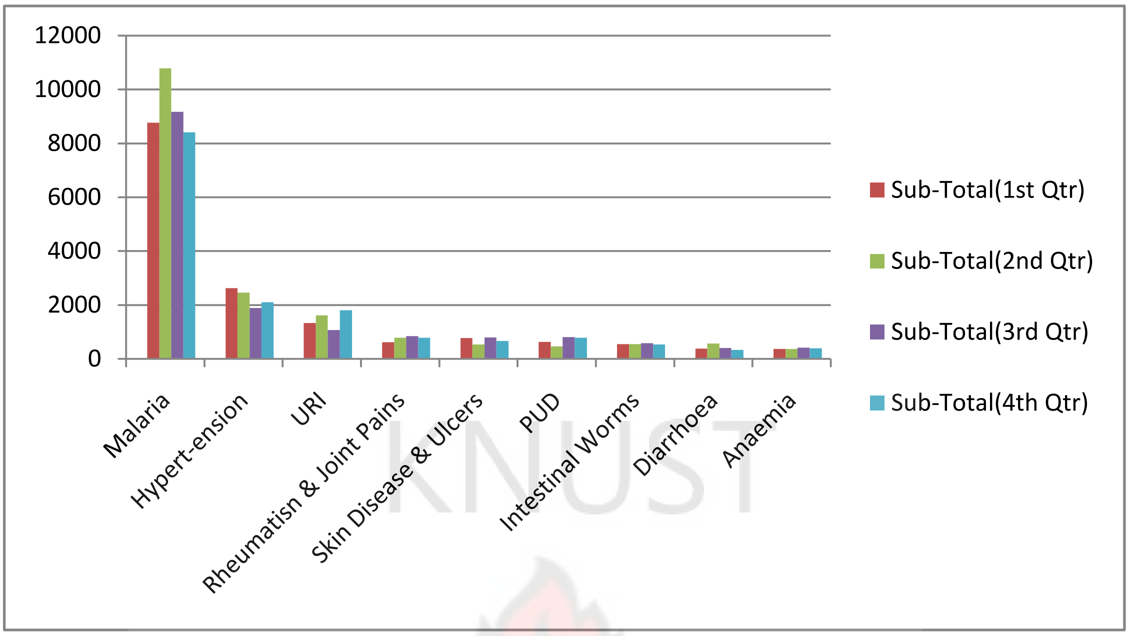


Figure 4.2 Reported Attendance of the top 9 Diseases in a Stacked Column Graph

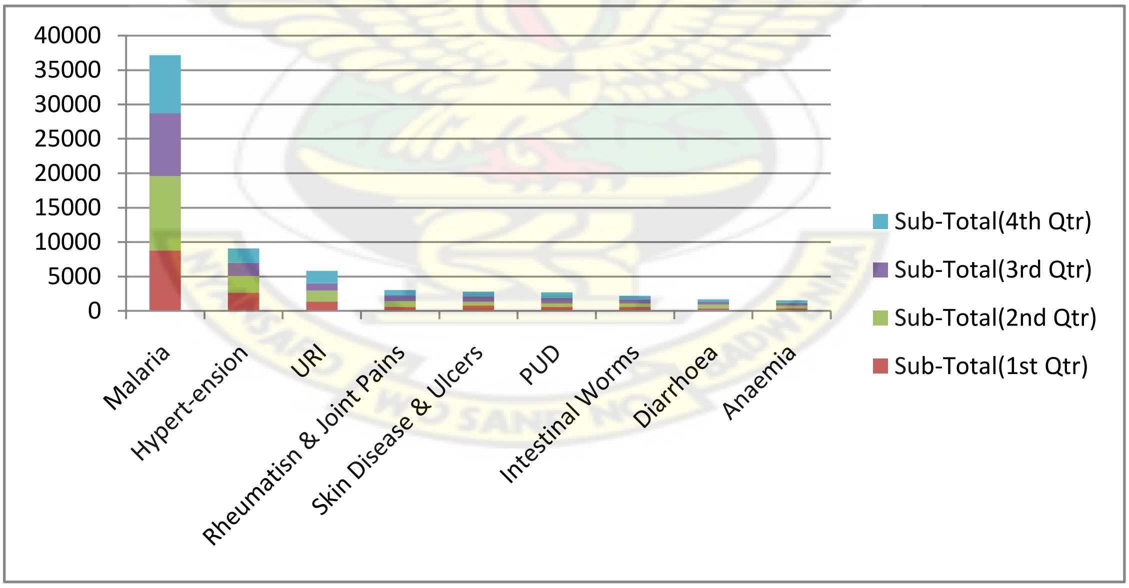
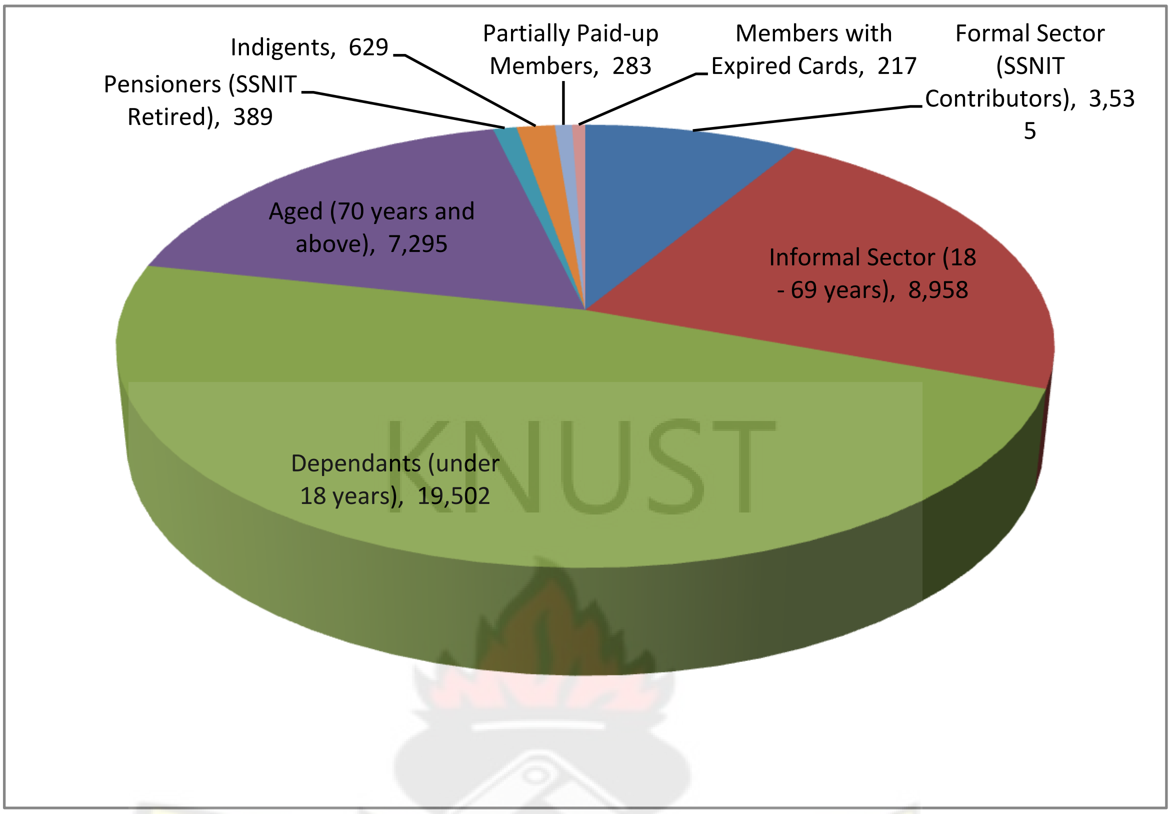


Figure 4.3 Graph of Membership of the Kpando District Mutual Health Insurance Scheme



Now, finding the various probabilities of the various categories of diseases will come out as;

Probability that someone from the entire target population, Kpando district will fall sick is given by;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ reported\ sick\ within\ the\ population)}{n(total\ no.of\ persons\ in\ the\ population)}$$

$$P(someone\ falls\ sick) = \frac{65{,}972}{73{,}778} = 0.8842$$

Average of reported malaria cases for the quarters is $\frac{8{,}774+10{,}789+9{,}175+8{,}416}{4} =$ 9,289

Probability that someone on the scheme falls sick of malaria is given as;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ malaria)}{n(no.of\ registered\ persons\ on\ the\ scheme)} = \frac{9{,}289}{40{,}808}$$

$$= 0.23$$

From the conditional probability;

$$P(malaria) = P(malaria/sick) \times P(sick) = 0.23 \times 0.8942 = 0.20$$

Then an estimated number of malaria cases in the district is given as;

$$= 0.20 \times 73{,}778 = 15{,}017$$

The cost of treatment for malaria is GHC19.17 for Out-Patient-Department (OPD) cases.

Then treatment of malaria for the estimated number from the population is given as;

$$= 15{,}017 \times 19.17$$

$$= \text{GHC } 287{,}877.23$$

Average number of reported Hypertension cases is $\frac{2{,}631+2{,}460+1{,}888+2{,}107}{4} = 2{,}272$

Probability that someone on the scheme falls sick of Hypertension is given as;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ hypertension)}{n(no.of\ registered\ persons\ on\ the\ scheme)} = \frac{2{,}272}{40{,}808}$$

$$= 0.06$$

From the conditional probability;

$$P(hypertension) = P(hypertension/sick) \times P(sick) = 0.06 \times 0.8942$$

$$= 0.0498$$

Then an estimated number of hypertension cases in the district is given as;

$$= 0.0498 \times 73{,}778 \;\; = \;\; 3{,}672$$

The cost of treatment for hypertension is GHC33.73 for OPD cases.

Then treatment of hypertension for the estimated number from the population is given as;

$$= 3{,}672 \times 33.73$$

$$= \text{GHC } 123{,}863.52$$

Average number of Upper Respiratory Infection (URI) cases is $\frac{1{,}326+1{,}610+1{,}064+1{,}808}{4} = 1{,}452$

Probability that someone from the target population falls sick of upper respiratory infection is given as;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ URI)}{n(no.of\ registered\ persons\ on\ the\ scheme)} = \frac{1{,}452}{40{,}808}$$

$$= 0.04$$

From the conditional probability;

$$P(URI) = P(URI/sick) \times \ P(sick) \;\; = \;\; 0.04 \times 0.8942 \;\; = 0.032$$

Then an estimated number of upper respiratory infection cases in the district is given as;

$$= 0.032 \times 73{,}778 \;\; = \;\; 2{,}347$$

The cost of treatment for upper respiratory infection is GHC15.11 for OPD cases.

Then treatment of malaria for the estimated number from the population is given as;

$$= 2{,}347 \times 15.11$$

$$= \text{GHC } 35{,}468.71$$

Average number of Rheumatism & Joint pains cases is $\frac{615+786+847+787}{4} = 759$

Probability that someone from the target population falls sick of rheumatism and joint pains is given as;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ rheumatism\ \&\ joint\ pains)}{n(no.of\ registered\ persons\ on\ the\ scheme)}$$

$$= \frac{759}{40,808} = 0.02$$

From the conditional probability;

$$P(rheumatism\ \&\ joint\ pains) = P(rheumatism\ \&\ joint\ pains/sick) \times P(sick)$$

$$= 0.02 \times 0.8942 = 0.0166$$

The estimated number of rheumatism and joint pains cases in the district is given as;

$$= 0.0166 \times 73,778 = 1,227$$

The unit cost of treatment for rheumatism and joint pains is GHC20.98.

Then treatment of rheumatism and joint pains for the estimated number from the population is given as; $= 1,227 \times 20.98$

$$= \text{GHC } 25,742.46$$

Average number of reported Skin disease & Ulcer cases is $\frac{775+537+796+667}{4} = 694$

Probability that someone on the scheme falls sick of skin disease and ulcers is given as;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ skin\ disease\ \&\ ulcers)}{n(no.of\ registered\ persons\ on\ the\ scheme)} = \frac{694}{40,808} = 0.02$$

From the conditional probability;

$$P(skin\ disease\ \&\ ulcers) = P(skin\ disease\ \&\ ulcers/sick) \times\ P(sick)$$

$$= \ 0.02 \times 0.8942\ = 0.0152$$

The estimated number of skin disease and ulcers in the district is;

$$= 0.0152 \times 73{,}778\ =\ 1{,}122$$

The cost of treatment for the skin disease and ulcers is GHC23.27.

Then treatment of skin disease and ulcers for the estimated number in the population is;

$$= 1{,}122 \times 23.27$$

$$= \text{GHC } 26108.94$$

Average number of reported Pelvic Ulcer Disease (PUD) cases is $\frac{625+458+806+787}{4} =$ 669

Probability that someone on the scheme from the population falls sick of PUD is;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ PUD)}{n(no.of\ registered\ persons\ on\ the\ scheme)} = \frac{669}{40{,}808}$$

$$= 0.016$$

From the conditional probability;

$$P(PUD) = P(PUD/sick) \times\ P(sick) = \ 0.016 \times 0.8942\ = 0.0147$$

The estimated number of PUD cases in the district is;

$$= 0.0147 \times 73{,}778\ =\ 1{,}082$$

The cost of treatment for PUD is GHC16.00 for OPD cases.

Then treatment of PUD for the estimated number from the district is given as;

$$= 1{,}082 \times 16.00$$

$$= \text{GHC } 17{,}312.00$$

Average number of reported Intestinal worms is $\frac{541+545+585+529}{4} = 550$

Probability that someone on the scheme from the population falls sick of intestinal worms is;

$$P(someone\ falls\ sick) = \frac{n(total\ no.of\ persons\ sick\ of\ intestinal\ worms)}{n(no.of\ registered\ persons\ on\ the\ scheme)} = \frac{550}{40{,}808}$$

$$= 0.013$$

From the conditional probability;

$$P(intestinal\ worms) = P(intestinal\ worms/sick) \times P(sick) = 0.013 \times 0.8942$$

$$= 0.0121$$

The estimated number of intestinal worm cases in the district is;

$$= 0.0121 \times 73{,}778 = 889$$

The cost of treatment for intestinal worms is GHC21.49

Then treatment of intestinal worms for the estimated number in the district is given as;

$$= 889 \times 21.490$$

$$= \text{GHC } 19{,}107.92$$

Average number of reported Diarrhoea cases is $\frac{377+576+408+328}{4} = 422$

Probability that someone on the scheme from the population falls sick of PUD is;

$$P(someone\ falls\ sick) = \frac{n(total\ no. of\ persons\ sick\ of\ diarrhoea)}{n(no. of\ registered\ persons\ on\ the\ scheme)} = \frac{422}{40{,}808}$$

$$= 0.010$$

From the conditional probability;

$$P(diarrhoea) = P(diarrhoea/sick) \times P(sick) = 0.010 \times 0.8942 = 0.0093$$

The estimated number of diarrhoea cases in the district is;

$$= 0.0093 \times 73{,}778 = 683$$

The cost of treatment for diarrhoea is GHC14.50

Then treatment of diarrhoea for the estimated number from the district is given as;

$$= 683 \times 14.50$$

$$= \text{GHC}9{,}903.50$$

Average number of reported Anaemia cases is $\frac{372+364+415+398}{4} = 387$

Probability that someone on the scheme from the population falls sick of anaemia is;

$$P(someone\ falls\ sick) = \frac{n(total\ no. of\ persons\ sick\ of\ anaemia)}{n(no. of\ registered\ persons\ on\ the\ scheme)} = \frac{387}{40{,}808}$$

$$= 0.0095$$

From the conditional probability;

$$P(anaemia) = P(anaemia/sick) \times P(sick) = 0.0095 \times 0.8942 = 0.0085$$

The estimated number of anaemia cases in the district is;

$$= 0.0085 \times 73{,}778 = 626$$

The cost of treatment for anaemia is GHC18.91 for OPD cases.

Then treatment of anaemia for the estimated number from the district is given as;

$$= 626 \times 18.91$$

$$= \text{GHC } 11.839.00$$

The treatment cost burden on the scheme is the sum of the treatment cost of the various diseases and this GHC 557,223.28 for a quarter which translates to GHC 2,228,893.12 To deduce the actual cost burden on the scheme, the loadings are then added to the estimated treatment cost. Loading is the additional premium insurance or higher rating incurred by items that are more valuable or at greater risk. The loadings here are;

- Overhead cost
- Administrative cost
- Allowances
- Profit margin

4.6 THE PREMIUM MODEL

The model for a premium is given as;

$$Premium = \frac{Expenditure + Loadings + Profit}{Total\ No.of\ exposure\ units\ (No.of\ persons\ paying\ premium)}$$

4.6.1 THE PURE NHIS PREMIUM MODEL

From the everyday life situation where actual cost on the insurance is handed down to the insured, management then decides to add a certain percentage of the premium as the profit element but this is not the case in this calculation, therefore the model becomes;

$$Premium = \frac{Expenditure + Loadings}{Total\ No.of\ exposure\ units\ (No.of\ persons\ paying\ premium)}$$

From the Implementation Budget, the loadings which also burden the scheme in the form of Overhead cost, Administration cost and Allowances for staff sums up to GHC966,434.12 and the government subsidy is GHC1,406,991.60. Since the calculations are assumed to be for a quarter, it is again assumed that the quarterly loadings and subsidy are a fourth of GHC966,434.12 and GHC1,406,991.60 which becomes GHC241,608.53 and GHC351,747.90 respectively.

Hence;

$$Premium = \frac{557{,}223.28 + 241{,}608.53}{12{,}993} = 61.48 \quad \cong 62.00$$

Thus, the Pure Premium is estimated as GHC62.00 per quarter.

On the other hand, the Government of the Republic of Ghana is determined to have the scheme work and succeed therefore has taken away the loadings and the profit element and then subsidising the premium to a great extent. Hence, the model now becomes;

$$Premium = \frac{Expenditure - Government\ Subsidy}{Total\ No.of\ Persons\ paying\ Premium}$$

$$Premium = \frac{557{,}223.28 - 351{,}747.90}{12{,}993} = 15.81 \quad \cong 16.00$$

Thus, the Social Premium is approximately GHC16.00 per quarter.

To translate these quarterly estimates into annual premiums the deduced estimates should be multiplied by four (4). Thus;

Type of Premium	**Quarterly Estimate**	**Annual Estimate**
Pure premium	GHC 62.00	GHC 248.00
Social premium	GHC 16.00	GHC 6 4.00

It is clear from estimates that the existing premium of GHC12.00 in the Kpandu district is woefully inadequate to sustain the scheme.

4.7 THE ACTUARIAL APPROACH

Suppose the 73,778 independent persons in the Kpandu district on day x purchase a continuous whole health annuity of 1 per day. If 0.001 and 0.02 are values of the forces of interest and morbidity respectively then the fund per individual person necessary to have a 90% chance of providing the annuity payments for the group on day (x) is deduced as;

For the mean,

$$E[Y] = a_x = \frac{1}{\mu+\delta} = \frac{1}{0.02 + 0.001} = \frac{1}{0.021} = 47.619$$

Then the variance of Y is

$$\text{var}(Y) = \frac{{}^2A_x - (A_x)^2}{\delta^2} = \frac{\left(\frac{\mu}{\mu+2\delta}\right) - \left(\frac{\mu}{\mu+\delta}\right)^2}{\delta^2}$$

$$= \frac{\left(\frac{0.02}{0.02+2(0.001)}\right)-\left(\frac{0.02}{0.02+0.001}\right)^2}{0.001^2} \quad = \frac{\left(\frac{0.02}{0.022}\right)-\left(\frac{0.02}{0.021}\right)^2}{0.000001} \quad = 2060.00$$

For the fund F per quarter and assume $z_{0.1} = 1.282$ the substitutions are effected as,

$$\frac{F}{n} = E[Y] + z_{\alpha}\sqrt{\frac{\mathrm{var}(Y)}{n}}$$

$$= 47.619 + 1.282\sqrt{\frac{2060.00}{73,778}}$$

$$\Rightarrow \frac{\mathrm{F}}{\mathrm{n}} = 49.06809$$

$$\therefore \ \mathrm{F} = 49.06809 \times \mathrm{n}$$

$$= 49.06809 \times 73{,}778 = \text{GHC } 3{,}620{,}146.10 \quad \text{per quarter}$$

Therefore the annual amount from the fund F is GHC 14,480,584.40 which points to the fact that the fund per person per annum is

$$= \frac{3{,}620{,}146.10 \times 4}{73{,}778} = \frac{14{,}480{,}584.40}{73{,}778} = \text{GHC}196.27 \cong \text{GHC}196.00$$

For the scheme to break even on the policy, i.e. Z = E[Z], with zero profit that is income being equal to expenditure, the premium should be quoted as GHC196.00 For the scheme to be able to take care of eventualities i.e. Z > E[Z]the premium should be a more than the GHC196.00.

The premium from the actuarial approach and the static probability approach confirm the need for some adjustments to be made for the sustainability of the scheme.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

Cash In-Flow for the year under review	
Total premium collected from informal sector	GHC 116,331.20
Total subsidy received from NHIA for exempt groups	GHC 1,047,962.85
Distress Fund from NHIA	GHC 359,028.75
Total registration & renewal fees collected	GHC 10,085.00
Total	GHC 1,533,407.80

It is patent from the above breakdown and deductions that, there is a deficit of GHC 173,871.33 for a quarter and GHC 695,485.32 annually which points to the fact that the cash out-flow is greater than the cash in-flow.

The actual cost burden on the scheme less the profit margin now becomes treatment cost burden on the scheme plus the other loadings which also burden the scheme

$$Actual\ cost\ burden = Treatment\ cost + Other\ loadings$$

$$= 557{,}223.28 + 241{,}608.53 \quad = 798{,}831.81$$

At this point the deficit now escalates to;

$$\text{Expenditure less Income} = \text{GHC}383{,}351.95 - 552{,}223.28$$

$$= \text{GHC}173{,}871.33$$

5.2 RECOMMENDATION

For the scheme to be sustained and the purpose of its implementation achieved, the following are measures that need to be adopted.

Stakeholders could increase the income of the scheme by having the courage and will power to

1. Increase the premium being paid now,
2. Increase the government subsidy to the scheme or
3. Increasing both the premium being paid and the government subsidy.

Within the Kpando District of the Volta Region of population 73,778 the health insurance premium should be at least GHC64.00 (without the loadings and the profit element). Thus, the existing GHC12.00 should be increased.

Considering the prevailing premium being paid in the district, it points to the fact that income is far less than expenditure. Therefore this measure will balance this anomaly (cancel out the deficit) but not allow for any profit for investment on the part of the scheme.

Government subsidy could be increased by a margin of GHC51.00 per person which is about 63% of the total revenue to take care of the difference between the existing GHC12.00 and the estimated GHC64.00

Stakeholders should also take a second look at the long process involved in paying claims which then pushes facilities to most often than not add an element of interest to their claims when submitting them.

Good accounting principles must be adopted to check the management of the Kpando District Mutual Health Insurance Scheme to reduce, if not eliminate entirely the financial misapplication.

A system should be put in place to check, detect and punish inflation of claims by the facilities.

5.2.1 STRATEGIC HEALTH MANAGEMENT SYSTEM

- Data collection should be scientific and representative of the nation. This should be collected weekly which will allow for the morbidity and mortality tables to computed for more robust exercises to be carried out in the future.
- A fund should be set aside solely for the health insurance scheme and well managed so that cash in-flow is periodic and actually paid into the fund. The fund should be targeted at growing from subsistence to a bigger fund that will have reserves.
- Insurance companies in the country could be taxed to contribute some percentage of the vehicle insurance premium collected into the national insurance fund.
- Food and Drugs Board (FDB) could tax companies and industries which produce items such as alcoholic beverages and cigarette that have indirect negative effect on the health of consumers and contribute this income to the health insurance fund.
- Environmental Protection Agency could also ask companies whose activities affect the environment to pay some tax which will go into the health insurance fund.

5.2.2 STRATEGIC HEALTH PROJECTS

When the health insurance fund is established and in operation;

- NHIA can support district assemblies to improve their sanitation conditions which will go a long way to reduce the spread and transmission of diseases.
- NHIA can also join in the malaria control program by distributing mosquito nets and discourage the sale of these nets.
- NHIA can move into the preventive measures of diseases other than concentrating on the curative only, thus move this public health education.

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