OPTIMIZATION OF PRODUCTION SCHEDULING AS LINEAR PROGRAMMING MODEL

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CERTIFICATION

I hereby declare that this submission is my own work towards the MSC and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due knowledge has been made in the text.

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ABSTRACT

Production levels must allow a manufacturing organization to meet demand for its production as well as sales targets which belong to key performance indicators, inspite of any prevailing constraints. Managerial decision making to achieve objectives carry along with it responsibility to interpret data to identify the way things work or should work. This includes the simultaneous considerations of both production and sales limitations. In a case study for a cable manufacturing company, optimization of production scheduling was treated as a linear programming problem. The simplex algorithm was applied to determine optimal schedules for each of the six fast selling copper products that can yield the total monthly copper budget at sales. In a sensitivity analysis, the impact of varying operational constraints on the linear programming model was examined to verify the effects of changes on outcomes of the model. Interpretations of the results from the analysis formed basis for recommendations to enhance managerial decisions.

Key words: Optimal schedules, production and sales target, linear objective function & constraints, simplex algorithm.

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LIST OF ABBREVIATIONS

The abbreviations used in the simplex tableau are defined as follows:

C_j = Objective function coefficients for variable

RHS = Right Hand Side value of the constraint set equations.

BV = Basic Variable. The basic variables have the value 1. Any value above or below it in the matrix must be zero.

C_B = Objective function coefficient of the basic variables.

Z_j = These are values calculated by multiplying the elements in the C_B column by the corresponding elements in the columns of the matrix and then summing them up.

$C_j – Z_j$ = Values in this row are determined by subtracting the appropriate Z_j value from the corresponding objective function coefficient C_j for that column. Each value in the $C_j – Z_j$ row represents the net contribution.

NCY = Designation for house wiring copper cables meaning: Standard cable size (N), produced from copper (C), and extruded with polyvinyl chloride (Y).

RS = Designation for conductor construction shape used for house wiring copper cables meaning: Round Stranded (RS) conductor.

DEDICATION

THIS WORK IS DEDICATED TO MY CHILDREN, AND SPECIALLY TO REBECCA ADDY FOR THE LOVE AND ENCOURAGEMENT GIVEN ME DURING MY POST GRADUATE STUDIES AND OVER THE PERIOD OF PREPARATION & WRITING OF THIS THESIS.

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CHAPTER 1

INTRODUCTION

BRIEF OVERVIEW

Production is the core process of every manufacturing organization, and so the efficiency and quality of decisions taken on the shop floor determines the performance of the organization’s quality management system. At planning level, a good production manager must be able to identify probable limitations in order to establish, prior to product realization processes, the appropriate preventive measures. The ultimate in every production scheduling solution must effect the provision and control of specified resources to accomplish the required manufacturing tasks with due considerations to right sequence of job methods and timing parameters. An optimized production scheduling process is therefore characterized by the availability of desired output of products in type and quantity within the planned time at minimum costs. This thesis seeks to discuss the application of the optimization concept at production scheduling in the dictionary of defined single objective and typical operational constraints.

1.1 BACKGROUND OF THE STUDY

Finding solution to production scheduling problem allows the production manager to handle the outcomes of the manufacturing system with a more conscientious efforts. Production scheduling belongs to one of the oldest and hardest problems in manufacturing systems.The combinatorial and dynamic nature of the scheduling process makes the problem a complex one. Except in cases of defined domain of the scheduling operations with specific assumptions and given conditions, exact or optimal algorithm of solutions vary from one system of operations to the other. Purpose of production scheduling optimization is to reduce production costs, or time, maximize profit or production output by rightly identifying what the production facility should produce, the correct machine operator, which machine can best perform the job and what sequence should the set of jobs be implemented. It aims at ultimately meeting customer delivery schedules and other expectations. Some powerful models proposed from the academic fields to optimize the scheduling process include the multi-objective optimization approach with the application of the transportation model and the linear programming models.

A wider scope of production program helps manufacturing organizations to respond readily to market trends. However, this results in a more complex production scheduling process. For instance, in the industrial yogurt production, there is a wide variety of products that differ in feature like fat content, whey used to produce the mixture, the flavor, the size of the container or language on the label. In such scheduling operations, inevitable constraints like satisfaction of multiple due dates, variable processing times, sequence dependent set up times, increased production costs and efforts to monitor inventory levels need to be efficiently handled in order to retain the required production objectives.

'In principle, the great diversity of processing methods and uniqueness of product features, characterize production planning, scheduling and controls in the manufacturing industries. These render the generic scheduling methodologies impractical for real life applications', Doganis et al (2008) Annals of operations research, Volume 159, pages 315-331.

This thesis seeks to determine the optimal schedules for production and sales. A referenced case study for a cable manufacturing company at Tema, in Ghana has been used to demonstrate the application of the intelligent concepts of linear programming to solve production scheduling problem, particularly in cable production firms.

'Production scheduling is a process to allocate appropriate resources for the required manufacturing tasks and to identify the sequence and timing parameters to accomplish these tasks',Graves (1981) Operations Research, Volume 29, no. 4. Optimal scheduling therefore refers to a schedule which yields the right quantity and type within the minimum expected time with the available resources to maximize profit or output. Inspite of all limitations in the production and sale processes, an optimal scheduling needs to be planned, with clear objectives and targets. Traditionally, high level planning and target setting is done at the beginning of each financial year of manufacturing organizations.

Knowledge of demand profile to the production manager assists in predictive scheduling. This facilitates planning and renders operations to be more target oriented in the realization of the right product types and quantities on the shop floor. Timely provision of input raw materials, identification of the appropriate machines for the job and provision of sufficient work instructions to operators contribute to optimal schedules. Input decision-making in the scheduling process is expected to be tapped from the planning level. An optimal schedule is expected to reduce strenuous efforts in the manufacturing process and to improve the competitiveness of products in terms of quantity and quality within targeted time schedules.

In hierarchical sequence, production planning, scheduling and control are directly interrelated. Planning is a predictive scheduling. The execution of planned schedules on the production shop floor to yield optimal scheduling output requires also the proportional combination of time, costs of labour and raw material, inspite of other limitations which may be ignored.

The extent of optimality of production scheduling output is usually constrained by plant capacity. The case study considers the sales pattern and volume for the relevant products of the company from the previous year transactions.

It is worth noting that, two or more identical machines exhibit different processing speed for the various types of house wiring cables under study in this thesis. Similarly, defect rates vary among identical machines. Efficient supervisory controls keep manufacturing constraints at minimum. The core of the scheduling problem in this case study relates basically timing parameters and realization of the optimized scheduling output such that the correct combination of products can be manufactured to meet market demand without excessive quantity remaining at stores, but at same time achieving targeted sales tonnage with the appropriate marketing strategy. This model computes the optimized quantity of the various products as a fore cast for 2013 production scheduling with strong determinative bearing on required marketing efforts.

In a flexible manufacturing system such as in cable production companies where inventory of both work-in-progress items and finished products are constantly kept at minimum with regular supply of the job specific raw materials, presentation of this model is an effective tool to manage deviation from ideal inventory levels.

1.2 PROBLEM STATEMENT

Manufacturing organizations engaged with the production of multi-items are invariably confronted with the problem of producing just enough of each item to satisfy demand, but at same time to maximize profit in terms of production costs, inventory costs, man-power limitations, production time and demand profile for the products. Usually the identification and manner of treatments of the associated production and sales constraints determine the extent of scheduling optimality. This becomes very necessary when output targets for both production and sales have been established as strict key performance indicators for the respective departments. Predominant production scheduling problems encountered belong to at least, one of the following:

1. Companies would always want to satisfy demand instantly, avoiding long term delivery schedules. The practice of turning away customers to make them come back later for supplies, causes losses of sales to competitors who may be located not far from work sites.

2. When sales team have identified customers preferences to combination of specific products, shortage of one product would eventually affect sales pattern. This implies that production scheduling would need to be optimized in such a way to ensure sufficiency of all product variants in the right proportions on stock.
3. There is the need to establish a referenced benchmark to measure and analyse performance at end of the month. Knowledge of required optimal scheduling output will facilitate inventory control.

In the case study for a cable manufacturing company, a monthly target of 92 tons copper needs to be produced and sold. The target resulted from 14.6 % increase on 2012 figure. The company manufactures more than one hundred and fifty types of copper products on production program. However, an optimal scheduling solution need to be identified such that, at least 70% of the total copper sales can proceed from the six fast selling products.

1.3 OBJECTIVE OF THE STUDY

The main objectives of this study are:

1.To construct a mathematical model which provides optimal scheduling solution for production output under normal operational environment.

2.To link the proposed modeling solution to a case study at a cable manufacturing company.

3.To interpret the outcomes of the applied model in a case study, using sensitivity analysis for various changes in the model due to impact from ignored constraint.

Annual targets usually set by production firms at the beginning of each year are geared towards the improvement of previous performance in order to maintain cash flow, withstand the turbulent economic trends and above all, to enhance customer satisfaction. The objectives of this study, if well achieved can be applied to other categories of production systems and products.

1.4 JUSTIFICATION OF THE STUDY

If production and sales targets can be conscientiously achieved at the end of the planned operational periods, then both production and commercial departments of manufacturing firms will have to establish key performance indicators against which performances can be monitored and measured. Treatment of the scheduling problem as a linear programming model has proved very efficient over the years in providing optimized values for scheduling outputs on the production shop floor on daily basis. Knowledge of how much of each product needs to be produced and sold enhances opportunities to identify early deviations from targets for corrective and preventive actions. In the referenced case study, for a cable production firm, a benchmark for production and sales scheduling performance has been necessary to be able to monitor performance trend. Results of the case study will improve the quality of decision making process. The production management team will become more objective than subjective.

In view of the fore knowledge of the optimized scheduling solution, control of inventory can be effective, especially when marketing functions identifies the right strategy to maintain demand.

1.5 METHODOLOGY FOR SOLVING THE SCHEDULING PROBLEM

Most industrial problems in manufacturing organizations can be expressed in terms of measurable parameters, with the objective to monitor and control relevant production processes for purposes of optimizing resources. For cases where the problems (production constraints) can be mathematically expressed as linear inequalities with the objective to optimize also in terms of a linear function, the production problem can be categorized as a linear optimization problem.

In business setting involving production scheduling problems, the application of linear optimization algorithms for solutions has provided insights into important and efficient managerial decision making. Optimization of production scheduling as linear programming model permits the optimal realization of objective based decisions to reduce production and inventory costs and maximize business profit, if all relevant constraints can be rightly identified.

There are at least, three basic linear programming algorithms in application. These include the simplex method, the ellipsoid method and the interior point method. These methods can be used to determine the feasibility of a linear program and hence optimize the linear objective function, if there exist such a feasible solution which satisfies all constraints. All three methods can solve linear programming problems in finite time. In worst cases, however, the simplex method runs in exponential time, while the other methods run in time polynomial in the size of the input parameters.

In practice, however, the simplex algorithm is usually very fast even on linear programs with large number of variable and constraints.

For the purpose of this study, we will apply the simplex algorithm to solve the production scheduling problem. Further discussions will be done on this algorithm at chapter three.

Now let's have a brief look at the historical background of the development of linear programming. The concept was exhibited as far back as 1820, when Fourier investigated approaches to solve systems of linear equations. In 1939, Kantorovich, a Russian mathematician developed the concept of linear programming to improve economic planning in the USSR and was awarded the Nobel prize winner in economic science, together with T.C Koopmans, who also applied the linear program solution to plan the optimal movement of ships across the Atlantic during the world war. In 1947, George Dantzig, who is today regarded by many as the author of the simplex method, solved linear programming problems for the U.S Air Force.
Today this mathematical tool is applied to solve a wide scope of industrial problems including the following areas:

Table 1.1: Some areas of application of linear program concept

No.	Industry	Problem
1	Manufacturing	Blending Production scheduling Farm planning Refinery problems Product mix
2	Supply chain	Product deployment Assignment problems
3	Time tabling	Man power planning
4	Transportation	Network flows

1.6 SCOPE OF THE STUDY

The scope of the study covers the industrial application of linear programming to determine the optimal scheduling solution for production and sales.

The study treats typical scheduling problems in manufacturing companies with special reference to a case study at Nexans Kabelmetal Ghana Limited, a production firm engaged in the production of house wiring cables. The manual computational approach of the simplex method was considered for the optimal solution. Example of a production scheduling problem in an automobile company was used to explain the application and the methodology. Reference was made to the case study for optimal solution. The constraints to the production and sales capacities in the problem formulations for the case study at Nexans were based on the company's activities over the period January-December, 2012. A production flow chart for the products concerned has been shown below.

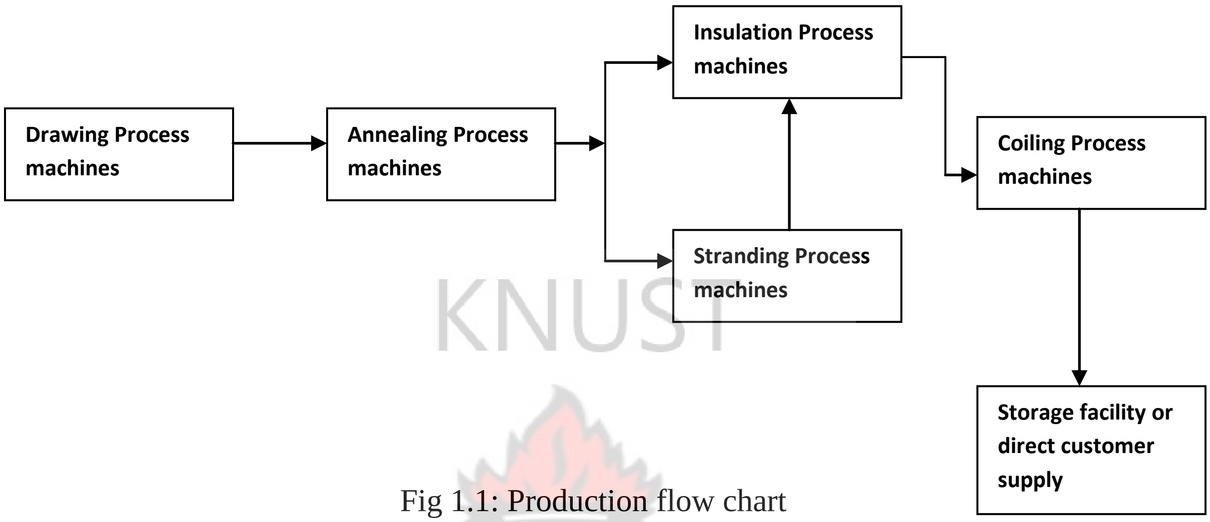

Fig 1.1: Production flow chart

1.7 LIMITATIONS OF THE STUDY

The computation of the optimal production scheduling output through the application of the simplex algorithm considers the assumption that parameters appearing in the linear programming model are constant. However, in real life situations, parameters are neither constant nor deterministic.

In the case study at Nexans Kabelmetal Ghana Limited, the assumption that the demand curve for the sale of house wiring cables in 2012 shall be similar to that in 2013 is very theoretical. Any performance gap that tends to show up in 2013 shall require rigorous approach of marketing functions to patch it up.

Delays in raw material supplies were not taken into account in the formulations of constraints.The management team at production, purchasing and commercial departments need careful coordination of customer orders and availability of raw materials to meet delivery dates. The model does not consider the effects of time on the constraints.

The application of the simplex algorithm to solve the scheduling problem permits the use of only a single objective. In real life situation other objectives like profit maximization could have been of equal importance.

Nexans customer orders for high copper content products like armoured or power cables are big advantages to achieve production targets on time. Application of the algorithm may then not be beneficial.

1.8 ORGANIZATION OF THE THESIS

The presentation of the thesis has been structured chapter 1 consisting of the following :
Section 1.1 provides a general background of the study, including definition for optimization of a scheduling process, reasons for complex scheduling problems,
the interactions between production planning, scheduling and control.
Section 1.2 discusses the problem statements of the study with regard to general production scheduling in manufacturing organizations.
Section 1.3 defines the objectives of the study.
Section 1.4 justifies the essence of the study.
Section 1.5 discusses the application of linear program algorithms in optimizing production scheduling. The simplex algorithm has been highlighted.
Section 1.6 gives a brief description of the scope of the study.
Section 1.7 identifies logical limitations to the application of the simplex method with reference to the case study.
Section 1.8 provides the organization structure of this thesis.
Chapter 2 gives the literature review of the study.
Chapter 3 looks at the basic concept of linear programming. Illustration and preview of the application of the simplex method with remarks on the general principles have been given.
Chapter 4 applies the linear programming model to solve a production scheduling problem in a case study. Sensitivity analysis of the outcomes of the model has been discussed.
Chapter 5 takes a look at the management of scheduling constraints and its impact. Recommendations and conclusions based on impact of operational changes on outcome of the model have been made. Statistics of actual scheduling output in view of the model has been briefly discussed.

CHAPTER 2

LITERATURE REVIEW

INTRODUCTION

Production scheduling problem has gained the attention of many researchers and academics over the years with proposals for different solution techniques. However, the diversity in product features and operation methods, renders the generic scheduling solution impractical. In this chapter, selected research results in the field of optimal scheduling of production processes will be discussed.

REVIEWS FROM RESEARCHERS

Hong-Sen et al (2003) addressed the closely related problems of production planning and scheduling on mixed model automobile assembly lines. They proposed an integrated solution, in which a production plan that was feasible with respect to aggregate capacity constraint was developed and then a sequence that was feasible with respect to this plan was sought. They proposed three tabu-search-based algorithms that explore the solution spaces for both problems to different degrees to find a combination of a production plan and schedule that were feasible and that approximately optimized the objective function (involving the overproduction and underproduction of finished automobiles, the set up cost, the idle times of work cells on the line, the makespan and the load deviations among work cells). Simulation was used to evaluate alternative schedules

Khmelnitsky et al (1993) considered a continuous time dynamic model of discrete scheduling problems for large class of manufacturing system.The realistic manufacturing based on multi-level bills of materials, flexible machines, controllable buffers and deterministic demand profiles was modeled in the canonical form of optimal control. Carrying buffer costs were minimized by controlling production rates of all machines that can be set up instantly. The maximum principle for the model was studied and properties of the optimal production regimes were revealed.

Khmelnitsky et al (2002) conducted a research on optimal control model which relaxes demand and inventory levels. Demand occasionally depends positively on the amount of displayed stock, especially in the case of novelty or impulse purchse items. Such inventory level dependence of demand rate was incorporated into some continuous review inventory control models, but only ones with demand rate which did not vary with time, no shortages and infinite production rate per unit time cost were minimized. They proposed an optimal control model which relaxed all these assumptions. The maximum principle was applied to obtain three possible singular regimes.

Many researchers have proposed different techniques to integrate production scheduling and preventive maintenance. These techniques have some drawbacks. For example, some of them are so intricate that one cannot easily implement them or some strongly exploit specific features of the original studied problem that one cannot apply them to other problems. Naderi et al (2009) proposed two techniques that are easy to understand and code and simple to adapt to any other machine scheduling problems. The study investigated job shop scheduling with sequence dependent set up times and preventive maintenance policies. The optimization criterion is to minimize makespan. Four meta-heuristics based on simulated annealing and genetic algorithms were employed to solve the problem.

Andre et al (2010) presented a research on an algorithm based on the principle of progressive optimality for determining the optimality of short term scheduling of multireservoir power systems. The method took into account water head variations, spilling and time delays between upstream and downstream reservoirs. The method was computationally efficient and had minimal storage requirements. The convergence was monotonic and a global solution was reached.

Xinyu et al (2008) focussed their studies on the integration problem of process planning and scheduling in a job shop environment. In an effort to integrate process planning and scheduling by taking advantage of the flexibility that alternative process plans offer. They designed a genetic algorithm based scheduling method. The performance of this study newly suggested genetic algorithm based method which was evaluated by comparing integrated scheduling with separated scheduling in a real company that had alternative process plans. Also a couple of benchmark cases were tested for performance evaluation.

Berrichi et al (2010) presented an algorithm based on Ant Colony Optimization paradigm to solve the joint production and maintenance scheduling problem. This approach was developed to deal with a model previously proposed. This was formulated according to a bi-objective approach to find tradeoff solutions between both objectives of production and maintenance. Reliability models were used to take account of the maintenance aspect. To improve the quality of solutions found in their previous study, an algorithm based on multi-objective Ant Colony Optimization approach was developed. The goal was to simultaneously determine the best assignment of production tasks to machines as ell as preventive maintenance. The experimental results showed that the proposed method outperformed the two well known multi-objective genetic algorithms.

Xing et al (2009) discussed an efficient search method for multi-objective flexible job scheduling problems. It was quite difficult to achieve an optimal solution to this problem due to the high computational complexity. They proposed an efficient search method for the multi-objective flexible job shop scheduling problems. The final experimental results showed that the proposed algorithm is a feasible and effective approach for the multi-objective flexible job shop scheduling problems.

Lei et al (2008) looked at the real life problems associated with several conflicting objectives at scheduling. The solutions of these problems provided deeper insights to the decision maker than those of single-objective problems. However, the literature of multi-objective scheduling is notably sparser than that of single-objective scheduling. Since the survey on multi-objective and bi-objective scheduling was conducted by Naderi et al in 1995, there has been an increasing interest in multi-objective production scheduling, especially in multi-objective deterministic problem. The goal of the study was to provide an extensive review of the literature on the scheduling problems with multi-objective in the past thirteen years.

Kogan et al (2007) described a new approach to continuous time optimal scheduling problems for a large class of manufacturing systems.This allowed for the traditional disadvantages of the two-level problem consideration (one level for defining the target production rates, and the other for scheduling the set up changes) to be avoided and stable control strategies to be obtained. Analysis of the maximum principle resulted in set up conditions of the optimal schedule and special regimes to which the optimal tends between subsequent set ups. Based on these results, a numerical method was developed to define the sequence of the special regimes and the timing for getting into and out of them.The problem of detailed scheduling of complex manufacturing systems was addressed by optimal flow control. A model problem of scheduling parallel machines was considered to obtain necessary set up conditions. Studying the conditions resulted in a new solution approach that took advantage of a juggling analogy of the production and set up scheduling. This analogy was used to direct construction of a solution method. The method searched for a globally optimal schedule by means of both a juggling strategy and a method of global optimization.The results obtained for a model problem are then generalized to systems with complex production and set up operations.

Bai et al (1995) studied a manufacturing system consisting of two machines separated by two intermediate buffers, and capable of producing two different products. Each product required a constant processing time on each of the machines. Each machine required a constant non-negligible set up change time from one product to the other. The demand rate for each product was considered to be piecewise constant. Each machine was prone to failure and repair. The time to repair and the time to failure were exponentially distributed random variables. Also the set up change and processing operations were resumable. The situation was modeled as a continuous flow process. An optimal control problem was formulated for the system to minimize the total expected discounted cost over an infinite horizon. To determine the optimal control policy structures, a discrete version of the problem was solved numerically using a dynamic programming formulation with a piecewise linear penalty function. A real time control algorithm was then developed with the objective of maintaining low work in-process inventory and keeping the production close to demand.

The job shop scheduling problem with sequence dependent set up times is an extension of the job shop scheduling problem that has interested researchers during the last years. Miguel et al (2006) confronted the job shop scheduling problem with sequence dependent set up times problem by means of a memetic algorithm. Two schedule generation schemas that are extensions of the well known G & T algorithm for the job shop scheduling. They reported from an experimental study that showed that proposed approaches produced similar results and that both of them were more efficient than other genetic algorithm.

Guerriero (2008) focused on heuristic approaches for solving deterministic job shop scheduling problem. More specifically, a new priority dispatch rule and hybrid rout algorithm were developed for approaching the problem under consideration. The proposed solution algorithms were tested on a set of instances and compared with other methods. The computational results validated the effectiveness of the developed solution approaches and showed that the proposed rollout algorithms are competitive with respect to several state-of-art heuristics for solving the job shop scheduling problem.

Zhang et al (2009) represented a decomposition based hybrid optimization algorithm for large scale job scheduling problems in which the total weighted tardiness must be minimized. In each iteration, a new subproblem was first defined by a simulated annealing approach and then solved using a genetic algorithm. A fuzzy inference system was constructed to calculate the job bottleneck characteristic values which depict the characteristic information in different optimization stages. This information was then utilized to guide the process of sub-problem-solving in an immune mechanism in order to promote the optimization efficiency. Numerical computational results show that the proposed algorithm is effective for solving large scale scheduling problems.

Zobolas et al (2008) presented a very important class of production scheduling problems and the main methods employed to solve them. After a brief description of single and parallel machines scheduling problems which constitute the basis of production scheduling search, the main shop scheduling problems were presented (flow shop, job shop, open shop, group shop and mixed shop) followed by analysis of their computational complexity. The most important exact heuristic and meta-heuristic methods were presented and classified. Finally a thorough review for each shop scheduling problem was conducted where the most important methods were proposed.

Tuncel et al (2007) approached the production scheduling problem with the use of Petri nets in conjunction with other methods. The study gave a comprehensive overview for production scheduling research that combines PN's with other methods. They discussed both theoretical developments and practical experiences and identified research trends.

Zobolas et al (2009) proposed a hybrid metaheuristic for the minimization of makespan in permutation flow shop scheduling problems. The solution approach was robust, fast, and simply structured, and comprised three components: an initial population generation method based on a greedy randomized constructive heuristic, a genetic algorithm for solution evolution and a variable neighbourhood search to improve the population the hybridization of the genetic algorithm and the variable neighbourhood search, combining the advantages of these two individual components was the key innovative aspect of the approach.

Weidenhiller et al (2007) addressed the production scheduling problem by presenting an extremely fast and simple method to find an optimal schedule in the case of a continuous time single machine problem with the goal of minimizing set up and holding costs. In the case where there were no set ups, this method always found the optimal solution.

Huang et al (2010) considered a single-machine scheduling problem with deteriorating jobs in which the due dates were determined by the equal slack method. Deteriorating job means the job's processing time is an increasing function of its starting time. They modeled job deteriorating as a function that was proportional to a linear function of time. The objective was to minimize the total weighted earliness penalty subject to no tardy jobs. They proved that two special cases of the problem remain polynomially solvable. The first case was the problem with equally weighted monotonous penalty objective function and the other case was the problem with weighted linear penalty objective function.

Lei (2008) described the real life scheduling problem as often having conflicting objectives. He was with the opinion that the solution of these problems can provide deeper insights to the decision maker than those of single-objective problems. However, the literature of multi-objective scheduling is notably sparser than that of single-objective scheduling. Since the survey paper on multi-objective and bi-objective scheduling was conducted by Nagar et al in 1995, there has been an increasing interest in multi-objective production scheduling, especially in multi-objective deterministic problem. The goal of this paper was to provide an extensive review of the literature on the scheduling problems with multi-objectives in the past 13 years.

Doganis et al (2008) conducted a research on optimal production scheduling for the dairy industry.The increasing variety of products offered by the food industry has helped the industry to respond to market trends, but at the same time has resulted in a more complex production process which requires flexibility and an efficient coordination of existing resources. Especially in industrial yogurt production there is a wide variety of products that differ in features like fat content, the whey used to produce the mixture, the flavor, the size of the container or the language on the label. The great diversification and the special features that characterize yogurt production lines (including :satisfaction of multiple due dates, variable processing times, sequence dependent set up times and costs and monitoring of inventory levels), render generic scheduling methodologies impractical for real world applications. Results of the study presented a customized mixed integer linear programming model for optimizing yogurt packaging lines that consist of multiple parallel machines. The model is characterized by parsimony in the utilization of binary variables and necessitates the use of only a small predetermined number of time periods. The efficiency of the proposed model was illustrated through its application to yogurt production.

TavaKKoli-Moghaddam et al (2006) discussed the attention gained by flow shop scheduling problems in the practical and academic fields. In this work they considered a multi-objective no-wait flow shop scheduling problem by minimizing the weighted mean completion time and weighted mean tardiness simultaneously. Since a flow shop scheduling problem has been proved to be NP-hard in a strong sense, an effective immune algorithm was proposed for searching locally the pareto optimal frontier for the given problem. To validate the performance of the proposed algorithm in terms of solution quality and diversity level, various test problems were carried out and the efficiency of the proposed algorithm, based on some comparison metrics was compared with a prominent multi-objective genetic algorithm. The computational results show that the proposed immune algorithm outperformed the above genetic algorithm, especially for large problems.

Jiang et al (2008) discussed weapon production using a single production stage with limited capability for production scheduling, and a parallel machine and a machine group dynamically flexible change strategy. An optimal schedule was created by integrating the schedule operation model into one-stage scheduling and combining it with a mathematical programming model. The study utilized Lindo 6.0 professional software. For verification and evaluation of computational results the software program implemented a mathematical programming model, concluding with a comparison of the first-come-first- serve technique. The proposed model yielded a favourable outcome and benefits, clearly assigning schedules for labor and production, thus obtaining the total least performance indicator for tardiness cost, earliness cost and machine group change over cost.

The research conducted by Neumann et al (2002) revealed that an advanced planning system (APS) offered support at all planning levels along the supply chain while observing limited resources. An APS for process industries consisting of the modules network designed for network planning was considered. A new solution approach was proposed in the case batch production, which solved much larger problems than the methods known so far.

Biskup et al (1999) focused their study on the analysis of learning in single-machine scheduling problems. It was surprising that the well known learning effects were never considered in connection with scheduling problems.It was shown in this study that even with the introduction of learning to job processing times, two important types of single-machine problems remain polynomially solvable.

Parunak et al (1991) discussed the difficulties in job shop scheduling with human intelligence. The push toward increased automation and flexibility in manufacturing led to a number of computerized schemes which addressed the problems with varying degree of success. These schemes often have little in common with one another. It was not clear whether they were addressing the same problem or how they should be extended or combined to advance the state of the art.The study developed a general context for the scheduling problem, as a framework for understanding existing approaches and as a roadmap for future development. They proposed a formal definition of a schedule and described five challenges to the computation of the schedules.The main objective of the study was to contribute to the solution of the scheduling problem in exhibiting all scheduling complexities.

From the computational point of view, the job shop scheduling is one of the notorious intractable NP-hard optimization problems. Chaoyong et al (2008) applied an effective hybrid genetic algorithm for the job shop scheduling problem. They proposed three novel features for this algorithm to solve the job shop scheduling problem. Firstly. A new full active schedule procedure based on the operation-based representation was presented to construct a schedule. After a schedule was obtained, a local search heuristic was applied to improve the solution.Secondly, a new cross-over operator, called the precedence operation cross-over operator, was proposed for the operation-based representation, which can preserve the meaningful characteristics of the previous generation. Thirdly, in order to reduce the disruptive effects of genetic operators, the approach of an improved generation alteration model was introduced. The proposed approaches were tested on some standard instances and compared with other approaches. The superior results validated the effectiveness of the proposed algorithm.

Kogan et al (2000) studied the problems of M-machine, J-product, N-time point preemptive scheduling in parallel and serial production systems. The objective was to minimize the sum of the costs related to inventory level and production rate along a planning horizon. Although the problem was NP-hard, the application of the maximum principle reduces it into a well tractable type of the two-point boundary value problem. As a result time complexities were developed for parallel and serial production systems.

Kim (2001) focused on production scheduling in a semiconductor wafer fab producing multiple product types that have different due dates and different process flows. In the wafer fab, wafer lots were processed on serial and batch processing workstations, each of which consisted of parallel identical machines. Machines in serial processing workstations processed wafer lots one by one, while those in batch processing workstations processed several wafer lots of the same recipe at the same time. What needed to be done for production scheduling were lot release control, lot scheduling, and batch scheduling. For these three decision problems, they developed several rules which used information such as order size (number of lots in orders) and processing status of the wafer lots. To evaluate these new rules, they used a simulation model in which the three decision problems were considered simultaneously. Simulation results show that the new rules work better than existing rules in terms of total tardiness of the orders.

Kim (1998) further, studied lot release control and scheduling problems in a semiconductor wafer fab producing multiple products that have different due dates and different process flows. For lot release control, it was necessary to determine the type of a wafer lot and the time to release wafers into the wafer fab, while it was necessary to determine sequences of processing waiting lots in front of workstations for lot scheduling. New dispatching rules were developed for lot release control and scheduling considering special features of the wafer fabrication process. Simulation experiments were carried out to test the dispatching rules. Results showed that lot release control and lot scheduling at photolithography workstations were more important than scheduling at other stations.

Lombardi et al (2012) worked on classical scheduling formulations which typically assume static resource requirements and focused on deciding when to start the problem activities, so as to optimize some performance metric. In many practical case the decision maker has the ability to choose the resource assignment as well as starting times. This is actually far from trivial task, with deep implication on the quality of the final schedule. Joint resource assignment and scheduling problems were incredibly challenging from computational point of view. Both approaches reported success and overall performed equally well. The hybrid approach was applied tio this class of problems. The main effort in the study was to identify key modeling and solution techniques.

Marius et al (1987) considered the design and analysis of algorithms for vehicle routing and scheduling problems with time window constraint. Given the intrinsic difficulty of this problem class, approximation methods seemed to offer the most promised for practical size problem. After decreasing a variety of heuristics, they conducted an extensive computational study of their performance. The problem set included routing and scheduling environments that differ in terms of the type of data used to generate the problems. They found that several heuristics performed well in different problem environments; in particular an insertion-type heuristic consistently gave very good results.

Gonzalez et al (2009) faced the job shop scheduling problem with sequence dependent set up times and makespan minimization. To solve this problem, they proposed a new approach that combined a Genetic algorithm with a Tabu Search method. They reported results from an experimental study across conventional benchmark instances showing that this hybrid approach outperformed the current state- of- the-art methods.

Kim (2010) studied the problem of scheduling wafer lots on diffusion workstations in a semiconductor wafer fabrication facility. In a diffusion workstation, there were multiple identical machines and each of them could process a limited number of wafer lots at a time. Wafer lots could be classified into several product families and wafer lots that belong to the product family could be processed together as a batch. Processing times and set up times for wafer lots of the same product family were the same but ready times of the wafer lots (at the diffusion workstation) could be different. They presented several heuristic algorithms for the problem with the objective of minimizing total tardiness. For evaluation of performance of the suggested algorithms, a series of computational experiments was performed on randomly generated test problems. Results showed that the suggested algorithms performed better than algorithms currently used in practice.

Qi (2003) validated a proposed job release methodology and experimentally investigated the impact of production control methodologies and system factors on wafer fab performance in terms of average cycle time, standard deviation of cycle time, average lateness, WIP inventory and fab output by simulating a wafer fab of chartered Semiconductor Manufacturing limited and statistically analyzing the experimental results using t-test and ANOVA. A full factorial design of experiment was conducted to evaluate the performance of three job release methodologies, three dispatching rules and three greedy levels of batching policy under different system environmental settings differentiated by fab output level and machine unreliability level.Based on the experimental results, the proposed job release methodology WIPLOAD control appeared to be very efficient to be able to potentially improve all the considered performance measures simultaneously. The advantage of WIPLOAD controls was robust to the change of system environmentally conditions. In contrast, the improvement brought by a dispatching rule on a certain performance measure could not cause the deterioration of other performances. Considering the relative impact on the fab performance, job release control appeared to be the most important production control factor in comparison with dispatching and batching policy, especially when the system was operating on high output level and/or with a high system variability level caused by machine unreliability.

Kim (2003) presented a real time scheduling methodology in a semiconductor wafer fab that produces multiple product types with different due dates. In the suggested real-time scheduling method, lot scheduling rules and batch scheduling rules were selected from sets of candidate rules based on information obtained from discrete event simulation. Since a rule combination that gave the best performance may vary according to the states of the fab, a selected rule combination was employed for a certain period of time and then a new combination was selected and employed. Since multiple simulation runs should be made in the simulation-based real time scheduling method, it may take excessively long computation time to react to unexpected events. To reduce response time, they suggested three techniques for accelerating rule comparison.These techniques were tested as well as other operational policies that could be used in the simulation-based real time scheduling method through computational experiments on a number of test problems. It is widely accepted that new production scheduling tools are playing a key role in flexible manufacturing systems to improve their performance by avoiding idleness of machines, while minimizing set up times penalties, reducing penalties for not delivering orders on time etc. Since manufacturing scheduling problems are NP-hard, there is a need of improving scheduling methodologies to get good solutions within low CPU time. Lagrange Relaxation is known for handling large scale separable problems, however, the convergence to the optimal solution can be slow. Lagrange Relaxation needs customized parametrization, depending on the scheduling problem, usually made by an expert user. It is interesting in the use of Lagrange Relaxation without being an expertise ie without difficulty in parameter tuning.

Ant colony optimization (ACO) algorithm is an evolutionary technology often used to resolve difficult combinatorial optimization problems, such as single machine scheduling problems, flow shop or job shop scheduling problems. Lin et al (2007) proposed a new ACO algorithm with an escape mechanism modifying the pheromone updating rules to escape local optimal solutions. The proposed method was used to resolve a single machine total weighted tardiness problem, a flow shop scheduling problem for makespan minimization and a job shop scheduling problem for makespan minimization. Compared with existing algorithms, the proposed algorithm will resolve the scheduling problems with less artificial ants and obtain better or at least the same solution quality. Production scheduling activities are common but complex. This leads to many different vies and perspectives of production scheduling. Each perspective has a particular scope, its own set of assumptions, and a different approach to improving production scheduling. Hermann (2003) described three important perspectives the problem-solving perspective, the decision-making perspective and the organizational perspective) and discussed the methodologies the these perspectives used. He finally presented a integrative strategythat was used to select in a, particular setting an approach for improving production scheduling.

Most of research in production scheduling is concerned with the optimization of a single criterion. However, the analysis of the performance of a schedule involves more than one aspect and therefore requires a multi-objective treatment. Louki et al (2003) designed a general method which was able to approximate the set of all the efficient schedules for a large set of models. The models were introduced to treat one machine, parallel machines and permutation flow shops. The multi-objective simulated annealing method was applied. The results indicated a superior and efficient scheduling mechanism.

Kogan et al (1998) presented an approach in tracking demands in optimal control of managerial systems with continuously divisible doubly constrained resources.The resources were allowed to be doubly constrained, so that both usage at every point of time and cumulative consumption over a planning horizon were limited as it was often the case in project and production scheduling. The objective was to tract changing in time demands for the activities as closely as possible. They proposed a general continuous time model that stated the problem in a form of the optimal control problem with non-linear speed-resource usage functions. With the aid of the maximum principle, properties of the solutions were derived to characterize optimal resource usage policies. On the basis of this analytical investigation, numerical scheduling methods were suggested.

Giffler et al (1960) discussed the algorithm for solving production scheduling problems. Algorithms are developed for solving problems to minimize the length of production schedules. The algorithms generate anyone, or all schedules of a particular subset of all possible schedules, called the active schedules. This subset contains, in turn, a subset of the optimal schedules. It was further shown in their study that every optimal schedule is equivalent to an active optimal schedule. Computational experience with the algorithm showed that this was practical, in problems of small size, to generate the complete bet of all active schedules and to pick the optimal schedules directly from this set and, when this was not practical, to random sample from the bet of all active schedules and thus to produce schedules that were optimal with a probability as close to unity as was desired.

Musikapun et al (2012) described the approach for solving multi-stage, multi-machine, multi-product scheduling problem, using Bat Algorithm. Bat algorithm is one of the recently developed nature inspired methods in the field of computational intelligence. Their studies presented the development of the Bat Algorithm based Scheduling Tool (BAST) that used to solve multi-stage, multi-machine, multi-product scheduling problems. The algorithm takes into account the Just-In -Time production philosophy by aiming to minimize the combination of earliness and tardiness penalty costs. The computational experiment on the BAST was conducted using data obtained from a collaborating company engaged in capital goods industry. The experimental results indicated that the Bat algorithm performance can be improved up to 8.37 % after adopting the appropriate parameters setting.

Bierwirthet al (1996) conducted a search into production scheduling and rescheduling with Genetic Algorithms. A general model for job shop scheduling was described which applies to static, dynamic and non-deterministic production environments. Next, a Genetic Algorithm was presented which solved the job shop scheduling problem. This Algorithm was tested in a dynamic environment under different workload situations. Thereby, a highly efficient decoding procedure was proposed which strongly improves the quality of schedules. Finally, this technique was tested for scheduling and rescheduling in a non-deterministic environment. It was shown by experiment that conventional methods of production control are clearly outperformed at reasonable runtime costs.

Zhang et al (1999) performed an extensive work in hybrid flow shop scheduling. The research study reviewed that the state of the art gave in details their contributions. The review was conducted with suggestions for future research directions.

Al-Hinai et al (2011) addressed the problem of finding robust and stable solutions for the flexible job shop scheduling problem with random machine breakdowns. A number of bi-objective measures combining the robustness and stability of the predicted schedule were defined and compared while the same used same rescheduling method. Consequently, a two-stage optimized the primary objective and minimized the makespan in this study, where all the data was considered to be deterministic with no expected disruptions. The second stage optimized the bi-objective function and integrated machine assignments and operations sequencing with the expected machine breakdown in the decoding space. An experimental study and analysis of variance was conducted to study the effect of different proposed measures on the performance of the obtained results. The results indicated that different measures have different significant effects on the relative performance of the proposed method. Furthermore the effectiveness of the current proposed method was compared against three other methods. Two were taken from the literature and the third was a combination of the former two methods.

Graves et al (1981) discussed the definition of production scheduling as the allocation of available production resources over time to best satisfy some set of criteria. Typically, the scheduling problem involves a set of tasks to be performed and the criteria may involve both tradeoffs between early and late completion of the task and between holding inventory for the task and frequent production changeovers. The intent of the study was to present a broad classification for various scheduling problems to review important theoretical developments for these problem classes and to contrast the currently available theory with the practice of production scheduling. The study highlighted the problem areas for which there was both a significant discrepancy between practice and theory.

Cheng et al (1996) identified the job shop scheduling problem as one of the well known hardest combinatorial optimization problems. During the last three decades, the problem had captured the interest of a significant number of researchers and a lot of literatures have been published, but no efficient solution algorithm had been found it to optimality in polynomial time. This has led to the recent interest in using genetic algorithm to address it. The purpose of the study and its companion was to give a tutorial survey of recent works on solving classical job shop scheduling problem using genetic algorithms. All of the techniques developed for the job shop scheduling problem were useful for the other scheduling problems in modern flexible manufacturing systems and other combinatorial optimization problem.

Sonke et al (2010) presented a study on resource constraint project scheduling problem RCP SP. This consisted of activities that must be scheduled subject to precedence and resource constraints such that the makespan was minimized. It has become a well known standard problem in the context of project scheduling which has attracted numerous researchers who developed both exact and heuristic scheduling procedures. However, it was a rather basic model with assumptions that were too restrictive for many practical applications. Consequently, various extensions of the basic resource constrained project scheduling problem was developed. This study gave an overview over these extensions. The extensions were classified according to the structure of the resource constrained project scheduling problem. They summarized generalizations of the activity concept of the precedence relations and of the resource constraints. Alternative objectives and approaches for scheduling multiple projects were discussed as well. In addition to popular variants and extensions such as multiple modes, minimal and maximal time lags, and net present valued based objectives, the study also provided a survey of many less known concepts.

Huang et al (2013) worked on some difficult production scheduling problem. Production scheduling is a rather difficult problem in virtual enterprises for the tasks of production which would be executed by some distributed and independent members. Factors such as the timing constraints of task and ability restrictions of the members were considered comprehensively to solve the global scheduling optimization problem. This study established a partner selection model based on an improved ant colony algorithm at first, then presented a production scheduling framework with two layers as global scheduling and local scheduling for virtual enterprise, and gave a global scheduling mathematical model with the smallest total production time based on it. An improved genetic algorithm was proposed in the model to solve the time complexity of virtual enterprise production scheduling. The presented experimental results validated the optimization of the model and the efficiency of the algorithm.

Jansen et al (2010) presented a study on a linear time approximation scheme. This was conducted for the non preemptive job shop scheduling problem, when the number of machines and the number of operations per job were fixed. They also showed how to extend the approximation scheme for the preemptive version of the problem.

Godinho et al (2012) reviewed the literature regarding Genetic Algorithms (GA) applied to flexible manufacturing system (FMS) scheduling. On the basis of this literature review, a classification system was proposed that encompassed six main dimensions. Flexible manufacturing system type, types of resource constraint, job description, scheduling problem, measure of performance and solution approach. The literature review found forty papers which were classified according to these criteria. The literature was analysed using the proposed classification system, which provided the following results regarding the application genetic algorithm to Flexible Manufacturing System scheduling.:(1) combinations of genetic algorithm and other methods were relatively important in the reviewed papers, (2) although most studies dealt with complex environments concerning both the routing flexibility and the job complexity, only a minority of papers simultaneously considered the variety of possible capacity constraints on a flexible manufacturing system environment including pallets and automated guided vehicles, (3) local search was rarely used, (4) makespan was the most widely used measure of performance.

Hwang et al (1970) were concerned with the search with optimal production planning and inventory control. The first problem was a multiperiod production scheduling problem in which the objective was to minimize the operating costs for a planned period. This costs was composed principally of the sum of the production costs and the inventory carrying costs. The second problem considered and inventory system with two decision variables in each planning period. These were the production schedule and the work force which were to be determined so as to minimize the operation costs which included the cost of changing the production rate, of changing the work force and of carrying the inventory. The maximum principle in the discrete form was used to reduce both the first problem which had N decision variables and the second problem which had 2N decision variables respectively to a series of two decision variables problems. The so-called sequential simplex pattern search technique was used to determine the optimal values of these two decision variables. Numerical examples were given to demonstrate the method.

Balasubramanian et al (2002) worked on the prevalent approach to the treatment of processing time uncertainties in production scheduling problems. This was done through the use of probabilistic models. Apart from requiring detailed information about probability distribution functions, this approach also had the drawback that the computational expense of solving these models was very high. In this study, they presented a non-probabilistic treatment of scheduling optimization under uncertainty, based on concepts from fuzzy set theory and interval arithmetic, to describe the impression and uncertainty in the task durations. They provided a brief review on the fuzzy set approach and compared it with the probabilistic approach. They presented models derived from applying this approach to two different problems namely the flow shop scheduling and the new product development process scheduling. Results indicated that theses MILP models were computationally tractable for reasonably sized problems. They also described the tabu search implementations in order to handle larger problems.

Camarinha-Matos et al (2009) worked on the concepts and practice in manufacturing enterprises. Participation in networks has nowadays become very important for any organization that strives to achieve a differentiated competitive advantage, especially if the company is small or medium sized. Collaboration is a key issue to rapidly answer market demands in a manufacturing company, through sharing competencies and resources. The Collaborative Networked Organizations (CNO) area focussed on this type of organizational models that use ICT for supporting the development of collaborative business opportunities. This study described the key concepts which were related to Collaborative Networked organizations and provided a high level classification of collaborative networks, and presented some application cases in the manufacturing industry. Finally a holistic research initiative addressing key challenges in the area was presented and a decision of the collaborative Network Organization paradigm contribution to the challenges faced by the manufacturing systems was made.

Zhang et al (2008) conducted a research on optimization algorithm for multi-objective flexible job shop scheduling. Flexible job shop scheduling problem (FJSP) is an extension of the classical job shop problem. Although the traditional optimization algorithms could obtain preferable results in solving the mono-objective flexible job shop scheduling problem. However, they are very difficult to solve multi objective flexible job shop scheduling problems very well. In this study, a particle swarm optimization (PSO) algorithm and a tabu search (TS) algorithm were combined to solve the multi-objective flexible job shop scheduling problem with several conflicting and incommensurable objectives. The particle swarm optimization algorithm which integrated local search and global search scheme possesses high search efficiency. And tabu search was a meta-heuristic which was designed for finding a near optimal solution of combinatorial optimization problems. Through reasonably hybridizing the two optimization algorithms, an effective hybrid approach for the multi-objective flexible job shop scheduling problem was proposed. The computational results proved that the proposed hybrid algorithm was efficient and effective and efficient approach to solve the multi-objective flexible job shop scheduling problem, especially for the problems on a large scale.

Wen Rui Jiang et al, (2011) conducted a search on game theoretic approach to job shop scheduling. The study proposed a non-operative game approach based on neural network to solve the job shop scheduling problem. Machines in manufacturing tasks were defined as players and strategies consisted of all the feasible programs which were selected by dispatching rules for minimizing the mean flow time. Strategies for the game model were generated from a backpropagation neural network, which selected combination of the rules for the machines. Cases study showed that the game approach based on neural network can be an effective approach to solve the job shop scheduling.

CHAPTER 3

METHODOLOGY OF THE STUDY

INTRODUCTION

Most industrial problems, including the optimization of production scheduling can be treated as optimization problem, once the problem is expressed as mathematical program. Optimization of a function Z over a given feasible set S involves finding the maximum or minimum value of Z in the feasible set. However, the type of optimization approach to apply is determined by the properties of Z and S. If Z is a linear function and the feasible set S is defined by a finite collection of linear inequalities and equalities, then the optimization problem is called linear program. If the optimization is a linear program and one however imposes that the feasible set S shall admit only integer values, then we have an optimization problem described as integer programming to solve. Similarly, if Z is defined by a nonlinear function and the feasible set S is defined by finite collection of nonlinear equations, we have an optimization problem called nonlinear programming on hand. In the case study in this thesis, the production scheduling problem will be treated as a linear programming model.

THEORY AND APPLICATION OF LINEAR PROGRAMMING

Production scheduling problems can be expressed as a collection of linear constraints with one basic linear objective function. This permits the handling of such industrial problems to be treated as linear optimization problem in which we have a collection of variables which can take real values. The assignment of values to these variables must satisfy the given collection of linear inequalities (linear constraints) and also maximize or minimize the given linear function. These are prerequisite general conditions which need to be satisfied, if there exist a feasible solution to the scheduling problems (from the established linear program)

As already discussed under section 1.5, we see that there are a number of linear programming algorithms which can be conveniently applied to solve production scheduling problems. Linear program needs firstly to be presented in a general standard form to display all properties required of a linear programming problem. This consist of a linear objective function f(x) such that, if in general $c_1,\ldots,c_n$ are real numbers, then the function f of real variables $x_1,\ldots, x_n$ can be defined as :

$$f(x) = c_1x_1 + \ldots + c_nx_n = \sum_{j=1}^{n} c_j\, x_j$$

Other properties include a linear constraint (which is one that is either a linear equation or linear inequality) and a non-negativity constraint. These can be written in mathematical notations as:

$$\sum_{j=1}^{n} a_{ij}\, x_j \le b_i \quad \forall\ i \in \{1, \ldots, m\} \qquad \text{(linear constraints)}$$

and $$x_j \ge 0 \quad \forall\ j \in \{1, \ldots, n\} \qquad \text{(non-negativity constraint)}$$

Hence every linear program in the standard form can be generally presented as:

$$Z = \max.\sum_{j=1}^{n} c_j\, x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le b_i \quad \forall\ i\ \in \{1, \dots, m\}$$

$$x_j \ge\ 0 \qquad \forall\ j \in \{1, \dots, n\}$$

If (x_1^*, x_2^*,…, x_n^*) satisfy all the constraints of linear program, then the assignment of values to these variables are called a feasible solution of the linear program. If a given set of feasible solution can optimize the value of the objective function Z (maximize or minimize depending on problem formulation) then, the optimal solution is said to have been obtained.

There are two logical steps to solve general optimization problems. These are namely modeling and solving. We will now consider these two steps and then provide a simple case illustration and preview of how the simplex method can be used to solve a linear programming problem.

Generally, modeling involves reading the problem presented in a verbal description to identify the decision variables. The objective and constraints of the problem can then be formulated in terms of the variables. This establishes the linear objective function and the linear constraints of the problem in a mathematical program. The modeled problem can then be solved using any of the algorithms which can appropriately provide accurate results.

Commercial software companies continually develop solvers which have in-built packages of optimization algorithms. We will use the option of manual calculations in this thesis for illustration and case study.

Some important scenarios in the application of the simplex method

There may be few pitfalls in the application of the simplex method. However, we will not deal with these aspects in detail. These include the following:

i. Initialization:

There are instances where one will not be able to start iterating. One of the reasons could be from the problem formulation or the problem has an infeasible origin. The problem with infeasible origin is that, one may not know whether a feasible solution exists at all. Such problems are often treated as auxiliary problems.

ii. Iteration:

In the cause of iterations there could be a tie between one or more variables entering the basis. It is indicated by these variables having exactly same values. It does not matter which variable is chosen for entry into the basis. The same conditions of tie breaking holds for leaving variables.

iii. Termination:

Cycling is one of the causes that makes termination of iteration process difficult or impossible. It occurs as follows: the arbitrary choice between the tied variables to be removed from the basis will always generate degenerate feasible solution in which all of the tied variables reach zero simultaneously as the entering basic variable is increased. If one of these basic variables retains its zero value until it is chosen at a later iteration to be a leaving variable, then the corresponding entering variable must then also enter at a value of zero. This means the value of the objective function would remain unchanged at that iteration and the simplex method then begins to cycle in a loop.

Problem:

An automobile manufacturer produces cars and trucks in a factory that is divided into two shops. Shop A which performs the basic operations must work 5 man-days on each truck but only 2 man-days on each car. Shop B which performs finishing operations must work 3 man-days for each car or truck that it produces. Because of men and machine limitations, shop A has 180 man-days per week available, while shop B has 135 man-days per week. If the manufacturer makes a profit of Rs.300 on each truck and Rs.200 on each car, how many of each should be produced to maximize profit of the organization.

Solution:	Shop A	Shop B	Profit
Cars	2 man-days	3 man-days	Rs.200
Trucks	5 man-days	3 man-days	Rs.300
Availability	180 man-days/week	135 man-days/week	

Modeling the problem

We model by identifying the variables and expressing the objective and constraints of the problem in terms of the variables.

Select decision variables

Let x_1 = no. of cars to be produced per week

x_2 = no. of trucks to be produced per week

Objective function

Maximize $Z = 200x_1 + 300x_2$

Constraints

Subject to $2x_1 + 5x_2 \leq 180$

$3x_1 + 3x_2 \leq 135$

$x_1, x_2 \geq 0$

In the standard form we have the following linear problem to solve;

Maximize $Z = 200x_1 + 300x_2$

Subject to $2x_1 + 5x_2 \leq 180$

$3x_1 + 3x_2 \leq 135$

$x_1, x_2 \geq 0$

Conversion of inequalities into equalities

We shall introduce the slack variables x_3 and x_4 to convert the linear inequalities into linear equations:

Hence

$x_3 = 180 - 2x_1 - 5x_2$ **EQ1**

$x_4 = 135 - 3x_1 - 3x_2$

$x_1, x_2, x_3, x_4 \geq 0$

Step 1:

We set the initial feasible solution at $x_1 = 0$, $x_2 = 0$ and $x_3, x_4 \geq 0$

This implies $x_3 = 180$ and $x_4 = 135$ and Z is evaluated at $Z = 0$

Step 2:

We can start the first iteration by increasing any of the variables x_1 or x_2. Let's increase x_1 and maintain $x_2 = 0$ and $x_3, x_4 \geq 0$

From the equations at **EQ1** we determine new value for x_1, setting $x_3 = 0$ we have

$x_1 = \min(90, 45) = 45$

Step 3:

We evaluate Z again with new set of feasible solution from step 2 as follows:

$x_1 = 45$, $x_2 = 0$, $x_3 = 90$, $x_4 = 0$ and $Z = 200(45) + 0 = 9{,}000$

Step 4:

We start the second iteration by identifying which decision variable is to be increased in order to increase value of Z further. To do this, we go to **EQ1** and express variables with non-zero values in terms of those with zero value (based on the last available feasible solution at step 3)

$x_1 = 45 - \frac{1}{3}\, x_4 \;-\; x_2\;\;)$

$x_3 = 90 + \frac{2}{3}\, x_4 \;-\; 3x_2$

$Z = 9000 - \frac{200}{3}\, x_4 \;+ 100x_2$

Step 5:

From step 4 only when x_2 increases can we increase Z. the question is: by how much can we increase x_2 ? To know this we determine values of x_2 from the two equations at step 4 and pick the minimum. Thus from the feasibility condition

$x_1, x_3 \geq 0$ and $x_4 = 0$ we have $x_2 = \min(45, 30) = 30$

Step 6:

Now we evaluate Z again with the new value of x_2 from step 5. Thus $x_1 = 15$, $x_2 = 30$,

$x_3 = 0$ and $x_4 = 0$ hence $Z = 9{,}000 – 0 + 100(30) = 9{,}000 + 3{,}000 = 12{,}000$

Step 7:

We start the third iteration by identifying which decision variable to increase. To do this, we express non-zero variables in terms of zero value variables (based on step 6 feasible solution)

$x_1 = 45 - \frac{1}{3}\, x_4 \;-\; x_2$ (we maintain this equation from step 4)

$x_2 = 30 + \frac{2}{9}\, x_4 \;-\; \frac{1}{3}\, x_3$

$Z = 12{,}000 - \frac{400}{9}\, x_4 \;\; - \frac{100}{3}\, x_3$

Since we cannot increase Z further by increasing any variable, we have obtained the optimal solution with **Z = Rs 12,000**

General principles in the application of the simplex method from the illustrated example

Let's consider a general presentation of linear program in the standard form:

$$Z = \max.\sum_{j=1}^{n} c_j\, x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \quad \forall\; i\; \in \{1, \ldots, m\}$$

$$x_j \geq 0 \qquad \forall\; j \in \{1, \ldots, n\}$$

where we have j=n as total decision variables with i = m as total constraints.

The first step in the application of the simplex method is to introduce slack variables $x_{n+1},\ldots,x_{n+m} \geq 0$. Thus every constraint gets one slack variable and the denotation of the slack variable starts or continues from the last decision variable x_n (j=n). Hence the inequality expressions (the constraint set) can be presented in equality forms and the linear program can be rewritten as:

$$x_{n+i} = b_i - \sum_{j=1}^{n} a_{ij}\, x_j \quad \forall\; i\; \in \{1, \ldots, m\}$$

$$Z = \sum_{j=1}^{n} c_j\, x_j$$

$$x_j \geq 0 \qquad \forall\; j \in \{1, \ldots, n, n+1, \ldots, n+m\}$$

The introduction of the slack variable converts the inequalities into equations. Graphically these equations establish boundary lines of the feasible region for the given linear programming problem, if this region ever exists. If a feasible solution exist for a linear programming problem, then by finite number of iterations, the optimum point in the region gives the objective function its maximum or minimum value.

The simplex method is an iterative procedure in which one searches for a feasible solution $\overline{x}_1,\ldots,\overline{x}_{n+m}$ in which the value of the objective function is evaluated higher, given an initial feasible solution $x_1,\ldots, x_{n+m}$. This implies

$$\sum_{j=1}^{n} c_j \overline{x}_j \;>\; \sum_{j=1}^{n} c_j\, x_j \quad \forall\; j \in \{1, \ldots, n, n+1, \ldots, n+m\}$$

Some basic terminology used in the application of the simplex method

From the example discussed above we see x_3 and x_4 (at step 1) initially in the basis as as basic variables. At step 3 x_1 and x_3 were the basic variables . Generally the non-basic variables are driven to zero in each iteration and they change from iteration to iteration.

1. Choice of entering variable is motivated by the fact that we want to increase the value of Z and we choose a variable that does that, to increase value of Z to possible maximum (in this example) or decrease to possible minimum. In the example discussed above, it was either x_1 or x_2. However, x_1 was chosen at step 2. Similarly at step 4 we chose x_2 as the entering variable finally.
2. Choice of leaving variable is motivated by the need to maintain feasibility. This is done by identifying the basic variable that poses the most stringent bound on the entering variable. In the example the leaving variable at first iteration was x_4 (see expression for Z at step 4). The next leaving variable was x_3 (see expression for Z at step 7).
3. The formular for the entering variable always appears in the pivot row. The process is called pivoting. In the discussed example we see the formular for the first entering variable at step 4 as

   $x_1 = 45 - \frac{1}{3}\, x_4 \;-\; x_2$ and $x_2 = 30 + \frac{2}{9}\, x_4 \;-\; \frac{1}{3}\, x_3$ at step 7. These

   equations are referred to as the pivot rows.

A basic concept must be noticed in the simplex method. That is: a set of feasible solution in previous equations is also a feasible solution to current system of equations. In the discussed linear programming problem above we see these illustrated at step1, step 3 and step 6.

This is precisely because we are just rewriting the system of equations using a different set of decision and slack variables in every iteration.

To avoid a repetitive writing of system of equations, we shall apply same concept of linear programming algorithm using the simplex tableau to solve same type of model in a case study at chapter 4.

CHAPTER 4

DATA COLLECTION, ANALYSIS AND RESULTS

INTRODUCTION

The practical application of the methodology discussed in chapter 3 will be the focus in this chapter. This involves the utilization of real production data from a manufacturing company in cable production, to establish a model whose outcome shall be examined and discussed, under varying operational constraints. Changes in the model and the corresponding outcomes impacted by these constraints, will be analysed for suitable recommendations and improvement opportunities.

DATA COLLECTION

Nexans Kabelmetal Ghana Limited manufactures, markets and supplies electrical and telephone cables, wires and conductors. The company produces for the local market as well as exports to neighbouring West African countries. The discovery and exploitation of new oil wells in the Western region of Ghana seem to be opening new marketing opportunities. The main metals used for conductors and wires are copper and aluminium, which are outsourced from European and South American suppliers. A budgeted total of 1,100 tons of copper is projected for production and sales over the year 2013. This target is about 14.6% increase on the 2012 performance in terms of production scheduling output and sales tonnage. It is already known from previous trend of records, that house wiring cables move relatively faster on stock than the other copper product categories. Management decision was therefore to intensify marketing operations with new strategies such that at least 70% of the total copper sales in 2013 can proceed from sales of house wiring cables. The remaining 30% will come from the other copper products.

The task in this case study is to determine the optimal scheduling output for each of the six products of house wiring cables involved through the application of a linear programming algorithm.

The following constraints were considered within the scope of normal operational conditions :

a) The copper sales target for 2012 was 960 tons. The actual results was well over 1,000 tons at end of December, 2012.Top management therefore decided to increase the 2012 copper budget by 14.6% to obtain new target for 2013. This yields a total of 1,100 tons copper, expected to be sold by end of December, 2013. However, the percentage of copper from house wiring cables out of the total copper sales for the year 2012 was about 70%. Management realized that this ratio can be maintained for 2013, if better marketing strategy can be implemented. This sets a minimum sales of 70% of the total monthly copper, as performance indicator for sales and production of house wiring cables. Hence on monthly basis a minimum total of 92 tons copper must be scheduled and a projected minimum of 65 tons out of the 92 tons must be accordingly copper proceeding from production and sales of house wiring cables.

Now the technical designations for the identification of the house wiring cables are as follows: NCY 1.5 RS, NCY 2.5 RS, NCY 4 RS, NCY 6 RS, NCY 10 RS and NCY 16 RS. Find the meaning of the abbreviations and numbers under "List of Abbreviations" at the front pages of this thesis.

b) The house wiring cable types NCY 1.5 RS and NCY 2.5 RS are commonly used for lighting and sockets wiring. They are needed mostly in every newly constructed building both in the private and public establishments as well as in every home with electric power supply. Comparatively higher quantities (in terms of number of packaged pieces) are therefore always in demand. The commercial department of Nexans has estimated that, if double the optimal average quantities for both types of cables can be sold in a month when the marketing team had realized high purchases from estates construction sites, then a margin of additional 10 tons can be achieved above the expected 65 tons per month. This implies twice the optimal schedules for NCY 1.5 RS and NCY 2.5 RS together with the other four products must sum up to maximum of 75 tons per month. This will ensure that there are always enough stock for NCY 1.5 RS and NCY 2.5 RS to avoid shortages. Any additional demand will be carefully planned and implemented.

c) NCY 4 RS and NCY 6 RS are cable types usually used for refrigerators and air conditioners depending on current ratings for equipments. Management deduced from the previous sales pattern that, if more hotels and other institutional building projects can be contacted, then a minimum of additional 23 tons can be sold above the expected 65 tons. Reason for this projection is based on comparatively higher copper content of these products in relations to NCY 1.5 RS & NCY 2.5 RS. Thus twice the optimal average quantities of NCY 4 RS and NCY 6 RS that can be sold together with the other four products must sum up to at least 88 tons per month. This comes closer to the overall total of 92 tons per month and therefore reduces the burden to meet the monthly target.

d) On the production shop floor, pairs of the products are often scheduled together due to their pattern of demand and convenience in processing methods as follows:
NCY1.5 RS and NCY 2.5 RS
NCY 4 RS and NCY 6 RS
NCY 10 RS and NCY 16 RS

e) From the 2012 commercial statistics, it was discovered that on the average production and sales of NCY 1.5 RS was about 70% of the total tonnage sales of NCY 2.5 RS on monthly basis. Similarly demand for NCY 4 RS was only 54% of that of NCY 6 RS on the average. Higher copper output for NCY 16 RS was realized as compared to NCY 10 RS in the proportion of 70%.

Modeling and Solving the problem

The mathematical program can now be derived from the verbal description of the constraints and objective. We shall apply the same concept of the simplex method illustrated with an example and relevant general principles at chapter three, to determine the optimal solution.

Select decision variables

Let x_1 = total output for NCY 1.5 RS and NCY2.5 RS in tons/month

x_2 = total output for NCY 4 RS and NCY 6 RS in tons/month

x_3 = total output for NCY10 RS and NCY16 RS in tons/month

Objective function

The objective is to optimize the total copper out from the production of house wiring cables per month, such that the optimal quantity to produce for each product pair within same period can be identified for production scheduling.

Let Z = the optimal total copper output due to house wiring cables in tons/month

Then the linear objective function is

Maximize $Z = x_1 + x_2 + x_3$

Constraints of the problem

a) In case of good marketing conditions for NCY 1.5 RS and NCY 2.5 RS, expected optimal quantity must double such that the expected total output for the month can be maximized to 75 tons (ie. 65 tons+ 10 tons)
That means: $2x_1 + x_2 + x_3 \leq 75$

b) Total copper budget for the year 2013 = 1,100 tons

Expected total copper to produce = 92 tons/month

70% of monthly output due to house wiring cables = 65 tons/month

This can be expressed as : $x_1 + x_2 + x_3 = 65$

c) Due to the relatively higher copper content for NCY 4 RS and NCY 6 RS products, favourable transactions over the month for these two products must yield an additional minimum of 23 tons to the expected output for the month, that is if the expected normal sales quantity can be doubled.

This can be mathematically expressed as : $x_1 + 2x_2 + x_3 \geq 88$

The problem can now be presented in the standard form for linear programming as:

Maximize $Z = x_1 + x_2 + x_3$

Subject to

$$2x_1 + x_2 + x_3 \leq 75$$

$$x_1 + x_2 + x_3 = 65$$

$$x_1 + 2x_2 + x_3 \geq 88$$

$$x_1, x_2, x_3 \geq 0$$

The above model can now be solved. We will introduce slack variables into each inequality to convert them to linear equations. From graphical point of view the linear equations become boundary lines for the feasible solution. Formation of a bounded segment by the linear equations will result in the location of an optimal point in the segment that gives the objective function its maximum value after finite number of iterations.

Conversion of the inequalities into equalities

Let's use the slack variables S_1, A_2, S_3, and A_3. In the standard form we have

Maximize $Z = x_1 + x_2 + x_3 + 0S_1 + 0S_3 - MA_2 - MA_3$

Subject to

$$2x_1 + x_2 + x_3 + S_1 = 75$$

$$x_1 + x_2 + x_3 + A_2 = 65$$

$$x_1 + 2x_2 + x_3 - S_3 + A_3 = 88$$

$$x_1, x_2, x_3, S_1, S_3, A_2, A_3 \geq 0$$

As already discussed at chapter three we will use the simplex tableau in this approach to avoid repetitive rewriting of system of equations in the iterations.

Setting up initial simplex tableau to iterate

Since we are maximizing, the coefficient of the artificial variables A_2 and A_3 must be negative M. The Simplex tableau can be constructed as follows:

Table 4.1: Simplex tableau 1

	C_j	1	1	1	0	0	-M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	θ ratio
0	S_1	2	1	1	1	0	0	0	75	75
-M	A_2	1	1	1	0	0	1	0	65	65
-M	A_3	1	2	1	0	-1	0	1	88	44
	Z_j	-2M	-3M	-2M	0	M	- M	-M		
	C_j- Z_j	1+2M	1+3M	1+2M	0	-M	0	0		

From the first tableau above;

1. The basic variables have the value of 1. Any value above or below it must be zero. Here S_1, A_2 and A_3 are the basic variables.
2. The entering variable is the variable with the most positive value in the C_j- Z_j row. Here in this tableau, the entering variable is x_2.
3. The leaving variable is the variable with the minimum non-negative θ $r$atio. Here in this tableau the leaving variable is A_3.
4. The next Simplex tableau provides new values for the variables (see table 4.2)

Table 4.2: Simplex tableau 2

	C_j	1	1	1	0	0	-M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	θ ratio
0	S_1	3/2	0	1/2	1	1/2	0	1/2	31	20.666
-M	A_2	1/2	0	1/2	0	1/2	1	-1/2	21	42
1	x_2	1/2	1	1/2	0	-1/2	0	1/2	44	88
	Z_j	1/2-M/2	1	1/2-M/2	0	-1/2-M/2	-M	1/2+M/2		
	C_j- Z_j	1/2+M/2	0	1/2+M/2	0	1/2+M/2	0	-1/2-3M/2		

From the above tableau:

1. The entering variable is the variable with the most positive value in the C_j- Z_j row. There is a tie for the entering variables. Here in this tableau the selected entering variable is x_1.
2. The leaving variable is S_1 (variable with the minimum non-negative Θ $r$atio.
3. The next Simplex tableau provides new values for the variables (see table 4.3).

Table 4.3: Simplex tableau 3

	C_j	**1**	**1**	**1**	**0**	**0**	**-M**	**-M**		
C_B	**BV**	x_1	x_2	x_3	s_1	s_3	A_2	A_3	**RHS**	**θ ratio**
1	**x_1**	1	0	1/3	2/3	1/3	0	-1/3	20.666	62
-M	**A_2**	0	0	1/3	-1/3	1/3	1	-1/3	10.666	32
1	**x_2**	0	1	1/3	-1/3	-2/3	0	2/3	33.666	101
	Z_j	1	1	2/3-M/3	1/3+M/3	-1/3-M/3	-M	1/3+M/3		
	C_j- Z_j	0	0	1/3+M/3	-1/3-M/3	1/3+M/3	0	1/3-4M/3		

From the above tableau:

1. The entering variable is the variable with the most positive value in the C_j- Z_j row. Here in this tableau the entering variable is x_3.
2. The leaving variable is A_2 (variable with the minimum non-negative Θ $r$atio.
3. The next Simplex tableau provides new values for the variables (see table 4.4).

Table 4.4: Simplex tableau 4

	C_j	1	1	1	0	0	M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	θ ratio
1	x_1	1	0	0	1	0	-1	0	10	
1	x_3	0	0	1	-1	1	3	-1	32	
1	x_2	0	1	0	0	-1/3	-1	1	23	
	Z_j	1	1	1	0	2/3	1	0		
	C_j- Z_j	0	0	0	0	-2/3	-M-1	-M		

All values in the C_j- Z_j row are either zero or negative. Table 4.4 therefore represents the final Simplex tableau with the following optimal scheduling solution:

x_1 = 10 tons/month **x_2 = 23 tons/month** **x_3 = 32 tons/month**

Evaluation of the Objective function

Maximize $Z = x_1 + x_2 + x_3$

= 10 + 23 + 32 tons/month

= **65 tons/month**

Table 4.5: Testing the optimal solution with constraints

No.	Verification	Status
1	$2x_1 + x_2 + x_3 \leq 75$ $\Rightarrow$ $2(10) + 23 + 32 = 75$	OK
2	$x_1 + x_2 + x_3 = 65$ $\Rightarrow$ $10 + 23 + 32 = 65$	OK
3	$x_1 + 2x_2 + x_3 \geq 88$ $\Rightarrow$ $10 + 2(23) + 32 = 88$	OK

Based on the commercial statistics we can deduce the required scheduling output per product type as follows:

x_1 = total tonnage for NCY1.5 RS and NCY2.5 RS = 10 tons/month

Let total tons/month for NCY1.5 RS = a

and total tons/month for NCY2.5 RS = b

then a/b = 0.70

if a + b = 10 tons/month then 0.70b + b = 10 tons/month

implies **b = 5.88 tons/month and a = 4.12 tons/month**

x_2 = total tonnage for NCY4 RS and NCY6 RS = 23 tons/month

Let total tons/month for NCY4 RS = c

and total tons/month for NCY6 RS = d

then c/d = 0.54

if c + d = 23 tons/month then 0.54d + d = 23 tons/month

implies **d = 14.94 tons/month and c = 8.06 tons/month**

x_3 = total tonnage for NCY10 RS and NCY16 RS = 32 tons/month

Let total tons/month for NCY10 RS = e

and total tons/month for NCY16 RS = f

then e/f= 0.70

if e + f = 32 tons/month then 0.70f + f = 32 tons/month

implies **f = 18.82 tons/month and e = 13.18 tons/month**

Table 4.6: Optimal solution to original scheduling problem

No.	Product	Application	Tons/month
1	NCY1.5 RS	Light wiring	4.12
2	NCY2.5 RS	Light wiring & wiring for electrical sockets	5.88
3	NCY4 RS	Wiring for refrigerator & electric cookers power supply sockets	8.06
4	NCY6 RS	Wiring for air conditioners power supply sockets.	14.94
5	NCY10 RS	Connection between meter and main switch board	13.18
6	NCY16 RS	Service connection from the local network	18.82
		TOTAL	**65**

ANALYSIS OF RESULTS

Parameters used in the model for the case study scheduling problem formulation have been assumed basically to remain constant throughout the year (2013). The projected performance obtained from solving the mathematical model (ie the derived optimal solution) can however, be affected by changes impacted from a number of constraint factors. These operational limitations may be dynamic in dimensions over time, and can have direct determinative bearing on scheduling outcomes of the management system. These include :

1. Effect of time on production costs (eg. in relations to water, electricity and labour due to changes in tarifs).

2. The stochastic nature of the expected sales volume over the time frame in question, due to changes in price and other economic developments.
3. The fluctuating costs of raw materials on the world market and likelihood of shipment delays coupled with management's obligations to maintain constant supply of appropriate raw materials for production.
4. The effect of preventive maintenance and machine breakdown hours on production capacity.

The constraints factors above have not been considered (or quantified) in the formulations for the model construction. It is worth noting, however, that they have the potential to create vast uncertainty on the outcomes of the model. The probable impacts of these ignored constraints form the basis in the analysis of the optimum results.

In order to examine the extent of deviations of the actual scheduling output from the projected optimal performance with regard to the impact of the ignored limitations mentioned above, we shall need to alter the original data used for the modeling, in systematic order. In simple mathematical terms we need to find out, if the optimum solution to the scheduling problem (both the values of the variables and the value of the objective function) is sensitive to small changes in any of the original problem coefficients (example: coefficients of the variables in the objective function or constraints, or the right hand side constants in the constraints) due to effect of changes in the operating environment.

The above case can be appropriately handled using sensitivity analysis to determine the impact of changes on the model. The importance of applying the sensitivity analysis include the following reasons:

i. to test the reliability of the results of the model in case of significant changes due to constraint factors
ii. to show the relationship that exist between input and output variables in the model
iii. to make recommendations more credible and persuasive to decision makers.

Sensitivity analysis is therefore the study of how sensitive solutions are to parameter changes. There are several approaches in the application of the sensitivity analysis. For large models one can apply the formal method of classical sensitivity analysis in which one relies on the relationship between the initial tableau and any later tableau to update the optimum solution when changes are made to the coefficients of the original tableau. In the fore going case study, the model is relatively small and so we shall use the brute force approach. In this method we shall simply change the initial data and solve the model again to see what results can be obtained.

Now let's consider the effect of uncertain circumstances on the model which may result in different coefficient values for the constraint sets and their right hand side values in comparison to the original problem.

Suppose the expected production schedules for NCY 1.5 RS and NCY 2.5 RS are affected by sudden machine breakdown for several days beyond the promised repair duration. This circumstance can delay customer delivery schedules and reduce subsequent sales for the month. Assuming sales is reduced by 10% (depending on breakdown hours), this will have effect on the constraint described at page 37 b). Demand for the two products shall reduce to 90%. We can therefore alter the coefficient of x_1 and reduce also the expected sales tonnage by 10% to obtain 67.5 tons as the right hand side constant for same constraint set. All other constraints remain the same as in the original problem.

We shall have the following formulations: $1.8x_1 + x_2 + x_3 \leq 67.5$

The resulting inequalities can be solved with same method applied at page 41.

With the same objective function

Max. $Z = x_1 + x_2 + x_3$

subject to:

$1.8x_1 + x_2 + x_3 \leq 67.5$

$x_1 + x_2 + x_3 = 65$

$x_1 + 2x_2 + x_3 \geq 88$

$x_1, x_2, x_3 \geq 0$

In the canonical form this is presented as:

Maximize $Z = x_1 + x_2 + x_3 + 0S_1 + 0S_3 - MA_2 - MA_3$

Subject to

$$
\begin{aligned}
1.8x_1 + x_2 + x_3 + S_1 &= 67.5 \\
x_1 + x_2 + x_3 + A_2 &= 65 \\
x_1 + 2x_2 + x_3 - S_3 + A_3 &= 88 \\
x_1, x_2, x_3, S_1, S_3, A_2, A_3 &\geq 0
\end{aligned}
$$

Table 4.7: Simplex tableau 5

	C_j	1	1	1	0	0	-M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	ө ratio
0	S_1	1.8	1	1	1	0	0	0	67.5	67.5
-M	A_2	1	1	1	0	0	1	0	65	65
-M	A_3	1	2	1	0	-1	0	1	88	44
	Z_j	-2M	-3M	-2M	0	M	-M	-M		
	C_j- Z_j	1+2M	1+3M	1+2M	0	-M	0	0		

i. The leaving variable in the above table is A_3 (it has the minimum non-negative Θ ratio)
ii. X_2 is the entering variable with the most positive value in the c_j-z_j row.
iii. The next simplex tableau for this problem tends to be as follows:

Table 4.8: Simplex tableau 6

	C_j	1	1	1	0	0	-M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	ө ratio
0	S_1	1.3	0	1/2	1	1/2	0	-1/2	23.5	235/13
-M	A_2	1/2	0	1/2	0	1/2	1	-1/2	21	42
1	x_2	1/2	1	1/2	0	-1/2	0	1/2	44	88
	Z_j	1/2-M/2	1	1/2-M/2	0	-1/2-M/2	-M	1/2-M/2		
	C_j- Z_j	1/2+M/2	0	1/2+M/2	0	1/2+M/2	0	-1/2+M/2		

i. The leaving variable in the above table is S_1 (it has the minimum non-negative Θ ratio)

ii. X_1 is the entering variable with the most positive value in the c_j-z_j row.
iii. The next simplex tableau for this problem tends to be as follows:

Table 4.9: Simplex tableau 7

	C_j	1	1	1	0	0	-M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	ɵ ratio
1	x_1	1	0	5/13	10/13	5/13	0	-5/13	235/13	47
-M	A_2	0	0	4/13	-5/13	4/13	1	-4/13	311/26	38.875
1	x_2	0	1	4/13	-5/13	-9/13	0	9/13	909/26	113.625
	Z_j	1	1	(9-4M)/13	(5+5M)/13	-4(1+M)/13	–M	4(1+M)/13		
	C_j- Z_j	0	0	4(1+M)/13	-5(1+M)/13	4(1+M)/13	0	-4/13-17M/13		

i. The leaving variable in the above table is A_2 (it has the minimum non-negative Ө ratio)
ii. X_3 is selected as the entering variable.
iii. The next simplex tableau provides the optimal solution as follows:
iv.

Table 4.10: Simplex tableau 8

	C_j	1	1	1	0	0	-M	-M		
C_B	BV	x_1	x_2	x_3	s_1	s_3	A_2	A_3	RHS	ɵ ratio
1	x_1	1	0	0	5/4	0	-5/4	0	3.125	
1	x_3	0	0	1	-5/4	1	13/4	-1	38.875	
1	x_2	0	1	0	0	- 1	-1	1	23	
	Z_j	1	1	1	0	0	9/4	0		
	C_j- Z_j	0	0	0	0	0	-M-9/4	-M		

All values in the c_j-z_j row are either zero or negative. Hence the optimal feasible solutions are given in the RHS column of simplex tableau 11 as:

x_1 = 3.125 tons/month, x_2 = 23 tons/month and x_3 = 38.875 tons/month.

Application of the commercial statistics provides the following optimal schedules per product shown in table 4.11:

Let u = tons/month for NCY1.5 RS, v = tons/month for NCY 2.5 RS

w = tons/month for NCY 4 RS, r = tons/month for NCY 6 RS

s = tons/month for NCY 10 RS, t = tons/month for NCY 16 RS

If u/v = 0.7 & u+v=3.125 then u=1.287 and v=1.838

w/r=0.54 & w+r=23 then w=8.06 and r=14.94

s/t=0.7 & s+t=38.875 then s=16.008 and t=22.867

Table 4.11 Results of first analysis due to impact of changes on original model

No.	Product	Tons/month
1	NCY.1.5 RS	1.287
2	NCY 2.5 RS	1.838
3	NCY 4 RS	8.06
4	NCY 6 RS	14.94
5	NCY 10 RS	16.008
6	NCY 16 RS	22.867
	TOTAL	**65**

The objective function is evaluated as Z = 3.125 + 23 + 38.875 = 65 tons/month.

Comparing table 4.6 with table 4.11, it can be observed that sales tonnage for NCY 4 RS & NCY 6 RS remained unchanged inspite of the machine breakdown.

Let's consider a second scenario involving production interruptions due to preventive maintenance work. The impact of this change on normal schedules is likely to lead to prioritization of fast production rate products like NCY 1.5 RS and NCY 2.5 RS. The product types NCY 10 RS and NCY 16 RS take relatively longer production time and with comparatively lower demand. If there are no urgent pending orders for these products, then the production manager will certainly like to implement at a time, schedules that satisfy higher demand products. Since it may not be certain if sales of the low copper content products NCY 1.5 RS & NCY 2.5 RS will yield the required target, we shall change the original constraint $x_1 + x_2 + x_3 = 65$ to $x_1 + x_2 + x_3 \leq 65$.

The impact of the preventive maintenance interruptions will therefore yield the formulations:

Maximize $Z = x_1 + x_2 + x_3$

Subject to:

$2x_1 + x_2 + x_3 \leq 75$

$x_1 + x_2 + x_3 \leq 65$

$x_1 + 2x_2 + x_3 \geq 88$

$x_1, x_2, x_3 \geq 0$

If the model is solved in similar approach as described between pages 38 and 45, then we obtain two possible solution as follows:

In the second simplex tableau for this solution, we have a tie between x_1 and x_3 for the entering variable into the basis. If x_3 enters the basis to break the tie, then final solution tends to be : $x_1 = 0$, $x_2 = 23$ and $x_3 = 42$.

However, if x_1 enters the basis to break the tie, then an alternative final solution to the model becomes: $x_1 = 10$, $x_2 = 23$ and $x_3 = 32$

Since NCY 1.5 RS and NCY 2.5 RS are always in demand, we shall select the solution $x_1 = 10$, $x_2 = 23$ and $x_3 = 32$ with the objective function evaluated as Z=10 + 23 + 32 = 65 tons/month as maximum output available for waiting customers.

Table 4.12 gives an overview of optimal scheduling output for each product.

Table 4.12 Results of second analysis due to impact of changes on original model

No	Product	Tons/month
1	NCY 1.5 RS	4.12
2	NCY 2.5 RS	5.88
3	NCY 4 RS	8.06
4	NCY 6 RS	14.94
5	NCY 10 RS	13.18
6	NCY 16 RS	18.82
	TOTAL	**65**

It is observed here in the results of the second analysis that the impact of preventive maintenance interruption hours does not effect any changes in scheduling output. Compare table 4.6 and 4.12.

We shall finally take a look at a worst case analysis of a production scheduling with a decrease of about 20% in the expected target.

To guard against raw material shortages, management has established a critical re-order level for raw materials. It sometimes happens however, that arrival dates far exceed the expected lead times and very little can be done to influence the movements of goods during shipments on the high seas. This affects delivery schedules for waiting and impatient customers who may decide to buy from competitors. If we reduce the expected sales volume to 80%, we shall have to change the RHS value of the original constraint set $x_1 + x_2 + x_3 = 65$ to $x_1 + x_2 + x_3 = 52$. Hence the original scheduling problem may be changed into following mathematical formulations:

Maximize $Z = x_1 + x_2 + x_3$

Subject to:

$2x_1 + x_2 + x_3 \leq 75$

$x_1 + x_2 + x_3 = 52$

$x_1 + 2x_2 + x_3 \geq 88$

$x_1, x_2, x_3 \geq 0$

Solving the above model with same simplex method as applied in previous cases we obtain following optimal scheduling output:

x_1 = 16 tons/month, x_2 = 36 tons/month and x_3 = 0 tons/month

The objective function can be evaluated as

Z = 16 + 36 + 0 = 52 tons/month

The optimal schedules per product for sales are indicated in table 4.13 below:

Table 4.13 Results of third analysis due to impact of changes on original model

No	Product	Tons/month
1	NCY.1.5 RS	6.59
2	NCY 2.5 RS	9.41
3	NCY 4 RS	12.63
4	NCY 6 RS	23.37
5	NCY 10 RS	0
6	NCY 16 RS	0
	TOTAL	**52**

The worst case scheduling interruptions has finally effected changes in the sales schedules for NCY 4 RS and NCY 6 RS. Due to the low copper content of NCY 1.5 Rs and NCY 2.5 RS and also the long production time for NCY 10 RS and NCY 16 RS, it will be expedient for the commercial department to identify more sales opportunities for NCY 4 RS and NCY 6 RS to make up the sales volume. Demand for NCY 4 RS and NCY 6 RS is relatively not quite high, so it is obvious that total sales volume falls below the expected 65 tons for the month (see table 4.13)

CHAPTER 5

CONCLUSION AND RECOMMENDATION

The procedures and algorithm for solving linear programming models are efficient and provide reliable outcomes that can contribute to making important managerial decisions. The main problem that really disturbs in practice is usually associated with getting sufficient data at all and getting these accurately. Some data used to construct mathematical models are inherently uncertain and so outcomes of models which are supposed to inform about profit maximization or production costs reduction lead to wrong results. Accurate and sufficient data provide magical models that can be reliable for the investigations of probable outcomes due to operational changes.

The data used for the case study were average values obtained from the previous year transactions. The operational environment for the previous year was relatively stable. The fact that the targeted copper tonnage of 960 tons was far exceeded to over 1,000 tons attests to this.

In many instances, multi-product manufacturing companies experience customers who would like to purchase products with different features in preferred combinations due to the nature of their application at users premises. It suggests that shortage of one product affects the sales of other available ones in stock. In the case study at the cable production company, preferences made by customers on product color variant impact changes on sales performance. Thus shortage of one product color can affect sales of other types (sometimes of same product). At planning level for production scheduling, supervisor or the planning team has the duty to consider color codes of same products available on buffer stock as well as those planned for production by customer specifications. This will ensure not only balanced availability of products color variants, but also identify the set up sequence on the machines, the appropriate supervisory controls, with regard to minimization of production time and wastes – example during set ups. This constraint can effect changes in the outcome of the proposed model, if not properly managed.

MANAGEMENT OF SCHEDULING CONSTRAINTS

Linear programming models providing optimal solution to business problems are usually associated with some constraints. Business managers are therefore required to focus attention on probable uncertainties rather than normal events. It is assumed in this application that the magnitude of operational limitations would remain ideally the same without any new challenges emerging. These assumptions permit the coefficients of the linear objective function and constraints as well as the right hand side values of the constraints to be taken as parameters for normal conditions of operation. In real life situations these are not so. The quantification of constraints like frequency of machine breakdown and their repair duration, increments in tariffs for water, electricity, fuel and labour cannot be easily considered simultaneously in the construction of linear programming models.

These limitations and other critical uncertainties require robust management approaches to prevent avoidable impacts on the manufacturing system. For instance, in the proposed model for the case study, production management has the duty to identify an index (as performance indicator) to be in a position to control water and electricity consumption. The indices can be expressed in litres per kilogram of copper and kilowatt hour per kilogram of copper respectively for water and electricity. If at one schedule 0.358 litres per kilogram and 0.525 kilowatt hour per kilogram were consumed, production management may need to compare these indices with those for optimal schedule outcomes or target values, to analyse performance and cost savings with respect to possible root causes and required remedial actions. Similar considerations can be given to additional labour costs in terms of manhours, if any. Serious constraints that can lead to business interruptions need to be resolved ahead of time.

OBSERVATIONS FROM SENSITIVITY ANALYSIS & RECOMMENDATIONS.

As earlier on mentioned in this chapter under ‘ Management of scheduling constraints’, it is obvious that operational conditions for scheduling process cannot remain the same, say for a period of one continuous year. The impact of varying constraints effect changes in the outcomes. If new values for the coefficients of the objective function and constraint variables as well as new values for the right hand side constants can be rightly identified, then an acceptable precision of outcomes in relations to changes in operational constraints can be expected.

In the case study, sensitivity analysis was used to examine the extent of changes in outcomes of the resulting model when subjected to impacts of operational limitations. Operation managers have interest in what happens when deviated from optimal input parameters. Besides the knowledge of optimal values of decision variables and the value of the objective function, production managers would like to have insight into how sensitive these optimal values or scheduling outcomes are to varying input parameters.

In the case study at a cable manufacturing company, following observations were made with their relevant recommendations in three cases of analysis:

CASE1: Sudden machine breakdown affecting schedules for NCY 1.5 RS and NCY 2.5 RS delayed customer delivery schedules. Total sales for the six products was reduced by 10 %. Outcome of the model indicated that output for NCY4 RS and NCY6 RS remained unchanged. Sales for NCY 1.5 RS and NCY 2.5 RS were reduced by about 68.75% with reference to the optimal values of the original problem. Sales for NCY 10 RS and NCY 16 RS needed similarly to be raised by 21.5% for each product to make up losses from NCY 1.5 RS and NCY 2.5 RS. Here difficulties will arise if there are no high demand for NCY 10 RS & NCY 16 RS at the time (see table 4.11). In a more efficient way, the storage of permissible buffer stock level for NCY

1.5 RS and NCY 2.5 RS must be sufficient always to take care of this limitations (see table 4.6 and table 4.11).

CASE 2: The second scenario considered production interruption due to preventive maintenance. Production management may decide to postpone planned preventive maintenance schedule to the week-end. This however, cannot be repeated for all cases throughout the year. There may be few instances where machines will have to be shut down for preventive maintenance work during normal working hours. In real world scheduling problems the impact of this changes on normal schedules usually lead to the application of the priority dispatch rules on the shop floor with regard to a) urgent customer orders, b) fast production rate products to achieve targets and perhaps c) satisfying special categories of customers etc, depending on planned interruption hours. It would therefore not be certain if the required minimum of 65 tons for the month could be achieved. The constraint $x_1 + x_2 + x_3 = 65$ would therefore be changed to $x_1 + x_2 + x_3 \leq 65$. The outcome of the model indicated that interruption of production schedules due to planned preventive maintenance has no effect on scheduling output. This can be possible when production is informed well ahead of time (see table 4.12).

CASE 3: The third instance considered a worst case analysis. This is a typical but serious business interruptions which top management would certainly like to prevent. It relates raw material shortages which affects both production capacity and sales at same time. Consequences are usually frustrations to waiting customers. The case considered a 20 % decrease in total sales and therefore altered the original constraint set from $x_1 + x_2 + x_3 = 65$ to $x_1 + x_2 + x_3 = 52$ (all other formulations remained the same). The outcome of the model suggested an increase of 60% of the optimal values from the original problem in the production schedules for NCY1.5 RS and NCY 2.5 RS and similarly about 56.4% increase for NCY 4 RS and NCY 6 RS. Thus the worst case scenario effected changes this time also on output for NCY 4 RS and NCY 6 RS. The model recommends that storage of permissible buffer stock level for NCY10 RS and NCY16 RS must be enough for supply in order to address such situations (see table 4.13) However, the effect of this constraint type on NCY 10 RS and NCY 16 RS may be less, if there are enough semi-finished items which can be processed into these products on the idling machines.

ADDRESSING THE PROBLEM STATEMENT OF THE STUDY

The problem statements presented in this thesis have been sufficiently addressed, with reference to the case study recommendations and the three cases sensitivity analysis. Products availability under changing operational constraints, identification of fast selling products and benchmark to serve as reference performance indicators have been well discussed in relations to the outcome of the original problem.

EXTENT OF ACHIEVING OBJECTIVE OF THE STUDY

With reference to the objective of this study indicated at page 4 of this thesis, a mathematical model constructed from the data available provides optimal scheduling solution. This is clearly demonstrated from the results of the linear programming model application for the case study, which indicates that the 65 tons target from production of the six house wiring cables can be feasible within prevailing constraints. Efficient management of constraints and emerging challenges in business operations can set the optimal solution as minimum output. Thus the optimal values can be further improved. Outcomes of the proposed model subjected to diverse changes in operational environment were interpreted to identify the appropriate recommendations for corrective and preventive measures.

CONCLUSION

Linear programming offers a practical methodology in solving scheduling problems. The concept behind the formulation of production scheduling problem into linear programming models is quite simple and provides an analytical methodology as basis for factual decision making in the industry. The application requires business data to be interpreted in a meaningful way and to be quantified as linear relationships existing between objective function and the operational constraints. Although this requirement in the application of linear programming may seem restrictive, many real-world business problems can be formulated in this manner.

Actual scheduling output for the case study is given in table 5.1 below :

Table 5.1: Actual scheduling output of copper from the case study (Production records – 2013)

i	Product	MARCH		APRIL		MAY		JUNE	
		per item	x_i	per item	x_i	per item	x_i	per item	x_i
		In tons		In tons		In tons		In tons	
1	NCY1.5 RS	15.677	47.559	25.684	52.725	15.821	37.381	22.097	49.078
	NCY2.5 RS	31.882		27.041		21.56		26.981	
2	NCY4 RS	3.556	6.49	9.04	14.956	10.554	17.533	8.729	12.622
	NCY6 RS	2.934		5.916		6.979		3.893	
3	NCY10 RS	6.716	13.928	9.548	20.236	4.002	15.071	10.547	25.736
	NCY16 RS	7.212		10.688		11.069		15.189	
Z: Value of objective function		**67.977**		**87.917**		**69.985**		**87.436**	
Total copper per month		**89.17**		**102.17**		**87.44**		**108.51**	

From the statistics in table 5.1 actual production scheduling output over the four months period considered were always above the required optimized value. This becomes the case when potential constraints likely to interrupt business operations are efficiently managed. Although in the months of March and May the total copper budget of 92 tons was not achieved, the required tonnages for house wiring cables (values of the objective function Z in table 5.1) were obtained. It is interesting to note from the table that values for x_1 were comparatively higher and values for NCY 10 RS were always below values for NCY16 RS as expressed in the constraints for the problem formulation . In general, the scheduling output for sales is determined by a combination of factors. The impact of these factors on the realization processes may also be dynamic. The implication is that although exact values of the variables from the linear optimization model may not be same as in the actual production results on the shop floor (see table 5.1), the calculated objective function value Z = 65 tons, come closer to the real life figures. It can be therefore reliably maintained, based on the actual measurements in table 5.1 and the outcome of the proposed model in table 4.6, that acceptable forecast of expected scheduling output for house wiring cables can be made with respect to the dynamics in operational constraints, if appropriate coefficients and Right Hand Side values can be identified. Outcomes of linear programming models established from very accurate business data can really assist important managerial decisions on optimization of production scheduling output.

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