KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY - KUMASI

SURVIVAL MODELLING OF TIME TO DEATH FOR RETIREES IN GHANA

By

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DECLARATION

I hereby declare that this submission is my own work towards the award of the M. Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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DEDICATION

This thesis is dedicated to my adorable parents, Mr. Rexman D.K. Boachie and Mad. Margaret Achiaa.

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ABSTRACT

Many countries have studied the lifetime of retirees, with special attention given to the quality of life of these people. In Ghana, few work has been done on the determinants of time to death after retirement. The average age of survival after retirement in Ghana is additionally not known. This study attempts to determine the life expectancy of a retiree in Ghana, both for men and women, by utilizing the life table methods. Data was acquired from the SSNIT yearly and monthly reports of Pensioners for the years 1966 to 2015 particularly on mortality levels. Survival Analysis was utilized to investigate the effect of gender, age category and pension category on survival time (i.e. time to death). The cox proportional hazard regression model was used to further explain the determinants of time to death after retirement in Ghana. This work demonstrated that there is a significant distinction between the survival times for male and female retirees, in truth showing that male retirees are 1.24 times more prone to die, contrasted with their female counterparts. This subsequently leading us to conclude that gender does indeed determine the chance of survival after retirement in Ghana. Socio-economic status and age-at-retirement were additionally observed to be significant determinants of time to death after retirement. This study concluded that early retirement does undoubtedly increase the likelihood of early death.

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INTRODUCTION

1.1 Background to the Study

In most developed countries, Mortality rates have been persistently decline over the past decades. In recent times, Ghana and other developing countries is undergoing much the same trend, which signifies that people are living longer years even after retirement.

According to Ghana Statistical Service (2010), life expectancy at birth which was estimated to be 33.6 years in the belated 1930s and beginning of 1940s has risen to 62.7 years within 2000 and 2007, incrementing by 87% through the seven decades, the life expectancy for both males and females in Ghana is 62.3 and 65.2 years respectively for a period of 2010-2015. This is projected to increase in the subsequent years up to 74.3 and 78.3 in 2040-2050 for males and females respectively.

The idea elderly alludes to a class of grown - ups who have achieved propelled years, 60 or 65. The UN utilizes age 60 to allude to the elderly. Developed nations where life expectancy is high and the time of retirement from active public financial work is 65 years, the elderly are characterized as persons aged 65 years and more. Notwithstanding, developing nations, since life expectancy is lower and the period of retirement is 60 years, the elderly are treated as people matured 60 years and more. The elderly are otherwise called the aged. The populace Aged 60 years and more increased from nearly 1 million in 2000 to 1.4 million in 2010 and it is anticipated (projected) to rise to 6.3 million in 2050. The population Aged 65 years and more will climb up to 4.3 million and that of the 80 years and above to more than 0.5 million by 2050 (Ghana Statistical service, 2010).

While this has been observed, this study seeks to quantify the life expectancy after retirement, to determine the average years lived after retirement by men and by women in Ghana.

There has been a sensational demographic change in Europe and numerous different parts of the world. Fundamental changes in societies have been troubled by ageing populations and therefore threaten the sustainability of pension systems. In other to meet this demographic challenge, the EU dispatch an open civil argument on the best way to nail down this issue. Actually, many European countries have effectively found a way to increment the age at retirement. In France it will increment from 60 to 62 by 2018 and in Germany from 65 to 67 by 2029. The UK

is significantly more aspiring: it arrangements to build the law-related retirement age from 65 to 68 by 2046 (Kuhn et al, 2012).

However, Kuhn et al (2012) also stated that, Employees and their political delegates emphatically disapprove such changes, given reasons that, in other to avert emerging health problems, employees who spent their entire life working in physically requesting occupations ought to be permitted to resign early. Retiring from toxic workplace is, all things considered, plainly helpful for good wellbeing. On the contrary, leaving the labour force forever may also have a health effect in the other way. In fact, retirement is not just connected with lower salary and less assets to put resources into one's wellbeing, additionally with less intellectual and physical action in addition with changes in day by day schedules and ways of life that are conceivably correlated with undesirable conduct. Consequently, the general outcomes of premature retirement on wellbeing could go in either bearing.

In other to retire comfortably at the normal retirement age of 60 and voluntary retirement age of 55 in Ghana, one needs to plan his or her career path and finances matter, so that he/she will experience long, cheerful and relaxation retirement life into the brilliant age of 80s and past.

More research have been conducted in the developed countries to buttress the idea of premature retirement, proposing to the fact that early retirees tend to die earlier, others have found that there was no substantial difference on mortality between the early retirees and the individuals who worked up to the ages of 65 and above.

Although studies on life after retirement shows that death in old age is associated with age at retirement, the likely determinants of death in retirees have been done in developed countries with few studies been supervised in developing countries. This is attributed to the way that impartial populace information are not easily obtainable for numerous nations.

When you plan your resources, it is much clear to accomplish a protected dream and comfortable retirement. Because we are living longer and healthier lives, we can suspect to spend extra more time in retirement than our grandparents did.

This research seeks to examine the issue of mortality after retirement in Ghana. We seek to assess the determinants to mortality and determine the life expectancy of individuals after retirement in Ghana.

Retirement ought to ordinarily be a time of rest from the anxiety and effort of work, when one invests the majority of his energy with the family, with outdated mates and, obviously, in bed (MacBean 2007, in Asamoah 2012). Normally, the most common form of cessation of ones working life is by retirement. Retirement therefore constitutes an inevitable stage in every worker's life. Workers in Ghana are not exempted from this experience.

Pension is an installment a man gets after retirement. It programmed as defined benefit or defined contribution. Government is the main employer especially for public sector workers, and for that matter employer's look up to the Government for economic bulwark. Government is customarily urged by the laws of a State to guarantee that economic security for its workers are given upon retirement. Based on this reason, Government everywhere throughout the world makes every effort to institute pension scheme for its workers. A pension might be a man's significant wellspring of wage at seniority when he/she is out of work. It is this cash will profit a retiree to pay for his bills, and keep up himself and the family. The season of benefits should not be a perfect time to work vivaciously again and experience stress. It ought to be a period of unwinding and appreciate what one has works for during his time of energy-consuming employment with the goal that one could be depicted as living a prosperous lifetime.

It ought to be a period that one will invest quality time with his family. Therefore, benefits plans ought to be outlined in a way that won't aggravate the worker off upon retirement. It ought to instead make the worker happier keeping in mind the end goal to eschew nonessential panics during retirement. Recently, nations everywhere throughout the world are incrementing their retirement age in other to make their pension schemes more captivating and rewarding.

It has been seen that the higher the period of retirement, the higher the rate of work. This sincerely betoken, work levels are conventionally high when the retirement age is high. This betokens a country's Pension ought to be composed in a manner that more individuals will work at any given time to reduce the caliber of unemployment. The reason is that State Pensions are visually perceived as poverty mitigation techniques to make the worker economically autonomous after retirement.

Retired employees bolster themselves either through pension or investment funds. As a rule the cash is given by the administration, however now and again conceded by private memberships to common assets. In this last case, subscription may be compulsory or intentional. The money related weight of arrangement pension on an administration spending plan is frequently substantial and is the purpose behind political verbal confrontations about retirement age. The expense of social insurance in retirement is extensive, on the grounds that individuals have a tendency to be sick all the more often in later life. Overall income after retirement originate from investments and private savings (owned housing, private pension funds), occupational pensions, state pensions, gifts: e.g. by kids and social advantages. On an individual level, the risen average cost for basic items amid retirement is a genuine worry to numerous more established grown-ups. Human services cost assumes an essential part.

Arisen challenge for numerous countries is populace maturing. As birth rates drop and life expectancy(L.E) increment, a perpetually astronomically immense segment of the populace is aged. This leaves less employees for every retired individual in numerous nations, this designates the administration and open area benefits could fall their economies unless annuity frameworks are changed or taxes are incremented. One technique for straightening out the annuity framework is to increment the retirement age.

Ghanaian workers retire after active help to the nation with great outlook that they would repose from their hard work and relish the fruit of their labour, this way various pensions and social security protection are paid to them by the body politic. Many Ghanaian workers are not able to vacate from the official bungalows given to them when they were in active service, due to improper preparation and poor condition of service. Others in the rented premises could hardly sustain payment thereby bringing out the anger and very terrible treatment of landlords.

This is attested by Lawoe (2010), who submitted that although there were some sort of activities aim at planning for retirement amongst tutors in public second cycle institutions in Ghana, the entire operation was incomplete.

Asamoah (2012) also found out that most Ghanaian employees know about retirement, however little is done about it as far as arrangement and adjustment. There is therefore the need to throw more light on retirement issues in Ghana.

1.2 Problem Statement

SSNIT Pension benefit is the primary source of retirement benefit among formal sector workers and recently informal sector workers in Ghana. Government in many countries pay particular attention to pension schemes because of pension funds enormous role in the development of the economy. Many Ghanaian workers do not do adequate preparation and therefore encounter a lot of problems during retirement from service such as the inability to afford utility bill, wards school fees, and meet basic social obligations. According to Ghana Statistical Service (2010), there has been a problem of Population Ageing as birth rates drop and life expectancy increases and this threatens the sustainability of the pension schemes in Ghana. This is because in an ageing population the number of people that contribute to a pension fund decreases relative to the number of people that receive a pension benefit. Moreover, there is also a problem of longevity risk due to the increasing life expectancy trends among pensioners and can results in payout levels that are higher than what the pension scheme originary accounts for. In view of this, this study intends to investigate the issues of mortality after retirement in Ghana.

1.3 Objectives of the Study

The principal objective of this survey is to investigate the determinants of time to death in a retiree in Ghana. Specifically, this study seeks to:

- Identify the significant determinants of time to death in retired Ghanaian.
- Investigate the difference in time to death between male and female retirees and the different pension categories, in the study population.

- Predict the life expectancy after retirement in Ghana.

1.4 Justification

Ageing of the individual means growing old; and old people retire from active employment. Ageing is a lifelong process and should be acknowledged as such. Moreover, Unpredictable capital markets, rising health consideration expenses, and low conservation rates among the populace have made the retirement behavior and policy a moving target (Clark and Mitchell, 2005). Preparation of the entire populace for the later phases of life ought to be an inbuilt segment of social policy and embrace physical, cultural, psychological, religious, spiritual, health, economic and other factors ensure their well-being (International Plan of Action on Ageing, [IPAA], 2001) (Lawoe, 2014).

Surveys have been performed on the impact of early retirement, with results denoting that self-perceived wellbeing, business level and employment fulfillment are all independent indicators of premature retirement. In Ghana, the normal retirement age is 60 and voluntary retirement age is 55 under the incipient pension scheme, whose impact hasn't been valued, therefore, the mean years of survival after retirement in Ghana is not experienced. The quality of liveliness of the retirees is also a hugely ignored matter. This study endeavor of finding the determinants of time to death, if any, of mortality in the retirement age and compulsory interventions put in position to assure that these cases are directed.

The life expectancy of male and female retirees will be very subsidiary in optically discerning the cessation of life annuities used customarily to define the remnant of the guarantee period. Besides, the life expectancy of retirees in Ghana will be real useful to Actuaries and Financial professionals in Ghana since they need them in estimating mortality and longevity risk for an insurance company especially in life annuity plans.

Again, determinations from the survey will be helpful to the partners in the livelihood division including government, occupation offices and associations, SSNIT and different partners to strategize an incipient and most ideal methods for guaranteeing that employees can adapt to retirement.

On the academic front, the study would add up to existing literature on the determinants of time to death of retirees.

1.5 Limitations

There are some limitations to this study which incorporates: The nature of the data for the study which is a secondary data and since the data accumulation was not done directly, the authenticity (reliability) of the data cannot be 100 percent guaranteed.

Additionally, all the retirees in various pension schemes in the country could not be studied because of the limited time available for the study and additionally because of lack of funding for this project (considered only retirees under the SSNIT Pension scheme).

Finally, the records of retirees appeared not properly kept, so the data on mortality levels for some particular years postured difficulties of retrieval for the study.

1.6 Organization of the Study

This survey will be exhibited in five chapters. The first Chapter is the introduction to the entire study and contains the background to the study, problem statement, objectives of the study and the significant of the work. Chapter two contains discussions of the suitable literature on the concepts and ideas on the subject matter. The third chapter is dedicated to issues of methodology. Chapter Four is devoted to the results of the fieldwork and the discussions thereof. The final chapter is where the summary of findings, conclusions drawn as a result of the findings, and recommendations made are written. Lastly, we have the list of references and Appendix.

Chapter 2

LITERATURE REVIEW

2.0 Introduction

The collection of literature on retirement issues and mortality is developing. So as to place this study in a legitimate connection, a portion of the current writing would be audited. The literature survey is conducted under different inscriptions such as the impacts of premature retirement on wellbeing, retirement planning, and etc.

2.1 Impacts of early retirement on wellbeing

There is a common believe that premature retirement is related to longer life expectancy and later retirement is connected with early demise. No agreement has been come out to on the relative mortality or survival of individuals who retire early or late. In the paper, Tsai et al., (2005), after analyzing data from past employees of Shell, utilizing the Kaplan Meier curves and cox proportional model, on comparison of survival curves on various retirement ages, found that age at retirement is not connected with increased or decreased survival. Nevertheless, result revealed that gender and socio-economic status of the employee was a determinant of survival after retirement.

In a standout amongst the most legitimately done investigations of retirement and wellbeing, scientists found that retirement had a contradictory effect on wellbeing. The survey visually examined 16,827 men in Greece who had not been determined to have a wellbeing condition, for example, diabetes, stroke, tumor or coronary illness. It contrasted men who resigned with men who were all the while working. The study found that the Pensioners had a 51% expansion in their danger of death as compared to those still working.(after controlling for variables, for example, riches, instruction, conjugal status, and so on). The vast majority of the expansion in death danger was connected with coronary illness and cardiovascular wellbeing, (Stibich, 2009). It would be consequential to evaluate the quality of life of the retirees. For instance, with the abundance of data assembled as at 50 years, is it reasonable to give up this individual or ought to there be alumni programs where the experience gained over decades is transferred to the youthful era? Programs like this would help both the youth and the older folk, enhancing incredibly their quality of life.

Brockmanna et al. (2009), utilized Kaplan Meier curves to figure out whether there was a distinction in the survival rates of different retirement ages, socioeconomic status and utilized the cox proportional models to determine the determinants to death after retirement. Results stated that retiring early affects long haul survival fundamentally. Lower survival probabilities were observed for people with weakness at premature retirement related with retirees who retire later. In any case, workers who leave the work showcase early and sound may have preferred survival probabilities over people who retire within the age intervals 61 and 65.

In a Comparisons of passings (cases) inside 5 years of retirement with survivors (controls) demonstrated that pre-retirement wellbeing status was the main critical indicator of survival after early retirement. Among typical pensioners, lower status employees were more liable to kick the bucket inside 3 years of retirement than higher status laborers, who were more noticeable among passings 4 to 5 years after retirement (Haynes, 1978).

Mwakala (2013), studied the determinants of time to death in Kanya taking into thought the age at retirement, socio-economic standing, got from the monthly pension paid to the retired person and gender. The study determined the life expectancy of a retiree in Kanya utilizing life table method. The cox PH model was utilized to determine the impact of gender, age at retirement and socio-economic status of retirees. The work demonstrated that there is a significant distinction between the survival times between male and female retirees, in truth showing that male retirees are 2.1 times more prone to die, contrasted with their female counterparts. Mwakala (2013), therefore concluded that gender does indeed determine the chance of survival after retirement in Kenya. Socio-economic status and age-at-retirement were additionally observed to be significant determinants to time to death after retirement.

It is questionable how late changes in labor power progress might have impacted the expanding quantities of individuals taking retirement industrialized nations. The Whitehall II survey gives a chance to inspect the indicators of premature retirement in one out of the biggest businesses in the UK. Mein et al. (2000) inspected the components anticipating premature retirement in a seven year follow up period starting from 1988 to 1995 utilizing longitudinal information on 2532 male and female London-based common workers within age intervals 50 and 59.5 years. Standard information on work grade and term of time employed in the

civil service were gotten from self-finished questionnaires. Essential variables analyzed involved wellbeing, employment qualities, inquiries on employment needs and job fulfillment and monetary unreliability, riches and material issues. Time before premature retirement was investigated. The Cox PH Regression model was use to fit the data. Of the 2532 common workers, 26.7% retired right on time amid the follow-up period. Mein et al. (2000) found that men and ladies in the more generously compensated livelihood reviews, those that had experienced sick health and those that were less fulfilled by their occupations will probably go on retirement early, though actual issues inclined to retain individuals working.

The outcomes demonstrate that that self-anticipated wellbeing, livelihood review and employment fulfillment are all autonomous indicators for ahead of schedule retirement. Subjective investigations might also add more to our comprehension of the withdrawal procedure.

People would need to hesitate over premature retirement. The reason is that, incipient Study has demonstrated a connection between premature retirement and sudden passing. Study by Kuhn et al.,(2012) revealed that males, specifically, had an expanded danger of death before age 67 when they resigned early. To demonstrate this, the scientists took an employees from Austria, conceived somewhere within the age intervals 1929 and 1941 and ascertain that a diminishment in the age at retirement causes a compelling increment in the danger of sudden passing for men, yet not for women, the research verbalized. ”The effect for males is not only statistically consequential but additionally quantitatively paramount. According to their estimates, one supplemental year of early retirement causes an incrementation in the risk of premature death of 2.4 percentage levels (a relative increment of about 13.4%, or 1.8 months as far as years of life lost)” (Kuhn et al., 2012).

Retirement sounds extraordinary: no supervisor, do what you need, travel, and so on, yet does retiring from work harmed your wellbeing and future? This is a perfect question, and the response relies on exactly what retirement is all about. A study by Stibich (2009), analyzed the survival of 839 workers that retired at age 55 and 1929 representatives that retired at age 60 and contrasted them with 900 workers that retired at age 65. Ladies built up just 11% of the research populace. The connection between premature retirement and sudden passing

was more prominent for males. Indeed, males who retired at age 55 had an 80% more prominent high hazard than ladies who retired at 55. Individuals who retired at
60 were the same as individuals who retired at age 65, as far as general danger of death is concern. It may be the case that individuals who retire at 55 have weakness and in this manner picked early retirement.

Another likewise research revealed that retiring at 60 appeared not to affect wellbeing. In the Whitehall II research scientists take after British Civil Servants. This provides them entry to an immense database of wellbeing and demographic data. There have been many studies from this database and one of them took a gender at the effect of retiring at 60 on wellbeing. Individuals that retired at 60 had exactly the same physical and emotional well-being capacity as individuals like them who were all the while working. Actually, the retirees' mental health enhanced fairly subsequent to retiring. So as indicated by this survey, retiring at age 60 is either not destructive or a bravo (Stibich, 2009).

A study directed by Sanlam Employees Benefit (in South Africa) as cited in Mwakala (2013), demonstrates that individuals are not sufficiently preserving for retirement. Dawie De Villiers of Sanlam's Structured Solution in his presentation said that of the overviewed retired people just 40% trusted that they have preserved enough money to keep going for whatever is left of their lives. Beneficiaries additionally highlighted that if there was anything they would have done another way is that they would have arranged better and preserve more before retiring. This backing the discoveries of the research, that is to say peoples are not adequately preserving for retirement. Most will need to work after they achieve their retirement age, not on the grounds that they need to, but rather on the grounds that they require supports, or live with their kids or depend on the social old age benefits. Results displayed demonstrate that this area of study requires more studies to be done to decide the determinants of longer or shorter survival years, after retirement.

Notwithstanding, it is paramount to specify that in spite of the fact that retirement can be agonizing, yet it is not deadly. It is a change much the same as those prior in life. While these causes are properly archived, information on the same is distracted; subsequent to these causes would insinuate the personal satisfaction of the retiree, which would have direct effect on the longevity of their life. Additionally, the vast majority who retire early amazing after retirement presumably had wellbeing issues afore they quit employment. Retirement had no thing to do with their end. This brings into inquiry the health status of the retirees shockingly; most pension schemes in Ghana don't get some information about the health status of the retiree in this manner it's difficult to make deduction on this as a reason for early death. Nonetheless, the age-at-retirement will be a variable to reveal some insight into in the event that it's a significant determinant reason for death. Leonette (2009) revealed in his article that the main association in the middle of retirement and premature passing might be because a few pensioners neglect to keep dynamic. They unwind to the limit that their bodies self-destruct. They surrender. They neglect to stay in control. There are numerous purposes behind premature retirement, and there are pretty much the same number of for retiring later. Be that as it may, maintaining a strategic distance from retirement arranging and remaining focused treadmill since you fear retirement will bring about early death and this ought not to be the situation.

2.2 Retirement Orchestrating (Planning)

In Ghana, there is frequently no obvious approach on retirement arranging. It is in perspective of this, that a survey by Asamoah (2012) was embraced in the Dormaa Municipality. The principle goal of the exploration was to discover how workers in Ghana modify in relation to retirement as far as economic, wellbeing, accommodation and leisure. The SPSS and a logistical relapse model were utilized to investigate the information which was gotten using structured and unstructured questionnaire. By and large, the workers in the Dormaa Municipality were revealed to change preferably in relation to retirement. They were in this manner observed to be very much arranged for retirement. Be that as it may, while the guys were preferably arranged in convenience, the women were preferably arranged economically. It was reasoned that majority of Ghanaian employees know about retirement, but little is effected concerning it as far as arrangement and change. Different suggestions were given with respect to how modification towards retirement should be possible. Asamoah (2012) further prescribed that the retirement date for every employee ought to be appended to their arrangement letters. Besides, all the public teaching of SSNIT authorities ought to initially focus on the businesses. This public teaching ought to be finished every employee when they have somewhere around ten to fifteen years to retire.

The reasons of the study was to investigate retirement planning, difficulties, and counseling among teachers of state funded schools in the Sekondi Circuit in the Western Region, Ghana. A sample of 50 teachers was chosen through convenience sampling. Only teachers who showed interest in taking part in the study were sampled. The fundamental instrument for gathering the data was questionnaire, which had a reliability co-efficient of $r = 0.81$. Descriptive and inferential statistical tools were utilized in analyzing the data collected. The aftereffects of the study showed that the significant retirement counseling that teachers in the Sekondi Circuit get is the talk on conditions of service and the retirement benefits in light of Social Security Pension Scheme. It was additionally found that poor state of mind towards savings, absence of knowledge in investment choices, lack of awareness of what to do with pension money, and trouble in time management were the primary difficulties confronted by teachers in the Sekondi Circuit in their retirement arranging. Moreover, it was found that without the Social Security Pension Scheme, teachers in the circuit do not have some other method for securing their future when they retire from active service. It was prescribed among others that the education directorate ought to advice every one of the educators they enlist once they offer them appointments. At long last, there is a need to fortify the Counseling Units of the Ghana Education Service, in order to counsel and aide teachers in their retirement arrangement and planning (Wilson and Aggrey, 2012).

This study examined the employment role affixment and retirement goals of state school tutors in Calabar, Nigeria. It was incentivized by the perception that most public school workers lack plans for retirement and accordingly don't get ready for it until it all of a sudden occurs to them. Couple of exact studies were looked into. Questionnaire was the primary instrument utilized for data gathering from a sample of 200 teachers. Independent t-test was utilized to check the expressed speculation at 0.05 level of significance. The outcome demonstrated that the committed (appended) workers have retirement aim to take a part-time work after retirement. The uncommitted (unattached) workers have expectation to retire earlier than those connected to their work. It was suggested that pre-retirement counseling ought to be received to help teachers to develop great retirement plans (Petters, & Asuquo, 2008).

The investigation of annuities, retirement or maturity pay bolster strategies in the advance world has been consigned to the foundation to a great extent on the grounds that such projects are constrained in degree and cover just a little division of the populace, generally formal labour market employees. However in light of the fact that pension strategies solve problems pertaining income security at the last phases of life, it is significant that we comprehend how they were created over time by policy makers whilst giving appropriate consideration to the domestic political elements that molded their opinions. Kpessa (2011) examined the advancement and change of retirement pay approach in Ghana. Contrary to the standard way of thinking whereby social security approaches were surrounded as by-results of advancement and industrialization, the study contends that formal retirement salary arrangements in Ghana have regularly been intended to financial and political change in different approaches. Past the exigency of retirement wage security, the study demonstrates that while seniority pay bolster arrangements, which concentrated normally in the formal work business sector were utilized to support, advance work profitability in the pilgrim benefit, the postcolonial directions of such strategies accentuated central accumulation and assembly of political backing.

2.2.1 Reasons for death in old age

Most studies partner death after retirement to:

1. Health problems: Ill health and disability; severe agonize or chronic; psychological decrease; harm to self-perception because of abscission or sickness.

2. Aloneness and desolation: Staying or living alone; a lessening group of friends because of death or migration; decremented versatility because of sickness or loss of driving chance.

3. Decreased upshot of motive: awareness of purposelessness or loss of personality because of retirement or physical constraints on movement.

4. Fears:Panic of dying or kicking the bucket; nervousness over budgetary issues or wellbeing issues.

5. Current loss: The passing of relatives, friends, and pets; the departure of a mate or accomplice (Mwakala, 2013).

2.2.2 Area distribution of pensioners (SSNIT)

The total number of pensioners as at 31, February 2015 was 145,054. Accra positioned the most elevated with 28 percent retired people took after by Koforidua with 20 percent and Kumasi with 18 percent. Sunyani region had 6 percent of the retirees which happens to be the least.

Table 2.1: No. of pensioners by areas

Area	No. of Pensioners	% of total
Accra	40,325	28%
Koforidua	28,723	20%
Kumasi	25,832	18%
Takoradi	20,977	14%
Tamale	11,019	7%
Tema	9,647	7%
Sunyani	8,531	6%
Total	145,054	100%

Source: SSNIT April news sheet, 2015.

2.2.3 Life Expectancy

The budgetary weight of maturing is an immediate capacity of the quantity of years that the person will live past retirement. A typical term that is identified with the issue is life expectancy. There are misinterpretations regarding its meaning. Therefore, we begin by clarifying this figure. Life expectancy estimates the mean period of life in a populace. In a steady populace, it would be an estimation of the mean age of departed individuals.

Life expectancy values are ascertained from the life table. Presume for a seconds that mortality has an unusual arrangement: all individuals are anticipated to die amid the year and to die immediately afore they reach the birthday cake just preceding to the following birthday. Actuaries can ascertain the aggregate number of years that the group of individuals displayed has lived from the life table, which is essentially the whole of all lx figures from age 0 to the end of the life table. In the event that this figures is divided by the number of individuals that were presumed to be alive at age 0, life expectancy is the outcome. Actuarial scientist do make a little adjustment to this figure: we have presumed that all individuals die at the end of the year, whereas individuals usually die throughout the year, so we have to subtract 0.5 from the mean value we obtained (Baranoff, et al., 2009).

2.2.4 Contingent Life Expectancy

Longevity risk identifies with the length of the postretirement time frame, and for that reason we have to consider contingent life expectancy (L.E). Life anticipation after retirement. that is, the L.E after retirement (or some other important age). These values are ascertained from the life table in a same manner to L.E during birth. For instance, to ascertain the contingent L.E at age 55, we need to add all lx figures in the life table from age fifty-five to the end of the table to obtain the number of years lived by the cohort of the individuals aged fifty-five. This sum should then be divided by l_{55}, the initial number of individuals in that age. Once more, because we utilized the rough presumption that all individuals die exactly at their birthdays, despite the fact that they die randomly throughout the year, we need to make a rectification. Simply subtracting half a year i.e 0.5 from the mean (average), we obtain the life expectancy (Baranoff, et al., 2009).

To assess the money related requirements to confront the danger of longevity, we should look extremely cautiously at the understanding of L.E. We will presume a extremely basic model, where individuals join the labor power at the age of twenty and retire at the age of sixty-five.

Furthermore, presume that L.E is seventy-five and is not anticipated that would change, and that interest rates can be overlooked. Presume likewise that the annual consumption of a retiree is identical to the individual working (a realistic presumption in perspective of surveys demonstrating that the aggregate consumption is quite permanent, in spite of the fact that its structure changes altogether with age). These fundamental parameters appear to create a straightforward retirement model: during forty-five (45) years of work, individuals should gather adequate funds to take care of an extra 10 years, that is, set aside around 10/45, or 22%, of their yearly income.

There are two major imperfections in this basic model, and either of them arise from a misinterpretation of the idea of life expectancy. To begin with, relying on an mean number, that is, L.E might be fine for a financial foundation that holds a large portfolio of numerous insured individual. A person, nevertheless, is not supposed to orchestrate her insurance wants in accordance with average values and should rather get ready for the extreme cases. L.E is an average value. A high percentage of individuals who are on retirement will live far longer than the presumed L.E (age seventy-five(75)).

What will every one these individuals do when the assets are exhausted toward the end of the expected ten year post-retirement period? Additionally, when individuals refer to L.E, they ordinarily refer to the published value, which is L.E at birth, that is, the average age of death. The remaining L.E is a complex function of age. If L.E at birth is seventy-five (75) years, the remaining L.E at the age of Sixty-five (65) might be significantly higher than 10 years in so far as the individuals who achieve age 65 have survived the childhood illnesses, the motorcycle accidents, the risks of military service, etc. The United State Vital Statistics, for instance, demonstrate that the L.E at birth of white males in 2005 was 75.2.

Notwithstanding, the remaining or conditional L.E at age seventy-five is not just ten years, but instead 17.2 years. The above values imply that when the United state white male in our model achieves retirement, he would be wise to have adequate funds for 17.2 years, on average, instead of just ten years, as the model has presumed. Setting aside accumulation (savings) for 17.2 years over 45 years of presumed work is nearly double the calculation that presumed a 10/45 ratio.

The issue of females is considerably more pressing in practically speaking. women are the naturally more resilient gender, and their L.E at birth is regularly greater than that of males. For instance, in 2005, L.E at birth for white United State women was 80.4 (related to just 75.2 for men). Nevertheless, females have a tendency to retire before men, say, at the age of sixty (60), and the remaining L.E at that age is twenty-four (24) years. Premature retirement, nevertheless, additionally implies a shorter working period, maybe just forty (40) years. Generally, numerous females additionally spend a few years out of the salary-earning work to bring up the family. The mean working time for ladies can really be decreased to 30 years or less. One should be a financial magician to have the capacity to amass adequate reserve to finance a post-retirement time of twenty-four (24) years more roughly 30 years of work (Baranoff, et al., 2009).

According to Cindy Levering, a retired consulting actuary, by age sixty-five (65), United State men in mean health have a forty(40) percent likelihood (chance) of living to age eighty-five (85) and females have a 53 percent likelihood of living to age 85. If the couple is married, there's a 72 percent likelihood that at least one of them will live until age 85. But if these same folks are "healthier" (25 percent healthier than the mean individual), then their chances of living to age 85 increase to 50 percent for the men, 62 percent for the women and 81 percent for no less than one of them. This depends on a static table from 2007 Social Security mortality data (Ebeling, 2012).

This article provides details regarding the risk of mortality of aging retired and crippled employee males over the decade after they got to be Social Security beneficiaries. Essential patterns of mortality rates are portrayed utilizing data from the 1982 New Beneficiary Survey connected to administrative records. The article looks at the relationship of increasing age, race, socioeconomic standing, private health insurance, and other demographic and health characteristics with the length of life between the two groups of men, utilizing statistical models. Throughout the decade, the hazard of death for retired employees significantly increased with aging and with low socioeconomic standing. The hazard of death for the disabled was significantly connected with being black (McCoy, et al., 1994).

Chapter 3

METHODOLOGY

3.0 Introduction

This study was done using data from Social Security and National Insurance Trust (SSNIT) in Ghana. Data was cleansed to engender survival data set and exploratory and inference done to explore the determinants of time to death in a retiree in Ghana. Kaplan Meier survival curves was acclimated to know the shape of the survival function. The life table method was acclimated to estimate the life expectancy of retirees in Ghana. The cox proportional hazard regression model was utilized in estimating the determinants of time to death in a retired Ghanaian. SPSS and Excel statistical software was acclimated to analyze the
data.

3.1 Data description

The dependent variable considered is survival time since retirement and the covariates: sex, age-at-retirement and categories of pension received monthly.

3.2 Exploratory data analysis

Exploratory data analysis was used to further describe the data. To understand the kind of individuals in the dataset that was used for this Thesis, it will be done, based on:

1. Gender
2. Pension categories
3. Age categories

3.3 Data Analysis

In any data analysis it is dependably an incredible thought to do some univariate investigation before continuing to more confounded models. In survival examination it is exceedingly suggested to look at the Kaplan Meier curves for all the categorical variables. We additionally look at the tests of equality across strata to examine regardless of whether to incorporate the indicator variable (covariate) in the final model. Survival Analysis was utilized to investigate the effect of gender, age category and pension category on survival time (i.e. time to death). Specifically, the Kaplan-Meier technique was employed for plotting the hazard and survival functions. Log rank test was employed to compare the survival curves, which is a non-parametric test. The Cox PH Model was employed to fit the data which is a semi parametric model. Log minus log plot was used to check for the proportionality assumption. That is, give an idea of regardless of whether the groups are proportional (that is to say, the survival functions are approximately
parallel).

3.3.1 Review on survival analysis

Survival analysis is normally defined as a set of techniques for examining information where the result variable is the time until existence of an event of interest, for instance death. Assume that T is the time span before a retiree dies. The haphazardness of T can be portrayed in three different approaches.

Survival function

This function signified by S(t), is characterized as the likelihood that a person survives longer than t:

$$S(t) = \frac{Number \quad of \quad retirees \quad surviving \quad longer \quad than \quad t}{Total \quad number \quad of \quad retirees} \tag{3.1}$$

$$S(t) = Pr(a \quad retiree \quad survives \quad longer \quad than \quad t) \tag{3.2}$$

$$S(t) = Pr(T > t) \tag{3.3}$$

$$= \int_t^\infty f(u)du \tag{3.4}$$

Where: T is the event time, t is the observed (Study) time and $f(u)$ is the Probability density function. S(t) is additionally called the cumulative survival rate.

Hypothetically, as t extents from 0 up to ∞ the survivor capacity can be diagramed as a smooth curve. As showed by the diagram beneath, where t shows the x-axis.

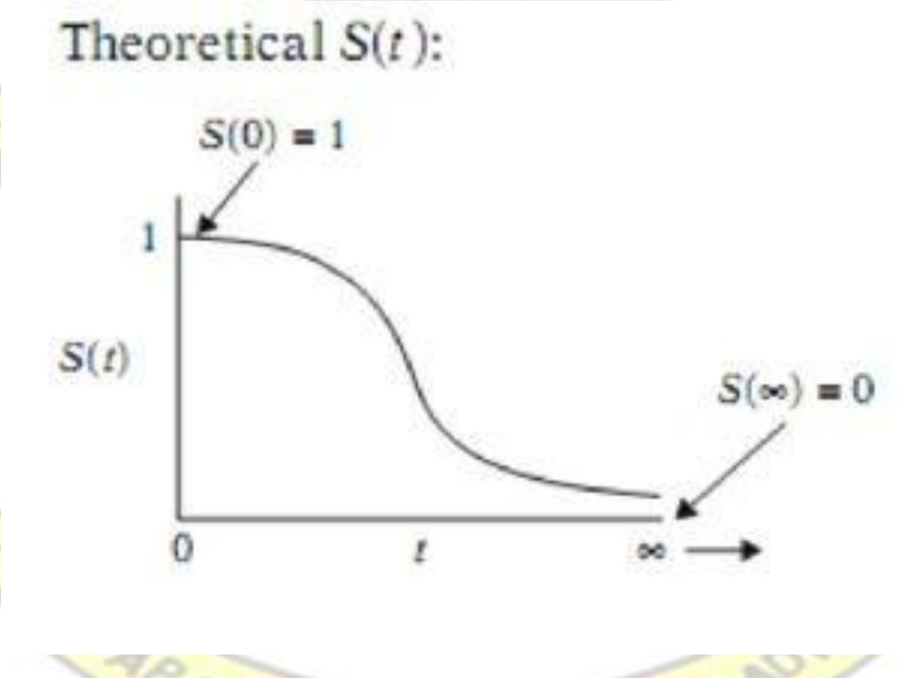

Figure 3.1: Theoretical graph of survivor function

In practice, when utilizing genuine information, we generally get charts that are step functions, as showed beneath, instead of smooth curves. Since the study time frame is never unending long, it is conceivable that not everybody concentrated experience the event. The evaluated survivor function, indicated by a caret over the S in the chart, therefore may not go the distance down to zero toward the end of the study. Cumulative distribution function $F(t)$ depicts the

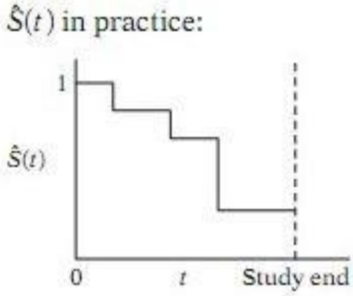

Figure 3.2: Practical graph of Survivor function

likelihood that, the time to event (T) is smaller or equal contrasted with a fixed time (t) and is given as:

$$F(t) = P(T \leq t) \tag{3.5}$$

From this, the survival function $S(t)$, is the probability that the time to event (T) is larger compared to a fixed time (t), can be derived as:

$$S(t) = 1 - P(a \quad retiree \quad dies \quad before \quad time \quad t) \tag{3.6}$$

When T is a continuous random variable, the survival function is equal to the complement of the cumulative distribution function, that is,

$$S(t) = P(T > t) = 1 - P(T \leq t) = \int_t^{\infty} f(t)dt \tag{3.7}$$

when T is discrete,

$$S(t) = P(T > t) = \sum_{T_j > t} P(T_j) \tag{3.8}$$

The hazard rate is defined by

$$h(t) = P(T = t_i | T \geq t_j) = \frac{P(t_j)}{S(t_{j-1})}, \quad j = 1, 2, .. \tag{3.9}$$

When T is a continuous random variable,then

$$h(t) = \frac{f(t)}{S(t)} = -\frac{dln\,[S(t)]}{dt} \tag{3.10}$$

Probability density function

This is the likelihood that the event time occurs at exactly time t (out of all possible times) and is given by:

$$f(t) = \lim_{\Delta t \to 0} \frac{P\,(\textit{a retiree dying in the time } \text{interval } (t, t+\Delta t))}{\Delta t} \tag{3.11}$$

$$f(t) = \lim_{\Delta t \to 0} \frac{P(t \leq T \leq t + \Delta t)}{\Delta t} \tag{3.12}$$

The probability density function is predicted as the portion of individuals dying in an interval per unit width:

$$f(t) = \frac{\textit{No. of retirees dying within the } \text{interval begining at time } t}{(\textit{Total no. of retirees})(\textit{Interval width})} \tag{3.13}$$

The probability density function is otherwise known as the unconditional failure rate.

Hazard function

And the last description is given by the hazard function . This is the probability that if a retiree survives up till time t, it will experience the event in the next instant and is given by:

$$h(t) = \lim_{\Delta t \to 0} \frac{Pr\,(\textit{a retiree dies in the time } \text{interval } (t, t+\Delta t) |\text{ aliveatt})}{\Delta t}$$

(3.14)

(3.15)

$$h(t) = \lambda(t) = \lim_{\Delta t \to 0} \frac{P(t \leq T \leq t + \Delta t / T > t)}{\Delta t} = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{S(t)} \quad)] \tag{3.16}$$

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{S(t)} = -\frac{d}{dt} In[S(t$$

In practice, the hazard function is evaluated as the portion of individuals dying in an interval per unit time, given that they have survived in the beginning of the interval:

$$h(t) = \frac{\textit{No. of retirees dying within the } \text{interval beginingattime } t}{(\textit{No. of retirees surviving at } t)(\textit{Interval width})} \tag{3.17}$$

The hazard function is additionally known as the instantaneous failure rate or conditional failure rate. The cumulative hazard function, H(t), is given by:

$$H(t) = \int_t^0 h(u)du = -In[S(t)] \tag{3.18}$$

The interpretation of the survival function $S(t)$ is very clear. It is a plot with on the left axis the portion of the populace still alive and on the x axis the time. The hazard function $h(t)$,is additionally known as incidence rate, instantaneous risk or force of mortality, is the event rate at t among those at risk at time t . It is normally plotted as the cumulative hazard rate on the y-axis and the time on the x-axis.

3.3.2 Censored Data

A typical issue in survival analysis is censored data or likewise called the missing data problem. The perfect dataset contains start and end dates of the considerable number of people in the study. Observations are known as censored when the information about their survival time is incomplete. Censoring happens when we have some information about individual survival time, yet we don't know the survival time precisely. Censoring is a critical problem in survival analysis, representing a specific kind of missing data. Censoring that is random and noninformative is generally recommended in order to avoid bias in a survival analysis.

Observed Survival:

A life table method (approach) to dealing with censored data from successive cohorts of individuals. Censoring implies that information on some aspect of time or duration of events of interest is absent (missing).

Three kinds of censoring usually encountered

- Right censoring
- Left censoring
- Interval censoring

Censoring implies that some important information needed to make a calculation is not accessible or available to us (Censored).

Right censoring $(T > t)$

Right censoring is the most well-known (common) concern. It implies that we are not certain what happened to individuals after some point in time. This happens when some individuals cannot be followed the whole or entire time because they died or were lost to follow-up. For instance, assume patients are followed in a study for five (5)years. A patient who does not encounter the event of interest for the period of the study is said to be right censored.

Left censoring $T < t$

Left censoring is when we are not sure (certain) what happened to individuals before some point in time. Commonest illustration is when individuals already have the disease of interest when the study begins.

Interval censoring $(t_i < T < t)$

Interval censoring is when we realize that something happened in an interval, that is, not before time x and not after time y, but do not know precisely when in the interval it happened. For instance, we know that the patient was well at time x (i.e. free from the disease) and was diagnosed with ailment (disease) at time y, so when did the malady (disease) actually start? All we know is the interval.
These types mentioned are represented in Figure 3.3.

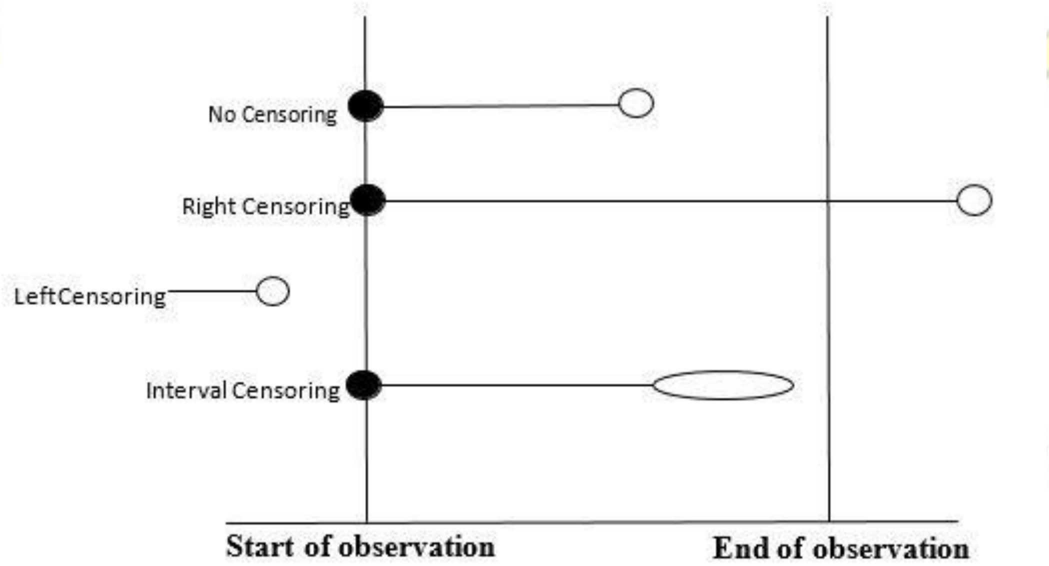

Figure 3.3: A diagram representing the three kinds of censoring

A critical presumption utilizing censored data is that the censoring is non-informative censoring. This presumption expresses that the censored retirees are at the same risk of subsequent failure as the individuals who are alive and uncensored. On the other hand at the end of the day retirees that drop out of the study ought to do as such because of reasons inconsequential to the study. In this study, censoring is thought to be non-informative.

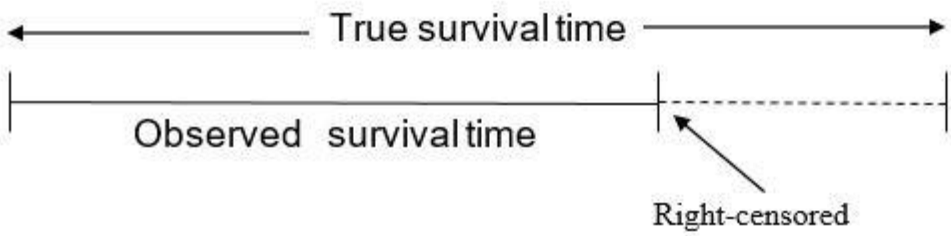

Figure 3.4: True and Observed survival times

3.4 Survival models

A Survival Model is a probabilistic model of a random variable that represents the waiting time until the occurrence of an unpredictable event. Survival Models are classified in two groups, namely; parametric and non-parameter models.

Survival Models can be applied in the modeling of:

- Time to death for retirees
- Time to onset (or relapse) of a disease
- Duration of a strike
- Money paid by health insurance
- The time until surrender of a policy holder's life insurance policy.
- The time until the full payment of a loan.
- The time until the death of a patient
- The waiting time until a woman gets married.
- The time until an insurance claim is paid or settled.

- The time until a claim is made on an automobile insurance policy.

- Viral load measurement

- Time until equipment failure
- Time until earthquake

- Time to finishing a doctoral dissertation!

3.4.1 Life (Actuarial) table analysis

Life tables refer to a statistical procedure which generates duration (time to event) distributions for a whole dataset or separately for each level of a factor. All things considered it is a form of nonparametric survival analysis. The essential output is a life table whereby the rows are researcher-defined time intervals and the columns be connected with counts, probabilities, and cumulative probabilities of the event of interest occurring within the given time interval. The Life Table technique was determined as the most appropriate technique to analyze the order of mortality for this data set. Life Table analysis gives estimates of probabilities of surviving a specific number of years after retirement. This method permits subjects to enter (that is, retire at different ages) or leave the study (that is, die) at different points in time and it employs all the data on incomplete exposure to the risk of dying. It is non-parametric and needs no assumptions about the distribution of the survival function. Here, Kaplan Meier approach will be utilize by grouping the event times into intervals (e.g. one interval for each year), and afterward counting up the number of deaths (events) in each to evaluate the probability of surviving the interval. The life table includes:
x = Age or duration; $0 \leq x \leq \omega$.

$l(x)$=No. of persons living at age x. $d(x)$ = No. of deaths

between age x and $x + 1$. $q(x)$ = The probability of dying between age x and $x + 1$. $p(x)$ = The probability of surviving

between age x and $x + 1$.

$L(x)$= Person years lived between age x and $x + 1$.

$T(x)$ = Total persons-years lived after age x.

$e(x)$ or e_x = Life expectation at age x,$(T(x)/l(x))$.

Construction of life (actuarial) tables

A life table is built (constructed) from a set of grouped or ungrouped failure data. The columns of the table are created or made utilizing a set of formulas. The rows of the table represent various time interims or intervals. We will now define each of the columns in the Actuarial table.

Time Interval

Each of the time interval is represented by $T_t \leq T < T_{t+1}$ *or* $[T_t, T_{t+1})$ *where* $t = 1,...,s.$. The interval is from T_i up to but excluding T_{i+1} The intervals are assumed to be fixed. The intervals do not have to be of equal (equivalent) length, but it is often convenient to make them so. The midpoint of the interval,

T_{mt}, is characterized as half way through the interval.

The width of the interims is b_t *where* $b_t = T_{t+1} - T_t$.

Conditional Proportion Dying

The conditional proportion dying, q_t, is an estimate of the conditional probability of death in the interval given exposure to the risk of death in the interval. It is computed using the formula $q_t = \frac{d_t}{n_t}$.

Conditional Proportion Surviving

The conditional proportion surviving, p_t , is an estimate of the conditional probability of surviving through the interval. It is computed using the formula $P_t = 1 - q_t$.

Cumulative Proportion Surviving

The cumulative proportion surviving,$S(T_i)$, is an estimate of cumulative survival rate at time . It is computed using the formula $S(T_t) = S(Tt - 1)P_{t-1}$ where $S\,(T_1) = 1$.

Hazard Rate

This is the estimated hazard rate function,$h(T_{mt})$. It is an estimate of the number of deaths per unit time isolated (divided) by the average number of survivors computed at the interval midpoint.

Standard Error

This is the large-sample estimate of standard error of the survival function. It is a measure of the precision of the survival estimate. Standard error of survival function: Greenwood formula

$$SE\left(S_t\right) = S_t\left[\sum_{i=1}^{i=t-1} \frac{q_i}{p_i R_i}\right]^{\frac{1}{2}} \tag{3.19}$$

Where: S_t is the survival function, R_i is the risk set, q_i is the probability of event, p_i is the probability of No event (Survival probability).

Lower and Upper Confidence Limits

The upper and lower confidence limits provide a pointwise confidence interval for the survival function. These limit are constructed so that the probability that the true survival probability lies within them is $1-\alpha$. Observe that these cutoff points (limits) are constructed for a single time point. Several of them cannot be utilized together to form a confidence band such that the whole or entire survival function lies within the band.

Confidence interval

One of the most widely used approaches for conducting statistical inference of life expectancy and other totaled mortality measures was presented (introduced) by Chin Long Chiang (1961, 1984). Chiang considered an unbiased estimate of likelihood of death, $\hat{q} = \frac{D}{N}$ where D signifies death number (number of events) within a particular age interval and cohort size (no. of trials) at the start of the same age interval. For estimation of the standard error of the likelihood of death, Chiang considered the plan of Bernoulli trials. Then the random death numbers would be distributed binomially around the mean value evaluated as and with the standard deviation assessed as

$$\sqrt{\hat{q}(1 - \hat{q}) \times N} \quad (3.20)$$

For this situation, standard error of likelihood of death is

$$S_{\hat{q}} = \sqrt{\frac{\hat{q}(1 - \hat{q})}{N}} \quad (3.21)$$

$$CI = \hat{q} \pm Z_{\alpha/2} S_{\hat{q}} \quad (3.22)$$

If the observed death number and populace size are higher than 15 to 20, the normal distribution can be considered as a good estimate (approximation) of the binomial distribution. For this situation, upper and lower certainty (confidence) limits for the confidence level can be expressed by the Wald formula.

Where $\alpha/2$ is the $100(1-\alpha/2)$ percentile of the standardized normal distribution. At last, Chiang deduced formulae connecting standard error of survival likelihood or probabilities and life expectancies with standard error of age-specific probabilities of death (Chiang, (1961 and 1984) as cited in Andreev and Shkolnikov (2010)).

Lamentably, the Wald formula is not relevant (applicable) when the likelihood or probability of death is close to zero or to one, or if the populace size N is too low. Condition $\min(D, N - D) \geq 5$ must be satisfied for Wald's formula to be applicable.

It is likewise unclear how age-specific death numbers that are equal to zero ought to be handled.

3.4.2 Actuarial life table assumptions and notations

In constructing or developing actuarial life tables, the following presumptions are often made: First, the events of interest (deaths) are presumed to happen toward the end of the interval and censored events are presumed to occur uniformly (or evenly) throughout the interval. Thus, an adjustment or alteration is often made to l to reflect the average number of individuals (participants) at risk during the interval, l^0 which is computed as follows: l_t = no. alive at the begin of the interval.

d_t = no. of deaths during the interval

C_t= no. censored during the interval.

$l'_t = l_t - \frac{C_t}{2}$ = adjusted no. at risk of the event in the interval.

$q_t = \frac{d_t}{l'}$ probability of dying during the interval.

$P_t = 1 - q_t$ = probability of surviving in the interval.

S_t or P_t = the proportion surviving surviving (or remaining event free) past interval t, this is occasionally refer to as the cumulative survival probability and its calculated as follows: First, the proportion of participants (retirees) surviving past time zero (0) i.e. the starting time is defined as $S_0 = 1$ (all retirees alive or event free at time zero or study start).

The number (proportion) surviving past each subsequent interval is calculated utilizing principles of conditional probability. Especially, the probability that a retiree survives past interval 1 is $S_t = P_t$

The probability that a retiree survives past interval 2 means that they had survive past interval 1 and through interval 2:

P_t = cumulative probability of surviving the beginning of the interval.

= probability of surviving at the end of the previous interval.

At the beginning of the study(Zero time), P(1)=1.0

$$P(t+1) = p_t \times P_t \tag{3.23}$$

for instance:

$$P_1 = 1.0 \tag{3.24}$$

$$P_2 = p_1 \times P_1 \tag{3.25}$$

$$P_3 = p_2 \times P_2 \tag{3.26}$$

$$P_4 = p_3 \times P_3 \tag{3.27}$$

3.4.3 Theory of survival distribution functions

The variable of interest (response variable) was the time to death after retirement or censoring time, with the study ending on the 31st February, 2015 meaning that all retirees who didn't experience the event of interest, death after retirement, were considered censored, with a censoring variable, Status, which indicated whether a retiree was censored or died, as another key variable. Hence in the dataset there were r happenings of the event of interest (death after retirement). Thus let t_i, be the time of death, for the ith death. Where $i = 1,2,3,...,r$. To achieve the objective of comparing the survival distribution functions for both males and females, there was need to estimate their respective survival functions utilizing the Kaplan-Meier estimator with the assumption that:

1. Probabilities for the event of interest will depend only on time after the initial event. They are presumed to be constant concerning absolute time. That is, cases that enter the study at various times (for instance, retirees who retired at various times) ought to behave similarly.

2. There ought to likewise be no efficient contrasts amongst censored and uncensored cases. For instance, in the event that, many of the censored cases are patients with more serious conditions, then the outcomes might be biased.

$$S(t) = \Pr(T > t) \tag{3.28}$$

The probability of survival beyond time t, i.e. the time to event (T) occurring after time t . Hence let n_i, be the no. of retirees at risk of death at time t_i, representing all retirees who have not died, including those who may be censored after time t_i. d_i be the no. of deaths observed at time t_i.

By description the conditional probability of surviving past time t_i given survival to that time is estimated by $\frac{(n_i - d_i)}{n_i}$ or simply: $1 - \frac{d_i}{n_i}$

Overall the unconditional probability of surviving beyond any time is estimated by the equation below;

$$\widehat{S}(t) = \prod\nolimits_{t_i \le t} \frac{(n_i - d_i)}{n_i} \tag{3.29}$$

Q represent the product of probability of survival throughout the observed time, t_i

The equation above is known as the Kaplan-Meier estimator which can be constructed using statistical software. The generation of the estimator is as indicated in the table 3.1 below:

The Kaplan-Meier estimator can be utilized to explain the event of death. Then

Table 3.1: Generation of Kaplan Meier estimator.

i	t_i	d_i	c_i	n_i	$(n_i - d_i)$	$\frac{(n_i - d_i)}{n_i}$	S(t)
0	t_0	d_0	c_0	n_0	$(n_0 - d_0)$	$\frac{(n_0 - d_0)}{n_0}$	$\frac{(n_0 - d_0)}{n_0}$
1	t_1	d_1	c_1	n_1	$(n_1 - d1)$	$\frac{(n_1 - d_1)}{n_1}$	$\frac{(n_0 - d_0)}{n_0} * \frac{(n_1 - d_1)}{n_1}$
.	.	.	.	.	.	.	.

k	t_k	d_k	c_k	n_k	$(n_k - d_k)$	$\frac{(n_k - d_k)}{n_k}$	$\frac{(n_0-d_0)}{n_0} * \frac{(n_1-d_1)}{n_1} * \cdots * \frac{(n_k-d_k)}{n_k}$

n_i is equal to the number of retirees in the study and d_i is the number of observed death at time i . When dealing with problems that use this estimator, the goal is to comprehend how n_i changes respect to censoring or truncation. If for instance a retiree leaves the study, it will affect the number at risk, but not on the observed death. This is the same for new retirees that enter at time t , they will be part of the risk set at time $t + 1$.

The KM estimator is equipped for utilizing stratification of variables. This idea comprises of splitting the populace into two or more groups based on some criteria. Plotting the KM estimator of both groups can give understanding in whether one group has a higher survival probability than the other.

The KM estimator has certain advantages starting with the simplicity: it can be computed by hand. Moreover the likelihood of incorporating stratification factors in order to evaluate various survival probabilities of various groups. The limitations of the KM estimator are that it is a mainly descriptive estimator and doesn't control for covariates. Besides no sensible interpretation of competing risks can be made.

3.4.4 Inference -Using Log Rank Test (Mantel-Cox test)

An intriguing issue in the clinical practice is the comparison of the survival functions of two (or more) groups of subjects. Classical statistical techniques such as T-test or Chi-square Test are not suitable to compare the proportion of survivors because they are not designed to take censored data into account. Instead, one could compare the proportions of survival on one or more pre-specified time points, yet this approach ignores the aggregate survival experience of the groups during the entire follow-up period. Additionally, the time points are arbitrary chosen. The most widely recognized technique used to compare survival curves is then a statistical hypothesis test known as log-rank test, which takes the entire follow-up period into account and does not need any assumption about the distribution of the survival function, which is unusually normally distributed and is often skewed because it often comprises numerous early events and only a few late ones. It is nonparametric test and appropriate suitable to utilize when the data are right censored. The null hypothesis for log-rank test is that there is no contrast (or difference) between the survivals of two or more populaces that are being compared (that is, the probability of the event of interest happening at any time point is the same for each population). The comparison depends on the difference between the observed number of events in every group and the expected number of events in the event of non-difference between the two groups.

The log-rank test is developed (constructed) by considering separately each death time in two groups that is Group I and Group II. Assume there are distinct death times $t_{(1)}<t_{(2)}<...t_{(r)}$ over the two groups and that at time $t_{1i}d_{1i}$ people in Group I and d_{2i} people in Group II die for $i = 1,2,...,r$. Unless two or more individuals in a group have the same recorded death time, the values of d_{1i} and d_{2i} will either be 0 or unit. Hence, at time there are deaths altogether out of $n_i = n_{1i} + n_{2i}$ people at risk.

To compare whether the survival distribution functions for the male and female retirees are statistically significantly different, there is need to test the hypothesis below. We would have to test the hypothesis below:

$$H_0 : S_{(a)} = S_{(b)} \qquad Versus \qquad H_1 : S_{(a)} =6\ S_{(b)} \tag{3.30}$$

Where $S_{(a)}$ is the survival function for male retirees while the $S_{(b)}$ is the survival function for the female retirees.

Hence let:

n_{mi} be the no. of retirees at risk of dying at time t_i, for the m^{th} group.

d_{mi} be the no. of dead retirees, whose death was observed at time t_i, for the m^{th} group.

Let X be the random variable that takes the values, d_{mi}.

Recall m = 1,2; there are two groups, strata, male and female retirees.

Table 3.2: Contingency table (2×2) to describe log- rank test for 2 strata (Male and Female)

Group	Number dead	Number Alive	Number at Risk
Males	d_{1i}	$n_{1i} - d_{1i}$	n_{1i}
Females	d_{2i}	$n_{2i} - d_{2i}$	n_{2i}
Total	d_i	d_i	n_i

The log-rank test statistic follows a hyper-geometric distribution. The hypergeometric distribution is a discrete likelihood distribution that portrays the probability of successes in a sequence of n draws without substitution from a finite populace of size N that contained m total successes. This is rather than the binomial distribution, which depicts the probability of m successes in n draws with replacement.

Suppose X is a random variable and follows the hypergeometric distribution then its probability mass function will be defined as:

$$P(X = k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}} \tag{3.31}$$

Where

N=the populace size, n=the no. of draws,

m=the no. of success states in the population,

k=the number of observed successes, $\binom{a}{b}$ =a binomial coefficient.

$$Mean = E[X] = \frac{nm}{N} \quad and \quad Variance = Var[X] = \frac{nm(N-n)(N-m)}{N^2(N-1)} \tag{3.32}$$

Therefore, the probability of the random variable X taking the value d_{mi}, would be given by

$$Pr(X) = d_{mi} = \frac{\binom{n_i}{d_{1i}}\binom{n_i}{d_{1i}}}{\binom{n_i}{d_i}} \quad (3.33)$$

While the expectation and variance of the random variable X are given as below:

$$E(X) = d_{1i} = \frac{d_i n_{1i}}{n_i} \quad (3.34)$$

$$Var(X) = d_{1i} = \frac{n_{1i} n_{2i} d_i (n_i - d_i)}{n_i^2 (n_i - 1)} \quad (3.35)$$

Let Y be the total number of dead male retirees

$$Y = \sum_{t_i} d_{1i} \quad (3.36)$$

Then

$$E(Y) = \sum_{t_i} E(d_{d1i}) \qquad and \qquad Var(Y) = \sum_{t_i} Var(d_{1i}) \quad (3.37)$$

Log-rank statistic for two groups =

$$\frac{(O_2 - E_2)^2}{Var(O_2 - E_2)} = \chi_i^2 \quad (3.38)$$

The log-rank test statistics is also given by:

$$Z = \frac{\sum_{i-1}^{r}(O_i - E_i)}{\sqrt{\sum_{i-1}^{K} V_j}} \sim N(0,1) \quad under \quad H_0 \quad (3.39)$$

Where: O_i=Observed, E_i Expected, and V_j = Variance. Let Z be defined as below, after standardization of the variable Y,

$$Z = \frac{Y - E(Y)}{\sqrt{Var(Y)}} \quad (3.40)$$

$E(Y) = 0$, $Var(Y) = 0$ and $Z \sim N(0,1)$

Substituting the values of Y, E(Y) and Var(Y) , we have Z defined as:

$$Z = \frac{\sum_{t_i} d_{1i} - \sum_{t_i} E(d_{1i})}{\sum_{t_i} Var(d_{1i})} \tag{3.41}$$

Then from probability distribution if consequently $Z \sim (0,1)$ then $Z \sim \chi^2_1$ Thus Z^2 is known as the log-rank test statistic and it follows a chi-square distribution with one degree of freedom.

Thus to conduct the hypothesis test for the hypotheses below:

$$H_1 : S_{(a)}\ 6 = S_{(b)}. \tag{3.42}$$

You obtain a critical value from the chi-square distribution tables at α level of significance. If the Test Statistic Z^2 is greater than the Critical Value then reject

$$H_0 : S_{(a)} = S_{(b)} \tag{3.43}$$

3.4.5 Wilcoxon Test (Breslow Test)

This test simply like the log rank test is utilized to test the null hypothesis that there is no difference in the survival function for two groups of survival data. The test statistics is $W_W = \frac{U_W^2}{V_W}$ which has χ^2 distribution with one degree of freedom when the hypothesis is true.

Where :

$$U_W = \sum_{i=1}^{r} n_i[d_{1i} - e_{1i}] \tag{3.44}$$

$$V_W = \sum_{i=1}^{r} n_i^2 V_{1i} \tag{3.45}$$

$$V_{1i} = \frac{n_{1i} n_{2i} d_i [n_i - d_i]}{n^2 [n_i - 1]} \tag{3.46}$$

3.4.6 Inference -using Cox-Proportional Regression Model

Cox model was used for the multivariate survival analysis.

Cox-Proportional Regression Model(Cox-PH Regression Model)

A more regular, alternative technique in mortality analysis is to examine patterns in mortality on the premise of exclusively decided patient survival times. Over here, the outcome or response variable is the time span until the event of interest takes place (e.g., death) or until some point in time when the patient is no longer followed (censored). Ordinarily, the investigation of patient survival times is done utilizing a Cox proportional hazards regression model. This is a semiparametric regression model that portrays survival time in a comparative sense where the complete description of survival time is not of essential significance. Rather, the attension is on how a particular risk factor changes survival experience in respect to not having the risk element.

The cox PH model extends the techniques for non-parametric KM estimates to regression type arguments taking into account the effects of censored observations. The principle of Cox PH model is to connect the survival time of a person to covariates. Case in point, in the medicinal area, when discovering which covariate has the most compelling effect on the survival time of an individual, Cox advances to forecast survival time in individual subjects by only using variables covarying with survival and overlooking the baseline hazard of the individuals. The model uses the assumption that the hazard functions of various people stayed relative and consistent after some time. At the point when there are several continuous explanatory variables, it is a great deal more helpful to utilize a regression technique such as Cox instead of a KM.

Assumptions of the Cox PH regression model:

1. Non-informative censoring. To fullfil this presumption, the design of the underlying study must guarantee that the mechanisms giving rise to

censoring of individual subjects are not connected to the probability of an event occurring.

2. Hazard ratio is independent of time.
3. Baseline hazard function not involved in the HR formula.
4. That the hazard functions are proportional over time. i.e. Hazard ratio for two X's are proportional:
5. No ties in the event times.

Cox's proportional hazards model is a distribution free model whereby predictors are related to lifetime multiplicatively.

To determine the determinants of time to death in the study, cox proportional hazards model regression analysis was computed to test whether the regression coefficients in the model pertaining to each of the predictor variables including the age-at-retirement are statistically significantly different from zero. The hypothesis tested was: $H_0 : \beta_i = 0$ Versus $H_1 : \beta_i 6= 0$.

The Cox PH regression model is the most well-known technique to modeling the predictors to estimate their effects on time to event.

It is a semi-parametric model that portrays survival time in a comparative sense where the complete depiction of survival time is not of essential significance. Rather, the emphasis is on how a specific risk factor modifies survival experience with respect to not having the risk factor. It is semi-parametric model because it takes no assumption of the likelihood (probability) distribution function for the time to event; the baseline hazard takes any form and the predictors enter the model through the linear predictor, n_l. It is based on the assumption that the hazards are proportional.

The cox model depends on the hazard function, $h(t,X)$ at time t . If we let $x_1,x_2,...,x_p$ be the values taken by the following covariates $X_1,X_2,...,X_p$, where p is the total number of predictor variables in a model.

Then the linear predictor, n_i is given as $n_l = \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p$ Consequently the cox proportional hazards regression model is expressed as follows

$$h(t,X) = h_0(t)exp\left(\sum_{i-1}^{p}\beta_i X_i\right) \tag{3.47}$$

h(t) is the baseline hazard function at time t.

β_i is the regression parameter for the cox PH model, while is predictor variable,

$X = (X_1, X_2, ..., X_P)$ are the explanatory/predictor variables.

Where $h(t) \geq 0$ and $\sum_{i-1}^{p} \geq 0$.

Recall $i = 1,2,...,p$ is the number of predictors in the cox proportional hazards model.

3.4.7 Relative risk type measure type of association

The basic interpretation of the measures of association given by the Cox model as hazard ratio or relative risk is extremely desirable in clarifying the risk of event for specific categories of predictors or exposures of interest. For instance, when a two-level indicator with a value of 0 = *no* and *yes* = 1 is observed, the hazard ratio becomes e^{β} where β is the parameter estimate for the regression. If the value of the coefficient $\beta = 1.099$, then $e^{1.099} = 3$. The measure is essentially saying that the subjects labeled with 1 (yes) are three times more prone to have an event than the subjects labeled with 0 (no). In this way we have a measure of association that gives understanding into the strength and direction of the relationship between our exposure and outcome.

One benefit of the cox PH model is that it takes care of censored observations aside from it being semi-parametric and ability to provide the relative risk measure for categorical covariates. The proportionality of the hazards is an important assumption of the cox proportional hazards model, it means that the hazard ratio is constant over time, or more precisely the hazard for an individual observation is proportional to the hazard for any other individual observation. he hazard ratio (HR) is obtained as shown from the ratio of hazard functions.

The Hazard Ratio is given by

$$HR = \frac{\hat{h}(t, X^*)}{\hat{h}(t, X)} \tag{3.48}$$

Interpretation of the hazard ratio

Hazard for one person divided by the hazard for a different person. For sake of

Interpretation, we usually need $HR \geq 1$ i.e. $\hat{h}(t, X^*) \geq \hat{h}(t,X)$

We therefore typically take

X^*:group with larger hazard X: group with smaller hazard

When Simplifying the Hazard Ratio, Baseline hazard cancels out

$$HR = \frac{\hat{h}(t, X^*)}{\hat{h}(t, X)} = \frac{\hat{h}_0(t)\exp\left(\sum_{i=1}^{p}\hat{\beta}_i X^*_i\right)}{\hat{h}_0(t)\exp\left(\sum_{i=1}^{p}\hat{\beta}_i X_i\right)} = \exp\left(\sum_{i=1}^{p}\hat{\beta}_i(X^*_i - X_i)\right) \tag{3.49}$$

Where $\hat{\beta}$ is the maximum likelihood estimate of β. Hazard functions should be strictly parallel. This suggests the proportion of the two hazards is a steady, which does NOT depend on time, t. As it were, the hazards of the two groups remain proportional over time.

$$H(t) = \frac{\lambda_1(t)}{\lambda_0(t)} = \ell \tag{3.50}$$

Hence when the value above for the group 1 and 2 is constant then the proportional hazards assumption is satisfied. We can construct $(1-\alpha)100\%$ confidence intervals for the HR as

$$\exp\left\{(X^* - X)\hat{\beta} \pm Z_{1-\alpha/2}\,\hat{se}\left((X^* - X)\hat{\beta}\right)\right\} \tag{3.51}$$

3.4.8 Steps to obtaining the Partial Likelihood

- Steps to obtaining the Partial Likelihood $t_1 < t_2 <,...,< t_k$

- Obtain the risk sets $R(t_1),R(t_2),...,R(t_k)$

- Then obtain the probability of an individual dying at time given risk set $R(t_i),t_i/R(t_i)$ which is given by $\frac{exp(\beta X_i)}{\sum r \in R(t) exp(\beta X)}$

Then finally obtaining the Partial Likelihood β

We shall assume that there are k different event of interest times such that there are no ties at the t_i be the time of the happening of the i^th ith event of interest.

Where $(t_1),R(t_2),...,R(t_k)$
Then the partial likelihood β can be obtained as shown below

$$L(\beta) = \prod_{i=1}^{k} \frac{h_i(t_i)}{\sum r \in R(t_i) exp h_r(t_r)} \quad (3.52)$$

$$L(\beta) = \prod_{i=1}^{k} \frac{exp(\beta X_i)}{\sum r \in R(t_i) exp(\beta X_r)} \quad (3.53)$$

Solving the equation above enables us to obtain (β) and β then .

- Obtain the partial likelihood $L(\beta)$

- Obtain the , $Log((\beta))$,natural log

- Then obtain the first derivative of the $Log((\beta))$ with respect to β, $\frac{dLog(\beta))}{d\beta}$
  With the below derivative
  $\frac{dLog(\beta))}{d\beta}$ = 0 we can obtain (β) and then β

Hence we can estimate the β_i the i^th regression parameter for the cox PH model.

Wald test statistic and Akaike information criterion (AIC)

To test the significance of the parameters obtained we can either use the Wald, Partial Likelihood ratio and Score Test. We shall use the Wald to test significance of the parameters, utilizing the hypothesis test underneath. The Wald test statistic is utilized to test the significance of a coefficient. If the data can be portrayed utilizing a statistical model with parameter that are estimated of a sample, the Wald test is utilized to test the true value of the parameter in light of the sample estimate. Fundamentally it test if there is a relation between the factor and the dataset. So there is no relationship $\beta = 0$ or there is a relationship $\beta \neq 0$. This test statistic is the ratio between the estimated coefficient and its estimated standard error.

The hypothesis of this test statistic are defined as:

$$H_0 : \beta = 0 \tag{3.54}$$

$$H_1 : \beta \neq 0 \tag{3.55}$$

The estimated standard error is defined as:

$$\hat{SE}(\hat{\beta}) = \sqrt{\hat{Var}(\hat{\beta})} = \sqrt{I(\hat{\beta})^{-1}} \tag{3.56}$$

The Wald test statistic is given by:

$$Z = \frac{\hat{\beta}}{\hat{SE}(\hat{\beta})} = N(0, 1) \tag{3.57}$$

where β is the coefficient value. Thus to conduct the hypothesis test for the hypotheses below

$$H_0 : \beta l = 0 \quad versus \quad H_1 : \beta l \neq 0 \tag{3.58}$$

We obtain a critical value from the normal distribution tables at level of significance. If the **Test Statistic (Z) is greater than the Critical Value** then reject $H_0 : \beta l = 0$ The endpoints of a $100(1-\alpha)$ % confidence interval for the coefficient are

$$\hat{\beta} \pm Z_{1-\alpha/2}\,\hat{SE}\left(\hat{\beta}\right). \tag{3.59}$$

The Akaike information criterion (AIC)

The AIC is a statistical test used to identify the model that best fits the populace from which the data were sampled. This measure creates a trade-off between the goodness of fit and complexity of the model. The complexity of the model is expressed in number of factors. Formula is defined by:

$$AIC = 2k - 2In(L) \tag{3.60}$$

Where:

- k represent the number of factors
- L represent the log likelihood

The goodness of fit is explained by the log likelihood and the number of factors is the measure of complexity. The most minimal AIC quality is the favored model. The utilization of this measure for choice of the model, prevents overfitting of the model.

3.4.9 Interpretation of Coefficients

The explanation of the exponentiated coefficients is the hazard ratio (risk ratio) of a one unit increment for the specific covariate. For instance, consider a one unit increase in the covariate.

The coefficient by itself therefore has the interpretation of the logarithm of the hazard ratio (risk ratio) of a unit increase for the specific covariate. Observe that the Cox Model does not estimate an intercept term. This is due to the fact that the parameter is unidentifiable, that is to say, we are unable to estimate its value. event will occur at time t, given that it has not yet occurred. The model equation of Cox PH model can be defined by $h(t,x) = h_0(t)exp(\beta_i x_i)$ where the baseline hazard, $h_0(t)$ characterizes the hazard function when $x = 0$ at time t. The baseline hazard rate $h_0(t)$ does not have to be specified. In fact, the actual form of baseline function is of little importance if the focus is on relative comparisons (e.g. hazard ratio, rate ratio). The hazard ratio is defined by:

$$\frac{h(t:x+1)}{h(t:x)} = \frac{h_0(t)exp(\beta(x+1))}{h_0(t)exp(\beta(x))} = exp(\beta) \tag{3.61}$$

This portrays how the hazard function changes as a function of covariate vector x. This is a semi-parametric multiplicative model with no intercept (i.e. the price one pays for semi-parametric model). If an intercept were present, it would correspond to the log baseline hazard function. The implication is that one cannot reconstruct group specific rates; only ratios can be estimated.

The hazard ratio can also be represented by the equation:

$$\hat{HR} = \frac{h(t)\exp\left(\hat{\beta}_1 x_1 + \ldots + \hat{\beta}_j (x_{j+1}) + \ldots + \hat{\beta}_p x_p\right)}{h(t)\exp\left(\hat{\beta}_1 x_1 + \ldots + \hat{\beta}_j (x_j) + \ldots + \hat{\beta}_p x_p\right)} = \exp(\hat{\beta}_j) \tag{3.62}$$

The Cox PH model is well suited in modeling a situation with unequal observational time. If there is a clear idea about the distribution of the survival data, parametric models such as the exponential regression model, log logistic regression model, or the Weibull regression model are preferred. Hazard ratio is considered an estimate for the relative risk of death. However, the Cox model makes no assumptions regarding what the shape the underlying hazard or death rate takes. As such it is reasoned that estimates of relative risk under the Cox model are more robust than what might otherwise be obtained utilizing a fully parametric model. This Cox proportional regression model was constructed to calculate the hazard ratios (HR) and confidence interval (CI) of risk factors for survival function. The Cox proportional assumption was tested via the log minus log. The coefficient by itself therefore has the interpretation of the logarithm of the hazard ratio (risk ratio) of a unit increment for the specific covariate. Observe that the Cox Model does not estimate an intercept term. This is on the grounds that the parameter is unidentifiable, that is, we are not able to estimate its value. Imagine that we added an intercept to this model. The model shall then look this:

$$h(t/x) = exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + K + \beta_p x_p) \tag{3.63}$$

$$= h_0(t)(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + K + \beta_p x_p) \tag{3.64}$$

Extended Cox Model

The cox model is suitable for handling explanatory variables, or covariates, that are fixed over time. This suggests that we can only use explanatory variables that are time-independent, such as sex. In cases where the covariates are time dependent, cox model will not be appropriate. Fortunately, several improvements and adjustments have been made to the Cox model in the last several years. There was an improvement made by Gill and Anderson. This adapted model is often called the extended Cox model. The extended Cox model includes the capacity to entertain left-censored data, time-varying covariates, and multiple events. The advantage is that the proportionality assumption does not have to hold in this case, because the covariates are dependent on time. The extended Cox model can be defined as

$$\lambda_1 = \lambda_0(t)exp\left(\sum_{i=1}^{p1}\beta \quad X_i + \sum_{j=1}^{p2}\beta \quad X_j(t)\right. \quad ! \tag{3.65}$$

Notice that covariates (or predictors) are now split in p1 time-independent predictors and p2 time-dependent predictors.

Chapter 4

ANALYSIS AND RESULTS

4.0 Introduction

In this section we utilized a non-parametric model called Kaplan-Meier technique to plot the survival and the Hazard functions. Log Rank and Breslow tests was utilized to test the equality of the survival curves. Then the Cox PH Model, which is a semi parametric model, was considered for the parametric analysis to select among the covariates (predictors), those that have a higher explanatory power on the Hazard rate of death of retirees in Ghana.

4.1 Description of data

The data analysis consists of three covariates; the gender of retirees, age category, and Pension category.

For this study, data was collected on retirees (Pensioners) in Ghana. The data spans 49 years from 1966 to 2015. This data is a secondary data obtained from the SSNIT Official website.

4.2 Classification of variables

The predictor variables selected to have the potential effect on the waiting time till a retiree dies are as follows:

Table 4.1: Characteristics of retirees

COVARIATES	LEVELS	NUMBER OF LEVELS
Gender	male female	2
Age categories (years)	Less than 55 55-59 Above 60	3
Pension category (GH₵)	0.50-100 100.01-500 500.01-1000 1000.01-2000 2000.01-5000 5000+	6

4.3 Exploratory analyses - Results

Data was imported into the analysis software, SPSS Version 17, and analysis done. The frequency of gender showed that 82.9% of the participants were male as shown in table 4.2 below.

4.3.1 Gender of retirees

Frequency based on Gender suggest that majority of retirees in this data set are Males (82.9%) as compare with their Female counterparts (17.1%) as shown below:

Table 4.2: Distribution based on gender

	Frequency	Percent	Valid Percent	Cumulative Percent
Valid Male	120236	82.9	82.9	82.9
Female	24818	17.1	17.1	100.0
Total	145054	100.0	100.0	

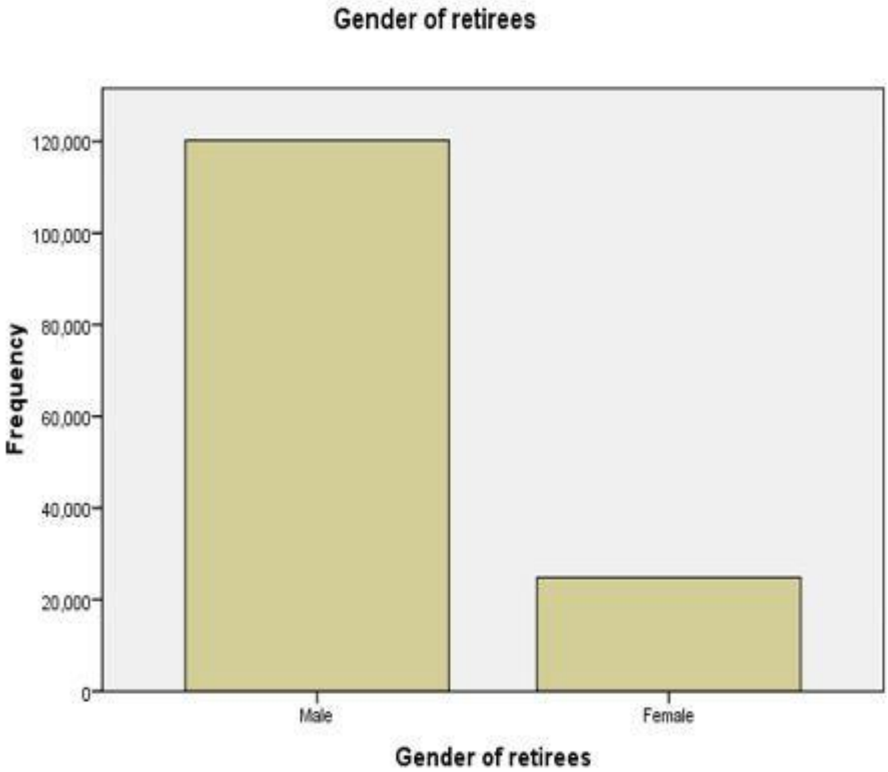

Figure 4.1: Pictorial representation of distribution per gender

4.3.2 Pension categories of retirees

Frequency based on categories according to monthly pension received suggest that majority of retirees earn between GH¢100.01 to GH¢500 while very few retirees earn more than GH¢5000 as shown below:

Table 4.3: Distribution based on pension categories

	Males	Females	Frequency	Percent	Valid %	Cumulative %
Valid 0.50-100	58928	10742	69670	48.0	48.3	96.3
100.01-500	57343	12646	69989	48.3	48.3	96.3
500.01-1000	2799	1238	4037	2.8	2.58	99.1
1000.01-2000	996	173	1169	.8	.8	99.9
2000.01-500	161	18	179	.1	.1	100.0
More than 5000	9	1	10	.0	.0	100.0

Total	120236	24818	145054	100.0	100.0	

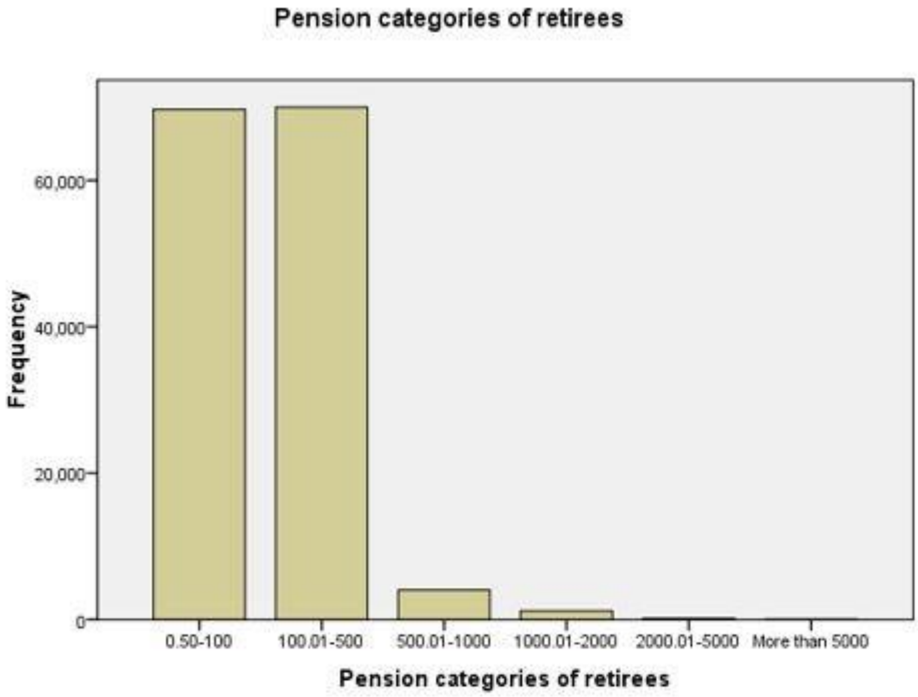

Figure 4.2: Pictorial representation of frequency per pension categories

4.3.3 Age categories of retirees

As expected, most of the retirees in this dataset retired at age 60 and above since the normal retirement age in Ghana is 60 years, with only 0.7% retirees retiring below 55 years.

Table 4.4: Frequency based on age categories

	Frequency	Percentage	Valid Percentage	Cummulative
Valid Less than 55 years	1073	.7	.7	.7
55-59 years	45591	31.4	31.4	32.2
60 and above	98390	67.8	67.8	100.0
Total	145054	100.0	100.0	

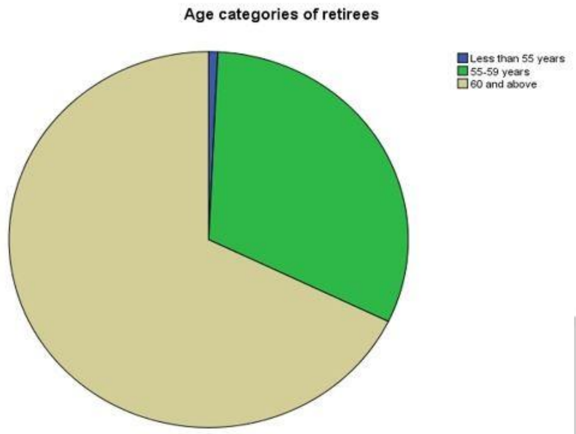

Figure 4.3: Pictorial representation of distribution per age categories

4.4 Life (Actuarial) Table-by gender

Table 4.5: Males' life table, demonstrating life expectancy of retirees

Time Interval [t,t+1)	alive at time t (No. Start Int'l) lt	No. Died qt	No. Lost (Censored) ct	No. Exp'd to Risk. (Risk set) l't	Conditional Probability of dying. tqx	Conditional Probability of Surviving. tpx	Survival function S(t)
[0-1)	120236	169	2926	118773	0.0014	0.9986	1
[1-2)	117141	502	2675	115803.5	0.0043	0.9957	0.9986
[2-3)	113964	889	2485	112721.5	0.0079	0.9921	0.9943
[3-4)	110590	912	3626	108777	0.0084	0.9916	0.9864
[4-5)	106052	608	2771	104666.5	0.0058	0.9942	0.9781
[5-6)	102673	819	2929	101208.5	0.0081	0.9919	0.9724
[6-7)	98925	1263	405	98722.5	0.0128	0.9872	0.9645
[7-8)	97257	2034	258	97128	0.0209	0.9791	0.9522
[8-9)	94965	2789	390	94770	0.0294	0.9706	0.9323
[9-10)	91786	2659	289	91641.5	0.029	0.971	0.9049
[10-11)	88838	4796	199	88738.5	0.054	0.946	0.8787
[11-12)	83843	1734	123	83781.5	0.0207	0.9793	0.8313
[12-13)	81986	1552	75	81948.5	0.0189	0.9811	0.8141
[13-14)	80359	2017	328	80195	0.0252	0.9748	0.7987
[14-15)	78014	1328	217	77905.5	0.017	0.983	0.7786
[15-16)	76469	1810	86	76426	0.0237	0.9763	0.7654
[16-17)	74573	1178	169	74488.5	0.0158	0.9842	0.7473
[17-18)	73226	1316	358	73047	0.018	0.982	0.7355
[18-19)	71552	2610	136	71484	0.0365	0.9635	0.7223
[19-20)	68806	2499	334	68639	0.0364	0.9636	0.6959
[20-21)	65973	2500	319	65813.5	0.038	0.962	0.6706
[21-22)	63154	2292	547	62880.5	0.0365	0.9635	0.6451
[22-23)	60315	1746	207	60211.5	0.029	0.971	0.6216
[23-24)	58362	1981	355	58184.5	0.034	0.966	0.6036
[24-25)	56026	1907	202	55925	0.0341	0.9659	0.5831
[25-26)	53917	3420	173	53830.5	0.0635	0.9365	0.5632
[26-27)	50324	1027	190	50229	0.0204	0.9796	0.5274
[27-28)	49107	2441	276	48969	0.0498	0.9502	0.5166
[28-29)	46390	4721	104	46338	0.1019	0.8981	0.4909
[29-30)	41565	3454	411	41359.5	0.0835	0.9165	0.4409
[30-31)	37700	1378	478	37461	0.0368	0.9632	0.4041
[31-32)	35844	1111	128	35780	0.0311	0.9689	0.3892
[32-33)	34605	2004	235	34487.5	0.0581	0.9419	0.3771
[33-34)	32366	3661	227	32252.5	0.1135	0.8865	0.3552
[34-35)	28478	813	268	28344	0.0287	0.9713	0.3149
[35-36)	27397	4738	176	27309	0.1735	0.8265	0.3059
[36-37)	22483	2276	515	22225.5	0.1024	0.8976	0.2528
[37-38)	19692	3583	126	19629	0.1825	0.8175	0.2269
[38-39)	15983	4198	90	15938	0.2634	0.7366	0.1855
[39-40)	11695	365	489	11450.5	0.0319	0.9681	0.1366
[40-41)	10841	892	280	10701	0.0834	0.9166	0.1322
[41-42)	9669	1822	166	9586	0.1901	0.8099	0.1212
[42-43)	7681	843	2	7680	0.1098	0.8902	0.0982
[43-44)	6836	2109	87	6792.5	0.3105	0.6895	0.0874
[44-45)	4640	1536	178	4551	0.3375	0.6625	0.0603
[45-46)	2926	692	32	2910	0.2378	0.7622	0.0399
[46-47)	2202	1110	20	2192	0.5064	0.4936	0.0304
[47-48)	1072	774	30	1057	0.7323	0.2677	0.015
48+	268	258	10	263	0.981	0.019	0.004
TOTAL	**2688766**	**93136**	**27100**	**2675216**	**5.3956**	**43.6044**	**26.6513**

From the Actuarial table above:

- The cumulative probability of a male retiree surviving at the beginning i.e. time 0 is 1.0
- The cumulative probability of a male retiree surviving to the beginning of the second year is 0.9986.
- The cumulative probability of a male retiree surviving at the end of the first year is 0.9986.
- The cumulative probability of a male retiree surviving to the beginning of year 3 is 0.9943.
- The cumulative probability of a male retiree surviving to the end of year 3 or beginning of year 4 is 0.9864.
- The cumulative probability of a male retiree surviving to the end of year 48 or beginning of year 49 is 0.004.

NOTE: The time interval [0-1) represents the ages [60-61) respectively with age 60 representing the normal retirement age in Ghana. The age 61 represent a one year period whilst the last interval [47-48) also represents ages [107-108).

Table 4.6: Continuation of Table 4.5 : Males' life table, demonstrating life expectancy of retirees

Time Interval [t, t+1)	Retiree-years lived between ages x and x+t (Mid-point survivorship). tLx	Retiree-years lived above age x. Tx	Expectation of life at age x. ex	tqx/ (tpx*risk set)	Standard error of survival function (GREENWOOD) SE(S(t))	[95% Conf. Int.]	
[0-1)	0.9993	26.1513	26.1513	1.18037E-08	0.0589	0.9984	0.9988
[1-2)	0.99645	25.152	25.1872622	3.72922E-08	0.0588	0.9953	0.9961
[2-3)	0.99035	24.15555	24.2940259	7.06423E-08	0.0585	0.9916	0.9926
[3-4)	0.98225	23.1652	23.4845904	7.78764E-08	0.0581	0.9911	0.9921
[4-5)	0.97525	22.18295	22.679634	5.57374E-08	0.0576	0.9938	0.9946
[5-6)	0.96845	21.2077	21.8096462	8.06864E-08	0.0572	0.9914	0.9924
[6-7)	0.95835	20.23925	20.9841887	1.31337E-07	0.0568	0.9866	0.9878
[7-8)	0.94225	19.2809	20.2487923	2.19773E-07	0.056	0.9783	0.9799
[8-9)	0.9186	18.33865	19.6703314	3.19622E-07	0.0549	0.9696	0.9716
[9-10)	0.8918	17.42005	19.2508012	3.25902E-07	0.0533	0.9701	0.9719
[10-11)	0.855	16.52825	18.8098896	6.43266E-07	0.0517	0.9447	0.9473
[11-12)	0.8227	15.67325	18.8539035	2.52294E-07	0.0489	0.9785	0.9801
[12-13)	0.8064	14.85055	18.2416779	2.35076E-07	0.0479	0.9803	0.9819
[13-14)	0.78865	14.04415	17.5837611	3.22357E-07	0.047	0.9739	0.9757
[14-15)	0.772	13.2555	17.0247881	2.21987E-07	0.0458	0.9823	0.9837
[15-16)	0.75635	12.4835	16.3097727	3.17632E-07	0.045	0.9754	0.9772
[16-17)	0.7414	11.72715	15.6926937	2.15518E-07	0.044	0.9835	0.9849
[17-18)	0.7289	10.98575	14.9364378	2.50933E-07	0.0433	0.9812	0.9828
[18-19)	0.7091	10.25685	14.200263	5.29947E-07	0.0425	0.9624	0.9646
[19-20)	0.68325	9.54775	13.7200029	5.50343E-07	0.041	0.9625	0.9647
[20-21)	0.65785	8.8645	13.2187593	6.00197E-07	0.0395	0.9609	0.9631
[21-22)	0.63335	8.20665	12.721516	6.02456E-07	0.038	0.9624	0.9646
[22-23)	0.6126	7.5733	12.1835586	4.9602E-07	0.0366	0.9701	0.9719
[23-24)	0.59335	6.9607	11.5319748	6.04915E-07	0.0355	0.965	0.967
[24-25)	0.57315	6.36735	10.9198251	6.31272E-07	0.0343	0.9649	0.9669
[25-26)	0.5453	5.7942	10.2879972	1.25961E-06	0.0331	0.9351	0.9379
[26-27)	0.522	5.2489	9.95240804	4.14598E-07	0.031	0.9788	0.9804
[27-28)	0.50375	4.7269	9.15001936	1.07027E-06	0.0304	0.949	0.9514
[28-29)	0.4659	4.22315	8.60287228	2.44857E-06	0.0289	0.8964	0.8998
[29-30)	0.4225	3.75725	8.52177364	2.20282E-06	0.0259	0.9149	0.9181
[30-31)	0.39665	3.33475	8.25228904	1.01989E-06	0.0238	0.9621	0.9643
[31-32)	0.38315	2.9381	7.54907503	8.971E-07	0.0229	0.9679	0.9699
[32-33)	0.36615	2.55495	6.77525855	1.78859E-06	0.0222	0.9406	0.9432
[33-34)	0.33505	2.1888	6.16216216	3.96966E-06	0.0209	0.8847	0.8883
[34-35)	0.3104	1.85375	5.88678946	1.04248E-06	0.0185	0.9704	0.9722
[35-36)	0.27935	1.54335	5.04527623	7.68689E-06	0.018	0.8244	0.8286
[36-37)	0.23985	1.264	5	5.13293E-06	0.0149	0.8959	0.8993
[37-38)	0.2062	1.02415	4.51366241	1.1373E-05	0.0134	0.8153	0.8197
[38-39)	0.16105	0.81795	4.40943396	2.24362E-05	0.0109	0.7341	0.7391
[39-40)	0.1344	0.6569	4.80893119	2.8777E-06	0.008	0.9671	0.9691
[40-41)	0.1267	0.5225	3.95234493	8.5028E-06	0.0078	0.915	0.9182
[41-42)	0.1097	0.3958	3.26567657	2.44857E-05	0.0071	0.8077	0.8121
[42-43)	0.0928	0.2861	2.91344196	1.60603E-05	0.0058	0.8884	0.892
[43-44)	0.07385	0.1933	2.21167048	6.62976E-05	0.0051	0.6869	0.6921
[44-45)	0.0501	0.11945	1.98092869	0.000111939	0.0035	0.6598	0.6652
[45-46)	0.03515	0.06935	1.73809524	0.000107214	0.0023	0.7598	0.7646
[46-47)	0.0227	0.0342	1.125	0.000468035	0.0018	0.4908	0.4964
[47-48)	0.0095	0.0115	0.76666667	0.002588008	0.0009	0.2652	0.2702
48+	0.002	0.002	0.5		0.0002	0.0182	0.0198
TOTAL	**26.1513**	**428.18005**	**573.08117**	**0.003463967**	**1.5684**		

The probability that a male retiree currently at age 60 will survive 15 years is denoted and given by:

$$ {}_{15}p_{60} = \frac{l_{60+15}}{l_{60}} = \frac{l_{75}}{l_{60}} = \frac{78,014}{120,236} = 0.6488 \quad (4.1) $$

Similarly, the probability that a male retiree currently at age 60 will die within 15 years is denoted ${}_{15}q_{60}$ and given by:

$$ {}_{15}q_{60} = 1 - {}_{15}p_{60} = 1 - \frac{l_{60+15}}{l_{60}} = \frac{l_{60} - l_{60+15}}{l_{60}} = \frac{l_{60} - l_{75}}{l_{60}} \quad (4.2) $$

$$ = \frac{120,236 - 78014}{120,236} = \frac{5,708}{120,236} = 0.3512 \quad (4.3) $$

The probability that a male retiree currently age 60 will survive for 20 years and then die within 5 years after is denoted ${}_{20|5}q_{60}$ and is given by

$$ {}_{20|5}q_{60} = \frac{l_{80} - l_{85}}{l_{60}} = \frac{68806 - 56026}{120236} = \frac{12780}{120236} = \frac{3195}{30059} = 0.1063 \quad (4.4) $$

Life table results implying that as at 15 years after retirement, 35.12% of the male retirees had died, having only 64.88% surviving.

The life expectancy for male retiree who retired at age 60 years is 26.15 years. This implies that a male retiree who retired at age 60 will survive up to approximately

86 years before dying.

Table 4.7: Females' life table, demonstrating life expectancy of retirees

Time Interval [t,t+1)	No. of retirees left alive at time t (No. Start Int'l) lt	No.Died qt	No. Lost (Censored) ct	No. Exp'd to Risk. (Risk set) l't	Conditional Probability of dying. tqx	Conditional Probability of Surviving. tpx	Survival function S(t)
[0-1)	24818	34	8	24814	0.0014	0.9986	1
[1-2)	24776	103	150	24701	0.0042	0.9958	0.9986
[2-3)	24523	183	111	24467.5	0.0075	0.9925	0.9944
[3-4)	24229	186	103	24177.5	0.0077	0.9923	0.9869
[4-5)	23940	124	59	23910.5	0.0052	0.9948	0.9793
[5-6)	23757	168	31	23741.5	0.0071	0.9929	0.9742
{6-7)	23558	259	14	23551	0.011	0.989	0.9673
[7-8)	23285	416	117	23226.5	0.0179	0.9821	0.9567
[8-9)	22752	571	70	22717	0.0251	0.9749	0.9396
[9-10)	22111	545	168	22027	0.0247	0.9753	0.916
[10-11)	21398	982	39	21378.5	0.0459	0.9541	0.8934
[11-12)	20377	355	32	20361	0.0174	0.9826	0.8524
[12-13)	19990	318	7	19986.5	0.0159	0.9841	0.8376
[13-14)	19665	413	142	19594	0.0211	0.9789	0.8243
[14-15)	19110	272	146	19037	0.0143	0.9857	0.8069
[15-16)	18692	371	169	18607.5	0.0199	0.9801	0.7954
[16-17)	18152	241	108	18098	0.0133	0.9867	0.7796
[17-18)	17803	269	6	17800	0.0151	0.9849	0.7692
[18-19)	17528	535	89	17483.5	0.0306	0.9694	0.7576
[19-20)	16904	512	81	16863.5	0.0304	0.9696	0.7344
[20-21)	16311	511	37	16292.5	0.0314	0.9686	0.7121
[21-22)	15763	469	81	15722.5	0.0298	0.9702	0.6897
[22-23)	15213	357	221	15102.5	0.0236	0.9764	0.6691
[23-24)	14635	406	52	14609	0.0278	0.9722	0.6533
[24-25)	14177	391	124	14115	0.0277	0.9723	0.6351
[25-26)	13662	798	262	13531	0.059	0.941	0.6175
[26-27)	12602	828	29	12587.5	0.0658	0.9342	0.5811
[27-28)	11745	311	32	11729	0.0265	0.9735	0.5429
[28-29)	11402	467	18	11393	0.041	0.959	0.5285
[29-30)	10917	607	61	10886.5	0.0558	0.9442	0.5068
[30-31)	10249	669	52	10223	0.0654	0.9346	0.4785
[31-32)	9528	823	132	9462	0.087	0.913	0.4472
[32-33)	8573	307	44	8551	0.0359	0.9641	0.4083
[33-34)	8222	693	302	8071	0.0859	0.9141	0.3936
[34-35)	7227	243	95	7179.5	0.0338	0.9662	0.3598
[35-36)	6889	771	37	6870.5	0.1122	0.8878	0.3476
[36-37)	6081	506	110	6026	0.084	0.916	0.3086
[37-38)	5465	502	94	5418	0.0927	0.9073	0.2827
[38-39)	4869	662	109	4814.5	0.1375	0.8625	0.2565
[39-40)	4098	323	149	4023.5	0.0803	0.9197	0.2212
[40-41)	3626	75	82	3585	0.0209	0.9791	0.2034
[41-42)	3469	640	19	3459.5	0.185	0.815	0.1991
[42-43)	2810	415	61	2779.5	0.1493	0.8507	0.1623
[43-44)	2334	233	190	2239	0.1041	0.8959	0.1381
[44-45)	1911	932	121	1850.5	0.5036	0.4964	0.1237
[45-46)	858	230	144	786	0.2926	0.7074	0.0614
[46-47)	484	227	39	464.5	0.4887	0.5113	0.0434
[47-48)	218	158	5	215.5	0.7332	0.2668	0.0222
48+	55	53	2	54	0.9815	0.0185	0.0059
TOTAL	**650761**	**20464**	**4354**	**648584**	**4.9977**	**44.0023**	**28.3634**

Table 4.8: Continuation of Table 4.7:demonstrating life expectancy of Female retirees

Time Interval [t,t+1)	Retiree-years lived between ages x and x+t (Mid-point survivorship). tLx	Retiree-years lived above age x. Tx	Expectation of life at age x. ex	tqx/ (tpx*risk set)	Standard error of survival function (GREENWOOD) SE(S(t))	[95% Conf. Int.]	
[0-1)	0.9993	27.8634	27.8634	5.64989E-08	0.1275	0.9981	0.9991
[1-2)	0.9965	26.8641	26.9017625	1.70751E-07	0.1274	0.995	0.9966
[2-3)	0.99065	25.8676	26.0132743	3.08845E-07	0.1268	0.9914	0.9936
[3-4)	0.9831	24.87695	25.2071638	3.20949E-07	0.1259	0.9912	0.9934
[4-5)	0.97675	23.89385	24.3989074	2.18614E-07	0.1249	0.9939	0.9957
[5-6)	0.97075	22.9171	23.5240197	3.01193E-07	0.1242	0.9919	0.9939
{6-7)	0.962	21.94635	22.688256	4.72266E-07	0.1234	0.9877	0.9903
[7-8)	0.94815	20.98435	21.9340964	7.84718E-07	0.122	0.9805	0.9837
[8-9)	0.9278	20.0362	21.3241805	1.13335E-06	0.1198	0.973	0.9768
[9-10)	0.9047	19.1084	20.8606987	1.14975E-06	0.1168	0.9734	0.9772
[10-11)	0.8729	18.2037	20.3757555	2.25031E-06	0.1139	0.9515	0.9567
[11-12)	0.845	17.3308	20.3317691	8.69708E-07	0.1087	0.981	0.9842
[12-13)	0.83095	16.4858	19.6821872	8.0839E-07	0.1068	0.9825	0.9857
[13-14)	0.8156	15.65485	18.9916899	1.10007E-06	0.1051	0.9771	0.9807
[14-15)	0.80115	14.83925	18.3904449	7.62066E-07	0.1029	0.9842	0.9872
[15-16)	0.7875	14.0381	17.6491074	1.09118E-06	0.1014	0.9784	0.9818
[16-17)	0.7744	13.2506	16.996665	7.44794E-07	0.0994	0.9853	0.9881
[17-18)	0.7634	12.4762	16.2197088	8.61321E-07	0.0981	0.9834	0.9864
[18-19)	0.746	11.7128	15.4604013	1.80547E-06	0.0966	0.9673	0.9715
[19-20)	0.72325	10.9668	14.9330065	1.85923E-06	0.0937	0.9675	0.9717
[20-21)	0.7009	10.24355	14.3849881	1.98975E-06	0.0908	0.9664	0.9708
[21-22)	0.6794	9.54265	13.8359432	1.95359E-06	0.088	0.9681	0.9723
[22-23)	0.6612	8.86325	13.2465252	1.60043E-06	0.0853	0.9745	0.9783
[23-24)	0.6442	8.20205	12.5547987	1.95735E-06	0.0833	0.9702	0.9742
[24-25)	0.6263	7.55785	11.9002519	2.01836E-06	0.081	0.9703	0.9743
[25-26)	0.5993	6.93155	11.2251822	4.63375E-06	0.0788	0.9381	0.9439
[26-27)	0.562	6.33225	10.8970057	5.5956E-06	0.0741	0.9311	0.9373
[27-28)	0.5357	5.77025	10.6285688	2.32086E-06	0.0692	0.9715	0.9755
[28-29)	0.51765	5.23455	9.90454115	3.75256E-06	0.0674	0.9565	0.9615
[29-30)	0.49265	4.7169	9.30722178	5.42853E-06	0.0646	0.9413	0.9471
[30-31)	0.46285	4.22425	8.82810867	6.845E-06	0.061	0.9315	0.9377
[31-32)	0.42775	3.7614	8.41100179	1.00708E-05	0.057	0.9095	0.9165
[32-33)	0.40095	3.33365	8.16470732	4.35467E-06	0.0521	0.9618	0.9664
[33-34)	0.3767	2.9327	7.45096545	1.16432E-05	0.0502	0.9106	0.9176
[34-35)	0.3537	2.556	7.10394664	4.87254E-06	0.0459	0.964	0.9684
[35-36)	0.3281	2.2023	6.33573072	1.83946E-05	0.0443	0.8839	0.8917
[36-37)	0.29565	1.8742	6.07323396	1.52179E-05	0.0394	0.9125	0.9195
[37-38)	0.2696	1.57855	5.58383445	1.88577E-05	0.0361	0.9037	0.9109
[38-39)	0.23885	1.30895	5.10311891	3.31125E-05	0.0327	0.8582	0.8668
[39-40)	0.2123	1.0701	4.83770344	2.17003E-05	0.0282	0.9163	0.9231
[40-41)	0.20125	0.8578	4.2173058	5.95429E-06	0.0259	0.9773	0.9809
[41-42)	0.1807	0.65655	3.29758915	6.56146E-05	0.0254	0.8102	0.8198
[42-43)	0.1502	0.47585	2.9319162	6.31418E-05	0.0207	0.8463	0.8551
[43-44)	0.1309	0.32565	2.35807386	5.18964E-05	0.0176	0.8921	0.8997
[44-45)	0.09255	0.19475	1.57437348	0.000548233	0.0158	0.4902	0.5026
[45-46)	0.0524	0.1022	1.66449511	0.000526243	0.0078	0.7017	0.7131
[46-47)	0.0328	0.0498	1.14746544	0.002057694	0.0055	0.5051	0.5175
[47-48)	0.01405	0.017	0.76576577	0.012752325	0.0028	0.2613	0.2723
48+	0.00295	0.00295	0.5		0.0008	0.0168	0.0202
TOTAL	**27.8634**	**480.2367**	**623.98086**	**0.016264491**	**3.617**		

The probability that a female retiree currently at age 60 will survive 15 years is denoted and given by:

$$ {}_{15}p_{60} = \frac{l_{60+15}}{l_{60}} = \frac{l_{75}}{l_{60}} = \frac{19110}{24,818} = 0.7700 \quad (4.5) $$

Similarly, the probability that a female retiree currently at age 60 will die within 15 years is denoted and given by:

$$ {}_{15}q_{60} = 1 - {}_{15}p_{60} = 1 - \frac{l_{60+15}}{l_{60}} = \frac{l_{60} - l_{60+15}}{l_{60}} = \frac{l_{60} - l_{75}}{l_{60}} \quad (4.6) $$

$$ = \frac{24,818 - 19110}{24,818} = \frac{5,708}{24,818} = 0.2300 \quad (4.7) $$

The probability that a female retiree currently age 60 will survive for 20 years and then die within 5 years after is denoted and is given by:

$$ {}_{20|5}q_{60} = \frac{l_{80} - l_{85}}{l_{60}} = \frac{16904 - 14177}{24818} = \frac{2727}{24818} = 0.1099 \quad (4.8) $$

Life table analysis suggests that as at 15 years after retirement, 23% of female retirees had died, having 77% surviving.

The life expectancy for female retirees who retired at age 60 (normal retirement) is 27.86 years. This implies that a female retiree who retired at age will survive up to approximately 83 years before dying.

In summary the estimated life expectancy for males and females who retired at age 60 is 26.15 and 27.86 years respectively. This signifies that female retirees live longer than males.

4.5 Non-Parametric Data Analysis

4.5.1 Kaplan-Meier (Product Limit) Estimator

We will look at the Kaplan-Meier curves for all categorical predictors. This will provide insight into the shape of the survival and the Hazard Functions. Then, we will evaluate whether or not K-M curves for two or more groups are statistically equivalent by using log-rank and Breslow tests.

4.5.2 Case Processing Summary for Gender of retirees

Table 4.9: Case Processing Summary for Gender

Gender of retirees	Total N	N of Events	Censored	
			N	Percent
Males	120236	93136	27100	22.5%
Females	24818	20464	4354	17.5%
Overall	145054	113600	31454	21.7%

The table above shows the Gender of retirees with males having the higher number of retirees (120236) as compare to Females (24818).

Table 4.10: Means and medians Survival Time on Gender bases of retirees

Gender of retirees	Mean[a]				Median			
			95% Confidence Interval				95% Confidence Interval	
	Estimate	Std. Error	Lower Bound	Upper Bound	Estimate	Std. Error	Lower Bound	Upper Bound
Female	34.513	.875	32.798	36.228	39.000	2.054	34.974	43.026
Male	31.709	.670	30.396	33.023	35.000	1.237	32.576	37.424
Overall	32.690	.529	31.653	33.727	37.000	.999	35.042	38.958

a. Estimation is limited to the largest survival time if it is censored.

From Table 4.10, the mean waiting time for a male and a female to die after retirement are 31.709 and 34.513 years respectively, and this gives a numerical comparison of the male and female retirees' average waiting time to die after retiring from work.

Since the CI for the difference between males and females does not contain zero, we can conclude that there is a statistically significant difference in the male and female values at the given level of confidence, it indicates that the average survival times of the two groups are not the same.

Similarly the median waiting time for a male and a female to die after retirement are 33.000 and 39.000 years respectively, and this gives a numerical comparison of the male and female retirees' medial waiting time to die after retiring from work.

Since there is much difference in the Confidence Intervals of the median survival times, it indicates that the medial survival times of the two groups are not the same.

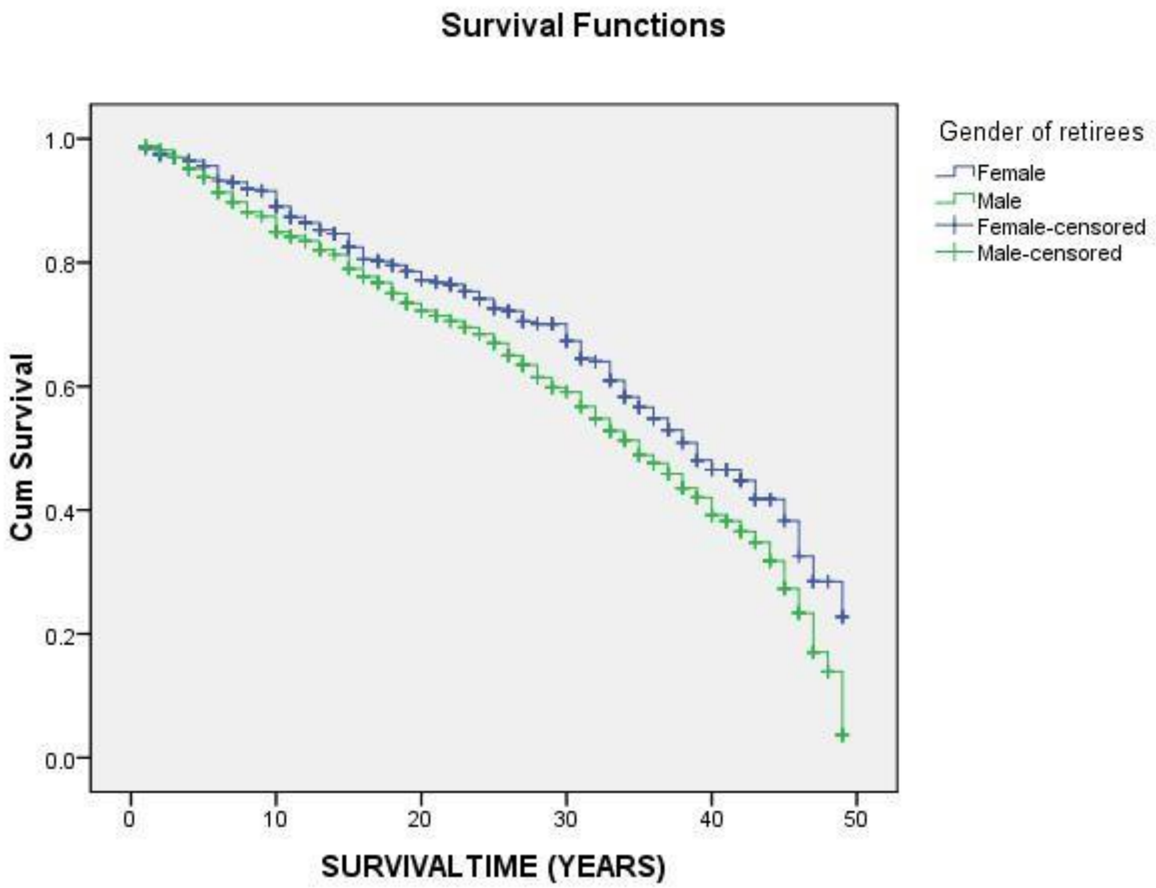

Figure 4.3: K-M Survival curves for Gender.

From Figure 4.3 above, the survival curve for the women lies consistently above that of the men throughout the study period. Therefore, the curve shows a significant difference between the survival times between men and women, with women having higher survival rates as compared to men. This indicates that the female retiree' probability of surviving these days is higher than that of males.

The plot below shows the hazard function which gives the potential of a male or female retiree dying. The green curve (male) which is above the blue curve (female) suggests that the males have a higher hazard rate of dying after retirement than that of the females.

To determine whether these pictorial differences are not due to chance, we take a look at the comparison table below where we have three different and independent

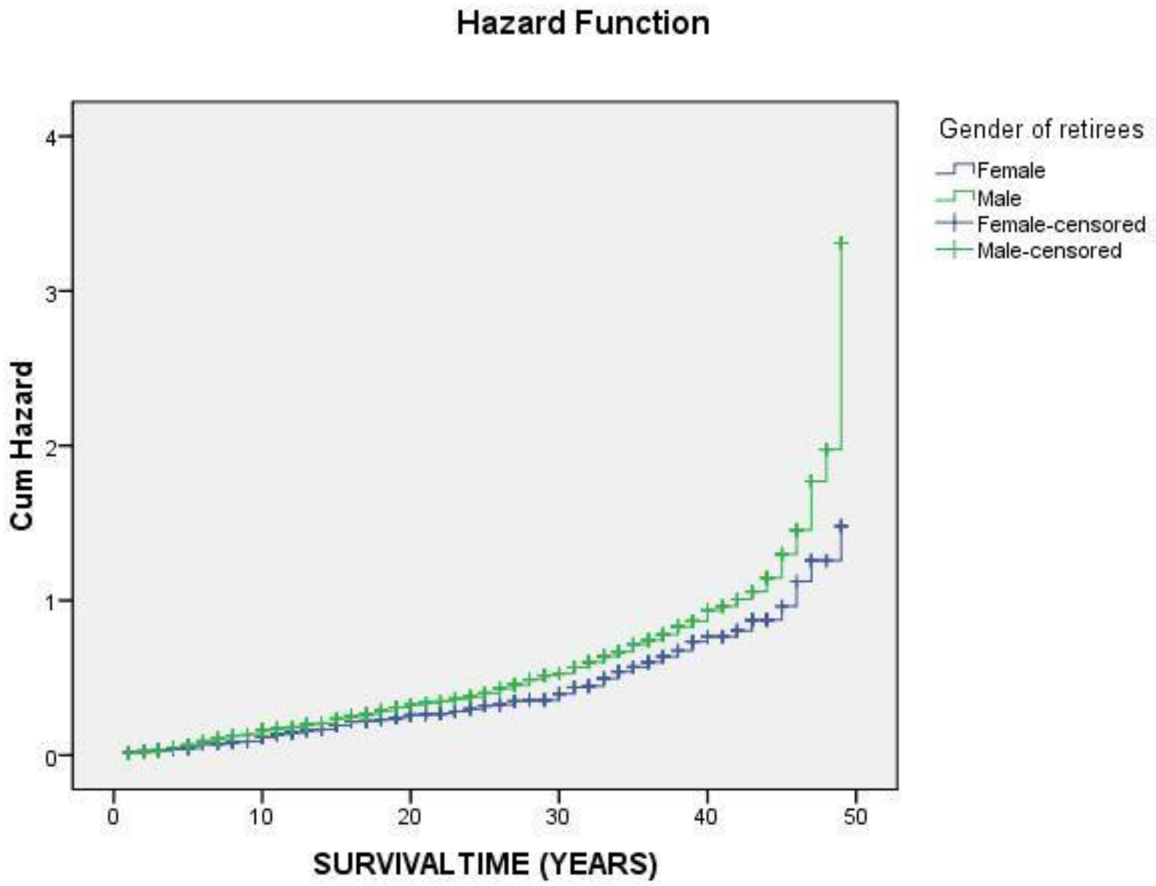

Figure 4.4: K-M Hazard Curves for Gender

tests (methods) of statistical comparison scales.

4.5.3 Overall Comparisons for Gender

Table 4.11: Test of equality of survival distributions for the different levels of Gender of retirees.

	Chi-Square	df	Sig.
Log Rank (Mantel-Cox)	8.356	1	.004
Breslow (Generalized Wilcoxon)	5.232	1	.022
Tarone-Ware	6.296	1	.012

The table 4.11 provides overall tests of the equality of the survival times of the male and female retirees. Since the significant values of the tests are all less than 5%, there is enough evidence supporting the null hypothesis that the two survival times are the same. Therefore, we conclude that there is significant difference between the survival times of male and female retirees in Ghana.

4.5.4 Case Processing Summary for Age categories of retirees

Table 4.12: Case Processing Summary for Age category

Age categories of retirees	Total N	N of Events	Censored	
			N	Percent
Less than 55 years	1073	1023	50	4.7%
55-59 years	45591	43310	2281	5.0%
60 and above	98390	69267	29123	29.6%
Overall	145054	113600	31454	21.7%

The table 4.12 shows the Age categories of retirees with category 60 and above having the highest number of retirees (98390), followed by category 55-59 with 45591 retirees. The category less than 55 having the lowest number of retirees (1073).

4.5.5 Means and medians Survival Time on Age category

bases of retirees

Table 4.13: Means and Medians for Survival Time for age category

Age categories of retirees	Mean[a]				Median			
	Estimate	Std. Error	95% Confidence Interval		Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound			Lower Bound	Upper Bound
Less than 55 years	30.818	.764	29.320	32.316	33.000	1.116	30.812	35.188
55-59 years	33.021	.948	31.163	34.879	39.000	1.359	36.336	41.664
60 and above	36.417	1.125	34.211	38.623	46.000	3.121	39.882	52.118
Overall	32.690	.529	31.653	33.727	37.000	.999	35.042	38.958

a. Estimation is limited to the largest survival time if it is censored.

From Table 4.13, the mean waiting time for retirees in age category less than 55 years, 55-59 years and 60 and above years to die after retirement are 30.818, 33.021 and 36.417 years respectively, and this gives a numerical comparison of Pensioners retiring in age category less than 55, 55-59 and 60 and above years average waiting time to die after retiring from work.

Since the CI for the difference between all the age categories does not contain zero, there is therefore much difference in the Confidence Intervals of the mean survival times, it indicates that the average survival times of the three groups are not the same.

Similarly the median waiting time for retirees in age category less than 55 years, 55-59 years and 60 and above years to die after retirement are 33.000, 39.000 and 46.000 years respectively, and this gives a numerical comparison of Pensioners retiring in age category less than 55 years, 55-59 years and 60 and above years average waiting time to die after retiring from work.

Since there is much difference in the Confidence Intervals of the median survival times, it signifies that the three groups have different medial survival times.

With the Kaplan-Meier Method we have been able to get a graphical representation of probability of survival against time in years. In the survival plot, drops in the survival curves occur whenever there is an indication of death after retirement.However, retirees who retired between the age range: less than 55 years, 55-59 years and 60 and above experienced death after retirement between successive years.

Comparing the three age categories from Fig.4.5, the survival value for all age categories at the start of the study is 1 and 0 at the end of the study.

In summary, the survival plots suggest that there is a high indication of retiree who retired at age 60 years and above (normal retirement age) surviving more than a retiree who retired 55-59 years (voluntary retirement), with those in category less than 55 years having the least survival rate.

Figure 4.6 above shows the hazard function which gives instantaneous potential of a retiree dying after retirement, given that he/she survived up to some time, t years (or has not yet died after retirement).The horizontal axis shows time in

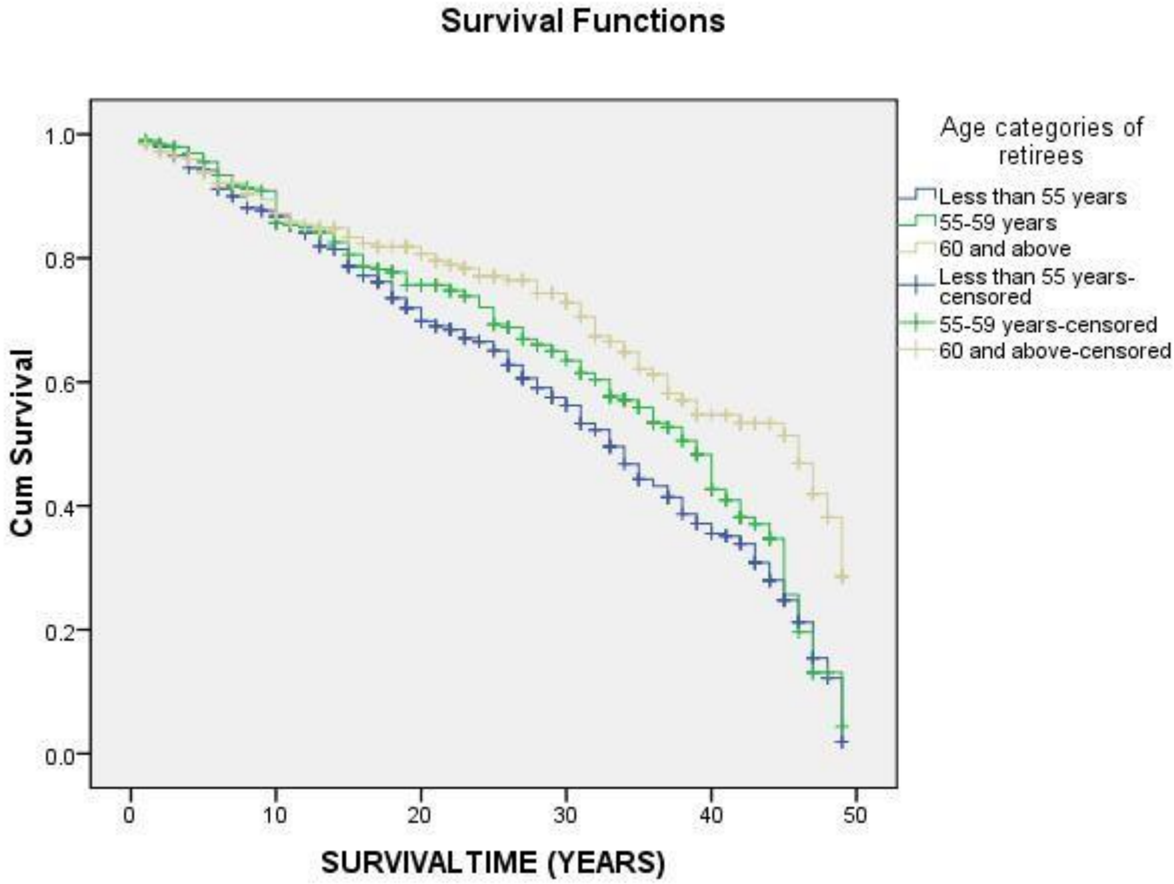

Figure 4.5: Kaplan-Meier survival plot of the three Age categories of retirees.

years for retiree to die after retirement, while the vertical shows the rate of a retiree dying after retirement. A rise in step of the plot shows that there is a drop in population of pensioners indicating death of a retiree.

From Figure 4.6, it is observed that the hazard curve for age categories 60 years and above is lower than that of 55-59 years. This indicates that the age category 55-59 years has the highest hazard rate (potential) of retirees dying after retirement as compared to 60 years and above.

4.5.6 Means for Survival Time on Pension categories bases

From Table 4.14, the mean waiting time for retirees in pension category GH¢5000+ is 35.682, which happens to be the highest with GH¢0.50-GH¢100 having the low-

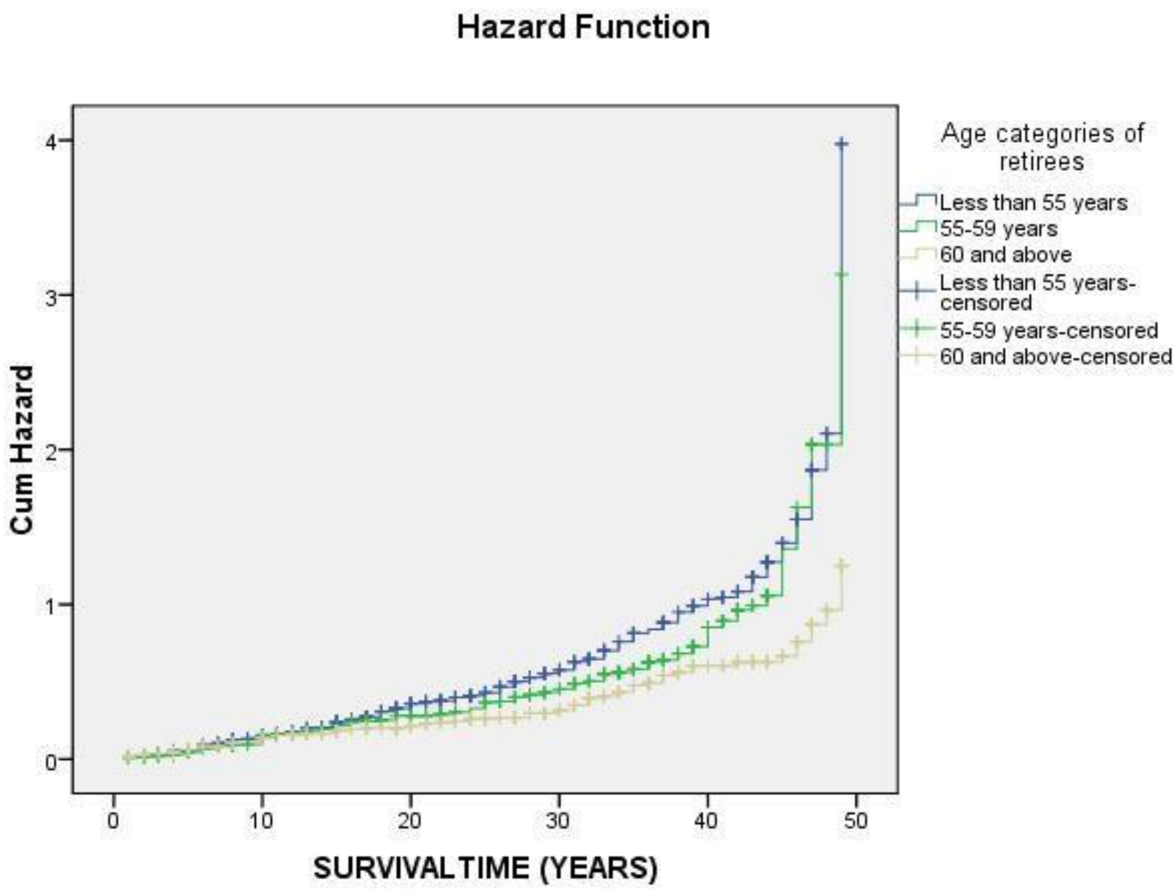

Figure 4.6: Kaplan-Meier Hazard plot of the three Age categories of retirees

Table 4.14: Means and Medians Survival Time for Pension categories of retirees

Pension Categories of retirees	Mean[a]				Median			
			95% Confidence Interval				95% Confidence Interval	
	Estimate	Std. Error	Lower Bound	Upper Bound	Estimate	Std. Error	Lower Bound	Upper Bound
0.50-100	29.182	1.156	26.916	31.448	32.000	1.861	28.353	35.647
100.01-500	33.159	1.148	30.909	35.410	35.000	1.560	31.943	38.057
500.01-1000	32.730	1.340	30.104	35.356	39.000	3.055	33.013	44.987
1000.01-2000	32.533	1.372	29.843	35.222	37.000	2.189	32.710	41.290
2,000.01-5,000	34.872	1.475	31.982	37.763	37.000	4.013	29.134	44.866
5000+	35.682	1.361	33.014	38.351	42.000	1.740	38.589	45.411
Overall	32.690	.529	31.653	33.727	37.000	.999	35.042	38.958

a. Estimation is limited to the largest survival time if it is censored.

est value of 29.182.

Similarly, the median waiting time for retirees in pension category GH¢5000+ is 42.000, which happens to be the highest with GH¢0.50-GH¢100 having the lowest value of 32.000.

The curve below shows a significant difference between the categories of pension

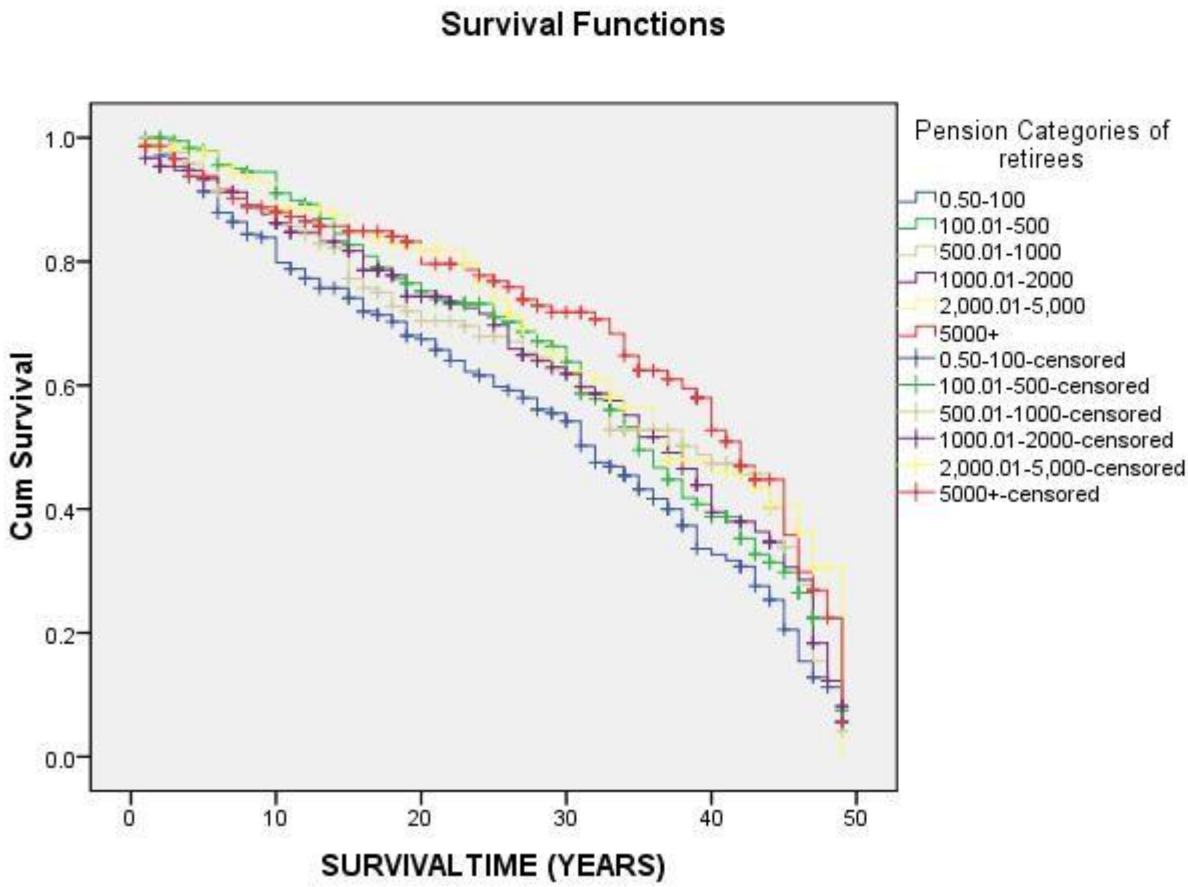

Figure 4.7: Survival curves comparing pension categories

received monthly, with the category GH¢5000+ having the higher survival time. Comparing the three Pension categories, the survival value for all Pension categories at the start of the study is 1 and 0 at the end of the study. In summary, the survival plots suggest that there is a high indication that, a retiree who received GH¢5000+ is surviving more than the rest in the other group with those receiving GH¢0.50-GH¢100 having the lowest survival rate.

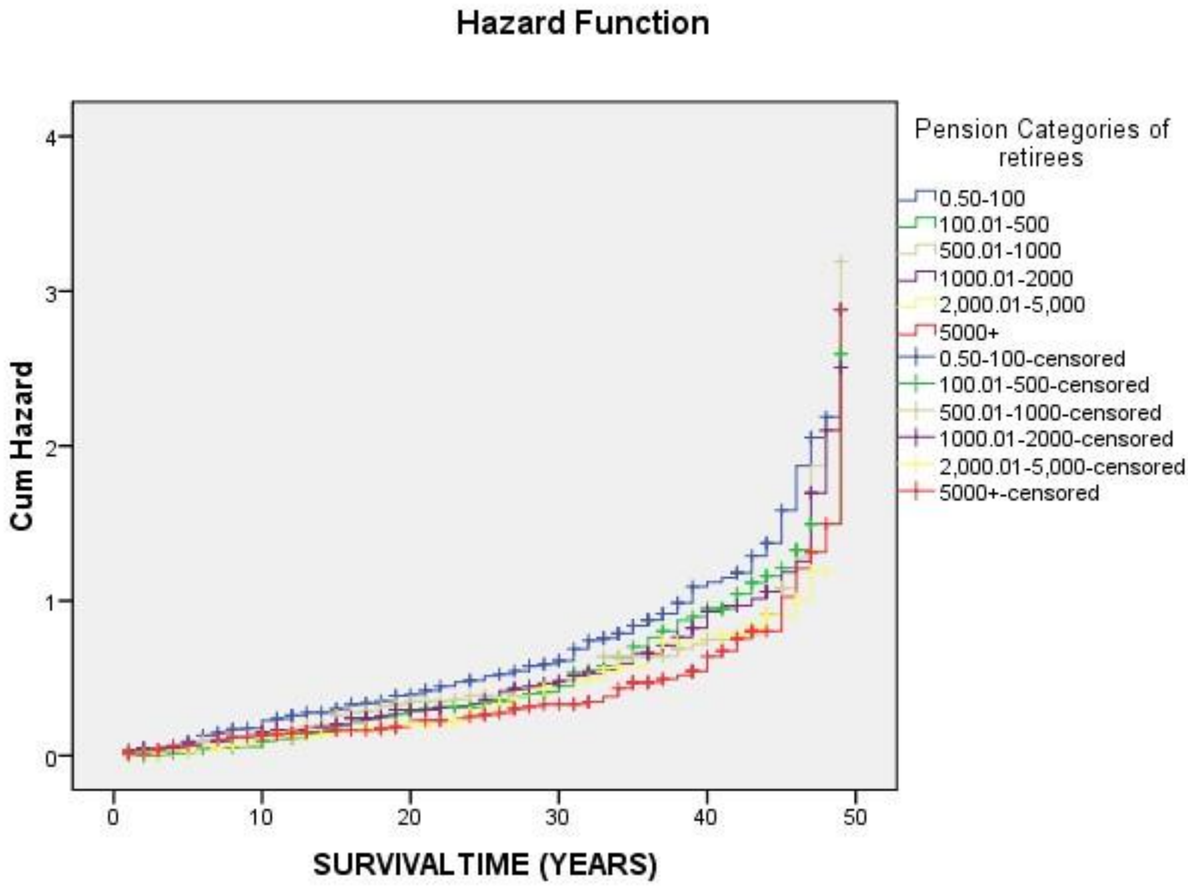

Figure 4.8: Hazard plot of Pension categories of retirees.

The figure above shows the hazard function which gives instantaneous potential of a retiree dying after retirement, given that he/she survived up to some time, t years (or has not yet died after retirement).

The horizontal axis shows time in years for retiree to die after retirement, while the vertical shows the rate of a retiree dying after retirement. A rise in step of the plot shows that there is a drop in population of pensioners indicating death of a retiree.

From Figure 4.8, it is observed that the hazard curve for pension category GH¢0.50GH¢100 is higher than the rest of the pension categories. This indicates that the pensioners receiving GH¢0.50-GH¢100 has the highest hazard rate (potential) of dying after retirement as compared to the rest.

4.5.7 Overall Comparisons Pension categories

Table 4.15: Test of equality of survival distributions for the different levels of Pension categories of retirees

	Chi-Square	df	Sig.
Log Rank (Mantel-Cox)	15.521	5	.008
Breslow (Generalized Wilcoxon)	16.404	5	.006
Tarone-Ware	16.916	5	.005

Note: The null hypothesis is that there is no difference between the survival curves.

The table above gives a significant level of 0.008 for the Log Rank (MantelCox) Test which indicates a significant difference between the Pension categories survival curves. Breslow (Generalized Wilcoxon) and Tarone-Ware also give significant levels of 0.006 and 0.05 respectively, which is less than 5% (0.05) ; and hence supporting the Log Rank Test conclusion.

4.5.8 Categorical Variable Coding of Data

The Categorical Variable Coding is a useful reference in the interpretation of the regression coefficients for categorical covariates.

a. Indicator Parameter (variable) Coding

b. The (0, 1) variable has been recoded or entered, so its coefficients will never be the same as for indicator (0, 1) coding.

c. Category variable: Pension Category (Pension categories of retirees)

d. Category variable: Age Category (Age categories of retirees)

e. Category variable: Sex (Gender of retirees)

Table 4.16: Categorical Variable Codings[c,d]

		Frequency	(1)[b]	(2)	(3)	(4)	(5)
Pension Category[a]	1=0.50-100	69670	1	0	0	0	0
	2=100.01-500	69989	0	1	0	0	0
	3=500.01-1000	4037	0	0	1	0	0
	4=1000.01-2000	1169	0	0	0	1	0
	5=2000.01-5000	179	0	0	0	0	1
	6=More than 5000	10	0	0	0	0	0
Age Category[a]	1=Less than 55 years	1073	1	0			
	2=55-59 years	45591	0	1			
	3=60 and above	98390	0	0			
Sex[a]	0=Female	24818	1				
	1=Male	120236	0				

a. Indicator Parameter (variable) Coding

4.6 Parametric Data Analysis and Modelling

4.6.1 Statistical Identification of the Model.

In this section, we assume a semi-parametric model called Cox Proportional Hazards Model which is sometimes known as the Cox Regression. This model is widely used because the results from utilizing the Cox Model will closely approximate the results for the correct parametric model.

For instance if correct model is Weibull; Cox Model will approximate Weibull. Also, if correct model is exponential; Cox Model will approximate exponential. Thus, when in doubt, the Cox Model is a safe choice of model and one does not need to worry about whether the wrong parametric model is chosen. As described previously, the formula of the Cox Model is given as:

$$h(t, X) = h_0(t) Exp \sum_{i=1}^{p} \beta_i X_i \tag{4.9}$$

with $h_0(t)$ =baseline hazard function

$X = (X_1, X_2, ..., X_p)$ is explanatory variables/covariates.
$i = 1,2,...,p$ is number of covariates. β_i is
the coefficiet of covariates.

The Cox PH Model selects among the covariates, those that have a higher influence on hazard rate of time to death after retirement.

In fitting the different levels of covariates to the model, we begin by estimating the parameters in the model; by using the Partial Maximum Likelihood Estimate method followed by determine the predictor variables in the model and finally, plots of Survival and Hazard curves for the mean of covariates.

Using the model:

$$h(t,X) = exp(\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3) = h_0(t)exp(\beta_1X_1+\beta_2X_2+\beta_3X_3) \quad (4.10)$$

We analyze the covariates, X, to determine their significance in determining the time to death, which is our dependent variable.

X1 is sex

X2 is age-at-retirement

X3 is pension (to measure the socio-economic status)

Results: Numbers of subjects observed were 145054, with 113600 experiencing the event, death after retirement. The p-value for all the covariates is less than 0.05.

Parameter Estimation and Model Selection

These are done together by the computer Software package; SPSS.

The Omnibus test is a measure of how well the model performs. The chi-square change from previous step is the difference between the negative second log likelihood model at the previous step and the current step. i.e. 2log likelihood model.

4.6.2 Case Processing Summary of Data

Table 4.17: Case Processing Summary of Data

		N	Percent
Cases available in analysis	Event[a]	113600	78.3%
	Censored	31454	21.7%
	Total	145054	100.0%
Cases dropped	Cases with missing values	0	.0%
	Cases with negative time	0	.0%
	Censored cases before the earliest event in a stratum	0	.0%
	Total	0	.0%
Total		145054	100.0%

a. Dependent Variable: SURVIVALTIME (YEARS)

The table above gives the total number of retirees (Pensioners) under study. Event cases are the number of retirees which experienced death after retirement. Censored cases are the retirees that did not experienced death after retirement. Censored cases are not utilized in the computation of the regression coefficient but are used in the computation of the baseline hazard function.

4.6.3 Parameter Estimation and model selection

These are done together by computer software package: SPSS. The omnibus test is a measure of how well the model performs. The Chi-square χ^2 change from previous step is the difference between the negative second log likelihood model (-2log likelihood model) at the previous step and the current Step.

4.6.4 Omnibus Tests of Model Coefficients

Table 4.18: Omnibus Test measures how well the model performs

Step	-2 Log Likelihood	Overall (score)			Change From Previous Step			Change From Previous Block		
		Chi-square	df	Sig.	Chi-square	df	Sig.	Chi-square	df	Sig.
1[a]	6116.492	21.902	1	.000	22.699	1	.000	22.699	1	.000
2[a]	6104.351	33.715	2	.000	12.141	1	.000	34.841	2	.000
3[c]	6099.227	38.648	3	.000	5.123	1	.024	39.964	3	.000

a. Variable(s) Entered at Step Number 1: Age at retirement

b. Variable(s) Entered at Step Number 2: Pension category

c. Variable(s) entered at Step Number 3: Sex

d. Beginning Block Number 0, Initial log likelihood function: -2 Log Likelihood: 6139.191

e. Beginning Block Number 1. Method = Forward Stepwise (Conditional LR)

The Chi-square values and a significant values less than α of 0.05 tells us that, our model as a whole fits significantly more than an empty model (i.e model with no covariate/predictor) The -2 log likelihood value can be use to compare nested models.

4.6.5 Variables in the Equation

Table 4.19: Explanatory Variable(s) in the model

		B	SE	Wald	df	Sig.	Exp(B)	95.0% CI for Exp(B)	
								Lower	Upper
Step 1	Age_at_retirement	-.268	.058	21.622	1	.000	.765	.683	.856
Step 2	Age_at_retirement	-.273	.058	22.220	1	.000	.761	.679	.853
	Pension_Category	-.090	.026	11.911	1	.001	.914	.868	.962
Step 3	Sex	.219	.098	4.989	1	.026	1.244	1.027	1.507
	Age_at_retirement	-.262	.058	20.293	1	.000	.769	.687	.862
	Pension_Category	-.087	.026	10.967	1	.001	.917	.871	.965

Table 4.20: Variables not in the Equation

		Score	df	Sig.
Step 1	Sex	5.946	1	.015
	Pension_Category	11.974	1	.001
Step 2	Sex	5.008	1	.025

a. Residual Chi Square = 16.943 with 2 df Sig. = 0.000

b. Residual Chi Square = 5.008 with 1 df Sig. = 0.025

The table variable not in the equation gives the results of a score test, which is also called the Lagrange Multiplier test. The column labelled score gives the estimated change in the model fit if a term is added to the model. The other two columns gives the degree of freedoms and the p-value (labelled sig) .

An SPSS output of Cox regression is displayed. B column displays the value

Table 4.21: Final model

		B	SE	Wald	df	Sig.	Exp(B)	95.0% CI for Exp(B)	
								Lower	Upper
Step 3	Sex	.219	.098	4.989	1	.026	1.244	1.027	1.507
	Age_at_retirement	-.262	.058	20.293	1	.000	.769	.687	.862
	Pension_Category	-.087	.026	10.967	1	.001	.917	.871	.965

of parameters. SE and df displays their standard error and degree of freedom respectively. The Wald column depicts the Importance of variable for this regression model. The higher the Wald value the more the important variable for the model. Exp(B) column gives the exponential estimation (value) of the beta coefficient, which is additionally called Odd's Ratio. The value of OR for gender is 1.244 demonstrating risk of retiree dying for male is about 1.244 times higher than female.

After series of variable selection processes the model selection included Sex in the Cox PH Model with parameter coefficient of 0.219. The Hazard ratio, $Exp(B)$ = 1.244 with its 95% CI as shown in the table above.

Sex, Age category and Pension category are all included in the model because they have a significant value less than 5% (0.05). This indicates that, Sex, age category and pension category have significant influence in retirees' death rate. In other words, it implies that according to this study, they all have enough effect on the rate (potential) of a retiree dying after retirement.

4.7 Interpreting the coefficients

Sex: X1

$$H_0 : \beta_0 = 0 \qquad vs \qquad H_1 : \beta_0 6= 0 \qquad (4.11)$$

At 95% confidence interval, α =0.05, since the p-value is 0.026, less than 0.05, we reject H_0 above, thereby making the test for variable Sex, significant since β 6= 0. Considering that Sex is a categorical variable, with female, coded as 0, considered as the reference group, the hazard ratio value, 1.244, will be interpreted as: Males are 1.244 times more likely to experience death after retirement as compared to females, Meaning, females live/survive 1.244 times longer than males.

Age-at-retirement: X2

$$H_0 : \beta_0 = 0 \qquad vs \qquad H_1 : \beta_0 6= 0 \qquad (4.12)$$

At 95% confidence interval, α=0.05, since the p-value is 0.000, which is less than 0.05, we reject H0 above, thereby making the test for variable Age-at-retirement significant, since β_1 6= 0. However, we therefore consider the result, since Ageat-retirement is a continuous variable; results will be measured in regards to a unit increase of the value. Hazard ratio, 0.769, being less than 1, we subtract the value from 1, to get 0.231. Result showing, that for every unit increase in the age at retirement, the chances of experiencing death are reduced by 23.1%.
Similarly, if we consider age at retirement as a categorical variable with category less than 55 years considered as a reference group, we can equally conclude that, retirees in age category less than 55 years are 23.1% times more likely to experience death as compare to those in other groups.

Pension Category (Amount paid as monthly pension, alluding to socio-economic status): X3

$$H_0 : \beta_0 = 0 \qquad vs \qquad H_1 : \beta_0 6= 0 \tag{4.13}$$

At 95% confidence interval, α=0.05, since the p-value is 0.001, much less than 0.05, we reject H0 above, thereby making the test for variable Pension, significant, since β_1 6= 0.

Considering that Pension Category is a categorical variable, with categories:

0.50 – 100 *coded as* 1

100.01 – 500 *coded as* 2

500.01 – 1000 *coded as* 3

1000.01 – 2,000 *coded as* 4

2000.01 – 5,000 *coded as* 5

More than 5,000 *coded as* 6

With Gh¢0.50- Gh¢100 coded 1, considered as the reference group.

The hazard ratio for this variable is, 0.917, being less than 1, we subtract the value from 1, to get a 8.3%. The result showing that the group receiving GH¢0.50GH¢100 monthly is 8.3% more likely to experience death, as compared to the other groups receiving more money than that.

4.8 Checking the Proportional hazard assumption with Log minus Log

One of the major presumptions of the Cox PH model is proportionality. There are several techniques for confirming that a model fulfills the assumption of proportionality.

Log-Log survival Probability plot is utilized to check proportionality hazard model. To meet the assumption, the two lines must not cross each other. That is, Log minus log plot of survival should give parallel lines if PH holds. If this assumption is violated, we must utilize Cox regression with time dependent covariate. I.e. the Extended Cox model.

This plot displays the log-minus-log of the survival function, versus the survival

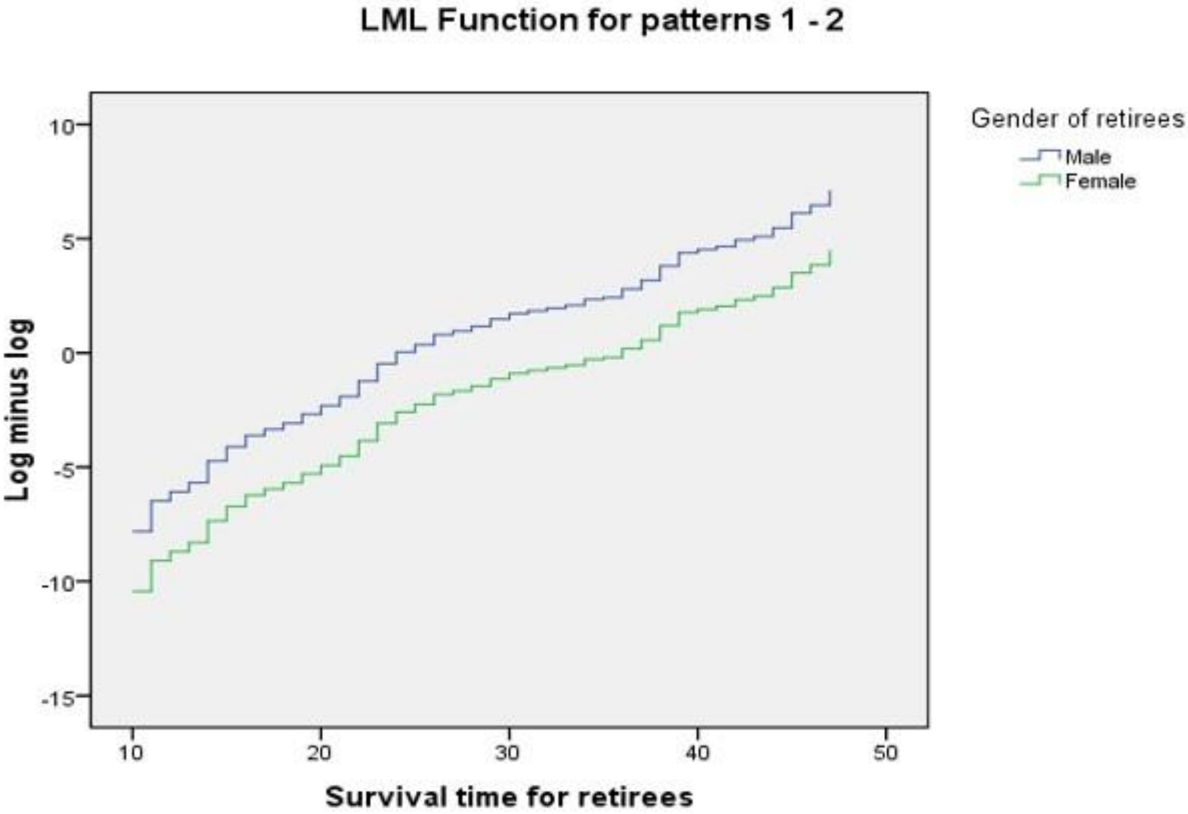

Figure 4.9: LML plot of gender of retirees

time. This particular plot displays a separate curve for each category of gender, and is a useful visualization of the effect of gender on the survival function. As seen in the parameter estimates table, it appears that the survival for Male retirees is distinguishable from the survival for Female retirees. From the plot above, we can conclude that PH assumption holds since the plot for gender is parallel and therefore proportional.

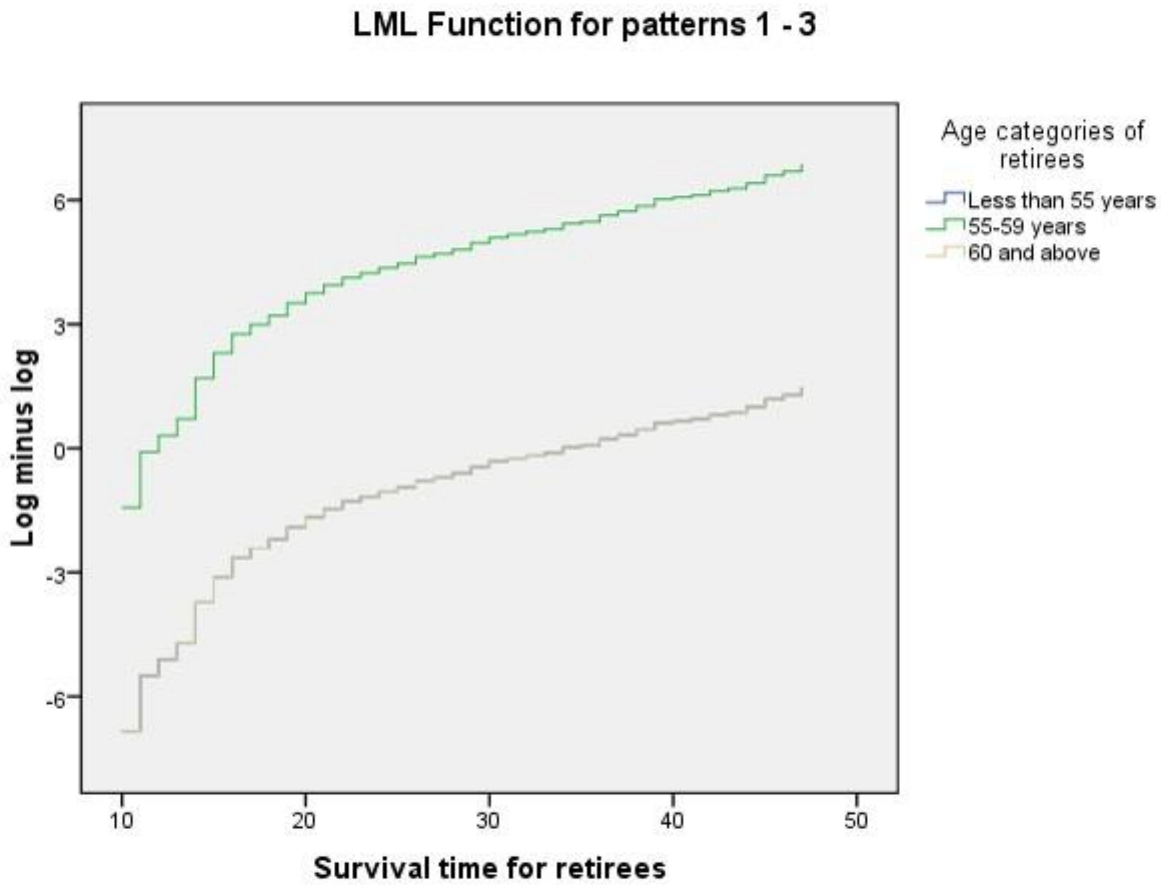

Figure 4.10: LML plot of Age categories of retirees.

Similarly, the plot above is parallel and hence proportional; it implies that PH assumption holds for age categories of retirees.

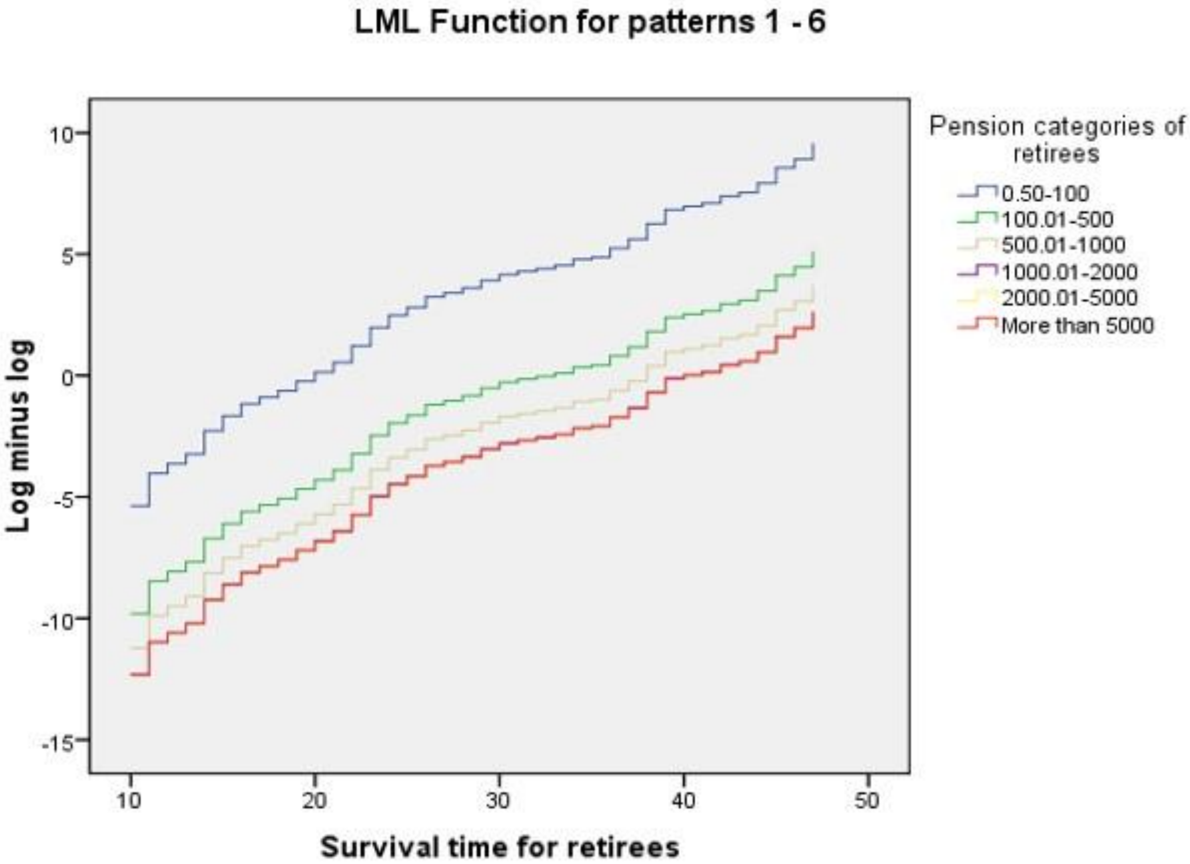

Figure 4.11: LML plot of Pension categories of retirees.

From the above, each pension category does not cross each other and therefore proportional, it signifies that PH assumption holds for Pension categories of retirees.

In summary, since the log minus log plot for all the covariates does not cross each other, they are parallel and therefore proportional. We can therefore conclude that PH assumption holds for all the covariates.

Chapter 5

CONCLUSIONS

5.0 Introduction

This chapter gives a general summary and conclusions of the study, and also recommendations for policy analysis.

5.1 Summary of Results

The following deductions are made from the research: From the Kaplan-Meier plots and life table analysis, there is indication of death after retirement in different years after retirement.

The study has determined life expectancy of male and female retirees. The results show that the life expectancy of the male retirees is 26.15 years while that of their female counterparts is 27.86 years. On running the regression models, utilizing the Cox PH model, we observed that male retirees were 1.244 times more likely to die after retirement as compared to their female counterparts. However, there is clear difference between survival times for males and females, with females having longer survival times, specifically, 1.244 times longer as compared to males.

The age-at-retirement test was significant, does determine the likelihood for survival, up to 23.1%, with reduction in the risk of death, or otherwise stated, increase in the probability of survival, up to 23.1% with every unit increase. This means that the longer one delays retirement, the more their chances of living longer, up to 23.1%, for every year. Brockmann (2009) attributed this increase in probability of survival to the health status of the retiree. He mentions that most probably people, who retire early, retire because they have to, due to their ill health. This therefore leaves a question, should more studies be done, with the health status of the retiree included in the model, to assess whether the health status is indeed a confounder?

There was a significant difference in the different probability of surviving among the retirees, based on the amount of monthly pension received, with the group receiving the least, GH¢0.50-GH¢100, being 8.3% more likely to experience death as compared to the groups earning more pensions, Haynes (1978) mentioned that the retirees of lower status were more likely to die compared to those of higher status, those earning more money. Therefore, we conclude that the amount of money paid monthly, as pension, is a significant factor on the time to death after retirement, probably because it has a direct implication on the living standards of individual.

In Ghana, it has been observed that after retirement, most people move to the rural areas, where life is very different, much slower, yet they were used to the urban areas, where life was extremely fast. Most retirees have been reported to become extremely inactive, thus reducing their days on earth. Their motivation has also been seen to reduce, since there is not much activity around them, this has also been seen to be a determinant to the time to death after retirement.

5.2 Conclusions

From the above observations and discussion of results we can make the following conclusions:

- Gender determine the chance of survival after retirement in Ghana. Socioeconomic status and age-at-retirement were additionally observed to be significant determinants of time to death after retirement.

- There is a significant distinction in time to death between male and female and different categories of pension received.

- The life expectancy of the male retirees is 26.15 years while that of their female counterparts is 27.86 years.

- Early retirement does indeed increase the chances of early death, though the study also suggests that more information on the health status of the early retirees should be used as a covariate during such a study, to eliminate the possibility that the early retirees were already sick, thus their health status rather than their age cause the early death.

- Death after retirement is approximately minimal for the first five years of retiring from work in Ghana. Hence, it implies that most of the retiree's survival rate at the time of retirement is higher at the beginning and reduces as the years passes.

5.3 Recommendations

In view of the crucial role pensioners plays in the advancement of society, every effort must be put to ensure that every retiree should be given the necessary education before going on retirement. The following recommendations are hereby made:

- Though, it is observed that there is significant difference between the death rate of male and female retirees, interventions by Actuaries, Policy Makers, the Government, NGOs, etc. to reduce time to death should not be focused on only male retirees but also on female retirees as well. This problem is recommended for further studies to identify the main cause(s).

- Employees should be advised to plan for their retirement since those retirees who earned less were 8.3% more likely to die than their counterparts who earned more. The standards of living after retirement can be boosted by a better savings plan that ensures that the life after retirement is catered for therefore employers should educate their employees on planning for this.

- Since females have been seen to live longer than males, it would be interesting to see the effect of marital status on time to death for male retirees.

- Health status should be associated with the age-at-retirement to adjust for the covariate of ill-health as a determinant to early death, after early retirement.

- Profession of the retirees, their residence before and after retirement should also be considered in other studies, to determine if this has a direct effect on mortality.

- Pension schemes and life annuity programs in insurance companies should be sensitized on the importance of comprehensive data, to be collected before enrolling individuals since lack of these variables was clearly a limitation to this study.

REFERENCES

1. Afosaa, V. (2009). Survival analysis of dropout rate at the basic school level:(Case study: Northern Ghana) (Doctoral dissertation, Kwame Nkrumah University of Science and Technology).

2. Asamoah, N. (2012). Adjustment towards Retirement (Doctoral dissertation, Institute of Distance Learning, Kwame Nkrumah University of Science and Technology).

3. Ashidam, B. N. A. (2011). The Problem of the Cap 30 Pension Scheme in Ghana (Doctoral dissertation, Institute of Distance Learning, Kwame Nkrumah University of Science and Technology).

4. Ata, N., & So¨zer, M. T. (2007). Cox regression models with nonproportional hazards applied to lung cancer survival data. Hacettepe Journal of Mathematics and Statistics, 36(2).

5. Baranoff, E., Brockett, P. L., & Kahane, Y. (2009). Enterprise and Individual Risk Management. Flat World Knowledge Inc., Washington, DC, USA.

6. Bonneux, L., Barendregt, J. J., Nusselder, W. J., & Van der Maas, P. J. (1998). Preventing fatal diseases increases healthcare costs: cause elimination life table approach. Bmj, 316(7124), 26-29.

7. Brockmann, H. Muller, R., & Helmert, H. (2009). Time to retire-Time to die? A prospective cohort study of the effects of early retirement on long-term survival. Social Science & Medicine 69 (2), 160-164.

8. Brown, R. L., & McDaid, J. (2003). Factors affecting retirement mortality. North American Actuarial Journal, 7(2), 24-43.

9. Cheng, E. Y., Ozerdemoglu, R. A., Transfeldt, E. E., & Thompson Jr, R. C.¨ (1999). Lumbosacral chordoma: prognostic factors and treatment. Spine, 24(16), 1639.

10. Cox, P. R. (1972). Life Tables. John Wiley & Sons, Inc..

11. Ebeling, A. (2012). Americans Clueless About Life Expectancy, Bungling Retirement Planning.

12. Ghana Statistical Service (2010). The elderly in Ghana. POPULATION & HOUSING CENSUS REPORT.

13. Harris, S. (2009). Additional Regression techniques.

14. Haynes, S. G., McMichael, A. J., & Tyroler, H. A. (1978). Survival after early and normal retirement. Journal of gerontology, 33(2), 269-278.

15. Ibrahim, J. G., Chen, M. H., & Sinha, D. (2005). Bayesian survival analysis. John Wiley & Sons, Ltd.

16. Kay, R. (1977). Proportional hazard regression models and the analysis of censored survival data. Applied Statistics, 227-237.

17. Kpessa, M. W. (2011). The politics of retirement income security policy in Ghana: Historical trajectories and transformative capabilities. African Journal of political science and international relations, 5(2), 92.

18. Kuhn, A., Wuellrich, J. P., & Zweimu¨ller, J. (2012). Fatal attraction? Access to early retirement and mortality. IZA DP No. 5160.

19. Kumado, K., & Gockel, A. F. (2003). Social security in Ghana.

20. Lawoe, B. A. K. (2014). An assessment of retirement planning and teachers well-being in the Tamale metropolitan public second cycle institutions.

21. Leonetti, M.E. (2009). Retirement Planning Myths.

22. Man, R. (2014). Survival analysis in credit scoring: A framework for PD estimation.

23. Mein, G., Martikainen, P., Stansfeld, S. A., Brunner, E. J., Fuhrer, R., & Marmot, M. G. (2000). Predictors of early retirement in British civil servants. Age and ageing, 29(6), 529-536.

24. McCoy, J. L., Iams, H. M., & Armstrong, T. (1994). Hazard of Mortality among Aging Retired-and Disabled-Worker Men: Comparative Socio demographic and Health Status Analysis, The. Soc. Sec. Bull., 57, 76.

25. Mwakala, M. N. (2013). Modeling time to death for retirees in Kenya (Doctoral dissertation, UNIVERSITY of Nairobi).

26. Oduro, F.T.; Lecture notes on Survival Models, 2013.

27. Petters, J. S., & Asuquo, P. N. (2008). Work-role attachment and retirement intentions of public school teachers in Calabar, Nigeria. Studies on Home and Community Science, 2(1), 11-17.

28. Social Security and National Insurance Trust (SSNIT) annual and monthly reports (1966-2015).

29. Stibich, M. (2009). Is retirement good for you?

30. Tsai, S. P., Wendt, J. K., Donnelly, R. P., de Jong, G., & Ahmed, F. S. (2005). Age at retirement and long term survival of an industrial population: prospective cohort study. BMJ, 331(7523), 995.

31. Wangari, M. F. (2013). ANALYSIS OF LONGEVITY RISK AND ITS EFFECT ON RETIREMENT BENEFITS (Doctoral dissertation, UNIVERSITY OF NAIROBI).

32. Wilson, K. N., & Aggrey, E. A. M. (2012). Retirement Planning and Counseling: Issues and Challenges for Teachers in Public Schools in the Sekondi Circuit. Online Submission.

33. Xin, X. (2011). A STUDY OF TIES AND TIME-VARYING COVARIATES IN COX PROPORTIONAL HAZARDS MODEL (Doctoral dissertation, The University of Guelph).

APPENDIX

Table 5.1: Covariate Means

	Mean
Sex	.603
Age Category	1.801
Pension Category	3.255

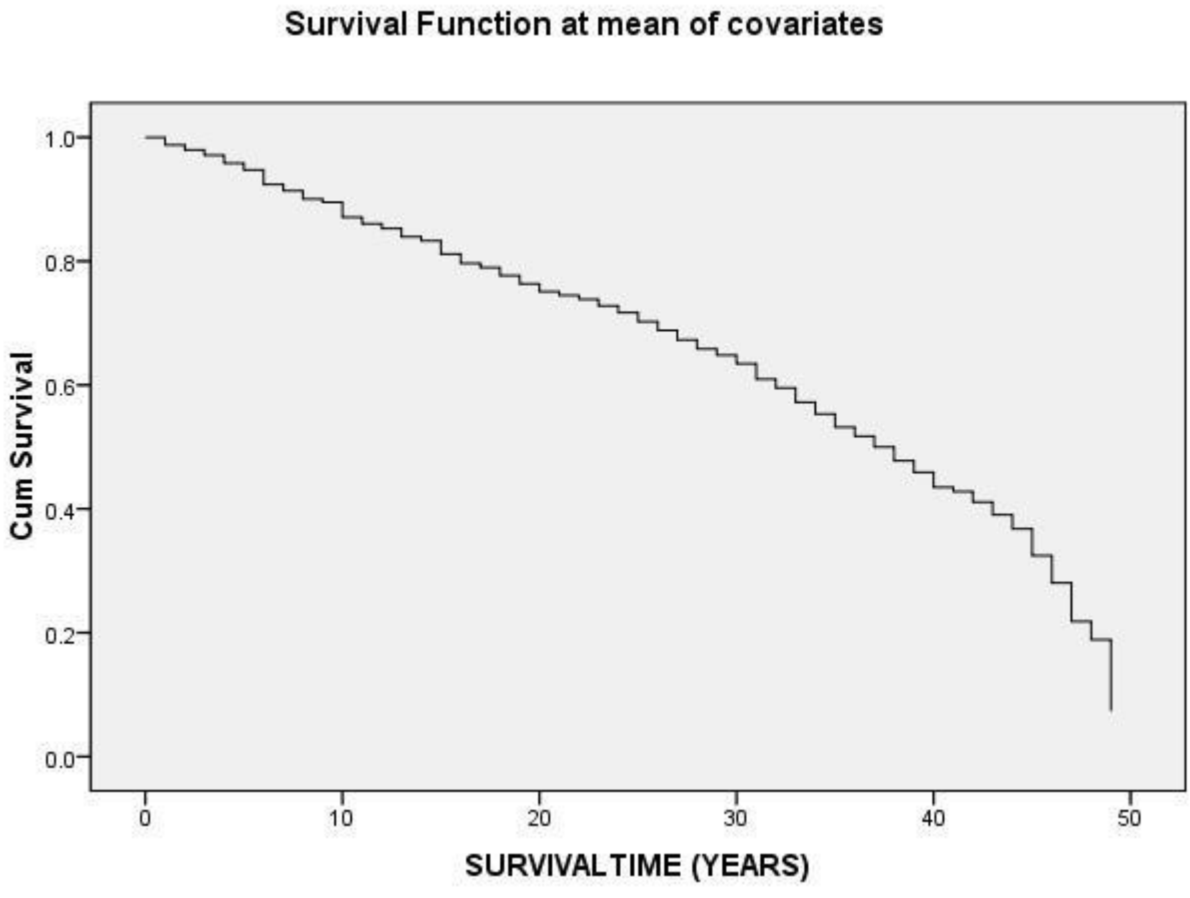

Figure 5.1: Survival Function at mean of covariates

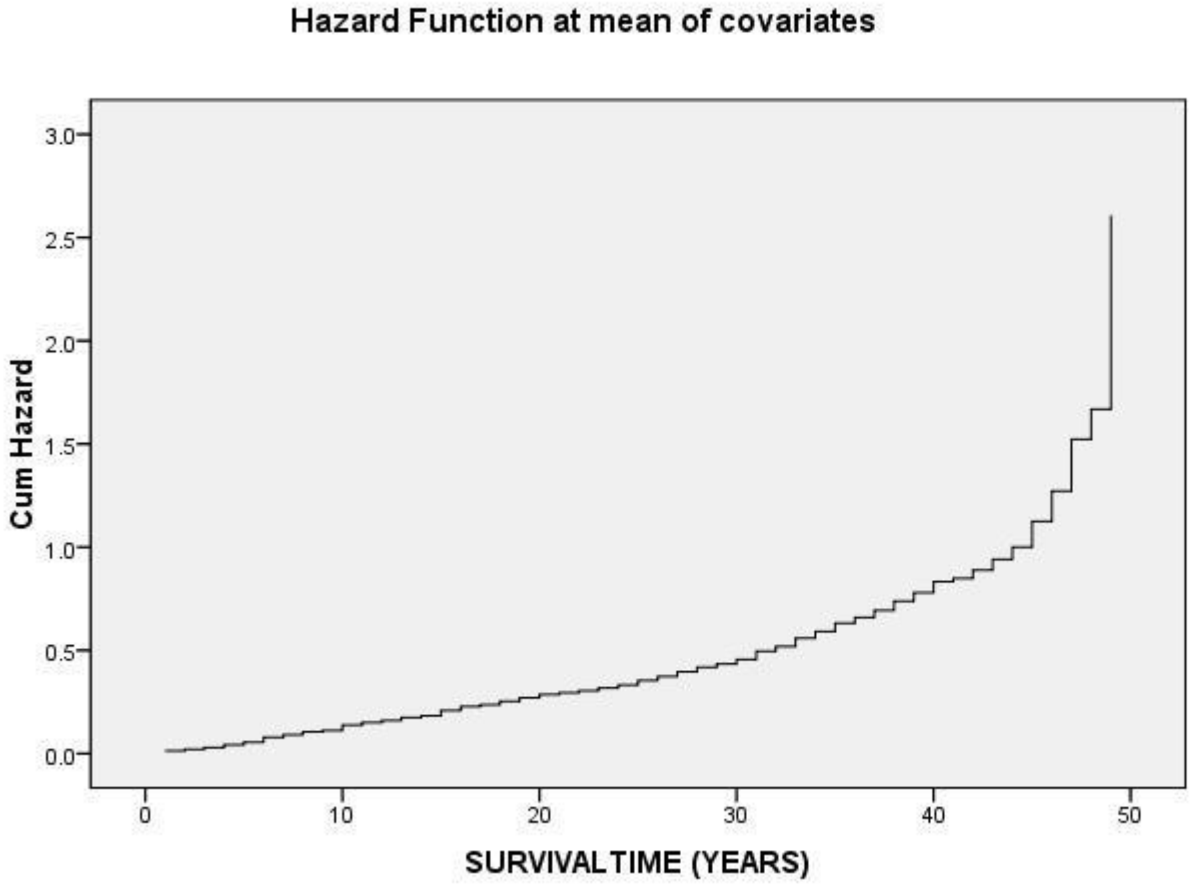

Figure 5.2: Hazard function at mean of covariates.

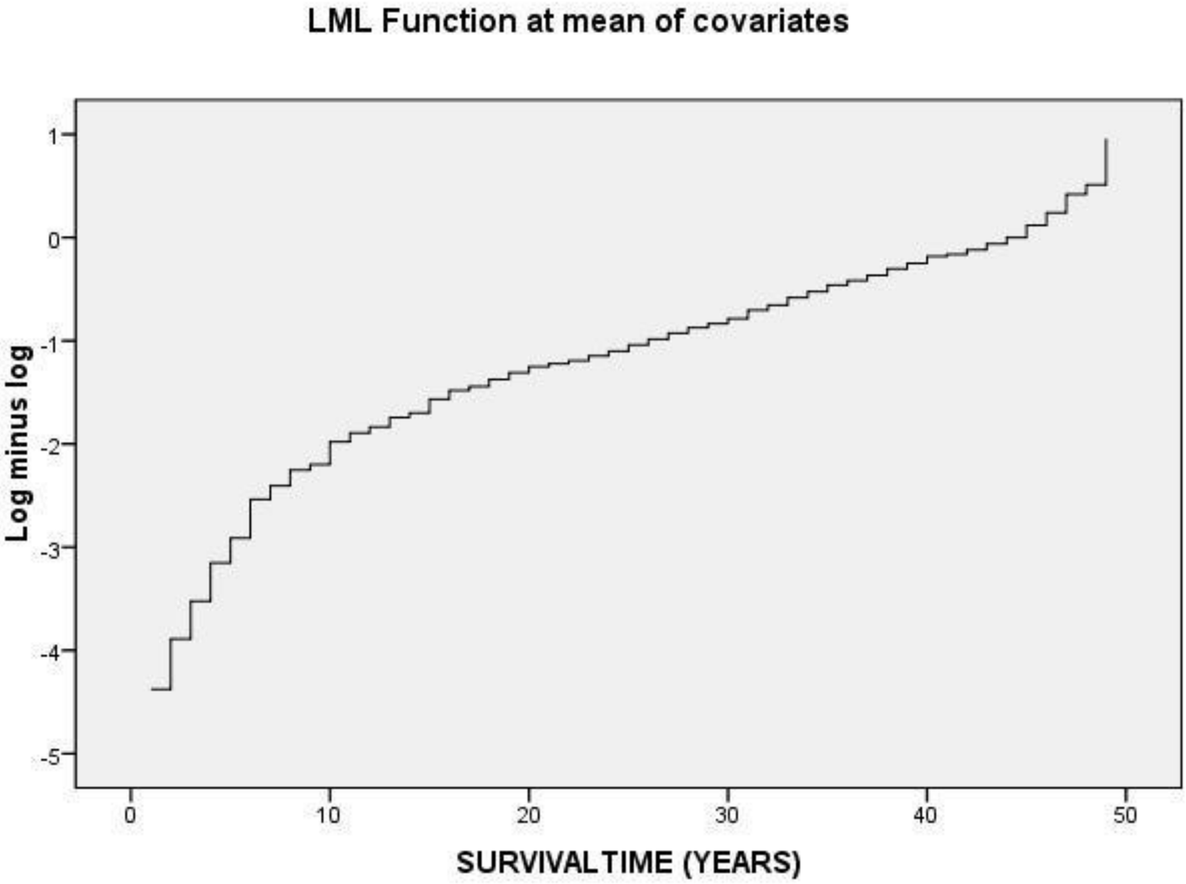

Figure 5.3: LML plot at mean of covariates.