KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY KUMASI, GHANA

COLLEGE OF SCIENCE

FACULTY OF PHYSICAL SCIENCE

DEPARTMENT OF MATHEMATICS

OPTIMAL LOCATION OF SEMI-OBNOXIOUS FACILITY

(CASE STUDY: LOCATION OF STUDENT CLINIC WITH AN AMBULANCE AT KNUST)

BY

FRANCIS KWADWO ADJEPONG,(BSc Hons)

JUNE, 2009

LOCATION OF SEMI-OBNOXIOUS FACILITY

(CASE STUDY: LOCATION OF STUDENT CLINIC WITH AN AMBULANCE AT KNUST)

BY

FRANCIS KWADWO ADJEPONG,(BSc Hons)

A Thesis submitted to the Department of Mathematics, Kwame Nkrumah University of Science and Technology

In partial fulfilment of the requirement for the degree of

MASTER OF SCIENCE

Department of Mathematics,

Faculty of Physical Sciences,

College of Science

June, 2009

DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge; it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

Francis Adjepong (PG96704060)	……………..	……………….
(Student's Name & Index No.)	Signature	Date

Certified by:

Mr. K. F. Darkwah	………………….	……………….
Supervisor	Signature	Date

Certified by:

Dr. S. K. Amponsah	……………………	…………………
Head of Department.	Signature	Date

ACKNOWLEDGEMENT

God has been good to me and my entire family. I give Him thanks and praise.

My sincerest thanks go to my supervisor, Mr. K. F. Darkwah for the patience extended to me in the course of writing my thesis.

My thanks also go to Dr S.K Amponsah, Degraft-Johnson Ankamah, Mr Andrews Oppong-Danquah of the Ministry of Health and Ebenezer Someah for the roles you played in my work.

Finally, I am grateful to all those who contributed to the completion and success of my work. May God replenish all your efforts. Amen

DEDICATION

I dedicate this work to my entire household, especially my mum, dad and brothers.

Most especially, I dedicate this to Miss. Cynthia Aniniwah Asimeng for her prayers and support.

Thank you all for your help and encouragement especially when it seemed impossible to complete this work. God bless you.

Abstract

This thesis seeks to locate students' clinic at a central position among the students' halls on the KNUST campus. The problem was considered to be a semi-obnoxious facility location problem and modelled as a Robust 1-center problem on a general network with demand weights. Robust 1-centre problem seeks to minimize the maximum weighted distance necessary for students to access the student clinic. We focus on the halls of residence and population at the hall spanning over three academic years.

We collected data on the road distance between the halls of residence together with total student population at the halls of residence on KNUST campus.

We developed a solution approach based on the decomposition of the network into basic intervals. We employ some methods of solution including the Floyd Warshal, Local Centre and Regret Analysis, to enable us find a location to place the student clinic.

In the end, our method found a location on the road link between Republic Hall and Independence Hall at a distance **105m** from Republic hall. The maximum weighted distance from the facility to the farthest node is **1,553,043** metres.

Contents

Introduction 1

1.1 Historical Background of Hospitals in Ghana 1

1.2 Government Hospital Developments & Goals of the Ministry of Health 3

1.3 Historical Background of Hospitals on KNUST Campus 3

1.4 Background Study 5

1.5 Problem Statement 6

1.6 Objective 6

1.7 Thesis Organisation 6

LITERATURE REVIEW 7

2.1 Non- Obnoxious Facilities 7

2.2 Seme- Obnoxious Facilities 8

2.3 Obnoxious Fcilities 8

2.4 LOCATION MODELS 9

2.4.1 The Location Break-Even Analysis 9

2.4.2 Centre of Gravity Method 11

2.4.3 The Factor Rationg Method 12

The factor rating method is a method used to find a suitable location for a facility considering a number of factors. 12

2.5 FACILITY NODES AND NETWORK, POPULATION CENTERS 14

2.5.1 Median Problem 14

2.5.2 Centre Problem 16

2.5.3 Vertex P-Center Problem formation 16

2.5.4 The Absolute 1-Center Problem on a Tree 18

2.6 MODELS OF 1-CENTRE PROBLEM ON A GRAPH 21

2.6.1 Absolute 1-Center Problem 21
2.6.2 Robust 1-Center Problem 22
2.6.3 Development of the Robust center problem 23
2.6.4 Finding the Absolute Centre 23
2.7 Computation using sets of weights $w^s : s = 1, 2, ..., h$ 27
2.8 REGRET ANALYSIS 28
2.9 An illustrative example 29
2.9.1 Construction of the upper envelope 34
2.9.2 CALCULATION OF REGRETS 36
Collection and Analysis of Data 47
3.1 Location of Local Centre 49
3.2 CALCULATION OF REGRETS 56
CONCLUSION 69
4.1 RECOMMENDATIONS 69
References; 70
APPENDIX A 73
APPENDIX B 76

Chapter one

Introduction

1.1 Historical Background of Hospitals in Ghana

A Hospital can be defined as a place where people who are ill are looked after by medical practitioners.

Until the era of Sir Frederick Gordon Guggisberg, the most illustrious British (Canadian born) colonial governor of the Gold Coast, nothing worth recognition in the area of public health infrastructure development for usage by indigenous Ghanaians had been done by any stakeholder or former Governor under the British rule spanning over 100 years.

According to Buah (1980), Governor Guggisberg's eight years of administration (1919-1927) were perhaps the most progressive years in the development of the Gold Coast. Besides other infrastructure such as railways and roads, he is remembered for constructing and establishing the Korle-Bu Teaching Hospital, the leading hospital in Ghana and one of the best in the west Coast of Africa. Guggisberg also extended medical service to other towns to cater for the indigenous population.

Before Governor Guggisberg, the few hospitals in the country were located in the bigger coastal towns/ cities such as Accra and Secondi- Takoradi which had substantial European populations. Secondi-Takoradi had the Harbour and other port facilities and, Accra was the seat of the British colonial administration. Indeed some of these hospitals were built exclusively for European patients and were referred to as ‘European Hospitals’. Examples were the Ridge Hospital in Accra and the Takoradi hospital.

In 1950, government hospitals throughout the country were less than 15, the rest were built and run by European missionaries who attached healing and education to conversion. Notable among these were the Catholic, Basel or Presbyterian and Methodist Missionaries. For instance the Methodist built the Wenchi Hospital in 1951. (*Acheampong, 1993)*

Attainment of independence on 6th March 1957 saw the development of infrastructures including roads and hospitals. Between 1957-1966, provision of hospitals by the government brought about the construction and initiation of some major hospitals such as the Tamale Hospital in the Northern Region of Ghana.

Health infrastructure development dwindled in the 1980s due to political and economic instability. In 1984 there was near collapse of the health care system.

Donor inflows and some improvements within the economy in the last 18 years have resulted in the state of the art renovation of some major hospitals including; Ho, Cape Coast and Sunyani Regional Hospitals and Sogakope, Ada and Begoro District Hospitals.

It should however be noted that public health infrastructure includes Hospitals, Clinics, Community Health Planning Services, Health Centers, Health Training Schools. Each of the ten regional capitals in Ghana has regional hospital, some also provide specialist services and some have health training institutions attached. The rest of the hospitals are found in the district capitals but some towns have hospitals and clinics. Some districts have more than one hospital whilst others have none.

1.2 Government Hospital Developments & Goals of the Ministry of Health

The development of hospitals in Ghana has been in line with the aims and objectives of the Ministry of Health and the Ghana Health Service. The five main strategic pillars as enshrined in the second health sector five years programme of work: 2002-2006 is as follow;

●To improve quality of health delivery.

●To increase geographical access to health

●To improve the efficiency of health services.

●To foster partnership in improving health.

●To improve financing of the health sector.

The development of hospitals is crucial to the attainment of the goals of the government. To attain quality health delivery, there should be provision of well designed infrastructures for both patients and staff. (*Oppong-Danquah, 2002)*

Similarly, geographical access will require proximity of location to the populace. As at 16th December, 2003 the total number of hospitals run by both the government and private sector was 177.

1.3 Historical Background of Hospitals on KNUST Campus

The Kwame Nkrumah University of Science and Technology (KNUST) Hospital is a full -fledge 100-bed hospital; second in status only to the Komfo Anokye Teaching Hospital (KATH) in the

Kumasi metropolis. It caters for about 150,000 people made up of staff, students and residents of the surrounding communities.

It is the medical arm of KNUST. It is located in the northwest part of the University campus and stretches along the Kumasi-Accra express wav. It was originally started in 1952 for the College of Technology as a dressing station.

In 1972, the female, children and male wards were constructed to enable the hospital receive more in-patients. The out-patient department and the theatre were added in 1973. The maternity ward was initially an isolated ward which was later converted for maternal purposes. In 1997, the hospital acquired ultramodern X-Ray equipment.

According to the quarterly newsletter of the University Hospital, ***(J Ebu-Sakyi, 2007)*** the KNUST Hospital was primarily set up to cater for the health needs of staff, their dependents and students of the University. However, it has now extended its services to the general public and provides health services to about 30 surrounding communities with a rapidly increasing population. Thus the KNUST Hospital is ranked as a District Hospital.

The University Hospital offers services in general care as well as specialist services. The vision of the hospital is to become a leading University Hospital with wider scope, general and specialist services comparable to renowned medical centers in Ghana and to make the KNUST Health Services a centre of excellence for quality health care, teaching and research.

The KNUST Hospital has a dental clinic which is well equipped with modern equipment. It was opened in June, 2005 with the objective of providing oral health care for students, staff, their dependants and the general public.

Currently the dental clinic is manned by a senior dental surgeon and three assistants. The ranges of services provided include the following;

• Filling (cosmetics, amalgam, indirect)

•Extractions

•Scaling, polishing.

In 2006 alone, about three thousand people were treated at the clinic.

The KNUST Hospital started the operation of the NHIS for staff and dependants on 1st of March, 2007. Like every institution, the KNUST Hospital faces a number of challenges including:

•Increasing student population

•Inadequate medical personnel e.g. Doctors and Nurses.

• Irregular University subvention.

• Inadequate computers to support network.

• Inadequate physical infrastructure

1.4 Background Study

In other to ease the pressure on the facilities available at the KNUST Hospital, which was serving so many people at a time, a Students' clinic was established on the 2nd of April, 2007 to enhance the health care delivery of students of the University. The clinic is located opposite the Ceramics Department, College of Arts and Social Sciences. The facilities at the clinic comprise of medical records unit, a Dispensary and a mini-laboratory, among others. The Clinic has a standby generator to generate power in case of any power outage. The student clinic project was commissioned by the vice chancellor of KNUST, *Prof. K. K Adarkwa.* ***(****J Ebu-Sakyi, 2007****)***

1.5 Problem Statement

Student clinic at consulting rooms 1 and 4 was moved from the main Hospital due to congestion at the hospital and resulting tension created among students.

The university authorities did not take the student demography into consideration while locating the student clinic

1.6 Objective

1. To locate a central site that will satisfy the location of the halls of residence and the student population.

2. To make recommendations to the university authorities.

1.7 Thesis Organisation

In the first chapter we look at a brief history of Hospitals in Ghana and KNUST.

In the second chapter, we have literature review.

In chapter three, we shall consider the data, its analysis and discussions.

In chapter 4, we have conclusion and recommendation.

CHAPTER 2

LITERATURE REVIEW

INTRODUCTION

In management, decision makers have to decide on the location of facilities for private and public service while emphasizing on accessibility of the people to these facilities. Examples of facilities are Schools, Hospitals, and Ambulance Services. Decision on location is based on a number of factors: it includes Physical, Economic, Social, Environmental or Political factors.

In the case of most medical emergencies, the risk of loss of life increases with respect to time or distance. Location problem is concerned with the location of one or more facilities in some space, so as to optimize some specified criteria. Often these criteria are linked with costs of providing optimal access for the customers of the facility in question. This does not necessarily follow however when facilities produce some undesirable or obnoxious effect. Here the risk to the local population far overweighs any benefit of close placement of the facility

In this modern society, the number of facilities available to the population often defines the quality of life. Dry-cleaner, garages, fire stations, football stadia, can be considered as physical entities that provide service. These facilities can be classified into three categories: desirable (non-obnoxious), semi-obnoxious and obnoxious.

2.1 Non- Obnoxious Facilities

Most services are provided by desirable or non-obnoxious facilities. There are facilities that bring comfort to customers and are pleasant in the neighbourhood.They may include supermarkets, warehouses, shops, garages, banks etc. As the customer needs access to the facility providing service, it is beneficial if these facilities are sited close to the customers who need their services

2.2 Semi- Obnoxious Facilities

Sometimes a facility that requires a high degree of accessibility provides a negative or undesirable effect. For example, a football stadium provides entertainment and so requires a large amount of access to enable supporters to attain a game. On the other hand, on a match day, local non-football fans will have to be content with the noise and traffic generated. The generation of noise and traffic will be unpleasant for locals who are not attending the match and who will therefore describe the facility as undesirable. The combination of the two makes this facility semi-obnoxious.

Another example is a hospital with an ambulance. Here access is needed for the treatment of the local population especially on emergency days. On the other hand the siren of the ambulance may be too noisy to others who might not need its service at the moment in time.

2.3 Obnoxious Fcilities

An obnoxious facility is one which is useful but has undesirable effect on the inhabitants and users in an area. Examples include equipment which emit pollutants such as noise and radiation or warehouses that contain flammable materials. Other obnoxious facilities include machines that are potential sources of hazards to nearby machines and workers. Further obnoxious facilities are the nuclear power stations, military installations. Although necessary for society, these facilities are undesirable and often dangerous to the surrounding inhabitants.

2.4 LOCATION MODELS

We discuss two approaches to siting a single facility on a road network.

1. Methods that do not make use of the road links;

a) Location Break Even Analysis

b) Centre of gravity

c) Factor rating method

2. Methods that make use of road links and which includes Median problems and Centre Problems.

2.4.1 The Location Break-Even Analysis

The location break-even analysis is the use of cost-volume analysis to make an economic comparison of location alternative. By identifying fixed and variable cost and graphing them for each location, we can determine which location provides the lowest cost. Location break-even analysis can be done mathematically or graphically. The graphical approach has the advantage of providing the range of volume over which each location is preferable.

The location break-even analysis method employs three steps, these are:

• Determine the fixed and variable cost for doing business at each location.

•Plot the cost for each location, with cost on the vertical axis of the graph and volume on the horizontal axis.

•Select the location that has the lowest total cost for the expected volume of business.

The location break-even analysis is determined by the equation;

$Y = ax + b$, where;

a = Variable cost

b = fixed cost

X= Volume of business, Y= Cost of business

Table (1.0) below illustrate an example where the fixed and variable cost for three potential manufacturing plant sites for a rattan chair waver.

Table (1.0): Fixed and variable cost for a manufacturing plant site

Site	Fixed cost(b)	Variable cost(a)
1	500	10
2	1,000	6
3	1,500	4

We relate the given table as;

$$Y_1 = 10x + 500$$

$$Y_2 = 6x + 1000$$

$$Y_3 = 4x + 1500$$

For a volume of $x = 125$, site 1 gives minimum cost of $y = 1,750$

2.4.2 Centre of Gravity Method

The centre of gravity method is a mathematical technique used for finding the location of a distribution center that will minimize distribution cost. For instance in the location of a market, the method takes into account the volume of goods shipped to those markets and shipping cost in finding the best location for the distribution centre.

The first step in the centre of gravity method is to place the locations on a coordinate system. The coordinates of each location must be carefully noted. The origin of the coordinate system is arbitrary, just as long as the relative distances are correctly represented. This can be done easily by placing a grid over an ordinary map of the location in question. The centre of gravity is determined by equations (1) and (2) below;

$$C_x = \frac{\sum d_{ix} W_i}{\sum W_i} \quad \text{.........................(1)}$$

$$C_y = \frac{\sum d_{iy} W_i}{\sum W_i} \quad \text{...................(2)}$$

Where,

C_x = X -coordinate of the centre of gravity

C_y = Y -coordinate of the centre of gravity

d_{ix} = X -coordinate of location i

d_{iy} = Y -coordinate of location i

W_i = volume of goods to or from location i

The centre of gravity is then determined by equation (1) and (2) above. Once x and y-coordinates have been obtained, we place that new location on the previously described map. If that particular location does not fall directly on a city, simply locate nearest city and place the new distribution there. In the case where there is more than one city that can be used as possible location, the factor rating method can be used to select one. This method could be implemented when locating a Library Complex on a network of towns/ cities.

2.4.3 The Factor Rationg Method

The factor rating method is a method used to find a suitable location for a facility considering a number of factors.

The factors include: labour cost (wages, unionization, and productivity), labour availability, proximity to raw materials and supplier, proximity to markets, state and local government fiscal policies, environmental regulations, utilities, site cost, transportation, and quality of life issues within the community, foreign exchange and quality of government. When using the factor rating method, the following six steps must be followed strictly.

These are:

● Develop a list of relevant factors.

● Assign a weight to each factor to reflect its relative importance in management’s objective.

● Develop a scale for each factor (for example, 1 to 10 or 1 to 100)

● Have management or related people score each relevant factor, using the scale developed above.

● multiply the score by the weight assigned to each factor and total the score for each location.

● Make a recommendation based on the maximum point score; considering the result of quantitative approaches as well.

Table (1.1) below gives an example of the map coordinates and shipping loads for a set of cities that we wish to connect trough a central 'hub'.

Table (1.1): Map coordinates and shipping loads for a set of cities

Factor No	Factor	Rating weight	Ratio of Rate	Location A	Location B	Location C
1	Proximity to port facilities	5	0.25	25	20	20
2	Power Source available and cost	3	0.15	12	10.5	15
3	Work force attitude and cost	4	0.2	6	12	14
4	Distance from Tema	2	0.1	1	8	6
5	Community Desirability	2	0.1	9	6	8
6	Equipment Suppliers in area	3	0.15	7.5	9	13.5
7	Economic activity	1	0.05	4.5	3	3
				65	**68.5**	**79.5**

Clearly from their respective aggregate scores, location C or site C would be recommended since it has the highest aggregate.

2.5 FACILITY NODES AND NETWORK, POPULATION CENTERS

Location simply refers to the strategy of putting a facility in place where it can be identified as serving inhabitants staying at population centres around the facility. The points of placement of the facilities are called facility nodes and the population centres are called demand nodes. These nodes are linked by paths or streets which are called edges. A node may simultaneously serve as facility node and demand node.

A graph is defined as G (V, E) consisting of a finite set of vertices (V) and a finite set of edges (E) such that V x V→ E. Example if $V_i, V_j \in$ V then $(V_i, V_j) \in$ E if there is an edge between V_i and V_j which implies there exist an edge distance $e(V_i, V_j) \neq 0$.

A network is a physical implementation of a graph. Example:

- A network of roads: The vertices are towns and the edges are road links.
- An electrical network: The vertices are junctions of resistors, inductors and capacitors and edges are wire links to junctions.

2.5.1 Median Problem

The median problem is to find the location of p facilities on a network so that the total cost is minimized. The cost of serving demands at node i is given by the product of the demand at node i and the distance ((d_{ij}) between demand node i and the nearest jth facility to node i. This problem may be formulated using the following notation:

Inputs

h_i = demand at node i

d_{ij} = distance between demand node i and candidate site j

P = number of facilities to locate

Decision variables

$$X_i = \begin{cases} 1, \text{ if we locate at candidate site } j \\ 0, \text{ if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1, \text{ if demands at node } i \text{ are served by a facility at node } j \\ 0, \text{ if not} \end{cases}$$

With this notation, the median problem may be formulated as follows:

Minimize $$\sum_i \sum_j h_i d_{ij} Y_{ij} \quad \ldots (1)$$

Subject to $$\sum Y_{ij} = 1 \quad \forall\, i \quad \ldots (2)$$

$$\sum_j X_j = P \quad \ldots (3)$$

$$Y_{ij} - X_i \leq 0 \quad \forall\, i, j \quad \ldots (4)$$

$$X_j = 0,1 \quad \forall\, j \quad \ldots (5)$$

$$Y_{ij} = 0,1 \quad \forall\, i, j \quad \ldots (6)$$

The objective function (1) minimizes the total demand-weighted distance between each demand node and the nearest facility. Constraint (2) requires each demand node i to be assigned to exactly one facility j. Constraint (3) states that exactly P facilities are to be located. Constraints (4) link the location variables, X_j and the allocation variables Y_{ij}. They state that demands at node i can only be assigned to a facility at location $j (Y_{ij} = 1)$ if a facility is located at node i $(X_j = 1)$. Constraints (5) and (6) are the standard integrality conditions.

The median formulation given above assumes that facilities are located on the nodes of the network. (Hamiki, 1995).

2.5.2 Centre Problem

The center problem is defined as the location of a number of facilities such that all the nodes are covered.

The center problem requires the model to minimize the coverage distance such that each demand node is covered by one of the facilities to be sited within the endogenously determined coverage distance. The center problem is a minimax problem.

The 1-center problem is a classical optimization problem that looks at the location of a single facility such that all the demand nodes are covered. Under the 1-center problem, we have the vertex center problem, which seeks to locate the facilities on the nodes of a network. There is also the absolute center problem that seeks to locate facilities at anywhere on the network.

2.5.3 Vertex P-Center Problem formation

Let a_{ij} = distance from demand node i to candidate facility site j

h_i = demand at node i

P = number of facilities to locate

Decision variables,

$$x_j = \begin{cases} 1, \text{ if we locate at candidate site } j \\ 0, \text{ if not} \end{cases}$$

Y_{ij} = fraction of demand at node i that is served by a facility at node j

W = maximum distance between a demand node and the nearest facility.

The problem is formulated as follows,

$$\text{Minimize } W \qquad 2.13a$$

Subject to

$$\sum_j Y_{ij} = 1 \quad \forall\ i \qquad 2.13b$$

$$\sum_j X_j = P \qquad 2.13c$$

$$Y_{ij} \le X_j \quad \forall\ i, j \qquad 2.13d$$

$$W \ge \sum_j a_{ij} Y_{ij} \quad \forall\ i \qquad 2.13e$$

$$x_j = 0,1 \quad \forall\ j \qquad 2.13\text{f}$$

$$Y_{ij} \ge 0 \qquad 2.13h$$

In some cases, the demand-weighted distance is considered and constraint $2.13\,e$ become

$W \geq h_i \sum_j a_{ij} Y_{ij} \quad \forall\ i \quad 2.13\mathrm{e}^{\mathrm{i}}$

2.5.4 The Absolute 1-Center Problem on a Tree

A tree is a network which has no loop in the connected nodes.

We have the absolute 1 center problem on a tree in which all of the demands are equal; this is called the un-weighted tree. We also have the weighted tree in which the weights associated with each of the nodes are not equal. Consider the weighted tree in figure 1.0 below;

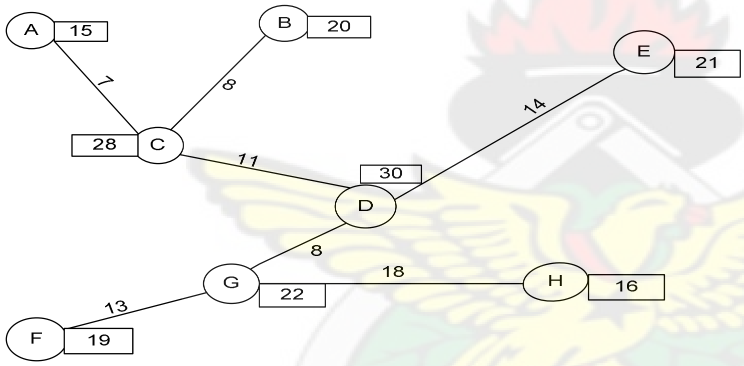

Figure 1.0. Example of tree network

The solution is computed with the steps below;

Let B_{ij} be the demand weighted distances between node i and j. This is computed as;

$$\beta_{ij} = \frac{h_i h_j d(i,j)}{h_i + h_j}$$

Where i and j are nodes. $d(i,j)$ is the distance between node i, j and h_i and h_j are respectively demand weights.

Step 1: Compute one row of β_{ij} elements

Step 2: Find the maximum element in the row that was just computed

Step 3: Compute the β_{ij} in the column in which the maximum β_{ij} element occurred in step 2.

Step 4: Find the maximum element in the column that was first computed

Step 5: Compute the elements β_{ij} in the row in which the maximum β_{ij} element occurred in Step 4.

Let F be candidate node and T be demand node. We find $B_{FT} = \max_{ij}(B_{ij})$. We further locate a point $[h_F/(h_F+h_T), d(F,T)]$ from node F on the unique path from F to T or equivalently, locate a point $[h_T/(h_F+h_T), d(F,T)]$ from node T on the unique path from T to F. When all β_{ij} are calculated according to figure 1.0we get the results as shown in table 1.2 below;

Table 1.2; B_{ij} Values

	A	B	C	D	E	F	G	H
A	0.00	128.57	68.37	180	280	326.91	231.89	340.65
B	128.57	0.00	93.33	22.8	338.05	389.74	282.86	400.00
C	68.37	93.33	0.00	159.31	300	362.21	234.08	376.73
D	180	228	159.31	0.00	175.94	244.29	101.54	271.3
E	280	338.05	300	175.94	0.00	349.13	236.37	508.54
F	326.91	389.74	362.21	244.29	349.13	0.00	132.54	269.26
G	231.89	282.86	234.08	101.54	236.37	132.54	0.00	166.74
H	340.65	400.00	376.73	271.3	508.54	269.26	166.74	0.00

We find $B_{FT} = \max_{ij}(B_{ij})$ and obtain

$B_{AE} = B_{FT} = B_{HE} = 508.54$

Point on $B_{HE} = \dfrac{H_H d(H,E)}{h_H + h_E}$

Point on $B_{HE} = \dfrac{16}{16+21}(40)$

$= 17.30$

The facility will be located 17.30 units from node H to node E.

2.6 MODELS OF 1-CENTRE PROBLEM ON A GRAPH

In this section, we present an optimal or near optimal location for an emergency semi-obnoxious facility such as a Clinic with an ambulance..

This chapter focuses on the quantitative analysis of discrete optimization problems on a network with demand nodes. The objective is to minimize the maximum cost of serving one of several clients in the centre problem modelled as the Minimax regret 1-cente problem on a network with discrete set of demands on each node. The cost is equivalent to the demand weighted distances between points on the network

2.6.1 Absolute 1-Center Problem

The absolute 1-center problem is to locate a facility on a network so as to minimize the maximum of the weighted distances between a facility node and the demand node. This model is suitable for the location of a hospital on a network whose nodes represent population centers of cities or the location of an ambulance station on a campus. Distance measure uses the shortest path between two nodes in the network and the weights of the nodes represent numbers of the population residing at the nodes.

The absolute 1-center problem was first defined and solved by Hakimi in 1964*(Hakimi, 1964).* Hakimi *et al.* implemented Hakimi's method for the weighted and the un-weighted case *(Schmeichel and Hakimi, 1978)*. Further refinements of the procedure were obtained by Kariv hxdand Hakimi, for the weighted and the un-weighted cases *(kariv and Hakimi, 1979).*

2.6.2 Robust 1-Center Problem

In planning, model parameters are usually uncertain and several values or scenarios have to be considered since they are based on estimates. In such a context, uncertain data render inappropriate the search for optimal solutions for absolute 1-centre problems and require the use of robust analysis. Unlike the absolute center problem which determines the best solution for one instance of values (or scenario), robust approaches try to find a solution or a set of solutions that is acceptable for a set of scenarios. In combinatorial optimization and particularly in location problems, the most used robustness criterion relies on *regret (Yu G and Kouvelis, 1997)*. A robust solution is one that minimizes the maximal regret among all scenarios. We recall that the *regret* is the difference between the resulting output under a given scenario and the best possible output under the same scenario.

The *minimax regret 1-center* problem was considered by many authors in the context of estimates of data where uncertain weights and/or uncertain distances are represented by estimates. In the case of estimated weights, Averbakh and Berman,(1997) developed an algorithm for the problem on a general network.

To model future demands at different nodes, a decision-maker will represent possible trends of the demographic evolution of the different cities through discrete scenarios instead of estimates. We consider the minimax regret 1-center problem on a network under uncertain demands. We assume that the set demands (or weights) at each node are modelled by a finite set S of possible scenarios, where q is the number of scenarios at the node.

2.6.3 Development of the Robust center problem

Let $G = (V, E)$ be a graph composed of a set $V = \{v_i,\ i = 1 \dots n\}$ of n nodes (or vertices) and a set E of m edges. We denote by $d\ (a,\ b)$ the minimum distance between two points a and b *of* G. A *point* of the graph corresponds either to a node or to any point along an edge. The length of each edge $e \in E$ is denoted by $c(p,q)$.

The matrix of shortest distances between nodes of G is calculated from the matrix of edge distances.

We assume that demands occur only at the nodes of the network and that they can be characterized by a weight vector $W = (w_1, w_2 \dots w_n)$ where w_i is the weight associated with node v_i for $i = 1, \dots, n$.

For a given point $x \in e$ on the edge, $e(p,q)$ the maximum of the weighted distance between x and all the nodes of G, is denoted by $D(x)$. It is also called the cost of x.

The cost $D(x)$ of x is then given by;

$$D(x) = \max_{1 \le i \le n} w_i d(x, v_i)$$

The local center on the edge $e(p,q)$ is given by the minimum of the maximum weighted distance for all points x on the edge $e(p,q)$. This gives;

$$\min_{x \in e} D(x)$$

2.6.4 Finding the Absolute Centre

A vertex is a designated node on a network or graph and an edge is a direct distance or link between two vertices. For two vertices p and q define $c(p,q)$, to be the edge cost or edge distance.

Consider the edge (p,q) with point x on it and a weight w_i on vertex V_i as shown in figure 1.1 below. Assuming we want to move from x to V_i, where V_i is any node on the network, we find the minimum cost of moving to V_i along the edge.

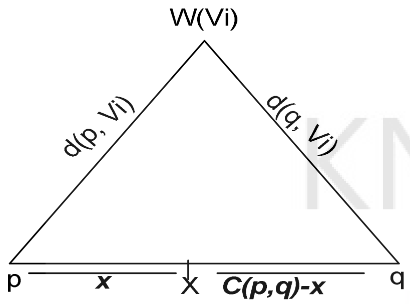

Figure 1.1: Edge (p,q) with point x on it, weight w_i and vertex V_i

Take p as the origin then the length (p,x) has its cost being x and (x,q) has a cost of $c(p,q)-x$. The movement from x to V_i can be done in two directions i.e. through the origin p or the other end vertex q. This gives rise to the cost equations:

$x+d(p,v_i)$ and $c(p,q)-x+d(q,v_i)$.

If w_i is the weight at the vertex V_i then the demand weighted cost or weighted distances are;

$y_1 = w_i(x+d(p,v_i))$ and $y_2 = w_1(c(p,q)-x+d(q,v_i))$ where y_1 is the distance from x to V_i through p and y_2 is the distance from x to V_i through q. As x moves along the edge (p,q) from p to q there will be a point when the two weighted distances or cost would be equal. At this point $y_1 = y_2$ and the point of intersection or the kink point could be found. Solving for the path of equal weighted cost we have;

$w(x+d(p,v_i)) = w(c(p,q) - x + d(q,v_i))$. Hence;

$$x_{kink} = \frac{d(q,v_i) - d(p,v_i) + c(p,q)}{2}$$

We note that y_1 does not hold for values of x beyond x_{kink} and y_2 does not hold for values of x below x_{kink}. The equations y_1 and y_2 are therefore used to draw graph for the edge (p,q) from which a local center can be determined.

We observe that, on a general network, the distance $d(x,v_i)$ between a point x varying on an edge $e=(p,q)$ and a given node V_i has three possible plots as shown in Figure 1.2.

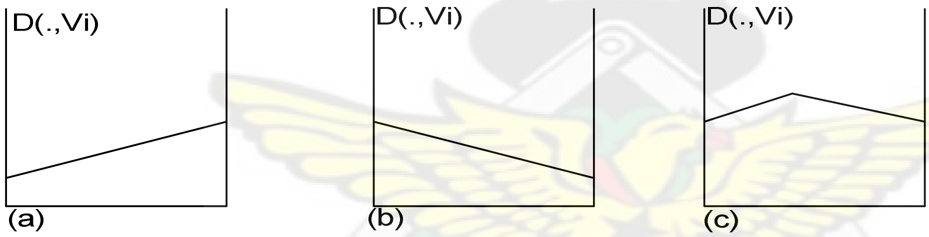

Figure 1.2: Plots of *d*(x, v_i) on a given edge (p,q)

In figure 1*a* and 1*b* x_{kink} coincide with an end vertex of the edge (p,q) or V_i is one of the vertices of (p,q). In figure 1*c* we have 0< $x_{kink} < c(p,q)$. Therefore, the function y is piecewise linear and continuous on each edge $c(p,q)$.

Local Centre

Proposition 1: For a set of all points x on a fixed edge (p,q), the maximum distance function $m(x)$ for the absolute centre problemis piecewise linear and its slope is always +*w* or -*w*.

To obtain the local centre on an edge (p,q), we use the equations;

$y_1 = w_i(\ x\ + d(p, v_i))$

$y_2 = w_i(c(p,q)\ - x\ + d(q, v_i))$

We solve for the kink points with respect to all vertices $V_i, i = 1,2,3,...n$ of G. For each vertex V_i, we contruct the piecewise linear graphs. From all the graphs, we contruct the upper envelop of the set of graphs. This is formulated as;

$$D\ (x) = \max_{1 \le i \le n} W_i\ d(x, v_i) \quad x \in (p,q)$$

The local centre x_i is the point $x \in (p,q)$ that corresponds to the minimum of the upper envelopes. This gives;

$$D^s(x) = \min_x \max_{1 \le i \le n} W_i^s\ d\ (x\ , \mathrm{v_i})\ \ x \in (p,q)$$

Proposition 2: For an edge (p,q) the local center satisfies

$M(X_1) \ge w\ \dfrac{\sum m(p) + m(q) - c(p,q)}{2}$, where $c(p,q)$ denotes the cost of edge (p,q) and w, a given scenario.

Absolute Centre

The local centre of an edge (p,q) is defined as a point x_i on (p,q) such that for every point y on $(p,\ q)$, (y may be on an edge of G), $m(x_i) \le m(y)$

Given that all the local centres for all the edges of the network have been determined, we pick the local centre that gives the minimum weighted cost and compare to the weighted cost of the node centre. We select the point or vertex that gives the minimum weighted cost.

2.7 Computation using sets of weights $w^s : s = 1, 2, ..., h$

For each of the set of weights $w^s = (w_1^s, w_2^s, ... w_n^s)$, we calculate the local centres to get h local centres for edge (p,q). We thus take each edge (p,q) of the network and compute the linear equations for each element of the set *S* or scenario of *S*. We then find the graph of its upper envelope with respect to the vertices and the local centre.

Define the values of x at which discontinuity occur in each upper envelope to be breakpoint ($x = t$), listing all the breakpoints of the h upper envelopes in an increasing order $0 = t_0 \prec t_1 \prec t_2 \prec t_n = c(p,q)$. Define $t_{ij} = [t_j - t_i]$ to be the t_{ij} basic interval satisfying $|t_{ij}| \succ 0$.

1). Let $a = t_i, b = t_j$ be two breakpoint on S_1 and $t \in [a,b]$, then for the upper envelope of the weight set w^{s1} we have;

$$D_e^{s1}(t) = D_e^{s1}(a) + \frac{D_e^{s1}(b) - D_e^{s1}(a)}{b - a}(t - a) \forall t \in [a,b] ....(1)$$

Equation (1) is the weighted cost at the point t.

$D_e^{s1}(a)$ is the weighted cost at a

$D_e^{s1}(b)$ is the weighted cost at b

$D_e^{s1}(t)$ is weighted cost at *t* and is a linear function of *t*.

2). Suppose $t = b$ is a breakpoint of the upper envelope of the weight set S_2 instead of S_1. We let $t = c$ be the next breakpoint of S_1 such that $a \prec b \prec c$ then;

$$D_e^{s1}(b) = D_e^{s1}(a) + \frac{D_e^{s1}(c) - D_e^{s1}(a)}{c-a}(b-a) \text{..........................}(2)$$

Using the above value for $D_e^{s1}(b)$ we have;

$$D_e^{s1}(t) = D_e^{s1}(a) + \frac{D_e^{s1}(b) - D_e^{s1}(a)}{b-a}(t-a) \forall t \in [a,b] \text{................}(1a)$$

For any current subinterval, the value of $D_e^{s1}(a)$ is taken to be the value of $D_e^{s1}(b)$ of the previous subinterval. For the first subinterval where $t = 0$, $D_e^{s1}(0)$ is obtained from the upper envelope.

We compute $D_e^{s1}(t)$ for all the basic intervals within $t \in [0, c(p,q)]$ and the regrets $R_e^{s1}(t)$

2.8 REGRET ANALYSIS

The regret of solution *x* also called *opportunity loss* or *absolute deviation* is the difference between the cost of *x* under scenario *s* and the cost of the best solution under the same scenario. It is given by:

$$R^s(t) = D^s(t) - D^s(x^{*s}) \text{...............}(3)$$

where x^{*s} is the local centre under scenario S .

The maximum of all the regrets for the entire basic interval under S_1 *is;*

$$R_e^{s1}(t) = \max R_e^{s1}(t) \quad 0 \le t \le c(p,q)$$

The minimum of all the maximum values $R_e^{s1}, R_e^{s2}, \ldots R_e^{sp}$ occur at the point t_R called the local robust centre. This is formulated as;

$$\min_{[a,b]\in e} \max_{0 \prec t \prec c(p,q)} R_e^s(t)$$

We describe below the different steps of our approach to solve the minmax regret 1- center problem under scenario-based uncertainty:

• We first use the Kariv and Hakimi's algorithm to solve q classical 1-center problems in order to compute the absolute centers x^{*s} for all $s \in S$.

• Then, we decompose each edge of the network into intervals called basic intervals.

• For each basic interval, we determine a local solution of the problem, using a procedure developed by Kouvelis and Yu.

• Finally, the minmax regret 1-center is determined among all the local solutions given by the previous step.

The minimax regret 1-centre is given by;

$$\min_{\substack{[a,b]\in e \\ e\in G}} \max_{0 \prec t \prec c(p,q)} R_e^s(t)$$

2.9 An illustrative example

Consider the graph G of figure 1.3 where values on edges represent lengths. Table 1.3 shows the road distance $c(p,q)=c(i,j)$ between nodes with node demands modelled by three scenarios S1, S2, S3 as shown in table 1.4

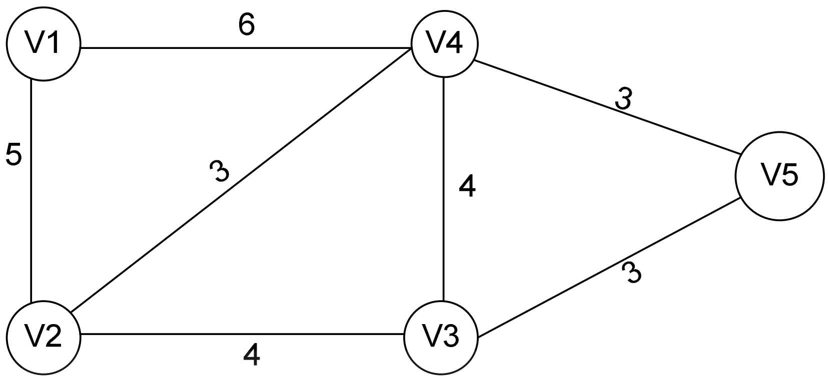

Figure 1.3: Example of a network

: ***Table 1.3: Table of edge matrix of direct road distances between nodes*** $c(i, j)$

	V_1	V_2	V_3	V_4	V_5
V_1	-	5	∞	6	∞
V_2	5	-	4	∞	∞
V_3	∞	4	-	4	3
V_4	6	∞	4	-	3
V_5	∞	∞	3	3	-

Table 1.4: Weights under the three scenarios S_1, S_2, S_3

Weight	W_1^s	W_2^s	W_3^s	W_4^s	W_5^s
W_i^{s1}	10	20	10	15	10
W_i^{s2}	10	15	20	10	10
W_i^{s3}	10	10	15	20	10

From the edge matrix of table 1.3, Floyd Warshall algorithm is used to compute shortest paths between all pairs of point. The result is shown in table 1.5 below;

Table1.5: All pairs shortest path distance Matrix

	V_1	V_2	V_3	V_4	V_5	Max
V_1	-	5	9	6	9	**9**
V_2	5	-	4	3	6	**6**
V_3	9	4	-	4	3	**9**
V_4	6	3	4	-	3	**6**
V_5	9	6	3	3	-	**9**

Vertex Center

From table 1.5, the maximum entries for all rows are given in column 7: Thus

$$m(V_1)=9 \qquad m(V_2)=6 \qquad m(V_3)=9 \quad m(V_4)=6 \qquad m(V_5)=9$$

The minimum of the row maximum occur at either V_2 and V_4. Thus the vertex centre is node 2 or node 4 with objective value of 6. The absolute centre of G may not coincide with the vertex center of G. Moreover, the absolute center of G may be located on an edge that is not incident to the vertex center of G.

1**. Location of local centre on edge $V_1 V_4$**

For given edge (p,q) we use the set of equation;

$$y_1 = w_i^s(x + d(v_i, p)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$y_2 = w_i^s(c(p,q) - x + d(q, v_i)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (2)$$

Location of local center for scenario S_1

Consider edge (V_1, V_4) using demand weights for scenario S_1, w^1=15 and $x \in [0, c(V_1, V_4)] = [0, 6]$

We set the origin at V_1. Hence p= V_1 and q= V_4

(i). For $V_i = V_4$ and demand at V_4 being w_4 =15

$d\,(p, V_i) = d\,(V_1, V_4) = 6$, $d\,(q, V_i) = (V_4, V_4) = 0$ and $c(p,q) = c\,(V_1, V_4)$ =6

Thus y_1= 15(x +6) and y_2 = 15(6- x). Solving for the point of equal cost, we have

15(x+6) = 15(6- x), $\Rightarrow$ x= 0 is the kink for the two equations; y_1, y_2

When x= 0, the equation y_1 falls outside the range [0, 6] and so is discarded and we are left with

$y = y_2 = 15(6 - x)$ $\qquad$ $0 \leq x \leq 6$……………………………….. (1)

(ii). When $V_i = V_1$, and demand at V_1 is w_1=10, then

$d\,(p, V_i)$ = d(V_1,V_1) =0 and d(q,V_i) = d(V_4,V_1) = 6

Thus y_1 = 10(x) and y_2 = 10(12-x), hence the kink point is x= 6.

The equation y_2 = 10(12- x) falls outside the range and so is discarded. Thus;

$y = y_1 = 10(x)$, $\qquad$ $0 \leq x \leq 6$…………………………………. (2)

(iii). For $V_i = V_2$ and $w_2 = 20$, then

$d(p,V_i) = d(V_1,V_2) = 5$ and $d(q,V_i) = d(V_4,V_2) = 3$

The resulting equations are;

$y_1 = 20(x+5)$ and $y_2 = 20(9-x)$ with a kink point $x=2$, thus we have;

$$y_1 = 20(x+5), \qquad 0 \le x \le 2 \ldots\ldots\ldots\ldots (3a)$$

$$y_2 = 20(9-x), \qquad 2 \le x \le 6 \ldots\ldots\ldots\ldots (3b)$$

(iv).For $V_i = V_3$, and $w_3 = 10$, then

$d(p,V_i) = d(V_1,V_3) = 9$ and $d(q,V_i) = d(V_4,V_3) = 4$

The resulting equations are;

$y_1 = 10(x+9)$ and $y_2 = 10(10-x)$ with the kink point, $x = ½$. Thus we have;

$$y_1 = 10(x+9), \qquad 0 \le x \le 1/2 \ldots\ldots\ldots\ldots (4a)$$

$$y_2 = 10(10-x), \qquad 1/2 \le x \le 6 \ldots\ldots\ldots\ldots (4b)$$

(v).For $V_i = V_5$ and $w_5 = 10$, then

$d(p,V_i) = d(V_1,V_5) = 9$ and $d(q,V_i) = d(V_4,V_5) = 3$

The resulting equations are;

$y_1 = 10(x+9)$ and $y_2 = 10(9-x)$ with the kink point $x=0$. We discard equation y_1 since it will fall outside the range of values. Thus we have;

$y =\ \ y_2 = 10(9\text{-}\,x),\qquad\qquad 0\leq x \leq 6$……………………………(5)

The resulting equations are then plotted on the same axes of of figure 1.4 as shown below;

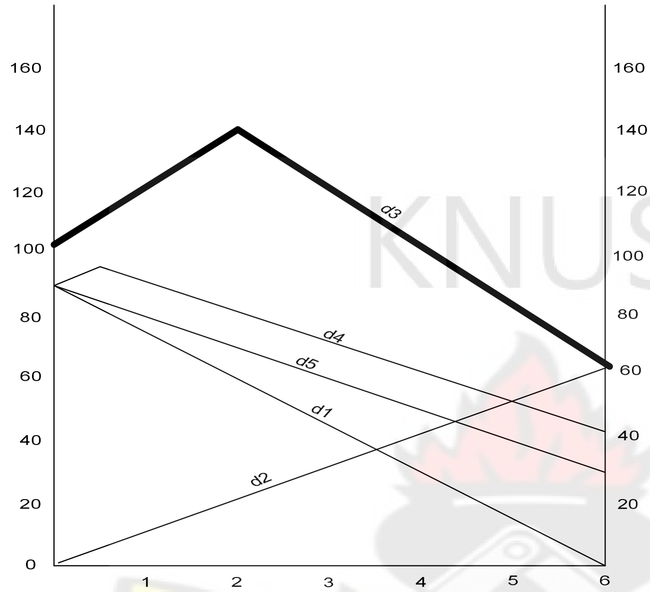

Figure 1.4: Edge V_1, V_4 for year 1

2.9.1 Construction of the upper envelope

Upper envelop for edge (V_1, V_4) under Scenario S_1

To construct the upper envelope, we trace all points of lines with maximum y value for a given x-value.beyond which there are no points with higher y values for the same x value. The trace is indicated by thick a line as shown in the figure 1.4 above. The local center on (V_1, V_4) is the x-value of the point on the envelope with minimum cost.

From the figure above, the local center of edge (V_1, V_4) for scenario S_1 is x^* =6 and the cost is $m(x^*) = 60$

Location of local center for Scenario S_2

We repeat the calculations for the local centers for edge (V_1, V_4) under scenario S_2 where the set of weights $w^{s2} = (10,15,20,10,10)$ are used.

Figure 1.5 below shows the plot and the upper envelop for edge (V_1, V_4) under scenario S_2 ,

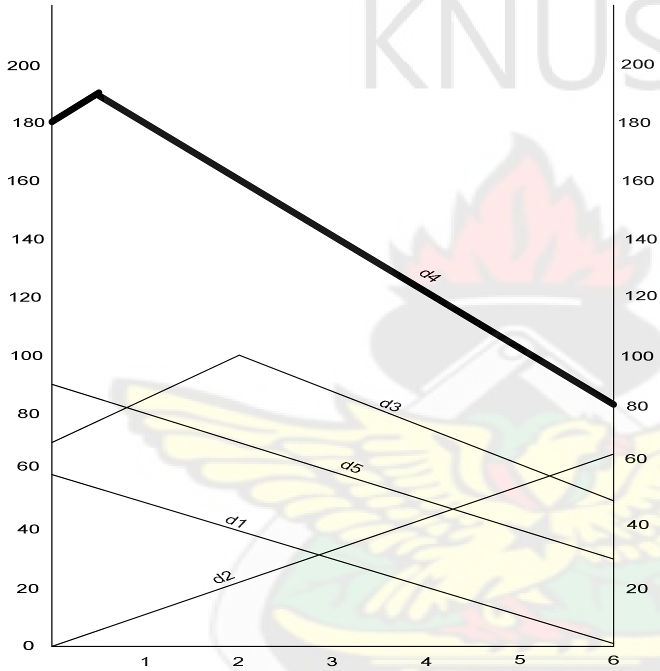

Figure 1.5: Edge V_1,V_4 for year 2

From figure 1.5, the local centre of edge (V_1, V_4) for scenario S_2 is x^* =6 and the cost is $m(x^*) = 80$.

Location of local center for Scenario S_3

Figure 1.6 below shows the plot and the upper envelope for edge (V_1, V_4) under scenario S3 where the set of weights $w^{s3} = (10,10,15,20,10)$ are used.

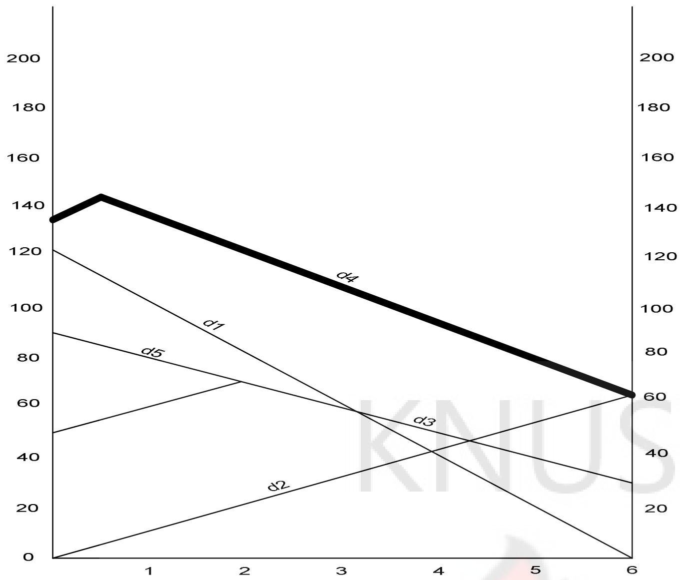

Figure 1.6: Edge V_1,V_4 for year 3

From figure 1.6, the local centre of edge V_1,V_4 for scenario S_3 is x^* =6 and the cost is $m(x^*) = 60$

2.9.2 CALCULATION OF REGRETS

Regret for edge (V_1,V_4)

The three upper envelops for the three scenarios S1, S2, S3 with respect to edge (V_1, V_4) are plotted on a single graph which is used to compute the regret for the given edge. Figure 1.7 is the plot of the three upper envelopes for edge (V_1, V_4)

EDGE V_1,V_4

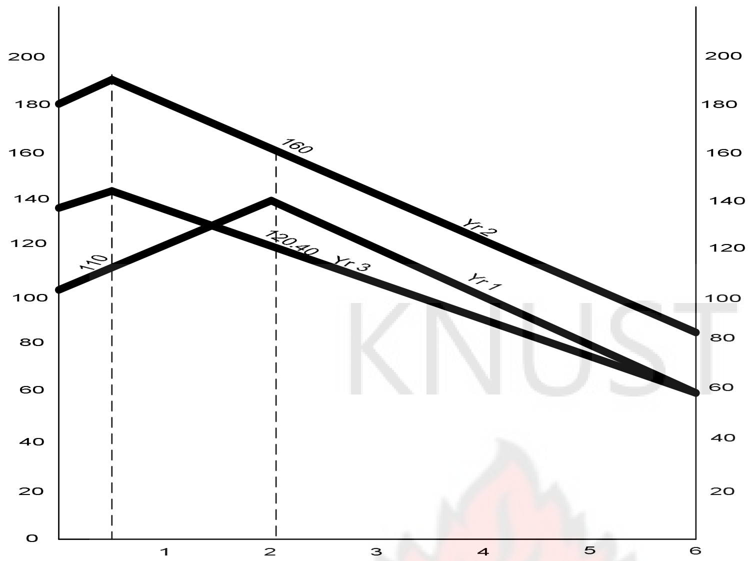

Figure 1.7: Combination of the upper envelopes for Edge V_1, V_4

The breakpoints of the graph are $t = 0, 0.5, 2$ and 6

From figure 1.7 above, we compute the regret of edge V_1,V_4.

For each basic interval $t \in [a,b]$ and scenario s,

$$D_e^s(t) = D_e^s(a) + \frac{D_e^s(b) - D_e^s(a)}{b-a}(t - a) \quad \forall [a,b] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$D_e^s(b)$ is the weighted cost at b for edge e if breakpoint exist at $x = b$ for scenario S otherwise,

for a breakpoint $x = c$ and $a \prec b \prec c$,

$$D_e^s(b) = D_e^s(a) + \frac{D_e^s(c) - D_e^s(a)}{c-a}(b - a) \ \forall \in [a,b] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

The regret is given by;

$$R_e(t) = D_e^s(t) - D_e^s(x^{*s}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

From previous calculations

$x^{*1}=6$ and $D_e^1(x^{*1})=60$ $x^{*2}=6$ and $D_e^2(x^{*2})=80$ $x^{*3}=6$ and $D_e^3(x^{*3})=60$

Regret for S_1 (year 1) for basic interval $t=[0,0.5]$

Consider year 1, for basic interval $t\in(t_0,t_1)=(0,0.5)$. The breakpoint $t_0=0$ is on year1 and $t\in(t_0,t_1)$, $D_e^1(0)$=100 but the breakpoint $t_1=b=0.5$ is on year 2. Take $t_2=c=2$ and use equation (ii) to find $D_e^1(t_1)$ as;

$$D_e^1(t_1)=D_e^1(t_0)+\frac{D_e^1(t_2)-D_e^1(t_0)}{t_2-t_0}(t_1\text{-}t_0)$$

$$=100+\frac{143-136}{0.5}(0.5\text{-}0)\Rightarrow D_e^1(t_1)=110$$

Thus from equation (i),

$$D_e^1(t)=D_e^1(t_0)+\frac{D_e^1(t_1)-D_e^1(t_0)}{t_1-t_0}(t\text{-}t_0)$$

$$R_e^1(t)=[D_e^1(t_0)-D_e^1(x^{*1})]-(\frac{D_e^1(t_1)-D_e^1(t_0)}{t_1-t_0})(t_0)+(\frac{D_e^1(t_1)-D_e^1(t_0)}{t_1-t_0})(t)$$

$$=100-60-(110\text{-}100)(0)+\frac{110-100}{0.5}t$$

$$R_e^1(t)=40+20t \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Regret for S_2 (year 2) for basic interval $t=[0,0.5]$

Consider year 2, t_0 $\in$year2 and t_1 $\in$ year 2. From equation (i);

$$D_e^2(t)=D_e^2(t_0)+\frac{D_e^2(t_1)-D_e^2(t_0)}{t_1-t_0}(t\text{-}t_0)$$

The regret is;

$R_e^2(t) = 100 + 20t$ ……………………………….. (b)

Regret for S_3 (year 3) for basic interval $t = [0, 0.5]$**,**

we have;

$R_e^3(t) = 136 - 60 + \dfrac{143 - 136}{0.5}\,t$

$R_e^3(t) = 76 + 14t$ ……………………………………(c)

Regret for basic interval $t = [t_1, t_2] = [0.5, 2]$

Let us consider the interval $[t_1, t_2]$. Consider year 1

$t_1 \in$ year1 and $t_2 \in$ year 1. The regret is;

$R_e^1(t) = 40 + 20(t)$ ……………………………………………………………..(a)

For year 2 ;

$t_1 \in$ year 2 and $t_2 \in$ year 1. The regret is;

$R_e^2(t) = 120 - 20t$ ……………………………………………………………. (b)

For year (3) $t_1 \in$ year3 and $t_2 \in$ year 3. The regret is;

$R_e^1(t) = 90.5 - 15.07t$ …………………………………… (c)

Table (1.6) below is a summary of computations of the entire breakpoints for edge (V_1, V_4)

Table (1.6): **Regret equations for subintervals of edge** ***(V_1, V_4)***

	Subintervals		
	[0, 0.5]	[0.5, 2]	[2, 6]
Year 1	$40 + 20t$	$40 + 20t$	$120 - 20t$
Year 2	$100 + 20t$	$120 - 20t$	$120 - 20t$
Year 3	$76 + 14t$	$90.5 – 15.07t$	$90.6 – 15.1t$

Since the regret equations are linear, the maximum values will occur at the endpoints of the basic intervals

Substituting the end point values of the interval in the given equations the results are shown in table (1.7) below:

Table (1.7); Regrets for the subintervals of edge (V_1, V_4)

	$t = 0$	$t = 0.5$	$t = 0.5$	$t = 2$	$t = 2$	$t = 6$
Year 1	40	50	50	**80**	80	0
Year 2	100	110	**110**	80	80	0
Year 3	76	**83**	82.965	60.36	60.4	0

The maximum regret for year1, year2, year3 is respectively 80, 110, and 83. The minimum of the maximum regrets is 80 of year 1 and it occurs at $t = 2$

EDGE V_4,V_5

Table (1.8) below is a summary of computations of the entire breakpoints for edge (V_4, V_5)

Table (1.8): **Regret equations for subintervals of edge** ***(V_4, V_5)***

	Subintervals			
	[0, 1]	[1, 1.2]	[1.2, 2]	[2, 3]
Year 1	$20t$	$20t$	$20t - 4$	$20t$
Year 2	$20t$	$40 - 20t$	$-32 + 40t$	$-20 + 10t$
Year 3	$15t$	$30 - 15t$	$10t$	$10t$

Substituting the end point values of the interval in the given equations the results are shown in table (1.9) below:

Table (1.9); Regrets for the subintervals of edge (V_4, V_5)

	$t=0$	$t=1$	$t=1$	$t=1.2$	$t=1.2$	$t=2$	$t=2$	$t=3$
Year 1	0	20	20	24	20	36	40	**60**
Year 2	0	20	20	16	16	**48**	0	10
Year 3	0	15	15	12	12	20	20	**30**

The maximum regret for year1, year2, year3 is respectively 60, 48, and 30. The minimum of the maximum regrets is 30 of year 3 and it occurs at $t = 3$

EDGE V_1,V_2

Table (2.0) below is a summary of computations of the entire breakpoints for edge (V_1, V_2)

Table (2.0): **Regret equations for subintervals of edge *(V_1, V_2)***

	Subinterval				
	[0, 0.3]	[0.3, 0.4]	[0.4, 1]	[1, 2.2]	[2.2, 5]
Year 1	$40 – 16.67t$	$32 + 10t$	$30 + 15t$	$60 - 15t$	$48.12 – 9.6t$
Year 2	$100 - 20t$	$100 - 20t$	$100 - 20t$	$100 - 20t$	$100 - 20t$
Year 3	$75 - 15t$	$75 - 15t$	$61.67+ 8.33t$	$100 - 20t$	$100 - 20t$

Substituting the end point values of the interval in the given equations the results are shown in table (2.1) below:

Table (2.1); Regrets for the subintervals of edge (V_1, V_2)

	$t=0$	$t=0.3$	$t=0.3$	$t=0.4$	$t=0.4$	$t=1$	$t=1$	t=2.2	t=2.2	t=5
Year 1	40	34.999	35	36	36	45	**45**	27	27	0.12
Year 2	100	94	**110**	92	92	100	100	56	56	0
Year 3	75	70.5	70.5	69	58.34	70	**80**	56	56	0

The maximum regret for year1, year2, year3 is respectively 45, 110, and 80. The minimum of the maximum regrets is 45 of year 1 and it occurs at $t = 1$

EDGE V_2,V_4

Table (2.2) below is a summary of computations of the entire breakpoints for edge (V_2, V_4)

Table (2.2): **Regret equations for subintervals of edge *(V_2, V_4)***

	Subintervals			
	[0, 0.5]	[0.5, 1.5]	[1.5, 2]	[2, 3]
Year 1	$5 - 10t$	$-5 + 10t$	$-5 + 10t$	$35 - 10t$
Year 2	$20t$	$20t$	$60 - 20t$	$60 - 20t$
Year 3	$16t$	$0.75 – 14.5t$	$45 - 15t$	$45 - 15t$

Substituting the end point values of the interval in the given equations the results are shown in table (2.3) below;

Table (2.3); Regrets for the subintervals of edge (V_2, V_4)

	$t = 0$	$t = 0.5$	$t = 0.5$	$t = 1.5$	$t = 1.5$	$t = 2$	$t = 2$	$t = 3$
Year 1	5	0	0	10	10	**15**	15	5
Year 2	0	10	10	30	**30**	20	20	0
Year 3	0	8	-6.5	-21	**22.5**	15	15	0

The maximum regret for year1, year2, year3 is respectively 15, 30, and 22.5. The minimum of the maximum regrets is 15 of year 1 and it occurs at $t = 2$

EDGE V_2 ,V_3

Table (2.4) below is a summary of computations of the entire breakpoints for edge (V_2, V_3)

Table (2.4): **Regret equations for subintervals of edge *(V_2, V_3)***

	Subinterval					
	[0, 0.5]	[0.5, 1]	[1, 2.5]	[2.5, 2.8]	[2.8, 3.65]	[3.65, 4]
Year 1	$10t$	$10 - 10t$	$15.33t - 15.33$	$64.67 - 16.67t$	$10.59t - 11.65$	$8.57t - 4.29$
Year 2	$20 - 20t$	$20 - 20t$	$-10+10t$	$-10+10t$	$10.59t - 11.65$	$8.57t - 4.29$
Year 3	$20t$	$20t$	$20t$	$100 - 20t$	$100 - 20t$	$-4.29 + 8.57t$

Substituting the end point values of the interval in the given equations the results are shown in table (2.5) below;

Table (2.5); Regrets for the subintervals of edge (V_2, V_3)

	$t = 0$	$t= 0.5$	$t= 0.5$	$t=1$	$t=1$	$t=2.5$	$t=2.5$	$t=2.8$	$t=2.8$	$t=3.65$	$t=3.65$	$t=4$
Year 1	0	5	5	0	0	22.995	22.995	17.99	18.00	27	26.99	**29.99**
Year 2	20	10	10	0	0	15	15	18	18	27	26.99	**29.99**
Year 3	0	10	10	20	20	50	**50**	44	44	27	26.99	29.99

The maximum regret for year1, year2, year3 is respectively 29.99, 29.99 and 50. The minimum of the maximum regrets is 29.99 of year 1 and it occurs at $t = 4$

EDGE V_3,V_5

Table (2.6) below is a summary of computations of the entire breakpoints for edge (V_3, V_5)

Table (2.6): **Regret equations for subintervals of edge *(V_3, V_5)***

	Subintervals				
	[0, 1]	[1, 1.5]	[1.5, 2.45]	[2.45, 2.5]	[2.5, 3]
Year 1	$10t$	$-10 + 60t$	$-10 + 20t$	$-10 + 20t$	$90 - 20t$
Year 2	$10t$	$10t$	$29.21 - 9.47t$	$-43 + 20t$	$42 - 14t$
Year 3	$10t$	$10t$	$29.21 - 9.47t$	$30.5 - 10t$	$33 - 11t$

Substituting the end point values of the interval in the given equations the results are shown in table (2.7) below;

Table (2.7); Regrets for the subintervals of edge (V_3, V_5)

	$t = 0$	$t = 1$	$t = 1$	$t = 1.5$	$t = 1.5$	$t = 2.45$	$t = 2.45$	$t = 2.5$	$t = 2.5$	$t = 3$
Year 1	0	10	50	**80**	20	39	39	40	40	30
Year 2	0	10	10	15	**15**	6	6	7	7	0
Year 3	0	10	10	15	**15**	6	6	5.5	5.5	0

The maximum regret for year1, year2, year3 is respectively 80, 15 and 15. The minimum of the maximum regrets is 15 of year 2 and year 3 and it occurs at $t = 1.5$

EDGE V_3,V_4

Table (2.8) below is a summary of computations of the entire breakpoints for edge (V_3, V_4)

Table (2.8):* Regret equations for subintervals of edge *(V_3, V_4)

	Subintervals				
	[0, 0.5]	[0.5, 0.7]	[0.7, 1.5]	[1.5, 3.35]	[3.35, 4]
Year 1	$30 + 10t$	$37.5 - 5t$	$20 + 20t$	$80 - 20t$	$80 - 20t$
Year 2	$23 + 10t$	$33 - 10t$	$33 - 10t$	$32.6 – 9.73t$	$-67 + 20t$
Year 3	$30 + 10t$	$40 - 10t$	$40 - 10t$	$39.6 – 9.73t$	$43.1 – 10.77t$

Substituting the end point values of the interval in the given equations the results are shown in table (2.9) below;

Table (2.9); Regrets for the subintervals of edge (V_3, V_4)

	$t = 0$	$t = 0.5$	$t = 0.5$	$t = 0.7$	$t = 0.7$	$t = 1.5$	$t = 1.5$	$t = 3.35$	$t = 3.35$	$t = 4$
Year 1	30	35	35	34	34	50	50	13	13	**80**
Year 2	23	28	**28**	26	26	18	18	0	0	13
Year 3	30	35	**35**	33	33	25	25	7	7	0

The maximum regret for year1, year2, year3 is respectively 80, 28 and 35. The minimum of the maximum regrets is 28 of year 2 and it occurs at $t = 0.5$

To find the solution to the above problem of figure 1.3, we compare all the regrets for the given edges and the most minimum becomes the solution to our problem. The values of the minimum of the maximal regret (R_e) on the various edges of the above network are listed in table (3.1) below.

Table (3.0); Values of the minimax regrets

$R_e(V_1,V_4)$	$R_e(V_3,V_4)$	$R_e(V_3,V_5)$	$R_e(V_2,V_3)$	$R_e(V_2,V_4)$	$R_e(V_1,V_2)$	$R_e(V_4,V_5)$
80	28	15	29.99	15	45	30

The minimum value is $R_e(V_3,V_5)=15$ and $R_e(V_2,V_4)=15$ corresponding respectively to points 1.5 and 2 on respective edges. The robust centre is the point x^* of edge (V_2,V_4) at a distance 2.0 from V_2.

In figure 1.8 we locate the upper envelope of edge (V_2,V_4) under year 1(S_1), the cost at $x=2$ is **65.**

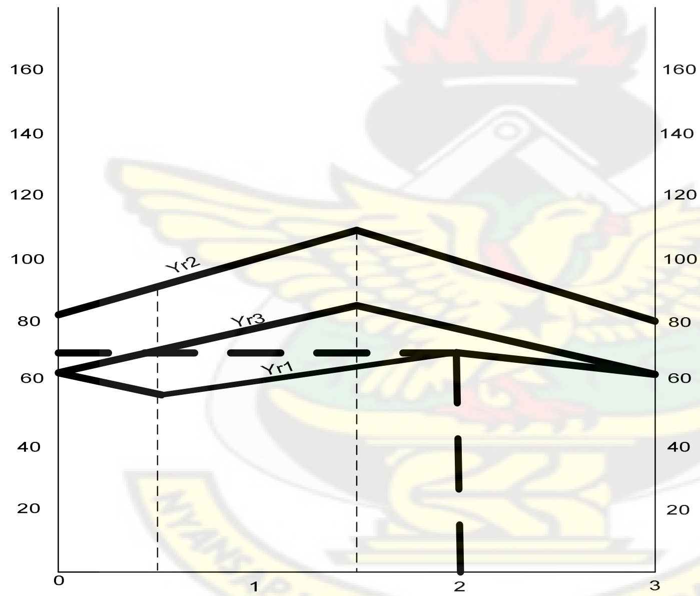

Figure 1.8: combination of the upper envelopes for Edge V_2,V_4

CHAPTER 3

Collection and Analysis of Data

We consider the six halls of residence of KNUST and a hostel in this thesis.The number of students in the various halls / hostel cover a three year period. Table 3.1 below is a table of the names of the halls / hostel and the number of students in them. The data was obtained from the hall secretaries of KNUST.

Table 3.1: Names of Halls / Hostel and the number of students in them

		No. of Students		
Halls/Hostel	**Node**	**Year 2005/2006(S_1)**	**Year 2006/2007(S_2)**	**Year 2007/2008(S_3)**
GUSS Hostel	**G**	967	1063	1389
University Hall	**B**	1280	1278	1063
Independence Hall	**I**	1144	1155	1139
Unity Hall	**U**	1836	1811	1925
Republic Hall	**R**	1173	1120	1208
Queens Hall	**Q**	1384	1165	1171
Africa Hall	**A**	741	682	712
Totals		***8,525***	***8,274***	***8,607***

The set of distances of roads linking the halls / hostel was collected from the Geomatic Department of the Kwame Nkrumah University of Science and Tecnology. This is presented in table 3.2 below

Table 3.2: Table of direct road distances between nodes $c(p,q)$

	Q	A	U	R	I	B	G
Q	-	375	∞	100	∞	950	∞
A	375	-	400	∞	∞	∞	∞
U	∞	400	-	380	340	∞	∞
R	100	∞	380	-	210	∞	∞
I	∞	∞	340	210	-	1050	∞
B	950	∞	∞	∞	1050	-	306
G	∞	∞	∞	∞	∞	306	-

The above data has been developed into a network of figure 1.9 below.

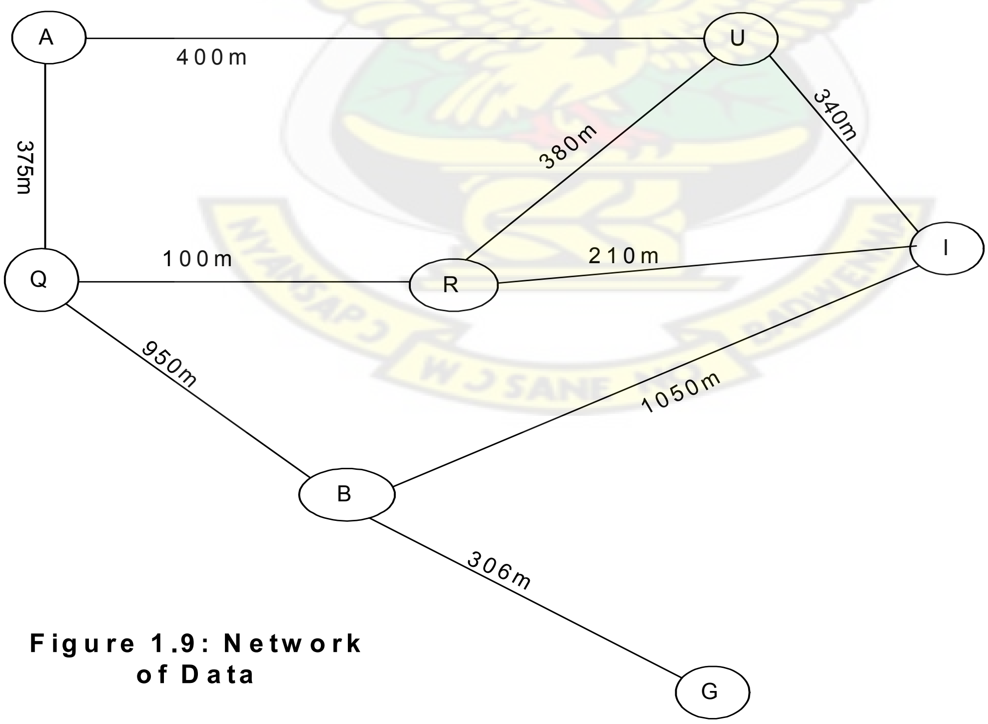

Figure 1.9: Network of Data

By using the Floyd-Warshall algorithm, the shortest path matrix or distance matrix for the above network was obtained as shown in table 3.3 below.

Table 3.3: All pairs shortest path distance matrix, $d(i,j)$

To / From	Q	A	U	R	I	B	G
Q	-	375	480	100	310	950	1256
A	375	-	400	475	685	1325	1631
U	480	400	-	380	340	1390	1696
R	100	475	380	-	210	1050	1356
I	310	685	340	210	-	1050	1325
B	950	1325	1390	1050	1050	-	306
G	1256	1631	1696	1356	1356	306	-

3.1 Location of Local Centre

We compute for each edge the local centre with respect to the demand weights for the various years.

Using the pair of equations equation;

$$y_1 = w_i^s(x + d(v_i, p)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$y_2 = w_i^s(c(p,q) - x + d(q, v_i)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (2)$$

$c(p,q)$ is the edge distance between the initial node p and the end node q.

$d(v_i, p)$ and $d(q, v_i)$ are respectively shortest paths between vertex v_i on the network and the nodes

x is a distance point on edge (p, q) measured from node p.

w_i^s is demand at vertex v_i for the year, S.

EDGE (A, U)

Location of local centre for year 1(S_1)

Set the origin at A, hence p = A and q = U, $c(p,q)$ = 400 with $0 \le x \le 400$. We use demand weight (S_1) of year 1.

(i).For V i = U=q and w_U = 1836

$y_1 = w(x + d(p, U))$ and $y_2 = w(c(p, q) - x + d(q, U))$

$y_1 = 1836(x + 400)$ and $y_2 = 1836(400 - x)$.

At the kink point, $y_1 = y_2$ and so $1836(x + 400) = 1836(400 - x)$.

. Hence $x = 0$. Equation y_1 is of positive slope ending on $x = 0$ and hence falls outside the range and so is discarded leaving the equation;

$y = y_2 = 1836(400 - x)$, $\quad\quad 0 \le x \le 400$………………………………….. (1)

(ii).For $V_i = A = p$ and $w_A = 741$

$y_1 = w\ (\ x + d\ (\ p\ , A\ ))$ and $y_2 = w(\ c(\ p\ ,q\ ) - x\ + d\ (\ q\ , A\ ))$

$y_1 = 741(\ x\ )$ and $y_2 = 741\ (800 - x\ )$

At the kink point, $741(\ x\ ) = 741\ (800 - x\ )$ and $x\ = 400$

Equation $y_2 = 741\ (800 - x\ )$ is of negative slope ending on $x = 400$ and hence falls outside the range and so is discarded. We have the equation;

$$y_1 = 741(\ x\ ), \qquad 0 \le x \le 400 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2)$$

$(\ iii\ )$. For $V_1 = Q$ and $w_Q = 1384$

$y_1 = w(\ x + d\ (\ p\ ,\ Q\ ))$ and $y_2 = w(c(\ p\ ,q\ ) - x\ + d(\ q\ ,Q\ ))$

$y_1 = 1384(\ x\ + 375)$ and $y_2 = 1384(880 - x\ )$.

At the kink $1384(\ x\ + 375) = 1384(880 - x\ )$. thus $x = 252.5$. We have the following equations.

$$y_1 = 1384(\ x\ + 375), \quad 0 \le x \le 252.5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (3a\ )$$

$$y_2 = 1384\ (880 - x\ ), \quad 252.5 \le x \le 400 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (3b\ )$$

$(\ iv\ )$. For $V_i = R$ and $w_R = 1173$

$y_1 = w(\ x + d\ (\ p\ , R))$ and $y_2 = w(c\ (\ p\ ,\ q\ ) - x\ + d(\ q\ , R))$

$y_1 = 1173(\ x + 475)$ and $y_2 = 1173(780 - x\ )$

At the kink 1173(x+475) = 1173(780 - x) thus x =152.5. We have the following equations;

y_1 = 1173($x + 475$), $0 \le x \le 152.5$………………………………………. (4a)

y_1 = 1173(780-x), $152.5 \le x \le 400$………………………………………. (4b)

(v). For V_i = I and w_I = 1144

$y_1 = w(x + d(p,I))$ and $y_2 = w(c(p,q) - x + d(q,I))$

y_1 = 1144(x + 685) and y_2 = 1144(740 $- x$)

At the kink point, 1144(x + 685) = 1144(740 $- x$) $\Rightarrow$ x = 27.5. We have the following equations;

y_1 = 1144 (x + 685), $0 \le x \le 27.5$ ………………………………………………. (5a)

y_2 = 1144 (740-x), $27.5 \le x \le 400$ ……………………………………………. (5b)

(vi). When V_i = B and w_B = 1280

$y_1 = w(x + d(p,B))$ and $y_2 = w(c(p,q) - x + d(q,B))$

y_1 = 1280 (x + 1325) and y_2 = 1280(1790 $- x$).

At the kink point, 1280 (x + 1325) = 1280(1790 $- x$). Thus x = 232. We have the following equations;

y_1 = 1280 (x + 1325) $0 \leq x \leq 232.5$ ………………………………………… $(6a)$

y_2 = 1280 (1790- x) $232.5 \leq x \leq 400$………………………………………… $(6b)$

(vii).For $V_i = G$ and $w_G = 967$

$y_1 = y_1 = w(x + d(p, G))$ and $y_2 = w(c(p, q) - x + d(q, G))$

$y_1 = 967(x + 1631)$ and $y_2 = 967(2096 - x)$.

At the kink point, $967(x + 1631) = 967(2096 - x)$. Thus $x = 232.5$. We have the following equations;

y_1 = 967(x + 1631), $0 \leq x \leq 232.5$……………………………………..………. $(7a)$

y_2 = 967 (2096- x) $232.5 \leq x \leq 400$…………………………………………… $(7b)$

We plot the graphs of equation (1) - (7) as shown in figure 2.0. We construct the upper envelope as the maximum y-value on a graph for each point of x :

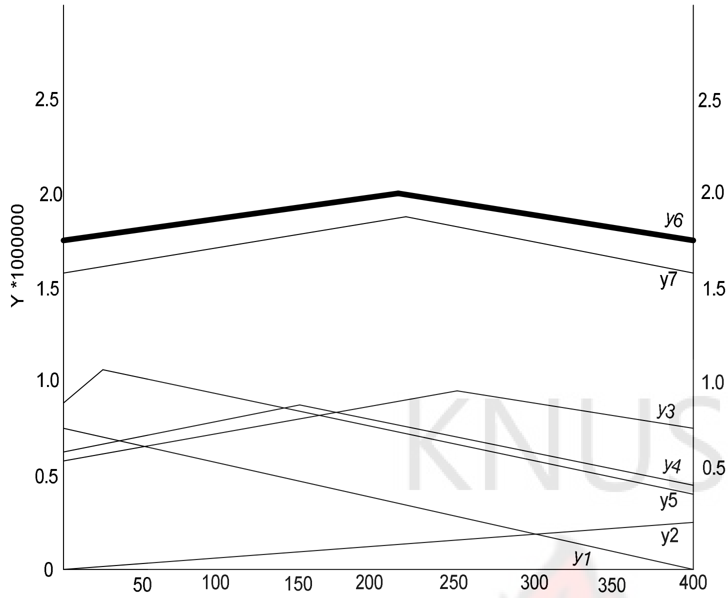

Figure 2.0: Edge (A, U) for year 1

From the figure 2.0 above, the local center of edge (A, U) for year 1 (S_1) is $x^* = 0$ and the cost is $m(x^*) = 1,696,000$

Location of local centre for year 2(S_2)

Figure 2.1 below show the plot and upper envelop for edge (A, U) under year 2.

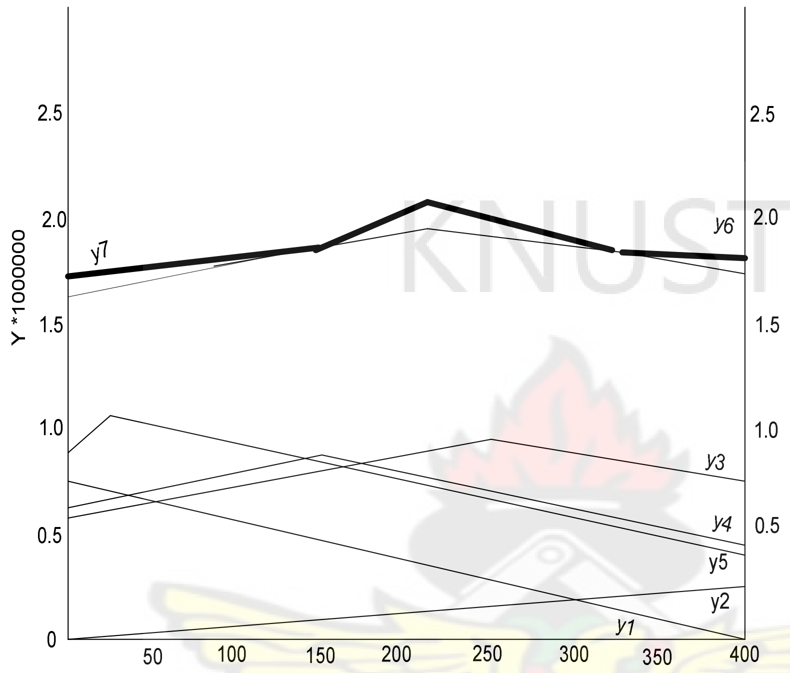

Figure 2.1: Edge (A, U) for year 2

From figure 2.1 above, the local centre of edge (A, U) for year 2 (S_2) is x^* =0 and the cost is $m(x^*)$ = 1,733,753

Location of local centre for year 3(S_3)

Figure 2.2 below show the plot and upper envelop for edge (A, U) under year 3.

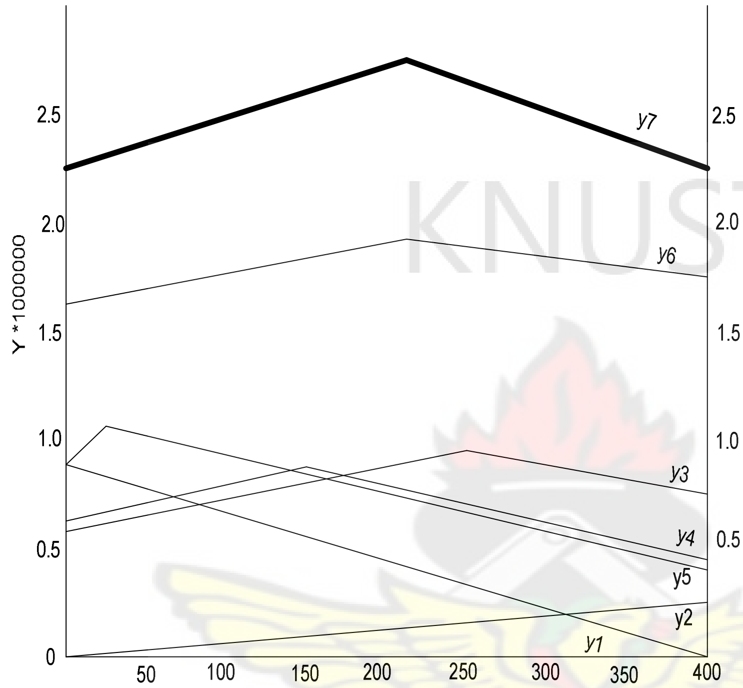

Figure 2.2: Edge (A, U) for year 3

From figure 2.2 above, the local center of edge (A, U) for year 3 (S_3) is $x^* = 0$ and the cost is $m(x^*) = 2{,}265{,}459$

3.2 CALCULATION OF REGRETS

The three upper envelopes of year 1, year 2 and year 3 with respect to edge (A, U) are plotted on a single graph which is used to compute the regret for the given edge. Figure 2.3 below is a plot of the three upper envelops for the edge.

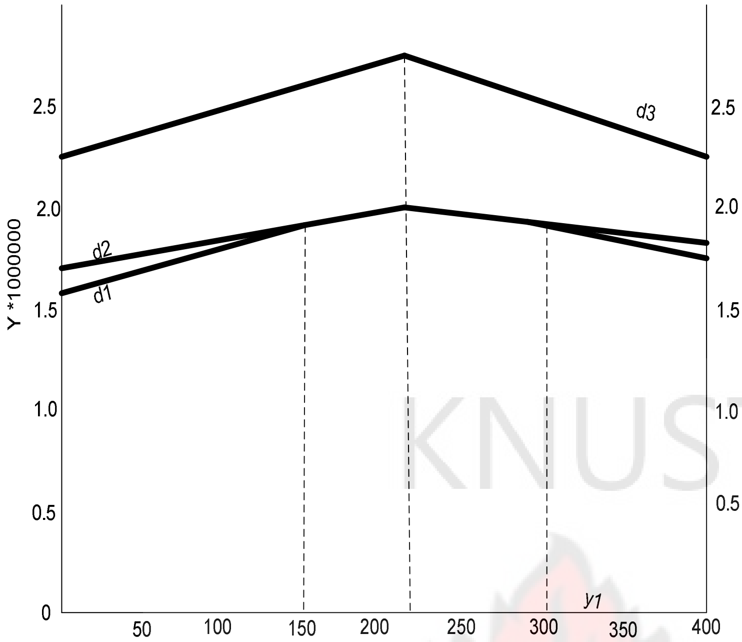

Figure 2.3: combination of upper envelopes for edge (A, U)

From figure 2.3, we identify the break points and their y-values as;

Table (i) for Year 1

X – $Values$	Y – $Values$
0	1,696,000
232.5	1,993,600
400	1,779,200

Table (j) for Year 2

$X - Values$	$Y - Values$
0	1,733,753
150	1,893,203
232.5	1,990,485
300	1,904,220
400	1,802,848

Table (k) for Year 3

$X - Values$	$Y - Values$
0	2,265,459
232.5	2,588,401.5
400	2,355,744

For each basic interval $t \in [a,b]$ and the given year (S_1, S_2, S_3), we have;

$$D_e^s(t) = D_e^s(a) + \frac{D_e^s(b) - D_e^s(a)}{b-a}(t - a) \;\forall [a,b] \qquad \text{(i)}$$

$$D_e^s(b) = D_e^s(a) + \frac{D_e^s(c) - D_e^s(a)}{c-a}(b-a) \;\forall \in [a,b] \qquad \text{(ii)}$$

$$R_e(t) = D_e^s(t) - D_e^s(x^{*s}) \qquad \text{(iii)}$$

Regret for year 1(S_1) for basic interval $t=[0,150]$

Consider year 1,

$t \in (t_0, t_1)$, $t_0 \in$ year 1 and $t_1 \in$ year 2.

Let $t=c$ be the next breakpoint of (S_1) such that ***a<b<c*** then from equation (ii) we have;

$$D_e^1(b) = D_e^1(a) + \frac{D_e^1(c) - D_e^1(a)}{c-a}(b-a)$$

$$D_e^1(t_1) = D_e^1(t_0) + \frac{D_e^1(t_2) - D_e^1(t_0)}{t_2 - t_0}(t_2 - t_0)$$

$$= 1{,}696{,}000 + \frac{1{,}993{,}600 - 1{,}696{,}000}{232.5 - 0}(150 - 0)$$

$$= 1{,}888{,}000$$

Thus from equation (i),

$$D_e^1(t) = D_e^1(t_0) + \frac{D_e^1(t_1) - D_e^1(t_0)}{t_1 - t_0}(t - t_0)$$

$$D_e^1(t) = 1{,}696{,}000 + \frac{1{,}888{,}000 - 1{,}696{,}000}{150}(t - 0)$$

$$= 1{,}696{,}000 + 1280(t)$$

$$R_e^1(t) = D_e^1(t) - D_e^1(x^{*1})$$

$$= 1{,}696{,}000 + 1{,}280(t) - 1{,}696{,}000$$

$R_e^1(t) = 1{,}280(t)$ ………………………………………………….. (a)

Regret for year 2(S_2) for basic interval $t=[0,150]$

Consider year 2, $t_0 \in$ year2 and $t_1 \in$ year 2. The regret is;

$R_e^2(t) = 1{,}711.55(t)$ ……………………………………………………. (b)

For year 3(S_3), $t \in (t_0, t_1)$, $t_0 \in$ year 3 and $t_1 \in$ year 2. Following the procedure used for the regret for year 1, we have the regret for year3 as;

$R_e^3(t) = 1{,}289t$ ……………………………..…………………………(c)

The regrets for the remaining subinterval of edge (A, U) are computed as above with the results shown 3.4 and table 3.5 below.

Table 3.4 below is a summary of computation of the entire breakpoints for edge (A,U)

Table 3.4: Regret equations for subintervals of edge (A,U)

	Subintervals			
	[0,150]	[150, 232.5]	[232.5, 300]	[300,400]
Year 1	$1{,}280t$	$1{,}280t$	$595{,}200 - 1{,}280t$	$595{,}200 - 1{,}280t$
Year 2	$1{,}711.55t$	$-40{,}403 + 1{,}278t$	$553{,}867 - 1{,}278t$	$474{,}583 - 1{,}013.72t$
Year 3	$1{,}389t$	$1{,}389t$	$645{,}885 - 1{,}389t$	$645{,}841 - 1{,}389t$

Substituting the end point values of the interval in the given equations the results are shown in table (3.5) below;

Table (3.5): Regrets for the subintervals of edge (A,U ***)***

	0	150	150	232.5	232.5	300	300	400
Year 1	0	192,000	192,000	**297,600**	297600	211,200	211,200	83,200
Year 2	0	**256,732.5**	151,297	256,732	256,732	170,467	170,467	69,095
Year 3	0	208,350	208,350	**322,942.5**	322,942.5	229,185	229,141	90,241

The maximum regret for year1, year2, year3 is respectively 297,600, 256,732.5 and 322,942.5. The minimum of the maximum regrets is 256,732.5 of year 1 and it occurs at $t = 150$

EDGE (Q, A)

Table 3.6 below is a summary of computation of the entire breakpoints for edge (Q, A).

Table 3.6: Regret equations for subintervals of edge (Q, A)

	Subintervals
	[0, 375]
Year 1	1,280(t)
Year 2	1,063(t)
Year 3	1,389(t)

Substituting the end point values of the interval in the given equations the results are shown in table (3.7) below;

Table (3.7): Regrets for the subintervals of edge **(Q, A)**

	0	375
Year 1	0	**480,000**
Year 2	0	**398,625**
Year 3	0	**520,875**

The maximum regret for year1, year2, year3 is respectively 480,000, 398,625 and 520,875. The minimum of the maximum regrets is 398,625 of year 2 and it occurs at $t = 375$

EDGE (Q, R)

Table 3.8 below is a summary of computation of the entire breakpoints for edge (Q, R)

Table 3.8: **Regret equations for subintervals of edge *(Q, R)***

	Subintervals
	[0, 100]
Year 1	1,280(t)
Year 2	1,063(t)
Year 3	1,389(t)

Substituting the end point values of the interval in the given equations the results are shown in table (3.9) below;

Table (3.9): Regrets for the subintervals of edge (Q, R)

	0	100
Year 1	0	**128,000**
Year 2	0	**106,300**
Year 3	0	**138,900**

The maximum regret for year1, year2, year3 is respectively 128,000, 106,300 and 138,900. The minimum of the maximum regrets is 128,000 of year 1 and it occurs at $t = 100$

EDGE (U, R)

Table 4.0 below is a summary of computation of the entire breakpoints for edge (U, R)

Table 4.0: **Regret equations for subintervals of edge** (U, R)

	Subintervals	
	[0, 20]	[20, 380]
Year 1	435,200 + 1,280t	486,400 – 1,280t
Year 2	361,420 + 1,063t	403,940 – 1,063t
Year 3	472,260 + 1,389t	527,820 – 1,389t

Substituting the end point values of the interval in the given equations the results are shown in table (4.1) below;

Table (4.1): Regrets for the subintervals of edge (U, R)

	0	20	20	380
Year 1	435,200	**460,800**	460,800	0
Year 2	361,420	**382,680**	382,600	0
Year 3	472,260	**500,040**	500,040	0

The maximum regret for year1, year2, year3 is respectively 460,800, 382,680 and 500,040. The minimum of the maximum regrets is 382680 of year 2 and it occurs at $t = 20$

EDGE (U,I)

Table 4.2 below is a summary of computation of the entire breakpoints for edge (U,I)

Table 4.2: **Regret equations for subintervals of edge *(U,I)***

	Subintervals
	[0, 340]
Year 1	$435{,}200 - 1{,}280t$
Year 2	$361{,}420 - 1{,}063t$
Year 3	$472{,}260 - 1{,}389t$

Substituting the end point values of the interval in the given equations the results are shown in table (4.3) below;

Table (4.3): Regrets for the subintervals of edge (U,I)

	0	340
Year 1	**435,200**	0
Year 2	**361,420**	0
Year 3	**472,260**	0

The maximum regret for year1, year2, year3 is respectively 435,200, 361,420 and 472,260. The minimum of the maximum regrets is 361,420 of year 2 and it occurs at $t = 0$

EDGE (R,I)

Table 4.4 below is a summary of computation of the entire breakpoints for edge (R,I)

Table 4.4: **Regret equations for subintervals of edge *(R,I)***

	Subintervals	
	[0,105]	[105, 210]
Year 1	$1{,}280t$	$268{,}800 - 1{,}280t$
Year 2	$1{,}063t$	$223{,}230 - 1{,}063t$
Year 3	$1{,}389t$	$291{,}690 - 1{,}389t$

Substituting the end point values of the interval in the given equations the results are shown in table (4.5) below;

Table (4.5): Regrets for the subintervals of edge (R,I)

	0	105	105	210
Year 1	0	**134,400**	134,400	0
Year 2	0	**111,615**	111,615	0
Year 3	0	**145,845**	145,845	0

The maximum regret for year1, year2, year3 is respectively 134,400, 111,615 and 145,845. The minimum of the maximum regrets is 111,615 of year 2 and it occurs at $t = 105$

EDGE (B,G)

Table 4.6 below is a summary of computation of the entire breakpoints for edge (B,G)

Table 4.6:* Regret equations for subintervals of edge *(B,G)

	Subintervals
	[0, 306]
Year 1	$1{,}836t$
Year 2	$1{,}811t$
Year 3	$1{,}925t$

Substituting the end point values of the interval in the given equations the results are shown in table (4.7) below;

Table (4.7): Regrets for the subintervals of edge (B,G)

	0	306
Year 1	0	**561,816**
Year 2	0	**554,166**
Year 3	0	**589,050**

The maximum regret for year1, year2, year3 is respectively 561,816, 554,166 and 589,050. The minimum of the maximum regrets is 554,166 of year 2 and it occurs at $t = 306$

EDGE (I, B)

Table 4.8 below is a summary of computation of the entire breakpoints for edge (I, B)

Table 4.8: **Regret equations for subintervals of edge** ***($I.B$)***

	Subintervals				
	[0,120]	[120,240]	[240,280]	[280,370]	[370,1050]
Year 1	264,828-1,280t	222,456-926.9t	-436,406.4+1,818.36t	-436,403.78+1,818.35t	-436,403.28+1,818.35t
Year 2	297,640-1,063t	297,640-1,063t	297,640 – 1,063t	-534,396.8 +1,908.56t	-482,112.28+1,767.25t
Year 3	513,930-1,389t	513,930-1,389t	513,930 – 1,389t	513,930 – 1,389t	-710,696 +1,920.8t

Substituting the end point values of the interval in the given equations the results are shown in table (4.9) below;

Table (4.9): Regrets for the subintervals of edge ($I.B$)

	0	120	120	240	240	280	280	370	370	1050
Year1	264,828	111,228	111,228	0	0	72,734.4	72,734.22	236,385.72	236,386.22	**1,472,864.**
Year 2	297,640	170,080	170,080	42,520	42,520	0	0	171,770.4	171,770.22	**1,373,500.**
Year 3	513,930	347,250	347,250	180,570	180,570	125,010	125,010	0	0	**1,306,144**

The maximum regret for year1, year2, year3 is respectively 1,472,864.22, 1,373,500 and 1,306,144. The minimum of the maximum regrets is 1,306,144 of year 3 and it occurs at $t = 1050$

EDGE (Q, B)

Table 5.0 below is a summary of computation of the entire breakpoints for edge (Q, B)

Table 5.0: **Regret equations for subintervals of edge (*Q, B*)**

	Subinterval					
	[0,70]	[70,120]	[120,165]	[165,245]	[245,930]	[930,950]
Year 1	117,488-1,280t	66,931.2-557.76t	1,839t-220,777.2	1,839.81t-220,779.09	1,839.81-220,776.91	3,197,728-1,836t
Year 2	175,395-1,063t	143,827.1-612.03t	258,073.09-1564.08t	1,821.93t-300,618.45	1,821.93t-300,618.39	3,078,007-1,811t
Year 3	340,305-1,389t	340,292.4-1,388.82t	340,305-1,389t	340,305-1,389t	1,912.34t-468,530.65	3,100,221-1,925t

Substituting the end point values of the interval in the given equations the results are shown in table (5.1) below;

Table (5.1): Regrets for the subintervals of edge (*Q, B*)

	0	70	70	120	120	165	165	245	245	930
Year 1	117,488	27,888	27,888	0	0	82,791.45	82,791.56	229,979.36	229,976.54	**1,490,246.39**
Year 2	175,827.10	100,985	100,985	70,383.5	70,383.49	0	0	145,754.4	145,754.46	**1,393,776.51**
Year 3	340,305	243,075	243,075	173,634	173,625	111,120	111,120	0	0	**1,309,945.55**

The maximum regret for year1, year2, year3 is respectively 1,490,248, 1,393,777 and 1,309,971. The minimum of the maximum regrets is 1,309,971 of year 3 and it occurs at $t = 930$

Minimax Regret

To find the solution to the thesis problem, we compare all the regrets for the given edges and the most minimum becomes the solution to our problem. The values of the minimum of the maximum regret (R_e) on the various edges of the above network are listed in table (5.3) below;

Table (5.2): Values of the minimax regrets

$R_e(Q,A)$	$R_e(A,U)$	$R_e(U,R)$	$R_e(Q,R)$	$R_e(U,I)$	$R_e(R,I)$	$R_e(Q,B)$	$R_e(I,B)$	$R_e(B,G)$
398,625	**256,732.5**	**382,680**	**128,000**	**361,420**	**111,615**	**1,309,771**	**1,306,144**	**554,166**

The minimum value is $R_e(R,I)$= 111,615. From tables 4.5 and 4.6, this occurs under year 2(S_2) on the edge. The robust centre is the point on edge (R,I) at a distance $x^* = 105$ from R .

From figure 2.4 of edge (R,I) under year 2(S_2) below, the weighted distance is **1,553,043m**

Thus the solution is $x^* = 105$ from vertex R on edge (R,I) with weighted distance of **1,553,043m.**

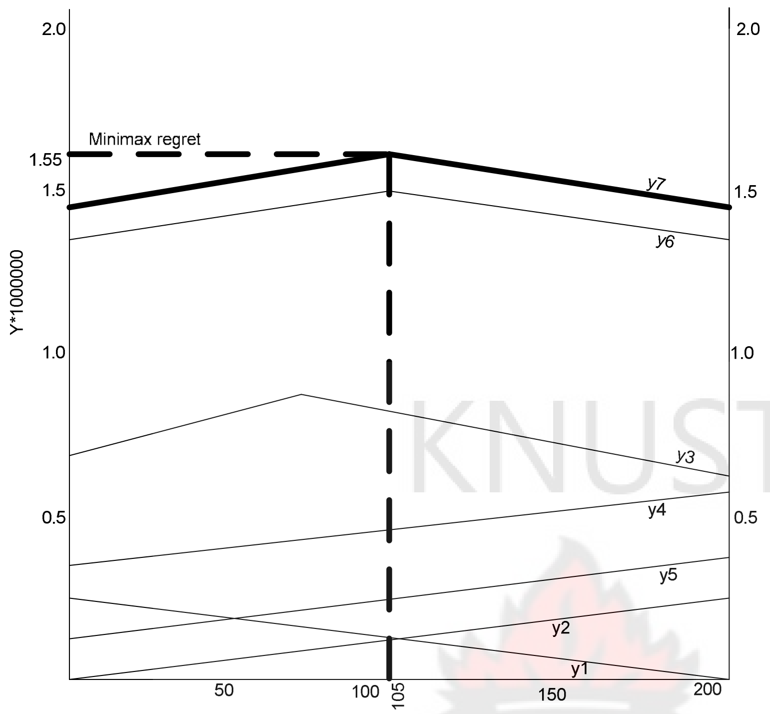

Figure 2.4: Edge R,I for year 2

CHAPTER 4

CONCLUSION

A suitable location for the student clinic should be between Republic Hall and Independence Hall, precisely *105m* from Republic Hall at a weighted distance of ***1,553,043m*.**

Aidoo (2008) located a Fire Hydrant between Independence Hall and the Administration block precisely 88m from Independence Hall without considering the demography of the students. His model was based on Absolute 1-Centre problem.

The use of Robust Analysis considers student demography and takes into account the planned student population in the halls and makes the location suitable for the demograph of students in the halls.

We therefore recommend this site as the best cite to locate a student clinic with an ambulance facility.

4.1 RECOMMENDATIONS

We recommend to the University Authorities to put up a permanent student clinic with an ambulance fully equipped with facilities and medical officers at our identified location.

We recommend the use of Robust Centre method in planning the siting.

.

References;

1. Acheampong E. 1993, *The History of the Gold Coast*.

2. Averbakh I. 2000, *Minimax regret solutions for minimax optimization problems with uncertainty. Operations Research Letters,* 27: 57-65.

3. Averbakh I., 2003, Complexity of robust single facility location problems on networks with uncertain edge length. *Discrete Applied Mathematics*, 127:505-522.

4. Averbakh I. and Berman O., 1997. *Minimax regret p-centre location on a network with demand uncertainty. Location Science, 5(4); 247-254*

5. Burkard R.E and Dollani H., 1997. A note on the Robust 1-centre problem on trees. *Annals of Operation Research,* 110:66-79.

6. Buah F. K, 1980, *Governance of Gold Coast under Governor Guggisberg*.

7. Daskin M.S., 1995 *Network and Discrete Location: Models, Algorithms and Applications*. 1

8 Ebu-Sakyi Y, (2007). *Brief History on KNUST Hospital.In: Bio Y(Ed):. KNUST Quarterly Newsletter of the University Hospital (January-March 2007 edition) Volume* 1, pages 5-8

9. Halfin S., 1974 On finding the absolute and vertex centres of a tree with distances. *Transportation Science,* 8:75-77.

10. Marianov V. and Revelle C., 1995. Siting emergency services. In Drezner Z., editor, Facility *Location: a survey of applications and methods*, pages 199-223.

11. Oppong-Danquah A., 2000*, (Ghana Health Service (2002) Procedures Manual for Procurement of Civil Works*).

APPENDIX A

(A) function [floyd_mat,pred_mat,thepath]=floyd_warshall(A,thestart,theend)

```
% A - This is the Floyd Matrix

%--Keep a copy of the ending node

new_theend=theend;

[r c]=size(A);

pred_mat=[];

if nargin<3

   disp('You need to enter STARTING and ENDING node for ROUTE')

elseif or(thestart,theend)>r

   disp('Non-existent node has been entered')

elseif or(thestart,theend)<0

   disp('Nodes can only be positive')

else

      %This is for the Pre Mat

      for i=1:r

         for j=1:r

            if A(i,j)~=0

               pred_mat(i,j)=i;

            else

               pred_mat(i,j)=0;
```

```
        end
    end
end
%Floyd-Warshall starts its work here
for k=1:r
    for i=1:r
        for j=1:r
          if (A(i,k)+A(k,j))<A(i,j)
              A(i,j)=A(i,k)+A(k,j);
              % We are improving the predecessor matrix
              pred_mat(i,j)=pred_mat(k,j);
          end
        end
    end
end
floyd_mat=A;
thepath=[];
count=1;
while(thestart~=theend)
    thepath(count)=pred_mat(thestart,theend);
    theend=pred_mat(thestart,theend);
```

```
        count=count+1;
    end
    thepath=fliplr(thepath);
end
% Let us add the last figure in the route
thepath(end+1)=new_theend;
% We are working on the number of output arguments
%
% if and(nargin==1,nargout>1)
 %     disp('Only one output argument must be entered')
% end
```

APPENDIX B

Graphs of combined upper envelop for given network.

1.0 *Edge Q, A*

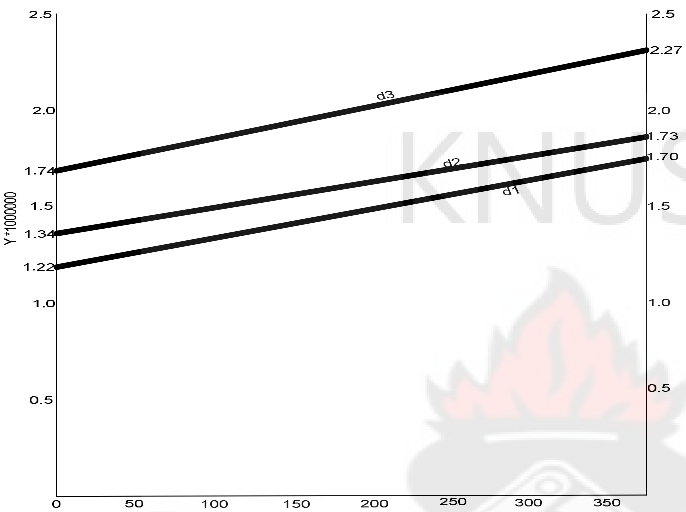

Figure 2.5. Combined Upper envelop for edge Q,A

2.0 *Edge A, U*

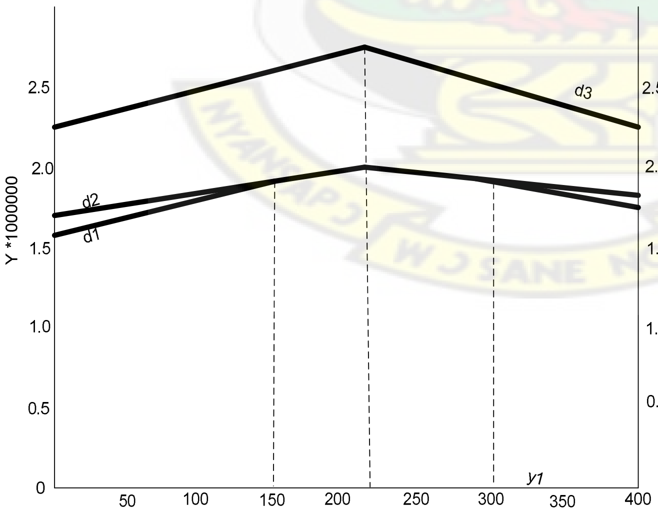

Figure 2.6. Combined upper envelop for edge A, U

3.0 ***Edge Q, R***

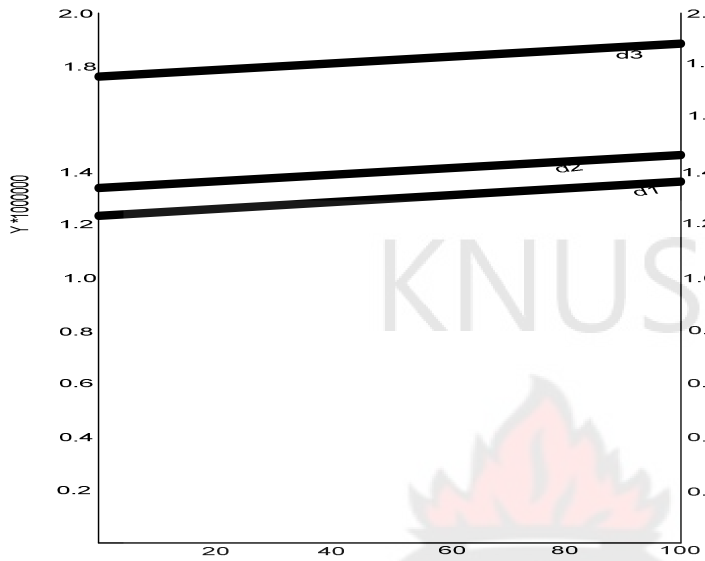

Figure 2.7. Combined upperenvelop for edge Q,R

4.0 ***Edge U, R***

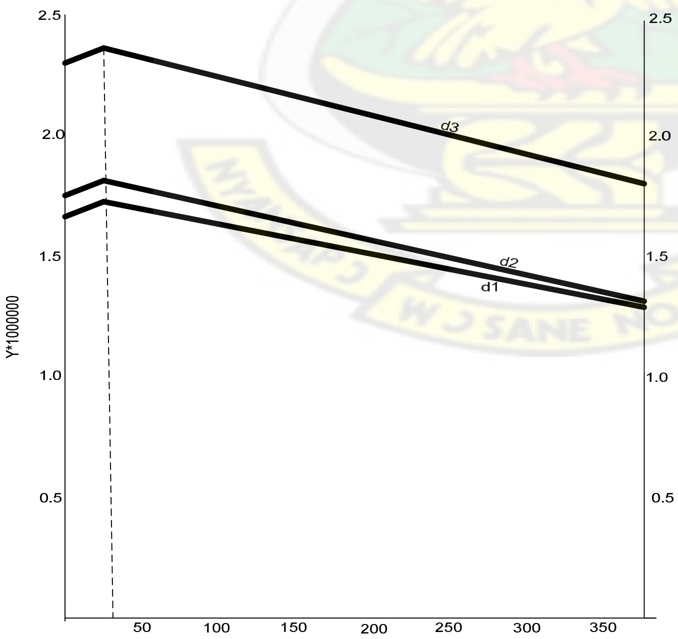

Figure 2.8 Combined upper envelop for edge U,R

5.0 ***Edge U, I***

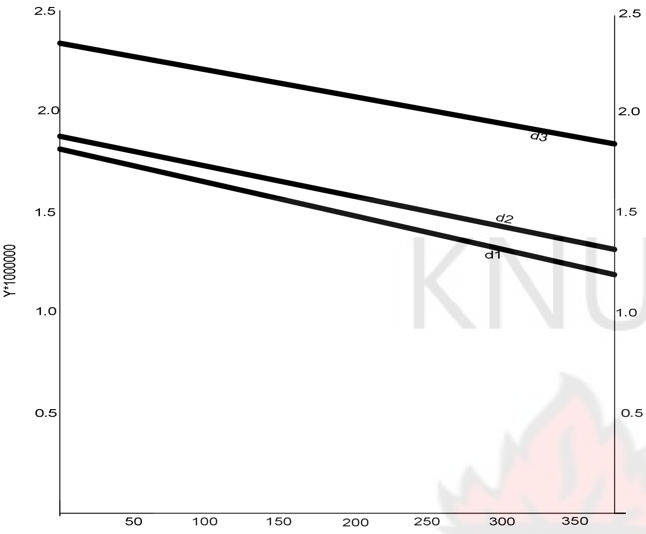

Figure 2.9 Combined upper envelop for edge U,I

6.0 ***Edge R, I***

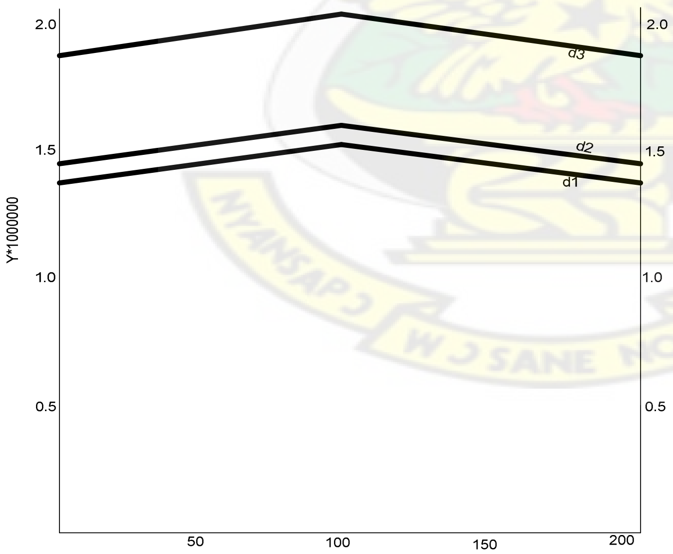

Figure 3.0 Combined upper envelop for edge R,I

7.0 ***Edge B, G***

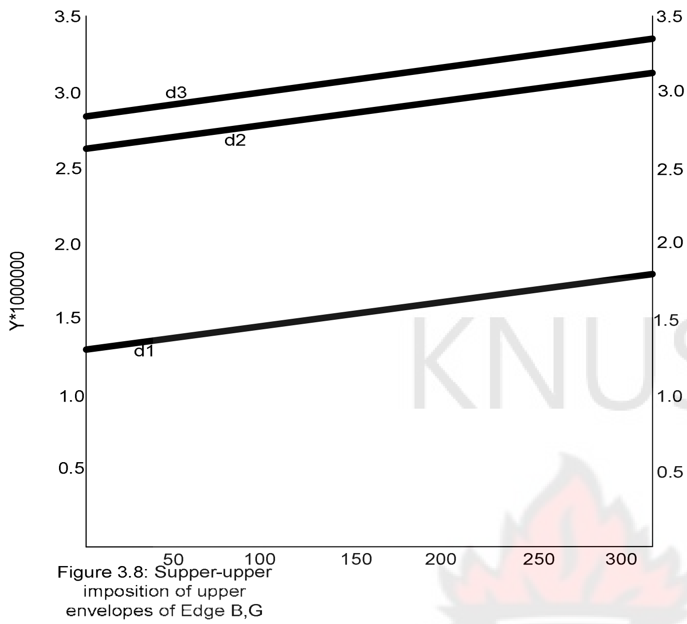

Figure 3.1 combined upper envelop for edge B,G

8.0 ***Edge I, B***

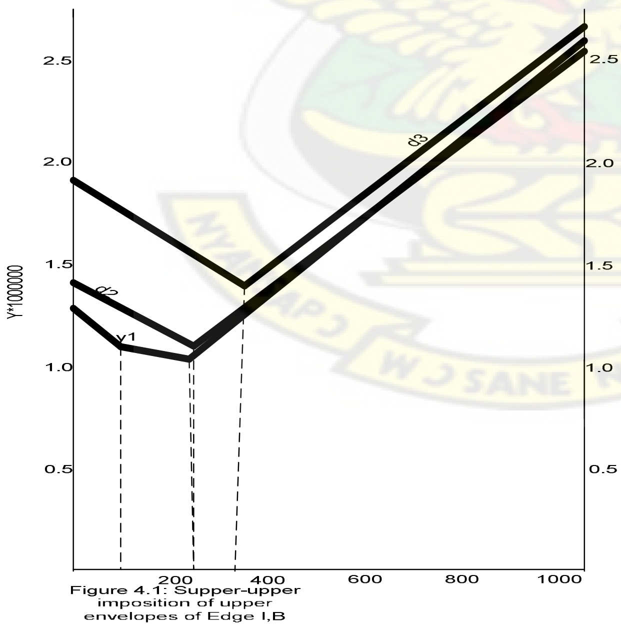

Figure 3.2 combined upper envelop for edge I,B

9.0 ***Edge Q, B***

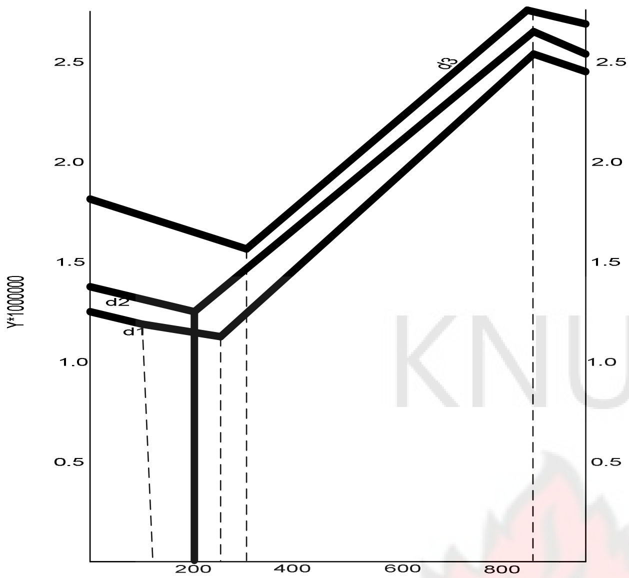

Figure 3.3 Combined upper envelop for edge Q,B