OPTIMAL LOCATION OF AN ADDITIONAL HOSPITAL IN EJURA – SEKYEDUMASE DISTRICT

(A CONDITIONAL P – MEDIAN PROBLEM)

By

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.

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ABSTRACT

This dissertation focuses mainly on conditional facility location problems on a network. In this thesis we discuss the conditional p – median problem on a network. Demand nodes are served by the closest facility whether existing or new.

The thesis considers the problem of locating a hospital facility (semi – obnoxious facility) as a conditional p – median problem, thus some existing facilities are already located in the district.

This thesis uses a new a new formulation algorithm for for the conditional p- median problem on a network which was developed by Oded Berman and Zvi Drezner (2008) to locate an additional hospital in Ejura – Sekyedumase district. A 25 – node network which had four existing hospital was used. The result indicated that additional hospital should be located at Frante (node 7) with an optimal objective function value of 113252. The additional facility at Frante will largely help reduce the pressure on the existing hospitals and improved the quality of service.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

DEDICATION

This work is dedicated to the Almighty Lord who is my source of strength and knowledge and to my beloved parents, Mr. and Mrs. Benefo who have always stood by me, inspired and supported me.

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CHAPTER 1

1.0 INTRODUCTION

Almost every public and private sector enterprise that we can think of has been faced with the problem of locating facilities. Government agencies need to determine locations of offices and other public services such as schools, hospitals, fire stations, ambulance bases, and so on. Industrial firms must determine locations for fabrication and assembly plants as well as warehouses. In these cases, the success or failure of facilities depends in part on the locations chosen for those facilities. Such problems are known as facility location problems.

In other words, facility location problems investigate where to physically locate a set of facilities (i.e. resources, servers) to satisfy some set of demand (i.e., customers, clients).
The goal is to place these facilities such that the quality of service provided is optimized. This optimization may vary depending on the particular objective function chosen. The function could be either: minimize average travel time or cost, minimize average response time, minimize maximum travel time or cost and or maximize net income (Amponsah, 2007).

A facility is considered as a physical entity that provides services. Facility location problems arise in a wide range of practical applications in different fields of study: economic, management, planning, production and many others. Welch et al (1997) also classified facilities into three categories: non – obnoxious (desirable), semi – obnoxious and obnoxious (non- desirable)

In most location problems we are interested in locating facilities that are desirable. Schools, hospitals, production plants post offices, ambulances and fire stations are all considered as facilities that are desirable. Facilities at times can produce an undesirable effect, which may be present even though a high degree of accessibility is required to the facility. If the undesirable effect outweighs the accessibility requirement, then the facility can be classified as obnoxious. Some examples of obnoxious facilities are the nuclear power stations, military installations, pollution produced by industrial plants and recycling centers. Although necessary for society, these facilities are undesirable and often dangerous to the surrounding inhabitants so lowering local house prices and quality
of life.

Sometimes though a facility produces a negative or undesirable effect, this effect may be present even though a high degree of accessibility is required by the facility. For example waste disposal sites and football stadium. These facilities are referred to as semi – obnoxious.(Brimberg and Juel, 1998)

This thesis aims to locate a hospital as an example of semi – obnoxious facility. Hospitals are useful and necessary for the community, but they are a source of negative effects, such as noise from the hospital‟s ambulance and also the solid waste materials from the hospitals that emits unpleasant smell which make it undesirable. The combination of the two makes the facility semi – obnoxious, (Gordillo et al, 2007).

In real life, we always encounter health issues. At times these occurrences need emergency attention within the shortest possible time; otherwise the result may turn out to be disaster with attendant morbidity and mortality. The hospital is always the best option in these cases and hence its location is always of interest

1.1 THE HOSPITAL

The word "hospital" comes from the latin word "hospes" which refers to either a visitor or the host who receives the visitor. From "hospes" came the latin "hospitalia", an apartment for strangers or guest and the medieval latin "hospitale" and the old French "hospital". It crossed the channel in the 14th century and in England began a shift in the 15th century to mean a home for the elderly or infirm.

Hospital only took on its modern meaning as "an institution where the sick or injured are given medical or surgical care" in the 16th century.

Hospital has been defined in the Macmillan English Dictionary as "a place where people stay when they are ill or injured and need a lot of care from doctors and nurses"

According to encyclopedia Britannica, Hospital is an institution for diagnosing and treating the sick or injured, housing them during treatment, examining patients and managing childbirth.

Hospitals may be public (government-owned) or private, profit-making or not- for- profit. In most nations except the United States most of the hospitals are public. Hospitals may also be general, accepting all types of medical or surgical cases, or special, examples are children"s hospitals, mental hospitals, limiting services to a single type of patients or illness.

1.1.1 TYPES OF HOSPITALS

Hospitals are classified according to the nature and purpose of the hospital.

GENERAL HOSPITAL

This type of hospital provides complete medical and surgical care to the sick and injured and maternity care and has:

i) An organized staff of qualified professionals, technical and administrative personnel and appropriate hospital department heads. ii) An approved laboratory with standardized equipment. iii) X-ray facilities, with the service of a consulting radiologist. iv) A separate surgical unit.

v)A separate maternity unit vi)

Dental unit.

CONTAGIOUS DISEASE HOSPITAL:

This institution devoted exclusively to the care of persons who have or are suspected of having, infectious, contagious, or communicable diseases.

CONVALESCENT HOSPITAL:

This type of hospital provides medical and nursing care for persons afflicted with a chronic disability resulting from injury.

MATERNITY HOSPITAL:

This institution provides service for maternity patients exclusively.

MENTAL HOSPITAL:

Mental hospital provides services exclusively to the care of mental patients.

ORTHOPEDIC HOSPITAL:

To operate as an orthopedic hospital an institution must be devoted exclusively to the care of orthopedic patients.

PEDIATRIC HOSPITALS:

This institution deals exclusively to the care and treatment of pediatric patients.

CHIROPRACTIC FACILITY:

It devotes exclusively to the treatment by adjustment with the hand or hands of the bony framework of the human body.

1.1.2 HISTORICAL BACKGROUND OF HOSPITALS IN GHANA

A Hospital can be defined as a place where people who are ill are looked after by medical practitioners. Until the era of Sir Frederick Gordon Guggisberg, the most illustrious British (Canadian born) colonial governor of the Gold Coast, nothing worth recognition in the area of public health infrastructure development for usage by indigenous Ghanaians had been done by any stakeholder or former Governor under the British rule spanning over 100 years.

According to Buah (1980), Governor Guggisberg‟s eight years of administration (19191927) were perhaps the most progressive years in the development of the Gold Coast. Besides other infrastructure such as railways and roads, he is remembered for constructing and establishing the Korle-Bu Teaching Hospital, the leading hospital in Ghana and one of the best in the West Coast of Africa. Guggisberg also extended medical service to other towns to cater for the indigenous population.

Before Governor Guggisberg, the few hospitals in the country were located in the bigger coastal towns cities such as Accra and Secondi- Takoradi which had substantial European populations. Secondi-Takoradi had the Harbour and other port facilities and, Accra was the seat of the British colonial administration. Indeed some of these hospitals were built exclusively for European patients and were referred to as „European Hospitals‟. Examples were the Ridge Hospital in Accra and the Takoradi hospital.

In 1950, government hospitals throughout the country were less than 15, the rest were built and run by European missionaries who attached healing and education to conversion. Notable among these were the Catholic, Basel or Presbyterian and Methodist Missionaries. For instance the Methodist built the Wenchi Hospital in 1951 (Acheampong, 1993).

Attainment of independence on 6th March 1957 saw the development of infrastructures including roads and hospitals. Between 1957-1966, provision of hospitals by the government brought about the construction and initiation of some major hospitals such as the Tamale Hospital in the northern Region of Ghana. Health infrastructure development dwindled in the 1980s due to political and economic instability. In 1984 there was near collapse of the health care system. Donor inflows and some improvements within the economy in the last 18 years have resulted in the state of the art renovation of some major hospitals including; Ho, Cape Coast and Sunyani Regional Hospitals, Sogakope, Ada and Begoro District Hospitals.

It should however be noted that public health infrastructure includes Hospitals, Clinics, Community Health Planning Services, Health Centers, Health Training Schools. Each of the ten regional capitals in Ghana has regional hospital, some also provide specialist services and some have health training institutions attached. The rest of the hospitals are found in the district capitals but some towns have hospitals and clinics. Some districts have more than one hospital whilst others have none.

1.2 BACKGROUND TO THE STUDY

Ejura – sekyedumase District was carved out of the former Sekyere and Offinso Disticts and was thus created as a results of the implementation of the decentralized progremmes on 29th November, 1988. The district is located within longitudes $1^{0}5W$ and $1^{0}39W$ and latitudes $7^{0}9N$ and $1^{0}36N$. It has a large land size of about 1782.2sq/km(690.781sq.miles) and is the fifth largest district in Ashanti region. Ejura – Sekyedumase district constitutes about 7.3% of the regions total land area with about one third of the land area lying in the Afram plains.

Ejura-Sekyedumase district is located in the northern part of the Ashanti region and is bounded in the north by Atebubu and Nkronza districts (both in the Brong/Ahafo region) on the west by Offinso district, on the East by Sekyere East district and the South by Mampong Municipal and Afigya Sekyere district.

The district is divided into four area councils, namely Ejura urban council, Sekyedumasi area council, Kasei area council and Dromankuma-Bonyon area council. Ejura is the district capital.

The Ejura – Sekyedumase District is made up of about 120 communities with the population and its size varying from each other. According to the 2000 Population and Housing Census all the communities have a population less than 6000 with the exception of Ejura and Sekyedumase which have population of 33907 and 11371 respectively. In 2005 the population of the district was estimated at 88753 living in over 120 settlements with a population density of 49.8 square kilometer. About 51.7% of the population is male and 48.3% females.

A study was conducted and the problems identified were:

1. Problem with environmental sanitation
2. Inadequate supply of portable water in the communities.
3. Inadequate health infrastructure
4. Inadequate staff accommodation

(Source: District Medium Term Development Plan. DMTDP, 2005-2010)

The district has seven health facilities all working to promote the health conditions of a population of 88753. Two out of the of the seven health facilities have attain the status of a hospital, that is the district hospital which is a referral hospital, located at the district capital, Ejura, and the other, which is private, at Kasei in the north – eastern part of the district.

Due to the large area to be covered and the long distances from the communities to the hospitals, it results in delays in accessing the hospitals. According to the district health service in the district about 239400 patients access the district hospital within the three year a period of 2008-2010. Table 1.0 gives the breakdown of Out-Patients summary in Ejura –Sekyedumase district hospital from 2008 – 2010.

Table 1.0: Statement of out–patient summary in Ejura-Sekyedumase district hospital from 2008-2010

Age Groups	2008		2009		2010	
	Male	**Female**	**Male**	**Female**	**Male**	**Female**
Below 1	1597	1237	2151	1845	2290	1969
1 to 4	6555	5222	6257	5380	7079	6174
5 to 9	3168	2801	3269	2920	3477	3217
10 to 14	1866	2015	1845	2005	2068	2236
15 to 17	1057	1279	1149	1698	1238	1898
18 to 19	684	1676	926	1927	1077	2374
20 to 34	5556	16600	4919	17300	5819	18426
35 to 49	4025	7228	3427	6927	3756	7825
50 to 59	1874	2362	1827	2453	2201	2906
60 to 69	1350	1450	1319	1809	1592	2015
70 and above	2441	3124	2511	3489	2887	4356
TOTAL	**30173**	**44994**	**29600**	**47753**	**33484**	**53396**

From table 1.0, 75167 people access the hospital in 2008 representing 68.59% of the total population. This reveals that majority of the people access their health needs in the district hospital. This also implies that there is a degree of pressure being posed on the existing facilities and health personnel in the district.

In 2009, 77353 representing 70.59% of the total population access the district hospital.

This shows clearly an increase in the people accessing the district hospital. This reveals that, still majority of the people access their health needs in the district hospital. Thus, the degree of pressure being posed on the existing facilities and personnel also increased.

In 2010, 86880 patients access the hospital. This also showed an overwhelming increase from the previous years. According to the district health director, the amount of pressure on the district hospital is too much and this makes personnel over- work. Thus the personnel feel reluctant to attend to emergency cases. It is therefore necessary and profitable to locate sites at Ejura - Sekyedumase to build a general hospital to serve the peoples in the district and its environs.

1.3 PROBLEM STATEMENT

Despite the fact that the two hospitals in the district have made effort to promote the health conditions of a population of about 88753, it can also be seen from Table 1.0 that majority of the people access their health needs within the district hospital because the community health centers and other health facilities in the district only give first aid to patients and refer most cases to the district hospital. This shows that there is a degree of pressure being imposed on the hospital. Based on the DMTDP, 2005 – 2010 report from the district health director, a second general hospital was recommended for district.

It is against this background that a mathematical model is needed to optimally locate sites for the establishment of additional hospital in the district to offset frustration in accessing the facilities.

1.4 OBJECTIVIES OF THE STUDY

The objectives of the study are:

(i) To model the location of an additional hospital using conditional p – median model.

(ii) To find the optimal location using Berman and Drezner algorithm.

1.5 METHODOLOGY

Location of facilities such as hospital can be considered a median problem or set covering problem. The problem at hand is a weighted graph. This could be solved by the Maximum Covering Location Model or the Conditional P-median Problem. The Conditional P- median model was used in this thesis, because the implementation of the Conditional P- median Model provides a systematic procedure for arriving at the necessary coverage distance based on choice of facility sites (P).

The objective of the study is to locate an additional hospital in the Ejura-Sekyedumase district using the conditional p-median problem. Data on road distances between major communities were collected from the district assembly office. Floyed Warshall algorithm was used to find the distance matrix, d(i, j) for all pairs shortest path.

Search on the internet was used to obtain related literature. The main library at KNUST was consulted in the course of the project.

1.6 JUSTIFICATION OF THE STUDY

In health care, the implication of poor location decisions extends well beyond cost and customer service consideration. If too few facilities are utilized and /or if they are not located well, increases in mortality and morbidity can result. Thus facility location modelling takes on an even greater importance when applied to the siting of health care facilities.

From the data above, it can be seen that the pressure and burden on the district hospital keep increasing every year as the population increases. This make people complain of inadequate facilities and personnel in the health institutions. Also because majority of the people access their health needs within the district there is always the tendency of very long queues in the hospital.

Patients at times can lose their lives if immediate care is not given and this is a loss of human resources to the country and also loss of bread – winners to some families.

With an additional hospital in the district, it would in turn help improve on the health status of the people in the district and in the country as a whole. It is hope that that the results of this study would help to inform the authorities in the Ejura-sekyedumase district about the right site to locate a hospital in the district.

1.7. ORGANIZATION OF THE THESIS

Chapter 1 looks at the introduction, the importance of general hospitals and types of hospitals. It also looks at the background of the study and the statement of the problem. It also briefly discusses the objectives of the study and the methodology used. Chapter 2 contains the literature review. Chapter 3 contains the methodology. Chapter 4 contains data analysis, modelling and results. The last chapter covers the conclusion and the recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

Undoubtedly, humans have been analyzing the effectiveness of locational decisions since they inhabited their first cave. The term "facility" is used here in its broadest sense. That it is meant to include entities such as air and maritime ports, factories, warehouses, schools, hospitals, just to mention a few. The long and voluminous history of location research results from several factors: some of these factors are physical, economical, social and environmental. Location decisions are frequently made at all levels of human organization from individuals and households to firms, government agencies and even international agencies. Such decisions are often strategic in nature. That is, they involve large sum of capitals resources and their economic effects are long term. In the private sector they have a major influence on the ability of a firm to compete in the market place. In the public sector they influence the efficiency by which jurisdictions provide public services and the ability of these jurisdictions to attract households and other economic activity (Daskin et al, 2001).

In locating a facility, usually we look for the best way to serve the demand points. This implies that we need to decide on:

i. The number and location of the facility to serve the demand ii. Size and capacity of each facility iii. The allocation of the demand points to open facilities iv. Optimizing some objective location function.

In general, facilities are divided into two groups, the first one are desirable to the nearby inhabitants, which try to have them as close as possible such as hospitals, fire stations, shopping stores and educational centers. The second group turns out to be undesirable for the surrounding population, which avoids them and tries to stay away from them such as garbage dump sites, chemical plants, nuclear reactors, military installations, prisons and pollution plants. Daskin (1995) discussed that Erkut and Neuman in 1989 distinguished betwwen Noxious (harzadeous to health) and Obnoxious (nuisance to lifestyle) facilities, although both can be simply regarded as undesirable. Moreover in the last decade, a new nomenclature has been develop to defined these oppositions: NIMBY (Not in my back yard), NIMNBY (Not in my neighbors back yard), and NIABY (Not in anyones back yard). (Capitivo and Climaco, 2008)

Another important way to measure the effectiveness of facility location is by evaluating the average (total) distance between the demand points and the facilities. When the average (total) distance decreases, the accessibility and effectiveness of the facilities increases. This relationship applies to both private and public facilities such as supermarkets, post offices as well as emergency service centers, for which proximity is desirable.

2.1 APPROACHES TO FACILITY LOCATION PROBLEMS

In Ogryczak and Malczewski (1990) the location of hospitals is formulated as a multiobjective optimization problem and an interactive approach DIN AS, Dynamimic interactive network analysis system (Ogryczak et al., 1989) based on the so called reference point approach (Wierzbicki,1982) is presented. A real application is presented, considering eight sites for potential location and at least four new hospitals to be built, originating in hundred and sixty three alternative location patterns each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision - making process making the final decision less subjective. They also observed that during the interactive process the decision – makers have gradually learned about the set of feasible alternatives and in consequence of this leaning process they have change their preference and priorities.

Christopher and Wills (1972) comprehensively present that whether the problem of depot location is static or dynamic, „Infinite Set‟ approaches and „Feasible Set‟ approach can be identified. The infinite set approach assumes that a warehouse is flexible to be located anywhere in a certain area. The feasible set approach assumes that only a finite number of known sites are available as warehouse locations. They believe the centre of gravity method is a sort of infinite set model.

Ballou (1998) states that exact centre of gravity approach is simple and appropriate for locating one depot in a region, since the transportation rate and the point volume are the only location factors. Given a set of points that represent source points and demand points, along with the volumes needed to be moved and the associated transportation rates, an optimal facility location could be found through minimizing total transportation cost. In principle, the total transportation cost is equal to the volume at a point multiplied by the transportation rate to ship to that point multiplied by the distance to that point. Furthermore, Ballou outlines the steps involved in the solution process in order to implement the exact centre of gravity approach properly. He discusses a selected number of facility location methods for strategic planning. He further classifies the more practical methods into a number of categories in the logistics network, which include single– facility location, multi–facility location, dynamic facility location, retail and service location.

Fonseca and Captivo (1996; 2006; 2007) study the location of semi obnoxious facilities as a discrete location problem on a network. Several bi-criteria models are presented considering two conflicting objectives, the minimization of obnoxious effect and the maximization of the accessibility of the community to the closest open facility. Each of these objectives is considered in two different ways, trying to optimize its average value over all the communities or trying to optimize its worst value. The Euclidean distance is used to evaluate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance between demand points and open facilities, and demand directly proportional to the population in each community. All the models are solved using Chalmet et al (1986), non- interactive algorithm for Bi-criteria Integer Linear Programming modified to an interactive procedure by Ferreira et al (1994). Several equity measures are computed for each non-denominated solution presented to the decision-maker, in order to increase the information available to the decision –maker about the set of possible solutions.

Ferreira et al (1996) present a bi-criteria mixed integer linear model for the facility location where the objectives are the minimization of total cost and the minimization of environmental pollution at facility sites. The interactive approach of Ferreira et al (1994) is used to obtain and analyze non-dominated solutions.

Giannikos (1998) presents a discrete model for the location of disposal or treatment facilities and transporting hazardous waste through a network linking the population centers that produce the waste and the candidate locations for the treatment facilities method to choose the location for a waste treatment facility

This sections begins with a review of three basic facility location models from which most other models are derived: the set covering model, p-center model and p-median model.

2.2 SET COVERING

The set covering problem seeks to minimize the number of facilities while locating them in order to cover all demands. In many covering problem, services that customers receive by facilities depend on the distance between the customer and facilities. In a covering problem the customer can receive service by each facility if the distance between the customer and facility is equal or less than a predefined number. This critical value is called convergence distance.

Church and Revelle (1974) model the maximization covering problem. Covering problem are divided into two branches, tree networks and general networks according to their graph. These problems are divided into two problems: Total covering and Partial covering problems, based on covering all or some demand points. The total covering problem is model by Toregas (1971). Up to the present time many developments have occurred about total covering and partial covering problems in solution technique and assumptions. Covering problems has many applications such as: designing of switching ciecuits, distributing products, warehouse locating and location emergency services (Francis et al. 1992).

According to Daskin et al (1988) there are circumstances where the provision of a service needs more than one "covering" facility, this occurs when facilities may not always be available. For example, assume that ambulances are being located at dispatching points in order to serve demand across an urban area, and the nearest ambulance is busy, then the next closest available ambulance will need to be assigned to a call when it is received. If the closest available ambulance is farther than the service standard then that demand/call for service is not provided service within the coverage standard. To handle such issues, models have been developed that seek multiple - coverage. Two examples of multiplecoverage exist, stochastic/probabilistic and deterministic.

Daskin (1983) formulated a probabilistic multiple cover model called the maximal expected coverage model. Hogan and ReVelle (1986) also formulated the simple back up covering model as a good example of a deterministic cover model that involves maximizing second-level coverage. Toregas (1971) was the first to recognise the possible need for multi-level coverage. Toregas defined the Multi-level Location Set Covering Problem (ML-LSCP) as a search for the smallest number of facility needed to cover each demand, a preset number of times, where the need for coverage might vary between demands.

2.3 THE P- CENTRE PROBLEM.

The P- centre model minimizes the maximum distance between any demand point and it nearest facility. This model is introduced under the title p-centre problem which is in fact a minimax problem. In this model the objective is to find locations of p-facilities so that all demands are covered and the nearest facility (coverage distance) is minimized. It can be said that we have relaxed the coverage distance (Daskin, 1995).

In the p-centre model, each demand point has a weight. These weights may have different interpretations such as time per unit distance, cost per unit distance or loss per unit distance (Daskin, 1995). So the problem would be seeking a centre to minimize a maximum time, cost or loss. In other words the concern is about the worst case and we want to make it as good as possible (Francis et al.1992).

Garfinkel et al. (1977) examined the fundamental properties of the P-centre problem in order to locate a given number of emergency facilities along a road network. He modelled the P-centre problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

ReVelle and Hogan (1989) formulated a P-centre to locate facilities so as to minimize the maximum distance within which the EMS is available with (alpha) reliability. System congestion is considered and a derived server busy probability is used to constrain the service reliability that must be satisfied for all demands.

Hochbaun and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods using the Stochastic P-centre models. The cost and distance between locations vary in each discrete time periods. The authors used k underlying networks to represent different periods and provided a polynomial-time, 3-approximation algorithm to obtain a solution for each problem.

Chen and Chen (2009), presented a new relaxation algorithm for solving the conditional continuous and discrete p-center problems. In the continuous p-center problem, the location of the service facilities can be anywhere in the two-dimensional Euclidean space. In the discrete variant there is a finite set of potential service points to choose from. An analogous representation of the discrete p-center problem is the p-center problem on networks. In the p-center problem on networks, both the demand points and the potential service points are located on a weighted undirected graph, and the distance between any two points is the cost of the shortest path between them. They assumed that, there are a finite number of values for the optimal solution of an unconditional p-center problem. They use the assumption to implement the subroutine Get- Next Bound (LowerBound) which returns the smallest value, among the possible values for the optimal solution, which is greater than Lower-Bound.

Hassin et al. (2003) introduce a local search strategy that suits combinatorial optimization problems with a min-max (or max-min) objective. According to this approach, solutions are compared lexicographically rather than by their worst coordinate. They apply this approach to the *p*-center problem. Based on a computational study, the lexicographic local search proved to be superior to the ordinary local search. This superiority was demonstrated by a worst-case analysis.

Cheng et al. (2005) worked on the Improved Algorithm for the p-Center Problem on Interval Graphs with Unit Lengths. They presented an O (n) time algorithm for the problem under the assumption that the endpoints of the intervals are sorted, which improves on the existing best algorithm for the problem that has a run time of O (pn). They modeled the network as a graph G = (V, E), where V is the vertex set with |V | = n and E is the edge set with |E| = m. it was assumed that, the demand points coincide with the vertices, and the location of the facilities was restricted to the vertices. Also they assumed that each edge of E has a unit length. It remains an interesting question whether they could develop an approximation algorithm for the p-center problem on interval graphs with general edge lengths.

2.4 THE P- MEDIAN PROBLEM

The p-median problem is one of the most widely used location models. Several facilities are to be located in an area to satisfied demand. Every demand point is serviced by the closest facility. The objective is to minimize the total transportation cost by the selection of the best sites for the new facilities. Distances between demand nodes and facilities are multiplied by a weight usually associated with the demand node. In the unweighted problem, all nodes are treated equally.

The p-median problem belongs to a class of formulations called minisum location models.

The problem is stated as:

Find the location of a fixed number of p facilities so as to minimize the weighted average distance of the system.

The first explicit formulation of the p-median problem is attributed to Hakimi (1964). Hakimi not only stated the formulation of the problem, but also proved that in a connected network, optimal locations can always be found on nodes. Later ReVelle and Swain (1970) formulated the *p*-median problem as a linear integer program and used a branch-and-bound algorithm to solve the problem.

Goldman (1971) provided simple algorithms for locating a single facility for both acyclic network (a tree) and a network containing exactly one cycle.

Beasly (1993) has also develop lagrangian heuristics for this p-median problem, based on lagrangian relaxation and subgradient optimization concepts. Pasamosca (1991), considers the interaction weights between the new facilities as well as the connection scheme as a tree. This case was treated as a problem of Euclidean distance multifacility location problem (EMFLP) on a large tree and its optimality were obtained using the optimality conditions of p problems of the type ESFL (Euclidean single facility location problem). Another type of variant involves placing the capacity restrictions on the facilities to be located. When the capacity is finite, the resulting problem is called a capacitated problem, otherwise the problem is uncapacitated.

Cavalier and Sherali (1986) presented exact algorithms to solve the p-median problem on a chain graph and the 2-median problem on a tree graph where the demand density functions are assumed to be piecewise uniform. For the uncapacitated p-median problem, Chiu (1987) address the 1-median problem on a general network as well as on a tree network. Dynamic location considerations on networks are address by Sherali (1991).

Francis et-al (1992) developed a median row-column aggration algorithm to slove largescale rectilinear distance p-median problems. Sherali and Nordai (1988) gave certain localization results and algorithms for solving the capacitated p-median problem or a chain graph and the 2-median problem on a tree graph.

Since its formulation, the p-median model has been enhanced and applied to a wide range of emergency facility location problems. Carbone (1974) formulated a deterministic *P*median model with the objective of minimizing the distance traveled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node is uncertain, Carbone, further extended the deterministic *P*-median model to a chance constrained model. The model seeks to maximize a threshold and meanwhile ensure the probability that the total travel distance is below the threshold is smaller than a specified level α.

Calvo and Marks (1973) constructed a *P*-median model to locate multi-level health care facilities including central hospitals, community hospitals and local reception centers. The model seeks to minimize distance and user costs, and maximize demand and utilization. Later, the hierarchical *P*-median model was improved by Tien et al. (1983) and Mirchandani (1987) by introducing new features and allowing various allocation schemes to overcome the deficient organization problem across hierarchies. Paluzzi (2004) discussed and tested a *P*-median based heuristic location model for placing emergency service facilities for the city of Carbondale, IL. The goal of this model is to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. The results were compared with the results from other approaches and the comparison validated the usefulness and effectiveness of the *P*-median based location model.

One major application of the *P*-median models is to dispatch EMS units such as ambulances during emergencies. Carson and Batta (1990) proposed a *P*-median model to find the dynamic ambulance positioning strategy for campus emergency service. The model uses scenarios to the demand conditions at different times. The ambulances are relocated in different scenarios in order to minimize the average response time to the service calls. Berlin et al. (1976) investigated two *P*-median problems to locate hospitals and ambulances. The first problem has a major attention to patient needs and seeks to minimize the average distance from the hospitals to the demand points and the average ambulance response time from ambulance bases to demand points. In the second problem, a new objective is added in order to improve the performance of the system by minimizing the average distance from ambulance bases to hospitals. Mandell (1998) developed a *P*-median model and used priority dispatching to optimally locate emergency units for a tiered EMS system that consists of advanced life-support (ALS) units and basic life-support (BLS) units. The model can also be used to examine other system parameters including the balance between ALS and BLS units, and different dispatch rules

Uncertainties have also been considered in many *P*-median models. Mirchandani (1980) examined a *P*-median problem to locate fire-fighting emergency units with consideration of stochastic travel characteristics and demand patterns. The author took into account the situations that a facility may not be available to serve a demand and used a Markov process to create a system in which the states were specified according to demand distribution, service and travel time, and server availability. Serra and Marianov (1999) implemented a *P*-median model and introduced the concept of regret and minmax objectives when locating fire station for emergency services. They explicitly addressed in their model the issue of locating facilities when there are uncertainties in demand, travel time or distance. In addition, the model uses scenarios to incorporate the variation of uncertainties and seeks to give a compromise solution by minimizing the maximum regret over the scenarios.

P-median models have also been extended to solve emergency service location problems in a queuing theory context. An example is the stochastic queue median (SQM) model due to Berman et al. (1985). The SQM model seeks to optimally dispatch mobile servers such as emergency response units to demand points and locate the facilities so as to minimize average cost of response.

2.4.1 CONDITIONAL LOCATION PROBLEM

Every application to the p-median problems becomes a conditional model when already there exist some facilities in the area of study.

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When q is equal to zero (q = 0), the problem is unconditional. In conditional p – median problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman, 2008). As an example, if one wants to locate p warehouses in an area, it is unconditional p-median problem. However, when q warehouses already exist in the area and we need to add up new warehouses it becomes a conditional location problem.

Handler and Mirchandani (1979) first studied conditional location location problems. In those references, the studied conditional problem was a conditional 1 – center problem (p = 1, and $q \geq 1$). Chen and Handler (1990, 1993) develop the conditional problem with $p \geq 1$ new facilities.

Drezner (1989) solves the conditional p – center problem by an algorithm that requires the solution of O(logn) unconditional p – centre problems (n being the number of demand nodes). The method proposed by Drezner is applicable to both planar and network configurations.

Berman and Simchi – Levi (1990) suggested to solve the conditional p – median and p – centre problems on a network by an algorithm that requires on time solution of an unconditional (p + 1) - median or (p + 1) – centre problem.

CHAPTER 3

METHODOLOGY

3.0 FACILITY LOCATION PROBLEMS

Facility location problems have occupied an important place in operation research since the early 1960"s. They investigate where to physically locate a set of facilities so as to optimize a given function subject to set of constraints.

Facility location models are used in a wide variety of applications. Examples include locating warehouses within a supply chain to minimize the average travel time to the markets, locating hazardous materials sites to minimize exposure to the public, locating railroad stations to minimize the variability of delivery schedules, locating automatic teller machines to best serve the banks customers, locating a coastal search and rescue station to minimize the maximum response time to maritime accidents and locating of hospitals to best serve the people in the area (Hale and Moberg, 2003).

There are different types of facility location problems. Some basic classes of facility location problems are listed below (Berman and Krass, 2002).

1. Discrete facility location problem: location problem where the sets of demand points and potential facility locations are finite.
2. Continuous facility location problem: location problem in a general space endowed with some metric, example l_p norm. Facilities can be located anywhere in the given space.
3. Network facility location problem: Location problem which is confined to the links and nodes of an underlying network.
4. Stochastic facility location problem: location problem where some parameters, example demand or travel time, are uncertain.

We can furthermore classify a model as capacitated as opposed to uncapacitated where the former term refer to the upper bound on the number of clients (or demand) that a facility can serve. Models are called dynamic (as opposed to static) if the time element is explicitly represented (Wesolowsky, 1973).

The problems on which this thesis is focus on can be characterised as discrete. Current et al (2002) listed several basic discrete network location models: Covering (including Set Covering and Maximal covering),*p*-center, *p*-dispersion, *p*-median, fixed charge, hub and maxisum. Distances or some related measures (e.g travel time or cost) are fundamental to such problems. Consequently , we classify them according to their consideration of distance . The *p*-center, *p*-dispersion and *p*-median are based on maximum distance where as the hub, and maxisum are based on total or average) distance.

3.1 TOTAL OR AVERAGE DISTANCE MODEL

Many facility location planning situations in the public and private sectors are concerned with the total travel distance between facilities and demand nodes. An example in the private sector might be the location of warehouses that receive their inputs from established sources by truckload deliveries. In the public sector, one might want to locate a network of service providers such as hospitals and schools in such a way as to minimize the total distance that people must traverse to reach their closet facility. This approach may be viewed as an "efficiency" objective as opposed to the "equity" objective of minimizing the maximum distance, which is mentioned in other models.

1. *P*-median problem: the *p*-median problem (Hakimi, 1964,1965) seeks to find the locations of *p* facilities to minimize the demand- weighted total distance between demand nodes and the facilities to which they are assigned

2. The maxisum location problem: the maxisum location problem seeks to location p-facilities (undesirable facilities) such that the total demand – weighted distance between demand nodes and the facilities to which they are assigned is maximized

3. 2 MAXIMUM DISTANCE MODELS

In some locations problems a maximum distance exists a priori. For example in many districts people within a mile of their hospital must walk to hospital. Transportation must be provided for those not within this maximum distance.

A district might want to locate hospitals to minimise the number to people who must be bussed at a cost.

In the facility location literature a priori maximum distances such as these are known as "covering" distances. Demand within the covering distance of its closest facility is considered "covered".

An underlying assumption of this measure of maximum distance is that demand is fully satisfied if the nearest facility is within the coverage distance and is not satisfied if the closest facility is beyond that distance. That is, being closer to a facility than the maximum distance does not improve satisfaction

1. Set Covering Location Model: The first location covering location problem was the set covering problem (Toregas et al, 1971). The objective is to locate the minimum number of facilities required to "cover" all of the demand nodes.

   The Set Covering model may be formulated mathematically using the following notations:

$$a_{ij} = \begin{cases} 1, & \text{if demand node } i \text{ is can be covered by facility at candidate site } j \\ 0, & \text{if not} \end{cases}$$

The Set Covering model attempts to minimise the cost of the facilities that are selected so that all demand nodes are covered. To formulate this model, we need the following additional sets and inputs:

I = set of demand nodes

J = set of candidate facility sites

f_j = fixed cost of locating a facilityat candidate site j In

addition, we need the following decision variable

$$X_{ij} = \begin{cases} 1, & \text{if we locate at candidate site } j \\ 0, & \text{if not} \end{cases}$$

With this notation, we can formulate the set covering problem as follows;

Minimise

$$\sum_{j \in J} f_j X_j \qquad (1)$$

Subject to

$$\sum_{j \in J} a_{ij} X_j \geq 1 \qquad \forall\, i \in I \qquad (2)$$

$$X_j \in \{0, 1\} \qquad \forall j \in J \qquad (3)$$

The objective function (1) minimises the total cost of all selected facilities. Constraint (2) stipulates that each demand node must be covered by at least one of the selected facilities. The left hand side of (2) represents the total number of selected facilities that can cover demand node i. In that case, the objective function becomes

Minimise $\sum_{j \in J} X_j$ (4)

To distinguish between these two model variants we will refer to the problem with (1) as the objective function as the set covering problem or model. When (4) is used, we will call the problem the location set covering problem.

In practice, at least two major problems occur with the set covering model. First, if (1) is used as the objective function, the cost of covering all demand is often prohibitive. If (4) is used as the objective function, the number of facilities required to cover all demands is often too large. Second, the model fails to distinguish between demand nodes that generate a lot of demand per unit time and those that generate relatively little demand.

2. Maximal covering location problem (MCLP). The maximal covering location problem (MCLP, Church and Revelle, 1974) was formulated to address planning situations which have an upper limit on the number of facilities to be sited.

The objective of the maximal covering location problem is to locate a predetermined number of facilities, p in such a way as to maximize the demand that is covered.

Thus, the maximal covering location problem assumed that there may not be enough facilities to cover all of the demand nodes. If not all nodes can be covered, the model seeks the sitting scheme that covers the most demand.

The maximal covering location problem was formulated by Church and ReVelle (1974) by defining the following additional inputs;

$h_i =$ demand at node i P = number of facilities to locate as well as the following additional decision

variable.

$$Z_i = \begin{cases} 1, & \text{if demand } i \text{ is covered} \\ 0, & \text{if not} \end{cases}$$

With this additional notation, the maximal covering location problem can be formulated as follows;

Maximize

$$\sum_{i \in I} h_i Z_i \qquad (5)$$

Subject to

$$Z_i - \sum_{j \in J} a_{ij} X_j \leq 0 \qquad \forall i \in I \qquad (6)$$

$$\sum_{j \in J} X_j = p \qquad (7)$$

$$X_j \in \{0,1\} \qquad \forall j \in J \qquad (8)$$

$$Z_i \in \{0,1\} \quad \forall i \in I \qquad (9)$$

The objective function (5) maximizes the number of covered demands. It is important to note that this model maximizes demands that are covered and not simply nodes. Constraint (6) states that demand node *i* cannot be counted as covered unless we locate at least one facility that is able to cover the demand node. Constraint (7) states that exactly p facilities are to be located and constraint (8) and (9) are standard integrality constraints.

3. The *p*-center problem: The *p*-center problem (Hakimi, 1964, 1965) addresses the problem of minimizing the maixmum distance, that demand is from its closest facility, given that we are sitting a pre-determined number of facilities.

4. The *p*-dispersion problem: For all of the models discussed, the concern is with the distance between demand and new facilities. The *p*-dispersion problem (PDP) differs from those problems in two ways (Kuby, 1987). First, it is concerned only with the distance between new facilities.

   Second, the objective is to maximize the minimum distance between any pair of facilities. An application of the p-dispersion problem is the siting of military installations where separation makes them difficult to attack

   To formulate this model we require an additional input (M) and a decision variable (D)

   M = a large constant (eg. $\max_{i\in I, j\in J}\{d_{ij}\}$)

   D = the minimum separation distance between any pair of facilities.

   With this notation, the p-dispersion model may be formulated as follows:

   Maximize D (10)

   Subject to:

$$\sum_{j\in J} X_j = p \quad (11)$$

$$D + (M - d_{ij})X_j + (M - d_{ij})X_j \leq 2M - d_{ij} \quad \forall\, i, j \in J, i < j \quad (12)$$

$$X_j \in \{0,1\} \;\forall\; j \in J \quad (13)$$

   The objective function (10) maximizes the distance between the two closet facilities. Constraint (11) requires that p facilities are located. Constraint (13) is a standard integrality constraint. Constraint (12) defines the minimum separation between any pair of open facilities. Note that if either X_i or X_j is zero then the constraint will not be binding. However , if both are equal to 1, then the constraint is equivalent to $D \leq d_{ij}$. Therefore, maximizing *D* has the effect of forcing the smallest inter- facility distance to be as large as possible

3.3 *P*-CENTER PROBLEM

The *p*-center problem which was also introduced first by Hakimi (1964-1965) is to find the facility locations such that the maximum distance between any demand point (customer) and its respective nearest facility is minimised

It has been used to model locations of emergency facilities such as ambulance stations and firehouses, the location of a helicopter to minimise the maximum time to respond to an emergency, and the location of a transmitter to maximise the lowest signal level received in a communication network (Caruso et al, 2003)

There are several possible variations of the basic model. If facility locations are restricted to the nodes of the network, the problem is referred to as a "vertex" *p*- center problem. Center problems which allow facilities to be located anywhere on the network are known as "absolute" *p*-centre problem. Both versions can be either weighted or unweighted. In the weighted problem, the distances between demand nodes and facilities are multiplied by a weight usually associated with the demand node. In the unweighted problem, all demand nodes are treated equally.

Given our previous definitions and the following decision variables

W= the maximum distance between a demand node and the facility to which it is assigned.

$$y_{ij} = \begin{cases} 1, & \text{if demand node } i \text{ is assigned to a facility} \\ 0, & \text{if not} \end{cases}$$

The p-center problem can be formulated as follows

Maximize W (1)

Subject to:

$$\sum_{j \in J} x_j = p \qquad (2)$$

$$\sum_{j \in J} y_{ij} = 1 \qquad \forall i \in I \qquad (3)$$

$$y_{ij} - x_j \leq 0 \qquad \forall i \in I, j \in J \qquad (4)$$

$$W - \sum_{j \in J} h_i d_{ij} y_{ij} \geq 0 \qquad \forall i \in I \qquad (5)$$

$$x_j \in \{0,1\} \qquad \forall j \in J \qquad (6)$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \qquad (7)$$

The objective function (1) minimises the maximum demand weighted distance between each demand node and its closest open facility. Constraint (2) stipulates that *p* facilities are to be located. Constraint set (3) required that each demand node be assigned to exactly one facility. Constraint set (4) restricts demand node assignments only to open facilities. Constraint (5) defines the lower bound on the maximum demand- weighted distance which is being minimised. Constraint set (6) established of the siting decision variable as binary. Constraint set (7) requires the demand at a node to be assigned to one facility only. Constraint set (7) can be replaced by $y_{ij} \geq 0 \;\; \forall i \in I, j \in J$ because

constraint set (4) guarantees that $y_{ij} = 1$. If some y_{ij} are fractional, we can simply assign node *i* to its closet open facility.

3.4 THE *P*-MEDIAN PROBLEM

The *p*-median problem is with no doubt, one of the most studied facility location models. Basically, the *p*-median problem seeks to find the location of *p* facilities to minimise the demand-weighted total distance between demand nodes and the facilities to which they are assigned. Therefore, demand is assigned to the closest facility. This model may be formulated as follows:

Minimise $\sum_{i\in I}\sum_{j\in J} h_i d_{ij} y_{ij}$ (1.1)

Subject to:

$$\sum_{j\in J} x_j = p \qquad (1.2)$$

$$\sum_{j\in J} y_{ij} = 1 \qquad \forall i \in I \qquad (1.3)$$

$$y_{ij} - x_j \le 0 \qquad \forall i \in I, j \in J \qquad (1.4)$$

$$x_j \in \{0,1\} \qquad \forall j \in J \qquad (1.5)$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \qquad (1.6)$$

Where:

i Index of demand point *j* Index of

potential facility sites

h_i Weight associated to each demand point

d_{ij} Distance between demand area *i* and potential facility at *j*

y_{ij} Variable that is equal to 1if demand area *i* is assigned to a facility at *j*, and 0 otherwise

x_j Variable that is equal to 1 if there is an open facility at *j*, and 0 otherwise

The objective function (1.1) minimises the demand-weighted total distance travelled. Constraint set (1.2) through (1.4) are identical to (2) through (4) of the p-center problem. Constraint sets (1.5) and (1.6) are identical to (6) and (7) of the p-center problem. Constraint set (1.6) can be eliminated following the same arguments as were usedfor constraint set (7). Toregas and ReVelle (1972) show that this formulation also minimises the average travel distance between the sited facilities and the demand.

This formulation (1.1 - 1.6) assumes that the potential facility sites are nodes on the network. Hakimi (1964) proved that relaxing the problem to allow facility locations on the arcs of the network would not reduced total travel cost. Consequently, this formulation will yield an optimal solution, even if the facilities could be located anywhere on the arc. Like the p-center problem, the p-median problem can be solved in polynomial time for fixed values of p, but is NP-hard for variable values of p.(Garey and Johnson, 1979)

3.5 THE CONDITIONAL *P*-MEDIAN PROBLEM

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When $= 0$, the problem is considered as unconditional. In the conditional p-median, once the new p locations are determined, a demand can be served either by one of the new facilities whichever is the closest facility to the demand.

Every application to the p-median problem becomes a conditional model when already there exist some facilities in the area under study. As an example, if one wants to locate p hospitals in an area, it is an unconditional median problem. However, when q hospitals already exist in the area and we need to add p new hospitals, it becomes a conditional pmedian problem.

FORMULATON OF THE PROBLEM

Let $G = (N, L)$ be a network with N being the set of nodes, $|N| = n$ and L being the set of links. Consider a non-negative number w_i that represent the demand weight at node $i \in N$. Let d_{xy} be the shortest distance between any two nodes.

Suppose that there is a set $Q(|Q| = q)$ of existing facilities. Let $Y = (Y_{1,} \ Y_{2,} \ Y_{3,} \ \dots\dots, Y_q)$ and $X = (X_1 \ X_{2,} \ X_{3,} \ \dots\dots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and X_i is the location of new facility i. Without any loss of generality we do not need to assume that $Y_i \in N$. The conditional p-median location problem can now be expressed as minimizing

$$f(x) = \sum_{i=1}^{n} wi \min\{d(X, \ i), d(Y, i)\}$$

Where $d(X, \ i)$ (or $d(Y, i)$) is the shortest distance from the closest facility in X (or Y) to node i.

3.5.1 THE ALGORITHM OF BERMAN AND SIMCHI- LEVI

The idea is to produce a new potential location representing all the existing facilities. If a demand point is utilizing the services of an existing facility, it will use the services of the closest existing facility. Thus, the distance between a demand point and the new location is the minimum distance among all the existing facilities.

Step 1

Let D be the shortest distance matrix with rows corresponding demands and columns corresponding to potential locations. In order to force the formation of a facility at the new location, a new demand point is considered with a distance of zero from the new potential location and a large distance from all the other potential locations.

Step 2

The new distance matrix, denoted by $\widehat{D}$, is constructed by adding a new location a_0 (a new column) to D which represents the Q existing locations and a new demand point v_0 with an arbitrary positive weight. For each regular demand point (node) *i*, we have

$d(i, a_0) = \min_{k \in Q}\{d_{ik}\}$ and $d(v_0, a_0) = 0$. For each regular potential location node j ,

$d(v_0, j) = M$, where *M* is a large number. Again the nodes in *Q* and in the potential locations *Q* are removed.

Step 3

Find the optimal new location by using the modified distance matrix $\widehat{D}$ for the network with objective function

$$min\left[f(x) = \sum_{i=1}^{n} w_i \, min\{d(X, i), \;\; d(Y, i)\}\right]$$

To illustrate the approach, we consider the network in figure 3.0 below, where the numbers next to the links are lengths and the numbers next to the nodes are weight. Suppose that the existing facilities are *Q* = {3, 5, 6} and only one facility is to be located.(*p* = 1)

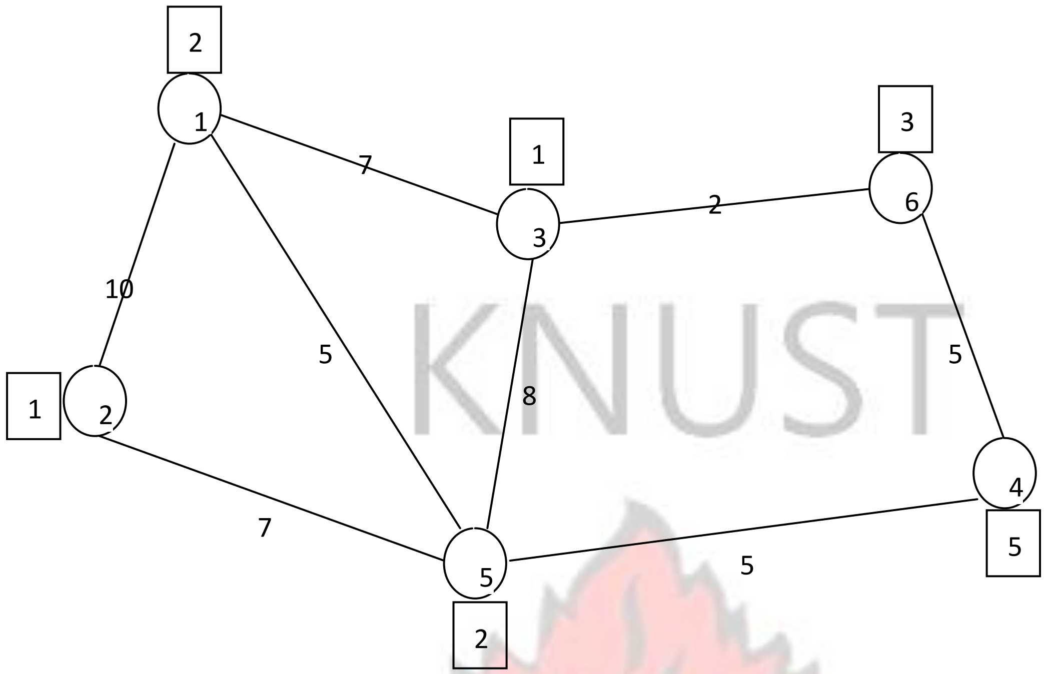

Figure 3.0: Sample network for p-median problem

STEP 1: Using Floyd-Warshall algorithm, we obtained the shortest distance matrix D, for the above network, with column 1 and row 1 representing the demand nodes and potential location respectively, and each other row represents the interconnected distances

Table 3.0 All pairs shortest path distance matrix, D.

Demand nodes	Potential location					
	1	2	3	4	5	6
1	0	10	7	10	5	9
2	10	0	15	12	7	17
3	7	15	0	7	8	2

4	10	12	7	0	5	5
5	5	7	8	5	0	10
6	9	17	2	5	10	0

STEP 2: The new distance matrix, denoted by $\widehat{D}$, is constructed by adding a new location a_0 (a new column) to D which represents the Q existing locations and a new demand point v_0 with an arbitrary positive weight. For each regular demand point (node) i, we have $d(i, a_0) = \min_{k\in Q}\{d_{ik}\}$ and $d(v_0, a_0) = 0$. For each regular potential location node j, $d(v_0, j) = M$, where M is a large number.

$$d(i, a_0) = \min_{k\in Q}\{d_{ik}\} \qquad Q = \{3, 5, 6\}$$

$i = 1$

$d(1, a_0) = \min\{d(1,3), d(1,5), d(1,6)\}$

$= \min\{7, 5, 9\} = 5$

$i = 2$

$d(2, a_0) = \min\{d(2,3), d(2,5), d(2,6)\}$

$= \min\{15, 7, 17\} = 7$

$i = 3$

$d(3, a_0) = \min\{d(3,3), d(3,5), d(3,6)\}$

$= \min\{0, 8, 2\} = 0$

$i = 4$

$d(4, a_0) = \min\{d(4,3), d(4,5), d(4,6)\}$

$= \min\{7, 5, 5\} = 5$

$i = 5$

$$d(5\,,a_0) = \min\{d(5,3), d(5,5), d(5,6)$$

$$= \min\{8, 0, 10\} = 0$$

$i = 6$

$$d(6\,,a_0) = \min\{d(6,3), d(6,5), d(6,6)$$

$$= \min\{2, 10, 0\} = 0$$

$d(v_0, a_0) = 0$ $\qquad$ $d(v_0, j) = M$

Table 3.1 The Modified Distance matrix, $\widetilde{\boldsymbol{D}}$

Demand nodes	**Potential location**						
	1	**2**	**3**	**4**	**5**	**6**	$\boldsymbol{a_0}$
1	0	10	7	10	5	9	5
2	10	0	15	12	7	17	7
3	7	15	0	7	8	2	0
4	10	12	7	0	5	5	5
5	5	7	8	5	0	10	0
6	9	17	2	5	10	0	0
$\boldsymbol{v_0}$	M	M	M	M	M	M	0

The nodes in Q representing existing facilities nodes are removed. This is shown in Table 3.2 below.

Table 3.2 Modified shortest path distance matrix, $\widehat{\boldsymbol{D}}$ with existing facility nodes removed.

Demand nodes	**Potential location**			
	1	2	4	$\boldsymbol{a_0}$
1	0	10	10	5
2	10	0	12	7

4	10	12	0	5
v_0	M	M	M	0

STEP 3: Find the optimal new location by using the modified distance matrix $\widehat{\boldsymbol{D}}$ and the objective function. Minimize

$$[f(x) = \sum_{i=1}^{n} w_i \, min\{d(X,i), \;\; d(Y,i)\}]$$

Find $\min\{d(X,i), d(Y,i)$

$X = \{1, 2, 4, \boldsymbol{a_0}\}$ Y = {3, 5, 6}

At X = 1 i = 1 i

= 2

$\min\{d(1,1), d(3,1), d(5,1)\; d(6,1)\}$ $\min\{d(1,2), d(3,2), d(5,2), d(6,2)\}$

$\min\{0, 7, 5, 9\} = 0$ $\min\{10, \; 15, \; 7, 17\} = 7$

i = 3 i = 4

$\min\{d(1,3), d(3,3), d(5,3)\; d(6,3)\}$ $\min\{d(1,4), d(3,4), d(5,4), d(6,4)\}$

$\min\{7, 0, 8, 2\} = 0$ $\min\{10, \; 7, \; 5, 5\} = 5$

i = 5 i = 6

$\min\{d(1,5), d(3,5), d(5,5)\; d(6,5)\}$ $\min\{d(1,6), d(3,6), d(5,6), d(6,6)\}$

$\min\{5, 8, 0, 10\} = 0$ $\min\{9, \; 2, \; 10, 0\} = 0$

At $X = 2$

$i = 1$

$\min\{d(2,1), d(3,1), d(5,1)\ d(6,1)\}$

$\min\{10, 7, 5, 9\} = 5$

$i = 2$

$\min\{d(2,2), d(3,2), d(5,2), d(6,2)\}$

$\min\{0,\ 15,\ 7, 17\} = 0$

$i = 3$

$\min\{d(2,3), d(3,3), d(5,3)\ d(6,3)\}$

$\min\{15, 0, 8, 2\} = 0$

$i = 4$

$\min\{d(2,4), d(3,4), d(5,4), d(6,4)\}$

$\min\{12,\ 7,\ 5, 5\} = 5$

$i = 5$

$\min\{d(2,5), d(3,5), d(5,5)\ d(6,5)\}$

$\min\{7, 8, 0, 10\} = 0$

$i = 6$

$\min\{d(2,6), d(3,6), d(5,6), d(6,6)\}$

$\min\{17,\ 2,\ 10, 0\} = 0$

At $X = 4$

$i = 1$

$\min\{d(4,1), d(3,1), d(5,1)\ d(6,1)\}$

$\min\{10, 7, 5, 9\} = 5$

$i = 2$

$\min\{d(4,2), d(3,2), d(5,2), d(6,2)\}$

$\min\{12,\ 15,\ 7, 17\} = 7$

$i = 3$

$\min\{d(4,3), d(3,3), d(5,3)\ d(6,3)\}$

$\min\{7, 0, 8, 2\} = 0$

$i = 4$

$\min\{d(4,4), d(3,4), d(5,4), d(6,4)\}$

$\min\{0,\ 7,\ 5, 5\} = 0$

$i = 5$

$\min\{d(4,5), d(3,5), d(5,5)\ d(6,5)\}$

$\min\{5, 8, 0, 10\} = 0$

$i = 6$

$\min\{d(4,6), d(3,6), d(5,6), d(6,6)\}$

$\min\{5,\ 2,\ 10, 0\} = 0$

The results are summarized and shown below in Table 3.3; with column 5 representing the new potential location.

Table 3.3 Optimal new location matrix using the modified shortest distance matrix

Demand node	Potential location		
	1	2	4
1	0	7	5
2	5	0	5
4	5	7	0

Finding the optimal new location using the modified shortest distance, $\widehat{D}$ and the objective function.

$$min\left[f(x) = \sum_{i=1}^{n} w_i\, min\{d(X,i),\ \ d(Y,i)\}\right]$$

At node 1 ***i*** **= 1**

$2(0) + 1(7) + 5(5) = \mathbf{32}$

At node 2 ***i*** **= 2**

$2(5) + 1(0) + 5(5) = 35$

At node 4 ***i*** **= 4**

$2(5) + 7(1) + 5(0) = 17$

From the above objective function values it can be easily be verify that the optimal new location using $\widehat{D}$ is node 4 with an objective function value of 17 because it is the minimum objective function value. Hence the new location for the facility is node 4.

3.5.2 BERMAN AND DREZNER'S ALGORITHM.

Berman and Drezner (2008) discuss a very simple algorithm that solves the conditional p-median problem on a network. This algorithm requires one-time solution of an unconditional p-median problem using an appropriate shortest distance matrix, rather than creating a new location for an artificial facility, and forcing the algorithm to locate a new facility, thereby creating an artificial demand point. Berman and Drezner"s algorithm just modify the shortest distance matrix.

Steps

1. Let D be the shortest path distance matrix with rows corresponding to demands and columns corresponding to potential locations.
2. Modified the shortest path distance matrix, from D to $\widehat{D}$. That is

   $\widehat{D}_{ij} = \min\{d_{ij}, \min\{d_{ik}\}\} \quad \forall\, i \in N, j \in N$ (median).

It should be noted that $\widehat{D}$ is not symmetric even when D is symmetric. The unconditional p-median problem using the appropriate $\widehat{D}$ solves the conditional p-median problem.

This is so since if the shortest distance from node i to the new p facilities is larger than $min_{k\in Q}\{d_{ik}\}$ then the shortest distance to the existing q facility is utilised.

3. Find the optimal new location by using the modified distance matrix $\widehat{D}$ for the network with objective function

$$min\left[f(x) = \sum_{i=1}^{n} w_i\, min\{d(X,i),\ d(Y,i)\}\right]$$

To illustrate the approach, we consider the network in figure 3.1 below, where the numbers next to the links are lengths and the numbers next to the nodes are weight. Suppose that the existing facilities are $Q = \{3, 5, 6\}$ and only one facility is to be located.($p = 1$)

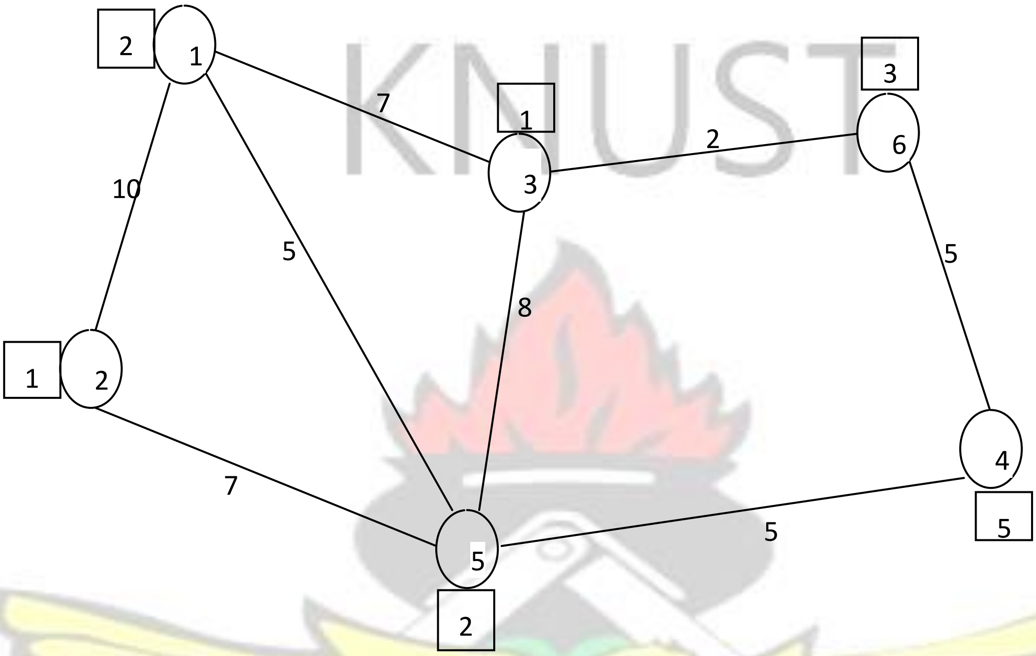

Figure 3.1 Sample network for p-median problem

STEP 1: Using Floyd-Warshall algorithm, we obtained the shortest distance matrix D, for the above network, with column 1 and row 1 representing the demand nodes and potential location respectively, and each other row represents the interconnected distances.

Table 3.4 All pairs shortest path distance matrix, D.

Demand nodes	**Potential location**					
	1	**2**	**3**	**4**	**5**	**6**
1	0	10	7	10	5	9
2	10	0	15	12	7	17
3	7	15	0	7	8	2
4	10	12	7	0	5	5
5	5	7	8	5	0	10
6	9	17	2	5	10	0

STEP 2: Determine a modified shortest distance matrix by:

$$\widehat{D}_{ij} = \min\left\{d_{ij,}\ min_{k\in Q}\{d_{ik}\}\right\} \quad \forall\, i \in N,\ j \in N$$

For node 1 Q = {3, 5, 6}

i = 1, *j* = 1

$$\widehat{D}_{11} = \min\{d_{11\,,}\ \ min\{d_{13}, d_{15}, d_{16}\}\}$$
$$= \min\{0,\ \ min\{7\,,5\,,\ 9\}\}$$
$$= \min\{0,\ \ 5\} = 0$$

i = 1, *j* = 2

$$\widehat{D}_{12} = \min\{d_{12\,,}\ \ min\{d_{13}, d_{15}, d_{16}\}\}$$
$$= \min\{10,\ \ min\{\ 7,5\,,\ 9\}\}$$
$$= \min\{10,\ \ 5\} = 5$$

i = 1, *j* = 3

$$\widehat{D}_{13} = \min\{d_{13\,,}\ \ min\{d_{13}, d_{15}, d_{16}\}\}$$
$$= \min\{7,\ \ min\{7\,,5\,,\ 9\}\}$$
$$= \min\{7,\ \ 5\} = 5$$

i = 1, *j* = 4

$$\widehat{D}_{14} = \min\{d_{14\,,}\ \ min\{d_{13}, d_{15}, d_{16}\}\}$$
$$= \min\{10,\ \ min\{7\,,5\,,\ 9\}\}$$
$$= \min\{10,\ \ 5\} = 5$$

i = 1, *j* = 5

$$\widehat{D}_{15} = \min\{d_{15\,,}\ \ min\{d_{13}, d_{15}, d_{16}\}\}$$
$$= \min\{5,\ \ min\{7\,,5\,,\ 9\}\}$$
$$= \min\{5,\ \ 5\} = 5$$

i = 1, *j* =

6

$$\widehat{D}_{16} = \min\{d_{16\,,}\ \ min\{d_{13}, d_{15}, d_{16}\}\}$$
$$= \min\{9,\ \ min\{7\,,5\,,\ 9\}\}$$
$$= \min\{9,\ \ 5\} = 5$$

At node 2

i = 2, *j* = 1

$$\widehat{D}_{21} = \min\{d_{21\,,}\ \ min\{d_{23}, d_{25}, d_{26}\}\}$$

i = 2, *j* = 2

$$\widehat{D}_{22} = \min\{d_{22\,,}\ \ min\{d_{23}, d_{25}, d_{26}\}\}$$

$= \min\{10,\ \ min\{15\ ,7\ ,\ 17\}\}$

$= \min\{10,\ \ 7\} = 7$

$= \min\{0,\ \ min\{15\ ,7\ ,\ 17\}\}$

$= \min\{0,\ \ 7\} = 0$

$i = 2, j = 3$

$$\widehat{D}_{23} = \min\{d_{23\,,}\ \ min\{d_{23}, d_{25}, d_{26}\}\}$$

$= \min\{15,\ \ min\{15\ ,7\ ,\ 17\}\}$

$= \min\{15,\ \ 7\} = 7$

$i = 2, j = 4$

$$\widehat{D}_{24} = \min\{d_{24\,,}\ \ min\{d_{23}, d_{25}, d_{26}\}\}$$

$= \min\{12,\ \ min\{15\ ,7\ ,\ 17\}\}$

$= \min\{12,\ \ 7\} = 7$

$i = 2, j = 5$

$$\widehat{D}_{25} = \min\{d_{25\,,}\ \ min\{d_{23}, d_{25}, d_{26}\}\}$$

$= \min\{7,\ \ min\{15\ ,7\ ,\ 17\}\}$

$= \min\{7,\ \ 7\} = 7$

$i = 2, j = 6$

$$\widehat{D}_{26} = \min\{d_{26\,,}\ \ min\{d_{23}, d_{25}, d_{26}\}\}$$

$= \min\{17,\ \ min\{15\ ,7\ ,\ 17\}\}$

$= \min\{17,\ \ 7\} = 7$

The results are then summarized and shown in Table 3.5 below with row 1 and column 1 represent potential location and demand node respectively. Other rows represent the interconnecting distances.

Table 3.5, Modified shortest path distance matrix, $\widehat{D}$

Demand nodes	Potential location					
	1	**2**	**3**	**4**	**5**	**6**
1	0	5	5	5	5	5
2	7	0	7	7	7	7
3	0	0	0	0	0	0
4	5	5	5	0	5	5
5	0	0	0	0	0	0
6	0	0	0	0	0	0

The existing facility nodes Q = {3, 5, 6} are removed from the modified shortest path distance matrix, $\widehat{D}$ and this is shown in Table 3.6 below.

Table 3.6 Modified shortest path distance matrix, $\widehat{D}$ with existing facility nodes removed.

Demand nodes	Potential location		
	1	2	4
1	0	5	5
2	7	0	7
4	5	5	0

STEP 3: Find the optimal new location using $\widehat{D}$ for the network with the objective function

Minimize

$$f(x) = \sum_{i=1}^{n} w_i \min\big(d(X,i), d(Y,i)\big)$$

Let X = {1, 2, 4} and Y = {3, 5, 6} At

***X* = 1**

i = 1

$\min\{d(1,1), d(3,1), d(5,1)\ d(6,1)\}$

$\min\{0,5,5,5\} = 0$

i = 2

$\min\{d(1,2), d(3,2), d(5,2), d(6,2)\}$

$\min\{7,\ 7,\ 7,7\} = 7$

i = 3

$\min\{d(1,3), d(3,3), d(5,3)\ d(6,3)\}$

$\min\{0,0,0,0\} = 0$

i = 4

$\min\{d(1,4), d(3,4), d(5,4), d(6,4)\}$

$\min\{5,\ 5,\ 5,5\} = 5$

i = 5

$\min\{d(1,5), d(3,5), d(5,5)\ d(6,5)\}$

$\min\{0,0,0,0\} = 0$

i = 6

$\min\{d(1,6), d(3,6), d(5,6), d(6,6)\}$

$\min\{0,\ 0,\ 0,0\} = 0$

At *X* = 2

i = 1

$\min\{d(2,1), d(3,1), d(5,1)\ d(6,1)\}$

$\min\{5,5,5,5\} = 5$

i = 2

$\min\{d(2,2), d(3,2), d(5,2), d(6,2)\}$

$\min\{0,\ 7,\ 7,7\} = 0$

i = 3

$\min\{d(2,3), d(3,3), d(5,3)\ d(6,3)\}$

$\min\{0,0,0,0\} = 0$

i = 4

$\min\{d(2,4), d(3,4), d(5,4), d(6,4)\}$

$\min\{5,\ 5,\ 5,5\} = 5$

$i = 5$

$\min\{d(2,5), d(3,5), d(5,5)\ d(6,5)\}$

$\min\{0,0,0,0\} = 0$

$i = 6$

$\min\{d(2,6), d(3,6), d(5,6), d(6,6)\}$

$\min\{0,\ 0,\ 0,0\} = 0$

At *X* = 4

$i = 1$

$\min\{d(4,1), d(3,1), d(5,1)\ d(6,1)\}$

$\min\{5,5,5,5\} = 5$

$i = 2$

$\min\{d(4,2), d(3,2), d(5,2), d(6,2)\}$

$\min\{7,\ 7,\ 7,7\} = 7$

$i = 3$

$\min\{d(4,3), d(3,3), d(5,3)\ d(6,3)\}$

$\min\{0,0,0,0\} = 0$

$i = 4$

$\min\{d(4,4), d(3,4), d(5,4), d(6,4)\}$

$\min\{0,\ 5,\ 5,5\} = 0$

$i = 5$

$\min\{d(4,5), d(3,5), d(5,5)\ d(6,5)\}$

$\min\{0,0,0,0\} = 0$

$i = 6$

$\min\{d(4,6), d(3,6), d(5,6), d(6,6)\}$

$\min\{0,\ 0,\ 0,0\} = 0$

The results are the summarized and shown in table 3.7 with row 1 representing potential location and column 1 representing demand nodes.

Table 3.7 Optimal Location Matrix, using $\widehat{\boldsymbol{D}}$

Demand nodes	**Potential location**		
	1	2	4

1	0	7	5
2	5	0	5
4	5	7	0

finding the optimal new location using the modified shortest distance, $\widehat{D}$ and the objective function.

$$min\left[f(x) = \sum_{i=1}^{n} w_i \, min\{d(X,i), \;\; d(Y,i)\}\right]$$

At node 1 ***i* = 1**

$$2(0) + 7(1) + 5(5) = \mathbf{32}$$

At node 2 ***i* = 2**

$$2(5) + 1(0) + 5(5) = \mathbf{35}$$

At node 4 ***i* = 4**

$$2(5) + 7(1) + 5(0) = \mathbf{17}$$

From the above objective function values it can be easily be verify that the optimal new location using $\widehat{D}$ is node 4 with an objective function value of 17 because it is the minimum objective function value. Hence the new location for the facility is node 4.

3.6 FACTOR RATING METHOD

The factor rating method is popular because a wide variety of factors, from education to labour skills to recreation can be objectively included. In using the factor rating method, the following steps must be followed:

i) Develop a list of relevant factors ii) Assign a weight to each factor to reflect its relative importance in the community.

iii) Develop a scale for each factor (for example 1 to 10 or 1 to 100 points).

iv) Have a related people score each relevant factor using the scale developed in iii above.

v) Multiply the score by the weight assigned to each factor and total the score for each location.

vi) Make a recommended based on the maximum point score, considering the result of qualitative approaches as well.

When a decision is sensitive to minor changes, further analysis of either the weighting or the points assigned may be appropriate. Alternatively, management may conclude that these intangible factors are not the proper criteria on which to base a location decision. Managers therefore place primary weight on the more quantitative aspects of the decision (Amponsah, 2007).

Table 3.8 illustrate an example of the factor rating analysis of which a construction company must decide among four sites for the construction of a health center. The company selected seven factors listed below as a basis for evaluation and has assigned rating weights on each factor.

Table 3.8 Rating weight of relevant factors and their respective location rate on a 1 to 100 basis.

Factor	Factor name	Rating Weight	Rating of sites			
			Location A	Location B	Location C	Location D
1	Land acquisition	5	100	70	80	90
2	Power-source availability and cost	4	80	80	100	80
3	Workforce attitude and cost	4	30	60	70	40
4	Population size	2	10	80	60	100
5	Community desirability	3	90	60	80	60
6	Equipment suppliers in area	2	50	50	90	50
7	Economic activities	1	90	50	60	50

Table 3.9 Relative scores on factors for the health center

Factor	Factor name	Rating Weight	Ratio of Rate	Rating of sites			
				Location A	Location B	Location C	Location D
1	Land acquisition	5	0.25	25	17.5	20	22.5

2	Power-source availability and cost	3	0.15	12	12	15	12
3	Workforce attitude and cost	4	0.2	6	12	14	8
4	Population size	2	0.1	1	8	6	10
5	Community desirability	3	0.15	13.5	9	12	9
6	Equipment suppliers in area	2	0.1	5	5	9	5
7	Economic activities	1	0.05	4.5	2.5	3	2.5
TOTAL				67	66	79	69

Clearly from their aggregate scores, **site C** would be recommended since it has the highest aggregate.

CHAPTER 4

DATA COLLECTION, ANALYSIS AND DISCUSSION OF RESULTS

4.0 INTRODUCTION

In this chapter a new formulation for the conditional p-median problem (Berman and Drezner, 2008) would be used to locate a new hospital (p =1) in twenty-five major towns at Ejura-Sekyedumase district. They are Bemi, Atramo, Anyinasu, Sekyedumase, Nkrampo, Drobon, Frante, Kobriti, Teacherkrom, Aframso, Nchensie, Ebuom, Bayere Nkwanta, Nyamebekyere, Ejura, Babaso, Nokwareasa, Bisiw(no.1), Sarakyi Akuraa, Ashakoko, Kyenkyenkura, Dromankoma, Boyon, Hiawoawo and Kasei.

The distict map of Ejura-Sekyedumase district will be used to draw a network for these major towns with the edges being the inter-town distances. The Floyed – warshall all pair shortest paths algorithm would be applied to the network to create the shortest path distance matrix and the Berman"s and Drezner"s algorithm would be followed through to solve the problem.

4.1 DATA COLLECTION

The shortest path distances connecting communities is of interest in this study. In view of this a map of Ejura-Sekyedumase District was obtained from the Planning and Engineering offices. Figure 1 in appendix shows the district map. The map was prepared in 2005. The major communities in the district were identified and ArcGIS software was used to calculate the distances between the major communities to obtain the interconnected distances between these communities

A network was formed out of the map. The twenty-five (25) nodes in the network are the towns or settlements. The access roads of these major communities are represented by the edges of the network. The numbers attached to the nodes are the respective population of the major communities. These populations depict the weights of each town. Figure 4.0 below shows the network. The key to the network is shown in Table 4.0.

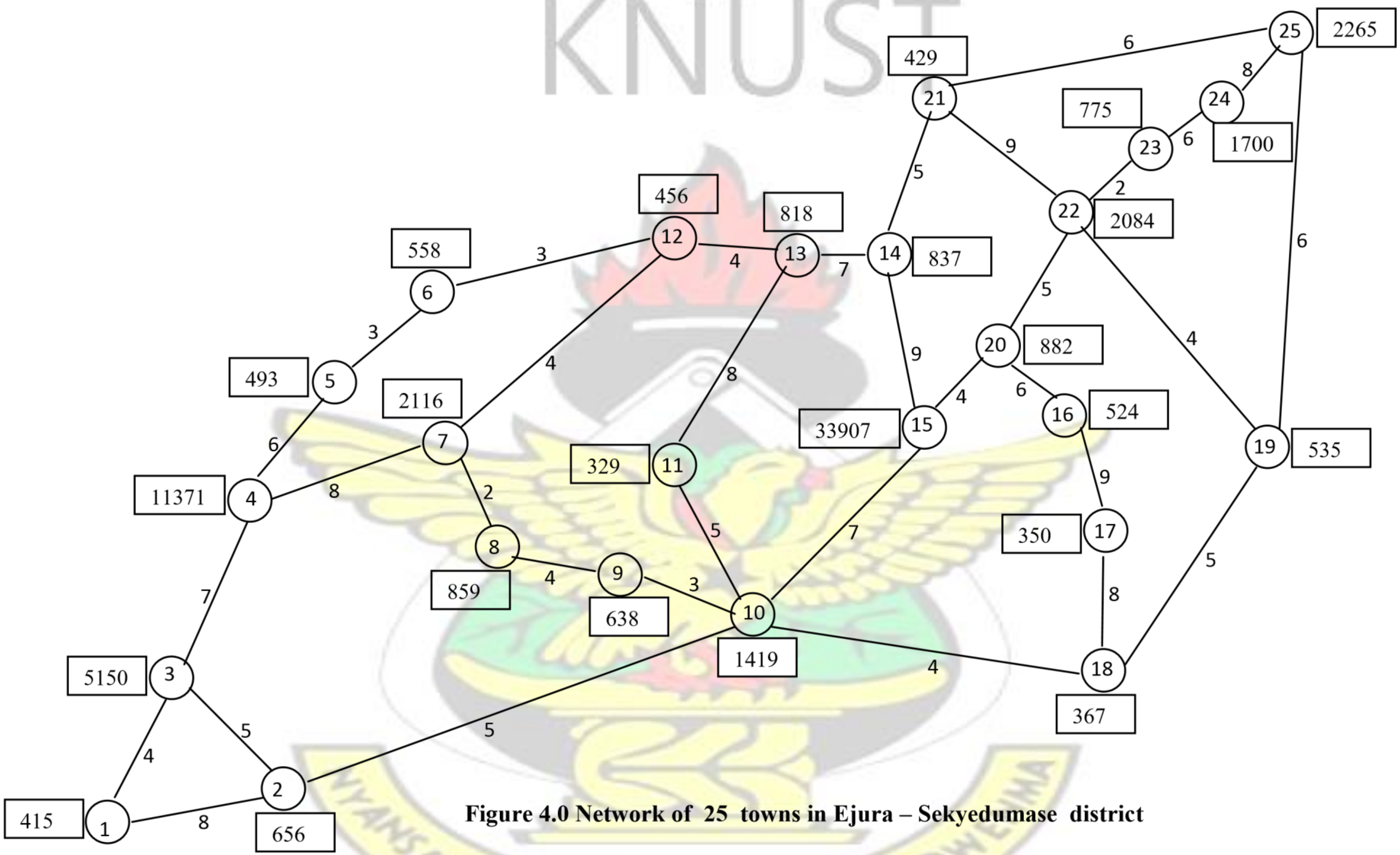

Figure 4.0 Network of 25 towns in Ejura – Sekyedumase district

Table 4.0 Major communities in Ejura-Sekyedumase District and their respective nodes

Town	Node	Town	Node	Town	Node
Bemi	1	Aframso	10	Sarakyi Akuraa	19
Atramo	2	Nchensei	11	Ashakoko	20
Anyinasu	3	Eboum	12	Kyenkyenkura	21
Sekyedumase	4	Bayere Nkwanta	13	Dromankoma	22
Nkrampo	5	Nyamebekyere	14	Boyon	23
Drobon	6	Ejura	15	Hiawoanwo	24
Frante	7	Babaso	16	Kasei	25
Kobriti	8	Nokwareasa	17		
Teachekrom	9	Bisiw (no 1)	18		

Table 4.1 Major communities in Ejura-Sekyedumase Municipal and their respective Populations.

Town	Population	Town	Population	Town	Population
Bemi	415	Aframso	1419	Sarakyi Akuraa	535
Atramo	656	Nchensei	329	Ashakoko	882
Anyinasu	5150	Eboum	456	Kyenkyenkura	429
Sekyedumase	11371	Bayere Nkwanta	818	Dromankoma	2084
Nkrampo	493	Nyamebekyere	837	Boyon	775
Drobon	558	Ejura	33907	Hiawoanwo	1700
Afrante	2116	Babaso	524	Kasei	2265
Kobriti	859	Nokwareasa	350		
Teachekrom	638	Bisiw (no 1)	367		

Source: Ghana‟s Census Reports (1960 – 2000) and Baseline Survey (2005).

The nodes of the network were developed in a matrix form. Communities which have direct road link are indicated with their respective distance, whereas communities with no direct road link are indicated with a dash. The matrix formed is a square matrix of order 25 by 25. Table 4.2 shows the raw data.

Table 4.2. Matrix of Network in Fig 4.0 Indicating Towns and their Pair of Distances.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	8	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	8	0	5	-	-	-	-	-	-	5	- -	-	- -	- -	- -	- -	- -	- -
3	4	5	0	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	-	7	0	6	-	8	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	6	0	3	-	-	-	-	- -	-	- -	- -	- -	- -	- -	- -
6	-	-	-	-	3	0	-	-	-	-	-	3	-	-	-	-	-	-
7	-	-	-	8	-	-	0	2	-	-	-	4	-	-	-	-	-	-
8	-	-	-	-	-	-	2	0	4	-	5	-	-	-	7	-	-	4
9	-	-	-	-	-	-	-	4	0	3	0	-	8	-	-	-	-	-
10	-	5	-	-	-	-	-	-	3	0	-	-	4	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	5	8	-	0	7	-	-	-	-
12	-	-	-	-	-	3	4	-	-	-	-	0	7	0	9	-	-	-
13	-	-	-	-	-	-	-	-	-	-	- -	4	- -	9	0	-	-	- -
14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	9	8
15	-	-	-	-	-	-	-	-	-	7	-	-	-	-	-	9	0	0
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	5
17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-	-	4	-	-	-	-	4	6	-	-
19	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	-
20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-		-		-	-	-	-	

22	-	-	-	-	-	-	-	-	-	-					-
23	-	-	-	-	-	-	-	-	-	-					-
24															
25															

4.2 DATA ANALYSIS

The Floyed – Warshall All Pair Shortest Path algorithm was applied to the matrix in Table 4.2 to obtain the all pairs shortest path distance matrix D, shown in Table 4.3. Row one and Column one represent the potential location and demand nodes respectively. The other rows also represent the inter – community distances. The MATLAB code for the Floyed – Warshall algorithm used to obtain the all pair shortest path distance is shown Appendix 4.0.

Table 4.3 Summary of Shortest Distance Matrix between Pair of Nodes, D

	1	2	3	4	5	6	.	.	.	12	13	14	15	.	.	.	20	21	22	23	24	25
1	0	8	4	11	17	20	.	.	.	23	26	29	20	.	.	.	24	34	26	28	34	28
2	8	0	5	12	18	21	.	.	.	18	18	21	12	.	.	.	16	26	18	20	26	20
3	4	5	0	7	13	16	.	.	.	19	23	26	17	.	.	.	21	31	23	25	31	25
4	11	12	7	0	6	9	.	.	.	12	16	23	24	.	.	.	28	28	30	32	38	32
5	17	18	13	6	0	3	.	.	.	6	10	17	26	.	.	.	30	22	31	33	36	28
6	20	21	16	9	3	0	.	.	.	3	7	14	23	.	.	.	27	19	28	30	33	25
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
12	23	18	19	12	6	3	.	.	.	0	4	11	20	.	.	.	24	16	25	27	30	22
13	26	18	23	16	10	7	.	.	.	4	0	7	16	.	.	.	20	12	21	23	26	18
14	29	21	26	23	17	14	.	.	.	11	7	0	9	.	.	.	13	5	14	16	19	11
15	20	12	17	24	26	23	.	.	.		16	9	0	.	.	.	4	14	9	11	17	19
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
20	24	16	21	28	30	27	.	.	.	2	20	.	.	.	.	.	0	14	5	7	13	15
21	34	26	31	28	22	19	.	.	.	0	12	.	.	.	.	.	14	0	9	11	14	6
22	26	18	23	30	31	28	.	.	.	.	21	13	4	.	.	.	5	9	0	2	8	10
23	28	20	25	32	33	30	.	.	.	.	23	5	14	.	.	.	7	11	2	0	6	12
	34	26	31	38	36	33	.	.	.	24	26	14	9	.	.	.	13	14	8	6	0	8
	28	20	25	32	28	25	.	.	.		18	16	11	.	.	.	15	6	10	12	8	0

24		.	16	19	17
25		.	25	11	19
		.	27		
		.	30		
		.	22		

4.3 MODEL FORMULATION

Berman and Drezner"s algorithm (2008) is used to solve the problem. This algorithm requires a one-time solution of an unconditional p- median problem using an approipriate shortest distance matrix.

We begin by formulating the conditional p- median problem as

$$min\left[f(x) = \sum_{i=1}^{n} w_i\, min\{d(X,i),\ \ d(Y,i)\}\right]$$

Let $G = (N, L)$ be a network with N being the set of nodes, $|N| = n$ and L being the set of links. Consider a non-negative number w_i that represent the demand weight at node $i \in N$. Let d_{xy} be the shortest distance between any two nodes $x, y \in G$.

Suppose that there is a set $Q(|Q| = q)$ of existing facilities. Let $Y = (Y_{1,}\ Y_{2,}\ Y_{3,}\ \dots\dots, Y_q)$ and $X = (X_1\ X_{2,}\ X_{3,}\ \dots\dots, X_p)$ be vectors of size *q* and *p* respectively, where Y_i is the location of existing facility *i* and X_i is the location of new facility *i*. where $d(X,i)$ and $d(Y,i)$ is the shortest distance from the closest facility in *X* and *Y* respectively to node *i*. Without any loss of generality we do not need to assume that $Y_i \in N$.

With existing hospitals at Ejura and Kasei, Clinics at Anyinasu and health centre at Sekyedumase. These communities form the set of existing facilities, thus node 15, node 25, node3, and node 4 respectively. This gives Y = { 3, 4, 15, 25}. The remaining nodes also form the set of potential location of new facilities.

Thus X= {1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24}.

Where

$$i = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25\}$$

4.4 ALGORITHM USED TO SOLVE THE PROBLEM

STEPS

1. Let D be the shortest path distance matrix with rows corresponding to demands and columns corresponding to potential locations.

2. Modified the shortest path distance matrix from D to $\widehat{D}$. That is

   $\widehat{D}_{ij} = \min\{d_{ij}, \min\{d_{ik}\}\} \quad \forall\, i \in N, j \in N$, where k belongs to the set of existing

facilities. It should be noted that $\widehat{D}$ is not symmetric even when D is symmetric.

3. Remove the nodes in Q and the Potential location in Q.

4. Find the optimal new location by using the modified distance matrix $\widehat{D}$. For the network with objective function

$$min\left[f(x) = \sum_{i=1}^{n} w_i\, min\{d(X,i),\ \ d(Y,i)\}\right]$$

4.5 COMPUTATION AND RESULTS

STEP 1: The Floyed-Warshall All pair shortest path algorithm was applied to the matrix in Table 4.2 to obtained the shortest distance matrix between each pair of node as displayed in Table 4.3 above. The matrix shows the length of the shortest path between respective nodes.

Step 2: A modified shortest distance matrix $\widehat{D}$ is determine by using the formulation

$\widehat{D}_{ij} = \min\{d_{ij}, \min\{d_{ik}\}\} \quad \forall\, i \in N, j \in N, k \in Q$, where Q = {3, 4, 15, 25} and *i*, *j* = {1, 2, 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25}. The MATLAB code used to obtain the modified shortest distance matrix is shown in Appendix 5.0.

For node 1

i = 1, *j* = 1

$$\widehat{D}_{11} = \min\{d_{11},\ min\{d_{13}, d_{1,4}, d_{1,15}\ d_{1,25}\}\}$$
$$= \min\{0,\ min\{4, 11, 20, 28\}\}$$
$$= \min\{0,\ 4\} = 0$$

i = 1, *j* = 2

$$\widehat{D}_{12} = \min\{d_{12},\ min\{d_{13}, d_{1,4}, d_{1,15}\ d_{1,25}\}\}$$
$$= \min\{8,\ min\{4, 11,20,28\}\}$$
$$= \min\{8,\ 4\} = 4$$

i = 1, *j* = 3

$$\widehat{D}_{13} = \min\{d_{11},\ min\{d_{13}, d_{1,4}, d_{1,15}\ d_{1,25}\}\}$$
$$= \min\{4,\ min\{4,11, 20,\ 28\}\}$$
$$= \min\{4,\ 4\} = 4$$

i = 1, *j* = 4

i = 1, *j* = 7

$$\widehat{D}_{17} = \min\{d_{17},\ min\{d_{1,3}, d_{1,4}, d_{1,15}\ d_{1,25}\}\}$$
$$= \min\{19,\ min\{4, 11,20,\ 28\}\}$$
$$= \min\{19,\ 4\} = 4$$

i = 1, *j* = 8

$$\widehat{D}_{18} = \min\{d_{18},\ min\{d_{1,3}, d_{1,4}, d_{1,15}\ d_{1,25}\}\}$$
$$= \min\{20,\ min\{4, 11, 20,28\}\}$$
$$= \min\{20,\ 4\} = 4$$

i = 1, *j* = 9

$$\widehat{D}_{19} = \min\{d_{19},\ min\{d_{1,3}, d_{1,4}, d_{1,15}\ d_{1,25}\}\}$$
$$= \min\{16,\ min\{4,11,20, 28\}\}$$
$$= \min\{16,\ 4\} = 4$$

i = 1, *j* = 10

$\widehat{D}_{14} = \min\{d_{14\,,}\, min\{d_{13}, d_{1,4}, d_{1,15}\; d_{1,25}\}\}$ $\widehat{D}_{1,10} = \min\{d_{1,10\,,}\, min\{d_{1,3}, d_{1,4}, d_{1,15}\; d_{1,25}\}\}$

$= \min\{11, \quad min\{4\,, 11{,}20{,}28\,\}\}$ $= \min\{13, \quad min\{4\,, 11, 20, 28\}\}$

$= \min\{11, \; 4\} = 4$ $= \min\{13, \; 4\} = 4$

$i = 1, j = 5$ $i = 1, j = 11$

$\widehat{D}_{15} = \min\{d_{15\,,}\, min\{d_{13}, d_{1,4}, d_{1,15}\; d_{1,25}\}\}$ $\widehat{D}_{1,11} = \min\{d_{1,11\,,}\, min\{d_{1,3}, d_{1,4}, d_{1,15}\; d_{1,25}\}\}$

$= \min\{17, \quad min\{4\,, 11{,}20, 28\}\}$ $= \min\{18, \quad min\{4{,}11, 20{,}28\,\}\}$

$= \min\{17, \; 4\} = 4$ $= \min\{18, \; 4\} = 4$

$i = 1, j = 6$ $i = 1, j = 12$

$\widehat{D}_{16} = \min\{d_{1,6\,,}\, min\{d_{13}, d_{1,4}, d_{1,15}\; d_{1,25}\}\}$ $\widehat{D}_{1,12} = \min\{d_{1,12\,,}\, min\{d_{1,3}, d_{1,4}, d_{1,15}\; d_{1,25}\}\}$

$= \min\{20, \quad min\{4\,, 11\,{,}20{,}28\,\}\}$ $= \min\{23, \quad min\{4, 11, 20{,}28\,\}\}$

$= \min\{20, \; 4\} = 4$ $= \min\{23, \; 4\} = 4$

The results are then summarized and shown in Table 4.4 below with row 1 and column 1 represent potential location and demand node respectively. Other rows represent the inter-communities distances

Table 4.4 Summary of Modified shortest path distance matrix, $\hat{D}$

Demand Nodes	Potential Location																				
	1	2	3	4	5	6	.	.	.	12	13	14	15	.	.	.	20	21	22	23	24	25
1	0	4	4	4	4		.	.	.	4	4	4	4	.	.	.	4	4	4	4	4	4
2	5	0	5	5	5		.	.	.	5	5	5	5	.	.	.	5	5	5	5	5	5
3	0	0	0	0	0	4 5 0 0	.	.	.	0	0	0	0	.	.	.	0	0	0	0	0	0
4	0	0	0	0	0	3	.	.	.	0	0	0	0	.	.	.	0	0	0	0	0	0
5	6	6	6	6	0	0	.	.	.	6	6	6	6	.	.	.	6	6	6	6	6	6
6	9	9	9	9	3	.	.	.	.	3	7	9	9	.	.	.	9	9	9	9	9	9
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	3 7	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
12	12	12	12	12	6	9	.	.	.	0	4	11	12	.	.	.	12	12	12	12	12	12
13	16	16	16	16	10	0	.	.	.	4	0	7	16	.	.	.	16	12	16	16	16	16
14	9	9	9	9	9	.	.	.	.	9	7	0	9	.	.	.	9	5	9	9	9	9
15	0	0	0	0	0	.	.	.	.	0	0	0	0	.	.	.	0	0	0	0	0	0
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	4 6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	9	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
20	4	4	4	4	4	11	.	.	.	4	4	4	4	.	.	.	0	4	4	4	4	4
21	6	6	6	6	6	8 0	.	.	.	6	6	5	6	.	.	.	6	0	6	6	6	6
22	9	9	9	9	9		.	.	.					.	.	.	5 7	9	0 2	2 0	8 6	9
23	11	11	11	11	11		.	.	.					.	.	.	8 0	11	8 0	6 0	0 0	11

24	8 8 8 8 8 / 0 0 0 0 0 · · · 9 9 9 9 · · · 8 / 0 8 / 0
25	· · · 11 11 11 11 · · · 8 8 8 8 · 0 0 0 0 ·

From the Table 4.4, it can be seen that the existing facility nodes Q = {3, 4, 15, 25} has a minimum road distance of zero between them. Hence the set of demand nodes and potential location of existing facility are removed from the modified shortest path distance matrix D and this is shown in table 4.5 below.

Table 4.5 Modified shortest path distance matrix $\widehat{D}$ with existing facilities removed

Demand node	Potential location																				
	1	2	5	6	7	8	9	10	11	12	13	14	16	17	18	19	20	21	22	23	24
1	0	4	4	4	4	4	4	4	4	4	4 4	4		4	4	4	4	4	4	4	4
2	5	0	5	5	5	5	5	5	5	5	5 5	5		5	5	5	5	5	5	5	5
5	6	6	0	3	6	6	6	6	6	6	6 6	6		6	6	6	6	6	6	6	6
6	9	9	3	0	7	9	9	9	9	3	7	9	9	9	9	9	9	9	9	9	9
7	8	8	8	8	0	2	6	8	8	4	8	8	8	8	8	8	8	8	8	8	8
8	10	10	10	9	2	0	4	7	10	6	10	10	10	10	10	10	10	10	10	10	10
9	10	8	10	10	6	4	0	3	8	10	10	10	10	10	7	10	10	10	10	10	10
10	7	5	7	7	7	7	3	0	5	7	7	7	7	7	4	7	7	7	7	7	7
11	12	10	12	12	12	12	8	5	0	12	8	12	12	12	9	12	12	12	12	12	12
12	12	12	6	3	4	6	10	12	12 0	4 11	12 8	4 0 7	16	12	12	12	12	12	12	12	12
13	16	16	10	7	8	10	14	13	9	9	7 9	0		16	16	16	16	12	16	16	16
14	9	9	9	9	9	9	9	9	10	10	10 0	10		9	9	9	9	5	9	9	9
16	10	10	10	10	10	10	10	10	17	19	19	19	9	9	10	10	6	10	10	10	10
17	19	17	19	19	19	19	15	12	9	11	11	11	11	0	8	13	15	19	17	19	19
18	11	9	11	11	11	11	7	4	6	6	6	6	6	8	0	5	11	11	11	11	11
19	6	6	6	6	6	6	6	6	4	4	4	4	4	6	5	0	6	6	4	6	6
20	4	4	4	4	4	4	4	4	6	6	6	5	6	4	4	4	0	4	4	4	4
21	6	6	6	6	6	6	6	6	9	9	9	9	9	6	6	6	6	0	6	6	6
22	9	9	9	9	9	9	9	9	11	11	11	11	11	9	9	4	5	9	0	2	8
23	11	11	11	11	11	11	11	11						11	11	6	7	11	2	0	6
	8	8	8	8	8	8	8	8						8	8	8	8	8	8	6	0

24	8	8	8	8	8

STEP 4: Find the optimal new location for the hospital using the modified distance matrix $\widehat{D}$ with existing facility nodes (Y= 3, 4, 15, 25) removed from the network with the objective function:

$$min\left[f(x) = \sum_{i=1}^{n} w_i\, min\{d(X,i),\ \ d(Y,i)\}\right]$$

Let i = {1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24} and X = {1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24}.

The optimal new location of the hospital is now found by using the modified shortest distance matrix and the objective function:

Minimise

$$f(x) = \sum_{i=1}^{n} w_i\, min\{d(X,i),\ \ d(Y,i)\}$$

Thus At X =1 (Potential location 1)

$$(10) + 1419(7) +$$
$$+ 367(11) + 535(6) +$$
$$415(0) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(10) + 350(19)$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{149252}$$

At X= 2 (Potential location 2)

$$(8) + 1419(5) +$$
$$+ 367(9) + 535(6) +$$
$$415(4) + 656(0) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(10) + 456(12) + 818(16) + 837(9) + 524(10) + 350(17)$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{141426}$$

$$(10) + 1419(7) +$$
$$+ 367(11) + 535(6) +$$

At X=5 (Potential location 5)

$$415(4) + 656(5) + 493(0) + 558(3) + 2116(8) + 859(10) + 638$$
$$329(12) + 456(6) + 818(10) + 837(9) + 524(10) + 350(19)$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{136962}$$

At X= 6 (Potential location 6)

$$415(4) + 656(5) + 493(3) + 558(0) + 2116(8) + 859(9) + 638(10) + 1419(7) + 367(11) + 535(6) +$$
$$329(12) + 456(3) + 818(7) + 837(9) + 524(10) + 350(19) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{132086}$$

At X= 7 (Potential location 7)

$$415(4) + 656(5) + 493(6) + 558(7) + 2116(0) + 859(2) + 638(6) + 1419(7) + 367(11) + 535(6) +$$
$$329(12) + 456(4) + 818(8) + 837(9) + 524(10) + 350(19) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{113252}$$

At X= 8 (Potential location 8)

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(2) + 859(0) + 638(4) + 1419(7) + + 367(11) + 535(6) +$$
$$329(12) + 456(6) + 818(10) + 837(9) + 524(10) + 350(19)$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{118154}$$

At X= 9 (Potential location 9)

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(6) + 859(4) + 638(0) + 1419(3) + + 367(7) + 535(6) +$$
$$329(8) + 456(10) + 818(14) + 837(9) + 524(10) + 350(15)$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{122738}$$

At X= 10 (Potential location 10)

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(7) + 638(3) + 1419(0) + + 367(4) + 535(6) +$$
$$329(5) + 456(12) + 818(13) + 837(9) + 524(10) + 350(12)$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{124160}$$

At X = 11 (Potential location 11)

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638(8) + 1419(5) + 367(9) + 535(6) +$$
$$329(0) + 456(12) + 818(8) + 837(9) + 524(10) + 350(17) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{134872}$$

At X = 12 (potential location 12)

$$415(4) + 656(5) + 493(6) + 558(3) + 2116(4) + 859(6) + 638(10) + 1419(7) + 367(11) + 535(6) +$$
$$329(12) + 456(0) + 818(4) + 837(9) + 524(10) + 350(19) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{120376}$$

At X = 13 (Potential location 13)

$(10) + 1419(7) +$
$(11) + 535(6) +$

$$415(4) + 656(5) + 493(6) + 558(7) + 2116(8) + 859(10) + 638$$
$$329(8) + 456(4) + 818(0) + 837(7) + 524(10) + 350(19) + 367$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{130070}$$

$(10) + 1419(7) +$
$+ 367(11) + 535(6) +$

At X = 14 (Potential location 14)

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(12) + 456(11) + 818(7) + 837(0) + 524(10) + 350(19)$$
$$882(4) + 429(5) + 2084(9) + 775(11) + 1700(8) = \mathbf{135132}$$

At X = 16 (Potential location 16)

$(10) + 1419(7) +$
$367(11) + 535(6) +$

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(0) + 350(9) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{142172}$$

At X = 17 (Potential location 17)

$(10) + 1419(7) +$
$367(8) + 535(6) +$

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(9) + 350(0) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{142637}$$

At X = 18 (Potential location 18)

$(7) + 1419(4) +$
$367(0) + 535(5) +$

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(9) + 456(12) + 818(16) + 837(9) + 524(10) + 350(8) +$$
$$882(4) + 429(6) + 2084(9) + 775(11) + 1700(8) = \mathbf{135332}$$

At X = 19 (Potential location 19)

$638(10) + 1419(7) +$
$) + 367(5) + 535(0) +$

$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) +$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(10) + 350(13$$
$$882(4) + 429(6) + 2084(4) + 775(6) + 1700(8) = \mathbf{129105}$$

At X = 20 (Potential location 20)

$$638(10) + 1419(7) +$$
$$+ 367(11) + 535(6) +$$
$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) +$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(6) + 350(15)$$
$$882(0) + 429(6) + 2084(5) + 775(7) + 1700(8) = \mathbf{132452}$$

At X = 21 (Potential location 21)

$$(10) + 1419(7) +$$
$$+ 367(11) + 535(6) +$$
$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) + 638$$
$$329(12) + 456(12) + 818(12) + 837(5) + 524(10) + 350(19)$$
$$882(4) + 429(0) + 2084(9) + 775(11) + 1700(8) = \mathbf{141718}$$

At X = 22 (Potential location 22)

$$638(10) + 1419(7) +$$
$$) + 367(11) + 535(4) +$$
$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) +$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(10) + 350(17$$
$$882(4) + 429(6) + 2084(0) + 775(2) + 1700(8) = \mathbf{124399}$$

At X = 23 (Potential location 23)

$$638(10) + 1419(7) +$$
$$) + 367(11) + 535(6) +$$
$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) +$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(10) + 350(19$$
$$882(4) + 429(6) + 2084(2) + 775(0) + 1700(6) = \mathbf{126056}$$

At X = 24 (Potential location 24)

$$638(10) + 1419(7) +$$
$$) + 367(11) + 535(6) +$$
$$415(4) + 656(5) + 493(6) + 558(9) + 2116(8) + 859(10) +$$
$$329(12) + 456(12) + 818(16) + 837(9) + 524(10) + 350(19$$
$$882(4) + 429(6) + 2084(8) + 775(6) + 1700(0) = \mathbf{131353}$$

4.6 DISCUSSION OF RESULTS

Considering the twenty – five node network depicted in figure 4.0 and solving the conditional 1 – median problem with Q = {3, 4, 15, 25} and P = 1. The optimal new location using the modified Shortest distance $\widehat{D}$ thus by using the Berman and Drezner‟s algorithm, the new optimal location of the hospital can be located at node 7,(thus Frante) with the minimum objective function value of **113252**.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

The objective of the study was to model the location of an additional hospital using the conditional p – median model and find an optimal location for the hospital in Ejura – Sekyedumase district. The data obtained from the district assembly was model into a conditional p – median problem and the Berman and Drezners algorithm (2008) was used to solve the problem.

The results as discussed in chapter 4, section 4.6 showed that the additional hospital should be located at Frante (node 7). The demand – weighted total (or average) distance using the Berman and Drezners algorithm is 113252 because it is the minimum objective function value.

The additional hospital will optimally serve the twenty – five major towns in the district as well as all the various communities in the district. The new hospital will largely help reduced the pressure on the existing hospitals. This will also help improve the quality of service provided to the residents of the district.

The facility has also been optimally located and this will give a fair travel distance to all persons who will patronize the services of the facility from all over the district. The additional hospital should be a general hospital so that it can provide a complete medical and surgical care to the sick and injured and maternity care. The hospital should have an organized staff of qualified professionals, an approved laboratory with standardized equipment , X – ray facilities, separate maternity unit, separate surgical unit and a dental unit.

Figure 5.0 below shows the site for the facility on the network of twons in the district.

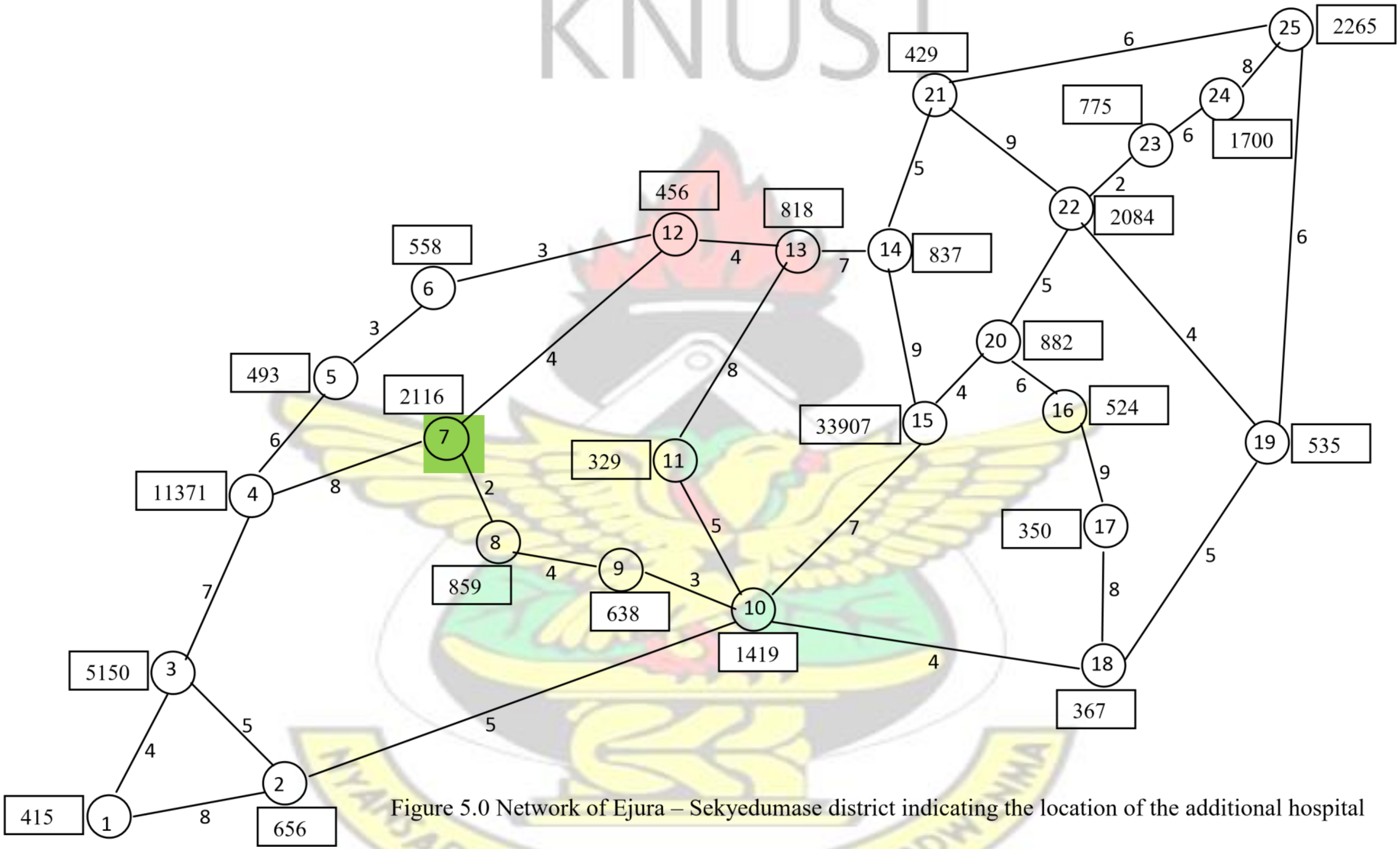

Figure 5.0 Network of Ejura – Sekyedumase district indicating the location of the additional hospital

5.2 RECOMMENDATION

In view of the results obtained in the study the following recommendation are made:

1. The Ejura – Sekyedumase district assembly is recommended to build an additional hospital based on this study at Frante to help reduce the pressures on the existing hospital facilities.
2. Private organizations who will like to invest in the establishment of a hospital in the district could use this study to optimally locate the hospital at Frante.
3. The siting of such emergency facilities should be done using more effective scientific approaches.

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APPENDIX 1.0

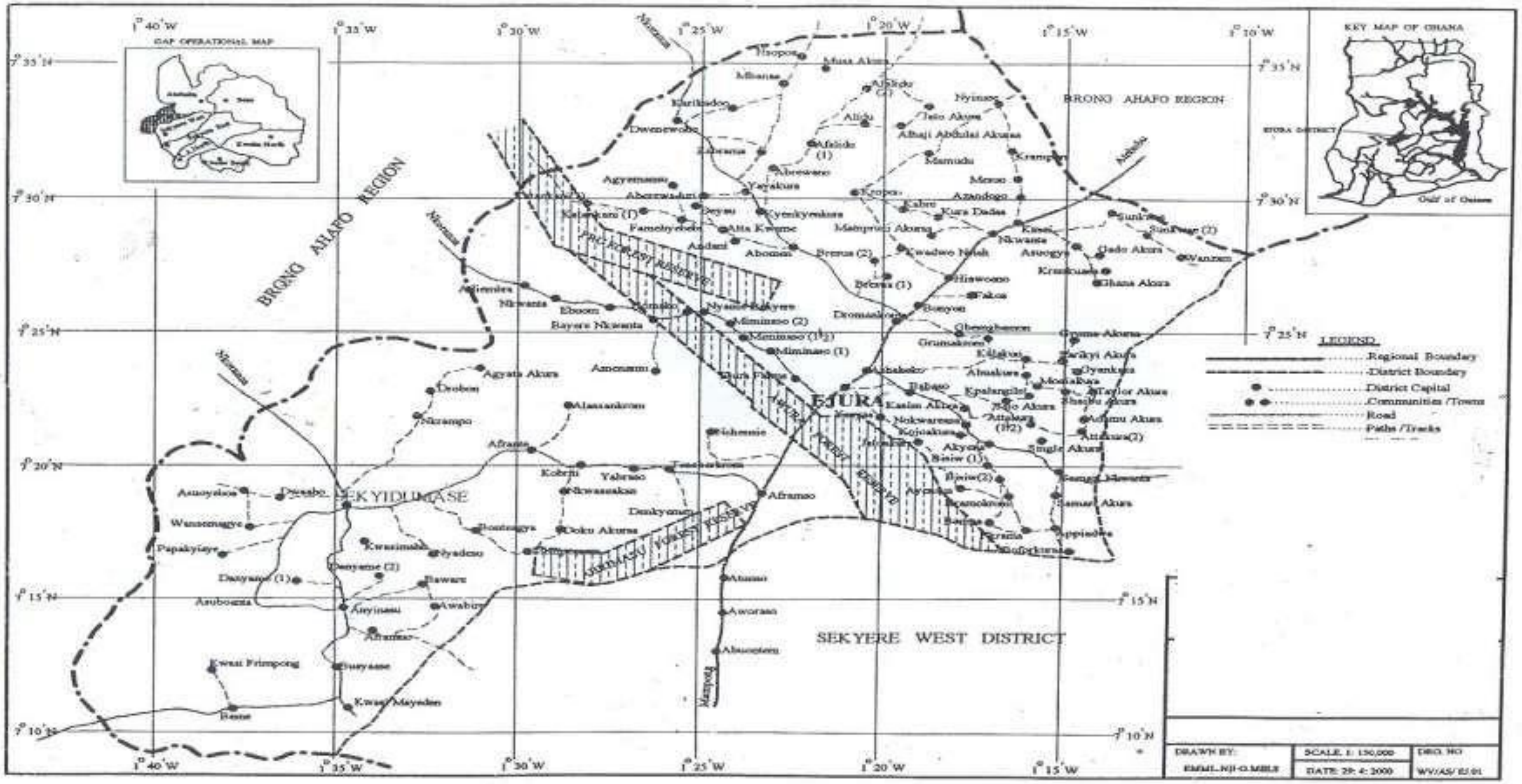

Figure A1.0 District Map of Ejura – Sekydumase.

APPENDIX 2.0

Table A2.0 All pair shortest path distance matrix D

Demand nodes	Potential location																							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
1	0	8	4	11	17	20	19	20	16	13	18	23	26	29	20	30	25	17	22	24	34	26	28	34	28
2	8	0	5	12	18	21	14	12	8	5	10	18	18	21		22	17	9	14	16	26	18	20	26	20
3	4	5	0	7	13	16	15	17	13	10	15		12			27	22	14	19	21	31	23	25	31	25
4	11	12	7	0	6	9	8	10	14	17	22	19	23	26		34	29	21	26	28	28	30	32	38	32
5	17	18	13	6	0	3	10	12	16	19	18		17			36	31	23	28	30	22	31	33	36	28
6	20	21	16	9	3	0	7	9	13	16	15	12	16	23	24	33	28	20	25	27	19	28	30	33	25
7	19	14	15	8	10	7	0	2	6	9	14	6	10	17	26	26	21	13	18	20	20	22	24	30	24
8	20	12	17	10	12	9	2	0	4	7	12	3	7	14		24	19	11	16	18	22	20	22	28	22
9	16	8	13	14	16	13	6	4	0	3	8		23			20	15	7	12	14	24	16	18	24	18
10	13	5	10	17	19	16	9	7	3	0	5	4	8	15		17	12	4	9	11	21	13	15	21	15
11	18	10	15	22	18	15	14	12	8	5	0		16			22	17	9	14	16	20	18	20	26	20
12	23	18	19	12	6	3	4	6	10	13	12	6	10	17	14	30	25	17	22	24	16	25	27	30	22
13	26	18	23	16	10	7	8	10	14	13	8	10	14	19	10	26	25	17	22	20	12	21	23	26	18
14	29	21	26	23	17	14	15	17	19	16	15	13	13	16	7	19	28	20	17	13	5	14	16	19	11
15	20	12	17	24	26	23	16	14	10	7	12	12	8	15	12	10	19	11	13	4	14	9	11	17	19
16	30	22	27	34	36	33	26	24	20	17	22	0	4	11	20	0	9	17	15	6	20	11	13	19	21
17	25	17	22	29	31	28	21	19	15	12	17	4	0	7	16	9	0	8	13	15	25	17	19	25	19
18	17	9	14	21	23	20	13	11	7	4	9	11	7	0	9	17	8	0	5	14	17	9	11	17	11
19	22	14	19	26	28	25	18	16	12	9	14	20	16	9	0	15	13	5	0	9	12	4	6	12	6
20	24	16	21	28	30	27	20	18	14	11	16	30	26	19	10	6	15	14	9	0	14	5	7	13	15
21	34	26	31	28	22	19	20	22	24	21	20	25	25	28	19	20	25	17	12	14	0	9	11	14	6
22	26	18	23	30	31	28	22	20	16	13	18	17	17	20	11	11	17	9	4	5	9	0	2	8	10
23	28	20	25	32	33	30	24	22	18	15	20	22	22	17	13	13	19	11	6	7	11	2	0	6	12
24	34	26	31	38	36	33	30	28	24	21	26	24	20	13	4	19	25	17	12	13	14	8	6	0	8
25	28	20	25	32	28	25	24	22	18	15	20	16	12	5	14	21	19	11	6	15	6	10	12	8	0
												25	21	14	9										

27	23	16	11
30	26	19	17
22	18	11	19

APPENDIX 3.0

Table A3.0 Modified shortest path distance matrix, $\hat{D}$

Demand nodes	Potential location																							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
1	0	4	4	4	4	4	4	4	4	4	4	4	4	4		4	4	4	4	4	4	4	4	4	4
2	5	0	5	5	5	5	5	5	5	5	5			4		5	5	5	5	5	5	5	5	5	5
3	0	0	0	0	0	0	0	0	0	0	0	5	5	5		0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0			5		0	0	0	0	0	0	0	0	0	0
5	6	6	6	6	0	3	6	6	6	6	6	0	0	0	0	6	6	6	6	6	6	6	6	6	6
6	9	9	9	9	3	0	7	9	9	9	9	0	0	0	0	9	9	9	9	9	9	9	9	9	9
7	8	8	8	8	8	8	0	2	6	8	8	6	6	6	6	8	8	8	8	8	8	8	8	8	8
8	10	10	10	10	10	9	2	0	4	7	10	3	7	9		10	10	10	10	10	10	10	10	10	10
9	10	8	10	10	10	10	6	4	0	3	8			9		10	10	7	10	10	10	10	10	10	10
	7	5	7	7	7	7	7	7	3	0	5	4	8	8		7	7	4	7	7	7	7	7	7	7
10	12	10	12	12	12	12	12	12	8	5	0			8		12	12	9	12	12	12	12	12	12	12
11	12	12	12	12	6	3	4	6	10	12	12	6	10	10	10	12	12	12	12	12	12	12	12	12	12
12	16	16	16	16	10	7	8	10	14	13	8	10	10	10	10	16	16	16	16	16	12	16	16	16	16
13	9	9	9	9	9	9	9	9	9	9	9	7	7	7	7	9	9	9	9	9	5	9	9	9	9
14	0	0	0	0	0	0	0	0	0	0	0	12	8	12	12	0	0	0	0	0	0	0	0	0	0
15	10	10	10	10	10	10	10	10	10	10	10	0	4	11	12	0	9	10	10	6	10	10	10	10	10
16	19	17	19	19	19	19	19	19	15	12	17	4	0	7	16	9	0	8	13	15	19	17	19	19	19
17	11	9	11	11	11	11	11	11	7	4	9	9	7	0	9	11	8	0	5	11	11	11	11	11	11
18	6		6	6	6	6		6								6	6		0	6	6	4	6	6	6

19	4	6	4	4	4	4	6	4	6	6	6	0	0	0	0	4	4	5	4	0	4	4	4	4	4
20	6	4	6	6	6	6	4	6	4	4	4	10	10	10	10	6	6	4	6	6	0	6	6	6	6
21	9	6	9	9	9	9	6	9	6	6	6	19	19	19	19	9	9	6	4	5	9	0	2	8	9
22	11	9	11	11	11	11	9	11	9	9	9	11	11	11	11	11	11	9	6	7	11	2	0	6	11
23	8	11	8	8	8	8	11	8	11	11	11	6	6	6	6	8	8	11	8	8	8	8	6	0	8
24	0	8	0	0	0	0	8	0	8	8	8	4	4	4	4	0	0	8	0	0	0	0	0	0	0
25		0					0		0	0	0	6	6	5	6			0							
												9	9	9	9										
												11	11	11	11										
												8	8	8	8										
												0	0	0	0										

APPENDIX 4.0

MATLAB CODE FOR FLOYD WARSHALL ALGORITHM

```
function floyd_mat = floyd_warshall(A,thestart,theend)

%close all; clc

%keeping a copy of the original ending node new_theend=theend;

%Obtaining the dimension of the matrix A
[r c] = size(A);

%creating an empty array to store the predecessor matrix pred_mat
= [];

if nargin < 3  %checking the number of input arguments
disp(' ') elseif or(thestart,theend) > r     disp('The node
you entered does not exist')  elseif or(thestart,theend) <
0     disp('Node can only be positive') else     for i = 1:r
for j = 1:r            if A(i,j) ~= 0              pred_mat(i,j) =
i;            else
              pred_mat(i,j) = 0;
end        end    end

  %Floyd_Warshall starts its work here
for k = 1:r        for i = 1:r           for j =
1:r                if (A(i,k) + A(k,j)) < A(i,j)
                 A(i,j) = A(i,k) + A(k,j);

                 % Update the predecessor matrix
pred_mat(i,j) = pred_mat(k,j);
end           end        end    end
```

floyd_mat = A;

%Array for storing the path
thepath = [];

while (thestart ~= theend)

thep = pred_mat(thestart,theend);
thepath = [thepath thep];

theend = pred_mat(thestart,theend);

end thepath =
fliplr(thepath);
%} end

% Let us add the last figure in the route
thepath(end+1) = new_theend; APPENDIX
5.0

MATLAB CODE FOR MODIFIED SHORTEST PATH DISTANCE MATRIX.

function [D hatD Dbar max_Dbar minimum_max_Dbar] = berman(A,ina,inb)
D = A; [n m] =
size(D); for i =
1:n for j = 1:m
D(i,j) = min(D(i,j), min(D(i,ina),D(i,inb)));
end end hatD = D;
% Deleting corresponding rows and column of the initial facility hatD([ina
inb],:) = []; hatD(:,[ina inb]) = [];
%initial facity is Y and the remaining nodes are contained in X
Y = [ina inb];

```
X = 1:n; X(:,Y) = [];
Dbar = zeros(n-2,m);
for k = 1:length(X)
kk = X(k); for i =
1:n
%Dbar is the optimal location
Dbar(k,i) = min([A(kk,i), A(ina,i), A(inb,i)]);
end end
Dbar(:,Y) = [];
%maximum of the optimal location max_Dbar
= max(Dbar');
%minimum of the maximum optimal location minimum_max_Dbar
= min(max_Dbar);
```