KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

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PORTFOLIO OPTIMIZATION USING THE MARKOWITZ MODEL: CASE STUDY OF SELECTED COMPANIES IN GHANA

BY

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. Industrial Mathematics programme, and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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DEDICATION

Dedicated to all my family and friends who have helped me in diverse ways to climb up to this level of education.

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The completion of this thesis would not have been possible but for the help of the Almighty God, who has brought me this far in attaining this level of secular education. He also used some individuals, who readily availed themselves, to help me accomplish this arduous task.

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God bless you all.

ABSTRACT

Over the past few years, there has been increased activity in the Ghanaian capital market. It is important that prospective investors and market watchers are able to determine the risk of listed companies. However, information regarding the beta of companies and what proportions to invest in portfolios to spread the risks for some expected returns are not readily available. Once an investor determines how risky listed companies are, he or she would be able to spread the risk by diversifying the investment, i.e. making the investment commensurate with the risk in order to maximize returns. Three data sets, one containing the Ghana Stock Exchange (GSE) All Share Index, the other containing the monthly beginning and closing stock prices of six of the most liquid stocks listed on the Ghana Stock Exchange and the Bank of Ghana 91-day Treasury bill rates for the period January, 1998 to December, 2002 were obtained from the Bank of Ghana. The sensitivity of the six selected companies was established by calculating their betas using regression analysis. The Markowitz Model was formulated and solved using the quadratic programming add-in and the Microsoft Excel Solver.

The results indicated that most of the six selected companies have negative betas, which implies that most of these companies listed on the Ghana Stock Exchange were less risky. The optimal solution to the Markowitz model indicated that in order to ensure diversification and good returns, Ghana Commercial Bank (GCB) stock should make up 19.64% of the portfolio, SG-SSB Bank stock should make up 8.93% of the portfolio, Standard Chartered Bank (SCB) stock should make up 17.86% of the portfolio, Home Finance Company (HFC) 10.71% of the portfolio, Enterprise Insurance (EIC) stock should make up 5.36% of the portfolio and Total Ghana Limited stock should make up 37.5% of the portfolio.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF STUDY

Investments play a vital role in the world's economy and can vary from small scale investment to large scale investment. A small scale investment may typically be that of an individual or a small company, whilst a large scale investment may be that of a large company or the government. Therefore, a typical investor can be an individual, a small organization, a fund-management company, a bank, a multi-national company or the government. For example, large fund-management companies in the UK invest several billions of pounds in investments across the globe and usually, these fund-management companies have a major role to play in making sure their investments are profitable and attractive as they often invest on behalf of unit trusts (mutual funds), pension funds, corporate bodies, individuals etc that collectively generate the money used by the fund management company for investment purposes. (Adedoyin, 2008)

Typically, an investor would have a collection of different assets (investments) in one place. This collection, in the financial investment world, is often referred to as a "Portfolio". An asset in a portfolio can represent a company's stock (shares) that is traded on stock markets, government bond, company bond, Treasury bills, etc.

Every asset is attributed with an expected return (gains) and an element of risk (although, some assets are risk-free as explained further below). The expected return and the risk (variance or standard deviation) form an elementary aspect of a portfolio and are used as basis for selecting assets into a portfolio.

The fundamental problem often faced by investors, which is known as the "Portfolio Selection problem", is "how" to distribute an investment amount across a number of potential assets (investments). Portfolio selection is an integral constituent of an investment strategy, which aims to identify a portfolio of assets that can provide the investor with high profits. However, an investment that produces high returns is often linked to a greater amount of risk. Every investor is aware of the risk-return trade-off and can make an investment decision depending on their risk-averse level. The only investments that can be considered as risk-free are government bonds, Government of Ghana Treasury bills and notes. On the other hand, company stocks (shares) are considered as risky assets because they usually bear an element of risk associated with their expected returns. An investor willing to invest in a portfolio that contains risky assets will opt to try and balance the odds of returns and risk. Stocks traded in stock markets do not always move in the same direction. Some stocks tend to move in the same direction (correlated), while some tend to move in the opposite direction (non-correlated) and some stocks tend not to have any relationship (neutral). The cross-correlation between stocks is vital to portfolio selection as it helps to find regularities in stocks, to gain understanding and to reduce risk. Using cross-correlation, i.e. the co-movement between the assets, an investor can reduce the overall risk of a portfolio by including some non-correlated stocks to balance the portfolio. A portfolio's performance may not always reflect the performance of individual assets that constitute the portfolio. The correlation patterns between the individual assets need to be taken into consideration to form a properly selected portfolio with a lesser risk than the sum of the risks of individual assets in the portfolio.

Portfolio selection decisions have been widely based on qualitative approaches and still continue to dominate the larger portion of approaches used. On the contrary, the formation of a correct portfolio cannot be achieved by human instinct only and requires modern and powerful quantitative approaches that can utilize the correlations, the expected returns, the risk and that can also take the investor's desired constraints into consideration. Quantitative approaches are becoming widely used as they possess the potential to produce a reliably well-formed and diversified portfolio that would meet the investor's expectations.

Markowitz (1959) was one of the major contributors to the portfolio selection problem by developing a mathematical quantitative framework to find the optimal portfolio that can produce the maximum portfolio return with a minimum portfolio risk simultaneously. The mathematical approach developed by Markowitz is often referred to as the "Mean-Variance (MV) model" and has formed the foundation of modern portfolio theory in finance. The MV model represents the portfolio selection problem as an optimization problem of real-valued variables with a quadratic objective function and linear constraints. The MV model uses the mean (average) of returns as the expected portfolio return and the variance of the portfolio as a risk measure. The overall mathematical problem can be formulated as various objective functions but to mention a few below:

1. Minimize risk for a specified expected return.
2. Maximize the expected return for a specified risk.
3. Minimize the risk regardless of the expected return.
4. Maximize the expected return regardless of the risk.
5. Maximize the expected return while minimizing the risk.
6. Maximize the expected return while minimizing the risk using a specified risk aversion factor.
7. Minimize risk below a specified threshold.
8. Maximize the expected return above a specified threshold.

The most common formulation is 5, which maximizes the returns while reducing the risk.

A low-risk bearer will usually opt for formulation 3, while a high-risk bearer will opt for formulation 4. The formulations can be used for comparison and benchmarking of various portfolios.

1.2 STATEMENT OF THE PROBLEM

Information regarding the risk level of companies and what proportions to invest in portfolios to spread the risks for some expected returns are not readily available. Once an investor determines how risky listed companies are, he or she would be able to spread the risk by diversifying the investment i.e. making the investment commensurate with the risk in order to maximize returns.

1.3 OBJECTIVES

The main objective of this paper is first to estimate how sensitive selected companies listed on Ghana Stock Exchange are relative to the market (GSE All Share Index) by running a regression of stock returns against market returns, formulate the Markowitz Model and apply it to the Ghana Stock Exchange for selected companies. A diversification or proportions that should be invested in those companies to ensure that investors not put all their eggs in one basket will also be found.

1.4 SIGNIFICANCE OF STUDY

It is essential that this research takes place because valid information on betas for listed companies in Ghana is not readily available to the public as well as to potential investors. Also, information on how potential investors can optimize their investment based on an expected yield and as to what proportions to invest in portfolios to spread their risk is not readily available.

Measuring the systematic risk of listed companies is of importance to both investors and investment advisors because investors more often than not channel their resources according to their risk profile. Therefore, if investors are risk averse then you would expect that they will invest their resources in companies that are less volatile in relation to the market, that is, companies with low betas or beta values lower than one. A company with a beta lower than one implies a relatively less risky share. However, betas greater than one are considered to be aggressive shares, therefore would be recommended for more risk tolerant investors.

With the completion of this study, instructors in universities across Ghana would be able to give practical examples in the field of optimization in the stock market and within our Ghanaian context in order to make the educational experience more meaningful to students.

This study is also particularly significant because with beta estimates investors as well as potential investors can calculate the expected return they would get as a result of holding onto a particular share.

1.5 METHODOLOGY

Only secondary data was used to acquire information to carry out the study. Secondary data contained information regarding dividends, month end prices of companies listed on stock exchange as well as monthly return of GSE All Share Index for the same period.

Data covering a 5 year period commencing January 1998 to December 2002 was obtained from the Bank of Ghana. Three data sets were provided – one containing the Ghana Stock Exchange (GSE) All Share Index for the stated period, the other containing the monthly beginning and closing stock prices of six most liquid stocks listed on the Ghana Stock Exchange and the Bank of Ghana 91-day Treasury bill rates for the same period.

The information gathered was used to run a regression on the return of a company and the market index using data analysis tool in Microsoft Excel. The information obtained was analyzed both qualitatively and quantitatively. Quantitative analysis was carried out in order to estimate the systematic risk of companies and also to test if a relationship exists between share price of a stock and its beta level. Qualitative analysis was executed to interpret the results gathered from the field.

The market return, the risk free rate and the excess return will be calculated from the data using excel. The means, standard deviations and the covariance matrix will also be calculated using excel. The Markowitz model will also be implemented in excel and it will be solved by using the quadratic programming add-in in Excel as well as the Excel solver to obtain the optimal portfolio which will in turn be used to calculated the total return for the portfolio.

1.6 ORGANIZATION OF WORK

The study is organized into five (5) main chapters. Chapter one deals with the background of the study, the statement of the problem, objectives, justification and methodology. Chapter two deals mainly with the literature review, all the different approaches to Portfolio Optimization are discussed before the discussion narrows down to the Markowitz model. Chapter three discusses the methodology of the work. The method used to solve the problem is discussed thoroughly. Chapter four discusses the analysis of data and how the results were obtained.

Finally, chapter five discusses the conclusions and recommendations for the thesis.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Portfolio selection is a very crucial problem arising in finance and economics. The mean variance (MV) optimization model, developed by Markowitz (1952), has become a very common quantitative model in finance today, which is used by investors to construct an optimal portfolio solution. The MV model allocates each asset in the portfolio a proportion of the investment amount by taking into consideration each asset's returns, risk and the correlations (movements) between the assets. The model assumes the normality of returns and also ignores some practical constraints such as cardinality constraints, proportion constraints etc. The model works under such assumptions but becomes more difficult to solve when integrating the practical real-world constraints which makes the whole model become a nonlinear mixed-integer programming problem.

Integrating real-world constraints into the portfolio selection problem requires the introduction of integer variables which transforms the problem into a nonlinear programming program that cannot be solved by the classical MV model [Cornuej´ols & T¨ut¨unc¨u (2006) and Scherer & Martin (2005)], although, it can be solved by specialized algorithms such as the simplex method as used by Wolfe (1959). Perold (1984) pointed out that as the number of assets increases the problem becomes more complex and more efficient algorithms are needed to take advantage of the covariance matrix.

This section explores various studies that have been carried out by researchers on the general approaches that have been taken to solve the portfolio selection problem.

Konno & Yamakazi (1991) developed a linear programming (LP) model using mean absolute deviation (MAD) as the risk function, thus replacing the variance in Markowitz's MV model. The LP model is however equivalent to the Markowitz's model when they possess a multivariate normal distribution of the asset returns. Konno & Suzuki (1995) used the mean-variance objective function and extended it to include skewness, while, Konno & Wijayanayake (1999) utilized a branch-and-bound algorithm to solve the MAD optimization model. Speranza (1993) was able to demonstrate that taking the linear combination of the mean semi-absolute deviations also resulted in a model corresponding to the MAD model. Subsequently, Speranza (1996) extended the model to accommodate cardinality constraints. A dynamic approach was introduced by Alexakis et al. (2007), to evaluate the performance of portfolio under risk conditions and also applied simulation to recommend the optimal portfolio solution.

Frangioni & Gentile (2006) developed a variant of the MV model by considering a set of buy-in threshold constraints and a new set of cutting planes to deal with the constraints.

They used up to 300 assets and reported satisfactory results. Bienstock (1996) also developed variants of the MV model by incorporating both buy-in threshold and cardinality constraints on problems with up to 3300 assets. He used a branch-and-bound algorithm and suggested the problem becomes NP-complete when cardinality constraint on the number of assets in the portfolio is imposed. Jobst et al. (2001) also developed a branch-and-bound algorithm for portfolio problems by considering buy-in threshold, round lot and cardinality constraints. Similarly, Borchers & Mitchell (1997) developed a variant of the MV model by incorporating constraints and solved problems using an interior point nonlinear method. Konno & Yamamoto (2005) also utilized a variant of the MV model by imposing fixed transaction cost and cardinality constraints on problems with up to 54 assets.

Several papers have discussed the use of metaheuristics to solve portfolio selection problems that have extensions which make the problem intractable with classical means.

Dueck and Scheuer (1990) introduced a heuristic optimization technique, called Threshold Accepting (TA). The TA technique is considered as a refined local search procedure which accepts solutions that are not worse than a specified threshold. The TA algorithm explores the neighbourhood with a fixed number of steps at every iteration and stops when a stopping criterion is met or until it reaches the maximum iteration. Winker (2000) provides an extensive exposure to the TA technique and is considered be a robust algorithm that can be applied to various optimization problems. Dueck and Winker (1992) were the first to apply the TA technique to the portfolio selection problem.

Subsequently, Gilli & Kellezi (2001) showed how the TA technique could be utilized to solve the complex portfolio selection problem by applying the technique to a non-convex optimization problem with integer variables and a set of constraints. They replaced Markowitz's MV model's risk measurement (variance) with Value-At-Risk and they applied both TA and quadratic programming (QP) on problems with up to 98 assets. Their findings suggested that TA provided an optimal solution than that produced by QP.

Gaspero & Schaerf (2003) devised a hybrid solution based on a local search meta-heuristic that utilizes a QP solver with a dual-active set algorithm for convex quadratic problems. Although still using Markowitz's objective function, they incorporated three additional constraints which include the Cardinality, Proportion (quantity) and Pre-assignment constraints. Their model worked for a less constrained problem formulation but was not tested on a general problem.

Bonami & Lejeune (2007) also considered extensions of the classic MV portfolio optimization by introducing a probabilistic constraint on the expected return of the constructed portfolio to exceed a specific return level with a high level of confidence. In addition, they introduced integer variables for handling real-world trading constraints, such as diversification of investments in a number of separate sectors, buying stock by lots and non-profitability of holding small positions. They proposed a branch-and-bound algorithm which features two new branching rules, a static rule (Idiosyncratic risk branching) and a dynamic rule (Portfolio risk branching). They showed the effectiveness of this new approach by using Bonami's mixed-integer nonlinear solver computational framework (Bonami et al. (2005)) to carry out computational experiments on 36 problem instances containing up to 200 assets.

Research shows that Polynomial Goal Programming (PGP), originally introduced by Tayi & Leonard (1988), has also been used to solve portfolio selection problems involving a significant degree of skewness in Lai (1991), Chunhachinda, et al. (1997), Qian & Yan (2003). Davies et al. (2005) incorporated investor preferences into a PGP optimization function which allowed them to solve for multiple competing hedge fund allocation objectives within a 4-moment framework. One of their conclusions suggests that introducing preferences for skewness and kurtosis in the portfolio decision-making process may result in portfolios that are different from the MV's model optimal portfolio which emphasizes the various tradeoffs involved. They show that PGP is well-suited for solving complex return distributions of hedge funds and can accommodate hedge funds practical institutional constraints.

2.2 SYSTEMATIC AND UNSYSTEMATIC RISK

Most assets (including real and financial assets) that investors choose to invest in have some exposure to risk. In finance, the total risk of a portfolio is the sum of its systematic (non diversifiable risk and unsystematic risk (diversifiable risk).

Unsystematic risk as defined by Van Horne and Wachowicz (2005) is "the risk component that is unique to a particular company or industry, as such, is independent of economic, political and other factors that affect all securities in a systematic manner". A typical example of this type of risk in a firm includes the quality of management.

By efficient diversification, this type of risk can be totally eliminated and, as such, is irrelevant when considering the risk of portfolio. The market does not provide extra compensation for bearing this type of risk.

Systematic risk on the other hand is that component of risk that comes as a result of factors that affect the overall market such as; changes in the nation's economy or a change in the world energy situation; for example an increase in oil prices or political factors. Systematic risk is therefore defined as the "variability of return on stocks or portfolios associated with changes in return on the market as a whole" (ibid). Investors who hold a well diversified portfolio are exposed only to this type of risk, as such would be compensated for bearing this type of risk.

The systematic risk of a security is determined by its beta coefficient, as such, Guilford C. Babcock, (1972) in his article "a note on justifying beta as a measure of risk" defines The Beta Coefficient of an individual security as simply a "measure of its volatility relative to the market rate of return".

Ambachtsheer defines beta as a "statistical proxy for a combination of fundamental company characteristics related to operating and financial risk" (Ambachtsheer, 1974).

William Sharpe also defines beta coefficient as “the slope term in the simple linear regression function where the rate of return on a market index is the independent variable and a security’s rate of return, the dependent variable” (Bowman, 1979).

Though the three academicians (Ambachtsheer, Sharpe and Babcock) define beta differently, a common theme running through all was the fact that beta is a measure of a firm’s risk. However, Ambachtsheer in his definition suggests that the fundamentals of a company must be taken into account when measuring its beta. Beta as defined by Babcock and Sharpe is sufficient because the fundamentals of a company would be reflected in its returns. If the fundamentals of a company are poor, its return would fall; on the other hand, if the fundamentals of a company are strong its return would rise.

The concept of beta arises because all stocks tend to move to some extent or degree with movements in the overall market. However, the returns of some stocks tend to move more aggressively than others when the market moves, hence it is important as academicians and investors to be able to measure the extent to which a stock’s return moves relative to the overall market index. This is achieved by measuring a stock’s beta coefficient.

According to Brenner and Smith (1972), an accurate estimation of beta is important for at least two reasons. Firstly, beta is important for understanding the risk – return or risk - reward relationship in capital market theory. This theoretical relationship can be established by analyzing the expected return – beta relationship as a reward – risk equation (Bodie et al, 2008). According to Bodie and others, “the beta of a security is the appropriate measure of its risk because beta is proportional to the risk that a security contributes to the optimal risky portfolio”. In the world of finance as in common reasoning, one would expect the reward or the risk premium on individual assets, to depend on the contribution of the individual asset to the risk of the portfolio. If the beta of a stock measures its contribution to the variance of the market portfolio then for any asset or security, the required risk premium or expected return should be a function of its beta; thus the higher the beta of a security the higher risk premium one should expect.

Secondly, an accurate estimation of beta is important because it aids in making investment decisions (Alexander and Chervany, 1980). Due to the fact that an understanding of a security's beta measures the effect of systematic risk on a particular security, beta is thus, an extremely useful tool for investors to understand how to create their own individual portfolios in accordance with their ability to take risk or in accordance to their risk profile. In addition, beta "is important in investment the decision process because it is very useful to a portfolio manager in assessing the downside risk of his portfolio during bear market (Ambachtsheer, 1974)".

Though beta estimates are widely used in estimating systematic risk, research revealed that one of its limitation as argued by critics is that, there is some level of confusion surrounding optimal estimation level interval. However, Basel in his article, "on the assessment of risk" concludes that, a forecaster or analyst would be better off using a longer estimation interval such as yearly or monthly interval when calculating or estimating beta as it provides a more stable beta estimate. The beta coefficient of the market model has nonetheless gained wide acceptance as a relevant measure of risk in portfolio and security analysis and as such is used to measure the risk profile of companies across different markets.

2.2.1 Beta: An Index of Systematic Risk

Beta as an index of systematic risk measures the sensitivity of a stock's returns to changes in returns on the market portfolio. The beta of a portfolio however, is a weighted average of the individual stock betas in the portfolio.

2.2.2 Adjusted Beta

Over time, there appears to be a tendency for measured betas of individual securities to converge towards the beta of the entire market index or toward the beta of the industry of which the company is part (Van Horne and Wachowicz, 2005). This tendency is due to economic factors affecting the operation and financing of the firm and to some extent statistical factors as well (ibid). To adjust for this tendency, an adjusted beta is calculated.

Merrill Lynch adjusts its calculated beta by taking the sample estimate of beta and averaging it with 1, using weights of two thirds and one – third (Bodie et al, 2008). Regression betas are past and betas do change over time. Nonetheless, there is a strong correlation between past betas and future betas (Ambachtsheer, 1974). As any forecaster would tend to agree that in order to predict the future accurately one has to look at past occurrences.

If a firm becomes very large and begins diversifying its product line, it would behave like the market and its beta would approach that of the market which is one (1). Thus, the future beta of a well managed expanding firm will lie somewhere between past beta and 1. Therefore, to have a correct estimate of its beta it is important to adjust a security's beta.

Due to the fact that different researchers and academicians calculate beta using subjective time periods, different return intervals, different market index, different researchers or services more often than not end up with different estimates so academicians have often resorted to adjusting a securities beta (Damodaran, 1999).

The problem however is that, the weights assigned in order to adjust beta remains the same for all companies regardless of the size of the firm. The assumption of keeping weights constant does not make theoretical sense because the degree to which various companies converge towards the market should be different because sizes of firms are different. Firms that are huge and tend to diversify aggressively should have their betas converge towards the market faster than firms that are not diversified or those that concentrate on a sole business.

Critics of beta adjustment argue that, since in the end all firms would converge towards one there is no need to adjust beta soon after estimation since in the long run they would eventually converge towards the market. Firms would eventually converge towards one in the long run because as they survive the competition and increase in size over time they would have the capacity to acquire more assets hence becoming more diversified and in the end pushing its beta towards one (Damodaran,1999) .

2.2.3 Ways of Estimating Beta

Characteristic Line

One way to determine the beta of a security is to find the slope of the line that describes the relationship between an individual security return and return on the market portfolio. The security return is the dependent variable represented on the y axis where as the return on the market is independent variable, represented on the x axis.

2.3 TYPES OF BETA

2.3.1 Implicit Beta

The implicit beta model was introduced by Andrew Siegel because he believed that the model based on the regression analysis of historical or past data introduced substantial statistical error into estimates of beta that cannot fully reflect current market conditions (Siegel, 1995).

Under this model, the beta of a firms stock is computed directly from observed option prices. The concept of implicit volatility was proposed by Latane and Redleman who observed that the Black-Scholes call option revealed the volatility of the underlying asset and because it is based on current market price of an option instead of series of past observation the implicit volatility solves some of the problem associated with historical volatility by providing an up to date volatility measure without the substantial statistical error associated with estimation of a standard deviation from a sample data" (Siegel, 1995).

2.3.2 Consumption Beta

Douglass Breeden (1989) developed a model in which a securities risk is measured by its sensitivity to changes in investor's consumption and this is termed consumption beta. The beta for consumption attempts to measure the covariance between an investor's ability to consume goods and services from investments, and the return from a market index.

2.4 INTERPRETATION OF BETA

2.4.1 Beta Greater than 1

A stock with a beta of more than one is termed as an aggressive stock. This is because the stocks excess return varies more than proportionally with the excess return of the market portfolio. In essence, this stock has more unavoidable risk than the market has a whole.

2.4.2 Beta less than 1

A stock with a beta of less than one means that, the stocks excess return varies less than proportionally with the excess return of the market portfolio. This type of stock is often termed as defensive share.

2.4.3 Beta equal to 1

A stock with a beta of one implies that, excess return for the stock varies proportionally with excess return of the market portfolio. This type of stock has the same systematic risk as the market as a whole.

2.4.4 Negative beta

A stock with a negative beta implies that excess return for stock is inversely related with the excess return of the market portfolio.

Research by Shapiro and others indicated that "high-beta firms did significantly better than low-beta firms in a rising market and significantly worse in a falling market, just as the capital asset pricing model predicts" (Lakonishok et al, 1984).

2.4.5 Uses of Beta

As explained by the capital asset pricing model and security market line, beta is used to determine expected return on security. The higher the beta of a security, the higher its risk premium and also its expected return. On the other hand, the lower the beta of a security, the lower its risk premium and also its expected return.

2.6 INPUTS REQUIRED FOR PORTFOLIO ANALYSIS

For performing the portfolio analysis using the Markowitz method, we need the expected return for the period of holding for each of the securities to be considered for inclusion in the portfolio. We also require the standard deviation of the return for each security. In addition we have to know the covariance (or correlation coefficient) between each pair of securities among all securities from which we have to form the portfolio.

The model proposed by Markowitz points out to the need for estimating expected returns in quantitative terms. But this line of enquiry (estimating expected returns over a period of time) was not pursued further adequately in the literature. That may be one of the reasons, why papers outlining the application of the model to real life data were in short supply. Analysts were giving their anticipation regarding the performance of various securities in twelve months or one year ahead even in 1920s. But Benjamin Graham (1940), known as Dean of Wall Street, was not in favor of such analysis. This analysis slowly developed into prediction of target prices 12 months ahead for many securities. These target price predictions can be used to determine the expected returns for one year holding period. Using the target price predictions to determine 12-month expected returns and then using these expected returns to form the optimal portfolios is a feasible and rational line of approach. This approach to quantitative investing is proposed and initiated in this paper.

To estimate standard deviations and covariances, past data can be used (Grinold and Kahn, 2004). The historical risk measures of securities are more stable in comparison to historical expected return measures.

2.7 RESEARCH ON TARGET PRICES

Research on target prices is of recent origin. Bradshaw (2002) has examined the frequency with which analysts have used target prices to justify their stock recommendations. He reported that in two thirds of the sample reports that were examined by him, analysts used target prices. The target prices were determined using price multiple heuristics, with PEG (price earnings growth ratio) as one of the important rule for specifying the price-earning (P/E) multiple.

Asquith et al. (2004) have examined the performance of target prices set by analysts of All-American Analyst award winners for the period 1997-99. They examined whether the price of the security crossed its target price within 12 months after the recommendation. When this definition of accuracy was used, the authors have found that 54% of the price targets were achieved or exceeded. Even in the case of remaining 46% of the securities or recommendations, on average 84% of the price target was found to be achieved. This performance is very creditable. But we have to notice that these price targets were targets of award winners, where the award itself was based on their performance. So, to generalize the findings, we require studies of more representative samples.

Bradshaw and Brown (2005) have examined the accuracy of 12-months-ahead target price forecasts over the period 1997-2002. They reported that on an average 24 to 45 percent of forecasts were met. Analysts have shown more skill in forecasting company earnings compared to forecasting target prices. This study generated interest in study of success rate of target price forecasts.

Gleason et al. (2006) have examined the performance of target prices over the period 1997-2003. According to this study, the buy recommendations have an average target return of 28 percent. They analyzed results over quintiles. In the most accurate quintile, 57% of the targets were achieved or exceeded within the 12 month period. In the least accurate quintile, the success rate was found to be 49%. The interesting finding of the study is that the return that would have been earned by selling each of the securities with buy recommendations at their maximum prices within the 12 months is 42.49% even in the case of lowest quintile. One needs to compare this 42.49% with average target return of 28%. These studies do provide evidence that target price estimates have utility to investors for their decision making. They also provide the evidence that investors, traders and fund managers are encouraging analysts to provide target prices and many analysts are providing them.

2.8 USE OF TARGET PRICES IN PORTFOLIO FORMATION

If target prices have information content that is useful to earn return over 12-month horizon, portfolios can be formed using the target prices as the basis. The expected return can be determined as the difference between the target price and the current market price on the date of portfolio analysis and this can be expressed as percentage of current market price on the date of portfolio formation. If the investor/trader has this information with him, an optimal portfolio can be specified for him using Markowitz portfolio analysis.

2.9 ASSUMPTIONS UNDERLYING MARKOWITZ THEORY

Portfolio theory in the shape of Markowitz Theory makes the following assumptions concerning the investment market and investors' behavior within those markets. We summaries these assumptions below:

1. Investors seek to maximize the expected return of total wealth.
2. All investors have the same expected single period investment horizon.
3. All investors are risk-adverse, that is they will only accept greater risk if they are compensated with a higher expected return.
4. Investors base their investment decisions on the expected return and risk (i.e. the standard deviation of assets historical returns).
5. All markets are perfectly efficient (e.g. no taxes and no transaction cost).

2.9.1 Risk and Expected Return

MPT assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists which has better expected returns.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

Portfolio optimization has been a highly researched area in Operations Research. This project will apply one of the foundations in portfolio optimization, the Markowitz portfolio selection model (Markowitz, 1959) to solve real world problems based on the daily returns of six (6) different companies.

One of the main components in the inputs to portfolio analysis is the correlation structure of the stocks. When the number of stocks to select from the portfolio is large, the estimation of the covariance can get very impractical for computation purposes. This project will look into the single index model which was discussed comprehensively by Elton and Gruber (1987) and analyze its applicability in solving the Markowitz optimization model using real world data.

The performance measures are the difference in returns by Markowitz model and the Ghana Stock Exchange (GSE) All Share returns. The non-parametric pair sign test will provide the required statistical analysis as the data of difference in returns will later be found to be not normally distributed. Additionally, some parameters of the Markowitz model will be customized to provide a broader view on the performance analysis of the single index model as a method of forecasting the correlation structures for portfolio optimization.

3.2 MARKOWITZ' MEAN-VARIANCE MODEL

According to Markowitz (1952), the inputs needed to create optimal portfolios are: expected returns for every asset, variances for all assets and covariances between all of the assets handled by the model.

Markowitz does not state exactly how these parameters should be estimated although his discussion of some alternatives is quite detailed. He sees past performances as one source of information, but he emphasizes that portfolio selection solely based on historical data assumes that past data are reasonable approximation of the future ditto. Instead, Markowitz (1991) prefers the “probability beliefs” of experts as inputs to the portfolio analysis. He compares the way a security analyst arrives at probability beliefs with the way a meteorologist arrives at a weather forecast and calls the security analyst the meteorologist of stocks and bonds. Markowitz also emphasizes that portfolio analysis begins where security analysis ends.

In Markowitz’ model, expected future returns are to be estimated as the expected return of every asset during the investment period. Investors specify the length of the investment period.

Risk, in the Markowitz model, as well as in many other financial models, is approximated by the variances and covariances of future returns. When considering only one asset, it is sufficient to estimate and evaluate only its expected future return and the future variance. When evaluating a portfolio of assets, however, we should consider how the assets within the portfolio covariate to be able to estimate the variance of the portfolio as a whole. The covariance is a measure of how the values of two random variables move up and down together. In this case the random variables are any pair of assets in a portfolio. The covariance is crucial to portfolio theory and increases the possibilities of getting a well-diversified portfolio.

3.3 FORMULATION OF THE MARKOWITZ MODEL

In portfolio theory, investors are assumed to want as high expected future return as possible but at a risk as low as possible. There are many other factors, which investors might consider, but risk and return are what this model focuses on.

We use the following notation:

w - the column vector of portfolio weights

w* - the Markowitz’ optimal portfolio

σ^2 - the variance of the portfolio

$\bar{r}_i$ - the expected return of asset number i

r_{rf} - the return of the risk free asset.

$\bar{r}$ - the expected return of the portfolio

w_{rf} - the weight of the risk free asset in percent of the portfolio as a whole

μ - the column vector of expected (excess) returns

Σ - the covariance matrix.

δ - the risk aversion parameter stated by the investors. States the trade-off between risk and return

We set:

$$\bar{r} = \begin{bmatrix} \bar{r}_1 \\ \vdots \\ \bar{r}_d \end{bmatrix} \qquad e = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Hence we get:

$$er_{rf} = \begin{bmatrix} r_{rf} \\ \vdots \\ r_{rf} \end{bmatrix}$$

To derive the set of attainable portfolios (derived from the expected return and the covariance matrix estimated by the investor) that an investor can reach, we need to solve the following problem:

$$\begin{cases} \min\limits_{w} \mathbf{w}^T \Sigma w \\ w^T \bar{\mathbf{r}} = \bar{r} \end{cases} \tag{3.1}$$

Or

$$\begin{cases} \max\limits_{w} \mathbf{w}^T \bar{r} \\ w^T \Sigma w = \sigma^2 \end{cases} \tag{3.2}$$

We minimize the variance of the portfolio given a certain level of expected return or we maximize the expected return of the portfolio for a certain level of risk (variance).

Assume we have d risky assets. The weight of the risk-less asset in the portfolio is hence:

$w_{rf} = 1 - \mathbf{e}^T\mathbf{w}$

The expected return of the portfolio, r_P is then

$r_P = \mathbf{w}^T \mathbf{r} + w_{rf}\, r_{rf}$ and we can write the expected return as

$r_P = \mathbf{w}^\mathbf{T}\mathbf{r} + (1 - \mathbf{w}^\mathbf{T}\mathbf{e})r_{rf} = \mathbf{w}^\mathbf{T}\,(\mathbf{r} - \mathbf{e}r_{rf}) + r_{rf}$

We define the vector of expected (excess) returns as:

$$\mu \equiv \mathbf{r} - \mathbf{e}r_{rf} = \begin{bmatrix} \overline{r}_1 - r_{rf} \\ \vdots \\ \overline{r}_d - r_{rf} \end{bmatrix}$$

Hence, the universe of available portfolios has been expanded and the efficient frontier is moved.

The new efficient frontier is a weighted combination of the risk-free asset and the portfolio in which a straight line drawn from the risk-free rate or return is a tangent to the efficient frontier when no risk-free asset is available. This is also quite reasonable because, in this model, we always want an expected return as high as possible when taking a certain level of risk or as low level of risk as possible for a certain level of expected return.

Let us introduce the parameter δ, often referred to as the risk-aversion parameter. This parameter is a measure of the risk the trade-off between risk and expected return of the portfolio. We are to solve the following problem:

$$\max_{w}\; r_{rf} + \mathbf{w}^T\mu - \frac{\delta}{2}\mathbf{w}^T\Sigma\mathbf{w}$$

(3.3)

Since r_{rf} is constant, we can exclude it and still get the same result. The problem to be solved is hence:

$$\max_{w}\; w^T\mu - \frac{\delta}{2} w^T \Sigma w$$

This problem is solved by setting:

$e_k^T = [00...010..0]$, number of elements equals number of assets

Differentiate the function and set it equal to zero:

$$e_k^T\mu - \frac{\delta}{2} e_k^T \Sigma w - \frac{\delta}{2} w^T \Sigma e_k = 0$$

$$e_k^T(\mu - \delta \Sigma w) = 0$$

This is true for all $k = 1,...,d \Rightarrow$

$$w^* = (\delta \Sigma)^{-1}\mu \qquad (3.4)$$

Where, **w*** represents the Markowitz optimal portfolio given the risk aversion coefficient, covariance matrix and vector of expected returns estimated by the investor.

Problem (3.4) is actually the same as solving problem (3.1). Hence:

$$\begin{cases} \max_{w} w^T\mu \\ w^T\Sigma w = \delta^2 \end{cases}$$

The Lagrange function is then:

$L = \mathbf{w}^T\mu - \lambda\,(\mathbf{w}^T\Sigma\mathbf{w} - \sigma^2)$

Differentiating, we get:

$$e_k^T\mu - 2\lambda e_k^T \Sigma w = 0$$

This is the same as differentiating (3.3), which is

Let $\lambda = \frac{\delta}{2}$ then

$$e_k^T\mu - \delta e_k^T \Sigma w = 0$$

$\mu = \delta\Sigma\mathbf{w}$

$$w = (\delta\Sigma)^{-1}\mu$$

$\sigma^2 = \mathbf{w}^T\Sigma\mathbf{w} = \delta^{-2}\mu T\Sigma^{-1}\Sigma\Sigma^{-1}\mu = \delta^{-2}\mu^T\Sigma^{-1}\mu$

This shows that when we select the value of the parameter σ the value of δ is given.

We can also choose a value of δ and we then get the value of σ.

$$\lambda = \frac{\delta}{2}$$

$\frac{\delta}{2}$ is thus just the Lagrange multiplier.

When:

$\mu = \delta\Sigma\mathbf{w}$

$\mathbf{w}^* = (\delta\Sigma)^{-1}\mu$

$\mu_P = \mathbf{w}^{*T}\mu = \mu^T(\delta\Sigma)^{-1}\mu = \delta^{-1}\mu^T\Sigma^{-1}\mu$

[illegible]

then:

$$\delta = \frac{\mu_P}{\sigma_P^2}$$

This is also consistent with Satchell and Scowcroft (2000). Economists would call this parameter the standard price of variance.

Hence the Markowitz optimized portfolio is:

$\mathbf{w}^* = (\delta\Sigma)^{-1}\mu$

3.3.1 ALTERNATIVE FORMULATION OF THE MODEL

Consider the following coordinate system of expected return and standard deviation of return. It will help us to plot all combinations of investments available to us. Some investments are riskless and some are risky. Our optimal portfolio will be somewhere on the ray connecting risk free investments R_F to our risky portfolio and where the ray becomes tangent to our set of risky portfolios or efficient set it has the highest possible slope, in Figure 3 this point is showed by *B*. Different points on the ray between tangent point and interception with expected return coordinate represents combination of different amounts possible to lend or borrow to combine with our optimal risky portfolio on intersection of tangent line and efficient set.

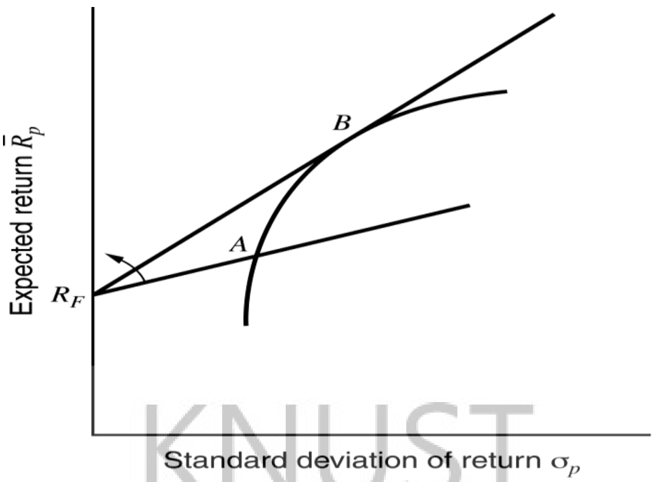


Figure 3.1 - Combinations of the riskless asset in a risky portfolio (Gruber et al.)

As we mentioned above, the ray discussed has the greatest slope. It can help us to determine

the ray. The slope is simply the return on the portfolio, R_P minus risk-free rate divided by standard deviation of the portfolio σ_p.Our task is to determine the portfolio with the greatest ratio of excess return to standard deviation θ. In mathematical terms, we should maximize the θ.(Later so called Sharpe ratio).

$$\theta = \frac{\bar{R}_P - R_F}{\sigma_P}$$

This function is subject to the constraint,

$$\sum_{i=1}^{N} X_i = 1$$

Where X_is are the samples members, also can be random variables. The constraint can be expressed in another way: Lintnerian, which considers an alternative definition for short sales. It assumes that when a stock is sold short, cash is not received but held as collateral. The constraint with Lintner definition of short sales is,

$$\sum_{i=1}^{N} |X_i| = 1$$

The above constrained problem can be solved by Lagrangian multipliers. We consider an alternative solution, by substituting the constraint in the objective function, where it will become maximized as in unconstrained problem. By writing R_F as R_F times 1,

$$R_F = R_F\left(\sum_{i=1}^{N} X_i\right) = \sum_{i=1}^{N} X_i R_F$$

By stating the expected return and standard deviation of the expected return in the general form we get,

$$\theta = \frac{\sum_{i=1}^{N} X_i\left(\bar{R}_i - R_F\right)}{\left[\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{i=1 \\ j\neq i}}^{N} X_i X_j \sigma_{ij}\right]^{1/2}}$$

Now we have the problem constructed and ready to solve. It is a maximization problem and solved by getting the derivatives of the function with respect to different variables and equating them to zero. It gives us a system of simultaneous equations,

$$1\frac{d\theta}{dX_1} = 0$$

$$2.\frac{d\theta}{dX_2} = 0$$

$$\vdots$$

$$N.\frac{d\theta}{dX_N} = 0$$

Let’s consider here the Lagrange theorem,

Let X be open in R^n and $f, g : X \to R$ be functions of class C. Let $S = \{x \in X \mid g(x) = c\}$ denote the level set of g at highest c. Then if $f|S$ (the restriction of f to S) has an extremum at a point $x_0 \in S$ such that $\nabla g(x_0) \neq 0$. There must be some scalar λ such that

$\nabla f(x_0) = \lambda \nabla g(x_0)$.

Where λ is called a Lagrange multiplier.

1. Form the vector equation, $\nabla f(x) = \lambda \nabla g(x)$

2. Solve the system,

$$\begin{cases} \nabla f(x) = \lambda \nabla g(x) \\ g(x) = c \end{cases} \quad \text{for} \quad x \quad \text{and} \quad \lambda > 0 \; .$$

By extension of this problem we have *n+1* equations in *n+1* unknowns $x_1, x_2, x_3, \ldots, x_n, \lambda$,

$$\begin{cases} f_{x_1}(x_1, x_2, ..., x_n) = \lambda g_{x_1}(x_1, x_2, ..., x_n) \\ f_{x_2}(x_1, x_2, ..., x_n) = \lambda g_{x_2}(x_1, x_2, ..., x_n) \\ f_{x_3}(x_1, x_2, ..., x_n) = \lambda g_{x_3}(x_1, x_2, ..., x_n) \\ \vdots \\ f_{x_n}(x_1, x_2, ..., x_n) = \lambda g_{x_n}(x_1, x_2, ..., x_n) \\ g(x_1, x_2, ... x_n) = c \end{cases}$$

Where the solution for $x \in (x_1, x_2, \ldots, x_n)$, along with any other point satisfying $\lambda\, g(x) = 0$, are candidate extrema for the problem.

3. Finally we determine nature of f (as maximum, minimum or neither) at the critical points found in step 2.

This method reduces a problem in n variables with k constraints to a solvable problem in $n+k$ variables with no constraint. This method introduces a new scalar variable, the Lagrange multiplier, for each constraint and forms linear combination involving the multipliers as coefficients.

Before we start to mention Lagrange theorem we got to the point that in order to solve the maximization problem we need to take derivatives of the ratio θ . We re-write θ in the following form;

$$\theta = \left[\sum_{i=1}^{N} X_i R_i - R_f\right] \left[\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij}\right]^{-\frac{1}{2}}$$

As it is written above, the ratio consists of multiplication of two functions. To differentiate this function, we need to use both the product rule and the chain rule of differentiation.

After applying the chain rule, we use product rule and we get,

$$\frac{d\theta}{dX_k} = \left[-\frac{1}{2}\left(\sum_{i=1}^{N} X_i(\bar{R}_i - R_F)\right)\left(\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_iX_j\sigma_{ij}\right)^{-3/2}\left(2X_k\sigma_k^2 + 2\sum_{\substack{j=1 \\ j\neq k}}^{N} X_j\sigma_{jk}\right)\right.$$

$$\left. + \left(\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_iX_j\sigma_{ij}\right)^{-1/2}\right]\left[(\bar{R}_k - R_F)\right] = 0$$

If we multiply the derivative by $\left(\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_iX_j\sigma_{ij}\right)^{1/2}$ and rearrange, then;

$$-\left[\frac{\sum_{i=1}^{N} X_i(\bar{R}_i - R_F)}{\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_iX_j\sigma_{ij}}\right]\left(X_k\sigma_k^2 + \sum_{\substack{j=1 \\ j\neq k}}^{N} X_j\sigma_{jk}\right) + \left[(\bar{R}_k - R_F)\right] = 0$$

Where we define λ as the Lagrange multiplier,

$$\frac{\sum_{i=1}^{N} X_i(\bar{R}_i - R_F)}{\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_iX_j\sigma_{ij}}$$

This yields

$$-\lambda\left(X_k\sigma_k^2 + \sum_{\substack{j=1 \\ j\neq k}}^{N} X_j\sigma_{jk}\right) + \left[(\bar{R}_k - R_F)\right] = 0$$

By multiplication,

$$-\left(\lambda X_k\sigma_k^2 + \sum_{\substack{j=1 \\ j\neq k}}^{N} \lambda X_j\sigma_{jk}\right) + \left[(\bar{R}_k - R_F)\right] = 0$$

Now, by extension

We use a mathematical trick, where we define a new variable $Z_k = \lambda X_k$. The X_k are fraction to invest in each security, and Z_k are proportional to this fraction. In order to simplify we substitute Z_k for λX_k and move variance covariance terms to the left,

3.4 MARKOWITZ MODEL – SOLUTION APPROACH

The basic solution approach to this problem is to implement the Markowitz model in finding an optimal portfolio selection in each forecast period. There are two objectives behind Markowitz model; to achieve high returns and to achieve stable returns with low uncertainty. In this project, the objective is function is to maximize total returns, constrained by maximum allowable risk level. The Markowitz optimization model can be modeled as follows:

Inputs

$r_i =$ Return on company i

$k =$ Maximum risk factor

$\text{cov}_{ij} =$ Covariance between company i and j

$N =$ Portfolio size (number of companies)

Decision Variables

$x_j =$ Fraction of portfolio to invest in industry j

Objective

Markowitz Total Returns : $\sum_{i \in 1} r_i * x_i$

Constraints

Budget constraint:

$$\sum_{i \in 1} x_i \leq 1$$

Maximum allowable risk:

$$\sum_{i \in 1} \sum_{j \in J} x_j * \text{cov}_{ji} * x_i \leq \frac{k}{N}$$

Two generally conflicting measures evaluate the portfolio, the expected return and the variance of the return. The latter represents the risk of the portfolio. The investor desires a portfolio that has a high return and low risk. Since the goals of maximizing return and minimizing risk are usually conflicting, we create a model that minimizes variance while satisfying a constraint on the return. By solving the model for a series of returns we obtain an efficient frontier of solutions.

Depending on the investor's risk tolerance, he or she should choose one of these solutions. Markowitz computes the variance of a portfolio using the Covariance matrix.

The Portfolio add-in in Excel creates the structure to hold historical data, constructs a mathematical programming model, provides for solving the model and provides for generating the efficient frontier.

The Math Programming add-in in Excel constructs the math programming model and the Solver add-in solves the model.

The variance of the Portfolio can be computed from the covariance matrix as shown below.

3.5 COMPUTING PORTFOLIO VARIANCE WITH COVARIANCE

We use statistical estimates from the historical data to compute the Covariance matrix used by the model.

a_{ij}: Covariance between security i and j

r_{ij}: Correlation between security i and j

σ_j: standard deviation of security j

$v_j = (\sigma_j)^2$: variance of security j

$a_{ij} = \sigma_i \sigma_j r_{ij}$

Q: Covariance matrix

x_j: proportion for security j in the portfolio

X: Column matrix of x_j values

Var = $\sum_{j=1}^{n} v_j x_j^2 + 2\sum_{j=1}^{n-1}\sum_{j=1}^{n} a_{ij} x_i x_j = \mathrm{x}^T Q \mathrm{x}$

The math programming model limits the total return while requiring that the security proportions sums to 1. The goal is to minimize portfolio variance.

3.5.1 Minimum Variance Model

P: Lower bound on the portfolio return.

μ_j: Mean return for security j.

Min. $Var = \sum_{j=1}^{n} v_j x_j^2 + 2\sum_{j=1}^{n-1}\sum_{j=1}^{n} a_{ij} x_i x_j = \mathrm{x}^T Q \mathrm{x}$

Subject to:

$$\sum_{i=1}^{n} x_j = 1$$

The project uses historical data to estimate the inputs to Markowitz model. These inputs

require estimate of the expected return on each stock r_i and the covariance between each possible pair of stocks for stocks under consideration. The estimation can get very complex as the portfolio size become large. For instance, if the number of stocks in a portfolio is 48, we need to estimate $\frac{N*(N-1)}{2} = \frac{(48*47)}{2} = 1128$ correlation coefficients. The large number of inputs can be computationally impractical due to the large number of estimates that have to be made. Part of the project was to develop a more efficient estimation model and assess its performance in terms of total returns created. For this objective, single index model was applied to simplify the inputs to the Markowitz model.

3.6 SINGLE INDEX MODEL

Single index model assumes that the co-movement between stocks is due to the large single common influence by market performance. Hence, the measure of this index can be found by relating the stock return to the return on a stock market index. The formulation for single index model can be shown below:

$r_i = a_i + \beta_i r_m$

Where

$r_i =$ Return on stock i

$a_i =$ Component of stock $i's$ return that is independent of the market's performance

$r_m =$ The rate of return on the market index

$\beta_i =$ A constant that measures the expected change in r_i given a change in r_m

The term a_i can be further broken down into α_i and e_i where α_i is the expected value of a_i and e_i is the random element in a_i.

The expected return, variance and covariance can be estimated as follows when they are used to represent the joint movement of stocks:

Mean return of stock, $\bar{r}_i = \alpha_i + \beta_i \bar{r}_m$

Variance of stock's return $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$

Covariance of returns between stocks i and j, $\sigma_{ij} = \beta_i \beta_j \sigma_m^2$

Where σ_m^2 =market variance and σ_{ei}^2 =unique risk factor

The single index model will need the estimates of mean return, variance of return and the beta for each stock, which amounts to $3N+2=(3*48+2)=146$ estimates, in the case of 48 companies. This is much easier to compute than the previously mentioned estimates of 1128 correlation coefficients. For the purpose of the Markowitz model, the mean return of each company , r_i and the market variance , σ_m^2 by calculating the average industry returns and variance of market returns over a specified period, respectively. Finally, we need to estimate beta for each stock in order to calculate the covariance needed in the Markowitz model. Beta is simply a measure of sensitivity of stock to market movement. There are 3 methods of estimating beta as forecasters of covariance:

i. Forecasts of covariance by estimating betas from prior historical period (unadjusted beta)

ii. Forecasts of covariance by estimating betas from the prior two periods and updating via Blume's technique (Blume's beta)

iii. Forecasts of covariance by estimating betas from prior historical period and updating via Vasicek's technique (Vasicek's beta).

3.6.1 Method 1: Unadjusted Beta

The first method simply estimates betas from historical data. The historical beta for each stock i can be obtained through regression analysis of stock return r_{it} against market return r_{mt} from a past period, $t=1$ to $t=T$. The calculation of beta for each stock is formally shown below. The estimation of historical beta is subjected to error and might deviate significantly from actual beta since actual beta is not perfectly stationary over time. The betas might change significantly from one period to another and large random error may lead to substantial error.

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\sum_{t=1}^{T}\left[\left(r_{it} - \overline{r}_{it}\right)\left(r_{mt} - \overline{r}_{mt}\right)\right]}{\sum_{t=1}^{T}\left(r_{mt} - \overline{r}_{mt}\right)^2}$$

3.6.2 Method 2: Blume's Beta

Blume's analysis on the behaviour of betas over time shows that there is a tendency of actual betas in the forecast period to move closer to one than the estimated betas from historical data. Blume's technique attempts to describe this tendency by correcting historical betas to adjust the betas towards one, assuming that adjustment in one period is a good estimate in the next period. Consider betas for all stocks i in period 0, β_{i0} and betas for the same stocks i in the successive period 1, β_{i1}. The betas for period 1 are then regressed against the betas for period 0 to obtain the following equation:

$$\beta_{i1} = k_1 + k_2\beta_{i0}$$

The relationship implies that the beta in period 1 is $k_1 + k_2$ times the beta in the period 0.

Therefore, if β_{i1} is A, the estimate of beta in the next period β_{i2} will be $(k_1 + k_2 * A)$ instead of A. This adjustment sets the average beta to undergo similar trend for subsequent forecast periods. If there is an increasing trend in average beta for period 1, average beta for period 2 will consequently increase. This might not reflect the actual beta movement from one period to another. Hence, Blume further modifies the average beta towards historical mean. This is done by first calculating the average beta of all stocks for period 1 and 2, $\overline{\beta}_1$ and $\overline{\beta}_2$. To adjust the mean of the forecasted beta towards historical mean, the new forecast of beta for each stock i β_{i2} is obtained by subtracting $\overline{\beta}_2$ from the previously forecast of beta and adding $\overline{\beta}_1$.

3.6.3 Method 3: Vasicek's Beta

As mentioned earlier, the average beta tends to move towards one over time. Another method to capture this tendency is via Vasicek's technique. Vasicek's technique adjusts past betas towards the average beta by modifying each beta depending on the sampling error about beta. When the sampling error is large, there is higher chance of larger difference from the average beta. Therefore, lower weight will be given to betas with larger sampling error. The following formula demonstrates this idea:

$$\bar{\beta}_{i2} = \frac{\sigma^2_{\beta 1i}}{\sigma^2_{\bar{\beta}1} + \sigma^2_{\beta 1i}}\bar{\beta}_1 + \frac{\sigma_{\bar{\beta}1}}{\sigma^2_{\bar{\beta}1} + \sigma^2_{\beta 1i}}\beta_{i1}$$

Where

$\beta_{i2} =$ forecast of beta for stock i for period 2(later period)

$\bar{\beta}_1 =$ average beta across the sample of stocks in period 1(earlier period)

$\beta^2_{\bar{\beta}1} =$ variance of the distribution of historical estimates of beta across the sample of stocks.

$\beta_{i1} =$ estimate of beta for stock i in period 1.

$\sigma^2_{\beta 1i} =$ variance of the estimate of beta for stock i in period 1

CHAPTER 4

ANALYSIS OF DATA

4.1 PRELIMINARY ANALYSIS

The Single Index Model assumes that the co-movement between stocks is due to the large single common influence by market performance. Hence, the measure of this index can be found by relating the stock return to the return on a stock market index. The formulation for single index model can be shown below:

$$r_i = a_i + \beta_i r_m$$

Where

$r_i =$ Return on stock i

$a_i =$ Component of stock $i's$ return that is independent of the market's performance

$r_m =$ The rate of return on the market index

$\beta_i =$ A constant that measures the expected change in r_i given a change in r_m

Snapshots of the preliminary analysis of the six companies which were done by regression runs in Excel have been displayed in the Appendices.

A summary of the regression runs for the six companies is shown below:

Table 4.1

Variable	Coefficients β_i	Standard Error	t Stat	$P-value$
GCB	1.690116584	0.215157072	7.855269	1.07E-10
SG-SSB	-99.81884494	16.63172254	-6.00171	1.35E-07
SCB	-102.3155424	12.4564041	-8.21389	2.69E-11
EIC	-90.63997934	24.56393666	-3.68996	0.000497
HFC	-80.8483	12.80099984	-6.31578	4.34954E-08
TOTAL GHANA	-145.534488	12.56731208	-11.5804	1.02E-16

4.1.1 Discussion of Regression Results for GCB

The model for the return on GCB's stock is given by $r_i = 0.0202 + 1.6901 r_m$.

(See regression output in Appendices).The a_i value of 0.0202 indicates that 20% of GCB's stock is independent of market performance. Also, for every one unit increase in the return of GCB stock, the rate of return on the market index (beta) increases by 1.6901. GCB's stock is thus considered very aggressive.

R^2 of 0.5155 indicates that 51.55% of the variation in the return on GCB's stock is explained by variation in the return on the GSE All Share Index as a whole, i.e. systematic risk explains about 51.55% of the total variability of GCB's stock.

4.1.2 Discussion of Regression Results for SG-SSB

The model for the return on SG-SSB's stock is given by $r_i = -0.1711 - 99.8188 r_m$. It is obvious that for every one unit increase in the return of this stock, the rate of return on the market index (beta) falls by 99.8188. This also means that when the rate of return on the market index is zero, the return on SG-SSB's stock falls by 17%.

R^2 of 0.3831 indicates that 38.31% of the variation in the excess return on SG-SSB's stock is explained by variation in the excess return on the GSE as whole i.e.: systematic risk explains about 38.31% of the total variability of SSB's stock.

4.1.3 Discussion of Regression Results for SCB

The equation for the return of SCB's stock is given by $r_i = -0.6806 - 102.3155 r_m$. It means for every unit increase in the return on the stock, the rate of return on the index falls by 102.3155. Hence SCB's stock is very volatile and less risky.

R^2 of 0.5377 indicate that 53.77% of variation in the excess return on SCB's stock is explained by variation in the excess return on the GSE as a whole i.e.: systematic risk explains about 53.77% of the total variability of SCB's stock.

4.1.4 Discussion of Regression Results for EIC

The equation for the return on EIC's stock is given by $r_i = -1.7303 - 90.6399 r_m$. This implies that for every unit increase in the return on EIC's stock, the rate of return is reduced by 90.6399.

R^2 of 0.1901 indicate that 19.01% of variation in the excess return on EIC's stock is explained by variation in the excess return on the GSE as a whole i.e. systematic risk explains about 19.01% of the total variability of EIC's stock.

4.1.5 Discussion of Regression Results for HFC

The equation for the return on HFC's stock is given by $r_i = -1.2165 - 80.8483 r_m$. This can be interpreted as for every unit increase in the return on this stock, the rate of return on the index is reduced by 80.8483.

R^2 of 0.4117 indicate that 41.17% of variation in the excess return on HFC's stock is explained by variation in the excess return on the GSE as a whole i.e.: systematic risk explains about 41.17% of the total variability of HFC's stock.

4.1.6 Discussion of Regression Results for TGL

The equation on the return on Total Ghana's stock is given by $r_i = 0.5896 - 145.5345 r_m$. This means that 58.96% of Total's mean return is independent on the market performance. This makes totals stock the best among the six stocks. This will be confirmed further with the optimal portfolio solution in the Excel worksheet.

R^2 of 0.6981 indicates that 69.81% of variation in the excess return on TGL'S stock is explained by variation in the excess return on the GSE as a whole ie: systematic risk explains about 69.81% of the total variability of TGL'S stock. This means that Total Ghana Ltd stock is a very high risk stock.

Summary of Results of Regression Runs

Stock	**Beta**	$\mathbf{R^2}$	$\mathbf{1 - R^2}$
GCB	1.69	0.5154	0.48
SSB	-99.81	0.3831	0.62
HFC	-80.85	0.4112	0.59
SCB	-102.32	0.5377	0.46
EIC	-90.64	0.1901	0.81
TGL	-145.53	0.6901	0.31

Table 5.1: Unsystematic risk underlying the various stocks

4.2 IMPLEMENTATION OF THE MODEL IN EXCEL

4.2.1 Market (Portfolio) Return (R_m)

The Market (Portfolio Return), R_m for each month is calculated from the GSE Stock Market Index (1998-2002) using the following formula;

$$Market\ \ Return(R_m) = \frac{Closing\ Index - Beginning\ Index}{Beginning\ Index} * 100\%$$

4.2.2 Security's Return (R_i)

A Security's Return (R_i) is calculated from stock prices of the various companies i.e. from Table 3.1, 3.3, 3.4, 3.5, 3.6, 3.7 using the following formula.

$$R_i = \frac{Ending\ Price - Beginning\ Price + Dividends}{Beginning\ Price}$$

4.2.3 Risk Free Rate (R_f)

The monthly Risk free rate (R_f) is calculated from the Bank of Ghana 91 – Day Treasury Bill Rates (Table 3.8).

$$\text{Monthly Risk Free Rate} = \frac{Annual\ \text{Rate}}{100*12}$$

The resultant table of returns or yield, using Excel, is given in Appendix IX.

4.3 COVARIANCE MATRIX

Statistical analysis in Excel computes the following averages, standard deviations and the covariance matrix. This is shown as follows:

Investments	1	2	3	4	5	6	7
Name	All-Share	GCB	SG-SSB	SCB	HFC	EIC	TOTAL GH
Return	-1.98	-3.89	-2.18	-2.70	-2.76	-3.52	-2.29
Standard Deviation	8.173568	18.67413	13.29148	11.40436	10.3051	16.99081	14.2371859
Select	No	No	No	No	No	No	No

The covariance matrix is given by a 7x7 matrix as follows:

Covariance Matrix

	All-Share	GCB	SG-SSB	SCB	HFC	EIC	TOTAL GH
All-Share	66.81	112.62	66.62	67.21	53.09	59.54	95.61
GCB	112.62	348.72	90.85	76.75	72.41	54.76	182.88
SG-SSB	66.62	90.85	176.66	50.62	40.47	231.63	82.77
SCB	67.21	76.75	50.62	130.06	67.97	63.82	89.50
HFC	53.09	72.41	40.47	67.97	106.20	59.51	91.54
EIC	59.54	54.76	231.63	63.82	59.51	288.69	55.43
TOTAL GH	95.61	182.88	82.77	89.50	91.54	55.43	202.70

The statistical analysis is entirely computed with Excel built-in functions. Cells on the main diagonal on the covariance matrix are computed with the VAR function. Cells above the diagonal are computed with the COVAR function and cells below the diagonal equal the corresponding values above the diagonal.

The covariance matrix is always symmetric and has the positive definite characteristic. This guarantees that the variance objective is strictly convex and that there is a single local minimum point that is also the global minimum. This simplifies the problem of finding the optimum solution.

The minimum variance solution is employed in the analysis. The Markowitz model is implemented in the worksheet screenshot below. Cell H13 computes the variance of the portfolio using matrix computations. Row 17 holds the constraint that the portfolio proportions must sum to 1. Row 18 holds the constraint that he average return be at least 0.75.

This is a nonlinear minimization problem with a convex objective function and linear constraints. With these conditions, there should be a unique local minimum (if there is a feasible solution).

As suggested by Markowitz, the optimum solution will often be diversified with several securities included in the portfolio. This is also suggested by the nonlinear-strictly convex nature of the objective function.

From the worksheet below, Total Ghana Limited with the greatest return has comprises 37.5% of the portfolio with the five other securities completing the portfolio. It can be observed that the character of the solution changes with the chosen value for the minimum return.

A snapshot of the worksheet before the model is solved is shown below. The inputs are entered in the solver as shown in the dialog box below:

<u>Setting up the inputs to the Markowitz Model</u>

Inputs

$r_i =$ Return on company i (as shown in the table of returns in Appendix X)

$k =$ Maximum risk factor (1)

$\text{cov}_{ij} =$ Covariance between company i and j

$N =$ Portfolio size (number of companies)

Decision Variables

$x_j =$ Fraction of portfolio to invest in industry j , where j=1,2,3…

Objective

Markowitz Total Returns: $\sum_{i \in 1} r_i * x_i$

Constraints

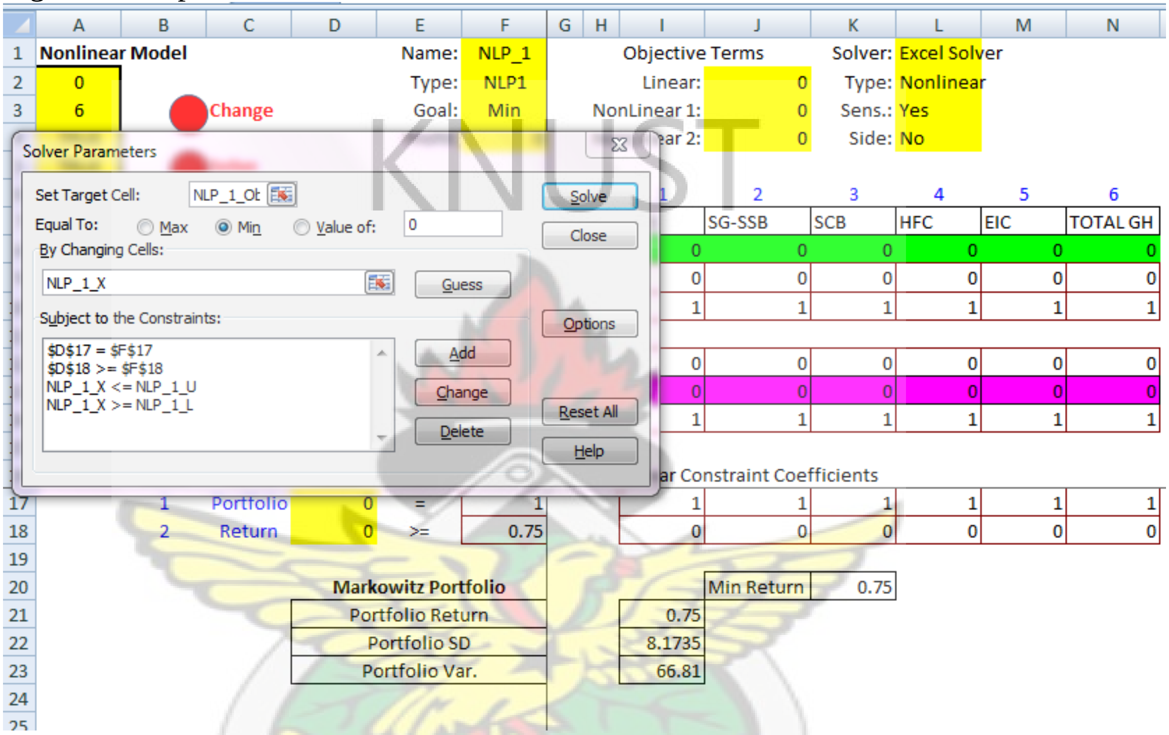
Budget constraint:

$$\sum_{i \in 1} x_i \leq 1$$

Maximum allowable risk:

$$\sum_{i \in 1} \sum_{j \in J} x_j * \text{cov}_{ji} * x_i \leq \frac{k}{N}$$

Figure 4.1 Implementation of the Model in Excel Solver



The feasible solution as found by the model is shown in the green cells below. GCB makes up 19.64% of the optimal portfolio, SG-SSB makes up 8.93% of the of the portfolio, SCB makes up 17.86% of the portfolio, HFC makes up 10.71% of the portfolio, EIC makes up 5.36% of the portfolio and Total Ghana Limited makes up 37.5% of the portfolio.

This means that for an investor to make *good* return on his investment, he or she should invest these proportions in the portfolio. This is when he expects a return of at least 75% of his investment.

Fig. 4.2 Optimal Solution found to the Markowitz Model

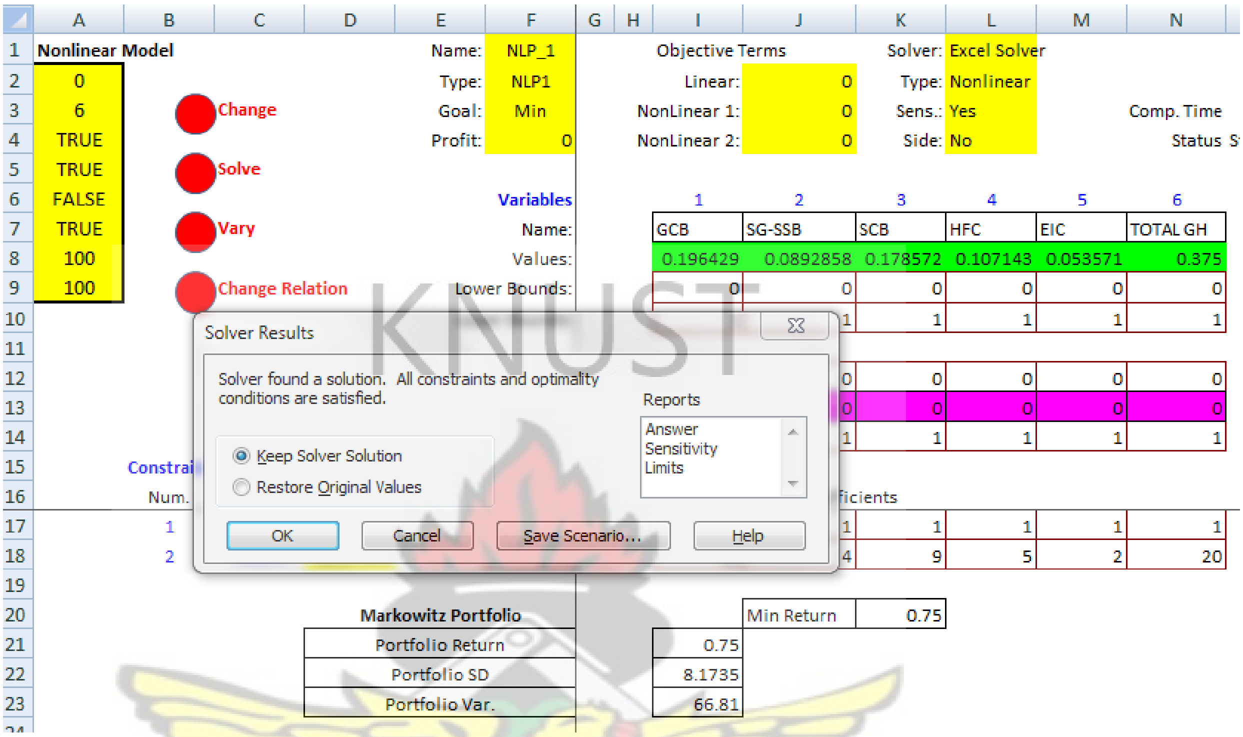


Figure 4.3 Solution showing proportion of shares to be invested to achieve high returns

B	C	D	E	F	G	H	I	J	K	L	M
Model			Name:	NLP_1		Objective Terms		Solver:	Excel Solver		
			Type:	NLP1		Linear:	0	Type:	Nonlinear		
	Change		Goal:	Min		NonLinear 1:	0	Sens.:	Yes		
			Profit:	0		NonLinear 2:	0	Side:	No		
	Solve										
				Variables		1	2	3	4	5	6
	Vary			Name:		GCB	SG-SSB	SCB	HFC	EIC	TOTAL GH
				Values:		0.196429	0.0892858	0.178572	0.107143	0.053571	0.375
	Change Relation			Lower Bounds:		0	0	0	0	0	0
				Upper Bounds:		1	1	1	1	1	1
				Linear Obj. Coef.:		0	0	0	0	0	0
				Nonlinear Obj. Terms:		0	0	0	0	0	0
				Nonlinear Obj. Coef.:		1	1	1	1	1	1
Constraints											
Num.	Name	Value	Rel.	RHS		Linear Constraint Coefficients					
1	Portfolio	1.000001	=	1		1	1	1	1	1	1
2	Return	12.07144	>=	0.75		10	4	9	5	2	20
		Markowitz Portfolio					Min Return	0.75			
			Portfolio Return			0.75					
			Portfolio SD			8.1735					
			Portfolio Var.			66.81					

CHAPTER 5

5.1 CONCLUSION AND RECOMMENDATIONS

An investor's approach to investing for the future should be no different from the approach to other important life decisions: use common sense, and remembering the old adage, "Don't put all your eggs in one basket". This best sums up the concept of diversification. All investing involves some degree of risk. Diversification is a simple way to manage those risks.

A well diversified portfolio is one's best bet for the growth of their investments.

GCB's stock is very aggressive and sensitive and good for risk-loving investors.

Total Ghana Stock is less risky hence Markowitz invested more in this stock, followed by GCB and the rest.

The Markowitz Model could be solved for a series of expected returns, which could be plotted against standard deviation of returns to produce what is called an efficient frontier

5.2 RECOMMENDATIONS

- Continuous historical data should be made accessible to all students who need them for research purposes.
- Future research could extend the historical period to ten or fifteen years.
- The number of companies could be increased to include major sectors of the economy like oil and gas, agric, banking and finance and services sector.
- Information on the efficient frontier of listed companies should be provided by the industry players on a regular basis so as to inform the general public about where to invest and at what risk. This will provide periodic and relevant information to prospective local investors.

- Government and policy-makers should include the study of finance and investment in the lower levels of the educational sector e.g. courses run by the Ghana Stock Exchange should be extended to schools.
- Companies must not be allowed to charge for data obtained for research and academic purposes

5.3 CONCLUSION

Ghana Commercial Bank's stock is a very sensitive and aggressive stock. A stock with a beta of more than one is termed as an aggressive stock. This is because the stocks excess return varies more than proportionally with the excess return of the market portfolio. In essence, this stock has more unavoidable risk than the market has a whole. The stock is very risky due to its high beta of 1.69.

Risk-loving investors who want higher returns could invest in this stock. This explains why it makes up 19.64% of the optimal portfolio.

The rest of the companies i.e. SG-SSB, SCB, HFC, EIC and TOTAL all have negative betas. A stock with a negative beta implies that excess return for stock is inversely related with the excess return of the market portfolio.

In conclusion, Total Ghana stock is the best among the six companies. This is followed by GCB which makes up 19.64% of the portfolio, followed by SCB which makes up 17.86% of the portfolio, followed by HFC which makes up 10.71% , SCB 8.93% and EIC 5.35%.

Hence an investor who invests in these proportions in the portfolio is assured of a good return on his or her investment.

For a series of expected returns, the proportions of the optimal portfolios could be achieved and plotted together. This will produce an efficient frontier which will show the relationship between the expected returns with the corresponding risks.

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APPENDICES

The following is the set of historical data to be used for the calculations. This comprises the beginning and ending prices of 6 companies trading on the Ghana Stock Exchange. Also included is the government of Ghana 91-day Treasury bill rates, which is considered to be the risk – free asset and the Ghana Stock Exchange (GSE) All-Share Index, from which will be calculated the market return and excess market returns.

The companies are Ghana Commercial Bank, SG-SSG Bank, Standard Chartered Bank, Enterprise Insurance Company Limited, Home Finance Company Limited and Mobil Oil (Now Total Ghana Limited).

The format of the data is as follows: Month, Year, Beginning Price, Closing Price, Dividends, Security Returns, Risk Free Rate, Excess Stock Return, Market Return and Excess Market Returns.

APPENDIX I - GSE STOCK MARKET RETURNS (1998 - 2002)

MONTH	YEAR	BEGINNING INDEX	CLOSING INDEX	MARKET RETURN R_M	EXCESS MARKET RETURN (R_m - R_f)
JANUARY	1998	511.74	562.83	0.099835854	0.066585854
FEBRUARY	1998	562.83	648.51	0.152230691	0.119314025
MARCH	1998	648.51	849.19	0.309447811	0.276531144
APRIL	1998	849.19	1184.58	0.394952837	0.362036171
MAY	1998	1184.58	1028.55	-0.131717571	-0.164634237
JUNE	1998	1028.55	970.18	-0.056749793	-0.085583127
JULY	1998	970.18	1012.19	0.043301243	0.015301243
AUGUST	1998	1012.19	980.2	-0.031604738	-0.059521405
SEPTEMBER	1998	980.2	854.29	-0.128453377	-0.15453671
OCTOBER	1998	854.29	784.41	-0.081798921	-0.104132254
NOVEMB	1998	784.41	776.32	-0.010313484	-0.032730151

ER					
DECEMBER	1998	776.32	868.35	0.118546476	0.096213142
JANUARY	1999	868.35	893.42	0.028870847	0.006537514
FEBRUARY	1999	893.42	862.03	-0.035134651	-0.056634651
MARCH	1999	862.03	828.61	-0.038768952	-0.060268952
APRIL	1999	828.61	820.88	-0.009328876	-0.030745543
MAY	1999	820.88	819.14	-0.002119676	-0.023036343
JUNE	1999	819.14	806.12	-0.015894719	-0.036311386
JULY	1999	806.12	787.81	-0.02271374	-0.043130407
AUGUST	1999	787.81	788.76	0.001205875	-0.019127459
SEPTEMBER	1999	788.76	759.93	-0.036551042	-0.056967709
OCTOBER	1999	759.93	759.41	-0.000684274	-0.02285094
NOVEMBER	1999	759.41	747.53	-0.015643723	-0.041643723
DECEMBER	1999	747.53	736.16	-0.015210092	-0.041460092
JANUARY	2000	736.16	741.66	0.007471202	-0.018778798
FEBRUARY	2000	741.66	739.73	-0.002602271	-0.028852271
MARCH	2000	739.73	763.1	0.031592608	0.005342608
APRIL	2000	763.1	869	0.138776045	0.112526045
MAY	2000	869	812.57	-0.064936709	-0.091436709
JUNE	2000	812.57	817.79	0.006424062	-0.024242605
JULY	2000	817.79	821.9	0.00502574	-0.028807593
AUGUST	2000	821.9	822.03	0.00015817	-0.033258497
SEPTEMBER	2000	822.03	855.51	0.040728441	0.008978441
OCTOBER	2000	855.51	863.84	0.009736882	-0.021846451
NOVEMBER	2000	863.84	866.25	0.002789868	-0.028876798
DECEMBER	2000	866.25	857.98	-0.009546898	-0.041213564
JANUARY	2001	857.98	858.52	0.000629385	-0.031037281
FEBRUARY	2001	858.52	879.12	0.023994782	-0.007671885
MARCH	2001	879.12	899.26	0.022909273	-0.009757394
APRIL	2001	899.26	897.88	-0.001534595	-0.035284595
MAY	2001	897.88	894.53	-0.003731011	-0.038314344
JUNE	2001	894.53	932.47	0.042413334	0.007580001
JULY	2001	932.47	1024.34	0.098523277	0.06360661
AUGUST	2001	1024.34	949.57	-0.072993342	-0.105326675

SEPTEMBER	2001	949.57	956.04	0.00681361	-0.02268639
OCTOBER	2001	956.04	961.01	0.005198527	-0.021468139
NOVEMBER	2001	961.01	958.54	-0.002570213	-0.028070213
DECEMBER	2001	958.54	955.95	-0.002702026	-0.025702026
JANUARY	2002	955.95	957.34	0.001454051	-0.020629282
FEBRUARY	2002	957.34	969.89	0.01310924	-0.007724093
MARCH	2002	969.89	1018.02	0.049624184	0.029957518
APRIL	2002	1018.02	1041.05	0.022622345	0.002789012
MAY	2002	1041.05	1132.68	0.088016906	0.068183573
JUNE	2002	1132.68	1223.69	0.08034926	0.05959926
JULY	2002	1223.69	1257.08	0.027286323	0.006286323
AUGUST	2002	1257.08	1309.71	0.041866866	0.020116866
SEPTEMBER	2002	1309.71	1310.67	0.000732987	-0.02093368
OCTOBER	2002	1310.67	1339.76	0.022194755	0.000444755
NOVEMBER	2002	1339.76	1362.65	0.01708515	-0.00466485
DECEMBER	2002	1362.65	1395.31	0.023968004	0.002051337

APPENDIX II - GHANA COMMERCIAL BANK - STOCK RETURNS (1998 - 2002)

MONTH	YEAR	BEGINNING PRICE	ENDING PRICE	DIVIDENDS	STOCK RETURN	EXCESS STOCK RETURN (R_i - R_f)
JANUARY	1998	800	838		0.0475	0.01425
FEBRUARY	1998	838	880		0.050119332	0.017202665
MARCH	1998	880	1,180		0.340909091	0.307992424
APRIL	1998	1,180	2,400		1.033898305	1.000981638
MAY	1998	2,400	1,300		-0.458333333	-0.49125
JUNE	1998	1,300	1,710		0.315384615	0.286551282
JULY	1998	1,710	1,670		-0.023391813	-0.051391813
AUGUST	1998	1,670	1,460		-0.125748503	-0.15366517
SEPTEMBER	1998	1,460	1,155		-0.20890411	-0.234987443
OCTOBER	1998	1,155	1,165		0.008658009	-0.013675325
NOVEMBER	1998	1,165	1,140		-0.021459227	-0.043875894
DECEMBER	1998	1,140	1,300	100	0.228070175	0.205736842
JANUARY	1999	1,300	1,100		-0.153846154	-0.176179487
FEBRUARY	1999	1,100	1,000		-0.090909091	-0.112409091
MARCH	1999	1,000	990		-0.01	-0.0315
APRIL	1999	990	1,000		0.01010101	-0.011315657
MAY	1999	1,000	1,000		0	-0.020916667

JUNE	1999	1,000	1,000		0	-0.020416667
JULY	1999	1,000	985		-0.015	-0.035416667
AUGUST	1999	985	900		-0.086294416	-0.10662775
SEPTEMBER	1999	900	850		-0.055555556	-0.075972222
OCTOBER	1999	850	840		-0.011764706	-0.033931373
NOVEMBER	1999	840	750		-0.107142857	-0.133142857
DECEMBER	1999	750	760	175	0.246666667	0.220416667
JANUARY	2000	760	770		0.013157895	-0.013092105
FEBRUARY	2000	770	735		-0.045454545	-0.071704545
MARCH	2000	735	905		0.231292517	0.205042517
APRIL	2000	905	980		0.082872928	0.056622928
MAY	2000	980	978		-0.002040816	-0.028540816
JUNE	2000	978	978		0	-0.030666667
JULY	2000	978	1,000		0.022494888	-0.011338446
AUGUST	2000	1,000	1,130		0.13	0.096583333
SEPTEMBER	2000	1,130	1,600		0.415929204	0.384179204
OCTOBER	2000	1,600	1,600		0	-0.031583333
NOVEMBER	2000	1,600	1,500		-0.0625	-0.094166667
DECEMBER	2000	1,500	1,505	250	0.17	0.138333333
JANUARY	2001	1,505	1,520		0.009966777	-0.021699889
FEBRUARY	2001	1,520	1,540		0.013157895	-0.018508772
MARCH	2001	1,540	1,600		0.038961039	0.006294372
APRIL	2001	1,600	1,600		0	-0.03375
MAY	2001	1,600	1,600		0	-0.034583333
JUNE	2001	1,600	1,600		0	-0.034833333
JULY	2001	1,600	1,600		0	-0.034916667
AUGUST	2001	1,600	1,550		-0.03125	-0.063583333
SEPTEMBER	2001	1,550	1,550		0	-0.0295
OCTOBER	2001	1,550	1,550		0	-0.026666667
NOVEMBER	2001	1,550	1,566		0.010322581	-0.015177419
DECEMBER	2001	1,566	1,570	400	0.25798212	0.23498212
JANUARY	2002	1,570	1,575		0.003184713	-0.01889862
FEBRUARY	2002	1,575	1,585		0.006349206	-0.014484127
MARCH	2002	1,585	1,690		0.066246057	0.04657939
APRIL	2002	1,690	1,787		0.05739645	0.037563116
MAY	2002	1,787	2,510		0.404588696	0.384755363
JUNE	2002	2,510	3,221		0.283266932	0.262516932
JULY	2002	3,221	3,262		0.012728966	-0.008271034
AUGUST	2002	3,262	3,310		0.014714899	-0.007035101
SEPTEMBER	2002	3,310	3,370		0.018126888	-0.003539778
OCTOBER	2002	3,370	3,510		0.041543027	0.019793027
NOVEMBER	2002	3,510	3,515		0.001424501	-0.020325499
DECEMBER	2002	3,515	3,516	500	0.142532006	0.120615339

APPENDIX III - SSB BANK LIMITED - STOCK RETURNS (1998 - 2002)

MONTH	YEAR	BEGINNING PRICE	ENDING PRICE	DIVIDENDS	STOCKRETURN(R_i)	EXCESS STOCK RETURN (R_i - R_f)
JANUARY	1998	1,700	1,775		0.044117647	0.010867647
FEBRUARY	1998	1,775	2,080		0.171830986	0.138914319
MARCH	1998	2,080	3,665		0.762019231	0.729102564
APRIL	1998	3,665	3,600		-0.017735334	-0.050652001
MAY	1998	3,600	2,790		-0.225	-0.257916667
JUNE	1998	2,790	2,610		-0.064516129	-0.093349462
JULY	1998	2,610	3,300		0.264367816	0.236367816
AUGUST	1998	3,300	3,000		-0.090909091	-0.118825758
SEPTEMBER	1998	3,000	2,000		-0.333333333	-0.359416667
OCTOBER	1998	2,000	1,990		-0.005	-0.027333333
NOVEMBER	1998	1,990	1,925		-0.032663317	-0.055079983
DECEMBER	1998	1,925	2,250	200	0.272727273	0.250393939
JANUARY	1999	2,250	2,280		0.013333333	-0.009
FEBRUARY	1999	2,280	2,000		-0.122807018	-0.144307018
MARCH	1999	2,000	1,840		-0.08	-0.1015
APRIL	1999	1,840	1,670		-0.092391304	-0.113807971
MAY	1999	1,670	1,664		-0.003592814	-0.024509481
JUNE	1999	1,664	1,910		0.147836538	0.127419872
JULY	1999	1,910	1,980		0.036649215	0.016232548
AUGUST	1999	1,980	2,000		0.01010101	-0.010232323
SEPTEMBER	1999	2,000	1,980		-0.01	-0.030416667
OCTOBER	1999	1,980	1,980		0	-0.022166667
NOVEMBER	1999	1,980	1,980		0	-0.026
DECEMBER	1999	1,980	1,984	240	0.123232323	0.096982323
JANUARY	2000	1,984	1,988		0.002016129	-0.024233871
FEBRUARY	2000	1,988	1,980		-0.004024145	-0.030274145
MARCH	2000	1,980	2,000		0.01010101	-0.01614899
APRIL	200	2,000	1,998		-0.001	-0.02725

	0					
MAY	2000	1,998	1,990		-0.004004004	-0.030504004
JUNE	2000	1,990	1,990		0	-0.030666667
JULY	2000	1,990	1,990		0	-0.033833333
AUGUST	2000	1,990	1,996		0.003015075	-0.030401591
SEPTEMBER	2000	1,996	2,105		0.054609218	0.022859218
OCTOBER	2000	2,105	2,200		0.045130641	0.013547308
NOVEMBER	2000	2,200	2,100		-0.045454545	-0.077121212
DECEMBER	2000	2,100	2,040	400	0.161904762	0.130238095
JANUARY	2001	2,040	2,080		0.019607843	-0.012058824
FEBRUARY	2001	2,080	2,300		0.105769231	0.074102564
MARCH	2001	2,300	2,300		0	-0.032666667
APRIL	2001	2,300	2,300		0	-0.03375
MAY	2001	2,300	2,300		0	-0.034583333
JUNE	2001	2,300	2,300		0	-0.034833333
JULY	2001	2,300	2,300		0	-0.034916667
AUGUST	2001	2,300	2,300		0	-0.032333333
SEPTEMBER	2001	2,300	2,300		0	-0.0295
OCTOBER	2001	2,300	2,300		0	-0.026666667
NOVEMBER	2001	2,300	2,300		0	-0.0255
DECEMBER	2001	2,300	2,200	600	0.217391304	0.194391304
JANUARY	2002	2,200	2,200		0	-0.022083333
FEBRUARY	2002	2,200	2,221		0.009545455	-0.011287879
MARCH	2002	2,221	2,704		0.217469608	0.197802942
APRIL	2002	2,704	2,900		0.072485207	0.052651874
MAY	2002	2,900	3,401		0.172758621	0.152925287
JUNE	2002	3,401	3,700		0.087915319	0.067165319
JULY	2002	3,700	3,801		0.027297297	0.006297297
AUGUST	2002	3,801	3,858		0.014996054	-0.006753946
SEPTEM	200	3,858	3,901		0.011145671	-0.010520995

BER	2					
OCTOBER	2002	3,901	3,953		0.013329915	-0.008420085
NOVEMBER	2002	3,953	3,963		0.002529724	-0.019220276
DECEMBER	2002	3,963	3,966	480	0.121877366	0.099960699

APPENDIX IV - HOME FINANCE COMPANY LTD - STOCK RETURNS (1998 - 2002)

MONTH	YEAR	BEGINNING PRICE(¢)	ENDING PRICE(¢)	DIVIDENDS(¢)	STOCKRETURN(¢)(Ri)	EXCESS STOCK RETURN (R_i -R_f) ¢
JANUARY	1998	235	245		0.042553191	0.009303191
FEBRUARY	1998	245	400		0.632653061	0.599736395
MARCH	1998	400	500		0.25	0.217083333
APRIL	1998	500	700		0.4	0.367083333
MAY	1998	700	765		0.092857143	0.059940476
JUNE	1998	765	750		-0.019607843	-0.048441176
JULY	1998	750	750		0	-0.028
AUGUST	1998	750	750		0	-0.027916667
SEPTEMBER	1998	750	750		0	-0.026083333
OCTOBER	1998	750	750		0	-0.022333333
NOVEMBER	1998	750	750		0	-0.022416667
DECEMBER	1998	750	750	24	0.032	0.009666667
JANUARY	1999	750	750		0	-0.022333333
FEBRUARY	1999	750	750		0	-0.0215
MARCH	1999	750	750		0	-0.0215
APRIL	1999	750	750		0	-0.021416667
MAY	1999	750	750		0	-0.020916667
JUNE	1999	750	750		0	-0.020416667
JULY	1999	750	750		0	-0.020416667
AUGUST	1999	750	750		0	-0.020333333
SEPTEMBER	1999	750	750		0	-0.020416667
OCTOBER	1999	750	750		0	-0.022166667
NOVEMBER	1999	750	750		0	-0.026
DECEMBER	1999	750	750	29	0.038666667	0.012416667
JANUARY	2000	750	750		0	-0.02625
FEBRUARY	2000	750	760		0.013333333	-0.012916667
MARCH	2000	760	770		0.013157895	-0.013092105
APRIL	2000	770	910		0.181818182	0.155568182
MAY	2000	910	910		0	-0.0265
JUNE	2000	910	950		0.043956044	0.013289377
JULY	2000	950	990		0.042105263	0.00827193
AUGUST	2000	990	950		-0.04040404	-0.073820707
SEPTEMBER	2000	950	950		0	-0.03175
OCTOBER	2000	950	950		0	-0.031583333
NOVEMBER	2000	950	952		0.002105263	-0.029561404
DECEMBER	2000	952	952	37	0.038865546	0.00719888

JANUARY	2001	952	952		0	-0.031666667
FEBRUARY	2001	952	952		0	-0.031666667
MARCH	2001	952	952		0	-0.032666667
APRIL	2001	952	952		0	-0.03375
MAY	2001	952	952		0	-0.034583333
JUNE	2001	952	952		0	-0.034833333
JULY	2001	952	952		0	-0.034916667
AUGUST	2001	952	952		0	-0.032333333
SEPTEMBER	2001	952	952		0	-0.0295
OCTOBER	2001	952	952		0	-0.026666667
NOVEMBER	2001	952	952		0	-0.0255
DECEMBER	2001	952	952	45	0.047268908	0.024268908
JANUARY	2002	952	952		0	-0.022083333
FEBRUARY	2002	952	950		-0.00210084	-0.022934174
MARCH	2002	950	950		0	-0.019666667
APRIL	2002	950	950		0	-0.019833333
MAY	2002	950	955		0.005263158	-0.014570175
JUNE	2002	955	955		0	-0.02075
JULY	2002	955	955		0	-0.021
AUGUST	2002	955	955		0	-0.02175
SEPTEMBER	2002	955	955		0	-0.021666667
OCTOBER	2002	955	955		0	-0.02175
NOVEMBER	2002	955	955		0	-0.02175
DECEMBER	2002	955	955		0	-0.021916667

APPENDIX V - STANDARD CHARTERED BANK LTD - STOCK RETURNS (1998 - 2002)

MONTH	YEAR	BEGINNING PRICE	ENDING PRICE	DIVIDENDS	STOCK RETURN(R_i)	EXCESS STOCK RETURN(Ri - Rf)
JANUARY	1998	8,100	9,500		0.172839506	0.139589506
FEBRUARY	1998	9,500	11,500		0.210526316	0.177609649
MARCH	1998	11,500	15,210		0.322608696	0.289692029
APRIL	1998	15,210	20,400		0.34122288	0.308306213
MAY	1998	20,400	22,000		0.078431373	0.045514706
JUNE	1998	22,000	20,000		-0.090909091	-0.119742424
JULY	1998	20,000	20,000		0	-0.028
AUGUST	1998	20,000	23,500		0.175	0.147083333
SEPTEMBER	1998	23,500	21,000		-0.106382979	-0.132466312
OCTOBER	1998	21,000	17,000		-0.19047619	-0.212809524
NOVEMBER	1998	17,000	19,400		0.141176471	0.118759804
DECEMBER	1998	19,400	24,000	2,500	0.365979381	0.343646048
JANUARY	1999	24,000	25,350		0.05625	0.033916667
FEBRUARY	1999	25,350	24,000		-0.053254438	-0.074754438
MARCH	1999	24,000	24,000		0	-0.0215
APRIL	1999	24,000	24,000		0	-0.021416667
MAY	1999	24,000	23,900		-0.004166667	-0.025083333
JUNE	1999	23,900	22,000		-0.079497908	-0.099914575
JULY	1999	22,000	20,000		-0.090909091	-0.111325758
AUGUST	1999	20,000	21,060		0.053	0.032666667

SEPTEMBER	1999	21,060	19,750		-0.062203229	-0.082619896
OCTOBER	1999	19,750	19,700		-0.002531646	-0.024698312
NOVEMBER	1999	19,700	19,700		0	-0.026
DECEMBER	1999	19,700	19,000	3,200	0.126903553	0.100653553
JANUARY	2000	19,000	19,200		0.010526316	-0.015723684
FEBRUARY	2000	19,200	19,200		0	-0.02625
MARCH	2000	19,200	19,500		0.015625	-0.010625
APRIL	2000	19,500	28,500		0.461538462	0.435288462
MAY	2000	28,500	22,000		-0.228070175	-0.254570175
JUNE	2000	22,000	22,000		0	-0.030666667
JULY	2000	22,000	22,000		0	-0.033833333
AUGUST	2000	22,000	21,000		-0.045454545	-0.078871212
SEPTEMBER	2000	21,000	22,100		0.052380952	0.020630952
OCTOBER	2000	22,100	22,000		-0.004524887	-0.03610822
NOVEMBER	2000	22,000	21,500		-0.022727273	-0.054393939
DECEMBER	2000	21,500	21,500	3,200	0.148837209	0.117170543
JANUARY	2001	21,500	21,500		0	-0.031666667
FEBRUARY	2001	21,500	21,550		0.002325581	-0.029341085
MARCH	2001	21,550	21,700		0.006960557	-0.02570611
APRIL	2001	21,700	21,700		0	-0.03375
MAY	2001	21,700	21,000		-0.032258065	-0.066841398
JUNE	2001	21,000	21,000		0	-0.034833333
JULY	2001	21,000	21,000		0	-0.034916667
AUGUST	2001	21,000	21,000		0	-0.032333333
SEPTEMBER	2001	21,000	20,500		-0.023809524	-0.053309524
OCTOBER	2001	20,500	20,500		0	-0.026666667
NOVEMBER	2001	20,500	20,500		0	-0.0255
DECEMBER	2001	20,500	20,500	4,200	0.204878049	0.181878049
JANUARY	2002	20,500	20,551		0.002487805	-0.019595528
FEBRUARY	2002	20,551	20,552		4.86594E-05	-0.020784674
MARCH	2002	20,552	20,500		-0.002530167	-0.022196834
APRIL	2002	20,500	20,500		0	-0.019833333
MAY	2002	20,500	20,500		0	-0.019833333
JUNE	2002	20,500	21,652		0.056195122	0.035445122
JULY	2002	21,652	21,802		0.006927766	-0.014072234
AUGUST	2002	21,802	26,005		0.192780479	0.171030479
SEPTEMBER	2002	26,005	26,023		0.000692175	-0.020974492
OCTOBER	2002	26,023	28,000		0.075971256	0.054221256
NOVEMBER	2002	28,000	28,002		7.14286E-05	-0.021678571
DECEMBER	2002	28,002	28,700	5,000	0.203485465	0.181568799

APPENDIX VI - ENTERPRISE INSURANCE CO. LTD. - STOCK RETURNS(1998 - 2002)

MONTH	YEAR	BEGINNING PRICE	ENDING PRICE	DIVIDENDS	STOCK RETURN	EXCESS STOCK RETURN (Ri - Rf)
JANUARY	1998	954	955		0.001048218	-0.032201782
FEBRUARY	1998	955	1,200		0.256544503	0.223627836
MARCH	1998	1,200	2,700		1.25	1.217083333

APRIL	1998	2,700	2,300		-0.148148148	-0.181064815
MAY	1998	2,300	2,300		0	-0.032916667
JUNE	1998	2,300	2,370		0.030434783	0.001601449
JULY	1998	2,370	2,450		0.033755274	0.005755274
AUGUST	1998	2,450	2,449		-0.000408163	-0.02832483
SEPTEMBER	1998	2,449	2,438		-0.004491629	-0.030574963
OCTOBER	1998	2,438	2,438		0	-0.022333333
NOVEMBER	1998	2,438	2,400		-0.015586546	-0.038003213
DECEMBER	1998	2,400	2,400	120	0.05	0.027666667
JANUARY	1999	2,400	2,300		-0.041666667	-0.064
FEBRUARY	1999	2,300	2,010		-0.126086957	-0.147586957
MARCH	1999	2,010	2,010		0	-0.0215
APRIL	1999	2,010	2,010		0	-0.021416667
MAY	1999	2,010	2,000		-0.004975124	-0.025891791
JUNE	1999	2,000	1,990		-0.005	-0.025416667
JULY	1999	1,990	1,800		-0.095477387	-0.115894054
AUGUST	1999	1,800	1,880		0.044444444	0.024111111
SEPTEMBER	1999	1,880	1,880		0	-0.020416667
OCTOBER	1999	1,880	1,880		0	-0.022166667
NOVEMBER	1999	1,880	1,880		0	-0.026
DECEMBER	1999	1,880	1,880	145	0.07712766	0.05087766
JANUARY	2000	1,880	1,880		0	-0.02625
FEBRUARY	2000	1,880	1,880		0	-0.02625
MARCH	2000	1,880	1,880		0	-0.02625
APRIL	2000	1,880	1,880		0	-0.02625
MAY	2000	1,880	1,880		0	-0.0265
JUNE	2000	1,880	1,880		0	-0.030666667
JULY	2000	1,880	1,880		0	-0.033833333
AUGUST	2000	1,880	1,880		0	-0.033416667
SEPTEMBER	2000	1,880	2,000		0.063829787	0.032079787
OCTOBER	2000	2,000	2,355		0.1775	0.145916667
NOVEMBER	2000	2,355	2,400		0.01910828	-0.012558386
DECEMBER	2000	2,400	2,700	180	0.2	0.168333333
JANUARY	2001	2,700	2,885		0.068518519	0.036851852
FEBRUARY	2001	2,885	2,895		0.003466205	-0.028200462
MARCH	2001	2,895	2,900		0.001727116	-0.030939551
APRIL	2001	2,900	2,890		-0.003448276	-0.037198276
MAY	2001	2,890	2,890		0	-0.034583333
JUNE	2001	2,890	2,890		0	-0.034833333
JULY	2001	2,890	2,890		0	-0.034916667
AUGUST	2001	2,890	2,900		0.003460208	-0.028873126
SEPTEMBER	2001	2,900	2,900		0	-0.0295
OCTOBER	2001	2,900	3,001		0.034827586	0.00816092
NOVEMBER	2001	3,001	3,006		0.001666111	-0.023833889
DECEMBER	2001	3,006	3,050	220	0.087824351	0.064824351

JANUARY	2002	3,050	3,061		0.003606557	-0.018476776
FEBRUARY	2002	3,061	3,121		0.019601437	-0.001231896
MARCH	2002	3,121	3,125		0.00128164	-0.018385026
APRIL	2002	3,125	3,500		0.12	0.100166667
MAY	2002	3,500	3,800		0.085714286	0.065880952
JUNE	2002	3,800	4,200		0.105263158	0.084513158
JULY	2002	4,200	4,200		0	-0.021
AUGUST	2002	4,200	4,500		0.071428571	0.049678571
SEPTEMBER	2002	4,500	4,520		0.004444444	-0.017222222
OCTOBER	2002	4,520	4,526		0.001327434	-0.020422566
NOVEMBER	2002	4,526	4,600		0.016349978	-0.005400022
DECEMBER	2002	4,600	4,600		0	-0.021916667

APPENDIX VII - MOBIL OIL GHANA LTD. - STOCK RETURNS (1998 - 2002)

MONTH	YEAR	BEGINNING PRICE	ENDING PRICE	DIVIDENDS	STOCK RETURN(Ri)	EXCESS STOCK RETURN (Ri - Rf)
JANUARY	1998	8,230	8,400		0.020656136	-0.012593864
FEBRUARY	1998	8,400	11,001		0.309642857	0.27672619
MARCH	1998	11,001	14,500		0.318061994	0.285145328
APRIL	1998	14,500	26,300		0.813793103	0.780876437
MAY	1998	26,300	18,000		-0.315589354	-0.34850602
JUNE	1998	18,000	16,400		-0.088888889	-0.117722222
JULY	1998	16,400	20,000		0.219512195	0.191512195
AUGUST	1998	20,000	19,900		-0.005	-0.032916667
SEPTEMBER	1998	19,900	16,500		-0.170854271	-0.196937605
OCTOBER	1998	16,500	13,300		-0.193939394	-0.216272727
NOVEMBER	1998	13,300	15,000		0.127819549	0.105402882
DECEMBER	1998	15,000	17,000	1,173	0.211533333	0.1892
JANUARY	1999	17,000	20,000		0.176470588	0.154137255
FEBRUARY	1999	20,000	17,000		-0.15	-0.1715
MARCH	1999	17,000	16,000		-0.058823529	-0.080323529
APRIL	1999	16,000	16,800		0.05	0.028583333
MAY	1999	16,800	16,800		0	-0.020916667

JUNE	1999	16,800	16,800		0	-0.020416667
JULY	1999	16,800	16,000		-0.047619048	-0.068035714
AUGUST	1999	16,000	16,000		0	-0.020333333
SEPTEMBER	1999	16,000	15,999		-0.0000625	-0.020479167
OCTOBER	1999	15,999	15,850		-0.009313082	-0.031479749
NOVEMBER	1999	15,850	15,700		-0.009463722	-0.035463722
DECEMBER	1999	15,700	13,800	1,820	-0.005095541	-0.031345541
JANUARY	2000	13,800	14,500		0.050724638	0.024474638
FEBRUARY	2000	14,500	14,500		0	-0.02625
MARCH	2000	14,500	14,550		0.003448276	-0.022801724
APRIL	2000	14,550	15,500		0.065292096	0.039042096
MAY	2000	15,500	17,100		0.103225806	0.076725806
JUNE	2000	17,100	17,500		0.023391813	-0.007274854
JULY	2000	17,500	17,500		0	-0.033833333
AUGUST	2000	17,500	17,500		0	-0.033416667
SEPTEMBER	2000	17,500	18,500		0.057142857	0.025392857
OCTOBER	2000	18,500	18,600		0.005405405	-0.026177928
NOVEMBER	2000	18,600	18,600		0	-0.031666667
DECEMBER	2000	18,600	18,600	2,500	0.134408602	0.102741935
JANUARY	2001	18,600	18,700		0.005376344	-0.026290323
FEBRUARY	2001	18,700	18,700		0	-0.031666667
MARCH	2001	18,700	18,700		0	-0.032666667
APRIL	2001	18,700	18,700		0	-0.03375
MAY	2001	18,700	19,000		0.016042781	-0.018540553
JUNE	2001	19,000	19,400		0.021052632	-0.013780702
JULY	2001	19,400	19,650		0.012886598	-0.022030069

AUGUST	2001	19,650	20,000		0.017811705	-0.014521628
SEPTEMBER	2001	20,000	20,000		0	-0.0295
OCTOBER	2001	20,000	18,001		-0.09995	-0.126616667
NOVEMBER	2001	18,001	18,200		0.011054941	-0.014445059
DECEMBER	2001	18,200	18,500	2,536	0.155824176	0.132824176
JANUARY	2002	18,500	18,500		0	-0.022083333
FEBRUARY	2002	18,500	18,502		0.000108108	-0.020725225
MARCH	2002	18,502	18,550		0.002594314	-0.017072353
APRIL	2002	18,550	18,810		0.014016173	-0.005817161
MAY	2002	18,810	18,820		0.000531632	-0.019301701
JUNE	2002	18,820	19,000		0.009564293	-0.011185707
JULY	2002	19,000	19,611		0.032157895	0.011157895
AUGUST	2002	19,611	19,700		0.004538269	-0.017211731
SEPTEMBER	2002	19,700	19,720		0.001015228	-0.020651438
OCTOBER	2002	19,720	19,721		5.07099E-05	-0.02169929
NOVEMBER	2002	19,721	19,721		0	-0.02175
DECEMBER	2002	19,721	19,730		0.000456366	-0.0214603

APPENDIX VIII - BANK OF GHANA 91 - DAY TREASURY BILL RATES (%)

MONTH	YEAR	RATE (%)	MONTHLY RATE(R_f)
JANUARY	1998	39.9	0.03325
FEBRUARY	1998	39.5	0.032916667
MARCH	1998	39.5	0.032916667
APRIL	1998	39.5	0.032916667
MAY	1998	39.5	0.032916667
JUNE	1998	34.6	0.028833333
JULY	1998	33.6	0.028
AUGUST	1998	33.5	0.027916667
SEPTEMBER	1998	31.3	0.026083333
OCTOBER	1998	26.8	0.022333333
NOVEMBER	1998	26.9	0.022416667

DECEMBER	1998	26.8	0.022333333
JANUARY	1999	26.8	0.022333333
FEBRUARY	1999	25.8	0.0215
MARCH	1999	25.8	0.0215
APRIL	1999	25.7	0.021416667
MAY	1999	25.1	0.020916667
JUNE	1999	24.5	0.020416667
JULY	1999	24.5	0.020416667
AUGUST	1999	24.4	0.020333333
SEPTEMBER	1999	24.5	0.020416667
OCTOBER	1999	26.6	0.022166667
NOVEMBER	1999	31.2	0.026
DECEMBER	1999	31.5	0.02625
JANUARY	2000	31.5	0.02625
FEBRUARY	2000	31.5	0.02625
MARCH	2000	31.5	0.02625
APRIL	2000	31.5	0.02625
MAY	2000	31.8	0.0265
JUNE	2000	36.8	0.030666667
JULY	2000	40.6	0.033833333
AUGUST	2000	40.1	0.033416667
SEPTEMBER	2000	38.1	0.03175
OCTOBER	2000	37.9	0.031583333
NOVEMBER	2000	38	0.031666667
DECEMBER	2000	38	0.031666667
JANUARY	2001	38	0.031666667
FEBRUARY	2001	38	0.031666667
MARCH	2001	39.2	0.032666667
APRIL	2001	40.5	0.03375
MAY	2001	41.5	0.034583333
JUNE	2001	41.8	0.034833333
JULY	2001	41.9	0.034916667
AUGUST	2001	38.8	0.032333333
SEPTEMBER	2001	35.4	0.0295
OCTOBER	2001	32	0.026666667
NOVEMBER	2001	30.6	0.0255
DECEMBER	2001	27.6	0.023
JANUARY	2002	26.5	0.022083333
FEBRUARY	2002	25	0.020833333
MARCH	2002	23.6	0.019666667
APRIL	2002	23.8	0.019833333
MAY	2002	23.8	0.019833333
JUNE	2002	24.9	0.02075
JULY	2002	25.2	0.021

AUGUST	2002	26.1	0.02175
SEPTEMBER	2002	26	0.021666667
OCTOBER	2002	26.1	0.02175
NOVEMBER	2002	26.1	0.02175
DECEMBER	2002	26.3	0.021916667

APPENDIX IX - SUMMARY OUTPUT - GCB

Regression Statistics	
Multiple R	0.689293694
R Square	0.475125797
Adjusted R Square	0.459252781
Standard Error	0.134709922
Observations	64

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1.034887398	1.034887	57.02876	2.3747E-10
Residual	63	1.143246067	0.018147		
Total	64	2.178133465			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A
BETA	1.664893031	0.216153997	7.702347	1.18E-10	1.232943474	2.09684259

APPENDIX X - SUMMARY OUTPUT - SSB

Regression Statistics	
Multiple R	0.572727248
R Square	0.3280165
Adjusted R Square	0.312143484
Standard Error	0.109644263
Observations	64

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.369700003	0.3697	30.7523	6.42724E-07
Residual	63	0.757377455	0.012022		
Total	64	1.127077458			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A
BETA	0.980740826	0.175933928	5.574484	5.54E-07	0.629164711	1.33231694

APPENDIX XI - SUMMARY OUTPUT - HFC

Regression Statistics	
Multiple R	0.625017485
R Square	0.390646857
Adjusted R Square	0.374773841
Standard Error	0.076618229
Observations	64

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.23709372	0.237094	40.38832	2.77011E-08
Residual	63	0.369832244	0.00587		
Total	64	0.606925965			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A
BETA	0.782647673	0.122940733	6.366057	2.51E-08	0.536970072	1.02832527

APPENDIX XII

TABLE OF RETURNS

Periods	All-Share Index	GCB	SG-SSB	SCB	HFC	EIC	TOTAL OIL	RISK FREE RATE
1	-9.98	-4.75	-4.41	-17.28	-4.26	-0.10	-2.07	0.03
2	-15.22	-5.01	-17.18	-21.05	-63.27	-25.65	-30.96	0.03
3	-30.94	-34.09	-76.20	-32.26	-25.00	-125.00	-31.81	0.03
4	-39.50	-103.39	1.77	-34.12	-40.00	14.81	-81.38	0.03
5	13.17	45.83	22.50	-7.84	-9.29	0.00	31.56	0.03
6	5.67	-31.54	6.45	9.09	1.96	-3.04	8.89	0.03
7	-4.33	2.34	-26.44	0.00	0.00	-3.38	-21.95	0.03
8	3.16	12.57	9.09	-17.50	0.00	0.04	0.50	0.03
9	12.85	20.89	33.33	10.64	0.00	0.45	17.09	0.03
10	8.18	-0.87	0.50	19.05	0.00	0.00	19.39	0.02
11	1.03	2.15	3.27	-14.12	0.00	1.56	-12.78	0.02

12	-11.85	-14.04	-16.88	-23.71	0.00	0.00	-13.33	0.02
13	-2.89	15.38	-1.33	-5.63	0.00	4.17	-17.65	0.02
14	3.51	9.09	12.28	5.33	0.00	12.61	15.00	0.02
15	3.88	1.00	8.00	0.00	0.00	0.00	5.88	0.02
16	0.93	-1.01	9.24	0.00	0.00	0.00	-5.00	0.02
17	0.21	0.00	0.36	0.42	0.00	0.50	0.00	0.02
18	1.59	0.00	-14.78	7.95	0.00	0.50	0.00	0.02
19	2.27	1.50	-3.66	9.09	0.00	9.55	4.76	0.02
20	-0.12	8.63	-1.01	-5.30	0.00	-4.44	0.00	0.02
21	3.66	5.56	1.00	6.22	0.00	0.00	0.01	0.02
22	0.07	1.18	0.00	0.25	0.00	0.00	0.93	0.02
23	1.56	10.71	0.00	0.00	0.00	0.00	0.95	0.03
24	1.52	-1.33	-0.20	3.55	0.00	0.00	12.10	0.03
25	-0.75	-1.32	-0.20	-1.05	0.00	0.00	-5.07	0.03
26	0.26	4.55	0.40	0.00	-1.33	0.00	0.00	0.03
27	-3.16	-23.13	-1.01	-1.56	-1.32	0.00	-0.34	0.03
28	-13.88	-8.29	0.10	-46.15	-18.18	0.00	-6.53	0.03
29	6.49	0.20	0.40	22.81	0.00	0.00	-10.32	0.03
30	-0.64	0.00	0.00	0.00	-4.40	0.00	-2.34	0.03
31	-0.50	-2.25	0.00	0.00	-4.21	0.00	0.00	0.03
32	-0.02	-13.00	-0.30	4.55	4.04	0.00	0.00	0.03
33	-4.07	-41.59	-5.46	-5.24	0.00	-6.38	-5.71	0.03
34	-0.97	0.00	-4.51	0.45	0.00	-17.75	-0.54	0.03
35	-0.28	6.25	4.55	2.27	-0.21	-1.91	0.00	0.03
36	0.95	-0.33	2.86	0.00	0.00	-12.50	0.00	0.03
37	-0.06	-1.00	-1.96	0.00	0.00	-6.85	-0.54	0.03
38	-2.40	-1.32	-10.58	-0.23	0.00	-0.35	0.00	0.03
39	-2.29	-3.90	0.00	-0.70	0.00	-0.17	0.00	0.03
40	0.15	0.00	0.00	0.00	0.00	0.34	0.00	0.03
41	0.37	0.00	0.00	3.23	0.00	0.00	-1.60	0.03
42	-4.24	0.00	0.00	0.00	0.00	0.00	-2.11	0.03
43	-9.85	0.00	0.00	0.00	0.00	0.00	-1.29	0.03
44	7.30	3.13	0.00	0.00	0.00	-0.35	-1.78	0.03
45	-0.68	0.00	0.00	2.38	0.00	0.00	0.00	0.03
46	-0.52	0.00	0.00	0.00	0.00	-3.48	10.00	0.03
47	0.26	-1.03	0.00	0.00	0.00	-0.17	-1.11	0.03
48	0.27	-0.26	4.35	0.00	0.00	-1.46	-1.65	0.02
49	-0.15	-0.32		-0.25	0.00	-0.36	0.00	0.02
50	-1.31	-0.63	-0.95	0.00	0.21	-1.96	-0.01	0.02
51	-4.96	-6.62	-21.75	0.25	0.00	-0.13	-0.26	0.02
52	-2.26	-5.74	-7.25	0.00	0.00	-12.00	-1.40	0.02

53	-8.80	-40.46	-17.28	0.00	-0.53	-8.57	-0.05	0.02
54	-8.03	-28.33	-8.79	-5.62	0.00	-10.53	-0.96	0.02
55	-2.73	-1.27	-2.73	-0.69	0.00	0.00	-3.22	0.02
56	-4.19	-1.47	-1.50	-19.28	0.00	-7.14	-0.45	0.02
57	-0.07	-1.81	-1.11	-0.07	0.00	-0.44	-0.10	0.02
58	-2.22	-4.15	-1.33	-7.60	0.00	-0.13	-0.01	0.02
59	-1.71	-0.14	-0.25	-0.01	0.00	-1.63	0.00	0.02
60	-2.40	-0.03	-0.08	-2.49	0.00	0.00	-0.05	0.02