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DETERMINATION OF TRANSFORMATION PARAMETERS FOR MONTSERRADO COUNTY, REPUBLIC OF LIBERIA

BY

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DECLARATION

I hereby declare that this submission is my own work towards the Masters of Science (MSc) and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

Spatial reference and geographic data integration has always been a crucial issue in positioning. To accomplish this, different referencing systems have been adopted by various countries for mapping and positioning purposes. This is due to the fact that no single ellipsoid could perfectly fit and model the undulating nature of the earth's surface, establishing reasons why countries adopt local ellipsoids and datum. On the other hand, GPS equipment which is the most popular and dominantly used equipment for positioning is based on the WGS 84 ellipsoid/spheroid. Henceforth, data integration between local systems and the global system remains an issue of concern. As a case study, the Liberian Geodetic System which is based on the Clarke 1880 spheroid was used in the scope of this project. Various transformation models, reference figures and map projection procedures were studied and reviewed giving an insight of the nature of work to be done. Data were collected for common points in Liberia and post-processed both in Liberia and Ghana using the Leica GeoOffice and the Spectrum Survey. For the corresponding coordinates of the common points in the Clarke 1880 system, the 1:50,000 map which was provided by the Ministry of Lands, Mines and Energy was used spatially to extract those coordinates with the aid of ArcMap 10.0. Matlab functions were written to compute the parameters using four transformation models (Abridged Molodensky - 5 parameters, Simple Three Parameters - 3 parameters, Bursa Wolf – 7 parameters and Molodensky-Badekas – 7 parameters). Statistics were also performed on the determined parameters. As part of the objectives, another application was written using the Visual Basic programming language which makes use of these parameters to transform coordinates between the two systems. From statistics on the determined parameters and considering its advantages, the Molodensky-Badekas (Seven Parameters) Transformation Model is the most suitable model to be used to transform coordinates between both systems in the study area.

LIST OF ACRONYMS

DEM	-	Digital Elevation Model (DEM)
DLSC	-	Department of Lands, Surveys and Cartography
DMA	-	Defense Mapping Agency
EPSG	-	European Petroleum Survey Group
ESRI	-	Environmental Systems Research Institute
GIS	-	Geographic Information System
GNSS	-	Global Navigation Satellite System
GPS	-	Global Positioning System
GRS 80	-	Gravity Reference System (1980)
ICSM	-	Inter-governmental Committee on Surveying & Mapping
KNUST	-	Kwame Nkrumah University of Science and Technology
LISGIS	-	Liberia Institute of Statistics & Geo-Information Services
MCC	-	Millennium Challenge Corporation
MLME	-	Ministry of Lands, Mines and Energy
NAD	-	North American Datum
NGA	-	National Geospatial-Intelligence Agency
RINEX	-	Receiver Independent Exchange Format
TRC	-	Liberia Truth and Reconciliation Commission
US	-	United States
UTM	-	Universal Transverse Mercator
WGS84	-	World Geodetic System (1984)

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CHAPTER 1: INTRODUCTION

In this chapter, the important issues which prompted this research would be discussed. Dominating these issues are background of the research, project relevance to Liberia as well as aims and objectives. The exact geographic location (Study Area) of the thesis is also made available. The scope of work and outline of the thesis have also been included in this chapter.

1.1 Background

Liberia is a West African nation situated on the Atlantic Ocean Coast. The country has an area of approximately 38,000 square miles (99,000 Square Kilometers) with population of around 3.5 million people and population density of 35 persons per square mile (90 persons per square kilometers (LISGIS, May 2008).

The country's geodetic network was established in 1971 by the US Army Topographic Command 72nd Engineering Battalion. This was based on the Clarke 1880 ellipsoid with fewer controls sparsely spread over the country. The network has not been in use as majority of the surveys done in the country use compass which has low accuracy. Also, this type of survey does not adequately define a point (Fosu, 2011).

Currently, there is no accurate and official published set of transformation parameters. However, the National Geospatial-Intelligence Agency (NGA) indicated in one of its publications that to convert coordinates from Liberia 1964 Datum to World Geodetic System 1984 (WGS 84) datum, three parameters (ΔX = –90m, ±15m, ΔY = +40m, ±15m, ΔZ = +88m, ±15m) could be used. This was based on collocation of four (4) points in 1987 (US Defense Mapping Agency, December, 1987)[1]. It is therefore necessary to determine transformation parameters between Clarke 1880 and WGS84 ellipsoid which will allow the

[1] The standard error of ±15m is not suitable for precise and accurate positioning

easy transformation of coordinates between the two systems. In addition, the integration of existing maps into the Universal Transverse Mercator System (UTM) mapping system will also be feasible.

1.2 Problem Statement

Liberia is one of Africa's oldest republics and by far one of the poorest countries in the world when it comes to infrastructure development and manpower capacity. All these occurred as a result of the country's prolonged civil unrest which lasted from 1990 to 2003. With the war over, there is a massive reconstruction and reestablishment of basic infrastructure in the County. All these post-war developments will prove futile if the country should experience another fracas of civil instability. Land tenure security is a critical issue in this post-conflict state and is widely recognized as a potential treat for further civil disturbances if not dealt with proactively (Frank Pichel et al, 2012).

It is from this perspective that stakeholders in the Liberian Peace Process deem it expedient that all variables which are potential causes of conflict in post-war development be addressed.[2] These variables include:

- ✓ Improving land rights and access;
- ✓ Increasing girls' access to primary education, and
- ✓ Improving Liberia's trade policy and practices.

To tackle the issue of '*improving land rights and accesses,* challenges associated with modern cadastral and surveying practices must be addressed. There is therefore an urgent need for the establishment of a densified reference network and a local datum in the country.

Again, a readily available set of transformation parameters will be needed to transform coordinates of control points that would be used to "tie" not only cadastral surveys to controls but all other surveys including engineering surveys done in the Country. With the presence of

[2] The Millennium Challenge Corporation (MCC) and the Government of Liberia

transformation parameters, spatial data (e.g. geographic information systems data and remote sensing imagery) relevant to Liberia but developed on the WGS84 coordinate system can be transformed to the local coordinate system with greater consistency.

Hence this project seeks to address the problem of the determination of transformation parameters for GPS Surveys in Montserrado County, Republic of Liberia.

1.3 Project Relevance to Liberia

There is no economic activity which is independent of land. To exploit the tremendous benefits of land, adequate positioning has to be considered. Government cannot continue to invest nation's resources into land mitigation issues when the situation can be handled proactively. This can be done with the establishment of a Survey and Mapping Bureau if not already established or the empowerment of said bureau if already established. The Survey and Mapping Bureau should be charged with the responsibility of producing maps based on the country's geodetic infrastructure at very high accuracy level. This is where the relevance of transformation parameters becomes apparent as most GPS equipment are based on the WGS84 ellipsoid. The GPS equipment has numerous application in engineering surveying, GIS, land related issues and navigation. Therefore, transformation parameters will be needed in transforming GPS coordinates to national coordinates (Liberian Grid).

1.4 Aims and Objectives

The main aim of this research is to determine Transformation Parameters in Montserrado County, Republic of Liberia for GPS Survey.

The specific objectives are to:

- ✓ Study the existing geodetic infrastructure of Liberia;

- ✓ Identify the most suitable transformation model and parameters for use in the study area;
- ✓ Write a computer software which will be used to transform and project coordinates between both systems.

1.5 Study Area

Montserrado County (figure 1) is located between Latitudes $6^{o}15'00''$N and $6^{o}30'00''N$ and Longitudes $10^{o}30'00''W$ and $10^{o}48'00''W$. Bounded to the North of Montserrado County is Gbarpolu County, the East is Margibi County, the West is the Bomi County and the South is the Atlantic Ocean.

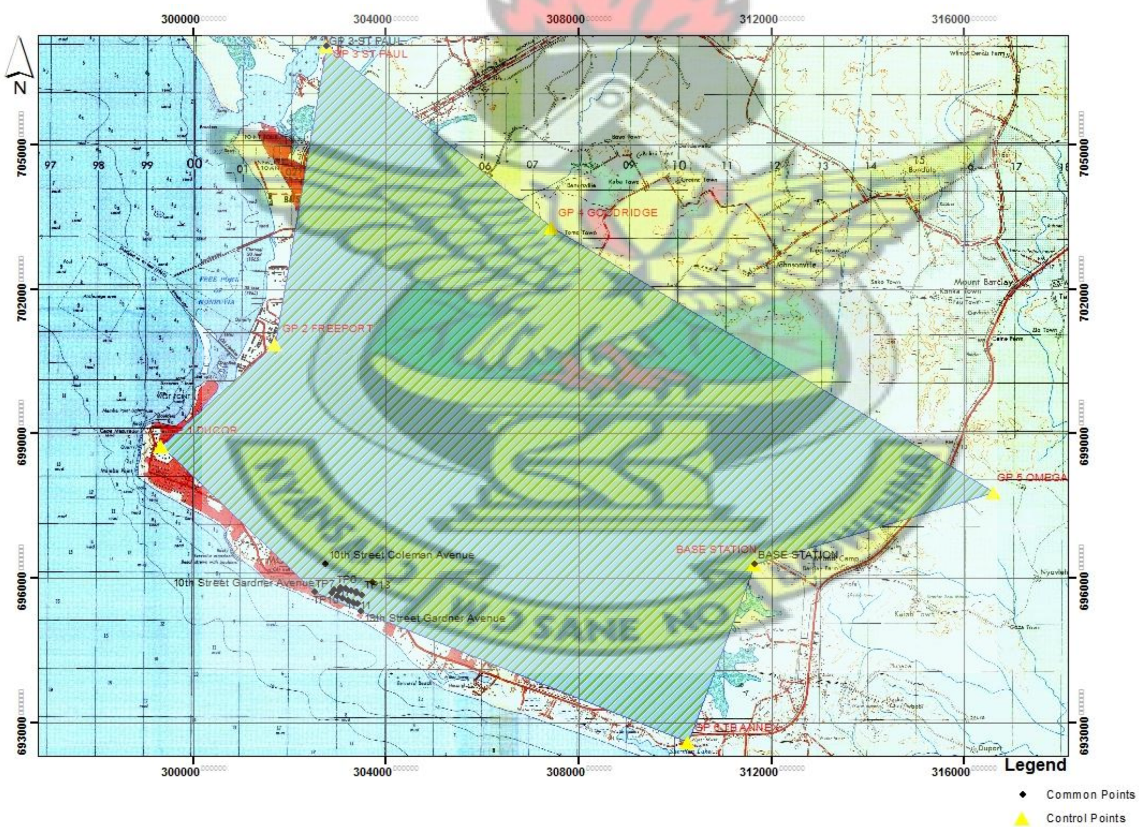


FIGURE 1: MAP OF LIBERIA SHOWING THE STUDY AREA WITH COMMON POINTS

1.6 Scope of Work

The project is divided into the following phases in chronological order:

- ✓ Desk Study

- ✓ Literature Review and Reconnaissance
- ✓ Design and Specification
- ✓ Monumentation
- ✓ Observation and Computation
- ✓ Analysis of Results
- ✓ Conclusion and Recommendation

1.7 Thesis Outline

This thesis has been categorized into five chapters (1 – 5). Chapter One introduces the nature of the Liberian Geodetic infrastructure and its present stage, the problem statement and also project relevance.

Chapter two gives an overview of what do be done in Liberia including design specification, datum and coordinate systems as relating to the topic, Models of transformation, concepts of map projection.

Chapter Three provides materials used during the project implementation, methodology to be applied, observational and computational procedures, details of the transformation model, statistics and testing of the results.

Chapter Four presents the results as obtained followed by individual discussions of the results including parameters, errors and residuals. Included in this chapter are statistical comparisons and graphical displays of results.

Chapter Five is a summary of the parameters from all models. The conclusion and recommendations for the project are also integral part of this chapter.

1.8 Concluding Remarks

After several analyses of the problem coupled with discussion and the gathering of relevant information appertaining to the subject including personal interaction with authorities, it has been established that there is a need for the project to be undertaken in the study area.

CHAPTER 2: DETERMINATION OF DATUM TRANSFORMATION PARAMETERS

In this chapter, the nature of the Liberian Land Sector, the Liberian Geodetic Infrastructure (Past and Present), different Transformation Models, Reference figures of the earth, Coordinate Systems, Datum and Coordinate Conversion methods as well as Map projections are discussed. Included in this chapter is an analysis of the various transformation models which consists of their relative advantages and disadvantages and the most suitable models chosen for this research.

The years of conflict in Liberia have left the land sector in complete disarray. There are few trained and experienced technical staffs and the agencies charged with the responsibilities of handling land related issues lack the tools and equipment needed to effectively accomplish the tasks required of them (Frank Pichel et al, 2012). This has contributed immensely to the downplay of survey activities in the country for which we have seen that the country does not have an established and recognized datum which is worth mentioning as a tangible geodetic infrastructure. Clifford J. Mugnier stated in a research entitled "Grids & Datums" that there was a Firestone Datum which was used by the world's largest single natural rubber operation in the past (Mugnier, 2011).

As mentioned earlier, one of the reasons for the irregularities in the land sector in Liberia is the result of the lack of a tangible geodetic infrastructure. As a result of the absence of this, the issue of positioning has always been a problem irrespective of technological advancement. President Ellen Johnson-Sirleaf indicated in her inaugural address on January 6, 2006 that, "*We can revisit our land system to promote more ownership and freeholding*[3] *for communities*". This was reemphasized in the Liberia Truth and Reconciliation Commission

[3] Freehold refers to the legal ownership of property; legal ownership of a property giving the owner unconditional rights, including the right to grand leases and take out mortgages

2008 final report that land disputes are a key threat to peace and if the issue is not addressed there is a strong likelihood of a return to violence (TRC, 2008). In an effort to address the numerous issues concerning land in the country, the Land Commission was established in 2009 with the mandate of formulating, proposing and advocating for reforms in the land sector in Liberia and also to lead the coordination of reforms by the various agencies involved in land issues.

Recently, a technical Group from the Lands, Mines and Energy Ministry and the Land Administration Advisory of Tetra Tech ARD is on reconnaissance activities for the construction of Survey Markers in ten different locations around the country. It was stated that the development of this Modern Geodetic Infrastructure of established survey monuments, will serve as references for all ground survey activities in Liberia, which is in fulfillment of one of the components of the Millennium Challenge Corporation (MCC) Threshold Program focusing on Restoring Confidence in the Land Administrative System[4].

In July of 2010 the Government of Liberia signed a Threshold Agreement with the United States Millennium Challenge Corporation (MCC). This Threshold Agreement is designed to assist Liberia in improving its performance on core indicators that must be reached in order for the country to become eligible for MCC compact. The Threshold Program focuses on three key areas, trade policy, girls' education and property rights; for which activities are designed to improve Liberia's performance. In the area of property rights, Liberia scored in the bottom 2% of its peer group according to the MCC assessment, and in the 2010 World Bank Ease of Doing Business Assessment, Liberia scored 176th out of 183 Countries, making property rights the indicator most in need of improvement. (Millennium Challenge Corporation, 2011)

[4] (http://allafrica.com/stories/201112020719.html, 2011)

All these efforts and developments in the Liberian Land Sector are done on the basis that if land tenures are not handled appropriately, there is a likelihood of occurrence of conflicts in the country in the nearby future. The biggest question remains as to how land tenures security can be enhanced in a country in the absence of an established geodetic infrastructure.

2.1 Liberia's Geodetic System

Very little is known about the existing geodetic network in Liberia. Information gathered from the Liberian Cartographic Service revealed that the Country's geodetic infrastructure was established in 1964 and was called the Liberia 1964 Datum based on the Clarke 1880 ellipsoid with the origin at Robertsfield Astro, Latitude = 6^{o}13’53.02 N”±0.07”, Longitude = 10^{o}21’35.44 W”± 0.08”, elevation = 8.23331 m, and azimuth = 195^{o}10’10.57”±0.14” to Roberts Field Astro Azimuth mark from South. The first order stations were monumented by concrete pillars with forced centering devices. There is little else information available about datum values such as projection type and false Easting and Northings and scale as well as location of and values of first order pillars in the Liberia Geodetic System. [5]

Most station points of the first order traverse are purported to ran across the country from the Roberts Field International Airport northeast to a point near the Guinea border in the Nimba Mountains. The observations were made in 1964 – 1965 by the 72nd Engineer Survey Liaison Detachment in the frame of the Joint Liberia-United States Mapping Project. First order astronomic position observations were performed in the Nimba Mountains, Zwedru, Foya Kama, Suakoko, Camp Ramrod, Robertfield and on the Southwest Cost at Nuon Point. At present, all these points are either damaged or inaccessible. Unfortunately, there is no diagram (map) for this network at the Ministry of Lands, Mines and Energy.

[5] This is an indication that the existing network does not meet the standards necessary for the foundations of a modern national spatial data infrastructure. The establishment of a new datum is one of the recommendations of the MCC.

2.2 Design Specifications for the New Liberian Datum

It was indicated in the Millennium Challenge Corporation's Liberia Threshold Program Report under the caption "Strategy for Modernizing of the Geodetic Infrastructure of Liberia" that a new Liberian Geodetic Datum[6] 2005 (LGD2005) should be based and aligned with, the International Terrestrial Reference System (ITRS) which is based on the ITRF2009 datum and the ellipsoid associated with this datum will be the Geodetic Reference System 1980 (GRS80)[7] (Millennium Challenge Corporation, 2011). The choice of the GRS80 ellipsoid is due to the fact that it is a three dimensional datum compatible with international geodetic systems, such as the International Reference System (ITRS) and the World Geodetic System 1984 (WGS84) and it is the most precise earth-centered earth-fixed terrestrial datum currently available (Altamimi et al, 2011).

2.3 Coordinate Systems

Adequately and uniquely defining a point position is a problem often faced by Geodesists. A point could represent an intersection, a building, etc. For the unique identification of points, a special pair of number is needed. These pairs of numbers form the basis of a *coordinate system*. Thus, a coordinate system can be defined as a set of numbers which uniquely identifies a position in space (Fosu, 2011). Speaking of a Coordinate System, one will consider three basic things: origin, directions of the axes and scale. Coordinate Systems can be categorized as one dimensional (eg. Number Line), two dimensional (eg. Cartesian-x,y) or three dimensional (Geographic-latitude, longitude, h).

Transformations are mathematical operations which takes the coordinates of a point in one coordinate system into the coordinates of the same point in a second coordinate system

[6] MCC's specification of the New Liberia Datum.
[7] WGS is currently the reference system being used by the GPS. It has as ellipsoidal shape defined by semi-major axis a = 6378137m and an inverse flattening = 298.257223563.

(Andrei, 2006). Different kinds of coordinates are used to position objects in a two or three dimensional space.

2.3.1 Geographic/Geodetic Coordinate System (ϕ, λ, h)

Geographic/Geodetic Coordinate System is the coordinate system which uses latitude, longitude and ellipsoidal height (ϕ, λ, h) (figure 2) to reference point on the ellipsoid or astronomical coordinates *(Φ, Λ, H)* to reference point on the geoid (Kennedy, 2000).

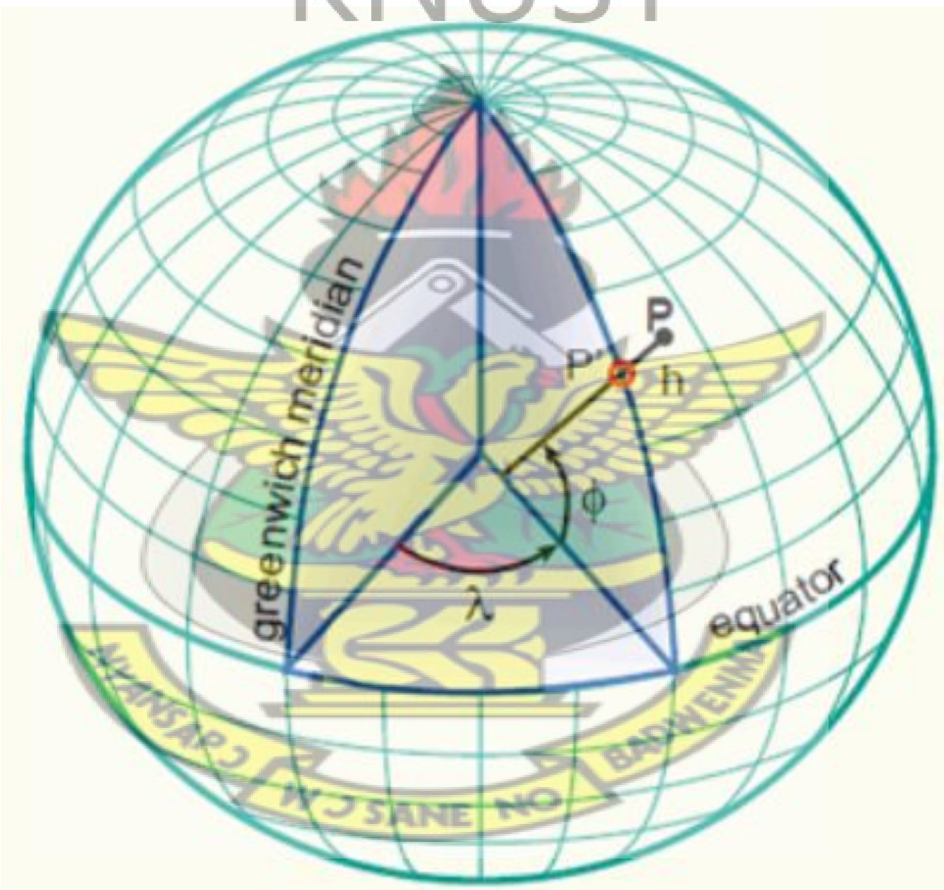


FIGURE 2: GEOGRAPHIC/GEODETIC COORDINATE SYSTEM (KENNEDY, 2000)

2.3.2 Cartesian Coordinate System (X,Y,Z)

3D positions on the surface of the Earth may be defined by means of geocentric coordinates (X,Y,Z), also known as 3D Cartesian Coordinates. The system of this definition has its origin at the mass-center of the Earth and the X- and Y-axes in the plane of the equator. The X-axis passes through the meridian of Greenwich, and the Z-axis coincides with the Earth's axis of rotation. The three axes are mutually orthogonal and form a right-handed system (figure 3).

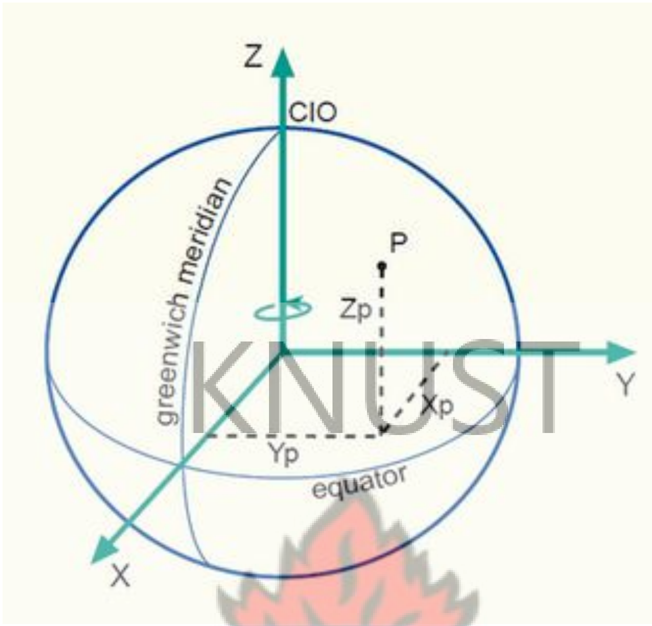


FIGURE 3: A TYPICAL 3D CARTESIAN COORDINATE SYSTEM

2.3.3 Conversion between Geographic and Cartesian Coordinates

Geodetic coordinates(ϕ, λ, h), for points in space which are related to the ellipsoid(a, f), can be mathematically converted to Cartesian Coordinates(x, y, z), if the Cartesian origin is at the ellipsoid center and the axes of the Cartesian Coordinates are mutually orthogonal along the minor axis and in the equator of the ellipsoid (Jekeli, 2006).

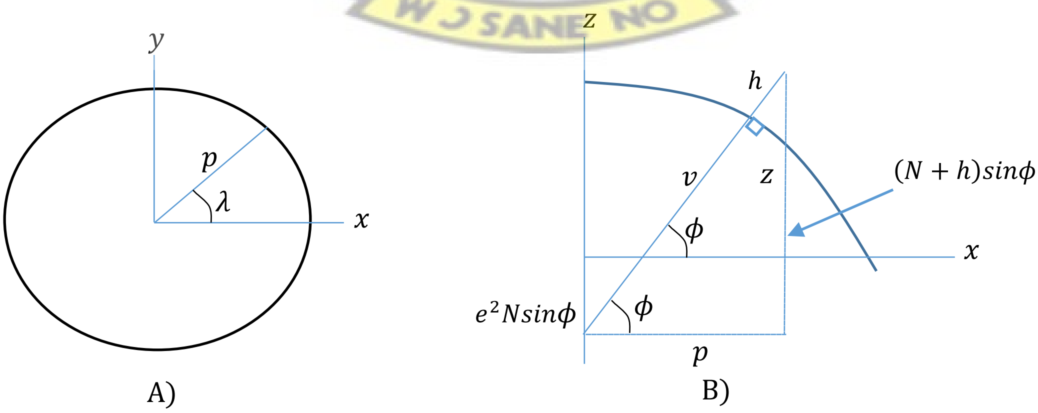


FIGURE 4: GEODETIC LATITUDE VS CARTESIAN COORDINATES

Ideally, given geodetic coordinates (ϕ, λ, h) and the ellipsoid to which they refer, the Cartesian Coordinates *(x,y,z),* (figure 4) are computed according to**:**

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (v + h)cos\phi cos\lambda \\ (v + h)cos\phi sin\lambda \\ (v(1 - e^2) + h)sin\phi \end{pmatrix} \qquad \ldots 1$$

Where, *v* is the radius of curvature in the prime vertical given as:

$$v = \frac{a}{(1 - e^2\, sin^2\, \phi)^{\frac{1}{2}}}$$

h is the height above the ellipsoid, and

e is the eccentricity of the ellipsoid given as:

$$e^2 = \frac{a^2 - b^2}{a^2} = 2f - f^2$$

This conversion as seen from the definition of the eccentricity e depended on the ellipsoidal flattening and semi-major axis.

The reverse transformation from Cartesian to geodetic coordinates is more complicated. The usual method is by iteration, but closed formulas also exist. They are summarized below:

$$\lambda = tan^{-1}\left(\frac{y}{x}\right) \quad \ldots\ldots\ldots 2$$

$$\boldsymbol{\phi = tan^{-1}\left(\frac{Z + \varepsilon b sin^3 q}{p - e^2\, a cos^3\, q}\right)} \quad \ldots\ldots\ldots 3$$

$$\boldsymbol{h = \left(\frac{p}{cos\phi}\right) - v} \quad \ldots\ldots\ldots 4$$

Where,

$\varepsilon = \frac{e^2}{1-e^2}$

$b = a(1 - f)$

$p = (X^2 + Y^2)^{\frac{1}{2}}$

$q = \tan^{-1}\left(\frac{Za}{pb}\right)$

2.4 Datum

The term datum refers to a chosen origin and accepted values for that origin from which the values of other points can be determined. In the case of a horizontal coordinate system, a datum definition should include the chosen ellipsoid defined through its shape and size (a, f), the origin point of the X,Y,Z, axis, and the orientation of the axes. The National Geodetic Survey defines a datum as “a set of constants specifying the coordinate system used for geodetic control, i.e., for calculating the coordinates of points on the Earth”. (Deakin, 2004) The main purpose of a datum is to provide the means by which the horizontal locations of points can be defined both mathematically and graphically figure 4. Countries and agencies adopt different datums as the basis for coordinate systems used to identify positions in geographic information systems, precise positioning systems, and navigation system.

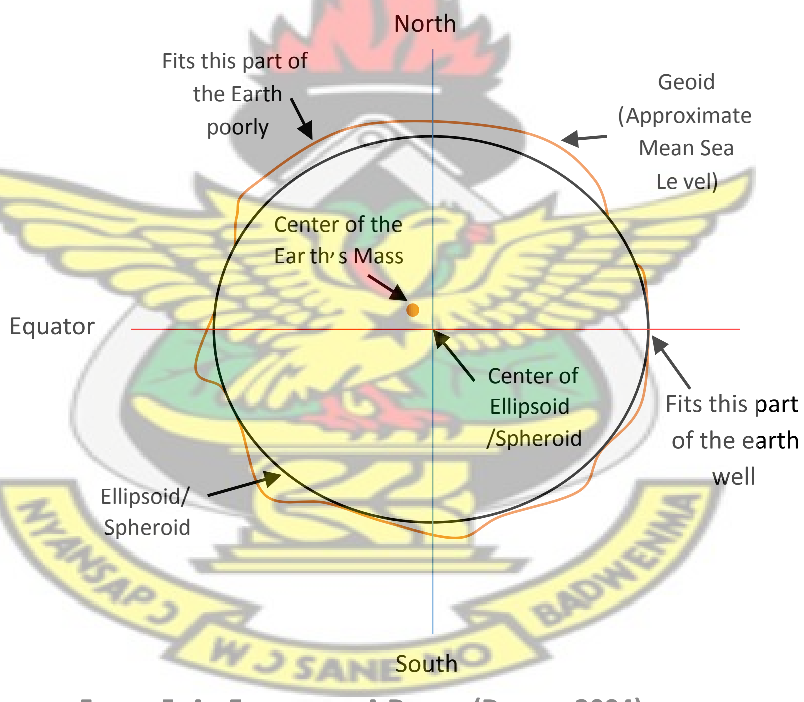


FIGURE 5: AN EXAMPLE OF A DATUM (DEAKIN, 2004)

2.4.1 Local Datum

Local Datums apply only to a region or local area of the earth’s surface. The definition of local datum is usually quite arbitrary, and the selection is subject only to convenience. The size and shape of the ellipsoid must first be defined by selection of a semi-major axis length, *a*, and a flattening, *f*. The parameters chosen have tended to depend on historic developments and international ellipsoids which have been agreed on from time to time. A local datum may also consist of the latitude and longitude of an initial point (origin); an azimuth of a line (direction) to some other triangulation station; the parameters (radius and flattening) of the ellipsoid selected for the computations; and the geoid separation at the origin.

2.4.2 Satellite Datum

A Satellite reference system is defined by the system in which the satellite ephemeris or orbit parameters are given. These orbit parameters are based on the adopted coordinates of a number of satellite tracking stations, an adopted geo-potential model for the earth's gravity field and a set of constants (Hoar, 1982). These constants are:

- The Gravitational Constant times the earth's mass, GM;
- The rotation of the earth with respect to the instantaneous equinox,
- The speed of light, c
- Clock corrections and oscillator drift rates at the tracking stations used for ephemeris calculation.

The concept of a reference ellipsoid which is a cornerstone for local datum definition is unnecessary in the definition of a satellite datum. The ellipsoid is not used in the orbit computation. However, an ellipsoid is usually associated with a satellite datum so that positions may be shown in geographic as well as a Cartesian coordinates. The ellipsoid is derived by a least squares fitting process.

Satellite datums can be broken into two categories, Precise Ephemeris Datum and Broadcast Ephemeris Datum.

For the scope of this project, parameters between a local datum (Clarke 1880) and a satellite datum (WGS84) will be determined for the project area.

2.5 Datum Transformation

Datum Transformation can then be defined as the transformation that is used to transform the coordinates of a point defined in one datum to coordinates in a different datum. Datum Transformations are required for converting coordinates determined with the use of satellite positioning equipment into local coordinates if these were defined on a different ellipsoid.

There are numerous transformation models. Some of these are: the Helmert Similarity Transformation, Bursa-Wolf, Multiple Regression, Molodensky-Badekas, Veis Model, Affine Transformation, etc. The choice of the most appropriate transformation model depends on the following factors:

- Whether the model is to be applied to a small area, or over a large region;
- The accuracy requirement;
- Whether transformation parameters are available, or they have to be determined.

Generally, a transformation in which the scale factor is the same in all directions is a similarity transformation or conformal transformation, and is by far the most widely used of the transformation models. It preserves the shape but not size. An Orthogonal Transformation is a similarity transformation in which the scale factor is unity. In this case, the shape and size of the network will not change, but the positions of points do. Similarity Transformations can be used when source and target coordinate reference systems have the following characteristics (EPSG, 2005):

- Each of them has orthogonal axes
- Each of them has same scale along both axes, and
- Both have the same units of measure.

2.6 Types of Datum Transformation

Datum Transformations may be broadly divided into two forms which can be subdivided further (Hoar, 1982):

- Two-Dimensional Transformation
- Three-Dimensional Transformation

2.6.1 Two-Dimensional Transformation (2D)

The two-dimensional transformation can be used to transform 2D Cartesian coordinates (x,y) from one system to another. Both systems should be in 2D. The complexity of this transformation range from similarity transformation between two sets of plane coordinates to more complex formulae giving change of latitude and longitude as functions of their corresponding changes with respect to some arbitrary point, change of orientation, and change of scale. Ideally, two-dimensional transformations are only valid over a very limited area and are of little use in Doppler[8] positioning problems except where there is very little or no height information available. Below are some of the two dimensional transformation models:

- The **conformal** transformation
- The **affine** transformation
- The **polynomial** transformation

[8] Doppler satellite surveying is a method of determining positions of points on the earth's surface by observing the Doppler shift of radio transmissions from satellites of the U.S. Navy Navigation Satellite System (NNSS). (http://www.ngs.noaa.gov/PUBS_LIB/Geodesy4Layman/TR80003D.HTM, Accessed, 2013)

2.6.1.1 The Conformal Transformation

A Conformal two dimensional Transformation (figure 6) is a linear (or first-order) transformation that relates two 2D Cartesian coordinate systems through a rotation, a uniform scale change, followed by a translation. The rotation is defined by one rotation angle (a) and the scale change by one scale factor (s). The translation is defined by two origin shift parameters (x_o,y_o). The equation is:

$$\begin{aligned} X' &= sXcos(a) - sYsin(a) + x_0 \\ Y' &= sXsin(a) + sYcos(a) + Y_0 \end{aligned} \quad \text{.......5}$$

The simplified equation is:

$$\begin{aligned} X' &= aX - bY + x_0 \\ Y' &= bX + aY + Y_0 \end{aligned}$$

Where $a = scos(a)$ and $b = ssin(a)$. The transformation parameters (or coefficients are *a, b,* x_o, *yo*).

2.6.1.2 The Affine Transformation

The Affine Transformation (figure 6B) is a linear (or first-order) transformation and relates two 2D Cartesian Coordinate systems through a rotation, a scale change in x and y direction, followed by a translation. The transformation function is expressed with 6 parameters: one rotation angle (a), two scale factors, a scale factor in x-direction (s_x) and a scale factor in the y-direction (s_y) and two origin shifts (x_o,y_o). The equation is:

$$\begin{aligned} X' &= s_xXcos(a) - s_yYsin(a) + x_0 \\ Y' &= s_xXsin(a) + s_yYcos(a) + Y_0 \end{aligned} \quad \text{..........6}$$

The simplified equation is:

$$\begin{aligned} X' &= aX - bY + x_0 \\ Y' &= cX + dY + Y_0 \end{aligned} \quad \text{........7}$$

Where the transformation parameters (or coefficients) are a,b,c,d,x_0,y_0.

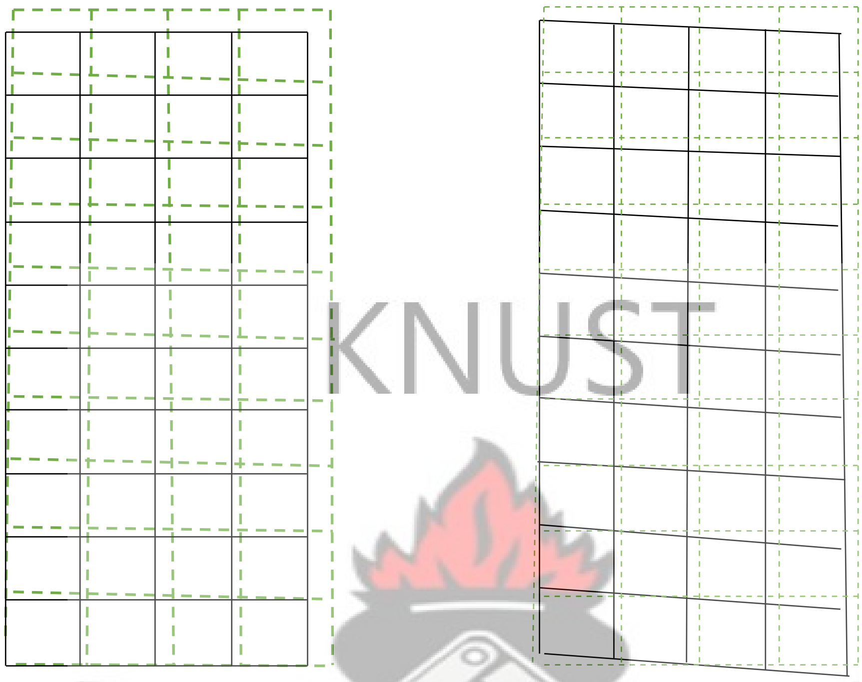


FIGURE 6 (A) CONFORMAL TRANSFORMATION (B) AFFINE TRANSFORMATION

The uniform scale change of the conformal transformation retains the shape of the original rectangular grid whereas the different scales in x and y-direction of the affine transformation changes the shape of the original rectangular grid, but the lines of the grid remain straight.

2.6.1.3 The Polynomial Transformation

A polynomial transformation is a non-linear transformation that relates two 2D Cartesian coordinate systems through a translation, a rotation and a variable scale change. The transformation function can have an infinite number of terms. The equation is:

$$\begin{matrix} \boldsymbol{X' = x_0 + a_1X + a_2Y + a_3XY + a_4X^2 + a_5Y^2 + a_6X^2Y + a_7XY^2 + a_8X^3 + \ldots\ldots} \\ \boldsymbol{Y' = Y_0 + b_1X + b_2Y + b_3XY + b_4X^2 + b_5Y^2 + b_6X^2Y + b_7XY^2 + b_8X^3 + \ldots\ldots} \end{matrix} \quad \ldots.8$$

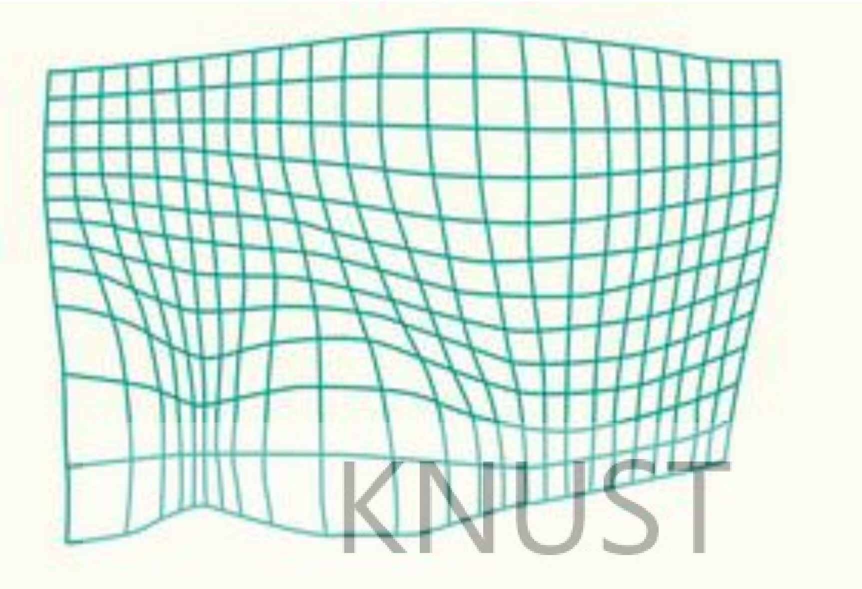


FIGURE 7 A POLYNOMIAL TRANSFORMATION MODEL

Polynomial Transformation are sometimes used to georeference uncorrected satellite imagery or aerial photographs or to match vector data layers that don't fit exactly by stretching or rubber sheeting them over the most accurate data layer. Figure 7 above shows a grid with no uniform scale distortions. It may occur in an aerial photograph, caused by the tilting of the camera and the terrain relief (topography). An approximate correction may be derived through a high-order polynomial transformation. The displacements caused by relief differences can be corrected using a Digital Elevation Model (DEM).

2.6.2 Three-Dimensional Transformation (3D)

Three-Dimensional Transformation (3D) are more suitable for positioning for a number of reasons. They are typically global in concept, they enable solutions for height as well as horizontal position, and they are mathematically rigorous. The complete three-dimensional transformation involves seven parameters that relate Cartesian coordinates in the two systems. There are three translation parameters to relate the origins of the two systems (ΔX, ΔY, ΔZ), three rotation parameters, one around each of the coordinate axes (R_x, R_y, R_z) to relate the orientation of the two systems, and one scale parameter (dS) to account for any difference in scale between the two systems.

2.6.2.1 The Helmert Transformation

For the transformation of the geocentric coordinates of a given point on a certain datum to the ones on another datum, (Wolf H. , 1963) suggested a simplified form of the three-dimensional Helmert transformation in which it is assumed that there are three types of differences between the two frames:

- ✓ The origin is different and a vector offset is necessary between them;
- ✓ There is a rotation about each axis, and
- ✓ There may be a scale change.

The notation for the transformation model relating coordinates of points in the X_B, Y_B, Z_B network to coordinates in the X_A, Y_A, Z_A network is of the form:

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix} + (1 + dS) \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \ldots\ldots 9$$

Where,

dS = is the scale factor and is expressed in parts per million (ppm) (10^{-6})

$\Delta(x, y, z)$ denotes the translation components

$R(x, y, z)$ denotes the three rotation angles respectively

$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$ = coordinates of the common point in the new system

$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}$ = coordinates of the common points in the old system

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix} \ldots\ldots 10$$

2.6.2.2 The Molodensky Transformation Models

One of the most common methods of directly transforming latitude, longitude, and height is the Molodensky transformation (Bowring, 1976). This model can be classified either as the standard seven (7) parameter or abridged five (5) parameter transformation. The seven parameter transformation comprise of three origin shifts (ΔX, ΔY, ΔZ), three rotations (Rx, Ry, Rz) and a scale (dS) of the geometrical centers of the reference ellipsoids associated with the datums. The five parameters model includes the three parameters with additional two parameters which are the changes in the semi-major axis (Δa) and the inverse flattening (Δf) of the two reference systems.

2.6.2.2.1 The Standard Molodensky-Badekas (7 parameters) Model

Below are the formulas for the standard Molodensky Transformation Model (Deakin, 2004):

$$\boldsymbol{\phi_w = \phi_A + \left[\left(\frac{1}{v+h}\right)(-\Delta X sin\phi cos\lambda - \Delta Y sin\phi sin\lambda + \Delta Z cos\phi) + \frac{\Delta a}{a}(ve^2 sin\phi cos\phi) + \Delta f\left(\frac{\rho a}{b} + \frac{vb}{a}\right) sin\phi cos\phi\right]} \quad \ldots\ldots\ldots\ldots \quad 11$$

$$\lambda_w = \boldsymbol{\lambda_A + \frac{-\Delta X sin\lambda + \Delta y cos\lambda}{(v+h)cos\phi}} \ldots\ldots\ldots\ldots 12$$

$$\boldsymbol{\Delta h = \Delta X cos\phi cos\lambda + \Delta Y cos\phi sin\lambda + \Delta Z sin\phi - \frac{a}{v}\Delta a + \frac{vb}{a} sin^2 \phi \Delta f} \quad \ldots\ldots\ldots\ldots \ 13$$

Where,

v = the radius of curvature in the prime vertical which is given as:

$$v = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}$$

ρ = the radius of curvature in the meridian and is given as:

$$\rho = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}}$$

2.6.2.2.2 The Abridged (5 parameters) Molodensky Transformation Model

The Abridged Molodensky Model is a complex formula for the shift in latitude, longitude and height and yields the result normally called a five parameter transformation. The abridged form is often given as:

$$\Delta\boldsymbol{\phi}" = \frac{\mathbf{1}}{\boldsymbol{\rho sin1}"}[-\Delta\boldsymbol{X sin\phi_1 \cos\lambda_1} - \Delta\boldsymbol{Y sin\phi_1 sin\lambda_1} + \Delta\boldsymbol{Z cos\phi_1} + (\boldsymbol{a_1}\Delta\boldsymbol{f} + \boldsymbol{f_1}\Delta\boldsymbol{a})\boldsymbol{sin2\phi_1}] \ldots\ldots\ldots\ldots 14$$

$$\Delta\lambda" = \frac{1}{v cos\phi_1 sin1"}[-\Delta X sin\lambda_1 + \Delta Y cos\phi_1] \ldots\ldots\ldots\ldots 15$$

$$\Delta h = \Delta X cos\phi_1 cos\lambda_1 + \Delta Y cos\phi_1 sin\phi_1 + \Delta Z sin\phi_1 + (a_1\Delta f + f_1\Delta a)\, sin^2\boldsymbol{\phi_1} - \Delta\boldsymbol{a} \ldots .16$$

$$\Delta f = f_1 - f_2$$

$$\Delta a = a_1 - a_2$$

Where,

$\Delta a, \Delta f$ = the difference between the ellipsoid parameters (Semi Major Axis and Inverse Flattening, respectively).

ϕ, λ, h = geodetic coordinates of the local geodetic system ellipsoid

$\Delta\phi, \Delta\lambda, \Delta h$ = corrections to transform local datum coordinates to WGS84 ϕ, λ, h

$\Delta X, \Delta Y, \Delta Z$ = corrections to transform local datum coordinates to WGS84 X,Y,Z

e = the eccentricity of the ellipsoid

$e^2 = 2f - f^2$

The Abridged Molodensky Transformation Model (figure 8) is mostly useful when the height of the local datum is unknown. This is an iterative method done by a computer program with the initial approximation that Δh = 0. This approximation of Δh leads to progressive refinements in the values of Δh and the corresponding $\Delta X, \Delta Y$ and ΔZ until convergence is reached (J. Ayer and T. Tiennah, 2008). The Abridged Molodensky Transformation Model has lower accuracy level (Kennedy, 2000).

The implementation of the Molodensky transformation is embedded in many geodetic programs and as such it is the most required and used (Newsome, 2008).

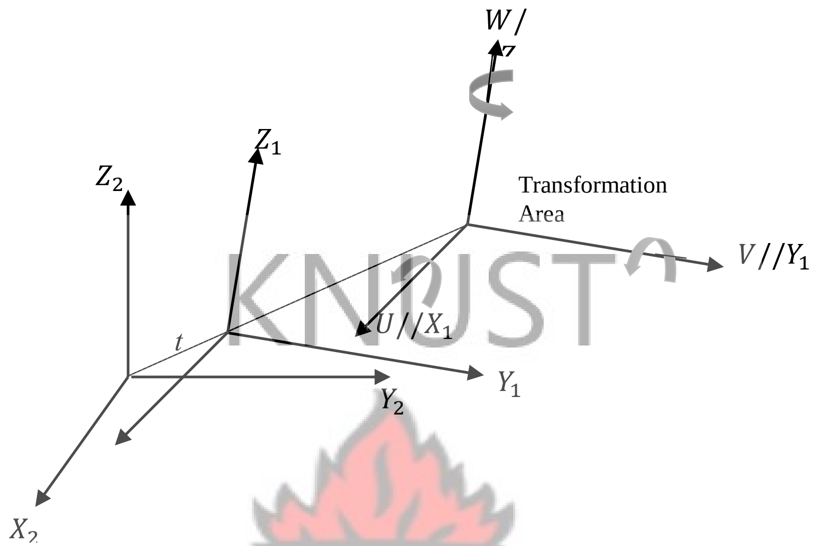


FIGURE 8: GEOMETRIC REPRESENTATION OF THE MOLODENSKY-BADEKAS TRANSFORMATION MODEL (KUTOGLU ET AL., 2002)

2.6.2.3 The Bursa-Wolf Transformation Model

The difference in application of the Molodensky and Bursa-Wolf Transformation Models is that the Bursa-Wolf model (figure 9) uses directly the coordinates provided while in the case of the Molodensky-Badekas model, the coordinates of the first system are shifted with respect to the centroidal coordinates of the common points (Deakin, 2006). In spite of this difference in application of the models the transformed coordinates produced by both models are unique. The translations and their root mean square values obtained from the computation of transformation, however, can be extremely different (Kutoglu, S. et al, 2002). Mathematically, this model is represented by:

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_Z \end{bmatrix} + (1+k) \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \quad \ldots\ldots 17$$

Ideally, the Bursa-Wolf and Molodensky-Badekas models should give the same results when the same data are used to determine the respective sets of transformation parameters.

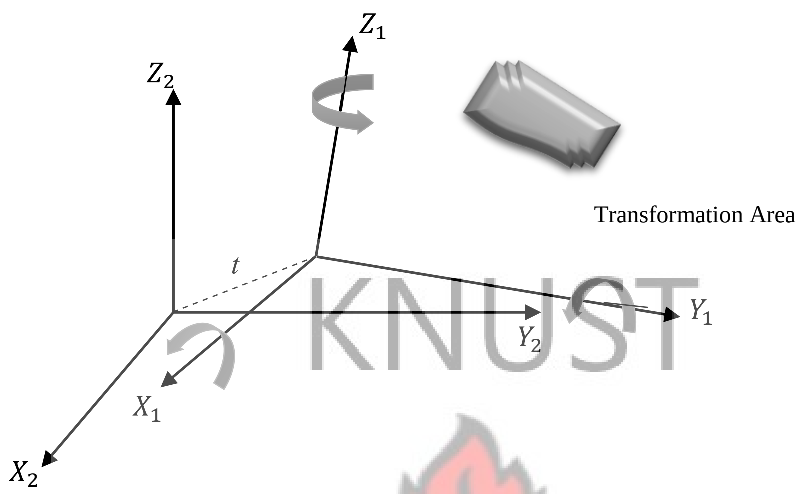


FIGURE 9: GEOMETRIC REPRESENTATION OF THE BURSA-WOLF TRANSFORMATION MODEL (KUTOGLU ET AL., 2002)

2.6.2.4 The Simple Three-Parameter Model

There are instances where the number of common points for the two datums is insufficient for the accurate determination of all seven parameters. If this occurs, the most logical solution is simply to determine the average translations (ΔX, ΔY, ΔZ) for the three points and be content with a three-Parameter transformation. Basically, the three parameter transformation implicitly assumes that there are no rotations and no scale change between the two systems. If rotations or a scale change do exist, their effects are being accommodated by the three translation parameters calculated by averaging ΔX, ΔY, ΔZ differences at the common points. If the residuals from this average three-parameter transformation are reasonable, it can be assumed that any rotations or a scale change are being adequately accounted for within the area of the common points. The limitation is that, this model cannot account for points outside the area of the common points. Mathematically, the three parameter transformation is represented below:

$$\begin{bmatrix} \boldsymbol{X_B} \\ \boldsymbol{Y_B} \\ \boldsymbol{Z_B} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_A} \\ \boldsymbol{Y_A} \\ \boldsymbol{Z_A} \end{bmatrix} + \begin{bmatrix} \Delta\boldsymbol{X} \\ \Delta\boldsymbol{Y} \\ \Delta\boldsymbol{Z} \end{bmatrix} \ldots\ldots.18$$

2.6.2.5 The Multiple Regression Method

The multiple regression equations (MRE) are ad hoc equations that provide for the shift in latitude and longitude as a function of position (Ayer, 2008). They take the form:

$$\Delta\boldsymbol{\varphi}" = \boldsymbol{A_o} + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + \cdots + A_{55}U^9V + A_{56}\boldsymbol{U^8V^2} + \cdots + \boldsymbol{A_{99}U^9V^9} \ldots\ldots\ldots\ldots 19$$

$$\Delta\boldsymbol{\lambda}" = \boldsymbol{B_o} + B_1U + B_2V + B_3U^2 + B_4UV + B_5V^2 + \cdots + B_{55}U^9V + B_{56}\boldsymbol{U^8V^2} + \cdots + \boldsymbol{B_{99}U^9V^9} \ldots\ldots\ldots\ldots 20$$

Where

A_0, B_0 = constant
$A_0, A_1 \ldots \ldots A_{nn};\ B_0, B_1 \ldots \ldots B_{nn}$= coefficients determined in the development
(φ, λ) = local geodetic latitude and local geodetic longitude (in degrees), respectively of the computation point.

The values of the independent variables, *U* and *V* are scaled latitude and longitude and are given by:

$$U = K(\varphi - \varphi_m)$$
$$V = K(\lambda - \lambda_m)$$

with K being a constant and (φ_m, λ_m) being a point near the middle of the area of validity. The regression model method can yield good accuracies but is useful within the sampled area only (Ayer, 2008).

2.6.3 Review of the Various Transformation Models

After comparison, analysis and study of the various transformation models, a table has been developed depicting the merits and demerits of each of the models.

TWO DIMENSIONAL (2D)			
NO.	**MODEL**	**ADVANTAGES**	**DISADVANTAGES**
1	Conformal Transformation	✓ Transform map coordinates to plane coordinates. ✓ Fewer points can be used. Minimum of two. ✓ Useful for re-gridding. ✓ Straight lines are preserved.	Only valid over a very limited area.
2	Affine Transformation	✓ Straight lines are preserved.	Only valid over a very limited area. Deforms the shape of polygons thus altering areas.
3	Polynomial Transformation	✓ Useful when one or both of the coordinate systems exhibits lack of homogeneity in orientation and scale. (EPSG, 2009)	Straight lines are changed. Deforms the size and shape of polygons.
THREE DIMENSIONAL (3D)			
NO.	**MODEL**	**ADVANTAGES**	**DISADVANTAGES**
1	Geocentric (Simple 3 Parameters)	✓ Simplest mathematical method to determine the parameters. ✓ Suitable if the number of common points in the two datums is insufficient for the accurate determination of all seven parameters.	Cannot account for points outside the common point Least accurate with accuracy of 5m to 10m (ICSM, 2002).
2	Abridged Molodensky (5 parameters) Model	✓ Can be used in the absence of height for local geodetic datum.	The absence of the height of local ellipsoid gives an approximation to the change in height.
3	Standard Molodensky (7 Parameters) Model	✓ Suitable for transformation from a satellite datum to a local datum. ✓ Eliminates high correlation between the translations and rotations in the derivation of the parameters as rotations are derived at a location within the points used in the determination.	Not suitable for use between two satellite datums
4	Bursa Wolf	✓ Uses Cartesian coordinates of the common points directly. ✓ Suitable for transformation between two satellite datums	Uses Cartesian Coordinate together with centroidal coordinates of the Common points.
5	Multiple Regression	✓ Better fit over continental size land areas (Featherstone et al., 2000).	

In selecting a particular model to be used for the transformation, the following should be considered (Fosu, 2011):

- ✓ Whether the model is to be applied to a small area, or to a larger area;
- ✓ Whether the network(s) have significant distortions;
- ✓ Whether the networks are three-dimensional (3-D), 2-D or even 1-D;
- ✓ The accuracy requirement.

In addition to the above listed, the geoid must be determined for accurate height measurement. Unfortunately, this is not the case in Liberia as the undulation is yet unknown.

The most common method for transforming coordinates between globally connected systems and national reference frames of an older type, in our case between WGS84 and Clarke 1880, is to use a similarity transformation in three dimensions (3D Transformation) (Reit, 2009).

2.7 Reference Figures of the Earth

The expression "*Reference Figure of the Earth"* has numerous meanings especially according to the way in which it is used coupled with the precision with which the Earth's size and shape is to be defined. Here, the term refers to the shape and size of the geometrical figure used to model (or represent) the Earth's physical or topographic surface (Dadzie, 2011). The surface of the earth is highly irregular and as a result constantly changing due to its rotation, deformation and other activities (especially geological) on the surface of the earth. These activities when resisted by the earth's gravity field cause changes on the earth surface (Dana, 1997).

2.7.1 Geoid

The Geoid is a model of the Earth's surface that represents the mean global sea level. Its shape passes through the Earth's crust and is determined from data collected all over the world about the Earth's gravity field. This means, measurements are made on the apparent or topographic surface of the Earth while computations are performed on the ellipsoid.

The geoid which is an equipotential surface of the earth's gravity field closely approximates the mean sea level and has the following characteristics (Fosu, 2011):

- ✓ Physical definition (surface);
- ✓ Complicated surface (irregular);
- ✓ Description by infinite number of parameters;
- ✓ Can be "sensed" by instruments;
- ✓ Allows users to know direction in which water flows.

Any point on the geoid is perpendicular to the direction of gravity (plumb line). The geoid is chosen as the reference level surface because every point on it has exactly the same potential, throughout the world.

2.7.2 Ellipsoid

An ellipsoid is a smooth mathematical surface which resembles a squashed sphere that is used to represent the earth's surface (Roman, 2007). One particular ellipsoid of revolution which of course is our concern, also called the "normal Earth" is the one having the same angular velocity and the same mass as the actual Earth, the potential U_o on the ellipsoid surface equal to the potential W_o of the geoid, and the center coinciding with the center of mass of the Earth (Li et al, 2001).

This mathematical surface (ellipsoid) that best approximates the earth has the following characteristics:

- ✓ Mathematical surface characterized by two constants (semi-major axes for direction and eccentricity or flattening for the shape);

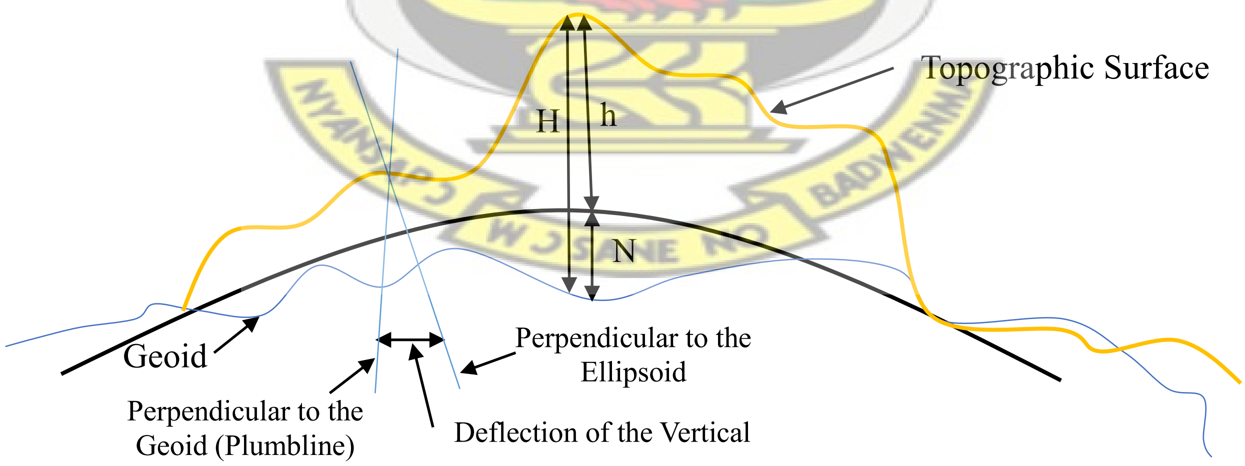
- ✓ Has several mathematical definitions;
- ✓ Can be described by two parameters;
- ✓ As opposed to the “geoid”, it cannot be “sensed” by instruments
- ✓ It has been chosen to be as close as possible (but not exactly) to the earth surface (or geoid) on a national regional or global point of view.

Table 1 below shows different ellipsoids in use in some countries.

TABLE 1: SOME COMMON ELLIPSOIDS IN USE

ELLIPSOID NAME	SEMI-MAJOR AXIS, *a* (*m*)	FLATTENING, *f*	WHERE USED
GRS 80 (1979)	6,378,137	1/298.26	Global
WGS 84(1984)	6,378,137	1/298.257223563	Global
Clarke (1880)	6,378,249	1/293.46	Liberia
War Office (1920)	6,378,300	1/296	Ghana
Everest (1830)	6,377,276	1/300.8	India
Clarke (1866)	6,378,206	1/294.98	North America
Bessel (1841)	6,377,397	1/299.15	Japan

2.7.3 Geoid/Ellipsoid Relationship



H = Orthometric Height
h = Ellipsoidal Height
N = Geoidal Height

$$H = h - N$$

FIGURE 10: DIAGRAM OF GEOID/ELLIPSOID RELATIONSHIP

The difference (figure 10) between the topographic elevation and the ellipsoid is called the ***ellipsoid height***. The vertical distance between the geoid and the ellipsoid is called the ***geoid height***. This height can be either negative of positive. Differences in height between the geoid and ellipsoid (geoid heights) range from roughly -100 to +100 meters. The difference between the topographic elevation and the geoid is called the ***orthometric heigh***t (Smith, 2006).

Topographic height is usually created using satellite or aerial photography and represents a more detailed model of the earth's surface. Elevation values are computed relative to the average local sea level.

2.7.4 Heights Relationship

Mathematically, the different heights mentioned above can be represented by the following basic equation (Dursun et al, 2002) :

$$\boldsymbol{h = H + N} \text{ ....21}$$

On the Clarke 1880 spheroid, the height is given as:

$$h_{Clarke} = H + N_{Clarke} = N_{WGS84} - \Delta h$$

The height on the WGS 84 spheroid is given as:

$$h_{WGs84} = H + N_{WGS84}$$

Finding the difference between the two equations and rearranging the resulting equations, the ellipsoidal heights above the Clarke 1880 spheroid can then be computed from the equation below:

$$\boldsymbol{h_{Clarke} = h_{WGS84} - \Delta h} \text{ ....22}$$

2.8 Map Projection

A map projection is the mathematical transformation of coordinates on a datum surface to coordinates on a projection surface (Deakin, 2004). This process is accomplished by the use of geometry (figure 11) or more commonly by mathematical formulas. Map Projection are necessary for creating maps. All map projections distort the surface in some fashion. Depending on the purpose of the map, some distortions are acceptable and others are not; therefore different map projections exist in order to preserve some properties of the sphere-like body at the expense of other properties. Different projections cause different types of distortions. Some projections are designed to minimize the distortion of one or two of the data's characteristics. A projection could maintain the area of a feature but alter its shape. There is no limit to the number of possible map projections (ESRI, 2010).

Map projection can be classified as: equal area projection which preserves the area of features, conformal projection which preserves the shape of features, equidistant projections which preserves distances (scale) to places from one point, or along one or more lines. True direction projections perverse bearings (azimuths) from center of map (Snyder, 1987).

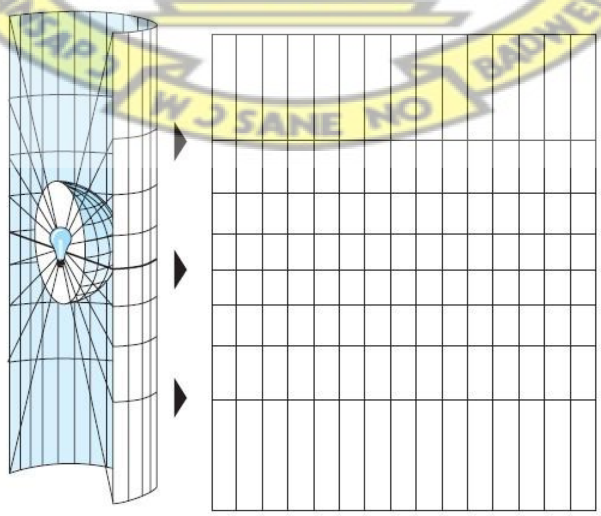


FIGURE 11: THE GRATICULE OF A GEOGRAPHIC COORDINATE SYSTEM IS PROJECTED ONTO A CYLINDRICAL PROJECTION SURFACE

2.8.1 Map Projection Parameters

A map projection grid is related to the geographical graticules of an ellipsoid through the definition of a coordinate conversion method and a set of parameters appropriate to that method. Different conversion methods may require different parameters. Any one coordinate conversion method may take several different sets of associated parameter values, each set related to a particular map projection zone applying to a particular country or area of the world.

The plane of the map and the ellipsoid surface may be assumed to have one particular point in common. This point is referred to as the ***natural origin***. It is the point from which the values of both the geographic coordinates on the ellipsoid and the grid coordinates on the projection are deemed to increment or decrement for computational purposes. Alternatively, it may be considered as the point which in the absence of application of false coordinates has grid coordinates of (0,0).

Since the natural origin may be at or near the center of the projection and under normal coordinate circumstances would thus give rise to negative coordinates over parts of the map, this origin is usually given false coordinates which are large enough to avoid this inconvenience. Hence each natural origin will normally have **False Easting**, **FE** and **False Northing**, **FN** values. For example, the false easting for the origins of all Universal Transverse Mercator zones is 500000m. As the UTM origin lies on the equator, areas north of the equator do not need and are not given a false northing but for mapping southern hemisphere areas the equator origin is given a false northing of 10,000,000m, thus ensuring that no point in the southern hemisphere will take a negative northing coordinate.

Longitudes are most commonly expressed relative to the **Prime Meridian** of Greenwich but some countries, particularly in former times, have preferred to relate their longitudes to a prime meridian through their national astronomic observatory, usually sited in or near their capital city. This meridian of the projection zone origin is known as the **Longitude of Origin**. For certain projection types it is often termed the **Central Meridian** or abbreviated as **CM** and provides the direction of the northing axis of the projection coordinate reference system.

In order to further limit the scale distortion within the coverage of the zone or projection area, some projections introduce a scale factor at the origin (on the central meridian for Transverse Mercator projections), which has the effect of reducing the nominal scale of the map here and making it have the nominal scale some distance away. For example in the case of the UTM and some other Transverse Mercator projections a scale factor of slightly less than unity is introduced on the central meridian thus making it unity on two roughly north-south lines either side of the central one, and reducing its departure from unity beyond these. The scale factor is a required parameter whether or not it is unity and is usually symbolized as k_o. (EPSG, 2005)

2.8.2 Conversion from Geographic Coordinates to UTM

There are situations when geographic coordinates should be converted to UTM and vice versa. The below formulas are used for such purpose. These formulas are accurate to within less than a meter with a given grid zone. The original formulas include a now obsolete term that can be handled more simply by merely converting radians to seconds of arc. That term is omitted.

$$\boldsymbol{S = A'(\phi) - B'\,sin(2\phi) + C'\,sin(4\phi) - D'\,sin(6\phi) + E'\,sin(8\phi)}$$

Here ϕ and all angles are expressed in radians, and

$$A' = a\left[1 - n + \frac{5}{4}(n^2 - n^3) + \left(\frac{81}{64}\right)(n^4 - n^5) \ldots.\right]$$

$$B' = \left[3\,tan\left(\frac{S}{2}\right)\right]\left(1 - n + \frac{7}{8}(n^2 - n^3) + \left(\frac{55}{64}\right)(n^4 - n^5) \ldots.\right)$$

$$C' = \left(15\, tan^2 \frac{S}{48}\right)\left[1 - n + \frac{11}{16}(n^2 - n^3) \dots\right]$$

$$D' = 35\left(tan^3\left(\frac{S}{48}\right)\right)\left[1 - n + \left(\frac{11}{16}\right)(n^2 - n^3) \dots ..\right]$$

$$E' = \left(315\, tan^4 \frac{S}{512}\right)[1 - n \dots .]$$

$$n = \frac{a - b}{a + b}$$

$$p = \lambda - \lambda_o \text{ (in radians)}$$

$$K_1 = S * sf$$

$$K_2 = vsin\phi cos\phi\left(\frac{sf}{2}\right)$$

$$K_3 = \frac{vsin\phi cos^3\phi}{24}(5 - tan^2\phi + 9e^2cos^2\phi + 4e^2cos^4\phi)sf$$

$$\textbf{Northing} = \boldsymbol{K_1 + K_2p^2 + K_3p^4} \dots \dots ...23$$

$$K_4 = sf(v)\cos(\phi)$$

$$K_5 = \left(\frac{sf(v)\, cos^3\, \phi}{6}(1 - tan^2\, \phi + e^2\, cos^2\, \phi)\right)$$

$$\textbf{Easting} = \boldsymbol{FE + K_4p + K_5p} \dots \dots \dots ...24$$

2.8.3 Conversion from UTM to Geographic Coordinates

$$Y = Northing - FN$$

$$M = \frac{Y}{sf}$$

$$\mu = \frac{M}{\left[a\left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256} \dots .\right)\right]}$$

$$e_1 = \frac{\left[1 - (1 - e^2)^{\frac{1}{2}}\right]}{\left[1 + (1 - e^2)^{\frac{1}{2}}\right]}$$

$$J1 = \left(\frac{3e_1}{2} - \frac{27e_1^3}{32} \dots \dots\right)$$

$$J2 = \left(\frac{21e_1^2}{16} - \frac{55e_1^4}{32} \ldots \ldots .\right)$$

$$J3 = \left(\frac{151e_1^3}{96} \ldots .\right)$$

$$J4 = \left(\frac{1097e_1^4}{512} \ldots \ldots\right)$$

$$FootPrint\ Latitude\ (FP) = \mu + J1\,sin(2\mu) + J2\,sin(4\mu) + J3\,sin(6\mu) + J4\sin(8\mu)$$

$$C1 = e'^2\,cos^2\,FP$$

$$T1 = tan^2(FP)$$

$$R1 = \frac{a(1-e^2)}{(1-e^2\sin^2 FP)^{\frac{3}{2}}}$$

$$N1 = \frac{a}{(1-e^2\sin^2 FP)^{\frac{1}{2}}}$$

$$D = \frac{\Delta x}{N1(sf)}$$

$$Q1 = \frac{N1\,tan(FP)}{R1}$$

$$Q2 = \frac{D^2}{2}$$

$$Q3 = (5 + 3T1 + 10C1 - 4C1^2 - 9e'^2)\left(\frac{D^4}{24}\right)$$

$$Q4 = (61 + 90T1 + 298C1 + 45T1^2 - 3C1^2 - 252e'^2)\left(\frac{D^6}{720}\right)$$

$$\boldsymbol{\phi = FP - Q1(Q2 - Q3 + Q4)} \ldots \ldots \ldots \ldots . . \ldots . 25$$

$$Q5 = D$$

$$Q6 = (1 + 2T1 + C1)\left(\frac{D^3}{6}\right)$$

$$Q7 = (5 - 2C1 + 28T1 - 2C1^2 + 8e'^2 + 24T1^2)\left(\frac{D^5}{120}\right)$$

$$\lambda = \boldsymbol{\lambda_0} + \frac{\boldsymbol{Q5 - Q6 + Q7}}{\boldsymbol{cos(FP)}} \dots\dots\dots\dots 26$$

In the above equation, λ_0 is the Central Meridian.

Additional Parameters

The Radius of Curvature in the Prime Vertical is given as:

$$v = \frac{a}{(1 - e^2 \, sin^2 \, \phi)^{\frac{1}{2}}}$$

The Radius of Curvature in the Prime Meridian is given as:

$$\rho = \frac{a(1 - e^2)}{(1 - e^2 \, sin^2 \, \phi)^{\frac{3}{2}}}$$

2.9 Concluding Remark

The state of the Liberian geodetic infrastructure has been investigated by taking a closer look at the datum used in the past and the design specification of the new Liberia Network which is yet to be established. Also, the various transformation models were reviewed and a study of map projection was done. For the scope of our work, the following transformation models were used to determine the parameters:

- ✓ Abridged Molodensky (Five parameters) Model
- ✓ Geocentric (Simple Three Parameters) Model
- ✓ Bursa Wolf (Seven Parameter) Model
- ✓ Molodensky Badekas (7 Parameters) Model.

CHAPTER 3: RESEARCH MATERIALS AND METHODS

The objective of this chapter is to give an in-depth analysis of the various processes involved in the determination of parameters as well as those materials and methods associated. The following methodologies will be adopted:

- ✓ *Desk Study*
- ✓ *Reconnaissance*
- ✓ *Monumentation*
- ✓ *Observational Procedures*
- ✓ *Computational Procedures*
- ✓ *Statistics of the Parameters*
- ✓ *Testing of the Parameters*

3.1 Materials

To ensure that the project is effectively implemented, the following materials were needed

- ✓ Topographic map of Montserrado County at a scale of 1:50,000 (1971).
- ✓ Google Earth Image of the project area.

The following software were used:

- ✓ Matlab R2012b
- ✓ Visual Studio 2012 (Visual Basic.net 2012)
- ✓ ArcGIS 10
- ✓ Selected Microsoft Office Suite (Microsoft Word, Microsoft Excel, Microsoft PowerPoint)
- ✓ Spectrum Survey
- ✓ Leica GNSS QC
- ✓ Leica Geo Office ©2011, version: 8.2.0.0
- ✓ Google Earth
- ✓ Teqc (Converter for Leica file format to RINEX)

Below are the instruments used in the project implementation:

- ✓ Leica Differential GPS (a complete set including two rovers and one base)
- ✓ Digital Camera
- ✓ Computers

3.2 Research Methodology

Datum transformation parameter determination between any two datums require the following procedures in chronological order:

1. Observation
2. Computation.

Prior to the period of data collection (March 10 – April 14, 2013), MLME stated that there were existing primary control points which were situated in the following communities:

- ✓ Ducor
- ✓ John F. Kennedy Memorial Hospital
- ✓ 72nd Camp Ramrod Barrack
- ✓ TB Annex
- ✓ Cemenco
- ✓ Battery Factory

Unfortunately, only two of the controls situated in these communities were accessible. The two include 72nd Camp Ramrod Barrack and the John F. Kennedy Memorial Hospital. Of these two, only the point at 72nd Camp Ramrod Barack has available coordinates.

Being cognizant of these developments, primary control points were planted in the following communities spread out as indicated on the map of the project area:

- ✓ Mamba Point
- ✓ Freeport
- ✓ St. Paul Bridge
- ✓ E.J Goodridge, Bardnersville
- ✓ Omega Tower Community
- ✓ TB Annex

These six Trigonometric Points (Common Points) [9] including a base station were used for the project. Not all these points could be adequately identified on the map (1:50,000) which is based on the Clarke 1880 Spheroid. Therefore, road interceptions of few communities were identified on the map together with two of the primary control points planted. They included:

Base Station

- ✓ 72nd Barrack

Road Interceptions

- ✓ 10th Street Coleman Avenue
- ✓ 10th Street Gardner Avenue
- ✓ 19th Street Coleman Avenue
- ✓ 19th Street Gardner Avenue

Control Point Planted

- ✓ St Paul Bridge

Additional Points Used

✓ TP19	TP18	TP17
✓ TP15	TP14	TP13
✓ TP12	TP11	TP10
✓ TP9	TP8	TP7
✓ TP6		

[9] Common Points are points that have known coordinates in both the local datum and the WGS 84.

3.2.1 Desk Study

A review of relevant information concerning the project was done. Topography maps, Google Earth image of the project area were acquired from the Ministry of Lands, Mines and Energy. During this stage, we also detailed on how the work will be done considering the prevailing circumstances in Liberia.

3.2.2 Reconnaissance

Site visitation was done to temporarily spot suitable areas for the planting of pillars. The various communities within the study area were visited and meetings were held with community leaders of these communities.

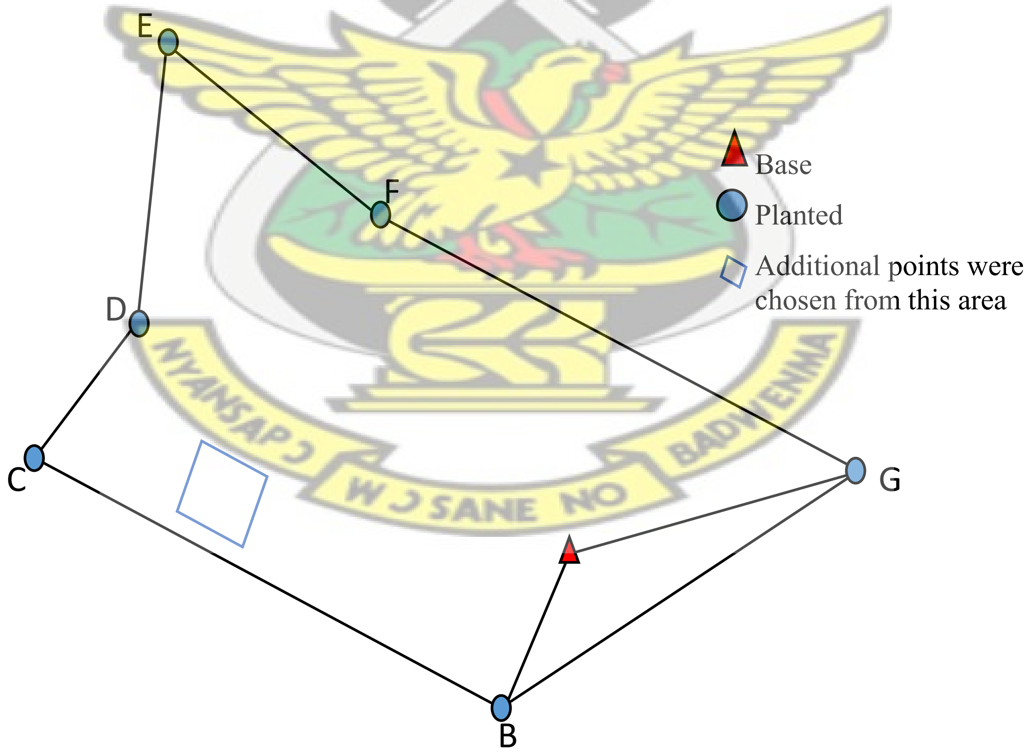


FIGURE 12 DIAGRAM OF SURVEY

3.2.3 Monumentation

Pillars were constructed in conformity with the design specification of the Ministry of Lands, Mines, and Energy (figures 13, 14, and 15). After construction, these pillars were planted in the six communities as indicated on page 39. Below is a photo of the pillars.

FIGURE 13 CONSTRUCTED PILLAR

FIGURE 14 PILLAR PLANTED AT FREEPORT

The pillars were constructed with pre-cast reinforced concrete. The monument specification was provided by the DLSC, while the Assistant Minister in person of George G. Miller provided the monument markers. The markers, made of brass, had the inscription: ***"Liberia Ministry of Lands, Mines, and Energy; Survey Marker; Do not disturb"***.

FIGURE 15 MARKER FOR PILLAR

3.2.4 Observational Procedure

For the determination of transformation parameters, it is better to use more common points in as this increases redundancy which subsequently leads to better results (Ghilani & Wolf, 2012). The observational procedure was initiated with the obtaining of raw GPS data of the common points (figure 12) over a period of three days (April 9 – 11, 2013) with an occupation period of three hours per station. This was done with the aid of the Leica Differential GPS. The nature of the data collected includes all forms which are convertible between different coordinate systems (Geodetic, Cartesian, UTM) including ellipsoidal heights. Additional data of more common points were obtained from previous survey done in August, 2012.

A base station was established at Camp Ramrod and with the method of *Precise Point Positioning*[10]. Since this station was intended to be used as the base, the occupation period was set to 10 hours. Afterwards, on each of the days, the base station was mounted together with two rovers for an occupational period of three (3) hours per station. This was done to establish a triangle at every stage of the data collection process for ease of adjustment. The observation was done in static mode enabling post processing. Raw data were post processed using the Leica Geo Office software in Liberia and the Spectrum Survey processing software in Ghana. The difference between the two processes were observed to be in percent of millimeter. Quality Checks were also done on all data obtained using the Leica GNSS QC to ensure data credibility.

As for the local data which should have been in the Clarke 1880 coordinate system, the map (1:50,000) was scanned and geo-referenced and the corresponding coordinates of the common points were extracted using spatial techniques in ArcGIS 10.0.

3.2.5 Computational Procedure

Transformation Parameters determination like any other geodetic computation requires the usage of computer program. At such, scripts were written in the Matlab Programming Language for this process. The choice of this software comes in the direction of its robustness in the handling of matrices of higher orders. These scripts follow the theory of Least Squares

[10] This is a Global navigation Satellite System (GNSS) positioning method to calculate very precise positions up to few centimeter lever using a single (GNSS) receiver in a dynamic and global reference framework like the International Terrestrial Frame (ITRF). (Witchayangkoon, 2000)

Adjustment to compute the transformation parameters (Fosu, 2006). In the wake of making the necessary statistical analyses, all models of the transformation parameters process were used. Details of the transformation models including formulas are indicated on page 44. Another software was written in Visual Basic 2012 to transform coordinates between the two systems. The choice comes in the wake that the software written in VB 2012 has a user-friendly interface which is backed by lots of "Help and Support" for end-users. This software makes use of the parameters computed from Matlab which are stored in a class in the Visual Basic Application.

Since the issue of height for the local coordinates in the Clarke 1880 system is paramount to the determination process, the Abridged Molodensky Model was used starting with equations (14, 15, 16) to compute ($\Delta X, \Delta Y$ and ΔZ). This is an iterative process done with the aid of MatLab with an initialization that the h = 0. The iteration continues until convergence is reached. The corresponding ellipsoidal heights of the common point in the Clarke 1880 coordinate system were computed using equation 21. The transformation parameter sets for each model was computed using their respective equations indicated on page 44 to 46.

Listed below are the results of the rearrangement of the various models in a matrix form. All the transformation models make use of the Least Square Solution,

$$\boldsymbol{X = (A^T * A)^{-1} * (A^T * L)} \ldots\ldots..27$$

This is the normalized form of the equation: $AX = L + V$. The vector of residuals is given by:

$$V = AX - L$$

3.2.5.1 The Abridged Molodensky (5 parameters) Model

Equations (14,15,16) were rearranged and organized and indicated below:

$$\boldsymbol{\rho sin1''\Delta\phi'' = [-\Delta x sin\phi_1 \cos\lambda_1 - \Delta Y sin\phi_1 sin\lambda_1 + \Delta Z cos\phi_1 + (a_1\Delta f + f_1\Delta a) sin2\phi_1]}$$

$$v cos\phi_1 sin1''\Delta\lambda'' = [-\Delta X sin\lambda_1 + \Delta Y cos\phi_1]$$

$$\Delta h = \Delta X cos\phi_1 cos\lambda_1 + \Delta Y cos\phi_1 sin\phi_1 + \Delta Z sin\phi_1 + (a_1\Delta f + f_1\Delta a)\, sin^2\, \phi_1 - \Delta a$$

Below is the resulting matrix which is of the form: $AX = L + V$

Where,

X is the matrix of unknown parameters, V is the matrix of residuals, A is the design matrix and L is the observation matrix.

$$\begin{bmatrix} sin\phi cos\lambda & sin\phi sin\lambda & -cos\phi \\ -sin\lambda & cos\phi & 0 \\ cos\phi cos\lambda & cos\phi sin\lambda & sin\phi \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} (a\Delta f + f\Delta a) sin2\phi - (\rho sin1")\Delta\phi" \\ \Delta\lambda"(v cos\phi sin1") \\ \Delta h - (a\Delta f + f\Delta a)\, sin^2\, \phi + \Delta a \end{bmatrix}$$

In this case,

$$A = \begin{bmatrix} sin\phi cos\lambda & sin\phi sin\lambda & -cos\phi \\ -sin\lambda & cos\phi & 0 \\ cos\phi cos\lambda & cos\phi sin\lambda & sin\phi \end{bmatrix},\ x = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix},\ L$$

3.2.5.2 The Molodenksy-Badekas (7 parameters) Model

Equations (11,12,13) were reviewed and the resulting matrix extracted:

$$A = \begin{bmatrix} 1 & 0 & 0 & \mu_x & 0 & -\mu_z & \mu_y \\ 0 & 1 & 0 & \mu_y & \mu_z & 0 & -\mu_x \\ 0 & 0 & 1 & \mu_z & -\mu_y & \mu_x & 0 \end{bmatrix}, x = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta S \\ R_x \\ R_y \\ R_z \end{bmatrix},\ L = \begin{bmatrix} X_{WGS84} - X_{Clarke} \\ Y_{WGS84} - Y_{Clarke} \\ Z_{WGS84} - Z_{Clarke} \end{bmatrix}$$

Where,

μ_x, μ_y, μ_z represent the below relation:

$$\begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix} = \begin{bmatrix} X_{Clarke} - X_m \\ Y_{Clarke} - Y_m \\ Z_{Clarke} - Z_m \end{bmatrix}$$

And

X_m, Y_m, Z_m is the centroid of the network based on the Clarke 1880 spheroid.

3.2.5.3 The Bursa-Wolf Transformation Model

Equation (17) was rewritten and reorganized as indicated below:

$$A = \begin{bmatrix} 1 & 0 & 0 & X_C & 0 & -Z_C & Y_C \\ 0 & 1 & 0 & Y_C & Z_C & 0 & -X_C \\ 0 & 0 & 1 & Z_C & -Y_C & X_C & 0 \end{bmatrix}, x = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta L \\ R_x \\ R_y \\ R_z \end{bmatrix}, \ L = \begin{bmatrix} X_{WGS84} - X_{Clarke} \\ Y_{WGS84} - Y_{Clarke} \\ Z_{WGS84} - Z_{Clarke} \end{bmatrix}$$

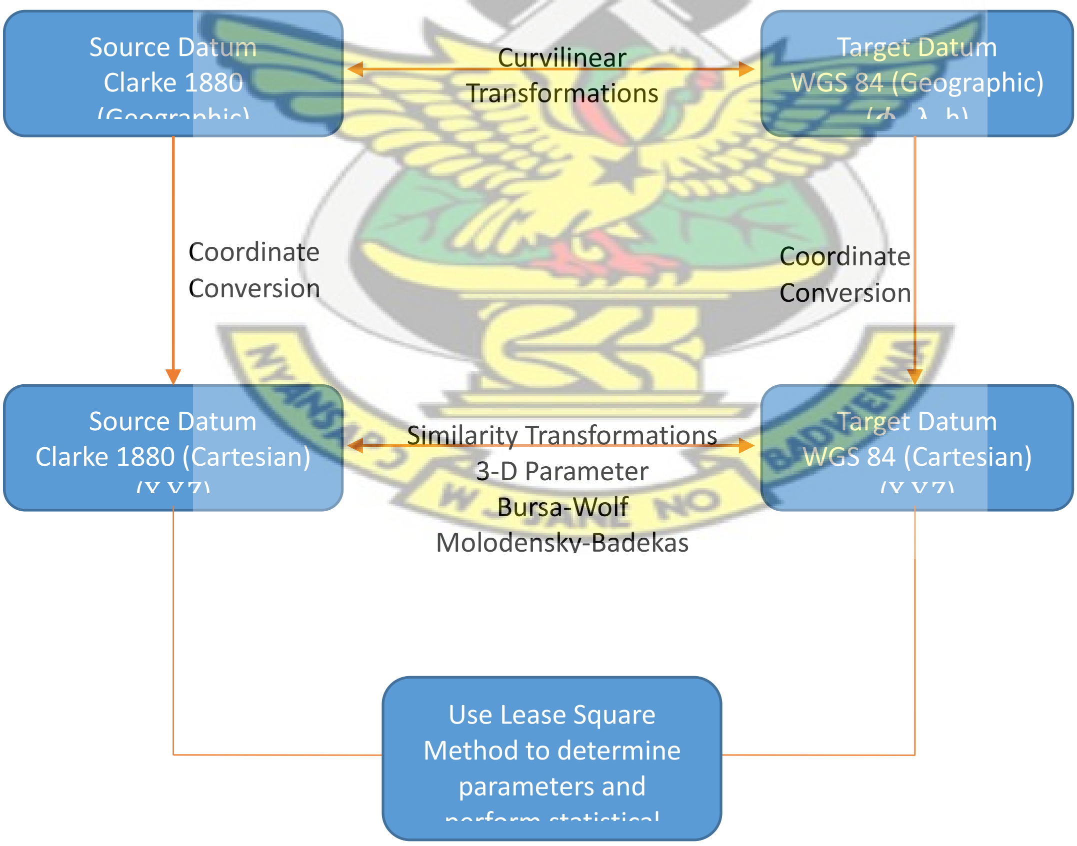
3.2.5.4 The Geocentric (Simple 3 Parameters) Model

To make use of this model, equation (18) was rewritten:

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix}$$

The least square solution becomes:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ x = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}, \ L = \begin{bmatrix} X_{WGS84} - X_{Clarke} \\ Y_{WGS84} - Y_{Clarke} \\ Z_{WGS84} - Z_{Clarke} \end{bmatrix}$$



The entire computation process is summarized in the following diagram.

FIGURE 16: DIAGRAM DEPICTING THE TRANSFORMATION PROCESS (FOSU, 2011)

3.2.6 Statistics of the "Determined Parameters"

After the determination of the parameters using different models as stated in chapter 2, statistics were performed on these models by computing the variance and standard deviation. The unit variance is given as: (Ghilana and Wolf, 2006)

$$\boldsymbol{\sigma^2 = \frac{V^T V}{n-u}} \ldots\ldots\ldots..28$$

Where n = number of measurements and u = number of unknown parameters. The standard deviation is also given by:

$$\boldsymbol{\sigma} = \sqrt{\boldsymbol{\sigma^2}} = \sqrt{\boldsymbol{\frac{V^T V}{n-u}}} \ldots\ldots\ldots..29$$

The standard deviation of the individual quantities is given by:

$$\boldsymbol{\sigma_x = \sqrt{\sigma^2 (q_{xx})}} \ldots\ldots\ldots\ldots30$$

Where q_{xx} is the diagonal matrix of the variance-covariance matrix which is denoted by (Ghilani & Wolf, 2012):

$$\boldsymbol{q_{xx} = (A^T A)^{-1}}$$

Another component of this work that tells us how correlated our data are is the residuals. The residuals plot allows us to determine if the regression model is a good fit of the data. When plotted, the residuals should (Decoursey, 2003):

- ✓ be horizontal. If the residuals are curved or have a slope, then your regression model is not accounting for all but the random variation in the data.

- ✓ have about the same width throughout the range. If they don't, then your model doesn't meet the requirement for equal variance.

- ✓ be uniformly scattered along the horizontal axis. If they aren't then your data are clustered and your regression model could be biased.

- ✓ be random. There should be no recognizable pattern. Good regression models give uncorrelated residuals.

3.2.7 Testing of the "Determined Parameters"

Since there were no known projected coordinates for the common in the Clarke 1880 spheroid based on the Liberian 1964 datum, the accuracies of this project work rely solely on the statistics of the determined parameters. The standard error which is based on the standard deviation of the data and the number of points is used to establish how close the parameters falls below or above the actual values (Montgomery et al, 2003). For each of the models, the standard deviation and individual standard error were computed. This is indicated in the results.

$$\boldsymbol{Standard\,Error\ (s.e) = \frac{\sigma}{\sqrt{n}}} \text{........31}$$

To test the validity of the determined transformation parameters, a hypothesis test for each of the parameters determined was performed. To accomplish this, a probability plot was done first to establish whether the residuals are normally distributed before the hypothesis test can be implemented.

The probability plot makes use of the index of the ordered data in ascending order (X_i) which is then used to compute the cumulative frequencies with the formula below. The corresponding standard z-scores of the cumulative frequency was then plotted.

$$\mathbf{c.f = \frac{j-0.5}{n} = P(Z \leq z_j) = \Phi(z_j)......} 32$$

To check the validity of the residuals, a straight line is drawn between the 25th and 75th percentile point. The probability plot is useful in identifying skewed distributions. When a sample is selected from a light-tailed distribution (such as the uniform distributions), the smallest and largest observations will not be as extreme as would be expected in a sample from a normal distribution. Thus if we consider the straight line drawn through the observations at the center of the normal probability plot, observations on the right side will tend to fall above the line. This will produce an S-shaped normal probability plot such as show in the figures in the results. (Lange, 2003)

After the probability test was done, the hypothesis test was performed for each of the transformation model determined. This test is to ensure that the determined parameters are statistically different from zero and therefore, significant. Below is the formula for the Hypothesis Test:

$$t = \frac{|\text{Parameter}|}{\text{SD}}$$

SD is the standard deviation of the individual parameters (Ghilana and Wolf, 2006).

This test was done at 5% level of significance. Therefore, α = 0.05. In this case, the null hypothesis becomes

$$H_0: \mu = 0 \text{ (Each parameter is not statistically different from zero)}$$

Whereas, the alternative hypothesis is:

$$H_0: \mu \neq 0 \text{ (Each parameter is statistically different from zero)}$$

The trump card behind this test is that null hypothesis (H_0) is rejected if $t > t_{\left(\frac{\alpha}{2},\ V\right)}$, where V is the degree of freedom. The value $t_{\left(\frac{\alpha}{2},\ V\right)}$ is the corresponding t-distribution critical value taken from the t-table. The degree of freedom is the difference between the sample space and the number of parameters for that model. Therefore,

$$V = n - p.$$

If the t-value of the parameter is greater than $t_{\left(\frac{\alpha}{2},\ v\right)}$, the null hypothesis of that parameter is rejected and vice versa.

3.3 Concluding Remark

Due to the damaging and/or inaccessibility of primary control points previously planted in Liberia, additional pillars for the project were planted. Readings were taken on the base station for 10 hours to enable Precise Point Positioning as there were no coordinates available for this station. Two GPS rovers were mounted on the points together with a base station each day for six days to collect data for an observational period of three (3) hours. Raw data were processed both in Liberia and Ghana and subsequently computations were done on the coordinates to determine the parameters. Statistics were done on the parameters to determine credibility and testing was also done to substantiate accuracies emanating from the individual models.

CHAPTER 4: RESULTS AND DISCUSSIONS

In this chapter, the results in a tabular format as determined from the computational procedures are presented and discussed. Included also are graphs of the residuals and standard errors, which adequately explain the results. The results are discussed in the light of statistics tests on each of the four models as well as probability plot and hypothesis testing.

4.1 Results

After various computations made on the common points which comprise of two control points and seventeen points chosen at random from the study area, the Abridged Molodensky Five-Parameter, the Geocentric-Three Sample Parameter, the Bursa-Wolf and the Molodensky-Badekas Seven Parameters yielded results as indicated in the following tables. This was done through the use of a least square approach which determined the transformation parameters, the residuals and the also conducted post statistics on the results.

Since there were no test data to work with, the research make use of the measure of central tendencies (Variance, Standard Deviation, Standard Error of individual parameters, etc) to approximate how far or how close they occur from the accepted value. Probability plot and hypothesis test were done on the residuals from each of the models. Moreover during the computation, all measurements were assumed to carry equal weight.

4.1.1 *Result from the Abridged Molodensky Model*

Table 2 below shows the parameters computed from the Abridged Molodensky Model. Statistically, it is evident that there are 68.56 percent probability that the parameters lie with the range of ±1.06m. It can be seen that the Standard Error in the shift parameters (**Δ** X and **Δ** Z) are the same whereas **Δ** Y is slightly different in the third decimal place.

TABLE 2: RESULTS FROM THE ABRIDGED MOLODENSKY (5 PARAMETERS) MODEL

Parameter	Value	Standard Error(m)
Δ X(m)	-131.8852143	0.243754298
Δ Y(m)	17.56531138	0.241006892
Δ Z(m)	221.1661789	0.243753737
Δ a(m)	-112	
Δ f	-5.48088E-05	
Standard Deviation	1.062497907	

4.1.2 *Result from the Geocentric (Simple Three Parameters) Model*

Table 3 below shows the parameters computed from the Geocentric Three Parameters Model. From the computation, there is a 68.82 percent probability that the parameters lie within the range of ±1.062m. The Standard Error is the same in each case for each of the shift parameters (**Δ** X, **Δ** Y, **Δ** Z).

Parameter	Value	Standard Error(m)
***Δ** X(m)*	-131.838583	0.243788408
***Δ** Y(m)*	17.76572162	0.243788408
***Δ** Z(m)*	220.9363015	0.243788408
Standard Deviation	1.062649033	

TABLE 3: RESULTS FROM THE GEOCENTRIC (SIMPLE 3 PARAMETERS) MODEL

4.1.3 Result from the Molodensky-Badekas (Seven Parameters) Model

Table 5 below shows the parameters computed from the Molodensky-Badekas (7 Parameters) Transformation Model. It can be seen that there are 68.86 percent that the parameters lie within the range of ±1.2169m. Additionally, the standards errors are the same throughout for the shift parameters.

Parameter	Value	Standard Error(m)
Δ X(m)	-131.838583	0.279176232
Δ Y(m)	17.76572162	0.279176232
Δ Z(m)	220.9363015	0.279176232
Rx(rad)	-0.000572572	8.78E-05
Ry(rad)	0.005887381	8.98E-05
Rz(rad)	-0.001138785	0.00018664
Δ S	6.39ppm	1.46ppm
Standard Deviation	1.216900985	

TABLE 4: RESULTS FROM THE MOLODENSKY-BADEKAS (7 PARAMETERS) MODEL

The figure below is the graph of all models with their Standard Error. This graph explains that the greater Standard Errors were obtained from the Molodensky Models whereas the Abridged and Geocentric Models have the same Standard Errors.

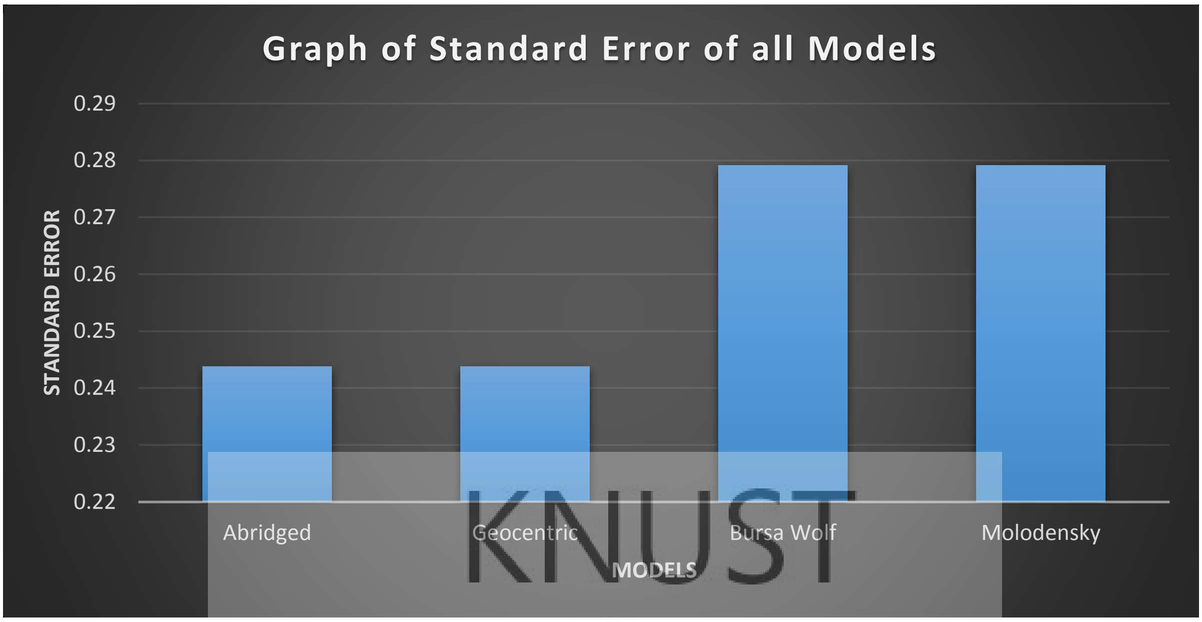


FIGURE 17 GRAPH OF STANDARD ERRORS

4.1.4 Scatter Diagram of Residuals of Common Points

FIGURES FIGURE 18 –

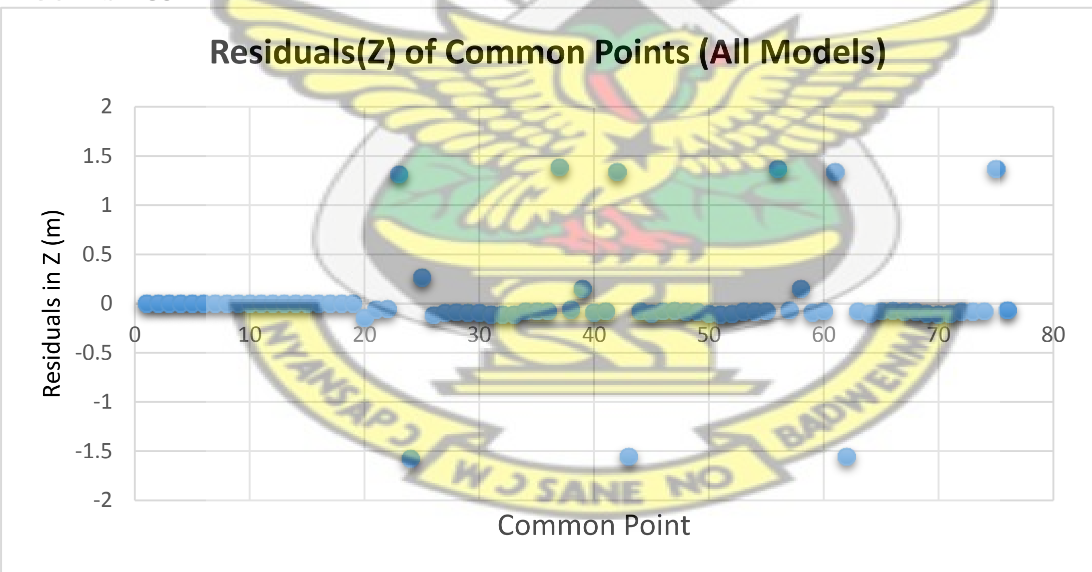


Figure 20 are scatter diagrams of the residuals generated from all models. These residuals tell us how the correlated or distributed our data are.

Figure 18 shows that except for few points in the Abridged Molodensky Transformation Model which are randomly distributed, the residuals in X (Vx) are highly correlated.

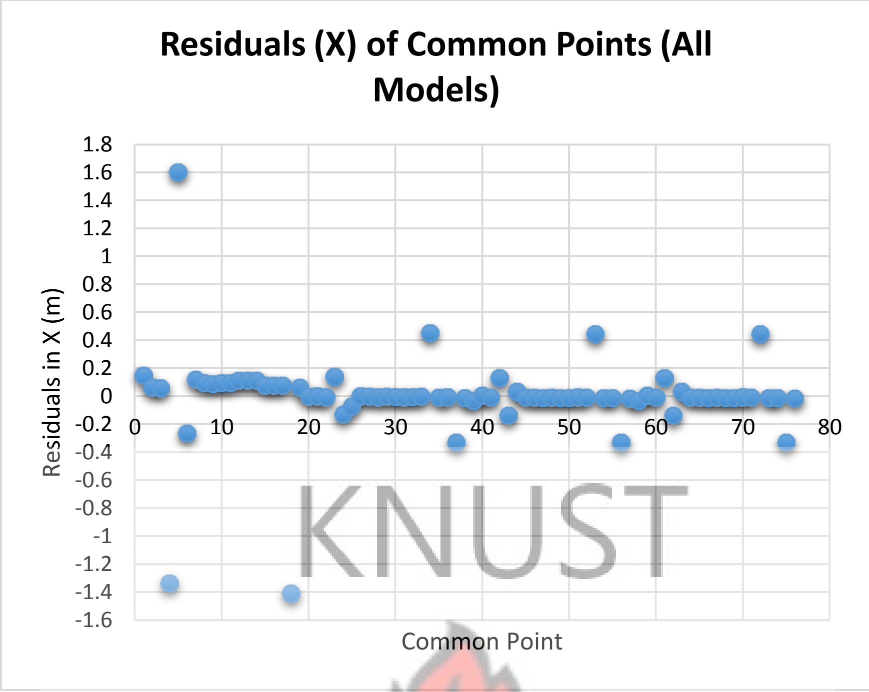


FIGURE 18: PLOTTED DIAGRAM OF THE RESIDUALS IN X

Figure

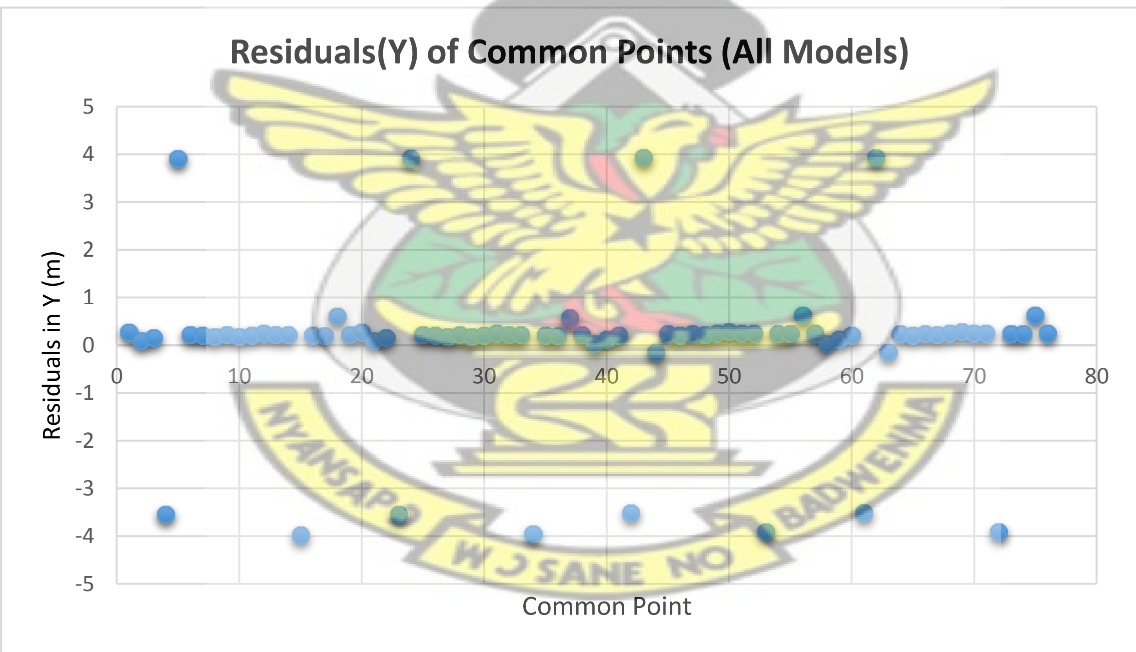


Figure 19 shows that the residuals in Y (Vy) are correlated except for few points in each model which are randomly distributed as indicated in the figure.

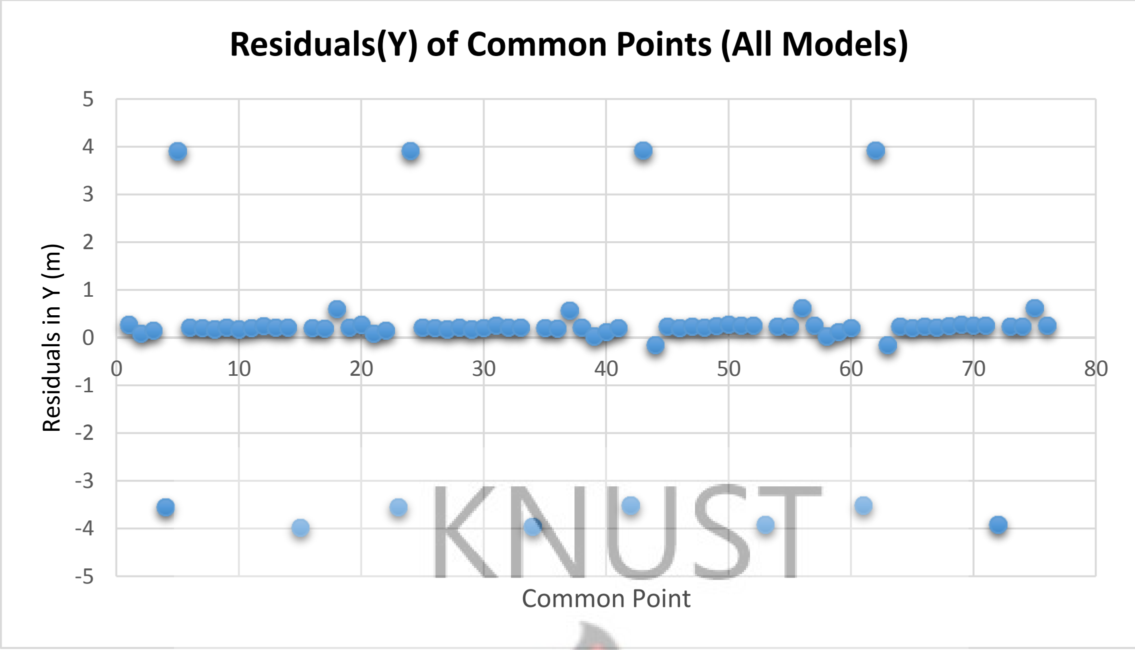


FIGURE 19: PLOTTED DIAGRAM OF THE RESIDUALS IN Y

FIGURE

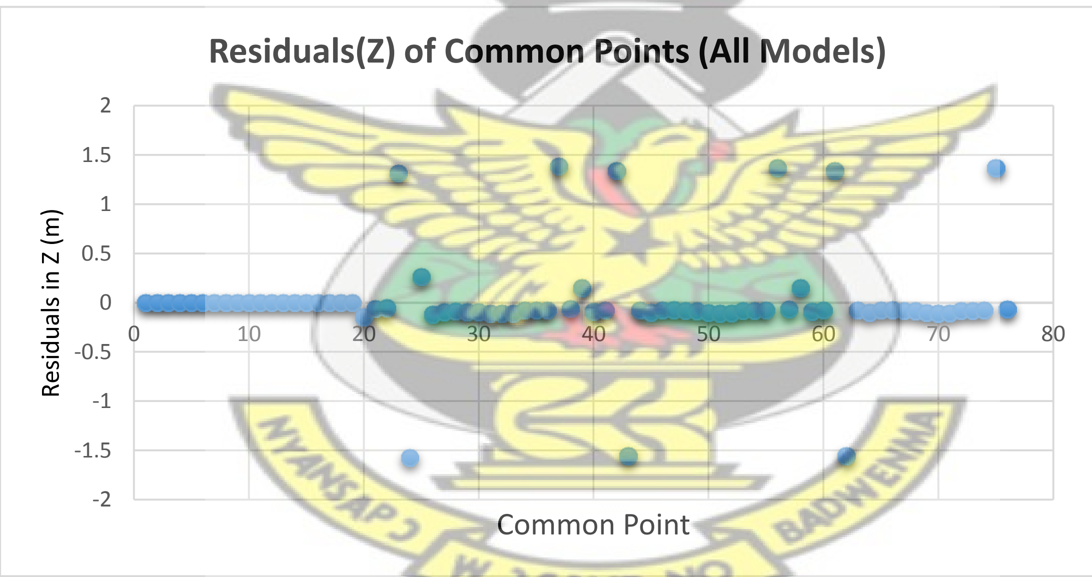


Figure 20 shows that, the residuals of the Abridged Molodensky Model are highly correlated while some points from the remaining three models are correlated and few are randomly distributed.

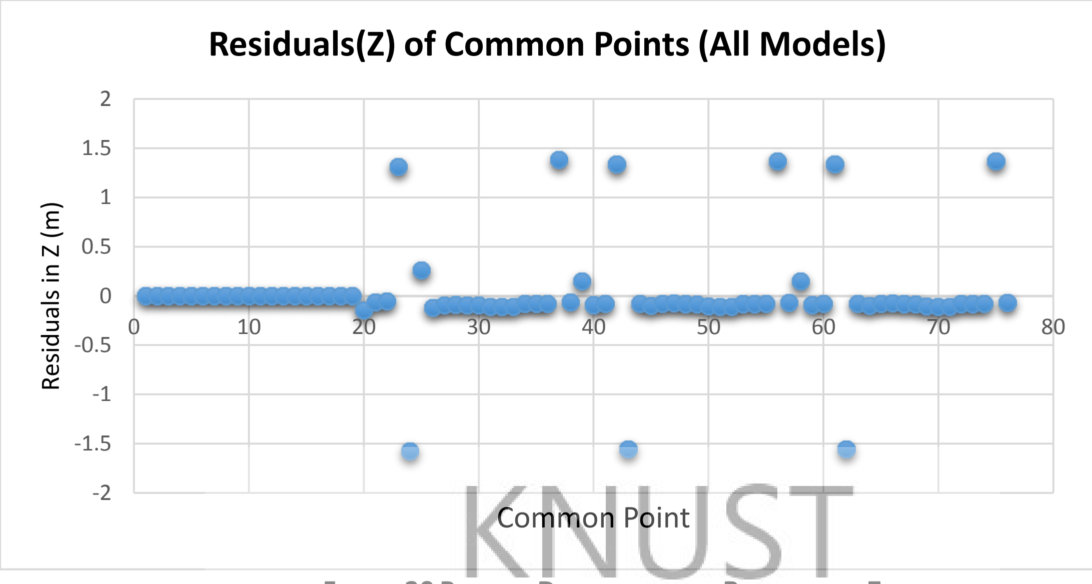


FIGURE 20 PLOTTED DIAGRAM OF THE RESIDUALS IN Z

Figures

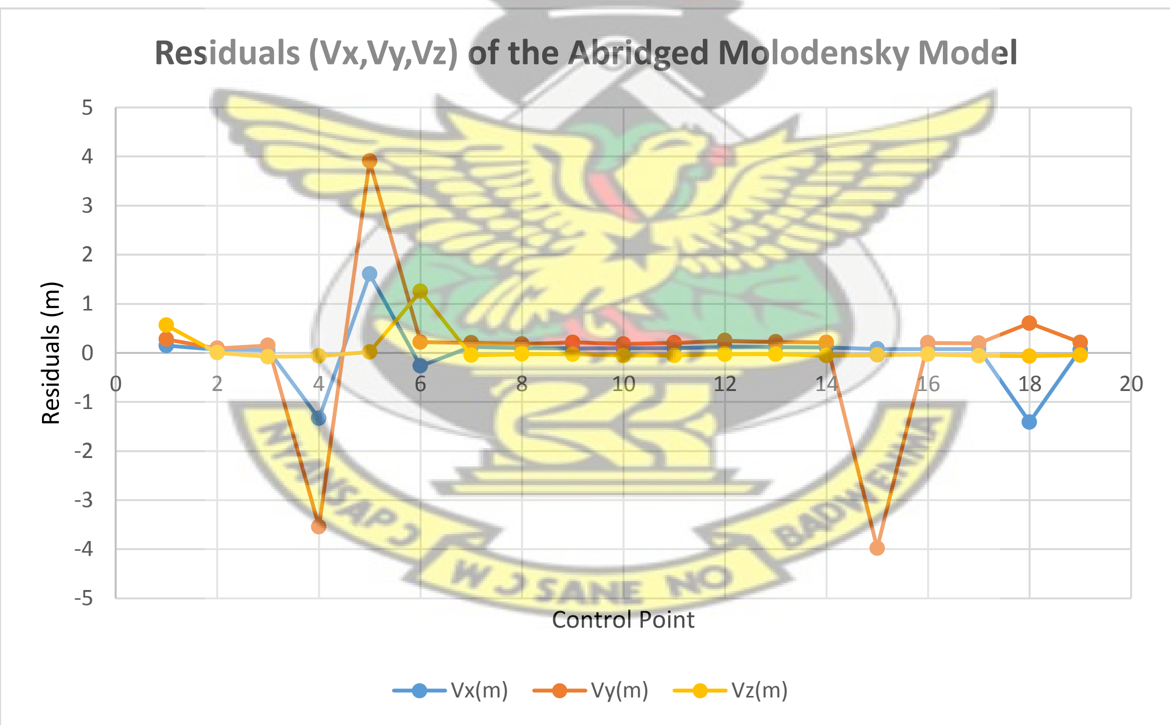


Figure 21 –

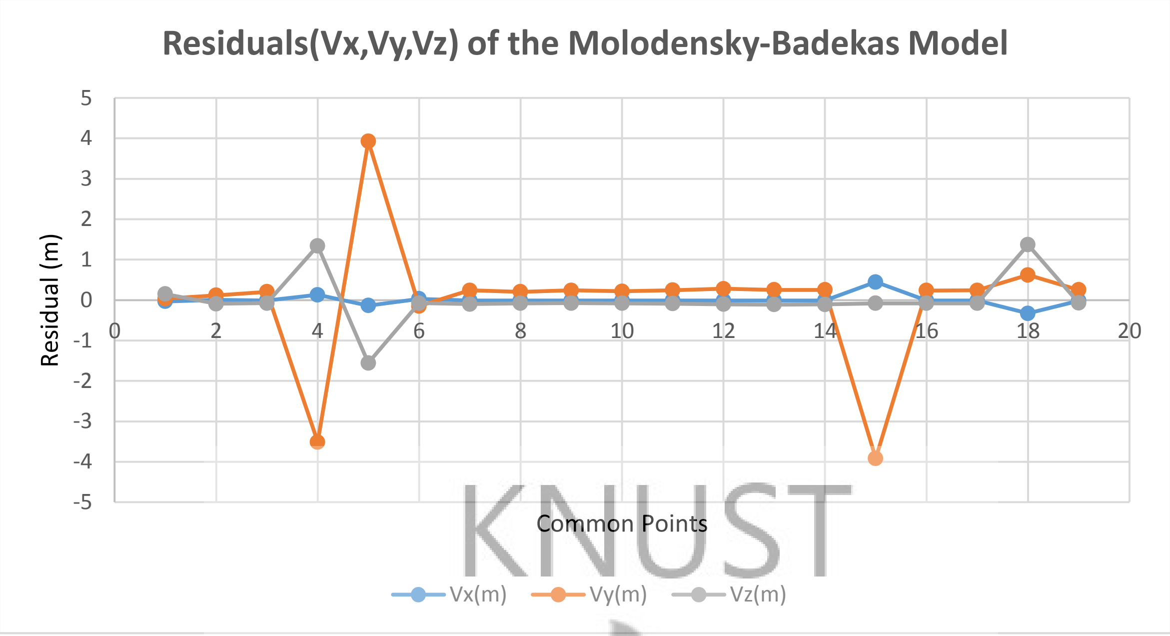


Figure 23 below are graphs showing the relationship between the residuals with the points. The Abridged Molodensky Model shows that the residuals in X (Vx) is comparable to that of Z (Vz) while the other graphs show that the residuals in X (Vx) are the least while those of Y (Vy) are the highest. For the other models, this means that the residuals in X (Vx) are correlating to the expected value and not randomly distributed.

The graph below shows that there was a significant rise at control point 5 (Vx) and a fall at control points 4 and 15 (Vy) while the residuals for the other control points are smoothly correlated.

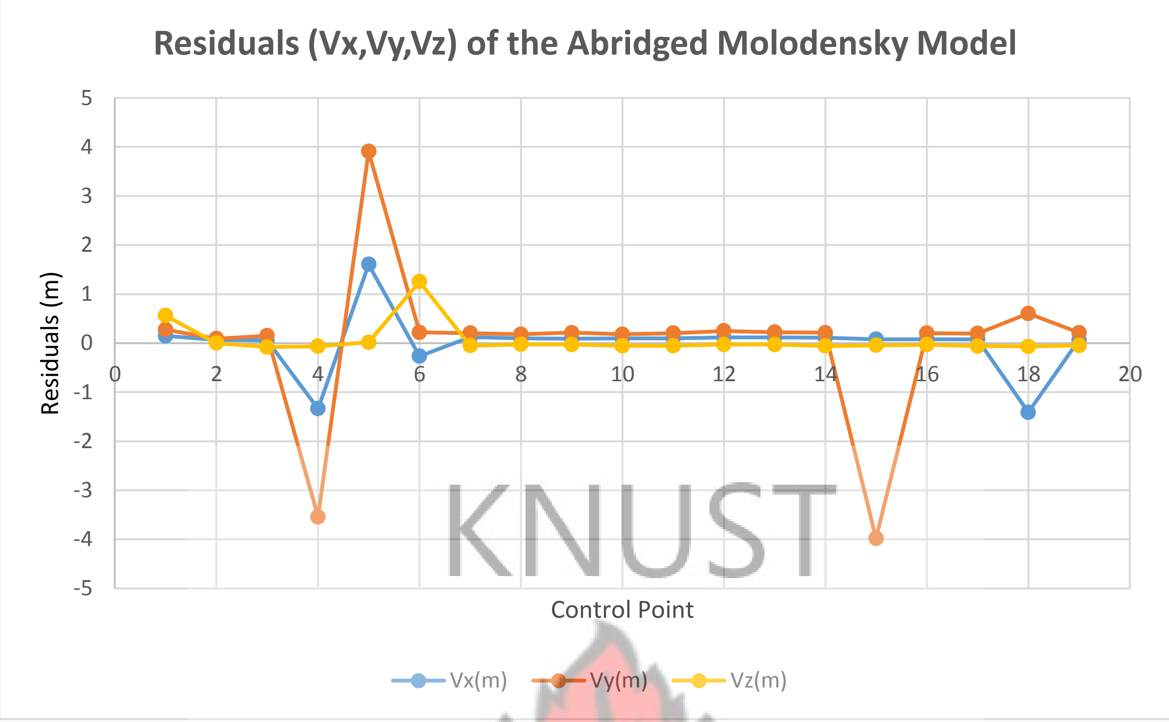


FIGURE 21: GRAPH OF THE RESIDUALS FOR THE ABRIDGED MOLODENSKY MODEL

The graph below shows that there is a little rise at control point 4 (Vz) and a significant rise at control point 5 (Vy) and a fall at control points 4 and 15 (Vy) while the residuals for the other control points are smoothly correlated.

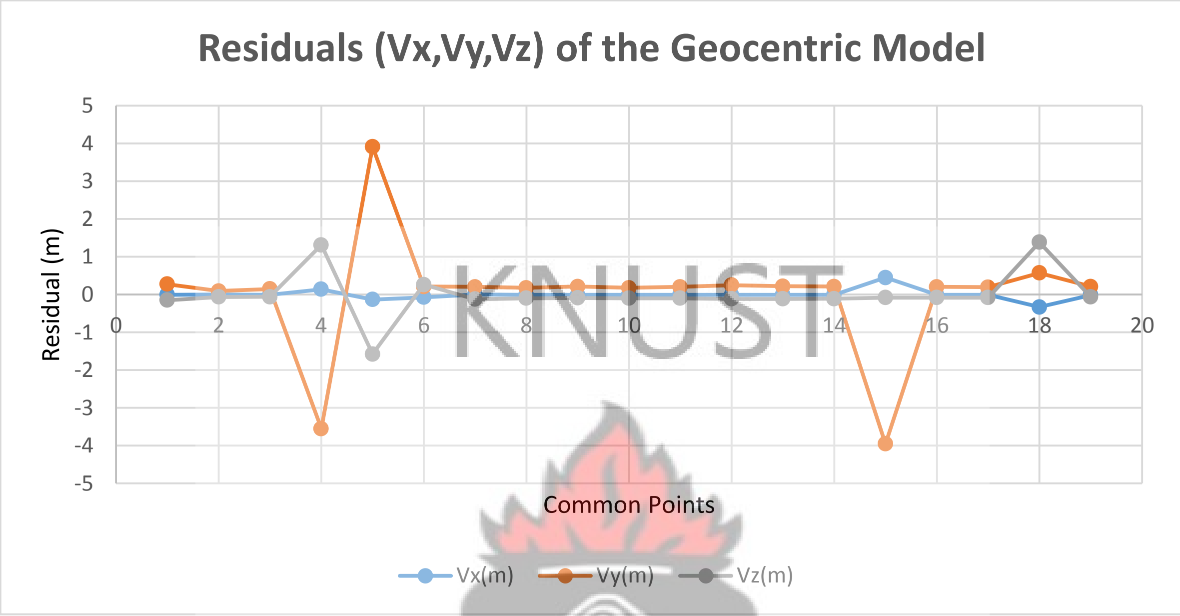


FIGURE 22: GRAPH OF THE RESIDUALS FOR THE GEOCENTRIC TRANSFORMATION MODEL

The graph below shows that there is a little rise at control point 4 (Vz) and a significant rise at control point 5 (Vy) and a fall at control points 4 and 15 (Vy) while the residuals for the other control points are smoothly correlated.

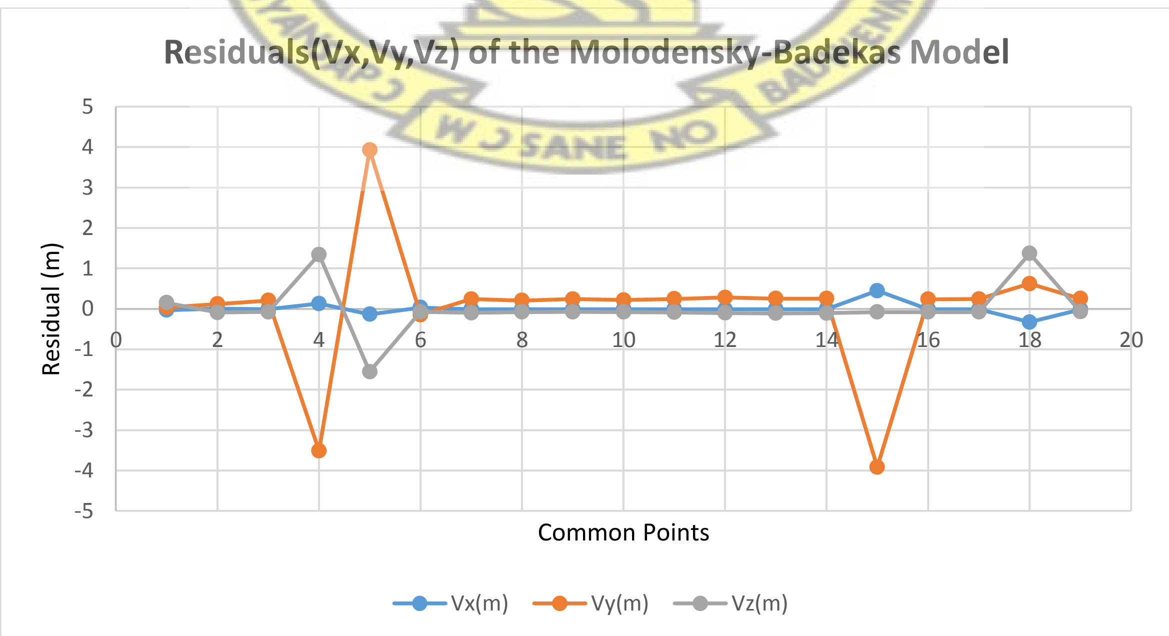


FIGURE 23: GRAPH OF THE RESIDUALS OF THE MOLODENSKY-BADEKAS MODEL

The figure below shows the residuals from all the models. Ideally, the residuals from all models are smaller except for those in Y (Vy).

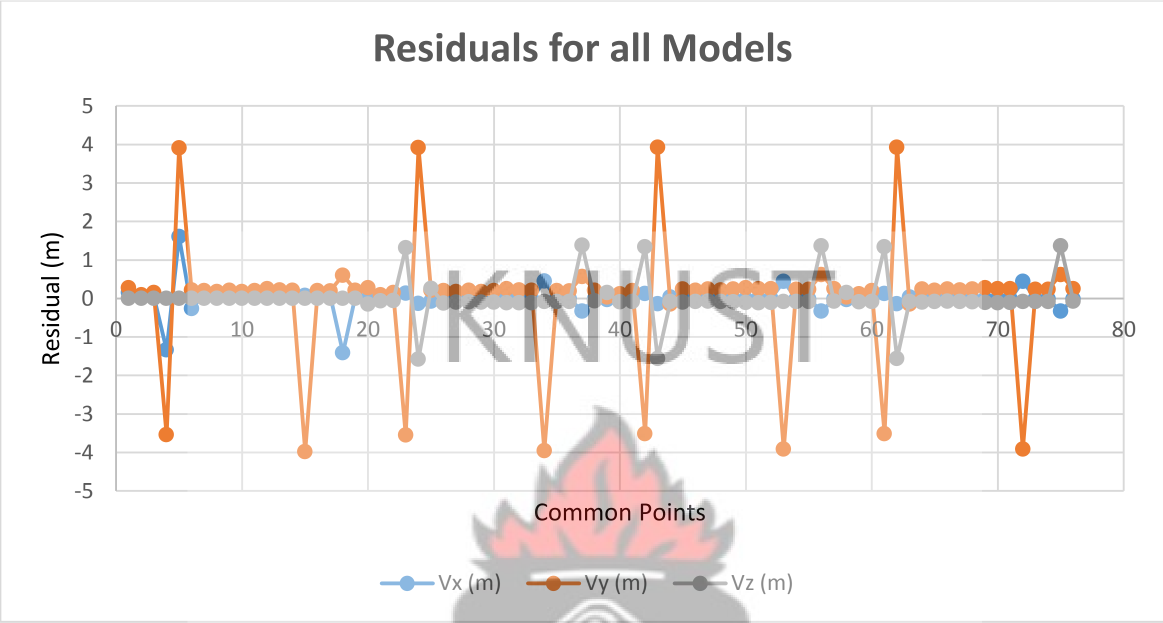


FIGURE 24: GRAPH OF THE RESIDUALS FOR ALL MODELS

4.2 Test Results

The residuals from all the models were tested using the normal probability plots. The graph consists of the residuals and the expected normal values corresponding the z-scores (standard scores) of the normal distribution. The importance of this test is to investigate whether processed data exhibit the standard normal "bell curve" or the Gaussian Distribution. Due to the chosen intervals from the plotting software and the variation between the dataset, the s-shape could not be adequate established.

4.2.1 Normal Probability Plots of the Residuals (Vx)

The graph below shows that the residuals in X (Vx) computed from all the models are normally distributed. This is due to the fact that the plotted points fit properly along the normal line. Since this is the major requirement of the Probability Plot, we can evidently suggest that our data are normal distributed.

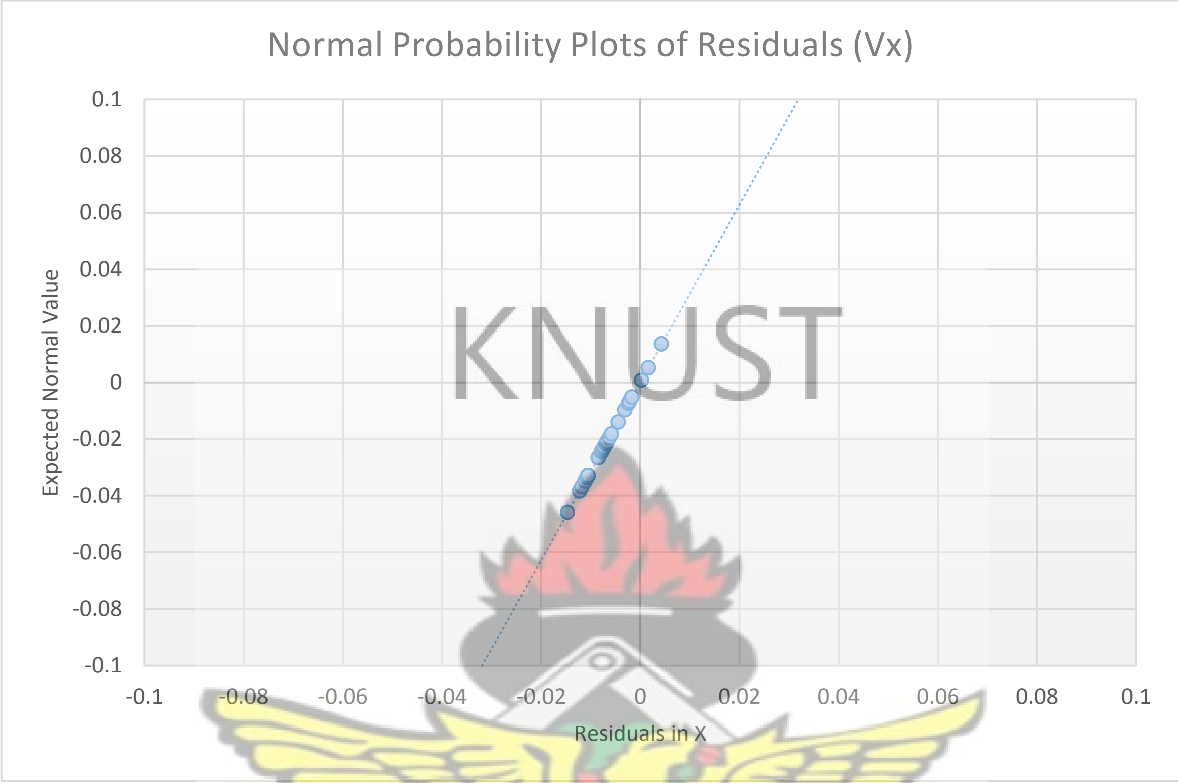


FIGURE 25 NORMAL PROBABILITY PLOTS OF RESIDUALS (VX)

4.2.2 Normal Probability Plots of the Residuals (Vy)

Again, we say our parameters as computed are normally distributed. This is indicated in the graph below. This is due to the fact that the plotted points fit properly along the normal line. Since this is the major requirement of the Probability Plot, we can evidently suggest that our data are normal distributed.

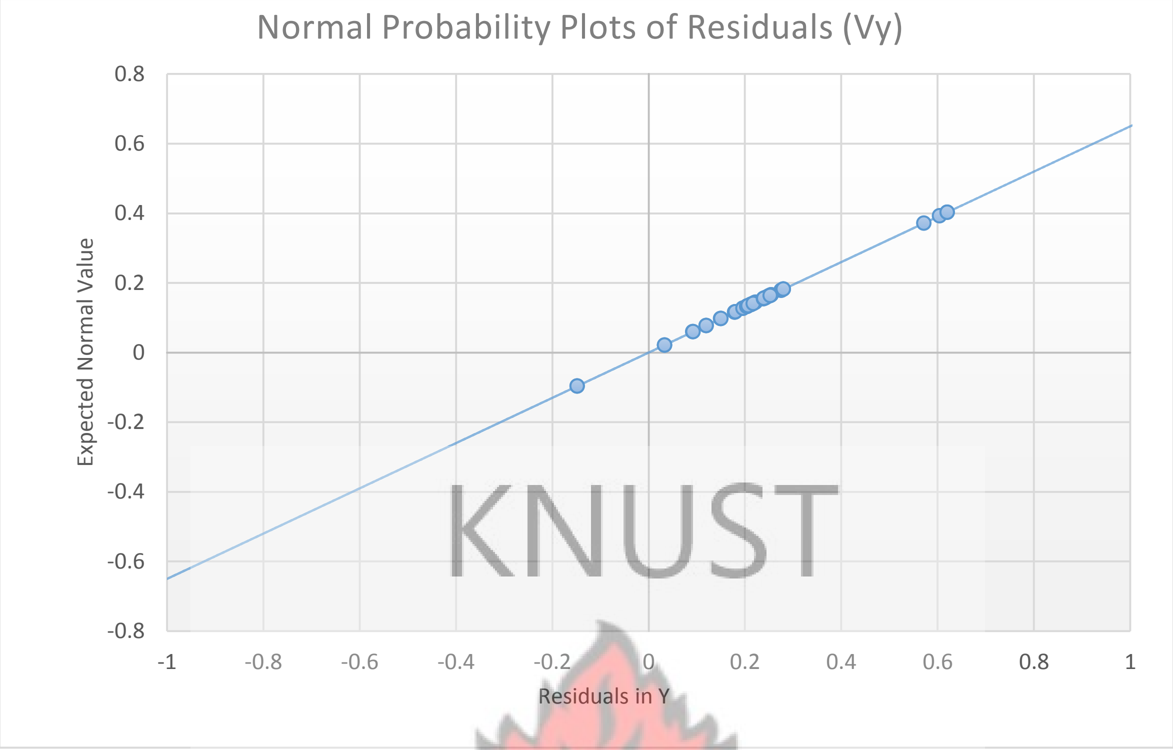


FIGURE 26 NORMAL PROBABILITY PLOT OF RESIDUALS IN Y (VY)

4.2.3 Normal Probability Plots of the Residuals (Vz)

As stated earlier, we say our parameters as computed are normally distributed. This is indicated in the graph as below. This is due to the fact that the plotted points fit properly along the normal line. Since this is the major requirement of the Probability Plot, we can evidently suggest that our data are normal distributed.

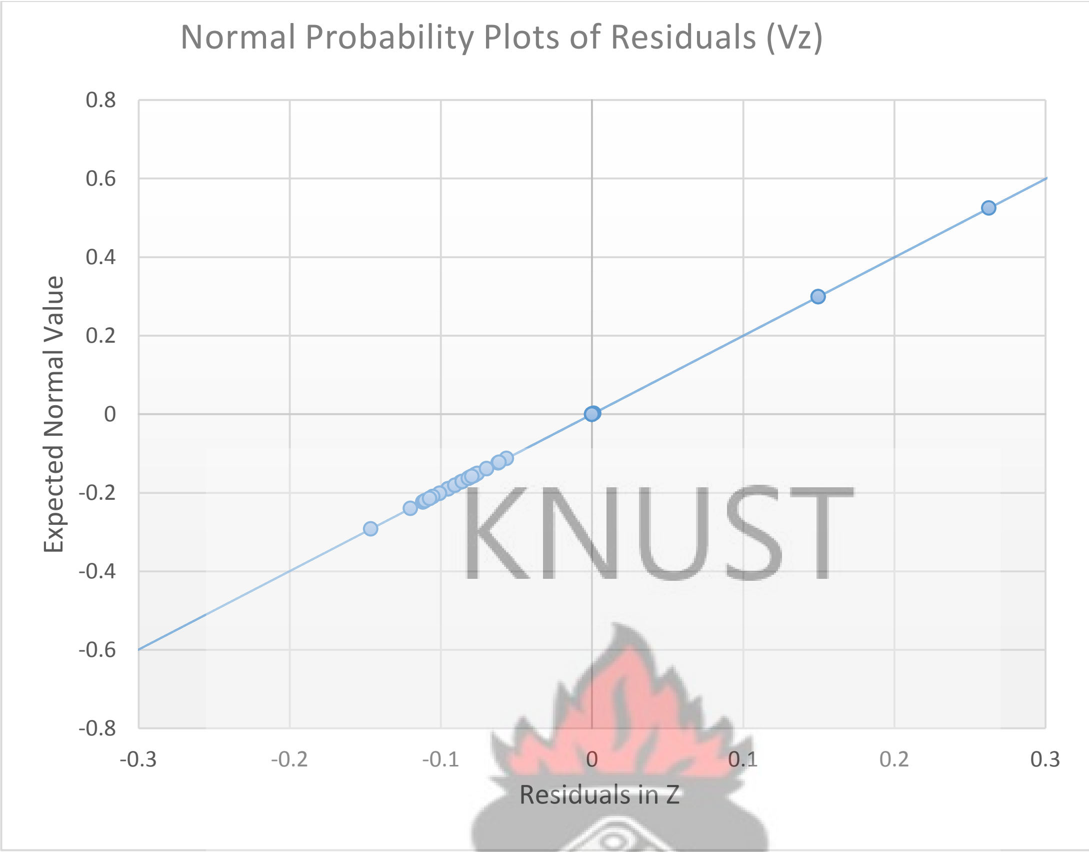


FIGURE 27

NORMAL PROBABILITY PLOT OF RESIDUALS IN Z (Vz)

4.2.4 T-Values of the Transformation Parameters

Table 6 below shows the t-values computed from the determined parameters whereas table 7 shows the t-distribution critical values. Since the values in the both tables are all greater than 0, it can be stated that all the parameters are statistically different from zero at 95% confidence level. Therefore, the null hypothesis of H_0: $\mu = 0$ is rejected for all models.

Also, by making a comparison between the two tables, we can say that the parameters computed from all models are statistically significant. Therefore, there is a non-zero relationship between the WGS84 and the Clarke 1880 spheroid.

T-Values of the Parameters							
Models	**tdx**	**tdy**	**tdz**	**tRx**	**tRy**	**tRz**	**tS**
Abridged Molodensky (5 parameters)	541.058	72.061	907.332				
Geocentric (Simple 3 parameters)	540.791	72.873	906.262				
Molodensky Badekas (7 parameters)	472.241	63.636	791.386	6.52	67.07	12.97	7.29

TABLE 5 T-VALUES OF THE TRANSFORMATION PARAMETERS (ALL MODELS)

Transformation Model	$t_{\left(\frac{\alpha}{2}, v\right)}$
Abridged Molodensky (5 parameters)	2.145
Geocentric (Simple 3 parameters)	2.120
Molodensky Badekas (7 parameters)	2.179

TABLE 6 T-DISTRIBUTION CRITICAL VALUES

CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In accordance with our objectives relative to the determination of Transformation Parameters for the project area between the WGS84 and the Clarke 1880 spheroid, the parameters were determined according to prescribed method. Initially, various transformation models were studied. On the basis relevance to the project and appropriation, four transformation models were reviewed and used. These models are frequently found in most geodetic transformation software.

Data were collected from points in Liberia totaling 19 common points. The corresponding Clarke 1880 coordinates of the common points were extracted from the Topographic Map (1:50,000) provided by the Department of Lands, Survey and Cartography (DLSC) using spatial techniques. Raw data were processed both in Liberia with the aid of the Leica GeoOffice and Spectrum Survey Processing Suite in Ghana.

A MatLab program was written to determine the parameters. This program follows the procedures of Least Square Adjustment to determine the parameters. Another program was written in Visual Basic 2012 which will be used to transform coordinates between the two systems using the parameters determined in MatLab. Below is a summary of the transformation parameters as determined by the four models.

After series of statistical test and analysis of the parameters, it was established that the Geocentric (Simple 3 parameters) and Molodensky-Badekas Seven (7) Parameter Model are the most suitable models for coordinate transformation and projection in the study area. However, the Molodensky-Badekas Seven (7) Parameter Model gives a better representation of the transformation due to the introduction of rotations.

PARAMETERS	ABRIDGED MOLODENSKY	GEOCENTRIC MODEL	MOLODENSKY BADEKAS MODEL
ΔX(m)	131.885 ±0.243	-131.838±0.243	-131.838±0.279
ΔY(m)	17.565±0.241	17.765±0.243	17.765±0.279
ΔZ(m)	221.166±0.243	220.936±0.243	220.936±0.279
Δa(m)	-112		
Δf	-5.48088E-05		
Rx(rad)			-0.000572572 ±0.000087
Ry(rad)			0.005887381 ±0.000087
Rz(rad)			-0.001138785 ±0.000108
ΔS (ppm)			(6.39 ±1.46)ppm
Standard Deviation	1.062	1.062	1.216

TABLE 7: SUMMARY OF RESULTS

The parameters showed that there is a negative displacement in the X-axis and positive displacement in the Y and Z axis for all the models. We have seen that the standard errors and standard are within acceptable range as their weights are relatively small than the parameters themselves. After the computation, it was seen that our unit variance (Standard Error of Unit Weight) was found to be unity. This means that standard errors are well estimated.

Of the parameters determined, the Molodensky Badekas Model gave suitable results for transformation of coordinates in the study area. This is due to the relative smaller error and standard deviation as compared to other models used. Moreover, when transforming from a satellite datum to a local datum, this model is more suitable. The Bursa wolf computations are neglected because the results obtained show marked deviations from all the rest of the models,

5.2 Recommendation

After the implementation of this project it is recommended that:

1. A research be done to determine the geoidal model of Liberia which will give a better representation of the transformation parameters taking into account the different height systems.
2. Since there were little or no information available in Liberia, another research be done on this topic to determine these parameters using more rigorous techniques.
3. There is a need for the parameters to be tested on test data which are unavailable at the moment.
4. The establishment of a local coordinate system of Liberia which will be based on a customized spheroid to suit the Liberian grid. This is something similar to the 'World Office' used in Ghana and the North American Datum (NAD) used in the USA.

REFERENCES

(Accessed, 2013). Retrieved from http://www.ngs.noaa.gov/PUBS_LIB/Geodesy4Layman/TR80003D.HTM.

Altamimi et al. (2011, April 07). Consistency evaluation of space geodetic techniques via ITRF combination.

Andrei, C.-O. (2006). *3D Affine Coordinate Transformation.* MSc Thesis.

Ayer, J. (2008). *Transformation Models and procedures for Framework Integration of the Ghana National Geodetic Network.*

Bowring, B. R. (1976). *Transformation from spatial to geographical coordinates.*

Dadzie, D. I. (2011). GE 353 - Geodetic Surveying I and 2.

Dana, P. (1997). Coordinate Systems Overview. .

Deakin, R. (2004). *A Guide to the Mathematics of Map Projections.* RMIT University, GPO Box 2476V, Melbourne VIC 3001.

Deakin, R. (2004). *The Standard and Abridged Molodensky Coordinate Transformation Formulae.*

Deakin, R. (2006). *A Note of the Bursa-Wolf and Molodensky-Badekas Transformations.*

Decoursey, W. J. (2003). *Statistics and Probability for Engineering Application.* Elsevier Science (USA).

Dursun et al. (2002). Orthometric Height Derivation from GPS Observations. In D. D. Yildirim.

EPSG. (2009). Coordinate Conversions and Transformation including Formulae (Revised).

EPSG, E. P. (2005). *Coordinate Conversions and Transformation including Formulae.*

ESRI, E. S. (2010). Map Projection.

Featherstone et al. (2000). The importance of using deviations of the vertical in the reduction of terrestrial survey data to a geocentric datum, the Trans-Tasman Surveyor.

Fosu, C. (2006). *Lectures Notes: Introduction to Geodesy.* Geomatic Engineering Department, KNUST, Kumasi.

Fosu, C. (2011, September). Introduction to Geodesy. *MSC Lecture Notes (Geodetic Models)*.

Frank Pichel et al. (2012). *Implementing an affordable, rapid deployment land record management solution for Liberia.*

Ghilana and Wolf. (2006). In P. P. Charles D. Ghilani, *Adjustment Computations - Spatial Data Analysis, Fourth Edition.* John Wiley & Sons, Inc.

Ghilani, C. D., & Wolf, P. R. (2012). *Elementary Surveying-An Introduction to Geomatics 13th Edition.*

Harvey, B. R. (1985). *Transformation of 3D Coordinates.*

Hoar, G. J. (1982). *Satellite Surveying, Theory, Geodesy, Map Projections.*

http://allafrica.com/stories/201112020719.html. (2011, December). Retrieved November 26, 2012, from http://allafrica.com/stories/201112020719.html

ICSM. (2002). *Geocentric Datum o Australia Technical Manual.* Retrieved from Inter-governmental Committee on Surveying & Mapping, Canberra: http://www.icsm.gov.au/icsm/gda/gdatm/index.html

J. Ayer and T. Tiennah. (2008). *Datum Transformations by Iterative Solution of the Abridging Inverse Molodensky Formulae.*

Jekeli, C. (2006). *Geometric Reference Systems in Geodesy.*

Kennedy, M. (2000). *Understanding Map Projections.* Environmental Systems research Institute, Inc.

Kutoglu et al. (2002). *A Comparison of Two Well-Known Models for 7 Parameters Transformation.*

Kutoglu, S. et al. (2002). *A Comparison of two well known models for 7-parameter transformation.*

Lange, K. (2003). Applied Probability.

Li et al. (2001). Ellipsoid, geoid, gravity, geodesy and geophysics. In X. L. Gotze.

LISGIS. (May 2008). *2008 National Population and Housing Census Final Results.* Liberia Institute for Geo-Information Statistics.

Millennium Challenge Corporation, M. (2011). *Strategy for Modernizing the Geodetic Infrastructure of Liberia.*

Montgomery et al. (2003). In G. C. Douglas C. Montgomery, *Applied Statistics and Probability for Engineers.* John Wiley & Sons, Inc.

Mugnier, C. J. (2011). Grids & Datums - Republic of Liberia.

Newsome, G. G. (2008). *GPS Coordinate Transformation Parameters for Jamaica.*

Reit, B.-G. (2009). *On Geodetic Transformations.*

Roman, D. R. (2007). *Datums, Heights and Geodesy.* Central chapter of the Professional Land Surveyors of Colorado.

Smith, R. B. (2006). *Orthorectification Using Rational Polynomials.*

Snyder, J. P. (1987). *Map Projection, Working Manual, U.S. Geological Survey Projessional Paper 1395.*

TRC. (2008). *Liberia Truth and Reconciliation Final Report.*

US Defense Mapping Agency. (December, 1987). *Department of Defense World Geodetic System 1984: Its Definition and Relationships with Local Geodetic Systems.*

Witchayangkoon, B. (2000). *Elements of GPS Precise Point Positioning.*

Wolf, B. (1962). The theory for the determination of the non-parallelism of the minor axis of the reference ellipsoid and the inertial polar axis of the Earth. Studia Geophysica et Geodetica.

Wolf, H. (1963). *Geometric Connection and Re-Orientation of three-dimensional triangulation nets.*

Yang, C. S. et al. (1997). *Test of Cadastral Survey by Use of GPS (in Korea)-Technical Report on Cadastral Technology Research Institute.* Korea Cadastral Survey Corporation, Seoul, Korea.

APPENDIX A: MATLAB CODE FOR THE DETERMINATION OF THE PARAMETERS

A.1 Abridged Molodensky (Five Parameters) Matlab Code

```
function [dx, dy, dz, variance] = Amolodensky(filename,awgs,invwgs,aclk,invclk,nIterations)

if ischar(filename)      %Check whether the filename is character
[pathstr, name, ext] = fileparts(filename) ; %Assignment the various parts of the file name
  if strcmpi(ext,'.xls') || strcmpi(ext,'.xlsx')
    [data txt]=xlsread(filename);%reading excel file
    ncol=size(data,2); %finding number of columns in data
    switch ncol
case 13 %First case is whether the data is dms for each point. If so, we convert to dd

 longwgs=-1*dms2degrees(data(:,1:3));
 latwgs=dms2degrees(data(:,4:6));
 hw=data(:,7);

 longclk=-1*dms2degrees(data(:,8:10));
 latclk=dms2degrees(data(:,11:13));

        case 5  %Whether the data is dd, no conversion at this stage

 longwgs=-1*data(:,1);
 latwgs=data(:,2);
 hw=data(:,3);

 longclk=-1*data(:,4);
 latclk=data(:,5);

    end

%CALCULATION STARTS HERE
 %-----------------------

 format long

latwgs1=deg2rad(latwgs);%converting latitude values in degrees to radian
longwgs1=deg2rad(longwgs);%converting longitude values in degrees to radian
latclk1=deg2rad(latclk);%converting latitude values in degrees to radian
longclk1=deg2rad(longclk);%converting longitude values in degrees to radian

fclk=1/invclk;% flattening
fwgs=1/invwgs;

e2clk=(2*fclk)-fclk.^2; %eccentricity square

sinlatclk = sin(latclk1);

V = aclk./sqrt(1-(e2clk.*sinlatclk.*sinlatclk)); % prime vertical radius of curvature(v)

P = (aclk.*(1- e2clk))./((sqrt(1- (e2clk.*((sin(latclk1)).^2)))).^3); %radius of curvature in
the

da=awgs-aclk;
df=fwgs-fclk;

dlat=(latwgs-latclk);      %Why was this done? Why we multiplied by 3600 when we are working in
radians
dlong=(longwgs-longclk);

% dlat=(latwgs-latclk);
% dlong=(longwgs-longclk);

%A MATRIX INPUTS. Formulation of the matrix
sinphicoslamda=sin(latclk1).*cos(longclk1);
sinphisinlamda=sin(latclk1).*sin(longclk1);y
cosphi=cos(latclk1);
sinlamda=sin(longclk1);
cosphi=cos(latclk1);
cosphicoslamda=cos(latclk1).*cos(longclk1);
cosphisinlamda=cos(latclk1).*sin(longclk1);
sinphi=sin(latclk1);
```

```
%L MATRIX INPUTS. Formulation of the matrix

T=aclk*df+fclk*da;
sin2phi=sin(2.*latclk1);
coslatclk=cos(latclk1);
sinlatclksq=sin(latclk1).^2;
%sin1sec=sin(dms2degrees([0 0 1]));  %why is it done this way?
sin1sec = sin(1);

dh=0;

n=size(latclk,1); %Check the size of the matrix using any column. In this case, we chose
column one.
 for k=1:nIterations

for i=1:n
      %Actual equations for the design and absolute matrix
A((3*i-2):3*i,1:3)=[sinphicoslamda(i) sinphisinlamda(i) -cosphi(i);-sinlamda(i) ...
                  cosphi(i) 0;cosphicoslamda(i) cosphisinlamda(i) sinphi(i)];
 %A(start index:End index of what rows, columns)
L((3*i-2):3*i,1)=[T.*sin2phi(i,1)-(sin1sec.*P(i,1)).*dlat(i,1);...
                  dlong(i,1).*(V(i,1)*coslatclk(i,1).*sin1sec); ...
                  dh-T.*sinlatclksq(i,1)+da          ];
format longg

                x=A\L;

                dx=x(1);
                dy=x(2);
                dz=x(3);

                %Compute the Residuals
                Residuals=((A*x)-L);

%          end
end
  end
                %compute the change in height
                dh=dx.*cosphicoslamda+dy.*cosphisinlamda+dz.*sinphi+T.*sinlatclksq-da

                   u=3;
                %Compute the Variance of unit weight
                variance = (Residuals'*Residuals)/(n - u)
                %n = number of measurements, u = number of unknown
                %parameters

                %compute the Standard Deviation
                sd = sqrt(variance)

                %Compute Standard Deviation of the individual quantities
                vXt = sd*sqrt(diag(inv(A'*A)))

                hc=hw-dh;

                %compute the standard Error
                se = sd/sqrt(n)
  else

      errordlg('File must be an excel file','error','modal')

  end

  return

end
```

A.2 Geocentric (Simple Three Parameters) Matlab Code

```
function [dx, dy, dz] = Geocentric(filename,awgs,invwgs,aclk,invclk)
if ischar(filename)      %Check whether the filename is character
[pathstr, name, ext] = fileparts(filename) ; %Assignment the various parts of the file name
  if strcmpi(ext,'.xls') || strcmpi(ext,'.xlsx')
    [data txt]=xlsread(filename);%reading excel file
    ncol=size(data,2); %finding number of columns in data
    switch ncol

case 14 %First case is whether the data is dms for each point. If so, we convert to dd

 longwgs=-1*dms2degrees(data(:,1:3));
 latwgs=dms2degrees(data(:,4:6));
 hw=data(:,7);
 hc=data(:,14);

 longclk=-1*dms2degrees(data(:,8:10));
 latclk=dms2degrees(data(:,11:13));

        case 5  %Whether the data is dd, no conversion at this stage

 longwgs=-1*data(:,1);
 latwgs=data(:,2);
 hw=data(:,3);
 hc=data(:,6);

 longclk=-1*data(:,4);
 latclk=data(:,5);
    end

%CALCULATION STARTS HERE
 %------------------------
format long

latwgs1=deg2rad(latwgs);%converting latitude values in degrees to radian
longwgs1=deg2rad(longwgs);%converting longitude values in degrees to radian
latclk1=deg2rad(latclk);%converting latitude values in degrees to radian
longclk1=deg2rad(longclk);%converting longitude values in degrees to radian

fclk=1/invclk;% flattening
fwgs=1/invwgs;

e2clk=(2*fclk)-fclk.^2; %Clarke eccentricity square
e2wgs=(2*fwgs)-fwgs.^2; %WGS eccentricity square

sinlatclk = sin(latclk1); %Sin of the latitude in clarke
sinlatwgs = sin(latwgs1); %Sin of the latitude in WGS

%Radii in the Clarke 1880 System
Vc = aclk./sqrt(1-(e2clk.*sinlatclk.*sinlatclk)); % prime vertical radius of curvature(v)
Pc = (aclk.*(1- e2clk))./((sqrt(1-(e2clk.*((sin(latclk1)).^2)))).^3); %radius of curvature in
the meridian

%Radii in the WGS84 coordinate system
Vw = awgs./sqrt(1-(e2wgs.*sinlatwgs.*sinlatwgs)); % prime vertical radius of curvature(v)
Pw = (awgs.*(1- e2wgs))./((sqrt(1- (e2wgs.*((sin(latwgs1)).^2)))).^3); %radius of curvature in
the meridian

da=awgs-aclk;
df=fwgs-fclk;

dlat=(latwgs-latclk);      %Why was this done? Why we multiplied by 3600 when we are working in
radians
dlong=(longwgs-longclk);

%A MATRIX INPUTS. Formulation of the matrix
%WGS X,Y,Z
cosphiw=cos(latwgs1);
coslambdaw = cos(longwgs1);
sinphiw = sin(latwgs1);
sinlambdaw = sin(longwgs1);

format longg

Xw=(Vw+hw).*cosphiw.*coslambdaw;
Yw=(Vw+hw).*cosphiw.*sinlambdaw;
```

```
Zw=((Vw-e2wgs*Vw)+hw).*sinphiw;

%Clarke X,Y,Z
cosphic=cos(latclk1);
coslambdac = cos(longclk1);
sinphic = sin(latclk1);
sinlambdac = sin(longclk1);

Xc=(Vc+hc).*cosphic.*coslambdac;
Yc=(Vc+hc).*cosphic.*sinlambdac;
Zc=((Vc-e2clk*Vc)+hc).*sinphic;

%L MATRIX INPUTS. Formulation of the matrix
%Difference between X,Y,Z in both system (WGS - Clarke)
xd=Xw-Xc;
yd=Yw-Yc;
zd=Zw-Zc;

n=size(latclk,1); %Check the size of the matrix using any column. In this case, we chose
column one.

for i=1:n

    %Actual equations for the design and absolute matrix
    A((3*i-2):3*i,1:3)=[1,0,0;0,1,0;0,0,1];
    format longg
 %A(start index:End index of what rows, columns)
L((3*i-2):3*i,1)=[xd(i);yd(i);zd(i)];

end

format longg
                x=A\L;
                dx=x(1);
                dy=x(2);
                dz=x(3);

                %Compute the Residuals
                 Residuals=((A*x)-L);

                 u=3; %This should be an input
                %Compute the Variance
                variance = (Residuals'*Residuals)/(n - u);
                %n = number of measurements, u = number of unknown
                %parameters

                %compute the Standard Deviation
                sd = sqrt(variance)

               %Compute Standard Deviation of the individual quantities
                vXt = sd*sqrt(diag(A'*A).^-1)

                %Compute the standard error
                se = sd/sqrt(n)
  else
      errordlg('File must be an excel file','error','modal')
  end

  return

end
```

A.3 Bursa Wolf (Seven Parameters) Matlab Code

```
function [dx, dy, dz,Rx,Ry,Rz,Scale] = BursaWolf(filename,awgs,invwgs,aclk,invclk)
if ischar(filename)      %Check whether the filename is character
[pathstr, name, ext] = fileparts(filename) ; %Assignment the various parts of the file name
  if strcmpi(ext,'.xls') || strcmpi(ext,'.xlsx')
    [data txt]=xlsread(filename);%reading excel file
    ncol=size(data,2); %finding number of columns in data
    switch ncol
case 14 %First case is whether the data is dms for each point. If so, we convert to dd

 longwgs=-1*dms2degrees(data(:,1:3));
 latwgs=dms2degrees(data(:,4:6));
 hw=data(:,7);
 hc=data(:,14);

 longclk=-1*dms2degrees(data(:,8:10));
 latclk=dms2degrees(data(:,11:13));

        case 5  %Whether the data is dd, no conversion at this stage

 longwgs=-1*data(:,1);
 latwgs=data(:,2);
 hw=data(:,3);
 hc=data(:,6);

 longclk=-1*data(:,4);
 latclk=data(:,5);


    end

%CALCULATION STARTS HERE
 %------------------------

 format long

latwgs1=deg2rad(latwgs);%converting latitude values in degrees to radian
longwgs1=deg2rad(longwgs);%converting longitude values in degrees to radian
latclk1=deg2rad(latclk);%converting latitude values in degrees to radian
longclk1=deg2rad(longclk);%converting longitude values in degrees to radian

fclk=1/invclk;% flattening
fwgs=1/invwgs;

e2clk=(2*fclk)-fclk.^2; %Clarke eccentricity square
e2wgs=(2*fwgs)-fwgs.^2; %WGS eccentricity square

sinlatclk = sin(latclk1); %Sin of the latitude in clarke
sinlatwgs = sin(latwgs1); %Sin of the latitude in WGS

%Radii in the Clarke 1880 System
Vc = aclk./sqrt(1-(e2clk.*sinlatclk.*sinlatclk)); % prime vertical radius of curvature(v)
Pc = (aclk.*(1- e2clk))./((sqrt(1-(e2clk.*((sin(latclk1)).^2)))).^3); %radius of curvature in
the meridian

%Radii in the WGS84 coordinate system
Vw = awgs./sqrt(1-(e2wgs.*sinlatwgs.*sinlatwgs)); % prime vertical radius of curvature(v)
Pw = (awgs.*(1- e2wgs))./((sqrt(1-(e2wgs.*((sin(latwgs1)).^2)))).^3); %radius of curvature in
the meridian

da=awgs-aclk;
df=fwgs-fclk;

dlat=(latwgs-latclk);     %Why was this done? Why we multiplied by 3600 when we are working in
radians
dlong=(longwgs-longclk);

%A MATRIX INPUTS. Formulation of the matrix
%WGS X,Y,Z
cosphiw=cos(latwgs1);
coslambdaw = cos(longwgs1);
sinphiw = sin(latwgs1);
sinlambdaw = sin(longwgs1);

format longg
```

```
Xw=(Vw+hw).*cosphiw.*coslambdaw;
Yw=(Vw+hw).*cosphiw.*sinlambdaw;
Zw=(Vw-e2wgs*Vw+hw).*sinphiw;

%Clarke X,Y,Z
cosphic=cos(latclk1);
coslambdac = cos(longclk1);
sinphic = sin(latclk1);
sinlambdac = sin(longclk1);

Xc=(Vc+hc).*cosphic.*coslambdac;
Yc=(Vc+hc).*cosphic.*sinlambdac;
Zc=(Vc-e2clk*Vc+hc).*sinphic;

%L MATRIX INPUTS. Formulation of the matrix
%Difference between X,Y,Z in both system (WGS - Clarke)
xd=Xw-Xc;
yd=Yw-Yc;
zd=Zw-Zc;

n=size(latclk,1); %Check the size of the matrix using any column. In this case, we chose
column one.

for i=1:n

    %Actual equations for the design and absolute matrix
    A((3*i-2):3*i,1:7)=[1,0,0,Xc(i),0,-Zc(i),Yc(i);0,1,0,Yc(i),Zc(i),0,-Xc(i);0,0,1,Zc(i),-
Yc(i),Xc(i),0];
    format longg
 %A(start index:End index of what rows, columns)
L((3*i-2):3*i,1)=[xd(i);yd(i);zd(i)];

end

format longg

                x=A\L;

                dx=x(1);
                dy=x(2);
                dz=x(3);
                Rx=x(4);
                Ry=x(5);
                Rz=x(6);
                Scale=x(7);

                %Compute the Residuals
                Residuals=((A*x)-L);

                 u=7; %This should be an input
                %Compute the Variance
                variance = (Residuals'*Residuals)/(n - u);
                %n = number of measurements, u = number of unknown
                %parameters

                %compute the Standard Deviation
                sd = sqrt(variance)

                %Compute Standard Deviation of the individual quantities
                vXt = sd*sqrt(diag(A'*A).^-1)
                %Compute the standard error
                se = sd/sqrt(n)
  else

      errordlg('File must be an excel file','error','modal')

  end

  return

end
```

A.4 Molodensky (Seven Parameters) Matlab Code

```
function [dx, dy, dz,Rx,Ry,Rz,Scale] = Molodensky(filename,awgs,invwgs,aclk,invclk)
if ischar(filename)     %Check whether the filename is character
[pathstr, name, ext] = fileparts(filename) ; %Assignment the various parts of the file name
  if strcmpi(ext,'.xls') || strcmpi(ext,'.xlsx')
    [data txt]=xlsread(filename);%reading excel file
    ncol=size(data,2); %finding number of columns in data
    switch ncol
      case 14 %First case is whether the data is dms for each point. If so, we convert to dd

 longwgs=-1*dms2degrees(data(:,1:3));
 latwgs=dms2degrees(data(:,4:6));
 hw=data(:,7);
 hc=data(:,14);

 longclk=-1*dms2degrees(data(:,8:10));
 latclk=dms2degrees(data(:,11:13));

        case 5  %Whether the data is dd, no conversion at this stage

 longwgs=-1*data(:,1);
 latwgs=data(:,2);
 hw=data(:,3);
 hc=data(:,6);

 longclk=-1*data(:,4);
 latclk=data(:,5);

    end
%CALCULATION STARTS HERE
 %------------------------
format long
latwgs1=deg2rad(latwgs);%converting latitude values in degrees to radian
longwgs1=deg2rad(longwgs);%converting longitude values in degrees to radian
latclk1=deg2rad(latclk);%converting latitude values in degrees to radian
longclk1=deg2rad(longclk);%converting longitude values in degrees to radian
fclk=1/invclk;% flattening
fwgs=1/invwgs;
e2clk=(2*fclk)-fclk.^2; %Clarke eccentricity square
e2wgs=(2*fwgs)-fwgs.^2; %WGS eccentricity square
sinlatclk = sin(latclk1); %Sin of the latitude in clarke
sinlatwgs = sin(latwgs1); %Sin of the latitude in WGS
%Radii in the Clarke 1880 System
Vc = aclk./sqrt(1-(e2clk.*sinlatclk.*sinlatclk)); % prime vertical radius of curvature(v)
Pc = (aclk.*(1- e2clk))./((sqrt(1-(e2clk.*((sin(latclk1)).^2)))).^3); %radius of curvature in
the meridian
%Radii in the WGS84 coordinate system
Vw = awgs./sqrt(1-(e2wgs.*sinlatwgs.*sinlatwgs)); % prime vertical radius of curvature(v)
Pw = (awgs.*(1- e2wgs))./((sqrt(1- (e2wgs.*((sin(latwgs1)).^2)))).^3); %radius of curvature in
the meridian
da=awgs-aclk;
df=fwgs-fclk;

dlat=(latwgs-latclk);     %Why was this done? Why we multiplied by 3600 when we are working in
radians
dlong=(longwgs-longclk);

%A MATRIX INPUTS. Formulation of the matrix
%WGS X,Y,Z
cosphiw=cos(latwgs1);
coslambdaw = cos(longwgs1);
sinphiw = sin(latwgs1);
sinlambdaw = sin(longwgs1);

format longg

Xw=(Vw+hw).*cosphiw.*coslambdaw;
Yw=(Vw+hw).*cosphiw.*sinlambdaw;
Zw=((Vw-e2wgs*Vw)+hw).*sinphiw;

%compute the mean for the WGS which is known as the centroid
Xwmu = mean(Xw);
Ywmu = mean(Yw);
Zwmu = mean(Zw);
```

```
%Clarke X,Y,Z
cosphic=cos(latclk1);
coslambdac = cos(longclk1);
sinphic = sin(latclk1);
sinlambdac = sin(longclk1);

Xc=(Vc+hc).*cosphic.*coslambdac;
Yc=(Vc+hc).*cosphic.*sinlambdac;
Zc=((Vc-e2clk*Vc)+hc).*sinphic;

%compute the mean for the Clarke which is known as the Centroid
Xcmu = mean(Xc);
Ycmu = mean(Yc);
Zcmu = mean(Zc);

%L MATRIX INPUTS. Formulation of the matrix
%Difference between X,Y,Z in both system (WGS - Clarke)
xd=Xw-Xc;
yd=Yw-Yc;
zd=Zw-Zc;

%Subtract the mean (centroid) from the clarke 1880 spheroid
muXc = Xc - Xcmu;
muYc = Yc - Ycmu;
muZc = Zc - Zcmu;


n=size(latclk,1); %Check the size of the matrix using any column. In this case, we chose
column one.
for i=1:n

    %Actual equations for the design and absolute matrix
    A((3*i-2):3*i,1:7)=[1,0,0,muXc(i),0,-muZc(i),muYc(i);0,1,0,muYc(i),muZc(i),0,-
muXc(i);0,0,1,muZc(i),-muYc(i),muXc(i),0];
    format longg
 %A(start index:End index of what rows, columns)
L((3*i-2):3*i,1)=[xd(i);yd(i);zd(i)];
end
format longg

                x=A\L;
                dx=x(1);
                dy=x(2);
                dz=x(3);
                Rx=x(4);
                Ry=x(5);
                Rz=x(6);
                Scale=x(7);

                %Compute the Residuals
               Residuals=((A*x)-L);

                 u=7; %This should be an input
                %Compute the Variance
                variance = (Residuals'*Residuals)/(n - u);
                %n = number of measurements, u = number of unknown
                %parameters

                %compute the Standard Deviation
                sd = sqrt(variance)

                %Compute Standard Deviation of the individual quantities
                vXt = sd*sqrt(diag(inv(A'*A)))
                %Compute the standard error
                se = sd/sqrt(n)
  else
      errordlg('File must be an excel file','error','modal')
  end
  return
end
```

APPENDIX B: RESIDUALS FROM THE DERIVATION OF THE PARAMETERS

TABLE 8: RESIDUALS FROM ABRIDGED MOLODENSKY (5 PARAMETERS) MODEL

Vx(m)	Vy(m)	Vz(m)
0.148970485	0.275368787	5.606956
0.063111318	0.091795226	-0.00814
0.0574398	0.14971944	-0.80943
-1.336965213	-3.542597042	-0.63484
1.603618657	3.907535732	0.164952
-0.267046113	0.21831956	12.51763
0.122157876	0.204335032	-0.51876
0.096782961	0.179108222	-0.25358
0.088272784	0.212452241	-0.27046
0.096882097	0.179089105	-0.5403
0.096743768	0.204344087	-0.55891
0.113859788	0.249147598	-0.28984
0.113723124	0.220944074	-0.30105
0.112969036	0.212634638	-0.57727
0.07929435	-3.978650521	-0.45406
0.079278077	0.204180335	-0.33383
0.079319752	0.195674042	-0.60456
-1.410688982	0.604022952	-0.67438
0.062407564	0.212459313	-0.48252

TABLE 9: RESIDUALS FROM THE GEOCENTRIC (SIMPLE 3 PARAMETERS) MODEL

Vx(m)	Vy(m)	Vz(m)
-0.002167451	0.276663	-0.14632
0.001653101	0.092568	-0.062
-0.005790777	0.150034	-0.05644
0.139612011	-3.54879	1.313574
-0.130578025	3.916965	-1.57566
-0.072747709	0.211459	0.262612
0.000271045	0.205634	-0.12002
-0.00166615	0.180037	-0.09509
-0.0068958	0.21301	-0.08673
-0.001662311	0.180023	-0.09519
-0.004463858	0.205125	-0.09506
-0.006175854	0.25001	-0.11187
-0.003104442	0.221975	-0.11173
-0.002359996	0.2137	-0.111
0.450904594	-3.95317	-0.07791
-0.007677467	0.204607	-0.07789
-0.006766098	0.196152	-0.07794
-0.328644065	0.571513	1.385975
-0.01174075	0.212491	-0.06132

TABLE 10: RESIDUALS FROM THE BURSA-WOLF (7 PARAMETERS) MODEL

Vx(m)	Vy(m)	Vz(m)
-0.03344	0.033316	0.149663
0.004312	0.119617	-0.09062
-0.00766	0.204279	-0.07701
0.130424	-3.51043	1.339506
-0.13528	3.928149	-1.55762
0.032275	-0.14803	-0.07719
-0.00726	0.241204	-0.10076
-0.0078	0.206563	-0.07834
-0.0122	0.241318	-0.07573
-0.00846	0.217374	-0.08081
-0.01049	0.244193	-0.08577
-0.0107	0.279908	-0.1051
-0.00669	0.253398	-0.11036
-0.00749	0.254579	-0.10728
0.447419	-3.91541	-0.08054
-0.01047	0.238301	-0.0818
-0.0111	0.23909	-0.07943
-0.33078	0.620161	1.368819
-0.0146	0.252417	-0.06965

TABLE 11: RESIDUALS FROM THE MOLODENSKY-BADEKAS (7 PARAMETERS) MODEL

Vx(m)	Vy(m)	Vz(m)
-0.03344	0.033316	0.149663
0.004312	0.119617	-0.09062
-0.00766	0.204279	-0.07701
0.130424	-3.51043	1.339506
-0.13528	3.928149	-1.55762
0.032275	-0.14803	-0.07719
-0.00726	0.241204	-0.10076
-0.0078	0.206563	-0.07834
-0.0122	0.241318	-0.07573
-0.00846	0.217374	-0.08081
-0.01049	0.244193	-0.08577
-0.0107	0.279908	-0.1051
-0.00669	0.253398	-0.11036
-0.00749	0.254579	-0.10728
0.447419	-3.91541	-0.08054
-0.01047	0.238301	-0.0818
-0.0111	0.23909	-0.07943
-0.33078	0.620161	1.368819
-0.0146	0.252417	-0.06965

APPENDIX C: GEOGRAPHY COORDINATES OF COMMON POINTS

		CLARKE 1880						WGS84						
	Location	Deg	Min	Sec	Deg	Min	Sec	Deg	Min	Sec	Deg	Min	Sec	Height
Base	72nd Barrack, Ramrod	10	42	10.339	6	17	45.251	10	42	9.12464	6	17	47.6257	53.141
1	10th Street Coleman Avenue	10	47	0.615	6	17	43.185	10	46	58.945	6	17	46.49	42.158
2	10th Street Gardner Avenue	10	47	7.898	6	17	24.706	10	47	6.199	6	17	27.717	42.886
3	19th Street Coleman Avenue	10	46	36.079	6	17	12.511	10	46	27.393	6	17	34.239	43.448
4	19th Street Gardner Avenue	10	46	28.716	6	17	30.911	10	46	34.771	6	17	14.873	43.037
5	St. Paul Bridge	10	47	2.9476	6	23	34.118	10	46	59.6212	6	23	36.78843	43.8431
6	TP19	10	46	38.478	6	17	17.041	10	46	37.106	6	17	19.775	41.891
7	TP18	10	46	35.961	6	17	23.101	10	46	34.462	6	17	25.963	43.401
8	TP17	10	46	39.815	6	17	24.696	10	46	38.273	6	17	27.389	45.86
9	TP15	10	46	41.866	6	17	18.256	10	46	40.367	6	17	21.118	42.36
10	TP14	10	46	45.339	6	17	19.599	10	46	43.839	6	17	22.333	43.161
11	TP13	10	46	42.779	6	17	25.744	10	46	41.365	6	17	28.251	42.428
12	TP12	10	46	46.338	6	17	27.34	10	46	44.923	6	17	29.99	41
13	TP11	10	46	49.112	6	17	21.11	10	46	47.693	6	17	23.802	42.957
14	TP10	10	46	51.2	6	17	25.673	10	46	49.612	6	17	49.612	40.786
15	TP9	10	46	50.152	6	17	28.47	10	46	48.564	6	17	31.205	40.731
16	TP8	10	46	52.796	6	17	22.324	10	46	51.208	6	17	25.102	42.558
17	TP7	10	47	3.656	6	17	26.202	10	46	54.598	6	17	26.91	48.53
18	TP6	10	46	55.181	6	17	27.014	10	46	53.508	6	17	29.707	47.31

TABLE 12 GEOGRAPHY COORDINATES OF COMMON POINTS

APPENDIX D: SCREENSHOTS OF LIBTRANS

FIGURE 28: LIBTRANS HOME SCREEN

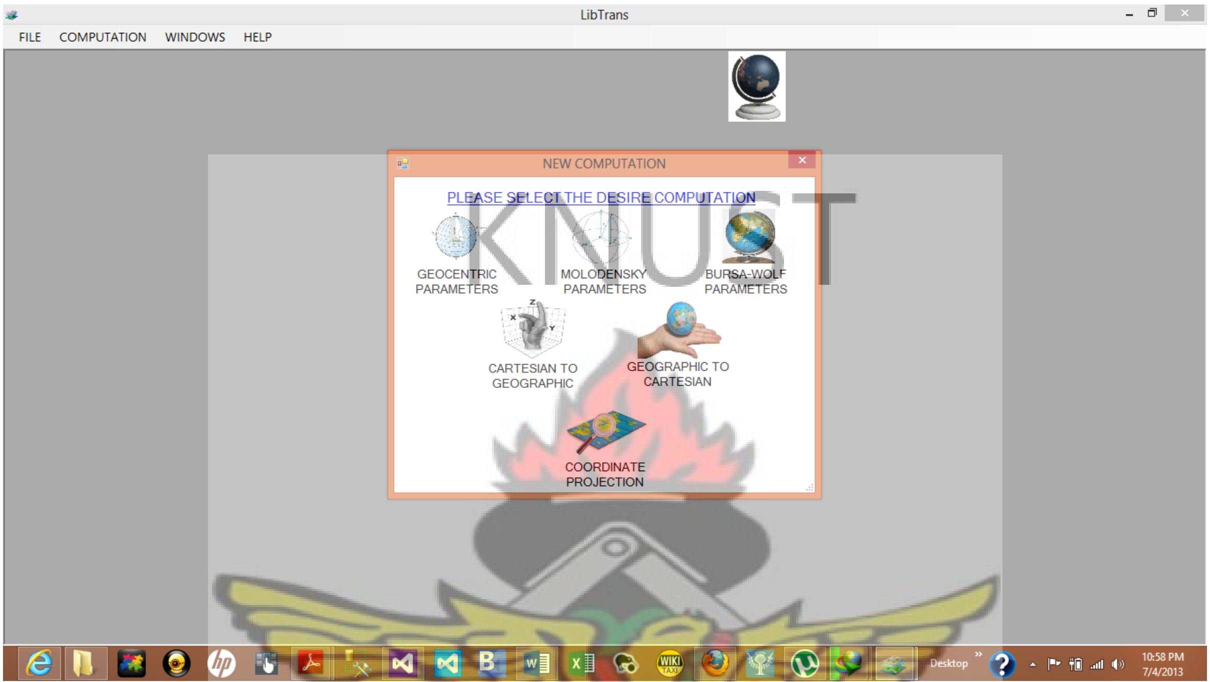


FIGURE 29: LIBTRANS COORDINATE CONVERSION FORM