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# A Two-Phase Method for Solving Transportation Models with Prohibited Routes

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**Abstract**

The Transportation Problem (TP) is a mathematical optimization technique which regulates the flow of items along routes by adopting an optimum guiding principle to the total shipping cost. However, instances including road hazards, traffic regulations, road construction and unexpected floods sometimes arise in transportation to ban shipments via certain routes. In formulating the TPs, potential prohibited routes are assigned a large penalty cost, $M$, to prevent their presence in the model solution. The arbitrary usage of the big $M$ as a remedy for this interdiction does not go well with a good solution. In this paper, a two-phase method is proposed to solve a TP with prohibited routes. The first phase is formulated as an All-Pairs Least Cost Problem (APLCP) which assigns respectively a non-discretionary penalty cost $M_{ij}^{\star} \leq M$ to each of $n$ prohibited routes present using the Floyd's method. At phase two, the new penalty values are substituted into the original problem respectively and the resulting model is solved using the transportation algorithm. The results show that, setting this modified penalty cost ($M^{\star}$) logically presents a good solution. Therefore, the discretionary usage of the $M \leq \infty$ is not a guarantee for good model solutions. The modified cost $M^{\star} \leq M$ so attained in the sample model, is relatively less than the Big $M(\leq \infty)$ and gives a good solution which makes the method reliable.

**Key Words:** Adjacency Matrix, All-pairs Least Cost Problem, Floyd's Method, Large Penalty Cost, Modified Penalty Cost, Shortest-Route Problems.

**Mathematical Subject Classification:** 90B06.

## 1. Background

The transport infrastructure is made up of nodes (such as cities, factories, warehouses, fuel stations, airports, railway stations) and arcs (such as cables, roads, pipelines, air, rail roads) which are used for the flow of resources using vehicles, to satisfy consumer needs. Items may be produced and transported from $m$ sources to $n$ destinations, or in-between cities to meet supply and demand at a total minimum cost. The TP was introduced in the twentieth century when Hitchcock (1941) formulated a model for resource allocation. Before Hitchcock's work, Monge (1781) formalized the transport theory for the study of optimal transportation and resource allocation. The distribution of goods across networks is controlled by selecting the best set of shipping routes that minimize total Transportation Cost (TC) according to Hitchcock (1941). A transportation model is balanced when the number of units supplied equals the number of units demanded and unbalanced otherwise. Several methods including the Dantzig's North-west Corner Rule (NCR), the Least Cost Method (LCM) and the Vogel's Approximation Method (VAM) developed by Reinfeld & Vogel (1958), help to find starting solutions to the problem whereas the stepping-stone method by Charnes & Cooper (1954), and the Modified Distribution (MODI) are used to find optimal solutions. Ackora-Prah et al. (2016) applied the Euclidean model in an optimal location of a power station in Accra-Ghana. The Euclidean model minimizes transportation cost $T = \sum_{i=1}^{\zeta=m} \Psi_i \rho_i d_i$ from a target location to $m$ locations with $(x_i, y_i)$ co-ordinates using $tial-$derivatives. Potential situations including traffic regulations, strike actions and poor road conditions arise to forbid routes and this affects real life activities including transportation. In view of this a large penalty (M$\leq \infty$) is assigned to forbidden routes if they appear in transportation models to control the solution process. The introduction of this penalty in the models removes forbidden routes from the solution. Li et al. (2019) addressed an outsourced routing decision to minimize total transportation cost in a vehicle routing problem using two algorithms which were claimed to be effective following their model results. Amaliah et al. (2019) also presented an initial feasible solution to find a near optimal solution to the TP. They compared their method with the VAM, the Juman and Hoque method and the Total Differences Method 1 (TDM1) presented by Hosseini (2017) using twenty-four data examples and the new method was claimed to yield better results. The shortest-path problem (Dijkstra , 1959) is about finding a path between vertices in a graph such that the total sum of the edge weights is minimum. Prasad & Singh (2020) stood on the LCM and MODI to propose a two-step exact algorithm (claimed to be effective) for solving balanced and unbalanced TPs. However, there was no strong conclusions drawn from the study results, though it was claimed to have been supported with numerical examples of large systems. The shortest-path problem can be defined for graphs which are undirected, directed, unweighted, weighted or mixed as presented by Aini & Salehipour (2012). These paths are constructed in time proportional to their lengths according to Seidel (1995). Amongst Dijkstra's algorithm shown by Dijkstra (1959), Phil (2010) and Amoako (2019), Bellman–Ford algorithm, Johnson's algorithm as shown by Cormen et al. (2001), and others is the Floyd's method by Floyd which was deliberately designed to solve shortest-route problems according to Khamami & Saputra (2019). The Floyd's algorithm by Floyd which is also also implemented by Ramadhan et al. (2018), simultaneously calculates minimum weights of routes that connect a pair of points and all pairs of points in a network and it is widely used in determining shortest paths in a graph according to Iheonu & Inyama (2016). Amoako (2019) defined a network as a set of points and a set of arcs connecting all pairs of the points where the arcs are associated with a flow. Khamami & Saputra (2019) used a modified version of the Floyd's method to develop a Shortest Path Determination Application (SPDA) to control transport routes in a city and the application yielded good results with accuracy, following a comparative analysis with results from the traditional Floyd's method. Kamal et al. (2021) presented a Multi-Objective TP with type-2 trapezoidal fuzzy objective functions. A two-phase technique was applied in transforming the trapezoidal objective function into an equivalent form and a Fuzzy Goal Programming (FGP) technique was employed to solve the problem for optimal decisions. Ekanayake et al. (2021) developed an alternative for finding optimum solutions to TPs in which an iterative scheme was ran for balanced and unbalanced problems. A comparative analysis was conducted with thirty-five (35) numerical examples such that, the new method was claimed to be effective. According to Lee et al. (2022), transportation activities undergo deterioration due to disasters bringing about a ban on shipments along various routes in and around our cities. Saleh & Shiker (2022) provided a new modification of the VAM to discover initial basic feasible solutions. The study was supported by numerical examples to yield near optimal results. Kane et al. (2022) applied the transportation algorithm to solving Fuzzy TPs (FTPs). The fuzzy problems were transformed using a proposed alternative approach, due to fuzziness (being considered) in rim values and the shipping costs per route, before they were solved. The proposed method was claimed to be an alternative approach to solving such FTPs with less uncertainty. In contrast to many other researches that has been done on transportation solution methods, there has not been a consideration of assigning a non-discretionary cost to prohibited routes in transportation models. This study is therefore motivated to propose a remedy for the large penalty cost on prohibited routes in TPs using a two-phase technique which comprises of Floyd and TP methods. This proposal follows the argument that, the big $M(\leq \infty)$ varies potentially up to infinity as seen in many literatures including Taylor (1999). The new method produced small costs (relative to M), leading to a good solution. The results show that if a TP has a set of prohibited routes, at least one of them potentially leaves the model solution. Furthermore, for all systems (TPs) of order $1 \times n$ or order $n \times 1$ with prohibited routes, each route becomes a candidate solution; this makes the 'non-discretionary cost' approach reliable. The proposed method, results and discussions are presented in the following sections 2 to 5.

## 2. The Floyd's Algorithm

Transportation and communication organizations encounter shortest-route problems due to instances where shortest distances are required. Finding a shorter route between two endpoints in a graph involves a detour via other points in that graph. The Floyd's algorithm shown by Floyd (1962) and Hougardy (2010) solves all-pairs shortest-route problem based on inductive arguments, following an application of a dynamic programming technique to attain an adjacency matrix (of all vertices) as indicated by Cormen et al. (1990) of shortest distances in $\mathtt{O}(\mathtt{n}^3)$ computations. For $i := 1, 2, \ldots, n$ and $j := 1, 2, \ldots, n$, let $d^i(i,j)$ denote (at the $i^{th}$ stage) the potential length of shortest path between two nodes $i$ and $j$ of $n$ nodes, subject to the condition that this path uses $n-1$ internal nodes. Then $d^{i\star}(i,j)$ denotes the actual shortest distance between $i$ and $j$. $d^i(i,j)$ is computed at the $i^{th}$ stage. The Floyd's algorithm first computes $d^i(i,j)$ for all pairs $(i,j)$ of nodes and terminates after it computes $d^{i\star}(i,j)$ for all pairs of nodes $(i,j)$. Therefore, given $d^i(i,j)$, we have:

$$d^{i\star}(i,j) = \min\left\{d^i(i,j),\ \left\{\sum\left(d^i(i,k), d^i(k,j)\right)\right\};\ k^{(\neq i,j)} \in [1,n]\right\};\ i,j \in [1,n], j \neq i \tag{1}$$

In this equation (1), $k$ represents the $k^{th}$ internal node in the adjacency matrix, $i$ denotes the immediate stage (row or column) of the matrix and the $\sum\left(d^i(i,k), d^i(k,j)\right)$ denotes a detour from the immediate vertex via all internal nodes which are connected to it in the network. This equation (1) can be applied to solving all-pairs least costs problems in transportation and other related fields.

## 3. The Two-phase Method

Variables which represent prohibited routes in TPs rarely appear in optimal solutions due to a large penalty transportation cost associated with them as illustrated in Table 1.

**Table 1: A prohibited route in model solution**

| $S_i$ \ $P_j$ | $P_3$ | $P_1$ | $P_2$ | **Supply** |
|---|---|---|---|---|
| $S_1$ | M<br>150 | 11 | 10<br>150 | 300 |
| $S_2$ | 7 | 6<br>250 | 12 | 250 |
| **Demand** | 150 | 250 | 150 | |

As an alternative, these variables can be excluded from the model formulation (which shall be discussed later) according to Taylor (1999). Equation (2) shows an introduction of the penalty cost $C^* = M$ for prohibited route into the objective function of the TP as reported by Iheonu & Inyama (2016).

$$Min.\ Z = \sum_{i=1}^{m}\sum_{j=m+1}^{n} C^*_{ij} X_{ij},\ C^*_{ij} = \begin{cases} M & for\ prohibited\ routes \\ C_{ij} & otherwise \end{cases} \tag{2}$$

where $C^*$ is the shipping costs associated with the transport routes and $X_{ij}$ represents the amount of units that can be shipped from source $i$ to destination $j$. From the equation (2) it follows that: The TP contains at least one prohibited route for some $C^* = \infty(\geq M)$. The penalty cost $M$ in equation (2) approximately equals $10^{10}$ according to Floyd (1962). The two-phase method is proposed to rectify problems of prohibited routes in transportation models. The method first replaces the cost M (for prohibited routes) with a modified (non-arbitrary) penalty costs calculated from the Floyd's method (using equation (1)) before solving the model.

### 3.1. The Two-Phase Algorithm

Input: A transportation matrix
Output: An optimal solution
Phase I

**Step 1:** Develop an adjacency matrix (from an APLCP) to contain nodes (vertices) and determine by equation (1), the minimum penalties for each prohibited route, using the equation (1).

**Step 2:** Replace all penalties in the original transportation model with corresponding minimum penalty costs.

Phase II

**Step 3:** Solve the resulting model for an optimal solution, using the transportation method.

### 3.2. Numerical Illustration

In Figure1 below is a network of seven nodes which comprises $S_1, S_2, S_3$ as sources, and $P_1, P_2, P_3$, $P_4$ as destinations. Each source $S_i$ $(i = 1, 2, 3)$ is linked to all destinations $\{P_j\}_{j=1}^{4}$ via $\overline{S_iP_j}$, a set of routes. Given that there are 1000 units available for supply and that, 1000 units are demanded by consumers, the problem's objective is to decide on the amount $X_{ij}$ (units) of a single commodity to distribute across the best set of routes in order to minimize the cost of transportation.

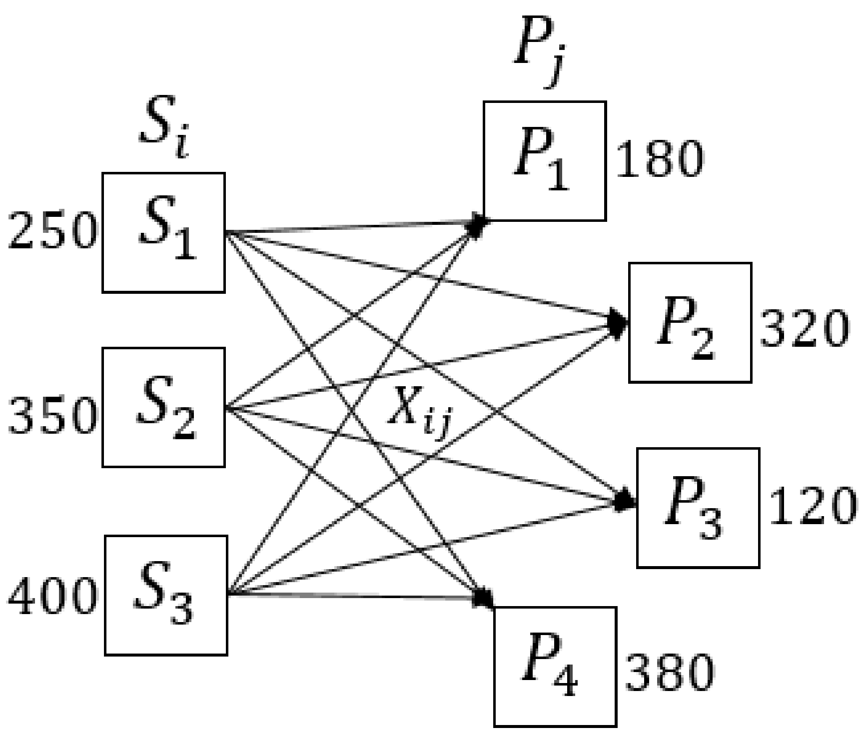


Figure 1: The Transportation Network.

Table 2 represents the problem with costs (for each route), supplies (250, 350, 400) at its sources and demands (180, 320, 120, 380) at its destinations.

**Table 2: Represents TP data with costs, supplies and demands.**

| $S_i$ \ $P_j$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | **Supply** |
|---|---|---|---|---|---|
| $S_1$ | 9 | 14 | 12 | 17 | 250 |
| $S_2$ | 11 | 10 | 6 | 10 | 350 |
| $S_3$ | 12 | 8 | 15 | 7 | 400 |
| **Demand** | 180 | 320 | 120 | 380 | |

The objective of this problem is defined as to:

$$
\begin{array}{ll}
\text{Min.Z=} & 9x_{11} + 14x_{12} + 12x_{13} + 17x_{14} + 11x_{21} + 10x_{22} + 6x_{23} + 10x_{21} + 12x_{31} + 8x_{32} + 15x_{33} + 7x_{34} \\
\text{S.t.:} & x_{11} + x_{12} + x_{13} + x_{14} = 250 \\
& x_{21} + x_{22} + x_{23} + x_{21} = 350 \\
& x_{31} + x_{32} + x_{33} + x_{34} = 400 \\
& x_{11} + x_{21} + x_{31} = 180 \\
& x_{12} + x_{22} + x_{32} = 320 \\
& x_{13} + x_{23} + x_{33} = 120 \\
& x_{14} + x_{21} + x_{34} = 380 \\
& x_{ij} \geq 0
\end{array} \tag{3}
$$

where $Z$ in the equation 3 is the objective function, the set of three lines in equation 3 denotes the supply constraints, the set of four lines in equation 3 represents the demand constraints, and the equation 3 denotes feasibility (non-negativity) constraints.

## 4. Results and Discussion

### 4.1. Optimal Solution to the Original problem

Table 3 shows the optimal solution (shipment list) to the original problem (with no prohibited routes), where $X_{14} = 180$ units (See Table 3, column 3 ) can be transported via the route $\overline{S_1 P_1}$ at a per unit cost of $9$ (See Table 3, column $4$ ) and total cost of $1,620$ (See Table 3, column $5$ ), in the same way with the other routes, yielding the optimal value as $8440$.

**Table 3: Represents Optimal solution.**

| From ( $S_i$) | To ($P_j$) | Shipment ($X_{ij}$) | Cost per unit ($C_{ij}$) | Shipment cost |
|---|---|---|---|---|
| $S_1$ | $P_1$ | 180 | 9 | $1,620$ |
| $S_1$ | $P_2$ | 70 | 14 | 980 |
| $S_2$ | $P_2$ | 230 | 10 | $2,300$ |
| $S_2$ | $P_3$ | 120 | 6 | 720 |
| $S_3$ | $P_2$ | 20 | 8 | 160 |
| $S_3$ | $P_4$ | 380 | 7 | $2,660$ |
| **Optimal value ($Z$):** | | | | **8440** |

### 4.2. Optimal Solution due to prohibited routes

#### 4.2.1. Solution due to interdiction along one route

Suppose that there is a ban on shipments along the route $\overline{S_1P_1}$ in the above problem (equation (3)). Then the shipping cost associated with this route is $M$(very large, as shown in Table 4) such that, the objective function is defined as to:

$$Min.\ Z = \boxed{M} x_{11} + 14x_{12} + 12x_{13} + 17x_{14} + 11x_{21} + 10x_{22} + 6x_{23} + 10x_{21} + 12x_{31} + 8x_{32} + 15x_{33} + 7x_{34} \quad (4)$$

subject to the model constraints in equation (3). According to phase I of the two-phase algorithm, Table 5 forms the associated adjacency matrix (for the APLCP) to this objective.

**Table 4: Represents TP data with prohibited route(s).**

| $S_i$ \ $P_j$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | **Supply** |
|---|---|---|---|---|---|
| $S_1$ | $M$ | 14 | 12 | 17 | 250 |
| $S_2$ | 11 | 10 | 6 | 10 | 350 |
| $S_3$ | 12 | 8 | 15 | 7 | 400 |
| **Demand** | 180 | 320 | 120 | 380 | |

**Table 5: Represents Adjacency Matrix associated with the model objective equation** (4)

| | $S_1$ | $S_2$ | $S_3$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | $\infty$ | $\infty$ | $\boxed{M}$ | 14 | 12 | 17 |
| $S_2$ | $\infty$ | 0 | $\infty$ | 11 | 10 | 6 | 10 |
| $S_3$ | $\infty$ | $\infty$ | 0 | 12 | 8 | 15 | 7 |
| $P_1$ | $\boxed{M}$ | 11 | 12 | 0 | $\infty$ | $\infty$ | $\infty$ |
| $P_2$ | 14 | 10 | 8 | $\infty$ | 0 | $\infty$ | $\infty$ |
| $P_3$ | 12 | 6 | 15 | $\infty$ | $\infty$ | 0 | $\infty$ |
| $P_4$ | 17 | 10 | 7 | $\infty$ | $\infty$ | $\infty$ | 0 |

Table 6 shows the transportation costs (including the modified penalty cost $M^{\star}_{11} = 29$) for each route which are calculated from equation (1).

**Table 6: Represents Shipping costs calculated from the first phase of Two-Phase Algorithm**

| | $S_1$ | $S_2$ | $S_3$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | 18 | 22 | $\boxed{29}$ | 14 | 12 | 17 |
| $S_2$ | 18 | 0 | 17 | 11 | 10 | 6 | 10 |
| $S_3$ | 22 | 17 | 0 | 12 | 8 | 15 | 7 |
| $P_1$ | $\boxed{29}$ | 11 | 12 | 0 | 20 | 17 | 19 |
| $P_2$ | 14 | 10 | 8 | 20 | 0 | 16 | 15 |
| $P_3$ | 12 | 6 | 15 | 17 | 16 | 0 | 16 |
| $P_4$ | 17 | 10 | 7 | 19 | 15 | 16 | 0 |

According to the second phase of the Two-Phase algorithm, Table 7 shows the optimal solution to the prohibited problem and the objective value is 9520. The forbidden route $\overline{S_1P_1}$ is excluded from this solution, since (from Table

7) shipments are banned from $S_1$ to $P_1$.

**Table 7: Represents Optimal solution due to interdiction on $\overline{S_1P_1}$.**

| From ( $S_i$) | To ($P_j$) | Shipment ($X_{ij}$) | Cost per unit ($C_{ij}$) | Shipment cost |
|---|---|---|---|---|
| $S_1$ | $P_2$ | 250 | 14 | 1350 |
| $S_2$ | $P_1$ | 180 | 11 | 1980 |
| $S_2$ | $P_2$ | 50 | 10 | 500 |
| $S_2$ | $P_3$ | 120 | 6 | 720 |
| $S_3$ | $P_2$ | 20 | 8 | 160 |
| $S_3$ | $P_4$ | 380 | 7 | 2660 |
| **Optimal value ($Z$):** | | | | **9520** |

The model equation (3) is solved using the same procedure with interdiction at different times, recorded in Table (8) where each route corresponds with a penalty cost ($M^\star$) and objective value, respectively below them from left to right. Each objective values were obtained using the Two-Phase algorithm.

**Table 8: Represents Optimal value to the same problem in equation (3) with interdiction at different times.**

| Time: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Prohibited route: | $\overline{S_1P_2}$ | $\overline{S_1P_3}$ | $\overline{S_1P_4}$ | $\overline{S_2P_1}$ | $\overline{S_2P_2}$ | $\overline{S_2P_3}$ | $\overline{S_2P_4}$ | $\overline{S_3P_1}$ | $\overline{S_3P_2}$ | $\overline{S_3P_3}$ | $\overline{S_3P_4}$ |
| Modified Penalty $\left(M^\star_{ij}\right)$: | 27 | 28 | 28 | 25 | 27 | 26 | 32 | 23 | 28 | 25 | 28 |
| Optimal value ($Z$) : | 8580 | 8440 | 8440 | 8440 | 8670 | 8980 | 8440 | 8440 | 8620 | 8440 | 9870 |

#### 4.2.2. Solution due to interdiction along two routes

Considering interdiction along $\overline{S_1P_2}$ and $\overline{S_2P_3}$ in model equation (3), the objective function becomes:

$$Min.\ Z = 9x_{11} + \boxed{M}x_{12} + 12x_{13} + 17x_{14} + 11x_{21} + 10x_{22} + \boxed{M}x_{23} + 10x_{21} + x_{31} + 8x_{32} + 15x_{33} + 7x_{34} \quad (5)$$

subject to the model constraints in equation (3), and Table 9 forms the adjacency matrix (representing the APLCP) for the problem.

**Table 9: Represents Adjacency matrix associated with the model equation (5)**

| | $S_1$ | $S_2$ | $S_3$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | $\infty$ | $\infty$ | 9 | $\boxed{M}$ | 12 | 17 |
| $S_2$ | $\infty$ | 0 | $\infty$ | 11 | 10 | $\boxed{M}$ | 10 |
| $S_3$ | $\infty$ | $\infty$ | 0 | 12 | 8 | 15 | 7 |
| $P_1$ | 9 | 11 | 12 | 0 | $\infty$ | $\infty$ | $\infty$ |
| $P_2$ | $\boxed{M}$ | 10 | 8 | $\infty$ | 0 | $\infty$ | $\infty$ |
| $P_3$ | 12 | $\boxed{M}$ | 15 | $\infty$ | $\infty$ | 0 | $\infty$ |
| $P_4$ | 17 | 10 | 7 | $\infty$ | $\infty$ | $\infty$ | 0 |

According to the Two-Phase algorithm, Table 10 gives the transportation costs for each route (with $M^\star_{12} = 29$ and $M^\star_{32} = 32$) , and Table 11 shows the optimal solution to the prohibited problem and the objective value is 8980.

**Table 10: Represents Shipping costs calculated from the first phase of the Two-Phase Algorithm.**

| | $S_1$ | $S_2$ | $S_3$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | 20 | 21 | 9 | $\boxed{29}$ | 12 | 17 |
| $S_2$ | 20 | 0 | 17 | 11 | 10 | $\boxed{32}$ | 10 |
| $S_3$ | 21 | 17 | 0 | 12 | 8 | 15 | 7 |
| $P_1$ | 9 | 11 | 12 | 0 | 20 | 21 | 19 |
| $P_2$ | $\boxed{29}$ | 10 | 8 | 20 | 0 | 23 | 15 |
| $P_3$ | 12 | $\boxed{32}$ | 15 | 21 | 23 | 0 | 22 |
| $P_4$ | 17 | 10 | 7 | 19 | 15 | 22 | 0 |

**Table 11: Represents Optimal solution due to interdiction on $\overline{S_1P_2}$ and $\overline{S_2P_3}$.**

| **From ( $S_i$)** | **To ($P_j$)** | **Shipment ($X_{ij}$)** | **Cost per unit ($C_{ij}$)** | **Shipment cost** |
|---|---|---|---|---|
| $S_1$ | $P_1$ | 130 | 9 | 1170 |
| $S_1$ | $P_3$ | 120 | 12 | 1440 |
| $S_2$ | $P_1$ | 50 | 11 | 550 |
| $S_2$ | $P_2$ | 300 | 10 | 3000 |
| $S_3$ | $P_2$ | 20 | 8 | 160 |
| $S_3$ | $P_4$ | 380 | 7 | 2660 |
| **Optimal value ($Z$):** | | | | **8980** |

#### 4.2.3. Solution due to interdiction along four routes

As another example, suppose an interdiction on the routes: $\overline{S_1P_1}$, $\overline{S_2P_3}$, $\overline{S_3P_2}$ and $\overline{S_3P_4}$ in the model equation (3) such that, its objective is defined as to:

$$Min.\ Z = \boxed{M}x_{11} + 14x_{12} + 12x_{13} + 17x_{14} + 11x_{21} + 10x_{22} + \boxed{M}x_{23} + 10x_{21} + 12x_{31} + \boxed{M}x_{32} + 15x_{33} + \boxed{M}x_{34} \quad (6)$$

subject to the model constraints in equation (3). Table 12 forms the adjacency matrix (representing the APLCP) for the problem, Table 13 gives the transportation costs for each route (with $M^{\star}_{11} = 35$, $M^{\star}_{23} = 36$, $M^{\star}_{32} = 33$ and $M^{\star}_{34} = 33$) , and Table 14 shows the optimal solution to the associated TP and the objective value is $14260$.

**Table 12: Represents Adjacency Matrix associated with the model objective equation** (6)**.**

| | $S_1$ | $S_2$ | $S_3$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | $\infty$ | $\infty$ | $\boxed{M}$ | 14 | 12 | 17 |
| $S_2$ | $\infty$ | 0 | $\infty$ | 11 | 10 | $\boxed{M}$ | 10 |
| $S_3$ | $\infty$ | $\infty$ | 0 | 12 | $\boxed{M}$ | 15 | $\boxed{M}$ |
| $P_1$ | $\boxed{M}$ | 11 | 12 | 0 | $\infty$ | $\infty$ | $\infty$ |
| $P_2$ | 14 | 10 | $\boxed{M}$ | $\infty$ | 0 | $\infty$ | $\infty$ |
| $P_3$ | 12 | $\boxed{M}$ | 15 | $\infty$ | $\infty$ | 0 | $\infty$ |
| $P_4$ | 17 | 10 | $\boxed{M}$ | $\infty$ | $\infty$ | $\infty$ | 0 |

**Table 13: Represents Shipping costs calculated from the first phase of the Two-Phase Algorithm.**

| | $S_1$ | $S_2$ | $S_3$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | 24 | 27 | $\boxed{35}$ | 14 | 12 | 17 |
| $S_2$ | 24 | 0 | 23 | 11 | 10 | $\boxed{36}$ | 10 |
| $S_3$ | 27 | 23 | 0 | 12 | $\boxed{33}$ | 15 | $\boxed{33}$ |
| $P_1$ | $\boxed{35}$ | 11 | 12 | 0 | 21 | 27 | 21 |
| $P_2$ | 14 | 10 | $\boxed{33}$ | 21 | 0 | 26 | 20 |
| $P_3$ | 12 | $\boxed{36}$ | 15 | 27 | 26 | 0 | 29 |
| $P_4$ | 17 | 10 | $\boxed{33}$ | 21 | 20 | 29 | 0 |

**Table 14: Represents Optimal solution due to interdiction on $\overline{S_1P_1}$, $\overline{S_2P_3}$, $\overline{S_3P_2}$ and $\overline{S_3P_4}$.**

| From ( $\mathbf{S_i}$) | To ($\mathbf{P_j}$) | Shipment ($\mathbf{X_{ij}}$) | Cost per unit ($C_{\mathbf{ij}}$) | Shipment cost |
|---|---|---|---|---|
| $S_1$ | $P_2$ | 250 | 14 | 3500 |
| $S_2$ | $P_4$ | 350 | 10 | 3500 |
| $S_3$ | $P_1$ | 180 | 12 | 2160 |
| $S_3$ | $P_2$ | 70 | 33 | 2310 |
| $S_3$ | $P_3$ | 120 | 15 | 1800 |
| $S_3$ | $P_4$ | 30 | 33 | 990 |
| **Optimal value ($Z$):** | | | | **14260** |

## 5. Conclusions

Recent studies showed that, the use of $M$ (very large) has widely been accepted as shipping cost in prohibited TPs. It has however been noticed that, this arbitrary usage is not a guarantee for good solutions. In this paper, the Two-Phase Method has been presented to discover an actual penalty value (of $M$) for this course. The method solves as a hybrid of Floyd-Warshall algorithm (for shortest-route problems), and the transportation algorithms. Implying from study results, the modified penalty cost ($M^\star$ - known) associated with routes in transportation is very useful for the management of supply-chain activities. Table 8 shows optimal value to the same problem in equation (3) with interdiction at different times. The results from this Table 8 imply that, prohibited routes are potential candidates in solutions and that, their presence in solution (as shown in Table 14) leads to some increase in the objective value. Hence, the provision of known penalties for prohibited routes in transportation is useful rather than the arbitrary use of large penalties. As another implication drawn from this study, the use of this known penalty costs ($M^\star$) minimizes the risk of attaining unbounded solutions to the problem. This study therefore provides insights about prohibited routes in transportation models and adds to the extant knowledge in the literature of network optimization, by modifying the penalty cost associated with prohibited routes in transportation models for a good solution.

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